The redistributive effects of bank capital regulation

Elena Carletti* Robert Marquez Silvio Petriconi

September 2017

Abstract

We build a general equilibrium model of banks' optimal capital structure, where investors are reluctant to invest in financial products other than deposits, and where bankruptcy is costly. We first show that banks raise both deposits and equity, and that investors are willing to hold equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it increases the cost of raising capital for banks, with the bulk of this cost accruing to depositors. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

Keywords: limited market participation, bank capital structure, capital regulation

JEL classifications: G18, G2, G21

*We thank Paolo Fulghieri, Wolf Wagner, and Andy Winton for helpful comments and suggestions. We also thank seminar participants at the Bank of Canada, the 2017 OxFIT conference, the 2017 FIRS conference, and the 2016 Baffi CAREFIN Annual International Banking Conference. Address for correspondence: Carletti: Bocconi University, IGIER and CEPR, Via Röntgen 1, 20136 Milan, Italy, elena.carletti@unibocconi.it. Marquez: University of California, Davis. 3410 Gallagher Hall. One Shields Avenue. Davis, CA 95616, USA, rsmarquez@ucdavis.edu. Petriconi: Bocconi University, IGIER and BIDSA, Via Röntgen 1, 20136 Milan, Italy, silvio.petriconi@unibocconi.it.

The redistributive effects of bank capital regulation

September 2017

Abstract

We build a general equilibrium model of banks' optimal capital structure, where investors are reluctant to invest in financial products other than deposits, and where bankruptcy is costly. We first show that banks raise both deposits and equity, and that investors are willing to hold equity only if adequately compensated. We then introduce (binding) capital requirements and show that: (i) it distorts investment away from productive projects toward storage; or (ii) it increases the cost of raising capital for banks, with the bulk of this cost accruing to depositors. These results hold also when we extend the model to incorporate various rationales justifying capital regulation.

Keywords: limited market participation, bank capital structure, capital regulation

JEL Classification Numbers: G18, G2, G21

1 Introduction

The regulation of financial institutions, and banks in particular, has been at the forefront of the policy debate for a number of years. Much of the concern over financial institutions relates to the perceived negative consequences associated with a bank's failure, and with how losses may be distributed across various stakeholders, such as borrowers (either corporate or individual) and creditors, including depositors and the government, with the ultimate bearers of the losses being households and shareholders.

A primary tool for bank regulation is the imposition of minimum capital standards, which amount to requirements that banks limit their leverage and issue at least a minimal amount of equity. Capital requirements thus act as a portfolio restriction on the liability side of banks' balance sheets,[1] and have two primary roles. First, by creating a junior security held by the ultimate shareholders of the bank, capital (i.e., equity) stands as a first line of defense against losses, with shareholders absorbing losses before those accrue to other bank claimants, thus triggering distress and ultimately bankruptcy. Second, by forcing shareholders to have "skin in the game", capital is seen as a way to control moral hazard or asset substitution problems that may otherwise arise as a result of investment decisions by levered banks. In fact, recent calls among regulators, policy makers, and even academics (see, e.g., Admati et al., 2013, for a discussion of this issue) have been for banks to dramatically increase the amount of capital they issue as way of reducing risk and ultimately increasing social welfare.

What is less understood in the discussion related to bank capital regulation is who bears the costs, to the extent that there are any, associated with requiring banks to increase their capitalization beyond their preferred levels. If banks', or bankers', primary goal is to maximize profits, and capital structure is chosen taking this objective into account, then the imposition of a leverage constraint, or any other restriction on banking activities, should lead to a reduction in bank profitability and consequently in the return available to bank claimants.

[1] Of course, the imposition of *risk weighted* capital requirements serve to place portfolio restrictions on the asset side as well, in addition to constraining how banks must finance themselves.

The regulatory perspective, of course, is that the possible loss in profitability associated with such constraints is more than compensated by increases in social welfare through other channels, such as a reduced social burden associated with government guarantees or the internalization of externalities that might arise when a bank fails. Nevertheless, at the bank level, the reduction in bank profitability should fall on the shoulders of its various claimants, such as depositors and shareholders, and there has been little study of which party bears the brunt of this burden. Do shareholders, as is commonly argued, primarily bear the costs associated with regulation, or are depositors and other creditors the parties more affected by tighter regulatory requirements? This is an important consideration for understanding the incidence of regulation. Moreover, if the aim of (capital) regulation is to protect specific agents, such as retail depositors and by extension the deposit insurance fund, an important question is whether capital regulation is likely to achieve this goal.

To tackle these questions, we present a general equilibrium model of banks' optimal capital structures where all parties are risk neutral, but investors are reluctant to participate in financial markets, preferring to invest only in safe assets, such as deposits. This key friction, which is well documented in the literature on household finance (see, e.g., Guiso and Sodini, 2013), implies that investors must be induced through additional compensation to hold anything other than a bank deposit. In particular, investors (or households, more generally) will be unwilling to become equity holders unless they receive compensation sufficient to overcome their reluctance.

Specifically, in our model banks exist to channel funds from investors, who have limited outside options for storing their savings, into productive but risky investments. Investors are naturally disinclined to invest in anything other than in storage or in "safe" bank deposits, but can be induced to hold equity if they view the equilibrium return to equity investment as being sufficiently high relative to that of holding a deposit. As investors are otherwise risk neutral, however, there is no premium for holding risk and thus effects related purely to leverage changes (as in Modigliani and Miller, 1958) are shut down.

Banks can finance themselves with either debt or equity, but using too much debt exposes them to default and, hence, to bankruptcy risk. Given that bankruptcy is costly, and that banks ultimately are owned by shareholders and thus try to maximize shareholder value, they endogenously limit the amount of debt financing to reduce expected bankruptcy costs. However, raising equity capital is difficult because investors face costs in becoming equity holders and thus need to be compensated, with greater compensation demanded the larger amount of equity capital the bank wants to raise. In other words, investors' participation in financial markets as holders of bank capital (i.e., equity) is endogenous, and will depend on the difference in the equilibrium returns of deposits versus equity. The trade-off between these two forces leads to an optimal capital structure, with banks always finding it optimal to raise some amount of debt financing (i.e., deposits), with the exact amount depending on how profitable their investment projects are, and how variable are their returns.

As a first important step, we characterize the equilibrium return to equity holders as well as to depositors. We show that, while the marginal holder of capital is indifferent between being an equity holder or a depositor, inframarginal equity holders earn a strictly positive rent as a result of investing in a bank's equity. The bank therefore creates value for investors by channeling funds from storage into real investment projects, allowing investors to earn a return that more than compensates them for the disutility they associate with being an equity market investor (i.e., their unwillingness to participate in financial markets). When the expected return to investment projects is sufficiently high, all available funds are put to productive use, with no funds going into storage. In this scenario, even depositors earn a premium in that their expected return is strictly higher than what they would earn in storage.

Despite the friction of (endogenous) limited market participation, the individually optimal capital structure decisions for banks in our baseline model are socially efficient, and the market solution yields no distortions that can be directly improved with capital regulation. This occurs because banks are competitive in our model, and all profits ultimately accrue to the banks' shareholders, who are the residual claimants. The model thus exhibits an efficient benchmark solution that provides us with an ideal laboratory in which to study how the distortion (on bank profits and, thus, social output) affects the claimants of the bank, namely depositors and shareholders who are, ultimately, represented by the households who are making portfolio decisions on how to allocate their savings.[2]

We identify two main sources of inefficiency associated with binding minimum capital requirements. One is that, when project returns are relatively low and not all funds are being invested in productive projects but are rather being held as storage, requiring banks to hold even greater amounts of capital reduces further the number of projects that are funded. This, in turn, reduces aggregate surplus since investment projects yield a higher surplus than investing in storage. As a result, capital requirements introduce a distortion away from productive investments toward storage. While bank deposits may become safer as a result, depositors don't benefit at all from the tighter regulation since their expected utility is pinned down by the outside option given by the return to the storage technology.

The second inefficiency arises at the other extreme, once all investment funds are being allocated to productive projects. Here, an increased requirement to hold capital raises the overall cost borne by investors. The reason is that satisfying the capital requirement necessitates that a larger number of investors be induced to become equityholders, and thus bear the disutility associated with holding equity. Much of this increased cost is ultimately borne by those investors who remain as depositors, since their equilibrium return goes down and they earn a lower return on their deposits relative to what they could get in the absence of a capital requirement. In other words, much of the incidence of the increased costs associated with satisfying the capital requirement falls on those investors who are least willing to be financial market participants, rather than on investors who, by holding equity, are the residual

[2] As described below, we later introduce various frictions that can be at least partly resolved through capital requirements, so that capital regulation plays a role in solving a social problem. We show that even in that context, where capital regulation can increase social welfare, the incidence of the regulation falls differentially on different classes of investors. In particular, it is not the households that are most inclined to be equity investors that bear the brunt of the regulatory burden.

claimants of the banks.

We then extend our baseline model to consider various market failures that may justify a need to introduce minimum capital requirements. We start by considering externalities that arise as a result of "fire sales" when a large number of banks fail, thus depressing recovery values on bank assets. In that setting, recovery values are lower the more banks are unable to meet their repayment obligations, so that a bank's failure has a negative externality on other banks, which ultimately is reflected in decreased overall value. By reducing the number of banks, capital regulation helps solve the inefficiency as it reduces the externality. Nevertheless, the burden of regulation is felt most strongly by households that would normally choose to be depositors rather than shareholders, as above.

Next, we study the implications of having deposit insurance. We first show that deposit insurance entails a trade-off. On the one hand, by reducing the interest rate that needs to be promised to depositors, deposit insurance can reduce the probability of costly bankruptcy and thus increase social welfare, all things equal. On the other hand, it introduces a distortion in banks' capital structure decisions since it leads banks to lever up as much as possible, using no equity and overly exposing themselves to bankruptcy risk. We then show that capital regulation can again help rectify the distortion and achieve greater surplus, but the primary benefit of the regulation accrues to shareholders rather than to households that hold deposits.

As a third scenario, we consider a situation where banks impose an externality on other sectors of the economy in that their failure entails a social cost which banks do not internalize. We show that in this case capital regulation lowers the equilibrium return to shareholders so that investors that choose to be shareholders would be inclined to oppose any capital increase. Even so, it is still depositors that bear most of its burden.

Finally, we introduce a standard limited-liability induced risk shifting problem where capital reduces the moral hazard problem for banks (see, e.g., Holmström and Tirole, 1997). In such a context, we first show that capital adequacy standards can increase social welfare while reducing the surplus in the banking sector. We then show that the return to shareholders is reduced with capital regulation but its incidence is again borne mostly by depositors. This suggests that the effect of capital regulation depends very much on the precise market failure that regulation should address.

Our work is related to various strands of literature. A sizable literature[3] has considered the role of bank capital as a buffer or as an incentive mechanism to control moral hazard. Most of these studies support the view that banks, if left unregulated, hold inefficiently low levels of capital because of market failures or the presence of externalities, which in turn renders them excessively prone to failure.[4] Thus, imposing minimum capital standards on banks should increase welfare, and much of the literature has focused on the mechanisms through which capital standards act. Far less explored, however, is the question of where the incidence of such regulatory intervention falls.

Our findings that higher capital standards may be to the detriment of depositors is related to the work by Besanko and Thakor (1992) and Repullo (2004). Both contributions discuss the consequences of tighter capital standards in a spatial model of imperfect bank competition, and show that deposit rates fall because capital regulation forces banks to substitute some deposits for equity, which in turn prompts lenders to compete less aggressively for depositors, even as returns on bank equity remain exogenously fixed. In contrast, in our model all markets are perfectly competitive, and equity returns, deposit returns, and the degree of equity market participation are all obtained endogenously in general equilibrium. This enables us to compare the incidence of regulation on all bank claimants. From this perspective, our model is closest to Allen, Carletti, and Marquez (2015), where limited participation in the equity market is the key friction, although it is still exogenously fixed. By contrast, in our model, the degree of participation is endogenous and is a function of the wedge in the expected return between equity and deposit markets, with greater participation, as called for

[3]See e.g., Holmström and Tirole (1997); Hellmann, Murdock, and Stiglitz (2000); Morrison and White (2005); Dell'Ariccia and Marquez (2006); Allen, Carletti, and Marquez (2011); Mehran and Thakor (2011); see Thakor (2014) for a survey.

[4]Berger and Bouwman (2013) document empirically that well-capitalized banks are more likely to withstand a financial crisis.

by higher regulatory capital standards, requiring banks to channel a larger fraction of surplus to equity investors rather than depositors.

Our finding that depositors, who are the investors with the least elastic demand for investment vehicles, bear most of the burden of capital requirements is reminiscent of studies that have investigated the incidence of taxation. For example, Huizinga, Voget, and Wagner (2014) have documented empirically that international corporate income taxation of banks is reflected in higher pre-tax interest rate margins, suggesting that the incidence of taxation falls primarily on bank customers and depositors rather than on shareholders.

By focusing on the endogenous degree of participation in the bank equity market, our paper is related to the literature analyzing households' limited participation in financial markets. Taking households' unwillingness to participate as given, an important body of literature has focused on the implications of this limited participation on the equilibrium pricing of assets. For instance, Allen and Gale (1994) study how limited market participation by potential investors can lead to amplified volatility of asset prices. Vissing-Jørgensen (2002) uses limited market participation by households to help explain part of the equity premium puzzle in financial economics. Several studies in the household finance literature also feature an endogenous degree of household participation in equity markets which arises due to heterogeneous household characteristics, just like in our model. For instance, Gomes and Michaelides (2005) calibrate a lifecycle model with equity market participation cost and heterogeneous risk aversion and show that it generates realistic stock market participation rates and household asset allocations. Lusardi, Michaud, and Mitchell (2017) show in a life-cycle model that the heterogeneous cost of acquiring financial knowledge limit stock market participation of less wealthy households and can account for a significant portion of wealth inequality. Our paper contributes to this literature by showing that investors' reluctance to hold risky assets not only affects household wealth, asset allocations, or stock volatility, it also implications for the capital structure of financial firms.

The paper proceeds as follows. The next section lays out the model. Section 3 contains the main analysis of the model, and the characterization of equilibrium. Section 4 looks at social welfare and studies the effects of a binding capital requirement. Section 5 extends the baseline model to include various settings where there is a social inefficiency which capital regulation can help address. Finally, Section 6 concludes. All proofs are relegated to Appendix A.

2 A frictionless benchmark

We develop a simple one period ($T = 0, 1$) benchmark model of financial intermediation with banks and investors that can provide funds in the form of equity capital or deposits. There exist two investment options: one is a safe *storage* technology which yields in $t = 1$ a return of one on every unit of funds invested at $t = 0$; the other is a *risky* investment which, for every unit of funds invested at $t = 0$, yields in $t = 1$ a risky return of r uniformly distributed in $[R - S, R]$, with $0 < S \leq R$ and $R > 2$. These conditions ensure that the expected return of the risky technology is at least as high as investing in the storage technology for any value of S.

There is a continuum of mass M of risk-neutral investors, endowed with one unit of wealth each. Investors may either invest directly in the storage technology, or they can place their wealth in a bank, either as depositors or as equity holders. However, investors are reluctant to hold anything other than a bank deposit, so that they have to be induced to hold risky, more junior securities such as bank equity. Specifically, each investor i incurs a non-pecuniary cost c_i to become *sophisticated* and be willing to hold bank equity. This cost may stand for the cost of acquiring the necessary skills to trade in equity markets or for heterogeneous taste for safe assets in the investor population, and a consequent disutility associated with holding junior, leveraged claims. The cost c_i, which is i.i.d. across investors and known to each investor i, is drawn from a uniform distribution on the support $[0, C]$. As a result, investors will only be willing to hold bank equity if the expected return from doing so exceeds the return from investing in either storage or a bank deposit by at least the cost c_i.

Banks are primarily vehicles that provide investors with access to the risky technology. Each bank finances itself with an amount of capital k and an amount of (uninsured) deposits $1-k$ and invests in the risky technology.[5] This implies that, by becoming shareholders in a bank, investors de facto take a position in the risky technology. We denote the promised per unit deposit rate as r_D, and the equilibrium expected return to bank deposits and to bank capital as u and ρ, respectively.

Banks are subject to bankruptcy if they are unable to repay their debt obligations. This occurs when $r < (1-k)r_D$, that is, when the realized return from the risky technology is lower than the total promised repayment to depositors. Bankruptcy is assumed to be costly. For simplicity, we make the extreme assumption that in the event of bankruptcy, all the project's return is dissipated.[6] Finally, we assume that the banking sector is perfectly competitive. Free entry reduces excess returns to zero, and banks behave as price takers with respect to the equilibrium expected return on bank capital ρ and bank deposits u.

3 Optimal capital structure

The equilibrium of the model is pinned down by the following conditions:

1. Investors optimally decide whether to become sophisticated, and invest to maximize their expected wealth;
2. Banks choose capital k and a promised deposit rate r_D to maximize expected excess returns;
3. The market for bank deposits clears;
4. The market for bank equity clears;
5. Free entry reduces bank excess returns to zero in equilibrium.

[5] Given there are constant returns to scale, normalizing the size of every bank to 1 is without loss of generality.

[6] We relax this assumption later in Section 5.1.

We start by analyzing investors' optimal investment strategy. At time $t = 0$, each investor i decides whether to become sophisticated based on how the difference in returns between equity and deposits, $\rho - u$, compares to the relevant cost c_i. Whenever $\rho - u < C$, there exists a marginal investor that is indifferent between earning the return u on deposits and earning the return ρ on bank capital minus his cost c_i. The marginal investor's cost must therefore satisfy $\widehat{c} = \rho - u$. For a given spread $\rho - u$, there is then a mass of investors K that choose to become sophisticated and hold bank equity, where

$$K = \begin{cases} M\frac{\rho-u}{C} & \text{if } 0 \leq \rho - u < C \\ M & \text{if } \rho - u \geq C. \end{cases}$$

The remaining mass of investors, $D = M - K$, find it optimal to either deposit their funds at a bank, or invest in storage.

Turning to the banks' problem, each bank chooses its capital k and its promised deposit rate r_D to solve the following problem:

$$\max_{k,r_D} E\left[\Pi_B\right] = \frac{1}{S}\int_{\max\{r_D(1-k),R-S\}}^{R} (r - (r_D(1-k))dr - \rho k \tag{1}$$

subject to

$$E\left[U_D\right] = \frac{1}{S}\int_{\max\{r_D(1-k),R-S\}}^{R} r_D\, dr \geq u \tag{2}$$

$$E\left[\Pi_B\right] \geq 0 \tag{3}$$

$$0 \leq k \leq 1. \tag{4}$$

Expression (1) represents the expected excess return of the bank. The first term captures the bank's return from investing in the risky technology, net of the payment $r_D(1-k)$ to depositors. Such a return is positive only when the bank does not go bankrupt, that is for $r \geq r_D(1-k)$ if $r_D(1-k) > R - S$ and for any r if $r_D(1-k) < R - S$. The second term

ρk reflects the return to shareholders for providing bank capital. Constraint (2) captures depositors' participation constraint. It requires that the payoff depositors receive when the bank remains solvent, that is for $r \geq \max\{r_D(1-k), R-S\}$, is at least equal to their opportunity cost u. Constraints (3) and (4) ensure that the bank is active and that the chosen capital structure lies within the feasible range. Given that there is free entry into the banking market, each bank's expected excess returns must be zero. This implies that the return to shareholders ρ adjusts so that $E[\Pi_B] = 0$.

Denote as N the equilibrium number of banks. We can now characterize the equilibrium.

Proposition 1. *The model has a unique equilibrium. There exist two boundaries, $\underline{R}(S) < \bar{R}(S)$, both increasing in S such that in equilibrium:*

i) For $R \in [S, \underline{R}(S))$ some funds remain in storage and banks are risky, with $k = \frac{4S}{R^2} - 1$, $r_D = \frac{R}{2}$, $\rho = \frac{2S}{4S-R^2}$, $u = 1$, and $N = MR^2 \frac{R^2-2S}{C(4S-R^2)^2} < M$.

ii) For $R \in \left[\max\{\underline{R}(S), S\}, \bar{R}(S)\right)$, all funds are invested in banks and banks are risky, with $k = \frac{R}{\sqrt{R^2+8CS}}$, $r_D = \frac{R}{2}$, $\rho = \frac{R}{4S}\left(R + \frac{R^2+4CS}{\sqrt{R^2+8CS}}\right)$, $u = \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right) > 1$, and $N = M$.

iii) For $R \geq \bar{R}(S)$, all funds are invested in banks, and banks are safe, with $k = 1 - \frac{R-S}{u^*}$, $r_D = u^*$, $\rho = \frac{Su^*}{2(u^*-R+S)}$, $u = u^*$, and $N = M$, where u^* is the unique positive solution to $\frac{2R-S}{2} - u^* - C\left(1 - \frac{R-S}{u^*}\right)^2 = 0$.

In all three cases, $\rho > E[r] > u$ holds.

The proposition shows that banks always find it optimal to raise some deposit financing, with the exact amount depending on the profitability of the risky technology, as measured by the boundaries for project returns R and S. Although banks are always levered, they do not always choose a capital structure that exposes them to bankruptcy risk. In particular, when R is very large, so that expected project returns are big, banks may choose a safe capital structure characterized by very low debt and no bankruptcy risk. The optimal level of capital in this case is simply the lowest level of capital consistent with zero bankruptcy risk.

In all other regions for R, the amount of leverage optimally chosen by the bank is such that the bank exposes itself, and hence its investors, to default risk since realized project returns may be insufficient to cover the promised payment to depositors, $r_D(1-k)$. The reason is that in these regions the project is not quite as profitable, but is still risky. Given this, using leverage increases the return to shareholders since deposits earn a lower equilibrium return, allowing a greater premium to be paid to investors willing to become equityholders. When project returns are relatively low (i.e., for $R \in [S, \underline{R}(S))$), at the optimum the amount of funds allocated to the banking sector, N, is less than the total funds available, M, and some households choose to keep their funds in storage. In this region, which we will refer to as *partial inclusion*, the equilibrium return to depositors is pinned down by the return to storage, 1, since households view deposits as equivalent to storage and must be indifferent between holding one or the other.[7]

For more intermediate levels for the project return (i.e., for $R \in [\max\{\underline{R}(S), S\}, \bar{R}(S))$, the investment projects are sufficiently profitable that the equilibrium allocation has all funds flowing to the banking sector, $N = M$, and households either hold deposits or become equityholders. Either way, in this region, which we will refer to as *full inclusion*, no household finds it optimal to put its funds into storage. In this region, the equilibrium return to depositors is no longer driven by the return to storage, but rather increases as project returns increase – u is increasing in R, the maximal return on the projects.

Finally, the proposition also shows that, irrespective of the specific capital structure banks choose, the equilibrium return for equityholders is always higher than the project's expected return, which itself is always higher than the return to depositors, that is $\rho > E[r] > u$. The

[7]The model presented above treats deposits as equivalent to storage from the perspective of households that are averse to participating in financial markets. It may be argued, however, that households' limited participation in financial markets stems partly from a demand for safe assets, and households are either suspicious or financially ignorant about more complex products which are subject to default risk. In appendix B we study an extension where households have a demand for *safe* assets and show that our results on the incidence of capital regulation are not affected by this assumption.

reason is that deposits earn a lower return given the alternative investment for investors to deposit is storage. This in turn allows for greater surplus to be distributed to bank equity holders.

Given the main features of the equilibrium, we can now pin down the expressions of the equilibrium bankruptcy probability as follows:

Corollary 1.1. *The equilibrium probability of bankruptcy is given by*

$$\Pr(\textit{bankr.}) = \begin{cases} 1 - \frac{2}{R} & \textit{if } R \in [S, \underline{R}(S)) \\ 1 - \frac{R}{2S} - \frac{R^2}{2S\sqrt{8CS+R^2}} & \textit{if } R \in [\max\{\underline{R}(S), S\}, \bar{R}(S)) \\ 0 & \textit{if } R \geq \bar{R}(S) \end{cases}$$

In the regions where $R < \bar{R}(S)$ and capital structure is risky, the probability of bankruptcy increases in R if $S \leq R < \underline{R}(S)$, while it decreases in R otherwise.

Proof. Because bankruptcy cost are 100%, we must have $u = (1 - p_B)r_D$ from which the statement follows immediately. □

The corollary establishes that when banks choose a risky capital structure, bankruptcy risk increases in project returns for low levels of R, and then decreases with the project's return for larger levels of R, when $N = M$. As R becomes very large, the bank's optimal capital structure converges to deliver zero risk of bankruptcy, consistent with the fact that at some point the equilibrium becomes safe and there is no longer any bankruptcy risk.

In what follows, we will concentrate the analysis on the regions with positive bankruptcy risk, which is the more empirically relevant case, rather than the region with very high project returns where the equilibrium is safe and involves no default risk. Given this, we next establish the following comparative statics.

Corollary 1.2. *The following comparative statics results hold in the region of the risky equilibrium, $R < \bar{R}(S)$:*

i) For given variance of the project returns, $var(r)$, an increase in the expected project return $E[r]$ decreases k and increases ρ for $R \in [S, \underline{R}(S))$, while it increases k, ρ and u for $R \geq \max\{\underline{R}(S), S\}$.

ii) For given $E[r]$, an increase in $var(r)$ increases k and decreases ρ for $R \in [S, \underline{R}(S))$, while it reduces k, ρ and u for $R \geq \max\{\underline{R}(S), S\}$.

iii) An increase in the average market participation cost, $E[c_i]$, leaves k and ρ unchanged and reduces the number of banks N for $R \in [S, \underline{R}(S))$, while it decreases k and u, increases ρ and leaves N unchanged for $R \geq \max\{\underline{R}(S), S\}$.

Part (i) of the corollary shows that an individual bank's capital is not monotonic in the expected project return, being counter-cyclical in recessions (when R is low) and pro-cyclical in boom phases (when R is larger). This result, which arises from the endogenous supply of funds by investors, contrasts with Allen, Carletti, and Marquez (2015) where both capital and the bankruptcy probability become fixed in the full inclusion region where all funds are invested in the banking sector since they assume markets are segmented and households cannot move from one investment into the other at any cost.

The corollary also shows that as $E[r]$ increases, shareholders always benefit through an increased expected return ρ, while depositors do so through an increased u only in the case of full inclusion, when $N = M$. In other words, when R is low so that $N < M$ and storage is in use, an increase in $E[r]$ creates a greater wedge between the return of the risky projects and that of storage. Here, as project returns increase, the bank finds it optimal to adjust its capital structure in order to maximize the return to equityholders. When R is relatively low, so that $N < M$, further increases in R increase the total amount of capital used in the banking sector, $K = kN$, but reduce the amount of capital at each individual bank, k. The reason is that it becomes optimal to operate more projects, so that N increases, with each bank trading off a riskier capital structure (i.e., higher leverage) with greater returns conditional on success of the projects.

Once R is larger and $N = M$, all funds are in use in the banking sector and it is no longer possible to allocate more funds to productive projects. For a given bankruptcy probability, a higher $E[r]$ increases the expected bankruptcy losses since project returns are greater. As a result, from Corollary 1.1, banks find it optimal to reduce their leverage and protect themselves through higher equity capital. This reduces bankruptcy risk and leads to an increase in the equilibrium return to capital holders, ρ, as well as in the aggregate amount of capital being used by the banking sector, K. The shift in expected return to investors, the number of banks, and capital holdings, individual as well as aggregate, is illustrated in Figure 1.

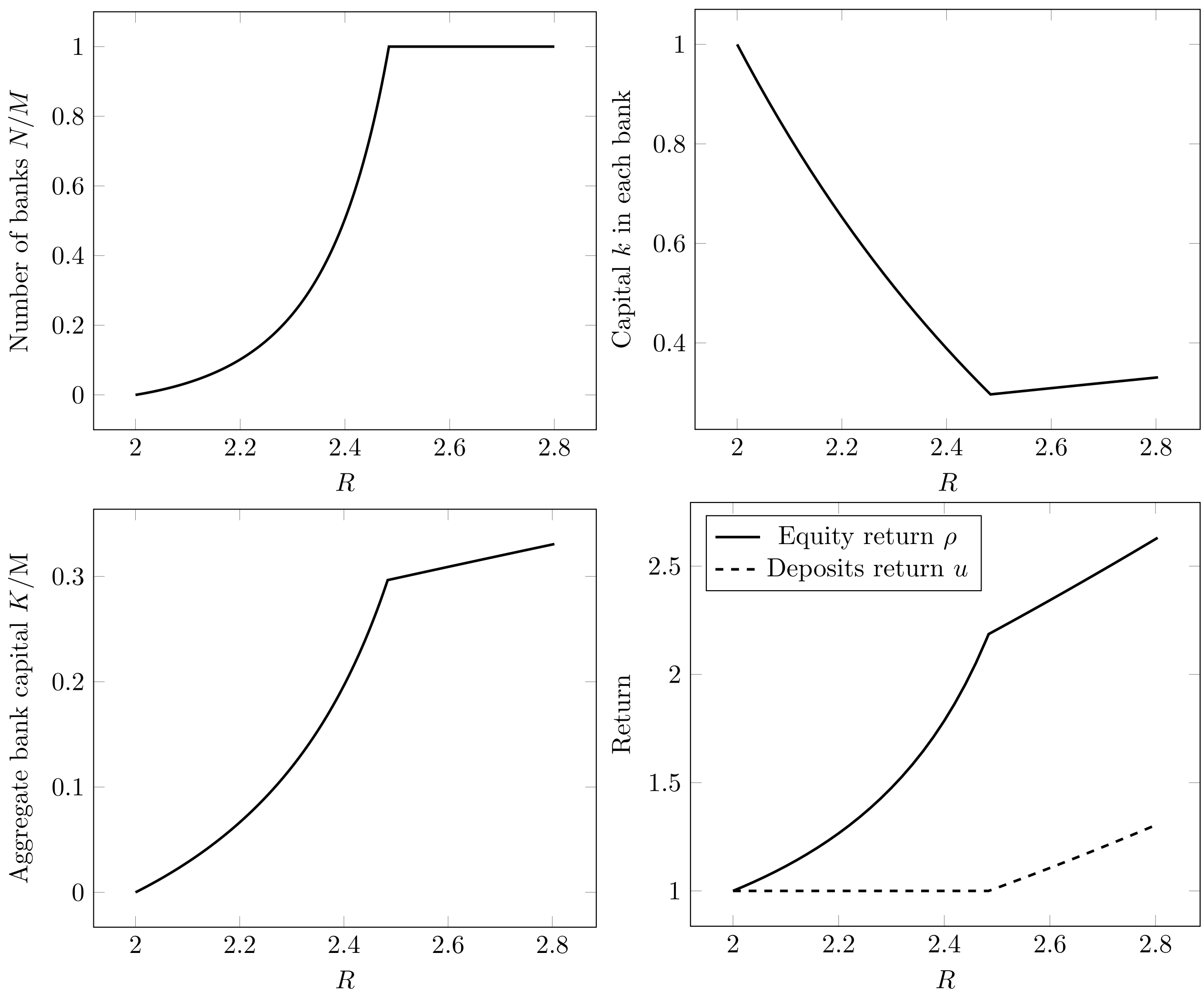

Figure 1: Equilibrium number of banks N, bank level capital k, aggregate capital K and equilibrium returns u, ρ as a function of R. Parametrization: $C = 4, S = 2$

Part (ii) of the corollary states that k, ρ and u react differently from part i) when a mean preserving spread (MPS) increases the variance of the project returns without affecting the average. Capital increases now in the region where $N < M$ and decreases when $N = M$, while ρ decreases throughout. With partial inclusion (i.e., $N < M$), a MPS increases the probability of bankruptcy, which, ceteris paribus, reduces the expected payoff to bank depositors. As a result, banks increase capital to reduce bankruptcy risk and retain depositors. This leads to fewer banks and less aggregate capital. A similar logic applies to the case with full inclusion (i.e., $N = M$) where $u > 1$, but now banks can offset part of the increased bankruptcy risk by reducing the equilibrium return to depositors without risking that depositors move back into storage. To see why capital decreases in this region, consider that an incremental unit of capital reduces bankruptcy risk and increases bank return by a factor of $\frac{1}{S}$, which corresponds to the density of the distribution of project returns around the default boundary.[8] As a MPS increases S, it reduces the marginal value of holding capital for banks, thus inducing them to reduce its use. Thus, k falls with a MPS, as do ρ and u, the equilibrium returns to capital holders and depositors, respectively.

Part (iii) is more intuitive since it relates to increases in the cost of becoming sophisticated. Suppose that there is an increase in the average cost of investors to hold equity. Under incomplete inclusion ($N < M$), u and, indirectly, also ρ are dictated by the storage outside option, so the spread $\rho - u$ remains constant. Therefore, there must be fewer investors who are willing to become capital providers under the higher cost.

[8] This can be seen directly from the maximization problem for the bank: $\max_{r_D,k} \frac{1}{S}\int_{r_D(1-k)}^{R} (r - r_D(1-k))\, dr - pk$, subject to depositors' participation constraint, $\frac{1}{S}\int_{r_D(1-k)}^{R} r_D dr = u$. Substituting the participation constraint into the bank's maximization problem yields

$$\max_k \frac{1}{S}\int_{r_D(1-k)}^{R} rdr - (1-k)\,u - pk.$$

The first order condition is $\frac{1}{S} r_D^2 (1-k) - (p-u) = 0$, or $k = 1 - \frac{S(p-u)}{r_D^2}$, which is decreasing in S for given ρ, u, and r_D, meaning that the marginal incentive to hold capital is lower.

4 The incidence of binding capital requirements

We now turn to the question how bank capital regulation affects the banks and, by extension, the households who represent banks' investors. We start with the frictionless benchmark we have studied so far to understand how, in the absence of any frictions, the burden imposed by regulation is distributed across the various parties. In other words, since in the absence of frictions capital regulation is distortionary, the framework so far provides an efficient benchmark to understand who primarily bears the burden of regulation. The possible benefits of regulation will be discussed below.

To study this issue, we first confirm that our benchmark model indeed provides a market solution that is coincident with what would be socially efficient, so that the banking sector fully internalizes the (social) cost borne by investors in overcoming their aversion to holding bank equity. To this end, we consider the case where a social planner chooses bank capital k_P to maximize social welfare (i.e., investors' aggregate returns net of aggregate participation costs), while deposit rates are set by the banks in order to maximize their expected excess return. This means that the planner solves

$$\max_{k^{FB}} SW = \rho K + uD - \int_0^{\rho - u} M \frac{c}{C}\, dc \tag{5}$$

subject to

$$r_D = \arg\max_{r_D} \frac{1}{S} \int_{r_D(1-k^{FB})}^{R} (r - (r_D(1-k^{FB}))dr - \rho k^{FB} \tag{6}$$

and (2), (3), and (4). The constraints are as in the market solution except constraint (6), which indicates that the deposit rate r_D is chosen by shareholders to maximize bank excess returns. We can now state the following result.

Proposition 2. *The socially optimal allocation of bank capital coincides with the competitive capital structure: $k_P = k^*$.*

Although banks behave as price takers in the competitive equilibrium and do not individually consider the impact of their capital structure choices on the equilibrium rate of return on capital, they end up issuing a socially optimal amount of bank capital. The reason is that there is no pecuniary externality in the bank equity market in our model. Banks maximize returns to the benefit of bank shareholders and, given the market for capital is competitive, they ultimately internalize investors' costs to become sophisticated and willing to hold bank equity. In other words, the market equilibrium is equivalent to the socially optimal solution.

Having shown that the market solution is efficient, so that there is no friction for capital regulation to overcome, we next study the issue of the incidence of regulation, and specifically of binding capital requirements. Note first that any capital requirement imposing $k^{reg} \leq k^*$ will not be binding since banks will prefer to choose k^* over the regulatory minimum. Therefore, we restrict our analysis to cases where $k^{reg} > k^*$.

Proposition 3. *Suppose that $k^{reg} > k^*$, and define K^* and K^{reg} as the aggregate amount of capital in the market and regulatory solution, and N^* and N^{reg} as the corresponding number of banks so that $K^* = N^* k^*$ and $K^{reg} = N^{reg} k^{reg}$.*

1. *Assume $N^{reg} < M$, with $u^{reg} = 1$. Then, $\frac{d\rho^{reg}}{dk^{reg}} < 0$ and $\frac{dN^{reg}}{dk^{reg}} < 0$ so that $K^{reg} < K^*$.*
2. *Assume $N^{reg} = M$, with $u^{reg} > 1$. Then, $\frac{du^{reg}}{dk^{reg}} < \frac{d\rho^{reg}}{dk^{reg}} < 0$ so that $K^{reg} > K^*$.*

The proposition establishes that binding capital requirements either lead to fewer banks operating, $N^{reg} < N^*$, or to the burden being borne disproportionately by depositors. The first result, which arises when $N^{reg} < M$, so that some investors use storage and not all funds are allocated to the banking sector, gives rise to an inefficiency since productive projects are funded through banks. Therefore, reducing the total funds that flow to the banking sector reduces the number of banks in equilibrium. In other words, the deadweight loss associated with capital requirements here is reflected in lower output being produced. Moreover, since $N^{reg} < N^*$, this implies that there is a region of parameter values R, S, and C such that for $N^{reg} < N^* = M$, depositors are made strictly worse off by binding capital requirements since without regulation they would earn an expected return of $u^* > 1$, whereas they earn only a return equal to 1 when banks are subject to capital regulation.

The second result highlights how binding capital requirements affect the different classes of investors. Since the market solution also maximizes aggregate output, a binding capital requirement leads to less total surplus. Since the number of projects that are financed remains constant with regulation (for local changes in the amount of capital around the market solution k^*), the deadweight loss arises from the increased participation costs borne by the additional investors that need to be induced to hold bank capital. While the equilibrium return to equity holders, ρ, decreases, the return to depositors, u, decreases *even more* because the difference between them, $\rho - u$, must increase in order for more investors to be willing to hold bank capital. Thus, while shareholders earn a lower return, reflecting the greater deadweight losses and lower aggregate output, the bulk of the losses are borne by depositors, who bear much of the burden of the increased capital requirement.[9] It is therefore the investors who in principle should be protected most by larger capital requirements (as indeed they are since bankruptcy risk is reduced) who ultimately pay for this protection through more than commensurate reductions in the return they earn in equilibrium.

5 Market frictions and bank capital regulation

In the baseline model with no frictions, banks' capital structure decisions are socially efficient so that there is no scope for capital regulation. As a consequence, any binding capital requirement leads to a social loss, which results in lower equilibrium returns to investors or lower aggregate output since fewer productive investments get funded. Now we extend the model to incorporate various frictions that can, at least partly, be resolved through capital

[9]Throughout the paper we measure the incidence of capital regulation in terms of the difference in the monetary returns between shareholders and depositors, i.e., $\rho - u$. An alternative is to assess the incidence in terms of households' utility, thus comparing the expected utility to shareholders, $\rho - \bar{c}$, to the utility u accruing to depositors, where $\bar{c}$ represents the average cost paid by households that become shareholders. Using $\rho - u = \widehat{c}$, we have $(\rho - \bar{c}) - u = \widehat{c} - \bar{c} = \frac{\widehat{c}}{2}$. Clearly, the imposition of capital requirements $k^{reg} > k^*$ increases the marginal cost more than the average cost of becoming shareholders, so that the wedge in utility goes up as well.

requirements.

Specifically, we study some canonical market failures associated with financial intermediaries. The first is the presence of externalities in the recovery value of assets that may arise when many banks fail at once – "fire sales" – and that may depress asset values. The second market failure derives from the introduction of deposit insurance, which provides an implicit or explicit subsidy for raising deposits rather than equity, and tilts banks' capital structures toward being excessively levered. Next, we study a situation where banks impose an externality on other sectors of the economy. Finally, we introduce a moral hazard (or risk shifting) problem induced by limited liability, which leads banks to exert too little effort and take excessive risk. We first establish that in all these instances capital regulation can help improve the market solution and thus increase social welfare. We then show that depositors are not the primary beneficiaries from such regulation. When it increases the output of the banking sector, as in the first two cases, capital regulation redistributes returns in favor of equity holders, and in fact allows these households to capture much of the welfare gains that come with it. When, by contrast, as in the last two cases we analyze, it reduces the surplus in the banking sector, capital regulation lowers the returns to the providers of funds to the banks but its incidence is borne mostly by depositors.

For simplicity, in what follows we assume that $S = R$, so that project returns are distributed uniformly in $[0, R]$. This simplifies calculations substantially, without affecting the main implications of the analysis related to the incidence of bank capital requirements.

5.1 Fire sale externalities

So far we have assumed that in the case of bankruptcy the entire project return is dissipated. Consider now a modification where liquidation yields a recovery value equal to a fraction $h < 1$ of the realized cash flow r and that such a value depends on how many other banks are in default and thus being liquidated. In other words, losses under bankruptcy are equal to $(1 - h)r$, where h decreases in the number of active banks N. This captures the idea that the failure of many banks at once depresses asset prices for all banks that are being liquidated – a "fire sale" externality.

As before, each bank chooses the amount of capital k that maximizes its expected excess returns, as given by (1). The only change to the bank's problem stems from depositors' participation constraint, which now incorporates that depositors may receive something in the event of bankruptcy, and is given by

$$E\left[U_D\right] = \frac{1}{R}\int_0^{r_D(1-k)} \frac{hr}{1-k}\,dr + \frac{1}{R}\int_{r_D(1-k)}^{R} r_D\,dr \geq u. \tag{7}$$

The recovery under bankruptcy is reflected in the first term, $\frac{hr}{1-k}$, while the second term is the promised repayment, which is made whenever $r \geq r_D(1-k)$, as before.

As is typical when there is an externality, banks choose their capital structure disregarding the effect of their choice on the equilibrium asset liquidation value. By contrast, a social planner would choose the amount of capital at each individual bank to maximize total surplus as given by

$$SW = N\frac{1}{R}\int_{r_D(1-k)}^{R} r dr - \int_0^{\widehat{c}} M\frac{c}{C}\,dc + M - N + N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr. \tag{8}$$

Denote as k^* and k^{reg} the solutions to the decentralized and the social planner's problems and as N^* and N^{reg} the number of banks in the respective case. We then have the following result.

Proposition 4. *In the case of fire sale externalities, we have $k^{reg} \geq k^*$ and $N^{reg} \leq N^*$, with the inequalities strict whenever $N^{reg} < M$.*

The proposition establishes that there is again a social value to requiring banks to hold more capital than what they are inclined to do as a way of reducing the externalities associated with fire sales in asset prices. By requiring banks to hold more capital, not only do banks face lower bankruptcy costs but, more importantly, the social planner succeeds in reducing the number of banks that will operate and, hence, possibly go bankrupt. This contraction in the number of banks reduces bankruptcy costs through greater recovery values as h is negatively correlated with N, and increases social welfare.

We now turn again to the question of who benefits from the increased surplus. Define ρ^* and ρ^{reg} as the return to shareholders and u^* and u^{reg} as the return to depositors in the market and in the social planner solutions, respectively. We have the following.

Corollary 4.1. *When there are "fire sale" externalities, so that recovery values under liquidation are decreasing in the number of banks that default, we have $\rho^{reg} - u^{reg} > \rho^* - u^*$ for $N < M$.*

The result establishes that shareholders are again the primary beneficiaries of the increased surplus generated by the introduction of capital regulation. The intuition is similar to that in the case of bank moral hazard. Capital regulation increases total surplus, which again must be allocated between households that become depositors and those that become equityholders. When $N < M$, the surplus accrues entirely to shareholders since depositors' expected return is $u = 1$ and moreover $1 - k$ decreases. Thus, ρ goes up, which also implies that a greater amount of capital on aggregate (K) flows to the banking sector despite the reduction in the number of banks. Once $N = M$, further increases in capital requirements no longer affect the number of banks that operate and can possibly fail, and so there is no further role for binding capital requirements. As a result, in that region the regulatory solution coincides with the market solution.

To sum up, the presence of fire sale externalities that may arise when banks are liquidated provides a rationale for the introduction of capital requirements. The increased social surplus banks generate leads to a greater use of capital in the banking system, although also to fewer active banks. As before, the increased surplus is entirely appropriated by shareholders through higher returns. Households that choose to remain depositors, while not made worse off, do not benefit from capital regulation that increases aggregate surplus.

5.2 Deposit Insurance

In the analysis above we assumed that deposits are not insured, so that the interest rate on deposits fully reflects the bank's risk of default. While this induces the bank to raise equity to control risk, it leaves some risk onto depositors. Suppose now instead that deposits are fully insured so that they become safe assets. As has often been argued (see, e.g. Boot and Greenbaum, 1993; Demirgüç-Kunt and Detragiache, 2002), the introduction of deposit insurance encourages excessive risk taking by reducing banks' incentives to raise capital, thus providing a motivation for capital regulation.

To study the effect of deposit insurance, we assume here that deposits are fully insured, so that even if the bank goes bankrupt depositors are repaid in full by the deposit insurance fund. For simplicity, we also assume that the insurance is financed from the proceeds of non-distortionary lump sum taxes.[10] In this case, the bank chooses capital k so as to maximize

$$\max_{k,r_D} \frac{1}{R}\int_{r_D(1-k)}^{R} \left(r - r_D(1-k)\right)\, dr - \rho k \tag{9}$$

subject to

$$E\left[U\right] = r_D = u. \tag{10}$$

The provision of insurance distorts banks' capital structure decisions, as shown in the following result.

Proposition 5. *In the presence of deposit insurance, the unique equilibrium is characterized by $k^* = 0$, $r_D^* = u^* = R$ and $N^* = M$.*

In equilibrium banks hold zero capital and promise depositors the highest possible return. Hence, they go bankrupt with probability one and, as a result, deposit insurance always pays a total of

$$DI = N\frac{1}{R}\int_0^{r_D(1-k)} r_D(1-k)\, dr = MR \tag{11}$$

[10] The results continue to hold if deposit insurance is priced such that it is actuarially fair from an ex-post perspective (see Allen, Carletti, and Marquez, 2015).

to depositors. This occurs because the rate on deposits is insensitive to the probability of bankruptcy and thus independent of the amount of leverage banks choose. It follows that banks have no incentive to raise capital as a way of reducing bankruptcy risk since their cost of borrowing (i.e., deposits) will not reflect this reduction in risk. In the end, banks' extreme leverage choices impose a clear social cost and reduce aggregate welfare.

We next show that minimum capital requirements can reduce the deadweight losses due to bankruptcy and increase welfare. To see this, suppose that a regulator imposes a minimum bank capital requirement which is chosen as to maximize welfare,

$$\max_{k^{reg}} SW = \rho K + uD - \int_0^{\rho - u} M \frac{c}{C}\, dc - DI. \tag{12}$$

subject to constraints (2)-(4) and (6). The primary difference between the regulator's objective function and that of the banks is that the regulator internalizes the cost of providing deposit insurance. Denote as k^* the market solution in Proposition (5) and as k^{reg} and r_D^{reg} the solutions in terms of the level of capital and the deposit rate in the social planner's problem in (12). We obtain the following.

Proposition 6. *When deposits are insured, we have $k^{reg} > k^* = 0$ and $u^{reg} < u^*$.*

The proposition shows that the optimal regulatory solution entails banks holding a strictly positive amount of capital as a way of reducing the capital structure distortion induced by deposit insurance. As a consequence, given that a sufficiently high return has to be paid to create incentives for some households to hold capital, under regulation the deposit rate will be lower than the maximum return R. This result also implies that capital regulation, while helping solve an inefficiency, leads to redistribution of surplus away from depositors and toward households that become equityholders.

In fact, the redistributive consequences of the regulatory intervention in the presence of deposit insurance are very similar to those that we have observed in the two cases studied above, but they are even more extreme. In equilibrium, the imposition of capital requirements leads to a fall in u, thus lowering the return to households with higher participation costs. In other words, whilst higher capital standards make banks more stable and reduce deposit insurance expenditures, shareholders again benefit disproportionately from the regulatory intervention.

5.3 Social costs of bank failure

Up to now, we have considered cases where the primary rationale for regulation stems from either a type of coordination failure that leads one bank's decisions to spill over onto other banks through fire sales, or through a distortion in the pricing of claims that leads to an inefficient capital structure. In both of these cases, imposing a capital requirement reduces the friction and leads to greater overall output, either at the bank level or at the industry level, leaving more surplus to be divided between the claimants, depositors and equityholders.

However, not all regulation is geared toward improving efficiency and output within the banking sector. Rather, regulation may be (and often is) motivated by a need to correct a social inefficiency that the banking sector does not internalize. For instance, bank failure may impose externalities on bank customers, who cannot easily shift to obtain credit from other sources and thus bear losses whenever their main bank goes out of business or contracts lending. Similarly, any public funds that are used either in the provision of deposit insurance for failed banks, or in the management of the bank resolution process, are likely to have a higher social cost. To study this aspect of regulation which is *external* to the output produced directly by the banking sector, we introduce a simple modification to the model that captures in a reduced form various alternative rationales for regulation. In particular, we assume that, upon bankruptcy, the social cost of failure is $\psi \geq 0$ for every unit that is lost. In other words, while as before bankruptcy destroys 100% of the project's return r, there is an additional social cost of ψr that a social planner would internalize and which reflects the various factors described above, and may be seen as a "shadow cost" of government funds as well as actual losses borne by other stakeholders.[11] With this addition, the social planner's objective function becomes

$$\max_{k_P} SW = N\frac{1}{R}\int_{r_D(1-k)}^{R} r dr - N\frac{1}{R}\int_0^{r_D(1-k)} \psi r\, dr - \int_0^{\widehat{c}} M\frac{c}{C}\, dc + M - N. \qquad (13)$$

The only difference between (13) and (5) is the addition of the term $-N\frac{1}{R}\int_0^{r_D(1-k)} \psi r\, dr$, reflecting the higher social costs of bank failure than what is borne privately. We can now state the following result.

Proposition 7. *For $\psi > 0$, the social planner optimally chooses a minimum capital requirement greater than in the market equilibrium: $k^{reg} > k^*$. The equilibrium return to shareholders is lower than in the market equilibrium: $\rho^{reg} < \rho^*$. For the case where $N = M$, we also have that $\rho^{reg} - u^{reg} > \rho^* - u^*$.*

The proposition shows that, when capital regulation is necessary to solve problems stemming from externalities created by bank failure, the equilibrium return to shareholders will go down (the number of banks will also weakly decrease). As a result, investors that choose to be shareholders would be inclined to oppose any proposals to increase capital requirements since the constraint imposed would reduce their return in equilibrium. Coupled with the results from the previous sections, the results here point out that how capital regulation affects shareholders very much depends on what problem such regulation is attempting to remedy. Here, we show that capital requirements may also hurt shareholders, rather than benefitting them as in the sections above. Nevertheless, as in Proposition 3, it is still depositors who bear the main burden of capital regulation.

[11] An alternative setup is to consider that there is a social cost $r_D(1-k)\psi$ proportional to the investments made by depositors. The parameter ψ captures the "shadow cost" of government funds that may be required to either repay depositors, or to provide unemployment benefits, etc., which may be claimed by the investors that have suffered losses. This alternative setting is qualitatively similar to the analysis in the text.

5.4 Moral hazard induced by limited liability

The final friction we study stems from limited liability, which is often argued to induce moral hazard or risk shifting, thus creating an inefficiency. To study such a setting, we modify the model slightly to allow the bank or, equivalently, bank shareholders to take a privately costly action a aimed at increasing project returns. Specifically, we assume that by putting in effort a at time $t = \frac{1}{2}$, the bank can increase the project's return by $a > 0$ but bears a cost of $\frac{\eta}{2}a^2$. This action is taken after the bank has chosen its capital structure and financed its project, so that all variables of interest – u, ρ, r_D, and k – are taken as fixed when choosing a. If the bank fails, there is a social cost ψ per unit that is lost, as in Section 5.3, so that default risk imposes an externality.

The model is solved backward. At $t = \frac{1}{2}$, the bank maximizes

$$\max_a E\left[\Pi_B\right] = \frac{1}{R}\int_{\max\{r_D(1-k),a\}}^{R+a} \left(r - r_D\left(1-k\right)\right) dr - \rho k - \frac{\eta}{2}a^2, \tag{14}$$

taking k, ρ, and r_D as given. Assuming that $r_D(1-k) > a$,[12] the FOC with respect to the action a is

$$\frac{1}{R}\left(R + a - r_D(1-k) - Ra\right) - \eta a = 0,$$

from which we obtain

$$a^* = \frac{R - r_D\left(1-k\right)}{\eta R - 1}. \tag{15}$$

Note that the optimal action a^* is increasing in the amount of capital k at the bank but remains below the level $a = \frac{R}{\eta R - 1} > a^*$ that maximizes project returns. This is a standard result of the limited liability effect, given that the bank chooses its action a taking the deposit rate r_D and the capital structure k as given – see, e.g., Allen, Carletti, and Marquez (2011).

Turning to $t = 0$, the bank chooses its capital k and the deposit rate r_D so as to maximize

[12] This assumption, which is satisfied for values of η large enough, ensures that the action a that will be chosen will be small relative to the amount of deposits $1-k$ so that the bank remains subject to bankruptcy risk. Although the derivation is slightly different, the analysis also extends to the case where, for small enough values of η, the bank chooses a high enough that default never occurs, i.e., so that $r_D(1-k) < a$).

(14) after substituting the expression for a^* in (15), subject to the depositors' participation constraint now given by

$$E\left[U_D\right] = \frac{1}{R}\int_{r_D(1-k)}^{R+a^*} r_D\, dr \geq u, \tag{16}$$

and the same constraints (3) and (4) as in the baseline model.

Consider now the case where the amount of capital at each individual bank is chosen by a social planner that maximizes total surplus as given by

$$\begin{aligned} SW &= N\frac{1}{R}\int_{r_D(1-k)}^{R+a} r\, dr - N\frac{1}{R}\int_{a}^{r_D(1-k)} \psi r\, dr - \int_0^{\widehat{c}} M\frac{c}{C}\, dc + M - N - \frac{\eta}{2}a^2 \\ &= N\frac{1}{R}\int_{r_D(1-k)}^{R+a} r\, dr - N\frac{1}{R}\int_{a}^{r_D(1-k)} \psi r\, dr - \frac{1}{2C}M\widehat{c}^2 + M - N - \frac{\eta}{2}a^2, \end{aligned} \tag{17}$$

where both a and r_D are chosen by the banks and thus are equal to, respectively, a^* as in (15) and the value of r_D that solves (16) with equality. The term $-N\frac{1}{R}\int_a^{r_D(1-k)} \psi r\, dr$, as above, represents the social losses associated with bank failure, and now incorporates how the bank's effort decision, a, affects the probability of failure and hence of a social loss. We have the following result.

Proposition 8. *Define by k^* and k^{reg} the market and the social planner solutions, respectively. For the case of a bank moral hazard problem, when $\psi > 0$ we have $k^{reg} > k^*$.*

The proposition shows that the social planner prefers a higher level of capital k at each bank than in the market solution as a way of inducing greater effort and reducing the bank's moral hazard problem. As in the previous section, all things equal capital reduces a bank's probability of default, lowering the expected social cost of bankruptcy. Additionally, the accompanying increase in effort further reduces default risk, lowering the social cost yet further. As a consequence, each bank winds up less levered and faces lower bankruptcy costs. Thus, capital regulation is socially valuable as it leads to binding capital requirements that increase social welfare.

In line with the previous section, the primary rationale for regulation here is to reduce the social cost associated with bank failure, and the requirement to raise more capital than is individually optimal implies a loss of surplus to the banking sector. We now turn to analyze who ultimately bears this cost.

Corollary 8.1. *When there is a bank moral hazard problem and bank failure is socially costly* ($\psi > 0$)*, the equilibrium return to shareholders is lower than in the market equilibrium:* $\rho^{reg} < \rho^*$*. For the case where* $N = M$*, we also have that* $\rho^{reg} - u^{reg} > \rho^* - u^*$*.*

The corollary presents a similar result to that in Proposition 7 and states that either depositors fail to benefit from capital regulation (for $N < M$) or bear the brunt of the cost (for $N = M$). Capital regulation is socially beneficial for the two reasons highlighted above: capital directly reduces bankruptcy risk and hence the social costs of bankruptcy, and it does so indirectly as well by encouraging greater effort by the bank. Nevertheless, the private costs of regulation are borne to a large extent by depositors.

6 Conclusion

This paper presents an analysis of banks' optimal capital structures in a setting where investors may be reluctant to participate in financial markets by holding anything other than safe assets, such as bank deposits, and have to be induced to do so through the promise of higher returns. The equilibrium amount of capital market participation in the banking sector is thus endogenous, and depends on the distribution of returns associated with the investment opportunity set available to banks. We use this framework to study the incidence of capital regulation, and shed light on whether requirements geared toward reducing bank failure and absorbing losses that would otherwise accrue to depositors and by extension the deposit insurance fund affect various classes of investors differently.

Our focus throughout the paper has been on the impact of capital regulation on the sources of financing for the bank, studying which types of investors primarily bear the brunt, or reap the benefits, of regulation. In other words, we have emphasized the right hand side of the bank's balance sheet. Of course, there are other parties that interact with the bank which are also likely affected by regulation. A salient example is bank borrowers, particularly those that are dependent on their main bank for most financing, and who ultimately may bear part of the cost (or benefit) of regulation through changes in interest rate margins, or through the availability of credit. Likewise, some of the costs and benefits may fall on bank employees. While these aspects are likely important for understanding the full consequences of changes to regulatory requirements, our main findings related to how the wedge between shareholder and depositor returns responds to stricter bank capital standards should remain.

Still on the liability side of banks, we have limited the analysis to the case where households' only alternative to storage is to hold deposits or invest in the financial sector through banks. In practice, of course, there are other institutions, including non-financial firms, with needs for funding and who may wish to raise debt or issue equity. Studying how capital regulation for banks affects the equilibrium distribution of investment and the returns to various financial instruments when such firms are included seems like a useful avenue for future research. Nevertheless, we expect that our main finding concerning the incidence of regulation should continue to hold given it is primarily driven by the need to compensate investors more heavily the greater the need for equity financing is.

Finally, in our analysis, we have explicitly sidestepped issues related to the interaction of risk and leverage that are present when systematic risk is priced by assuming risk neutrality. Therefore, the standard results stemming from the work by Modigliani and Miller (1958) are not present, allowing us to isolate the effects stemming from limited market participation and capital regulation. An interesting issue, however, would be to consider how risk aversion, coupled with the existence of systematic risk, interact with the results we obtain here. At present, investors' reluctance to invest in equities implies in our framework that greater need or desire to issue capital (i.e., equity) by banks requires that investors earn a higher return in order to induce them to participate. By contrast, the usual logic of risk aversion and systematic risk implies that greater leverage makes equity riskier on a systematic basis, and increases its required return. The study of this issue introduces additional complexities to understand the exact source of households' unwillingness to participate in financial markets, and is left for future research.

A Proofs

Proof of Proposition 1: We consider banks' optimal choice of capital k. Define $\kappa \equiv 1-\frac{R-S}{r_D}$. There are two fundamentally distinct scenarios regarding the level of bank capital: if banks choose any capital stock $k \geq \kappa$, they are fully protected against bankruptcy because banks owe less to depositors than the lowest possible state, $r_D(1-\kappa) = R - S$. We will turn to this case later. If banks choose $k \in (0, \kappa)$, bankruptcy will occur with positive probability. Solving $E[U_D] = u$ for k we see that the bank chooses a capital stock of

$$k = 1 - \frac{R - S(u/r_D)}{r_D}, \tag{18}$$

which is smaller than κ if and only if $u < r_D$. Back-substituting (18) into (1), we find

$$E\left[\Pi_B\right] = \frac{Su^2 - 2Su\rho + 2\rho R r_D - 2r_D^2\rho}{2r_D^2}. \tag{19}$$

Maximization for r_D results in

$$r_D^* = \frac{Su(2\rho - u)}{\rho R}, \tag{20}$$

which gives a total maximum excess return of $E\left[\Pi_B\right] = \frac{\rho\left(\rho R^2 - 2Su(2\rho-u)\right)}{2Su(2\rho-u)}$. There is free entry, so in equilibrium excess returns are zero. Solving for ρ yields

$$\rho = \frac{2Su^2}{4Su - R^2}. \tag{21}$$

Substituting this expression into (20) and collecting terms yields

$$r_D = \frac{R}{2}. \tag{22}$$

Substituting r_D in (18), we obtain

$$k = \frac{4Su}{R^2} - 1. \tag{23}$$

Market clearing in the bank capital market commands that

$$M\frac{\widehat{c}}{C} = Nk, \tag{24}$$

which alternatively can be written as

$$\rho - u = \frac{NCk}{M}. \tag{25}$$

Let us for now assume that there is full inclusion, i.e., no storage is used and all investors invest in either equity or deposits: $N = M$. Substituting (23) and (21) in (25) and solving for u, we find two possible roots, $u = \frac{R^2}{4S}\left(1 \pm \sqrt{\frac{R^2}{R^2+8CS}}\right)$. However, back-substituted into (23), one can see that the only economically sensible solution is

$$u = \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right) \tag{26}$$

because it implies a nonnegative capital stock of $k = \sqrt{\frac{R^2}{R^2+8CS}}$. Combining these results with (25), we find

$$\rho = \frac{R^2}{4S} + \frac{R\left(4CS + R^2\right)}{4S\sqrt{8CS + R^2}}.$$

If $\widehat{u} \equiv \frac{R^2}{4S}\left(1 + \sqrt{\frac{R^2}{R^2+8CS}}\right) \geq 1$, this full participation equilibrium also satisfies the depositor participation constraint and is the unique equilibrium of the game. However, in case $\widehat{u} < 1$, the full participation equilibrium is not feasible because it violates the depositor participation constraint: at full participation returns, depositors would strictly prefer storage over deposits. The unique equilibrium must in this case exhibit incomplete (partial) participation, with some funds going to storage, and the return of deposits matching this storage outside option, thus

$u = 1$. We obtain the remaining equilibrium variables

$$k = \frac{4S}{R^2} - 1 \tag{27}$$

$$\rho = \frac{2S}{4S - R^2} \tag{28}$$

$$K = Nk = M\frac{2S - R^2}{C(R^2 - 4S)} \tag{29}$$

directly from equations (21), (23) and (24). This concludes our analysis of the risky bank equilibrium.

We now turn to the safe equilibrium that is obtained whenever banks choose capital $k \geq \kappa$. The expected utility of depositors becomes $E[U_D] = r_D = u$, whereas the expected return of shareholders is pinned down by the equation

$$E[\Pi_B] = E[r] - (1-k)u - k\rho = E[r] - k(\rho - u) - u = 0,$$

which implies that $\rho = \frac{E[r]-(1-k)u}{k} = \frac{R-S/2-(1-k)u}{k}$. Assume $\rho > u$ (which must hold in equilibrium to attract a nonzero capital stock). Then maximization of excess returns dictates that banks choose the smallest possible k which still attains safety, $k = 1 - \frac{R-S}{u}$. The return to capital becomes then $\rho = \frac{R-S/2-(R-S)}{1-\frac{R-S}{u}} = \frac{Su/2}{u-R+S}$.

The equilibrium value of u is determined by market clearing (24), with $M = N$ and $\widehat{c} = \rho - u$. Substituting previous results, we find $\frac{Su/2}{u-R+S} - u = C\left(1 - \frac{R-S}{u}\right)$, which can be transformed to read

$$\frac{Su^2 - 2u^2(u-R+S) - 2C(u-R+S)^2}{2(u-R+S)^2} = 0$$

$$\Rightarrow (2R - S - 2u)u^2 - 2C(u-R+S)^2 = 0.$$

The latter is the defining equation for u^* in the safe regime, and the remaining allocations are obtained by inserting u^* into the previously derived equations. Note that a safe equilibrium can never be optimal when an "interior" risky equilibrium with $k < \kappa$ exists: by choosing k arbitrarily close to, and just marginally below, κ, banks can attain any safe equilibrium return in a risky setting. Whenever the risky equilibrium choice does not feature such corner solution, it is thus clear that the benefit from the risky equilibrium must be strictly higher.

Next, we show that the two conditions $1 \leq \widehat{u} < \frac{R}{2}$ for the full inclusion risky equilibrium and $\widehat{u} \geq \frac{R}{2}$ for the safe equilibrium can equivalently be expressed in terms of thresholds $\underline{\mathrm{R}}(S)$ and $\bar{R}(S)$. For given S, $\widehat{u} = \frac{R^2}{4S}\left(1+\sqrt{\frac{R^2}{R^2+8CS}}\right)$ is a continuous and strictly monotonic function of R, and the same holds for $\widehat{v} \equiv \widehat{u}/\frac{R}{2} = \frac{R}{2S}\left(1+\sqrt{\frac{R^2}{R^2+8CS}}\right)$. Moreover, it is easy to show that $\{1\}$ is contained both in the image of the open interval $(0,\infty)$ under $\widehat{u}$ and in the image of the open interval (S,∞) under $\widehat{v}$. Define $\underline{\mathrm{R}}(S) \equiv (\widehat{u})^{-1}(1)$, and $\bar{R}(S) \equiv (\widehat{v})^{-1}(1)$. Then, both thresholds are well-defined and unique, $\underline{\mathrm{R}}(S) < \bar{R}(S)$ must hold for all $R > 2$, the thresholds must coincide at $R = 2$, and using the implicit function theorem it is immediate to verify that both $\underline{\mathrm{R}}(S)$ and $\bar{R}(S)$ are increasing in S. By monotonicity, $\underline{\mathrm{R}}(S)$ and $\bar{R}(S)$ separate the different equilibrium regimes in the sense that for all $R < \underline{\mathrm{R}}(S)$ we have $\widehat{u} < 1$ and thus a partial inclusion equilibrium, whereas for all R with $\underline{\mathrm{R}}(S) \leq R < \bar{R}(S)$ we have that $1 \leq \widehat{u} < \frac{R}{2}$ and full inclusion and the risky equilibrium regime prevails. Finally, for $R \geq \bar{R}(S)$, we have $\widehat{u} \geq \frac{R}{2}$ and the equilibrium must be safe. A minor qualification is necessary, however, as $\underline{\mathrm{R}}(S)$ can become smaller than S when S is sufficiently large, and no partial inclusion equilibrium exists that satisfies $R \geq S$. Restricting to the admissible parameter space $\{(R,S) : R \geq S\}$ yields the modified conditions $S \leq R < \underline{\mathrm{R}}(S)$ for partial inclusion, and $\max\{\underline{\mathrm{R}}(S), S\} \leq R < \bar{R}(S)$ for full inclusion.

It now only remains to show that $\rho > E\left[r\right] > u$. For this purpose, we first establish the following result:

Lemma A.1. *The per-unit return to shareholders ρ is maximized at the capital level k^* of the market solution, k^*.*

Proof: For the safe equilibrium, $E\left[\Pi_B\right] = E[r] - \rho k$. Since $E[\Pi_B] = 0$ and the market solution chooses the lowest feasible k within the zero default risk regime, it is immediate that

k^* maximizes shareholder return in this region. In the regions where the equilibrium is risky, bank excess returns are given by $E[\Pi_B] = \frac{1}{S}\int_{\widehat{r}_D(1-k)}^{R}(r - \widehat{r}_D(1-k))\,dr - \rho k$, where we use $\widehat{r}_D$ to represent the deposit rate that satisfies depositors' participation constraint, (2). Clearly, $\widehat{r}_D$ is a function of k. For ease of notation, define $F(k) = \frac{1}{S}\int_{\widehat{r}_D(1-k)}^{R}(r - \widehat{r}_D(1-k))\,dr$, and note that F is increasing and strictly concave in k. This allows us to write bank excess returns as $E[\Pi_B] = F(k) - \rho k$ where, as always, ρ is chosen such that $E[\Pi_B] = 0$ in equilibrium. The implicit function theorem tells us that

$$\frac{d\rho}{dk} = -\frac{\frac{\partial E[\Pi_B]}{\partial k}}{\frac{\partial E[\Pi_B]}{\partial \rho}} = -\frac{F'(k) - \rho}{-k} = \frac{F'(k) - \rho}{k}. \tag{30}$$

The first order condition for the market solution is $F'(k^*) = \rho$, and from strict concavity of F it follows that $F'(k) > \rho$ for all $k < k^*$, and $F'(k) < \rho$ for all $k > k^*$. Combined with (30) this proves that ρ is maximal at $k = k^*$. □

We can now show that $\rho > E[r] > u$. If banks were to choose $k = 1$, they could offer a return of $\rho = E[r]$ to shareholders. In the market solution, however, banks choose a level of capital k^* strictly below 1, and Lemma A.1 establishes that the market solution strictly maximizes shareholder returns per unit, ρ. Hence, $\rho > E[r]$ must hold. However, payoffs to depositors and shareholders can at most sum up to the return of the risky technology, $(1-k)u + k\rho \leq E[r]$. Therefore, this also implies that $u < E[r]$. □

Proof of Corollary 1.2: i) For the partial inclusion ($u = 1$) case, it is immediate that $\frac{d\rho}{dR} = \frac{4RS}{(4S-R^2)^2} > 0$ and $\frac{dk}{dR} = -\frac{8S}{R^3} < 0$. Market clearing then implies that $\frac{dN}{dR} > 0$. Under full inclusion, $\frac{dk}{dR} = \frac{8CS\sqrt{\frac{R^2}{8CS+R^2}}}{8CRS+R^3} > 0$. As a product of two increasing functions, $u = \frac{R^2}{4S}(1+k)$ must be increasing in R as well, and market clearing (24) then dictates that $\frac{d\rho}{dR} > 0$.

ii) A mean preserving spread dX comprises both of an increase in S and of a corresponding increase in R such that the mean is preserved, $\tilde{E}[r] = \frac{2(R+dX/2)-(S+dX)}{2} = E[r] = \frac{2R-S}{2}$. Consider first partial inclusion with $N < M$. Straightforward calculations yield $\frac{dk}{dX} = \frac{1}{2}\frac{\partial k}{\partial R} +$

$\frac{\partial k}{\partial S} = \frac{4(R-S)}{R^3} > 0$ and $\frac{d\rho}{dX} = \frac{1}{2}\frac{\partial \rho}{\partial R} + \frac{\partial \rho}{\partial S} = -\frac{2R(R-S)}{(R^2-4S)^2} < 0$. For the full inclusion regime where $M = N$, we obtain $\frac{dk}{dX} = -\frac{4CR(R-S)}{k(R^2+8CS)^2} \leq 0$, $\frac{du}{dX} = -\frac{R(R-S)\left(R^2(1+k)+4CS(3k+2)\right)}{4S^2(R^2+8CS)} \leq 0$, from which follows $\frac{d\rho}{dX} \leq 0$ due to the simplified market clearing condition $\rho = u + Ck$.

iii) Eqs (18) and (23) show that k and ρ do not depend on C directly, but only indirectly via u. Thus, for $u = 1$ (partial inclusion) both ρ and k are constant, and thus N must be decreasing in C to satisfy market clearing condition (24). For $u > 1$, one can immediately verify by differentiation that $\frac{dk}{dC} < 0$. It follows that $\frac{du}{dC} < 0$ since $u = \frac{R^2}{4S}(1 + k)$, and finally market clearing allows us to conclude that $\frac{d\rho}{dC} < 0$. □

Proof of Proposition 2: To establish the result, it is useful to start with the problem of bank excess return maximization, which can be expressed as

$$\max_k E\left[\Pi_B\right] = \frac{1}{S}\int_{r_D(1-k)}^{R} (r - r_D(1-k))\, dr - \rho k$$

subject to the same constraints as above for the social planner's problem. Note that the first order condition implied by (6), for any level of capital k, is always negative, meaning that the bank (and the social planner) always finds it optimal to choose the lowest deposit rate r_D that is consistent with satisfying depositors' participation constraint. Therefore, depositors' participation constraint will always be satisfied with equality, allowing us to substitute the constraint into the bank's maximization problem to obtain

$$\max_k E\left[\Pi_B\right] = \frac{1}{S}\int_{r_D(1-k)}^{R} r\, dr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{S}\left(r_D^2\,(1-k) - \frac{\partial r_D}{\partial k}\,(1-k)^2\, r_D\right) - (\rho - u) = 0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained from the constraint. This first order condition must be satisfied in equilibrium whatever the values for ρ and u, which are obtained from market clearing as in

Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively be written as

$$\begin{aligned}\max_{k_P} SW &= N\frac{1}{S}\int_{r_D(1-k)}^{R} r\,dr - \int_0^{\widehat{c}} M\frac{c}{C}\,dc + M - N \\ &= N\frac{1}{S}\int_{r_D(1-k)}^{R} r\,dr - \frac{1}{2C}M\widehat{c}^2 + M - N,\end{aligned}$$

reflecting the fact that maximizing the return to all stakeholders is equivalent to maximizing aggregate output, $N\frac{1}{S}\int_{r_D(1-k)}^{R} rdr$, since all output is allocated to either depositors or capital holders. The last term, $M-N$, represents the funds that are not invested in the banking sector but rather held as storage, to the extent that M may be strictly greater than N. Recall now the market clearing condition (24), $M\frac{\widehat{c}}{C} = kN$, which implies that $\widehat{c} = \frac{C}{M}kN$, or that $\widehat{c}^2 = \left(\frac{C}{M}kN\right)^2$, and which is taken into account by the social planner. We can thus write the maximization problem above as

$$\begin{aligned}\max_{k_P} SW &= N\frac{1}{S}\int_{r_D(1-k)}^{R} r\,dr - \frac{1}{2C}M\left(\frac{C}{M}kN\right)^2 + M - N \\ &= N\frac{1}{S}\int_{r_D(1-k)}^{R} r\,dr - \frac{1}{2}\frac{C}{M}k^2N^2 + M - N.\end{aligned}$$

For an interior solution, the necessary first order condition to this problem is

$$N\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - \frac{C}{M}kN^2 + \frac{\partial N}{\partial k}\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N\frac{\partial N}{\partial k} - \frac{\partial N}{\partial k} = 0.$$

Grouping terms obtains

$$N\left(\frac{1}{S}\left(r_D^2(1-k) - \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right) - \frac{C}{M}kN\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N - 1\right) = 0.$$

Since at equilibrium $\rho - u = \widehat{c} = \frac{C}{M}kN$, we can further rewrite as

$$N\left(\frac{1}{S}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \left(\rho - u\right)\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} r dr - k\left(\rho - u\right) - 1\right) = 0$$

Now observe that $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1$ since, if $u > 1$, all funds are being used in the banking sector, so a marginal increase in k cannot change $N = M$.

Consider first the case that $u > 1$, so that $\frac{\partial N}{\partial k} = 0$. We are then left with only

$$N\left(\frac{1}{S}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \left(\rho - u\right)\right) = 0,$$

which is the same condition as must be satisfied for the bank's problem.

Alternatively, suppose that $u = 1$, which allows us to express the first order condition for the social planner as

$$N\left(\frac{1}{S}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \left(\rho - 1\right)\right) + \frac{\partial N}{\partial k}\left(\frac{1}{S}\int_{r_D(1-k)}^{R} r dr - k\rho - \left(1-k\right)\right) = 0.$$

Note now that term in the parentheses of the second line, $\frac{1}{S}\int_{r_D(1-k)}^{R} r dr - k\rho - (1-k)$, is simply $E\left[\Pi_B\right]$ for the case where $u = 1$, which in equilibrium is equal to zero, with all rents going to shareholders through ρ. This leaves the term below, after eliminating the N:

$$\frac{1}{S}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \left(\rho - 1\right) = 0,$$

again exactly as in the bank's problem for the case $u = 1$. Therefore, for all cases the necessary condition to be satisfied is identical to that which maximizes excess returns. Given that the market clearing condition that pins down the equilibrium returns u and ρ is the same across both maximization problems, we can conclude that both problems must have the same solution. □

Proof of Proposition 3: To see that $\frac{d\rho}{dk} < 0$ for both cases (1) and (2) in the proposition, we refer to Lemma A.1, which shows that ρ is maximal at the market solution and thus $\frac{d\rho}{dk} < 0$ must hold for $k^{reg} > k^*$.

To establish the rest of the result, consider first the case where $N < M$. Market clearing implies

$$\rho - u = Ck\frac{N}{M}.$$

Given the result above that ρ is lower for larger k, and the fact that $u = 1$, the RHS must decrease to satisfy market clearing. Given that we are considering an increase in k, N must fall, and it must fall enough that even kN must be lower. Since $K = kN$, it follows that $K^{reg} < K^*$.

Consider next the case where $N = M$ and therefore $u > 1$. For a marginal increase in k, market clearing simplifies to

$$\rho - u = kC.$$

The RHS must be larger than in the market solution because $k > k^*$. Given that ρ is lower, per the argument above, we must have that u decreases more than proportionally. That is, $0 > \frac{d\rho}{dk} > \frac{du}{dk}$, as desired. Finally, $K^{reg} = Nk > Nk^* = K^*$, which concludes the proof. $\square$

Proof of Proposition 4: The proof follows a similar approach as that of Proposition 2. Since at equilibrium (7) will be satisfied with equality, we can rewrite (1) as

$$\max_k E\left[\Pi_B\right] = \frac{1}{R}\int_{r_D(1-k)}^{R} rdr + \frac{1}{R}\int_0^{r_D(1-k)} hr\,dr - u(1-k) - \rho k.$$

The necessary first order condition that must now be satisfied is

$$\frac{1}{R}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - (\rho - u) + h\frac{1}{R}\left(\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D - r_D^2\left(1-k\right)\right) = 0,$$

or

$$\frac{1}{R}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)(1-h) - (\rho - u) = 0,$$

where $\frac{\partial r_D}{\partial k}$ is obtained directly from (7). This first order condition must be satisfied in equilibrium whatever the values for ρ and u, which are obtained from market clearing as in Proposition 1.

Consider now the maximization problem for the social planner, which can alternatively be written as

$$\begin{aligned}\max_{k^{FB}} SW &= N\frac{1}{R}\int_{r_D(1-k)}^{R} rdr - \int_0^{\widehat{c}} M\frac{c}{C}\,dc + M - N + N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr \\ &= N\frac{1}{R}\int_{r_D(1-k)}^{R} rdr - \frac{1}{2}\frac{C}{M}k^2N^2 + M - N + N\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr.\end{aligned}$$

For an interior solution, the necessary first order condition to this problem is

$$\begin{aligned}&N\frac{1}{R}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right) - \frac{C}{M}kN^2 + \frac{\partial N}{\partial k}\frac{1}{R}\int_{r_D(1-k)}^{R} rdr - \frac{C}{M}k^2N\frac{\partial N}{\partial k} - \frac{\partial N}{\partial k} \\ &+ Nh\frac{1}{R}\left(\frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D - r_D^2\left(1-k\right)\right) + \frac{\partial N}{\partial k}\frac{1}{R}\int_0^{r_D(1-k)} hr\,dr + N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr = 0.\end{aligned}$$

Grouping terms obtains

$$\begin{aligned}&N\left(\frac{1}{R}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right) - \frac{C}{M}kN\right) \\ &+ \frac{\partial N}{\partial k}\left(\frac{1}{R}\int_{r_D(1-k)}^{R} rdr + \frac{1}{R}\int_0^{r_D(1-k)} hr\,dr - \frac{C}{M}k^2N - 1\right) + N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr = 0.\end{aligned}$$

Since at equilibrium $\rho - u = \widehat{c} = \frac{C}{M}kN$, we can further rewrite as

$$\begin{aligned}&N\left(\frac{1}{R}\left(r_D^2\left(1-k\right) - \frac{\partial r_D}{\partial k}\left(1-k\right)^2 r_D\right)\left(1-h\right) - \left(\rho - u\right)\right) \\ &+ \frac{\partial N}{\partial k}\left(\frac{1}{R}\int_{r_D(1-k)}^{R} rdr + \frac{1}{R}\int_0^{r_D(1-k)} hr\,dr - k\left(\rho - u\right) - 1\right) + N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr = 0\end{aligned}$$

Now observe that $\frac{\partial N}{\partial k} \neq 0 \Rightarrow u = 1$ since, if $u > 1$, all funds are being used in the banking sector, so a marginal increase in k cannot change $N = M$.

Consider now the market solution for the case that $u > 1$, so that $\frac{\partial N}{\partial k} = 0$. In that case, $\frac{\partial h}{\partial k} = 0$ as well, and there is thus no scope for capital regulation.

Alternatively, suppose that under the market solution $u = 1$, and again substitute these values into the social planner's first order condition. This would give us

$$N\left(\frac{1}{R}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right)(1-h) - (\rho - 1)\right)$$
$$+\frac{\partial N}{\partial k}\left(\frac{1}{R}\int_{r_D(1-k)}^{R} r\,dr + \frac{1}{R}\int_0^{r_D(1-k)} hr\,dr - k\rho - (1-k)\right) + N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr.$$

Under the k, r_D, u, and ρ implied by the market solution, the term in the parentheses, $\frac{1}{R}\int_{r_D(1-k)}^{R} r dr + \frac{1}{R}\int_0^{r_D(1-k)} hr\,dr - k\rho - (1-k)$, is simply $E[\Pi_B]$ for the case where $u = 1$, which in equilibrium is equal to zero, with all rents going to shareholders through ρ. This leaves the term below:

$$N\left(\frac{1}{R}\left(r_D^2(1-k) + \frac{\partial r_D}{\partial k}(1-k)^2 r_D\right)(1-h) - (\rho - 1)\right) + N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr.$$

At the market solution, the first term is equal to zero. This means that, evaluated at the market solution, the derivative with respect to k of the social planner's objective function is $N\frac{1}{R}\int_0^{r_D(1-k)}\frac{\partial h}{\partial k}r\,dr > 0$. Therefore, the market solution involves banks holding too little capital relative to what a social planner would like and, consequently, too many banks. $\square$

Proof of Proposition 5: Note that the optimization problem in (9) can be rewritten as

$$E[\Pi_B] = \frac{1}{2R}\left(R - r_D(1-k)\right)^2 - \rho k, \tag{31}$$

which is a convex function of k (as verified by $\frac{d^2}{dk^2}E[\Pi_B] = \frac{r_D^2}{R} > 0$) with a minimum at $k = 1 - R\frac{1-(\rho/r_D)}{r_D}$ which is no less than 1 for $\rho \geq r_D = u$. Since $\rho \geq u$ holds for any $k \geq 0$, the unique maximum of $E[\Pi_B]$ is attained by choosing the lowest capital level in the $[0,1]$ interval. Therefore, $k^* = 0$, and all M investors become depositors: $N^* = M$. Since bank excess returns (31) in equilibrium are zero, we finally obtain $u = r_D = R$. $\square$

Proof of Proposition 6: The social welfare function can be written as

$$
\begin{aligned}
SW &= -N(1-k)\frac{1}{R}\int_0^{r_D(1-k)} r_D\, dr + \rho K + uD - M\int_0^{\rho-u}\frac{c}{C}\, dc \\
&= -\underbrace{Nr_D(1-k)}_{=Dr_D}\underbrace{\frac{r_D(1-k)}{R}}_{=\Pr(\text{bankr.})} + \rho K + uD - M\frac{(\rho-u)^2}{2C}.
\end{aligned}
$$

Applying the market clearing condition for capital, $K = M\frac{\rho-u}{C}$, to the last term and substituting $r_D = u$ from the depositor participation constraint we obtain

$$
SW = uD\left(1 - \Pr(\text{bankr.}) + \frac{K}{2}\left(\rho + u\right).\right.
$$

Taking derivatives wrt k at the point of the unregulated equilibrium $k = 0$, we get

$$
\begin{aligned}
\frac{d}{dk}SW = &\frac{dD}{dk}\underbrace{u(1-\Pr(\text{bankr.}))}_{=0} + D\frac{d}{dk}\left[u(1-\Pr(\text{bankr.}))\right] \\
&+ \frac{dK}{dk}\underbrace{\left(\frac{\rho+u}{2}\right)}_{\geq 0} + \underbrace{K}_{=0}\frac{d}{dk}\left(\frac{\rho+u}{2}\right).
\end{aligned}
$$

The term $D\frac{d}{dk}\left[u(1-\Pr(\text{bankr.}))\right]$ must be positive because

$$
\frac{d}{dk}\left[u(1-\Pr(\text{bankr.}))\right] = \frac{du}{dk}\underbrace{(1-\Pr(\text{bankr.}))}_{=0} + u\underbrace{\left(-\frac{d}{dk}\Pr(\text{bankr.})\right)}_{>0} > 0,
$$

and hence social welfare must increase strictly in k at $k = 0$. The planner therefore optimally chooses a strictly positive amount of k to maximize welfare. $\square$

Proof of Proposition 7: We first note that the solution to (5), which does not incorporate an externality, is the same as the market solution. The first order condition to the problem now contains the additional term $\frac{d}{dk}\left(-N\frac{1}{R}\int_0^{r_D(1-k)} r\psi dr\right)$, which is clearly positive since more capital reduces the deposit rate r_D as well as the bankruptcy threshold directly, thus lowering the social cost of bankruptcy. Therefore, $k^{reg} > k^*$.

The argument that $\rho^{reg} < \rho^*$ follows directly from Proposition 3: since binding capital requirements do not lead to increased output from the banking sector, they must reduce the return to shareholders, exactly as in the prior result. Likewise, the argument that $\rho^{reg} - u^{reg} > \rho^* - u^*$ when $N = M$ derives from the fact that $K = kN$ must increase. □

Proof of Proposition 8: For $\psi = 0$, a minor extension of the result from Proposition 2 continues to hold, and the bank's problem is equivalent to that of the social planner, implying that the solutions for k are the same as well. For $\psi > 0$, the social planner's problem differs from the bank's problem by the term

$$-N\frac{1}{R}\int_{a^*}^{r_D(1-k)} r\psi \, dr,$$

which can be seen to be increasing in k directly because the upper limit $r_D(1-k)$ is decreasing in k (the entire term is negative), and because a^* is increasing in k. This leads to $k^{reg} > k^*$, as desired. □

B Household demand for safe assets

In the baseline model presented in the paper, we assume that households view bank deposits as equivalent to storage as long as the deposits earn an equilibrium expected return at least as high as the return to storage. One possible concern, however, is that in the absence of insurance, deposits offered by a bank may be subject to default, and indeed our primary focus is on the parameter region where in equilibrium banks choose risky capital structures. As such, an argument can be made that households that are averse to participating in financial markets may view any instrument, even a bank deposit, as exposing them to risk, and may demand additional compensation for being exposed to this risk. In other words, households may exhibit a demand for *safe*, or at least safer, assets (Golec and Perotti, 2017; Gorton, 2017).

To study this issue, we extend the model as follows. Consider that a deposit contract can be viewed as an investment with a binary return: with Pr (no default), the investment pays off r_D per unit of deposit. With complementary probability, Pr (default), it repays zero. We model households' aversion to default as a disutility that they incur whenever the deposit contract defaults, and we denote this disutility as α. Higher values of α can therefore be interpreted as households demanding safer assets, and in the limit, as $\alpha \to \infty$, households would only be willing to hold deposits that are perfectly safe. With this, we can write the depositors' expected utility as

$$\begin{aligned} E\left[U_D\right] &= \Pr\left(r \geq r_D\left(1-k\right)\right) r_D - \alpha \Pr\left(r < r_D\left(1-k\right)\right) \\ &= \int_{\max\{R-S, r_D(1-k)\}}^{R} r_D dr - \alpha \int_{\min\{R-S, r_D(1-k)\}}^{\max\{R-S, r_D(1-k)\}} dr, \end{aligned}$$

with the usual constraint that the expected utility must be at least as high as the opportunity cost: $E\left[U_D\right] \geq u$. Note, however, that u is no longer the expected return to the deposit contract, but rather the expected utility for depositors given that they incur a cost α whenever the deposit contract defaults.

We can now state the bank's maximization problem as maximizing abnormal returns as in (1), subject to the usual constraints that $E\left[\Pi_B\right] \geq 0$, $0 \leq k \leq 1$, and $E\left[U_D\right] \geq u$, where $E\left[U_D\right]$ is as defined above. The solution to this problem will determine, as before, the bank's optimal capital structure k^*, the optimal deposit rate r_D^*, the equilibrium returns to equityholders and depositors, ρ^* and u^*, respectively, and the number of banks that operate in equilibrium, N^*.

Since our primary goal is to show that the main implications concerning the incidence of capital regulation are unchanged by introducing a demand for safe assets on the side of households, we omit the characterization of equilibrium and instead focus on the incidence of regulation. Suppose therefore that capital regulation is imposed, which is binding: $k > k^*$. We divide our analysis into two cases:

1. Suppose that R is sufficiently low that $N < M$. In that case, $E[U_D] = 1$ since the expected utility of depositors must equal the return to storage given that some households, $M - N > 0$, choose to hold their wealth in storage rather than in the banking sector. For this region, the results are analogous to those studied in previous sections: a marginal increase in the capital requirement does not change $E[U_D] = u = 1$. Therefore, if there is a benefit to regulation, it must accrue to shareholders. Conversely, if there is a cost, it must also be borne by shareholders, so that the number of banks that are formed must decrease.

2. If R is sufficiently large that $N = M$, then we must have that $E[U_D] = u > 1$ since all households strictly prefer to hold claims against the banking sector. Now, an increase in the capital requirement at each bank requires that $K = kN$ increase. In order for this to happen, $\rho - u$ must increase, similar to before. So either depositors benefit less from the tightened capital requirement, or they bear more of the cost, whichever the case may be. This is true even if the increased capital requirement for each bank reduces the bank's default risk and thus, ceteris paribus, increases the expected utility of depositors.

Conclusion: The results on the incidence of capital regulation obtained in the baseline model, as well as in our extensions with frictions, extend to the case where households demand safe assets and view risky deposits as somewhat inferior to (safe) storage.

References

Admati, A. R., DeMarzo, P. M., Hellwig, M. F., and Pfleiderer, P. Fallacies, irrelevant facts, and myths in the discussion of capital regulation: Why bank equity is not socially expensive. Working paper, Stanford University, October 2013.

Allen, F. and Gale, D. Limited market participation and volatility of asset prices. *American Economic Review*, 84(4):933–55, 1994.

Allen, F., Carletti, E., and Marquez, R. Credit market competition and capital regulation. *Review of Financial Studies*, 24(4):983–1018, 2011.

Allen, F., Carletti, E., and Marquez, R. Deposits and bank capital structure. *Journal of Financial Economics*, 118(3):601–619, 2015.

Berger, A. and Bouwman, C. How does capital affect bank performance during financial crises? *Journal of Financial Economics*, 109:146–176, 2013.

Besanko, D. and Thakor, A. Banking deregulation: Allocational consequences of relaxing entry barriers. *Journal of Banking and Finance*, 16:909–932, 1992.

Boot, A. and Greenbaum, S. I. Bank regulation, reputation and rents: theory and policy implications. In Mayer, C. and Vives, X., editors, *Capital Markets and Financial Intermediation*, pages 262–285. Cambridge University Press, 1993.

Dell'Ariccia, G. and Marquez, R. Competition among regulators and credit market integration. *Journal of Financial Economics*, 79(2):401–430, 2006.

Demirgüç-Kunt, A. and Detragiache, E. Does deposit insurance increase banking system stability? An empirical investigation. *Journal of Monetary Economics*, 49(7):1373–1406, 2002.

Golec, P. and Perotti, E. Safe assets: a review. Working paper 2035, European Central Bank, 2017.

Gomes, F. and Michaelides, A. Optimal life-cycle asset allocation: Understanding the empirical evidence. *Journal of Finance*, 60(2):869–904, 2005.

Gorton, G. The history and economics of safe assets. *Annual Review of Economics*, 9: 547–586, 2017.

Guiso, L. and Sodini, P. Household finance: An emerging field. In George M. Constantinides, M. H. and Stulz, R. M., editors, *Handbook of the Economics of Finance*, volume 2, Part B of *Handbook of the Economics of Finance*, chapter 21, pages 1397 – 1532. Elsevier, 2013.

Hellmann, T. F., Murdock, K. C., and Stiglitz, J. E. Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review*, 90(1):147–165, 2000.

Holmström, B. and Tirole, J. Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–691, 1997.

Huizinga, H., Voget, J., and Wagner, W. International taxation and cross-border banking. *American Economic Journal: Economic Policy*, 6(2):94–125, 2014.

Lusardi, A., Michaud, P.-C., and Mitchell, O. Optimal financial knowledge and wealth inequality. *Journal of Political Economy*, 125(2):431–477, 2017.

Mehran, H. and Thakor, A. Bank capital and value in the cross-section. *Review of Financial Studies*, 24:1019–1067, 2011.

Modigliani, F. and Miller, M. H. The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, 48(3):261–297, 1958.

Morrison, A. and White, L. Crises and capital requirements in banking. *American Economic Review*, 95:1548–1572, 2005.

Repullo, R. Capital requirements, market power, and risk-taking in banking. *Journal of Financial Intermediation*, 13(2):156–182, 2004.

Thakor, A. Bank capital and financial stability: An economic tradeoff or a faustian bargain? *Annual Review of Financial Economics*, 6:185–223, 2014.

Vissing-Jørgensen, A. Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, 110(4):825–853, 2002.