Redemption Fees and Information-Based Runs*

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Abstract

We study how the imposition of a redemption fee affects runs on financial institutions when investors are asymmetrically informed about fundamentals. Although the fee eliminates the first-mover advantage by internalizing the payoff externality and, therefore, discourages runs by informed investors, it also alters the information externality by influencing uninformed investors' learning and may, thereby, either increase or decrease overall run potential. Additionally, the fee may create a last-mover advantage for the informed, resulting in a wealth transfer from uninformed to informed investors. These effects render the fee's impact on welfare and the investors' propensity to run preemptively ambiguous.

Keywords: Runs; Information externality; Payoff externality; Last-mover advantage

JEL: D8, G2

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1 Introduction

Runs on financial institutions have plagued markets for centuries. Due to the negative welfare consequences associated with runs, banking regulators have devised several mechanisms over time in an attempt to prevent runs on banks, including deposit insurance, suspension of convertibility, and other instruments. While these mechanisms have been relatively successful at preventing runs on traditional banks during recent history, they leave many other types of systemically important "shadow-banking" institutions, such as money market mutual funds ("MMMFs"), which are not subject to banking regulations, vulnerable to runs. As discussed by Schmidt et al. (2016), this vulnerability was exploited by investors during the 2008 financial crisis, resulting in many MMMFs experiencing heavy, run-like redemptions.

In response to the heavy redemptions that occurred during the financial crisis, the U.S. Securities and Exchange Commission ("SEC") adopted new regulations in 2014 designed to stabilize MMMFs and reduce run potential.[1] These new regulations embrace a novel approach to mitigating run potential by enabling MMMFs to impose a redemption fee on withdrawals during periods of stress. Specifically, under the new regulations, a MMMF is permitted to impose a fee of up to 2% on redemptions in situations wherein the MMMF's liquidity level is sufficiently low.[2] Many believe that a redemption fee should diminish the potential for runs by forcing withdrawing investors to internalize the liquidity costs that they generate.[3] However, when investors are asymmetrically informed, imposing a redemption fee may also change the information environment faced by investors and, hence, their inclination to run. To the best of our knowledge, there are no formal economic analyses that evaluate the impact of a redemption fee through this information channel. We attempt to fill this gap.

[1] Hanson et al. (2015) evaluate various policy proposals. The SEC also adopted regulations designed to reduce risk and increase transparency of MMMFs in 2010, but these are not pertinent to our analysis.

[2] SEC Rule 2a-7 allows a MMMF to impose a fee of up to 2% of the value of shares redeemed if its weekly liquid assets fall below 30% of total assets. Furthermore, a MMMF must impose a fee if its weekly liquid assets drop below 10% of total assets unless the board of directors determines that imposing a fee is not in the best interests of the fund. The 2014 amendments to Rule 2a-7 also enable a MMMF to impose a gate (i.e., a suspension of convertibility) under similar conditions. Rule 2a-7 still permits investors in retail (but not institutional) MMMFs to buy and sell MMMF shares at a stable net asset value (NAV) of $1.00 per share.

[3] See, e.g., "Money Market Fund Reform; Amendments to Form PF," 79 Federal Register 157 (2014).

We analyze a multi-stage deposit withdrawal game to study the effects of a redemption fee on the potential for runs and investor welfare. In our model, investors are asymmetrically informed about fundamentals and may face liquidity shocks, so uncertainty and learning play a key role in shaping the dynamics of runs. A single fund invests all of its deposits in a risky asset at $t = 0$. The investment matures at $t = 2$, when the asset generates a random payoff $\tilde{v}$. Premature liquidation at $t = 1$ is costly. There is a continuum of risk-neutral investors who are identical ex ante. As time progresses, investors are differentiated into three groups based on whether they obtain private information about $\tilde{v}$ and/or experience a liquidity shock.

The first group of investors encounters a liquidity shock at $t = 1$ and must withdraw their deposits before the investment matures, regardless whether they are informed about $\tilde{v}$. Withdrawals are dynamic and can occur at two stages at $t = 1$. This first group withdraws immediately at the first stage. The second group of investors receives an identical and private informative (but noisy) signal, s, on $\tilde{v}$ at the first stage. They have no urgent liquidity needs but may also withdraw at the first stage if s indicates a low $\tilde{v}$ (i.e., if s is below some endogenously determined signal threshold $\hat{s}_1^{\phi}$). The third group consists of investors who are uninformed and do not experience a liquidity shock. They observe the aggregate first-stage withdrawals by the other two groups (i.e., withdrawals for either liquidity or information reasons), update their beliefs about $\tilde{v}$ accordingly, and may withdraw at the second stage if their posterior beliefs are sufficiently low. The second group of investors also may withdraw at the second stage if they did not previously withdraw at the first stage, and such a possibility is factored into the determination of their first-stage withdrawal threshold $\hat{s}_1^{\phi}$.[4]

As a benchmark against which to evaluate the effects of a redemption fee, we analyze a setting in which investors may withdraw early (at either stage of $t = 1$) without paying a fee. Because the fund must liquidate a portion of its investment to satisfy redemption requests, early withdrawers generate liquidation costs that are borne by late withdrawers.

[4] This two-stage setting is meant to capture the dynamic nature of runs that exists in reality (e.g., the two runs on Washington Mutual in July and September 2008 as documented by He and Manela (2016)). As discussed below, this setting also permits us to evaluate the impact of a redemption fee on preemptive runs. In practice, investors can observe net withdrawals because many MMMFs' total assets under management are publicly available (from sources such as Morningstar) on a daily basis.

This creates a payoff externality and, hence, a first-mover advantage among investors. To avoid potential losses due to this externality, investors without liquidity needs may withdraw too early relative to the first-best outcome, thereby leading to socially inefficient allocations. Premature withdrawals also create an information externality because uninformed investors update their beliefs about $\tilde{v}$ based on the size of the first-stage withdrawal.

When a redemption fee is imposed on withdrawals made at each stage of $t = 1$,[5] the fee is deducted from the amount distributed to early withdrawers and retained by the fund. Thus, the fee reduces the amount received by investors who withdraw prematurely and enables the fund to liquidate a smaller portion (compared to the benchmark setting) of its investment to meet early-withdrawal demands. This alters the tradeoffs faced by investors when making their withdrawal decisions by (at least partially) eliminating the first-mover advantage. More subtly, but importantly, it also influences how uninformed investors interpret a given first-stage withdrawal size and update their beliefs about the risky asset's return. That is, the fee changes the information externality, which is the main novelty of our model.

To succinctly highlight how a redemption fee affects the information externality, we focus on a setting wherein the size of the fee is chosen to exactly offset the liquidation cost and, thus, fully internalize the payoff externality, but the qualitative results and intuition are robust to alternative settings with an arbitrary fee size. The liquidation costs generated by early withdrawers in this setting are borne entirely by themselves rather than being imposed on late withdrawers, so the first-mover advantage vanishes. As a result, investors without liquidity needs make withdrawal decisions based solely on their beliefs about the risky asset's return. Because the fee in this setting fully internalizes the payoff externality, any undesirable effects of the fee must stem from its impact on information and learning.

Compared to the no-fee benchmark, the lowest signal for which informed investors without liquidity needs do not withdraw at the first stage of $t = 1$, $\hat{s}_1^{\phi}$, is lower when they must pay a fee to withdraw. Thus, informed depositors are less likely to withdraw early. This influences learning by uninformed investors and, hence, alters the information externality in two dis-

[5] We also examine an extension of the model in which only second-stage withdrawals are subject to a fee.

tinct ways. On the one hand, for a given first-stage withdrawal size, the posterior likelihood that informed investors remain invested increases simply because, as explained above, the fee reduces the informed investors' tendency to withdraw. This shifts the uninformed investors' beliefs about s upward and, thus, causes them to become more optimistic about the asset return $\tilde{v}$ because, *ceteris paribus*, informed investors maintain their deposits only if $s \geq \hat{s}_1^{\phi}$. We call this the "likelihood effect." On the other hand, the lower signal threshold also causes uninformed investors to become more pessimistic about $\tilde{v}$: (i) a non-withdrawal by informed investors in the presence of a fee conveys less optimism about the fundamental because the informed investors' lower signal threshold reduces the average expected asset return for which they maintain their deposits, i.e., $\mathbb{E}[\tilde{v}|s \geq \hat{s}_1^{\phi}]$ decreases as $\hat{s}_1^{\phi}$ decreases; and (ii) an early withdrawal by informed investors conveys greater pessimism because the lower signal threshold reduces the average expected return given a withdrawal, i.e., $\mathbb{E}[\tilde{v}|s < \hat{s}_1^{\phi}]$ also decreases as $\hat{s}_1^{\phi}$ decreases. We call this the "distribution effect." Depending on whether the distribution effect or the likelihood effect dominates, a fee may either raise or lower the tendency of uninformed investors to withdraw, thereby either increasing or decreasing the occurrence of large premature withdrawals (i.e., runs). We show that the distribution effect is more likely to dominate the likelihood effect (hence, imposing a redemption fee is more likely to destabilize the fund) when the cost of premature liquidation is lower.

Although the fee eliminates the informed investors' first-mover advantage by internalizing the payoff externality, it simultaneously creates a last-mover advantage by enabling informed investors to maintain their deposits when they would otherwise withdraw absent a fee. Specifically, when the informed investors' signal indicates a high asset return, the fee may effectively create a wealth transfer from early withdrawers (which may include uniformed investors without liquidity needs) to informed investors for two reasons. First, informed investors are not forced to abandon profitable investments (as they sometimes are in the absence of a fee due to payoff externalities) when early withdrawers bear their own liquidation costs. Second, fees paid by early withdrawers enable the fund to maintain a larger investment in the high-return asset, which benefits informed investors who maintain their deposits. Notably, this last-mover advantage, like the first-mover advantage, stems from the informed investors' information advantage. Hence, the informed investors' information advantage is not eliminated by the fee but is merely manifested in a different way.

By altering the investors' withdrawal decisions, the fee also affects social welfare, measured as the net surplus generated by fund investment. Notably, when the fee increases overall run potential (by making uninformed investors more prone to withdraw early), it raises (lowers) welfare in states wherein the asset's expected return is low (high). Thus, in periods of economic stress with low expected returns, a fee may yield a social benefit if it destabilizes MMMFs but harm investors if it stabilizes MMMFs. In contrast, when the fee reduces early withdrawals, it increases aggregate fund investment and, therefore, has an amplifying effect on welfare, lowering (raising) welfare in states wherein the asset's expected return is low (high).

Our two-stage framework of the withdrawal game allows us to also evaluate the impact of a redemption fee on preemptive runs. While our main analysis (discussed above) assumes that the fee applies to withdrawals at both the first and second stages of $t = 1$, we also consider an extension wherein only second-stage withdrawals are subject to a fee. This captures the practical notion that fees may be imposed after realized withdrawals reduce a fund's liquidity reserves. One might expect this to precipitate preemptive runs at the first stage (e.g., Cipriani et al., 2014), but our analysis reveals that, quite the contrary, this may actually strengthen the informed investors' incentives to remain invested until the fund's investment matures at $t = 2$. The reason is that informed investors enjoy a last-mover advantage when their private signal indicates a high asset return, and second-stage withdrawals by uninformed investors generate a wealth transfer to informed investors through the redemption fees paid by the former.

At a broad level, our analysis shows that regulatory actions may interfere with information structures and influence learning by economic agents, which may render well-intentioned regulations less effective. Viewed in this way, a redemption fee that internalizes only the payoff externality but not the information externality constitutes a suboptimal Pigouvian tax policy. In a similar vein, Cong et al. (2016) show that government liquidity injections aimed at mitigating coordination failures may generate information externalities.

Our model is more closely related to the literature on the role of information in bank runs, though, to the best of our knowledge, there are no existing models that evaluate the impact of a redemption fee when investors are asymmetrically informed.[6] Chari and Jagannathan (1988) consider a simultaneous-move game in which uninformed depositors may misinterpret large liquidity withdrawals as being caused by adverse information about bank assets, which may trigger a panic run even when no one has any adverse information about future returns. They show that suspension of convertibility can prevent panic runs and improve social welfare. In a modified version of the classic Diamond and Dybvig (1983) model, Gu (2011) analyzes a multi-stage game in which depositors make withdrawal decisions sequentially, and depositors at later stages learn from observed earlier-stage withdrawals. She finds that imperfect learning may lead to herding, in which case a long sequence of withdrawals persuade informed depositors at later stages to join the withdrawal queue even if they receive a positive private signal. He and Manela (2016) study the timing of runs in a dynamic model with endogenous information acquisition. They show that rumors of illiquidity can motivate depositors to acquire information and lead to runs even when depositors receive neutral signals because there is a possibility that others may receive more negative signals and withdraw before those who receive a neutral signal. Goldstein and Pauzner (2005) assume that depositors receive private noisy signals about bank fundamentals. Using a global game approach, they derive a unique equilibrium in which the occurrence of runs is determined by fundamentals.

Runs on mutual funds have also been studied by others. Parlatore (2016) examines the effects of sponsor support on MMMF stability and the underlying asset market liquidity. Zeng (2016) analyzes runs on mutual funds when all investors are symmetrically informed.

[6] We discuss only a few studies that are closely related to our paper. For other important contributions, see, e.g., Gorton (1985), Jacklin and Bhattacharya (1988), Alonso (1996), Allen and Gale (1998), Chen (1999), and Ennis and Keister (2009). Another strand of the bank-run literature focuses on panic runs in the classic Diamond and Dybvig (1983) setting wherein depositors are symmetrically informed about bank fundamentals. An essential element of the theory is a sequential service constraint, which generates payoff externalities among depositors. Green and Lin (2003) and Peck and Shell (2003), among others, further explore the sequential service constraint and study the dynamic nature of runs within the Diamond and Dybvig (1983) setting.

Although his main focus is on floating NAV, the analysis also touches upon redemption fees. In contrast to our results, he finds that redemption fees reduce runs, but there is no learning by investors in his model. Cipriani et al. (2014) study the potential for redemption fees to lead to preemptive runs, but there is no learning in their model, either.

Empirically, Kacperczyk and Schnabl (2013) demonstrate that MMMFs hold risky assets and are, therefore, vulnerable to runs. Schmidt et al. (2016) document run-like behaviors in MMMFs during the 2008 financial crisis. Chernenko and Sunderam (2014) show that MMMFs are systemically important and that runs on MMMFs can have spillover effects on firms' abilities to raise capital. More generally, Chen et al. (2010) and Goldstein et al. (2016) show, respectively, that equity and corporate-bond mutual funds that hold less liquid assets experience greater outflows in response to poor performance, which can precipitate runs.

The article is organized as follows. We describe the model in Section 2 and analyze how a redemption fee affects deposit withdrawals in Section 3. We evaluate welfare in Section 4 and study preemptive runs in Section 5. Section 6 concludes. Proofs are in the Appendix.

2 Model

We consider an economy comprising a single fund and a continuum of risk neutral investors of mass one. There are three dates indexed by $t \in \{0, 1, 2\}$; date 1 is divided into two stages.

Preferences. Each investor has one unit of account on deposit at the fund at $t = 0$. Investors derive utility from consuming wealth. A random fraction $\tilde{\lambda}$ of investors are impatient, whereas the rest are patient. Impatient investors derive utility only from consumption at $t = 1$. Patient investors consume at $t = 2$; if a patient investor withdraws her deposit at $t = 1$, she can costlessly store what is received from the fund and consume at $t = 2$. No individual investor knows her own consumption type at $t = 0$, but each investor privately learns whether she is patient or impatient at the first stage of date 1. At that stage, the mass of impatient investors λ (i.e., the realized value of $\tilde{\lambda}$) is also determined but is not directly observable to anyone. We stipulate that λ is drawn from some distribution that admits a continuous density function $g(\lambda)$ with support $[\underline{\lambda}, \bar{\lambda}] \subseteq [0, 1)$. Without loss of generality, we normalize $\underline{\lambda}$ to 0.

Technology. The fund invests all deposits in an infinitely divisible risky asset. Each unit invested at $t = 0$ returns either a random amount $\tilde{v} \in \{H, L\}$ at $t = 2$, where $H > 1 > L > 0$, or $(1 + \gamma)^{-1} \in (L, 1)$ at $t = 1$, where $\gamma \in (0, L^{-1} - 1)$ represents a liquidation cost. The common prior belief at $t = 0$ is that $\Pr(\tilde{v} = H) = \pi \in (0, 1)$ and $\Pr(\tilde{v} = L) = 1 - \pi$, which satisfies $\pi H + (1 - \pi)L > 1$. Thus, fund investment offers a higher expected long-run return than storage but is illiquid in the short run.

Information. At the first stage of date 1, a fraction α of investors privately receive an identical and informative signal $s \in [0, 1]$ about the prospective date-2 asset return $\tilde{v}$, where α is a constant and common knowledge. We refer to investors who receive s as informed and to those who do not as uninformed. No individual at $t = 0$ knows whether she will become informed. The signal s is realized according to some payoff-dependent distribution functions $F_v(s)$ with corresponding continuous densities $f_v(s)$. We assume that F_H dominates F_L in the Monotone Likelihood Ratio order, so $\frac{f_H(s)}{f_L(s)}$ is strictly increasing in s. The unconditional distribution function of s is denoted by $F(s)$ with density $f(s) = \pi f_H(s) + (1 - \pi)f_L(s)$.

After observing s, informed investors update their beliefs about $\tilde{v}$ (using Bayes' rule) to

$$\pi(s) \equiv \Pr(\tilde{v} = H|s) = \frac{\pi f_H(s)}{\pi f_H(s) + (1 - \pi)f_L(s)}. \tag{1}$$

Clearly, $\partial\pi(s)/\partial s > 0$. We further assume that $f_H(1) > 0$, $f_L(0) > 0$, and $f_H(0) = f_L(1) = 0$, so the signal is fully revealing at the boundaries, i.e., $\pi(0) = 0$ and $\pi(1) = 1$. Informed investors' conditional expectations about the risky asset's return are then given by

$$V(s) \equiv \mathbb{E}[\tilde{v}|s] = \pi(s)H + (1 - \pi(s))L. \tag{2}$$

Note that $V(s)$ is continuous and strictly increasing on $[0, 1]$, with $V(0) = L$ and $V(1) = H$.

Together, these imply the existence of a unique interior cutoff $s^\gamma \in (0,1)$ satisfying

$$V(s^\gamma) = \frac{1}{1+\gamma}, \tag{3}$$

such that it is ex post efficient to liquidate the asset at $t = 1$ if and only if $s \in [0, s^\gamma)$. There exists another unique interior cutoff $s^* \in (s^\gamma, 1)$ such that $V(s^*) = 1$.

Redemption fee. The fund imposes a redemption fee $\phi \in [0, \Phi]$, where $\Phi \equiv \frac{\gamma}{1+\gamma}$, on early withdrawals (i.e., withdrawals at date 1). An investor who requests a withdrawal at date 1 receives $1 - \phi$, provided that the fund possesses enough resources (from asset liquidation) to satisfy all withdrawal requests. Because the fund invests all deposits in the risky asset at date 0 and each unit of the asset returns only $\frac{1}{1+\gamma}$ if liquidated at date 1, the fund must liquidate $(1+\gamma)(1-\phi)$ units of its investment for each unit of deposit withdrawn early. If fund resources are insufficient to meet all withdrawal demands, then the fund is liquidated, and the liquidation value is equally distributed among all withdrawers, who each receive less than $1-\phi$. A higher fee lowers liquidation costs by allowing the fund to liquidate a smaller fraction of its investment to satisfy a given early-withdrawal demand. The upper bound on ϕ ensures that $(1+\gamma)(1-\phi) \geq 1$, so fees paid by early withdrawers merely reduce the associated liquidation costs but do not provide additional capital for the fund's remaining investors.[7] Note that there is a unique cutoff $s^\phi \in [s^\gamma, s^*]$ such that $V(s^\phi) = 1 - \phi$. The redemption fee applies to withdrawals at both the first and second stages at date 1 but not to distributions at date 2. Section 5 considers an extension in which the fee applies only to second-stage withdrawals.

To streamline the exposition and highlight the fee's impact on deposit withdrawals and welfare via the information channel, we focus much of our analysis on two limiting cases of ϕ. The first limiting case is $\phi = 0$, which corresponds to a setting wherein investors may

[7] According to the SEC, the redemption fee should be "high enough ... so that funds can recoup costs of providing liquidity to redeeming shareholders ... but low enough to permit investors who wish to redeem ... to receive their proceeds without bearing disproportionate costs," and "a default fee of 1% strikes this balance" ("Money Market Fund Reform; Amendments to Form PF," 79 Federal Register 157, 2014). The size of 1% was determined by analyzing the increase in liquidation costs, i.e., bid-ask spreads, during the 2008 financial crisis, as outlined in the DERA Memorandum Regarding Liquidity Cost During Crisis Periods, dated March 17, 2014. Hence, it appears that the size of the default fee was designed to internalize the payoff externality.

withdraw without paying a fee. Such a setting serves as a benchmark for evaluating the effects of a fee. The second limiting case is $\phi = \Phi$, wherein the fund liquidates exactly 1 unit of the asset for each unit of deposit withdrawn early because $(1+\gamma)(1-\Phi) = 1$. A fee equal to Φ thus fully internalizes the payoff externality generated by early withdrawers by exactly offsetting the associated liquidation costs. In other words, when $\phi = \Phi$, the liquidation costs generated by early withdrawers are borne *entirely* by themselves instead of being imposed on late withdrawers either completely (if $\phi = 0$) or partially (if $\phi \in (0, \Phi)$). Consequently, such a fee completely eliminates the first-mover advantage that would otherwise exist among investors. This setting cleanly demonstrates how the fee alters the information externality and, thereby, affects investor learning, which is the main novelty of our model.

Withdrawal game at date 1. As stated above, date 1 is divided into two stages. We model a two-stage game because runs tend to evolve over time and have important feedback effects (e.g., He and Manela (2016) and Schmidt et al. (2016)). At each stage, investors independently decide whether to withdraw their deposits, so withdrawals may occur at the first stage, the second stage, or both stages of date 1. For simplicity, we assume that investors generally do not make a withdrawal if they are indifferent between withdrawing and not withdrawing. However, in cases wherein investors know with certainty at the first stage that they will withdraw before date 2 but are indifferent between withdrawing at the first or second stage of date 1, we assume that they withdraw at the first stage rather than waiting until the second. Figure 1 illustrates the sequence of events at date 1, which we describe below.

First stage. At the beginning of the first stage, each investor privately learns her consumption type (patient or impatient). A fraction α of investors also become privately informed; they receive the signal s and update their beliefs about the date-2 fund return to $V(s)$. The other $1-\alpha$ fraction remain uninformed; they do not observe s and are unaware, at this first stage, that some other investors have become informed. As explained below, this unawareness assumption simplifies the analysis but does not affect the qualitative results. All investors then decide simultaneously and independently whether to withdraw. Impatient investors (mass λ),

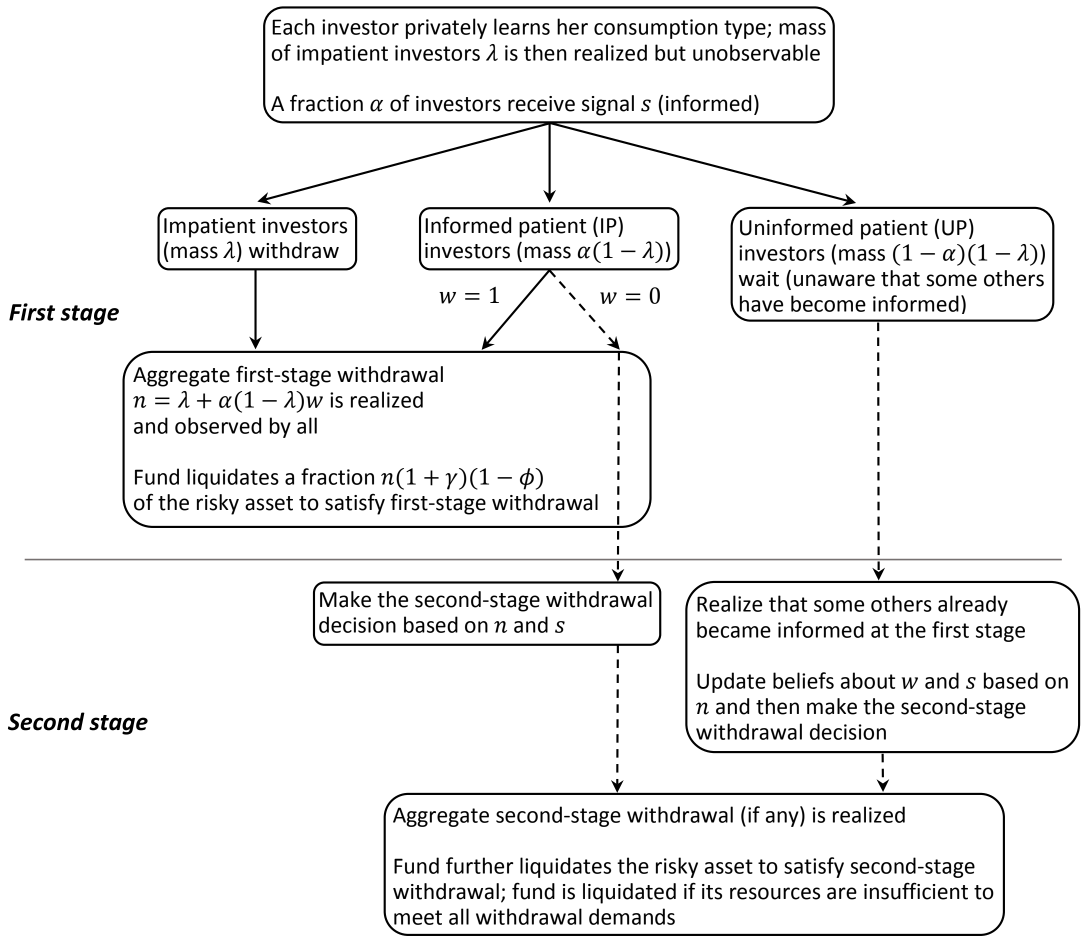

Figure 1: Timing of the withdrawal game at date 1.

regardless of being informed or uninformed, must withdraw at date 1. They withdraw at the first stage rather than the second because waiting until the second stage may result in a lower payment if additional investors withdraw at the second stage and, thereby, prevent the fund from being able to fully satisfy all of the second-stage withdrawal requests due to additional liquidation costs. In contrast, no uninformed patient ("UP") investors (mass $(1-\alpha)(1-\lambda)$) withdraw at the first stage. Being unaware that others have become informed, a UP investor's available information at this stage is the same as that at date 0. Thus, if a UP investor were to withdraw at this stage, she would not have invested in the fund in the first place at date 0.

Informed patient ("IP") investors (mass $\alpha(1-\lambda)$) make their decisions based on the signal s. Formally, each IP investor chooses $w \in \{0, 1\}$, where $w = 1$ if she withdraws at the first stage and $w = 0$ otherwise. An IP investor who does not withdraw at the first stage may still withdraw at the second stage or keep her deposit until date 2.

After all investors have made their decisions, the aggregate first-stage withdrawal

$$n = \lambda + \alpha(1 - \lambda)w \tag{4}$$

is realized and observed by all. Liquidity withdrawals λ and individual IP investors' choices of w, however, are not directly observable. Note that $n \in [0, \bar{\lambda} + \alpha(1 - \bar{\lambda})]$. In practice, investors can observe daily net capital flows for many MMMFs because total assets under management are publicly available (from sources such as Morningstar) at a daily frequency.

Second stage. After observing n, all remaining investors infer that the amount of deposits remaining at the fund is $1 - n(1 + \gamma)(1 - \phi)$. They again decide simultaneously and independently whether to withdraw at the second stage or wait until date 2. UP investors, who were unaware at the first stage that a fraction α of investors became informed, now realize this fact. The realization could be due to the circulation of rumors, as in He and Manela (2016). Based on n, UP investors (who did not withdraw at the first stage) update their beliefs about the decisions made by IP investors at the first stage (i.e., w), which in turn affects their beliefs about the signal s and the prospective asset return $\tilde{v}$. Then, they decide whether to withdraw.

IP investors make their decisions at the second stage based on the signal s and the realized first-stage withdrawal n (assuming that they did not already withdraw at the first stage, i.e., $w = 0$). These types of investors infer that the first-stage withdrawals were all made by impatient investors (i.e., $n = \lambda$) because they know that $w = 0$.

Assuming that UP investors are unaware at the first stage that some others have become informed ensures that they do not withdraw at the first stage and, thereby, trivialize the second-stage game. Instead, if UP investors realized their information disadvantage at the first stage, then they could choose to make a first-stage withdrawal under certain parameterizations even though ex ante they were willing to invest at date 0. In that case, the signal extraction problem faced by UP investors at the second stage and the impact of a redemption fee on such inference, which are the cornerstones of our analysis, would be rendered moot: the two-stage dynamic withdrawal game essentially would degenerate into a one-stage game with simultaneous moves. The unawareness assumption is stronger than required because, depending on parameters, UP investors could choose to maintain their deposits even if they realized their information disadvantage at the first stage. However, determining such parameterizations would require us to compute a UP investor's expected payoff at the first stage conditional on her beliefs about the strategies played by others at the first *and* second stages. Such an analysis would be tremendously complex without providing much additional insight — after all, we only require that UP investors do not withdraw at the first stage, and this can be achieved by imposing an unawareness assumption. Similar assumptions are employed in Abreu and Brunnermeier (2003) and He and Manela (2016).[8]

Parametric Restrictions. We make the following two assumptions to ensure that the signal extraction problem faced by UP investors at the second stage is non-trivial and informative.

Assumption 1. *There exist possible realizations of the aggregate first-stage withdrawal n upon which UP investors face a non-trivial inference problem at the second stage:*

$$\alpha < \bar{\lambda} < \min\left\{1 - \frac{\gamma}{(1-\alpha)(1+\gamma)}\,,\; 1 - \frac{\gamma H}{\alpha[(1+\gamma)H - 1]}\right\}. \tag{5}$$

As stated above, all possible realizations of n lie in the region $[0, \bar{\lambda} + \alpha(1-\bar{\lambda})]$. If $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1-\bar{\lambda})]$, then UP investors infer unambiguously that IP investors already withdrew at the first stage (i.e., $w = 1$) because the largest possible first-stage withdrawal by impatient investors alone is $\bar{\lambda}$. If $n \in [0, \alpha)$, then UP investors know for sure that IP investors did not withdraw at the first stage (i.e., $w = 0$); if they had withdrawn, then the smallest possible first-stage withdrawal would be α. The lower bound on $\bar{\lambda}$ in (5) ensures the existence of

[8]In their analysis of rumor-based bank runs, He and Manela (2016) also assume that uninformed investors realize with a delay (which they refer to as an "awarenes window") that others may be informed and remain fully deposited before they become aware of their information disadvantage. While they provide parametric restrictions under which such an outcome emerges in equilibrium, the parametric restrictions required in our framework, if we were to revoke the unawareness assumption, would be much more complex.

a "confounding region" wherein $n \in [\alpha, \bar{\lambda}]$. In this region, UP investors face a non-trivial inference problem because they cannot ascertain whether $w = 0$ or $w = 1$ based on n.

To focus on the role of information, we also assume that $\bar{\lambda}$ is bounded from above as in (5). The first upper bound, $1 - \frac{\gamma}{(1-\alpha)(1+\gamma)}$, ensures that the combined first-stage withdrawals by impatient and IP investors (if $w = 1$) do not exhaust fund resources at the first stage for any $\phi \in [0, \Phi]$. This ensures that UP investors are not fully exploited by first-stage withdrawers and, hence, have a meaningful second-stage game to play. The second upper bound, $1 - \frac{\gamma H}{\alpha[(1+\gamma)H-1]}$, excludes the possibility of second-stage "panic runs" in which the combined liquidation costs generated by withdrawals made by impatient investors (first stage) and UP investors (second stage) are so large that IP investors who do not make a first-stage withdrawal always withdraw at the second stage regardless of their signal s for any $\phi \in [0, \Phi]$. As we make clear in Section 3.1 (see footnote 11), this allows us to focus on information-based runs at the second stage.

Assumption 2. *In the confounding region wherein the realized first-stage withdrawal $n \in [\alpha, \bar{\lambda}]$, UP investors infer a greater likelihood that IP investors withdrew at the first stage when n is larger, but the marginal change in the conditional likelihood is relatively modest:*

$$0 < \frac{\partial \log\left(\frac{g(\frac{n-\alpha}{1-\alpha})}{g(n)}\right)}{\partial n} \leq \frac{1}{1-n} \qquad \forall n \in [\alpha, \bar{\lambda}]. \tag{6}$$

The first inequality in (6) ensures that the first-stage withdrawal $n \in [\alpha, \bar{\lambda}]$ is informative to UP investors: a larger n implies a higher probability that IP investors made a first-stage withdrawal (i.e., $w = 1$). Because $n = \lambda + \alpha(1-\lambda)w$, the likelihood of observing a first-stage withdrawal of size n conditional on $w = 1$ is $g(\frac{n-\alpha}{1-\alpha})$, whereas the likelihood of observing n conditional on $w = 0$ is $g(n)$. The inequality, which states that the ratio $g(\frac{n-\alpha}{1-\alpha})/g(n)$ is strictly increasing in n, means that a larger realization of n assigns a strictly greater probability mass on $w = 1$ relative to $w = 0$ in the sense of the Monotone Likelihood Ratio order.

The second inequality states that the likelihood ratio $g(\frac{n-\alpha}{1-\alpha})/g(n)$ does not increase too fast in n. That is, although the likelihood that $w = 0$ (relative to $w = 1$) decreases in n, it does not decrease too much at the margin. The implication of this assumption is as follows. At the second stage, UP investors not only face the aforementioned signal extraction problem but may also suffer from a payoff externality generated by IP investors who did not withdraw at the first stage (if $w = 0$) but who might withdraw at the second stage. Obviously, the externality does not exist if $w = 1$. The assumption here ensures that the likelihood of such an externality being imposed on UP investors at the second stage does not diminish too fast at the margin. This condition is stronger than necessary, but, as we make clear in Section 3.2, it provides a tractable and interpretable restriction on the parameter space that guarantees the existence of an equilibrium while maintaining the generality of distribution assumptions.

3 Equilibrium

We restrict attention to symmetric pure-strategy Perfect Bayesian Equilibria ("PBE"), wherein investors of the same type choose the same equilibrium strategies. A PBE of the two-stage game at date 1 for a given ϕ, denoted by a triplet $\{\hat{s}_1^\phi, \hat{s}_2^\phi(n), \bar{n}^\phi\}$, consists of IP and UP investors' strategies at each stage and their beliefs, with the following specifics.[9]

1. There exists a signal threshold $\hat{s}_1^\phi \in (0, 1)$ such that IP investors withdraw at the first stage (i.e., $w = 1$) if and only if $s < \hat{s}_1^\phi$. This strategy maximizes an IP investor's expected payoff conditional on the information available to her at the first stage (i.e., the density $g(\lambda)$ and the signal s) and her beliefs about other IP investors' first-stage strategies and the strategies played by investors at the second-stage subgame (including herself if she chooses not to withdraw at the first stage, i.e., $w = 0$).

2. There exists a signal threshold $\hat{s}_2^\phi(n) \in (0, 1)$, which is a function of the realized first-stage withdrawal n, such that IP investors who chose $w = 0$ withdraw at the second stage if and only if $s < \hat{s}_2^\phi(n)$. This strategy maximizes a remaining IP investor's expected

[9]The equilibrium definition does not specify any beliefs in response to out-of-equilibrium moves from the first stage. This is because there are no detectable out-of-equilibrium moves from the first stage given that deviation by a single atomistic investor has no consequence on the realization of the first-stage withdrawal n.

payoff at the second-stage subgame given the information available to her at that stage (i.e., n and s) and her beliefs about the second-stage strategies played by UP investors (i.e., $\bar{n}^{\phi}$, as described below) and other remaining IP investors.

3. There exists a withdrawal threshold $\bar{n}^{\phi}$ such that UP investors withdraw at the second stage if and only if $n > \bar{n}^{\phi}$. This strategy maximizes a UP investor's expected payoff given the information available to her at the second stage (i.e., n) and her beliefs about other UP investors' strategies and the strategies played by IP investors at the first stage (i.e., $\hat{s}_1^{\phi}$) and the second stage (i.e., $\hat{s}_2^{\phi}(n)$) if she believes $w = 0$.

4. Investors' beliefs are updated (whenever possible) according to Bayes' rule, taking others' strategies as given, and they are consistent with those strategies played in equilibrium.

In Section 3.1, we derive the equilibrium strategies for IP investors, who may withdraw at either stage based on a combination of their private signal and the payoff externality generated by withdrawing impatient investors (first stage) and possibly UP investors (second stage). In Section 3.2, we derive the equilibrium strategy for UP investors, who may withdraw at the second stage based on the information they extract from the realized first-stage withdrawal and the payoff externality generated by withdrawing impatient investors (first stage) and possibly IP investors (either stage). We discuss equilibrium uniqueness in Section 3.3.

3.1 IP Investors

We use backward induction to characterize the IP investors' strategies, taking the UP investors' strategy $\bar{n}^{\phi}$ as given. We first determine the IP investors' second-stage strategy $\hat{s}_2^{\phi}(n)$ conditional on not withdrawing at the first stage in Section 3.1.1. In Section 3.1.2, we characterize their first-stage strategy $\hat{s}_1^{\phi}$.

3.1.1 IP Investors' Second-Stage Strategy

At the beginning of the second stage, the remaining investors consist of IP investors if they did not withdraw at the first stage and UP investors. After observing the realized first-stage withdrawal n, these investors infer that the amount of deposits remaining at the fund is $1 - n(1+\gamma)(1-\phi)$. Provided that an IP investor did not already withdraw at the first stage, she withdraws at the second stage when the expected payoff from doing so exceeds that from keeping her deposit until date 2. The latter payoff depends on whether UP investors also keep their deposits until date 2, which in turn depends on the first-stage withdrawal n. We focus on the case wherein $n \in [0, \bar{\lambda}]$ because, as discussed in Section 2, there are no IP investors remaining at the second stage if $n > \bar{\lambda}$. Furthermore, we conjecture (and verify in Lemma 2 in Section 3.2) that UP investors always withdraw at the second stage if they know for sure that IP investors already withdrew at the first stage (i.e., if $n > \bar{\lambda}$). Thus, the UP investors' second-stage decision threshold satisfies $\bar{n}^{\phi} \in [0, \bar{\lambda}]$.

If $n \leq \bar{n}^{\phi}$, then UP investors do not withdraw at the second stage in the conjectured equilibrium. As a result, each individual IP investor also does not withdraw if and only if

$$\frac{1 - n(1+\gamma)(1-\phi)}{1-n} V(s) \geq 1 - \phi. \tag{7}$$

Equation (7) can be understood as follows. Given that UP investors do not withdraw at the second stage, IP investors know that if they also do not withdraw, then the amount of deposits remaining at the fund until date 2 is $1 - n(1+\gamma)(1-\phi)$, resulting in a date-2 fund value of $[1 - n(1+\gamma)(1-\phi)]V(s)$. This value is equally distributed among all remaining investors at date 2, the mass of which is $1 - n$. Instead, if an atomistic IP investor deviates from the equilibrium strategy and withdraws at the second stage, then she receives $1 - \phi$.

IP investors face a slightly different tradeoff if $n > \bar{n}^{\phi}$ because UP investors withdraw at the second stage. IP investors also withdraw in this case unless

$$\frac{1 - [n + (1-\alpha)(1-n)](1+\gamma)(1-\phi)}{\alpha(1-n)} V(s) \geq 1 - \phi. \tag{8}$$

Equation (8) can be interpreted similarly as (7), with one difference. When $n > \bar{n}^{\phi}$, IP investors know that if they do not withdraw at the second stage, then the amount of remaining deposits is $1-[n+(1-\alpha)(1-n)](1+\gamma)(1-\phi)$, given that UP investors (mass $(1-\alpha)(1-n)$) withdraw their deposits. The expected date-2 fund value is equally divided among IP investors who keep their deposits until date 2 (mass $\alpha(1-n)$). The ensuing theorem characterizes $\hat{s}_2^\phi(n)$.

Theorem 1. *If $\phi < \Phi$, then for any realized first-stage withdrawal $n \in [0, \bar{\lambda}]$ and the UP investors' second-stage withdrawal threshold $\bar{n}^\phi \in [0, \bar{\lambda}]$, there exists a unique signal threshold $\hat{s}_2^\phi(n) \in (s^\gamma, 1]$, characterized by*

$$V(\hat{s}_2^\phi(n)) = \begin{cases} \dfrac{(1-n)(1-\phi)}{1-n(1+\gamma)(1-\phi)} & \text{if } n \in [0, \bar{n}^\phi] \\ \dfrac{\alpha(1-n)(1-\phi)}{1-[1-\alpha(1-n)](1+\gamma)(1-\phi)} & \text{if } n \in (\bar{n}^\phi, \bar{\lambda}], \end{cases} \tag{9}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $s < \hat{s}_2^\phi(n)$; furthermore, $\hat{s}_2^\phi(n)$ is strictly increasing in n. If $\phi = \Phi$, then $\hat{s}_2^\Phi(n) = s^\gamma$.

Impact of a redemption fee on the IP investors' second-stage strategy. Theorem 1 describes $\hat{s}_2^\phi(n)$ for any generic fee $\phi \in [0, \Phi]$. To succinctly highlight the novel learning channel through which a redemption fee affects $\hat{s}_2^\phi(n)$, we focus our discussion on the two limiting cases described in Section 2: $\phi = 0$ and $\phi = \Phi$. As discussed there, the payoff externality among investors is strongest when $\phi = 0$, but such externality is fully internalized and investors base their decisions solely on their beliefs about fundamentals when $\phi = \Phi$. Although we explicitly discuss only the two limiting cases, the intuition developed is robust to any $\phi \in [0, \Phi]$. In particular, it is straightforward to show that as ϕ increases over $[0, \Phi]$, investors place more emphasis on their beliefs about the asset payoff and less on the potential payoff externality when making their withdrawal decisions.

Case with $\phi = 0$. Figure 2, which illustrates $\hat{s}_2^\phi(n)$ when $\phi = 0$, demonstrates two main results in Theorem 1 that hold in general for any $\phi < \Phi$.[10] First, IP investors who did not make a first-stage withdrawal withdraw at the second stage if and only if their signal is below

[10]Figure 2 plots $V(\hat{s}_2^\phi(n))$ against n, which is convenient due to the explicit functional relation between the two (see (9)). Because $V(s)$ is continuous and strictly increasing in s, the relation between $\hat{s}_2^\phi(n)$ and n is isomorphic to the plotted relation between $V(\hat{s}_2^\phi(n))$ and n.

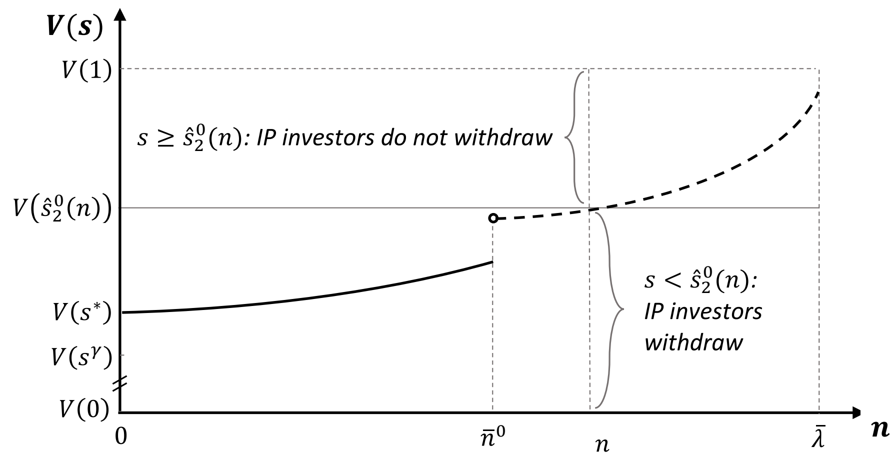

Figure 2: IP investors' second-stage signal threshold $\hat{s}_2^\phi(n)$ when $\phi = 0$. $V(\hat{s}_2^0(n))$, which is isomorphic to the (remaining) IP investors' second-stage signal threshold $\hat{s}_2^0(n)$, is plotted as a function of the first-stage withdrawal n. The solid curve represents the threshold if UP investors do not withdraw at the second stage (i.e., if $n \leq \bar{n}^0$), whereas the dashed curve represents the threshold if UP investors withdraw (i.e., if $n > \bar{n}^0$).

some threshold $\hat{s}_2^0(n)$. The threshold is strictly increasing in n because IP investors demand a greater expected asset return (equivalently, higher s) to maintain their deposits when a larger mass of investors withdraw at the first stage and, thereby, impose greater liquidation costs on those who remain invested. Second, $\hat{s}_2^0(n)$ jumps upward as n crosses the UP investors' withdrawal threshold $\bar{n}^0$. This is because IP investors bear only the liquidation costs imposed by impatient investors' first-stage withdrawals when $n \leq \bar{n}^0$, in which case UP investors do not withdraw at the second stage, but must absorb the *additional* liquidation costs caused by UP investors' second-stage withdrawal when $n > \bar{n}^0$. Consequently, IP investors, *ceteris paribus*, require a higher s to maintain their deposits when $n > \bar{n}^0$.[11]

IP investors' withdrawal decisions are socially suboptimal when $\phi = 0$ (note that $\hat{s}_2^\phi(n) > s^\gamma$ for any $\phi < \Phi$; recall, s^γ is the signal cutoff that defines the ex post efficient liquidation threshold in (3)), so they may withdraw even when the asset's expected payoff exceeds its liquidation value. This inefficiency arises because IP investors who do not withdraw must bear the liquidation costs generated by other investors who withdraw if the fee does not fully

[11]The assumption that $\bar{\lambda} < 1 - \frac{\gamma H}{\alpha[(1+\gamma)H-1]}$ made in (5) ensures that $\hat{s}_2^\phi(\bar{\lambda}) < 1$ for any $\phi \in [0, \Phi]$. Thus, if $s \in [\hat{s}_2^\phi(\bar{\lambda}), 1]$, then IP investors remain invested until date 2 (if $w = 0$) even if impatient and UP investors withdraw at date 1.

internalize the payoff externality. Moreover, an IP investor who withdraws when $\phi = 0$ does not bear the liquidation costs generated by her own withdrawal.

Case with $\phi = \Phi$. In contrast, an IP investor's second-stage strategy when $\phi = \Phi$ is independent of both the first-stage withdrawal n and the UP investors' second-stage actions because the fees paid by withdrawing investors fully internalize the liquidation costs generated by those investors. This not only allows IP investors to focus solely on their signal and ignore others' actions, but it also eliminates the first-mover advantage by forcing IP investors to bear their own liquidation costs. As a result, IP investors' second-stage decisions are purely information-based and socially optimal: they withdraw if and only if their signal indicates that the asset's expected payoff $V(s)$ is below its liquidation value $\frac{1}{1+\gamma}$ (i.e., $s < s^{\gamma}$).

3.1.2 IP Investors' First-Stage Strategy

IP investors decide whether to withdraw at the first stage based on their signal s and their beliefs about the strategies played by investors at the second-stage subgame, i.e., $\hat{s}_2^{\phi}(n)$ and $\bar{n}^{\phi}$. They withdraw at the first stage if and only if the payoff from withdrawing, $1-\phi$, exceeds the expected payoff from keeping their deposits at the first stage.

Theorem 1 states that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $s < \hat{s}_2^{\phi}(n)$, which is equivalent to $n > \hat{n}^{\phi}(s)$, where the threshold $\hat{n}^{\phi}(s)$ is determined by inverting $\hat{s}_2^{\phi}(n)$ and is explicitly characterized by Lemma A.1 in Appendix A. Figure 3 depicts $\hat{n}^{\phi}(s)$ when $\phi = 0$, but the intuition is robust to any $\phi < \Phi$.[12] The figure shows that IP investors who maintain their deposits at the first stage withdraw at the second stage if and only if the realized first-stage withdrawal n (equivalently, λ, if only impatient investors withdraw at the first stage) exceeds $\hat{n}^0(s)$. The threshold is weakly increasing in s because the remaining IP investors at the second stage are willing to absorb greater liquidation costs imposed by a larger first-stage withdrawal when s (hence, $V(s)$) is higher. Depending on s, $\hat{n}^{\phi}(s)$ may be less than, equal to, or greater than the UP

[12] Like Figure 2, Figure 3 plots $\hat{n}^{\phi}(s)$ against $V(s)$, but the relation between $\hat{n}^{\phi}(s)$ and s is isomorphic to the relation between $\hat{n}^{\phi}(s)$ and $V(s)$.

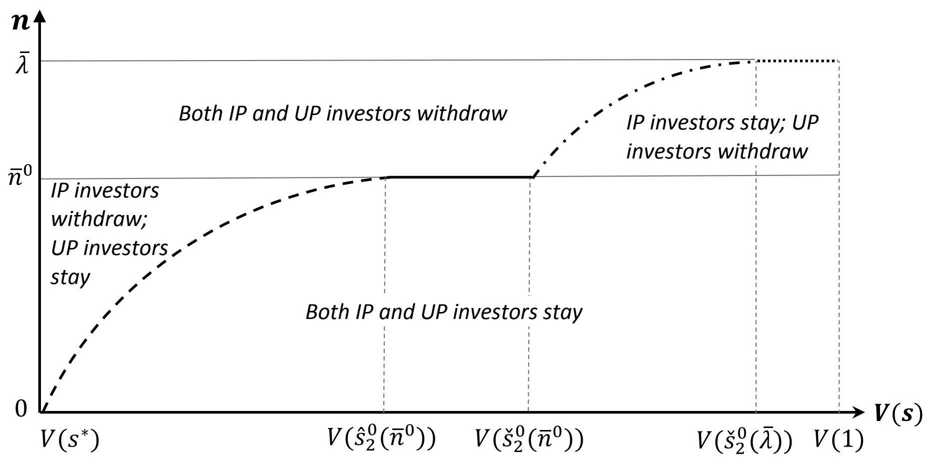

Figure 3: IP investors' second-stage withdrawal threshold $\hat{n}^{\phi}(s)$ when $\phi = 0$. The (remaining) IP investors' second-stage withdrawal threshold $\hat{n}^0(s)$ is plotted as a function of $V(s)$ (which is isomorphic to s). The dashed curve represents the thresholds if $s \in [s^*, \hat{s}_2^0(\bar{n}^0)]$ wherein IP investors are more likely to withdraw than UP investors (i.e., $\hat{n}^0(s) < \bar{n}^0$), the solid curve represents the threshold if $s \in (\hat{s}_2^0(\bar{n}^0), \check{s}_2^0(\bar{n}^0)]$ wherein IP and UP investors' withdrawal decisions are identical (i.e., $\hat{n}^0(s) = \bar{n}^0$), and the dashed-dotted curve represents the threshold if $s \in (\check{s}_2^0(\bar{n}^0), \check{s}_2^0(\bar{\lambda})]$ wherein IP investors are less likely to withdraw than UP investors (i.e., $\hat{n}^0(s) > \bar{n}^0$). In the region with the dotted curve wherein $s > \check{s}_2^0(\bar{\lambda})$, IP investors do not withdraw at the second-stage regardless of the first-stage withdrawal size n. Lemma A.1 in Appendix A characterizes $\check{s}_2^0(n)$.

investors' withdrawal threshold $\bar{n}^{\phi}$. Thus, for a given signal s, the relation between $\bar{n}^{\phi}$ and $\hat{n}^{\phi}(s)$ determines which of the following four scenarios an IP investor may encounter at the second stage (conditional on not withdrawing at the first stage, of course).[13]

1. *Neither IP nor UP investors withdraw at the second stage.* In this scenario, there are no additional liquidation costs beyond those imposed by the first-stage liquidity withdrawals λ. Thus, an IP investor's expected payoff from not withdrawing is $\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda}V(s)$.

2. *IP investors withdraw at the second stage but UP investors do not.* In this case, IP investors receive $1 - \phi$ from withdrawing at the second stage.

3. *UP investors withdraw at the second stage but IP investors do not.* In this case, IP investors must bear the additional liquidation costs imposed by withdrawing UP investors (mass $(1-\alpha)(1-\lambda)$). Hence, an IP investor's expected payoff from not withdrawing is $\frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)(1-\phi)}{\alpha(1-\lambda)}V(s)$.

[13]If $\hat{n}^{\phi}(s) < \bar{n}^{\phi}$, then scenario 3 is impossible. If $\hat{n}^{\phi}(s) = \bar{n}^{\phi}$, then neither scenario 2 nor 3 is possible. If $\hat{n}^{\phi}(s) > \bar{n}^{\phi}$, then scenario 2 is impossible. If $\hat{n}^{\phi}(s) = \bar{\lambda}$, then neither scenario 2 nor 4 is possible.

4. *Both IP and UP investors withdraw at the second stage.* If this happens, then the fund must liquidate all of its investment remaining after the first-stage liquidity withdrawals, $1-\lambda(1+\gamma)(1-\phi)$, and evenly distribute the liquidation proceeds among the IP and UP investors (mass $1-\lambda$), who each receive $\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)}$.

The specific scenario encountered by IP investors at the second stage depends on the realized liquidity withdrawal λ, which affects n. However, IP investors do not know λ when making their first-stage decisions. Therefore, they must take into account their uncertainty about λ when determining their first-stage strategy. The following theorem describes the IP investors' first-stage strategy $\hat{s}_1^\phi$, which is characterized in Appendix A by (A8).

Theorem 2. *If $\phi < \Phi$, then given the UP investors' second-stage withdrawal threshold $\bar{n}^\phi$, there exists a unique signal threshold $\hat{s}_1^\phi \in (s^\gamma, 1)$ such that IP investors withdraw at the first stage if and only if $s < \hat{s}_1^\phi$; furthermore, $\hat{s}_1^\phi$ is decreasing in $\bar{n}^\phi$. If $\phi = \Phi$, then $\hat{s}_1^\Phi = s^\gamma$.*

Impact of a redemption fee on the IP investors' first-stage strategy. We again focus our discussion on the two limiting cases, $\phi = 0$ and $\phi = \Phi$, to emphasize the information channel through which the fee affects $\hat{s}_1^\phi$.

Case with $\phi = 0$. Figure 4 illustrates $\hat{s}_1^\phi$ when $\phi = 0$, but the intuition holds in general for any $\phi < \Phi$. The four regions of $V(s)$ in the figure correspond to the four regions in Figure 3, and $V(\hat{s}_1^0)$ is the point at which the curve, which represents the ex ante expected payoff to a non-withdrawing IP investor at the first stage, intersects 1 (note that $1-\phi = 1$ in this limiting case). The signal threshold $\hat{s}_1^0$ is strictly greater than s^* (recall, $V(s^*) = 1$) due to the expected liquidation costs imposed on a non-withdrawing IP investor by other investors who may withdraw before date 2. Additionally, $\hat{s}_1^0$ is decreasing in $\bar{n}^0$. As $\bar{n}^0$ increases, UP investors become less likely to make a second-stage withdrawal, which reduces the expected liquidation costs imposed on non-withdrawing IP investors. Anticipating this, IP investors do not require as high of a signal to maintain their deposits at the first stage, so $\hat{s}_1^0$ decreases.

When $\phi = 0$, the IP investors' first-stage and second-stage signal thresholds, $\hat{s}_1^0$ and $\hat{s}_2^0(n)$,

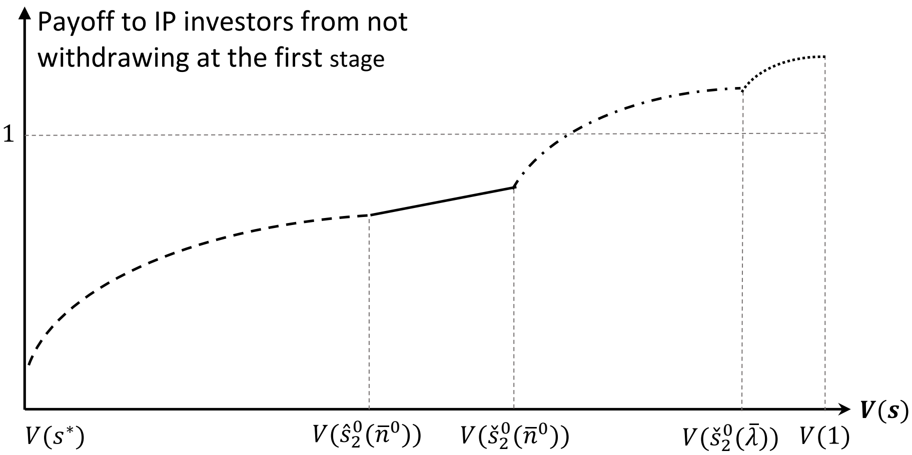

Figure 4: IP investors' first-stage signal threshold $\hat{s}_1^\phi$ when $\phi = 0$. The IP investors' expected payoff before the realized liquidity withdrawal λ is observed is plotted as a function of $V(s)$ (which is isomorphic to s). The dashed curve represents the payoff in scenarios wherein IP investors are more likely to withdraw at the second stage than UP investors, the solid curve represents the payoff in scenarios wherein IP and UP investors are equally likely to withdraw at the second stage, the dashed-dotted curve represents the payoff in scenarios wherein UP investors are more likely to withdraw at the second stage than IP investors, and the dotted curve represents the payoff in scenarios wherein IP investors do not withdraw regardless of the first-stage withdrawal size n. Lemma A.1 in Appendix A characterizes $\check{s}_2^0(n)$.

generally differ: $\hat{s}_1^0$ may be either greater or less than $\hat{s}_2^0(n)$, depending on n. The difference between the two thresholds arises from the fact that without a fee, IP investors' withdrawal decisions are based on not only their information but also the potential payoff externality. While the former remains constant from the first to the second stage, the latter generally does not. At the first stage, IP investors face uncertainty regarding both the size of the first-stage liquidity withdrawals λ and the potential for second-stage withdrawals by UP investors. This uncertainty disappears at the second stage because IP investors can infer both λ and whether UP investors will make a second-stage withdrawal given any realized first-stage withdrawal n.

Case with $\phi = \Phi$. When $\phi = \Phi$, however, the UP investors' second-stage strategy $\bar{n}^\Phi$ and the uncertainty about the liquidity withdrawal λ are irrelevant to the IP investors' first-stage decisions because the fees paid by withdrawing investors fully compensate the fund for the associated liquidation costs. As a result, an IP investor's expected payoff from keeping her deposit until date 2 is always $V(s)$, and her payoff from withdrawing at either stage of date 1 is always $1-\Phi = \frac{1}{1+\gamma}$. Consequently, IP investors' first-stage strategies are entirely information-based and identical to their second-stage strategies (i.e., $\hat{s}_1^{\Phi} = \hat{s}_2^{\Phi} = s^{\gamma}$), which is in sharp contrast to the outcome with no fee. Therefore, if IP investors withdraw at date 1, then they will withdraw at the first stage; if they do not withdraw at the first stage, then they will remain invested until date 2.[14] Hence, IP investors make no second-stage withdrawals in equilibrium when $\phi = \Phi$.

In sum, IP investors become less inclined to withdraw when a redemption fee is imposed. Moreover, their withdrawal decisions are socially optimal when $\phi = \Phi$ but are socially suboptimal when $\phi = 0$.

3.2 UP Investors

We now examine the UP investors' strategy $\bar{n}^{\phi}$, taking as given the IP investors' first-stage and second-stage signal thresholds, $\hat{s}_1^{\phi}$ and $\hat{s}_2^{\phi}(n)$. UP investors do not observe the signal s received by IP investors, but they can update their beliefs about s, and hence the asset return $\tilde{v}$, based on the first-stage withdrawal n.

As discussed in Section 2, if $n \in [0, \alpha)$, then UP investors know with certainty that IP investors did not make a first-stage withdrawal (i.e., $w = 0$) and, consequently, infer that $s \geq \hat{s}_1^{\phi}$. Conversely, if $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1-\bar{\lambda})]$, then UP investors infer unambiguously that IP investors withdrew at the first stage (i.e., $w = 1$) and, hence, $s < \hat{s}_1^{\phi}$. In the confounding region wherein $n \in [\alpha, \bar{\lambda}]$, UP investors cannot disentangle whether the first-stage withdrawal is entirely due to impatient investors withdrawing for liquidity reasons or partially due to IP investors withdrawing for information reasons. Thus, they are uncertain about w, but they infer the conditional likelihood that $w = 1$ based on n and update their beliefs about s accordingly. Specifically, because $w = 1$ only when $s < \hat{s}_1^{\phi}$ and the density of liquidity-driven withdrawals $g(\cdot)$ satisfies the Monotone Likelihood Ratio Property (see Assumption 2), the probability that $s < \hat{s}_1^{\phi}$ conditional on n, denoted by $F(\hat{s}_1^{\phi}|n)$, is increasing in n. Consequently,

[14] Recall, our model assumes that an investor withdraws at the first stage rather than waiting until the second if she withdraws early but is indifferent between withdrawing at the first or second stage of date 1.

UP investors' beliefs about $\tilde{v}$ are more pessimistic when n is larger. The following lemma characterizes $F(\hat{s}_1^\phi|n)$ and UP investors' conditional expectations of $\tilde{v}$, denoted by $\mathbb{E}[\tilde{v}|n,\hat{s}_1^\phi]$.

Lemma 1. *Conditional on a first-stage withdrawal n, UP investors' posterior beliefs about the likelihood that IP investors already withdrew at the first stage (hence $s < \hat{s}_1^\phi$) are given by*

$$F(\hat{s}_1^\phi|n) = \begin{cases} 0 & \text{if } n \in [0,\alpha) \\ \dfrac{F(\hat{s}_1^\phi)g\left(\frac{n-\alpha}{1-\alpha}\right)}{F(\hat{s}_1^\phi)g\left(\frac{n-\alpha}{1-\alpha}\right) + (1-F(\hat{s}_1^\phi))g(n)} & \text{if } n \in [\alpha,\bar{\lambda}] \\ 1 & \text{if } n \in (\bar{\lambda}, \bar{\lambda}+\alpha(1-\bar{\lambda})], \end{cases} \tag{10}$$

and their conditional expectations of $\tilde{v}$ are given by

$$\mathbb{E}[\tilde{v}|n,\hat{s}_1^\phi] = \begin{cases} \mathbb{E}[\tilde{v}|s \geq \hat{s}_1^\phi] & \text{if } n \in [0,\alpha) \\ \dfrac{F(\hat{s}_1^\phi)g\left(\frac{n-\alpha}{1-\alpha}\right)\mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi] + (1-F(\hat{s}_1^\phi))g(n)\mathbb{E}[\tilde{v}|s \geq \hat{s}_1^\phi]}{F(\hat{s}_1^\phi)g\left(\frac{n-\alpha}{1-\alpha}\right) + (1-F(\hat{s}_1^\phi))g(n)} & \text{if } n \in [\alpha,\bar{\lambda}] \\ \mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi] & \text{if } n \in (\bar{\lambda}, \bar{\lambda}+\alpha(1-\bar{\lambda})]. \end{cases} \tag{11}$$

Furthermore, $F(\hat{s}_1^\phi|n)$ is non-decreasing in n and monotonically increasing in n on $[\alpha,\bar{\lambda}]$. Conversely, $\mathbb{E}[\tilde{v}|n,\hat{s}_1^\phi]$ is non-increasing in n and monotonically decreasing in n on $[\alpha,\bar{\lambda}]$.

After updating their beliefs, UP investors decide whether to withdraw: they do so when the expected payoff from withdrawing exceeds that from maintaining their deposits until date 2. This decision is complicated by the fact that the payoff from maintaining their deposits also depends on whether the remaining IP investors (if any) withdraw their deposits at the second stage because that will impose additional liquidation costs (beyond those by any first-stage withdrawal) on non-withdrawing UP investors when $\phi < \Phi$.

Further complicating the matter, because UP investors observe only the first-stage withdrawal n but not the signal s, they generally cannot ascertain: (i) whether IP investors already withdrew at the first stage (when $n \in [\alpha,\bar{\lambda}]$); and (ii) whether IP investors who did not make a first-stage withdrawal will withdraw at the second stage (when $n \in [0,\bar{\lambda}]$). Nevertheless, UP investors can infer the likelihood of (i) based on n, as indicated by Lemma 1. Moreover, as we discuss below, UP investors can also use n to infer the likelihood of (ii).

Before moving forward, we first verify an earlier conjecture (made in Section 3.1.1) that UP investors always withdraw at the second stage if they can ascertain that IP investors already withdrew at the first stage (i.e., if $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})]$).

Lemma 2. *If $n \in (\bar{\lambda}, \bar{\lambda} + \alpha(1 - \bar{\lambda})]$, then UP investors withdraw at the second stage.*

We now characterize the UP investors' withdrawal threshold $\bar{n}^{\phi}$. Lemma 2 implies that $\bar{n}^{\phi} \in [0, \bar{\lambda}]$. To compute the expected payoff to a UP investor who does not withdraw at the second stage, we consider three cases, depending on whether IP investors (i) withdraw at the first stage, (ii) withdraw at the second stage, or (iii) do not withdraw at either stage.

First, suppose IP investors already withdrew at the first stage (i.e., $s < \hat{s}_1^{\phi}$). Conditional on n, UP investors believe that this outcome occurs with probability $F(\hat{s}_1^{\phi}|n)$. In this case, a UP investor's expected payoff from maintaining her deposit until date 2 is

$$\frac{1 - n(1+\gamma)(1-\phi)}{1-n} \mathbb{E}[\tilde{v}|s < \hat{s}_1^{\phi}]. \tag{12}$$

The numerator in (12), $1 - n(1+\gamma)(1-\phi)$, represents the remaining investment in the risky asset after the first-stage withdrawal n, and the denominator, $1 - n$, represents the mass of remaining investors at date 2. In this case, no additional liquidation costs beyond those generated by the first-stage withdrawal n are imposed on UP investors who maintain their deposits at the second stage because all IP investors already withdrew at the first stage.

Second, if IP investors did not withdraw at the first stage (i.e., $s \geq \hat{s}_1^{\phi}$), then they may make a second-stage withdrawal, which will impose additional liquidation costs on UP investors who do not withdraw at the second stage. According to Theorem 1, this happens if $s \in [\hat{s}_1^{\phi}, \hat{s}_2^{\phi}(n))$, which, from the UP investors' perspectives, occurs with probability $(1 - F(\hat{s}_1^{\phi}|n)) \frac{\max\{F(\hat{s}_2^{\phi}(n)) - F(\hat{s}_1^{\phi}), 0\}}{1 - F(\hat{s}_1^{\phi})}$. Here, $1 - F(\hat{s}_1^{\phi}|n)$ is the likelihood that $s \geq \hat{s}_1^{\phi}$ conditional on n, and $\frac{\max\{F(\hat{s}_2^{\phi}(n)) - F(\hat{s}_1^{\phi}), 0\}}{1 - F(\hat{s}_1^{\phi})}$ is the likelihood that $\hat{s}_1^{\phi} \leq s < \hat{s}_2^{\phi}(n)$ conditional on both $s \geq \hat{s}_1^{\phi}$

and n. In this case, the expected payoff to a non-withdrawing UP investor is

$$\frac{1-[n+\alpha(1-n)](1+\gamma)(1-\phi)}{(1-\alpha)(1-n)}\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \leq s < \hat{s}_2^\phi(n)]. \tag{13}$$

In (13), the term $\alpha(1-n)(1+\gamma)(1-\phi)$ in the numerator reflects the additional liquidation costs imposed by IP investors who withdraw at the second stage, and the denominator represents the mass of UP investors who keep their deposits until date 2.

The third case occurs when IP investors do not withdraw at either stage (i.e., when $s \geq \max\{\hat{s}_1^\phi, \hat{s}_2^\phi(n)\}$). From a UP investor's perspective, this occurs with probability $(1 - F(\hat{s}_1^\phi|n))\frac{1-\max\{F(\hat{s}_2^\phi(n)),F(\hat{s}_1^\phi)\}}{1-F(\hat{s}_1^\phi)}$. Similar to the second case, $1 - F(\hat{s}_1^\phi|n)$ is the likelihood that $s \geq \hat{s}_1^\phi$ conditional on n, and $\frac{1-\max\{F(\hat{s}_2^\phi(n)),F(\hat{s}_1^\phi)\}}{1-F(\hat{s}_1^\phi)}$ is the likelihood that $s \geq \max\{\hat{s}_1^\phi, \hat{s}_2^\phi(n)\}$ conditional on both $s \geq \hat{s}_1^\phi$ and n. Here, a non-withdrawing UP investor's expected payoff is

$$\frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|s \geq \max\{\hat{s}_1^\phi, \hat{s}_2^\phi(n)\}]. \tag{14}$$

Like the first case (wherein IP investors withdrew at the first stage), non-withdrawing UP investors are not exposed to further liquidation costs when the remaining IP investors do not withdraw at the second stage, so the coefficient in (14), $\frac{1-n(1+\gamma)(1-\phi)}{1-n}$, is the same as in (12).

The UP investors' withdrawal threshold $\bar{n}^\phi$ is determined by aggregating these three cases. Theorems 3 characterizes $\bar{n}^\phi$ and shows it to be unique. Note that $\mathbb{1}_{\{\cdot\}}$ is an indicator function.

Theorem 3. *Given the IP investors' respective first-stage and second-stage signal thresholds $\hat{s}_1^\phi$ and $\hat{s}_2^\phi(n)$, there exists a unique withdrawal threshold $\bar{n}^\phi$, characterized by*

$$\bar{n}^\phi = \sup\left\{n \in [0, \bar{\lambda}] : 1-\phi \leq \beta_1(\phi) - \beta_2(\phi)\right\}, \tag{15}$$

where

$$\beta_1(\phi) \equiv \frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi] \tag{16}$$

$$\beta_2(\phi) \equiv \mathbb{1}_{\{\hat{s}_1^\phi < \hat{s}_2^\phi(n)\}}(1-F(\hat{s}_1^\phi|n))\frac{F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi)}{1-F(\hat{s}_1^\phi)}\frac{\alpha[\gamma-(1+\gamma)\phi]\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \leq s < \hat{s}_2^\phi(n)]}{(1-\alpha)(1-n)}, \tag{17}$$

such that UP investors do not withdraw at the second stage if and only if $n \leq \bar{n}^\phi$.

The left-hand side (LHS) of the condition in (15), $1-\phi$, represents an atomistic individual UP investor's payoff from deviating from the equilibrium strategy by withdrawing while others do not, and the right-hand side (RHS) captures her expected payoff from not withdrawing. The first term on the RHS, $\beta_1(\phi)$, represents a UP investor's expected payoff from keeping her deposit until date 2, given that IP investors do not withdraw at the second stage (which could happen because either they already withdrew at the first stage or their signal s is high enough that they choose to keep their deposits until date 2). In this case, UP investors know that if they also do not withdraw, then the amount of deposits remaining invested until date 2 is $1-n(1+\gamma)(1-\phi)$. Thus, the expected date-2 fund value is $[1-n(1+\gamma)(1-\phi)]\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$, which will be equally divided among all remaining investors at date 2 (mass $1-n$).

The second term, $\beta_2(\phi)$, captures the potential additional liquidation costs imposed on UP investors by IP investors who may withdraw at the second stage. This occurs when $s \in [\hat{s}_1^\phi, \hat{s}_2^\phi(n))$, the probability of which is determined as follows. Conditional on a first-stage withdrawal n, the probability that IP investors did not withdraw at the first stage is $1-F(\hat{s}_1|n)$; and given that, the probability that IP investors withdraw at the second stage is $\mathbb{1}_{\{\hat{s}_1^\phi < \hat{s}_2^\phi(n)\}}\frac{F(\hat{s}_2^\phi(n))-F(\hat{s}_1^\phi)}{1-F(\hat{s}_1^\phi)}$. If IP investors make a second-stage withdrawal (i.e., if $s \in [\hat{s}_1^\phi, \hat{s}_2^\phi(n))$), then non-withdrawing UP investors must bear additional liquidation costs whose expected value is $\frac{\alpha[\gamma-(1+\gamma)\phi]\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \leq s < \hat{s}_2^\phi(n)]}{(1-\alpha)(1-n)}$, which is the difference between the expected payoffs to non-withdrawing UP investors when IP investors do not make a second-stage withdrawal and when they do: $\left[\frac{1-n(1+\gamma)(1-\phi)}{1-n} - \frac{1-[n+\alpha(1-n)](1+\gamma)(1-\phi)}{(1-\alpha)(1-n)}\right]\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \leq s < \hat{s}_2^\phi(n)]$.

Impact of a redemption fee on the UP investors' strategy. Theorem 3 characterizes

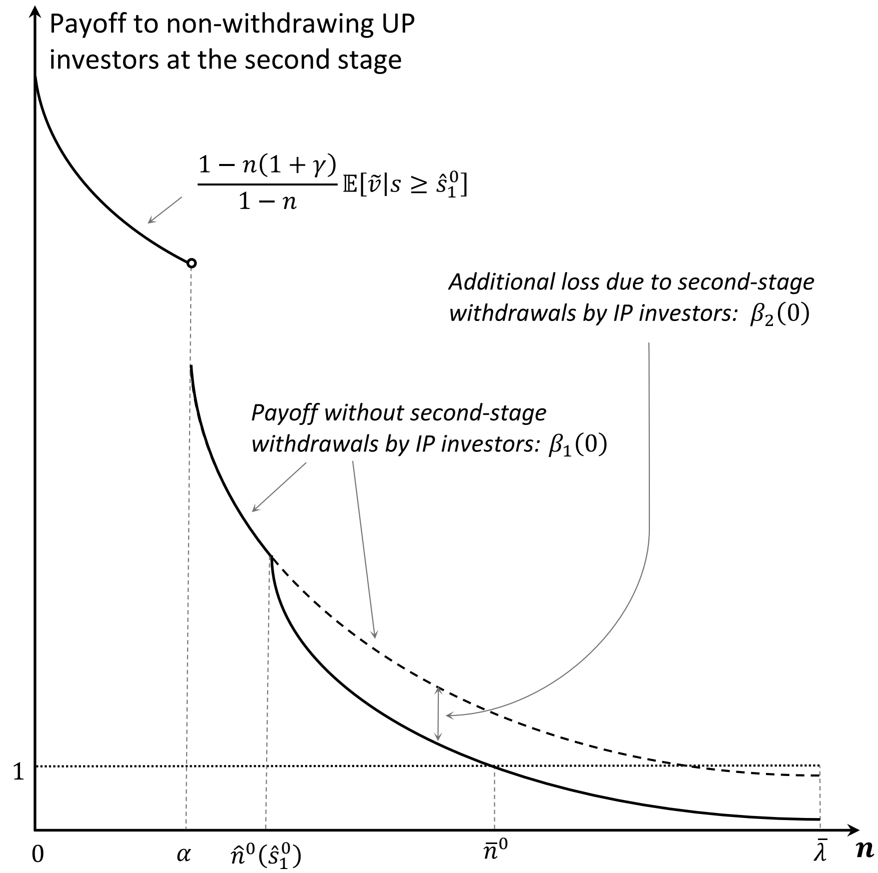

Figure 5: UP investors' second-stage withdrawal threshold $\bar{n}^\phi$ when $\phi = 0$. The UP investors' expected payoff from maintaining their deposits until date 2 (solid curve) is plotted as a function of n. The UP investors' withdrawal threshold $\bar{n}^0$ is the point at which the curve crosses 1 (dotted line). The expected additional loss due to potential second-stage withdrawals by IP investors is captured by the distance between the solid curve and the dashed curve, which represents the expected payoff to non-withdrawing UP investors if IP investors do not withdraw at the second stage.

$\bar{n}^\phi$ for any generic fee $\phi \in [0, \Phi]$. To highlight the role of information and learning, we again focus our discussion on the two limiting cases: $\phi = 0$ and $\phi = \Phi$.

Case with $\phi = 0$. Figure 5 illustrates $\bar{n}^\phi$ when $\phi = 0$, but the implications hold in general for any $\phi < \Phi$. The figure decomposes a non-withdrawing UP investor's expected payoff into two components: the expected payoff conditional on there being no additional withdrawals at the second stage (i.e., $\beta_1(0)$) and the expected loss due to potential second-stage withdrawals by IP investors (i.e., $\beta_2(0)$). The potential for additional losses kicks in when the signal threshold $\hat{s}_2^0(n)$ used by the remaining IP investors (if any) in making their second-stage withdrawal decisions exceeds their first-stage signal threshold $\hat{s}_1^0$. Because $\hat{s}_2^0(n)$ is strictly increasing in the realized first-stage withdrawal n, these additional losses become a possibility for UP investors when n exceeds the IP investors' withdrawal threshold $\hat{n}^0(\hat{s}_1^0)$.[15]

Case with $\phi = \Phi$. Imposing a redemption fee internalizes the payoff externality among investors, so they place more weight on their beliefs about the asset payoff and less on the potential payoff externality when making their withdrawal decisions. That is, $\beta_1(\phi)$ converges to $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\Phi]$ while $\beta_2(\phi)$ converges to 0 as ϕ approaches Φ. Hence, like it does for IP investors, the complete internalization of the payoff externality enables UP investors to base their decisions solely on their conditional expectations of the asset payoff when $\phi = \Phi$. Liquidation costs generated by prior (first-stage) or contemporaneous (second-stage) withdrawals are irrelevant because such costs are fully internalized by those who generate them.

Information externality. A redemption fee alters the information externality because it does not eliminate information asymmetry among investors but changes how UP investors interpret a given first-stage withdrawal. Before proceeding, note that $\bar{n}^\Phi \geq \alpha$. The reason is as follows. When a fee $\phi = \Phi$ is imposed, UP investors base their withdrawal decisions solely on their beliefs about the asset payoff $V(s)$ (discussed above). If $\bar{n}^\Phi$ were less than α, then UP investors would withdraw when $n \in (\bar{n}^\Phi, \alpha)$ even though they would know with certainty that IP investors did not withdraw at the first stage (note that $s \geq \hat{s}_1^\Phi = s^\gamma$ almost surely, $\forall n \in [0, \alpha)$; see Lemma 1). Consequently, their expected payoff from keeping their deposits ($V(s)$) would strictly exceed that from withdrawing ($1 - \Phi = \frac{1}{1+\gamma}$), which is a contradiction. To demonstrate the possibility that imposing a fee $\phi = \Phi$, despite fully internalizing the payoff externality, may still increase the UP investors' inclination to run relative to the no-fee case, i.e., $\bar{n}^\Phi < \bar{n}^0$, in the following analysis we restrict attention to the case wherein $\bar{n}^0 \geq \alpha$.

[15] Recall from Section 3.1.2 that $\hat{n}^\phi(s)$, characterized by Lemma A.1, is the inversion of $\hat{s}_2^\phi(n)$. If $n > \hat{n}^\phi(\hat{s}_1^\phi)$, then $\hat{s}_2^\phi(n) > \hat{s}_1^\phi$, which means that there is a possibility that IP investors make a second-stage withdrawal. Figure 5 depicts $\hat{n}^0(\hat{s}_1^0)$ at a point smaller than $\bar{n}^0$ but larger than α, but this need not be the case that emerges in equilibrium. The relations between the magnitudes of α, $\hat{n}^\phi(\hat{s}_1^\phi)$, and $\bar{n}^\phi$ depend on the values of the underlying parameters. The curve jumps downward at $n = \alpha$ because UP investors infer that $w = 0$ for certain if $n < \alpha$ but are uncertain about w if $n \in [\alpha, \bar{\lambda}]$, leading to a jump in beliefs about s around $n = \alpha$.

We now formally analyze the fee's impact on the information externality. The fee changes the UP investors' inference problem by lowering the IP investors' first-stage signal threshold $\hat{s}_1^\phi$ (Theorem 2). Recall from Lemma 1 that a UP investor's conditional expectation about the date-2 asset return is

$$\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi] = F(\hat{s}_1^\phi|n)\mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi] + (1 - F(\hat{s}_1^\phi|n))\mathbb{E}[\tilde{v}|s \geq \hat{s}_1^\phi]. \tag{18}$$

By lowering $\hat{s}_1^\phi$, the fee affects $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$ in two distinct ways. On the one hand, for a given n, a lower $\hat{s}_1^\phi$ increases the posterior likelihood that IP investors did not make a first-stage withdrawal, $F(\hat{s}_1^\phi|n)$, which in turn elevates $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$, holding the conditional expectations $\mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi]$ and $\mathbb{E}[\tilde{v}|s \geq \hat{s}_1^\phi]$ fixed. We refer to this as a "likelihood effect." On the other hand, a lower $\hat{s}_1^\phi$ also causes UP investors to be more pessimistic about $\tilde{v}$, *regardless* of their beliefs about whether IP investors withdrew at the first stage, as $\mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi]$ and $\mathbb{E}[\tilde{v}|s \geq \hat{s}_1^\phi]$ both decrease when $\hat{s}_1^\phi$ falls. This lowers $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$ for a given n, holding $F(\hat{s}_1^\phi|n)$ fixed. We refer to this as a "distribution effect."

The likelihood effect captures the intuitive notion that a lower signal threshold shifts the UP investors' posterior beliefs about s toward the higher distribution (i.e., $s \geq \hat{s}_1^\phi$) and, thus, raises $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$. The distribution effect captures the perhaps more subtle consequence that the UP investors' posterior beliefs about $\tilde{v}$, conditional on s being either below or above the threshold, decrease and, thus, $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$ declines as the threshold $\hat{s}_1^\phi$ decreases. Hence, by lowering $\hat{s}_1^\phi$, for a given n a redemption fee may either raise UP investors' conditional expectations of the date-2 asset return if the likelihood effect dominates or lower their expectations if the distribution effect dominates.

If the fee's effect on the information externality is sufficiently strong relative to its effect on the payoff externality, then imposing a fee can lower the UP investors' withdrawal threshold $\bar{n}^\phi$, making them more inclined to withdraw early. The following proposition highlights the fee's impact through the learning channel by comparing the UP investors' withdrawal threshold for the two limiting cases, $\phi = 0$ and $\phi = \Phi$.

Proposition 1. *Compared to the case wherein investors may withdraw without paying a fee, i.e., $\phi = 0$, imposing a redemption fee $\phi = \Phi$ decreases the UP investors' withdrawal threshold (i.e., $\bar{n}^{\Phi} < \bar{n}^{0}$) if*

$$\frac{g(\alpha)}{g(0)} < \frac{\int_0^{s^\gamma} \left(\frac{1}{1+\gamma} - V(s)\right) f(s)\, ds}{\int_{s^\gamma}^1 \left(V(s) - \frac{1}{1+\gamma}\right) f(s)\, ds}. \tag{19}$$

Furthermore, the set of density functions $g(\cdot)$ for which $\bar{n}^{\Phi} < \bar{n}^{0}$ becomes larger as γ decreases.

Proposition 1 provides a sufficient condition (in terms of exogenous parameters) for which imposing a redemption fee lowers the UP investors' withdrawal threshold; the necessary condition (in terms of the endogenous variable $\bar{n}^0$) is

$$\frac{g(\bar{n}^0)}{g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right)} < \frac{\int_0^{s^\gamma} \left(\frac{1}{1+\gamma} - V(s)\right) f(s)\, \mathrm{d}s}{\int_{s^\gamma}^1 \left(V(s) - \frac{1}{1+\gamma}\right) f(s)\, \mathrm{d}s}. \tag{20}$$

Essentially, the condition in (20) states that, given a realized first-stage withdrawal n equal to $\bar{n}^0$ (i.e., the largest withdrawal for which a UP investor maintains her deposit when $\phi = 0$), when $\phi = \Phi$ a UP investor's expected loss from maintaining her deposit strictly exceeds her expected gain.[16] Thus, when a fee $\phi = \Phi$ is imposed, UP investors strictly prefer to withdraw when $n = \bar{n}^0$. Therefore, it must be the case that $\bar{n}^{\Phi} < \bar{n}^0$.

The ratio on the LHS of (20), $\frac{g(\bar{n}^0)}{g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right)}$, represents the likelihood effect: the likelihood that IP investors maintained their deposits at the first stage relative to the likelihood that they withdrew when $\phi = 0$ and $n = \bar{n}^0$. Although the imposition of a fee increases this ratio and, therefore, strengthens the likelihood effect for a given n (as discussed above), the highest possible likelihood ratio is $\frac{g(\alpha)}{g(0)} \geq \frac{g(\bar{n}^0)}{g(\frac{\bar{n}^0-\alpha}{1-\alpha})}$ because $\frac{g(n)}{g(\frac{n-\alpha}{1-\alpha})}$ is monotonically decreasing

[16] Given $n = \bar{n}^0$, a UP investor believes: (i) with probability $g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right)$ IP investors withdrew at the first stage, which implies $s < s^\gamma$ and, hence, an expected loss of $\int_0^{s^\gamma} \left(\frac{1}{1+\gamma} - V(s)\right) f(s)\, \mathrm{d}s$ if she maintains her deposit instead of withdrawing; and (ii) with probability $g(\bar{n}^0)$ IP investors did not withdraw at the first stage, which implies $s \geq s^\gamma$ and, hence, an expected gain of $\int_{s^\gamma}^1 \left(V(s) - \frac{1}{1+\gamma}\right) f(s)\, \mathrm{d}s$ if she maintains her deposit rather than withdrawing (note that if the UP investor withdraws, she obtains $1 - \Phi = \frac{1}{1+\gamma}$, whereas her expected payoff from maintaining her deposit is $V(s)$). The condition in (20) ensures that the UP investor's expected loss would strictly exceed her expected gain if she were to maintain her deposit when $n = \bar{n}^0$ and $\phi = \Phi$.

in n, $\forall n \in [\alpha, \bar{\lambda}]$ (Assumption 2). Thus, (19) is a sufficient condition to ensure (20) and, hence, $\bar{n}^{\Phi} < \bar{n}^{0}$. The RHS of (19) (equivalently, (20)), which imposes an upper bound on the likelihood ratio, is the ratio of a UP investor's expected loss to expected gain from maintaining her deposit instead of withdrawing when $\phi = \Phi$. This ratio is related to the distribution effect, which causes UP investors to become more pessimistic about the asset's payoff and, thereby, increases the expected loss-to-gain ratio from maintaining rather than withdrawing their deposits. The inequality in (19), therefore, essentially provides a sufficient condition for the distribution effect to dominate the likelihood effect so that the imposition of a fee generates a sufficiently negative information externality that outweighs the benefits stemming from fee's removal of the payoff externality.

It is straightforward to show that the RHS of (19) is strictly decreasing in the liquidation cost γ, which implies that the distribution effect is more likely to dominate the likelihood effect when γ is smaller.[17] Thus, imposing a fee $\phi = \Phi$ is more likely to increase the UP investors' inclination to withdraw prematurely when the fund's liquidation cost γ is smaller. The economic intuition is as follows. The loss-to-gain ratio (RHS of (19)), *ceteris paribus*, captures the UP investors' propensity to withdraw when $\phi = \Phi$; the higher the ratio, the stronger the UP investors' incentive to withdraw. However, their belief about the likelihood that IP investors did not withdraw at the first stage (and, hence, $s \geq s^{\gamma}$), which increases when a fee is imposed, curbs the UP investors' inclination to withdraw; the less likely that IP investors withdrew, the more likely that UP investors gain from maintaining their deposits rather than withdrawing. The condition in (19) ensures that the former effect dominates the latter, so UP investors become more inclined to withdraw when a fee is imposed. As the liquidation cost γ becomes smaller, the corresponding fee Φ decreases, which, *ceteris paribus*, raises UP investors' withdrawal incentive. In contrast, the cap on the likelihood effect (LHS of (19)) is independent of γ. Therefore, provided that the condition in (19) holds, the imposition of a fee is more likely to induce UP investors to run when γ is smaller.

[17] Note that $s^{\gamma} = V^{-1}\left(\frac{1}{1+\gamma}\right)$ is decreasing in γ.

3.3 Equilibrium Uniqueness

We now discuss issues related to equilibrium uniqueness, beginning with the second-stage subgame. According to Theorems 1 and 3, $\hat{s}_2^{\phi}(n)$ is uniquely determined for a given $\bar{n}^{\phi}$, and vice versa. However, this does not necessarily imply uniqueness of the pair, which requires that there are not multiple pairs of $\bar{n}^{\phi}$ and $\hat{s}_2^{\phi}(n)$ that are consistent with each other for a given $\hat{s}_1^{\phi}$. The following corollary states that the pair is indeed unique.

Corollary 1. *Given the IP investors' first-stage signal threshold $\hat{s}_1^{\phi}$, the pair of second-stage strategies characterized in Theorems 1 and 3, $\{\hat{s}_2^{\phi}(n), \bar{n}^{\phi}\}$, is unique.*

While the second-stage subgame equilibrium is unique for a given $\hat{s}_1^{\phi}$, determining the uniqueness of the equilibrium for the overall two-stage game is less straightforward. Although Theorems 2 and 3 indicate that $\hat{s}_1^{\phi}$ is unique given $\bar{n}^{\phi}$, and vice versa, we are unable to ascertain whether the pair of strategies is unique without making additional parametric assumptions, which we refrain from doing to maintain generality. Thus, there may be multiple triplets, $\{\hat{s}_1^{\phi}, \hat{s}_2^{\phi}(n), \bar{n}^{\phi}\}$, each constituting a PBE for the overall two-stage game. As usual, multiple equilibria are possible due to the *endogenous* relations between $\hat{s}_1^{\phi}$ and $\bar{n}^{\phi}$.

Nevertheless, the number of equilibria is not essential to our analysis. Whenever there are multiple equilibria, they can be Pareto ranked. Because $\hat{s}_1^{\phi} > s^{\gamma}$ in any PBE if $\phi < \Phi$ (Theorem 2), we can select the "socially best" equilibrium, i.e., the one with the lowest $\hat{s}_1^{\phi}$ and, correspondingly, the highest $\bar{n}^{\phi}$. As Theorem 2 shows, the unique first-stage signal threshold for IP investors when $\phi = \Phi$ is s^{γ}. Thus, regardless of the potential for multiple equilibria when $\phi < \Phi$, a redemption fee $\phi = \Phi$ always lowers the IP investors' first-stage threshold.

4 Welfare

Although a redemption fee may lead to socially optimal actions by IP investors, it influences learning by UP investors and may, therefore, lead to either an improvement or a deterioration in outcomes. In this section, we examine the effects of the fee on both social welfare and the welfare of individual investors. To highlight the learning channel, we again focus on the two limiting cases: $\phi = 0$ and $\phi = \Phi$. Note first that impatient investors, who always withdraw at the first stage of date 1, suffer a welfare loss when $\phi = \Phi$ because they receive only $1 - \Phi$ instead of 1 (as when $\phi = 0$). The analysis, therefore, focuses on IP and UP investors.

4.1 IP Investors' Welfare and the Last-Mover Advantage

An IP investor's payoff hinges solely on the asset's fundamental value $V(s)$ when $\phi = \Phi$ because the fee fully internalizes the payoff externality. If $V(s) < V(s^*) = 1$ (i.e., if $s < s^*$), then IP investors always withdraw at the first stage of date 1 and receive 1 when $\phi = 0$, but their expected payoff is strictly less than 1 when $\phi = \Phi$ (regardless of their withdrawal decisions). Conversely, if $V(s) > V(s^*) = 1$ (i.e., if $s > s^*$), then IP investors remain invested until date 2 with an expected payoff $V(s) > 1$ when $\phi = \Phi$, but their expected payoff is strictly less than $V(s)$ when $\phi = 0$ (regardless of whether they withdraw). Therefore, the fee causes IP investors to suffer a loss when the asset's conditional expected payoff $V(s)$ is less than 1 but enjoy a gain when $V(s)$ is greater than 1.

Proposition 2. *Compared to the case wherein investors may withdraw without paying a fee, i.e., $\phi = 0$, imposing a redemption fee $\phi = \Phi$ decreases (increases) IP investors' welfare when $s < s^*$ ($s > s^*$). The fee has no impact on IP investors' welfare when $s = s^*$.*

Notably, the fee changes the nature of the IP investors' competitive advantage over UP investors. When $\phi = 0$, IP investors possess a strong first-mover advantage: upon receiving a low signal, they withdraw before UP investors and inflict *all* the associated liquidation costs on them. The disappearance of this first-mover advantage when $\phi = \Phi$ explains the IP investors' welfare loss when $s < s^*$. At the same time, the fee creates a *last-mover advantage* for IP investors: they remain invested until date 2 upon receiving a high signal and benefit from the fees paid by early withdrawers (including UP investors in some cases, as shown below in Section 4.2). The rise of this last-mover advantage explains the IP investors' welfare gain when $s > s^*$, in which case the fee essentially creates a wealth transfer from early withdrawers to (non-withdrawing) IP investors. Thus, importantly, while the fee eliminates the IP investors' first-mover advantage in bad states (i.e., low s), it creates a last-mover advantage for them in good states (i.e., high s).[18]

4.2 UP Investors' Welfare and a Wealth Transfer

Besides internalizing the payoff externality, the fee also alters the nature of the information externality for UP investors, changing their beliefs about the date-2 asset return. As discussed in Section 3.2, this may change UP investors' withdrawal decisions, depending on the realized first-stage withdrawal n and whether the UP investors' withdrawal threshold with a fee, $\bar{n}^{\Phi}$, is greater or less than the corresponding threshold without a fee, $\bar{n}^{0}$. Like with IP investors, the fee's effect on UP investors' welfare also depends the signal s. There are four cases.[19]

If the fee converts UP investors from non-withdrawers to withdrawers (i.e., if $n \in (\bar{n}^{\Phi}, \bar{n}^{0}]$), then they benefit from the fee when s is sufficiently low but suffer a loss when s is sufficiently high. The reason is that if s is sufficiently low, by converting from non-withdrawers to withdrawers, UP investors avoid holding a low-valued asset; whereas if s is higher, they suffer a detriment because they pay a fee to rid themselves of a high-valued asset. Conversely, if the fee converts UP investors from withdrawers to non-withdrawers (i.e., if $n \in (\bar{n}^{0}, \bar{n}^{\Phi}]$), then they benefit from the fee when the signal s is sufficiently high because they retain a high-valued asset but are harmed when s is sufficiently low because they hold a low-valued asset.

Alternatively, UP investors' withdrawal decisions may be unaffected by the alteration of the information externality, in which case the effect of the fee on UP investors' welfare stems solely from internalizing the payoff externality. If they withdraw regardless of the fee (i.e., if

[18]Strictly speaking, the last-mover advantage pertains to $s \in (s^*, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$. Within this range of signals, IP investors withdraw at date 1 when $\phi = 0$ but maintain their deposits when $\phi = \Phi$. That is, the fee converts IP investors from first movers to last movers. For higher signals, IP investors maintain their deposits regardless of the fee arrangement. In any event, the fee effectuates a wealth transfer from early withdrawers to (non-withdrawing) IP investors when $s > s^*$ because the fee internalizes the payoff externality which would otherwise reduce an IP investor's expected payoff from maintaining her deposit. Note that this wealth transfer arises even though the fee does not add additional capital to the fund but merely offsets the liquidation costs.

[19]We organize the analysis in this section according to the endogenous variable n because UP investors base their decisions on n. Alternatively, the analysis could be organized based on the exogenous variable λ.

$n > \max\{\bar{n}^0, \bar{n}^\Phi\}$), then the fee is beneficial to UP investors when s is sufficiently low because they benefit from the fees paid by withdrawing impatient and IP investors,[20] but the fee is detrimental when s is sufficiently high because they pay a fee to rid themselves of a high-valued asset. If, however, UP investors keep their deposits until date 2 irrespective of the fee (i.e., if $n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$), then they benefit from the fee regardless of the signal s because the fees paid by early withdrawers offset the associated liquidation costs.

Proposition 3. *Compared to the case wherein investors may withdraw without paying a fee, i.e., $\phi = 0$, imposing a redemption fee $\phi = \Phi$ increases (decreases) UP investors' welfare when: (i) UP investors do not withdraw regardless of the fee, i.e., $n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$; (ii) the fee converts UP investors from non-withdrawers to withdrawers, i.e., $n \in (\bar{n}^\Phi, \bar{n}^0]$, and the signal is sufficiently low (high); (iii) the fee converts UP investors from withdrawers to non-withdrawers, i.e., $n \in (\bar{n}^0, \bar{n}^\Phi]$, and the signal is sufficiently high (low); or (iv) UP investors withdraw regardless of the fee, i.e., $n > \max\{\bar{n}^0, \bar{n}^\Phi\}$, and the signal is sufficiently low (high).*

As alluded to in Section 4.1, the fee generates a potential wealth transfer from UP to IP investors. Specifically, in cases (ii) and (iv) in Proposition 3, when s is sufficiently high UP investors withdraw and pay a fee while IP investors maintain their deposits. In these cases, although the fees paid by UP investors do not add additional capital to the fund, they nonetheless effectuate a wealth transfer to IP investors by offsetting the liquidation costs generated by early withdrawals. As a result, UP investors receive less from their withdrawals, and IP investors own more units of a high-valued asset.

4.3 Social Welfare

We now analyze the impact of a redemption fee on social welfare, measured as the aggregate amount received by all investors from the fund over both dates 1 and 2. We denote social welfare in the absence and presence of a fee by Ω^0 and Ω^Φ, respectively.

[20] Note that the entire fund is liquidated if $n > \max\{\bar{n}^0, \bar{n}^\Phi\}$ and s is sufficiently low because all investors withdraw at date 1. Nonetheless, the fee benefits UP investors because it eliminates the first-mover advantage.

Investment efficiency drives social welfare. Recall that it is socially efficient to liquidate the fund's investment in the risky asset at date 1 if and only if $s < s^\gamma$ (see (3)). Thus, conditional on s, each unit remaining invested in the asset until date 2 generates a social welfare gain of $V(s) - V(s^\gamma)$ if $s > s^\gamma$ but a social welfare loss of $V(s^\gamma) - V(s)$ if $s < s^\gamma$. Therefore, the change in social welfare resulting from the imposition of a redemption fee $\phi = \Phi$ is given by

$$\Omega^\Phi - \Omega^0 = (V(s) - V(s^\gamma))\Theta, \tag{21}$$

where Θ denotes the change in aggregate investment in the risky asset caused by the fee. A positive (negative) Θ indicates that the fee decreases (increases) early liquidation.

The fee's effect on social welfare may vary, depending on how it changes the actions of IP and UP investors, which in turn depends on n and s (note that impatient investors withdraw at date 1 regardless of the fee). There are a total of twelve distinct scenarios.[21] Table 1 lists the change in aggregate investment Θ along with the sign of $\Omega^\Phi - \Omega^0$ (i.e., [+] or [−]) for each scenario. The fee's impact on UP investors' actions depends on n, which varies across the four rows, whereas its effect on IP investors' behaviors depends on s, which varies across the three columns. We derive Θ for each scenario in Appendix B. Here, we discuss the intuition.

The fee affects welfare by internalizing the payoff externality and altering the information externality. Table 2 decomposes the effect on Θ (corresponding to each scenario in Table 1) into that arising from the internalization of the payoff externality (Panel A) and that resulting from the alteration of the information externality (Panel B). The latter effect reflects the change in investment due to the fee's alteration of the UP investors' beliefs about the asset payoff and, hence, their withdrawal decisions. The former effect reflects the change in investment due to reduced asset liquidation, holding the UP investors' withdrawal decisions fixed.

Payoff externality effects. As Panel A shows, internalizing the payoff externality always (weakly) increases aggregate investment (i.e., $\Theta \geq 0$), but the magnitude depends on the relations between n, $\bar{n}^0$, and s. If UP investors maintain their deposits and, hence, generate

[21] Again, we organize the analysis around n rather than λ because UP investors base their decisions on n.

no payoff externality when $\phi = 0$ (i.e., if $n \leq \bar{n}^0$; top two rows in Panel A), then the fee's impact on welfare from internalizing the payoff externality stems solely from reducing asset liquidation associated with withdrawals made by impatient and IP investors. Recall, the fund must liquidate $1 + \gamma$ units of the asset for each unit of deposit withdrawn at date 1 when $\phi = 0$, provided that the entire fund is not liquidated. In contrast, the fund must liquidate only $(1+\gamma)(1-\Phi) = 1$ for each unit of deposit withdrawn early when $\phi = \Phi$. Therefore, the fee provides liquidation-cost savings of γ for each unit of deposit withdrawn at date 1.

For high signals (3rd column), IP investors keep their deposits regardless of the fee, so internalizing the payoff externality increases investment by an amount equal to the liquidation-cost savings that result from the fees paid by impatient investors, $\lambda\gamma$. For low signals (1st column), IP investors withdraw irrespective of the fee, so investment increases by an amount equal to the liquidation-cost savings resulting from the fees paid by impatient and IP investors, $[\lambda + \alpha(1-\lambda)]\gamma$. For intermediate signals (2nd column), the fee causes IP investors to maintain their deposits when they would otherwise withdraw without a fee. In this case, internalizing the payoff externality increases investment by an amount equal to the liquidation-cost savings from the fees paid by impatient investors, $\lambda\gamma$, plus the increase in deposits resulting from IP investors converting from withdrawers to non-withdrawers, $\alpha(1-\lambda)(1+\gamma)$, which comprises the IP investors' original deposits and the associated liquidation-cost savings.

If UP investors withdraw and, hence, generate a payoff externality when $\phi = 0$ (i.e., $n > \bar{n}^0$; bottom two rows in Panel A), then for high signals (3rd column) internalizing the payoff externality increases investment by an amount equal to the liquidation-cost savings that stem from the fees paid by impatient and UP investors, $[\lambda + (1-\alpha)(1-\lambda)]\gamma$, because IP investors maintain their deposits irrespective of the fee.[22] For low and intermediate signals (1st and 2nd columns, respectively), however, the entire fund is liquidated when $\phi = 0$ because all investors withdraw early. Therefore, the marginal withdrawal decreases fund deposits by 1 instead of $1+\gamma$, so the fee does not result in any liquidation-cost savings. Rather, the internalization of

[22] Any change in investment resulting from UP investors changing their withdrawal decisions stems from the alteration of the information externality and is, therefore, recorded in Panel B.

the payoff externality increases investment only when it alters IP investors' withdrawal decisions. This occurs for intermediate signals, in which case internalizing the payoff externality converts IP investors from withdrawers to non-withdrawers and, thus, increases investment by an amount equal to the IP investors' deposits, $\alpha(1-\lambda)$.

Information externality effects. While internalizing the payoff externality always (weakly) increases aggregate investment, the alteration of the information externality may cause investment to either rise or fall. Because only UP investors are affected by the information externality, the impact on investment resulting from the fee's alteration of the information externality is independent of the signal s (which UP investors do not observe). The information externality reduces investment by 1 for each UP investor converting from a non-withdrawer to a withdrawer (2nd row in Panel B) but increases investment by 1 for each UP investor converting from a withdrawer to a non-withdrawer (3rd row in Panel B).[23] Thus, the magnitude of the effect on investment from altering the information externality equals the mass of UP investors, $(1-\alpha)(1-\lambda)$. Altering the information externality does not affect investment when UP investors do not change their withdrawal decisions (1st and 4th rows in Panel B).

Aggregating the effects from the internalization of the payoff externality and the alteration of the information externality yields the fee's total effect on investment Θ, which is catalogued in Table 1. The fee's effect on social welfare is summarized in the following proposition.

Proposition 4. *Compared to the case wherein investors may withdraw without paying a fee, i.e., $\phi = 0$, imposing a redemption fee $\phi = \Phi$ may either raise or lower social welfare.*

1. *If the fee does not convert UP investors from non-withdrawers to withdrawers, then the fee decreases (increases) social welfare when $s < s^\gamma$ ($s > s^\gamma$).*
2. *If the fee converts UP investors from non-withdrawers to withdrawers, then the fee:*
   a. *increases (decreases) social welfare when $s < s^\gamma$ ($s > s^\gamma$) if $\alpha < \frac{1-\lambda(1+\gamma)}{(1-\lambda)(2+\gamma)}$; but*
   b. *increases (decreases) social welfare when $s < \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$ ($s > \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$) if $\alpha > \frac{1-\lambda(1+\gamma)}{(1-\lambda)(2+\gamma)}$.*

[23] The liquidation-cost savings, γ, for each UP investor converting from a withdrawer to a non-withdrawer arises from the internalization of the payoff externality and is, therefore, catalogued in Panel A.

The fee has no effect on social welfare when $s = s^{\gamma}$.

Proposition 4 highlights two noteworthy effects of the redemption fee on social welfare. First, as long as it does not convert UP investors from non-withdrawers to withdrawers, the fee amplifies the effects of the risky asset's return on social welfare, raising welfare in states wherein it is ex post efficient to continue the investment in the asset at date 1 (i.e., $s > s^{\gamma}$) but lowering welfare in states wherein early liquidation is socially efficient (i.e., $s < s^{\gamma}$). This occurs because the fee reduces the potential for runs and forces withdrawing investors to internalize the liquidation costs that they generate, which increases aggregate investment in the risky asset. Second, the fee raises social welfare in states wherein early liquidation is socially efficient only if it converts UP investors from non-withdrawers to withdrawers. That is, the redemption fee enhances social welfare in bad states (i.e., low s) precisely when it increases the potential for runs. In good states (i.e., high s), however, the fee is socially detrimental when it increases the UP investors' inclination to run.

5 Preemptive Runs

In this section, we examine whether the prospect of a redemption fee being levied in the future may lead to a "preemptive run" in which investors withdraw preemptively before the fee is imposed. Much of the analysis here mirrors that in Section 3, so we place the derivations in Appendix C and only discuss the intuition underlying the results.

We assume that only second-stage withdrawals at date 1 are subject to a fee. To streamline the analysis, we further assume that the fee equals Φ. Investors know at the first stage that a fee will be imposed at the second stage. Although this setup is simplistic, it nonetheless captures many of the tradeoffs associated with an impending fee without any loss of tractability. Again, we conjecture and verify the existence of an equilibrium like the one characterized in Section 3. Here, we use a superscript "φ" to denote the investors' equilibrium strategies, $\{\hat{s}_1^{\varphi}, \hat{s}_2^{\varphi}(n), \bar{n}^{\varphi}\}$, when they must pay a fee to withdraw at the second (but not the first) stage.

A second-stage fee internalizes the liquidation costs generated by second-stage withdrawers but *not* those by first-stage withdrawers, and the absence of a fee at the first stage allows first-stage withdrawers to impose the liquidation costs (generated by their withdrawals) entirely on second-stage investors. Thus, although second-stage withdrawals do not produce any *additional* liquidation costs that must be borne by investors who maintain their deposits until date 2, a payoff externality still exists among second-stage investors because their second-stage actions determine how liquidation costs generated by first-stage withdrawals are shared among them. Second-stage investors share these costs if they all remain invested until date 2, but one group of investors must bear the entire burden if they keep their deposits at the second stage while the other group withdraws.

The IP and UP investors' second-stage strategies, as characterized by Theorems C.1 and C.3 in Appendix C, are similar to their corresponding strategies when a fee exists at both stages (Section 3), but the precise thresholds $\hat{s}_2^{\varphi}(n)$ and $\bar{n}^{\varphi}$ generally differ from $\hat{s}_2^{\phi}(n)$ and $\bar{n}^{\phi}$ because the tradeoffs are different with just a second-stage fee. Importantly, though, a second-stage fee still affects the IP investors' first-stage strategy (as discussed below), which alters the UP investors' inference problem and, therefore, may either increase or decrease the UP investors' withdrawal threshold for the same reasons discussed in Section 3.2.

The IP investors' first-stage strategy is characterized by Theorem C.2 in Appendix C. With only a second-stage fee, an IP investor's uncertainty regarding the mass of impatient investors λ affects her first-stage strategy because she may end up bearing the liquidation costs generated by impatient investors if she does not withdraw at the first stage. In contrast, as Theorem 2 shows, when a fee equal to Φ is present at both stages, IP investors' first-stage decisions are based solely on their signal because the fee fully internalizes the liquidation costs at both stages. This leads to socially optimal decisions (i.e., $\hat{s}_1^{\phi} = s^{\gamma}$ when $\phi = \Phi$) and a lower likelihood of a first-stage withdrawal compared to when $\phi = 0$, as discussed in Section 3.1.2. Conversely, when a fee is present only at the second stage, IP investors must account for the premature withdrawals by impatient and (possibly) UP investors when making their first-stage decisions. While this results in socially suboptimal decisions (i.e., $\hat{s}_1^{\varphi} > s^{\gamma}$ when $\varphi = \Phi$), as the following proposition shows, the likelihood of IP investors making a first-stage withdrawal may either rise or fall relative to the case wherein $\phi = 0$ at both stages, depending on the underlying parameters.

Proposition 5. *Compared to the case wherein investors may withdraw without paying a fee at either stage of date 1, i.e., $\phi = 0$, imposing a redemption fee $\varphi = \Phi$ only on second-stage withdrawals may either increase or decrease the likelihood of IP investors withdrawing at the first stage.*

Proposition 5 states that imposing only a second-stage redemption fee may either increase or decrease the likelihood of preemptive runs by IP investors at the first stage relative to the case wherein the fee is zero at both stages. On the one hand, a second-stage fee may raise the inclination for IP investors to make a first-stage withdrawal because they receive their entire deposit, 1, if they withdraw at the first stage but only $1 - \Phi$ if they withdraw at the second stage. On the other hand, a second-stage fee may lower the incentive for IP investors to make a first-stage withdrawal because, with a second-stage fee, any potential second-stage withdrawals by UP investors generate a wealth transfer from UP to IP investors if s is sufficiently high and IP investors keep their deposits until date 2. The reason is the same as that discussed in Section 4.2: UP investors who withdraw at the second stage must pay a fee to rid themselves a high-valued asset, which benefits those who remain invested until date 2. If the latter effect dominates, then $\hat{s}_1^{\varphi} < \hat{s}_1^0$, in which case the second-stage fee, by creating a wealth transfer from UP to IP investors, prevents a preemptive run by IP investors for $s \in [\hat{s}_1^{\varphi}, \hat{s}_1^0)$. The fact that a redemption fee may lower the probability of a preemptive run contrasts with claims by Cipriani et al. (2014), Hanson et al. (2015), and others that redemption fees will precipitate preemptive runs.

6 Conclusion

We study how redemption fees affect runs on financial institutions when investors are asymmetrically informed about fundamentals. By internalizing the payoff externality, fees mitigate the informed investors' first-mover advantage and, thus, reduce their tendency to run. However, the decline in the informed investors' inclination to run alters the information externality by influencing uninformed investors' learning, which may cause them to become more pessimistic about fundamentals and, hence, more prone to run. Therefore, the fee's net effect on runs is generally ambiguous. Our welfare analysis reveals that redemption fees may induce a wealth transfer from uninformed to informed investors. Rather than leveling the playing field, the fee transforms the informed investors' first-mover advantage (in the absence of a fee) to a last-mover advantage. Furthermore, social welfare falls in good states (i.e., high fundamentals) when the fee increases the overall run potential. Thus, imposing a redemption fee that fully internalizes the payoff externality but fails to account for the information externality is akin to misguided Pigouvian taxation.

Our analysis presents many interesting avenues for future research. For instance, because current regulations permit MMMFs to impose a redemption fee only during periods of stress, a fee could affect fund investments, possibly leading funds to invest in less risky assets. Additionally, redemption fees may affect the potential for financial contagion across funds. A direct extension of our model would be to consider multiple funds holding correlated assets. The initiation of a redemption fee in one fund would not only influence learning by uninformed investors in that particular fund, but also those in other funds. This could trigger a contagion through the learning channel across different funds. We leave these for future explorations.

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Appendix A: Proofs

Proof of Theorem 1. We consider the relevant case in which IP investors did not withdraw at the first stage. When $n \le \bar{n}^{\phi}$, IP investors do not withdraw at the second stage if and only if

$$V(s) \ge \frac{(1-n)(1-\phi)}{1-n(1+\gamma)(1-\phi)}, \tag{A1}$$

which follows from (7). Similarly, (8) implies that IP investors do not withdraw at the second stage when $n > \bar{n}^{\phi}$ if and only if

$$V(s) \ge \frac{\alpha(1-n)(1-\phi)}{1-[1-\alpha(1-n)](1+\gamma)(1-\phi)}. \tag{A2}$$

If $\phi < \Phi$, then the results follow from the following facts: (i) $V(s^{\gamma}) = \frac{1}{1+\gamma}$ and $V(s)$ is continuous and monotonically increasing in s; (ii) the RHSs of both (A1) and (A2) are greater than $\frac{1}{1+\gamma}$ and are increasing in n; and (iii) the RHS of (A1) is strictly less than the RHS of (A2) for a given n. If $\phi = \Phi$, then the RHSs of both (A1) and (A2) reduce to $\frac{1}{1+\gamma}$. □

Lemma A.1. *If $\phi < \Phi$, then for any realized signal $s \in [s^{\phi}, 1]$ and the UP investors' second-stage withdrawal threshold $\bar{n}^{\phi}$, there exists a unique threshold $\hat{n}^{\phi}(s) \in [0, \bar{\lambda}]$, characterized by*

$$\hat{n}^{\phi}(s) = \begin{cases} \dfrac{V(s)-(1-\phi)}{[(1+\gamma)V(s)-1](1-\phi)} & \text{if } s \in [s^{\phi}, \hat{s}_2^{\phi}(\bar{n}^{\phi})] \\ \bar{n}^{\phi} & \text{if } s \in (\hat{s}_2^{\phi}(\bar{n}^{\phi}), \check{s}_2^{\phi}(\bar{n}^{\phi})] \\ 1 - \dfrac{[\gamma-(1+\gamma)\phi]V(s)}{\alpha[(1+\gamma)V(s)-1](1-\phi)} & \text{if } s \in (\check{s}_2^{\phi}(\bar{n}^{\phi}), \check{s}_2^{\phi}(\bar{\lambda})] \\ \bar{\lambda} & \text{if } s \in (\check{s}_2^{\phi}(\bar{\lambda}), 1], \end{cases} \tag{A3}$$

where $\check{s}_2^{\phi}(n)$ satisfies

$$V(\check{s}_2(n)) = \frac{\alpha(1-n)(1-\phi)}{1-[1-\alpha(1-n)](1+\gamma)(1-\phi)}, \tag{A4}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $n > \hat{n}^{\phi}(s)$; furthermore, $\hat{n}^{\phi}(s)$ is continuous and non-decreasing in s. If $\phi = \Phi$, then $\hat{n}^{\Phi}(s) = \bar{\lambda}$ and IP investors who do not withdraw at the first stage do not withdraw at the second stage, either.

Proof of Lemma A.1. If $s < s^{\phi}$, then the highest possible expected payoff for an IP investor who does not withdraw at the first stage is $V(s) < V(s^{\phi}) = 1-\phi$. Thus, IP investors always withdraw at the first stage if $s < s^{\phi}$. The remainder of the proof is predicated on IP investors not withdrawing at the first stage (i.e., $s \geq s^{\phi}$).

If UP investors do not withdraw at the second stage (i.e., if $n \leq \bar{n}^{\phi}$), then it follows from (7) that IP investors do not withdraw at the second stage if and only if

$$n \leq \frac{V(s)-(1-\phi)}{[(1+\gamma)V(s)-1](1-\phi)}. \tag{A5}$$

Similarly, (8) implies that if UP investors make a second-stage withdrawal (i.e., if $n > \bar{n}^{\phi}$), then IP investors do not withdraw at the second stage if and only if

$$n \leq 1-\frac{[\gamma-(1+\gamma)\phi]V(s)}{\alpha[(1+\gamma)V(s)-1](1-\phi)}. \tag{A6}$$

Suppose $\phi < \Phi$. The RHS of (A5) is greater than the RHS of (A6) for a given s. Additionally, the RHSs of both (A5) and (A6) are increasing in s. First, suppose $s \in [s^{\phi}, \hat{s}_2^{\phi}(\bar{n}^{\phi})]$. In this case, the RHSs of both (A5) and (A6) are less than or equal to $\bar{n}^{\phi}$. Because UP investors do not make a second-stage withdrawal in this case, $\hat{n}^{\phi}(s)$ is given by the RHS of (A5). Second, suppose $s \in (\hat{s}_2^{\phi}(\bar{n}^{\phi}), \check{s}_2^{\phi}(\bar{n}^{\phi})]$. In this case, $1-\frac{[\gamma-(1+\gamma)\phi]V(s)}{\alpha[(1+\gamma)V(s)-1](1-\phi)} \leq \bar{n}^{\phi} < \frac{V(s)-(1-\phi)}{[(1+\gamma)V(s)-1](1-\phi)}$. Consequently, neither (A5) nor (A6) defines $\hat{n}^{\phi}(s)$ because $1-\frac{[\gamma-(1+\gamma)\phi]V(s)}{\alpha[(1+\gamma)V(s)-1](1-\phi)}$ is an appropriate threshold only if UP investors make a second-stage withdrawal (i.e., if $n > \bar{n}^{\phi}$), and $\frac{V(s)-(1-\phi)}{[(1+\gamma)V(s)-1](1-\phi)}$ is an appropriate threshold only if UP investors do not make a second-stage withdrawal (i.e., if $n \leq \bar{n}^{\phi}$). Instead, $\hat{n}^{\phi}(s)$ equals $\bar{n}^{\phi}$. On the one hand, if $n > \bar{n}^{\phi}$ in this region, then n is greater than the RHS of (A6), and IP investors withdraw. On the other hand, if $n \leq \bar{n}^{\phi}$, then n is less than or equal to the RHS of (A5), and IP investors do not withdraw. Thus, $\hat{n}^{\phi}(s) = \bar{n}^{\phi}$. Third, suppose $s \in (\check{s}_2^{\phi}(\bar{n}^{\phi}), \check{s}_2^{\phi}(\bar{\lambda})]$. In this case, the RHSs of both (A5) and (A6) are greater than $\bar{n}^{\phi}$. Because UP investors withdraw at the second stage in this case, $\hat{n}^{\phi}(s)$ is given by the RHS of (A6). Fourth, suppose $s \in (\check{s}_2^{\phi}(\bar{\lambda}), 1]$. In this case, IP investors never withdraw at the second stage because the signal s is high enough to offset the largest possible liquidation costs imposed by the aggregate withdrawals of impatient and UP investors. Thus, $\hat{n}^{\phi}(s) = \bar{\lambda}$. Note that $\hat{n}^{\phi}(\hat{s}_2^{\phi}(\bar{n}^{\phi})) = \bar{n}^{\phi}$, $\hat{n}^{\phi}(\check{s}_2^{\phi}(\bar{n}^{\phi})) = \bar{n}^{\phi}$, and $\hat{n}^{\phi}(\check{s}_2^{\phi}(\bar{\lambda})) = \bar{\lambda}$.

Hence, $\hat{n}^{\phi}(s)$ is continuous in s. Furthermore, because the RHSs of both (A5) and (A6) are increasing in s, $\hat{n}^{\phi}(s)$ is non-decreasing in s. Finally, suppose $\phi = \Phi$. In this case, the RHSs of both (A5) and (A6) equal 1. Hence, IP investors never make a second-stage withdrawal. □

Proof of Theorem 2. The payoff to IP investors in the four scenarios discussed in Section 3.1.2 combined with the respective withdrawal thresholds for IP and UP investors, $\hat{n}^{\phi}(s)$ and $\bar{n}^{\phi}$, imply that IP investors do not withdraw at the first stage if and only if the following condition is satisfied:

$$1-\phi \leq \int_0^{\min\{\hat{n}^{\phi}(s),\bar{n}^{\phi}\}} \frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda} V(s)g(\lambda)\,\mathrm{d}\lambda + \int_{\min\{\hat{n}^{\phi}(s),\bar{n}^{\phi}\}}^{\bar{n}^{\phi}} (1-\phi)g(\lambda)\,\mathrm{d}\lambda$$
$$+ \int_{\bar{n}^{\phi}}^{\max\{\hat{n}^{\phi}(s),\bar{n}^{\phi}\}} \frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)(1-\phi)}{\alpha(1-\lambda)} V(s)g(\lambda)\,\mathrm{d}\lambda$$
$$+ \int_{\max\{\hat{n}^{\phi}(s),\bar{n}^{\phi}\}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)} g(\lambda)\,\mathrm{d}\lambda. \quad \text{(A7)}$$

Suppose $\phi < \Phi$. First, we show that the RHS of (A7) is continuous and monotonically increasing in s. Note that $\frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)(1-\phi)}{\alpha(1-\lambda)} < \frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda} \leq 1$ and $\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)} < \frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda} \leq 1$ (with strict inequality when $\lambda > 0$). It follows that the RHS is less than $1-\phi$ unless $V(s) > 1-\phi$. Hence, $\hat{s}_1^{\phi} > s^{\phi} > s^{\gamma}$. Additionally, Lemma A.1 implies: (i) $\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda} V(s) \geq 1-\phi$ for $\lambda \leq \hat{n}^{\phi}(s) = \frac{V(s)-(1-\phi)}{[(1+\gamma)V(s)-1](1-\phi)}$ when $s \in [s^{\phi}, \hat{s}_2^{\phi}(\bar{n}^{\phi})]$; (ii) $\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda} V(s) \geq 1-\phi$ for $\lambda \leq \bar{n}^{\phi}$ when $s \in (\hat{s}_2^{\phi}(\bar{n}^{\phi}), 1]$; and (iii) $\frac{1-[\lambda+(1-\alpha)(1-\lambda)](1+\gamma)(1-\phi)}{\alpha(1-\lambda)} V(s) \geq 1-\phi$ for $\lambda \leq \hat{n}^{\phi}(s) = 1 - \frac{[\gamma-(1+\gamma)\phi]V(s)}{\alpha[(1+\gamma)V(s)-1](1-\phi)}$ when $s \in (\check{s}_2^{\phi}(\bar{n}^{\phi}), 1]$. Thus, because $V(s)$ is continuous and increasing in s and $\hat{n}^{\phi}(s)$ is continuous and non-decreasing in s, the RHS is continuous and monotonically increasing in s.

Next, we show that the RHS of (A7) crosses the LHS at some $s \in (s^{\gamma}, 1)$. As stated above, the RHS is strictly less than $1-\phi$ if $s = s^{\phi} > s^{\gamma}$. Conversely, if $s = 1$, then $V(s) = H > \frac{\alpha(1-\bar{\lambda})}{1-[\bar{\lambda}+(1-\alpha)(1-\bar{\lambda})](1+\gamma)(1-\phi)}$ (Assumption 1) and $\hat{n}^{\phi}(s) = \bar{\lambda}$ (Lemma A.1), so the integrands in the first and third terms on the RHS are strictly greater than $1-\phi$, whereas the second and fourth terms are zero. Hence, the RHS is strictly greater than 1. Therefore, there exists a unique $\hat{s}_1^{\phi} \in (s^{\gamma}, 1)$ such that IP investors make a first-stage withdrawal if and only if $s < \hat{s}_1^{\phi}$.

It then follows immediately from (A7) that

$$V(\hat{s}_1^\phi) = \begin{cases} \dfrac{1-\phi-\int_{\hat{n}^\phi(\hat{s}_1^\phi)}^{\bar{n}^\phi}(1-\phi)g(\lambda)\,\mathrm{d}\lambda - \int_{\bar{n}^\phi}^{\bar{\lambda}}\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)}g(\lambda)\,\mathrm{d}\lambda}{\int_0^{\hat{n}^\phi(\hat{s}_1^\phi)}\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda}g(\lambda)\,\mathrm{d}\lambda} & \text{if } \Psi \le V(\hat{s}_2^\phi(\bar{n}^\phi)) \\ \dfrac{1-\phi-\int_{\bar{n}^\phi}^{\bar{\lambda}}\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)}g(\lambda)\,\mathrm{d}\lambda}{\int_0^{\bar{n}^\phi}\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda}g(\lambda)\,\mathrm{d}\lambda} & \text{if } V(\hat{s}_2^\phi(\bar{n}^\phi)) < \Psi \le V(\check{s}_2^\phi(\bar{n}^\phi)) \\ \dfrac{1-\phi-\int_{\hat{n}^\phi(\hat{s}_1^\phi)}^{\bar{\lambda}}\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)}g(\lambda)\,\mathrm{d}\lambda}{\int_0^{\hat{n}^\phi(\hat{s}_1^\phi)}\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda}g(\lambda)\,\mathrm{d}\lambda - \int_{\bar{n}^\phi}^{\hat{n}^\phi(\hat{s}_1^\phi)}\frac{(1-\alpha)[\gamma-(1+\gamma)\phi]}{\alpha(1-\lambda)}g(\lambda)\,\mathrm{d}\lambda} & \text{if } \Psi > V(\check{s}_2^\phi(\bar{n}^\phi)), \end{cases} \tag{A8}$$

where

$$\Psi \equiv \frac{1-\phi-\int_{\bar{n}^\phi}^{\bar{\lambda}}\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)}g(\lambda)\,\mathrm{d}\lambda}{\int_0^{\bar{n}^\phi}\frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda}g(\lambda)\,\mathrm{d}\lambda}.$$

Because $\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)} < 1-\phi$, the top equation in (A8) is decreasing in $\bar{n}^\phi$. Similarly, the middle equation (and hence Ψ) is decreasing in $\bar{n}^\phi$ because $\frac{1-\lambda(1+\gamma)(1-\phi)}{(1-\lambda)(1+\gamma)} < \frac{1-\lambda(1+\gamma)(1-\phi)}{1-\lambda}$. The bottom equation in (A8) is obviously decreasing in $\bar{n}^\phi$. Finally, because the top equation is weakly less than the middle equation, which is weakly less than the bottom equation, and Theorem 1 implies that both $V(\hat{s}_2^\phi(\bar{n}^\phi))$ and $V(\check{s}_2^\phi(\bar{n}^\phi))$ are increasing in $\bar{n}^\phi$, $\hat{s}_1^\phi$ is monotonically decreasing in $\bar{n}^\phi$.

Alternatively, suppose $\phi = \Phi$. In this case, $\hat{n}^\Phi(s) = \bar{\lambda}$ (Lemma A.1), and (A7) reduces to $\frac{1}{1+\gamma} \le V(s)$. Hence, $\hat{s}_1^\Phi = s^\gamma$. □

Proof of Lemma 1. The results for the cases wherein $n \notin [\alpha, \bar{\lambda}]$ follow immediately from the fact that the first-stage withdrawal $n = \lambda + \alpha(1-\lambda)w$ fully reveals whether $w = 0$ or $w = 1$ in these cases. When $n \in [\alpha, \bar{\lambda}]$, n does not fully reveal w. If $w = 1$, then $n = \lambda + \alpha(1-\lambda)$, and the likelihood of observing n given $w = 1$ is $g\left(\frac{n-\alpha}{1-\alpha}\right)$. If $w = 0$, then $n = \lambda$, and the probability of observing n conditional on $w = 0$ is $g(n)$. Thus, $F(\hat{s}_1^\phi|n)$, according to Bayes' rule, is given by (10). Note that

$$\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi] = F(\hat{s}_1^\phi|n)\mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi] + (1 - F(\hat{s}_1^\phi|n))\mathbb{E}[\tilde{v}|s \ge \hat{s}_1^\phi]. \tag{A9}$$

Substituting (10) into (A9) yields (11).

Next, provided that $n \in [\alpha, \bar{\lambda}]$, differentiating (10) with respect to n gives

$$\frac{\partial F(\hat{s}_1^{\phi}|n)}{\partial n} = \frac{F(\hat{s}_1^{\phi})(1 - F(\hat{s}_1^{\phi}))\left[g(n)\frac{\partial g\left(\frac{n-\alpha}{1-\alpha}\right)}{\partial n} - g\left(\frac{n-\alpha}{1-\alpha}\right)\frac{\partial g(n)}{\partial n}\right]}{\left[F(\hat{s}_1^{\phi})g\left(\frac{n-\alpha}{1-\alpha}\right) + \left(1 - F(\hat{s}_1^{\phi})\right)g(n)\right]^2} \tag{A10}$$

$$\propto \left[g(n)\frac{\partial g\left(\frac{n-\alpha}{1-\alpha}\right)}{\partial n} - g\left(\frac{n-\alpha}{1-\alpha}\right)\frac{\partial g(n)}{\partial n}\right]$$

$$> 0,$$

where the inequality follows from Assumption 2. It follows that

$$\frac{\partial \mathbb{E}[\tilde{v}|n, \hat{s}_1^{\phi}]}{\partial n} = \left(\mathbb{E}[\tilde{v}|s < \hat{s}_1^{\phi}] - \mathbb{E}[\tilde{v}|s \geq \hat{s}_1^{\phi}]\right)\frac{\partial F(\hat{s}_1^{\phi}|n)}{\partial n} < 0,$$

given $\mathbb{E}[\tilde{v}|s < \hat{s}_1^{\phi}] < \mathbb{E}[\tilde{v}|s \geq \hat{s}_1^{\phi}]$. If $n \notin [\alpha, \bar{\lambda}]$, then both $F(\hat{s}_1^{\phi}|n)$ and $\mathbb{E}[\tilde{v}|n, \hat{s}_1^{\phi}]$ are constants and, therefore, unaffected by changes in n. □.

Proof of Lemma 2. Proof is by contradiction. Suppose that UP investors do not always withdraw at the second stage when $n > \bar{\lambda}$. This supposition implies that UP investors never withdraw when $n \in [0, \bar{\lambda}]$. Given this strategy by UP investors, IP investors understand that if they do not withdraw at the first stage, then $n \in [0, \bar{\lambda}]$, which means that UP investors will never withdraw at the second stage. Consequently, an IP investor's expected payoff at the second stage is bounded from below by $1 - \phi$, which is the amount she receives if she withdraws at the second stage and UP investors keep their deposits until date 2. Hence, an IP investor will make a first-stage withdrawal if and only if the expected return on the risky asset is less than $1 - \phi$ or, equivalently, $s < s^{\phi}$. As a result, if $n > \bar{\lambda}$, then UP investors infer that $s < s^{\phi}$ because the IP investors' actions are fully revealed. Conditional on $n > \bar{\lambda}$, the expected payoff to a UP investor at the second stage is $\frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|s < s^{\phi}]$, which is strictly less than $1 - \phi$ because $\frac{1-n(1+\gamma)(1-\phi)}{1-n} \leq 1$ and $V(s^{\phi}) = 1 - \phi$. Therefore, UP investors always make a second-stage withdrawal when $n > \bar{\lambda}$, which is a contradiction. □

Proof of Theorem 3. A UP investor does not withdraw if and only if

$$
\begin{aligned}
1-\phi \leq\ & F(\hat{s}_1^{\phi}|n)\frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|s<\hat{s}_1^{\phi}] + (1-F(\hat{s}_1^{\phi}|n)) \\
& \times \Bigg(\frac{\max\{F(\hat{s}_2^{\phi}(n))-F(\hat{s}_1^{\phi}),0\}}{1-F(\hat{s}_1^{\phi})}\frac{1-[n+\alpha(1-n)](1+\gamma)(1-\phi)}{(1-\alpha)(1-n)}\mathbb{E}[\tilde{v}|\hat{s}_1^{\phi}\leq s<\hat{s}_2^{\phi}(n)] \\
& + \frac{1-\max\{F(\hat{s}_1^{\phi}),F(\hat{s}_2^{\phi}(n))\}}{1-F(\hat{s}_1^{\phi})}\frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|s\geq \max\{\hat{s}_1^{\phi},\hat{s}_2^{\phi}(n)\}]\Bigg). \qquad \text{(A11)}
\end{aligned}
$$

The LHS, $1-\phi$, is an individual atomistic UP investor's payoff from deviating from the equilibrium strategy by withdrawing while others do not. The RHS, obtained by aggregating (12), (13), and (14), is a probability-weighted expectation of the payoff from not withdrawing. The fact that

$$
\begin{aligned}
\mathbb{E}[\tilde{v}|n,\hat{s}_1^{\phi}] =\ & F(\hat{s}_1^{\phi}|n)\mathbb{E}[\tilde{v}|s<\hat{s}_1^{\phi}] + \frac{1-F(\hat{s}_1^{\phi}|n)}{1-F(\hat{s}_1^{\phi})} \\
& \times \Big(\max\{F(\hat{s}_2^{\phi}(n))-F(\hat{s}_1^{\phi}),0\}\mathbb{E}[\tilde{v}|\hat{s}_1^{\phi}<s<\hat{s}_2^{\phi}(n)] \\
& + (1-\max\{F(\hat{s}_1^{\phi}),F(\hat{s}_2^{\phi}(n))\})\mathbb{E}[\tilde{v}|s>\max\{\hat{s}_1^{\phi},\hat{s}_2^{\phi}(n)\}]\Big) \qquad \text{(A12)}
\end{aligned}
$$

allows (A11) to be rewritten as

$$1-\phi \leq \beta_1(\phi)-\beta_2(\phi). \qquad \text{(A13)}$$

We first show that the RHS of (A13) is monotonically decreasing in n. If $\hat{s}_1^{\phi}\geq \hat{s}_2^{\phi}(n)$ or $\phi=\Phi$, then $\beta_2(\phi)=0$. Differentiating $\beta_1(\phi)$ with respect to n yields

$$\frac{1-n(1+\gamma)(1-\phi)}{1-n}\frac{\partial \mathbb{E}[\tilde{v}|n,\hat{s}_1^{\phi}]}{\partial n} - \frac{[\gamma-(1+\gamma)\phi]\mathbb{E}[\tilde{v}|n,\hat{s}_1^{\phi}]}{(1-n)^2},$$

which is non-positive $\forall n\in[0,\bar{\lambda}]$ and negative $\forall n\in[\alpha,\bar{\lambda}]$ if $\phi=\Phi$ because $\partial\mathbb{E}[\tilde{v}|n,\hat{s}_1^{\phi}]/\partial n\leq 0$ with strict inequality if $n\in[\alpha,\bar{\lambda}]$ (Lemma 1); if $\phi<\Phi$, then the expression is negative. If $\hat{s}_1^{\phi}<\hat{s}_2^{\phi}(n)$ and

$\phi < \Phi$, then differentiating $\beta_2(\phi)$ with respect to n yields

$$-\frac{\alpha[\gamma-(1+\gamma)\phi]}{(1-\alpha)(1-n)(1-F(\hat{s}_1^\phi))}\Bigg[(1-F(\hat{s}_1^\phi|n))\Bigg(\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]\frac{\partial F(\hat{s}_2^\phi(n))}{\partial n} + (F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi))\frac{\partial \mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]}{\partial n} + \frac{(F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi))\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]}{1-n}\Bigg) - (F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi))\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]\frac{\partial F(\hat{s}_1^\phi|n)}{\partial n}\Bigg],$$

which is negative because $\partial F(\hat{s}_2^\phi(n))/\partial n > 0$ (Theorem 1), $\partial \mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]/\partial n \ge 0$,[24] and

$$\begin{aligned}
\frac{\partial F(\hat{s}_1^\phi|n)}{\partial n} &= \frac{F(\hat{s}_1^\phi)(1-F(\hat{s}_1^\phi))\Big[g(n)\frac{\partial g(\frac{n-\alpha}{1-\alpha})}{\partial n} - g\big(\frac{n-\alpha}{1-\alpha}\big)\frac{\partial g(n)}{\partial n}\Big]}{\big[F(\hat{s}_1^\phi)g\big(\frac{n-\alpha}{1-\alpha}\big) + (1-F(\hat{s}_1^\phi))g(n)\big]^2} \\
&= \frac{F(\hat{s}_1^\phi)(1-F(\hat{s}_1^\phi|n))\Big[g(n)\frac{\partial g(\frac{n-\alpha}{1-\alpha})}{\partial n} - g\big(\frac{n-\alpha}{1-\alpha}\big)\frac{\partial g(n)}{\partial n}\Big]}{g(n)\big[F(\hat{s}_1^\phi)g\big(\frac{n-\alpha}{1-\alpha}\big) + (1-F(\hat{s}_1^\phi))g(n)\big]} \\
&= \frac{(1-F(\hat{s}_1^\phi|n))\Big[\frac{\partial g(\frac{n-\alpha}{1-\alpha})/\partial n}{g(\frac{n-\alpha}{1-\alpha})} - \frac{\partial g(n)/\partial n}{g(n)}\Big]}{1 + \frac{(1-F(\hat{s}_1^\phi))g(n)}{F(\hat{s}_1^\phi)g(\frac{n-\alpha}{1-\alpha})}} \\
&< (1-F(\hat{s}_1^\phi|n))\frac{\partial \log\left(\frac{g(\frac{n-\alpha}{1-\alpha})}{g(n)}\right)}{\partial n} \\
&\le \frac{1-F(\hat{s}_1^\phi|n)}{1-n},
\end{aligned}$$

[24]The conditional expectation can be written as

$$\mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)] = \int_{\hat{s}_1^\phi}^{\hat{s}_2^\phi(n)} \frac{V(s)f(s)}{F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi)}\,\mathrm{d}s.$$

Differentiating this expression with respect to $\hat{s}_2^\phi(n)$ yields

$$\begin{aligned}
\frac{\partial \mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]}{\partial \hat{s}_2^\phi(n)} &= \frac{V(\hat{s}_2^\phi(n))f(\hat{s}_2^\phi(n))}{F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi)} - \int_{\hat{s}_1^\phi}^{\hat{s}_2^\phi(n)} \frac{V(s)f(s)f(\hat{s}_2^\phi(n))}{[F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi)]^2}\,\mathrm{d}s \\
&\propto V(\hat{s}_2^\phi(n)) - \int_{\hat{s}_1^\phi}^{\hat{s}_2^\phi(n)} \frac{V(s)f(s)}{F(\hat{s}_2^\phi(n)) - F(\hat{s}_1^\phi)}\,\mathrm{d}s \\
&= V(\hat{s}_2^\phi(n)) - \mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)] \\
&> 0,
\end{aligned}$$

where the inequality follows from the fact that $\partial V(s)/\partial s > 0$. Then, $\partial \mathbb{E}[\tilde{v}|\hat{s}_1^\phi \le s < \hat{s}_2^\phi(n)]/\partial n \ge 0$ follows from the fact that n enters the conditional expectation only through $\hat{s}_2^\phi(n)$ and, according to Theorem 1, $\hat{s}_2^\phi(n)$ is strictly increasing in n if $\phi < \Phi$.

where the first equality follows from (A10), the second equality follows from (10), the third equality and the first inequality follow from algebra, and the second inequality follows from Assumption 2. Additionally, $\mathbb{1}_{\{\hat{s}_1^\phi<\hat{s}_2^\phi(n)\}}$ is non-decreasing in n because $\partial\hat{s}_2^\phi(n)/\partial n \geq 0$ (Theorem 1). Together, these imply that the RHS of (A13) is strictly decreasing in n if $\phi < \Phi$; the RHS is non-increasing $\forall n \in [0, \bar{\lambda}]$ and strictly decreasing $\forall n \in [\alpha, \bar{\lambda}]$ if $\phi = \Phi$.

Next, we show that the RHS of (A13) crosses the LHS at some unique $n \in [0, \bar{\lambda}]$. Note that $\lim_{n\to 0^+} V(\hat{s}_2^\phi(n)) = 1-\phi$ (see (9)), which implies that $\lim_{n\to 0^+} \hat{s}_2^\phi(n) \leq \hat{s}_1^\phi$ because $V(\hat{s}_1^\phi) \geq 1-\phi$. Thus, $\lim_{n\to 0^+} \beta_2(\phi) = 0$. Additionally, $\lim_{n\to 0^+} \frac{1-n(1+\gamma)(1-\phi)}{1-n} = 1$ and $\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi] = \mathbb{E}[\tilde{v}|s \geq \hat{s}_1^\phi] > V(\hat{s}_1^\phi) \geq 1-\phi$ when $\alpha \geq n \to 0^+$ (Lemma 1), so $\lim_{n\to 0^+} \beta_1(\phi) > 1-\phi$. Hence, the RHS of (A13) approaches a value strictly greater than $1-\phi$ as $n \to 0^+$. If $n > \bar{\lambda}$, then Lemma 1 implies that the RHS reduces to $\frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|s < \hat{s}_1^\phi]$, which must be strictly less than $1-\phi$ for the reason discussed in the proof of Lemma 2. Therefore, the RHS must cross the LHS (i.e., $1-\phi$) at some $n \in [0, \bar{\lambda}]$. This argument holds as long as the RHS is continuous in n. However, Lemma 1 implies that $\lim_{n\to\alpha^-} \frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi] > \lim_{n\to\alpha^+} \frac{1-n(1+\gamma)(1-\phi)}{1-n}\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$. Thus, if $\lim_{n\to\alpha^+} \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi] < 1-\phi \leq \lim_{n\to\alpha^-} \frac{1-n(1+\gamma)}{1-n}\mathbb{E}[\tilde{v}|n, \hat{s}_1^\phi]$, then the threshold is α. Taking this discontinuity into account, the threshold $\bar{n}^\phi$ is defined as in (15). Uniqueness of $\bar{n}^\phi$ follows from the fact that the RHS of (A13) is non-increasing in n and strictly decreasing if $\phi < \Phi$. $\square$

Proof of Proposition 1. The UP investors' strategy, as given by (15), reduces to

$$\bar{n}^\Phi = \sup\left\{n \in [0, \bar{\lambda}] : (1+\gamma)^{-1} \leq \mathbb{E}[\tilde{v}|n, \hat{s}_1^\Phi]\right\}$$

when $\phi = \Phi$. Thus, imposing a fee $\phi = \Phi$ decreases the UP investors' withdrawal threshold (i.e.,

$\bar{n}^\Phi < \bar{n}^0$) if and only if $\mathbb{E}[\tilde{v}|\bar{n}^0, \hat{s}_1^\Phi] < \frac{1}{1+\gamma}$. This inequality can be rewritten as (note that $\hat{s}_1^\Phi = s^\gamma$)

$$
\begin{aligned}
\frac{1}{1+\gamma} &> \mathbb{E}[\tilde{v}|\bar{n}^0, s^\gamma] \\
&= F(s^\gamma|\bar{n}^0)\mathbb{E}[\tilde{v}|s < s^\gamma] + (1 - F(s^\gamma|\bar{n}^0))\mathbb{E}[\tilde{v}|s \geq s^\gamma] \\
&= \frac{F(s^\gamma)g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right)\mathbb{E}[\tilde{v}|s < s^\gamma] + (1 - F(s^\gamma))g(\bar{n}^0)\mathbb{E}[\tilde{v}|s \geq s^\gamma]}{F(s^\gamma)g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right) + (1 - F(s^\gamma))g(\bar{n}^0)} \\
&= \frac{g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right)\int_0^{s^\gamma} V(s)f(s)\,\mathrm{d}s + g(\bar{n}^0)\int_{s^\gamma}^1 V(s)f(s)\,\mathrm{d}s}{g\left(\frac{\bar{n}^0-\alpha}{1-\alpha}\right)\int_0^{s^\gamma} f(s)\,\mathrm{d}s + g(\bar{n}^0)\int_{s^\gamma}^1 f(s)\,\mathrm{d}s},
\end{aligned}
$$

where the first equality follows from (18) and the second equality follows from Lemma 1. Rearranging terms yields (20). Because $\frac{g(n)}{g(\frac{n-\alpha}{1-\alpha})}$ is decreasing in n (see Assumption 2), $\frac{g(\frac{\bar{n}^0-\alpha}{1-\alpha})}{g(\bar{n}^0)} \leq \frac{g(0)}{g(\alpha)}$; thus, (19) is a sufficient condition for (20). Finally, note that the RHS of (19) is clearly decreasing in γ, so a larger set of density functions $g(\cdot)$ satisfies the inequality in (19) when γ is smaller. □

Proof of Corollary 1. According to Theorems 1 and 3, $\hat{s}_2^\phi(n)$ is uniquely determined for a given $\bar{n}^\phi$, and vice versa. Theorem 1 also implies that $\hat{s}_2^\phi(\bar{n}^\phi)$ is strictly increasing in $\bar{n}^\phi$. However, $\bar{n}^\phi$ is non-increasing in $\hat{s}_2^\phi(\bar{n}^\phi)$ (given $\hat{s}_1^\phi$), as implied by the RHS of the condition in (15). Thus, conditional on $\hat{s}_1^\phi$, the pair of second-stage strategies must be unique. □

Proof of Proposition 2. There are three cases. First, if $s < s^\gamma$, then IP investors make a first-stage withdrawal regardless of the fee arrangement. They receive 1 when $\phi = 0$, but only $1 - \Phi$ when $\phi = \Phi$.

Second, if $s \in [s^\gamma, \hat{s}_1^0)$, then IP investors withdraw at the first stage when $\phi - 0$ and receive 1. When $\phi = \Phi$, they remain invested until date 2 and expect to receive $V(s)$. Thus, the fee changes an IP investor's welfare by $V(s) - 1$, which is negative if $s \in [s^\gamma, s^*)$ but positive if $s \in (s^*, \hat{s}_1^0]$. The fee has no impact on an IP investor's welfare when $s = s^*$ because $V(s^*) = 1$.

Third, if $s \geq \hat{s}_1^0$, then IP investors keep their deposits until date 2 when $\phi = \Phi$ and expect to receive $V(s)$. When $\phi = 0$, they do not withdraw at the first stage but may make a second-stage withdrawal. Regardless of whether an IP investor withdraws at the second stage, her expected payoff in the no-fee case is strictly less than $V(s)$ due to the liquidation costs generated by others who withdraw, as $\lambda > 0$ almost surely. □

Proof of Proposition 3. The first case wherein $n \leq \{\bar{n}^0, \bar{n}^\Phi\}$ is described in the text. For the second case wherein $n \in (\bar{n}^\Phi, \bar{n}^0]$, the effect of the fee on UP investors' welfare depends on s.

- If $s < \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$, then IP investors withdraw at date 1 when $\phi = 0$, and the expected payoff to a non-withdrawing UP investor is $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)}V(s)$. Setting this expression equal to $(1+\gamma)^{-1}$ defines a threshold $\acute{s}(\lambda) \in (s^\gamma, 1]$, characterized by

$$V(\acute{s}(\lambda)) = \frac{(1-\alpha)(1-\lambda)}{1+\gamma-[\lambda+\alpha(1-\lambda)](1+\gamma)^2}, \tag{A14}$$

  such that $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)}V(s)$ is less than, equal to, and greater than $1-\Phi$ for $s < \acute{s}(\lambda)$, $s = \acute{s}(\lambda)$, and $s > \acute{s}(\lambda)$, respectively.

- If $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$, then IP investors also do not withdraw when $\phi = 0$, and a UP investor's expected payoff is $\frac{1-\lambda(1+\gamma)}{1-\lambda}V(s)$. This payoff is greater than $1-\Phi$, which follows from the definition of $\hat{s}_1^0$ and the fact that $s \geq \hat{s}_1^0$. Hence, the fee hurts UP investors.

Thus, the fee benefits UP investors when s is sufficiently low, i.e., $s < \min\big\{\max\{\hat{s}_1^0, \hat{s}_2^0(n)\}, \acute{s}(\lambda)\big\}$, but harms them when s is sufficiently high, i.e., $s > \min\big\{\max\{\hat{s}_1^0, \hat{s}_2^0(n)\}, \acute{s}(\lambda)\big\}$. For the third case wherein $n \in (\bar{n}^0, \bar{n}^\Phi]$, the effect of the fee on UP investors' welfare again depends on s.

- If $s < \hat{s}_1^0$, then IP investors withdraw at the first stage when $\phi = 0$ and the fund liquidates when UP investors withdraw at the second stage. Thus, UP investors receive $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$ because they must bear the liquidation costs generated by impatient and IP investors in addition to those generated by themselves. There exists a threshold $\grave{s}(\lambda) \in (0, s^\gamma)$, defined by

$$V(\grave{s}(\lambda)) = \frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}, \tag{A15}$$

  such that a UP investor's payoff from not withdrawing when $\phi = \Phi$, $V(s)$, is less than, equal to, and greater than $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$ for $s < \grave{s}(\lambda)$, $s = \grave{s}(\lambda)$, and $s > \grave{s}(\lambda)$, respectively. Note that $\grave{s}(\lambda) < \hat{s}_1^0$.

- If $s \geq \hat{s}_1^0$, then IP investors do not withdraw at the first stage when $\phi = 0$ and UP investors (who withdraw at the second stage) receive either 1 if IP investors do not withdraw at the second stage or $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} < 1$ if IP investors withdraw at the second stage. Because $V(s) > V(s^*) = 1$ given $s \geq \hat{s}_1^0$, UP investors benefit from the fee.

Hence, the fee benefits UP investors in this case when s is sufficiently high, i.e., $s > \grave{s}(\lambda)$, but harms them when s is sufficiently low, i.e., $s < \grave{s}(\lambda)$. For the fourth case wherein $n > \max\{\bar{n}^0, \bar{n}^\Phi\}$, UP investors' welfare depends on s.

- If $s < \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$, then IP investors withdraw at date 1 and the fund is liquidated. If IP investors withdraw at the first stage (i.e., if $s < \hat{s}_1^0$), then a UP investor receives $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$, which is equal to each UP investor's pro rata share of the remaining deposits after both impatient and IP investors withdraw, discounted by the liquidation costs. Conversely, if IP investors withdraw at the second stage (i.e., if $s \in [\hat{s}_1^0, \hat{s}_2^0(n))$), then each UP investor receives her discounted pro rata share of the remaining deposits after impatient investors withdraw, $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$. Because $1 - \Phi$ is greater than both $\frac{1-[\lambda+\alpha(1-\lambda)](1+\gamma)}{(1-\alpha)(1-\lambda)(1+\gamma)}$ and $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$, UP investors benefit from the fee in both cases.
- If $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$, then IP investors do not withdraw and the fund is not liquidated when UP investors make a second-stage withdrawal. Consequently, UP investors receive 1 in the absence of a fee but only $1 - \Phi$ in the presence of a fee.

Therefore, the fee is beneficial to UP investors in this case when s is sufficiently low, i.e., $s < \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$, but is detrimental when s is sufficiently high, i.e., $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$. □

Appendix B: Derivation of Social Welfare Effects

In this appendix, we derive the Θs in Table 1 and prove Proposition 4. The derivations are organized based on the realized first-stage withdrawal n and signal s.

Case 1: $n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$. UP investors do not withdraw regardless of the fee arrangement. However, the fee may affect IP investors' actions. Depending on s, there are three possible outcomes.

1.1 $s < s^\gamma$. IP investors withdraw at the first stage regardless of the fee. In this case,

$$\Omega^0 = \lambda + \alpha(1-\lambda) + \big(1 - [\lambda + \alpha(1-\lambda)](1+\gamma)\big)V(s), \tag{B1}$$

$$\Omega^\Phi = \lambda(1-\Phi) + \alpha(1-\lambda)(1-\Phi) + (1-\alpha)(1-\lambda)V(s). \tag{B2}$$

The first, second, and third terms in each expression represents the respective amounts received by impatient, IP, and UP investors. Subtracting (B1) from (B2) yields,

$$\Omega^{\Phi} - \Omega^{0} = \gamma[\lambda + \alpha(1-\lambda)](V(s) - V(s^{\gamma})) < 0. \tag{B3}$$

1.2 $s \in [s^{\gamma}, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$. IP investors withdraw when $\phi = 0$ but do not withdraw when $\phi = \Phi$. Here, Ω^0 is given by (B1), and

$$\Omega^{\Phi} = \lambda(1-\Phi) + (1-\lambda)V(s). \tag{B4}$$

The first term represents the amount received by impatient investors and the second term represents the amount received by IP and UP investors. Subtracting (B1) from (B4) yields

$$\Omega^{\Phi} - \Omega^{0} = (\gamma[\lambda + \alpha(1-\lambda)] + \alpha(1-\lambda))(V(s) - V(s^{\gamma})) \geq 0, \tag{B5}$$

which holds with strict inequality when $s \neq s^{\gamma}$.

1.3 $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$. IP investors do not withdraw regardless of the fee arrangement. In this case, Ω^{Φ} is given by (B4) and

$$\Omega^{0} = \lambda + [1 - \lambda(1+\gamma)]V(s) \tag{B6}$$

because only impatient investors withdraw. Subtracting (B6) from (B4) gives

$$\Omega^{\Phi} - \Omega^{0} = \gamma\lambda(V(s) - V(s^{\gamma})) > 0. \tag{B7}$$

Case 2: $n \in (\bar{n}^{\Phi}, \bar{n}^{0}]$. UP investors withdraw when $\phi = \Phi$ but not when $\phi = 0$. IP investors may also alter their actions, depending on the signal s. There are three possible outcomes.

2.1 $s < s^{\gamma}$. IP investors withdraw at the first stage regardless of the fee, so Ω^0 is given by (B1) but

$$\Omega^{\Phi} = 1 - \Phi \tag{B8}$$

because all investors withdraw early and receive $1 - \Phi$. Subtracting (B1) from (B8) yields

$$\Omega^{\Phi} - \Omega^{0} = ([\lambda + \alpha(1-\lambda)](1+\gamma) - 1)(V(s) - V(s^{\gamma})) > 0, \tag{B9}$$

where the inequality follows from Assumption 1 (which implies $1 - [\lambda + \alpha(1-\lambda)](1+\gamma) < 0$) and the fact that $V(s) < V(s^{\gamma})$.

2.2 $s \in [s^{\gamma}, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$. IP investors convert from withdrawers when $\phi = 0$ to non-withdrawers when $\phi = \Phi$. In this case, Ω^0 is again given by (B1), and

$$\Omega^{\Phi} = \lambda(1-\Phi) + (1-\alpha)(1-\lambda)(1-\Phi) + \alpha(1-\lambda)V(s). \tag{B10}$$

The first, second, and third terms in this expression represent the respective amounts received by impatient, UP, and IP investors. Subtracting (B1) from (B10) gives

$$\Omega^{\Phi} - \Omega^{0} = (\gamma[\lambda + \alpha(1-\lambda)] + \alpha(1-\lambda) - (1-\alpha)(1-\lambda))(V(s) - V(s^{\gamma})). \tag{B11}$$

Clearly, $\Omega^{\Phi} - \Omega^{0} > 0$ if and only if $\alpha > \frac{1-\lambda(1+\gamma)}{(1-\lambda)(2+\gamma)}$.

2.3 $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$. IP investors do not withdraw regardless of the fee. In this case, Ω^{Φ} is again given by (B10), but Ω^0 is given by (B6) because only impatient investors withdraw when $\phi = 0$. Subtracting (B6) from (B10) yields

$$\Omega^{\Phi} - \Omega^{0} = [\gamma\lambda - (1-\alpha)(1-\lambda)](V(s) - V(s^{\gamma})) < 0, \tag{B12}$$

where the inequality follows from Assumption 1 and the fact that $V(s) > V(s^{\gamma})$.

Case 3: $n \in (\bar{n}^0, \bar{n}^{\Phi}]$. The fee converts UP investors from withdrawers to non-withdrawers. Depending on s, IP investors' behaviors may also change. Again, there are three possible outcomes.

3.1 $s < s^{\gamma}$. IP investors withdraw at the first stage regardless of the fee. Because all investors withdraw when $\phi = 0$,

$$\Omega^0 = \frac{1}{1+\gamma}. \tag{B13}$$

When $\phi = \Phi$, only UP investors maintain their deposits until date 2, so Ω^Φ is given by (B2). The effect of the fee on social welfare, which is derived by subtracting (B13) from (B2), is

$$\Omega^\Phi - \Omega^0 = (1-\alpha)(1-\lambda)(V(s) - V(s^\gamma)) < 0. \tag{B14}$$

3.2 $s \in [s^\gamma, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$. IP investors withdraw at the first stage when $\phi = 0$ but do not withdraw when $\phi = \Phi$. Here, Ω^0 and Ω^Φ are given by (B13) and (B4), respectively. Then,

$$\Omega^\Phi - \Omega^0 = (1-\lambda)(V(s) - V(s^\gamma)) \geq 0, \tag{B15}$$

which holds with strict inequality when $s \neq s^\gamma$.

3.3 $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$. IP investors do not withdraw. In this case, Ω^Φ is given by (B4) but

$$\Omega^0 = \lambda + (1-\alpha)(1-\lambda) + (1 - [\lambda + (1-\alpha)(1-\lambda)](1+\gamma))V(s). \tag{B16}$$

The first, second, and third terms in this expression represent the amounts received by impatient, UP, and IP investors, respectively. Subtracting (B16) from (B4) yields

$$\Omega^\Phi - \Omega^0 = [\gamma\lambda + (1-\alpha)(1-\lambda)(1+\gamma))(V(s) - V(s^\gamma)) > 0. \tag{B17}$$

Case 4: $n > \max\{\bar{n}^0, \bar{n}^\Phi\}$. UP investors withdraw regardless of the fee, but the fee may alter IP investors' behaviors. There are three possible outcomes, depending on s.

4.1 $s < s^\gamma$. IP investors withdraw irrespective of the fee. Here, Ω^0 and Ω^Φ are given by (B13) and (B8), respectively, because all investors withdraw regardless of the fee. Therefore, $\Omega^\Phi - \Omega^0 = 0$.

4.2 $s \in [s^\gamma, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$. IP investors convert from withdrawers to non-withdrawers. In this case, Ω^0 is given by (B13) and Ω^Φ is given by (B10). Subtracting (B13) from (B10) gives

$$\Omega^\Phi - \Omega^0 = \alpha(1-\lambda)(V(s) - V(s^\gamma)) \geq 0, \tag{B18}$$

which holds with strict inequality when $s \neq s^\gamma$.

4.3 $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$. IP investors maintain their deposits until date 2. Here, Ω^0 and Ω^Φ are given by (B16) and (B10), respectively. Subtracting (B16) from (B10) yields

$$\Omega^\Phi - \Omega^0 = \gamma[\lambda + (1-\alpha)(1-\lambda)](V(s) - V(s^\gamma)) > 0. \tag{B19}$$

Appendix C: Derivations for Preemptive Runs

In this appendix, we derive the investors' strategies when a fee is present only at the second stage. We omit formal proofs, which are analogous to those in Appendix A.

At the beginning of the second stage, IP investors who chose $w = 0$ and all UP investors remain invested in the fund. These remaining investors observe the first-stage withdrawal n and infer that the amount of deposits remaining at the fund at the beginning of the second stage is $1 - n(1+\gamma)$.

If $n \leq \bar{n}^\varphi$, then UP investors do not withdraw, and IP investors also do not withdraw at the second stage if and only if

$$\frac{1-n(1+\gamma)}{1-n} V(s) \geq 1 - \Phi = \frac{1}{1+\gamma}, \tag{C1}$$

which is identical to (7) except that the lack of a first-stage fee results in higher liquidation costs. Conversely, if $n > \bar{n}^\varphi$, then UP investors withdraw. IP investors also withdraw in this case unless

$$\frac{1-n(1+\gamma)-(1-\alpha)(1-n)}{\alpha(1-n)} V(s) \geq 1 - \Phi = \frac{1}{1+\gamma}. \tag{C2}$$

This expression is similar to (8), except that (i) impatient investors do not pay a fee when withdrawing at the first-stage, which increases first-stage liquidation costs, and (ii) the fees paid by withdrawing UP investors enable the fund to liquidate only $(1-\alpha)(1-n)$ units of its investment, instead of $(1-\alpha)(1-n)(1+\gamma)(1-\phi)$ as in (8), to meet UP investors' withdrawal demand. The following theorem characterizes the IP investors' second-stage signal threshold $\hat{s}_2^\varphi(n)$.

Theorem C.1. *For any realized first-stage withdrawal* $n \in [0, \bar{\lambda}]$ *and the UP investors' second-stage withdrawal threshold* $\bar{n}^\varphi \in [0, \bar{\lambda}]$*, there exists a unique signal threshold* $\hat{s}_2^\varphi(n) \in [s^\gamma, 1]$*, characterized by*

$$V(\hat{s}_2^\varphi(n)) = \begin{cases} \dfrac{1-n}{[1-n(1+\gamma)](1+\gamma)} & \text{if } n \in [0, \bar{n}^\varphi] \\ \dfrac{\alpha(1-n)}{[\alpha(1-n)-\gamma n](1+\gamma)} & \text{if } n \in (\bar{n}^\varphi, \bar{\lambda}], \end{cases} \tag{C3}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $s < \hat{s}_2^\varphi(n)$*. Furthermore,* $\hat{s}_2^\varphi(n)$ *is strictly increasing in* n*.*

At the first stage, (C3) can be inverted to find a second-stage withdrawal threshold for IP investors that depends on the realized first-stage withdrawal n.

Lemma C.1. *For any realized signal* $s \in [s^\gamma, 1]$ *and the UP investors' second-stage withdrawal threshold* $\bar{n}^\varphi$*, there exists a unique threshold* $\hat{n}^\varphi(s) \in [0, \bar{\lambda}]$*, characterized by*

$$\hat{n}^\varphi(s) = \begin{cases} \dfrac{(1+\gamma)V(s)-1}{(1+\gamma)^2 V(s)-1} & \text{if } s \in [s^\gamma, \hat{s}_2^\varphi(\bar{n}^\varphi)] \\ \bar{n}^\varphi & \text{if } s \in (\hat{s}_2^\varphi(\bar{n}^\varphi), \check{s}_2^\varphi(\bar{n}^\varphi)] \\ 1 - \dfrac{\gamma(1+\gamma)V(s)}{\alpha[(1+\gamma)V(s)-1]+\gamma(1+\gamma)V(s)} & \text{if } s \in (\check{s}_2^\varphi(\bar{n}^\varphi), \check{s}_2^\varphi(\bar{\lambda})] \\ \bar{\lambda} & \text{if } s \in (\check{s}_2^\varphi(\bar{\lambda}), 1], \end{cases} \tag{C4}$$

where $\check{s}_2^\varphi(n)$ *satisfies*

$$V(\check{s}_2^\varphi(n)) = \frac{\alpha(1-n)}{[\alpha(1-n)-\gamma n](1+\gamma)}, \tag{C5}$$

such that IP investors who do not withdraw at the first stage withdraw at the second stage if and only if $n > \hat{n}^\varphi(s)$*. Furthermore,* $\hat{n}^\varphi(s)$ *is continuous and non-decreasing in* s*.*

There are four relevant scenarios for IP investors at the second stage. First, if both IP and UP investors maintain their deposits until date 2, then an IP investor's expected payoff is $\frac{1-\lambda(1+\gamma)}{1-\lambda}V(s)$. Second, if IP investors withdraw and UP investors do not, then an IP investor receives $\frac{1}{1+\gamma}$. Third, if UP investors withdraw and IP investors do not, then an IP investor's expected payoff is $\big(1 - \frac{\gamma n}{\alpha(1-n)}\big)V(s)$ because the liquidation costs generated by impatient investors are borne entirely by IP investors rather than being shared with UP investors. Fourth, if both IP and UP investors make a second-stage withdrawal, then the fund is liquidated and an IP investor receives $\frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)}$. These four scenarios can be combined such that IP investors do not withdraw at the first stage if and only if the following condition is satisfied:

$$1 \leq \int_0^{\min\{\hat{n}^\varphi(s),\bar{n}^\varphi\}} \frac{1-\lambda(1+\gamma)}{1-\lambda} V(s)g(\lambda)\,\mathrm{d}\lambda + \int_{\min\{\hat{n}^\varphi(s),\bar{n}^\varphi\}}^{\bar{n}^\varphi} \frac{1}{1+\gamma} g(\lambda)\,\mathrm{d}\lambda$$
$$+ \int_{\bar{n}^\varphi}^{\max\{\hat{n}^\varphi(s),\bar{n}^\varphi\}} \left(1 - \frac{\gamma\lambda}{\alpha(1-\lambda)}\right) V(s)g(\lambda)\,\mathrm{d}\lambda + \int_{\max\{\hat{n}^\varphi(s),\bar{n}^\varphi\}}^{\bar{\lambda}} \frac{1-\lambda(1+\gamma)}{(1-\lambda)(1+\gamma)} g(\lambda)\,\mathrm{d}\lambda. \quad \text{(C6)}$$

The following theorem characterizes the IP investors' first-stage withdrawal decisions when a fee is present only at the second stage.

Theorem C.2. *Given the UP investors' second-stage withdrawal threshold $\bar{n}^\varphi$, there exists a unique signal threshold $\hat{s}_1^\varphi \in (s^*, 1)$ such that IP investors withdraw at the first stage if and only if $s < \hat{s}_1^\varphi$. Furthermore, $\hat{s}_1^\varphi$ is decreasing in $\bar{n}^\varphi$.*

UP investors update their beliefs based on n according to Lemma 1, with $\hat{s}_1^\varphi$ in place of $\hat{s}_1^\phi$. If IP investors do not make a second-stage withdrawal (either because they already withdrew at the first stage or because they maintain their deposits until date 2), then a non-withdrawing UP investor's expected payoff is

$$\frac{1-n(1+\gamma)}{1-n} \mathbb{E}[\tilde{v}|n, \hat{s}_1^\varphi]. \quad \text{(C7)}$$

If, however, IP investors withdraw at the second-stage, then a non-withdrawing UP investor's expected payoff is

$$\frac{1-n(1+\gamma)-\alpha(1-n)}{(1-\alpha)(1-n)} \mathbb{E}[\tilde{v}|\hat{s}_1^\varphi \leq s < \hat{s}_2^\varphi(n)]. \quad \text{(C8)}$$

The following theorem characterizes the UP investors' withdrawal threshold $\bar{n}^\varphi$.

Theorem C.3. *Given the IP investors' respective first-stage and second-stage signal thresholds $\hat{s}_1^\varphi$ and $\hat{s}_2^\varphi(n)$, there exists a unique withdrawal threshold $\bar{n}^\varphi$, characterized by*

$$\bar{n}^\varphi = \sup\left\{ n \in [0, \bar{\lambda}] : \frac{1}{1+\gamma} \leq \frac{1-n(1+\gamma)}{1-n} \mathbb{E}[\tilde{v}|n, \hat{s}_1^\varphi] \right.$$
$$\left. - \mathbb{1}_{\{\hat{s}_1^\varphi < \hat{s}_2^\varphi(n)\}} (1 - F(\hat{s}_1^\varphi|n)) \frac{F(\hat{s}_2^\varphi(n)) - F(\hat{s}_1^\varphi)}{1 - F(\hat{s}_1^\varphi)} \frac{\alpha\gamma n \mathbb{E}[\tilde{v}|\hat{s}_1^\varphi \leq s < \hat{s}_2^\varphi(n)]}{(1-\alpha)(1-n)} \right\}, \quad \text{(C9)}$$

such that UP investors do not withdraw at the second stage if and only if $n \leq \bar{n}^{\varphi}$.

As discussed in Section 3.2, imposing a redemption fee changes the UP investors' posterior expectation of the risky asset's payoff. Consequently, a second-stage fee may either increase or decrease the UP investors' withdrawal threshold.

Table 1: Social welfare. The change in aggregate investment Θ along with the sign of the change in social welfare (in brackets) are listed for each scenario. The rows represent changes in UP investors' actions: when $n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$, UP investors do not withdraw regardless of the fee arrangement (1st row); when $\bar{n}^\Phi < \bar{n}^0$ and $n \in (\bar{n}^\Phi, \bar{n}^0]$, the fee converts UP investors from non-withdrawers to withdrawers (2nd row); when $\bar{n}^\Phi > \bar{n}^0$ and $n \in (\bar{n}^0, \bar{n}^\Phi]$, the fee converts UP investors from withdrawers to non-withdrawers (3rd row); and when $n > \max\{\bar{n}^0, \bar{n}^\Phi\}$, UP investors withdraw at the second stage irrespective of the fee (4th row). The columns represent changes in IP investors' actions: when $s < s^\gamma$, IP investors withdraw at the first stage regardless of the fee (1st column); when $s \in [s^\gamma, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$, the fee converts IP investors from withdrawers to non-withdrawers (2nd column); and when $s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$, IP investors do not withdraw irrespective of the fee (3rd column).

	$V(s) - V(s^\gamma) < 0$	$V(s) - V(s^\gamma) \geq 0$	
	$s < s^\gamma$	$s \in [s^\gamma, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$	$s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$
$n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$	$[\lambda + \alpha(1-\lambda)]\gamma$ [−]	$\lambda\gamma + \alpha(1-\lambda)(1+\gamma)$ [+]	$\lambda\gamma$ [+]
$n \in (\bar{n}^\Phi, \bar{n}^0]$	$\gamma - (1-\alpha)(1-\lambda)(1+\gamma)$ [+]	$\lambda\gamma + \alpha(1-\lambda)(1+\gamma) - (1-\alpha)(1-\lambda)$ [+/− if α big/small]	$\lambda\gamma - (1-\alpha)(1-\lambda)$ [−]
$n \in (\bar{n}^0, \bar{n}^\Phi]$	$(1-\alpha)(1-\lambda)$ [−]	$1 - \lambda$ [+]	$\lambda\gamma + (1-\alpha)(1-\lambda)(1+\gamma)$ [+]
$n > \max\{\bar{n}^0, \bar{n}^\Phi\}$	0 [no change]	$\alpha(1-\lambda)$ [+]	$[\lambda + (1-\alpha)(1-\lambda)]\gamma$ [+]

Table 2: Payoff and information externalities. Panel A lists the portion of the change in aggregate investment Θ arising from the internalization of the payoff externality. Panel B lists the portion of the change in aggregate investment Θ arising from the alteration of the information externality. The rows and columns are as described in Table 1.

	$V(s) - V(s^\gamma) < 0$	$V(s) - V(s^\gamma) \geq 0$	
	$s < s^\gamma$	$s \in [s^\gamma, \max\{\hat{s}_1^0, \hat{s}_2^0(n)\})$	$s \geq \max\{\hat{s}_1^0, \hat{s}_2^0(n)\}$
	Panel A: Payoff Externality		
$n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$	$[\lambda + \alpha(1-\lambda)]\gamma$	$\lambda\gamma + \alpha(1-\lambda)(1+\gamma)$	$\lambda\gamma$
$n \in (\bar{n}^\Phi, \bar{n}^0]$	$[\lambda + \alpha(1-\lambda)]\gamma$	$\lambda\gamma + \alpha(1-\lambda)(1+\gamma)$	$\lambda\gamma$
$n \in (\bar{n}^0, \bar{n}^\Phi]$	0	$\alpha(1-\lambda)$	$[\lambda + (1-\alpha)(1-\lambda)]\gamma$
$n > \max\{\bar{n}^0, \bar{n}^\Phi\}$	0	$\alpha(1-\lambda)$	$[\lambda + (1-\alpha)(1-\lambda)]\gamma$
	Panel B: Information Externality		
$n \leq \min\{\bar{n}^0, \bar{n}^\Phi\}$	0	0	0
$n \in (\bar{n}^\Phi, \bar{n}^0]$	$-(1-\alpha)(1-\lambda)$	$-(1-\alpha)(1-\lambda)$	$-(1-\alpha)(1-\lambda)$
$n \in (\bar{n}^0, \bar{n}^\Phi]$	$(1-\alpha)(1-\lambda)$	$(1-\alpha)(1-\lambda)$	$(1-\alpha)(1-\lambda)$
$n > \max\{\bar{n}^0, \bar{n}^\Phi\}$	0	0	0