Bargaining and News

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Motivation

A central issue in the bargaining literature

- Will trade be (inefficiently) delayed?

What is usually ignored

- If trade is in fact delayed, new information may come to light...

This paper = Bargaining + News

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Application 1: Catered Innovation

Consider a startup that has "catered" its innovation to Google

- This exit strategy has become increasingly common (Wang, 2015)
  - Alphabet alone has made over 200 acquisition
  - Nest, Waze, Android, Picasa, YouTube, DropCam
- The longer the startup operates independently, the more Google will learn about the value of the innovation
- But delaying the acquisition is inefficient because Google can leverage economies of scale

Questions

- How does Google's ability to delay acquisition and acquire more information affect its bargaining power?
- How does the exit strategy affect incentives for innovation?

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Application 2: Due Diligence

"Large" transactions typically involve a due diligence period:

- Corporate acquisitions
- Commercial real estate transactions

This information gathering stage is inherently dynamic.

- e.g., Verizon's acquisition of Yahoo

Questions: How does the acquirer's ability to conduct due diligence and renegotiate the terms

- Initial terms of sale? Eventual terms of sale?
- Profitability of acquisition? Likelihood of deal completion?

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A canonical setting

- An indivisible asset (e.g., firm, project, security)
  - Asset value is privately known by one player

- One informed player (seller), one uninformed player (buyer)
  - The uninformed player makes price offers
  - Common knowledge of gains from trade
  - Efficient outcome: trade immediately

- Infinite horizon; discounting; no commitment

- News: information about the asset is gradually revealed

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Preview of Results

- The buyer's ability to extract more surplus is remarkably limited.
  - A negotiation takes place and yet the buyer gains nothing from it.
  - Coasian force overwhelms buyer's access to information.

- Buyer engages in a form of costly experimentation
  - Makes offers that are sure to lose money if accepted, but generate information if rejected
  - Seller benefits from buyer's incentive to experiment

- Introducing competition can lead to worse outcomes.
  - Under certain conditions, seller's payoff is higher and/or the outcome is more efficient with a single buyer than with competing ones.

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Setup: Players and Values

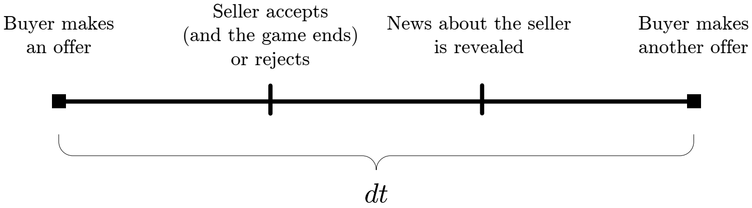
Players: seller and buyer

- Seller owns asset of type $\theta \in \{L, H\}$
- θ is the seller's private information

Values:

- Seller's reservation value is K_θ, where $K_H > K_L = 0$
- Buyer's value is V_θ, where $V_H \geq V_L$
- Common knowledge of gains from trade: $V_\theta > K_\theta$
- "Lemons" condition: $K_H > V_L$

Setup: Timing and Payoffs



- Both players are risk neutral and discount at rate r

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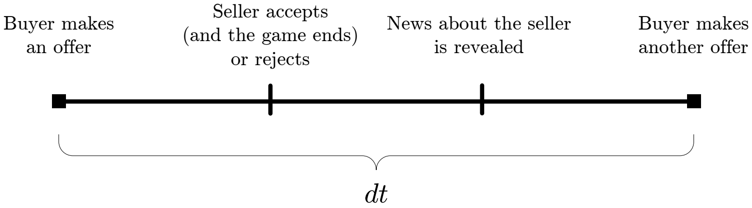

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Complete Information Outcome

Suppose θ is public information.

- The buyer has all the bargaining power.
- The buyer extracts all the surplus.
- Offers K_θ at $t = 0$ and the seller accepts.
- Payoffs:

$$\text{Buyer payoff} = V_\theta - K_\theta$$
$$\text{Seller payoff} = 0$$

Clearly, knowing θ is beneficial to the buyer.

- What happens if buyer only learns about θ gradually?

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Setup: News

- Represented by a publicly observable process:

$$X_t(\omega) = \mu_\theta t + \sigma B_t(\omega)$$

  where B is standard B.M. and without loss $\mu_H > \mu_L$

- The quality of the news is captured by the signal-to-noise ratio:

$$\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$$

Equilibrium objects

1. Offer process, $W = \{W_t : 0 \leq t \leq \infty\}$
2. Seller stopping times: τ^θ for each $\theta \in \{L, H\}$
   - Allow for seller mixing
   - Let $S_t^\theta = P(\tau^\theta \leq t|\text{buyer's information})$
3. Buyer's belief process, $Z = \{Z_t : 0 \leq t \leq \infty\}$

We look for equilibria that are stationary in the buyer's beliefs:

- Z is a time-homogenous Markov process
- Offer is a function that depends only on the state, $W_t = w(Z_t)$

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Buyer's beliefs

Buyer starts with a prior $P_0 = \Pr(\theta = H)$

- At time t, buyer conditions on
  - (i) the path of the news,
  - (ii) seller rejected all past offers

- Using Bayes Rule, the buyer's belief at time t is

$$P_t = \frac{P_0 f_t^H(X_t)(1 - S_{t-}^H)}{P_0 f_t^H(X_t)(1 - S_{t-}^H) + (1 - P_0) f_t^L(X_t)(1 - S_{t-}^L)}$$

- Define $Z \equiv \ln\left(\frac{P_t}{1-P_t}\right)$, we get that

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Seller's problem

Seller's Problem

Given (w, Z), the seller faces a stopping problem

$$\sup_{\tau} E_z^{\theta} \left[e^{-r\tau} \left(w \left(Z_{\tau} \right) - K_{\theta} \right) \right]$$

Let $F_{\theta}(z)$ denote the solution.

Buyer's problem

In any state z, the buyer has essentially three options:

1. Wait: Make a non-serious offer that is rejected w.p.1.
2. Screen: Make an offer $w < K_H$ that only the low type accepts with positive probability
3. Buy/Stop: Offer $w = K_H$ and buy regardless of θ

Let $F_B(z)$ denote the buyer's value function.

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Details

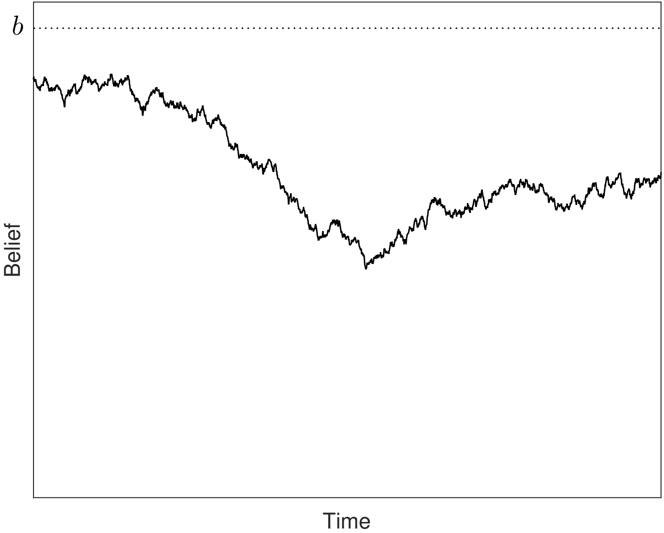
Equilibrium Characterization

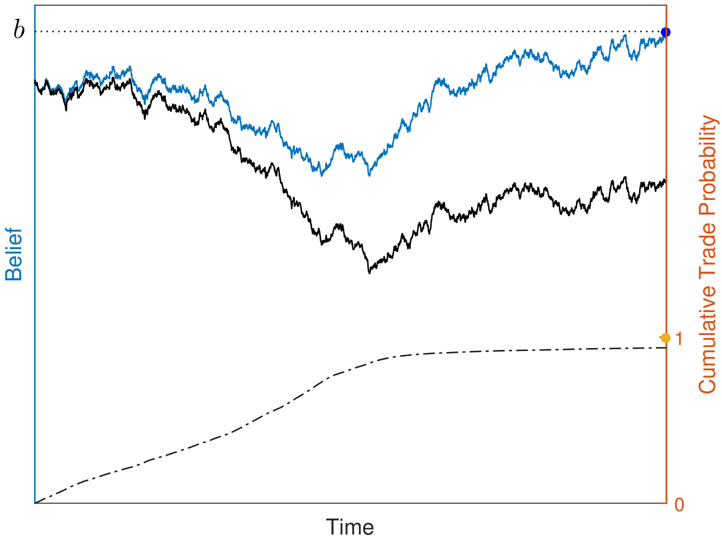
Theorem

There exists a unique equilibrium. In it,

- *For $P_t \geq b$, trade happens immediately: buyer offers K_H and both type sellers accept.*
- *For $P_t < b$, trade happens "smoothly": only the low-type seller trades and with probability that is proportional to dt.*
  - *i.e., $dQ_t = \dot{q}(Z_t)dt$*

Equilibrium: sample path




b
Belief
Cumulative Trade Probability
1
0
Time

Equilibrium construction

Conjecture the equilibrium is "smooth"

1. Buyer's problem is linear in the rate of trade: $\dot{q}$
   - Derive F_B (independent of F_L)
2. Given F_B, what must be true about F_L for smooth trade to be optimal?
   - Derive F_L, which implies w
3. Low type must be indifferent between waiting and accepting
   - Indifference condition implies $\dot{q}$

Summary: Smooth $\implies F_B \implies F_L \implies \dot{q}$

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A bit more about Step 1

$$rF_B(z) = \underbrace{\frac{\phi^2}{2}\left(2p(z)-1\right)F_B'(z) + \frac{\phi^2}{2}F_B''(z)}_{\text{Evolution due to news}}$$

$$+ \dot{q}(z)\underbrace{\Big((1-p(z))\big(V_L - F_L(z) - F_B(z)\big) + F_B'(z)\Big)}_{\Gamma(z)=\text{net-benefit of screening at } z}$$

- Buyer's value is linear in $\dot{q}$
- For "smooth" trade to be optimal, it must be that $\Gamma(z) = 0$
  $\Rightarrow$ F_B is independent of $\dot{q}$ and evolves as if $\dot{q} = 0$
- Therefore, buyer does not benefit from screening!
  $\Rightarrow$ Pins down exactly how expensive it must be to buy L, i.e., $F_L(z)$

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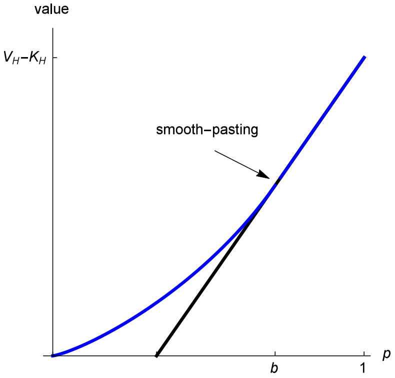
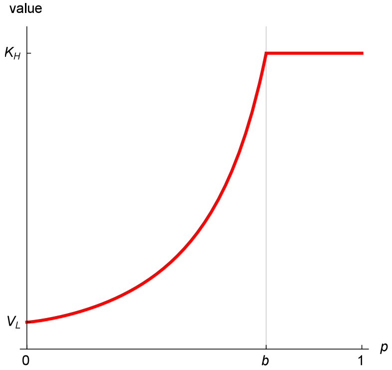
Equilibrium payoffs

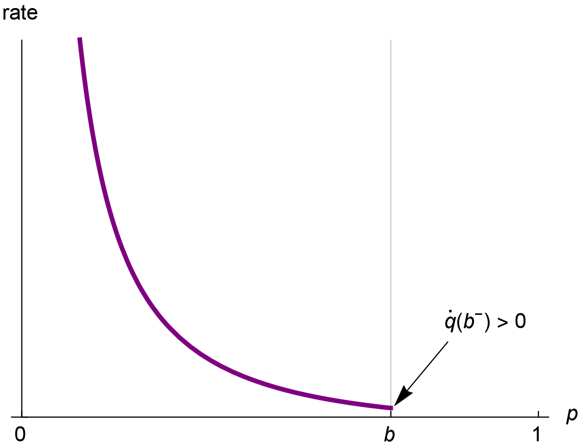


Step 1: Buyer value, F_B



Step 2: Low-type value, F_L

Equilibrium rate of trade



Step 3: Rate of trade, $\dot{q}$

Interesting Predictions?

1. Buyer does not benefit from the ability to negotiate the price.
   - Though she *must* negotiate in equilibrium.

2. The buyer is guaranteed to lose money on any offer below K_H that is accepted.
   - A form of costly experimentation.
   - Seller benefits from experimentation.

3. Incentive for experimentation eliminated by competition among buyers.
   - Competition may be both less efficient and worse for the seller.

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Who Benefits from the Negotiation?

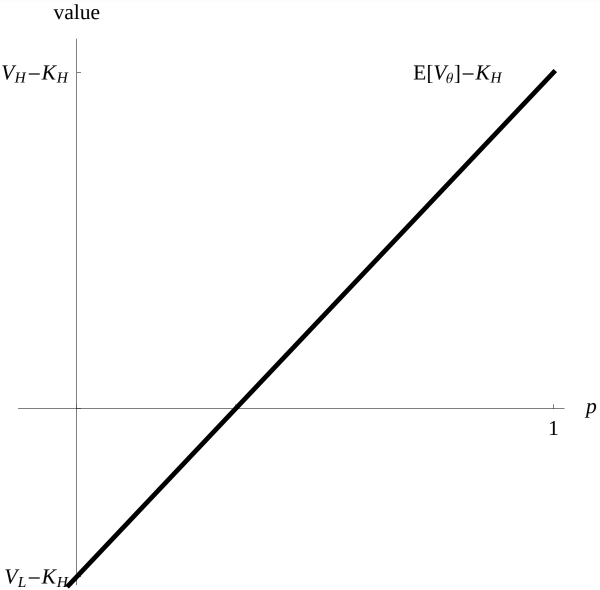
Suppose the price is exogenously fixed at K_H.

- The buyer can conduct due diligence (observes $\hat{Z}$) and decides when and whether to actually complete the deal.
- Buyer's strategy is simply a stopping rule, where the expected payoff upon stopping in state z is

$$E_z[V_\theta] - K_H$$

- Call this the due diligence game.
  - NB: it is not hard to endogenize the initial terms.

Due Diligence Game



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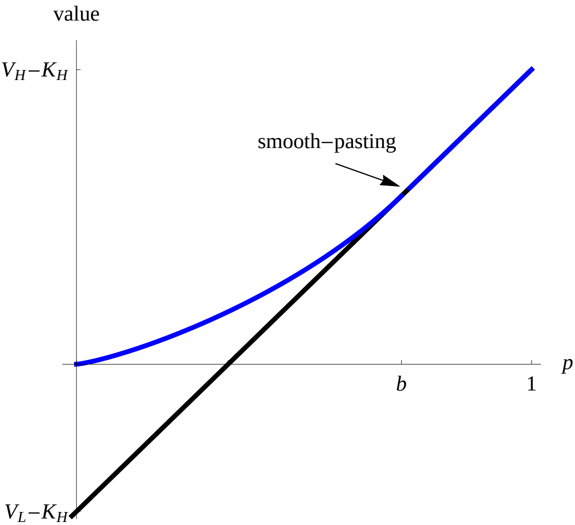


Who Benefits from the Negotiation?

Result

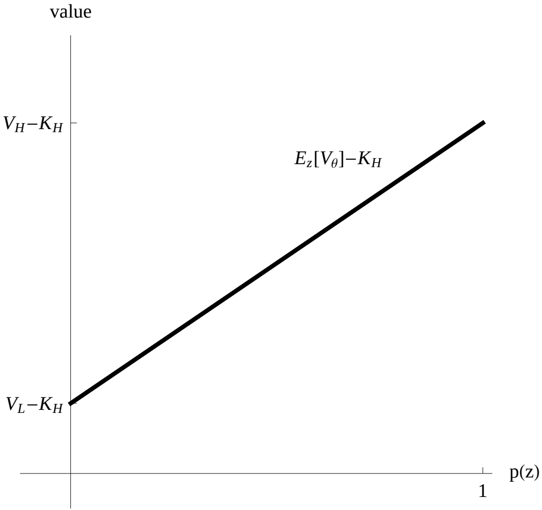
In the equilibrium of the bargaining game:

1. The buyer's payoff is identical to the due diligence game.
2. The (L-type) seller's payoff is higher than in the due diligence game.

Total surplus higher with bargaining, but fully captured by seller.

- Despite the fact that the buyer makes all the offers.

No Lemons $\implies$ No Learning



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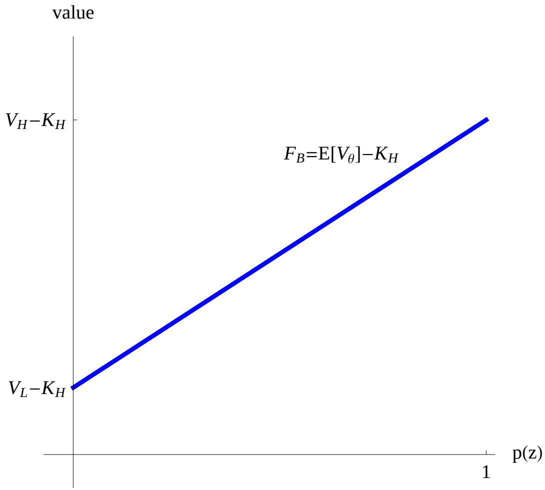

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Result

When $V_L \geq K_H$, unique equilibrium is immediate trade at price K_H.

- Absent a lemons condition, the Coasian force overwhelms the buyer's incentive to learn.

Experimentation and regret

Below b, the buyer is making an offer that:

(1) will ONLY be accepted by the low type

(2) will make a loss whenever accepted

Why?

- One interpretation: costly experimentation
- Buyer willing to lose money today (if offer accepted) in order to learn *faster* (if rejected)
- Both news and lack of competition necessary for this feature to arise

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Remarks

- One implication is that acquisitions that take place at a price below the initial terms add less value for the acquirer.
  - In fact, they necessarily lose value for the acquirer.
  - A downward renegotiation of the acquisition price should negatively affect acquirer's share price.
  - E.g., when Verizon announced the Yahoo merger is going through but at a price $300M below the original bid.
- Competition among buyers reduces the incentive to experiment.
  - Let's explore the effect of competition in a bit more detail.

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Competition and the Coase Conjecture

The buyer's desire to capture future profits from trade leads to a form of intertemporal competition.

- Seller knows buyer will be tempted to increase price tomorrow
- Which increases the price seller is willing to accept today
- Buyer "competes" against future self

Coase Conjecture: Absent some form of commitment, the outcome with a monopolistic buyer will resemble the competitive outcome.

Question: How does learning/news affect the Coase conjecture?

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Competitive equilibrium

Remarks

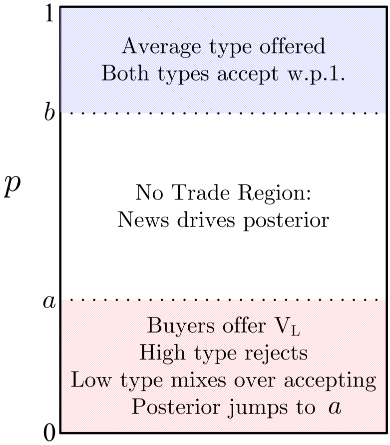
1. Competitive equilibrium $\neq$ Monopolistic equilibrium
2. Buyer competition eliminates incentive for experimentation!

Competitive equilibrium

Theorem (Daley and Green, 2012)

There is a unique equilibrium satisfying a mild refinement on off-path beliefs. In it,

- *For $P_t \geq b$: trade happens immediately, buyers offer $V(P_t)$ and both type sellers accept*
- *For $P_t < a$: buyers offer V_L, high types reject w.p.1. Low types mix such that the posterior jumps to a*
- *For $P_t \in (a, b)$: there is no trade, buyers make non-serious offers which are rejected by both types.*



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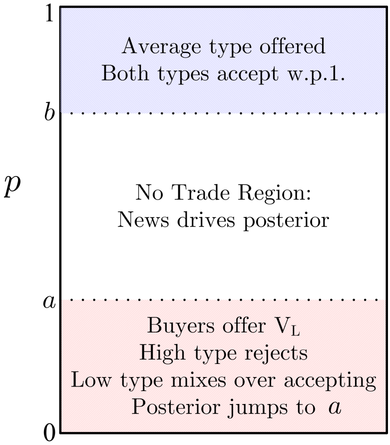

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Effect of competition

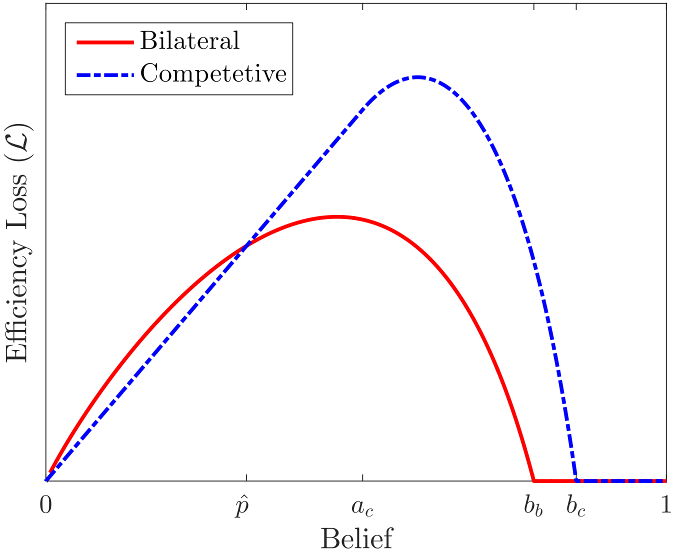
Result

- Efficient trade requires a higher belief in a competitive market:

$$b_b < b_c$$

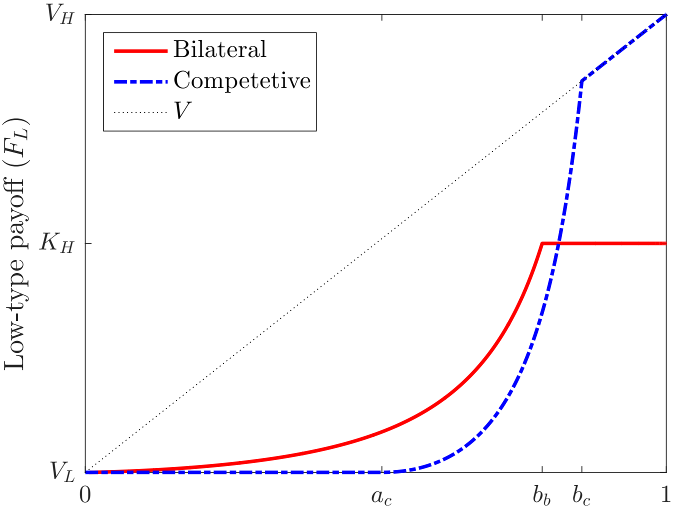
- There exists a $\hat{p}$ such that the competitive equilibrium is strictly less efficient for $p \in (\hat{p}, b_c)$.

Efficiency

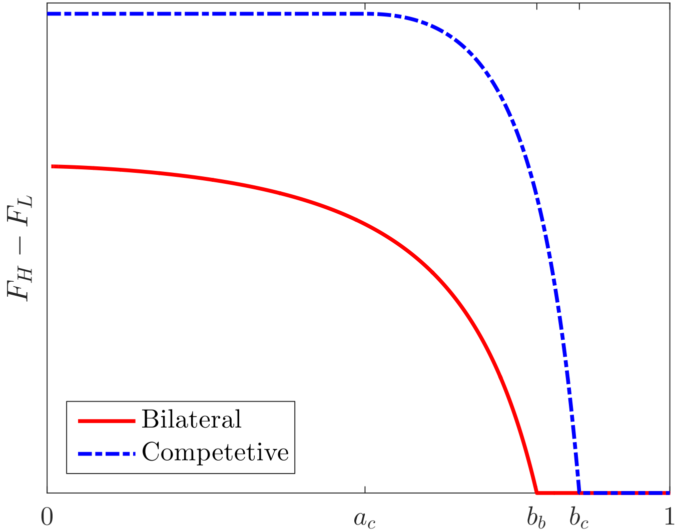


Bilateral
Competetive
Efficiency Loss ($\mathcal{L}$)
0
$\hat{p}$
a_c
b_b
b_c
1
Belief

Low-type value



Incentives for Innovation



Additional Results

- Uniqueness
  - Why trade must be smooth below β with a single buyer

- The effect of news quality
  - The no-news limit differs from Deneckere and Liang (2006)

- Extensions/Robustness
  1. Costly investigation
  2. "Lumpy" information arrival

  Robust finding: buyer does not benefit from ability to negotiate.
  - Solve analogous due diligence game first ($F_B \implies F_L \implies \dot{q}$)
  - Useful heuristic for constructing equilibria with frequent offers

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Summary

We explore the effect of news in a canonical bargaining environment

- Construct the equilibrium (in closed form).
- Buyer's ability to leverage news to extract surplus is remarkably limited.
  - Buyer negotiates based on new information in equilibrium, but gains nothing from doing so.
  - The robust implication of the Coasian force
- Relation to the competitive outcome
  - Competition eliminates the Coasian force, may reduce both total surplus and seller payoff.
  - But competition also provides stronger incentives for innovation.

Other equilibria?

We focused on the (unique) smooth equilibrium. Can other stationary equilibria exist?

- No

By Lesbegue's decomposition theorem for monotonic functions

$$Q = Q_{abs} + Q_{jump} + Q_{sing}$$

To sketch the argument, we will illustrate how to rule out:

1. Atoms of trade with L (i.e., jumps)
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Uniqueness

Suppose there is some z_0 such that:

- Buyer makes offer w_0
- Low type accepts with atom

Let α denote the buyer's belief conditional on a rejection. Then

1. $F_L(z_0) = F_L(\alpha) = w_0$, by seller optimality
2. $F_L(z) = w_0$ for all $z \in (z_0, \alpha)$, by buyer optimality

Therefore, starting from any $z \in (z_0, \alpha)$, the belief conditional on a rejection jumps to α.

- If there is an atom, the behavior must resemble the competitive-buyer model...

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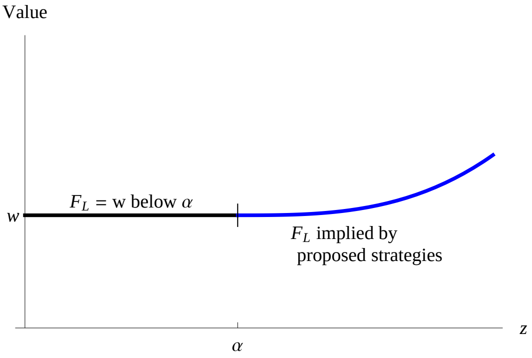
Why trade must be smooth with a single buyer



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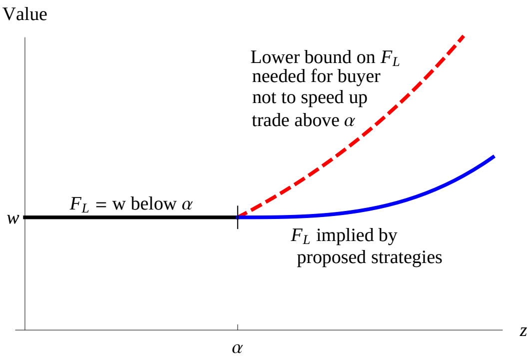

Intuitively,

- L is no more expensive to trade with at $z = \alpha + \epsilon$ than at $z = \alpha$.
- If the buyer wants to trade with L at price w below $z = \alpha$, he will want to extend this behavior above $z = \alpha$ as well.

Effect of news quality

Proposition (The effect of news quality)

As the quality of news increases:

1. *Both β and F_B increase*
2. *The rate of trade, $\dot{q}$, decreases for low beliefs but increases for intermediate beliefs*
3. *Total surplus and F_L increase for low beliefs, but decrease for intermediate beliefs*

Two opposing forces driving 3.

- Higher ϕ increases volatility of $\hat{Z} \implies$ faster trade
- Higher β (and/or) lower $\dot{q} \implies$ slower trade

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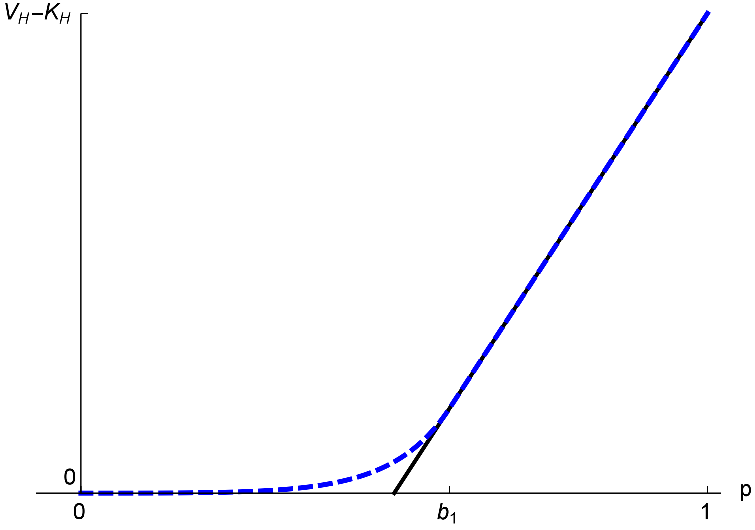
Effect of news on buyer payoff



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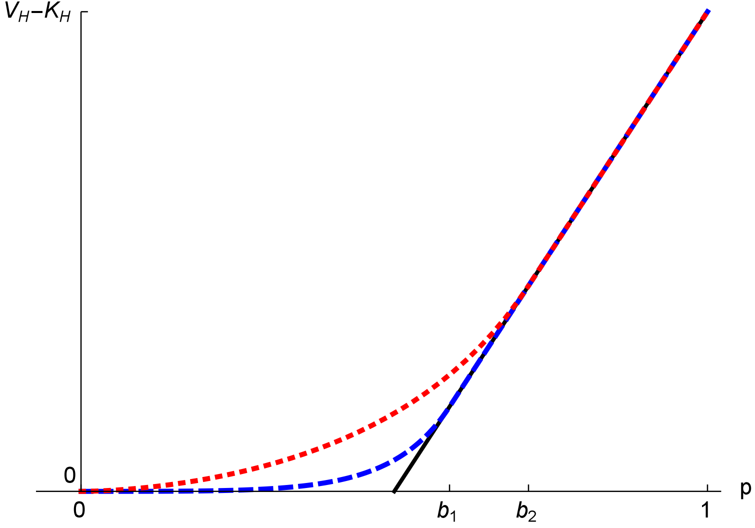

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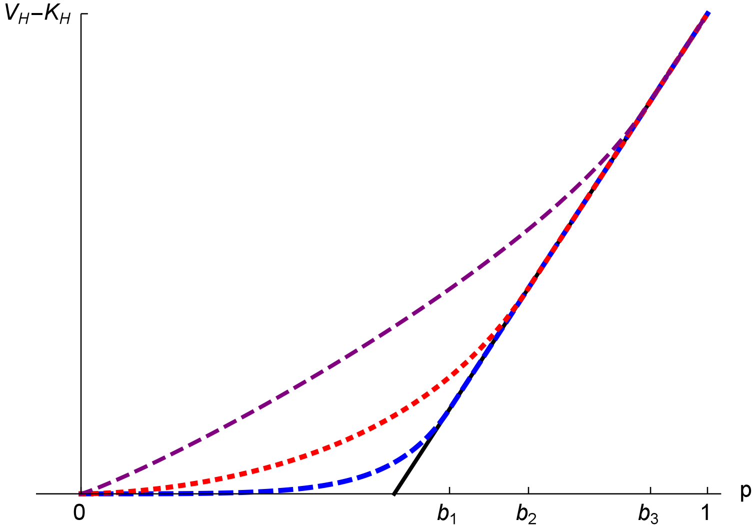

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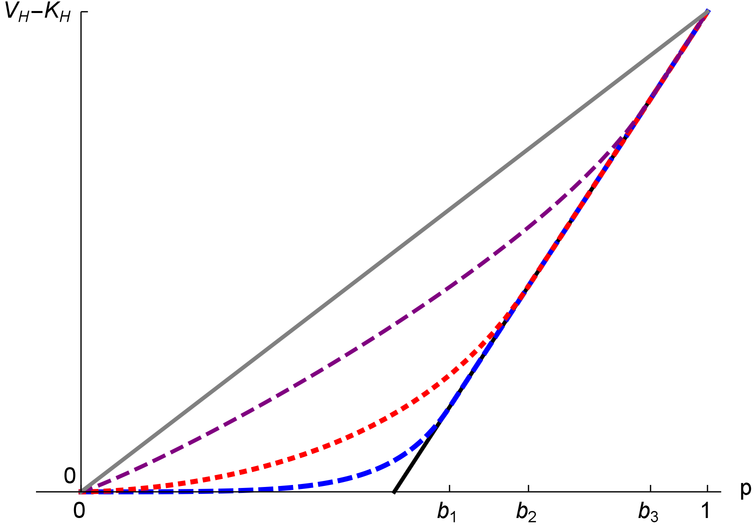
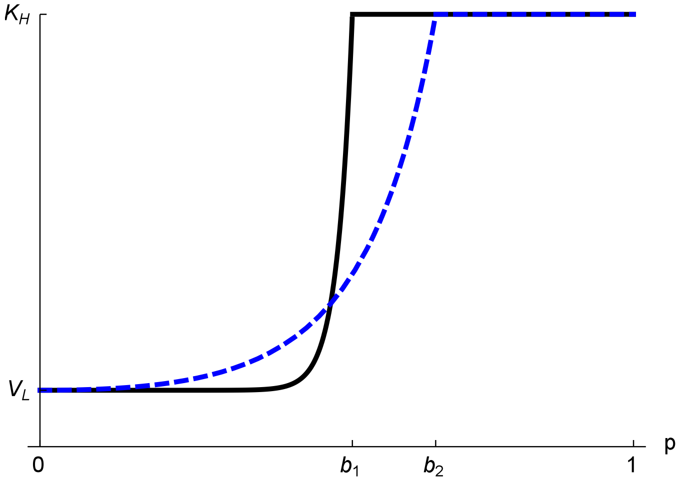


Effect of news on low-type payoff



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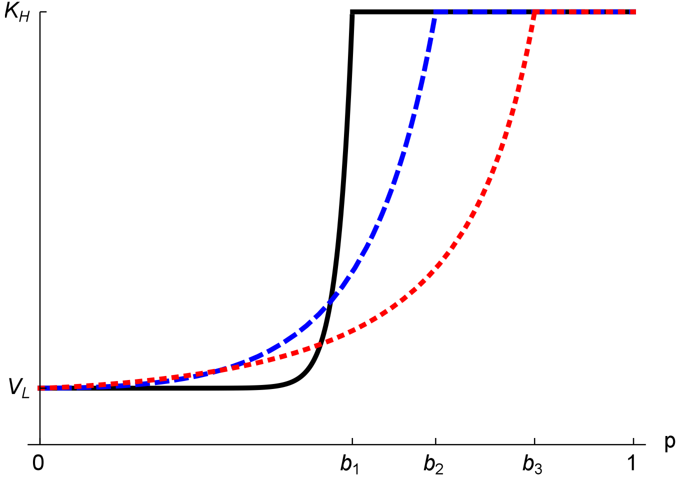
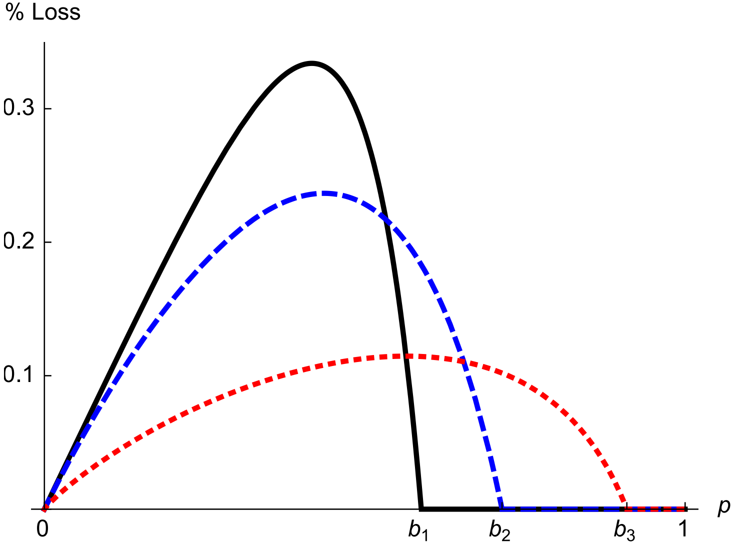


(In)efficiency



Arbitrarily high quality news

Result

As news quality becomes arbitrarily high ($\phi \to \infty$):

1. $\beta \to \infty$ (i.e., $b \to 1$)
2. $F_B \stackrel{u}{\to} p(z)(V_H - K_H)$
3. $F_L \stackrel{pw}{\to} V_L$
4. $\dot{q} \stackrel{pw}{\to} \infty$

Note that buyer waits until certain that $\theta = H$ before offering K_H

- Captures full surplus from trade with high type
- But NONE of the surplus from trade with low type

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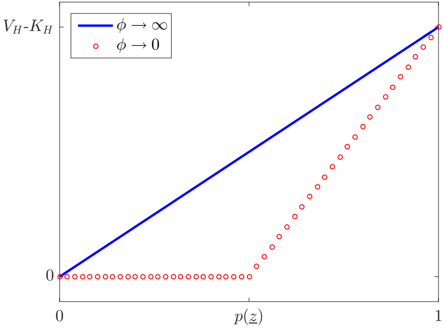
Arbitrarily low quality news

Result

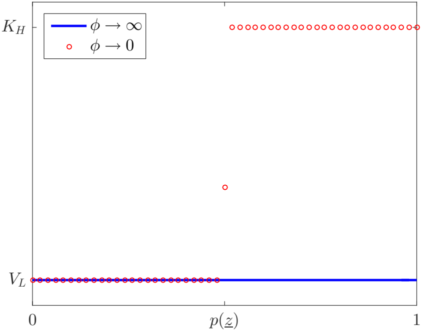
As news quality becomes arbitrarily low ($\phi \to 0$):

1. $\beta \to \underline{z}$
2. $F_B \xrightarrow{u} \max\{0, V(\underline{z}) - K_H\}$
3. $$F_L \xrightarrow{pw} \begin{cases} V_L & \text{if } z < \underline{z} \\ \frac{e-1}{e} V_L + \frac{1}{e} K_H & \text{if } z = \underline{z} \\ K_H & \text{if } z > \underline{z} \end{cases}$$
4. for all $z < \underline{z}$, $\dot{q}(z) \to \infty$, but $\dot{q}(\underline{z}) \to 0$

Limiting payoffs



Buyer payoff



Low type payoff

Effect of news

Our $\phi \to 0$ limit differs from Deneckere and Liang (2006)

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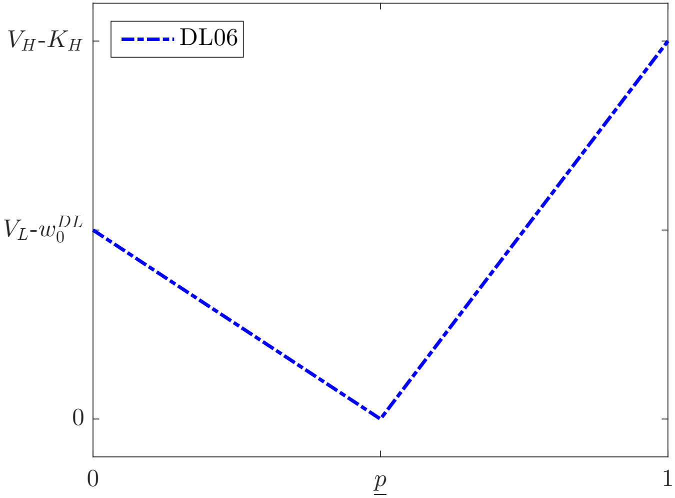

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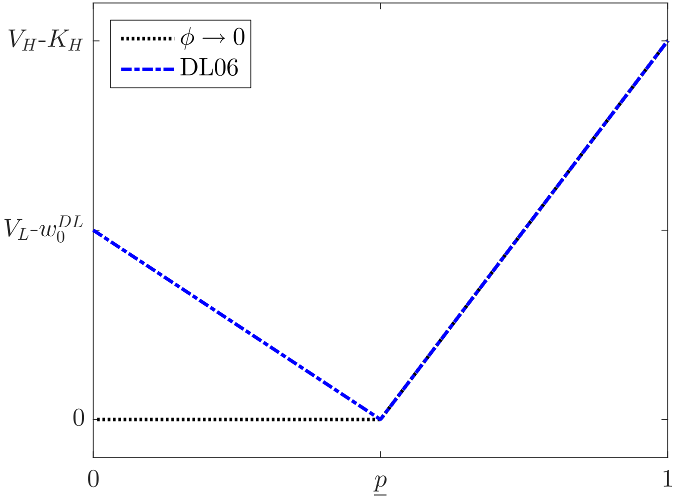

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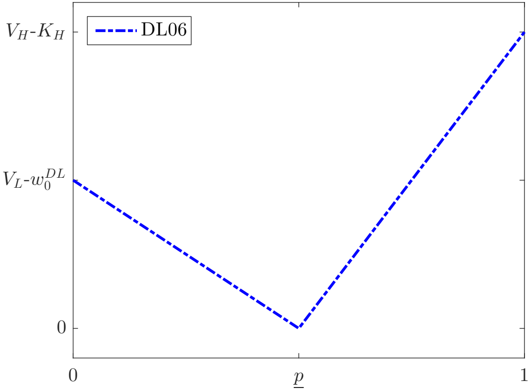
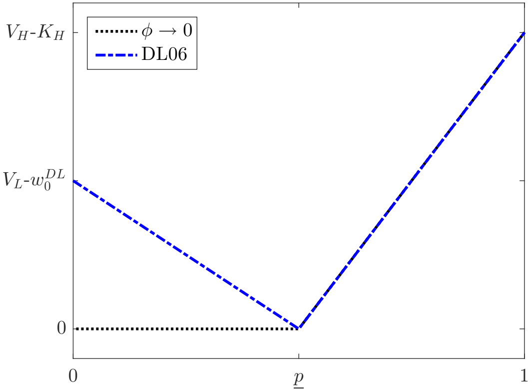


Intuition for DL06:

- Coasian force disappears at precisely $Z_t = \underline{z}$
- Buyer leverages this to extract concessions from low type at $z < \underline{z}$

Effect of news



With news, his belief <u>cannot</u> just "sit at $\underline{z}$", so this power evaporates.

- Even with arbitrarily low-quality news!

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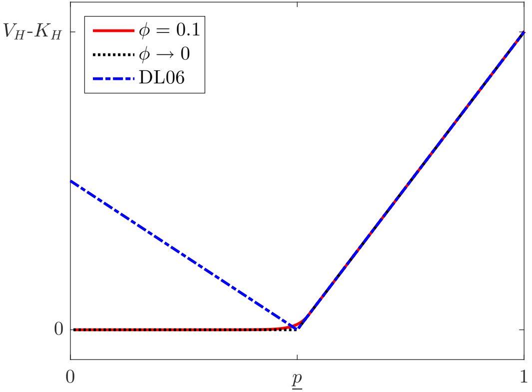

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▸ back

Stochastic control problem

The buyer must decide:

- How quickly to trade with only the low type (i.e., choose Q given F_L)
- When to "buy the market" (i.e., choose T at which to offer K_H)

Buyer's Problem

Choose (Q, T) to solve, for all z,

$$\sup_{Q,T} \left\{ (1-p(z)) E_z^L \left[\int_0^T e^{-rt} (V_L - F_L(\hat{Z}_t + Q_t)) e^{-Q_{t^-}} dQ_t \right.\right.$$
$$\left.\left. + e^{-(rT+Q_T)}(V_L - K_H) \right] + p(z) E_z^H \left[e^{-rT}(V_H - K_H) \right] \right\}$$

Let $F_B(z)$ denote the solution.

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Back

Buyer's problem

Lemma

For all z, $F_B(z)$ satisfies:

Option to wait: $$rF_B(z) \geq \frac{\phi^2}{2}\left(2p(z)-1\right)F_B'(z) + \frac{\phi^2}{2}F_B''(z)$$

Optimal screening: $$F_B(z) \geq \sup_{z'>z}\left\{\left(1-\frac{p(z)}{p(z')}\right)(V_L - F_L(z')) + \frac{p(z)}{p(z')}F_B(z')\right\}$$

Option to buy: $$F_B(z) \geq E_z[V_\theta] - K_H$$

where at least one of the inequalities must hold with equality.

Back

Equilibrium construction

1. For $z < \beta$, $w(z) = F_L(z)$ and the buyer's value is

$$F_B(z) = \left(V_L - F_L(z)\right)\left(1 - p(z)\right)\dot{q}(z)dt + \left(1 - \frac{\dot{q}(z)}{1+e^z}dt\right) E_z\left[F_B(z + dZ_t)\right]$$

and $dZ_t = d\hat{Z}_t + \dot{q}(Z_t)dt$. So,

$$rF_B(z) = \underbrace{\frac{\phi^2}{2}\left(2p(z)-1\right)F_B'(z) + \frac{\phi^2}{2}F_B''(z)}_{\text{Evolution due to news}}$$

$$+ \dot{q}(z)\underbrace{\left(\left(1-p(z)\right)\left(V_L - F_L(z) - F_B(z)\right) + F_B'(z)\right)}_{\Gamma(z) = \text{net-benefit of screening at } z}$$

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Equilibrium construction

2. Observe that the buyer's problem is linear in $\dot{q}$

$$rF_B(z) = \underbrace{\frac{\phi^2}{2}\left(2p-1\right)F_B' + \frac{\phi^2}{2}F_B''}_{\text{Evolution due to news}}$$

$$+ \sup_{\dot{q}\geq 0} \dot{q} \underbrace{\left((1-p)\big(V_L - F_L - F_B\big) + F_B'\right)}_{\Gamma(z)=\text{net-benefit of screening}}$$

Hence, in any state $z < \beta$, either

(i) the buyer strictly prefers $\dot{q} = 0$, or

(ii) the buyer is indifferent over all $\dot{q} \in \mathbb{R}_+$

Equilibrium construction

3. In either case

$$\dot{q}(z)\Gamma(z) = 0$$

4. This simplifies the ODE for F_B to just

$$rF_B = \frac{\phi^2}{2}(2p-1)F_B' + \frac{\phi^2}{2}F_B''$$

→ F_B does not depend on $\dot{q}$

→ Buyer gets same value he would get from $\dot{q} = 0$

→ Buyer gains nothing from the ability to screen using prices!

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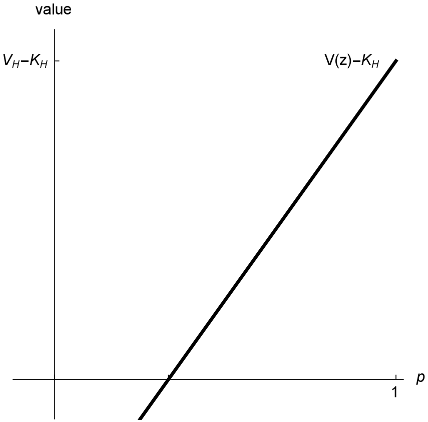
Equilibrium construction

Using the appropriate boundary conditions, we find $F_B(z) = C_1 \frac{e^{u_1 z}}{1+e^{z}}$,

- where $u_1 = \frac{1}{2}\left(1 + \sqrt{1 + 8r/\phi^2}\right)$ and C_1 solves VM and SP at $z = \beta$.



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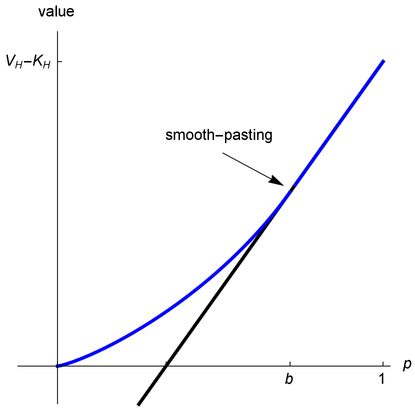

Equilibrium construction

Next, conjecture that $\dot{q}(z) > 0$ for all $z < \beta$. Then, it must be that

$$\Gamma(z) = 0$$

Or equivalently

$$F_L(z) = (1 + e^z)F_B'(z) + V_L - F_B(z)$$

This pins down exactly how "expensive" the low type must be for the buyer to be indifferent to the speed of trade (i.e., F_L).

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Equilibrium construction

For $z < \beta$, the low-type must be indifferent between accepting $w(z)$ and waiting.

The waiting payoff is

$$F_L(z) = \mathbb{E}_z^L \left[e^{-rT(\beta)} K_H \right]$$

which evolves as

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So, $\dot{q}(z)$ must satisfy

$$\dot{q}(z) = \frac{rF_L(z) + \frac{\phi^2}{2} F_L'(z) - \frac{\phi^2}{2} F_L''(z)}{F_L'(z)}$$

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Back