An Informational Rationale for Action over Disclosure

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August 27, 2017

Abstract

The past two decades have seen a considerable increase in the amount of public information provided by policy makers. This paper proposes a novel argument against such disclosure. Unlike other providers of public information, a policy maker can condition a policy instrument, such as a tax or an interest rate, on his information. This option allows the policy maker to control the influence of his information on market outcomes without the added obfuscation associated with partial disclosure. As a result, the exclusive use of a policy instrument is preferable in simple models in which externalities render full disclosure suboptimal. I show how this argument extends from an abstract game to a micro-founded macroeconomic model in which firms learn from market prices.

JEL codes: E52, D82, D83

Keywords: Public information, optimal policy, endogenous learning

1 Introduction

The past two decades have witnessed a considerable increase in the amount of public information provided by monetary and fiscal authorities about the state of the economy. A simple estimate based on data from wire services points to over a two-fold increase in the case of the US.[1]

Despite the prevalence of such policy maker releases, and the considerable resources devoted to their production, these statements are not without noise. The published *Federal Reserve* forecasts are, for instance, often imprecise. Commentary, such as those from the *Treasury* about the state of the economy, is harder to quantify but the noise seems similar. Should we be concerned that releasing such noisy information may cause confusion and lower welfare? Do such disclosures help households and firms make better choices? Or would it be preferable to instead use such information to aptly set a *policy instrument,* such as a tax or an interest rate?

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I am indebted to Ryan Chahrour, Anezka Christovova, Per Krusell, Jennifer La'O, Kristoffer Nimark, Alessandro Pavan, Torsten Persson, Daniel Quigley, Donald Robertson, Robert Ulbricht, Ansgar Walther and seminar and conference participants at Boston College, the Barcelona GSE Summer Forum 2015, the Federal Reserve Board, the 11th World Congress of the Econometric Society, the Riksbank, SED 2015, Toulouse School of Economics and Aarhus University for providing invaluable comments, improving the quality of this draft.

[1]This is based on Bloomberg news summary data. The precise number of releases is 453 in 2014 and 213 in 1995. These values include speeches, comments and documents by the President, Federal Reserve Presidents, senior US Treasury officials, and members of the CEA and the CBOE about the current state of the US economy.

A substantial debate since Morris and Shin's (2002) influential contribution has attempted to provide exact conditions for the social value of public information releases.[2] Despite frequent mentions of policy maker communication, much of the debate, however, initially abstracted from the presence of policy instruments. This abstraction is not without cost. Angeletos and Pavan (2009), Angeletos *et al.* (2016) and others show how a tax or an interest rate based on *ex-post* information about realized fundamentals or aggregate activity alters people's emphasis on public information, and thus profoundly shapes the social value of additional public releases.[3]

This paper sharpens the focus onto policy makers: it contrasts disclosure with a policy instrument's capacity to utilize the same noisy information that a policy maker has.[4] To do so, it develops a simple, dispersed information model in which a policy maker can affect individual actions either by disclosing his information or by using it to set a policy instrument. Similar to other extensions of Lucas' (1972) framework, full disclosure does not achieve efficient use of private and public information (see, for instance, Hellwig 2005 and Lorenzoni 2010). The core contribution of this paper is to show how this commonplace feature delivers a novel rationale for the active use of policy instruments: changing a tax or an interest rate is a better means to use a policy maker's information than direct disclosure; it allows a policy maker to utilize his information without the added obfuscation associated with partial communication.

My main results build on the ability of the conditional use of a policy instrument to substitute for disclosure. Consider the example of a firm that wants to set a price in line with the unobserved demand for its product. A policy maker can here improve the firm's welfare by two means: He can either disclose his own information that is salient to the firm's demand. Or he can use that information to instead set an interest rate to stabilize what ultimately matters for the firm, the unobserved demand for its product. Both means allow the policy maker to decrease the firm's uncertainty and improve welfare.

This substitutability between disclosure and the conditional use of a policy instrument has important consequences. Suppose that full disclosure does not attain the efficient outcome. Imagine, for instance, that it causes the firm to erroneously downweigh its own private information, to the detriment of welfare. With the interest rate, the policy maker can, in that case, simply use his information to stabilize the firm's demand by less. By contrast, with disclosure, the policy maker can only decrease the use of his own information by releasing an *obfuscated*, *noisier* version of his news. This warping of a signal that enters the firm's decisions yet further distorts its choice of price, a welfare cost which the use of the interest rate completely avoids.

This simple distinction – that a policy instrument allows the policy maker to *himself directly* control the influence of his own information instead of having to *induce others* to change their emphasis with added noise – is at the core of my argument for the use of policy instruments.[5]

[2] This includes Svensson (2006), Morris *et al.* (2006) and Angeletos and Pavan (2007), who all consider *beauty contest* games à la Morris and Shin (2002). Hellwig (2005), Angeletos and La'O (2014) and Angeletos *et al.* (2016) explore the degree to which the results from these simple games extend to workhorse macroeconomic models.

[3] See also, for instance, Lorenzoni (2010), Angeletos and La'O (2012) and Paciello and Wiederholt (2013).

[4] See, furthermore, Walsh (2007), Baeriswyl and Cornand (2010), James and Lawler (2011) and the discussion in the related literature section below about the conditional use of policy maker information.

[5] Alan Greenspan's (1987) *dictum* that: "*Since becoming a central banker, I have learned to mumble with great*

Indeed, across the baseline model and its extensions, any disclosure by the policy maker merely results in more uncertainty, which reduces welfare except in knife-edge cases. This holds true irrespective of the efficacy of responses to the underlying state or the extent of direct strategic interactions; factors that elsewhere have been shown to be critical for the optimal disclosure of public information (*cf.* Angeletos and Pavan, 2007).

The analysis in this paper first centers on a simple, linear-quadratic prediction game with dispersed information. In the model, a large population of individuals have access to both public and private information about an underlying fundamental. Each individual's payoff depends on how appropriate her action is to a combination of said underlying fundamental and a policy maker's instrument, which is set in part based on the policy maker's own imperfect beliefs about the fundamental. Although reduced form, the model enables a simple, two-step welfare decomposition which precisely illustrates the added cost of communication.

A variety of different mechanisms have been shown to result in the inefficiency of full disclosure. The baseline model focuses for concreteness on *the learning externality* that is present when people observe *endogenous public statistic*s, such as market prices (*cf.* Vives 1997; 2016, Morris and Shin 2005, Amador and Weill 2010). This externality causes people to rely excessively on public information as they do not take into account that the resultant lower emphasis on their own private information decreases the information content of market outcomes. Because of the learning externality, the policy maker can improve welfare by moving the economy away from full disclosure and towards more emphasis on private rather than public information.

The basic rationale for instrument policy, however simple, extends much beyond economies with market-based information. In fact, as I demonstrate, only the weaker condition that the limit of full disclosure does not achieve efficient use of private and public information is necessary for the rationale to exist. Other drivers towards less emphasis on public information, such as *excessive direct strategic complementarity* (Morris and Shin, 2002), *inefficient fundamentals* (Angeletos and Pavan, 2007) or the *destruction of insurance possibilities* (Hirshleifer, 1971) essentially deliver the same basic reason. Indeed, so too do certain cases with *insufficient direct strategic complementarity*, which result in too little influence of public information and are pertinent to business cycle models with sticky prices (Hellwig, 2005).[6]

This robustness of my main argument is of key importance. I use it to show how the rationale for instrument policy extends to a workhorse, micro-founded macroeconomic model with nominal frictions, in which workers and firms learn from the prices that they observe and both efficient and inefficient disturbances drive the economy. The abstract policy maker is here made concrete and equal to the central bank. This macroeconomic model closely resembles those recently proposed by Adam (2007), Amador and Weill (2010), Paciello and Wiederholt (2013) and Angeletos *et al.* (2016) to study the social value of public information. But I here introduce the two critical components that constitute the back-bone of my analysis: a policy maker who conditions his policy instrument on his own private information and learning from market outcomes. The *Online Appendix* to this paper considers alternative models where I, for instance, instead identify the

incoherence.", in other words, seems quite pertinent.

[6]Full disclosure here prevents instrument policy from further increasing the influence of public information.

policy maker with the economy-wide tax authority.

Combined, these macroeconomic extensions provide a bridge between the baseline model and common frameworks for two of the most important providers of policy maker information: the central bank and the treasury. This extendibility of the baseline model is a crucial step: without the discipline of specific micro-foundations to support the main mechanism, other combinations of payoff assumptions and information structures than those considered could perceivably rationalize most uses of a policy maker's information. Angeletos and Pavan (2007), for instance, demonstrate how the welfare results of Morris and Shin (2002) depend on payoff assumptions that are invalid in certain workhorse macroeconomic models.

Context and Contribution: My rationale for the use of policy instruments – that a policy instrument, unlike disclosure, allows a policy maker to control the influence of his information without the addition of noise to the information structure – contributes to the recent debate about the social value of public information started by Morris and Shin (2002).

Angeletos *et al.* (2016) provide a unifying taxonomy for much of the earlier literature. Nevertheless, like earlier work, Angeletos *et al.* (2016) condition policy instruments on the true, realized fundamentals of the economy, in addition to *ex-post* information about economy-wide activity. A hypothetical policy maker is thus able to equate the equilibrium use of information with the socially optimal. This, in turn, ensures that the release of additional public information about efficient fundamentals, like productivity shocks in workhorse macroeconomic models, is optimal.[7] By contrast, this paper presupposes that policy instruments themselves are conditioned on the imperfect information that a policy maker considers to release. This simple but salient difference helps explain why my results deviate from those prescribed by the taxonomy.

James and Lawler (2011, 2012) have independently studied how the presence of policy instruments affects the social value of public information when the emphasis on new information is suboptimal, similar to this paper. But their contribution focuses exclusively on the relative emphasis accorded to public and private information within Morris and Shin's (2002) beauty contest framework. This focus is somewhat misplaced; it abstracts from the true, principal mechanism and assumptions that underlie their main results. In fact, the same two-step decomposition that I develop below illustrates that what separates instrument from communication policy within their framework is *not* the ability to achieve a better emphasis on public information *per se*. Indeed, communication policy can replicate any emphasis on public and private information that is relevant. Rather, it is instrument policy's ability to alter the relative emphasis without the introduction of noise to the information structure that is important. Their contribution can therefore be seen as one key, non-microfounded example of how the basic mechanism that this paper proposes extends to other drivers of the inefficiency of full disclosure – here suboptimal direct strategic complementarity à la Morris and Shin (2002).

A distinct mechanism to the one I examine below has recently been suggested by Walsh (2007) and Baeriswyl and Cornand (2010).[8] Their contributions study the benefits of central bank

[7] Paciello and Wiederholt (2013) consider a related point within their model of Rational Inattention.

[8] Melosi (2016) documents how such *signaling effects* of monetary policy can also account for why inflation expectations in the *Survey of Professional Forecasters* appear to respond with a delay to monetary impulses and

disclosure within a simplified variant of the New-Keynesian model when imperfectly informed firms learn from how monetary policy is set. Opacity is shown to potentially be preferable to disclosure because it allows the central bank through its policy instrument to better modify firm beliefs about the mix of (efficient *vs.* inefficient) shocks to the economy. My results add to this analysis: both papers overlook the crucial role of expected policy, the capacity of, for instance, future interest rates to shape uncertainty and welfare today. I demonstrate below how my argument for the conditional use of policy instruments, which depends upon this central role of expected policy, continues to hold when people also in part learn about the policy maker's beliefs from how his policy instrument is currently set.

Also related to this paper is the sizable literature that studies the informational effects of policy instruments. King (1982) and Weiss (1982) provide early important contributions that show how a policy instrument can influence the relative emphasis on private and public information. More recently, Angeletos and Pavan (2009), Lorenzoni (2010) and Wiederholt (2015) analyze how to optimally set policy instruments in settings with market-based information. But as with the earlier contributions, they do not consider the dual use of instrument and communication policy that provides the basic premise behind my analysis.

Organization: The plan for the rest of this paper is as follows: I introduce and solve the baseline model in the next section. The crux of the paper is in Sections 3 and 4, which detail the dominance of a policy instrument. I also here discuss a number of extensions, whose purpose is to delve deeper into the underlying mechanism for the theoretical results. Section 5 maps the insights from the baseline model into a micro-founded macroeconomic model with a central bank. I conclude in Section 6. Additional extensions and all proofs are in the *Appendix*.

2 A Prediction Game with a Policy Maker

2.1 Model Assumptions

I start with a simple model with dispersed information and learning from an economy-wide outcome in which individual actions are influenced by a policy maker's instrument as well as his information disclosure. This baseline model will later provide an accurate template for the optimal use of policy maker information within a micro-founded macroeconomic model.

There is a continuum of measure one of private sector individuals, each with imperfect information about an underlying fundamental θ drawn from a uniform distribution over the real line. I assume that all individuals in the economy have the same quadratic utility function,

$$\mathcal{U}_i = -\frac{1}{2}\mathbb{E}_i \left[a_i - (\theta - m)\right]^2, \tag{2.1}$$

where $a_i \in \mathbb{R}$, $i \in [0, 1]$, denotes an individual's action and m the policy maker's instrument. The manner in which policy affects utility in (2.1) is, in a way, natural: it allows the policy maker, through an appropriate adjustment of the policy instrument, to completely offset the

remain disanchored. See also Gosselin *et al.* (2008), Hahn (2011) and Tamura (2016) for related work.

welfare effects of changes in θ, similar to the ability of policy in workhorse business cycle models. Unlike the related beauty contest models by Morris and Shin (2002) and James and Lawler (2011), there are no direct strategic complementarities in (2.1): other people's decisions do not directly influence an agent's payoff from her own action. In essence, this implies that (2.1) boils down to a simple *prediction game*, where the welfare of each individual depends only on how well aligned her action is to the difference between the underlying fundamental and the policy stance $(\theta - m)$ – the difference which I for brevity call *the effective state of the economy*. Direct strategic complementarity is introduced later in Section 5, where I show that it does not alter the main insights from the simple prediction game.

The policy maker, like an individual agent in the economy, has imperfect information about the unobserved fundamental. I assume that he sets his policy instrument m as a linear function of his expectation of θ, summarized by the unbiased signal z,

$$m = \phi \mathbb{E}_G [\theta] + \epsilon_m = \phi z + \epsilon_m, \quad z = \theta + \epsilon_z, \tag{2.2}$$

where G denotes the policy maker (the *G*overnment) and $\phi \in [0, 1]$ the publicly known level of policy activism.[9] Variations in the policy instrument that are orthogonal to the policy maker's beliefs are captured by ϵ_m.[10] The error terms $\epsilon_z \sim \mathcal{N}(0, 1/\tau_z)$ and $\epsilon_m \sim \mathcal{N}(0, 1/\tau_m)$ are assumed independent of θ, and τ_z denotes the precision of the policy maker's beliefs. I thus characterize instrument policy in terms of a commitment to a linear rule. Because of the quadratic form of (2.1), this assumption by itself does not prevent policy from achieving the efficient outcome.[11]

To complete the description of the model, it is necessary to specify the information structure. I assume that all individuals in the economy observe a combination of private and public information. The private information of individual i is summarized by the noisy signal x_i,

$$x_i = \theta + \epsilon_x^i, \quad \epsilon_x^i \sim \mathcal{N}(0, 1/\tau_x),$$

where τ_x denotes the common precision of the type-specific signal and ϵ_x^i is assumed independent of all other random variables with $\mathbb{C}\text{ov}\left[\epsilon_x^i, \epsilon_x^j\right] = 0$ for all $i \neq j$. I follow convention and assume that $\int_0^1 \epsilon_x^i di = 0$ almost surely (a.s.). In addition to their private information, individuals in the economy observe two distinct public signals: *(i)* a noisy realization of the economy-wide action

[9] I here bound $\phi \in [0, 1]$. But, as I show below, ϕ will optimally always be within this range.

[10] The introduction of ϵ_m serves two purposes: First, it allows the policy instrument to respond to another factor than the policy maker's beliefs, similar to the policy shocks frequently attached to monetary and fiscal policy rules. This coarseness will, in turn, illustrate how my main results derive solely from the policy maker's ability to control how much his policy instrument responds to his own information, and not from the lack of noise or other factors in his policy rule (see Section 4). Second, ϵ_m also serves a technical purpose: In *Online Appendix D*, ϵ_m prevents agents from learning the true value of z simply from the observation of m.

[11] In (2.2), I have 'hard-wired' the beliefs of the policy maker to equal the noisy signal z. This simplifying assumption is stark, but it allows me to focus on how the setting of the policy instrument affects the behavior of the private sector without having to internalize how their responses, in turn, affect the policy maker's knowledge about the state of the economy, and hence his policy choice. Generalizing the framework to include this feedback between individual decisions and public policy, dealing with the associated multiplicity of equilibria, is a worthwhile extension, and one that I tackle in related work (Kohlhas, 2014).

$\bar{a} = \int_0^1 a_i di$; and *(ii)* a (potentially) noisy signal of the policy maker's beliefs z. The signal of the economy-wide action, my stand-in for endogenous market-based information, equals,

$$a = \int_0^1 a_i di + \epsilon_a, \quad \epsilon_a \sim \mathcal{N}(0,\, 1/\tau_a)\,, \tag{2.3}$$

where ϵ_a is assumed independent of all other random disturbances with precision τ_a.[12] The signal of the policy maker's own beliefs is, by contrast, given by,

$$\omega = z + \epsilon_\omega, \quad \epsilon_\omega \sim \mathcal{N}(0,\, 1/\tau_\omega)\,, \tag{2.4}$$

where ϵ_ω is independent of θ, ϵ_z, ϵ_a and ϵ_x^i for all i. The case of full disclosure here corresponds to the limit $\tau_\omega \to \infty$, while complete opacity is equivalent to the situation where the policy maker's communication contains no valuable information, $\tau_\omega \to 0$. Partial disclosure refers to the interim case. Neither the policy maker nor private sector agent i can observe other agents' actions a_j, $j \neq i$. The information structure can therefore be summarized by the following information sets:[13]

$$\Omega_i = \{x_i,\, a,\, \omega\} \quad \forall i \in [0,\, 1]\,, \tag{2.5}$$

$$\Omega_G = \{z\}\,. \tag{2.6}$$

The advantage of the chosen approach to model communication policy is that it allows, in a comparatively simple way, for a meaningful discussion of different, intermediate levels of partial disclosure (see, for instance, Cukierman and Meltzer, 1986). This advantage, of course, rests on the ability of the policy maker to commit to a disclosure rule such as (2.4). Without this commitment, the policy maker could announce anything following the realization of his private information, and the only values that would be consistent with equilibrium would be the limits of full disclosure and complete opacity. I demonstrate below how the main results from my analysis still remain valid in such a case.

A stark feature of (2.5) is that individuals do not observe the policy instrument. Individual choices can instead be considered pre-set and made *before* the realization of m. In reality, however, both prospective and current policy matter for individual choices, and changes to current instruments often provide an indicator of the policy maker's beliefs. However, as I emphasize in Section 5 and in *Online Appendix C* and *D* within a multi-period economy, the observation of current instruments does not meaningfully alter my main results. All that is necessary is that *(1)* individual actions also in part depend upon expectations of a policy instrument;

[12] The reason for the introduction of the shock ϵ_a in (2.3) is purely technical: the important role it plays is to limit individuals' ability to infer the true value of θ from the observation of a. The use of "non-invertibility" shocks like ϵ_a to maintain imperfect information in a rational expectations model is common. A similar modeling device is used in, for instance, Lorenzoni (2009).

[13] The inclusion of a or ω into the policy maker's information set (2.6) would seem natural. However, neither alters the set of possible welfare outcomes. The influence of a or ω on m can be perfectly predicted and thus offset by the private sector. I therefore for simplicity exclude a and ω from (2.6).

and that (*2*) the policy maker's disclosure provides some additional information about the policy maker's beliefs beyond what could be learned from the observation of current instruments. Combined, these characteristics ensure a central premise behind my argument: that both communication and expected instrument policy can influence agents' uncertainty.[14] Blinder *et al.* (2008) provide empirical evidence in support of the second assumption; that policy maker communications, more precisely central bank disclosures, provide additional information about the policy maker's beliefs.

Obviously, the information structure in (2.5) and (2.6) is very stylized. Nevertheless, it does allow people to simultaneously learn from both an endogenous public signal, like a financial market price or a statistical release, and from a policy maker's communication of his own beliefs about the state of the economy. The interaction between these two different sources of information will be critical for the suboptimality of full disclosure.

2.2 Equilibrium Characterization

We can now proceed to analyze the equilibrium of the economy described above. In accordance with the literature on noisy rational expectations, I restrict myself to symmetric linear Bayesian equilibria. The standard approach to find linear equilibria is the method of undetermined coefficients. Here, that involves three steps: First, one computes individual expectations of θ and z, using the conjecture that the equilibrium solution of a_i is linear in the elements of Ω_i. Then, using these expectations, one derives an individual's optimal action, which from (2.1) equals,

$$a_i \quad = \quad \mathbb{E}_i\left[\theta - m\right] = \mathbb{E}_i\left[\theta\right] - \phi \mathbb{E}_i\left[z\right]. \tag{2.7}$$

Consistent with the initial conjecture, the resulting action is a linear function of the elements in Ω_i, but where the coefficients now depend on those from the conjecture. Clearly, in equilibrium, the two sets of coefficients have to be the same. Last, solving this set of fixed-point conditions results in the unique equilibrium detailed in Proposition 1.

Proposition 1. *The unique linear Bayesian equilibrium action for agent $i \in [0\,,1]$ equals,*

$$a_i = k_0 x_i + k_1 a + k_2 \omega,$$

where $a = \int_0^1 a_i di + \epsilon_a$. Furthermore, $k_0 \in (0,\,1)$, $k_1 = \frac{\tau_a k_0^2}{\tau_x + \tau_a k_0^2}$ and $k_2 = (1-\phi)\frac{\tau_x}{\tau_x + \tau_a k_0^2} - k_0$.

In the rest of this section, I provide a simple derivation of the central coefficient k_0 in Proposition 1. Although this derivation differs from that outlined above, it helps build intuition about what ultimately determines the informativeness of the signal of the economy-wide outcome: how much emphasis individuals place onto their own private information.[15]

[14]Suppose, by contrast, that expected policy instruments are irrelevant and that current policy *perfectly* reveals the policy maker's beliefs. Then, neither communication nor instrument policy affect agents' uncertainty: additional disclosure provides no new information since there is in effect always full disclosure; and since expected instrument policy is irrelevant to agents' choices it is also irrelevant to the uncertainty that surrounds them.

[15]The link between the weight on private information and the informativeness of endogenous public signals

2.3 The Informativeness of the Economy-wide Outcome

The coefficient k_0 is the main endogenous variable in our study of how to best exploit the policy maker's information. It directly determines the informativeness of the signal of the economy-wide outcome, and hence how much weight agents attach to the various elements of their information sets. After combining terms and subtracting ω from,

$$a \quad = \int_0^1 a_i di + \epsilon_a = \quad k_0\theta + k_1 a + k_2\omega + \epsilon_a, \tag{2.8}$$

it follows that observing the noisy signal of the economy-wide outcome is equivalent to observing $k_0\theta + \epsilon$ or $\theta + \frac{1}{k_0}\epsilon_a$, where $k_0 > 0$. I denote the latter signal for reference by $y = \theta + \frac{1}{k_0}\epsilon_a$. When k_0 is large, this signal is very informative about the fundamental, and contrariwise when small.

Corollary 1. *The informativeness of the signal of the economy-wide outcome, k_0, is determined by the unique solution to,*

$$k_0 = \frac{\tau_x\left[\tau_\omega + (1-\phi)\,\tau_z\right]}{(\tau_z + \tau_\omega)\left(\tau_x + \tau_a k_0^2\right) + \tau_z\tau_\omega}, \qquad \phi \in [0,\,1]\,. \tag{2.9}$$

The fixed-point condition (2.9) has a natural interpretation: it describes the equilibrium link between the weight that agents attach to their own private information and the informativeness of the signal of the economy-wide outcome. To see this link, notice that both the expectation of the fundamental and of the policy maker's information, the two sole determinants of an agent's action in (2.7), can be split into,

$$\mathbb{E}_i[\theta] = \mathbb{E}[\theta \mid x_i,\,\omega,\,y] = w_x\mathbb{E}[\theta \mid x_i] + (1-w_x)\,\mathbb{E}[\theta \mid \omega,\,y]$$
$$\mathbb{E}_i[z] = \mathbb{E}[z \mid x_i,\,\omega,\,y] = v_x\mathbb{E}[z \mid x_i] + (1-v_x)\,\mathbb{E}[z \mid \omega,\,y]\,,$$

where $w_x = \frac{\tau_x(\tau_z+\tau_\omega)}{(\tau_z+\tau_\omega)\left(\tau_x+\tau_a k_0^2\right)+\tau_z\tau_\omega}$ and $v_x = \frac{\tau_x\tau_z}{(\tau_z+\tau_\omega)\left(\tau_x+\tau_a k_0^2\right)+\tau_z\tau_\omega}$. The term $\mathbb{E}[\theta \mid x_i] = \mathbb{E}[z \mid x_i] = x_i$ here describes an agent's truly *private forecast* of θ and z, while $\mathbb{E}[\theta \mid \omega,\,y]$ and $\mathbb{E}[z \mid \omega,\,y]$ detail the corresponding *public forecasts* based only on public information. Using these expressions combined with (2.7) shows that an agent's equilibrium action can be written as,

$$a_i = (w_x - \phi v_x)\,x_i + \left\{(1-w_x)\,\mathbb{E}[\theta \mid \omega,\,y] - \phi\,(1-v_x)\,\mathbb{E}[z \mid \omega,\,y]\right\},$$

and thus that the signal of the economy-wide outcome equals,

$$a = (w_x - \phi v_x)\,\theta + \left\{(1-w_x)\,\mathbb{E}[\theta \mid \omega,\,y] - \phi\,(1-v_x)\,\mathbb{E}[z \mid \omega,\,y]\right\} + \epsilon_a. \tag{2.10}$$

Here is where the distinction between private and public forecasts becomes important. All private forecasts are conditionally independent and equal to agents' own private information x_i. Their simple cross-sectional average therefore equals θ in (2.10). Public forecasts, by contrast, can be computed by everyone. Each agent can thus subtract their influence on the signal of the

has been studied elsewhere. See, for instance, Vives (2010) or Amador and Weill (2010) for an exposition which resembles that in Subsection 2.3.

economy-wide outcome. Combined, these two features imply that observing a is equivalent to observing $(w_x - \phi v_x)\,\theta + \epsilon_a$ or $y = \theta + \frac{1}{k_0}\epsilon_a$ since $k_0 = w_x - \phi v_x$. This demonstrates Corollary 1.

In essence, all that anyone can hope to learn from the observation of a is the sum of individuals' private information, that is the only truly new information contained in the endogenous signal. This, in turn, helps explain why the weight attached to x_i ultimately determines a's informativeness. Yet, it also demonstrates the fixed-point problem inherent to our analysis: the informativeness of the signal of the economy-wide outcome depends on the equilibrium weight that individuals attach to their own private information, but that weight in turn depends critically on the informativeness of the economy-wide outcome.[16]

3 The Uncertainty Trade-Off

The policy maker has two means of exploiting his information about the state of the economy in the above model. He can either directly disclose (a potentially noisy version of) his signal or he can use his information to set the policy instrument. In this section, I detail how both of these means involve the same basic trade-off. Both allow the policy maker to use the additional information contained within his signal to, all else equal, decrease agents' uncertainty about their correct action, and hence allow for more informed choices. However, both policies also come at the indirect cost of decreasing the weight that agents attach to their own private information, thereby reducing the informativeness of the endogenous economy-wide outcome.

3.1 Welfare Criterion

To demonstrate how both communication and instrument policy face this trade-off between exploiting the policy maker's information and agents' own private information, it will be useful for us to first define a welfare criterion. I here use utilitarian welfare and define the *social welfare loss function* as minus the *ex-ante* expected average level of utility in the population,

$$\mathcal{W}(\phi,\, p_\omega) = -\mathbb{E}\int_0^1 \mathcal{U}_i di = \frac{1}{2}\mathbb{E}\left[\Delta_i\right]^2, \tag{3.1}$$

where,

$$\Delta_i \equiv a_i - (\theta - m) = \mathbb{E}_i\left[\theta\right] - \theta + \phi\left(z - \mathbb{E}_i\left[z\right]\right) + \epsilon_m \tag{3.2}$$

denotes *the ex-post deviation of an agent's action from the effective state of the economy.* This quantity will be of considerable importance later. It follows from Proposition 1 and Corollary 1 that $\mathcal{W}$ can, after some straightforward but tedious manipulations, compactly be written as,

$$\mathcal{W}(\phi,\, \tau_\omega) = \frac{1}{2}\frac{\phi^2\left[\tau_x + \tau_a k_0\left(\phi,\, \tau_\omega\right)^2 + \tau_z\right] - 2\phi\tau_z + \tau_\omega + \tau_z}{\left(\tau_\omega + \tau_z\right)\left[\tau_x + \tau_a k_0\left(\phi,\, \tau_\omega\right)^2\right] + \tau_\omega\tau_z} + \frac{1}{2}\tau_m^{-1}. \tag{3.3}$$

[16] For ease of exposition, I will sometimes refer to "the informativeness of the economy-wide outcome". By that, I always mean "the informativeness of *the signal* of the economy-wide outcome".

3.2 The *Direct Decrease* in Uncertainty

Let us start with the benefit of using the policy maker's information. Both communication and instrument policy can use the additional information contained within the policy maker's signal to decrease agents' uncertainty about their correct action, and hence allow for more informed choices. To make this point most clearly, I here hold the informativeness of the economy-wide outcome constant. This simplifies the exposition by side-stepping the adverse changes in k_0 caused by the use of communication and instrument policy. I consider the more pertinent case where k_0 is endogenous in the next subsection.

To see the direct benefit of communication policy notice that, with k_0 held constant, increasing the precision of the policy maker's disclosure always decreases $\mathcal{W}$, $\frac{\partial \mathcal{W}}{\partial \tau_\omega} \leq 0$ in (3.3).[17] This is the intuitive beneficial effect of providing more public information: it directly lowers agents' uncertainty about both the unobserved fundamental and the policy maker's instrument, allowing them to better determine the effective state of the economy, and thus their own action.

That said, the use of a policy instrument can also decrease agents' uncertainty. In fact, it can always replicate the direct decrease in uncertainty about the effective state of the economy achieved by communication policy. This substitutability between communication and instrument policy when holding k_0 constant is a result of both being able to use the policy maker's information to better line-up agents' actions with the effective state of the economy.

Proposition 2. *Increases in the precision of the policy maker's disclosure, $\tau_\omega \in \mathbb{R}_+$, or in instrument policy, $\phi \in \left[0, \hat{\phi}\right]$, $\hat{\phi} \equiv \frac{\tau_z}{\tau_x + \tau_a k_0^2 + \tau_z}$, decrease social welfare loss, $\mathcal{W}$, when keeping k_0 constant. All welfare outcomes attainable with communication policy can also be achieved by instrument policy with $\phi \leq \hat{\phi}$, $\mathcal{W}(0, \mathbb{R}_+) = \mathcal{W}\left(\left[0, \hat{\phi}\right], \tau_\omega \to 0\right)$.*

This result is intuitive. A policy maker can choose to either directly disclose (some of) his information or commit to use that information to stabilize what ultimately matters for agents' choices – in this case, the effective state of the economy. But, to some extent, these two options are equivalent. Both policies directly decrease agents' uncertainty about what action to take and thereby improve welfare. This is either by providing agents with more information about the effective state of the economy or by directly stabilizing it, thus making it easier to predict. Indeed, by stabilizing the effective state of the economy, the policy maker can always replicate any improvement in welfare he could also achieve with communication policy.[18]

An important example of this equivalence between instrument and communication policy is the full disclosure case. This outcome represents the maximum decrease in $\mathcal{W}$ attainable when keeping k_0 constant. I will later use this example to derive the optimal policy.

[17] Specifically, $\frac{\partial \mathcal{W}}{\partial \tau_\omega} = -\frac{1}{2}\left[\frac{\phi\left(\tau_x + \tau_a k_0^2 + \tau_z\right) - \tau_z}{\left(\tau_\omega + \tau_z\right)\left(\tau_x + \tau_a k_0^2\right) + \tau_\omega \tau_z}\right]^2 \leq 0$.

[18] The proof is simple: $\mathcal{W}$ in continuous and decreasing in both $\phi \leq \hat{\phi}$ and τ_ω, and ranges from $\mathcal{W}_{max} = \frac{1}{\tau_x + \tau_a k_0^2}$ to $\mathcal{W}_{min} = \frac{1}{\tau_x + \tau_a k_0^2 + \tau_z}$ in either argument. Hence, $\mathcal{W}(0, \mathbb{R}_+) = \mathcal{W}\left(\left[0, \hat{\phi}\right], \tau_\omega \to 0\right)$.

Example 1. *The full disclosure case when k_0 is held constant:* Under full disclosure, an agent's information set equals $\Omega_i^d = \{x_i,\, y,\, z\}$ since $\tau_\omega \to \infty$. This, in turn, implies that the deviation from the effective state of the economy, the term that matters for $\mathcal{W}$ in (3.1), simplifies to $\Delta_i^d = a_i^d - (\theta - m) = \mathbb{E}_i^d[\theta] - \theta + \epsilon_m$. All agents know z and hence are able to perfectly predict its influence on the policy instrument in (3.2). We are now able to decompose,

$$\begin{aligned}\Delta_i^d = \mathbb{E}_i[\theta \mid x_i,\, y,\, z] - \theta + \epsilon_m &= \mathbb{E}_i[\theta \mid x_i,\, y] + w_\omega^d \left(z - \mathbb{E}_i[\theta \mid x_i,\, y]\right) - \theta + \epsilon_m, \\ &\equiv \mathbb{E}_i^o[\theta] - \theta + w_\omega^d \left(z - \mathbb{E}_i^o[\theta]\right) + \epsilon_m, \end{aligned} \tag{3.4}$$

where $w_\omega^d = \hat{\phi} = \frac{\tau_z}{\tau_x + \tau_a k_0^2 + \tau_z}$ and $\mathbb{E}_i^o[\theta] = \mathbb{E}[\theta \mid x_i,\, y]$ denotes the expectation of θ under the complete opacity information set, $\Omega_i^o = \{x_i,\, y\}$ in which $\tau_\omega \to 0$. But if we now compare (3.4) with Δ_i under complete opacity, $\Delta_i^o = \mathbb{E}_i^o[\theta] - \theta + \phi\left(z - \mathbb{E}_i^o[\theta]\right) + \epsilon_m$, we see that when $\phi = w_\omega^d = \hat{\phi}$ the two expressions are identical, and hence achieve the same level of welfare. A coefficient of instrument policy equal to $\hat{\phi}$ thus replicates the full disclosure outcome.[19]

3.3 The *Indirect Increase* in Uncertainty

The use of communication or instrument policy to directly decrease agents' uncertainty about what action to take is, nevertheless, costly. In both cases, it comes at the indirect cost of decreasing the weight that agents attach to their own private information, thereby reducing the informativeness of the economy-wide outcome. This indirect effect tends to increase agents' posterior uncertainty about what action to take, all else equal increasing welfare loss, $\frac{\partial \mathcal{W}}{\partial k_0} < 0$.[20]

Proposition 3. *Increases in the precision of the policy maker's disclosure, $\tau_\omega \in \mathbb{R}_+$, decrease the informativeness of the endogenous public signal, k_0, when $\phi \leq \hat{\phi} = \frac{\tau_z}{\tau_x + \tau_a k_0^2 + \tau_z}$. Increases in instrument policy, $\phi \in [0,\, 1]$, likewise decrease k_0. All values of k_0 achievable with communication policy can also be obtained by instrument policy with $\phi \leq \hat{\phi}$, $k_0(0, \mathbb{R}_+) = k\left(\left[0,\, \hat{\phi}\right],\, \tau_\omega \to 0\right)$.*

Proposition 3 follows directly from the total differentiation of the fixed-point relation (2.9).[21] The mechanics of how increases in the precision of public information or in the responsiveness of instrument policy decrease k_0 are well-known and have, for instance, recently been discussed by Amador and Weill (2010), Lorenzoni (2010) and Angeletos and La'O (2014). When the policy maker's disclosure becomes more precise or when the policy maker commits to use his information to stabilize the effective state of the economy more, he directly decreases agents' uncertainty about what matters to them, the effective state of the economy (Proposition 2). However, in

[19] The decomposition used in the top-line of (3.4) can here be seen as a consequence of the *Projection Theorem*. Section 4 and Ericson (1969) discuss how this decomposition extends well beyond the linear-normal case. Whether we (a) take the expectation of θ based on $\Omega_i^d = \{x_i,\, y,\, z\}$ directly or (b) first project θ onto z, using the projection coefficient $w_\omega^d = \hat{\phi}$ from (a), take the expectation error from that projection and then compute the expectation of $\theta - \hat{\phi} z$ based on $\Omega_i^o = \{x_i,\, y\}$ results in *exactly* the same expectation error.

[20] It follows from (3.3) and Corollary 1 that $\frac{\partial \mathcal{W}}{\partial k_0} = -\left[\frac{\tau_\omega + \tau_z - \phi \tau_z}{(\tau_\omega + \tau_z)(\tau_x + \tau_a k_0^2) + \tau_\omega \tau_z}\right]^2 \tau_a k_0 = -\frac{\tau_a k_0^3}{\tau_x^2} < 0$.

[21] Since k_0 is continuous and decreasing in both $\phi \leq \hat{\phi}$ and τ_ω, and ranges from $k_{0,\max} = \frac{\tau_x}{\tau_x + \tau_a k_0^2}$ to $k_{0,\min} = \frac{\tau_x}{\tau_x + \tau_a k_0^2 + \tau_z}$ in either argument, it also follows that $k_0(0, \mathbb{R}_+) = k\left(\left[0,\, \hat{\phi}\right],\, \tau_\omega \to 0\right)$.

each case, the resultant decrease in uncertainty comes at a cost; the cost of making agents place less weight onto their own private information when updating their beliefs. The added use of the policy maker's information effectively crowds-out how much individuals have to rely on their own private signals. Because agents now use less their own private information when determining their own actions, the economy-wide outcome reflects less the combined independent private information, the truly new information that agents could learn from one another. Consequently, the information content of the economy-wide outcome falls. This, in turn, tends to, all else equal, increase uncertainty and thus increase social welfare loss.[22]

The similarity between the adverse effect of communication and instrument policy detailed in Proposition 3 can usefully be demonstrated by the full disclosure case, the case that achieves the maximum decrease in $\mathcal{W}$ when keeping k_0 constant.

Example 2. *The full disclosure case and k_0:* With full disclosure, k_0 equals the unique solution to (see Corollary 1),

$$k_0^d = \frac{\tau_x}{\tau_x + \tau_a \left(k_0^d\right)^2 + \tau_z}. \tag{3.5}$$

But this level of informativeness of the endogenous public signal can also be obtained under complete opacity with $\phi = \hat{\phi}$, $k_0\left(\phi, \tau_\omega \to \infty\right) = k_0\left(\hat{\phi}, \tau_\omega \to 0\right)$. A coefficient of instrument policy equal to $\hat{\phi}$ thus replicates the level of k_0 achieved under full disclosure.

The cornerstone behind this adverse effect of communication and instrument policy is, in essence, a *learning externality*: When deciding on how to respond to their own private information, agents do not internalize the informativeness of the endogenous public statistic, and hence how much others are able to learn from it. This causes them to over-emphasize public information to the detriment of private in equilibrium. The next section demonstrates how this learning externality is also, at heart, what causes the exclusive use of instrument policy to dominate.

4 A Informational Rationale for Instrument Policy

We have seen how both instrument and communication policy are subject to a trade-off between, on the one hand, the direct benefit of using the policy maker's information to decrease individuals' uncertainty and, on the other hand, the indirect cost of decreasing the information content of the economy-wide outcome. The question then arises whether instrument or communication policy is a better tool to balance this trade-off? I here demonstrate how instrument policy always provides a better means of using the policy maker's information than directly disclosing it. I then discuss the basic mechanism behind this preference for the conditional use of a policy instrument and how it extends across variations of the baseline model.

[22]Contrary to, for instance, Amador and Weill (2010) increases in the precision of the policy maker's disclosure can here also make the economy-wide outcome *more* informative rather than only *less*. This happens in the overlooked case when $\phi > \hat{\phi}$. But while this effect is of separate interest and occurs because of an increase in the commonality of the policy maker's beliefs (Kohlhas, 2014), the policies where $\phi > \hat{\phi}$ and τ_ω finite are all strictly dominated by the full disclosure outcome (*Appendix A*). Both the direct decrease in uncertainty obtained *for a given* k_0 and the informativeness of the economy-wide outcome are greater in the full disclosure case. I therefore, for brevity, choose to set aside the $\phi > \hat{\phi}$ case and focus on the key welfare trade-off that arises when $\phi \leq \hat{\phi}$.

4.1 The Dominance of a Policy Instrument

A convenient approach to solve for the optimal use of the policy maker's information is *the primal approach*: First, one starts by substituting out for ϕ in $\mathcal{W}$ in (3.3), using the mapping between k_0 and ϕ from Corollary 1. Next, one minimizes the resulting expression for $\mathcal{W}$ with respect to k_0 and τ_ω. Since one has already substituted out for ϕ, there is no need to also internalize this variable's influence. Last, one uses Corollary 1 once more to translate the optimal k_0 coefficient back into the level of ϕ that it entails. Following these steps results in:

Theorem 1. *The unique optimal policy is complete opacity, $\tau_\omega^\star \to 0$, combined with active instrument policy,*

$$\phi^\star = \frac{\tau_z}{\tau_x + (1+\alpha)\,\tau_a k_0^{\star,2} + \tau_z} \in \left(0,\, \hat{\phi}\right), \tag{4.1}$$

where $\alpha \equiv \frac{\tau_x + \tau_a k_0^{\star,2}}{\tau_x + 2\tau_a k_0^{\star,2}} > 0$. The informativeness of the endogenous public signal under the optimal policy is always larger than that attained with full disclosure, $k_0^\star > k_0^d$.

The sharpness of Theorem 1 follows from two inherent properties of our setup: First, that the learning externality makes full disclosure suboptimal, with excessive emphasis on policy maker relative to private information. And second, that instrument policy can increase the weight on private information without the introduction of noise to the information structure, unlike the alternative of partial disclosure. Let me now turn to how these properties lead to Theorem 1.

The Suboptimality of Full Disclosure: Because agents do not internalize the informativeness of public statistics when deciding on how much emphasis to place onto their own private information, full disclosure is not optimal. At full disclosure, the benefit of a more informative economy-wide outcome that internalizes the learning externality outweighs the (second-order) loss from incomplete use of the policy maker's information. Theorem 1 directly shows that $k_0^\star > k_0^d$. The policy maker can improve welfare by moving the economy away from the full disclosure outcome and towards more emphasis on private rather than policy maker information.

The Added Cost of Noise: But why is the policy instrument the better means to increase the emphasis on private information? Suppose that the policy maker starts at the full disclosure outcome. Examples 1 and 2 illustrate that he can achieve this outcome by two distinct means: either (a) by communication policy $\{\phi = 0,\, \tau_\omega \to \infty\}$ or (b) by instrument policy $\left\{\phi = \hat{\phi},\, \tau_\omega \to 0\right\}$. Both means achieve the same k_0 as well as the same level of welfare $\mathcal{W}$. Next, consider a small decrease in τ_ω in case (a) or ϕ in case (b) that pushes emphasis from policy maker onto private information (Proposition 3). This results in a deviation from the effective state of the economy, the term that matters for welfare in (3.1) and (3.2), equal to,

$$\Delta_i^a = a_i - (\theta - m^a) = \mathbb{E}_i^o[\theta] - \theta + w_\omega^a \left(z - \mathbb{E}_i^o[\theta]\right) + \epsilon_m + w_\omega^a \epsilon_\omega \tag{4.2}$$

$$\Delta_i^b = a_i - \left(\theta - m^b\right) = \mathbb{E}_i^o[\theta] - \theta + \phi^b \left(z - \mathbb{E}_i^o[\theta]\right) + \epsilon_m, \tag{4.3}$$

where I have used that $\mathbb{E}_i^a[\theta] = \mathbb{E}_i[\theta \mid x_i, y, \omega] = \mathbb{E}_i[\theta \mid x_i, y] + w_\omega^a \left(\omega - \mathbb{E}_i[\theta \mid x_i, y]\right)$ to arrive at (4.2). The expectation under the complete opacity case (b) is once more denoted by $\mathbb{E}_i^b[\theta] = \mathbb{E}_i[\theta \mid x_i, y] = \mathbb{E}_i^o[\theta]$. Otherwise, I follow the exact same steps used in Example 1.

Equations (4.2) and (4.3) are central to Theorem 1. The reason is that if we set $\phi^b = w_\omega^a < \hat{\phi}$ they show that the only difference between the two policies is the noise term $w_\omega^a \epsilon_\omega$ from the policy maker's partial disclosure $(\omega = z + \epsilon_\omega)$.[23] *Communication and instrument policy are equivalent only up to a noise term.* Squaring this term and taking *ex-ante* expectations illustrates the additional welfare cost that communication policy entails – added noise.

When the policy maker uses the policy instrument to stabilize the effective state of the economy less than what would replicate full disclosure, he induces agents to rely more on their own private information. The direct increase in prior uncertainty makes agents update their beliefs more based on their own information, and hence increases the informativeness of the economy-wide outcome. This, in turn, alleviates the failure to internalize the learning externality. When the policy maker, by contrast, chooses to use communication policy, he can only induce agents to attach more emphasis onto private rather than policy maker information by releasing a nosier signal of his own beliefs. That is the only mechanism by which communication policy can internalize the learning externality and make the economy-wide outcome more informative. However, this additional noise comes at an added welfare cost; the cost of making the policy maker's signal worse. This warping of a signal that agents use to base their decisions on further distorts their actions, a welfare cost which the use of the policy instrument completely avoids.

We can summarize the above discussion in the following Lemma, which can also be used to provide an alternative proof of Theorem **1**. The Lemma, furthermore, shows how the identified distinction between the two means to use a policy maker's information naturally extends to also any policy that mixes the conditional use of a policy instrument with partial disclosure.[24]

Lemma 1. *Consider any partial disclosure policy $\tau_\omega \in \mathbb{R}_+$ with instrument policy $\phi \in \mathbb{R}$. Then, there exists a complete opacity policy (o) $\tau_\omega^o \to 0$ with $\phi^o = w_\omega + \phi(1 - v_\omega)$ that has the same emphasis on policy maker information in social welfare $\mathcal{W} = \mathbb{E}\left[\Delta^2\right]$, but with a welfare benefit proportional to the variance of the noise in the policy maker's partial disclosure, τ_ω^{-1}.*

4.2 Extensions, Variations and Discussion

Theorem 1 and the related Lemma 1, in essence, show how the exclusive use of a policy instrument is optimal because it allows the policy maker to *himself directly* decrease the use of his own information. With communication policy, by contrast, the policy maker has to decrease how much *other agents* use his information, and he can only do so by warping the signal that he sends of his own beliefs at an added welfare cost. This distinction is at the core of Theorem **1** and extends well beyond the focus of the baseline model.

Alternative Shock and Information Structures: The linear-normal solution of the model provides an attractive illustration of the above results. Yet, the general conclusions are robust

[23] Note that $k_0^a = k_0^b$ when $\phi^b = w_\omega^a$. This equivalence follows from the weight attached to $\mathbb{E}_i^a[\theta]$ and hence x_i being identical in the two cases (see also Corollary 1).

[24] Specifically, $\Delta_i = \mathbb{E}_i^o[\theta] - \theta + [w_\omega + \phi(1 - v_\omega)](z - \mathbb{E}_i^o[\theta]) + \epsilon_m + (w_\omega - \phi v_\omega)\epsilon_\omega$.

to alternative specifications. Ericson (1969), DeGroot (1970) and Vives (2010), for instance, illustrate how the main decomposition used to arrive at (4.2), and hence Theorem 1, extends much beyond the linear-normal case. Indeed, the decomposition holds for many of the most commonly used distributions when combined with natural priors.[25] *Online Appendix C* provides a simple example with binary signals and a beta-distributed fundamental. Furthermore, the decomposition also extends, for any information structure, to the important case where we restrict agents to only construct linear-best predictors (see Brockwell and Davis, 2009). Thus, neither the normality nor the improper prior are essential to Theorem 1.

Other Payoff Assumptions: The Suboptimality of Full Disclosure: The learning externality provides one example mechanism for why full disclosure is suboptimal. Nevertheless, as the derivation of (4.2) and (4.3) shows, the precise mechanism is not key; the main results rest only on the weaker condition that full disclosure does not achieve the efficient outcome.

Online Appendix C considers several extensions of the baseline model where different payoff assumptions cause full disclosure not to achieve the efficient outcome. Specifically, it shows how other drivers towards less emphasis on public information, such as *excessive direct strategic complementarity* (Morris and Shin, 2002), *inefficient fundamentals* (Angeletos and Pavan, 2007) or the *destruction of insurance possibilities* (Hirshleifer, 1971) essentially deliver the same results. Indeed, so too do certain cases with *insufficient direct strategic complementarity*, which result in too little influence of public information and are pertinent to business cycle models with sticky prices (Hellwig, 2005). Full disclosure here prevents instrument policy from further increasing the emphasis on policy maker information (see Section 5). The precise mechanism by which full disclosure is suboptimal is hence not critical to the main insights from Theorem 1.

Other Welfare Assumptions: **Macroeconomic Extensions:** The above robustness of Theorem 1 is of key importance. I use it in Section 5 to show how my rationale for the conditional use of a policy instrument extends to a workhorse, micro-founded business cycle model in which firm prices are strategic complements and both efficient and inefficient disturbances drive the economy. The abstract policy maker is made concrete and equal to the central bank. *Online Appendix D* considers another, related application in which the policy maker instead corresponds to the economy-wide tax authority and firm output choices are strategic substitutes.[26] Importantly, unlike the baseline model, social welfare in both cases depends on the deviation of the sum of individual actions from an unobserved fundamental, in addition to their cross-sectional dispersion. This illustrates how the conclusions from Theorem 1 extend to a popular, micro-founded class of social welfare functions.

Combined, these macroeconomic extensions provide a bridge between the baseline model and a common framework for two of the most important providers of policy maker information: the

[25]The decomposition, for example, holds for affine information structures with beta-binomial or gamma-poisson combinations of prior and likelihood. Other cases are when the observations are negative binomial, gamma or exponential when assigned natural conjugate priors.

[26]This application also shows how the main results carry over to cases where the policy instrument only influences the economy through its effects on individual choices.

central bank and the treasury. This extendibility of the baseline model is crucial: without the discipline of specific micro-foundations to support the insights from Theorem 1, other combinations of payoff and social welfare assumptions than those considered could perceivably rationalize most uses of policy maker information. Angeletos and Pavan (2007), for instance, demonstrate how the welfare results of Morris and Shin (2002) depend on social welfare assumptions that are invalid in certain workhorse macroeconomic models (*cf.* Angeletos *et al.*, 2016).

Policy Maker Information vs. Public Information: The insights from Theorem 1 contrast public information from policy makers with that which, for instance, comes from a statistical office or a news service. Absent the policy instrument, full disclosure is preferable to complete opacity or partial disclosure, despite the presence of the learning externality and hence the inefficiency of the full disclosure outcome. Indeed, it is straightforward to show that $\mathcal{W}(\phi = 0,\, \tau_\omega \to \infty) < \mathcal{W}(\phi = 0,\, \tau_\omega \in \mathbb{R}_+)$. The added cost of noise associated with partial disclosure, in this case, outweighs any benefits of internalizing the learning externality. Theorem 1 thus, in essence, shows how policy maker information differs from other sources of public information precisely because a policy maker can instead condition his policy instrument on his information.

Noisy Policy vs. Noisy Communication: A central characteristic of the above results is that they hold in spite of the policy instrument itself being noisy. The policy instrument responds to the noise shock ϵ_m in (2.2) and hence to another factor than the policy maker's beliefs. This coarseness of the policy instrument, in turn, underscores how Theorem 1 derives solely from the ability of the policy maker to directly control how much his policy instrument responds to his own information and not from the lack of noise or additional factors in his policy rule.[27]

Relative Emphasis on Public and Private Information: I end this subsection with a brief discussion of how Theorem 1 relates to recent work that studies how the conditional use of policy instruments affects the social value of public information. Angeletos and Pavan (2009), Lorenzoni (2010), Angeletos *et al.* (2016) and others have shown how a tax or an interest rate based on *ex-post* information about realized fundamentals or aggregate activity can alter agent's emphasis on private and public information, and thus profoundly shape the social value of additional public releases.[28] James and Lawler (2011), for instance, couch their contribution in terms of how "the policymaker [via the policy instrument can] determine the relative weights accorded to alternative information sources" (p.1,570).

Theorem 1, by contrast, builds on a novel equivalence between communication and instrument policy's capacity to alter the economy-wide emphasis on private and policy maker information. This equivalence, in turn, arises when the policy instrument is conditioned on the same noisy information that the policy maker considers to disclose. Indeed, Proposition 2 and 3 show how both communication and instrument policy can replicate any emphasis on private and

[27] *Online Appendix C* expands on the irrelevance of noise in the policy maker's instrument rule. This *Appendix* also illustrates how the main conclusions extend to cases where the policy instrument is "crude", in the sense that it uses the policy maker's information to achieve several objectives. The *Appendix* then relates such objectives to the above discussion of micro-founded social welfare functions. See also Section 5.

[28] See, for instance, also Angeletos and La'O (2012) and Paciello and Wiederholt (2013).

policy maker information that is relevant. This substitutability underscores that what drives Theorem 1 is not the ability of the policy instrument to achieve a better emphasis on private and policy maker information *per se*. But rather, it is the policy instrument's ability to alter the relative emphasis without the introduction of additional noise to the information structure that is important.[29]

4.3 The Constrained Efficient Outcome

The presence of the learning externality makes the market solution inefficient. But does the optimal instrument rule completely overcome this inefficiency? The answer to this question turns out to be no. Even under the optimal policy, the economy does not attain the constrained efficient outcome. The source of inefficiency in this economy cannot be fully alleviated by the use of the policy instrument alone (or its combination with communication policy).

To speak meaningfully about deviations from a constrained efficient outcome, I first need to establish an appropriate welfare benchmark. Such a benchmark is provided by *the team solution*: the social planner problem in which individuals internalize collective welfare but must still rely on their own information sets when making their own choices (Radner, 1979 and Vives, 1988). The team efficient solution thus internalizes the learning externality. Proposition 4 characterizes the constrained efficient outcome. In this subsection, I also set the variance of the noise ϵ_m in the policy maker's optimal instrument rule equal to zero to sharpen the welfare comparison.

Proposition 4. *There exists a unique, linear constrained efficient outcome ("team solution"),*

$$a_i = c_0^\star x_i + c_1^\star a + c_2^\star z, \quad \forall i \in [0,\, 1]$$

where $c_1^\star = \frac{\tau_a c_0^{\star,2}}{\tau_z + \tau_a c_0^{\star,2}}\,(1 - c_0^\star)$ *and* $c_2^\star = 1 - c_0^\star - c_1^\star$. *At this constrained efficient outcome, the social welfare loss is lower than the corresponding welfare loss under the optimal policy,* $\mathcal{W}_{TS}^\star < \mathcal{W}^\star$*, and the informativeness of the public signal higher,* $c_0^\star > k_0^\star$.

The team solution always attains a smaller social welfare loss than that achieved under the optimal policy. This inefficiency of the market outcome under the optimal policy is yet another consequence of the learning externality. Suppose, for instance, that the level of instrument policy ϕ is set such as to equate the loading onto the policy maker's information with the team solution case. This equivalence requires that $\phi = c_2^\star$ since the entire weight onto the policy maker's information stems from the policy instrument under complete opacity (see 4.3). Because of the learning externality, this policy would, however, result in less weight onto private information, and hence a less informative economy-wide outcome, than under the team solution that internalizes

[29] As also discussed in the introduction, while James and Lawler's (2011) contribution focuses on the relative weight placed on public information, their main results in essence derive from the above identified distinction: that instrument policy, unlike communication policy, can alter the weight on private and policy maker information without the introduction of additional noise. Their results can therefore be seen as one important example of how the basic mechanism that is studied in this paper extends beyond the learning externality, to other drivers of the suboptimality of full disclosure – in this case, the suboptimal direct strategic complementarity that exists in Morris and Shin's (2002) beauty contest model (see also *Online Appendix C*).

the externality, $k_0 < c_0^\star$. Under the optimal policy, the policy maker therefore trades off the loading onto his information with the information content of the economy-wide outcome, and hence achieves an outcome where $\phi^\star < c_2^\star$ and $k_0 < k_0^\star < c_0^\star$.

In essence, with only the policy instrument, the policy maker cannot achieve an efficient use of both private and his own information. And since communication policy merely offers a more inefficient trade-off between the two, the policy maker cannot here achieve the constraint efficient outcome. This underscores a notable difference between our setup, with its combination of endogenous public information and an imperfectly informed policy maker, and much of the related literature that is able to attain the efficient outcome (James and Lawler (2011), Angeletos and La'O (2014) and Angeletos *et al.* (2016), for instance).[30]

5 A Business Cycle Application

The analysis that we have covered shows within an abstract framework why the conditional use of a policy instrument is a better means to use a policy maker's information than direct disclosure. I now demonstrate how the above rationale for instrument policy generalizes to a more concrete, workhorse setup.

To this end, I show how the prediction framework can be used to guide welfare analysis in a micro-founded business cycle model, in which firms set prices under incomplete dispersed information. This application is particularly relevant since it allows me to capture the important case where we identify the policy maker with the central bank, the most common provider of policy maker information. It also conveniently allows me to illustrate how several of the above mentioned extensions merely reinforce the main insights from Theorem 1. Specifically, how the presence of (*1*) direct strategic interactions and inefficient disturbances, factors that elsewhere have been shown critical for the benefits of public information disclosure; and (*2*) additional information about prospective policy from, for instance, the information that current policy instruments provide do not alter my basic argument (see also *Online Appendix C* and *D*). The *Online Appendix* considers another related application, in which both firms and workers make their employment decisions under incomplete information and the policy maker instead corresponds to the tax authority. Prices are, by contrast, in this example market-clearing.

5.1 A Monetary Economy with Pre-set Prices

Apart from the introduction of an imperfectly informed policy maker and learning from market prices, I base the analysis on an amended version of the model described by Angeletos *et al.* (2016). The specification of monetary policy follows Hellwig (2005).

[30]The information structure in Angeletos and La'O (2014) is, like in this paper, endogenous. But Angeletos and La'O (2014) allow policy to be set on what here would correspond to a *perfect* realization of the economy-wide outcome$\int_0^1 a_i di$. This added information above what the private sector uses combined with the additional term in the instrument rule allows the policy maker to completely eliminate the wedge between what is attained under the market outcome and the first best under full information (see also Angeletos and Pavan, 2009).

The economy consists of a representative household comprised of a consumer and a continuum of workers. There is a continuum of islands, indexed by $i \in [0, 1]$, which delineate local labor markets and partition the information structure. On each island, a continuum of firms $j \in [0, 1]$ specialize in the production of differentiated goods. Each period is comprised of two stages. In the first stage, firms pre-set their prices. At this stage, firms receive information about local productivity but have imperfect information about the productivity of, and hence demand from, other islands. After prices are set, the economy transitions to the second stage, where all information that was previously dispersed becomes publicly known. Workers are now sent to each island, where they produce what is demanded of island goods at stated prices. The local wage adjusts to clear the market. Commodity markets open and the household consumes.

Households: The utility of the representative household equals,

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{1}{1+\eta} \int_0^1 \int_0^1 L_{ijt}^{1+\eta} dj di \right], \tag{5.1}$$

where β denotes the household discount factor; L_{ijt} the number of hours worked by the worker who is sent to firm j on island i in the second stage of period t; η parametrizes the Frisch elasticity of labor supply; and C_t the level of aggregate consumption. The latter equals,

$$C_t = \left(\int_0^1 C_{it}^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}}, \quad C_{it} = \left(\int_0^1 C_{ijt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \tag{5.2}$$

where C_{ijt} is the quantity the household consumes in period t of the goods produced by firm j on island i and $\rho > 1$. The use of a two-level CES structure is here convenient; it allows me to later incorporate mark-up shocks (real demand disturbances) into the analysis. P_t denotes the associated welfare-based price index and P_{it} the index of prices set by firms on island i:

$$P_t = \left(\int_0^1 P_{it}^{1-\rho} di \right)^{\frac{1}{1-\rho}}, \quad P_{it} = \left(\int_0^1 P_{ijt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \tag{5.2'}$$

Since the representative household receives all labor income and profits, its per-period budget constraint is,

$$\int_0^1 \int_0^1 P_{ijt} C_{ijt} dj di + M_t^d \leq \int_0^1 \int_0^1 \Pi_{ijt} dj di + \int_0^1 \int_0^1 W_{it} L_{ijt} dj di + M_{t-1}^d + T_t, \tag{5.3}$$

where Π_{ijt} is the profit of the representative firm j on island i; M_t^d the household's demand for nominal balances; W_{it} the nominal wage on island i; and T_t lump-sum transfers. Household consumption is, in addition to (5.3), restricted by a *cash-in-advance constraint* after receiving nominal transfers,

$$\int_0^1 \int_0^1 P_{ijt} C_{ijt} dj di \leq M_{t-1}^d + T_t, \quad T_t = M_t - M_{t-1}. \tag{5.4}$$

The household seeks to maximize its utility (5.1) subject to the budget constraint (5.3) and the cash-in-advance constraint (5.4).

Firms: The representative firm j on island i produces output in accordance with the linear production function,

$$Y_{ijt} = X_{it} L_{ijt}, \tag{5.5}$$

where L_{ijt} denotes the labor input used and X_{it} the level of total factor productivity on island i. Island productivity is made up of two distinct components,

$$x_{it} = \theta_t + \epsilon^i_{xt},$$

where lower-case letters denote the log of upper-case letters, $\theta_t \sim \mathcal{WN}\left(0,\, 1/\tau_\theta\right)$ unobserved aggregate productivity and $\epsilon^i_{xt} \sim \mathcal{WN}\left(0,\, 1/\tau_x\right)$ a purely island-specific productivity shock. Both θ_t and ϵ^i_{xt} are assumed independent of each other and all other stochastic disturbances. The representative firm's objective is to set its price P_{ijt} to maximize its expectation of the household's valuation of its profits using the discount factor $1/\left(P_t C_t\right)$. Profits at time t equal,

$$\Pi_{ijt} = P_{ijt} Y_{ijt} - W_{it} L_{ijt}.$$

Central Bank: The policy maker in this economy, the central bank, has access to two separate tools: *(i)* the money supply M_t and *(ii)* the precision of its disclosure ω_t about its own beliefs z_t about aggregate productivity. Similar to the baseline model, the money supply is set in accordance with,

$$m_t - m_{t-1} = \phi_0 + \phi z_t + \epsilon_{mt}, \tag{5.6}$$

where $z_t = \theta_t + \epsilon_{zt}$ and ϕ_0 is always set such that $\delta \equiv \beta e^{-\phi_0 + \frac{1}{2}\phi^2\left(\frac{\tau_z + \tau_\theta}{\tau_\theta \tau_z}\right)} < 1$. I will later describe how this inequality ensures that the cash-in-advance constraint always binds, and hence allows the central bank to effectively control nominal demand in the economy. The error terms ϵ_{zt} and ϵ_{mt} are white noise normal with precision τ_z and τ_m and independent of all other random disturbances. Communication policy is set as in the baseline model.

Information Structure: To close the model, I need to specify the information structure. Analogous to the baseline model, island i firms' information sets equal,

$$\Omega_{it} = \left\{\chi_{i\tau},\, \omega_\tau,\, p_\tau,\, \Delta\hat{m}_\tau,\, m_{\tau-1}\right\}_{\tau=-\infty}^{\tau=t}, \tag{5.7}$$

where $\chi_{it} = \theta + \epsilon^i_{\chi t}$ and $p_t = \log(P_t) + \epsilon_{at}$.[31] Both $\epsilon^i_{\chi t}$ and ϵ_{at} are white noise normal with mean zero and precision τ_χ and τ_a, respectively. I hence assume that firms observe a noisy signal of the economy-wide price level and use this signal to estimate demand from other islands. As in Lorenzoni (2010), the noise in p_t can be attributed to the statistical error that occurs when firms

[31] Equation (5.7) allows firms to also have imperfect information about local productivity. That is, for $\chi_{it} \neq x_{it}$. However, neither of my results would change if instead $\chi_{it} = x_{it}$ (see *Online Appendix D* and Section 5.3).

observe only a random sample of other island prices, instead of the full distribution.[32]

Unlike the baseline model, firms in (5.7) also observe the noisy signal $\Delta\hat{m}_t = \phi z_t + \epsilon_{st}$, where ϵ_{st} is white noise normal with precision τ_s, in addition to last period's money supply m_{t-1}. The purpose of the additional signal $\Delta\hat{m}_t$ is to provide a summary measure that captures all other sources of information available about the money supply in a tractable manner. *Online Appendix D* provides a pertinent example in which such other sources of information include the observation of a policy instrument that is isomorphic to the observation of $\Delta\hat{m}_t$. The more responsive monetary policy is, that is the larger $\phi > 0$ is, the more informative $\Delta\hat{m}_t$ is of the central bank's beliefs, and hence about the expected money supply. The presence of $\Delta\hat{m}_t$ thus in a reduced form manner captures the *signaling effects* of more active monetary policy discussed in, for instance, Baeriswyl and Cornand (2010) (see Section 5.2 and *Online Appendix D*). Because of ϵ_{st}, however, $\Delta\hat{m}_t$ does not perfectly reveal the central bank's beliefs. This is important; it allows communication policy to still provide additional information about the central bank's beliefs, and thus for a meaningful choice between communication and monetary policy.

Equilibrium Characterization: I proceed in two steps: First, I solve the representative household's problem, imposing market clearing, to derive an equilibrium relationship between the wage on island i and nominal demand in the economy. Then, I use this relationship to derive a simple expression for island i firm prices. Lemma 2 details the first step.

Lemma 2. *When $\delta < 1$, the cash-in-advance constraint always binds and the wage rate on island $i \in [0, 1]$ satisfies: $w_{it} - \eta l_{ijt} = m_t - \log(\delta)$.*

We can now use Lemma 2 with ϕ_0 set such that $\delta < 1$ to derive a simple expression for the solution to a firm's problem. This allows me to show that island firm prices are determined by:

Lemma 3. *The unique linear equilibrium price that firms on island $i \in [0, 1]$ set satisfies,*

$$p_{it} \quad = \quad \mathbb{E}_{it}\left[\xi\bar{p}_t + (1-\xi)\left(m_t - x_{it}\right)\right] = \kappa_0\chi_{it} + \kappa_1 p_t + \kappa_2\omega_t + \kappa_3\hat{m}_t, \tag{5.8}$$

where $\bar{p}_t = \log\left(P_t\right)$ and $\xi \equiv \frac{\eta(\rho-1)}{1+\rho\eta} \in (0, 1)$ determines the extent of strategic complementarity. The coefficients κ_1, κ_2 and κ_3 are functions of $\kappa_0 \in \mathbb{R}$ and the parameters of the model.[33]

Equation (5.8) closely resembles the decision rule analyzed in Sections 2 to 4; the main difference being the presence of direct strategic complementarity between firm prices.

Corollary 2 now follows from a similar approach to that used in Section 3.

Corollary 2. *The informativeness of the economy-wide price level, p_t, is equal to $\tau_a\kappa_0^2$, where κ_0 is determined by the unique solution to,*

$$\kappa_0 = -(1-\xi)\tau_\chi \frac{1 - \phi\frac{\tau_z}{\tau_\omega+\phi^2\tau_s+\tau_z}}{\tau_\theta + (1-\xi)\tau_\chi + \tau_a\kappa_0^2 + \tau_z\frac{\tau_\omega+\phi^2\tau_s}{\tau_\omega+\phi^2\tau_s+\tau_z}}. \tag{5.9}$$

[32] The best an individual firm can do in this case is to average the prices it observes. This can result in the expression used for p_t, and also explains why only p_t enters in (5.7) (see, for instance, Lorenzoni, 2010).

[33] For simplicity, I in this Section ignore unimportant constant and pre-determined terms (see *Appendix B*).

Both increases in τ_ω when $\phi \in \left[0,\, \hat{\phi}\right]$ and increases in $\phi \in \left[0,\, 1\right]$ decrease the informativeness of the price level. The expression for $\hat{\phi}$ is now given by $\hat{\phi} \equiv \frac{\tau_z}{\tau_\theta + \tau_x(1-\xi) + \tau_a \kappa_0^2 + \tau_z}$.

5.2 The Dominance of a Policy Instrument Revisited

The previous subsection characterized equilibrium prices across our economy. Let me now turn to the optimal use of the central bank's information. The central bank here seeks to maximize the *ex-ante* expected utility of the representative household by adjusting its two tools: the money supply and the precision of the signal that it sends about its own beliefs about economy-wide productivity. Because of the two-stage structure of the model, conditional on an optimal production subsidy, welfare in the economy takes the following particularly simple form:

Lemma 4. *There exists a function $f : \mathbb{R}_+ \to \mathbb{R}$ such that the ex-ante level of welfare equals,*

$$\mathcal{W} = f\left(\Lambda\right), \quad \Lambda = \mathbb{E}\left[y_t - y_t^\star\right]^2 + b\mathbb{E}\left[p_{it} - \bar{p}_t\right]^2 \tag{5.10}$$

$$= \mathbb{E}\left[p_{it} - \left(m_t - \theta_t\right)\right]^2 + d\mathbb{E}\left[p_{it} - \bar{p}_t\right]^2 \equiv \mathbb{E}\left[\Delta_{it}\right]^2 + d\mathbb{E}\left[\Theta_{it}\right]^2, \tag{5.11}$$

where $y_t^\star = \theta_t$ denotes the first-best level of output, $b \equiv \frac{\rho}{1-\xi}$ and $d \equiv b - 1 > 0$. Moreover, $\mathcal{W}$ attains its maximum (the first best level) at $\Lambda = 0$ and is strictly decreasing in Λ.

Equation (5.10) is a well-known expression for social welfare. Similar to, for instance, Woodford (2002), $\mathcal{W}$ is determined by a convex combination of the variance of the output gap and the cross-sectional dispersion of relative prices. This expression for social welfare can, however, also be re-written to closely resemble that used in our baseline model. In fact, with the exception of the dispersion term $\mathbb{E}\left[\Theta_{it}\right]^2$, the two are identical in (5.11). While of interest, this additional term which determines the desire for more coordination between island prices will nevertheless not affect our rationale for instrument policy. But before I arrive at that important point, let me first detail the optimal policy. Using once more the primal approach shows that:

Proposition 5. *The optimal central bank policy is complete opacity, $\tau_\omega \to 0$, combined with active monetary policy, $\phi = \phi^\star_{mp} > 0$. The informativeness of the price level, p_t, under the optimal policy is either greater or smaller than that achieved with full disclosure, $\left|\kappa_0^\star\right| \gtreqless \left|\kappa_0^d\right|$.*

As in the baseline model, the combination of complete opacity and active instrument policy is here optimal. However, in contrast to Theorem 1, we cannot always conclude that the informativeness of the price level under the optimal policy is above that attained with full disclosure. This indeterminacy is due to two offsetting forces.[34]

Learning Externality vs. Insufficient Coordination: On the one hand, *the learning externality* induces the central bank to push towards less use of its own information, more of private information, and hence towards a more informative price level. However, on the other hand,

[34]Because of these offsetting forces, there exist parameters $(\rho - 1)\left[\tau_\theta + \tau_\chi\left(1-\xi\right)\rho + \tau_z\right] = \tau_a\left(1-\xi\right)\tau_x$ such that $\kappa_0^\star = \kappa_0^d$. With the exception of this knife-edge case, the sole use of monetary policy is *uniquely* optimal.

the direct strategic complementarity between island firm prices is here *conditionally insufficient*: Conditional on the informativeness of the price level, the equilibrium degree of coordination between firm prices is below the socially optimal.[35] Provided that κ_0 is held constant, firm prices are too disperse. As Hellwig (2005) makes clear, this excess dispersion is due to the precise nature of monopolistic competition implied by (5.2). More emphasis on central bank information and less on private information may therefore all else equal be beneficial (*cf.* Angeletos and Pavan, 2007). It would help firms better coordinate prices, lowering the welfare loss due to excessive dispersion. Which of these two effects ultimately dominates determines whether the price level will be more or less informative under the optimal policy than under full disclosure.[36]

That said, despite the conditional insufficient coordination between firm prices, the exclusive use of monetary policy is always optimal in Proposition 5. This can seem unexpected. After all, absent monetary policy and keeping the informativeness of the price level constant, (5.8) and (5.11) make public disclosure invariably beneficial.

The reason is that the policy instrument, nominal demand, here has two distinct advantages over communication policy. First, it is able to control the weight on private and central bank information without the introduction of noise to the information structure, the advantage from our baseline model. And second, it can increase the equilibrium weight on central bank information, even beyond the full disclosure case. This allows the central bank to create even more coordination between firm prices than with full disclosure. I now turn to how these two benefits interact to create Proposition 5. This will also allow me to demonstrate how the advantage from our baseline model is generically present, in stark contrast to the second advantage.

The Added Cost of Noise: Suppose, to start, that *the learning externality dominates the lack of coordination between firm prices* such that $|\kappa_0^\star| > \left|\kappa_0^d\right|$. Consider now once more the two distinct policies from Section 4 that attempt to push more emphasis onto private information than in the full disclosure case: *(a)* the exclusive use of communication policy $\{\phi = 0,\, \tau_\omega = \tau_\omega^a \in \mathbb{R}_+\}$ and *(b)* the sole use of instrument policy $\left\{\phi = \phi^b < \hat{\phi},\, \tau_\omega \to 0\right\}$. We can then use Lemma 3 and 4 to decompose the two components that make up social welfare $\mathcal{W}$ in (5.11).

Lemma 5. *The two components of social welfare $\mathcal{W}$, the deviation from the effective state of the economy Δ_{it} and the dispersion in firm prices Θ_{it}, equal under the two policies (a) and (b),*

$$\Delta_{it}^a = \kappa \mathbb{E}_{it}^o[\theta_t] + \theta_t + \beta_\omega^a \left[z_t - (1-\xi)\,\mathbb{E}_{it}^o[\theta_t]\right] + \epsilon_{mt} + \beta_\omega^a \epsilon_{\omega t}, \qquad \Theta_{it}^a = \kappa_0 \epsilon_{\chi t}^i \tag{5.12}$$

$$\Delta_{it}^b = \kappa \mathbb{E}_{it}^o[\theta_t] + \theta_t + \phi^b \left[z_t - (1-\xi)\,\mathbb{E}_{it}^o[\theta_t]\right] + \epsilon_{mt}, \qquad \Theta_{it}^b = \kappa_0 \epsilon_{\chi t}^i, \tag{5.13}$$

where $\kappa \equiv \xi - 1 + \xi\kappa_0$ and $\beta_\omega^a \equiv (\xi - 1 + \xi\kappa_0)\frac{w_\omega^a}{1-\xi}$. The weight firms attach to the policy maker's signal ω in (a) when forming their expectations about productivity θ_t is denoted by w_ω^a.[37]

[35] The socially optimal amount of coordination is proportional to $\rho > 1$ times the equilibrium amount.

[36] *Online Appendix C* considers an example of *insufficient direct strategic substitutability* and shows how the lack of substitutability reinforces the insights from Theorem 1.

[37] The expression for w_ω^a is: $w_\omega^a = \frac{\tau_\omega^a \tau_z}{(\tau_\omega^a + \phi^2\tau_s)(\tau_\theta + \tau_\chi + \tau_a\kappa_0^2 + \tau_z) + \tau_z(\tau_\theta + \tau_\chi + \tau_a\kappa_0^2)}$.

Combined, (5.12) and (5.13) show how the added welfare cost of noise from our prediction framework carries over to the business cycle case. Indeed, the two expressions are almost identical to those derived in Section 4. If we set $\phi^b = \beta^a_\omega$, the only difference between Δ^a_{it} and Δ^b_{it} is once more the noise term $\beta^a_\omega \epsilon_{\omega t}$ from the central bank's partial disclosure. The level of κ_0 and hence the dispersion terms are by contrast equal from (5.9), $\Theta^a_{it} = \Theta^b_{it}$. Squaring the added noise term and taking ex-ante expectations therefore once more illustrates the additional welfare cost that the use of communication policy entails.

The Benefit of More Coordination: Now, suppose instead the converse; that the *lack of coordination between firm prices dominates the learning externality.* Then, unlike in our baseline model, the emphasis on the central bank's information under full disclosure is below the socially optimal. That is, to alleviate the lack of coordination, one needs more use of the central bank's information and less of private information than in the full disclosure case.

The best means for the central bank to achieve an outcome where $|\kappa^\star_0| < \left|\kappa^d_0\right|$ is, however, still exclusively with instrument policy. Only with $\phi > \lim_{\tau_\omega \to \infty} \beta^a_\omega = \hat{\phi}$ and $\tau_\omega \to 0$ can the central bank increase the emphasis on its own information, and thereby decrease that on private information, beyond the full disclosure case without the noise associated with partial disclosure.

To see this, notice how increases to ϕ^b in (5.13) can arbitrarily increase the equilibrium loading on z_t under complete opacity. By contrast, with communication policy in (5.12) the central bank cannot push the equilibrium loading on z_t above the full disclosure limit, $\lim_{\tau_\omega \to \infty} \beta^a_\omega = \hat{\phi}$. Furthermore, the level of nominal demand is fully known when the central bank perfectly discloses its information. So even if the central bank tried in this case to use nominal demand to further increase the equilibrium loading onto its information, it would not be able to do so. Equation (5.9) shows the corollary of this limitation: increases to ϕ decrease the emphasis on private information $|\kappa_0|$, but only when the policy maker's information is not fully disclosed – that is, when $\tau_\omega \nrightarrow \infty$. The combination of complete opacity and active instrument policy is therefore once more optimal because it avoids the noise associated with partial disclosure. It is the only means by which the central bank can decrease $|\kappa_0|$ below $\left|\kappa^d_0\right|$ without the added welfare cost that noisy, partial disclosure entails.

Thus, while the lack of coordination does not affect our basic rationale for the use of policy instruments, its existence does provide instrument policy with yet another advantage. When the learning externality is relatively strong ($|\kappa^\star_0| > \left|\kappa^d_0\right|$), the exclusive use of monetary policy is optimal *only because* it avoids the introduction of noise to the information structure. When the lack of coordination, by contrast, is relatively severe ($|\kappa^\star_0| < \left|\kappa^d_0\right|$), the sole use of monetary policy is optimal *partially also* because it can increase coordination between firm prices beyond the full disclosure case. Comparing $|\kappa^\star_0|$ with $\left|\kappa^d_0\right|$ shows which case we are in.

Further Reasons for $|\kappa^\star_0| > \left|\kappa^d_0\right|$: That said, there exist two important caveats that bring the results in Proposition 5 even closer to those from our baseline model.

First, whether *nominal or real firm choices* are based on incomplete dispersed information matters critically for the conditional efficiency of firm choices (see Angeletos *et al.*, 2016). *On-*

line Appendix D shows that when firms instead of setting prices make their labor input decisions before the resolution of demand, firm choices are *conditionally efficient*. This is despite the presence of direct strategic interactions between firms. The only inefficiency in the use of information that remains in this case is the learning externality. All results in *Online Appendix D* therefore precisely mirror those from our baseline model – specifically $|\kappa_0^\star| > |\kappa_0^d|$ for all parameter values.

Second, the insufficiency in the coordination between firm prices is influenced by the exact instrument rule used by the central bank (*cf.* Angeletos and Pavan, 2009 and Angeletos and La'O, 2012). Assume, for instance, that when the central bank sets nominal demand, it also eventually observes a different signal of the price level than firms. Let us for simplicity assume that it perfectly observes $\bar{p}_t$ and sets $\Delta m_t = \phi_0 + \phi z_t + \psi \bar{p}_t + \epsilon_{mt}$.[38] Then, by setting $\psi = \hat{\psi} \equiv 1 - \rho$ the central bank can make the coordination between firm prices *conditionally efficient*. This, in turn, brings us back to a situation that resembles the baseline model. Given $\psi = \hat{\psi}$, optimally $|\kappa_0^\star| > |\kappa_0^d|$. Clearly, the exact details of the policy depend here on the fairly unrealistic assumption that the central bank *perfectly* observes the price level. But any future central bank information about $\bar{p}_t$ will tend to push the optimal policy closer to a situation where $|\kappa_0^\star| > |\kappa_0^d|$.[39]

Signaling Effects of Monetary Policy: Similar to Theorem 1, Proposition 5 builds on how expectations about monetary policy alter firm choices. A common indicator of the central bank's beliefs, and thus about expected monetary policy, is in reality often the current level of the central bank's policy instrument. I have above accommodated for such signaling effects of current policy in a reduced form manner within our repeated one-shot framework in which only the expected money supply matters for firm prices; I have done so through the summary statistic $\Delta \hat{m}_t$. The more responsive monetary policy is, that is the larger ϕ is, the more informative $\Delta \hat{m}_t$ is of the central bank's beliefs, and hence about the expected money supply. This is similar to the decrease in uncertainty that would occur in a fully dynamic model when the current policy instrument would to a larger extent reflect the central bank's information, and hence its future intensions (*cf*. Melosi 2016). *Online Appendix D* provides a pertinent example of a dynamic setup in which the additional information that exists about future policy comes in part from the observation of a current policy instrument. *Online Appendix C* instead considers in depth a simple extension of the static prediction framework, which mirrors the information structure used in (5.7).

In all three cases, the main insights from Theorem 1 continue to hold. The reason is simple: Suppose that $|\kappa_0^\star| > |\kappa_0^d|$. A decrease in ϕ from $\hat{\phi}$, where $\hat{\phi}$ once more replicates the full disclosure outcome, still increases the informativeness of the price level, alleviating the consequences of the learning externality without the introduction of additional noise (see 5.12 and 5.13). Clearly, decreases in ϕ now also come at the cost of less information about the central bank's beliefs through a decrease in the informativeness of $\Delta \hat{m}_t$. But for a small decrease in ϕ below $\hat{\phi}$, the

[38] This assumption is, of course, extreme: if the central bank *perfectly* observes $\bar{p}_t$, then it can *perfectly* back-out θ_t and achieve the full-information first best with $\psi \to -\infty$. But it is here simply meant to demonstrate the importance of the exact instrument rule used by the central bank for the coordination between firm prices.

[39] I here compare $|\kappa_0^\star|$ with the full disclosure level $|\kappa_0^d|$ where ψ has been set optimally. Additional central bank information about θ_t has similar consequences to that about $\bar{p}_t$.

first order benefit of alleviating the learning externality still outweighs the second order cost from a distorted use of the central bank's information. Thus, although the optimal value of ϕ is closer to $\hat{\phi}$ than in an economy without such signaling effects, the main conclusions from Theorem 1 remain.

5.3 Monetary Policy with Inefficient Disturbances

We have seen how the insights from Theorem 1 extend to a micro-founded business cycle model in which direct strategic complementarities between firm prices matter for the optimal use of central bank information. I now turn to how these insights also continue to hold when responses to the underlying fundamental themselves are inefficient, such as with real demand shocks, even under full information. Hellwig (2005) and Angeletos and Pavan (2007) demonstrate how the inefficiency of the underlying disturbance provides yet another reason for why full disclosure does not attain the constrained efficient outcome.

I now introduce such an *inefficient disturbance*, a mark-up shock, into our business cycle framework. I do so by assuming that the elasticity of substitution σ in (5.2) is island-specific and stochastic such that,

$$\mu_{it} \equiv \log \mathcal{M}_{it} = \mu_t + \epsilon^i_{\chi t}, \quad \mathcal{M}_{it} \equiv \frac{\sigma_{it}}{\sigma_{it} - 1}, \tag{5.14}$$

where $\mathcal{M}_{it}$ denotes a firm's stochastic markup and both μ_t and $\epsilon^i_{\chi t}$ are white noise normal with mean zero and precision τ_μ and τ_χ, respectively. To keep matters simple, I assume that island productivity is constant and equal to $x_{it} = \theta_t = \bar{\theta} \in \mathbb{R}$. I make two further changes to the model: First, I assume that firms observe their island-specific mark-up before they set prices, $\chi_{it} = \mu_{it}$. And second, I assume that both the central bank's information and disclosure pertain to the economy-wide mark-up shock, μ_t. The rest of the model is identical to before.

Lemma 6. *The equilibrium price that island $i \in [0, 1]$ firms set when mark-up shocks drive the economy equals,*

$$p_{it} = \mathbb{E}_{it}\left[\xi \bar{p}_t + (1-\xi)\, m_t\right] + \mu_{it} = \underline{\kappa}_0 \mu_{it} + \underline{\kappa}_1 p_t + \underline{\kappa}_2 \omega_t + \underline{\kappa}_3 \hat{m}_t, \tag{5.15}$$

where $\underline{\kappa}_1$, $\underline{\kappa}_2$ and $\underline{\kappa}_3$ are functions of $\underline{\kappa}_0 \in \mathbb{R}$ and the parameters of the model. The informativeness of the economy-wide price level, p_t, once more equals $\tau_a \underline{\kappa}_0^2$.

By contrast to the efficient productivity shock, even if firms here perfectly observe the mark-up disturbance, responses to it are still suboptimal. If $\mu_t = \mu_{it} \in \Omega_{it}$, the equilibrium response is $\frac{1}{1-\xi}\mu_t$, while the socially optimal is zero. Firms overreact compared to what is socially optimal.

This overreaction provides yet another reason for full disclosure to be suboptimal. Even when monetary policy is mute and keeping the informativeness of the price level constant, the optimal disclosure policy can equal $\tau_\omega \to 0$.[40] Providing more information, in this case, merely

[40] This occurs when $(1+\xi)\frac{\xi\tau_x}{\tau_\mu+\tau_x+\tau_a\underline{\kappa}_0^2} < 1 - \rho\,(1-\xi)$.

exacerbates the excess response to the mark-up disturbance, making any disclosure suboptimal.

Once we allow for the active use of monetary policy and internalize the informativeness of the economy-wide price level, it is thus hardly surprising that the fully optimal policy once more features complete opacity combined with active monetary policy.

Proposition 6. *The optimal central bank policy when mark-up shocks drive the economy is complete opacity combined with active monetary policy, $\phi = \phi^{\star}_{mp} < 0$. The informativeness of the economy-wide price level, p_t, under the optimal policy is either greater or smaller than that achieved under full disclosure, $\underline{\kappa}_0^{\star} \gtreqless \underline{\kappa}_0^{d}$.*

Since mark-up shocks are inefficient, firms tend to all else equal overreact to new information about them. This, in turn, tends to push the optimal emphasis on central bank information to be below that under full disclosure, and hence the emphasis on private information to be above. Similar to the learning externality, the presence of mark-up shocks thus tends towards an optimal policy in which $\underline{\kappa}_0^{\star} > \underline{\kappa}_0^{d}$. The presence of mark-up shocks therefore provides yet another reason for monetary policy to be optimal solely because it avoids the introduction of noisy information.

6 Conclusion

The past two decades have seen a considerable increase in the amount of public information provided by policy makers. This paper has proposed a novel argument against such disclosure that rests on the alternative means that a policy maker has to use his information: he can either disclose his news or condition a policy instrument on its realization.

At the core of my results has been a simple distinction: With the conditional use of a policy instrument, a policy maker can directly control the influence of his own information on market outcomes. With disclosure, by contrast, a policy maker can only decrease other agents' reliance on his information by obfuscating his news at an added welfare cost. This paper has shown how this basic distinction provides a novel rationale for the exclusive use of policy instruments within the context of models in the spirit of Lucas (1972). Indeed, if the policy instrument is set optimally, by also disclosing his information the policy maker merely succeeds in lowering welfare.

This preference for the conditional use of a policy instrument has been demonstrated to hold irrespective of whether the equilibrium amount of coordination is above or below the socially optimal. Because of this robustness, my rationale has carried over from an abstract prediction game to a micro-founded business cycle model, in which firms learn from market prices and either a central bank or the tax authority sets economy-wide policy.

Contrary to public information from policy makers, full disclosure from a statistical office or a news service would have been preferable to complete opacity across the models that I have considered, either for some parameter values or uniformly. My results have thus underscored how policy maker information differs from other sources of public information – precisely because a policy maker can always use his information to set a policy instrument instead.

Finally, in this paper I have relied on abstract and simple frameworks that do not internalize non-informational reasons for setting a policy instrument, such as considerations of equity or menu-costs. I have thus, for clarity, abstracted from the challenge of how to balance a policy's informational role with its other objectives. Such trade-offs could perceivably reintroduce an active role for communication policy. That said, I believe that the core informational effects studied above would continue to extend to such cases; the ability of a policy maker to condition a policy instrument on his own information would still be central for his optimal disclosure. I would therefore like to view this analysis as a step towards a comprehensive picture of optimal policy under imperfect information.

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Appendix A: The Basic Model

This Appendix details the proofs of the Propositions and Theorems in Sections 2 to 4.

Proof of Proposition 1: I follow the three step procedure outlined in Subsection 2.2. To solve for the linear rational expectations equilibrium, I conjecture that a_i follows:

$$a_i \quad = \quad \mathbb{E}_i[\theta - m] = \mathbb{E}_i[\theta] - \phi\mathbb{E}_i[z] \tag{A1}$$

$$\quad = \quad k_0 x_i + k_1 a + k_2 \omega. \tag{A2}$$

Given that $\int_0^1 \epsilon_x^i di = 0$, it follows that the signal of the economy-wide action equals,

$$a \quad = \int_0^1 a_i di + \epsilon_a = \quad k_0\theta + k_1 a + k_2\omega + \epsilon_a. \tag{A3}$$

But since a partially depends on itself, in addition to the policy maker's communication, it is cumbersome to solve individuals' signal extraction problem using (A3). Instead, it is much simpler to consider the signal,

$$\frac{1}{k_0}\left(a - k_1 a - k_2\omega\right) = \theta + \frac{1}{k_0}\epsilon_a. \tag{A4}$$

I denote this signal by y, and notice that y is independent of the other signals in Ω_i conditional on θ.

To verify the solution in (A2), we need to compute expressions for individuals' expectations of both the underlying fundamental and the policy maker's beliefs. Because of the assumptions about our information structure, these are given by,

$$\mathbb{E}_i[\theta] = w_x x_i + w_y y + w_\omega \omega, \quad w_x \equiv \frac{\tau_x(\tau_z + \tau_\omega)}{(\tau_z + \tau_\omega)(\tau_x + \tau_a k_0^2) + \tau_z\tau_\omega}, \tag{A5}$$

and

$$\mathbb{E}_i[z] = v_x x_i + v_y y + v_\omega \omega, \quad v_x \equiv \frac{\tau_x\tau_z}{(\tau_z + \tau_\omega)(\tau_x + \tau_a k_0^2) + \tau_z\tau_\omega}, \tag{A6}$$

where w_j and v_j, $j = \{y, \omega\}$ are implicitly defined and $\sum_{j=x,y,\omega} w_j = \sum_{j=x,y,\omega} v_j = 1$.

Now, inserting (A5) and (A6) into (A1), we find that,

$$a_i \quad = \quad (w_x - \phi v_x)x_i + (w_y - \phi v_y)y + (w_\omega - \phi v_\omega)\omega, \tag{A7}$$

$$\quad = \quad (w_x - \phi v_x)x_i + (w_y - \phi v_y)\frac{1-k_1}{k_0}a + \left[(w_\omega - \phi v_\omega) - \frac{k_2}{k_0}(w_y - \phi v_y)\right]\omega$$

which verifies our conjecture *iff.* there exists a solution to the system of equations,

$$k_0 = w_x - \phi v_x, \quad k_1 = \frac{\tau_a k_0^2}{\tau_x + \tau_a k_0^2}, \quad k_2 = (1-\phi)\frac{\tau_x}{\tau_x + \tau_a k_0^2} - k_0, \tag{A8}$$

where $k_j \in \mathbb{R}$, $j = \{0, 1, 2\}$ and I have repeatedly used that $\sum_{j=x,y,\omega} w_j - \phi v_j = 1 - \phi$. Because all coefficient equations depend only on k_0, all that, however, needs to be shown is that the equation for k_0 has a solution.

The equation determining k_0 is,

$$k_0 = \frac{\tau_x[\tau_\omega + (1-\phi)\tau_z]}{(\tau_z + \tau_\omega)(\tau_x + \tau_a k_0^2) + \tau_z\tau_\omega}. \tag{A9}$$

Since $\phi \in [0,\, 1]$, it is clear that any solution must lie within the unit interval. Furthermore, since the right-hand side of (A9) is always decreasing in k_0 towards zero from a positive value, while the left-hand side equals the 45°-line, there must in fact exist a *unique* solution to (A9) within $k_0 \in (0,\, 1)$.

□

Proof of Proposition 2: The social welfare loss function *when keeping* k_0 *constant at* $\bar{k}_0$ equals,

$$\mathcal{W}(\phi,\, p_\omega) = \frac{1}{2}\frac{\phi^2\left(\tau_x + \tau_a \bar{k}_0^2 + \tau_z\right) + \tau_\omega + \tau_z - 2\phi\tau_z}{\left(\tau_z + \tau_\omega\right)\left(\tau_x + \tau_a \bar{k}_0^2\right) + \tau_z\tau_\omega} + \frac{1}{2}\tau_m^{-1}. \tag{A10}$$

Differentiating (A10) shows that:

- $\frac{\partial \mathcal{W}}{\partial \tau_\omega} = -\frac{1}{2}\left[\frac{\phi\left(\tau_x+\tau_a\bar{k}_0^2+\tau_z\right)-\tau_z}{\left(\tau_\omega+\tau_z\right)\left(\tau_x+\tau_a\bar{k}_0^2\right)+\tau_\omega\tau_z}\right]^2 < 0$ for all $\phi \neq \hat{\phi}$ and that $\frac{\partial \mathcal{W}}{\partial \tau_\omega} = 0$ when $\phi = \hat{\phi}$.
- $\frac{\partial \mathcal{W}}{\partial \phi} = \frac{\phi\left(\tau_x+\tau_a\bar{k}_0^2+\tau_z\right)-\tau_z}{\left(\tau_\omega+\tau_z\right)\left(\tau_x+\tau_a\bar{k}_0^2\right)+\tau_\omega\tau_z} \lesseqgtr 0$ for all $\phi \lesseqgtr \hat{\phi}$ and that $\frac{\partial \mathcal{W}}{\partial \phi} = 0$ when $\phi = \hat{\phi}$.

All that remains to show is that $\mathcal{W}(0,\, \mathbb{R}_+) = \mathcal{W}\left(\left[0,\, \hat{\phi}\right],\, \tau_\omega \to 0\right)$. The limit of $\mathcal{W}$ when $\tau_\omega \to \infty$ equals: $\lim_{\tau_\omega\to\infty}\mathcal{W} = \frac{1}{\tau_x+\tau_a k_0^2+\tau_z} + \frac{1}{2}\tau_m^{-1} \equiv \hat{\mathcal{W}}$. Evaluating $\mathcal{W}$ at the critical point $\left\{\hat{\phi},\, \tau_\omega \to 0\right\}$ shows that $\mathcal{W}\left(\hat{\phi},\, \tau_\omega \to 0\right) = \hat{\mathcal{W}}$. Combined with that $\mathcal{W}$ is continuous and decreasing in both τ_ω and $\phi \leq \hat{\phi}$, this equivalence between the optimum outcome that can be attained with either communication or instrument policy alone shows that $\mathcal{W}(0,\, \mathbb{R}_+) = \mathcal{W}\left(\left[0,\, \hat{\phi}\right],\, \tau_\omega \to 0\right)$.[1]

□

Proof of Proposition 3: The precision of the signal y conditional on the fundamental θ equals $\tau_a k_0^2$. A smaller value of $k_0 \geq 0$ thus, all else equal, implies a less informative public signal.

Totally differentiation of (A9) with respect to ϕ shows that,

$$\frac{dk_0}{d\phi} = \left(1 - \frac{\partial k_0^{RHS}}{\partial k_0}\right)^{-1}\frac{\partial k_0^{RHS}}{\partial \phi},$$

where k_0^{RHS} denotes *the right-hand side* of (A9). But since $\frac{\partial k_0^{RHS}}{\partial k_0} \leq 0$ and $\frac{\partial k_0^{RHS}}{\partial \phi} < 0$ for all $k_0 \geq 0$ and $\phi \in [0,\, 1]$, $\frac{dk_0}{d\phi} < 0$. Increases in $\phi \in [0,\, 1]$ always lead to decreases in the equilibrium value of k_0.

Total differentiation of (A9) with respect to τ_ω, by contrast, yields,

$$\frac{dk_0}{d\tau_\omega} = \left(1 - \frac{\partial k_0^{RHS}}{\partial k_0}\right)^{-1}\frac{\partial k_0^{RHS}}{\partial \tau_\omega}, \tag{A11}$$

where,

$$\frac{\partial k_0^{RHS}}{\partial \tau_\omega} = \frac{-\tau_x\tau_z\left[\tau_z - \phi\left(\tau_x + \tau_a k_0^2 + \tau_z\right)\right]}{\left[\left(\tau_z + \tau_\omega\right)\left(\tau_x + \tau_a k_0^2\right) + \tau_z\tau_\omega\right]^2}.$$

This latter expression (and hence $\frac{dk_0}{d\tau_\omega}$) is equal to zero when τ_ω is finite *iff.* $\hat{\phi}$ solves,

$$\hat{\phi} = \frac{\tau_z}{\tau_x + \tau_a k_0^2\left(\hat{\phi},\, \tau_\omega\right) + \tau_z} \in (0,\, 1)\,. \tag{A12}$$

[1] It furthermore follows from (A10) that the full set of optimal policies when keeping k_0 constant equals the union of $\{\phi \in [0,\, 1]\,,\, \tau_\omega \to \infty\}$ and $\left\{\phi = \hat{\phi},\, \tau_\omega \in \mathbb{R}_+\right\}$ since $\mathcal{W}([0,\, 1]\,,\, \tau_\omega \to \infty) = \mathcal{W}\left(\hat{\phi},\, \tau_\omega\right) = \hat{\mathcal{W}}$.

But such a value always exists since $\hat{\phi}$ solves (A12) *iff.* $\hat{\phi}$ solves,

$$\hat{\phi} = \frac{\tau_z}{\tau_x} k_0 \left(\hat{\phi},\, \tau_\omega \right), \tag{A13}$$

where I have used (A9). Because $\frac{dk_0}{d\phi} < 0$, (A13) always has a unique positive solution; the right-hand side is strictly decreasing in $\phi \in [0,\, 1]$ from a positive value, while the left-hand side equals the 45°-line. Moreover, this solution is independent of τ_ω. Taking the derivative of (A13) with respect to τ_ω immediately shows that $\frac{d\hat{\phi}}{d\tau_\omega} = 0$ since $\frac{\partial k_0}{\partial \tau_\omega}_{\phi=\hat{\phi},\, \tau_\omega \in \mathbb{R}_+} = \frac{dk_0}{d\tau_\omega}_{\phi=\hat{\phi},\, \tau_\omega \in \mathbb{R}_+} = 0$ from (A11). It thus follows that $\phi \lesseqgtr \hat{\phi}$ implies that $\frac{\partial k_0^{RHS}}{\partial \tau_\omega} \lesseqgtr 0$, and hence from (A11) that $\frac{dk_0}{d\tau_\omega} \lesseqgtr 0$.

It remains to show that $k_0\left(0,\, \mathbb{R}_+\right) = k\left(\left[0,\, \hat{\phi}\right],\, \tau_\omega \to 0\right)$. This, however, follows almost directly since $\lim_{\tau_\omega \to \infty} k_0\left(0,\, \tau_\omega\right) = k_0\left(\hat{\phi},\, \tilde{\tau}_\omega\right)\ \forall \tilde{\tau}_\omega \in \mathbb{R}_+$, including when $\tilde{\tau}_\omega \to 0$. Combined with that k_0 is continuous and decreasing in both τ_ω and $\phi \leq \hat{\phi}$, this demonstrates the last part of the Proposition.

□

Proof of Theorem 1: The proof proceeds in three steps: The first step uses Proposition 1 and Corollary 1 to derive a convenient expression for $\mathcal{W}$ as a function of k_0 and τ_ω. The second step then uses that expression to find the unique, optimal values of these two parameters. Last, we use Corollary 1 once more to translate the optimal k_0 coefficient back into the level of instrument policy that it entails.

Step 1: Equilibrium welfare.

The deviation of a person's action from the effective state of the economy equals,

$$\begin{aligned}\Delta_i &= a_i - (\theta - m)\\ &= k_0 x_i + \tilde{k}_1 y + \tilde{k}_2 \omega - \theta + \phi z + \epsilon_m = k_0 \epsilon_x^i + \frac{\tilde{k}_1}{k_0}\epsilon_a + \left(\phi + \tilde{k}_2\right)\epsilon_z + \tilde{k}_2 \epsilon_\omega + \epsilon_m,\end{aligned}$$

where from (A7) and Corollary 1, $\tilde{k}_1 \equiv w_y - \phi v_y = \frac{\tau_a k_0^2}{\tau_x} k_0$, $\tilde{k}_2 \equiv w_\omega - \phi v_\omega$ and $k_0 + \tilde{k}_1 + \widetilde{k_2} = 1 - \phi$. Thus, after a few, simple derivations,

$$\mathcal{W} = \frac{1}{2}\mathbb{E}\left[\Delta_i\right]^2 = \frac{1}{2}\left[\frac{\tau_x + \tau_a k_0^2}{\tau_x^2} k_0^2 + \left(\frac{\tau_x + \tau_a k_0^2}{\tau_x^2} k_0 - 1\right)^2 \frac{1}{\tau_z} + \tilde{k}_2^2 \frac{1}{\tau_\omega} + \frac{1}{\tau_m}\right]. \tag{A14}$$

Step 2: The unique optimal values of k_0 and τ_ω.

Consider now, to start, the only term that depends on τ_ω in (A14),

$$\tilde{k}_2{}^2 \frac{1}{\tau_\omega} = \tau_\omega \frac{\left[\tau_z - \phi\left(\tau_x + \tau_a k_0^2 + \tau_z\right)\right]^2}{\left[\left(\tau_z + \tau_\omega\right)\left(\tau_x + \tau_a k_0^2\right) + \tau_z \tau_\omega\right]^2} \geq 0.$$

This term attains its global minimum when either $\tau_\omega \to 0$, $\tau_\omega \to \infty$ or $\phi = \hat{\phi}$. But when $\tau_\omega \to \infty$ or $\phi = \hat{\phi}$, k_0 is always equal to its full disclosure value, k_0^d. This follows from Example 2 and Corollary 1. Thus, if $k_{\star,0} \equiv \arg\min_{k_0} \mathcal{W}\left(k_0,\, \tau_\omega \to 0\right) \neq k_0^d$, then we know that complete opacity is uniquely optimal.

Minimizing (A14) when $\tau_\omega \to 0$ shows that this is indeed the case; the unique optimal (complete opacity) policy is characterized by,[2]

$$k_{\star,0} = D\left(k_{\star,0}\right) \equiv \frac{\tau_x}{\tau_x + \tau_a k_0^2 + \tau_z\left(\frac{\tau_x + 2\tau_a k_{\star,0}^2}{\tau_x + 3\tau_a k_{\star,0}^2}\right)} \in \left(k_0^d,\, k_0^o\right), \tag{A15}$$

[2]The social welfare loss function in (A14) is strictly pseudo-convex when $\tau_\omega \to 0$. A solution to $\frac{d\mathcal{W}}{dk_0} = 0$ [or (A15)] is therefore the unique global minimum. But such a solution $k_{\star,0} > 0$ always exists since $D(0) = \frac{\tau_x}{\tau_x + \tau_z} > 0$ and $\lim_{k_0 \to \infty} D\left(k_0\right) = 0$, which combined with the continuity of D secures a crossing with the 45^o-line at a positive point.

where k_0^o equals the *complete opacity, no-policy case*, $k_0^o = \frac{\tau_x}{\tau_x+\tau_a\left(k_0^o\right)^2}$.

Step 3: The optimal level of instrument policy, $\phi_\star$.

This optimal value of k_0 can, in turn, uniquely be implemented by,

$$\phi_\star = 1 - \frac{\tau_x + \tau_a k_{\star,0}^2}{\tau_x} k_{\star,0} = \frac{\tau_z}{\tau_x + (1+\alpha)\,\tau_a k_0^{\star,2} + \tau_z} \in \left(0,\, \hat{\phi}\right), \quad \alpha \equiv \frac{\tau_x + \tau_a k_0^{\star,2}}{\tau_x + 2\tau_a k_0^{\star,2}} > 0, \tag{A16}$$

where I once more have used Corollary 1. This completes the Proof.

Proof of Proposition 4: The proof proceeds in two steps: I first show that there exists a unique constrained efficient outcome and characterize the solution. I then demonstrate that $c_{\star,0} > k_{\star,0}$.

Step 1: The unique constrained efficient outcome.

The constrained efficient outcome is given by the solution to the program,

$$\begin{aligned} \min_{c_0,\,c_1,\,c_2} \mathcal{W}_{TS} &= \frac{1}{2}\mathbb{E}\int_0^1 \mathbb{E}_i\left[a_i - \theta\right]^2 di \\ & \quad s.t. \\ a_i &= c_0 x_i + c_1 y + c_2 z, \quad \forall i \in [0,\,1] \\ y &= \theta + \frac{1}{c_0}\epsilon_a, \end{aligned}$$

where it is a property of the solution that $c_0 + c_1 + c_2 = 1$. I have here written a_i in terms of y instead of a, like in (A7), to simplify the subsequent derivations. Inserting the constraints into the objective function and minimizing the resulting strictly pseudo-convex function with respect to c_0 and c_1 shows that the constrained efficient outcome is characterized by the solution to the equations,

$$c_{\star,0} = J\left(c_{\star,0}\right), \quad J\left(c_{\star,0}\right) \equiv \tau_x \frac{\tau_z + \tau_a c_{\star,0}}{\tau_x\left(\tau_z + \tau_a c_{\star,0}\right) + \left(\tau_z + \tau_a c_{\star,0}^2\right)^2} \tag{A17}$$

$$c_{\star,1} = \left(1 - c_{\star,0}\right)\frac{\tau_a c_{\star,0}^2}{\tau_z + \tau_a c_{\star,0}^2}. \tag{A18}$$

Equation (A17) [and hence (A18)] has a unique (strictly positive) solution *iff.*,[3]

$$P\left(c_0\right) \equiv \tau_a c_0^5 + 2\tau_z c_0^3 + \tau_x c_0^2 - \tau_x c_0 = \frac{\tau_z}{\tau_a}\left[\tau_x - \left(\tau_x + \tau_z\right)c_0\right], \tag{A19}$$

has a unique (strictly positive) solution. But since,

$$\frac{\partial^2 P}{\partial c_0^2} = 20\tau_a c_0^3 + 12\tau_z c_0 + 2\tau_x > 0 \quad \forall c_0 \geq 0,$$

it follows that P is strictly convex for all $c_0 \geq 0$.

Combining the convexity of P with $P(0) = 0$ and that $\frac{\tau_z}{\tau_a}\left[\tau_x - \left(\tau_x + \tau_z\right)c_0\right]$ is linearly decreasing in $c_0 \geq 0$ from $\frac{\tau_x\tau_z}{\tau_a} > 0$ completes the proof of the first step (see Figure A1).[4]

Step 2: The informativeness of public signals, $c_{\star,0} > k_{\star,0}$.

I consider two cases: *(i)* $\tau_x \leq \tau_z$ and *(ii)* $\tau_x > \tau_z$.

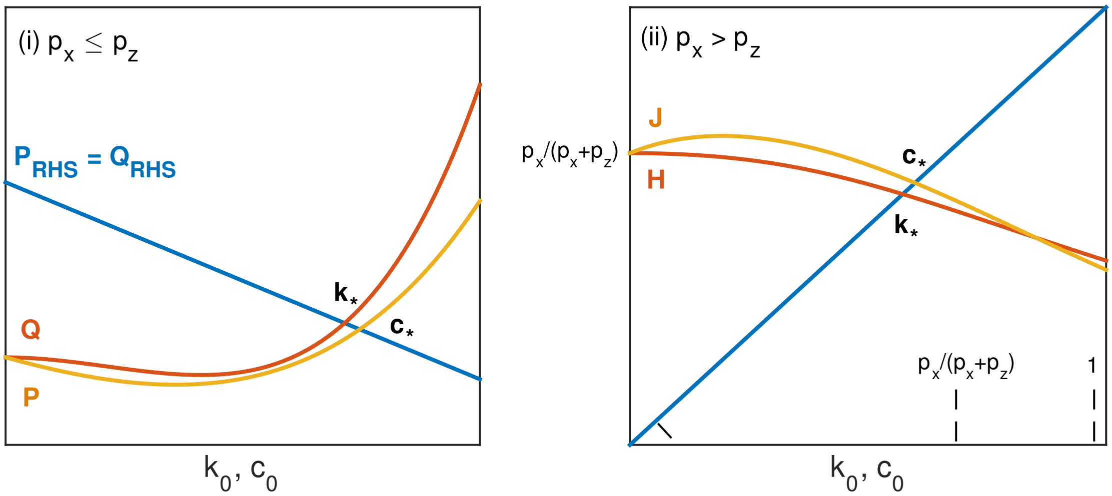
[3]It immediately follows from (A17) and (A18) that $c_{\star,0} \in (0,\,1)$ and $c_{\star,1}, > 0$.

[4]This may seem like a roundabout way to prove uniqueness. After all, the social welfare loss function,

$$\mathcal{W}_{TS} = \frac{1}{2}\left[c_0^2\frac{1}{\tau_x} + \left(\frac{c_1}{c_0}\right)^2\frac{1}{\tau_a} + \left(c_0 + c_1 - 1\right)^2\frac{1}{\tau_z}\right]$$

is strictly pseudo-convex and a solution to the first-order conditions [or (A17) and (A18)] always exists. So uniqueness is guaranteed. However, the more indirect approach that I here adopt will be of use for *Step 2*.

Figure A1: The Team Solution vs. The Optimal Policy



The left-hand side of Figure A1 depicts Q, P and $\frac{p_z}{p_a}\left[p_x - \left(p_x + p_z\right) h\right]$ for $h \geq 0$; the right-hand side exhibits J and H.

(AD i) The equation determining $k_{\star,0}$ can be rewritten as,

$$Q\left(k_0\right) \equiv \left(\frac{\tau_z}{\tau_x}\right)\left[3\tau_a k_0^5 + 2\tau_z k_0^3 + 4\tau_x k_0^3 - 3\tau_x k_0^2\right] = \frac{\tau_z}{\tau_a}\left[\tau_x - \left(\tau_x + \tau_z\right) k_0\right], \tag{A20}$$

where $Q(h) > P(h)\ \forall h > 0$ since $4h^3 - 3h^2 \geq h^2 - h\ \forall h > 0$ and $Q(0) = P(0) = 0$.

But since Q, like P, is strictly convex (with a unique minimum for $h \geq 0$) and $\frac{\tau_z}{\tau_a}\left[\tau_x - \left(\tau_x + \tau_z\right) h\right]$ is linearly decreasing in $h \geq 0$ from $\frac{\tau_z \tau_x}{\tau_a} > 0$, it directly follows that $c_{\star,0} > k_{\star,0}$ (see Figure A1).[5]

(AD ii) Equation (A15) can be restated as,

$$k_{\star,0} = H\left(k_{\star,0}\right), \quad H\left(k_{\star,0}\right) \equiv \frac{\tau_x\left(\tau_x + 3\tau_a k_{\star,0}^2\right)}{\tau_z\left(\tau_x + 2\tau_a k_{\star,0}^2\right) + \left(\tau_x + 3\tau_a k_{\star,0}^2\right)\left(\tau_x + \tau_a k_{\star,0}^2\right)}, \tag{A21}$$

where $H\left(\frac{\tau_x}{\tau_x+\tau_z}\right) < J\left(\frac{\tau_x}{\tau_x+\tau_z}\right) < \frac{\tau_x}{\tau_x+\tau_z}$, $H\left(0\right) = J\left(0\right) = \frac{\tau_x}{\tau_x+\tau_z}$ and $H\left(1\right) > J\left(1\right)$.

Differentiating H directly yields that (see Figure A1),

$$\frac{\partial H}{\partial k_0} = \frac{-2k_0 \tau_a \tau_x\left(9\tau_a^2 k_0^4 + 6\tau_a \tau_x k_0^2 + \tau_x^2 - \tau_x \tau_z\right)}{\left[\tau_z\left(\tau_x + 2\tau_a k_{\star,0}^2\right) + \left(\tau_x + 3\tau_a k_{\star,0}^2\right)\left(\tau_x + \tau_a k_{\star,0}^2\right)\right]^2} < 0, \quad \forall k_0 > 0.$$

J, by contrast, has a unique maximum for $c_0 \geq 0$. This follows from the first-order condition having a unique solution ($c_{J,0} > 0$),

$$\frac{\partial J}{\partial c_0} = 0 \Leftrightarrow -3\tau_a^2 c_0^4 - 4\tau_a \tau_z c_0^3 - 2\tau_a \tau_z c_0^3 - 4\tau_z^2 c_0 + \tau_z^2 = 0,$$

where uniqueness follows from *Descartes' Rule of Signs,* and that $\frac{\partial J}{\partial c_0} \gtrless 0$ for $c_o \geq 0$ *iff.* $c_0 \lessgtr c_{J,0}$.

Thus, it follows that $c_{\star,0} > k_{\star,0}$, and hence that $\mathcal{W}_{TS}^{\star} < \mathcal{W}^{\star}$ – even if we were to disregard the influence of the added noise component ϵ_m caused by the policy instrument.

□

[5]The existence of a unique minimum for $\{k_0,\, c_0\} > 0$ for Q and P is apparent from their first derivatives in conjunction with *Descartes' Rule of Signs.*

Appendix B: Business Cycle Application

This Appendix derives the results discussed in Section 5.

Proof of Lemma 2: The Lagrangian of the household's problem is,

$$\begin{aligned}\mathcal{L} &= \mathbb{E}_0 \sum_t \beta^t \left[\log C_t - \frac{1}{1+\eta} \int_0^1 \int_0^1 L_{ijt}^{1+\eta} dj di - \mu_t \left(P_t C_t - M_{t-1}^d - T_t \right) \right. \\ &- \left. \lambda_t \left(P_t C_t + M_t^d - \int_0^1 \int_0^1 \Pi_{ijt} dj di - \int_0^1 \int_0^1 W_{it} L_{ijt} dj di - M_{t-1}^d - T_t \right) \right],\end{aligned}$$

with corresponding sufficient first-order conditions,

$$C_t : \quad \tfrac{1}{C_t} - P_t \left(\lambda_t + \mu_t \right) = 0, \qquad \frac{1}{P_t C_t} = \lambda_t + \mu_t \tag{A22}$$

$$L_{ijt} : \quad -L_{ijt}^{\eta} + \lambda_t W_{it} = 0, \qquad L_{ijt}^{\eta} = \lambda_t W_{it} \tag{A23}$$

$$M_t^d : \quad -\lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} + \mu_{t+1} \right] = 0, \quad \lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} + \mu_{t+1} \right]. \tag{A24}$$

Now, (A22) and (A23) show that,

$$\mu_t = \frac{1}{P_t C_t} - \frac{L_{ijt}^{\eta}}{W_{it}} = (1 - \delta) \frac{1}{M_t^s} > 0,$$

which proves the first part of the Lemma.

To find δ, notice that (A24) directly implies that,

$$\frac{\delta}{M_t^s} = \beta \mathbb{E}_t \left[\frac{\delta}{M_{t+1}^s} + \frac{1-\delta}{M_{t+1}^s} \right] = \beta \mathbb{E}_t \left[\frac{1}{M_{t+1}^s} \right] = \beta e^{-m_t^s - \phi_0 + \frac{1}{2}\phi^2 \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_z} \right)}.$$

This, in turn, shows that $\delta = \beta e^{-\phi_0 + \frac{1}{2}\phi^2 \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_z} \right)}$.

□

Proof of Lemma 3: Because of the two-stage CES structure, firm (i, j)'s demand equals,[6]

$$Y_{ij} = \left(\frac{P_{ij}}{P_i} \right)^{-\sigma} Y_i, \quad Y_i = \left(\frac{P_i}{P} \right)^{-\rho} Y.$$

The representative firm's problem is therefore to,

$$\max_{p_{ij}} \mathbb{E}_i \left[\frac{1}{PC} \left\{ (1 - T_s) P_{ij} Y_{ij} - W_i L_{ij} \right\} \right] =$$

$$\max_{p_{ij}} \mathbb{E}_i \left[(1 - T_s) P_{ij}^{1-\sigma} P_i^{\sigma-\rho} P^{\rho-1} - \frac{W_i}{X_i} P_{ij}^{-\sigma} P_i^{\sigma-\rho} P^{\rho-1} \right].$$

where $1 - T_s = -\frac{\sigma}{1-\sigma}$. The sufficient first-order condition to this problem implies that,

$$\mathbb{E}_i \left[\left(\frac{P_i}{P} \right)^{-\rho} \frac{1}{P} \right] = \mathbb{E}_i \left[\left(\frac{P_i}{P} \right)^{-\rho-1} \frac{W_i}{P} \frac{1}{X_i P} \right]$$

[6] I here for convenience drop time subscripts.

since $Y_i = X_i L_i$ and island firm prices are symmetric. Thus,

$$P_i = \mathbb{E}_i \left[P^{\rho} \left(\frac{W_i}{X_i P} \right) \right] \left(\mathbb{E}_i \left[P^{\rho - 1} \right] \right)^{-1}. \tag{A25}$$

To show Lemma 2, and hence characterize the equation describing firm prices, we thus need to derive expressions for the different terms in (A25). To do so, I first conjecture and later verify that,

$$p_i = \tilde{\kappa} + \kappa_0 \chi_i + \kappa_1 y^s + \kappa_2 \omega + \kappa_3 \tilde{m}, \tag{A26}$$

where $k_j \in \mathbb{R}$, $j = \{\tilde{\cdot}, 0, 1, 2, 3\}$, $y^s = s^p = \theta + \frac{1}{\kappa_0}\epsilon_a$ denotes the orthogonalized version of $p = \log(P_t) + \epsilon_a$ and $\tilde{m} = z + \frac{1}{\phi}\epsilon_s$. It follows from (A26) and (5.2) that P is log-normal. We can thus re-write (A25) as,[7]

$$\begin{aligned} p_i &= \frac{1}{2} \left(\mathbb{V}\left[\rho \bar{p} + mc_i \right] - \mathbb{V}\left[\bar{p} \right] \right) + \mathbb{E}_i \left[\bar{p} + mc_i \right] \\ &\equiv \underline{\kappa} + \mathbb{E}_i \left[\bar{p} + mc_i \right], \end{aligned} \tag{A27}$$

where $mc_i = w_i - x_i - \bar{p}$ denotes island i's marginal cost and $\bar{p} = \log(P)$.

Now, equating labor supply with labor demand (using Lemma 1) shows that,

$$\frac{W_i}{P} = \frac{1}{\delta} \left(\frac{P_i}{P} \right)^{-\rho\eta} X_i^{-\eta} \left(\frac{M}{P} \right)^{1+\eta},$$

or in terms of logs,

$$mc_i = -\log(\delta) - \rho\eta \left(p_i - \bar{p} \right) - (1+\eta) x_i + (1+\eta)(m - \bar{p}). \tag{A28}$$

Last, inserting (A28) into (A27) shows that,

$$\begin{aligned} p_i &= \left(\frac{\underline{\kappa} - \log(\delta)}{1 + \rho\eta} \right) + \mathbb{E}_i \left[\frac{\eta(\rho - 1)}{1 + \rho\eta} \bar{p} + \frac{1+\eta}{1+\rho\eta}(m - x_i) \right], \\ &\equiv \bar{\kappa} + \mathbb{E}_i \left[\xi \bar{p} + (1 - \xi)(m - x_i) \right], \end{aligned} \tag{A29}$$

which is the equation provided in Lemma 2.

It remains to check (A26) and to characterize the coefficients in the solution. Solving firms' signal extraction problem yields,

$$\begin{aligned} \mathbb{E}_i[\theta] &= w_\chi \chi_i + w_y y^s + w_m \tilde{m} + w_\omega \omega, \quad w_x = \frac{\tau_\chi \left(\tau_\omega + \tau_z + \phi^2 \tau_s \right)}{\left(\tau_\omega + \phi^2 \tau_s \right)\left(\tau_\theta + \tau_\chi + \tau_a \kappa_0^2 + \tau_z \right) + \tau_z \left(\tau_\theta + \tau_\chi + \tau_a \kappa_0^2 \right)} \\ \mathbb{E}_i[z] &= v_\chi \chi_i + v_y y^s + v_m \tilde{m} + v_\omega \omega, \quad v_x = \frac{\tau_\chi \tau_z}{\left(\tau_\omega + \phi^2 \tau_s \right)\left(\tau_\theta + \tau_\chi + \tau_a \kappa_0^2 + \tau_z \right) + \tau_z \left(\tau_\theta + \tau_\chi + \tau_a \kappa_0^2 \right)}. \end{aligned}$$

Using these expressions in conjunction with (A26), (A29) and (5.2) shows that,

$$\begin{aligned} p_i &= \bar{\kappa} + \frac{1 - \rho}{2} \xi \mathbb{V}[\bar{p}] - (1 + \xi) \mathbb{E}_i[\theta] \\ &+ \mathbb{E}_i \left[\xi \left(\tilde{\kappa} + \kappa_0 \theta + \kappa_1 y^s + \kappa_2 \omega + \kappa_3 \tilde{m} \right) + (1 - \xi)(m_{-1} + \phi_0 + \phi z) \right], \end{aligned}$$

[7] Throughout this Appendix, I use that if $Q \sim LN(\mu, \sigma^2)$ then $\mathbb{E}[Q^\delta] = \mathbb{E}[Q]^\delta \, exp\left[\frac{\delta}{2}(\delta - 1)\sigma^2 \right]$.

and hence that,

$$\begin{aligned} p_i &= \bar{\kappa} + \frac{1-\rho}{2}\xi \mathbb{V}[\bar{p}] + (1-\xi)(m_{-1}+\phi_0) + \xi(\tilde{\kappa} + \kappa_1 y^s + \kappa_2 \omega + \kappa_3 \tilde{m}) \\ &+ (\xi - 1 + \xi\kappa_0)\mathbb{E}_i[\theta] + (1-\xi)\phi\mathbb{E}_i[z]. \end{aligned}$$

Thus,

$$\kappa_0 = (\xi - 1 + \xi\kappa_0) w_\chi + (1-\xi)\phi v_\chi \tag{A30}$$

$$\kappa_1 = \xi\kappa_1 + (\xi - 1 + \xi\kappa_0) w_y + (1-\xi)\phi v_y \tag{A31}$$

$$\kappa_2 = \xi\kappa_2 + (\xi - 1 + \xi\kappa_0) w_\omega + (1-\xi)\phi v_\omega \tag{A32}$$

$$\kappa_3 = \xi\kappa_3 + (\xi - 1 + \xi\kappa_0) w_m + (1-\xi)\phi v_m \tag{A33}$$

$$\tilde{\kappa} = \xi\tilde{\kappa} + \bar{\kappa} + \xi\frac{1-\rho}{2}\mathbb{V}[\bar{p}] + (1-\xi)(m_{-1}+\phi_0), \tag{A34}$$

where the uniqueness of κ_0, and thus of the solution to (A30) to (A34), follows from an analogous argument to that used in the Proof of Proposition 1.[8]

□

Proof of Lemma 4: See Angeletos et al (2016).
The first best full information level of local-island and economy-wide output equal, respectively,[9]

$$y_i^\star = \varsigma x_i + (1-\varsigma) y^\star, \quad y^\star = \theta.$$

where $\varsigma \equiv \frac{(1+\eta)\sigma}{1+\eta\sigma} > 1$. The same steps as in Angeletos et al (2016) can then be used to show that,

$$\mathcal{W} = f(\Lambda), \quad \Lambda = \mathbb{E}[y - y^\star]^2 + \frac{1}{\varsigma}\mathbb{E}[(y_i - y_i^\star) - (y - y^\star)]^2,$$

where f is strictly decreasing in Λ and $\mathcal{W}$ attains its maximum (the first best level) at $\Lambda = 0$. Equation (5.2) and Lemma 3 can then be combined to show that,

$$\Lambda = \rho(1-\xi)\frac{1}{\tau_x} + \mathbb{E}[y - y^\star]^2 + b\mathbb{E}[p_i - \bar{p}]^2.$$

The last equation in the lemma follows from that $\bar{y} = m - p$, $y^\star = \theta$ and,

$$\mathbb{E}[m - p - \theta]^2 = \mathbb{E}[m - p_i - \theta + p_i - p]^2 = \mathbb{E}[m - p_i - \theta]^2 - \mathbb{E}[p_i - p]^2.$$

This completes the statement.

□

[8] The equation for κ_0 is,

$$\kappa_0 = -(1-\xi)\tau_\chi \frac{1 - \phi\frac{\tau_z}{\tau_\omega + \phi^2\tau_s + \tau_z}}{\tau_\theta + (1-\xi)\tau_\chi + \tau_a\kappa_0^2 + \tau_z\frac{\tau_\omega + \phi^2\tau_s}{\tau_\omega + \phi^2\tau_s + \tau_z}}.$$

[9] I here ignore some unimportant constant terms for brevity.

Proof of Proposition 5: The approach used closely resembles those applied to the "*Noisy, Observable Instrument*" and the "*Direct Strategic Complementarity*" extensions in *Appendix C*. I therefore only provide a brief sketch of the Proof here.

Step 1: Equilibrium Welfare: Using the expression for Λ combined with (A26) and (A30) to (A34) shows after some routine algebra that,

$$\begin{aligned}\Lambda \quad &= \quad \frac{\rho}{1-\xi}\left[\frac{1}{\tau_\chi}\kappa_0^2+(1-\xi)^2\frac{1}{\tau_x}\right]+\frac{\tau_\theta+\tau_a\kappa_0^2}{\tau_x^2(1-\xi)^2}\kappa_0^2 \\ &+ \quad \left[1+\frac{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2}{(1-\xi)\tau_x}\kappa_0\right]^2\frac{1}{\tau_z}+\kappa_2^2\frac{1}{\tau_\omega}+\left(\frac{\kappa_3}{\phi}\right)^2\frac{1}{\tau_s}+\frac{1}{\tau_m}.\end{aligned} \tag{A35}$$

Step 2 and 3: The Weak Optimality of Opacity: Repeated use of the equilibrium conditions (A30), (A32) and (A33) combined with the loss function in (A35) then shows that:

- All *partial disclosure* policies where $\phi^p \neq \frac{\tau_z}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2+\tau_z}$ are strictly dominated by the *complete opacity* policy $\phi^o=\frac{1-\xi-\xi\kappa_0}{1-\xi}\beta_\theta+\phi^p\left(1-\beta_z\right)$, where β_θ and β_z denote the projection coefficients of θ and z onto $\mathcal{F}_\omega=\left\{\omega-\mathcal{P}_{\Omega_i^o}\omega\right\}$, respectively.
- Those *partial disclosure* policies where $\phi^p=\frac{\tau_z}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2+\tau_z}$, by contrast, achieve the same level of welfare loss as any *full disclosure* policy, which outcome can also be replicated under complete opacity by setting $\phi^o=\frac{\tau_z}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2+\tau_z}$.

Step 4: The Strict Optimality of Opacity: It remains to check when $\phi_{mp}^{o,\star}\neq\frac{\tau_z}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2+\tau_z}$, or equivalently when $\kappa_0^\star\neq\kappa_0^d=-\frac{(1-\xi)\tau_\chi}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2+\tau_z}$. To do so, consider Λ when $\tau_\omega\to 0$,

$$\Lambda=f\left(\kappa_0\right)+\left(\frac{\kappa_3}{\phi}\right)^2\frac{1}{\tau_s},$$

where I have used that $\kappa_2^2\frac{1}{\tau_\omega}\to 0$ when $\tau_\omega\to 0$ and f is the strictly pseudo-convex function,

$$f\left(\kappa_0\right)\equiv\frac{\rho}{1-\xi}\left[\frac{1}{\tau_\chi}\kappa_0^2+(1-\xi)^2\frac{1}{\tau_x}\right]+\frac{\tau_\theta+\tau_a\kappa_0^2}{\tau_x^2(1-\xi)^2}\kappa_0^2+\left[1+\frac{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2}{(1-\xi)\tau_x}\kappa_0\right]^2\frac{1}{\tau_z}.$$

Consider now the unique value of κ_0 that minimizes f,

$$\kappa_0^{\star,f}\;=\;-\frac{(1-\xi)\tau_\chi}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\left(\kappa_0^{\star,f}\right)^2+\tau_z\frac{\tau_\theta+\rho(1-\xi)\tau_\chi+2\tau_a\left(\kappa_0^{\star,f}\right)^2}{\tau_\theta+(1-\xi)\tau_\chi+3\tau_a\left(\kappa_0^{\star,f}\right)^2}}\lesseqgtr\kappa_0^d, \tag{A36}$$

where the inequality in (A36) depends on whether $(\rho-1)\left(\tau_\theta+\rho\tau_\chi\left(1-\xi\right)+\tau_z\right)^2\lesseqgtr\tau_a(1-\xi)\tau_\chi$. Because $\left|\kappa_0^{\star,f}\right|\gtreqless\left|\kappa_0^d\right|$, an identical argument to that used in the "*Noisy, Observable Instrument*" extension in *Appendix C* then establishes that $\left|\kappa_0^\star\right|\gtreqless\left|\kappa_0^d\right|$ *iff.* $(\rho-1)\left(\tau_\theta+\rho\tau_\chi\left(1-\xi\right)+\tau_z\right)^2\lesseqgtr\tau_a(1-\xi)\tau_\chi$.[10]

□

[10]We still need to check that $\phi_{mp}^{o,\star}>0$. This, however, follows from that either $\left|\kappa_0^\star\right|\in\left(\left|\kappa_0^d\right|,\left|\kappa_0^{\star,f}\right|\right)$ or $\left|\kappa_0^\star\right|\in\left(\left|\kappa_0^{\star,f}\right|,\left|\kappa_0^d\right|\right)$, depending on our parametric condition, and that κ_0 under complete opacity equals:

$$\kappa_0=-(1-\xi)\tau_\chi\frac{1-\phi\frac{\tau_z}{\phi^2\tau_s+\tau_z}}{\tau_\theta+(1-\xi)\tau_\chi+\tau_a\kappa_0^2+\tau_z\frac{\phi^2\tau_s}{\phi^2\tau_s+\tau_z}}.$$

Proof of Lemma 5: Consider island i's optimal price in case (a),

$$\begin{aligned} p_i^a = \mathbb{E}_i^a\left[\xi\bar{p} + (1-\xi)(m - x_i)\right] &= \mathbb{E}_i^a\left[\xi\left(\kappa_0\theta + \kappa_1 y^s + \kappa_2\omega + \kappa_3\Delta\hat{m}\right) + (\xi - 1)x_i\right] \\ &= \left(\xi - 1 + \xi\kappa_0\right)\mathbb{E}_i^a\left[\theta\right] + \xi\left(\kappa_1 y^s + \kappa_2\omega + \kappa_3\Delta\hat{m}\right), \end{aligned}$$

where I for convenience have dropped constant and pre-determined terms.

Using once more the decomposition,

$$\mathbb{E}_i^a\left[\theta\right] = \mathbb{E}_i^o\left[\theta\right] + w_\omega\left(\omega - \mathbb{E}_i^o\left[\theta\right]\right),$$

in addition to the equilibrium condition for κ_2 then shows that,

$$p_i^a = \left(\xi - 1 + \xi\kappa_0\right)\mathbb{E}_i^o\left[\theta\right] + \beta_\omega^a\left[z - (1-\xi)\,\mathbb{E}_i^o\left[\theta\right]\right] + \beta_\omega^a\epsilon_\omega + \bar{\kappa},$$

where $\beta_\omega^a = \kappa_2$ and $\bar{\kappa} \equiv \xi\kappa_1 y^s + \xi\kappa_3\Delta\hat{m}$.

We can likewise show that island i's optimal price in case (b) can be written as,

$$p_i^b = \left(\xi - 1 + \xi\kappa_0\right)\mathbb{E}_i^o\left[\theta\right] + (1-\xi)\,\phi^b\mathbb{E}_i^o\left[\theta\right] + \bar{\kappa}.$$

Lemma 4 then follows from inserting the expressions for p_i^a and p_i^b, respectively, into Lemma 3.

$\square$

Proof of Lemma 6: A representative firm $(i,\, j)$'s problem now equals,

$$\max_{p_{ij}} \mathbb{E}_i\left[\frac{1}{PC}\left\{(1-T_s)\,P_{ij}Y_{ij} - W_i L_{ij}\right\}\right] =$$

$$\max_{p_{ij}} \mathbb{E}_i\left[(1-T_s)\,P_{ij}^{1-\sigma_i}P_i^{\sigma_i-\rho}P^{\rho-1} - \frac{W_i}{X_i}P_{ij}^{-\sigma_i}P_i^{\sigma_i-\rho}P^{\rho-1}\right].$$

Thus,

$$P_i = \mathcal{M}_i\mathbb{E}_i\left[P^\rho\left(\frac{W_i}{X_i P}\right)\right]\left(\mathbb{E}_i\left[P^{\rho-1}\right]\right)^{-1},$$

where $\mathcal{M}_i = \frac{\sigma_i}{(1-T_s)(\sigma_i-1)}$ is now stochastic with $\log\mathcal{M}_i \equiv x_i = \mu + \epsilon_\mu^i$ and I have once more used the symmetry of island prices. The rest of the proof then follows the exact same steps used in the proofs of Lemma 2 and 3. The equilibrium coefficients equal,[11]

$$\underline{\kappa_0} = 1 + \xi\underline{\kappa_0}\beta_x + (1-\xi)\,\phi\gamma_x \tag{A37}$$

$$\underline{\kappa_1} = \xi\underline{\kappa_1} + \xi\underline{\kappa_0}\beta_y + (1-\xi)\,\phi\gamma_y \tag{A38}$$

$$\underline{\kappa_2} = \xi\underline{\kappa_2} + \xi\underline{\kappa_0}\beta_\omega + (1-\xi)\,\phi\gamma_\omega \tag{A39}$$

where the solution to $\underline{\kappa_0}$ (and thus $\underline{\kappa_1}$ and $\underline{\kappa_2}$) is unique because,

[11] I here ignore unimportant constant and pre-determined terms, akin to $\tilde{\kappa}$ in Lemma 3.

$$\beta_x = \frac{\tau_x\left(\tau_\omega + \tau_z\right)}{\left(\tau_\omega + \tau_z\right)\left(\tau_{\mathcal{M}} + \tau_x + \tau_a \underline{\kappa_0}^2\right) + \tau_\omega \tau_z}, \qquad \gamma_x = \frac{\tau_x \tau_z}{\left(\tau_\omega + \tau_z\right)\left(\tau_{\mathcal{M}} + \tau_x + \tau_a \underline{\kappa_0}^2\right) + \tau_\omega \tau_z},$$

such that (A37) can be re-written as,

$$\underline{\kappa_0} = 1 + \tau_x \frac{\xi\left(\tau_\omega + \tau_z\right) + \phi\left(1-\xi\right)\tau_z}{\left(\tau_\omega + \tau_z\right)\left(\tau_{\mathcal{M}} + (1-\xi)\tau_\chi + \tau_a \underline{\kappa_0}^2\right) + \tau_\omega \tau_z}. \tag{A40}$$

Equation (A40) has a unique solution for all parameter values since the right-hand side is continuous, symmetric around zero and ranges from 1 to $1 + \frac{\xi\tau_x(\tau_\omega+\tau_z)+\phi(1-\xi)\tau_x\tau_z}{(\tau_\omega+\tau_z)(\tau_{\mathcal{M}}+\tau_x)+\tau_\omega\tau_z} \lesseqgtr 0$ in k_0.

□

Proof of Proposition 6: The approach taken resembles that applied to the "*Direct Strategic Complementarity*" extension in *Appendix C*. I therefore only provide a sketch of the Proof here.

Step 1: Equilibrium Welfare: Using the expression for the social welfare loss function combined with (A37) to (A39) shows after some straightforward but tedious algebra that,

$$\begin{aligned}\Lambda \;=\; & \frac{\rho}{\tau_x(1-\xi)}\underline{\kappa_0}^2 + \left[\frac{1}{1-\xi} - \frac{\tau_{\mathcal{M}}}{\tau_x\left(1-\xi\right)}\left(\underline{\kappa_0}-1\right)\right]^2 \frac{1}{\tau_{\mathcal{M}}} \\ + \; & \frac{\tau_a \underline{\kappa_0}^2}{\tau_x^2\left(1-\xi\right)^2}\left(\underline{\kappa_0}-1\right)^2 + \left[\frac{1}{1-\xi} - \underline{\kappa_0} - \frac{\tau_{\mathcal{M}} + \tau_a \underline{\kappa_0}^2}{(1-\xi)\tau_x}\left(\underline{\kappa_0}-1\right)\right]^2 \frac{1}{\tau_z} + \underline{\kappa_2}^2 \frac{1}{\tau_\omega} + \frac{1}{\tau_m}.\end{aligned} \tag{A41}$$

Step 2 and 3: The Unique Optimal Values of $\underline{\kappa_0}$, τ_ω and ϕ: Consider the only term that depends on τ_ω in (A41), $\underline{\kappa_2}^2 \frac{1}{\tau_\omega}$. Because this term is similar to that from Theorem 1, it once more follows that complete opacity combined with active instrument policy is *uniquely optimal iff.* $\underline{\kappa_0^\star} \equiv \arg\min_{\underline{\kappa_0}} \Lambda\left(\underline{\kappa_0},\, \tau_\omega \to 0\right) \neq \underline{\kappa_0^d}$, where $\underline{\kappa_0^d}$ solves $\underline{\kappa_0^d} = 1 + \frac{\xi\tau_x}{\tau_{\mathcal{M}}+\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{d,2}}+\tau_z}$. All that remains is to characterize $\underline{\kappa_0^\star}$. Minimizing Λ when $\tau_\omega \to 0$ shows that $\underline{\kappa_0^\star}$ equals the unique solution to,[12]

$$\underline{\kappa_0^\star} = D\left(\underline{\kappa_0^\star}\right) \equiv 1 + \tau_x \frac{\xi + \tau_z \frac{1-\rho(1-\xi)}{\tau_{\mathcal{M}}+\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{\star,2}}+2\tau_a\underline{\kappa_0^\star}(\underline{\kappa_0^\star}-1)}}{\tau_{\mathcal{M}} + \tau_x(1-\xi) + \tau_a\underline{\kappa_0^{\star,2}} + \tau_z \frac{\tau_{\mathcal{M}}+\rho\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{\star,2}}+\tau_a\underline{\kappa_0^\star}(\underline{\kappa_0^\star}-1)}{\tau_{\mathcal{M}}+\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{\star,2}}+2\tau_a\underline{\kappa_0^\star}(\underline{\kappa_0^\star}-1)}} \gtreqless \underline{\kappa_0^d}, \tag{A42}$$

which is implemented by $\phi^\star_{mp} = \frac{\tau_{\mathcal{M}}+\tau_x+\tau_a\underline{\kappa_0^{\star,2}}}{1-\xi}\left(\underline{\kappa_0^\star}-1\right) - \frac{\xi}{1-\xi}\underline{\kappa_0^\star}$.[13]

□

[12]The uniqueness of $\underline{\kappa_0^\star}$ follows from the strict pseudo-convexity of Λ in (A41) when $\tau_\omega \to 0$. A solution to $\frac{d\Lambda}{d\underline{\kappa_0}} = 0$ [or (A42)] is therefore the unique global minimum. But such a solution $\underline{\kappa_0^\star}$ always exists since $\lim_{\underline{\kappa_0}\to\pm\infty} D\left(k_0\right) = 0$, which combined with the continuity of D ensures a crossing with the 45^o-line.

[13]We still need to check that $\phi^\star_{mp} < 0$. This, however, follows immediately from (A40) when $\tau_\omega \to 0$ when compared to $\underline{\kappa_0^\star}$ in (A42), where $\underline{\kappa_0^\star} \in \left(0,\, \underline{\kappa_0^{max}}\right)$ and $\underline{\kappa_0^{max}} > 1$ solves,

$$\underline{\kappa_0^{max}} = 1 + \tau_x \frac{\xi + \tau_z \frac{\xi}{\tau_{\mathcal{M}}+\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{max,2}}+2\tau_a\underline{\kappa_0^{max}}(\underline{\kappa_0^{max}}-1)}}{\tau_{\mathcal{M}} + \tau_x(1-\xi) + \tau_a\underline{\kappa_0^{max,2}} + \tau_z \frac{\tau_{\mathcal{M}}+\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{max,2}}+\tau_a\underline{\kappa_0^{max}}(\underline{\kappa_0^{max}}-1)}{\tau_{\mathcal{M}}+\tau_x(1-\xi)+\tau_a\underline{\kappa_0^{max,2}}+2\tau_a\underline{\kappa_0^{max}}(\underline{\kappa_0^{max}}-1)}}.$$