Endogenous Rare Disaster Risk: Solution for Counter-Cyclical Excess Return and Volatility?

Ville K. Savolainen

Hanken School of Economics and Harvard University

Abstract

This study proposes a model to endogenously generate counter-cyclical dividend and return volatilities, dividend/price ratios and expected excess returns from business cycle fluctuations in rare disaster framework, with constant stochastic discount factor parameters and rational agents exhibiting standard power utility. Contrary to orthodoxy, I assume monopolistic competition and increasing returns to scale. These two assumptions cause the riskiness of dividend stream to fluctuate over business cycle: when demand for goods is lower the firms operate on a more steeper part of their average cost curve, and therefore the dividends are more sensitive to conventional and rare disaster shocks. Consequently, this generates counter-cyclicality, and decreases the hurdle for any stochastic discount factor to generate historically observed risk premium. I estimate the fit of aggregate average cost curve from corporate profits using structural regression, employing it to generate the salient features of stock returns.

Correspondence
Ville K. Savolainen
Doctoral Candidate, Hanken School of Economics (Helsinki, Finland)
Visiting Fellow, Harvard University (Cambridge, MA)
email: vsavolainen@g.harvard.edu
+1-617-407-5908; +358 45 136 5687

Introduction

Price-to-dividend and price-to-earnings ratios vary procyclically, and predict stock returns (Campbell and Shiller 1988a, b). This variation is not explained by changes in dividend growth or interest rates, hence the expected excess return is countercyclical (Cochrane, 1991). Volatility (Shiller (1981)) and magnitude (Mehra and Prescott, 1985) of expected excess return, in a standard model, are incomprehensible with the observed real risk-free rate which implies a more stable discount factor (see e.g. Jermann (1998)) and low levels of risk aversion (Weil, 1989). If variation in expected excess return is caused by changes in a stochastic discount factor the historical risk-free rate should be more volatile. Suggesting, that the observed fluctuations in real risk-free rate imply that expected excess returns are not completely explained by fluctuations in risk appetite, expected growth rate or its volatility. In addition, return volatility is counter cyclical (see e.g. Brandt and Kang (2004)). Implying that the changes in expected excess return are larger in contractions than in expansions. Therefore, a question: "Which mechanisms, unrelated to the stochastic discount factor, could cause riskiness to vary over business cycle?" naturally develops.

This paper derives the mechanism by deviating from two orthodox assumptions: decreasing returns to scale and perfect competition. In the model economy, strictly increasing returns to scale and monopolistic competition generate a dividend stream that is more sensitive to shocks during contractions than expansions. Therefore, even if shocks have a time invariant expected magnitude, they exhibit countercyclical volatility; and because of increased sensitivity, they are riskier during contractions implying a counter cyclical expected excess returns and P/D ratios.

Effects of increasing returns to scale and monopolistic competition have been discussed in the macro literature. Hall (1988) argues that the inability of macro models to match the time varying capital share is a consequence of assuming perfect competition and decreasing returns to scale. However, to my best knowledge the literature has been silent of the possible implications for asset prices. Generally, firms have been demonstrated to poses market power if e.g. there are increasing returns to scale, barriers of entry, product diversification, or intellectual property rights.

Increasing returns to scale are modelled with a strictly convex downward sloping average cost curve; and monopolistic competition ensures that firms are economically viable in defiance of this condition. The model is constructed with standard power utility, constant risk aversion, constant growth rate of consumption and its standard deviation. In the language of a Lucas tree world: the expected excess return of the tree is constant over time. The only parameter that varies is the effective slope of the average cost curve, this variation is endogenously generated by business cycle fluctuations. Thus, greater the harvest higher and less risky the share of equity holders.

To introduce increasing returns to scale I add fixed costs to the production function. The aggregate level of fixed costs increases deterministically with same rate as mean expected consumption growth. And for parsimony, I will assume constant marginal costs and a constant mark-up rule in pricing, implying a constant price. Therefore, variable costs are a constant proportion of sales; whereas, the proportion of fixed costs fluctuates with business cycle. As the proportion of fixed costs is smaller during an expansion than during a contraction, firms' profits are pro-cyclical.

The components of firms' profits can be priced separately: Growth rate of fixed costs is deterministic, and hence uncorrelated with the consumption growth. Therefore, it is priced as a risk-free asset. Whereas, growth profits before variable costs is perfectly correlated with the consumption growth. The price of an asset is a sum of prices of these two components. Because, unit price of future fixed costs is larger than price of sales excluding marginal costs; the price normalized by dividends is decreasing and expected excess return is increasing in the ratio of fixed cost to sales excluding variable costs. During a recession the demand is lower, and consequently firms operate on a steeper part of their average cost curve. Accordingly, the profits are more sensitive to shocks. Whereas, during expansions, because of a more gradual effective slope of the average cost curve, a change in demand has a milder effect on profits. Therefore, stocks are a riskier investment in recessions. Consequently, price/dividend ratio is pro-cyclical and expected excess returns are countercyclical. To a certain extent, the level of fixed costs may be considered as a strike price and the fruits as an underlying asset, hence equity resembles an option to a Lucas tree.

By the virtue of endogenous cyclical sensitivity to shocks, the mechanism generates a large and countercyclical return volatility, with constant stochastic discount factor parameters. The stocks are more volatile because a shock simultaneously affects all the future cash flows (dividend volatility); and because, a shock affects the effective slope of the average cost curve and therefore sensitivity to future shocks, therefore shifting the required rate of return and price (price volatility). Therefore, we can generate large variations in price-to-earnings ratio with a constant stochastic discount factor parameters and stable risk-free rate. This is the model's explanation for the volatility puzzle. In addition, because a shift in the effective slope of the average cost curve is associated with a negative return and an increase in volatility the model exhibits a leverage effect, without any changes in actual financial leverage ratio. Contrary to many previous papers, I simultaneously endogenize return and dividend volatility and excess returns.

In addition, the increased sensitivity to shocks is a novel source of additional risk. Therefore, diminishing the hurdle for any form of stochastic discount factor to match the observed risk premium. The numerical calibration of the model is conducted in a rare disaster framework of Rietz (1988) and Barro (2006), however the analytical findings are general. In the calibrations, I demonstrate how the historical expected excess return is matched with low risk aversion and lower rare disaster severity and probability than in most of the previous literature.

As explained, modelling dividend process separately is generally beneficial because it circumvents the generation of contradictory volatile risk-free rate by a volatile stochastic discount factor; the importance of separate modelling is also recognized by e.g. Longstaff and Piazzessi (2004), and Santos and Veronesi (2006). Closest to my work Gabaix (2012) introduces time varying firm level resilience for disasters and demonstrates that this time varying shock sensitivity can explain the volatility puzzle, and predictability of risk premium by time varying price/dividend ratio. This paper contributes to previous literature, by deriving dividends' time varying sensitivity to shocks directly from business cycle fluctuations. This generates additional value by linking the volatility and the expected excess return and their fluctuations to business cycles.

Model

Economy

In the model, economy consists of households, end producers and intermediate goods producers. End producers employ labor and intermediate goods to produce end products. Intermediate goods producers have monopoly power. Each intermediate good is produced on a specific production line which has fixed (denoted ψ) and variable costs. Hence, the aggregate profit flow has two components: fixed costs and difference of sales and variable costs (the difference is denoted as ω). The aggregate fixed costs grow deterministically with the number of production lines; whereas the difference between sales and variable costs is a constant share of aggregate production.

Economic growth is composed of two parts, deterministic and variable. Variable part: shocks, rare and conventional, affect the labor productivity which has a zero-expected mean. This is convenient, because the demand for intermediate goods and growth rate of economy are linear in labor productivity shocks. The deterministic part of the economy grows by exogenous vertical innovations in intermediate products. Consequently, the economy, dividends and the fixed costs have the same expected growth rate.

The end producers operate in a competitive market and therefore have zero profits. The production function is of Cobb-Douglas form:

1. $$Y_i = AL_i^{1-\alpha} \cdot \sum_{j=1}^{N} (X_{ij})^{\alpha}$$

Where $0 < \alpha < 1$; Y, L and A are end product, labor, and total factor productivity; X_j is the jth intermediate good used. Because all intermediate goods enter the production function symmetrically they are employed in same quantity. Hence, the production function simplifies to

2. $$Y_i = AL_i^{1-\alpha} \cdot (NX_i)^{\alpha} \cdot N^{1-\alpha}$$

Where N is the number of intermediate products in the economy.

The demand of jth input is derived from its marginal product, yielding

3. $$X_{ij} = L_i \cdot (A\alpha/P_j)^{\alpha/(1-\alpha)}$$

The intermediate goods sector transforms one unit of end product Y to one unit of intermediate good X, this implies a constant marginal cost. In addition, each production line j has a fixed cost ψ. Solving profit flow for optimal price yields a constant mark-up price $1/\alpha$. For production to be feasible it must hold that $(1/\alpha\text{-}1) \cdot X - \psi \geq 0$.

By writing down the profit function and summing over all X_j and substituting for solved P and X we may express the aggregate profit flow as

4. $$N\pi = \left\{\frac{1}{\alpha} - 1\right\} A^{1/(1-\alpha)} \alpha^{2/(1-\alpha)} LN - N\psi$$

where $\omega \equiv \left\{\frac{1}{\alpha} - 1\right\} A^{1/(1-\alpha)} \alpha^{2/(1-\alpha)} L$ is the difference between aggregate sales and variable costs.

If we substitute in the expression for optimal demand on X and for markup price P we can express aggregate output as

5. $$Y = A^{1/(1-\alpha)}\alpha^{2\alpha/(1-\alpha)}LN$$

We may observe that $\frac{\omega N}{Y} = \alpha\{1-\alpha\}$ which is a constant.

I specify the labor productivity process as

6. $$l\,n(L_{t+1}) \ = \ l\,n(L_t) \ + \ u_{t+1} \ + \ v_{t+1} + c$$

where u_{t+1} is i.i.d. normal with mean 0 and variance σ^2; v_{t+1} captures the rare disaster risk; and *c* is a constant ensuring that $e^{E(L_{t+1}-L_t)} = 0$. As innovations in ω are linear in *L* the innovations in labor productivity process directly affect the aggregate profit flow; a percentage change in ω is related with a $\frac{1}{1-\frac{\psi}{\omega}}$ percentage change in dividends. Higher the relative fixed costs larger the effect and vice versa.

The growth process of N is exogenously given by

7. $$ln(N_{t+1}) \ = \ ln(N_t) + \gamma$$

Where $\gamma > 0$ is a constant. Therefore

8. $$ln(Y_{t+1}) \ = \ ln(Y_t) + \gamma + \ u_{t+1} \ + \ v_{t+1} + c$$

The aggregate expected growth rate of the economy, dividends and fixed costs is therefore given by

9. $$E[1+g] = e^{\gamma}$$

Estimating Fixed and Variable Costs

The ω and ψ cannot be directly observed from data, but we may estimate them. The expectation of next period's dividend at *t* is

10. $$E_t[(\omega_{t+1} - \psi)N_{t+1}] = (\omega_t - \psi)N_t \cdot e^{\gamma}$$

The realized dividend growth is the expected dividend growth and the effect of labor productivity shock, which can be expressed in terms of expected aggregate growth and deviations from it:

11. $$\frac{(\omega_{t+1}-\psi)N_{t+1}}{(\omega_t-\psi)N_t} = e^{\gamma}\left\{1 + \frac{\omega_t}{\omega_t-\psi}\left(e^{g_{\omega,t+1}} - 1\right)\right\}$$

where $g_{\omega,t+1}$ is the realized log labor productivity shock. Because dividend is a component of two parts and only ω_{t+1} is affected by labor productivity shock, the effect of this shock on dividend at *t+1* is $\frac{\omega_t}{\omega_t-\psi}\left(e^{g_{\omega,t+1}} - 1\right)$.

By multiplying and dividing $\frac{\omega_t}{\omega_t-\psi}$ with $\frac{N_t}{Y_t}$ we have

12. $$\frac{(\omega_{t+1}-\psi)N_{t+1}}{(\omega_t-\psi)N_t} - 1 \approx E(g) + \left\{\frac{\omega_t\frac{N_t}{Y_t}}{(\omega_t-\psi)\frac{N_t}{Y_t}}\left(g_{t+1} - E(g)\right)\right\}$$

where $\omega_t\frac{N_t}{Y_t}$ is an unknown constant (to be estimated from data) and $(\omega_t - \psi)\frac{N_t}{Y_t}$ is a changing proportion of corporate profits of the GDP, which can be observed for every t. And g_{t+1} is the realized GDP growth.

Assuming that the expected growth rate and aggregate sales excluding variable costs over aggregate production $\left(\omega_t\frac{N_t}{Y_t}\right)$ are constant, as predicted by the model, we can estimate $\omega\frac{N}{Y}$ by running an OLS regression of the form

13. $$\%\,Change\ in\ Corporat\ Profits_{t,t+1} \approx \delta + \hat{\beta}\left\{\frac{1}{(\omega_t-\psi)\frac{N_t}{Y_t}}\left(g_{t+1} - E(g)\right)\right\}$$

where $\hat{\beta} = \omega\frac{N}{Y} = \alpha\{1-\alpha\}$; and δ is a constant. *E*(*g*) is the expected percentage change of GDP and g_{t+1} is a realized percentage change.

The average proportion of fixed costs is given by

14. $$\psi\frac{N}{Y} = \omega\frac{N}{Y} - \frac{A}{Y}$$

where A/Y is a sample average of $\frac{(\omega-\psi)N}{Y}$.

I employ annual US corporate business profits before tax without inventory valuation and capital consumption adjustment and real GDP from Federal Reserve Economic Data (FRED) for time period of 1935-2016 to estimate the average $\psi\frac{N}{Y}$, $\omega\frac{N}{Y}$, and $\frac{A}{Y}$. The average $\frac{A}{Y}$ is 0.089 (t Stat 35.6) and $\hat{\beta} = 0.244$ (t stat 6,86). These imply $\alpha = 0.42\ or\ 0.58$ and an average $\psi\frac{N}{Y} = 0.155$.

TABLE I $\%\ Change\ in\ Corporat\ Profits_{t,t+1} \approx \delta + \hat{\beta}\left\{\frac{1}{(\omega_t-\psi)\frac{N_t}{Y_t}}(g_{t+1} - E(g))\right\}$

Sample	δ	$\hat{\beta}$	adjR^2	$Mean\left(\frac{(\omega-\psi)N}{Y}\right)$
1935-2016	0.064 (3.7)	0.244 (6.9)	0.36	0.089 (35.6)
1947-2016	0.056 (3.5)	0.219 (4.0)	0.18	0.087 (35.2)
1935-2006	0.062 (3.4)	0.26 (7.1)	0.41	0.88 (31.7)
1947-2006	0.056 (3.6)	0.26 (4.6)	0.26	0.086 (30.8)

Table presents results of the above ordinary least squares regression, and t-Stats in parentheses.

Expected Returns of Two Components

The households own all firms and have utility functions of standard power utility form:

15. $$u(c) = \frac{c^{1-\theta}-1}{1-\theta}$$

where θ is a constant risk aversion parameter, and c is consumption.

Dividend stream is composed of two different streams. Therefore, stock return is a compensation for two different components of risk: the value of future fixed costs is implicitly sold short and the value of expected discounted difference of future sales and variable costs (ω) is purchased.

Therefore, the expected return of any asset is determined by the proportions and expected returns of these two components.

The analytical results of the model are general; nonetheless, for numerical results the expected returns are calibrated in a rare disaster framework of Rietz (1988) and Barro (2006) with a constant disaster risk. Rare disaster framework is chosen for its traceability and ability to generate a large enough Γ which is crucial for the model. However, because the stocks are riskier in this model, the required disaster severity and level of risk aversion are lower than in previous studies.

The aggregate profit flow can be separated to fixed costs and a difference between aggregate sales and variable costs. To price these two different streams, I will follow Barro (2006). In a continuous time limit the expected log return for the variable and fixed parts can be respectively written as

16. $$lnE(R^{\omega}) = \rho + \theta(\gamma + c) - \left(\frac{1}{2}\right)\theta^2\sigma^2 + \theta\sigma^2 - p\left[E(1-b)^{(1-\theta)} - 1 + Eb\right]$$

,

17. $$lnE(R^{\psi}) = \rho + \theta(\gamma + c) - \left(\frac{1}{2}\right)\theta^2\sigma^2 - p\left[E(1-b)^{(-\theta)} - 1\right]$$

where, ρ, and θ are time preference, risk-aversion parameters, p, and *b* are the disaster probability and severity respectively. In a case of a disaster the labor productivity is expected to contract by E(b). Note that the productivity growth is linear in labor productivity shocks.

There is a bond which during a disaster has a *q* probability of defaulting. In a case of default the face value of the bond is reduced by an amount *b*. The expected log bond return in a continuous time limit is given by

18. $$lnE(R^{b}) = \rho + \theta(\gamma + c) - \left(\frac{1}{2}\right)\theta^2\sigma^2 - p\left[(1-q)E(1-b)^{(-\theta)} + qE(1-b)^{(1-\theta)} + qE(b) - 1\right]$$

where, *q* is the probability of default conditional on the disaster event. Following Barro (2006) I assume that the proportion lost in default is *b*.

By combining 16. and 18. we observe that the expected log excess return of a variable part is given by

19. $$lnE(R^{e,\omega}) = \theta\sigma^2 - p(1-q)\left[E(1-b)^{-\theta} - E(1-b)^{(1-\theta)} - Eb\right]$$

Stock Price

The price of an asset is a sum of two parts, price of discounted stream of future ψ and ω. Given that the aggregate shares and the expected returns are known we can express price of any stock as:

20. $$P_t = P_{t,\omega} - P_{t,\psi} = \left\{1 - \frac{E_t\psi_{t+1}}{E_t\omega_{t+1}}\Gamma\right\} P_\omega$$

Where ψ, ω, $P_\psi \equiv \frac{E(1+g)[\psi]}{E[r_\psi]-E[g]}$, $P_\omega \equiv \frac{E(1+g)[\omega]}{E[r_\omega]-E[g]}$ are fixed costs, price excluding marginal cost multiplied by demand for intermediate good *j*, and their discounted prices respectively. $\Gamma \equiv \frac{E[r_\omega]-E(g)}{E[r_\psi]-[g]}$, multiplying and dividing P_ψ with P_ω yields $P_\psi = \frac{\psi}{\omega}\Gamma P_\omega$ because $E\left[\frac{\psi}{\omega}\right] = \frac{\psi}{\omega}$; $E[r_\omega]$ and $E\left[r_\psi\right]$ are the expected returns for one unit of ω and ψ respectively. To ensure strictly positive price I will assume that $E[r_\omega] > E[g]$ and $E\left[\frac{\psi}{\omega}\right]\Gamma < 1$. As $\Gamma > 1$ always hold, for risk averse agents this implies that $\psi < \omega$. These restrictions ensure that the expected return and variance are finite. The stock price is therefore a price of a portfolio consisting of two different assets with the relative weights and prices determined by Γ and $\frac{\psi}{\omega}$. The relative amount of fixed costs varies through business cycle, causing the price to vary[1].

Counter Cyclical Expected Excess Returns

The simple one period expected return is given by $r = \frac{P_\omega r_\omega - P_\psi r_\psi}{P_\omega - P_\psi}$ by substituting in $P_\psi = \frac{\psi}{\omega}\Gamma \cdot P_\omega$ and rearranging this simplifies to

21. $$r = \frac{\omega}{\omega - \psi\Gamma} \cdot \left\{r_\omega - r_\psi\right\} + r_\psi$$

expected stock return has two components with different weightings. From the equation, we may observe that the expected return may vary independently of the expected growth rate, its standard

[1] To alleviate unnecessary complexities the model assumes that stock price can never reach zero and that this truncation of the distribution is not priced, the actual price of non-negativity could be formulated by considering stock as a barrier option.

deviation, risk-aversion, time preference, rare disaster risk probability or severity, or any other parameter related to stochastic discount factor of any form. Deviations from the growth path alter the expected return through the variation in relative weights of ω and ψ. An increase in demand for intermediate goods decreases the relative amount of ψ.

The expected excess return is given by

22. $$r_e = \frac{\omega}{\omega - \psi\Gamma} \cdot \{ r_\omega - r_\psi \} + r_\psi - r_b$$

note that because of this internal leverage the stocks are riskier than in any standard model.

By rearranging we can express the expected excess return as a function of excess return of r_ω

23. $$r_e = \left\{ \frac{\omega}{\omega - \psi\Gamma} \right\} \cdot r_{e,w} + \frac{\psi\Gamma}{\omega - \psi\Gamma} \cdot \{ r_b - r_\psi \}$$

where the $\frac{\psi\Gamma}{\omega - \psi\Gamma} \cdot \{ r_\psi - r_b \}$ adjusts for the difference between risk-free rate and expected government bond return.

Given $\rho = 1\ \%,\ \theta = 3,\ \gamma = 2\ \%, \sigma = 2\ \%,\ c = 0.42\ \%,\ p = 1.7\ \%,\ and\ b = 0.26$ the expected returns for variable and fixed components are 5.6 % and 6.4 % respectively. Implying $\Gamma = 1.21$, an expected return of 8.9 %, given the expected return on bond 5.9 % the expected excess return is 3.1 %. Expected excess return without fixed costs $(r_{e,w})$ is 0.5 %; and we may interpret $r_{e,w}$ as expected excess return of a Lucas tree yielding aggregate consumption.

The expected return is a function of two variables ω and ψ. A percentage deviation of ω, given the mean level of $\frac{\omega}{\psi} = \frac{0.244}{0.089} = 2.7$, is associated with a $\frac{\omega}{\omega - \psi\Gamma} = 4.4\ \%$ change in valuation. Hence stock prices are very sensitive to deviations from expected growth rate. A 5 % negative deviation from expected growth rate imply that the stock market contracts by 22 % from its expectation, regardless of how fast the deviation occurs. Even though labor shocks, in the model, are considered to be i.i.d. deviations from the mean expected growth path can be loosely interpreted as business cycle fluctuations.

Price to Earnings Ratio and Return Predictability

Using the fact that dividend of a stock (A) is $\omega - \psi$ and $P_\omega = \frac{(1+g)\omega}{E[r_\omega]-E(g)}$ the price normalized by expected dividend is

24. $$\frac{P}{A} = \frac{\omega-\psi\Gamma}{\omega-\psi} \cdot \frac{1}{E[r_\omega - g]}$$

For the abovementioned parameter values the average P/A is 14.4. From Figure 1, we may observe how deviations from the expected growth alter the P/A, after a five percent negative (positive) deviation P/A is 13. (15.4).

The relationship between dividend yield and return can be expressed as

25. $$r = \frac{A}{P} + g = \frac{\omega-\psi}{\omega-\psi\Gamma} r_\omega - \frac{\psi(\Gamma-1)}{\omega-\psi\Gamma} g$$

as g, r_ω and Γ are constant the return variation is determined by fluctuations in ψ/ω, lager the ratio lower the expected return.

Countercyclical Volatility

Realized return is a function of expected return and the only stochastic variable ω. Realized one period return is given by

26. $$1 + r_{t,t+1} = \frac{P_{t+1}+A_{t+1}}{P_t} = E_t[1+r] + \frac{(1+E[r_\omega])\omega_t}{\omega_t-\psi\Gamma}(e^{g_{\omega,t+1}} - 1)$$

Given that the expected returns of the two components and growth rates are constant, the return volatility is driven by shocks to ω. The return volatility is given by:

27. $$Standard\ Deviation(e^{g_\omega}) \cdot \frac{\omega_t(1+E[r_\omega])}{\omega_t-\psi\Gamma}$$

Where expected variance for small time intervals is given by

28. $$\sigma^2 + pE[b^2]$$

For the $\sigma = 2\ \%,\ p = 1.7\ \%,\ and\ b = 0.26$ this implies an annual standard deviation of 3.9 %, which happens to be the standard deviation of aggregate economic growth. We can immediately observe that the volatility is an increasing and convex function of ψ/ω for the relevant range ($\frac{\psi}{\omega} < \Gamma^{-1}$). Hence, as the relative proportion of ω is increasing in positive innovations of general economic conditions the volatility is counter-cyclical. For average ψ/ω the expected annual return volatility is 18.2 %. A five percent negative (positive) deviation from the expected growth path generates a 22 % increase (14 % decrease) in volatility. Figure 1 demonstrates how the expected volatility fluctuates along business cycles.

Calibration

I calibrate the model in a following manner: All the values are annual. The fixed and variable costs are as estimated above. On average fixed costs are estimated to be 15.5 % of production, corporate profits before taxes are 8.9 % and the average profits before fixed costs are 24.4 %. I set the average dividend payout ratio to 0.55. The calibration inputs are summarized in Table II. Table III-V and Figure I and II summarizes the main results. I assume a constant disaster probability *p* = 1.7 %, following Barro (2006). I set relative risk aversion $\theta = 3$, and time preference $\rho = 1\,\%$. The expected growth rate $\gamma = 2$ % and $\sigma = 2$ %. In case of a disaster, the economy contracts by *b*. The value of contraction is obtained by solving b for annual dividend volatility $\frac{\omega}{\omega-\psi}\sqrt{\sigma^2 + pE[b^2]} = 11\%$, yielding $b = 0.26$. 11 % dividend volatility is obtained from Campbell and Cochrane (1999). I assume that occurring disasters are of constant magnitude. Hence, the risk adjusted probability of disasters is 4.2 %. Growth rate of economy, dividends and consumption during normal times is, $g + c = 2.42$ %. The standard deviation of aggregate consumption growth is 3.9 % which is in the same ball park with estimates of Barro (2006).

TABLE II

Variables	Values
Growth rate of consumption, aggregate production and dividends	$\gamma + c = 2\ \% + 0.42\ \%$
Standard deviation of aggregate consumption growth	$\sigma = 2\ \%; p = 1.7\ \%; E(b^2) = 0.067;$ $\sqrt{\sigma^2 + pE[b^2]} = 3.9\ \%$
Risk aversion, time-preference	$\theta = 3, \rho = 1\ \%$
Average corporate profits over production, dividend payout ratio, average profits before fixed costs over production	$0.089\ (35.6);\ 0.55, 0.244\ (6.9)$
Disaster probability	$p = 1.7\ \%$

Expected stock returns are given by equation 21, which yields 8.9%, which is on a reasonable level. However, the expected bond return (5.9 %), given by equation 18, is higher than in previous rare disaster calibrations, this is because the risk adjusted disaster probability is approximately four times lower. Because of the excess sensitivity to shocks, generated by increasing returns to scale, the implied risk premium conditional on disaster occurring is 3.1 % which is approximately six times larger than $r_{e,w} = 0.5\ \%$. By considering increasing returns to scale and monopolistic competition the model is able to generate endogenously a higher risk premium. This result is not specific to rare disaster specification, any model that generates a large enough Γ with the given $\frac{\omega}{\psi}$ is able to generate high risk premium. For e.g. $g = 2\ \%$, $r_\omega = 2.48\ \%, r_\psi = r_f = 2.2\ \%$ implies $\Gamma = 1{,}4$, given $\frac{\omega}{\psi} = 2.7$ the expected excess return is 7.2 %.

TABLE III

Variables	Values
Expected stock return	8.9 %
Expected return fixed	5.6%
Expected return variable	6.4 %
Expected bond return	5.9 %
Expected equity premium	3.1 %

Figure I demonstrates, how the expected return fluctuates with realized production's deviations from expected mean. We may immediately observe that deeper the recession larger the effect of any future shock to the stock price. The expected stock return is also countercyclical, without any changes in the stochastic discount factor parameter values. Therefore, the risk-free rate and expected bond-returns are constant in the model. I conclude that this may be a prominent endogenous explanation for countercyclical fluctuations in expected excess return.

The stock return volatility is given by equations 27. and 28. and is 18.2 %, very close to the historical average e.g. Campbell (2003) report it to be 18. The model volatility is generated from the business cycle fluctuations affecting the slope of effective average cost curve; the expected volatility varies trough the business cycle as demonstrated by Figure II. Hence, I am willing to conclude that the model can endogenously generate excess volatility from business cycle fluctuations.

Figure I

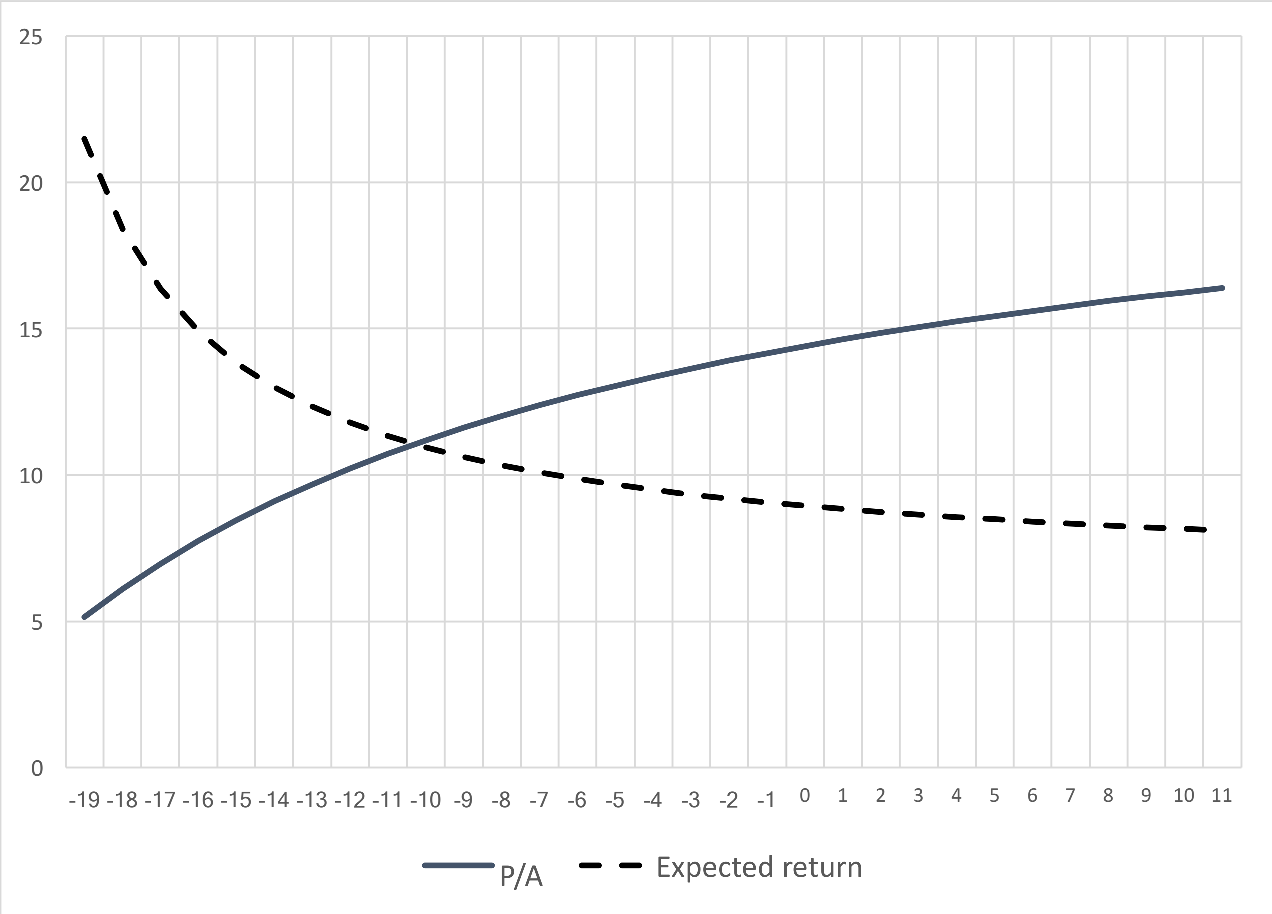


TABLE IV

Variables	Values
Stock return volatility	18.3 %
Expected total wealth portfolio excess return $r_{e,w}$ and volatility	0.45 %; 2.3 %
Dividend volatility	11.0 %
Disaster Severity	$b = 0.239$

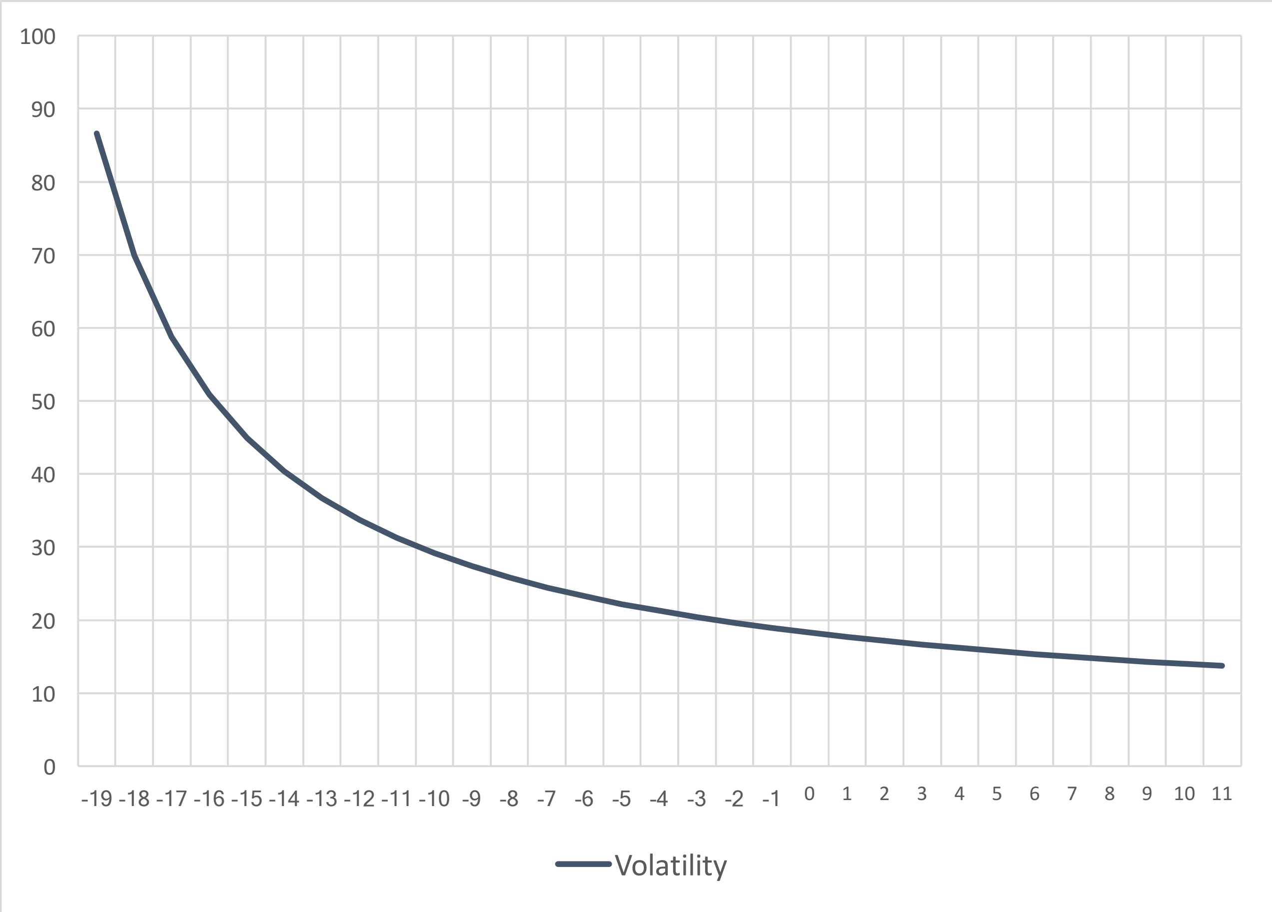
Volatility is counter cyclical as demonstrated in Figure II. Furthermore, it is a decreasing and convex function of deviations from the expected growth path explaining why in disastrous situations, e.g. 1929-1933 the economy contracted by a cumulative 30 % and the corporate profits' share of the production dropped to 3 % from 11 %, the expected volatility and subsequent stock returns were extraordinary high. This drop would correspond to an approximately 19-23 % decrease from the expected level of production in the model. The model is able to explain the observation that after a disastrous event the stocks are riskier and therefore bare a higher premium, without an increase in probability of a subsequent shocks or investors' risk appetite.

TABLE V

Variables	Values
Stock return volatility	18.3 %
Price/Dividend ratio	26.2
Price/Earnings	14.4

Price to Earnings and Price to Dividend ratios 14.4 and 26.2 are relatively close to historical averages, by construction they do not predict future dividends, nonetheless they do predict excess returns and they vary over the business cycle as demonstrated in Figure II. The P/A is the price to earnings ratio, we may observe that after a negative deviation from the mean expected return the P/A decreases and vice versa for a positive deviation. The effect is concave, hence larger the expansion less effect a productivity shock has on expected returns and vice versa for contractions. The Theoretical minimum of P/A is given by the P/A of total wealth portfolio, i.e. a firm without fixed costs, and it is 23. P/A and P/D fluctuate along with the business cycle.

Figure II



Conclusion

This paper builds an endogenous correspondence between time-varying severity of disaster risk and business cycle fluctuations. Therefore, linking the first and second moments of stock returns, earnings and dividend growth, and P/D ratio to business cycle fluctuations. This endogeneity is generated by introducing fixed costs in production function. The calibration result demonstrate that the model can feasibly explain the salient features of asset return fluctuations: equity premium and risk-free rate puzzles (Mehra and Prescott 1985; Weil 1989). The model is also able to generate counter-cyclical variation in the expected excess return observed by Fama and French (1989). The model offers an explanation for predictive power of price-dividend ratio (Campbell and Shiller 1988a,b). The model is able to generate countercyclical volatility (see e.g. Ghysels, Harvey, and Renault (1996) for a survey) and explain the magnitude of expected volatility, generating endogenously the excess volatility puzzle of Shiller (1981).

References

Barro, R.J. 2006, "Rare disasters and asset markets in the twentieth century", The Quarterly Journal of Economics, vol. 121, no. 3, pp. 823-866.

Brandt, M.W. and Kang, Q., 2004. On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach.

Campbell, J.Y., “Consumption-Based Asset Pricing,” in *Handbook of the Economics of Finance*, George M. Constantinides, Milton Harris, and ReneM. Stulz, eds. (Amsterdam: Elsevier, North Holland, 2003).

Campbell, J.Y. & Shiller, R.J. 1988, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies,* vol. 1, no. 3, pp. 195-228.

Campbell, J.Y. & Shiller, R.J. 1988, "Stock Prices, Earnings, and Expected Dividends", *The Journal of Finance,* vol. 43, no. 3, pp. 661-676.

Cochrane, J.H., 1991. Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations. *The Journal of Finance*, 46(1), pp. 209-237.

Fama, E.F. & French, K.R. 1989, *Business conditions and expected returns on stocks and bonds*.

Gabaix, X, Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance , *The Quarterly Journal of Economics*, Volume 127, Issue 2, 1 May 2012, Pages 645–700,

Ghysels, E., Harvey, A.C. & Renault, E. 1996, "5 Stochastic volatility", *Handbook of statistics,* vol. 14, pp. 119-191.

Hall, R.E., 1988. The Relation between Price and Marginal Cost in U.S. Industry. *Journal of Political Economy*, 96(5), pp. 921-947.

Jermann, U.J., 1998. Asset pricing in production economies. Journal of Monetary Economics, 41(2), pp. 257-275.

Longstaff, F.A. and Piazzesi, M., 2004. Corporate earnings and the equity premium. Journal of Financial Economics, 74(3), pp. 401-421

Mehra, R. & Prescott, E.C. 1985, "The equity premium: A puzzle", *Journal of Monetary Economics,* vol. 15, no. 2, pp. 145-161.

Rietz, T.A. 1988, The equity risk premium a solution, *Journal of Monetary Economics*, vol. 1, no. 22, pp. 117-131.

Santos T., Veronesi, P., Labor Income and Predictable Stock Returns, *The Review of Financial Studies*, Volume 19, Issue 1, 1 March 2006, Pages 1–44,

Shiller, R. J.: 1981, “Do stock prices move too much to be justified by subsequent changes in dividends?”, The American Economic Review 71(3), 421–436.

Weil, P. 1989, "The equity premium puzzle and the risk-free rate puzzle", *Journal of Monetary Economics,* vol. 24, no. 3, pp. 401-421.