High-Frequency Cross-Market Trading: Model Free Measurement and Applications

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Abstract

We propose a set of intuitive model-free measures of cross-market trading activity based on publicly available trade and quote data with sufficient time stamp granularity. By virtue of capturing the offset at which co-activity peaks, as well as its magnitude and dispersion, the measures allow us to shed new light on the distinct features of the high-frequency cross-market linkages in US Treasury and equity markets. First, the measures avoid reliance on noisy return series often used in the literature and demonstrate sharp identification of the prevailing lead-lag relationships between trading activity across markets. Second, we show how the measures can be used to examine price impact and liquidity provision in (near) arbitrage linked markets. In particular, we provide new evidence pointing to the fact that price discovery in US Treasury, equity and EUR/USD FX markets primarily takes place in futures rather than cash markets and we give a strong rationale for considering the cross-market price impact between arbitrage linked markets. Finally, we show that our measures of cross-market activity are closely linked with observed market volatility even after controlling for commonly used measures of individual market activity such as trading volume and number of transactions. Overall, our empirical findings suggest that accounting for cross-market trading activity is important when studying volatility and liquidity in US Treasury and equity markets.

Keywords: High-frequency trading, cross-market activity, lead-lag relationships, liquidity provision, order flow, price impact, price discovery, realized volatility, S&P500, US Treasuries, FX markets

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1. Introduction

Market activity and volatility in the most liquid US fixed income and equity markets have long been recognized as important barometers of policy expectations and economic conditions that are scrutinized by market participants and policymakers alike. However, trading activity in today's leading financial markets is no longer dominated by long-term investors expressing their views about risk reward trade-offs. Instead, the majority of market activity has become associated with fast-speed automated trading strategies aiming to optimally place orders and quickly take advantage of short lived trading opportunities. Much of this activity is taking place across closely related trading venues such as the leading cash and futures platforms that are seen as distinct liquidity pools allowing investors to manage substantially similar risk exposures.

As such, there has been much debate on the impact of high-frequency trading (HFT) on liquidity provision and its role in increased price co-movements between different markets. This is at least in part because of difficulties with identifying the precise footprint of alleged changes in cross-market activity despite the unprecedented amount of market data available for many markets today. Motivated by the need for improved data-driven inference, this paper develops a simple new approach for sharp identification of arbitrary patterns of cross-market activity in the form of a set of intuitive model-free measures of cross-market trading activity based on publicly available trade and quote data with sufficient time stamp granularity. By virtue of capturing the time offset at which co-activity peaks, as well as the magnitude and dispersion of cross-market activity, the measures allow us to shed new light on the distinct features of the high-frequency cross-market linkages in US Treasury and equity markets.

Accordingly, the starting point of our analysis is that a core feature of the rise in HFT over the past decade has been the increasingly faster and by now almost instant order placement and execution in multiple markets.[1] Moreover, such high-frequency cross-market trading activity is known to take place in response not only to news about fundamentals but also to brief dislocations in relative values or market activity itself. This leads to four important observations and our main contributions in this paper.

First, while various types of cross-market trading, e.g. basis trading, have historically always been an important component of trading activity, the new market structure is dominated by automated trading which gives rise to high-frequency cross-market linkages

[1]By adopting fiber optic and microwave tower technology, order and market information transmission speeds have rapidly been approaching the speed of light.

subject to technologically determined time lags usually measured in milliseconds or microseconds. This allows the new cross-market activity measures we propose to reliably pin down key characteristics of the lead-lag relationships between different markets such as the prevailing timing offset, the magnitude of peak cross-activity and the dispersion around it due to market heterogeneity or platform technology. In this regard, our measures provide a useful complement to popular approaches for studying lead-lag relationships and cross-market price discovery through return correlation measures and price impact regressions.[2] In contrast to such existing alternatives, though, our measures utilize only time stamps with sufficient granularity (but no prices or quantities) and are exceedingly simple and cheap to compute in terms of CPU cycles, while also offering sharper identification in terms of timing.

Second, while the positive developments in market functioning due to HFT have been widely acknowledged,[3] we argue that the (price) efficiency gains associated with high-frequency cross-market trading come at the cost of making the real-time assessment of market liquidity across multiple venues more difficult as order placement and execution in one market can affect liquidity provision across related markets almost instantly. In particular, we show how to specialize the proposed cross-market activity measures for studying the co-movement of order books, sometimes referred to as the "liquidity mirage", reflecting the challenges for large investors to accurately assess available liquidity based on displayed market depth across different trading venues.[4] We stress that such adjustment of quotes is consistent with prudent market making behavior and is nothing new per se although the near simultaneity of quote adjustments across venues is enabled by market makers deploying cutting edge technology. Our findings in this regard support the notion that the modern market structure implicitly involves a trade-off between increased price efficiency and heightened uncertainty about the overall available liquidity in the market for many investors.

Third, the strong relationship between trading and quoting in cash and futures markets uncovered by our model-free measure of cross-market activity suggests that price impact and trading costs in general should not be studied for each market in isolation in the context of arbitrage linked markets. The importance of this insight is illustrated through the strong cross-market price-impact regression results we obtain by relating US Treasury

[2] See for example, Benos, Brugler, Hjalmarsson, and Zikes (2015), Chan (1992), Hasbrouck (1995), Huth and Abergel (2014), Laughlin, Aguirre, and Grundfest (2014), among many others.

[3] Notable studies include Brogaard, Hendershott, and Riordan (2014),Hasbrouck and Saar (2013), Hendershott and Riordan (2013), Menkveld (2008), O'Hara (2015), ...

[4] This approach also allows for data-driven analysis of anecdotal examples of fleeting liquidity such as the one famously described by Lewis (2014).

and equity index returns to volume imbalances in both cash and futures markets in a simple extension of the classical price-impact regression framework along the lines of Breen, Hodrick, and Korajczyk (2002). In particular, we find that volume imbalances in the futures market are much more important for predicting returns than the similar cash market quantities. These findings are entirely consistent with the strong asymmetry in the prevailing lead-lag patterns for high-frequency trading and quoting activity in futures versus cash markets captured by our model-free measure of cross-market activity and support the hypothesis that price discovery in Treasury, equity and EUR/USD FX markets primarily takes place in the futures market. More generally, our cross-market price impact results provide strong reasons why arbitrage-linked markets should preferably be analyzed jointly rather than independently of each other.

Fourth, while the close relationship between market volatility and trading activity is a long-established fact in financial markets, the new market structure appears to be further associated with HFT-related surges in cross-market activity in relation to the increased presence of short-lived arbitrage opportunities during periods of heightened volatility. This motivates us to look empirically for a separate link between cross-market activity and volatility in both U.S. Treasuries (between the ten-year Treasury note cash and futures markets) and equities (between the S&P 500 cash ETF and E-mini futures markets) over more than a decade from January 1, 2004 to September 30, 2015. In particular, we document that with the rise in HFT in recent years our model-free measure of cross-market activity expressed as the peak number of cross-active milliseconds (across all offsets) has become more strongly associated with volatility than trading volume and the number of trades in each market. This observation may simply reflect the fact that volatility can create brief dislocations in relative values spurring bursts of cross-market activity by high-frequency traders seeking to exploit these trading opportunities. When liquidity is ample, cross-market activity can therefore capture incremental information about market volatility beyond traditional measures of overall market activity such as trading volume and the number of transactions. By contrast, the existing voluminous empirical findings and alternative theories regarding the relationship between trading activity and volatility do not seem to readily account for such potential high-frequency feedback effects between volatility and the cross-market component of overall trading activity.[5] We further note that while studies such as Andersen et al. (2015), Ané and Geman (2000), Clark (1973), or

[5]The vast literature on the subject listed in chronological order includes Ying (1966), Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), Karpoff (1987), Schwert (1989), Harris (1987),Jones, Kaul, and Lipson (1994), Andersen (1996), Bollerslev and Jubinski (1999), Ané and Geman (2000), Kyle and Obizhaeva (2013), Andersen, Bondarenko, Kyle, and Obizhaeva (2015), among many others.

Kyle and Obizhaeva (2013) aim to rectify a particular theory-implied form of the relationship between volatility and trading activity in a given single market, our main focus is to show that the cross-market component of high-frequency trading activity between closely related markets contains extra information about market volatility unspanned by trading volume and the number of trades in each market. Thus, we help establish cross-market activity as an increasingly more important driver of the evolving link between trading activity and volatility. Overall, our findings suggest that there is a potential need to account for volatility-induced surges in trading due to cross-market activity when modeling the relationship between trading and volatility in arbitrage-linked markets.

The paper proceeds as follows. Section 2 defines the proposed new class of model-free measures of cross-market activity based on timings instead of returns and discusses key features such as location, dispersion, magnitude and robustness. Section 3 carries out three different empirical applications of the proposed measures to studying lead-lag relationships between markets, capturing "liquidity mirage" phenomena, and establishing the strong link between cross-market activity and volatility. Section 4 summarizes our main findings and discusses possible extensions.

2. Model-Free Measurement of Cross-Market Activity Based on Timings Instead of Returns

We measure cross-market activity using transaction-level data with millisecond or higher precision for a pair of related markets such as the ones for benchmark U.S. Treasury notes traded on BrokerTec versus Treasury futures contracts traded on the CME or the ones for the SPDR S&P 500 ETF traded on NYSE versus E-Mini S&P 500 futures traded on the CME. This allows us to first identify "active" milliseconds in each market when activity (e.g. trades) occur and then compare the timing offset (millisecond lead or lag) of activity for two distinct instruments or markets. In particular, we define a measure of cross-market activity at each offset as the proportion of milliseconds with coincident trading at that offset relative to the total number of active milliseconds in the less active of the two markets. We further correct our measure for the expected proportion of such coincident activity if all trading were independent across markets. The measure can therefore be interpreted as the excess proportion of trading (relative to the least active market) accounted for by cross-market activity.

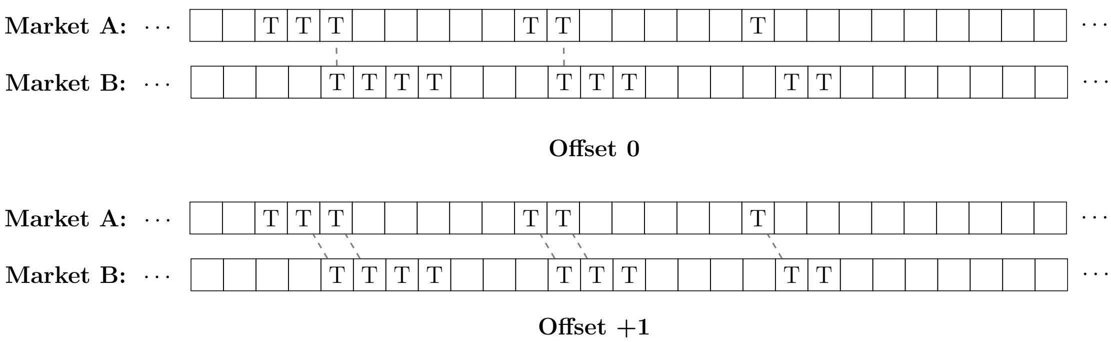

FIGURE 1: **Model-Free Measures of Cross-Market Activity**

2.1. A Timing-Based Measure of Cross-Market Activity

To be specific, consider two markets, A and B, that are observed for N time periods, $i = 1, \ldots, N$. The incidence of cross market activity is then simply obtained by counting the number of times that both markets are active during time buckets that are concurrent or suitably offset in order to capture potential lead-lag relationships between the two markets. The raw cross-market activity at offset t is given by

$$\mathcal{X}_t^{\text{raw}} = \sum_{i=|t|}^{N-|t|} \mathbb{1}_{\{\text{market } A \text{ active in period } i\} \cap \{\text{market } B \text{ active in period } i+t\}} \tag{1}$$

The diagram in Figure 1 illustrates the computation where six active time buckets, marked with "T", are observed in market A along with nine active time buckets in market B. Applying the definition, at zero offset we find $\mathcal{X}_0^{\text{raw}} = 2$, while at offset $+1$ (using the convention that the offset is applied to the first market) we find $\mathcal{X}_1^{\text{raw}} = 5$ in this example.

For any one of the markets, the raw cross market activity measure above can be further decomposed into a product of the number of active time buckets times the fraction of time buckets that were cross active. The latter ratio represents a measure of relative cross-market activity which is interpretable as the fraction of activity in that market (potentially) related to cross market activity at that offset. For market A, the relative activity measure at offset t is

$$\mathcal{X}_t^{\text{rel,A}} = \frac{\sum_{i=|t|}^{N-|t|} \mathbb{1}_{\{\text{market } A \text{ active in period } i\} \cap \{\text{market } B \text{ active in period } i+t\}}}{\sum_{i=|t|}^{N-|t|} \mathbb{1}_{\{\text{market } A \text{ active in period } i\}}}$$

When analyzing a pair of markets which may have very different levels of activity across time and markets, the measure is typically calculated for the least active market because this leads to a ratio which by definition ranges from 0 to 1:

$$\mathcal{X}_t^{\text{rel}} = \frac{\sum_{i=|t|}^{N-|t|} \mathbb{1}_{\{\text{market } A \text{ active in period } i\} \cap \{\text{market } B \text{ active in period } i+t\}}}{\min\left[\sum_{i=|t|}^{N-|t|} \mathbb{1}_{\{\text{market } A \text{ active in period } i\}}, \sum_{i=|t|}^{N-|t|} \mathbb{1}_{\{\text{market } B \text{ active in period } i+t\}}\right]} \tag{2}$$

Applied to the example given in Figure 1, $\mathcal{X}_0^{\text{rel}} = \frac{1}{3}$ while $\mathcal{X}_1^{\text{rel}} = \frac{5}{6}$. As should be clear from this example, both the raw and relative measures of cross-market activity given in (1)-(2) require only time stamps and are exceedingly simple and cheap (in CPU cycles) to compute. Importantly, the measures do not require a bandwidth choice beyond the time granularity of the data analyzed which ideally should be slightly higher than the fastest transmission time between markets.

In order to account for the level of coincidental cross-market activity that may occur when analyzing two highly active markets, one may adjust the relative cross market activity measure by subtracting off a term which is the average cross-market activity measure for very large positive and negative offsets.

$$\mathcal{X}_t = \mathcal{X}_t^{\text{rel}} - \mathcal{X}_\infty^{\text{rel}}, \; \mathcal{X}_\infty^{\text{rel}} = \frac{1}{2(T_2 - T_1)} \sum_{|t|=T_1+1}^{T_2} \mathcal{X}_t^{\text{rel}} \tag{3}$$

This non-parametric adjustment, for large $T_2 > T_1$, is robust to clustering and arbitrary local arrival rates of market activity which ensures that the cross market activity measure (3) can be interpreted as a measure of excess activity associated with cross market activity.

2.2. Key Features: Location, Dispersion and Magnitude of Peak Cross-Activity

The three main features of our measure are location, dispersion and magnitude of the peak cross-market activity across different offsets. We illustrate each feature below using a recent representative data sample over the period from Jul 1, 2014 to Dec 31, 2014 for the five-year and ten-year Treasury notes traded on the BrokerTec platform and the corresponding Treasury futures traded on the Chicago Mercantile Exchange.

2.2.1. Location

Figure 2 shows the relative cross-market activity, $\mathcal{X}_t$, for the ten-year and five-year Treasury notes. Based on our measure, cross-market trading accounts for around 8 percent of activity in the cash Treasury market on normal days but almost double than that on

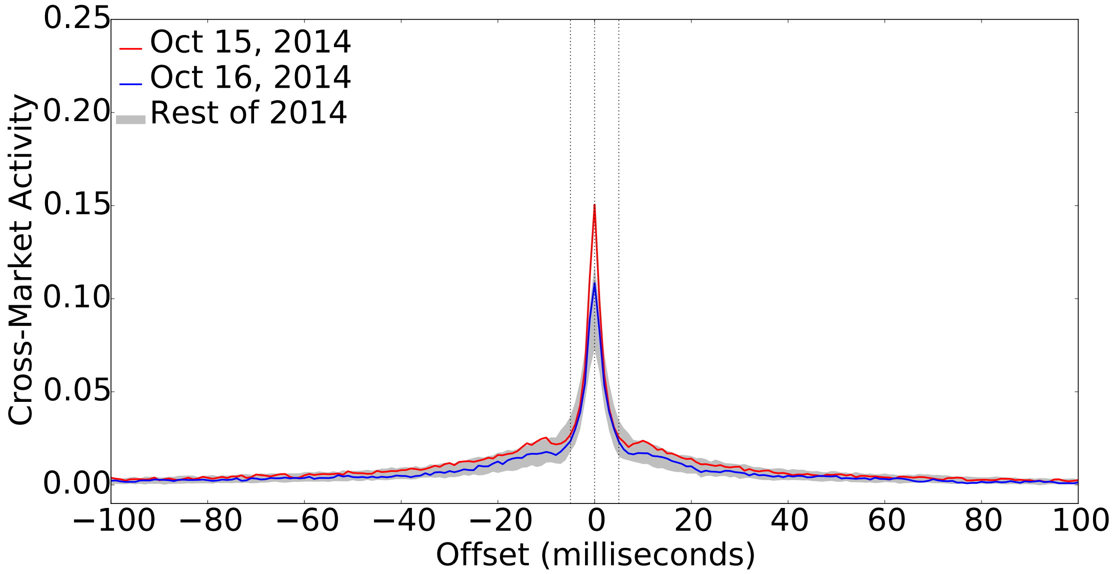

FIGURE 2: **Cross-Market Trading Activity Between 10-Year and 5-Year Treasury Cash Markets. Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

October 15, 2014, known to be an exceptionally volatile day in Treasury markets.[6] The location of the peak at zero is intuitive since the Treasury notes are traded on the same platform, and the sharpness of the spike indicates that random latency (i.e. the time it takes the platform to process trading instructions known to be around 0.2 milliseconds on the Brokertec cash platform) is small compared to the millisecond resolution at which the analysis is carried out.

2.2.2. Dispersion

The cross-market activity between five- and ten-year Treasury futures also exhibits a spike at zero offset (figure 3), indicating a similarly significant amount of nearly instantaneous trading in these instruments. However, the spike is much more diffuse: it spreads over a much wider range of offsets. This may reflect a wider range of available connectivity options to CME as well as some occasional platform latency associated with Treasury futures trading.[7] Comparing figures 2 and 3 shows how the defined non-parametric mea-

[6]See "Joint Staff Report: The U.S. Treasury Market on October 15, 2014", `http://www.treasury.gov/press-center/press-releases/Documents/Joint_Staff_Report_Treasury_10-15-2015.pdf`.

[7]For more discussion of latency in futures trading, see the "Joint Staff Report: The U.S. Treasury Market on October 15, 2014", `http://www.treasury.gov/press-center/press-releases/Documents/Joint_Staff_Report_Treasury_10-15-2015.pdf`.

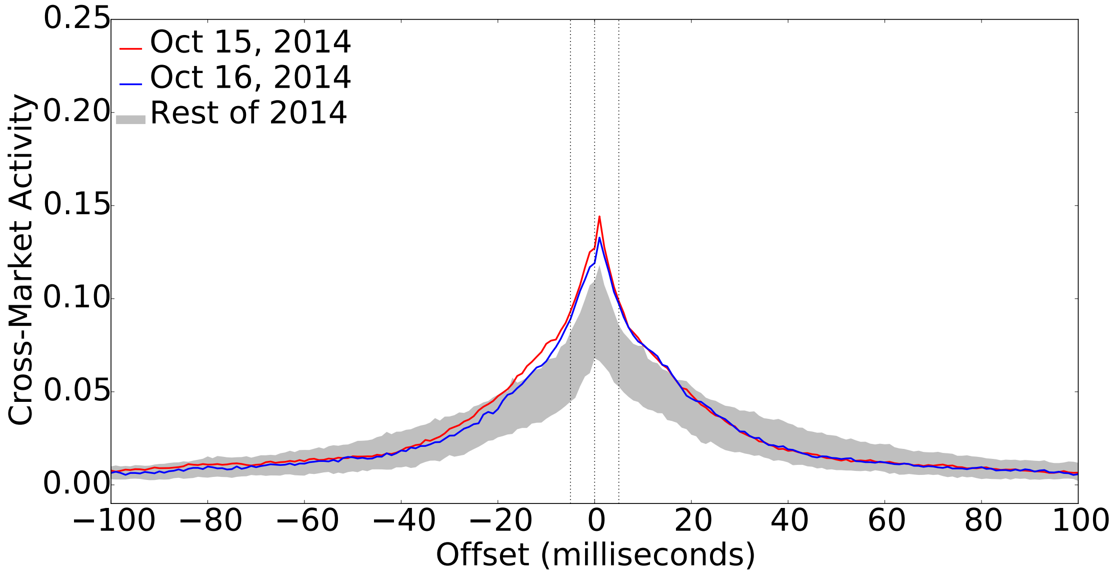

FIGURE 3: **Cross-Market Trading Activity Between 10-Year and 5-Year Treasury Futures Markets. Data Sources:** Nanotick (CME futures market data).

sure of cross-market activity can be used to study the impact of market structure and technology differences on the observed trading behaviors.

2.2.3. Magnitude

To show how the magnitude of cross-market activity can be used to gauge the evolution of high-frequency cross-market trading along the yield curve, in figure 4 we plot the historical levels of cross-market trading at zero millisecond offset in the ten-year and five-year Treasury note on BrokerTec over the past decade. Early on, during 2004-2006, there was essentially no simultaneous trading in the two markets. Starting with a systems upgrade of the cash platform in March 2006, minor evidence of cross-market activity appears. In March 2012, a major upgrade to the BrokerTec market data feeds and matching engine significantly reduced platform latency, and synchronized trading activity more than doubled.[8] A more gradual but larger increase followed by mid to late 2013, as a number of further upgrades to order submission and market data protocols took place, with synchronized trading activity reaching an average of around 10 percent of total trading

[8] Specifically, BrokerTec introduced a new trading system for its U.S. based products using a modified version of the NASDAQ OMX Genium INET system, with changes taking effect on March 26, 2012. The upgraded platform enabled BrokerTec to increase order volume tenfold and decrease order input latency by up to 50 times. The platform's average latency dropped to less than 200 microseconds, with previous average latency approximately 10 milliseconds. Source: "BrokerTec Brings World Class Trading Platform To U.S. Fixed Income Market", NASDAQ Inc. press release, April 18, 2012.

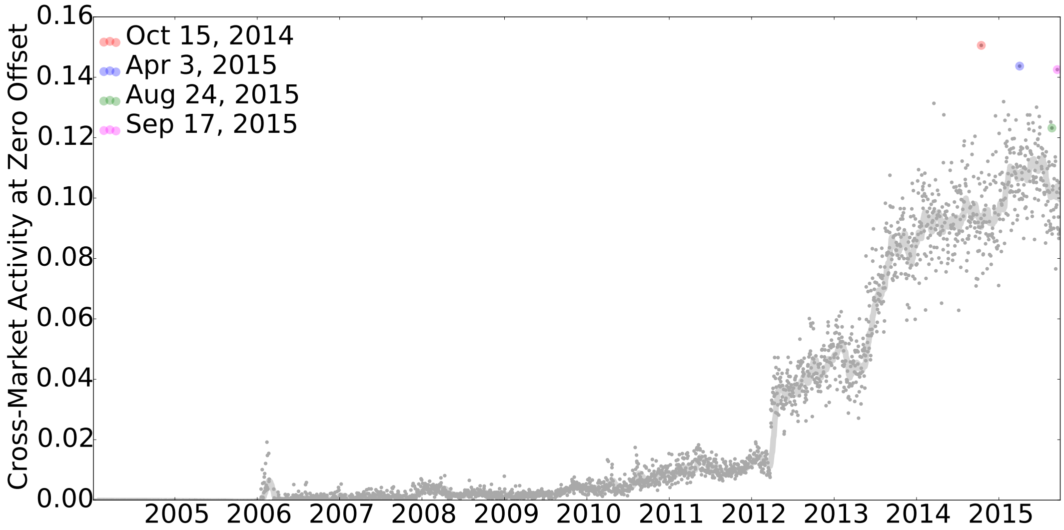

Figure 4: **Evolution of Cross-Market Trading Activity Between 10-Year and 5-Year Treasury Cash Markets. Data Sources:** BrokerTec (cash market data).

activity.[9] This example shows how the non-parametric measure of cross-market activity can be used to track changes in trends and patterns of trading behaviors resulting from discrete technological change.

2.3. Robustness

A key element of the proposed measure is the accurate measurement of when activity occurred. In practice, this can be difficult for a number of reasons, including random variations in the latency with which gateways or matching engines process incoming messages, or latency associated with the market data feed.[10] Any random variation in latency will tend to blur the measure of cross-market activity as seen in the example of CME latency above and may further cause the spike in cross-market activity to occur at a spurious offset. However, in all cases we have encountered in practice, the magnitude of the peak cross-market activity measure across all offsets remains an informative measure

[9]In particular these upgrades included the introduction of the ITCH (mid to late 2013) and OUCH (November 2012) protocols. OUCH is a low-level high performance native protocol that allows participants to enter, replace, and cancel orders and receive executions in an automated fashion on the Genium INET platform. ITCH is a high performance direct data feed protocol that displays all public orders and trades that occur on the platform.

[10]The special theory of relativity holds that it is impossible to determine (in an absolute sense) that two distinct events occurred at the same time if those events were separated in space. Thankfully, on the scale of milliseconds/microseconds, this is not yet a binding constraint.

of cross-market activity. The appendix explores this issue in the context of alternative high-frequency data feeds with vastly different latencies incurred at the point of data capture.

3. Empirical Illustrations and Applications

In this section, we apply the proposed model-free measures of cross-market activity to analyse various aspects of cross-market activity between the cash markets and the associated derivatives markets for U.S. Treasuries and U.S. Equitiy indices. While these relationships have been studied extensively in the literature by comparing observed (often noisy) market returns, we are able to shed some new light on the matter by relying instead solely on the timing of activity based on millisecond or higher resolution time stamps.

The Chicago Mercantile Exchange matching engine is located in Aurora Illinois and is connected via fiber optics and microwave technology to the cash market exchanges. The Brokertec platform is located in Secaucus New Jersey, roughly 4.7 milliseconds from Aurora at the speed of light, which puts a natural lower bound on the offset at which cross-market activity may be expected to take place in recent years.[11] The main US equity exchanges are primarily located in northern New Jersey with negligibly different distance to Aurora.[12] As we shall see throughout the analyses presented below, the roughly 5ms offset is clearly visible in our analysis and confirms huge impact of technology on trading behaviors observed in practice.

3.1. A New Analysis of the Lead-Lag Relation between Cash and Futures Markets

We analyze the link between cross-market activity and volatility in both US Treasuries (10-Year Treasury note cash and futures) and equities (S&P500 E-mini and SPY ETF) over the six-month period from July 1, 2014 to Dec 31, 2014. We measure cross-market activity on each trading day as the number of cross-active milliseconds at its peak across all offsets and restrict attention to the most active US portion of the electronic trading hours for each pair: 7:00-16:00 ET for the 10-Year T-Note and 9:30-16:00 ET for S&P500. While we carry out the analysis at millisecond frequency, it trivially generalizes to any other frequency with adequate time resolution for meaningful cross-activity measurements at different offsets.

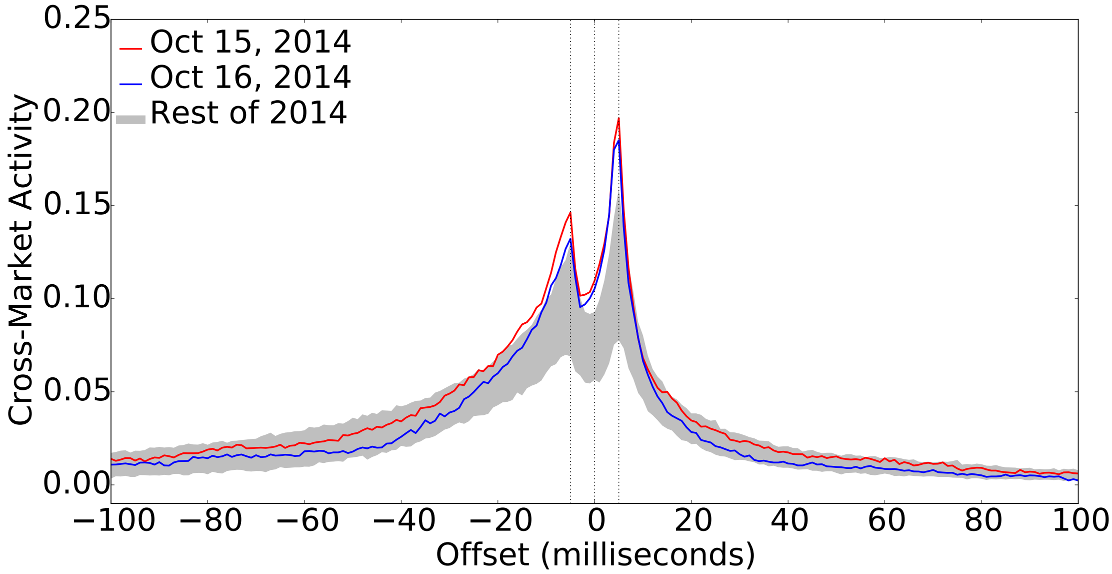

FIGURE 5: **Cross-Market Trading Activity Between 10-Year Treasury Cash and Futures Markets. Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

3.1.1. The US Treasury Cash and Futures Markets

Figure 5 displays the cross-market activity measure for the cash and futures ten-year Treasury note markets. The measure is shown for October 15, 2014, a day with extraordinarily high volatility and trading; for October 16, 2014, another day with somewhat heightened volatility and trading; and for the rest of the other, more typical, trading days in October 2014.

The cash market platform (in Secaucus, New Jersey) and the futures market exchange (in Aurora, Illinois) exhibit very pronounced cross-market trading activity at an offset of +/-5 milliseconds, strikingly consistent with the time it takes to transmit information between the two trading venues using current microwave tower technology. Spikes occur at both negative and positive offsets, suggesting that sometimes Treasury futures lead cash Treasuries (+5 millisecond offset) and other times the cash market leads futures (-5 millisecond offset), a remarkable fact since it indicates that price discovery takes place in both markets as they can both lead each other. The dip at zero is expected because coordinated trading within a millisecond is not possible (although the local leg of a trade could purposely be delayed). However, the measure does not reach zero since trading

[11]Since the refractive index of glass is around 1.5, communication via fiber-optic technology leads to a lower bound of roughly 7 ms.

[12]BATS in Weehawken, DirectEdge in Secaucus, Nasdaq in Carteret, and NYSE in Mahwah, NJ.

activity tends to occur in short bursts and some amount of spurious cross-market activity may be picked up at very small offsets.

Overall, the large share of cross-market activity probably helps explain the extremely tight link between Treasury cash and futures markets. Consistent with this, the two highest volatility days in October 2014, the 15th and 16th, also had the most cross-market activity.

3.1.2. The S&P 500 E-Mini Futures and Cash Markets

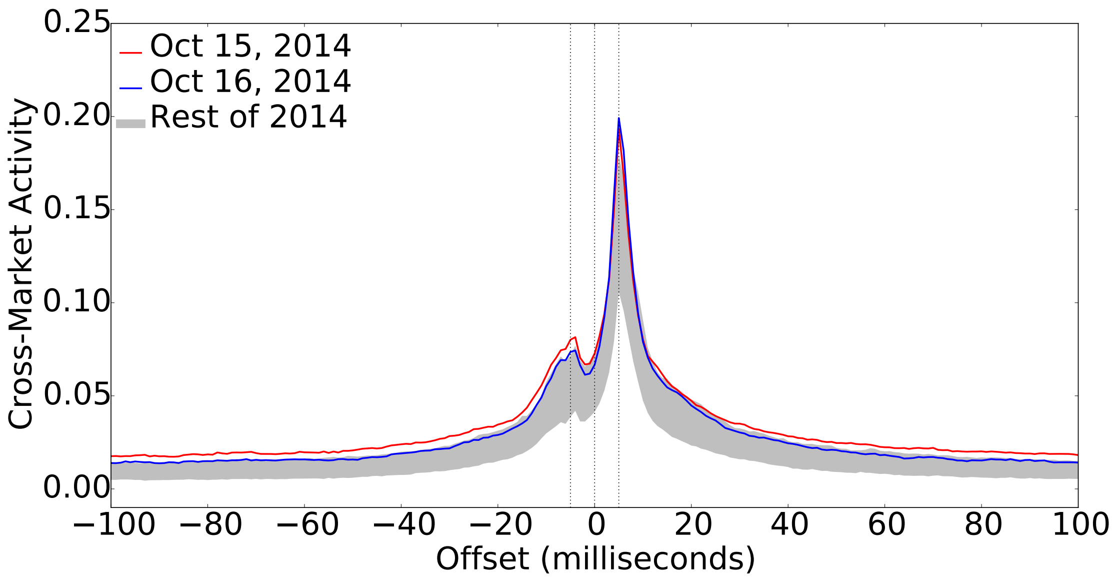

FIGURE 6: **Cross-Market Trading Activity Between S&P 500 Cash and Futures Markets. Data Sources:** Thomson Reuters Tick History (cash market data); Nanotick (CME futures market data).

Figure 6 displays the average measure of cross-market activity for the S&P 500 cash and futures markets. Of note is the pronounced asymmetry of the spike in the measure at +5 milliseconds for the S&P 500 compared to the 10-Year US Treasury. The much higher spike for the positive 5ms offset is consistent with the often cited dominant role played by the S&P futures in price discovery. Unlike prior studies based on returns and correlation measures, though, our measure successfully uncovers this relationship solely based on trading activity time stamps and pins down the precise offset with very high precision.

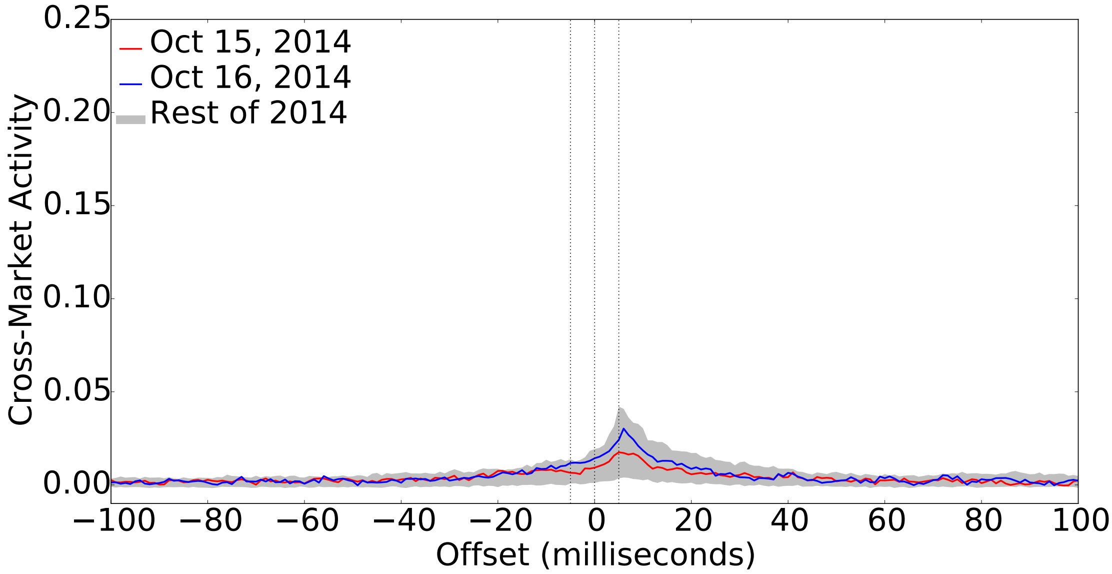

FIGURE 7: **Cross-Market Trading Activity Between 10-Year Treasury Cash and E-Mini S&P 500 Futures Markets. Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

3.1.3. The US Treasury Cash and S&P 500 E-Mini Futures Markets

A different pattern emerges when we compare trading activity across substantially different highly active markets, underscoring the fact that the cross-market activity measure is not simply picking up spurious coactivity. For example, overall cross-market activity in the ten-year Treasury notes and E-mini S&P 500 futures is much lower (see figure 7) and indicates that, to the extent that cross-market activity does occur, the E-mini market leads the cash Treasury market (+5 millisecond offset) but not vice versa.

3.2. Capturing the Liquidity Mirage

Market efficiency is often pointed to as a main benefit of automated and high-frequency trading (HFT) in U.S. Treasury markets. Fresh information arriving in the market place is reflected in prices almost instantaneously, ensuring that market makers can maintain tight spreads and that consistent pricing of closely related assets generally prevails. While the positive developments in market functioning due to HFT have been widely acknowledged, we argue that the (price) efficiency gain comes at the cost of making the real-time assessment of market liquidity across multiple venues more difficult. This situation, which we term the liquidity mirage, arises because market participants respond not only to news about fundamentals but also market activity itself. This can lead to order placement and execution in one market affecting liquidity provision across related markets almost instantly. The modern market structure therefore implicitly involves a trade-off between increased price efficiency and heightened uncertainty about the overall available liquidity in the market.

To illustrate this trade-off, we consider the trading activity in the ten-year U.S. Treasury note and futures in the month of October 2014 during which the October 15 "flash rally" occurred. Trading in the cash market takes place on the BrokerTec and eSpeed interdealer platforms (both located in Secaucus, New Jersey), while the corresponding front-month U.S. Treasury futures contract trades on the Chicago Mercantile Exchange (CME). All three venues feature anonymous electronic central limit order books with trading largely dominated by principal trading firms and bank dealers that often employ automated and low-latency trading techniques, as documented in the recently released Joint Staff Report on October 15, 2014.

To an investor in the U.S. Treasury market, the eSpeed, BrokerTec, and CME platforms represent distinct liquidity pools which typically exhibit tight spreads and significant depth at the top of the order book. Thanks to low-latency cross-market trading activity, the prices on all three platforms are likely to be competitive (market efficiency at work!). However, the liquidity effectively available to the investor at any point in time is highly unlikely to be the sum of top-of-book depths across the platforms. To see why, consider a New York-based investor submitting a buy order to all three venues. Given the short distance to the interdealer platforms, the investor's orders will reach one of them first. Suppose the BrokerTec order is matched first and the trader gets the requested quantity at the best offer. As soon as the BrokerTec transaction is observed in the market data feed, colocated low-latency market participants may immediately seek to cancel top-of-book offers on eSpeed and CME or submit competing buy orders to eSpeed and CME.

The former would be consistent with prudent risk management by market makers while the latter would be an example of opportunistic trading ahead of anticipated order flow. Due to random fluctuations in network latency and the close proximity between eSpeed and BrokerTec, the investor's order may or may not arrive in time to get filled at the expected best offer on eSpeed. However, low-latency traders will almost certainly be able to preempt the order's arrival at CME, leaving it in a position where it may not be filled at all. Hence, the displayed market depth across distinct liquidity pools can convey a misleading impression of the aggregate available liquidity.

3.2.1. Causes of the Liquidity Mirage: Prudent Market Making

To investigate how low-latency liquidity providers respond to incoming market data, we study the order book reactions to trades across platforms based on cash market data

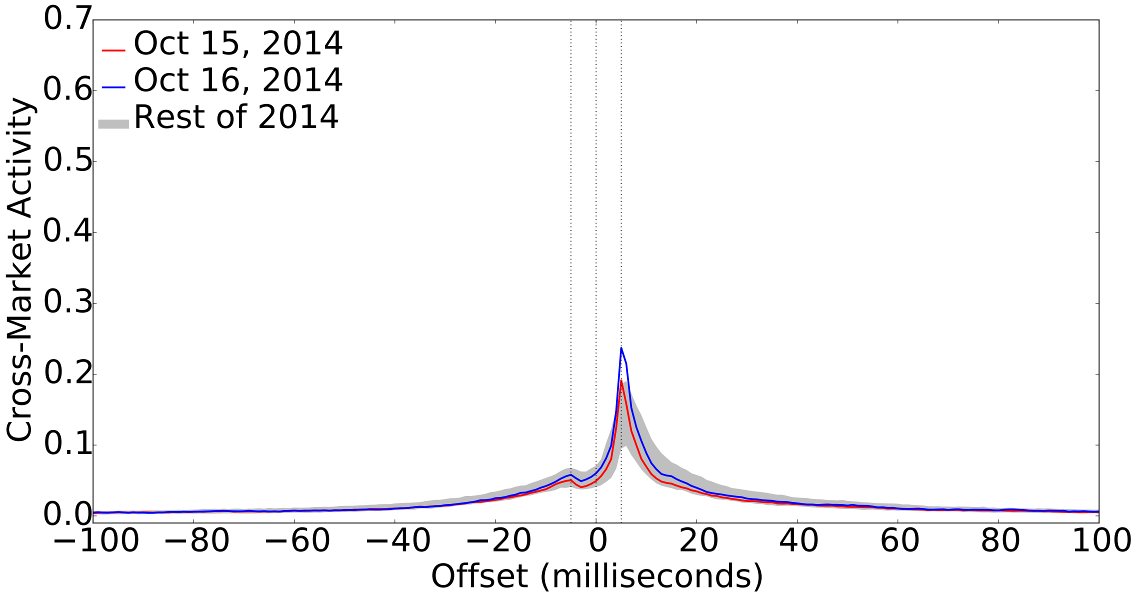

FIGURE 8: **CME Trades versus Depth Reductions on BrokerTec.** Cross-market activity is expressed as a share of futures trades and reflects the incidence rate of a trade at the bid (offer) on CME coinciding with a reduction in depth at the best bid (offer) on BrokerTec. A positive offset indicates that a CME trade happens prior to a depth reduction on BrokerTec. The gray area represents the 2.5th to 97.5th interpercentile range of the daily cross-market activity measures over the second half of 2014. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

from BrokerTec and CME futures market data from Nanotick. We focus our analysis on the extent to which CME trades may cause a reduction in depth on BrokerTec, as captured by the cross-market trading activity measure $\mathcal{X}_t$. In this case, the measure can be interpreted as the proportion of CME trades associated with top-of-book depth reduction on BrokerTec in excess of what might be expected by pure chance.

Figure 8 shows that on average as much as 20 percent of Treasury futures trades at the bid (offer) are associated with depth reduction at the bid (offer) on the BrokerTec platform. Moreover, this BrokerTec order book reaction to CME trades peaks at a roughly 5 millisecond delay, which matches the current shortest possible transmission time between the two venues using cutting-edge microwave transmission technology. The evidence therefore supports the hypothesis that rapid depth reduction by low-latency liquidity providers contributes to the liquidity mirage. It is also worth highlighting that October 15, while exhibiting extreme price volatility, is not an outlier in this regard. As such, we did not find any evidence that the liquidity mirage was more pronounced on October 15 compared with our control days.

An even stronger link exists between large CME trades (those with a trade size greater

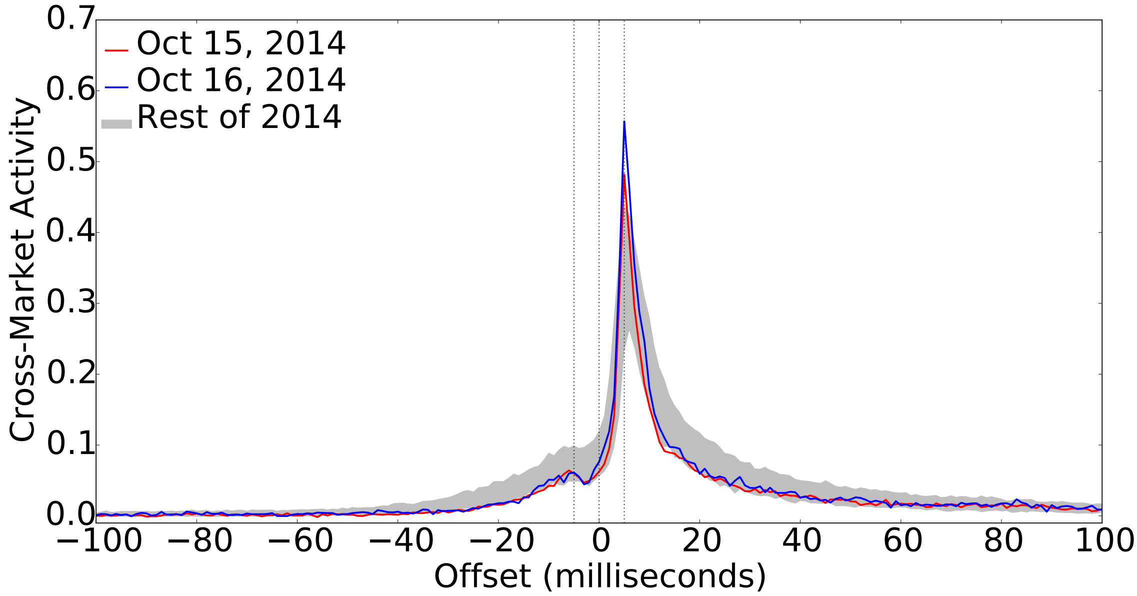

FIGURE 9: **Large CME Trades versus Depth Reductions on BrokerTec.** Cross-market activity is expressed as a share of futures trades and reflects the incidence rate of a trade at the bid (offer) on CME coinciding with a reduction in depth at the best bid (offer) on BrokerTec. A positive offset indicates that a CME trade happens prior to a depth reduction on BrokerTec. The gray area represents the 2.5th to 97.5th interpercentile range of the daily cross-market activity measures over the second half of 2014. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

than fifty contracts) and top-of-book depth changes on BrokerTec. Figure 9 displays a remarkable spike indicating that nearly 60 percent of large CME trades are followed by BrokerTec depth reductions with an offset of 5 milliseconds. These findings underscore the fact that the total liquidity available to an investor at a given point in time in practice may tend to be closer to the depth of the market that gets accessed first rather than the sum of the depths in each market.

We stress that the patterns displayed above are entirely consistent with prudent market making in an anonymous central limit order book environment where the informational advantage of market makers does not lie in having proprietary access to customer flows (as New York Stock Exchange specialists once did), but rather in their speed and ability to process complex market data.

3.2.2. Causes of the Liquidity Mirage: Opportunistic Trading

A second potential cause for the liquidity mirage is that traders with a speed advantage may attempt to trade ahead of (expected) incoming order flow. In this scenario, they would react to buy (sell) orders at CME by immediately submitting buy (sell) orders on

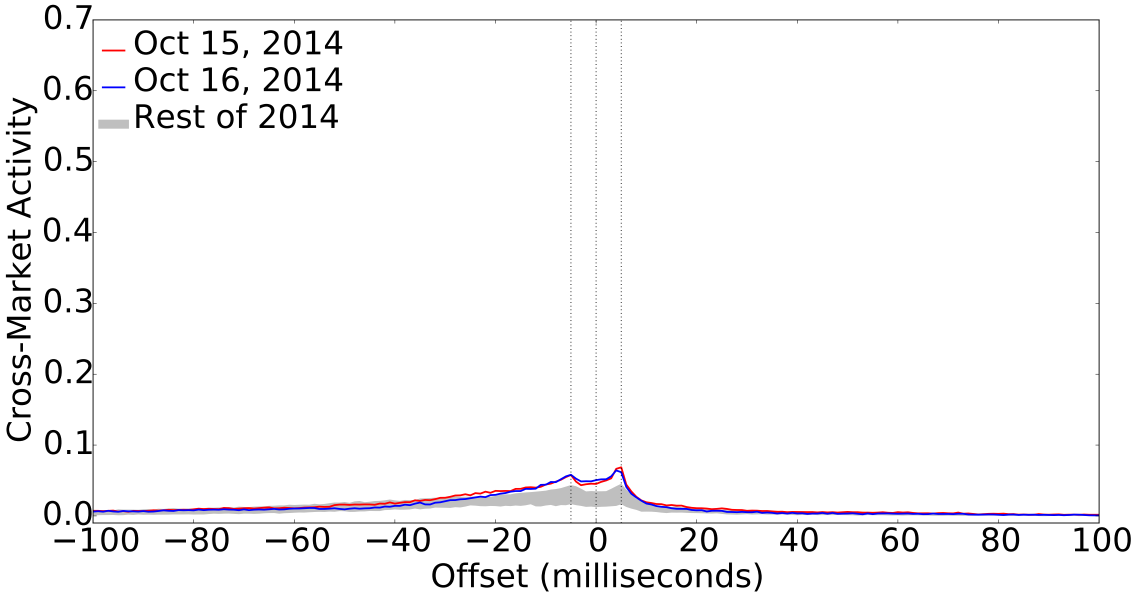

FIGURE 10: **CME Trades versus BrokerTec Trades.** Cross-market activity is expressed as a share of cash trades and reflects the incidence rate of a trade at the offer (bid) on CME coinciding with a trade at the offer (bid) on BrokerTec. A positive offset indicates that a CME trade happens prior to a BrokerTec trade. The gray area represents the 2.5th to 97.5th interpercentile range of the daily cross-market activity measures over the second half of 2014. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

BrokerTec in anticipation that prices may tick up (down) due to incoming order flow. However, the evidence for this behavior turns out to be quite weak in the data, based on our cross-market activity measure.

Figure 10 shows the cross-market activity measure for buy and sell trades on CME and BrokerTec. There is modest evidence that buy (sell) trades take place in a coordinated fashion at an offset of +/-5 milliseconds, but this accounts for very little of the overall trading activity on the BrokerTec platform (the less active of the two markets). For large orders (not shown here), the evidence is even weaker.

3.3. Implications for Cross-Market Price Impact

The strong relationship between cash and futures trades and between trades and quotes across the market pairs studied above suggests that measures of price impact and trading costs should not be constructed for each market in isolation. As captured by our cross-market activity measure, the tick data time stamps indicate that a trade in one market may move quotes in the other even in absence of trading in the latter market. Moreover, the asymmetry evident in Figure 6 and, to a lesser degree, in Figure 5, point to futures markets as being potentially more important in price discovery on average. To investigate this hypothesis, we build on the classic price impact regression framework of Breen et al. (2002) specialized to US Treasury and equity index cash and futures markets.

In the case of a single market, the imbalance Δv_t between buyer-initiated and seller-initiated trading volume during a short time interval tends to explain the contemporaneous return r_t, leading to the following popular price-impact regression studied by Breen et al. (2002), among many others:

$$r_t = \lambda \Delta v_t + \varepsilon_t \tag{4}$$

In the context of arbitrage linked markets, in view of the uncovered lead-lag patterns in high-frequency cross-market order placement and execution activity, we instead consider the following extension to a pair of cross-market price-impact regressions for US Treasury returns and volume imbalance on both CME and BrokerTec:

$$r_t^{BTEC} = \lambda^{CME} \Delta v_t^{CME} + \lambda^{BTEC} \Delta v_t^{BTEC} + \varepsilon_t \tag{5}$$

$$r_t^{CME} = \tilde{\lambda}^{CME} \Delta v_t^{CME} + \tilde{\lambda}^{BTEC} \Delta v_t^{BTEC} + \tilde{\varepsilon}_t \tag{6}$$

where the returns $\{r_t^{CME}, r_t^{BTEC}\}$ differ slightly due to the duration difference between the instruments and intraday changes in the cash-futures basis.[13]

From the regression results for pooled five-minute series reported in Table 1, it is clear that the loading on volume imbalance for both markets is highly significant but that CME volume appears to be much more informative than BrokerTec volume regardless of whether one is predicting CME or BrokerTec returns (as mentioned the returns are strongly but not perfectly correlated). Moreover, adding BrokerTec volume in a bivariate regression only marginally increases the R-squared of the regression, especially for CME returns. These results are remarkably robust when running the regressions day by day (i.e. without pooling the five minute series across days), as further reported in table A.3. Overall, this is consistent with the prevailing lead-lag patterns for CME versus BrokerTec trading and quoting activity and supports the hypothesis that price discovery in the Treasury markets primarily takes place in the futures market.

The strong cross-market price impact is not unique to US Treasury markets. As seen from Figure 6 vis-à-vis Figure 5, the tendency for futures to lead cash markets is even more pronounced in the case of the S&P500 equity index. This leads us to consider the

[13]Cross-market price impact in other such settings has recently been studied by Pasquariello and Vega (2015), Back and Crotty (2015), and Boulatov, Hendershott, and Livdan (2013). Multivariate extensions of Kyle's (1985) lambda have previously been considered also by Caballé and Krishnan (1994), among others.

r^{BTEC}				r^{CME}			
Const	3.68E-06	4.29E-05	2.54E-05	**Const**	4.69E-06	2.27E-05	1.56E-05
	[0.120]	[1.334]	[0.894]		[0.192]	[0.835]	[0.662]
Δv^{CME}	1.12E-06		8.35E-07	Δv^{CME}	9.03E-07		7.39E-07
	[93.296]		[66.201]		[94.329]		[70.687]
Δv^{BTEC}		1.81E-05	1.09E-05	Δv^{BTEC}		1.27E-05	6.32E-06
		[75.825]	[45.408]			[62.67]	[31.795]
Adj R^2	0.44	0.34	**0.51**	**Adj R^2**	0.44	0.26	**0.48**

Table 1: **Cross Market Price Impact Regressions for Ten-Year US Treasury Cash and Futures Markets.** The left panel shows the results of different regression specifications with BrokerTec five-minute returns for the ten-year Treasury Note as the dependent variable while the right hand panel shows the results using CME five-minute returns for the ten-year Treasury futures as the dependent variable. In each case, we run two univariate specifications as in (4) using each market's net volume as well as a bivariate specification as in (5)-(6). T-stats are given is square brackets under each coefficient estimate. The five-minute return series and corresponding net order flow measures are pooled across all days for the sample period from July 1, 2014 to December 31, 2014. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

r^{NYSE}				r^{CME}			
Const	1.42E-04	-1.17E-03	-2.08E-04	**Const**	1.42E-04	-1.15E-03	-1.75E-04
	[2.277]	[-13.392]	[-3.226]		[2.236]	[-12.960]	[-2.647]
Δv^{CME}	2.99E-06		2.78E-06	Δv^{CME}	3.03E-06		2.84E-06
	[117.21]		[100.25]		[116.55]		[100.19]
Δv^{NYSE}		2.45E-08	7.22E-09	Δv^{NYSE}		2.41E-08	6.53E-09
		[48.487]	[17.692]			[46.831]	[15.654]
Adj R^2	0.54	0.17	**0.55**	**Adj R^2**	0.54	0.16	**0.55**

Table 2: **Cross Market Price Impact Regressions for S&P 500 Cash and Futures Markets.** The left panel shows the results of different regression specifications with NYSE five-minute returns for the SPY ETF as the dependent variable while the right hand panel shows the results using CME five-minute returns for the E-mini futures as the dependent variable. In each case, we run two univariate specifications as in (4) using each market's net volume as well as a bivariate specification as in (7)-(8). T-stats are given is square brackets under each coefficient estimate. The five-minute return series and corresponding net order flow measures are pooled across all days for the sample period from July 1, 2014 to December 31, 2014. **Data Sources:** Nanotick (CME futures market data); Thomson Reuters Tick History (cash market data).

following pair of cross-market price-impact regressions for S&P 500 returns and volume imbalance on CME (as captured by E-mini futures) and NYSE (as captured by the SPY ETF):

$$r_t^{NYSE} = \lambda^{CME}\Delta v_t^{CME} + \lambda^{NYSE}\Delta v_t^{NYSE} + \varepsilon_t \tag{7}$$

$$r_t^{CME} = \tilde{\lambda}^{CME}\Delta v_t^{CME} + \tilde{\lambda}^{NYSE}\Delta v_t^{NYSE} + \tilde{\varepsilon}_t \tag{8}$$

The pooled regression results reported in Table 2 along with the corresponding day by day regression results in Table A.4 reveal that, for the S&P 500, the net order flow in the futures market alone explains the S&P 500 five-minute returns and that adding the net order flow in the cash market has a negligible impact.[14] As such, our cross-market price impact findings reveal that the futures market plays a far more dominant role for S&P 500 than in the case of US Treasuries consistent with the pronounced one-sided peak in our measure of high-frequency cross-market trading for S&P 500 (Figure 6 above). By contrast, the two-sided peak in our measure for US Treasuries (Figure 5 above) with a secondary peak at -5 milliseconds suggests that cash Treasury markets can sometimes also lead the Treasury futures markets and can thus have a somewhat more informative net order flow compared to S&P 500 cash versus futures markets. In this light, our results provide a strong rationale for explicitly considering the cross-market price impact between arbitrage linked markets as a testable implication of the prevailing high-frequency lead-lag relationship between them that is identified by our model-free measure of cross-market trading activity.

We further analyze the evolution of high-frequency cross-market price impact over the period 2004-2015 by repeating the above regressions for five-second instead of five-minute returns and plotting the partial R^2 representing the improvement in the explanatory power of the bivariate specification (7)-(8) over the corresponding univariate specification (4) for each market. In the case of S&P 500, Figure 11 shows that futures market order flows have been providing substantial additional explanatory power for S&P 500 returns over cash market order flows at the five-second frequency since at least 2007 but increasingly less so going further back to 2004 when HFT is known to have been at very early stages. Moreover, in line with the pronounced one-sided peak in our measure of high-frequency cross-market trading for S&P 500 (Figure 6 above), there appears to be almost no ad-

[14]The intraday returns for the S&P500 and SPY ETF are almost perfectly correlated at the 5 min frequency and the regression results for the two markets are therefore near identical.

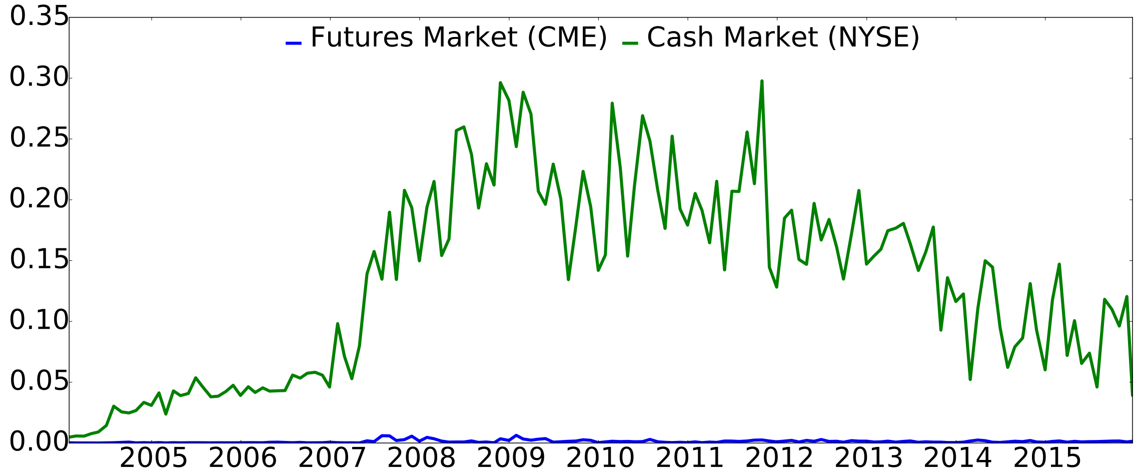

FIGURE 11: **Evolution of Partial R^2 for Bivariate versus Univariate Price Impact Regressions for S&P 500 Cash and Futures Markets.** We run daily price impact regressions of five-second returns on corresponding net order flows. The green line represents the improvement in explanatory power as measured by partial R^2 for the bivariate specification (7)-(8) over the corresponding univariate specification (4) for the SPY ETF traded on NYSE. The blue line represents the improvement in explanatory power as measured by partial R^2 for the bivariate specification (7)-(8) over the corresponding univariate specification (4) for the S&P 500 E-mini Futures traded on CME. The plotted values represent monthly medians of the corresponding daily partial R^2 values in each month over the period 2004-2015. **Data Source:** Thomson Reuters Tick History (CME futures market data and NYSE cash market data).

ditional explanatory power when augmenting futures market order flows by cash market order flows to explain S&P 500 returns. By contrast, in line with the two-sided peak in our measure for US Treasuries (Figure 5 above), Figure 12 shows that from 2008 onwards US Treasury cash-market order flows have started to provide some additional explanatory power on top of US Treasury futures order flows, which otherwise have still had much greater explanatory power on top of cash market order flows also for US Treasury returns at the five-second frequency.[15].

Importantly, the same rationale for considering cross-market price impact regressions extends to many other arbitrage-linked markets even in cases when coarser than millisecond time stamp granularity or other venue-specific data collection features could make it difficult to apply our model-free measure of cross-market activity. For example, in the case of the EUR/USD FX rate, trading activity on the EBS platform has traditionally been

[15]Qualitatively similar conclusions can be obtained also by repeating the regression analysis at other even higher frequencies (e.g. one-second) that we omit reporting in order to save space.

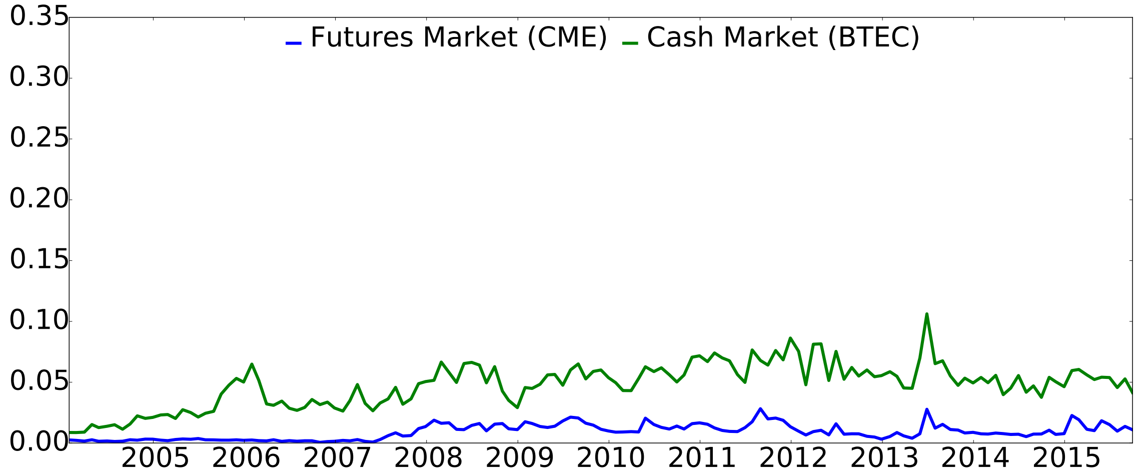

FIGURE 12: **Evolution of Partial R^2 for Bivariate versus Univariate Price Impact Regressions for Ten-Year US Treasury Cash and Futures Markets.** We run daily price impact regressions of five-second returns on corresponding net order flows. The green line represents the improvement in explanatory power as measured by partial R^2 for the bivariate specification (7)-(8) over the corresponding univariate specification (4) for the Ten-Year U.S. Treasury Note traded on BrokerTec. The blue line represents the improvement in explanatory power as measured by partial R^2 for the bivariate specification (7)-(8) over the corresponding univariate specification (4) for the Ten-Year U.S. Treasury Futures traded on CME. The plotted values represent monthly medians of the corresponding daily partial R^2 values in each month over the period 2004-2015. **Data Sources:** BrokerTec (cash market data); Thomson Reuters Tick History (CME futures market data).

reported as an aggregate over 100 millisecond buckets, thereby precluding precise millisecond identification of the lead-lag relationship with respect to the EUR/USD futures markets. Nonetheless, the recorded order flow still suffices to study the differential importance of cross-market price impact in the cash versus futures market over time, thereby refining and extending the price impact findings in the extant literature which has focused on a single market in isolation.[16] In line with the analysis of Treasury and equity markets above, we consider the following bi-variate price impact regression specification:

$$r_t^{EBS} = \lambda^{CME} \Delta v_t^{CME} + \lambda^{EBS} \Delta v_t^{EBS} + \varepsilon_t \tag{9}$$

$$r_t^{CME} = \tilde{\lambda}^{CME} \Delta v_t^{CME} + \tilde{\lambda}^{EBS} \Delta v_t^{EBS} + \tilde{\varepsilon}_t \tag{10}$$

Figure 13 reveals that there was a largely symmetric relationship between cash and

[16]See for example Berger, Chaboud, and Hjalmarsson (2009).

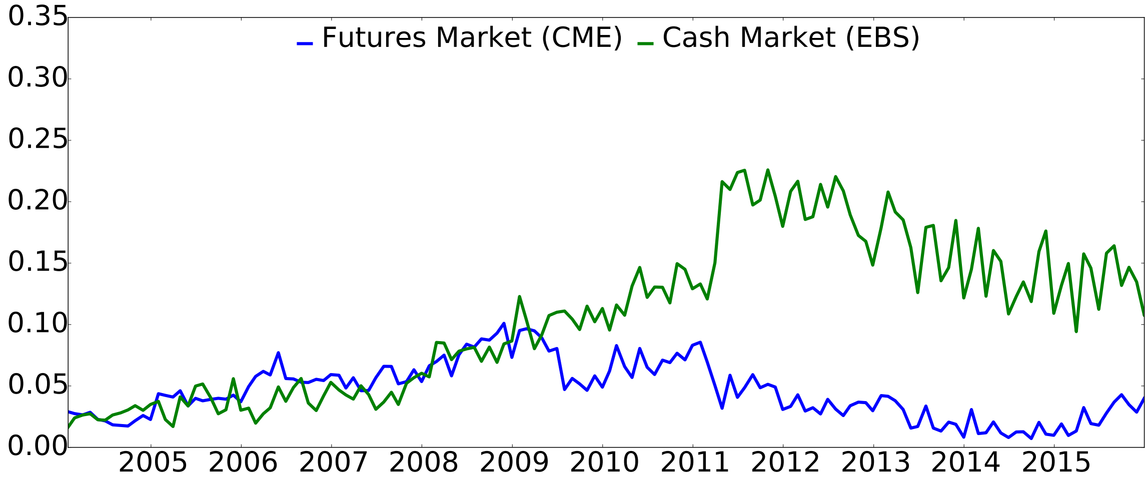

FIGURE 13: **Evolution of Partial R^2 for Bivariate versus Univariate Price Impact Regressions for EUR/USD FX Cash and Futures Markets.** We run daily price impact regressions of five-second returns on corresponding net order flows. The green line represents the improvement in explanatory power as measured by partial R^2 for the bivariate specification (7)-(8) over the corresponding univariate specification (4) for the EUR/USD FX rate traded on EBS. The blue line represents the improvement in explanatory power as measured by partial R^2 for the bivariate specification (7)-(8) over the corresponding univariate specification (4) for the EUR/USD FX Futures traded on CME. The plotted values represent monthly medians of the corresponding daily partial R^2 values in each month over the period 2004-2015. **Data Sources:** EBS (cash market data); Thomson Reuters Tick History (CME futures market data).

futures markets in terms of the incremental informativeness of order flows until 2009. Subsequently, a considerable divergence took place with futures market order flows appearing to gain considerably more explanatory power relative to cash market order flows. This differential jumped dramatically in response to the March 2011 decision by EBS to introduce price decimalization (cutting the tick size by a factor of 10) that was also blamed for impairing liquidity on the EBS platform by many market participants. In 2012 EBS partially reversed the change (increasing the tick size to half of what it had been pre March 2011) but our analysis indicates that EBS never regained its pre-2009 importance as a venue for price discovery. However, our analysis does indicate that the gap between EBS and CME in this regard appears to have narrowed since 2015.

Overall, the documented evolution in the additional explanatory power of cross-market versus single-market price impact regressions highlights the importance of properly accounting for high-frequency cross-market trading when studying price discovery in many arbitrage-linked markets as exemplified by the cash and futures markets for US Treasuries, equities, and FX rates.

3.4. Cross-Market Activity and Volatility

The close relationship between market volatility and trading activity is a long-established fact in financial markets.[17] On the one hand, a large body of empirical findings and compelling alternative theories advocate that trading volume comoves with measures of within-day price variability. On the other hand, in recent years much of the trading in U.S. Treasury and equity markets has been associated with nearly simultaneous trading between the leading cash and futures platforms. Thus, the striking cross-activity patterns we uncover in both high-frequency cross-market trading and related cross-market order book changes naturally lead to the question of how the cross-market component of overall trading activity is related to volatility.

Leaving aside the pronounced asymmetry in figure 6 versus figure 5, the spikes in cross-market activity on October 15 and 16, 2014, stand out as being well-aligned with the heightened volatility and trading observed on those days.[18] Cross-market trading and quoting activity thus appears to be related to variations in market volatility, which can create (short-lived) dislocations in relative valuations as market participants respond to news about fundamentals or market activity itself. We further demonstrate empirically that the peak number of cross-active milliseconds (across all offsets) comoves more strongly with market volatility than generic market-activity proxies such as trading volume and the number of transactions. This pattern is consistent with a positive feedback effect by which an increase in volatility can spur additional trading activity by creating cross-market trading opportunities. This observation stands in contrast to the more conventional view in the finance and economics literature which holds that trading activity predominantly causes volatility but not vice versa.

We study the evolution of the link between cross-market activity and volatility in both U.S. Treasuries (ten-year Treasury note cash and futures) and equities (S&P 500 E-mini and SPY ETF) over more than a decade from January 1, 2004 to September 30, 2015. Our analysis starts with a closer look at intraday patterns and the degree of alignment between volatility and cross-market activity over our representative recent sample period from July 1, 2014 to December 31, 2014 before proceeding with the full

[17] A survey of the early literature on the subject can be found in Karpoff (1987), while notable later examples include Schwert (1989), Ané and Geman (2000), Kyle and Obizhaeva (2013) and Andersen et al. (2015), among many others.

[18] See "Joint Staff Report: The U.S. Treasury Market on October 15, 2014", `http://www.treasury.gov/press-center/press-releases/Documents/Joint_Staff_Report_Treasury_10-15-2015.pdf`.

decade-long sample. We measure cross-market activity on each trading day as the number of cross-active milliseconds at its peak across all offsets and restrict attention to the most active electronic U.S. trading hours for each pair: from 7:00 to 16:00 ET for the ten-year Treasury note and from 9:30 to 16:00 ET for the S&P 500. While we carry out the analysis at millisecond frequency, it trivially generalizes to any other frequency with adequate time resolution for meaningful cross-activity measurements at different offsets.

3.4.1. Intraday Patterns in Cross-Market Trading Activity and Volatility

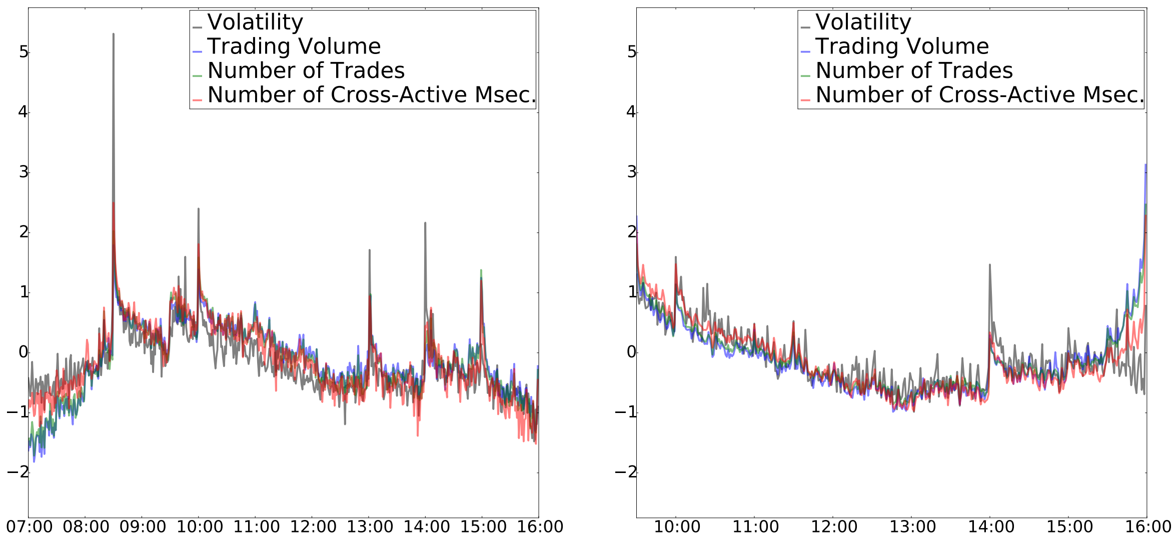

FIGURE 14: **Intraday Patterns in Different Trading Activity Measures and Volatility.** We plot the minute-average peak number of cross-active milliseconds between cash and futures markets (red line), number of trades (blue line) and trading volume (green line) against the minute-average realized volatility (gray line) in the cash market. The averages are taken over the period from July 1, 2014 to December 31, 2014 and the values are brought to the same scale by representing them as the binary logarithm of the ratio to their intraday mean. Zero indicates equality to the intraday mean; positive/negative one indicates twice/half the intraday mean. **Left Panel:** Ten-Year U.S. Treasury Note. **Right Panel:** S&P 500. **Data Sources:** BrokerTec; Nanotick; Thomson Reuters Tick History.

Figure 14 shows that for both the ten-year Treasury note and S&P 500, the prevailing intraday volatility pattern is matched very closely by the diurnal pattern in cross-market activity as measured by the number of cross-active milliseconds between the cash and futures markets. The biggest volatility spikes for U.S. Treasuries (left panel of figure 14) occur at 8:30, 10:00, 13:00, and 14:00 ET around known times of news announcements, Treasury auctions, and the release of Federal Open Market Committee announcements and meeting minutes. For the S&P 500, only the spikes at 10:00 and 14:00 ET stand out to a lesser degree (right panel of figure 14). For U.S. Treasuries, there is also a notable peak around 15:00 ET corresponding to the CME market close for all pit-traded interest rate options.

Furthermore, in terms of correlation, the number of cross-active milliseconds is tracking the intraday volatility pattern somewhat more closely than either trading volume or the number of trades. However, the tight range of most observed values within negative and positive one (excluding the extremes) suggests that interday as opposed to intraday variation may provide a better measure of the degree to which the different activity series relate to volatility.

3.4.2. Day-to-Day Variations in Volatility and Cross-Market Trading Activity

Figure 15 shows changes in daily logarithmic realized volatility plotted against changes in each daily logarithmic activity measure over the July 1 to December 31, 2014 sample period. For both the ten-year Treasury (left column) and the S&P 500 (right column) markets, the day-to-day changes in volatility appear to be more closely associated with the day-to-day changes in the peak number of cross-active milliseconds (bottom row) than with the changes in the number of trades (middle row) or trading volume (top row). In particular, the R squared for the number of cross-active milliseconds exceeds by more than 0.1 the R squared for trading volume and about half that amount the R squared for the number of trades.

Moreover, the number of cross-active milliseconds often appears to crowd out both trading volume and the number of trades if included jointly as regressors for volatility. This result is quite remarkable since it establishes that the number of cross-active milliseconds subsumes both trading volume and the number of trades in terms of information content that is available about volatility. It is also worth highlighting that while October 15 (in red) and October 16 (in blue) are known to have exhibited extreme volatility, they are not large outliers in terms of the strong linear relationship observed between changes in volatility and changes in the number of cross-active milliseconds (bottom row).

3.4.3. Evolution of the Relationship between Cross-Market Trading Activity and Volatility

To better assess the extent to which market volatility has become more closely associated with high-frequency cross-market trading activity, table A.5 reports historical R squared measures of goodness of fit for each year from Jan 1, 2004 to Sep 30, 2015 obtained by regressing year by year the daily changes in logarithmic realized volatility on the daily changes in logarithmic trading volume, number of trades, or number of cross-active milliseconds for the 10-Year US Treasury (left part of the table) and S&P 500

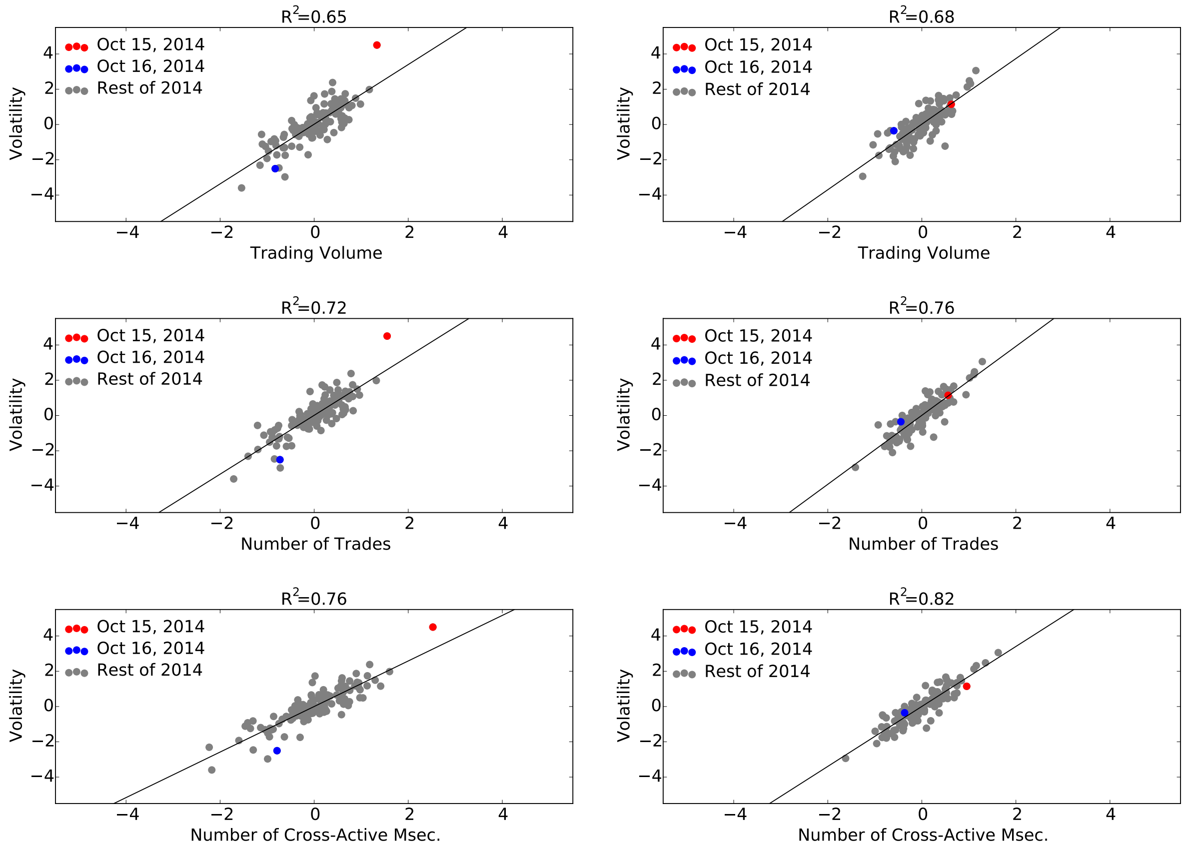

FIGURE 15: **Degree of Association between Different Trading Activity Measures and Volatility.** Daily changes in logarithmic trading volume (top row), number of trades (middle row) and peak number of cross-active milliseconds between cash and futures markets (bottom row) are plotted and regressed against daily changes in logarithmic realized volatility over the period from July 1, 2014 to December 31, 2014. The R squared measure of goodness of fit is reported above each panel. **Left Column:** Ten-Year U.S. Treasury Note. **Right Column:** S&P 500. **Data Sources:** BrokerTec; Nanotick; Thomson Reuters Tick History.

(right part of the table). In particular, measuring the peak number of cross-active milliseconds day by day and taking logarithmic differences of the daily measures during each twelve-month period limits the impact of secular trends in latency and trading practices over the past decade resulting from technological improvements and the related evolution in high-frequency trading. Table A.5 thus strongly indicates that with the rise in high-frequency trading in recent years, cross-market activity as measured by the peak number of cross-active milliseconds between the cash and futures markets has typically been more tightly linked to volatility than standard activity measures such as overall trading volume or the number of trades in either the cash or futures markets.

Even more strikingly, the proposed model-free measure of cross-market activity appears to contain extra information about volatility beyond what is implied by naively combining the trading volumes and counts for both the cash and futures markets. Table A.6 shows the high statistical significance of the loading on the peak number of cross-active milliseconds in yearly OLS regressions including also the trading volumes and number of trades for both the cash and futures markets. The regression specification takes the form:

$$\Delta rv_t = const + \beta[n_1] \cdot \Delta n_{1,t} + \beta[n_2] \cdot \Delta n_{2,t} + \beta[v_1] \cdot \Delta v_{1,t} + \beta[v_2] \cdot \Delta v_{2,t} + \beta[x_{12}] \cdot \Delta x_{12,t} + \epsilon_t \ ,$$

where daily changes in logarithmic realized volatility (rv_t) are regressed on daily changes in the logarithmic number of trades and trading volume in both the futures market ($n_{1,t}$ and $v_{1,t}$) and the cash market ($n_{2,t}$ and $v_{2,t}$) as well as daily changes in the logarithmic measure of cross-market activity between the two markets ($x_{12,t} = \log(\max_s \mathcal{X}^{\mathrm{raw}}_{s,t})$) given by the peak number of cross-active milliseconds across all different offsets. In addition to the high statistical significance of the model-free measure of cross-market activity, the resulting improvements in R squared are most visible in the years when high-frequency trading has become prevalent for each market pair (starting from 2011 for the ten-year US Treasury Note and from 2009 for S&P 500). We further note that while other studies such as Andersen et al. (2015), Ané and Geman (2000), Clark (1973), or Kyle and Obizhaeva (2013) aim to rectify a particular theory-implied form of the relationship between volatility and trading activity in a given single market, our main goal is to show that the cross-market component of high-frequency trading activity between closely related markets contains extra information about market volatility unspanned by trading volume and the number of trades in each market.

To address possible concerns stemming from the high degree of multi-collinearity between the trading activity measures, we further consider an alternative specification that brings all cross-correlations below 0.5 by way of suitable transformations. Table A.7 shows the OLS regression results for one such rearrangement in which both the cross-market activity and the number of trades in the futures market are taken relative to the number of trades in the cash market, while the trading volume in each market is transformed to average trade size (daily trading volume taken relative to trade count). By design this yields the same R squared measures of goodness of fit while better revealing the statistical significance of cross-market activity in comparison to the rest of the activity variables. In particular, it becomes clear that while most of the explanatory power stems from the chosen proxy for the baseline trading activity level (the number of trades in the cash markets $n_{2,t}$), the proposed measure of cross-market activity relative to the baseline activity level $(x_{12,t} - n_{2,t})$ has consistently been highly statistically significant, especially after high-frequency cross-market trading activity is known to have grown up in the considered cash and futures markets.

The resulting increase in terms of the adjusted R squared measure of goodness of fit is more evident when regressing volatility residuals on cross-market activity residuals after partialling out the number of trades and trading volume for each market from both volatility and cross-market activity (denoted as relative increase in adjusted R squared in the last column of tables A.6 and A.7). Thus, for the ten-year U.S. Treasury Note cash and futures markets at least 5% of the variability in volatility unexplained by the number of trades and trading volume in both cash and futures markets appears to be consistently associated with cross-market activity from 2011 onwards when high-frequency cross-market trading is known to have become prevalent in these markets. Even more strikingly, with the rise in high-frequency cross-market trading between S&P 500 cash and futures markets from 2009 onwards, the proposed measure of cross-market activity consistently can explain as much as 10% of the variability in volatility that remains unexplained by the number of trades and trading volume in both markets.

4. Conclusion

We propose a set of intuitive model-free measures of cross-market trading activity based on publicly available trade and quote data with sufficient time stamp granularity. By virtue of capturing the offset at which co-activity peaks, as well as the magnitude and dispersion of cross-market activity, the measures allow us to shed new light on the distinct features of the high-frequency cross-market linkages in US Treasury and equity markets. First, the measures avoid reliance on noisy return series often used in the literature and demonstrate sharp identification of the prevailing lead-lag relationships between trading activity in closely related markets. Second, we show how the measures can also be used to study cross-market linkages of liquidity provision as illustrated by an application to examining the existence of so called liquidity mirages. In this regard, we further provide new evidence pointing to the fact that price discovery in US Treasury, equity and EUR/USD FX markets primarily takes place in futures rather than cash markets and we give strong rationale for considering the cross-market price impact between arbitrage linked markets. Finally, as we uncover significant presence of high-frequency cross-market trading, we reexamine the close relationship between trading activity and volatility. In particular, we show that our measures provide additional explanatory power for observed market volatility, beyond commonly used measures of overall market activity such as trading volume and number of transactions. This observation may reflect the fact that volatility can create brief dislocations in relative values spurring bursts of cross-market activity by high-frequency traders seeking to exploit these trading opportunities. When liquidity is ample, cross-market activity can therefore capture incremental information about market volatility beyond traditional measures of overall market activity such as trading volume and the number of transactions.

Overall, our findings suggest that accounting for cross-market trading activity is directly relevant when studying volatility, liquidity provision, and transaction cost analysis. In particular, from this new perspective there is a potential need to account for volatility-induced surges in trading due to cross-market activity when modeling the relationship between trading and volatility in arbitrage-linked markets.

References

Andersen, T. G., 1996. Return volatility and trading volume: An information flow interpretation of stochastic volatility. The Journal of Finance 51, 169–204.

Andersen, T. G., Bondarenko, O., Kyle, A. S., Obizhaeva, A. A., 2015. Intraday trading invariance in the E-mini S&P 500 futures market. Working Paper.

Ané, T., Geman, H., 2000. Order flow, transaction clock, and normality of asset returns. The Journal of Finance 55, 2259–2284.

Back, K., Crotty, K., 2015. The informational role of stock and bond volume. The Review of Financial Studies 28, 1381–1427.

Benos, E., Brugler, J. A., Hjalmarsson, E., Zikes, F., 2015. Interactions among high-frequency traders. Bank of England Working Paper No. 523.

Berger, D., Chaboud, A., Hjalmarsson, E., 2009. What drives volatility persistence in the foreign exchange market? Journal of Financial Economics 94, 192 – 213.

Biais, B., Foucault, T., Moinas, S., 2015. Equilibrium fast trading. Journal of Financial Economics 116, 292 – 313.

Bollerslev, T., Jubinski, D., 1999. Equity trading volume and volatility: Latent information arrivals and common long-run dependencies. Journal of Business and Economic Statistics 17, 9–21.

Boulatov, A., Hendershott, T., Livdan, D., 2013. Informed trading and portfolio returns. The Review of Economic Studies 80, 35–72.

Breen, W. J., Hodrick, L. S., Korajczyk, R. A., 2002. Predicting equity liquidity. Management Science 48 (4), 470483.

Brogaard, J., Hendershott, T., Riordan, R., 2014. High-frequency trading and price discovery. Review of Financial Studies.

Budish, E., Cramton, P., Shim, J., 2015. The high-frequency trading arms race: Frequent batch auctions as a market design response. The Quarterly Journal of Economics.

Caballé, J., Krishnan, M., 1994. Imperfect competition in a multi-security market with risk neutrality. Econometrica 62, 695–704.

Chan, K., 1992. A further analysis of the lead-lag relationship between the cash market and stock index futures market. Review of Financial Studies 5, 123–152.

Clark, P. K., 1973. A subordinated stochastic process model with finite variance for speculative prices. Econometrica 41, 135–155.

Epps, T. W., Epps, M. L., 1976. The stochastic dependence of security price changes and transaction volumes: Implications for the mixture-of-distributions hypothesis. Econometrica 44, 305–321.

Godfrey, K., 2014. Detecting pairs trading (or any related trading) in two financial securities. Working Paper, University of Western Australia.

Harris, L., 1987. Transaction data tests of the mixture of distributions hypothesis. The Journal of Financial and Quantitative Analysis 22, 127–141.

Hasbrouck, J., 1995. One security, many markets: Determining the contributions to price discovery. The Journal of Finance 50, 1175–1199.

Hasbrouck, J., Saar, G., 2013. Low-latency trading. Journal of Financial Markets 16, 646–679.

Hayashi, T., Yoshida, N., 2005. On covariance estimation of non-synchronously observed diffusion processes. Bernoulli 11, 359–379.

Haynes, R., Roberts, J., 2015. Automated trading in futures markets. CFTC White Paper.

Hendershott, T., Riordan, R., 8 2013. Algorithmic trading and the market for liquidity. Journal of Financial and Quantitative Analysis 48, 1001–1024.

Huth, N., Abergel, F., 2014. High frequency lead/lag relationships empirical facts. Journal of Empirical Finance 26, 41 – 58.

Jones, C. M., Kaul, G., Lipson, M. L., 1994. Transactions, volume, and volatility. Review of Financial Studies 7, 631–651.

Karpoff, J. M., 1987. The relation between price changes and trading volume: A survey. The Journal of Financial and Quantitative Analysis 22, 109–126.

Kyle, A. S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1335.

Kyle, A. S., Obizhaeva, A. A., 2013. Market microstructure invariance: Theory and empirical tests. Working Paper.

Laughlin, G., Aguirre, A., Grundfest, J., 2014. Information transmission between financial markets in Chicago and New York. Financial Review 49, 283–312.

Lee, C. M. C., Ready, M. J., 1991. Inferring trade direction from intraday data. The Journal of Finance 46, 733–746.

Lewis, M., 2014. Flash Boys: A Wall Street Revolt. W. W. Norton.

Menkveld, A. J., 2008. Splitting orders in overlapping markets: A study of cross-listed stocks. Journal of Financial Intermediation 17, 145 – 174.

O'Hara, M., 2015. High frequency market microstructure. Journal of Financial Economics 116, 257 – 270.

Pasquariello, P., Vega, C., 2015. Strategic cross-trading in the u.s. stock market*. Review of Finance 19, 229–282.

Schwert, G. W., 1989. Why does stock market volatility change over time? The Journal of Finance 44, 1115–1153.

Tauchen, G. E., Pitts, M., 1983. The price variability-volume relationship on speculative markets. Econometrica 51, 485–505.

Ying, C. C., 1966. Stock market prices and volumes of sales. Econometrica 34, 676–685.

Appendix A. Tables

	5%	10%	25%	50%	75%	90%	95%
Panel A: Daily regressions of BrokerTec five-minute returns on net order flow							
$\Delta \mathbf{v}^{BTEC}$	0.15	0.19	0.22	0.31	0.39	0.44	0.49
$\Delta \mathbf{v}^{CME}$	0.18	0.24	0.33	0.41	0.47	0.54	0.57
$\Delta \mathbf{v}^{BTEC}, \Delta \mathbf{v}^{CME}$	0.26	0.33	0.41	0.50	0.56	0.62	0.65
Panel B: Daily regressions of CME five-minute returns on net order flow							
$\Delta \mathbf{v}^{BTEC}$	0.07	0.10	0.17	0.23	0.31	0.39	0.44
$\Delta \mathbf{v}^{CME}$	0.21	0.25	0.32	0.42	0.50	0.55	0.60
$\Delta \mathbf{v}^{BTEC}, \Delta \mathbf{v}^{CME}$	0.25	0.29	0.36	0.47	0.55	0.60	0.63

Table A.3: **Daily Regressions of Intraday Returns on Net Order Flow for the Ten-Year US Treasury Cash and Futures Markets.** The table reports percentiles of adjusted R^2 for daily regressions of five-minute returns on net order flow for the Ten-Year U.S. Treasury Note traded on BrokerTec (Panel A) and the Ten-Year U.S. Treasury Futures traded on CME (Panel B) based on the univariate specifications (4) using each market's net order flow as well as the bivariate specification (5)-(6) using the net order flow in both markets. The percentiles are taken across all days for the sample period from July 1, 2014 to December 31, 2014. **Data Source:** BrokerTec; Nanotick.

	5%	10%	25%	50%	75%	90%	95%
Panel A: Daily regressions of NYSE five-minute returns on net order flow							
$\Delta \mathbf{v}^{NYSE}$	0.05	0.08	0.13	0.20	0.27	0.32	0.36
$\Delta \mathbf{v}^{CME}$	0.33	0.42	0.52	0.59	0.68	0.74	0.75
$\Delta \mathbf{v}^{NYSE}, \Delta \mathbf{v}^{CME}$	0.33	0.44	0.54	0.61	0.70	0.75	0.77
Panel B: Daily regressions of CME five-minute returns on net order flow							
$\Delta \mathbf{v}^{NYSE}$	0.03	0.06	0.12	0.18	0.25	0.30	0.36
$\Delta \mathbf{v}^{CME}$	0.36	0.42	0.51	0.58	0.66	0.73	0.75
$\Delta \mathbf{v}^{NYSE}, \Delta \mathbf{v}^{CME}$	0.38	0.43	0.52	0.59	0.68	0.74	0.75

Table A.4: **Daily Regressions of Intraday Returns on Net Order Flow for the S&P 500 Cash and Futures Markets.** The table reports percentiles of adjusted R^2 for daily regressions of five-minute returns on net order flow for the SPY ETF traded on NYSE (Panel A) and the E-mini Futures traded on CME (Panel B) based on the univariate specifications (4) using each market's net order flow as well as the bivariate specification (7)-(8) using the net order flow in both markets. The percentiles are taken across all days for the sample period from July 1, 2014 to December 31, 2014. **Data Sources:** Nanotick; Thomson Reuters Tick History.

	Historical Degree of Association between Different Trading Activity Measures and Volatility					
	Ten-Year US Treasury Note			**S&P 500**		
Year	**Trading Volume**	**Number of Trades**	**Number of Cross-Active Msec.**	**Trading Volume**	**Number of Trades**	**Number of Cross-Active Msec.**
2004	0.51	0.52	**0.55**	0.55	**0.65**	0.63
2005	0.64	0.66	**0.67**	0.60	**0.65**	0.61
2006	0.61	**0.65**	0.61	0.59	**0.71**	0.67
2007	0.50	0.57	**0.64**	0.63	**0.76**	0.76
2008	0.41	**0.44**	0.43	0.54	0.69	**0.69**
2009	0.45	**0.47**	0.43	0.44	0.56	**0.63**
2010	0.48	0.52	**0.62**	0.58	0.62	**0.67**
2011	0.52	0.58	**0.64**	0.51	0.68	**0.72**
2012	0.56	0.60	**0.65**	0.52	**0.60**	0.59
2013	0.64	0.72	**0.75**	0.61	0.71	**0.73**
2014	0.63	0.69	**0.75**	0.65	0.71	**0.76**
2015	0.56	0.67	**0.73**	0.57	0.63	**0.67**

TABLE A.5: **Historical Degree of Association between Different Trading Activity Measures and Volatility.** The table reports yearly R squared measures of goodness of fit obtained by regressing year by year the daily changes in logarithmic realized volatility on the daily changes in logarithmic trading volume, number of trades, and peak number of cross-active milliseconds over the period from Jan 1, 2004 to Sep 30, 2015. The numbers in bold represent the largest values obtained separately for each year for the Ten-Year U.S. Treasury Note and S&P 500. **Data Sources:** BrokerTec; Nanotick; Thomson Reuters Tick History.

Regression of Volatility on Cash and Futures Market Trading Activity Measures

Year	OLS Coefficients						Student's t-statistics						Adjusted R^2			
	const	n_1	n_2	v_1	v_2	x_{12}	const	n_1	n_2	v_1	v_2	x_{12}	Without x_{12}	With x_{12}	Absolute Increase	Relative Increase
Panel A: Ten-Year U.S. Treasury Note																
2004	0.0	3.1	-1.0	-0.6	0.5	-0.3	0.8	**8.2**	-1.4	-1.4	0.9	-1.4	0.74	0.74	0.00	0.01
2005	0.0	1.6	0.3	-0.5	-0.4	0.6	0.1	**4.2**	0.8	-1.8	-1.5	**5.9**	0.74	0.75	0.01	0.05
2006	0.0	1.5	1.0	-0.6	-0.5	0.1	0.4	**4.8**	**2.5**	**-2.5**	-1.5	0.8	0.74	0.74	0.00	0.01
2007	0.0	1.8	0.8	-1.0	-0.8	0.7	-0.2	**6.4**	1.8	**-3.2**	**-2.6**	**3.4**	0.76	0.78	0.02	0.10
2008	0.0	1.6	0.0	-0.2	-0.1	0.1	0.0	**5.0**	-0.1	-0.9	-0.2	1.4	0.68	0.68	0.00	0.01
2009	0.0	2.9	-0.4	-1.3	0.4	0.0	-0.3	**5.3**	-0.3	**-4.2**	0.4	-0.3	0.60	0.60	0.00	0.00
2010	0.0	2.4	-0.3	-0.6	-0.3	0.3	-0.7	**7.8**	-0.6	**-2.5**	-0.6	**2.1**	0.74	0.74	0.01	0.03
2011	0.0	1.0	0.5	0.1	-0.9	0.7	-0.8	1.5	1.0	0.1	**-2.2**	**2.9**	0.69	0.70	0.02	0.06
2012	0.0	0.5	0.1	-0.1	0.0	0.8	-0.9	1.9	0.2	-0.2	0.1	**4.0**	0.67	0.70	0.03	0.10
2013	0.0	0.8	1.4	-0.7	-0.9	0.8	0.3	1.8	**3.3**	-1.8	-1.9	**4.1**	0.77	0.78	0.01	0.06
2014	0.0	1.1	0.7	-0.1	-1.2	0.8	-0.4	**4.3**	**2.3**	-0.6	**-4.0**	**5.4**	0.79	0.81	0.01	0.07
2015	0.0	0.5	1.6	0.1	-1.6	0.6	0.5	1.3	**3.4**	0.3	**-5.2**	**3.8**	0.78	0.79	0.01	0.06
Panel B: S&P 500																
2004	0.0	1.1	0.5	-0.7	0.0	0.3	-0.7	**2.9**	**3.3**	**-3.0**	-0.1	**2.1**	0.65	0.66	0.01	0.02
2005	0.0	0.7	0.0	-0.2	0.1	0.4	-0.1	**3.1**	0.3	-1.8	**2.0**	**2.2**	0.65	0.66	0.01	0.03
2006	0.0	1.6	0.2	-0.7	0.1	0.1	-0.2	**5.2**	1.6	**-3.8**	0.5	0.7	0.75	0.75	0.00	0.00
2007	0.0	1.8	-0.6	-1.2	0.2	1.1	-1.0	**4.8**	**-3.0**	**-9.1**	1.5	**3.8**	0.80	0.82	0.02	0.11
2008	0.0	2.7	0.4	-1.9	0.0	0.3	-0.2	**5.3**	**2.0**	**-4.9**	-0.2	**2.2**	0.81	0.81	0.00	0.03
2009	0.0	1.3	0.4	-1.3	0.0	0.8	-1.7	**3.1**	1.8	**-6.4**	-0.1	**2.0**	0.68	0.72	0.04	0.13
2010	0.0	0.1	0.7	-0.6	-0.1	1.0	-0.4	0.3	**2.2**	**-2.8**	-0.4	**2.1**	0.71	0.74	0.03	0.10
2011	0.0	1.0	0.6	-1.3	-0.6	1.3	-0.7	**3.6**	**2.1**	**-6.3**	**-2.4**	**4.6**	0.74	0.77	0.03	0.12
2012	0.0	1.6	0.1	-1.3	-0.1	0.7	0.7	**3.2**	0.3	**-3.6**	-0.4	**2.2**	0.64	0.65	0.01	0.04
2013	0.0	1.2	0.3	-1.2	-0.3	1.2	-0.8	**2.9**	0.9	**-3.9**	-1.7	**4.4**	0.72	0.76	0.03	0.12
2014	0.0	1.0	-0.1	-1.1	-0.2	1.4	-0.2	**2.2**	-0.2	**-3.1**	-1.5	**4.4**	0.76	0.81	0.05	0.20
2015	0.0	0.1	0.7	-0.1	-1.1	1.6	0.1	0.4	1.6	-0.4	**-3.8**	**4.9**	0.69	0.75	0.05	0.17

TABLE A.6: **Regression of Volatility on Cash and Futures Markets Trading Activity Measures.** The table reports results for yearly OLS regressions $\Delta rv_t = const + \beta[n_1] \cdot \Delta n_{1,t} + \beta[n_2] \cdot \Delta n_{2,t} + \beta[v_1] \cdot \Delta v_{1,t} + \beta[v_2] \cdot \Delta v_{2,t} + \beta[x_{12}] \cdot \Delta x_{12,t} + \epsilon_t$ of daily changes in logarithmic realized volatility (rv_t) on daily changes in logarithmic number of trades and trading volume in both the futures market ($n_{1,t}$ and $v_{1,t}$) and the cash market ($n_{2,t}$ and $v_{2,t}$) as well as daily changes in the model-free logarithmic measure of cross-market activity between the two markets ($x_{12,t}$) given by the peak number of cross-active milliseconds across all different offsets. Student's t-statistics in bold are significant at 0.05 level after Newey-West adjustment with 22 lags. The relative increase in adjusted R squared is computed by regressing volatility residuals on cross-market activity residuals after partialling out the number of trades and trading volume for each market from both volatility and cross-market activity. The sample period is from Jan 1, 2004 to Sep 30, 2015 and gray boxes indicate when high-frequency cross-market trading is known to have become prevalent in each of the considered market pairs. **Panel A:** Ten-Year U.S. Treasury Note. **Panel B:** S&P 500. **Data Sources:** BrokerTec; Nanotick; Thomson Reuters Tick History.

Regression of Volatility on Transformed Cash and Futures Market Trading Activity Measures

Year	OLS Coefficients						Student's t-statistics						Adjusted R^2			
	const	n_1-n_2	n_2	v_1-n_1	v_2-n_2	x_{12}-n_2	const	n_1-n_2	n_2	v_1-n_1	v_2-n_2	x_{12}-n_2	Without x_{12}-n_2	With x_{12}-n_2	Absolute Increase	Relative Increase
Panel A: Ten-Year U.S. Treasury Note																
2004	0.0	2.5	1.7	-0.6	0.5	-0.3	0.8	**9.5**	**9.3**	-1.4	0.9	-1.4	0.74	0.74	0.00	0.01
2005	0.0	1.2	1.6	-0.5	-0.4	0.6	0.1	**3.0**	**23.0**	-1.8	-1.5	**5.9**	0.74	0.75	0.01	0.05
2006	0.0	0.9	1.4	-0.6	-0.5	0.1	0.4	**3.9**	**14.5**	**-2.5**	-1.5	0.8	0.74	0.74	0.00	0.01
2007	0.0	0.8	1.4	-1.0	-0.8	0.7	-0.2	**4.2**	**14.4**	**-3.2**	**-2.6**	**3.4**	0.76	0.78	0.02	0.10
2008	0.0	1.3	1.4	-0.2	-0.1	0.1	0.0	**7.3**	**27.1**	-0.9	-0.2	1.4	0.68	0.68	0.00	0.01
2009	0.0	1.6	1.6	-1.3	0.4	0.0	-0.3	**5.0**	**12.6**	**-4.2**	0.4	-0.3	0.60	0.60	0.00	0.00
2010	0.0	1.8	1.5	-0.6	-0.3	0.3	-0.7	**12.2**	**18.8**	**-2.5**	-0.6	**2.1**	0.74	0.74	0.01	0.03
2011	0.0	1.1	1.4	0.1	-0.9	0.7	-0.8	**4.6**	**17.3**	0.1	**-2.2**	**2.9**	0.69	0.70	0.02	0.06
2012	0.0	0.4	1.3	-0.1	0.0	0.8	-0.9	1.9	**19.4**	-0.2	0.1	**4.0**	0.67	0.70	0.03	0.10
2013	0.0	0.2	1.4	-0.7	-0.9	0.8	0.3	0.5	**14.2**	-1.8	-1.9	**4.1**	0.77	0.78	0.01	0.06
2014	0.0	1.0	1.3	-0.1	-1.2	0.8	-0.4	**4.9**	**20.1**	-0.6	**-4.0**	**5.4**	0.79	0.81	0.01	0.07
2015	0.0	0.6	1.2	0.1	-1.6	0.6	0.5	**2.2**	**10.2**	0.3	**-5.2**	**3.8**	0.78	0.79	0.01	0.06
Panel B: S&P 500																
2004	0.0	0.4	1.3	-0.7	0.0	0.3	-0.7	**2.0**	**11.2**	**-3.0**	-0.1	**2.1**	0.65	0.66	0.01	0.02
2005	0.0	0.5	1.1	-0.2	0.1	0.4	-0.1	**2.5**	**21.7**	-1.8	**2.0**	**2.2**	0.65	0.66	0.01	0.03
2006	0.0	0.9	1.3	-0.7	0.1	0.1	-0.2	**5.1**	**18.6**	**-3.8**	0.5	0.7	0.75	0.75	0.00	0.00
2007	0.0	0.5	1.3	-1.2	0.2	1.1	-1.0	1.8	**14.5**	**-9.1**	1.5	**3.8**	0.80	0.82	0.02	0.11
2008	0.0	0.8	1.5	-1.9	0.0	0.3	-0.2	**3.4**	**14.3**	**-4.9**	-0.2	**2.2**	0.81	0.81	0.00	0.03
2009	0.0	0.0	1.2	-1.3	0.0	0.8	-1.7	0.1	**9.0**	**-6.4**	-0.1	**2.0**	0.68	0.72	0.04	0.13
2010	0.0	-0.5	1.1	-0.6	-0.1	1.0	-0.4	-1.6	**9.4**	**-2.8**	-0.4	**2.1**	0.71	0.74	0.03	0.10
2011	0.0	-0.2	1.1	-1.3	-0.6	1.3	-0.7	-0.9	**11.0**	**-6.3**	**-2.4**	**4.6**	0.74	0.77	0.03	0.12
2012	0.0	0.3	1.0	-1.3	-0.1	0.7	0.7	0.8	**7.3**	**-3.6**	-0.4	**2.2**	0.64	0.65	0.01	0.04
2013	0.0	0.0	1.1	-1.2	-0.3	1.2	-0.8	-0.2	**13.9**	**-3.9**	-1.7	**4.4**	0.72	0.76	0.03	0.12
2014	0.0	-0.2	1.0	-1.1	-0.2	1.4	-0.2	-0.4	**11.3**	**-3.1**	-1.5	**4.4**	0.76	0.81	0.05	0.20
2015	0.0	0.0	1.2	-0.1	-1.1	1.6	0.1	-0.1	**10.9**	-0.4	**-3.8**	**4.9**	0.69	0.75	0.05	0.17

TABLE A.7: **Regression of Volatility on Transformed Cash and Futures Markets Trading Activity Measures.** The table reports results for yearly OLS regressions $\Delta rv_t = const + \beta[n_1] \cdot \Delta(n_{1,t} - n_{2,t}) + \beta[n_2] \cdot \Delta n_{2,t} + \beta[v_1] \cdot \Delta(v_{1,t} - n_{1,t}) + \beta[v_2] \cdot \Delta(v_{2,t} - n_{2,t}) + \beta[x_{12}] \cdot \Delta(x_{12,t} - n_{2,t}) + \epsilon_t$ of daily changes in logarithmic realized volatility (rv_t) on daily changes in the logarithmic number of trades in futures markets relative to cash markets ($n_{1,t} - n_{2,t}$), logarithmic number of trades in cash markets ($n_{2,t}$), logarithmic average trade size in futures markets ($v_{1,t} - n_{1,t}$) and cash markets ($v_{2,t} - n_{2,t}$), as well as daily changes in the model-free logarithmic measure of cross-market activity between the two markets given by the peak number of cross-active milliseconds across all different offsets relative to the number of trades in the cash market ($x_{12,t} - n_{2,t}$). Student's t-statistics in bold are significant at 0.05 level after Newey-West adjustment with 22 lags. The relative increase in adjusted R squared is computed by regressing volatility residuals on cross-market activity residuals after partialling out the number of trades and trading volume for each market from both volatility and cross-market activity. The sample period is from Jan 1, 2004 to Sep 30, 2015 and gray boxes indicate when high-frequency cross-market trading is known to have become prevalent in each of the considered market pairs. **Panel A:** Ten-Year U.S. Treasury Note. **Panel B:** S&P 500. **Data Sources:** BrokerTec; Nanotick; Thomson Reuters Tick History.

Appendix B. Description of High Frequency Data Sources and Timestamp Precision

We obtain high frequency market data for both U.S. Treasuries (ten-year Treasury note and futures) and equities (S&P 500 E-mini futures and SPDR SPY ETF) with millisecond or higher precision of the timestamps over a recent representative six-month sample period from July 1, 2014 to December 31, 2014. In particular, for the ten-year Treasury cash market we rely on Brokertec data with platform-provided microsecond precision of the timestamps. For the S&P 500 cash market represented by the SPDR ETF we use data available through Thomson Reuters Tick History (TRTH), which contains exchange-provided NYSE timestamps recorded with millisecond precision. Finally, for the CME Treasury and S&P 500 futures market we rely on high-frequency market data captured on-site by Nanotick with sub micro-second resolution of the timestamps. As such, the timestamp precision is more than adequate enough for constructing reliable cross-market activity measures at millisecond frequency.

We further note that while measuring the prevailing offset and dispersion of cross-market activity can get affected by timestamp distortions due to extra latency incurred at data capture, measuring relative changes in the magnitude of cross-market activity tends to be a lot more robust across alternative data sources. This inherent robustness greatly expands the ability to extract meaningful information from the proposed cross-market activity measures across a wider range of data sources and time periods. In particular, it allows us to study changes in the magnitude of cross-market activity over the past decade from January 1, 2004 to September 30, 2015 by using the longer history of futures market data available through TRTH despite the somewhat coarser than millisecond accuracy of most TRTH-provided timestamps in view of the extra time lag incurred due to centralized off-site data processing in London.

Appendix C. Robust Realized Volatility Estimation via Model-Free Quote-Filtering of the Bid-Ask Bounce in Trade Prices

Plotting the mean daily realized volatility estimates over a given date range of interest as a function of the underlying intraday sample frequency is a time-honored way for identifying distortions due to various market macrostructure effects such as the bid-ask bounce in consecutive trade prices or the quote activity at the top of the limit order book. The flat region in these so called volatility signature plots is typically used to identify the highest sample frequency that can be used for reliable estimation of the volatility of the unobserved efficient price process. As an example, figure C.16 shows annualized realized volatility signature plots computed from unfiltered trade prices for the ten-year Treasury futures and cash markets over the second half of 2014.

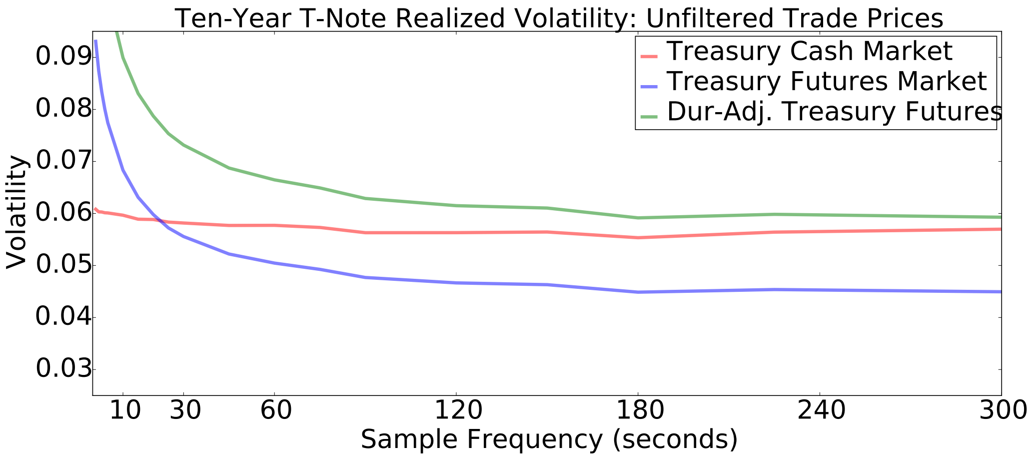

FIGURE C.16: **Ten-Year Treasury Note Realized Volatility Signature Plots for Unfiltered Trade Prices.** We plot average realized volatility as a function of sample frequency over the second half of 2014 for the ten-year Treasury note (in red), the ten-year Treasury futures (in blue) and the duration-adjusted ten-year Treasury futures (in green) based on *unfiltered trade prices* in each market. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

Semi-martingale properties and the near-arbitrage relationship between the Treasury futures and cash markets dictate that, apart from staying flat across different frequencies, the volatility of the ten-year Treasury futures contract should be proportional to the volatility of the ten-year Treasury note, with proportionality constant given to a first order of approximation by the ratio of the corresponding modified durations on each day over the considered date range.[19] Therefore, the realized volatility of the ten-year Treasury note (represented by the red line) should be closely approximated by the realized volatility of the duration-adjusted ten-year Treasury futures (represented by the green line). However, the plotted realized volatility estimates clearly violate this condition even at the five-minute frequency (denoted on the chart as 300 seconds) used in many studies as a rule of thumb to avoid distortions by microstructure noise. Another salient violation of the relationships in place is that as the sample frequency goes up, the average realized volatility for the ten-year Treasury futures without duration adjustment (represented by

[19] The modified duration of the futures contract is dictated by the so called cheapest to deliver bond and the corresponding embedded optionality in the futures contract. The average ratio of the modified duration of the ten-year Treasury futures to the modified duration of the ten-year Treasury note over the second half of 2014 is about 0.76 and the ratio varies between 0.69 and 0.82 throughout this period.

the blue line) first surpasses the corresponding average implied volatility level of 5.1% (registered by the CBOE Ten-Year U.S. Treasury Note Volatility Index) and then quickly becomes higher even than the realized volatility in the Treasury cash market.

Most important of all, though, is the observation that even the seemingly flat region of the signature plots for sample frequencies from three to five minutes on figure C.16 does not satisfy the near-arbitrage relationship in place between Treasury cash and futures markets. This suggests that the close links between such markets may naturally offer extra power to detect distortions of the corresponding realized volatility estimates due to noise, given that only the volatility of the efficient price (but not the noise component) scales in accordance with the constant of proportionality implied by no arbitrage.

To alleviate such apparent biases in the realized volatility estimates even at frequencies often deemed to be safe to choose, we devise a simple model-free procedure that filters out the bid-ask bounce in trade prices. The proposed filter is extremely simple to apply and operates by inferring the mid-quote price corresponding to each trade price after adding or subtracting half the bid-ask spread. In particular, relying solely on the available record for the buy/sell side of each trade, the mid-quote price is inferred as follows:[20]

$$P_t^{Mid} = \begin{cases} P_t^{Trade} + \frac{1}{2}(P_t^{Ask} - P_t^{Bid}) & \text{for seller initiated trade at time t} \\ P_t^{Trade} - \frac{1}{2}(P_t^{Ask} - P_t^{Bid}) & \text{for buyer initiated trade at time t} \end{cases}$$

In the highly liquid ten-year Treasury Note futures and cash markets over the considered date range the filter can be further simplified by setting the bid-ask spread $P_t^{Ask} - P_t^{Bid}$ to its minimum possible value of 1/64 cents prevailing in these markets.[21] Furthermore, we do not need to match trades against quotes to distinguish between buyer-initiated and seller-initiated transactions as our data sources for both the futures and cash markets include an indicator for the direction of each trade. This makes it trivial to apply the proposed procedure for filtering out the bid-ask bounce in Treasury futures and cash market trade prices.

The resulting realized volatility signature plots for the Treasury futures and cash markets are shown on figure C.17. They are considerably more stable and remain strikingly consistent between each other for sample frequencies as high as 30 seconds. The remaining

[20]When a buy/sell side indicator for each trade is not available, one can instead rely on a suitable adaptation of the Lee and Ready (1991) algorithm for matching trades against quotes to determine the direction of each trade.

[21]For markets where the size of the bid-ask spread is highly unstable, an alternative simplification would be to take the unfiltered prices only for the observed portion of seller (or buyer) initiated trades and completely discard the information for the remaining portion of buyer (or seller) initiated trades.

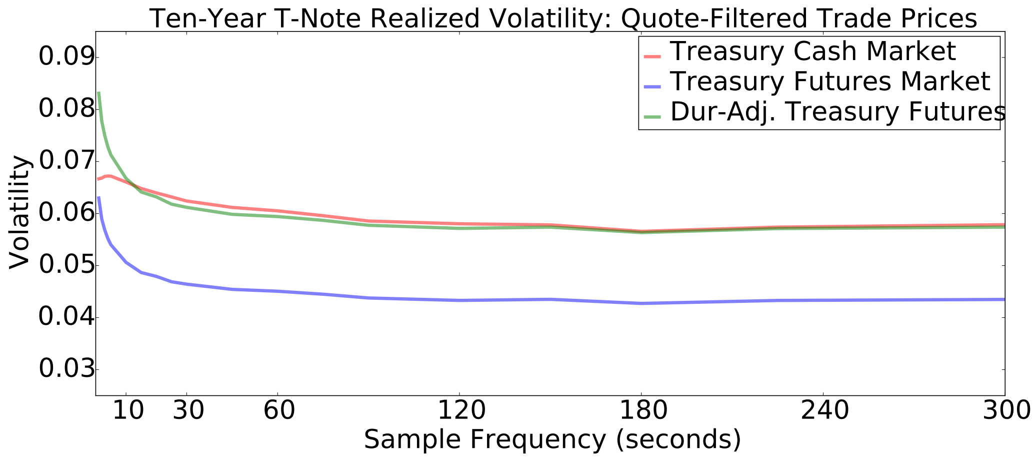

FIGURE C.17: **Ten-Year Treasury Note Realized Volatility Signature Plots for Quote-Filtered Trade Prices.** We plot average realized volatility as a function of sample frequency over the second half of 2014 for the ten-year Treasury note (in red), the ten-year Treasury futures (in blue) and the duration-adjusted ten-year Treasury futures (in green) based on *quote-filtered trade prices* in each market. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

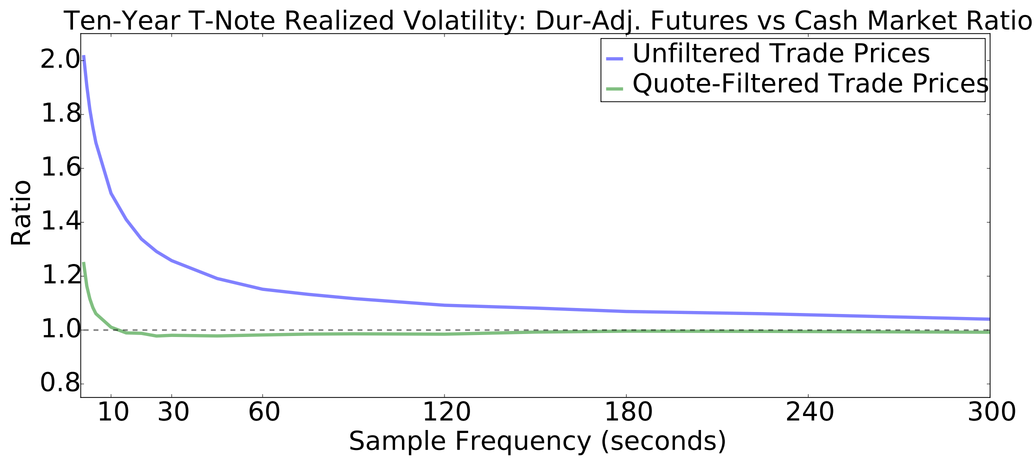

FIGURE C.18: **Signature Plots of the Ratio of Ten-Year Treasury Note Realized Volatility for the Duration-Adjusted Futures vs Cash Market.** We plot the ratios for the average realized volatility of the duration-adjusted ten-year Treasury futures versus the ten-year Treasury note as a function of sample frequency over the second half of 2014 based on either *unfiltered trade prices* (in blue) or *quote-filtered trade prices* (in green). The dashed line (in black) represents the no-arbitrage value of the ratio equal to one. **Data Sources:** BrokerTec (cash market data); Nanotick (CME futures market data).

visible distortion for frequencies higher than 10-15 seconds reflects occasional mid-quote price under-correction with the imposed constant minimum bid-ask spread adjustment as well as possible activity patterns and bid/ask asymmetries on top of the order book in relation to trades.

Figure C.18 further compares the ratio of the realized volatility between the duration-adjusted ten-year Treasury futures and the ten-year Treasury note for unfiltered trade prices (represented by the blue line) versus quote-filtered trade prices (represented by the green line). When relying on the proposed filter as opposed to using unfiltered trade prices, the ratio is remarkably stable around one for almost the entire frequency range, indicating perfect agreement with the underlying arbitrage-related restrictions between Treasury futures and cash markets. Thus, the introduced simple method for quote-filtering of trade prices allows reliable estimation of realized volatility in these markets up to sample frequencies as high as 30 seconds without the need to apply more complex procedures to achieve sufficient robustness to microstructure noise.

The main advantage of the proposed approach in comparison to standard approaches for noise-robust estimation in the realized volatility literature (e.g. kernel based estimation, pre-averagaing, multi-scale estimation and the like) is its model-free nature and simplicity. In particular, there is no need to use more complex robust extensions of the simple realized volatility estimator that involve at least some extra statistical assumptions on the noise component in the observed prices due to market microstructure effects. As another contribution, we show how arbitrage-related restrictions for closely related markets such as those for Treasury notes and futures can be exploited for improved empirical assessment of the quality of realized volatility estimators at different sample frequencies.