

Quantile relationships between standard, diffusion and jump betas across Japanese banks

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Abstract:

How the banking sector absorbs news is critical to disbursing information to financial markets and the real economy. Using high frequency financial data and quantile regression techniques we characterise some stylised facts about standard betas, diffusion betas, jump betas and the relationships between them for Japanese banking stocks and bank portfolios. Jump betas, which relate to the arrival of unexpected news, are on average, higher and more dispersed than the diffusion betas across the banking sector. While on average, the standard beta is a weighted average of the diffusion and jump betas, the magnitudes of the weights differ significantly across the quantiles, indicating non-linearity in how jump information is incorporated. On average, small bank portfolios have smaller diffusion betas and smaller jump betas than large bank portfolio. While there are no significant differences between the jump-diffusion beta ratios when conditioned by market capitalisation, during times of financial crisis, small bank portfolios have significantly higher jump beta-diffusion beta ratios than large bank portfolios; indicating that during time of financial crisis, small Japanese banks face much higher relative jump risks than larger Japanese banks.

Key words: Beta; Jumps; High-frequency data; Quantile regression; Japanese Banks.

JEL Classification: G12, G21, C58

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I. Introduction

In the one factor capital asset pricing model (CAPM), systematic risk, measured by beta, is determined by the asset's covariance with the market over the market variance ([Sharpe 1963](#); [Lintner 1965](#)). The traditional way of estimating the asset's constant beta has been by linear regression, typically based on 5 years of monthly data. However, the advent of more powerful computers and easy access to high frequency data has made alternative non-parametric approaches realistic. Non-parametric approaches reduce the scale of the calculation problem and avoid many of the assumptions necessary for parametric modelling. The use of high frequency data results in statistically superior beta estimates and the computation of continuously time varying realized betas, providing a simple and robust estimator for measurement of time varying systematic risk. (see [Wang et al. \(2013\)](#)).

From a pricing perspective, the empirical failure of the unconditional Capital Asset Pricing Model (CAPM) has led to three possible approaches to relaxing the overly restrictive CAPM assumptions. The first is to use additional systematic factors, as in [Merton \(1973\)](#), allowing extra-market factors to capture additional systematic risks; such as the three-factor model of [Fama and French \(1993\)](#) and the four-factor model of [Carhart \(1997\)](#). The second approach is to relax the static relationship between expected return and risk by allowing time variation in the systematic factors. In that sense, [Jagannathan and Wang \(1996\)](#), [Lettau and Ludvigson \(2001\)](#) and [Petkova and Zhang \(2005\)](#) find that betas of assets with different characteristics move differently over the business cycle and [Campbell and Vuolteenaho \(2004\)](#), [Fama and French \(1996\)](#) and [Ferson and Harvey \(1999\)](#) show that time-variation in betas helps to explain anomalies such as value, industry and size. However, this conditional time-varying framework does not seem to be enough to improve the weak fit of the CAPM, as shown by [Lewellen and Nagel \(2006\)](#).

The third approach is the use of dual or conditional betas whereby the market beta is conditioned on market states i.e. bullish or bearish or positive or negative market returns. [Bhardwaj and Brooks \(1993\)](#), [Howton and Peterson \(1998\)](#) and [Pettengill et al. \(1995\)](#) among others have investigated the relationship between beta risk and stock market conditions. [Fabozzi and Francis \(1977\)](#) first tested the stability of betas over the "bull" and "bear" markets; [Pettengill et al. \(1995\)](#) observe that larger firms experience larger betas in down market conditions than in up market conditions, with the reverse being true for smaller firms, Using an alternative return decomposition method, [Campbell and Vuolteenaho \(2004\)](#)

decompose CAPM betas into discount rate betas and cash flow betas which [Botshekan et al. \(2012\)](#) follow to construct a return decomposition distinguishing cash flow and discount rate betas in up and down markets. They find that for larger companies, the priced components of risks become more symmetric (both upside and downside market).

In each of the above three approaches, the various beta estimates assume a continuous data generation process, while in fact the empirical papers in high frequency literature support the occurrence and persistence of jumps in the observed data generation process. A large body of literature shows that both theoretically and empirically jumps explain many of the dynamic features of stylized facts documented in asset prices. Studies on stochastic behaviour of the stock market generally agree that stock returns are generated by a mixed process with a diffusion component and a jump component. In this case the standard CAPM beta is at best a ‘summary proxy’ for the systematic risk of a mixed-process, i.e. a weighted average of the diffusion component and the jump component. By separating the standard beta into two component betas we can capture the two risks separately: one for continuous and small changes (diffusion beta) and the other for discrete and large changes (jump beta). [Todorov and Bollerslev \(2010\)](#) provide a theoretical framework for disentangling and estimating the sensitivity towards systematic diffusive and jump risk in the context of factor models. They focus on the decomposition of systematic risk by recognizing jump occurrence at the aggregate market level and show that diffusion and jump betas with respect to aggregate market portfolio differ significantly and substantially. The use of high frequency data enables both betas to be time-varying.

The key contribution of this paper is to examine the relationship between standard, diffusion and jump betas across the quantiles of observed returns. Although the continuous returns and jump returns are orthogonal by the [Todorov and Bollerslev \(2010\)](#) decomposition, the three realised betas (i.e. standard, diffusion and jump betas) are not restricted nor expected to be orthogonal. In fact, a simple correlation test indicates some dependencies. The rich cross-sectional and time-series heterogeneity in our estimates of monthly betas enables us to study how standard beta, diffusion beta and jump betas vary both across quantiles and over time. We adopt a quantile regressions (QR) approach to model the relationship between standard betas and diffusion and jump betas not just for the mean of the conditional distribution, but also across the distribution (e.g. [Koenker and Hallock \(2001\)](#)).

Our empirical investigations are based on high-frequency stock data of 50 Japanese banks included in the Nikkei 225 index over the 2002-2012 sample period. We begin by estimating

two separate betas; the diffusion and jump betas as well as a standard CAPM beta for each of the individual stocks on a monthly basis over the whole sample period. We rely on 5-minute intraday sampling frequency for the beta estimation, with the frequency chosen to guard against the market microstructure complications that arise at the higher frequencies. We regress the standard beta against the diffusion and jump beta and we find that the quantile regression relations between standard beta and diffusion and jump beta varies widely depending on the quantile level of standard beta.

We find that on average the standard beta is weighted more on the diffusion beta component than the jump beta component. The relationship holds across all the quintiles considered. However, the actual magnitude of the weights differ across the quintiles. In general, the weights are jointly lower for low standard betas, increasing around the 50th-75th quintiles and dropping again post 75th quantile.

Sorting bank portfolios based on the size, we find that large banks have high betas and small banks have low betas. The results hold for all the three betas; indicating that larger Japanese banks are more sensitive to both market movements than smaller institutions, regardless of whether these movements occur through a jump or not. However, the relative effects do not remain the same. The ratio of the standard betas for large equity versus small equity portfolios is 2.81, while the ratio of diffusion betas for large equity versus small equity portfolios is 5.82. In contrast, the ratio of jump betas for large equity versus small equity portfolios is 1.65.

This study also investigates the empirical beta-relationship between the jump-diffusion model and the conventional CAPM. We find that under certain market conditions, particularly during crisis periods, the hypothesis that the standard beta or systematic risk is the weighted average of the diffusion and jump betas (i.e. both the diffusion and jump (market) risks) is not rejected at standard significance levels.

The rest of the paper is organised as follows. In Section II, we present our theoretical framework. Section III presents the methodology used in this study. Section IV describes the data. The empirical results are present in Section IV. Section V concludes the paper.

II. Theoretical Framework

A. Capital Asset Pricing Model

The standard capital asset pricing model (CAPM) is given as:

$$r_{i,t} = \alpha_i + \beta_{it}r_{m,t} + \varepsilon_{i,t} \quad (1)$$

where $r_{i,t}$ is the monthly excess stock return on stock i , and $r_{m,t}$ is the aggregate market returns at time t ; α_i is the asset specific constant, and the error term $\varepsilon_{i,t}$ is the idiosyncratic risk of stock i , which is uncorrelated with r_m or the idiosyncratic risk of any other stock under CAPM assumptions. The slope coefficient, $\beta_{i,t}$, in equation (1), commonly known as the standard beta, is the systematic risk of asset i , and measures the responsiveness of the changes in stock's prices to changes in market prices. According to the CAPM, the equilibrium expected return on a risky asset is a function of its covariance with the market portfolio.

Standard beta, in CAPM is defined as,

$$\beta_{i,t} = \frac{\text{Cov}(r_{i,t}, r_{m,t})}{\text{Var}(r_{m,t})} \quad (2)$$

The CAPM model basically depends on stock and market returns, which in turn, depend on the underlying prices of individual stocks. It is now widely agreed in the literature that financial return volatilities and correlations are time-varying and returns follow the sum of a diffusion process and a jump process.¹

Consider the case where the log-price (p_t) process of an asset at time t follows a continuous-time jump-diffusion process defined by the stochastic differential equation as follows:

$$dp_t = \mu_t dt + \sigma_t dW_t + k_t dJ_t \quad (3)$$

where μ_t is the instantaneous drift of price process and σ_t is the diffusion process, and W_t is standard Brownian motion. The first two terms correspond to the diffusion part of the total variation process, interpreted as responsible for the usual day-to-day price movement. Changes in stock prices may be due to variation in capitalization rates, a temporary imbalance between supply and demand, or the receipt of information which only marginally affects stock prices. The final term, $k_t dJ_t$ refers to the jump component of the total process, where j_t is a counting process such that $dJ_t = 1$ indicates a jump at time t and $dJ_t = 0$ otherwise, and k_t is the size of jump at time t if a jump occurred. The jump part is assumed to be due to the receipt of any important information that causes a more than marginal change

¹ See, for example, [Press \(1967\)](#), [Merton \(1976\)](#), and [Ball and Torous \(1983\)](#) and among others.

(i.e. abnormal change) in the price of stock. The arrival of this kind of information is random and the intensity of information arrival follows a Poisson process.

If the return of stocks should be divided into jump part and diffusion part certainly the risk associated with returns of securities should be similarly decomposed. The presence of jump variations in both individual assets and aggregate market affect co-variance estimation and consequently estimates of realized beta and systematic risk. Thus it would be prudent to disentangle the jump component and the diffusion component in prices because they are basically two quite different sources of risk; see, e.g. [Bates \(2000\)](#), [Eraker \(2004\)](#), [Pan \(2002\)](#) and [Todorov \(2009\)](#).

B. Decomposing Systematic Risk: Diffusion and Jump components

Our framework motivating the different betas and the separate pricing of diffusion and jump market price risk and relies on the approach originally developed by [Todorov and Bollerslev \(2010\)](#) for decomposing market returns into two components: one associated with diffusion price movement and another associated with jumps. Hence, in the presence of both components, equation (1) becomes:

$$r_{i,t} = \alpha_i + \beta_{i,t}^c r_{m,t}^c + \beta_{i,t}^j r_{m,t}^j + \varepsilon_{i,t} \quad (4)$$

where $r_{i,t}$ is the monthly excess stock return on stock i , α_i is its drift term and the total market risk ($r_{m,t}$) is modelled as a combination of a diffusion ($r_{m,t}^c$) and jump component ($r_{m,t}^j$). The parameters $\beta_{i,t}^c$ and $\beta_{i,t}^j$ denote the responsiveness of each stock's movement to the diffusion and jump components of market risk and ε_i denotes the idiosyncratic term -- which is also made up of a continuous and jump component. This decomposition is pertinent because standard single factor models of risk implicitly assume that an asset's systematic risk is unaffected by any market jumps (i.e. there are no market jumps because stock jumps are diversified away at the aggregate level or the assets diffusion and jump betas are the same). Equation (1) does not distinguish between the diffusion and jump components of total return, but does decompose total returns into systematic ($\beta_{i,t} r_{m,t}$) and non-systematic ($\alpha_i + \varepsilon_{i,t}$) components. Any market jump is embedded within $r_{m,t}$, while any non-systematic jump unique to firm i is included in the error term. When the systematic risks exposure of a firm to both diffusion and jump price movements are identical, i.e. $\beta_{i,t}^c = \beta_{i,t}^j$, the two-factor dual-beta market model collapses to the usual one-factor single-beta market model, which relates

the stock return $r_{i,t}$ to the total market return $r_{m,t} = r_{m,t}^c + r_{m,t}^j$. The restriction that $\beta_{i,t}^c = \beta_{i,t}^j$ implies that the asset responds in the same manner to market diffusion and jump price changes, i.e. intuitively that the asset and the market co-move in the same manner during “normal” times and periods of “abrupt” market moves. If, on the other hand, $\beta_{i,t}^c$ and $\beta_{i,t}^j$ differ, empirical evidence for which is provided below, the cross-sectional variation in the diffusion and jump betas may be used to determine any separate pricing behaviour. The extant literature suggests that the two betas are not the same.

Chen (1996) showed that under the usual assumptions of CAPM, relaxing only the normality of asset returns, the jump-diffusion model includes a diffusion beta, which measures the systematic risk when no jumps occurs and jump beta, which measures the systematic risk when jumps take place in the market. The dual beta jump-diffusion model is defined as follows:

$$r_{i,t} = \alpha_i + r_{m,t}[(1 - \phi)\beta_{i,t}^c + \phi\beta_{i,t}^j] + \varepsilon_{i,t} \quad (5)$$

The right side of equation (5) is the weighted average of two betas, with weighting parameter ϕ . $\beta_{i,t}^c$ is the diffusion beta as defined by $\beta_{i,t}^c = \frac{\text{cov}(r_{i,t}r_{m,t}^c)}{\text{var}(r_{m,t}^c)}$; $\beta_{i,t}^j$ is the jump beta as defined by $\beta_{i,t}^j = \frac{\text{cov}(r_{i,t}r_{m,t}^j)}{\text{var}(r_{m,t}^j)}$. Two special cases apply. If there are no jumps in the market, this

implies $\phi = 0$, and equation (5) collapses to the conventional CAPM equation,

$$r_{i,t} = \alpha_i + r_{m,t}[\beta_{i,t}^c] + \varepsilon_{i,t} \quad (5a)$$

On the other hand, if asset returns are generated by a pure jump process, $\sigma^2(r_m) = 0$ which implies $\phi = 1$, then equation (5) reduces to a pure jump CAPM equation,

$$r_{i,t} = \alpha_i + r_{m,t}[\beta_{i,t}^j] + \varepsilon_{i,t} \quad (5b)$$

III. Methodology

The possibility of a two-way decomposition of the standard beta prompts us to investigate the relationship between the standard betas and the decomposed diffusion and jump beta components. We now consider the relationship between standard beta, diffusion beta and jump beta across Japanese banks.

A. Realized Beta

Standard betas are not directly observable. The traditional approach for estimating standard betas relied on rolling linear regressions, typically requiring sample sizes of up to 5 years of monthly data to satisfy sample size requirements.² However, the advent of readily available high frequency data in recent years, have now made it possible to estimate realized betas over much shorter sample sizes.

Realized beta or high frequency standard beta is the ratio of realized covariance of stock and market to the realized market variance. [Andersen et al. \(2005\)](#) argue that realized beta is a more accurate measurement of the standard beta because it employs more information than the traditional regression on monthly returns. The estimate of realized beta for an individual stock, $\hat{\beta}_{i,t}^s$ is defined as:

$$\hat{\beta}_{i,t}^s = \frac{RCOV_{i,t,s}^s}{RV_{m,t,s}^s} = \frac{\sum_{s=1}^n r_{i,t,s} r_{m,t,s}}{\sum_{l=1}^n (r_{m,t,s})^2} \quad (6)$$

Despite the advantages of realized beta, equation (6) still defines the standard beta in a one-factor CAPM model. The same readily high frequency data that makes possible the computation of the realized betas also enables the disentangling of these realized betas into diffusion betas and jump betas, effectively giving rise to a two-factor CAPM model for pricing assets which follow not only a diffusion process but also a jump process. Hence, the two-factor CAPM model in this paper is also a jump-diffusion CAPM model.

B. Diffusion and Jump betas

The decomposition of the returns for the market into separate diffusion and jump components that formally define the $\beta_{i,t}^c$ and $\beta_{i,t}^j$ in equations (4) are, of course, not directly observable. Instead, we assume that prices are observed at discrete time grids of length $1/s$ over the active trading day $[0, T]$. Empirical studies rely on discretely sampled returns denoted as

$$r_{t,s} = p_{t,s} - p_{t,s-1}, \quad s = 1, \dots, n; t = 1, \dots, T \quad (7)$$

where $p_{t,s}$ refers to the s th intra-day log-price for day t ; T is the total number of days in the sample and s is the (regular) sampling frequency.

² see, e.g., the classical work by [Fama and MacBeth \(1973\)](#).

Assuem that the intraday stock price processes for the aggregate market index, denoted by $dp_{m,t}$, and the i th stock, denoted by $dp_{i,t}$, each follow a general diffusion-time process. To allow for the presence of jumps in the price process [Todorov and Bollerslev \(2010\)](#) considered the following specification for stock i and aggregate market m .³

For the market,

$$r_{m,t,s} \equiv dp_{m,t} = \alpha_{m,t}dt + \sigma_{m,t}dW_{m,t} + k_{m,t}dJ_{m,t} \quad (8)$$

and for the stock,

$$r_{i,t,s} \equiv dp_{i,t} = \alpha_{i,t}dt + \beta_{i,t}^c \sigma_{m,t}dW_{m,t} + \beta_{i,t}^j dJ_{m,t} + \sigma_{i,t}dW_{i,t} + k_{i,t}dJ_{i,t}, i = 1, \dots, N \quad (9)$$

where, $W_{m,t}$ and $W_{i,t}$ are standard Brownian motions for the market and asset i ; $\alpha_{m,t}$ and $\alpha_{i,t}$ denote the diffusive volatility of the aggregate market and stock i , respectively; and $J_{m,t}$ and $J_{i,t}$ refer to the pure jump Levy processes in the aggregate market and stock i , respectively. $\beta_{i,t}^c$ and $\beta_{i,t}^j$ then measure the responsiveness of an individual stock to the diffusion and jump component of market risk. Within this framework, $[\beta_{i,t}^c, \beta_{i,t}^j]$ is assumed constant throughout each month but can change from month to month.

In order to disentangle $\beta_{i,t}^c$ and $\beta_{i,t}^j$, [Todorov and Bollerslev \(2010\)](#) propose a non-parametric beta estimation technique using a multi-power covariation/variation formulation between the returns of individual stocks and the market portfolio for the diffusion and jump components. By expressing the co-variation between the continuous components of $r_{i,t}$ and $r_{m,t}$ as $[r_{i,t}^c, r_{m,t}^c]_{[0,T]} = \beta_{i,t}^c \int_{t-1}^t \sigma_{m,s}^2 ds$, and the variation of continuous component of $r_{m,t}$ as $[r_{m,t}^c, r_{m,t}^c]_{[0,T]} = \int_{t-1}^t \sigma_{m,s}^2 ds$ in the continuous-time model, they show that the diffusion beta of the i^{th} asset, $\beta_{i,t}^c$ can be expressed as:

$$\beta_{i,t}^c = \frac{[r_i^c, r_m^c]_{[0,T]}}{[r_m^c, r_m^c]_{[0,T]}}, \quad i = 1, \dots, N. \quad (10)$$

³ The notation here is simplified relative to that in [Todorov and Bollerslev \(2010\)](#) see their article for more details.

In reality, price data are not observed continuously. The estimator $\hat{\beta}_{i,t}^c$ takes the following form in the discrete-time setting;

$$\hat{\beta}_{i,t}^c = \frac{\sum_{s=1}^n r_{i,t,s} r_{m,t,s} \mathbb{1}_{\{|r_{t,s}| \leq \theta\}}}{\sum_{s=1}^n (r_{m,t,s})^2 \mathbb{1}_{\{|r_{t,s}| \leq \theta\}}}, \quad i = 1, \dots, N. \quad (11)$$

where, $\mathbb{1}_{\{|r_{t,s}| \leq \theta\}}$ is the indicator function, based on the truncation level, θ , for the diffusion component,

$$\mathbb{1} = \begin{cases} 1 & \text{if } \{|r_{t,s}| \leq \theta\} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

The observed high-frequency returns may contain both diffusive and jump risk components. Raising the high-frequency returns to powers of orders greater than two, has the effect of making the diffusion components negligible, so that systematic jumps dominate asymptotically for $n \rightarrow \infty$.⁴ As formally shown in [Todorov and Bollerslev \(2010\)](#), the following estimator is consistent for jump beta when there is at least one significant jump in the market portfolio for the given estimation window for $n \rightarrow \infty$.

$$\begin{aligned} & \hat{\beta}_{i,t}^j \\ &= \text{sign} \left\{ \sum_{s=1}^n \text{sign}\{r_{i,t,s} r_{m,t,s}\} |r_{i,t,s} r_{m,t,s}|^\tau \right\} \times \left(\frac{|\sum_{s=1}^n \text{sign}\{r_{i,t,s} r_{m,t,s}\} |r_{i,t,s} r_{m,t,s}|^\tau|}{\sum_{s=1}^n (r_{m,t,s})^{2\tau}} \right)^{\frac{1}{\tau}}, \quad (13) \end{aligned}$$

Here, the power $\tau \geq 2$ so that the diffusion price movements do not matter asymptotically. The sign in equation (13) is taken to recover the signs of jump betas that are eliminated when taking absolute values.

Following [Todorov and Bollerslev \(2010\)](#) and [Alexeev et al. \(2017\)](#) we set the parameter values for θ , ϖ , and α estimate the $\hat{\beta}_{i,t}^c$ and $\hat{\beta}_{i,t}^j$ on monthly basis. For estimating the $\hat{\beta}_{i,t}^c$ and $\hat{\beta}_{i,t}^j$, the truncation threshold, $\theta = \alpha \Delta_n^{\varpi}$, uses $\varpi = 0.49$ and $\alpha \geq 0$, suggesting that the threshold values may vary across stocks and across different estimation windows. We implement the threshold, $\theta = \alpha_i^c = 3 \sqrt{BV_i^{[0,T]}}$ for $\hat{\beta}_{i,t}^c$ suggesting that the diffusion

⁴ The basic idea of relying on higher orders powers of returns to isolate the jump component of the price has previously been used in many other situations, both parametrically and nonparametrically; see e.g., [Barndorff-Nielsen and Shephard \(2003\)](#) and [Ait-Sahalia \(2004\)](#).

components are contained within three standard deviation from mean, where, $BV_i^{[0,T]}$ is the bi-power variation of the i -th stock at time $[0, T]$; implemented with $\tau = 2$ for equation (13).

III. Sample and Data

The sample consists of publicly-traded Japanese bank stocks from January 2001 through December 2012, including varying phases of the business cycle. Our final sample consists of 50 (of the 63) commercial banks listed on the Tokyo Stock Exchange (TSE) as the remaining banks did not have sufficient data availability. All the high-frequency data are extracted from the Thompson Reuters Tick history (TRTH) database available via the SIRCA. We use the Nikkei 225 index as the proxy of the market portfolio. Following the standard high-frequency literature, we sample at a 5-minute frequency for all data, reflecting a trade-off between using all available high-frequency data and avoiding the impact of market microstructure effects, such as infrequent trading or nonsynchronous trading. Unlike the more commonly investigated US and European markets, daily trading on the TSE is interrupted by a lunch break, trading between 09:00 am - 11:00 am and 12:30 pm- 3:00 pm local time. We sample prices from 9:05 am-11:00 pm and 12.35 pm-3.00 pm, with overnight and over-lunch returns excluded from the data set. Missing data at 5-minute intervals is filled with the previous price; when no actual trade occurs during a time interval, it is logical to assume that a stock price carries the same price of the previous particular time interval. [Hansen and Lunde \(2006\)](#) show that the previous tick method is a sensible way to sample prices in calendar time. Consequently, we have 53 intra-day observations for 2866 active trading days over a 12 year period (144 months).

IV. Empirical Results

A. Betas

Our main empirical results are based on monthly standard, diffusion and jump beta estimates for each of the stocks in the sample. Table 1 presents the means and standard deviations of the time varying betas for period 2001-2012 and three sub periods; a pre-crisis period from Jan 2001 to June 2007, a crisis period consistent with the Global Financial Crisis (GFC) from July 2007 to May 2009 and a post-crisis period from June 2009 to the end of the sample.⁵ The statistics show that across the entire sample the jump beta has a higher mean (volatility)

⁵ We use the crisis period identified in [Dungey and Gajurel \(2014\)](#).

of 0.912 (0.626), relative to that of the standard beta 0.501(0.280) and the diffusion beta of 0.324 (0.309). The difference in means of diffusion beta and jump beta over the full sample period is statistically significant (t-value = -78.219***) based on the pooled variance t-test of difference. These relative sizes are retained in the three sub-periods, while the standard, diffusion and jump betas are each generally larger and more volatile in crisis period compared to pre-crisis and post-crisis periods.

Table 1: Summary Statistics for Standard, Diffusion and Jump Betas

The table summarizes of the time varying betas averages as estimated. The statistics include mean and standard deviations (in parentheses) are summarized by the full sample periods averages and three sub-periods. We include the pooled variance t-test of the difference between the two sample means for the Standard Beta, Diffusion Beta and Jump Beta. The t-statistics are shown in parentheses. * denotes significance at 10 % level; ** denotes significance at 5 % level, and *** denotes significance at 1 % level

	Standard Beta	Diffusion Beta	Jump Beta
Full-sample Period			
Mean	0.501	0.280	0.912
Std.Dev	0.324	0.309	0.626
t-test of difference		-78.219***	
Pre-crisis Period			
Mean	0.390	0.223	0.759
Std.Dev	0.276	0.276	0.572
t-test of difference		-53.434***	
Crisis Period			
Mean	0.702	0.452	1.095
Std.Dev	0.342	0.321	0.746
t-test of difference		-29.376***	
Post-crisis Period			
Mean	0.548	0.248	1.042
Std.Dev	0.306	0.308	0.552
t-test of difference		-57.495***	

Figure 1 plots the kernel density estimates of the unconditional distributions of the three different betas averaged across time and stocks. The jump betas are somewhat larger on average, while the diffusion betas are the least dispersed of the three betas across time and stocks. Part of the dispersion in the betas maybe be due to estimation errors.⁶

Figure 2 shows the time series of equally weighted portfolio betas, based on monthly quintile sorts for each of the three different betas and all of the individual stocks in the sample. The figure suggests that the variation in the standard beta and diffusion beta sorted portfolios (in Panels A and B) are clearly fairly close, as would be expected. The plots for the jump beta

⁶ Based on the expressions derived in [Todorov and Bollerslev \(2010\)](#), [Bollerslev et al. \(2015\)](#) report that the asymptotic standard errors for diffusion and jump betas averaged across all of the stocks and months in the sample equal 0.06 and 0.12, respectively, compared with 0.14 for the conventional OLS- based standard errors for the standard beta estimates.

quintile portfolios in Panel C, are distinctly different and more dispersed than the standard and diffusion betas quintile portfolios. Jump beta is significantly different from diffusion and standard beta. Motivated by these and to address the significant heterogeneity observed across Japanese banking sector stocks, we depart from the previous literature and employ the quantile regression analysis to estimate the relationship between standard, diffusion and jump betas.

Figure 1: Distributions of Betas

The figure displays kernel density estimates of the unconditional distributions of the three different betas averaged across firms and time.

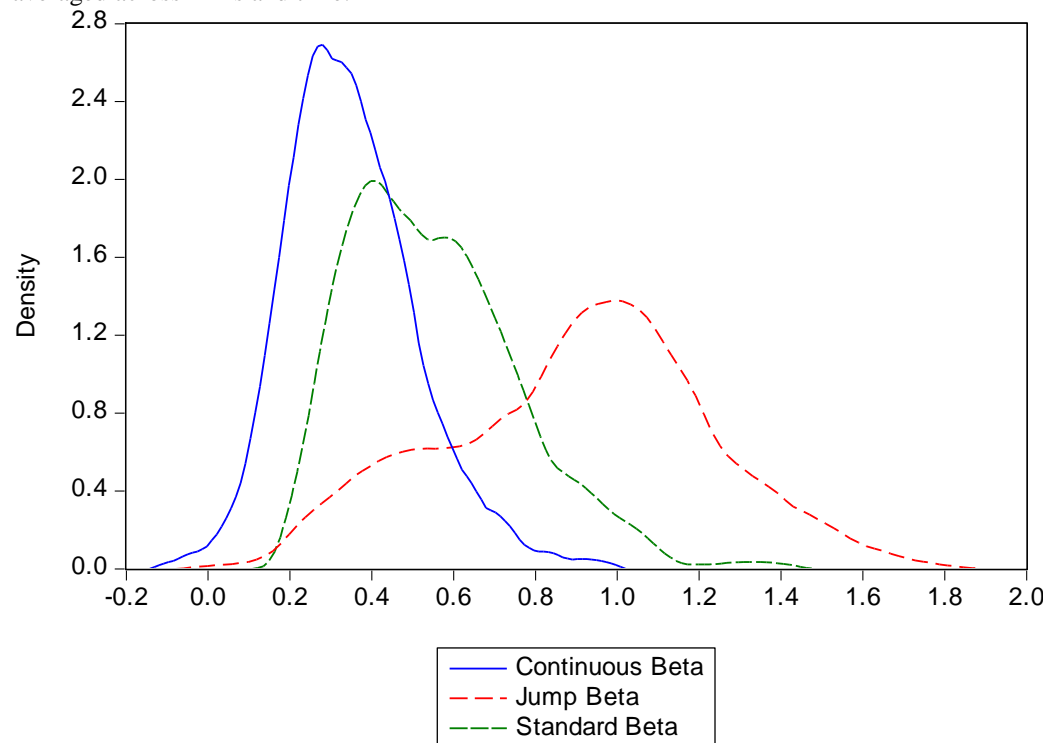
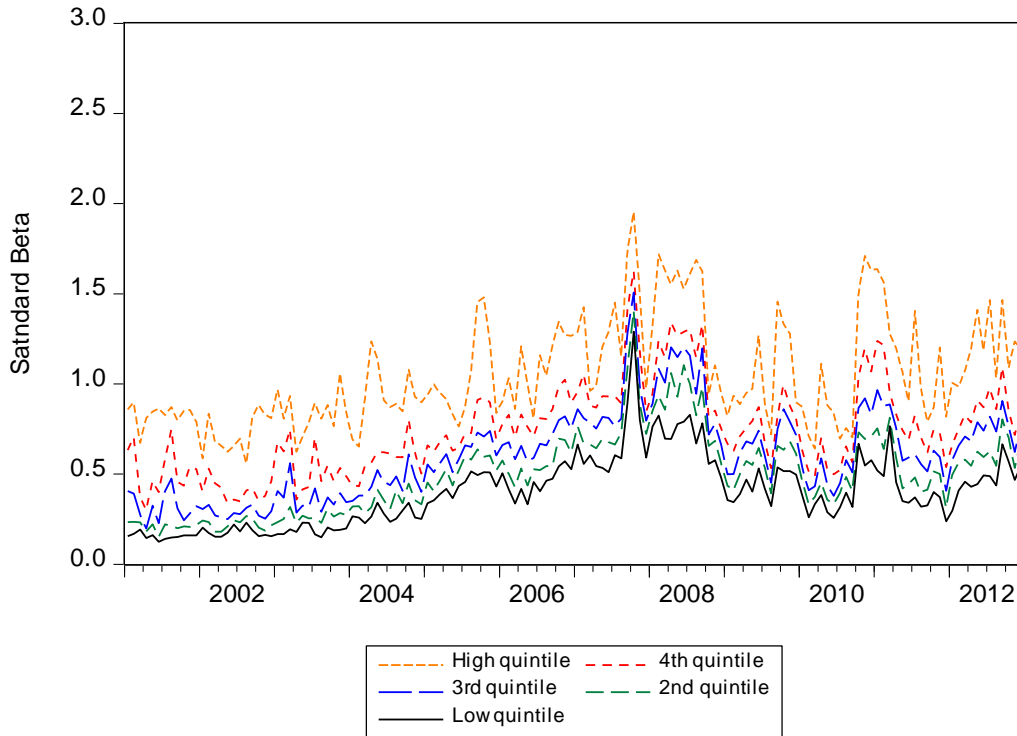


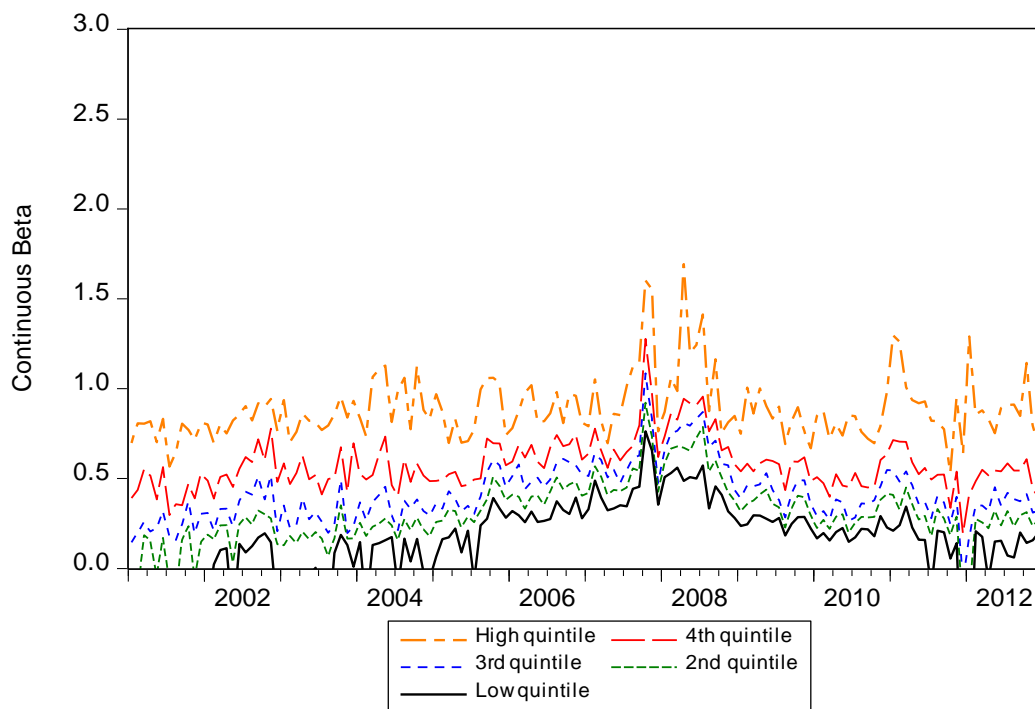
Figure 2: Time series plots of betas

The figure displays the time series of betas for equally weighted beta-sorted quintiles portfolios. Panel A shows the result for the standard beta sorted portfolios, Panel B the diffusion beta sorted portfolios and Panel C the jump beta sorted portfolios.

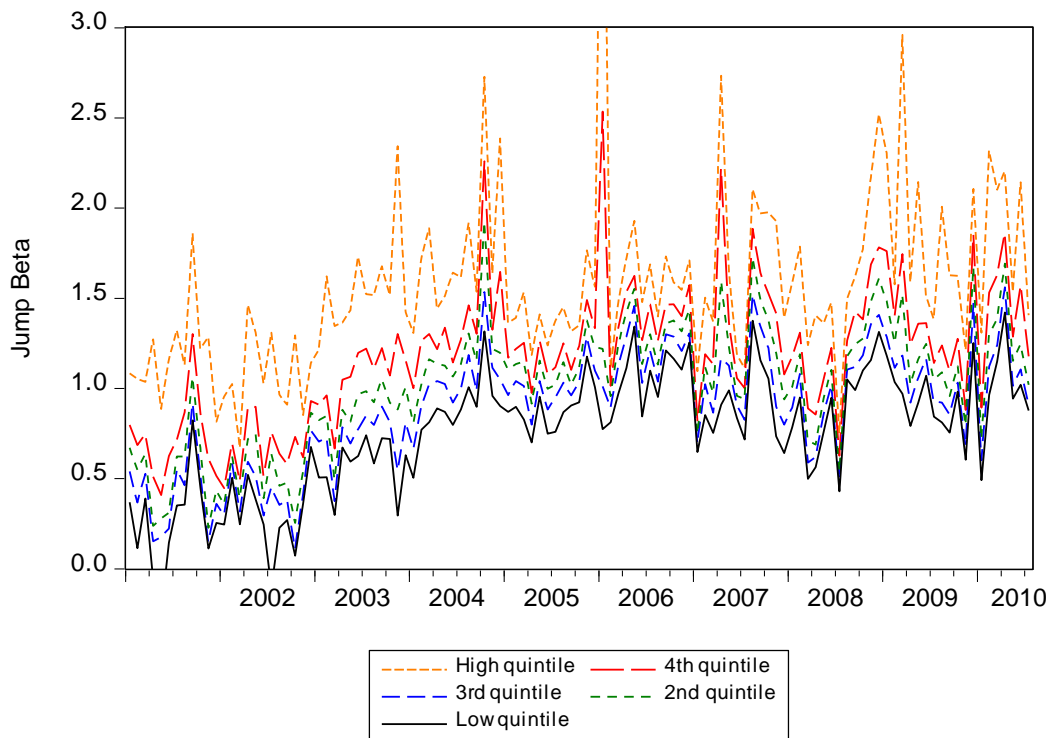
Panel (A): Standard Beta



Panel (B): Diffusion Beta



Panel (C): Jump Beta



B. Quantile regression model

Ordinary least squares regression determines the average relation between the dependent and a set of relevant explanatory variable, focussing on the estimation of the conditional mean. Quantile regression (QR) model allows us to estimate the relationship at any specific quantiles; and helps in examination of non-linearity and the tail behaviour of a distribution. Quantile regression is more robust to the effects of heteroskedasticity, skewness and leptokurtosis, each of which are reported stylised features of financial data (Koenker and Xiao 2006).

The quantile regression approach has been widely used in many areas of applied economics and econometrics; such as the investigation of wage structure (Buchinsky 1994) earnings mobility (Trede 1998; Eide and Showalter 1999), and educational quality issues (Eide and Showalter 1998; Levin 2001). There is also growing interest in employing quantile regression methods in the financial literature. Applications in this field include work on Value at Risk (Taylor 1999; Chernozhukov and Umansev 2001; Engle and Manganelli 2004), option pricing (Morillo 2000), and the analysis of the cross section of stock market returns (Barnes and Hughes, 2002), return distributions (Allen et al. 2013), mutual fund investment styles

([Bassett Jr and Chen 2002](#)), the investigation of hedge fund strategies ([Meligkotsidou et al. 2009](#)), the return-volume relationship in the stock market ([Chuang et al. 2009](#)), and the diversification and firm performance relationship ([Lee and Li 2012](#)).

The quantile regression takes the following form

$$y_i = x'_i b^\tau + \varepsilon_i^\tau \quad (14)$$

where y_i is the dependent variable of interest and x_i the vector of predictor variables. The parameter vector b^τ is associated with the τ -quantile while ε_i^τ is the error term, allowed to have a different distribution across quantiles. Note that the local effect of x_i on the τ -quantile is assumed to be linear. The slope coefficient vector b^τ differs across quantiles and the estimator for b^τ is obtained from

$$\begin{aligned} \min \sum_{i:\varepsilon_i^\tau > 0} \tau \times |\varepsilon_i^\tau| + \sum_{i:\varepsilon_i^\tau < 0} (1 - \tau) \times |\varepsilon_i^\tau| \\ = \sum_{i:y_i - x'_i b^\tau \geq 0} \tau \times |y_i - x'_i b^\tau| + \sum_{i:y_i - x'_i b^\tau < 0} (1 - \tau) \times |y_i - x'_i b^\tau| \end{aligned} \quad (15)$$

The quantile function is estimated by minimizing a weighted sum of absolute residuals, where the weights are functions of the quantiles of interest. The coefficient estimates are computed by using linear programming methods (for more details, see [Koenker \(2005\)](#)). For $\tau = 0.5$, i.e., the conditional median of x , the problem collapses to the (well known) Least Absolute Deviation (LAD) estimation. The value of b is obtained using linear programming algorithms and standard errors via bootstrapping techniques. We conduct the minimization procedure at quantiles of $\tau = 0.05, 0.25, 0.50, 0.75, 0.95$.

C. Quantile Regression Analysis

As a benchmark regression, we first explore what OLS regression has to say about the relationships of three beta across Japanese banks. Table 2 presents the results from OLS regressions to explain the cross-sectional and time series variation in the standard betas as a function of the variation in the two other betas, diffusion and jump betas. Model (1) in Table 2 shows that the diffusion beta exhibits the highest explanatory power for standard beta, with an average adjusted R-squared of 0.64. To provide an impression on the contribution of jump betas, we include model (2). The jump beta explains 48% variation in standard beta. When we add the diffusion beta and jump beta as in model (3), we see that altogether, 80 % of the variation in standard beta may be accounted for by the component betas, with diffusion beta having by far largest and most significant effect. The OLS regression results are consistent with Figures 1 and 2.

Table 2: The relationship between Standard, Diffusion and Jump betas across Japanese Banks

This table presents the pooled OLS regression results between Standard beta, Diffusion beta and Jump beta across different banks. **Standard errors** are displayed in parentheses below the **coefficients**. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Dependent Variable=Standard Beta	Models		
	(1)	(2)	(3)
Diffusion Beta	0.874*** (0.029)		0.678*** (0.027)
Jump Beta		0.362*** (0.022)	0.229*** (0.011)
Constant	0.257*** (0.013)	0.164*** (0.015)	0.107*** (0.008)
R-squared	0.64	0.48	0.80

The OLS estimator focuses only on the central tendency of distributions. We implement QR analysis to investigate how the standard, diffusion and jump betas are inter-related at their various quantiles.⁷ The quantile regression procedures yield a series of quantile coefficients, one for each sample quantile. We test whether standard beta responds differently to changes in the independent variables depending on whether the beta is in the left tail of distribution

⁷ We proceed to examine the relationship between standard beta, diffusion beta and jump beta across Japanese bank using the following quantile regression model:

$$Q(\tau)_{\beta^s}(\beta_{i,t}^s) = a_0(\tau) + b_1(\tau)\beta_{i,t}^c + b_2(\tau)\beta_{i,t}^j + \varepsilon_{i,t}$$

The variable of primary interest is the coefficient of diffusion and jump betas on the standard betas. The slopes of the regressors are estimated at five different quantiles τ –the 5th, 25th, 50th, 75th, 95th– using the same set of explanatory variables for each quantile.

(low risk bank) or in the right tail of the distribution (high risk bank). In Table 3, we present the parameter estimates for selected quantiles ranging from 0.05 to 0.95. The relationship between standard, diffusion and jump betas changes in magnitude across the distribution quantiles. For example, while the response rates for diffusion beta and jump beta at the 5th quantile are, respectively, 0.55 and 0.16, at the median they are 0.71 and 0.28, and at the 95th quantile they are 0.68 and 0.22. All coefficients are strongly statistically different from zero. Additionally, our results show that the conditional mean approach is also misleading in terms of goodness-of-fit.

Table 4 presents the results for F-tests of the null hypothesis of equal slopes across quantiles to formally test whether the slopes of explanatory variables change across quantiles where a bootstrap procedure was extended to construct a joint distribution across pairs of quantiles ([Chuang et al. 2009](#)). These results indicate that the coefficients are significantly different from each other between all quintiles. Further, we observe that there are significant differences between the coefficient of 5th quantile and 95th quantile, supporting the notion that at low and high of standard betas within the Japanese banking sector the relationships between standard, diffusion and jumps betas differ significantly. Our results indicate that the relationship may be far more complex (i.e. non-linear) than can be described using least-squares regression. The relationships between standard betas, diffusion betas and jump betas for Japanese banking stock are non-linear across quantiles and the relationships at tail quantiles are quite different from those at central quantiles.

Table 3: The relationship between Standard beta, Diffusion beta and Jump beta different quantiles

This table presents the regression results between Standard beta, Diffusion beta and Jump beta across different quantiles. **Standard errors** are displayed in parentheses below the **coefficients**. Standard errors are obtained by bootstrapping with 100 replications. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Dependent Variable= Standard Beta	Standard Beta				
	5th quant	25th quant	50th quant	75th quant	95th quant
Diffusion Beta	0.555*** (0.025)	0.689*** (0.012)	0.709*** (0.010)	0.684*** (0.012)	0.677*** (0.028)
Jump Beta	0.157*** (0.006)	0.245*** (0.004)	0.281*** (0.006)	0.291*** (0.010)	0.222*** (0.018)
Constant	-7.77e-16 (0.003)	-3.28e-15 (0.000)	0.0410*** (0.005)	0.120*** (0.008)	0.376*** (0.018)
Pseudo R-squared	0.48	0.58	0.61	0.60	0.53

Table 4: Post estimation linear hypothesis testing.

The table presents F-test for testing whether coefficients between different the quintiles are equal. Quantiles were estimated by simultaneous regression analysis. Standard errors were obtained by bootstrapping with 100 replications. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Panel A:

H0: Test whether Diffusion beta and Jump beta coefficients are equal across different quantiles	
H0: Q5=Q25	F(2, 5401) =214.87*** Prob > F = 0.0000
H0: Q25=Q50	F(2, 5401) = 48.98*** Prob > F = 0.0000
H0: Q50=Q75	F(2, 5401) = 3.18** Prob > F = 0.0417
H0: Q75=Q95	F(2, 5401) = 10.92*** Prob > F = 0.0000
H0: Q05=Q95	F(2, 5401) = 22.42*** Prob > F = 0.0000
H0: Q25=Q75	F(2, 5401) = 23.40*** Prob > F = 0.0000

Panel B:

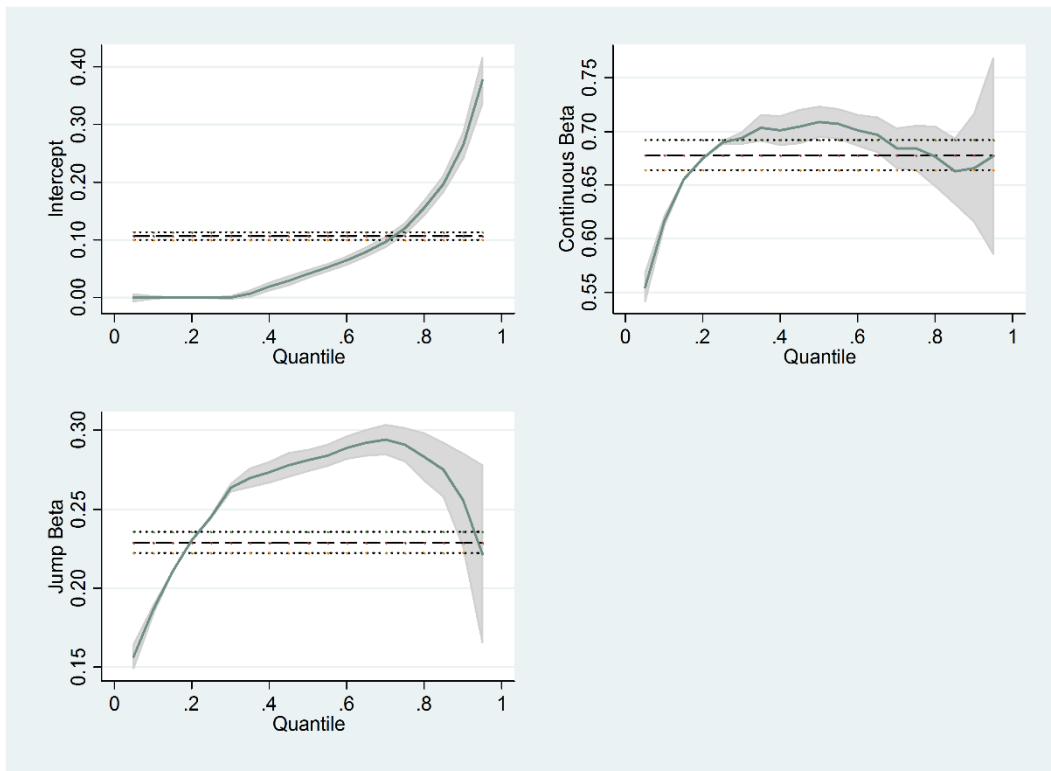
H0: Test whether Diffusion beta and Jump beta coefficients are equal across different quantiles	
H0: Q5=Q05	F(2, 5401) =1536.52*** Prob > F = 0.0000
H0: Q25=Q25	F(2, 5401) = 2907.22*** Prob > F = 0.0000
H0: Q50=Q50	F(2, 5401) = 4544.71*** Prob > F = 0.0000
H0: Q75=Q75	F(2, 5401) = 3309.56*** Prob > F = 0.0000
H0: Q95=Q95	F(2, 5401) = 541.05*** Prob > F = 0.0000

Figure 3 shows how the beta values vary across quantiles, depicting the point estimates of the slope of explanatory variable and 95% pointwise confidence band. If assumptions for the standard linear regression model hold, the quantile slope estimates should trace a constant and horizontal line across the quantiles, with only the intercept parameters systematically increasing with τ . However, none of the slope estimates shown could be described as constant and horizontal. In fact, the quantile slope estimates of the variables such as diffusion beta and jump beta followed a non-linear pattern with low values in the left tail and high values in the right tail. It is apparent that the slope of regression changes across the quantiles and is clearly not constant, as presumed in OLS. The results indicate that on average the jump betas for a quantile are higher than the corresponding diffusion betas. However, companies with low quantile standard betas are less sensitive to market jumps in comparison to companies with high quantiles standard betas.

Figure 4 shows the scatter plots of the monthly standard betas versus diffusion betas and monthly standard beta versus jump betas for each quantile. The scatter plot in panel A of figure 4 suggests heteroskedasticity in the dataset, given that the dispersion of results seems somewhat narrower at the higher tail end of the distribution. The best-fit lines for the 5th and 95th quantiles shown in the panel A indicate that for higher diffusion betas at the tails the dependency relationships of the standard betas with the diffusion betas are very similar i.e. they have similar gradients. For the intermediate quantiles (25th, 50th and 75th quantiles) the best-fit lines tends to converge at higher diffusion betas, indicating the relationship between standard beta and diffusion beta is dissimilar at low diffusion betas. In the case of jump betas (see Figure 4, panel B), the gap between the 5th and 95th quantiles is higher on the right side of the graph; in other words, among those firms- the jump betas for the 5th and 95th quantiles are diverging. Thus, at the tails, the diffusion betas and jump betas behave differently from each other.

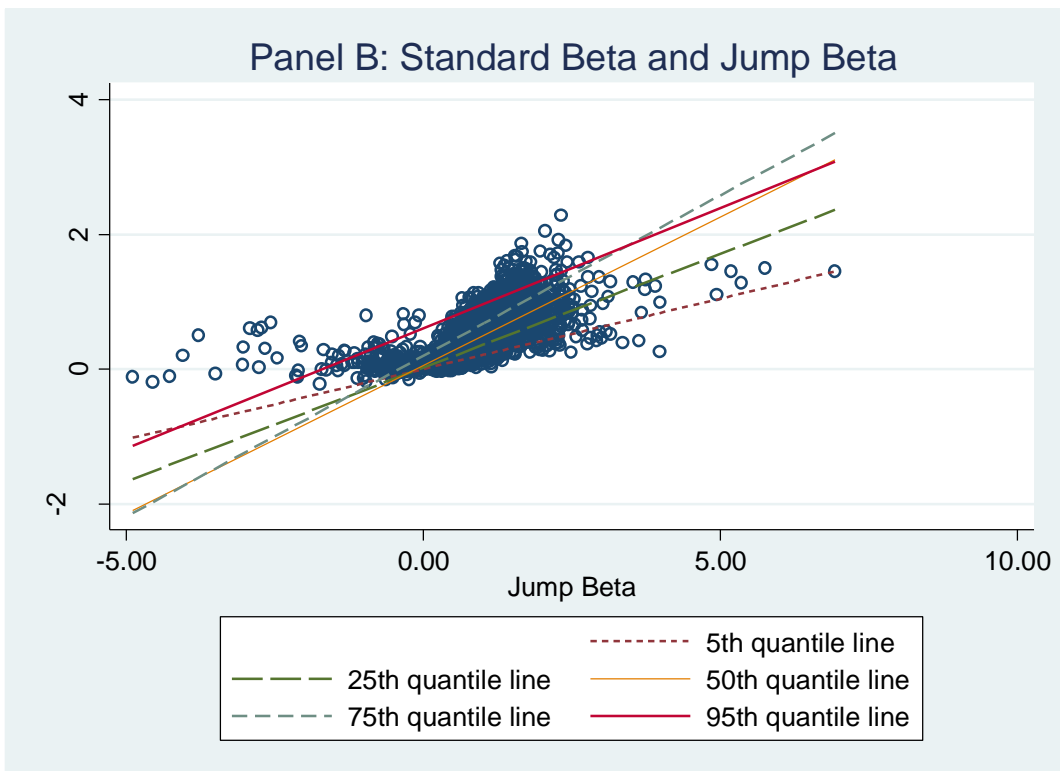
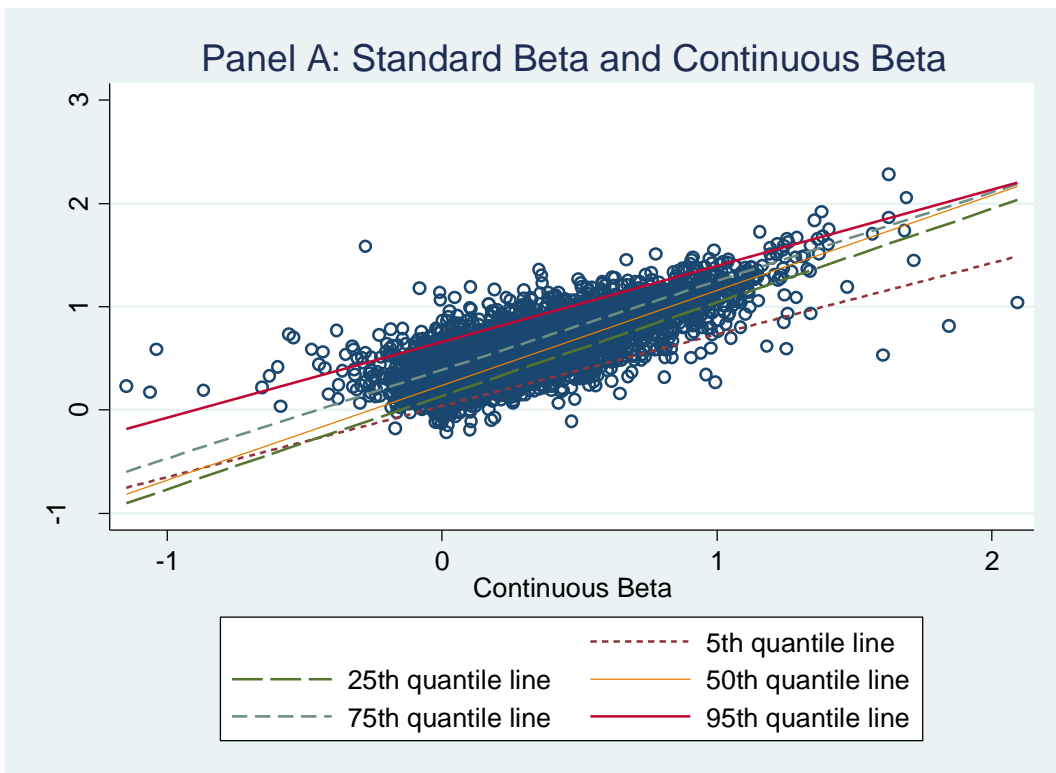
Figure 3: Quantile plot of estimated slopes and 95% confidence interval

The solid line gives the coefficients of diffusion beta estimates from the quintile regression, with the shaded grey area depicting a 95% confidence interval. The dashed line gives the OLS estimate of mean effect, with two dotted lines again representing a 95% confidence interval for this coefficient.



The general conclusion that can be drawn is that there exists a wide disparity in behaviour between high risk firms and low risk firms when receiving diffusion and jump shocks.

Figure 4: Scatterplot of Betas across different quantiles



D. Size-sorted portfolios

To control for possible size effects, we test the relationship between standard beta, diffusion beta and jump beta using five subsample portfolios constructed by sorting the data with respect to size. The banks are grouped into five benchmark portfolios ranked by size and based on market capitalization at the end of each year t . Tables 5 and 6 report the results for portfolios sorted on stock size and rebalanced each year. Portfolio 1 includes the smallest banks in the group and portfolio 5 includes largest banks in the sample. Table 5 shows a clear size effect for the OLS-estimated coefficients. The diffusion beta coefficients are all statistically significant with higher loadings for the smaller portfolios. For jump betas, all the corresponding coefficients are also statistically significant but now the larger portfolios have the higher loadings. Irrespective of the relative loadings, however, the diffusion coefficient is consistently larger than the jump beta coefficient for each of the five size-sorted portfolios.

Table 5: The OLS relationship between Standard beta, Diffusion beta and Jump beta across for size-sorted stock portfolios

This table presents the pooled OLS regression results between Standard beta, Diffusion beta and Jump beta across different banks. **Standard errors** are displayed in parentheses below the **coefficients**. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively

Dependent Variable= Standard Beta	Standard Beta				
	Small	2	3	4	Large
Diffusion Beta	0.600*** (0.043)	0.685*** (0.066)	0.740*** (0.047)	0.469*** (0.063)	0.573*** (0.026)
Jump Beta	0.192*** (0.015)	0.199*** (0.018)	0.215*** (0.021)	0.271*** (0.020)	0.203*** (0.016)
Constant	0.103*** (0.013)	0.112*** (0.012)	0.107*** (0.013)	0.161*** (0.026)	0.242*** (0.019)
R-squared	0.67	0.71	0.79	0.70	0.73

We apply a quantile regression methodology to the size-sorted portfolios and report in Table 6 the quantile relationships between the three betas. We obtain results similar to those reported in Table 3 with one noticeable difference. For the largest portfolio in Table 6, the quantile regression lines converge, whereas as for the remaining portfolios the quantile regression lines diverge as is the case in Table 3.

For small bank portfolios the diffusion components are loaded relatively higher than jump components. For the large bank portfolios, however, the two components are loaded more evenly. These findings lead us to conclude not only that smaller bank portfolios have lower jump betas relative to the larger bank portfolios, but that the effective contributions of jump betas to standard betas also differ significantly across size sorted portfolios.

Table 6: The Quantile relationship between Standard beta, Diffusion beta and Jump beta for size-sorted stock portfolios

This table presents the regression results between Standard beta, Diffusion beta and Jump beta across different quantiles. **Standard errors** are displayed in parentheses below the **coefficients**. Standard errors are obtained by bootstrapping with 100 replications. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Dependent Variable= Standard Beta					
	5th quant	25th quant	50th quant	75th quant	95th quant
	Small				
Diffusion Beta	0.329*** (0.049)	0.510*** (0.042)	0.632*** (0.049)	0.638*** (0.032)	0.665*** (0.052)
Jump Beta	0.170*** (0.012)	0.195*** (0.010)	0.224*** (0.013)	0.225*** (0.019)	0.185*** (0.036)
Constant	-0.019** (0.009)	0.029*** (0.006)	0.064*** (0.009)	0.133*** (0.015)	0.308*** (0.036)
Pesudo R-squared	0.39	0.40	0.42	0.44	0.45
	2				
Diffusion Beta	0.498*** (0.048)	0.624*** (0.053)	0.638*** (0.042)	0.717*** (0.035)	0.656*** (0.044)
Jump Beta	0.142*** (0.015)	0.205*** (0.012)	0.248*** (0.010)	0.259*** (0.019)	0.192*** (0.032)
Constant	0.006 (0.005)	0.030*** (0.005)	0.063*** (0.006)	0.122*** (0.014)	0.354*** (0.029)
Pesudo R-squared	0.40	0.47	0.50	0.49	0.47
	3				
Diffusion Beta	0.558*** (0.064)	0.690*** (0.045)	0.762*** (0.031)	0.736*** (0.030)	0.760*** (0.047)
Jump Beta	0.157*** (0.017)	0.223*** (0.017)	0.244*** (0.012)	0.262*** (0.020)	0.209*** (0.026)
Constant	-0.006 (0.008)	0.022** (0.011)	0.071*** (0.008)	0.141*** (0.015)	0.357*** (0.021)
Pesudo R-squared	0.45	0.51	0.56	0.57	0.56
	4				
Diffusion Beta	0.265*** (0.060)	0.463*** (0.040)	0.540*** (0.027)	0.524*** (0.026)	0.512*** (0.053)
Jump Beta	0.213*** (0.025)	0.276*** (0.016)	0.316*** (0.016)	0.339*** (0.026)	0.292*** (0.034)
Constant	0.040*** (0.014)	0.058*** (0.011)	0.081*** (0.013)	0.163*** (0.024)	0.397*** (0.041)
Pseudo R-squared	0.38	0.47	0.48	0.49	0.49
	Large				
Diffusion Beta	0.630*** (0.043)	0.591*** (0.033)	0.590*** (0.022)	0.570*** (0.031)	0.540*** (0.048)
Jump Beta	0.170*** (0.027)	0.257*** (0.028)	0.260*** (0.021)	0.225*** (0.024)	0.170*** (0.025)
Constant	-3.33e-16 (0.002)	0.081** (0.033)	0.158*** (0.021)	0.296*** (0.028)	0.563*** (0.038)
Pseudo R-squared	0.54	0.52	0.50	0.48	0.45

E. Size Effect and the Betas

The evidence for the effect of size on bank systematic risk is mixed; whilst [Demsetz and Strahan \(1997\)](#) find that large banks tend to diversify their business more efficiently and are less prone to bankruptcy, [Saunders et al. \(1990\)](#) and [Anderson and Fraser \(2000\)](#) find that bank systematic risk increases with bank size as large banks could be more sensitive to general market movements than small banks. We test if the time varying betas are related to the market capitalisation or size of the portfolios and over non-crisis and crisis periods. Table 7 presents the mean and standard deviations of the standard, diffusion and jump betas for small and large portfolios. We report the t-statistics for the test of the hypothesis that there is no difference in the beta averages and ratios between small and large portfolios. In all cases we obtain reject the null hypothesis and conclude that larger banks are more sensitive to market movements than the smaller banks, regardless of whether they occur through a jump or not.

Although the betas of large bank portfolios are larger than the small bank portfolios, the jump-diffusion beta ratios between the two portfolios do not differ significantly. However, there is one exception. During the crisis period there is a statistically significant difference in the jump-diffusion beta ratios, with smaller portfolios exhibiting relatively larger jump beta increases compared with the corresponding diffusion beta increases. This is corroborated by the larger magnitudes of the estimated intercepts for large portfolios than small portfolios (see Tables 5 and 6). Small portfolio equities are more sensitive to large surprises than the large portfolio equities during times of crisis. An explanation for this phenomenon is that small bank equities are riskier than large bank equities because less information is available about the former than about the latter. Consequently, small bank portfolios react more severely to surprises than do the large bank portfolios. [Reinganum and Smith \(1983\)](#) have pointed out that for the differential information explanation to hold, the additional risk caused by the relative lack of information must not be idiosyncratic. That is, the lack of information must be a source of systematic risk that cannot be diversified away.

Table 7: Betas in large and small equity portfolios

The beta statistics include mean and standard deviations (in parentheses) are summarized by the full sample periods and three sub-periods. We report the betas for two size-sorted equity portfolios (large size equity beta portfolio, and small size equity beta portfolio). We include the pooled variance t-test of the difference between the two sample means for the Standard Beta, Diffusion Beta and Jump Beta and also the size-sorted equity portfolio. The t-statistics are given in parentheses. * denotes significance at 10 % level; ** denotes significance at 5 % level, and *** denotes significance at 1 % level

	Large equity portfolio				Small equity portfolio			
	Std Beta	Dfu Beta	Jump Beta	Jmp-Dfu Beta Ratio	Std Beta	Dfu Beta	Jmp Beta	Jmp-Dfu Beta Ratio
Full-sample Period								
Mean	0.814	0.576	1.165	6.355	0.290	0.099	0.707	5.718
Std.Dev	0.282	0.319	0.630		0.203	0.173	0.595	
t-test of difference	-48.466***	-40.478***	- 16.927 ***	- 0.0793				
Pre-crisis Period								
Mean	0.720	0.528	1.080	8.032	0.159	0.036	0.443	8.718
Std.Dev	0.252	0.300	0.508		0.109	0.101	0.513	
t-test of difference	- 39.583 ***	- 28.828 ***	- 18.501 ***	0.040				
Crisis Period								
Mean	0.988	0.752	1.251	1.796	0.438	0.226	0.868	5.724
Std.Dev	0.266	0.254	0.955		0.230	0.211	0.712	
t-test of difference	- 23.610***	- 24.0423***	- 4.856 ***	1.956 **				
Post-crisis Period								
Mean	0.888	0.527	1.306	6.525	0.316	0.073	0.830	2.904
Std.Dev	0.267	0.363	0.449		0.181	0.152	0.516	
t-test of difference	- 32.545 ***	- 20.664 ***	- 11.607 ***	- 0.460				

IV. Difference between the Jump-Diffusion Model and the CAPM

Theoretically the jump-diffusion APM is related to the CAPM. Based on the CAPM single index model, stock returns can be formulated as follows⁸:

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t} \quad (16)$$

The covariance of a pair of assets, 1 and 2, can then be defined as:

$$Cov(r_1, r_2) = Cov(\alpha_1 + \beta_1 r_m + \varepsilon_1, \alpha_2 + \beta_2 r_m + \varepsilon_2) \quad (17)$$

where α is constant and we assume

$$Cov(r_m, \varepsilon_1) = Cov(r_m, \varepsilon_2) = Cov(\varepsilon_1, \varepsilon_2) = 0 \quad (18)$$

Equation (17) then becomes

$$Cov(r_1, r_2) = Cov(\beta_1 r_m, \beta_2 r_m) \quad (19)$$

Since β_1 and β_2 are both constants, Equation (19) then becomes

$$Cov(r_1, r_2) = \beta_1 \beta_2 Cov(r_m, r_m) = \beta_1 \beta_2 \sigma_m^2 \quad (20)$$

Using the return decomposition argument in this paper, the market returns can be broken into a diffusion returns component and a jump returns component i.e.

$$r_{m,t} = r_{m,t}^c + r_{m,t}^j \quad (21)$$

Thus the single index model can be rewritten as

$$r_{i,t} = \alpha_i + \beta_i (r_{m,t}^c + r_{m,t}^j) + \varepsilon_{i,t} \quad (22)$$

However, Equation (22) assumes that $\beta_i^c = \beta_i^j = \beta_i$.

For the general case where $\beta_i^c \neq \beta_i^j$, then the above relationship can be further rewritten as

$$r_{i,t} = \alpha_i + \beta_i^c r_{m,t}^c + \beta_i^j r_{m,t}^j + \varepsilon_{i,t} \quad (23)$$

The covariance of two assets 1 and 2, is then

$$Cov(r_1, r_2) = Cov(\alpha_1 + \beta_1^c r_m^c + \beta_1^j r_m^j + \varepsilon_1, \alpha_2 + \beta_2^c r_m^c + \beta_2^j r_m^j + \varepsilon_2) \quad (24)$$

We correspondingly drop the constant α and the error term ε in Equation (24) and only keep the remaining variables. Therefore,

$$Cov(r_1, r_2) = Cov(\beta_1^c r_m^c + \beta_1^j r_m^j, \beta_2^c r_m^c + \beta_2^j r_m^j) \quad (25)$$

⁸ The proof as shown is referenced from [Liu \(2014\)](#).

$$\begin{aligned} Cov(r_1, r_2) = & Cov(\beta_1^c r_m^c, \beta_2^c r_m^c) + Cov(\beta_1^j r_m^j, \beta_2^c r_m^c) + Cov(\beta_1^c r_m^c, \beta_2^j r_m^j) \\ & + Cov(\beta_1^j r_m^j, \beta_2^j r_m^j) \end{aligned} \quad (26)$$

$$\begin{aligned} Cov(r_1, r_2) = & \beta_1^c \beta_2^c Cov(r_m^c, r_m^c) + \beta_1^j \beta_2^c Cov(r_m^j, r_m^c) + \beta_1^c \beta_2^j Cov(r_m^c, r_m^j) \\ & + \beta_1^j \beta_2^j Cov(r_m^j, r_m^j) \end{aligned} \quad (27)$$

The second and third terms of Equation (27) are zero as the diffusion returns and jump returns are uncorrelated. Therefore,

$$Cov(r_1, r_2) = \beta_1^c \beta_2^c Cov(r_m^c, r_m^c) + \beta_1^j \beta_2^j Cov(r_m^j, r_m^j) \quad (28)$$

$$Cov(r_1, r_2) = \beta_1^c \beta_2^c \sigma_m^2(c) + \beta_1^j \beta_2^j \sigma_m^2(j) \quad (29)$$

The standard CAPM beta is defined as

$$\beta_{i,t} = \frac{Cov(r_1, r_2)}{\sigma_m^2} \quad (30)$$

Using Equations (29) and (30), we can rewrite the Equation (30)

$$\beta_{i,t} = \frac{\beta_1^c \beta_2^c \sigma_m^2(c) + \beta_1^j \beta_2^j \sigma_m^2(j)}{\sigma_m^2} = \frac{\beta_1^c \sigma_m^2(c) + \beta_2^j \sigma_m^2(j)}{\sigma_m^2}; \text{ where } \beta_m^c = \beta_m^j = 1.0 \text{ by definition.}$$

Consequently, the standard beta is the weighted average of diffusion and jump betas

$$\beta_{i,t} = \frac{\sigma_{m,t}^2(c)}{\sigma_{m,t}^2} \beta_{i,t}^c + \frac{\sigma_{m,t}^2(j)}{\sigma_{m,t}^2} \beta_{i,t}^j \quad (31)$$

Equation (31) implies that the standard beta (in the conventional CAPM) is the weighted average of the jump beta and diffusion beta (in the jump diffusion asset pricing model). This hypothesis can be tested empirically using following regression equation:

$$\beta_{i,t}^S = c_0 + c_1 \beta_{i,t}^c + c_2 \beta_{i,t}^j + \varepsilon_{i,t} \quad (32)$$

The testable hypotheses are:

$$c_1 + c_2 = 1 \quad (33)$$

$$c_0 = 0 \quad (34)$$

In Table 8 (Full Sample Analysis), we report the F-tests of whether the systematic risk is a weighted average of diffusion and jump betas. For bank stocks, Panel A shows that the hypothesis is not rejected in either the OLS regression with No Constant (NC) case or Quantile regression at the median (50th quantile). For bank portfolios, Panel B shows more mixed results. The results for the three sub-periods are shown in Table 9. In the crisis period the bank stocks, Panel A, do not reject both hypotheses in OLS regression with No Constant (NC) case and Quantile regression at the median (50th quantile). For all other periods, the results are mixed for both bank stocks (Panel A) and bank portfolios (Panel B).

These results suggest that during crisis periods, there is a higher decoupling of the market returns. That is, the diffusion returns and jump returns are independent from each other and consequently, the diffusion and jump betas are also not independent. During other periods, these component betas are seemingly correlated resulting in the mixed rejections of the two hypotheses. We conclude that under market conditions where the component market returns are not strongly correlated, such as crisis periods, the hypothesis that the standard beta or systematic risk on an asset is the weighted average of the diffusion and jump betas, that is both the diffusion and jump (market) risks, is not rejected.

Table 8: Testing Distinction between the Jump-Diffusion Model and the CAPM: Full sample Analysis

The table presents F-test for testing whether the beta in the conventional CAPM is the weighted average of the jump beta and diffusion beta in the jump-Diffusion model. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively. NC: No Constant.

H0: Test whether the beta in conventional CAPM is the average of diffusion beta and jump beta in the jump-diffusion model							
H0: C1+C2=1							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	18.63***	0.53	172.21***	47.85***	1.71	8.5**	18.07***
P-value	0.000	0.471	0.000	0.000	0.192	0.004	0.000
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	17.77***	3.60*	138.93***	31.43***	8.36***	24.04***	10.37***
	0.001	0.077	0.000	0.000	0.003	0.000	0.004
2	3.69	0.83	69.53***	13.77***	10.73***	1.01	23.56***
	0.065	0.370	0.000	0.001	0.001	0.316	0.000
3	1.55	4.46*	27.79***	8.76***	0.04	0.00	0.92
	0.225	0.044	0.000	0.003	0.838	0.962	0.337
4	14.80***	3.66*	86.87***	57.37***	25.95***	29.60***	9.52***
	0.001	0.066	0.000	0.000	0.000	0.000	0.002
Large	114.88***	2.62	77.04***	16.38***	44.28***	71.64***	35.82***
	0.000	0.123	0.000	0.000	0.000	0.000	0.000
H0: C0=0							
Panel A: Individual Stocks							
	OLS	OLS(NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	179.51***	-	0.00	0.00	71.25	222.96***	452.40***
P-value	0.000		1.000	1.000	0.192	0.000	0.000
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	66.79***		4.66***	20.66***	49.32***	128.25***	71.21***
	0.000		0.031	0.000	0.000	0.000	0.000
2	82.22***		1.71	30.48***	102.66***	67.12***	112.18***
	0.000		0.192	0.000	0.000	0.000	0.000
3	70.15***		0.62	4.59**	98.27***	120.23***	262.67***
	0.000		0.430	0.032	0.000	0.000	0.000
4	37.38***		5.89***	24.16***	39.23***	40.63***	98.42***
	0.000		0.015	0.000	0.000	0.000	0.000
Large	156.58***		0.00	5.19***	44.45***	121.15***	219.35***
	0.000		1.000	0.023	0.000	0.000	0.000

Table 9: Testing Distinction between the Jump-Diffusion Model and the CAPM: Sub Sample Analysis

The table presents F-test for testing whether the beta in the conventional CAPM is the weighted average of the jump beta and diffusion beta in the jump-Diffusion model as in Table 2, but for different subsamples. The asterisks *, **, and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

H0: Test whether the beta in conventional CAPM is the average of diffusion beta and jump beta in the jump-diffusion model							
H0: C1+C2=1							
Pre-crisis Period							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	53.86***	13.34***	305.47***	165.09***	47.27***	39.94***	70.04***
P-value	0.000	0.000	0.000	0.000	0.192	0.004	0.000
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	59.42***	20.93***	138.93***	80.99***	40.61***	30.12***	24.08***
	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	92.60***	54.06***	47.02***	92.98***	36.87***	66.93***	13.43***
	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	32.11***	9.04***	101.59***	130.22***	151.99***	36.10***	19.33***
	0.000	0.006	0.000	0.000	0.000	0.000	0.000
4	98.64***	40.39***	109.12***	163.20***	81.87***	58.61***	30.64***
	0.000	0.000	0.000	0.000	0.000	0.000	0.002
Large	46.48***	3.12	53.42***	56.66***	29.53***	33.74***	116.99***
	0.000	0.105	0.000	0.000	0.000	0.000	0.000
Crisis Period							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	0.04	0.58	0.47	0.28	0.21	0.02	1.31
P-value	0.839	0.495	0.495	0.594	0.646	0.895	0.253
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	11.90***	4.08*	19.45***	2.24	2.37	1.99	1.55
	0.004	0.065	0.000	0.136	0.125	0.159	0.214
2	0.27	22.06***	0.91	0.96	5.76**	2.88*	4.85**
	0.610	0.000	0.341	0.328	0.017	0.091	0.029
3	7.00**	73.12***	3.77	40.09***	5.25**	16.92***	0.68
	0.016	0.000	0.532	0.000	0.022	0.000	0.410
4	9.83***	4.50*	2.55	5.02**	13.61***	2.95*	1.44***
	0.006	0.072	0.112	0.026	0.000	0.087	0.232
Large	5.35**	93.00***	3.54**	8.83***	3.57*	1.36	0.50

	0.033	0.000	0.061	0.003	0.060	0.244	0.481
Post-crisis Period							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	11.95***	11.95	104.93***	51.92***	22.11***	16.36***	22.15***
P-value	0.001	0.001	0.000	0.000	0.000	0.000	0.000
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	29.80***	8.79***	51.27***	113.13***	33.82***	25.46***	6.52***
	0.000	0.010	0.000	0.000	0.000	0.000	0.011
2	5.88**	0.13	6.19***	10.38***	14.25***	15.41***	6.99***
	0.027	0.718	0.013	0.001	0.000	0.000	0.009
3	4.18*	1.12	4.34**	3.86**	0.30	2.14	1.95
	0.062	0.309	0.039	0.051	0.586	0.146	0.164
4	40.18***	10.87***	164.27***	58.70***	73.62***	42.23***	42.23***
	0.000	0.005	0.000	0.000	0.000	0.000	0.000
Large	107.63***	15.72***	23.19***	27.62***	36.79***	24.69***	14.51***
	0.000	0.003	0.000	0.000	0.000	0.000	0.000
H0: C0=0							
Pre-crisis Period							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	68.86***	-	0.23	0.23	41.00***	223.30***	367.52***
P-value	0.000		0.631	0.631	0.000	0.000	0.000
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	59.42***		2.75*	37.74***	255.64***	147.74***	61.21***
	0.000		0.098	0.000	0.000	0.000	0.000
2	92.60***		0.77	40.73***	81.15***	264.92***	65.75***
	0.000		0.379	0.000	0.000	0.000	0.000
3	32.11***		0.00	16.17***	183.65***	112.57***	130.15***
	0.000		1.000	0.000	0.000	0.000	0.000
4	98.64***		0.18	40.12***	76.57***	76.57***	72.28***
	0.000		0.670	0.000	0.000	0.000	0.000
Large	46.48***		0.25	0.00	17.76***	32.75***	286.52***
	0.000		0.615	1.000	0.000	0.000	0.000
Crisis Period							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	0.47	-	0.50	45.57***	63.54***	103.73***	540.93***

P-value	0.495	0.48	0.000	0.000	0.0000	0.0000	
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	11.90***		3.46*	4.15**	7.06***	28.87***	71.82***
	0.004		0.064	0.042	0.008	0.000	0.000
2	0.27		0.48	5.70	22.79***	12.25***	68.04***
	0.610		0.488	0.181	0.000	0.000	0.000
3	7.00**		0.33	0.46	12.23***	28.73***	66.80***
	0.016		0.569	0.496	0.001	0.000	0.000
4	9.83***		6.21***	11.62**	26.15***	7.65***	12.86***
	0.006		0.013	0.001	0.000	0.006	0.000
Large	5.35**		8.31***	39.08***	69.10***	40.62***	78.48***
	0.033		0.004	0.000	0.000	0.000	0.000
Post-crisis Period							
Panel A: Individual Stocks							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
F-stat	44.98***	-	1.47	7.77***	29.61***	52.77***	115.06***
P-value	0.000		0.226	0.005	0.000	0.000	0.000
Panel B: Portfolios							
	OLS	OLS (NC)	5th quant	25th quant	50th quant	75th quant	95th quant
Small	29.80***		0.88	11.66***	26.29***	43.82***	79.69***
	0.000		0.350	0.000	0.000	0.000	0.000
2	5.88**		0.10	9.70***	11.86***	24.27***	20.12***
	0.027		0.754	0.002	0.001	0.000	0.000
3	4.18*		2.14	4.24**	3.86**	50.15***	22.03
	0.062		0.145	0.041	0.051	0.000	0.000
4	40.18***		8.49***	12.25***	37.75***	30.85***	84.02***
	0.000		0.004	0.000	0.000	0.000	0.000
Large	107.63***		9.52***	16.42***	24.56***	23.80***	36.62***
	0.000		0.002	0.000	0.000	0.000	0.000

V. Conclusion

In this paper, we used high-frequency data and a novel method of decomposing a security's systematic risk into two components to estimate diffusion beta and the jump beta components. We empirically test for any relationship between the standard betas, diffusion betas and jump betas across a portfolio of Japanese banking stocks using quantile regression techniques to allow for non-linearity.

Using high-frequency data of the Japanese banks from 2001-2012, we find that the relationship between standard, diffusion and jump betas is different (i.e. non-linear) across quantiles. More precisely, we find that the standard beta, on average and as expected, is weighted more by the diffusion component than the jump component, though the actual magnitudes of the weights differ significantly across quantiles. The relationship holds for both bank stocks and bank portfolios.

Past empirical studies have shown that standard betas vary systematically across firm size. A close look at our results indicates that, on average, large banks have larger betas whereas small banks have smaller betas i.e. larger Japanese banks are more sensitive to both types of market movements than smaller institutions, regardless of whether these movements are continuous or jumps. However, in our study the smaller bank portfolios exhibit larger jump-diffusion beta ratios than the larger bank portfolios during times of crisis, suggesting that the jump betas are disproportionately larger than the corresponding diffusion betas in the small portfolios, indicating an additional size-cum-crisis effect. The results suggest that, during times of crisis, the jump-diffusion beta asymmetry could be more severe for smaller banks than larger banks in Japan.

Our findings also indicate that during crisis periods the diffusion returns and jump returns are independent and consequently, the diffusion and jump betas are also uncorrelated. During other period, these component betas are seemingly correlated leading to mixed rejections of the hypotheses around whether standard betas can be expressed as a weighted combination of diffusion and jump beta. Thus, we can say that under market conditions where the component market returns are not strongly correlated, such as crisis periods, the hypothesis that the standard beta or systematic risk on an asset is the weighted average of the diffusion and jump betas i.e. both the diffusion and jump (market) risks is not rejected.

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Appendix:

A1. Jump Test

We apply the nonparametric jump-detection methods proposed by [Barndorff-Nielsen and Shephard \(2006\)](#), hereafter, BNS, to detect jumps in the Nikkei 225 index. BNS propose two general measures based on realized power variations to test for jumps and to estimate the contribution of jumps to total variation- realized variance (RV) and bi-power variation (BV). The realized variance (RV) is defined as the sum of squared intraday-returns,

$$RV_t = \sum_{s=1}^n r_{t,s}^2, \quad t = 1, \dots, T \quad (A.1)$$

where n is the sampling total sample (usually daily/monthly) and $r_{t,s}$ is the intraday logarithmic return. Note that equation (A.1) uses only returns from within each trading day (intraday returns), discarding any overnight returns (intraday-returns). As a result, any jumps resulted from overnight returns are excluded from realized variance. When M goes to zero, realized variance converges to integrated variance plus the jumps ([Barndorff-Nielsen and Shephard 2004](#); [Andersen and Bollerslev 1998](#)). We can re-write this as:

$$RV_t \xrightarrow{p} \int_{t-1}^t \sigma_s^2 ds + \sum_{s=q_{t-1}}^{q_t} k_s^2, \quad t = 1, \dots, T \quad (A.2)$$

Where, $M =$ sampling frequency, σ_s^2 is the time-diffusion intergrade variance function and k_s^2 is the squared discrete jump term. It is clear that realized variance is not a robust measure of the variance σ_s^2 in the presence of jumps.

Therefore, to improve the robustness of variance estimation in the presence of jumps, BNS propose bi-power variation (BV)

$$BV_t = \mu_1^{-2} \frac{n}{n-1} \sum_{s=2}^n |r_{t,s}| |r_{t,s-1}|, \quad t = 1, \dots, T \quad (A.3)$$

where $\mu_1 = \sqrt{2/\pi}$. ([Barndorff-Nielsen and Shephard 2004](#)), show that BV consistently estimates the diffusion true or integrated variance (i.e. jump free) when the sampling frequency goes to zero. Intuitively, in the presence of any jump, one of the two consecutive returns is bound to be larger. The product of the smaller return and the larger returns, however, will be small and thus neutralize the effect of the jumps. Therefore,

$$BV_t \rightarrow \int_{t-1}^t \sigma_s^2 ds, \quad \text{for } M \rightarrow 0 \quad (A.4)$$

Combining equations (A.2) and (A.3), for $M \rightarrow 0$

$$RV_t - BV_t \rightarrow \sum_{s=q_{t-1}}^{q_t} k_s^2, \quad t = 1, \dots, T \quad (A.5)$$

Thus, the difference between the RV_t and BV_t consistently estimates the jump contribution to the total variation.

Following [Huang and Tauchen \(2005\)](#), we define the jump ratio statistic

$$RJ_t = \frac{RV_t - BV_t}{RV_t}, \quad (A.6)$$

which converges to a standard normal distribution when scaled by its asymptotic variance in the absence of jumps. That is

$$ZJ_t = \frac{RJ_t}{\sqrt{\left[\left(\frac{\pi}{2}\right)^2 + \pi - 5\right] \frac{1}{M} \max\left(1, \frac{DV_t}{BV_t^2}\right)}} \xrightarrow{d} N(0,1) \quad (A.7)$$

where DV_t is the quad-power variation robust to jumps as shows in [Barndorff-Nielsen and Shephard \(2004\)](#) and [Andersen et al. \(2007\)](#). The quad-power variation is defined as

$$DV_t = n\mu_1^{-4} \left(\frac{n}{n-3}\right) \sum_{s=4}^n |r_{t,s-3}| |r_{t,s-2}| |r_{t,s-1}| |r_{t,s}|, \quad t = 1, \dots, T \quad (A.8)$$

The ZJ_t statistic in equation (A.7) can be applied to test the null hypothesis that there is no jump in the return process during a trading day, t . [Huang and Tauchen \(2005\)](#) show that this test has very good size and power properties and is quite accurate for detecting jumps. Significant jumps are identified by the realizations of ZJ_t in excess of the 99.9% critical value ϕ_α .

$$J_{t,\alpha} = I[Z > \phi_\alpha] \cdot [RV_t - BV_t] \quad (A.9)$$

where I refers to the indicator function equal to one if a jump occurs and zero otherwise.