Aggregating Firm-Level Estimates of Business Investment Policies *

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Abstract

This paper proposes a simple approach to aggregate firm-level estimates of capital distortions. This approach takes general equilibrium effects into account, is consistent with a large class of models of firm dynamics, and does not require a structural estimation. We first lay out a standard dynamic general equilibrium model with heterogeneous firms subject to generic capital frictions: adjustment costs, taxes and financing constraints. A subset of the firms in the model receive an empirical treatment that changes the parameters governing these frictions. The econometrician observes the firm-level response to this treatment. The aggregation of this evidence consists of calculating macroeconomic outcomes following a generalization of this treatment to all firms in the economy. We derive simple aggregation formulas based on moments of the distribution of revenue to capital ratios and provide sufficient conditions for the validity of these formulas. These conditions are satisfied for a large class of standard macro-finance models. We investigate the robustness of this approach to changes in the aggregation framework.

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1 Introduction

Governments around the world have a wide range of policies to facilitate business investment. A burgeoning empirical literature seeks to evaluate the effectiveness of these policies using firm-level data and well-identified empirical settings. Some papers look at financial market liberalizations (see for instance Aghion et al. (2007), Bertrand et al. (2007), Larrain and Stumpner (2017)). Others analyze firm response to the availability of subsidized credit (e.g. Lelarge et al. (2010), Banerjee and Duflo (2014), Brown and Earle (2017)), or changes in bank lending behavior (Fraisse et al. (2017), Blattner et al. (2017)). Another set of papers analyses the effect of capital taxes or subsidies on firm investment and hiring (Yagan (2015), Zwick and Mahon (2015), Rauh (2006), Rotemberg (2017)). By comparing treated firms to a plausibly exogenous control group, these papers can quantify the relative effect of these policy interventions on *treated* firms. However, they remain silent on how these firm-level effects would aggregate were the intervention generalized to a broader set of firms. In this paper, we offer simple formulas to estimate such an aggregate counterfactual using firm-level evidence. This approach does not require the estimation of a structural model of firm behavior and, in particular, does not require that the empiricist precisely knows how the intervention affects firm-level distorsions, i.e. how it maps into the various parameters governing firm-level frictions.

Aggregating firm-level responses to these policies is a non-trivial exercise because it requires an explicit modeling of how firms and workers interact with one another. First, standard general equilibrium (GE) effects will typically dampen firm-level responses: for instance, if a growth-enhancing policy is extended to a larger set of firms, labor demand increases, which in turn raises the equilibrium wage and mitigates the initial direct effect. Second, extending the policy to a larger scale reallocates inputs across firms: as distortions are reduced, capital and labor flow from firms with low marginal productivity to firms with high marginal productivity, which leads to an increase in aggregate productivity.

To account for such equilibrium effects and map firm-level estimates into macro-economic outcomes, we proceed in three steps. First, we set up a general equilibrium (GE) model with heterogeneous firms who face stochastic productivity shocks and are subject to several forms of distortions: adjustment costs, taxes and financing frictions. We relate aggregate output and total factor productivity (TFP) to the economy-wide distribution of revenue to capital ratios. The distribution of the revenue to capital ratio, commonly called "capital wedge" in the misallocation literature, captures the extent of distorsions in the economy, i.e. the distance of firms' investment policies to a frictionless benchmark: a firm with a relatively high revenue to capital ratio is a firm that invest too little, because of adjustment costs, financing constraints or taxes. The distribution of revenue to capital ratios is not a deep structural parameter, in the sense that it depends on firms' histories and choices. But the effect of a given policy on this distribution can easily be estimated using standard datasets and a well-designed experimental setting.

In a second step, we introduce such an empirical setting into our model. We assume that a policy intervention targets a random subset of firms in the economy. This treatment affects a collection of the parameters governing firm-level frictions, although we do not need to specify which parameters in particular. Using firm-level data, we assume that an econometrician estimates the effect of such a treatment on the distribution of revenue to capital ratios (e.g., its mean and variance, or its covariance with firm-level productivity). A natural solution to the aggregation exercise we consider – computing the effect of a generalization of the policy to all firms in the economy – is then to simply plug in the estimated treatment effect of the policy on the distribution of revenue to capital ratios into the aggregation formulas described above. But this solution, adopted in one form or another by a few recent papers (Larrain and Stumpner (2017), Rotemberg (2017) and Blattner et al. (2017)) and which follows the insight of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), essentially takes the distribution of revenue to capital ratios as independent of the market equilibrium.

However, in principle, the distribution of these capital wedges can depend on the market equilibrium. For instance, with a higher wage in the labor market, firms are less profitable and hence more financially constrained, which may result in a different distribution of revenue to capital ratios. This scale-dependence is a significant threat to the methodology outlined above. To see this, consider a policy that relaxes financing constraints for a small subset of firms in the economy. The econometrician estimates a significant reduction in the mean and variance of revenue to capital ratio for treated firms, implying that the policy boosts firm-level efficiency and improves the allocation of capital within treated firms. These treatment effects are the firm-level estimates that one would like to plug in the aggregation formulas. However, as this policy is generalized to more firms in the economy, labor demand goes up, which leads to an increase in the equilibrium wage in the labor market. Since firms are on average more constrained, the firm-level statistics estimated in the economy where only a small number of firms are treated could not be valid anymore.[1] The bottom-line is that the revenue to capital ratios measured in a particular experiment

[1]for instance, reducing firm-level frictions may lead to a greater reduction in the average revenue to capital ratio when the equilibrium wage is higher

are only observed *conditional* on current equilibrium conditions, and may not be used in alternative counterfactuals.

A key contribution in this paper is to provide conditions under which the revenue to capital ratio is *independent* of general equilibrium conditions. This property allows us to safely use the estimated effect of the policy into the formulas for GE outcome. It relies on two key assumptions. First, the sources of distorsions (financing frictions, tax schedules, adjustment costs) are assumed to be homogeneous of degree 1. The intuition for this is that homogeneity guarantees that frictions remain on average constant on a size-adjusted basis. Hence, a change in general equilibrium, which affects firm size, will not affect distorsions. This assumption is only mildly restrictive and consistent with most models of firm dynamics used in macro-finance. The second assumption is the firm-level Cobb-Douglas production function, which is another common assumption in the literature.

The formulas we obtain for aggregate output and TFP combine parameters of the macroeconomic model (labor share, goods substitutability, labor supply elasticity) and three sufficient statistics for the joint distribution of log productivity and log revenue to capital. The first statistic is the effect of the treatment on average log wedge. It captures the extent to which the treatment affects the aggregate amount of savings available to firms. The second statistic is the treatment effect on the variance of log wedges. It measures how the treatment distorts the allocation of capital across firms. The final statistic is the treatment effect on the covariance of log wedges and log productivities. Intuitively, if the treatment reduces this covariance, it will make the productive firms relatively less distorted which is good for aggregate output.

We then consider a series of relevant extensions to the basic setup and show how our aggregation formulas extend to these different settings. First, our formulas can easily accomodate the situation where the aggregation exercise is *partial*, in the sense that in the counterfactual, only a larger subset of firms – instead of all firms – receive the treatment. This is a relevant extension since in many settings (e.g., small firms subsidies), the policy focuses by design on a subset of firms, so that the relevant aggregation should be done within this subset. Second, we allow for a more realistic market structure, where firms imperfectly compete within industries, which are allowed to be heterogeneous in terms of their labor shares, price elasticity of demand, and total output shares. Third, we allow for decreasing returns to scale in production. Finally, we provide formulas that make no parametric assumptions about the distribution of revenue to capital ratios.

Our paper is first and foremost of interest for the growing literature that empirically analyzes firm-level distorsions using experimental-like settings. Many of the papers cited above estimate the *firm-level* effect of policies promoting business investment but do not speak to how these policies would affect macroeconomic outcome were the policy extended to all firms in the economy. Our paper provides a simple framework to answer this question using similar identification strategies but focusing on a set of sufficient statistics typically not computed in these studies (mean log revenue to capital ratio, its variance, and its covariance with log productivity). Recent exceptions are Blattner et al. (2017), Rotemberg (2017) and Larrain and Stumpner (2017), who consider an aggregation framework somewhat similar to ours, but in which the distribution of revenue to capital ratios are assumed to be exogenous, and in particular independent of aggregate conditions. One of our contributions is to provide sufficient conditions under which endogenous capital wedges are in fact independent of the market equilibrium. Our paper is also related to the rising literature that seeks to bridge reduced form analysis and structural approach by isolating simple "sufficient statistics" that help measure aggregate outcomes out of simple firm or household-level statistics. For instance, Davila (2016) writes down a model of household borrowing with costs of bankrupcty. He derives the optimal bankruptcy exemption as a function of sufficient statistics that can be observed, in particular, the reduction of consumption measured among bankrupt households and the probability of defaulting. Also related to this paper is Baqaee and Farhi (2017), who compute sufficient statistics to evaluate the sensitivity of macroeconomic fluctuations to microeconomic shocks.

Section 2 lays out the economic model. Section 3 develops our methodology. Section 4 shows that the assumptions of Section 2, which are necessary to this result, are consistent with most of the literature on firm dynamics. Section 5 investigates the robustness of our formulas to various extensions to the basic set-up. The last Section concludes.

2 The Economic Model

2.1 Set-up

The economy is dynamic ($t = 0,\ 1, \dots, \infty$), but there is no aggregate uncertainty and the economy is assumed to be in steady state. We first consider a simple market structure and extend the analysis to include heterogenous industries in Section 5. At each date t, a continuum of monopolists produce imperfectly substitutable intermediate goods in quantity y_{it} at a price p_{it} (Dixit and Stiglitz (1977)). There is a perfectly competitive final good market, which aggregates intermediate output according to a CES technology:

$$Y = \left(\int_i y_{it}^{\theta} di \right)^{\frac{1}{\theta}}, \tag{1}$$

where we omit the t subscript for aggregate output because the economy is in steady state. We use the final good as the numeraire. Profit maximization in the final good market implies that the demand for product i is given by: $p_{it} = \left(\frac{Y}{y_{it}}\right)^{1-\theta}$ and $-\frac{1}{1-\theta}$ is the price elasticity of demand.

To produce, firms combine labor and capital according to a Cobb-Douglas production function: $y_{it} = e^{z_{it}} k_{it}^{\alpha} l_{it}^{1-\alpha}$, where k_{it} is firm i's capital stock in period t, l_{it} is the labor input in period t, α is the capital share and z_{it} is firm i's idiosyncratic productivity shock in period t. With monopolistic competition and the demand system in Equation (1), firm i revenue in period t is $p_{it} y_{it} = Y^{1-\theta} y_{it}^{\theta}$. We assume that there is no adjustment costs to labor so that labor is a static input. If w is the steady state wage, static labor optimization implies that firm i's profit becomes:

$$\pi_{it} = p_{it} y_{it} - w l_t = \kappa_0 \left(\frac{Y}{w^{\frac{1-\alpha}{\alpha} \frac{\phi}{1-\phi}}} \right)^{1-\phi} e^{\frac{\phi}{\alpha} z_{it}} k_{it}^{\phi},$$

where $\phi = \frac{\alpha\theta}{1-(1-\alpha)\theta} < 1$. κ_0 is a function of α and ϕ. Productivity shocks (z_{it}) are markovian and $H(z_{it+1}|z_{it})$ is the c.d.f of z_{it+1} conditional on z_{it}.

The capital good consists of final good – so that its price is also 1 – and it depreciates at a rate δ. Capital investment in period t is subject to a one period time-to-build. Firms can finance investment using the profits they realize from operations or through external financing. The first source of outside financing is debt. b_{it+1} is the total real payment due to creditors in period $t+1$. To simplify notations, we define $\boldsymbol{x}_{it} = (k_{it}, k_{it+1}, b_{it}, b_{it+1})$.

We introduce $\boldsymbol{\Theta_i}$, a vector containing all the model's parameters for firm i (for now firms may follow different models). $r_{it} = r(z_{it}, \boldsymbol{x_{it}}; \boldsymbol{\Theta_i}, w, Y)$ is the interest rate on the loan granted at date t, so that $\frac{b_{it+1}}{1+r_{it}}$ is the proceed from debt financing received in period t. As will become clear, this flexible function is designed to allow for risky debt and loss given default. We allow the firm's investment and debt financing at date t to be subject to adjustment costs $\Gamma(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y)$. We also assume that firms pay taxes and receive subsidies: $\mathcal{T}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y)$ corresponds to the net tax paid by the firm.

The second source of outside funding is equity. The firm can raise funds from shareholders in the equity market, or distribute excess funds to shareholders: e_{it} is the equity issuance (if negative) or distribution (if positive) made by firm i in period t: it corresponds to the financing gap left after all other sources of financing have been used:

$$\begin{aligned} e_{it} &= \pi_{it} - (k_{t+1} - (1-\delta)k_t) - \Gamma\left(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y\right) \\ &+ \left(\frac{b_{it+1}}{1 + r(z_{it}, \boldsymbol{x_{it}}; \boldsymbol{\Theta_i}, w, Y)} - b_{it}\right) - \mathcal{T}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y) \\ &= e(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y) \end{aligned}$$

We consider generic financing frictions. First, equity issuance may be costly, and we note $\mathcal{C}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y)$ these equity issuance costs. Second, the amount of outside financing may be constrained, a friction that we capture through a vector of constraint: $\boldsymbol{M}(z_{it}, \boldsymbol{x}_{it}; \boldsymbol{\Theta_i}, w, Y) \leq 0$.

The timing is standard in models of firm dynamics. At the beginning of period t, productivity z_{it} is realized. The firm then combines capital in place k_{it} with labor l_{it} to produce and receive the corresponding profits. It then selects the next period stock of capital k_{it+1}, pays the corresponding adjustment costs, reimburse its existing debt b_{it} and receive the proceed from debt issuance $\frac{b_{it+1}}{1+r_{it}}$

Omitting the it index and denoting with prime next-period variables, we can represent firms optimization problem through the following Bellman equation:

$$V(z, k, b; \boldsymbol{\Theta}, w, Y) = \left| \begin{array}{l} \max\limits_{k', b'} e(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) - \mathcal{C}(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) + \beta^F \mathbb{E}_{z'}[V(z', k', b'; \boldsymbol{\Theta}, w, Y)|z], \\ \boldsymbol{M}(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) \leq 0 \end{array} \right. \tag{2}$$

where β^F is the firm's discount rate. In what follows, we assume the standard conditions on the cost functions and the constraints so that the contraction mapping theorem applies to this Bellman equation and there is a unique value function.

The household side of the economy is stripped down to its essentials. A representative household has linear preferences over consumption and leisure: $u(c_t, l_t) = c_t - \frac{l_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} \frac{\bar{w}}{\bar{L}^{\frac{1}{\epsilon}}}$, where c_t is period t consumption, l_t is period t labor supply, ϵ is the Frisch elasticity and ($\bar{w}$, $\bar{L}$) are constant. The representative household owns all firms in the economy. β^H is the representative household's discount rate. In the absence of aggregate uncertainty, optimal consumption and labor supply decisions imply that $L_t^s = \bar{L}\left(\frac{w}{\bar{w}}\right)^\epsilon$ and $\beta^H = \frac{1}{1+r}$, where r is the exogenous risk-free rate. Note that since households portfolios are well diversified across firms, we also have $\beta^F = \frac{1}{1+r}$ even though the model potentially allows for firms' default.

2.2 Introducing capital wedges

Instead of solving the model explicitly, we will characterize its equilibrium as a function of the distribution of objects defined as capital wedges τ which vary over time and across firms. These wedges are defined as the ratio of a firm's marginal revenue product of capital to some frictionless user cost of capital R for firm i in period t. R is arbitrary but fixed throughout the analysis. Wedges measure how much firms' capital stock deviates from frictionless optimization. In our model, firms potentially deviate from frictionless optimum for three reasons: financing frictions, adjustment costs, and taxes.

Definition 1 (Definition of wedges)**.**

$$\begin{aligned} 1+\tau(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) &= \frac{1}{R}\frac{\partial py}{\partial k}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) \\ &= \frac{\alpha\theta}{R}\frac{py(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)}{k} \end{aligned}$$

As previously noted in the literature, in this Cobb-Douglas framework, wedges are easy to measure since they are proportional to the ratio of revenue to capital. Both revenue and capital can be approximated using standard firm-level datasets containing financial statements. A priori, wedges are complicated functions of productivity, firm-level policies variables (debt and capital), model parameters Θ and aggregate conditions (w, Y). In the following, we will summarize the information on wedges through the joint distribution of log wedges $\log(1+\tau)$ and log productivity z, whose c.d.f. we note $F(z,\tau;\Theta,w,Y)$. This distribution reflects the fact that otherwise similar firms may have different wedges because of different histories embedded in their state variables. For instance, firms who experienced a long sequence of adverse productivity shocks have little capital and therefore little ability to borrow, even if the current productivity goes up. The wedge will be bigger. The c.d.f. $F(z,\tau;\Theta,w,Y)$ also reminds us that the wedge distribution is a priori conditional on the aggregate state of the economy (firms in a bigger economy may be taxed more, or less constrained) and the parameters governing their behavior.

2.3 Equilibrium

We now solve the competitive equilibrium of this economy in the steady state as a function of the distribution of the capital wedges defined in Definition 1. In this simple model, the steady state equilibrium corresponds to an equilibrium wage w that clears the labor market and aggregate output Y that clears the final good market.

Given the definition of capital wedges τ, the static FOC in labor, we can write down the market clearing conditions in labor and final good. We note $F(z, \tau; \Theta, w, Y)$ the c.d.f. of log wedges $\log(1+\tau)$ and log productivities z.

To simplify exposition, we assume that all firms follow the same model so there Θ is the same for all firms. This assumption is not necessary but makes equations clearer. This leads to the following formulas describing aggregate production and TFP, as in Hsieh and Klenow (2009) and Midrigan and Xu (2014):

$$Y \propto \left(\int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \right)^{(1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta}} \tag{3}$$

$$TFP = \frac{\left(\int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) \right)^{\frac{1-(1-\alpha)\theta}{\theta}}}{\left(\int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta} z}}{(1+\tau)^{\frac{1-(1-\alpha)\theta}{1-\theta}}} dF(z, \tau; \Theta, w, Y) \right)^{\alpha}} \tag{4}$$

Clearly, the above equations do not solve for (w, Y), since analytically solving the model is in general not feasible. These two equations offer relationships between macro outcomes (output and TFP) and the distribution of wedges, which can be observed. To make these equation easier to apprehend, we now introduce a simplification.

2.4 Small Perturbation Approximation

These equations show how TFP and output can be computed as functions of the joint distribution of log productivity z and wedges τ. To clarify exposition, we will now focus on the following case

Assumption 1. *z and $\log(1+\tau)$ have small deviations around their respective means.*

In this multiplicative set-up, assumption 1 is equivalent to assuming that $\log(1+\tau)$ and z are jointly normally distributed (which is the assumption made for instance in Hsieh and

Klenow (2009)). Since $log\left(\frac{py}{k}\right) = \ln(1+\tau) + \text{cst}$, Assumption 1 implies that the log revenue to capital ratio is also normally distributed. We test the relevance of this assumption using data from BvD AMADEUS Financials for the year 2014. As in Gopinath et al. (2015), we measure $p_i y_{it}$ as the value added of the firm, i.e. the difference between gross output (operating revenue) and materials. We measure the capital stock, k_{it}, with the book value of fixed tangible and intangible. For 9 countries in our sample (France, Italy, Spain, UK, Portugal, Croatia, Sweden, Bulgaria and Romania), we report in Figure 1 normal probability plots, i.e. plots of the empirical c.d.f. of the standardized log revenue to capital ratios against the c.d.f. of a normal distribution. Figure 1 shows that the log-normality assumption is a reasonable one.

Neither of these assumptions (small deviations or log normality) is necessary to our approach and we provide in our robustness section formulas that do not rest on it. But assumption 1 proves useful to clarify the logic of our approach, as it summarizes the distribution of wedges in a handful of moments.

The second order Taylor expansion of equations (3-4) in $\log(1+\tau)$ and z around their means leads to simple formulas summarized in the following proposition:

Proposition 1. *Assumption 1 holds. Then, at equilibrium, output and aggregate TFP can be written as simple functions of three moments of the joint distribution of log wedges and log productivities.*

$$\log Y = \frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\mu_\tau(\Theta,w,Y) + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\sigma_\tau^2(\Theta,w,Y) - 2\sigma_{\tau z}(\Theta,w,Y)\right)\right) + cst \quad (5)$$

$$\log(TFP) = -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sigma_\tau^2(\Theta,w,Y) \quad (6)$$

where $\mu_\tau(\Theta,w,Y)$ and $\sigma_\tau^2(\Theta,w,Y)$ are the mean and variance of the distribution of log capital wedges $\log(1+\tau)$. $\sigma_{\tau z}(\Theta,w,Y)$ is the covariance between log productivity and log wedges.

Proof. See Appendix B.1. □

These formulas illustrate forces already discussed in the literature. Dispersion of wedges impairs aggregate efficiency because it creates capital misallocation (Hsieh and Klenow (2009)). A positive correlation between productivity and wedges also hurts aggregate production: output is lower when the most productive firms experience the largest distortions (Hopenhayn (2014)). However, in our setting, such a correlation does not affect aggregate TFP. This result emanates from the small deviation assumption (or alternatively, from log normality). For instance, it does not hold in Restuccia and Rogerson (2008), who use a binary distribution for the distribution of distorsions.

The above formulas suggest a very simple methodology to aggregate micro evidence: Measure first the effect of a policy experiment on the three moments of a subgroup of treated firms (mean and variance of log wedges, and covariance of log wedges with log productivities). These three moments are easy to compute using firm-level data since log wedges are equal to log sales to capital ratios up to a constant. Then, plug these wedges into formulas (5-6). This would lead to the aggregate effect (on output and TFP) of generalizing the experiment to all firms in the economy. This approach is originally the one of Hsieh and Klenow (2009) who use the variance of log sales to capital ratios of the US to investigate the TFP loss of Indian firms. It has since been taken on in a few recent papers based on quasi-experimental frameworks (Blattner et al. (2017), Larrain and Stumpner (2017), Rotemberg (2017)).

But such methodologies face a challenge, well illustrated by the above formulas. The distribution of wedges is only observed *conditional* on the economy's equilibrium (w, Y). Hence, an experiments affecting only a fraction of firms may a priori have a different distribution of wedges than its extention to all firms in the economy. Since through aggregation we expect w and Y to change, the distribution of wedges in the counterfactual economy may a priori differ from what was observed in small scale experiment. The next Section makes clear what this exercise looks like in our set-up, and more importantly, when it is valid.

3 Inference and Aggregation of Policy Experiments

In this section, we show how to perform aggregate counterfactuals using firm-level estimates. To simplify the exposition, we assume in this section that the empirical exercise consists of a simple binary treatment, where a random subset of firms are treated. But our approach can easily be generalized to continuous treatments.

3.1 Definition of the Empirical Treatment

An econometrician owns data on an infinite number of firms, of which a subset is subject to an empirical treatment. The treatment is binary: Each firm i is either treated ($T_i = 1$) or untreated ($T_i = 0$). This treatment is a policy that affects the parameters Θ: financing constraints, recovery rates in bankruptcy or taxes for example. Θ_0 (Θ_1) are the parameters of non-treated (treated) firms. We assume that the econometrician is ignorant about how exactly these parameters are affected by the policy. She does estimate the structural model of equation (2). We assume, however, that the treatment leaves the following three parameters unchanged: The capital share in production α, the price elasticity of demand θ, and the labor supply elasticity ϵ.

In this simplified setting, we do not need to assume that the treatment is "exogenous", because all firms are identical. Heterogeneity only stems from temporary productivity shocks. We could add firm-level heterogeneity, for instance in the form of long-term productivity differences. In this case, our results would carry through, but would require that the treatment is random, in the sense that it affects a representative sample of firms, i.e. firms with the same distribution of long-term productivity as in the population. We do not develop this point here to clarify exposition.

3.2 Aggregating the Treatment

Our goal is to aggregate the evidence coming from the experiment. This aggregation exercise consists of measuring the effect of generalizing the policy from the subset of treated firms to *all* firms in the economy. An alternative version of our aggregation exercise consists of extending the policy to just a larger fraction of firms in the economy. We focus on a complete aggregation here for the sake of clarity, but explore partial aggregation in Section 5. We summarize the total aggregation exercise in the paragraph below.

Objective 1 (Aggregation of Policy Treatment)**.** *The aggregation of policy treatment consists of computing the effect on output and TFP from moving from an economy where all firms are* non-treated *(*$\Theta = \Theta_0$*) to an economy where all firms are* treated *(*$\Theta = \Theta_1$*). We call* (w_0, Y_0, TFP_0) *(resp.* (w_1, Y_1, TFP_1)*) the equilibrium quantities where all firms are treated (resp. untreated). The aggregate effect of moving from one economy to the other is given by:*

$$\Delta \log Y = \frac{\alpha(1+\epsilon)}{1-\alpha}\left(-\Delta\mu_\tau + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\Delta\sigma_\tau^2 - 2\Delta\sigma_{\tau z}\right)\right) \tag{7}$$

$$\Delta \log(TFP) = -\frac{\alpha}{2}\left(1 + \frac{\alpha\theta}{1-\theta}\right)\Delta\sigma_\tau^2 \tag{8}$$

where:

$$\Delta\mu_\tau = \mu_\tau(\Theta_1, w_1, Y_1) - \mu_\tau(\Theta_0, w_0, Y_0)$$
$$\Delta\sigma^2_\tau = \sigma^2_\tau(\Theta_1, w_1, Y_1) - \sigma^2_\tau(\Theta_0, w_0, Y_0)$$
$$\Delta\sigma_{z\tau} = \sigma_{z\tau}(\Theta_1, w_1, Y_1) - \sigma_{z\tau}(\Theta_0, w_0, Y_0)$$

The equations above make clear the challenge faced by the econometrician. The econometrician does not directly observe the distribution of wedges and productivities *conditional* on the new equilibrium (w_1, Y_1), since this new equilibrium is by definition not observed.

To see this, assume that the econometrician can directly measure, using an infinitely large firm-level dataset, moments of the joint wedge-productivity distribution. As we previously mentionned, by definition the log wedge is equal to the log revenu to capital ratio, which can directly observed in standard firm datasets. The measurement of z is more challenging but there is a large literature devoted to this question, so we take it as given here in this paper. Using the data, the econometrician can thus compare the three moments μ_τ, σ^2_τ and $\sigma_{z\tau}$ between treated and non-treated firms. Focus for instance on the mean of the log wedge. In this case, the ecnometrician observes:

$$\begin{aligned}
\widehat{\Delta\mu_\tau} &= \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0\right) \\
&= \mu_\tau(\Theta_1, w^*, Y^*) - \mu_\tau(\Theta_0, w^*, Y^*) \\
&\neq \Delta\mu_\tau
\end{aligned}$$

where (w^*, Y^*) are the aggregate variables in the (partially) treated economy.

The above equation makes it clear that the econometrician only observes the wedge differential *conditional* on the current equilibrium. As shown in Objective 1, this is a priori not the statistic needed to perform the aggregation exercise. To implement this exercise, the econometrician needs to use the wedge differential conditional on the hypothetical equilibrium, which can by assumption not be observed. This problem is not specific to the mean but is relevant for all moments of the wedge-productivity joint distribution. The next Section defines a set of assumptions for which this problem disappears.

3.3 Scale Invariance of the Wedge Distribution

Here comes the main result of this paper. We show here that under certain assumptions, the distribution of wedges does not depend on the equilibrium quantities w and Y. As we detail below, the assumptions necessary to obtain this result are satisfied in a large class of models of firm dynamics.

Proposition 2 (Distribution of wedges)**.**
We note $S = \frac{Y}{w^{\frac{(1-\alpha)\phi}{(1-\phi)\alpha}}}$ *the steady state "scale of the economy", and make the following two assumptions:*

1. *Each one of the following four functions: (1) the adjustment cost function* $\Gamma()$*, (2) the tax function* $\mathcal{T}()$*, (3) the vector of funding constraint function* $\boldsymbol{M}()$ *and, (4) the equity issuance cost function* $\mathcal{C}()$*, satisfies the following property:*

$$\forall \ (z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y), \quad Q(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) = S \times Q\left(z, \frac{\boldsymbol{x}}{S}; \boldsymbol{\Theta}, 1, 1\right) \tag{9}$$

   where Q *represents each one of the four functions.*

2. *The interest rate function* $r()$ *satisfies the following property:*

$$\forall \ (z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y), \quad r(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y) = r\left(z, \frac{\boldsymbol{x}}{S}; \boldsymbol{\Theta}, 1, 1\right)$$

Then:

$$F(z, \tau; \boldsymbol{\Theta}, w, Y) \equiv F(z, \tau; \boldsymbol{\Theta})$$

Proof. See Appendix B.2 □

This proposition shows that, given parameters Θ, the ergodic distribution of capital wedges does depend on the size of the economy. It is the key result of the paper: It establishes that the wedge distribution observed in a set of homogenous firms will always be the same, whatever the parameters that govern the behavior of other firms in the economy (which will affect the firm via equilibrium prices Y and w). This result will help us implemeting the aggregation exercise of moving from a micro experiment to a macro counterfactual. It rests on two key assumptions. The first one is the Cobb-Douglas technology, whose multiplicative propety allows us to extract the scale factor S of the economy. The second assumption is summarized in the proposition: Frictions have to satisfy properties that resemble homogeneity in firm policies k and b. Our assumption is a bit more general as it only requires homogeneity with respect to S, an assumption that firm's operating profit satisfy – although they may not be strictly homogeneous. As it turns out, these assumptions are valid in a large class of existing models of firm dynamics. We discuss the validity of these homogeneity assumption in greater detail in Section 4.

3.4 Taking Stock: Aggregation Formulas

In this Section, we summarize our findings. To fix idea assume that the econometrician has a dataset with an infinite number of firms. More importantly, assume also that Proposition 2 holds, in this case, the econometrician can proceed in two steps. First, the econometrician would use her firm-level data to measure the effect of the treatment on the three moments. To see how this works, let us first focus on the mean of log wedges, which the econometrician can measure as:

$$
\begin{aligned}
\widehat{\Delta\mu_\tau} &= \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right) \\
&= \mathbb{E}\left(\log\left(1+\tau\right)|T_i=1\right) - \mathbb{E}\left(\log\left(1+\tau\right)|T_i=0\right) \\
&= \mu_\tau(\Theta_1, w*, Y^*) - \mu_\tau(\Theta_0, w*, Y^*) \\
&= \mu_\tau(\Theta_1, w_1, Y_1) - \mu_\tau(\Theta_0, w_0, Y_0) \\
&= \Delta\mu_\tau
\end{aligned}
$$

As the equations above make clear, the differential log sales to capital ratio does measure the statistic relevant to the aggregation exercise now. This is because of the result in Proposition 2 which makes the distribution of wedges independent of aggregate variables (w, Y). The same reasoning applies to the two other relevant moments, which can be estimated through:

$$
\begin{aligned}
\widehat{\Delta\sigma^2_\tau} &= \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=1\right) - \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i=0\right) = \Delta\sigma^2_\tau \\
\widehat{\Delta\sigma_{z\tau}} &= \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=1\right) - \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i=0\right) = \Delta\sigma_{z\tau}
\end{aligned}
$$

In a second step, the econometrician then plugs the above three moments in the aggregation formulas (7-8). They require prior knowledge of only three parameters: the labor share α, the extent of competition θ and the labor supply elasticity ϵ.

To obtain a sense of the orders of magnitude implied by these aggregation formulas, let us fix $\alpha = .3$, $\theta = .8$ and $\epsilon = 0$. Then, consider a hypthetical policy which reduces the revenue per capital ratio by 1 percentage point at the firm level – this is the econometrician's estimate of the empirical treatment. In this Cobb-Douglas framework, this corresponds to an increase in investment by about 1.8%.[2] Furthermore, assume to simplify that this policy has no effect on the variance or covariance of the log wedges. Using formula (7), we obtain that generalizing this policy to *all* firms in the economy would lead to an increase in output by $\frac{\alpha(1+\epsilon)}{1-\alpha} = .4\%$. Overall then, a policy treatment which raises the capital stock of targeted firms by 1.8% in micro estimate will raise aggregate GDP by .4%. This is of course not accounting for changes in allocation captures by the two other moments $\sigma_z\tau$ and σ_τ^2.

The method suggested here rests on several assumptions. The various cost functions satisfy the assumptions in Proposition 2. The aggregation exercise aims to comppute aggregate output and TFP is *all* firms are treated. The economy structure is simple: Each industry only contains one firm, and they are all identical. The rest of the paper explores the importance of these assumptions.

4 Validity of Proposition 2 in Standard Models of Firm Dynamics

Proposition 2 requires homogeneity assumptions in the various cost functions of the firm model. In this Section, we show that these assumptions are satisfied in a large class of standard models of firm dynamics: We discuss in turn adjustment costs, financial frictions and taxes.

[2]The revenue to capital ratio is given by:

$$\Delta \log \frac{py}{k} = -\frac{1-\theta}{1-\theta(1-\alpha)} \Delta \log k$$

4.1 Adjustment Costs

Consider first the case of adjutsment costs. Quadratic adjustment costs to capital, linear adjustment costs of capital, fixed costs of adjustments that scale either with production, output and capital or discount for capital resale all satisfy the assumptions in Proposition 2. For instance, if $\Gamma()$ is given by:

$$\Gamma(z, \boldsymbol{x}; \Theta, w, Y) = \gamma_1 \frac{(k' - (1-\delta)k)^2}{k} + \gamma_2 k + \mathbb{1}_{\{k'-(1-\delta)k \neq 0\}} \left(\gamma_3 y + \gamma_4 py + \gamma_5 k\right) + \gamma_6 k \mathbb{1}_{\{k'-(1-\delta)k<0\}},$$

then, since $y(z, k; \Theta, w, Y) = S \times y(z, \frac{k}{S}; \Theta, 1, 1)$ and $py(z, k; \Theta, w, Y) = S \times py(z, \frac{k}{S}; \Theta, 1, 1)$, it is trivial to show that $\Gamma(z, \boldsymbol{x}; \Theta, w, Y) = S \times \Gamma(z, \frac{\boldsymbol{x}}{S}; \Theta, 1, 1)$

4.2 Financing Frictions

Second, consider the financing side of the model. Our formulation encompass standard models of financing constraints and investment.

Let us start with the interest rate function. For instance, in Michaels et al. (2016) or Gilchrist et al. (2014), debt is risky and in the event that the firm is unable to repay, the lender can seize a fraction $1 - \zeta$ of the firm's fixed assets k. The firms's future market value is not collateralizable, so that a firm's access to credit is mediated by a net worth covenant, which restrains the firm's ability to sell new debt based on its current physical assets and liabilities. Concretely, default is triggered when net worth reaches 0, which defines a threshold value for productivity $\hat{z}$ such that:

$$0 = \kappa_0 S^{1-\phi} e^{\frac{\phi}{\alpha}\hat{z}} k^{\phi} - b + c^k (1-\delta) k, \tag{10}$$

where c_k is the second-hand price of capital, which we treat as a technological parameter. As in Michaels et al. (2016), the right side of the previous equation represents the resources that the firm could raise in order to repay its debt just prior to bankruptcy, which is why its capital is valued at the second-hand price c_k. The wage bill is absent from the previous equation because labor is paid in full, even if the firm subsequently defaults. Finally, the face value of debt discounted at the interest rate $r(z, \boldsymbol{x}; \boldsymbol{\Theta}, w, Y)$ must equal the debt holder's expected payoff discounted at the risk-free rate:

$$\frac{1}{1+r}\left[\int_0^{\hat{z}}\left(\kappa_0 S^{1-\phi}e^{\frac{\phi}{\alpha}z'}k'^{\phi}+(1-\zeta)(1-\delta)k'\right)dH(z'|z)+(1-H(\hat{z}|z))b'\right]=\frac{b'}{1+r(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)} \tag{11}$$

Equations (10) and (11) provides the joint definition for the interest rate function $r(z,\boldsymbol{x};\Theta,w,Y)$, which satistfies our needed assumption in Proposition 2. Note first that equation (10) can be rewritten as: $0=\kappa_0 e^{\frac{\phi}{\alpha}\hat{z}}\frac{k}{S}^{\phi}-\frac{b}{S}+c^k(1-\delta)\frac{k}{S}$. As a result, it is clear that $\hat{z}(k,b;\Theta,w,Y)=\hat{z}(\frac{k}{S},\frac{b}{S};\Theta,1,1)$. Also, we can rewrite Equation (11) as:

$$\frac{1}{1+r}\left[\int_0^{z(\frac{k'}{S},\frac{b'}{S};\Theta,1,1)}\left(\kappa_0 e^{\frac{\phi}{\alpha}z'}\frac{k'}{S}^{\phi}+(1-\zeta)(1-\delta)\frac{k'}{S}\right)dH(z'|z)+(1-H(\hat{z}|z))\frac{b'}{S}\right]=\frac{\frac{b'}{S}}{1+r(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)},$$

so that $r(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y)=r(z,\frac{\boldsymbol{x}}{S};\boldsymbol{\Theta},1,1)$.

Similarly, the specification of debt renegotiation in Hennessy and Whited (2007) would also satisfy these assumptions. More generally, these models make the probability of default independent of the scale of the economy S, and the loss given default proportional to S. These properties ensure our assumption about $r()$ in Proposition 2 is satistified. Obvisouly, models of risk-free debt, such as Midrigan and Xu (2014), also satisfy our assumption.

Our assumption on the cost of equity is also verified in Michaels et al. (2016) and Gilchrist et al. (2014), who posit that equity issuances are subject to an underwriting fees such that there is a positive marginal cost to issue equity:

$$\mathcal{C}(z,\boldsymbol{x};\Theta,w,Y)=\lambda|e(z,\boldsymbol{x};\Theta,w,Y)|\mathbb{1}_{\{e(z,\boldsymbol{x};\Theta,w,Y)<0\}}$$

Given that $e(z,\boldsymbol{x};\Theta,w,Y)=Se(z,\frac{\boldsymbol{x}}{S};\Theta,1,1)$, it is obvious that $\mathcal{C}(z,\boldsymbol{x};\Theta,w,Y)=S\times\mathcal{C}(z,\frac{\boldsymbol{x}}{S};\Theta,1,1)$. Thus, the financing frictions specified in Gilchrist et al. (2014) and Michaels et al. (2016) satisfy the assumptions of Proposition 2. Additionally, it is obvious to see that fixed or quadratic issuance costs would satisfy our assumptions as long as they are appropriately scaled with the size of the firm. For instance, $\psi\frac{e_{it}^2}{k_{it}}\mathbb{1}_{e_{it}<0}$, or $\iota k_{it}\mathbb{1}_{e_{it}<0}$ would work.

Finally, our formulation of financing frictions also encompasses debt constraints as for instance in Midrigan and Xu (2014) or Catherine et al. (2017). In Midrigan and Xu (2014), debt is assumed to be risk-free through full collateralization: $b'\leq\xi k'$ so that $r(z,\boldsymbol{x};\Theta,w,Y)=r_f$ and producers can only issue claims to a fraction χ of their future profits: $e(z, \boldsymbol{x}; \Theta, w, Y) \geq -\chi V(z, \boldsymbol{x}; \Theta, w, Y)$. In this case, the vector $\boldsymbol{M}(z, \boldsymbol{x}; \Theta, w, Y)$ consists of the last two inequalities, and it is direct to see that both $\boldsymbol{M}$ and $r()$ satisfy the assumptions of Proposition 2. Of course, any combination of the constraints in Midrigan and Xu (2014) and Hennessy and Whited (2007) would also satisfy these assumptions. Note also that our model also encompasses debt constraints where debt financing is limited by existing or future cash flows ($b \leq \iota e(z, \boldsymbol{x}; \Theta, w, Y)$).

4.3 Taxes

Standard specifications for the corporate income taxe satisfy the assumption of Proposition 2: $\mathcal{T}(z, \boldsymbol{x}; \Theta, w, Y) = \tau \max\left(0, \pi(z, \boldsymbol{x}; \Theta, w, Y) - \delta k - b\right)$. However, a progressive tax system would violate our assumptions.

5 Robustness and Extensions

This section discusses the robustness of our approach to various modelling assumptions that we made in Section 3.

5.1 Partial Aggregation

In many settings, the full aggregation exercise discussed in Section 3 is not meaningful. Take for instance the case of a loan subsidy program targeted at small firms. The idea behind these programs is that small firms are more likely to be constrained. The natural experiment provides the econometrician with a subset of small firms (for instance small firms below a given threshold). In this case, a meaningful aggregation exercise would be to extend the policy to all small firms. In this Section, we investigate this question and show that our approach is essentially unchanged.

We start with the same model as in Section3. Some firms are treated ($T_i = 1$), others belong to the control group ($T_i = 0$, which may correspond to just a fraction of remaining firms). Like before, the econometrician uses her large dataset to compare the distribution of wedges across the two groups. But the aggregation exercise changes: The econometrician wants to investigate the effect on aggregate output and TFP of going from a situation where all firms are untreated (i.e. have Θ_0) to a situation where a fraction λ of firms is treated, while $1 - \lambda$ remain untreated.

In this case, the two step approach that the econometrician needs to follow is summarized in the following proposition:

Proposition 3 (Partial Aggregation Formulas)**.** *Assume that all assumptions made in Section 3 apply. In addition, assume that the effect of the treatment is small.*
The econometrician wants to use the micro inference to estimate the effect of moving from 0% to λ% firms treated (partial aggregation). In this case, the econometrician needs to proceeds in three steps:

1. *First, the econometrician needs to use the empirical treatment to compare the usual three moments:*

$$\widehat{\Delta\mu_\tau} = \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0\right)$$
$$\widehat{\Delta\sigma^2_\tau} = \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1\right) - \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0\right)$$
$$\widehat{\Delta\sigma_{z\tau}} = \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 1\right) - \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 0\right)$$

2. *Then, adapt these moments to the partial aggregation setting, which involves the following transformations:*

$$\widetilde{\Delta\mu_\tau} = \lambda\widehat{\Delta\mu_\tau}$$
$$\widetilde{\Delta\sigma^2_\tau} = \lambda\widehat{\Delta\sigma^2_\tau} + \lambda(1-\lambda)(\widehat{\Delta\mu_\tau})^2$$
$$\widetilde{\Delta\sigma_{z\tau}} = \lambda\widehat{\Delta\sigma_{z\tau}}$$

3. *Finally, plug the above adapted moments into the aggregation formulas (7-8).*

Proof. See Appendix □

The above proposition says that the formulas developed in Section 3 still apply under partial aggregation, but the data moments have to be adapted to take into account the fact that not all firms are treated. Quite intuitively, the effect of the differential mean wedge has to be multiplied by the fraction actually treated in the aggregation counterfactual (λ). The same intuition directly applies to the covariance betwen log productivity and log wedges. The variance of log wedges is, however, affected in a subtler way as the result of two effect. The first effect comes from the fact the the experiment itself increase the variance of wedges by $\Delta\sigma^2_\tau$, so the overall variance increases by λ times that amount. The second effect is specific to partial aggregation: treating only a fraction λ of firms will generate additional misallocation between treated and non treated. This effect does not happen in the full aggregation case.

5.2 Heterogeneous Industries

In this section, we consider a more general aggregation model than the one presented in Section 2: the economy features industries that are heterogeneous in (1) their output share in total output (2) their labor share (3) the degree of competition between firms within the industry (4) the parameters that govern the firm-level dynamics of investment and hiring and (5) potentially the treatment they receive.

More precisely, let S be the number of industries and M_s the set of firms operating in industry s. Firms in each industry produce in monopolistic competition as in Section 2, and the price-elasticity of demand $-\frac{1}{1-\theta_s}$ can be industry specific: $Y_s = \left(\int_i y_{is}^{\theta_s}\right)^{\frac{1}{\theta_s}}$. The final good market produces by combining each industry output according to the following Cobb-Douglas production function:

$$\ln(Y) = \sum_{k=1}^{S} \phi_s \ln(Y_s) \qquad \text{and} \qquad \sum_{s=1}^{S} \phi_s = 1. \tag{12}$$

Within industry s, the production function is given by: $y_{it} = e^{z_{it}} k_{it}^{\alpha_s} l_{it}^{1-\alpha_s}$, i.e. the labor share is assumed to be industry-specific, while we assume that the distribution of idiosyncratic productivity shocks is common across industries. Beyond ϕ_s, α_s and θ_s, an industry is also characterized by the vector of parameters that govern the firm-level optimization problem 2: Θ_s is the set of parameters under which firms in industry $s \in [1, S]$ operate. All the other assumptions leading to our main results are the same as in Section 3. In particular, we assume that log wedges and log productivity experience small deviations around their means.

Finally, we consider, as before, a binary treatment (again, generalization to a continuous treatment is straightforward) captured by the dummy variable T_i. The difference is now that we allow treatment effects to vary across sectors. In sector s, untreated firms have parameters Θ_s^0 while treated firms have Θ_s^1. We also assumes that the econometrician can observe an infinite number of firms in each industry, and can thus observe the distribution of capital wedges for treated and untreated firms in each sector.

In this new setting, the econometrician then wants to perform the following aggregation exercise. From the industry-level moments that she observes, she wants to infer the effect on aggregate output and TFP of increasing, *in each industry* s, Θ_0^s to Θ_s^1 for all firms of that industry.

The following proposition describes the methodology and the formulas:

Proposition 4 (Aggregating Heterogeneous Industries)**.** *Assume that all assumptions made in Section 3 apply, except for the industry structure described above, and for the fact that the treatment has an heterogeneous effect across sectors. We also assume that the effect of the treatment is small.*

Assume an econometrician observes the result of an empirical micro treatment within each industry s. Her goal is to estimate the effect on aggregate TFP and output of expanding the empirical treatment to all firms.

Then, the result of proposition 2 apply: The joint wedge-productivity distribution is independent of industry equilibrium. Thus, the econometrician should implement the following three step procedure:

1. *First, compute, for each industry S, the following three moments:*

$$\widehat{\Delta\mu_\tau}(s) = \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1, s_i = s\right) - \mathbb{E}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0, s_i = s\right)$$
$$\widehat{\Delta\sigma_\tau^2}(s) = \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 1, s_i = s\right) - \mathrm{Var}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right)|T_i = 0, s_i = s\right)$$
$$\widehat{\Delta\sigma_{z\tau}}(s) = \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 1, s_i = s\right) - \mathrm{Cov}\left(\log\left(\frac{p_{it}y_{it}}{k_{it}}\right), z_{it}|T_i = 0, s_i = s\right)$$

2. *Then, use the following output formula to aggregate these moments:*

$$\Delta \ln Y = \sum_s \left(\frac{\alpha_s\theta_s\phi_s(1+\epsilon)}{\sum_{s'}(1-\alpha_{s'})\theta_{s'}\phi_{s'}}\right)\left(-\widehat{\Delta\mu_\tau}(s) + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha_s\widehat{\Delta\sigma_\tau^2}(s) - 2\widehat{\Delta\sigma_{z\tau}}(s)\right)\right)$$

   If, in addition, $\alpha_s = \alpha$, $\theta_s = \theta$, and the share of each industry is small, we obtain:

$$\Delta \ln Y = \frac{\alpha(1+\epsilon)}{1-\alpha}\sum_s \phi_s\left(-\widehat{\Delta\mu_\tau}(s) + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\widehat{\Delta\sigma_\tau^2}(s) - 2\widehat{\Delta\sigma_{z\tau}}(s)\right)\right)$$
$$\Delta \log TFP = -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\widehat{\Delta\sigma_\tau^2} - \alpha(1+\alpha\theta)\sum_s(\kappa_s - \phi_s)\left(-\widehat{\Delta\mu_\tau}(s) + \frac{1}{2}\frac{\theta}{1-\theta}\left(\alpha\widehat{\Delta\sigma_\tau^2}(s) - 2\widehat{\Delta\sigma_{z\tau}}(\right.\right.$$

In particular, if the treatment is homogeneous across sectors, the aggregation formulas (7-8) *are unchanged.*

Proof. See Appendix B.4. □

Three remarks about this proposition. First, this proposition shows that, provided the treatment is homogeneous across sectors, and capital shares and mark-ups are identical across sectors, our main formulas are unchanged. This is in spite of assuming several industries and heterogeneous industry shares. Second, if the treatment has an heterogeneous effect across industries, our formulas remain quite close to our main expressions (7-8). They are a straightforward aggregation of these formulas using industry-specific treatment effects weighted by industry shares ϕ_s. The only important difference is the appearance of a second, first order term, in the TFP formula, resulting from the fact that across industry distorsions may be present in the baseline economy. The intuition for this second, potentially large effect, is that some industries may be "too big" compared to the undistorted optimum ($\kappa_s > \phi_s$): if distorsions decrease more in these industries, aggregate TFP may increase as a result. Last, if we allow the capital share and mark-ups to differ across industries, the output formula remains a straightforward aggregation of our main formula (7).

5.3 Decreasing returns to scale

In this Section, we show that our formulas do not change when we allow for decreasing returns to scale. We return to the simple framework of Section 2, but assume that firms' production function exhibits technological decreasing returns to scale (span of control): $y_{it} = e^{z_{it}}(k_{it}^{\alpha} l_{it}^{1-\alpha})^{\nu}$ where $\nu < 1$. Then results in Proposition 1 and 2 carry through with marginal modifications:

Proposition 5. *With decreasing technological returns to scale ν, Proposition 2 remains valid: at the steady-state of the economy, the c.d.f. of the joint distribution of wedges and productivities z_i is independent of the aggregate equilibrium. Thus, the econometrician can use the same methodology as in Section 3, except that the aggregation formulas are now:*

$$\Delta \log(Y) = \frac{\alpha\nu\,(1+\epsilon)}{(1-\alpha)\nu + (1+\epsilon)(1-\nu)} \left(-\widehat{\Delta\mu_\tau} + \frac{1}{2}\frac{\nu\theta}{1-\nu\theta}\left(\alpha\widehat{\Delta\sigma_\tau^2} - 2\widehat{\Delta\sigma_{z\tau}}\right)\right)$$

$$\Delta \log(TFP) = -\frac{\alpha\nu}{2}\left(1 + \frac{\alpha\nu\theta}{1-\nu\theta}\right)\widehat{\Delta\sigma_\tau^2}$$

Proof. See Appendix B.5. □

The modifications introduced by decreasing returns to scale are marginal. Proposition 5 makes clear that our approach also applies to models of perfect competition ($\theta = 1$) and decreasing returns to scale such as Hopenhayn (2014) or Midrigan and Xu (2014). It also makes clear that the modifications induced by decreasing returns to scale $\nu < 1$ will quantitatively be small, since ν is typically close to 1. Take for instance the case where $\alpha = .3$, $\epsilon = 0$ and $\nu = .95$. In this case, the coefficient in front of the output formula will be equal to $.95 \times .3/(1 - .95 \times .3) = .40$ while assuming $\nu = 1$ it would be $.42$. A mistake made about the decreasing nature of returns to scale will be at most minor quantitatively.

5.4 Non-parametric Formulas

Finally, we explore here the effect of relaxing the assumption that distorsions and productivity vary little around their means. It turns out that simple formulas, similar to (7-8) can be developped. The other advantage of this alternative approach is that we avoid estimating firm-level log TFP z, by making more intense use of the Cobb Douglas assumption. We thus come back to the framework of Section 2 and derive the following proposition:

Proposition 6.
Assume the empirical treatment observed by the econometrician is small: The measure of firms for which $T_i = 1$ is zero. Further, let $\widehat{\Delta l}$ and $\widehat{\Delta k}$ be the following sufficient statistics estimated by the econometrician:

$$\begin{cases} \widehat{\Delta l} = \ln\left(\mathbb{E}[l_{it}|T_i = 1\right) - \ln\left(\mathbb{E}[l_{it}|T_i = 0]\right) \\ \widehat{\Delta k} = \ln\left(\mathbb{E}[k_{it}|T_i = 1]\right) - \ln\left(\mathbb{E}[k_{it}|T_i = 0]\right) \end{cases}$$

Then, the effect of generalizing the treatment to all firms in the economy on aggregate output and TFP is given by the following formulas:

$$\begin{cases} \Delta \log Y = \dfrac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \times \widehat{\Delta l} \\ \Delta \log TFP = \left(\dfrac{1}{\theta} - (1-\alpha)\right) \times \widehat{\Delta l} - \alpha \times \times \widehat{\Delta k} \end{cases}$$

Proof. See Appendix B.6. □

The intuition behind this proposition is that the reaction of firm's employment and investment are natural sufficient statistics whne it comes to aggregating effect of distorsion-reducing treatments. The multiplicative coefficient in the output equation is easy to interpret and accounts for general equilibrium effects. First, the micro impact of the treatment on a firm's employment is smaller if output substitution is stronger (θ closer to 1). This happens because, in this case, resources are massively reallocated to the least distorted firms, which produce all the goods in the economy. Reducing distorsions in this case has little impact on aggregate outcomes because allocation is nearly optimal. Second, the micro impact of the treatment is bigger is the supply of input is more elastic to efficiency, i.e. ϵ is larger. Overall, these formulas use natural sufficient statistics, which are potential alternatives to the log revenu to assets ratio we emphasize here. The downside of this approach, however, is that it is sensitive to outliers, since it involves taking the mean of an unscaled variable. This may prove to be problematic in practical applications.

6 Conclusion

This paper develops a simple sufficient statistics framework to aggregate well-identified evidence of capital distortions. The methodology proceeds in two steps: (1) the empirical experiment is used to recover causal estimates of sufficient statistics that characterize how distortions are affected by an empirical "treatment" (2) using these estimated statistics within a general equilibrium model to infer changes in aggregate outcomes that would result from extending the treatment to all firms in the economy. We investigate the robustness of these formulas to changes in the economic framework: a more complicated industry structure, partial instead of full aggregation, decreasing returns to scale, large variance of productivity and distorsions.

Variants of this methodology have been used in recent applied work, but our framework makes clear that is is not always correct to do so. Specifically, the methodology can only be applied when the distribution of capital wedges is independent of general equilibrium quantities. We show this is the case in a generic class of macro-finance models where (1) intermediate inputs are combined with (nests of) CES aggregators (2) production takes place according to a Cobb-Douglas technology with labor and capital (3) capital adjustment costs, financing frictions and taxes satisfy a version of homogeneity property which is satisfied in most off-the-shelf models of firm dynamics.

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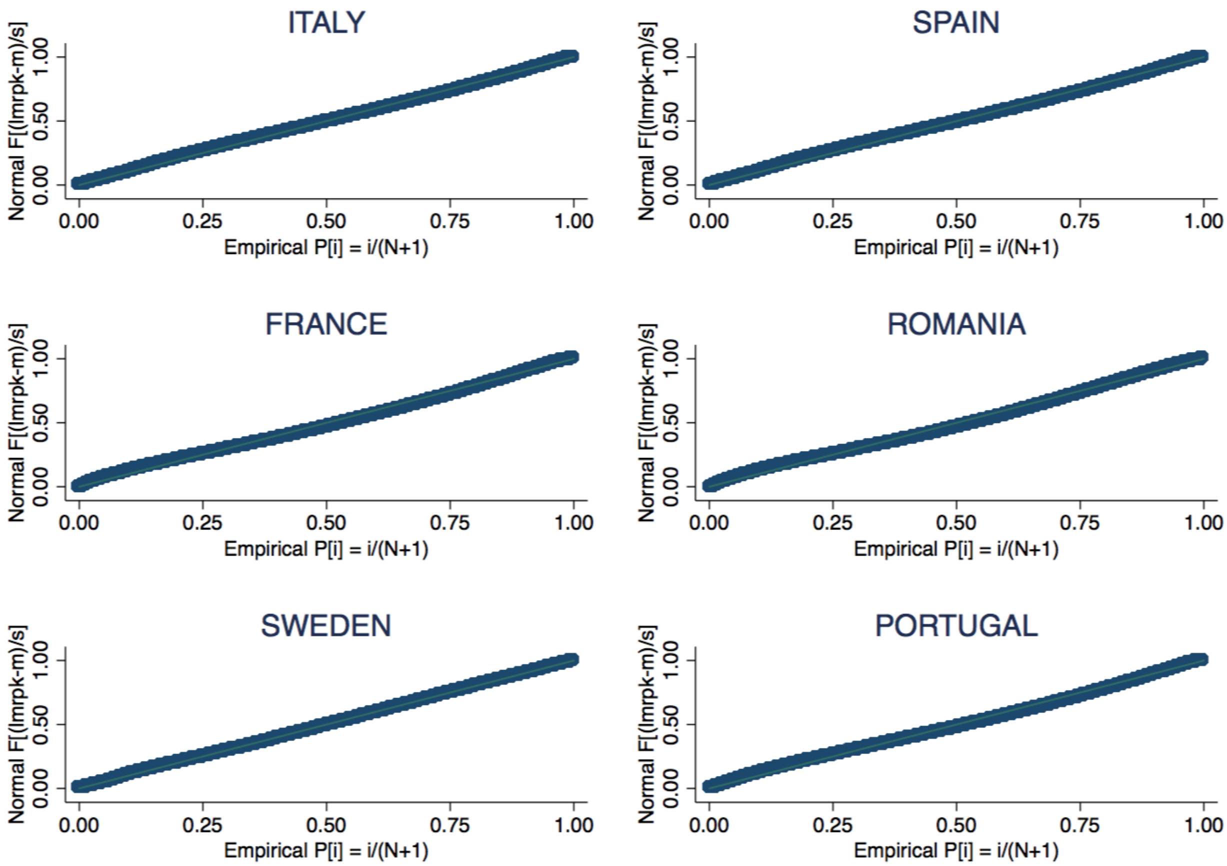
A Figures and Tables

Figure 1: Normal probability plot of log-MRPK for firms in Amadeus



Source: BvD AMADEUS Financials, 2014. Note: This figure shows normal probability plots for 6 OECD countries (France, Spain, Italy, Portugal, Romania and Sweden) for the distribution of log-MRPK. Log-MRPK is computed as the ratio of value added (operating revenue minus materials) and total fixed assets.

B Proofs

B.1 Proof of Proposition 1

First, note $\mu_\tau = \mathbb{E}\tau$, $\sigma_\tau^2 = \text{Var}\tau$ and $\sigma_{z\tau} = \text{Cov}(z, \tau)$. Since the distribution of wedges is a function of Θ and the aggregate equilibrium (w, Y), so are these moments, but we omit the dependence to ease notations.

We start with aggregate production (3):

$$\log Y = (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log \int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha}\right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y)$$

Note $\delta_\tau = \log(1+\tau) - \mu_\tau$, and $u = \frac{\theta}{1-\theta}(z - \alpha\delta_\tau)$. Then:

$$\begin{aligned}
\int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha}\right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) &= \mathbb{E}\left(e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau + u}\right) \\
&= e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau u} \\
&\approx e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \mathbb{E}\left(1 + u + \frac{u^2}{2}\right) \\
&\approx e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \left(1 + \frac{\text{Var}u}{2}\right) \\
&\approx e^{-\frac{\alpha\theta}{1-\theta}\mu_\tau} \left(1 + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2)\right)
\end{aligned}$$

so that:

$$\begin{aligned}
\log \int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha}\right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y) &\approx -\frac{\alpha\theta}{1-\theta}\mu_\tau + \log\left(1 + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2)\right) \\
&\approx -\frac{\alpha\theta}{1-\theta}\mu_\tau + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2 (\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2)
\end{aligned}$$

which leads to the result. Computation of the TFP formula follows the same logic.

B.2 Proof of Proposition 2

Remember that equity issuance / distributions are given by:

$$\begin{aligned} e_{it} &= \frac{\alpha}{\alpha+(1-\alpha)\phi}\left(\frac{(1-\alpha)\phi}{\alpha+(1-\alpha)\phi}\right)^{\frac{1-\alpha}{\alpha}\phi} S_t^{1-\phi} e^{\frac{\phi}{\alpha}z_{it}} k_{it}^{\phi} - (k_{t+1}-(1-\delta)k_t) - \Gamma\left(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t\right) \\ &\quad + \left(\frac{b_{it+1}}{1+r(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t)} - b_{it}\right) - \mathcal{T}(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t), \end{aligned}$$

where $S_t = \frac{Y_t}{w_t^{\frac{(1-\alpha)\phi}{\alpha(1-\phi)}}}$. By combining the different assumptions in Proposition 2, we get that:

$$\begin{aligned} e_{it} &= S_t\left(\frac{\alpha}{\alpha+(1-\alpha)\phi}\left(\frac{(1-\alpha)\phi}{\alpha+(1-\alpha)\phi}\right)^{\frac{1-\alpha}{\alpha}\phi} e^{\frac{\phi}{\alpha}z_{it}}\left(\frac{k_{it}}{S_t}\right)^{\phi} - (\frac{k_{t+1}}{S_t}-(1-\delta)\frac{k_t}{S_t})\right. \\ &\quad \left. - \Gamma\left(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1\right) + \left(\frac{\frac{b_{it+1}}{S_t}}{1+r(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\Theta,1,1)} - \frac{b_{it}}{S_t}\right) - \mathcal{T}(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1)\right) \end{aligned}$$

Therefore, $e(z_{it},\boldsymbol{x}_{it};\boldsymbol{\Theta},w_t,Y_t) = S_t e(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1)$. Since the equity issuance cost $\mathcal{C}()$ also satisfies property 9, the flow variable in the Bellman equation 2 can be rewritten as:

$$e(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) - \mathcal{C}(z,\boldsymbol{x};\Theta,w,Y) = S\times\left(e(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1) - \mathcal{C}(z_{it},\frac{\boldsymbol{x}_{it}}{S_t};\boldsymbol{\Theta},1,1)\right)$$

We now consider the steady-state of this economy: $w_t = w_{t+1} = w$ and $Y_t = Y_{t+1} = Y$. The Bellman equation 2 becomes:

$$V(z,k,b;\boldsymbol{\Theta},w,Y) = \left| \begin{array}{l} \max\limits_{k',b'} e(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) - \mathcal{C}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) + \dfrac{\mathbb{E}_{z'}[V(z',k',b';\boldsymbol{\Theta},w,Y)|z]}{1+r_f} \\ \boldsymbol{M}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) \leq 0 \end{array}\right.$$

Let $\mathcal{B}$ be the Bellman operator:

$$\mathcal{B}f(z,k,b;\boldsymbol{\Theta},w,Y) = \left| \begin{array}{l} \max\limits_{k',b'} e(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) - \mathcal{C}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) + \dfrac{\mathbb{E}_{z'}[f(z',k',b';\boldsymbol{\Theta},w,Y)|z]}{1+r_f} \\ \boldsymbol{M}(z,\boldsymbol{x};\boldsymbol{\Theta},w,Y) \leq 0 \end{array}\right.$$

Consider the set of functions $\mathcal{F}$ such that for all $(z,k,b;\boldsymbol{\Theta},w,Y)$, $f(z,k,b;\boldsymbol{\Theta},w,Y) = S\times f(z,\frac{k}{S},\frac{b}{S};\boldsymbol{\Theta},1,1)$.

If $f \in \mathcal{F}$, then $\mathcal{B}f \in \mathcal{F}$:

$$
\begin{aligned}
\mathcal{B}f(z,k,b;\mathbf{\Theta},w,Y) \quad &= \quad \left| \begin{array}{l} \max\limits_{k',b'} S \times \left(e(z, \frac{\boldsymbol{x}}{S}; \mathbf{\Theta}, 1, 1) - \mathcal{C}(z, \frac{\boldsymbol{x}}{S}; \mathbf{\Theta}, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', \frac{k'}{S}, \frac{b'}{S}; \mathbf{\Theta}, 1, 1)|z]}{1+r_f} \right) \\ \boldsymbol{M}(z, \frac{\boldsymbol{x}}{S}; \mathbf{\Theta}, 1, 1) \leq 0 \end{array} \right. \\
= S \times &\left| \begin{array}{l} \max\limits_{k',b'} e(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \mathbf{\Theta}, 1, 1) - \mathcal{C}(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \mathbf{\Theta}, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', \frac{k'}{S}, \frac{b'}{S}; \mathbf{\Theta}, 1, 1)|z]}{1+r_f} \\ \boldsymbol{M}(z, \frac{k}{S}, \frac{k'}{S}, \frac{b}{S}, \frac{b'}{S}; \mathbf{\Theta}, 1, 1) \leq 0 \end{array} \right. \\
= S \times &\left| \begin{array}{l} \max\limits_{k',b'} e(z, \frac{k}{S}, k', \frac{b}{S}, b'; \mathbf{\Theta}, 1, 1) - \mathcal{C}(z, \frac{k}{S}, k', \frac{b}{S}, b'; \mathbf{\Theta}, 1, 1) + \frac{\mathbb{E}_{z'}[f(z', k', b'; \mathbf{\Theta}, 1, 1)|z]}{1+r_f} \\ \boldsymbol{M}(z, \frac{k}{S}, k', \frac{b}{S}, b'; \mathbf{\Theta}, 1, 1) \leq 0 \end{array} \right. \\
= \quad &S \times \mathcal{B}f(z, \frac{k}{S}, \frac{b}{S}; \mathbf{\Theta}, 1, 1)
\end{aligned}
$$

Since the contraction mapping theorem applies and $\mathcal{F}$ is a compact space, this implies that the value function V also belongs to $\mathcal{F}$:

$$V(z,k,b;\mathbf{\Theta},w,Y) = S \times V(z, \frac{k}{S}, \frac{b}{S}; \mathbf{\Theta}, 1, 1).$$

The previous equations also show that, in an economy with scale (w, Y), if (k', b') are the optimal policies for a firm with state variable (z, k, b), then $(\frac{k'}{S}, \frac{b'}{S})$ are the optimal policies for a firm with state variables $(z, \frac{k}{S}, \frac{b}{S})$ and in the economy with scale ($w = 1$, $Y = 1$). As a result, the ergodic distribution of $\frac{k}{S}$ in the economy (w, Y) is equal to the ergodic distribution of k in the economy $(1, 1)$.

Remember that, by definition in the steady-state, capital wedges are equal to:

$$(1+\tau_{it}) = \frac{\alpha\theta}{r_f+\delta} \frac{p_{it}y_{it}}{k_{it}} = \frac{\alpha\phi}{(\alpha+(1-\alpha)\phi)(r_f+\delta)} e^{\frac{\phi}{\alpha}z_{it}} \left(\frac{k_{it}}{S}\right)^{\phi}$$

Since the ergodic distribution of $(\frac{k}{S})$ in the economy (w, Y) is the same as the ergodic distribtion of k in the economy $(1, 1)$ and since the distribution of z is independent of (w, Y), this implies that, in the steady state, the distribution of wedges τ_{it} does not depend on (w, Y) and can be written $G(\tau_i; \Theta)$.

B.3 Proof of Proposition 3

With heterogenous firm models, formulas (3-4) change a little bit. Let λ be the fraction of firms with Θ_1 and $1 - \lambda$ the fraction of firms with Θ_0. Then, aggregate output writes:

$$\log Y = (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log\left(\lambda M(\Theta_1, w, Y) + (1-\lambda) M(\Theta_0, w, Y)\right)$$

where we note:

$$\log M(\Theta, w, Y) = \int_{z,\tau} \left(\frac{e^z}{(1+\tau)^\alpha} \right)^{\frac{\theta}{1-\theta}} dF(z, \tau; \Theta, w, Y)$$
$$= -\frac{\alpha\theta}{1-\theta}\mu_\tau + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2 \left(\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2\right)$$

Since:

$$\log M(\Theta_1, w, Y) = \log M(\Theta_0, w, Y) + \underbrace{-\frac{\alpha\theta}{1-\theta}\mu_\tau + \frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2 \left(\sigma_z^2 - 2\alpha\sigma_{z\tau} + \alpha^2\sigma_\tau^2\right)}_{\Delta}$$

we have that:

$$\log Y = (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log\left((1-\lambda)e^{\log M(\Theta_0, w, Y)} + \lambda e^{\log M(\Theta_1, w, Y)}\right)$$
$$= \log Y_0 + (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \log\left(1 - \lambda + \lambda e^{\Delta}\right)$$
$$= \log Y_0 + (1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} \left(\lambda\Delta + \frac{\lambda(1-\lambda)}{2}\Delta^2\right)$$

where we make use of the assumption that $\Delta \ll 1$, which is stated in the proposition as "the effect of the treatment is small".

We then substitute the expression of Δ, neglect terms beyond order 2, and obtain the formula for GDP in the proposition. The proof for TFP follows the same logic.

B.4 Proof of Proposition 4

First, notice that profit maximization of the final sector leads to:

$$p_s Y_s = \phi_s Y$$

and the FOC in labor nicely aggregates into:

$$wL = \left(\sum_s \theta_s \phi_s (1 - \alpha_s)\right) Y$$

while industry-level aggregator leds to the following firm-level demand curve:

$$p_i = p_s Y_s^{1-\theta_s} y_i^{\theta_s - 1}$$

We then start with the firm-level relations and omit the time subscripts:

$$l_i = \frac{\theta_s(1-\alpha_s)p_i y_i}{w}$$
$$k_i = \frac{\theta_s \alpha_s p_i y_i}{R(1+\tau_i)}$$

which we plug back into the definition of revenue to obtain:

$$p_i y_i \propto p_s^{\frac{1}{1-\theta_s}} Y_s \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{w^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} (1+\tau_i)^{\frac{\alpha_s \theta_s}{1-\theta_s}}}$$

Aggregating at the industry level, we obtain that:

$$Y_s \propto Y J_s^{1-\theta_s} \frac{1}{w^{(1-\alpha_s)\theta_s}}$$

where:

$$\log J_s = \log\left(\int \frac{e^{\frac{z\theta_s}{(1-\theta_s)}}}{(1+\tau_i)^{\frac{\alpha_s\theta_s}{1-\theta_s}}} dF(z,\tau)\right)$$
$$\approx \frac{\theta_s \alpha_s}{1-\theta_s} \underbrace{\left(-\mu_\tau + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha\sigma_\tau^2 - 2\sigma_{z\tau}\right)\right)}_{A_s}$$

assuming small deviations of τ and z around their means.

We then reaggregate at the economy level using $\log Y = \sum_s \phi_s \log Y_s$ and the fact that $Y \propto wL$ so as to obtain:

$$\log \frac{Y}{L} = \textsf{cst} + \sum_s \frac{\phi_s \alpha_s \theta_s}{\sum_{s'} \phi_{s'} \theta_{s'}(1-\alpha_{s'})} A_s$$

differentiating and taking into account the fact that $Y \propto w^{1+\epsilon}$ leads to the equation for Y.

We now turn to aggregate TFP. We start from the following natural definition:

$$\log TFP = \left(\sum_s \phi_s(1-\alpha_s)\right)\log\frac{Y}{L} + \left(\sum_s \phi_s \alpha_s\right)\log\frac{Y}{K}$$

We thus need to focus on $\log\frac{Y}{K}$. Start from the fact that:

$$k_i \propto \frac{p_i y_i}{1+\tau_i} \propto p_s^{\frac{1}{1-\theta_s}} Y_s \frac{e^{\frac{\theta_s}{1-\theta_s} z_i}}{w^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} (1+\tau_i)^{1+\frac{\alpha_s \theta_s}{1-\theta_s}}}$$

which we aggregate at the sector level into:

$$K_s \propto Y_s \frac{I_s}{J_s}$$

where:

$$\begin{aligned}
\log I_s &= \log \left(\int \frac{e^{\frac{z\theta_s}{(1-\theta_s)}}}{(1+\tau_i)^{1+\frac{\alpha_s\theta_s}{1-\theta_s}}} dF(z,\tau) \right) \\
&\approx \left(1 + \frac{\theta_s \alpha_s}{1-\theta_s}\right) \left(-\mu_\tau + \frac{1}{2}\frac{\theta_s}{1-\theta_s}\left(\alpha\sigma_\tau^2 - 2\sigma_{z\tau}\right) + \frac{1}{2}\sigma_\tau^2\right) \\
&\approx \left(1 + \frac{\alpha_s\theta_s}{1-\theta_s}\right)\left(A_s + \frac{\sigma_\tau^2}{2}\right)
\end{aligned}$$

Hence:

$$\log \frac{K_s}{Y} = cst + \left(1 + \frac{\alpha_s\theta_s}{1-\theta_s}\right)\frac{\sigma_\tau^2}{2} + (1+\alpha_s\theta_s)\,A_s - (1-\alpha_s)\theta_s \sum_{s'} \frac{\phi_{s'}\alpha_{s'}\theta_{s'}}{\sum_{s''}\phi_{s''}\theta_{s''}(1-\alpha_{s''})} A_{s'}$$

Differentiating and linearizing $\log \frac{K}{Y}$, we obtain:

$$\Delta \log \frac{K}{Y} \approx \sum_s \frac{K_s^0}{K^0}\left(\Delta \log \frac{K_s}{Y} + \frac{1}{2}\left(1 - \frac{K_s^0}{K^0}\right)\left(\Delta \log \frac{K_s}{Y}\right)^2\right)$$

which lead to a large, ugly formula for TFP.

Assume that $\alpha_s = \alpha$, $\theta_s = \alpha$. Note $\kappa_s = \frac{K_s^0}{K^0}$ the capital share of each sector in the control economy where no firm is treated. In this case:

$$\Delta \log \frac{Y}{L} = \frac{\alpha}{1-\alpha} \sum_s \phi_s \Delta A_s$$

and:

$$\begin{aligned}
\Delta \log \frac{K}{Y} = &-\frac{1}{2}\left(1 + \frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma_\tau^2 + \alpha(1+\alpha\theta)\sum_s(\phi_s - \kappa_s)\Delta A_s \\
&+ \frac{1}{2}\sum_s \kappa_s(1-\kappa_s)\left((1+\alpha\theta)\Delta A_s - \alpha\theta\sum_{s'}\phi_{s'}\Delta A_{s'}\right)^2
\end{aligned}$$

We further assume that industries are small, which allows us to neglect the last term:

$$\Delta \log TFP = -\frac{\alpha}{2}\left(1 + \frac{\alpha\theta}{1-\theta}\right)\Delta\sigma_\tau^2 + \alpha(1+\alpha\theta)\sum_s(\phi_s - \kappa_s)\Delta A_s$$

which leads to the formula in the proposition.

B.5 Proof of Proposition 5

We first show that Proposition 2 still holds with decreasing returns to scale.

With monopolistic competition and decreasing returns to scale, for a firm i with a stock of capital k_{it}, operating profits after optimizing labor demand are given by:

$$\begin{aligned} p_i y_i - w l_i &= (1-(1-\alpha)\nu\theta)\left(\frac{(1-\alpha)\nu\theta}{w}\right)^{\frac{(1-\alpha)\nu\theta}{1-(1-\alpha)\nu\theta}} Y^{\frac{1-\theta}{1-(1-\alpha)\nu\theta}} e^{z_i \frac{\theta}{1-(1-\alpha)\nu\theta}} k_i^{\frac{\alpha\nu\theta}{1-(1-\alpha)\nu\theta}} \\ &= S \times (1-(1-\alpha)\nu\theta)\left((1-\alpha)\nu\theta\right)^{\frac{(1-\alpha)\nu\theta}{1-(1-\alpha)\nu\theta}} e^{z_i \frac{\theta}{1-(1-\alpha)\nu\theta}} \left(\frac{k_i}{S}\right)^{\frac{\alpha\nu\theta}{1-(1-\alpha)\nu\theta}} \end{aligned}$$

where $S = \frac{Y^{\frac{1-\theta(1-\alpha\nu)}{1-(1-\alpha)\nu\theta}}}{w^{\frac{\nu\theta}{1-(1-\alpha)\nu\theta}}}$.

It follows directly from the proof of Proposition 2 that in this economy, and under the assumptions of Proposition 2, the ergodic joint distribution of capital wedges and productivity is independent of (w, Y) and depend only on the parameters Θ. Let $F(z, \tau; \Theta)$ denote this distribution as before.

With decreasing returns to scale ν, profit maximization for firm i in industry s as a function of a capital wedge τ_{is} leads to:

$$\begin{cases} k_i \propto \left(\frac{1}{w}\right)^{\frac{\nu(1-\alpha)\theta}{1-\nu\theta}} Y^{\frac{1-\theta}{1-\nu\theta}} e^{z_i \frac{\theta}{1-\nu\theta}} \left(\frac{1}{(1+\tau_i)}\right)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}} \\ l_i \propto \left(\frac{1}{w}\right)^{\frac{1-\alpha\nu\theta}{1-\theta}} Y^{\frac{1-\theta}{1-\nu\theta}} e^{z_i \frac{\theta}{1-\nu\theta}} \left(\frac{1}{(1+\tau_i)}\right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} \end{cases}$$

Firm i output at the optimum is given by:

$$p_i y_i \propto Y^{\frac{1-\theta}{1-\nu\theta}} \left(\frac{1}{w}\right)^{\frac{(1-\alpha)\nu\theta}{1-\nu\theta}} \left(\frac{1}{1+\tau_i}\right)^{\frac{\alpha\nu\theta}{1-\nu\theta}} e^{z_i \frac{\theta}{1-\nu\theta}} \tag{13}$$

Omiting the i subscripts, equilibrium on the product market implies that:

$$w \propto Y^{-\frac{1-\nu}{(1-\alpha)\nu}} \left(\int_{z,\tau} \frac{e^{z\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta)\right)^{\frac{1-\nu\theta}{(1-\alpha)\nu\theta}} \tag{14}$$

Equilibrium on the labor market implies that $Y \propto w^{1+\epsilon}$

Combining these two equations provides the following expression for aggregate output:

$$Y \quad \propto \quad \left(\int_{(z,\tau)} \frac{e^{z\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta)\right)^{\frac{(1+\epsilon)(1-\nu\theta)}{\theta((1-\alpha)\nu+(1+\epsilon)(1-\nu))}}$$

which then leads to the expression in the proposition after Taylor expansion.

Finally, aggregate TFP admits a simple expression:

$$
\begin{aligned}
TFP &= \frac{Y}{K^{\alpha\nu} L^{(1-\alpha)\nu}} \\
&= \left(\int_{z,\tau} \frac{z^{\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{\alpha\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta) \right)^{\frac{1-(1-\alpha)\nu\theta}{\theta}} \left(\int_{z,\tau} \frac{z^{\frac{\theta}{1-\nu\theta}}}{(1+\tau)^{\frac{1-(1-\alpha)\nu\theta}{1-\nu\theta}}} dF(z,\tau;\Theta) \right)^{-\alpha\nu}
\end{aligned}
$$

which then leads to the formula in the proposition after straightforward Taylor expansion.

B.6 Proof of Proposition 6

Optimal labor demand as a function of firm-level capital wedge is:

$$
l_i \propto \left(\frac{1}{w}\right)^{\frac{1-\alpha\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1}{(1+\tau_i)}\right)^{\frac{\alpha\theta}{1-\theta}}
$$

Assume treated firms are a zero-measure set. Then, the following sufficient statistic can be computed for both the treatment and control groups:

$$
\mathbb{E}[l_i | T_i = T] \propto \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_T)
$$

where $T \in \{0,1\}$.

We now introduce the log difference in mean employment:

$$
\begin{aligned}
\widehat{\Delta l} &= \log\left(\mathbb{E}[l_i|T_i=1]\right) - \log\left(\mathbb{E}[l_i|T_i=0]\right) \\
&= \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_1) - \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_0)
\end{aligned}
$$

Given the output equation (3), it follows directly that

$$
\Delta \log Y = \frac{(1+\epsilon)(1-\theta)}{(1-\alpha)\theta} \widehat{\Delta_l}
$$

We now compute TFP, which requires calaculating the capitak stock. Similarly, optimal capital demand implies that:

$$
k_i \propto \left(\frac{1}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1}{1+\tau_i}\right)^{\frac{1-(1-\alpha)\theta}{1-\theta}}
$$

Like for employment, we use this to compute the new capital sufficient statistic:

$$
\begin{aligned}
\widehat{\Delta k} &= \log\left(\mathbb{E}[k_i|T_i=1]\right) - \log\left(\mathbb{E}[k_i|T_i=0]\right) \\
&= \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{1+\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_1) - \log \int_{z,\tau} \frac{e^{\frac{\theta}{1-\theta}}}{(1+\tau)^{1+\frac{\alpha\theta}{1-\theta}}} dF(z,\tau,\Theta_0)
\end{aligned}
$$

Given the TFP formula (4), $\widehat{\Delta l}$ and $\widehat{\Delta k}$ can be straightforwardly combined into the formula given in the proposition.