Deferred Acceptance with Compensation Chains

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Matching Problem

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Each man and each woman has a strict preference ordering over the other side of the market (and being unmatched).

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- everyone prefers the partner to being unmatched,
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The theory of stable matchings plays a big role in real-life problems: school choice (assigning students to schools), residency match (NRMP, assigning doctors to hospitals), other centralized labor markets.

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The (men-proposing) DA algorithm:

- Each man proposes to his most-preferred woman;
- Each woman temporarily accepts the best offer, rejects remaining offers;
- Rejected men propose to the next woman on their preference list;
- These steps are repeated until all men are either matched or have already proposed to all acceptable women.

The outcome is the (men-optimal) stable matching.

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What happens when both sides of the market propose?

Will we still reach a stable matching? **Can we generate all stable matchings?**

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1) Equivalence between DA and stability

2) New proof technique for showing convergence (based on a potential function)

3) A procedurally fair implementation of a stable matching

Related Literature (selection)

Deferred Acceptance and other stable algorithms: Gale and Shapley (1962), McVitie and Wilson (1971), Roth and Vande Vate (1990), Ma (1996) Blum, Roth and Rothblum (1997), Blum and Rothblum, (2002), Kesten (2004), Martinez, Masso, Neme, Oviedo (2004)

Fixed-point algorithms: Adachi (2000), Fleiner (2003), Hatfield and Milgrom (2005), Echenique and Oviedo (2006).

Median matching and fair algorithms: Ma (1996), Teo and Sethuraman (1998), Romero-Medina (2005), Klaus and Klijn (2006), Cheng (2008), Schwarz and Yenmez (2011), Kuvalekar (2015).

Computation: Irving and Leather (1986).

Linear programming formulation: Vande Vate (1989).

THE MODEL

The standard one-to-one matching model

M - a finite set of men.

W - a finite set of women.

Each man m has a strict preference relation $\succ^m$ over W and being unmatched.

Each woman w has a strict preference relation $\succ^w$ over M and being unmatched.

Standard definitions of matching and stability.

Agents are non-strategic.

The DACC algorithm

Deferred Acceptance with Compensation Chains

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Agent i *divorces* agent j if:

- i and j are temporarily matched,
- i breaks the match to become matched with someone else.

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A budget B_i for agent i is any subset of agents on the other side of the market.

Agent i *divorces* agent j if:

- i and j are temporarily matched,
- i breaks the match to become matched with someone else.

Agent i *deceives* agent j if:

- i divorces j,
- i proposed to j before.

Deferred Acceptance with Compensation Chains

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- In round k, agent $\Phi(k)$ proposes to the most preferred partner in his/ her budget set.
- An agent receiving a proposal chooses to match with the more preferred partner (proposer versus current match partner).

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Adjustment to budget sets:

- Whenever i rejects or divorces j, remove i from j's budget set.
- Whenever i proposes to j, add i to j's budget set.

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Adjustment to budget sets:

- Whenever i rejects or divorces j, remove i from j's budget set.
- Whenever i proposes to j, add i to j's budget set.

The algorithm stops when each agent is matched to the best partner in his/ her budget set.

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If agent j is deceived, we compensate agent j by letting him/her propose out of order.

A compensation chain (starting at *j*):

Agent j proposes. If

- agent j is rejected, j proposes again (if j has acceptable options in his/her budget);
- agent j is accepted by k who thereby deceives some agent i, i proposes;
- otherwise, terminate the CC.

Deferred Acceptance with Compensation Chains - example

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

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$m1$
$m2$
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$w1$
$w2$
$w3$

$w1$: $m3 > m2$
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Matches

Deferred Acceptance with Compensation Chains - example

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$

$m2$

$m3$

$w1$

$w2$

$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$

$m2$

$m3$

Matches

Budgets

$w1$

$w2$

$w3$

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$m1$: $w3 > w2$
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$m3$: $w2 > w1$

$m1$

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$w1$

$w2$

$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, w2, w3$

Matches

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	$w3$

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$m1$: $w3 > w2$
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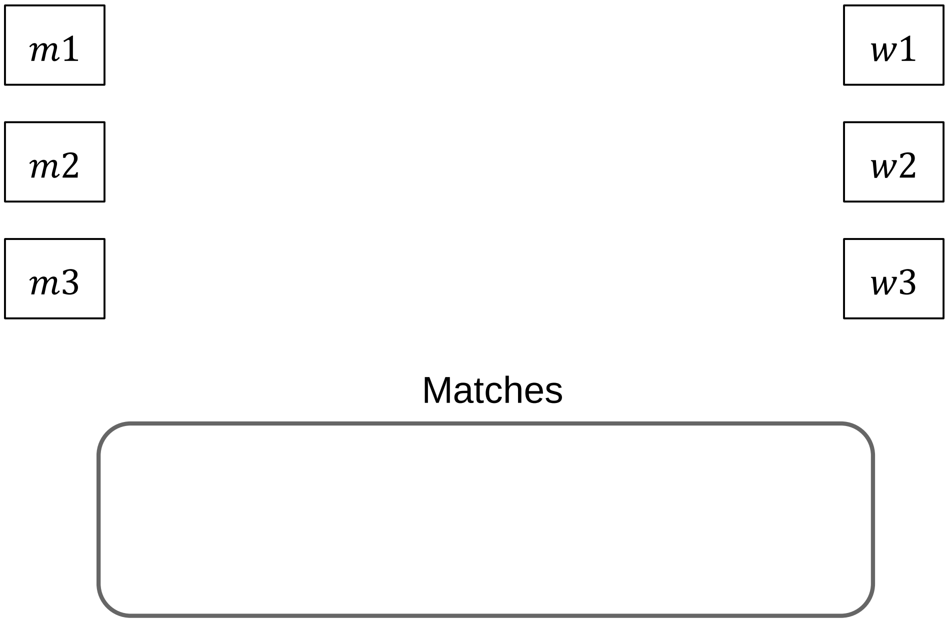

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, w2, w3$

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	$w3$

Order: $w2, m2, m3, w3, m3, w3, m2, w2, m1, w1,$ ...

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Budgets

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Matches

Budgets

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Order: $\boxed{w2}, m2, m3, w3, m3, w3, m2, w2, m1, w1, \ldots$

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Matches

$w2$ | $m1$

Budgets

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Budgets

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Matches

$w2$	$m1$

Budgets

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Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, w2, w3$

Matches

$w2$	$m1$	$m2$	$w1$

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
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$m3$	$w1, w2, w3$

Matches

$w2$	$m1$	$m2$	$w1$

Budgets

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$m2$ $w1, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w2$ $m1$
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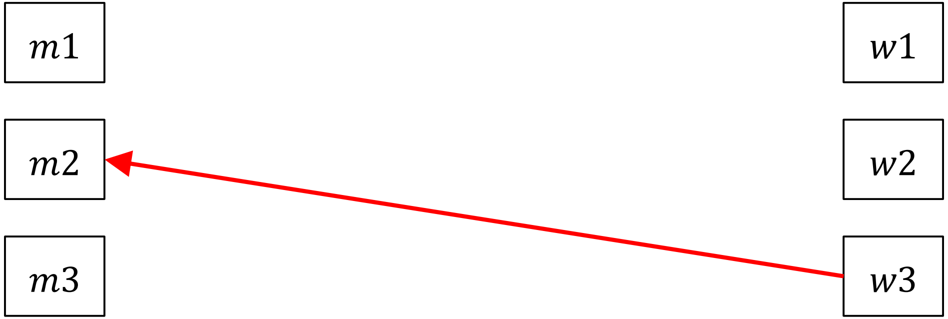

$m1$
$m2$
$m3$

$w1$
$w2$
$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $w1, w2, w3$
$m3$ $w1,$ ~~$w2$~~$, w3$

Matches

$w2$ $m1$
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Budgets

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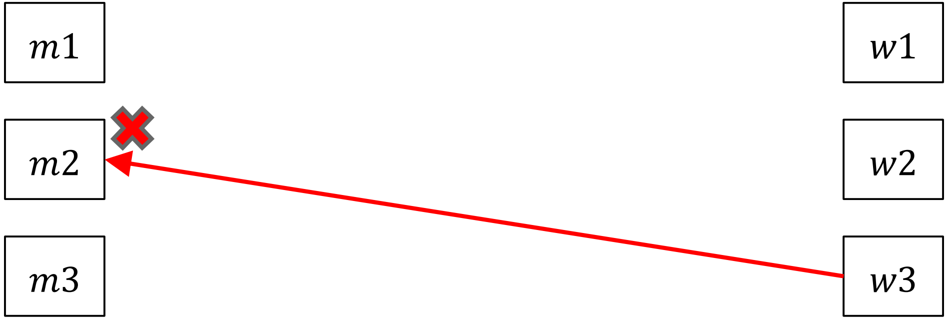

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Budgets

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Matches

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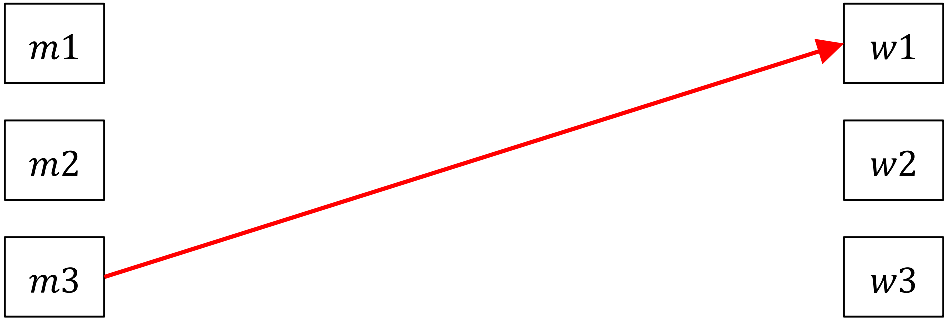

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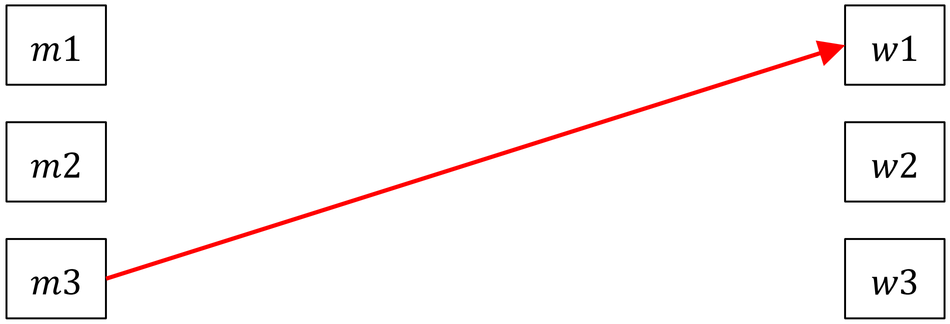

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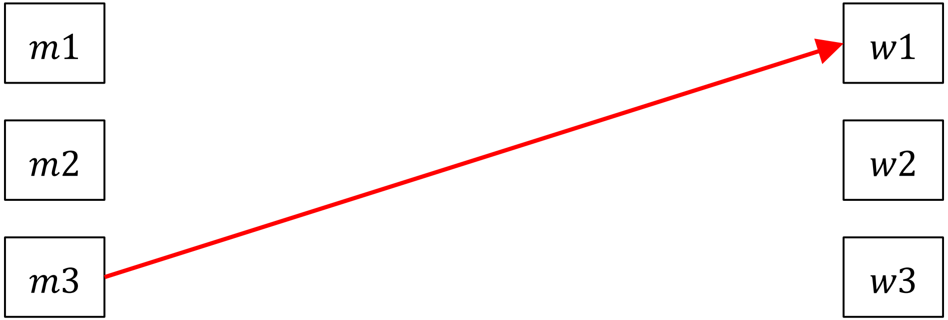

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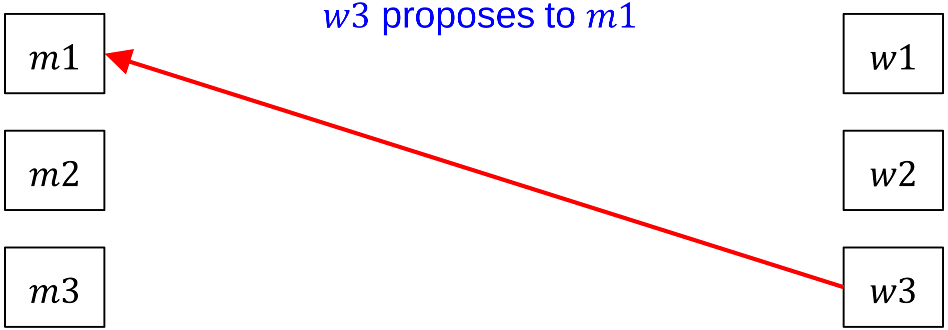
$w3$ proposes to $m1$



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Matches

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$m3$ $w1$

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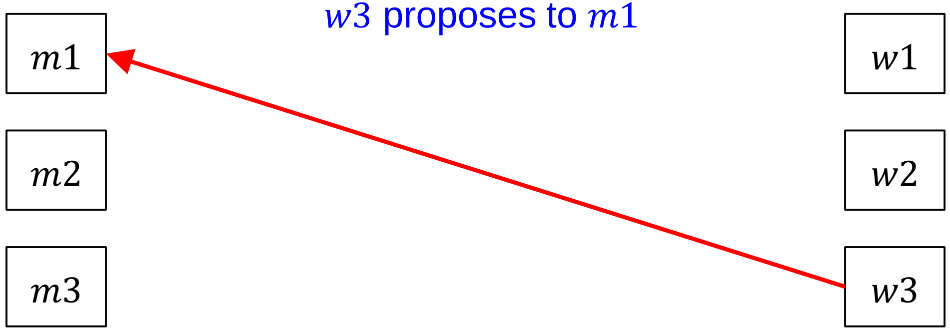
Deferred Acceptance with Compensation Chains - example

$w3$ proposes to $m1$



$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w2$ $m1$
$w3$ $m1$
$m3$ $w1$

Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
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Order: $w2, m2, m3, w3, m3, \boxed{w3,} m2, w2, m1, w1,$ ...

Deferred Acceptance with Compensation Chains - example

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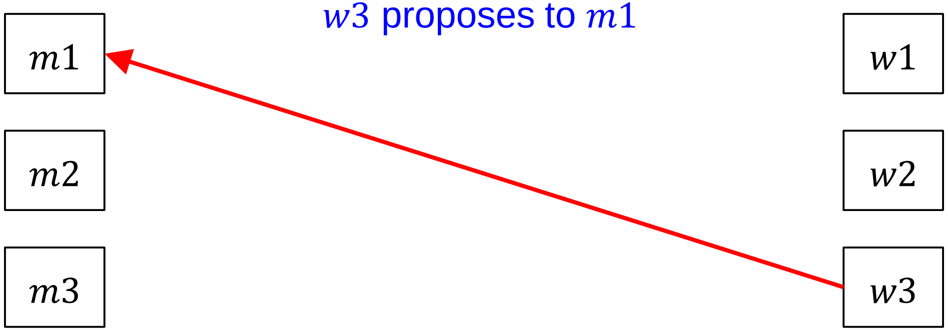

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Budgets

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Matches

$w3$	$m1$	$m3$	$w1$

Budgets

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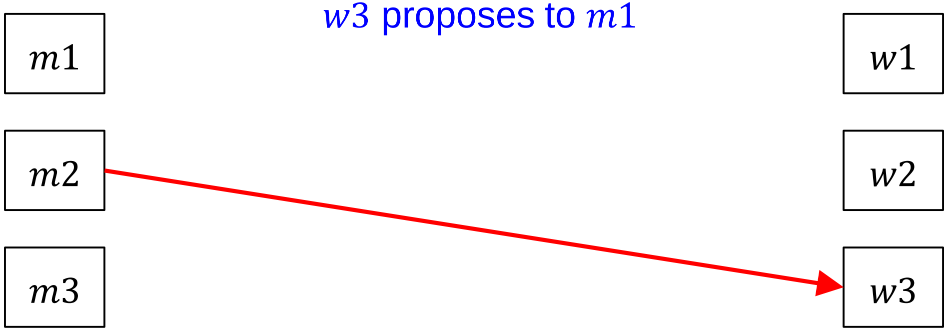

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Matches

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Budgets

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Deferred Acceptance with Compensation Chains - example

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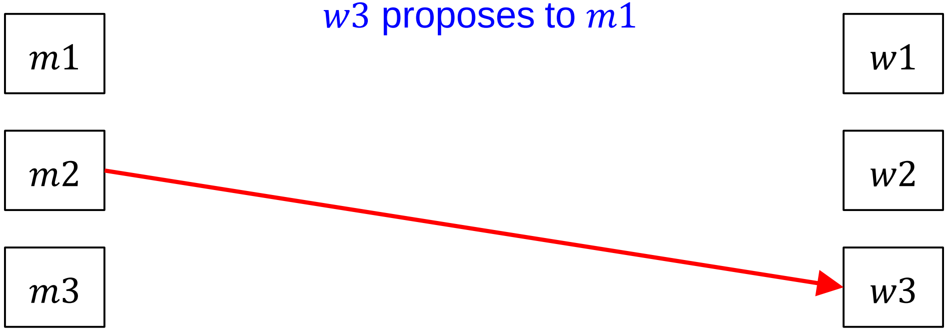

$w1$: $m3 > m2$
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Matches

$w3$ $m1$ $m3$ $w1$

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Deferred Acceptance with Compensation Chains - example

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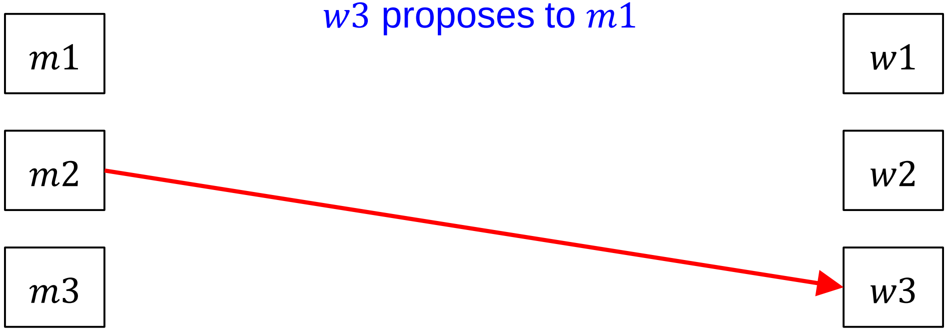

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Matches

$m2$	$w3$	$w3$	$m1$	$m3$	$w1$

Budgets

$m1, m2, m3$	$w1$
$\cancel{m1}, m2, m3$	$w2$
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Order: $w2, m2, m3, w3, m3, w3, \boxed{m2}, w2, m1, w1,$ ...

Deferred Acceptance with Compensation Chains - example

$w3$ divorces $m1$

$m1$: $w3 > w2$
$m2$: $w1 > w3$
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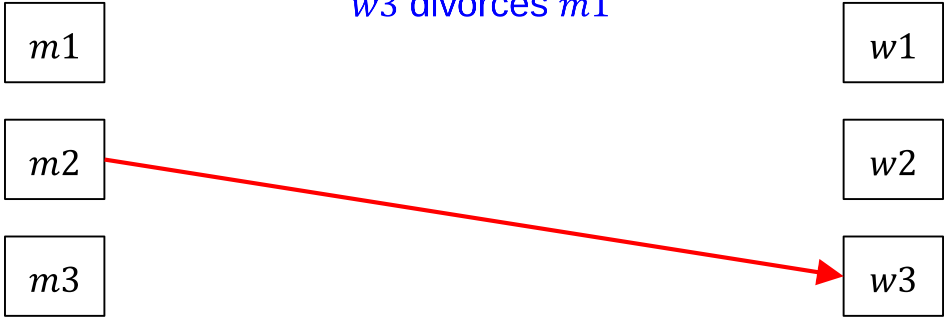

Budgets

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Matches

$m2$	$w3$
$m3$	$w1$

Order: $w2, m2, m3, w3, m3, w3, \boxed{m2}, w2, m1, w1, \ldots$

Deferred Acceptance with Compensation Chains - example

$m1$ is deceived!

$m1$: $w3 > w2$
$m2$: $w1 > w3$
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Deferred Acceptance with Compensation Chains - example

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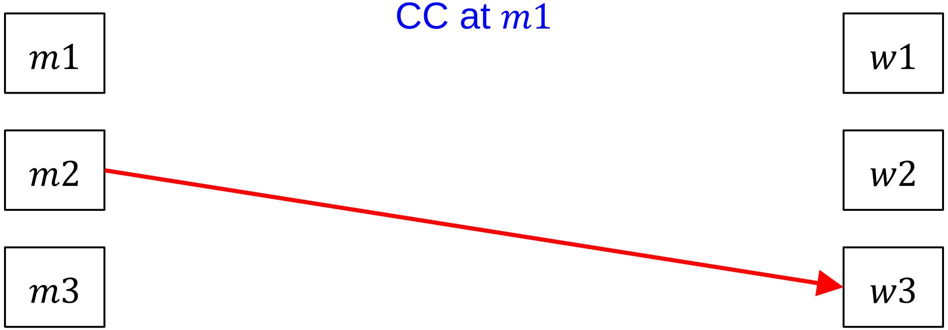
CC at $m1$



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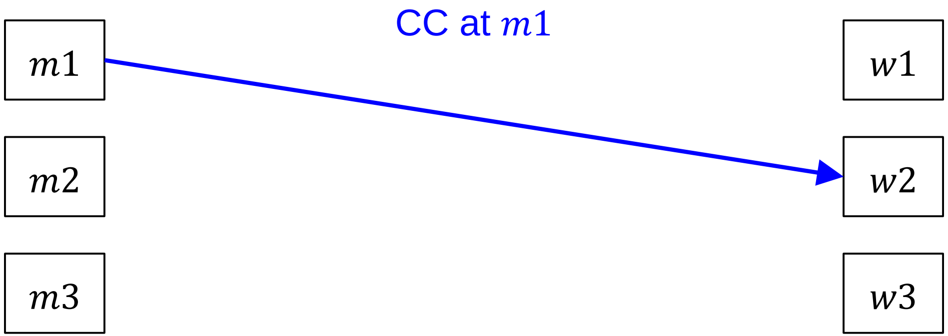

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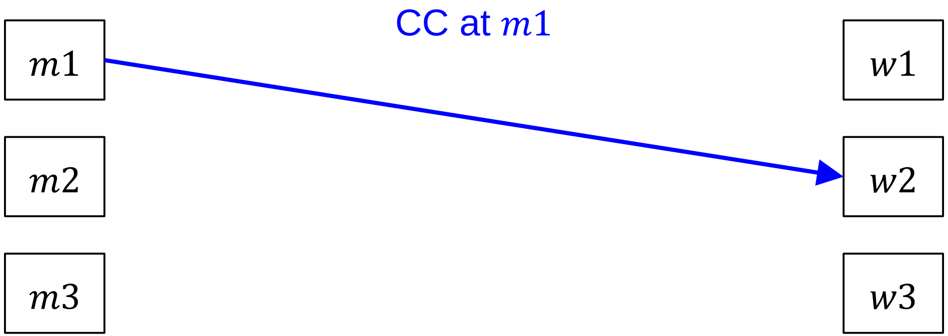

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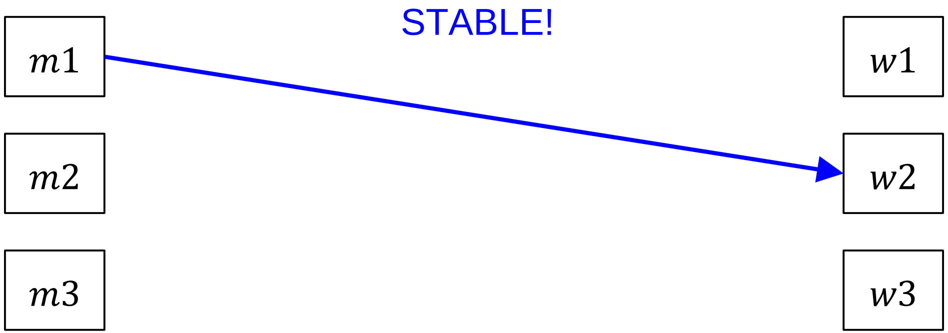

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Deferred Acceptance with Compensation Chains

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If only men appear in the sequence Φ *for sufficiently many rounds, then the DACC is equivalent to the men-proposing DA.*

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Once a stable matching is reached, all subsequent offers are rejected in DACC.

There are no compensation chains (because the proposing side never receives offers).

Deferred Acceptance with Compensation Chains

Theorem

For each sequence Φ*, the DACC algorithm converges to a stable matching in finitely many rounds .*

Deferred Acceptance with Compensation Chains

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By the stopping criterion, i is matched to the best partner in his/her budget – contradiction!

Deferred Acceptance with Compensation Chains

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number of agents in i's budget set that i prefers to the current match

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Deferred Acceptance with Compensation Chains

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- For agents who propose, d_i goes strictly down.
- For agents who receive offers, d_i goes weakly down.
- So we only worry about divorces. After sufficiently many rounds, every divorce leads to a CC.
- And for agents who propose in a CC, $d_i = 0$ when the CC is over.

Deferred Acceptance with Compensation Chains

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Theorem

For any stable matching μ, there is an ordering of agents such that μ is the outcome of the DACC with that order.

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We construct the ordering recursively. Start with an arbitrary agent. Suppose we have already chosen k agents. We choose the $(k+1)st$:

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- If agent $\Phi(k)$ was rejected, choose him/her again.
- If agent $\Phi(k)$ was accepted by his/her μ-partner, choose an any agent who is not yet matched to his/her μ-partner yet.
- If agent $\Phi(k)$ was accepted by j who is not his/her μ-partner, choose j as the next agent in the sequence.

Deferred Acceptance with Compensation Chains

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Corollary

A matching is stable if and only if it is an outcome of a DACC algorithm.

Deferred Acceptance with Compensation Chains

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Two key elements:

- Budget Sets – crucial for stability - Two-sided Deferred Acceptance
- Compensation Chains – crucial for convergence - Budget-Based Deferred Acceptance

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5) Arrival of agents. Agents arrive gradually to the market. DACC extends easily and retains all its properties.

Conclusions and final comments

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The DACC algorithm:

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The DACC algorithm:

- Always converges to a stable matching, without relying on the monotonicity of the offer process – analogy to the tâtonnement process for prices in GE.
- Achieves all stable matchings – unlike previous deferred acceptance algorithms in the literature.
- Establishes an equivalence between deferred acceptance procedures and stability.
- Does not distinguish between the two sides of the market – admits a fair implementation.

Deferred Acceptance with Compensation Chains

Thank you!

Appendix

Appendix

The 2DA algorithm

Two-sided Deferred Acceptance

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Given Φ, run a direct extension of the DA algorithm, i.e.

- In round $k,$ agent $i = \Phi(k)$ proposes to the most preferred partner that hasn't rejected i yet (if better than the current match partner).
- An agent receiving an offer, chooses to match with the more preferred partner (proposer versus current match partner).

Two-sided Deferred Acceptance

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- In round k, agent $i = \Phi(k)$ proposes to the most preferred partner that hasn't rejected i yet (if better than the current match partner).
- An agent receiving an offer, chooses to match with the more preferred partner (proposer versus current match partner).

The algorithm stops when there are no new proposals.

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Every stable matching can be achieved (same proof as for DACC).

Two-sided Deferred Acceptance

If a stable matching is reached, all subsequent proposals will be rejected.

If only men appear in the beginning of the sequence Φ for a sufficiently long time, this boils down to the men-proposing DA.

Every stable matching can be achieved (same proof as for DACC).

However, the outcome may fail to be stable for some choices of Φ.

Two-sided Deferred Acceptance - failure of stability

Consider the following market with 3 people on each side:

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Two-sided Deferred Acceptance - failure of stability

Consider the following market with 3 people on each side:

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

The unique stable matching is

$$\boldsymbol{m1 - w1}, \quad \boldsymbol{m2 - w2}, \quad \boldsymbol{m3 - w3}$$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

$m1$
$m2$
$m3$

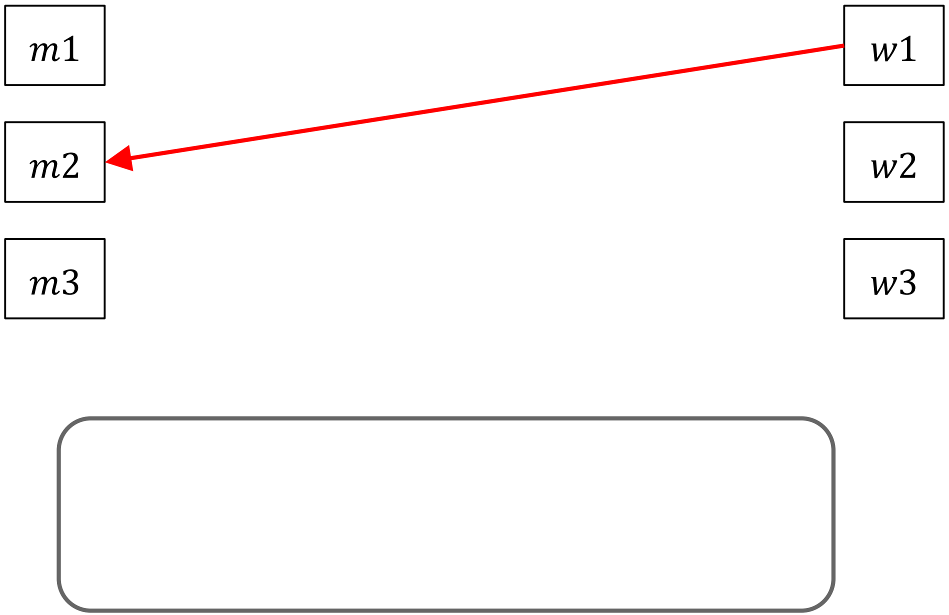
$w1$
$w2$
$w3$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$



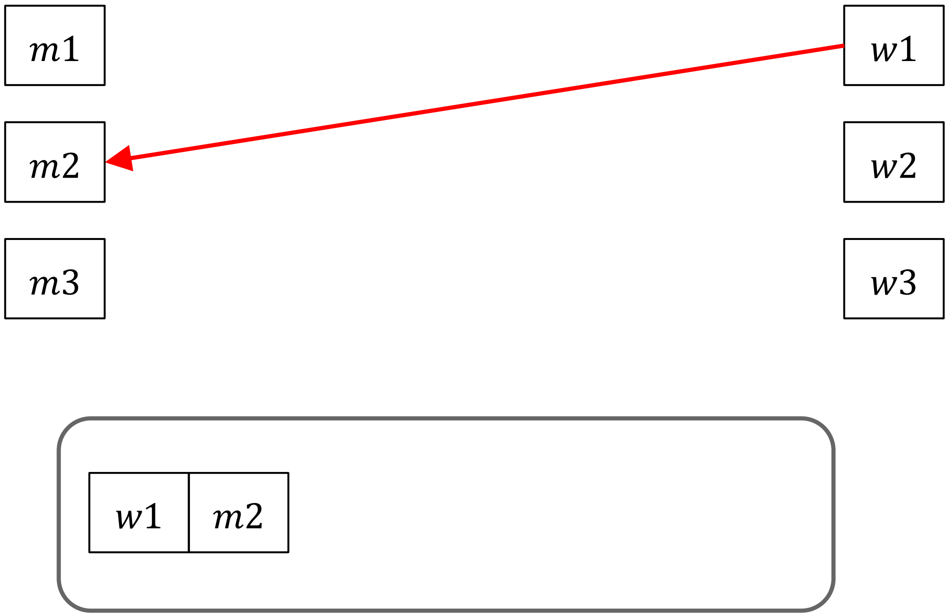
$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $\boxed{w1}, m2, m1, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1\colon w3 > w1$
$m2\colon w2 > w1$
$m3\colon w3 > w2$

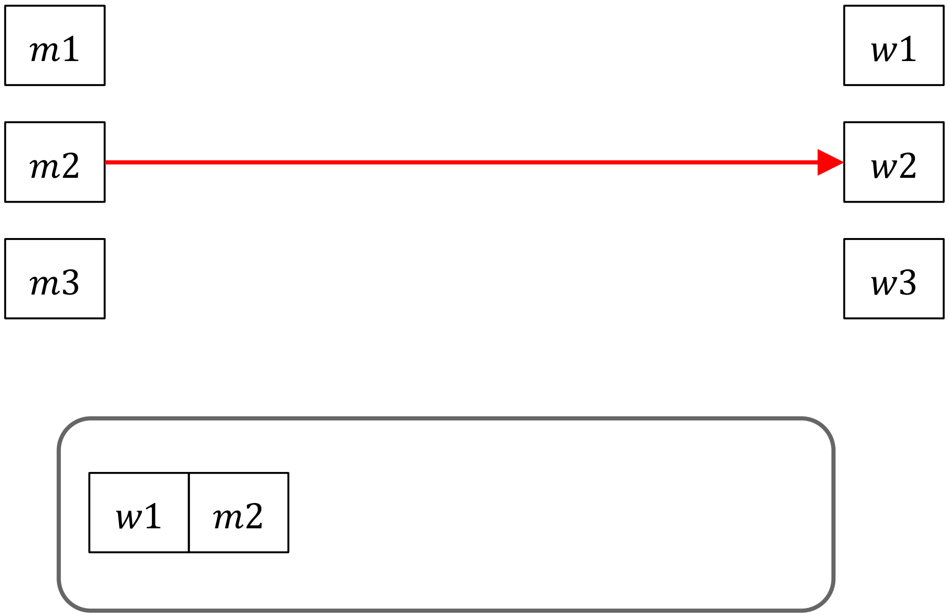
$w1\colon m2 > m1$
$w2\colon m3 > m2$
$w3\colon m3 > m1$



Order: $\boxed{w1}, m2, m1, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

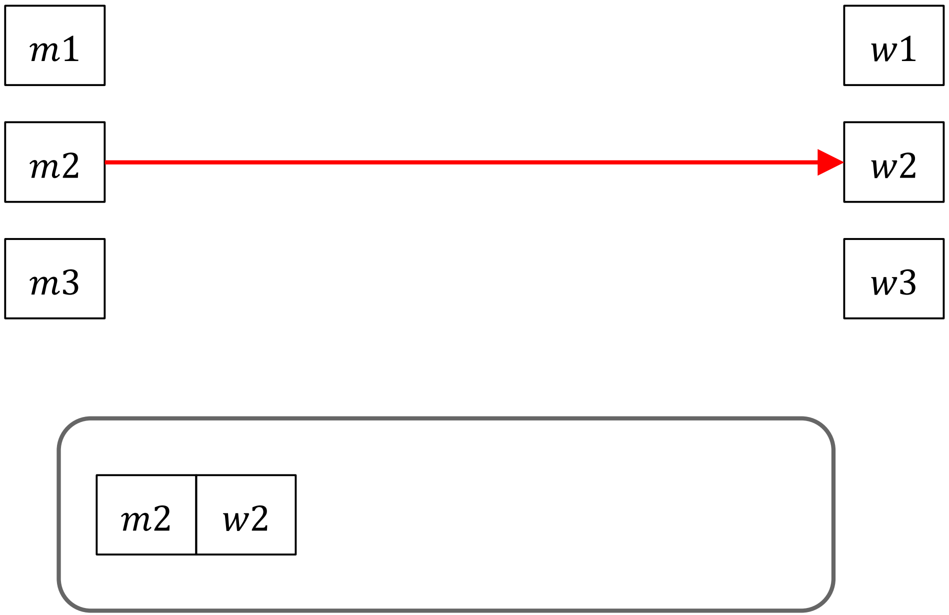


$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, \boxed{m2}, m1, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1\text{: } w3 > w1$
$m2\text{: } w2 > w1$
$m3\text{: } w3 > w2$

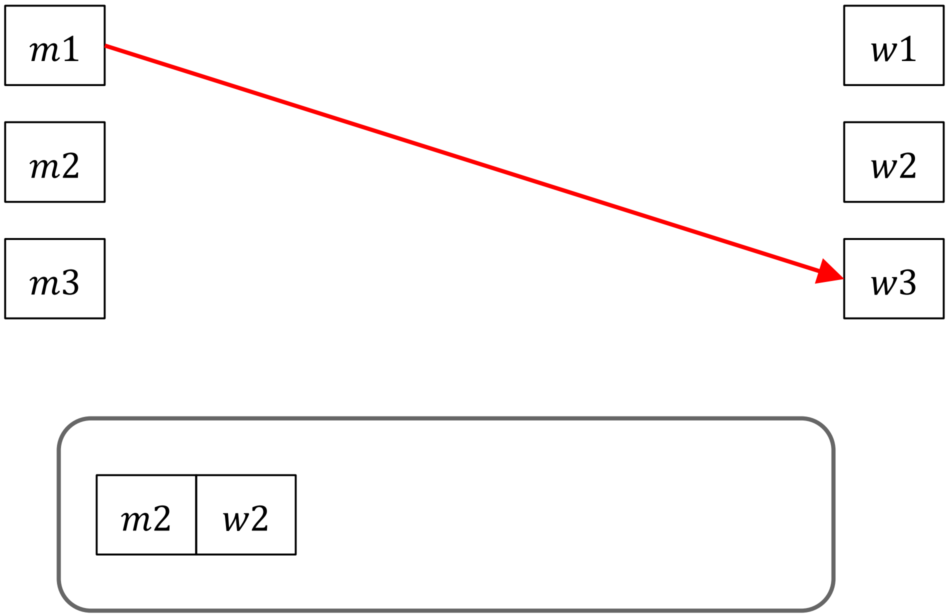


$w1\text{: } m2 > m1$
$w2\text{: } m3 > m2$
$w3\text{: } m3 > m1$

Order: $w1, \boxed{m2}, m1, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

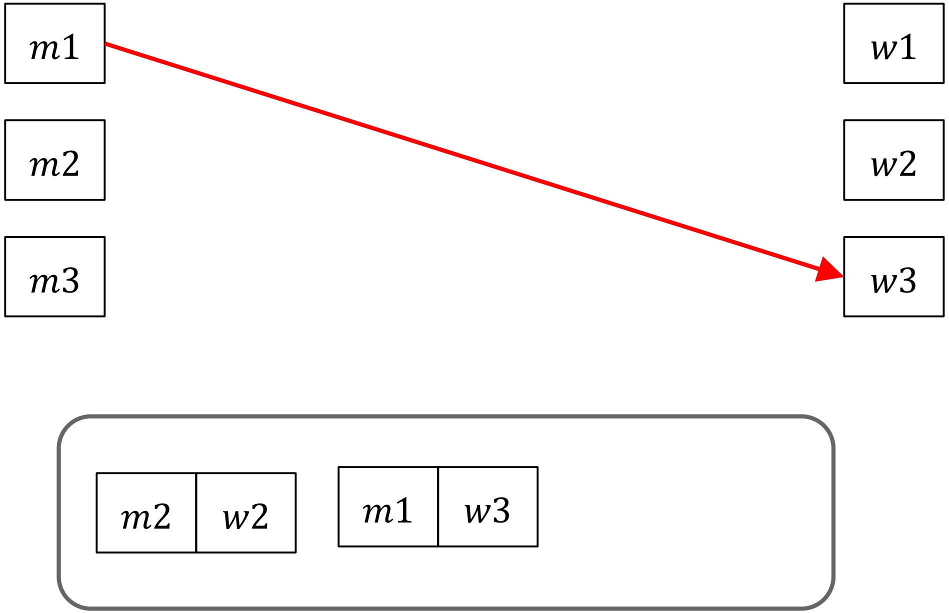


$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, \boxed{m1}, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1\colon w3 > w1$
$m2\colon w2 > w1$
$m3\colon w3 > w2$

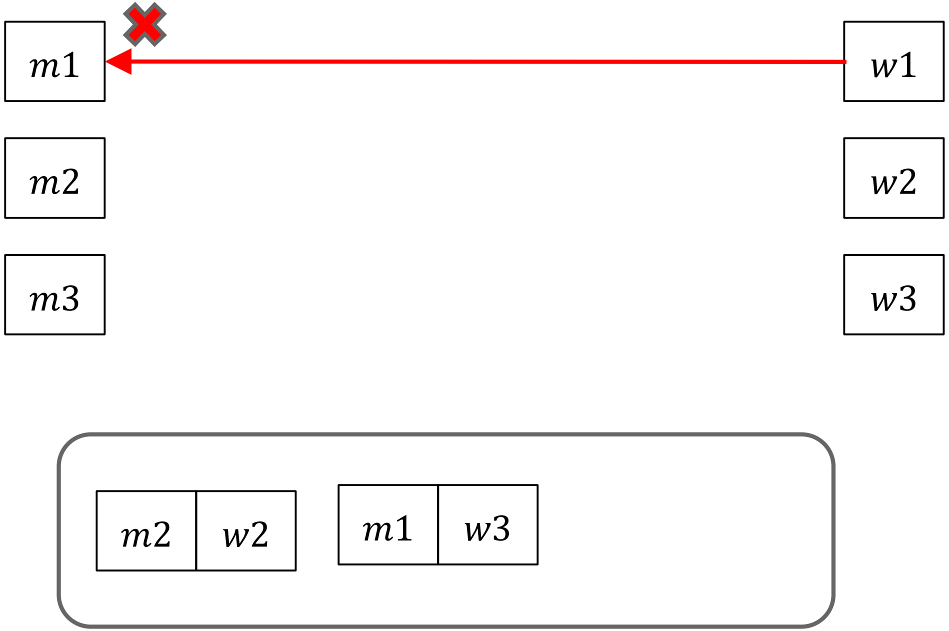


$w1\colon m2 > m1$
$w2\colon m3 > m2$
$w3\colon m3 > m1$

Order: $w1, m2, \boxed{m1}, w1, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$



$m1$ $w1$

$m2$ $w2$

$m3$ $w3$

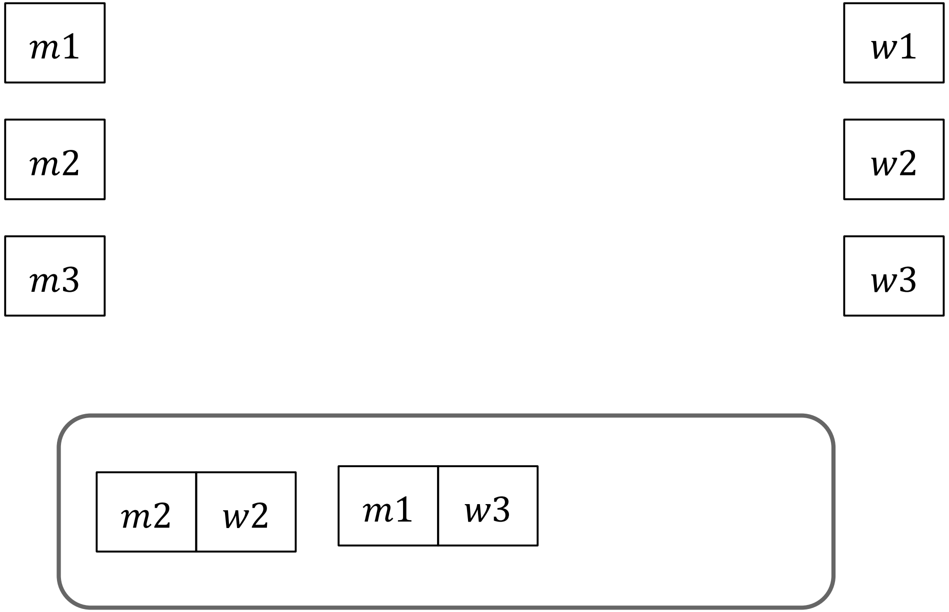
$m2$ $w2$ $m1$ $w3$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, \boxed{w1}, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$


$m1$ $m2$ $m3$

$w1$ $w2$ $w3$

$m2$ $w2$

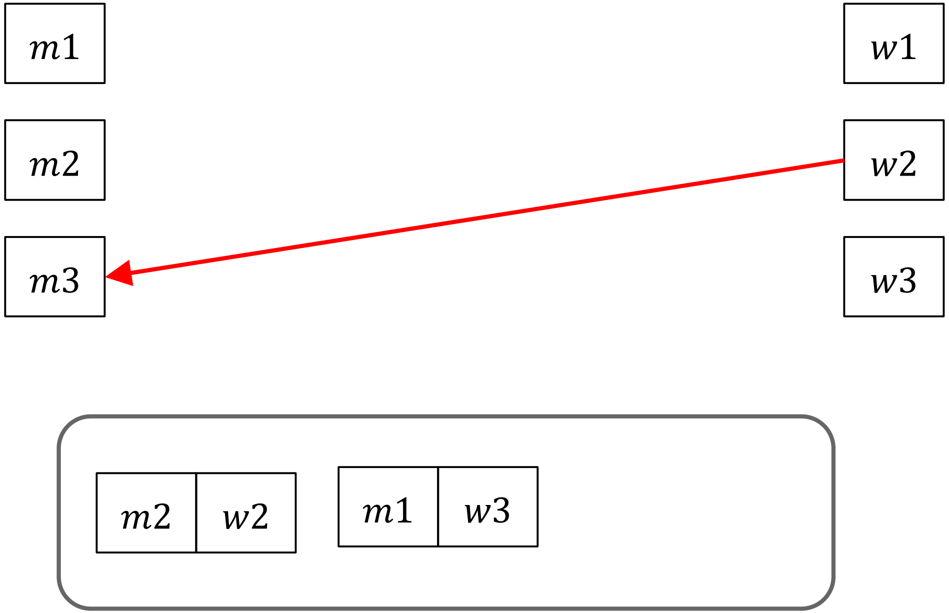
$m1$ $w3$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, \boxed{w1}, w2, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$



$m1$
$m2$
$m3$

$w1$
$w2$
$w3$

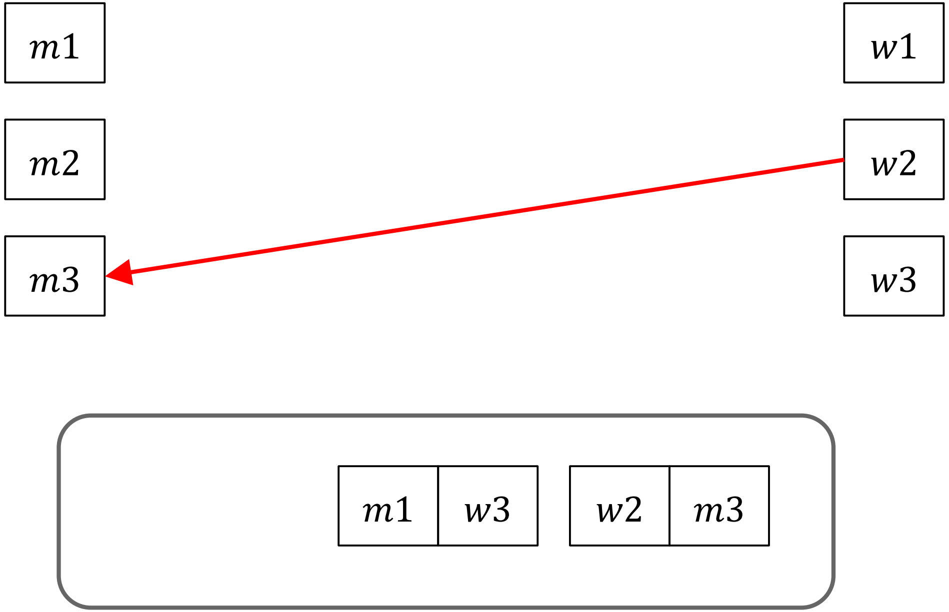
$m2$ $w2$
$m1$ $w3$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, w1, \boxed{w2}, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

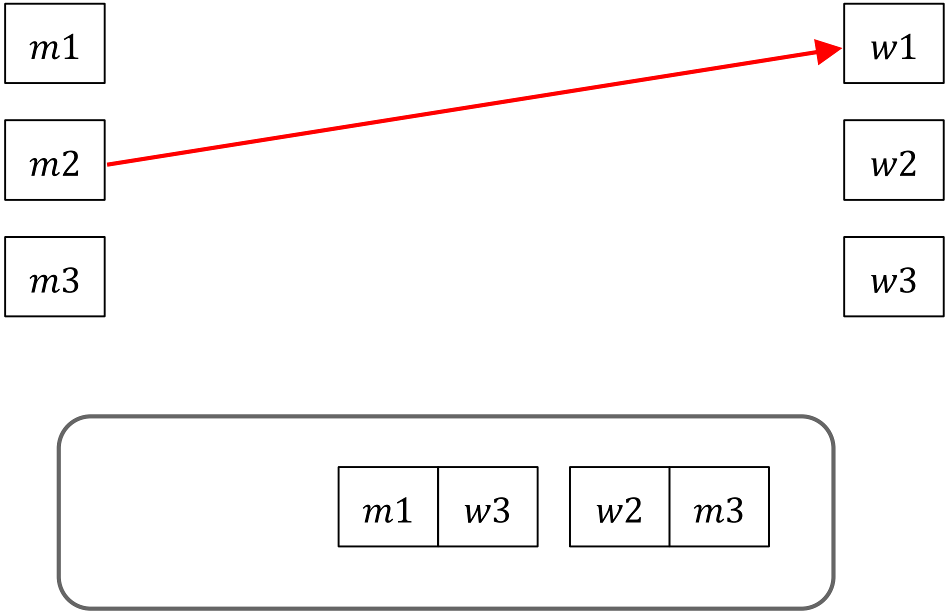


$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, w1, \boxed{w2}, m2, w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

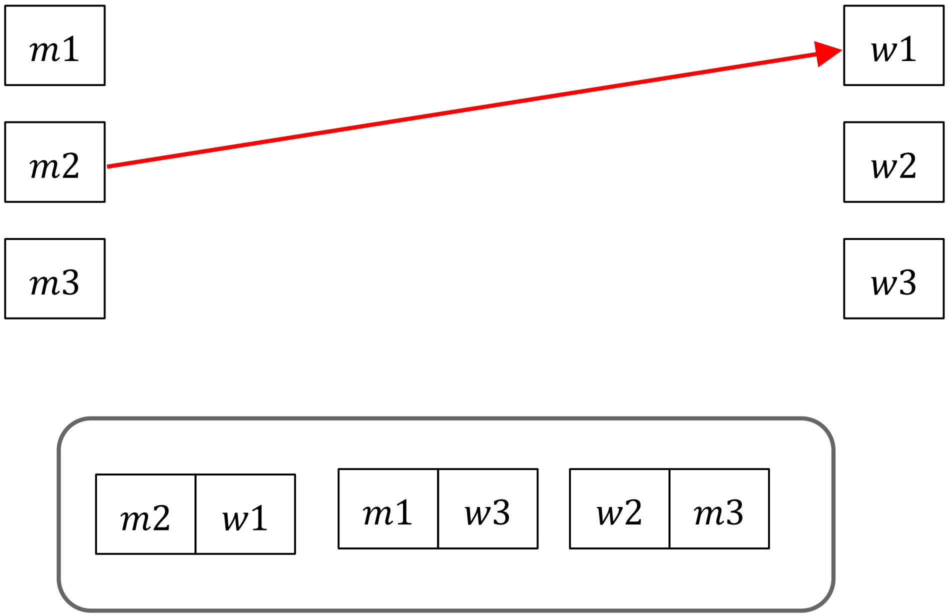


$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

Order: $w1, m2, m1, w1, w2, \boxed{m2,} w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1\colon w3 > w1$
$m2\colon w2 > w1$
$m3\colon w3 > w2$



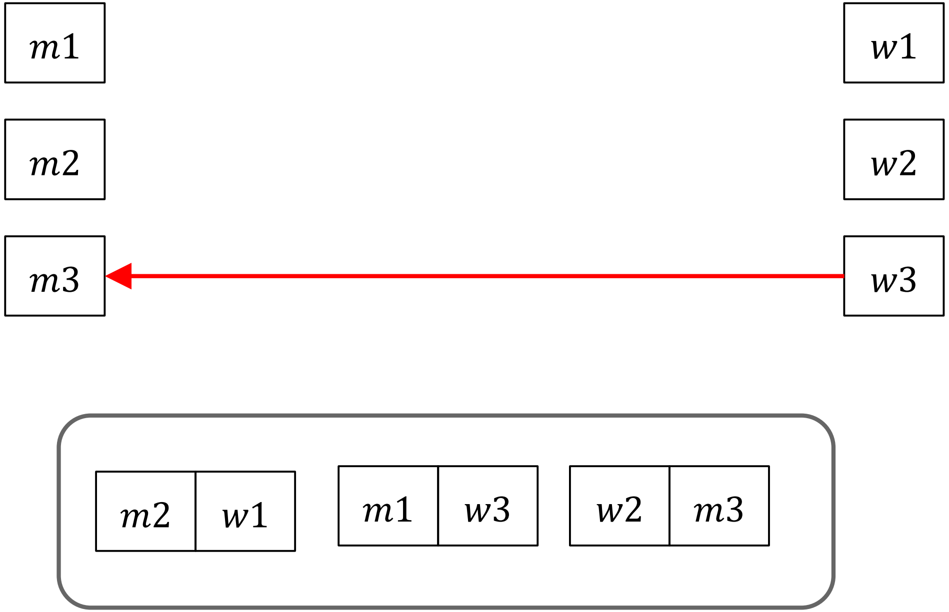
$w1\colon m2 > m1$
$w2\colon m3 > m2$
$w3\colon m3 > m1$

Order: $w1, m2, m1, w1, w2, \boxed{m2,} w3, m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1\colon w3 > w1$
$m2\colon w2 > w1$
$m3\colon w3 > w2$

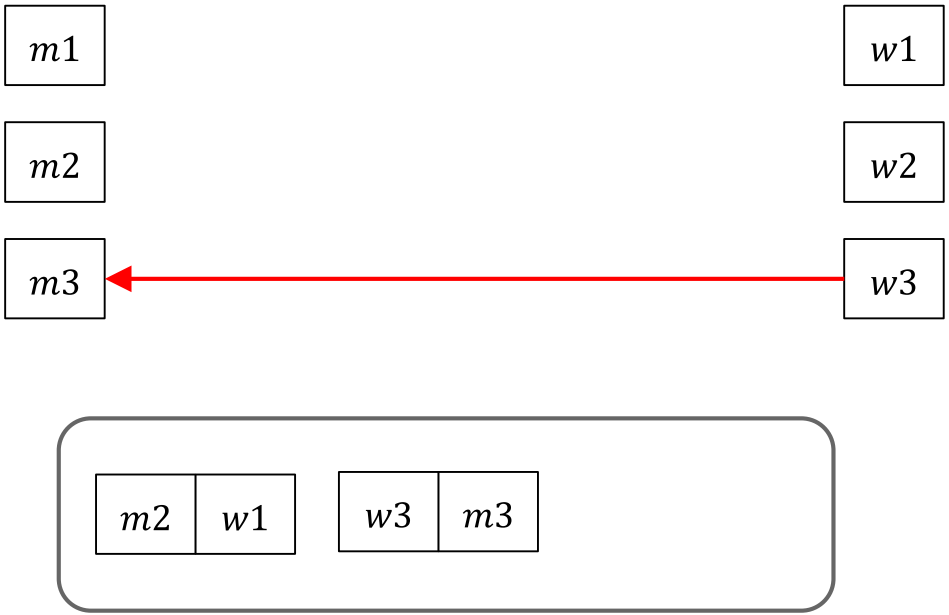
$w1\colon m2 > m1$
$w2\colon m3 > m2$
$w3\colon m3 > m1$



Order: $w1, m2, m1, w1, w2, m2, \boxed{w3,} m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$



$m1$
$m2$
$m3$
$w1$
$w2$
$w3$

$m2$ $w1$
$w3$ $m3$

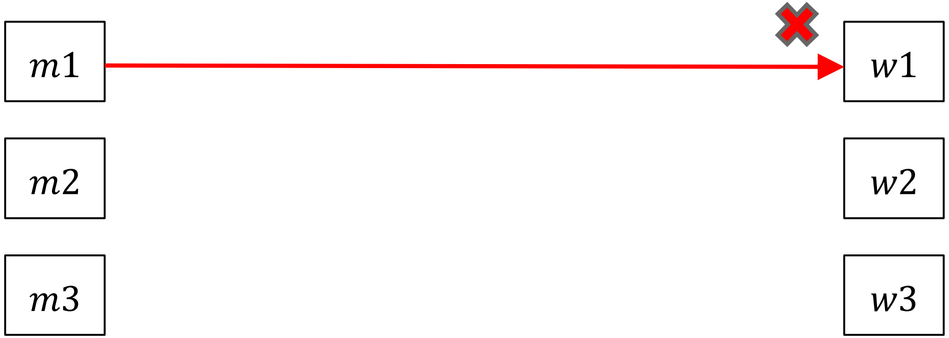
$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

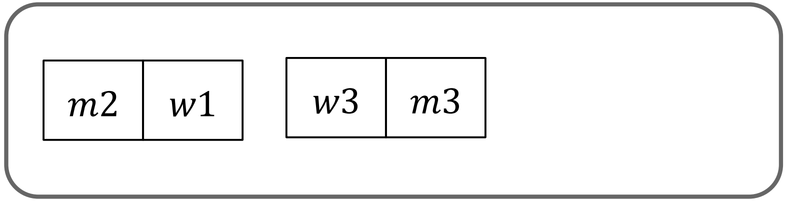
Order: $w1, m2, m1, w1, w2, m2, \boxed{w3,} m1, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1\colon w3 > w1$
$m2\colon w2 > w1$
$m3\colon w3 > w2$

$w1\colon m2 > m1$
$w2\colon m3 > m2$
$w3\colon m3 > m1$



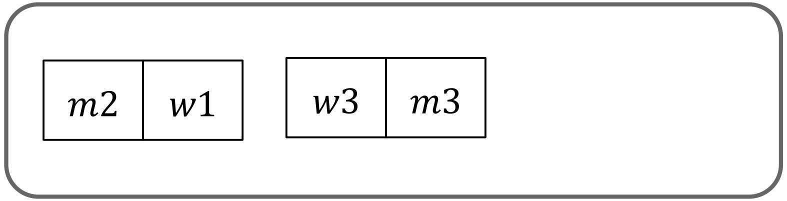


Order: $w1, m2, m1, w1, w2, m2, w3, \boxed{m1}, w2, \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

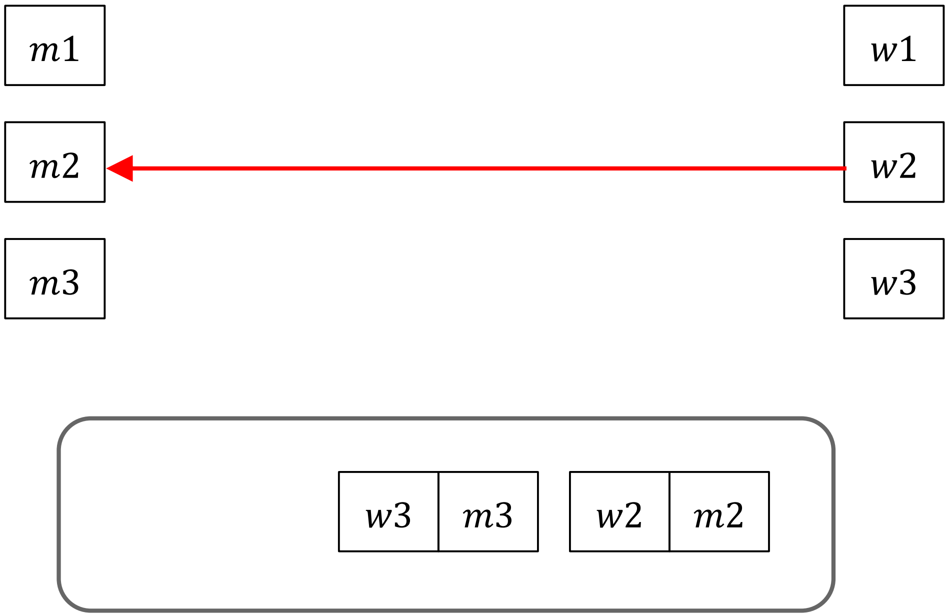
$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$



Order: $w1, m2, m1, w1, w2, m2, w3, m1, \boxed{w2,}$ ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$



$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

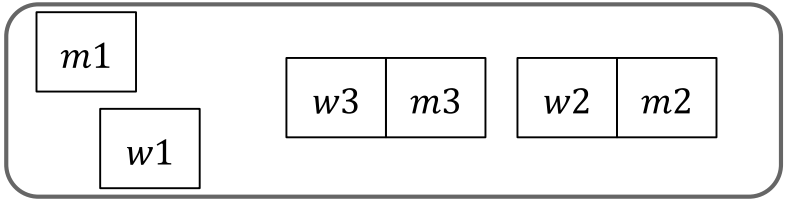
Order: $w1, m2, m1, w1, w2, m2, w3, m1, \boxed{w2,}$ ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w1$
$m2$: $w2 > w1$
$m3$: $w3 > w2$

$w1$: $m2 > m1$
$w2$: $m3 > m2$
$w3$: $m3 > m1$

not stable



Order: $w1, m2, m1, w1, w2, m2, w3, m1, w2,$ ...

Budget-Based Deferred Acceptance

The B2DA algorithm

Budget-Based Deferred Acceptance

The B2DA algorithm

Budget-Based Deferred Acceptance

The B2DA algorithm

Identical to DACC except that we do not run the compensation chains.

Budget-Based Deferred Acceptance

The B2DA algorithm

Identical to DACC except that we do not run the compensation chains.

Still possible to achieve all stable matchings (same proof).

Budget-Based Deferred Acceptance

The B2DA algorithm

Identical to DACC except that we do not run the compensation chains.

Still possible to achieve all stable matchings (same proof).

It may cycle.

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$
$m2$
$m3$

$w1$
$w2$
$w3$

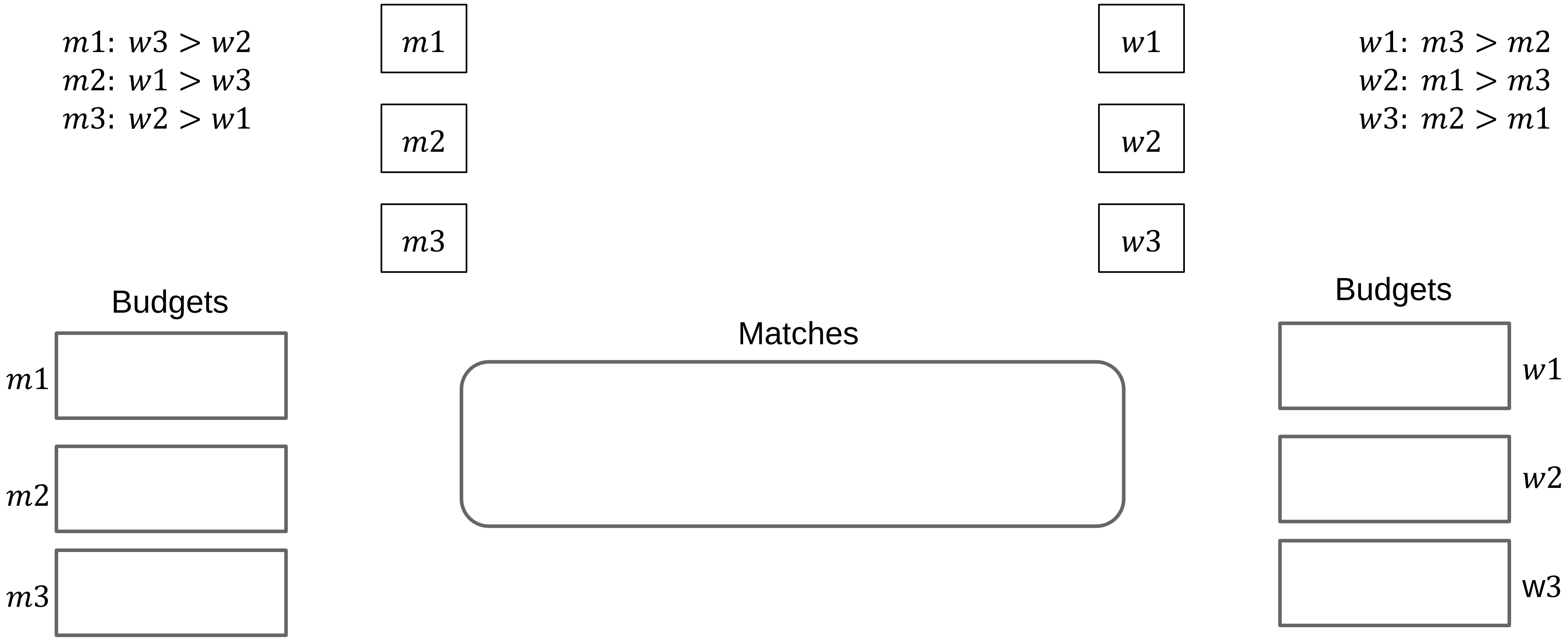
$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Matches

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$



Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$

$m2$

$m3$

$w1$

$w2$

$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $w1, w2, w3$
$m3$ $w1, w2, w3$

Matches

Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, m2, m3$ w3

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$
$m2$
$m3$

$w1$
$w2$
$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $w1, w2, w3$
$m3$ $w1, w2, w3$

Matches

Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, m1, w1), \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $w1, w2, w3$
$m3$ $w1, w2, w3$

Matches

Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $\boxed{w2}, m2, m3, w3, (m3, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$
$m2$
$m3$

$w1$
$w2$
$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, w2, w3$

Matches

$w2$	$m1$

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $\boxed{w2}, m2, m3, w3, (m3, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, w2, w3$

Matches

$w2$	$m1$

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, \boxed{m2}, m3, w3, (m3, w3, m2, w2, m1, w1), \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$

$m2$

$m3$

$w1$

$w2$

$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $w1, w2, w3$
$m3$ $w1, w2, w3$

Matches

$w2$	$m1$
$m2$	$w1$

Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, \boxed{m2}, m3, w3, (m3, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, w2, w3$

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Matches

$w2$	$m1$
$m2$	$w1$

Order: $w2, m2, \boxed{m3}, w3, (m3, w3, m2, w2, m1, w1), \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$w2$	$m1$	$m2$	$w1$

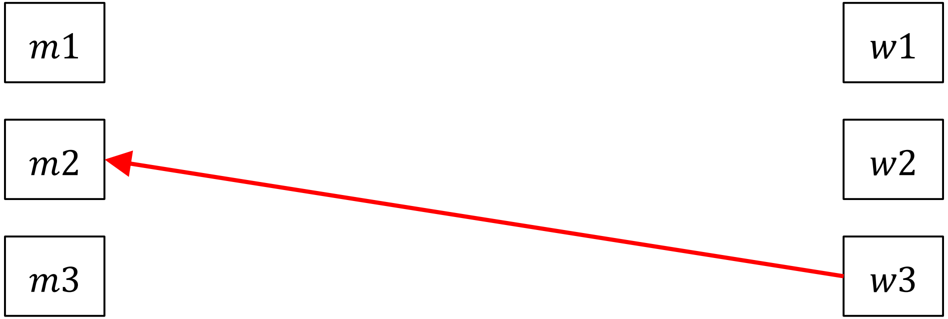
Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, \boxed{m3}, w3, (m3, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$w2$	$m1$	$m2$	$w1$

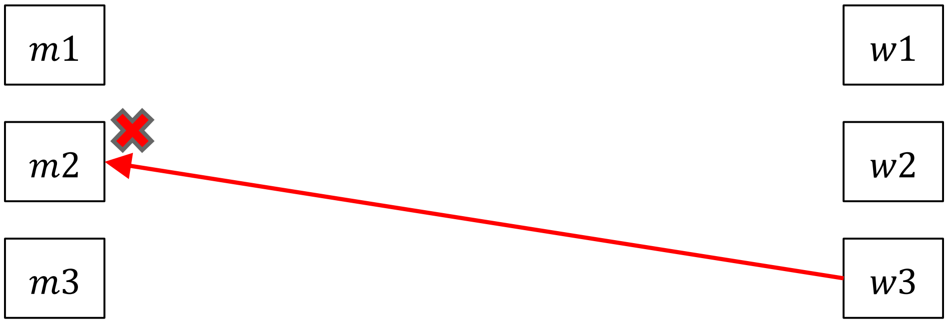
Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, \boxed{w3,}$ $(m3, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$w2$ $m1$ $m2$ $w1$

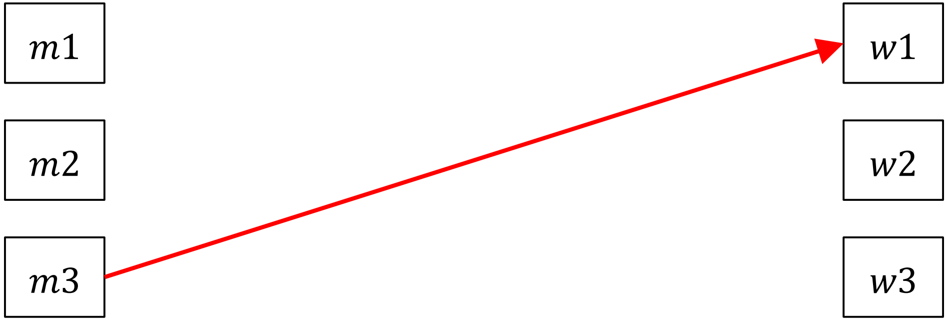
Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, \cancel{m2}, m3$	w3

Order: $w2, m2, m3, \boxed{w3}, (m3, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $w1, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w2$ $m1$
$m2$ $w1$

Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (\boxed{m3}, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$w1, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$w2$	$m1$
$m2$	$w1$
$m3$	$w1$

Budgets

$m1, m2, m3$	$w1$
$m1, m2, m3$	$w2$
$m1, \cancel{m2}, m3$	w3

Order: $w2, m2, m3, w3, (\boxed{m3}, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$\boxed{w2}\boxed{m1}$ $\boxed{m3}\boxed{w1}$

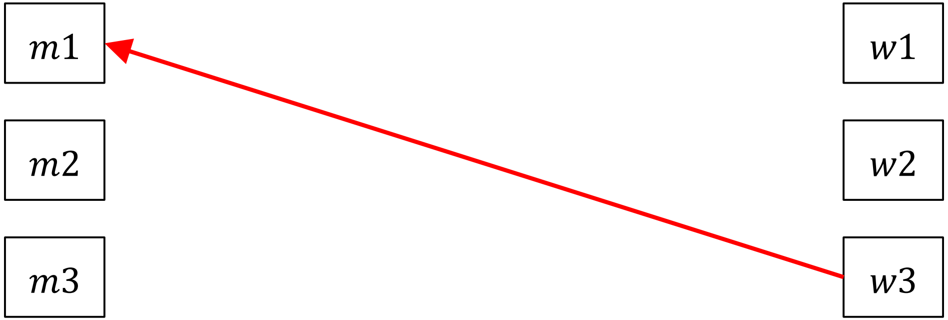
Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (\boxed{m3}, w3, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w2$ $m1$
$m3$ $w1$

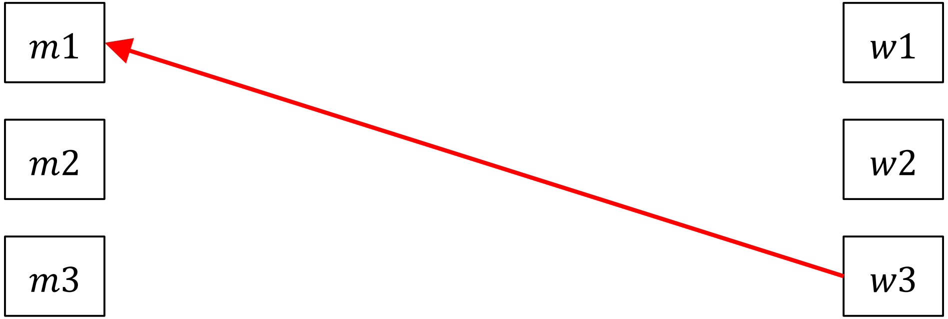
Budgets

$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (m3, \boxed{w3}, m2, w2, m1, w1), \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1: w3 > w2$
$m2: w1 > w3$
$m3: w2 > w1$



$w1: m3 > m2$
$w2: m1 > m3$
$w3: m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w2$ $m1$
$w3$ $m1$
$m3$ $w1$

Budgets

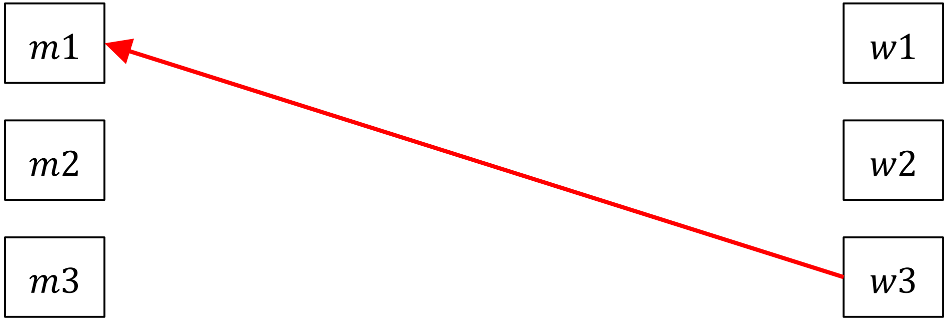
$m1, m2, m3$ $w1$
$m1, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (m3, \boxed{w3}, m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$



Budgets

$m1$	$w1, w2, w3$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Budgets

$m1, m2, m3$	$w1$
$\cancel{m1}, m2, m3$	$w2$
$m1, \cancel{m2}, m3$	w3

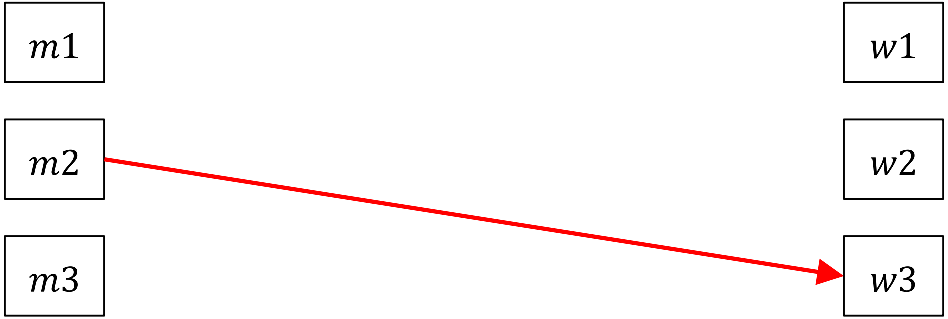
Matches

$w3$	$m1$	$m3$	$w1$

Order: $w2, m2, m3, w3, (m3, \boxed{w3,} m2, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, w3$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w3$ $m1$
$m3$ $w1$

Budgets

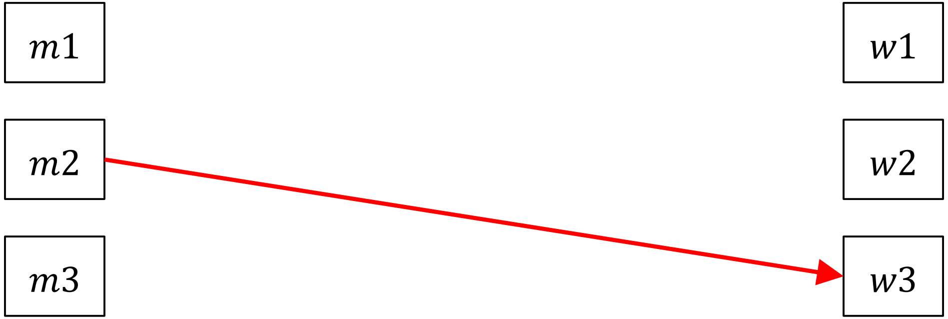
$m1, m2, m3$ $w1$
$\cancel{m1}, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, \boxed{m2}, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$



Budgets

$m1$ $w1, w2, w3$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w3$ $m1$
$m3$ $w1$

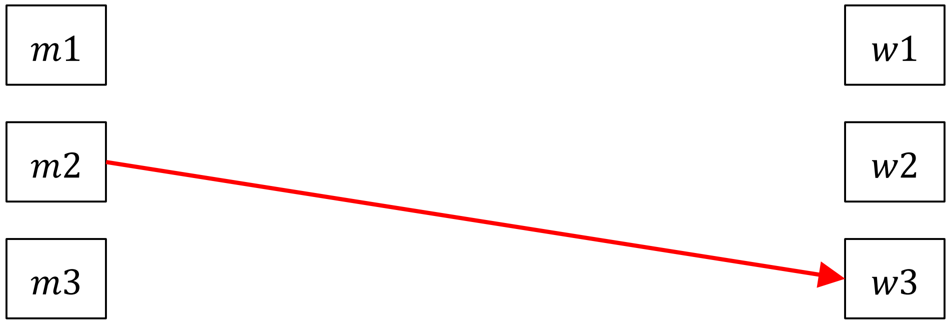
Budgets

$m1, m2, m3$ $w1$
$\cancel{m1}, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, \boxed{m2}, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, w3$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$m2$	$w3$
$w3$	$m1$
$m3$	$w1$

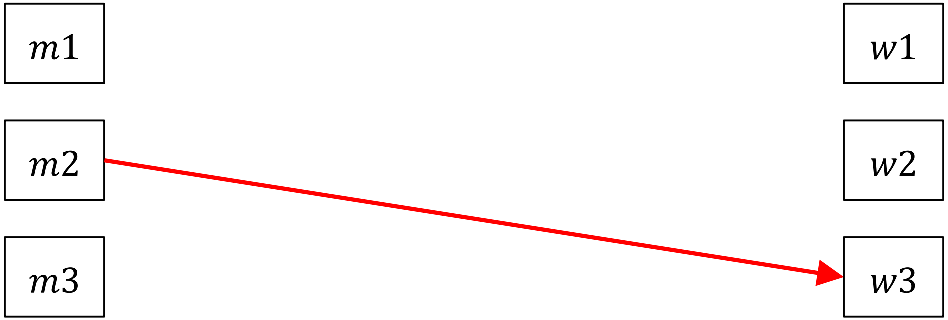
Budgets

$m1, m2, m3$	$w1$
$\cancel{m1}, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, \boxed{m2}, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$



$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, \cancel{w3}$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$m2$ $w3$
$m3$ $w1$

Budgets

$m1, m2, m3$ $w1$
$\cancel{m1}, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, \boxed{m2}, w2, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, \cancel{w3}$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$m2$	$w3$
$m3$	$w1$

Budgets

$m1, m2, m3$	$w1$
$\cancel{m1}, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, m2, \boxed{w2,} m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, \cancel{w3}$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, w2, w3$

Matches

$m2$	$w3$
$m3$	$w1$

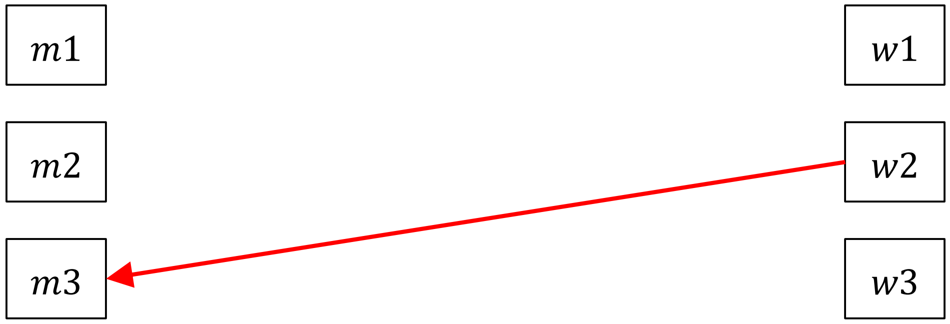
Budgets

$m1, m2, m3$	$w1$
$\cancel{m1}, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, m2, \boxed{w2}, m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1\text{: } w3 > w2$
$m2\text{: } w1 > w3$
$m3\text{: } w2 > w1$



$w1\text{: } m3 > m2$
$w2\text{: } m1 > m3$
$w3\text{: } m2 > m1$

Budgets

$m1$	$w1, w2, \cancel{w3}$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, w2, w3$

Matches

$m2$	$w3$	$w2$	$m3$	$m3$	$w1$

Budgets

$m1, m2, m3$	$w1$
$\cancel{m1}, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, m2, \boxed{w2,} m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$: $w1, w2, \cancel{w3}$
$m2$: $\cancel{w1}, w2, w3$
$m3$: $w1, w2, w3$

Matches

$m2$ $w3$

$w2$ $m3$

Budgets

$m1, m2, \cancel{m3}$ $w1$
$\cancel{m1}, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, m2, \boxed{w2,} m1, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, \cancel{w3}$
$m2$ $\cancel{w1}, w2, w3$
$m3$ $w1, w2, w3$

Matches

$m2$ $w3$
$w2$ $m3$

Budgets

$m1, m2, \cancel{m3}$ $w1$
$\cancel{m1}, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, \boxed{m1}, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, \cancel{w3}$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, w2, w3$

Matches

$m2$	$w3$	$w2$	$m3$

Budgets

$m1, m2, \cancel{m3}$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, \boxed{m1}, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, \cancel{w3}$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, w2, w3$

Matches

$m2$	$w3$	$w2$	$m3$	$m1$	$w2$

Budgets

$m1, m2, \cancel{m3}$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, \boxed{m1}, w1), \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$	$w1, w2, \cancel{w3}$
$m2$	$\cancel{w1}, w2, w3$
$m3$	$w1, \cancel{w2}, w3$

Matches

$m2$ $w3$

$m1$ $w2$

Budgets

$m1, m2, \cancel{m3}$	$w1$
$m1, m2, m3$	$w2$
$m1, m2, m3$	w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, \boxed{m1}, w1)$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$: $w1, w2, \cancel{w3}$
$m2$: $\cancel{w1}, w2, w3$
$m3$: $w1, \cancel{w2}, w3$

Matches

$m2$	$w3$
$m1$	$w2$

Budgets

$m1, m2, \cancel{m3}$ $w1$
$m1, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, m1, \boxed{w1}), \ldots$

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$: $w1, w2, \cancel{w3}$
$m2$: $w1, w2, w3$
$m3$: $w1, \cancel{w2}, w3$

Matches

$m2$ | $w3$
$m1$ | $w2$

Budgets

$w1$: $m1, m2, \cancel{m3}$
$w2$: $m1, m2, m3$
w3: $m1, m2, m3$

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, m1, \boxed{w1})$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, \cancel{w3}$
$m2$ $w1, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$m2$	$w3$
$w1$	$m2$
$m1$	$w2$

Budgets

$m1, m2, \cancel{m3}$ $w1$
$m1, m2, m3$ $w2$
$m1, m2, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, m1, \boxed{w1})$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, \cancel{w3}$
$m2$ $w1, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w1$ $m2$
$m1$ $w2$

Budgets

$m1, m2, \cancel{m3}$ $w1$
$m1, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (m3, w3, m2, w2, m1, \boxed{w1})$, ...

Two-sided Deferred Acceptance - failure of stability

$m1$: $w3 > w2$
$m2$: $w1 > w3$
$m3$: $w2 > w1$

$m1$
$m2$
$m3$

$w1$
$w2$
$w3$

$w1$: $m3 > m2$
$w2$: $m1 > m3$
$w3$: $m2 > m1$

Budgets

$m1$ $w1, w2, \cancel{w3}$
$m2$ $w1, w2, w3$
$m3$ $w1, \cancel{w2}, w3$

Matches

$w1$ $m2$
$m1$ $w2$

Budgets

$m1, m2, \cancel{m3}$ $w1$
$m1, m2, m3$ $w2$
$m1, \cancel{m2}, m3$ w3

Order: $w2, m2, m3, w3, (\boxed{m3}, w3, m2, w2, m1, w1)$, ...