Trading Complex Risks

Felix Fattinger

Brain, Mind and Markets Lab
Department of Finance, University of Melbourne

January, 2018

Where I start from ...

That economic decisions are made without certain knowledge of the consequences is pretty self-evident.

Kenneth J. Arrow

Roadmap

1. What do I mean by ‘complex’ risks?

2. How to derive theoretical predictions?

3. How does the theory hold up against the experimental data?

My Terminology: Simple vs. Complex Risks

- The aim is to study the effects of complexity on the trading and pricing of consumption risk in a *well-defined* environment.

- I therefore rely on the following distinction:

  Simple risks: Agents possess perfect information about the underlying objective probabilities.

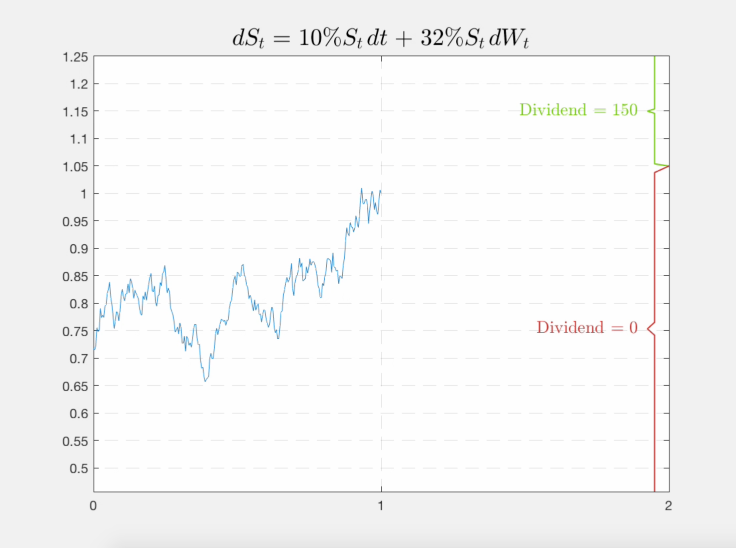
  Complex risks: Agents only have access to imperfect information about the underlying objective probabilities.

- In the context of complex risks, the quality of agents' information depends on the cognitive resources at their disposal.

An Example

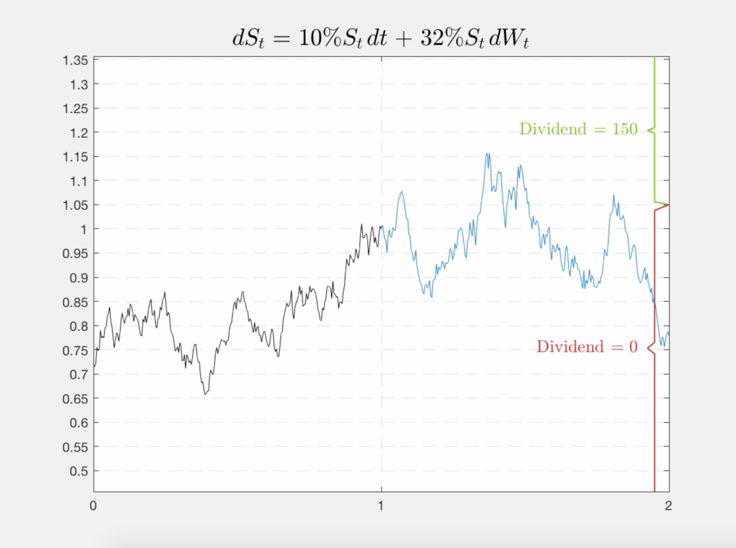
Trading Complex Risks: An Example

What is the probability π of receiving a dividend X equal to 150?



Trading Complex Risks: An Example (cont'd)

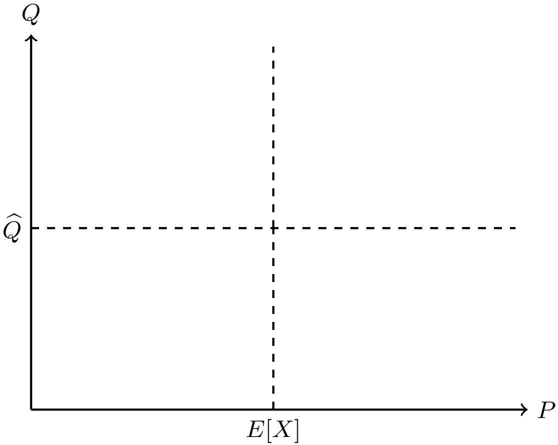
What is the probability π of receiving a dividend X equal to 150? ▸ solution



Theory in a Nutshell (Intuition!)

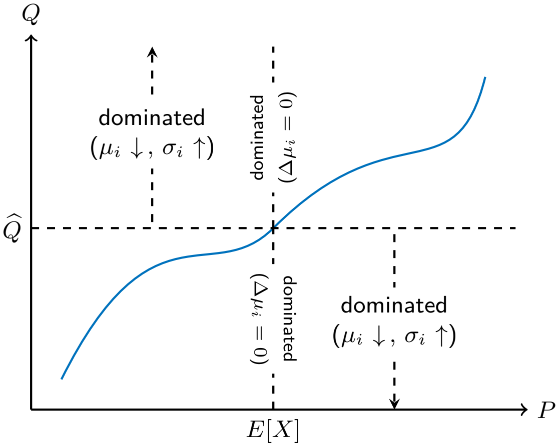
Trading Simple Risks (Benchmark)

Agent i's expected utility from consumption depends on π, μ_i, and σ_i.



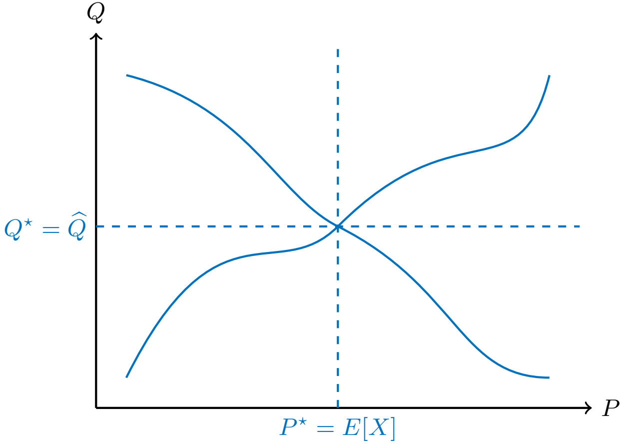
Trading Simple Risks (Benchmark)

Agent i's expected utility from consumption depends on π, μ_i, and σ_i. ▸ def.



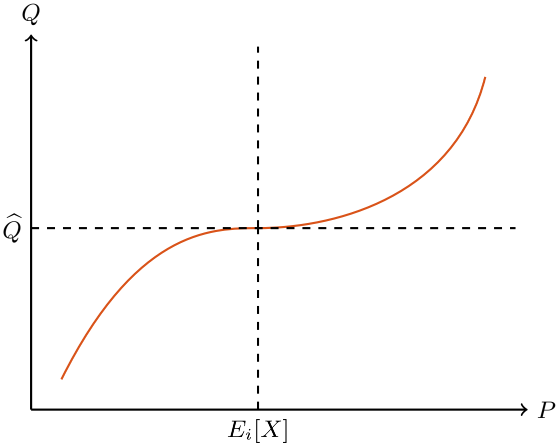
Equilibrium for Simple Risks (Benchmark)

In the absence of aggregate risk (if $\exists\,\widehat{Q}$), market completeness implies:



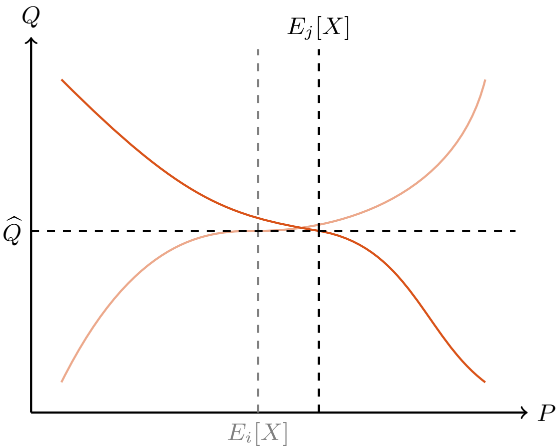
Trading Complex Risks

If risks are complex, ambiguity-averse agents are more reluctant to bear them.



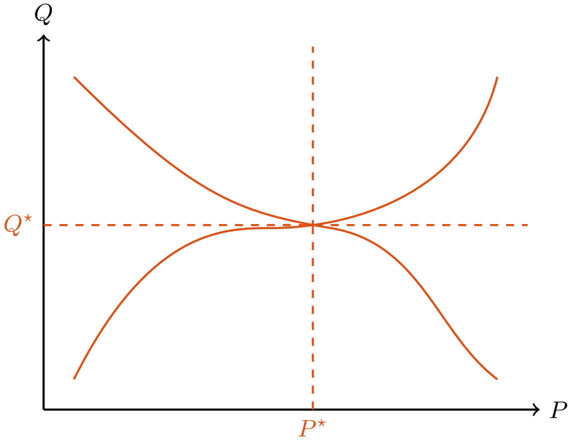
Trading Complex Risks

If risks are complex, agents likely have different beliefs.



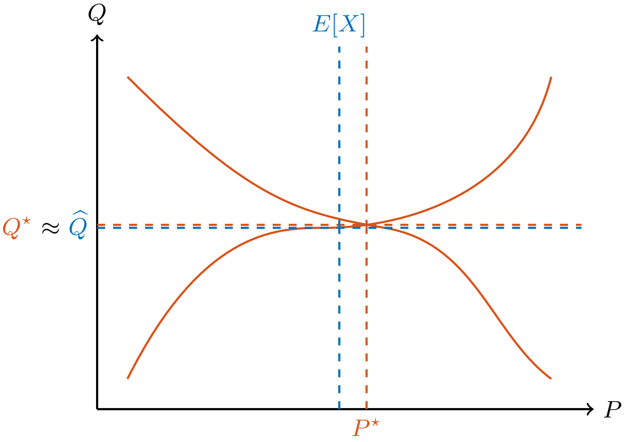
Equilibrium for Complex Risks

If risks are complex, market outcomes are a function of agents' beliefs.



Equilibrium for Complex Risks

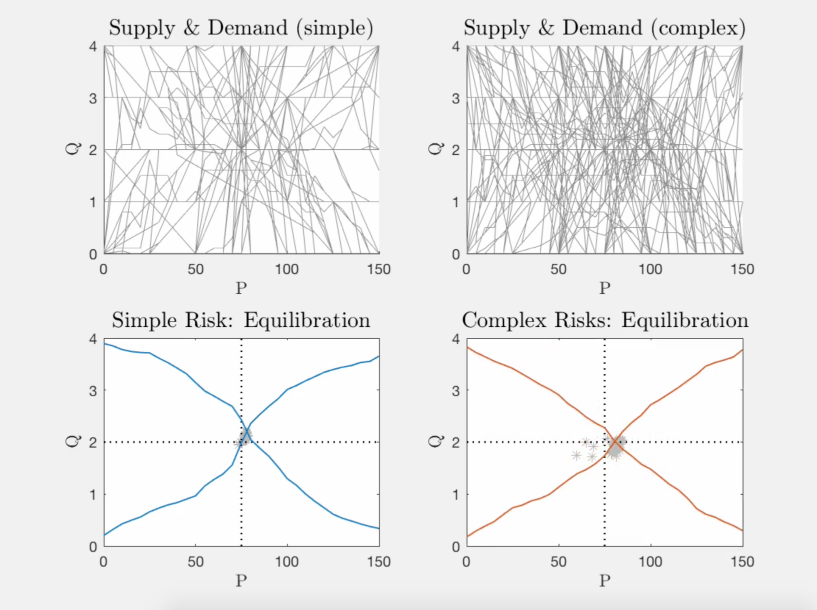
If agents are ambiguity-averse, efficient risk sharing prevails under complexity.



Results on a First Glance

▸ overview

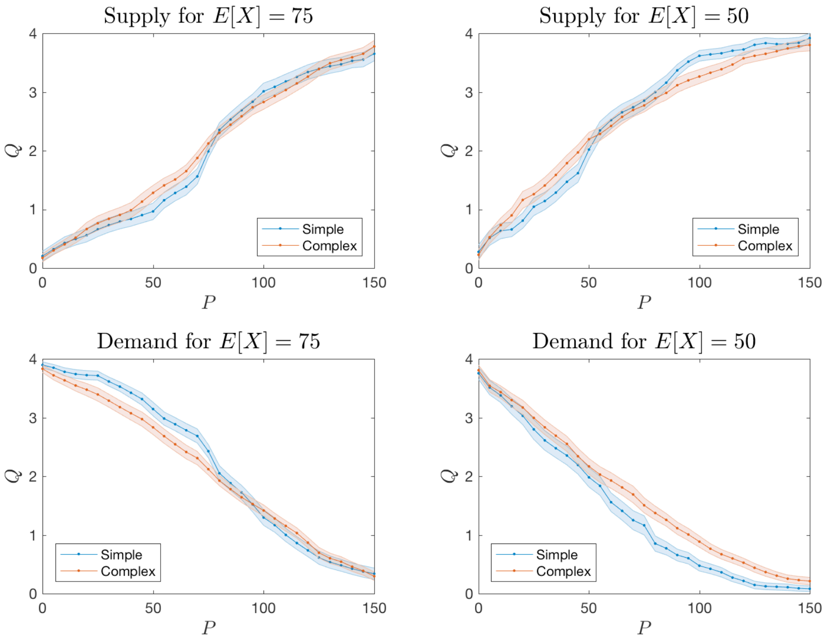
The Beauty of Aggregation (for $\widehat{Q} = 2$ and $\pi = 1/2$, i.e., $E[X] = 75$)



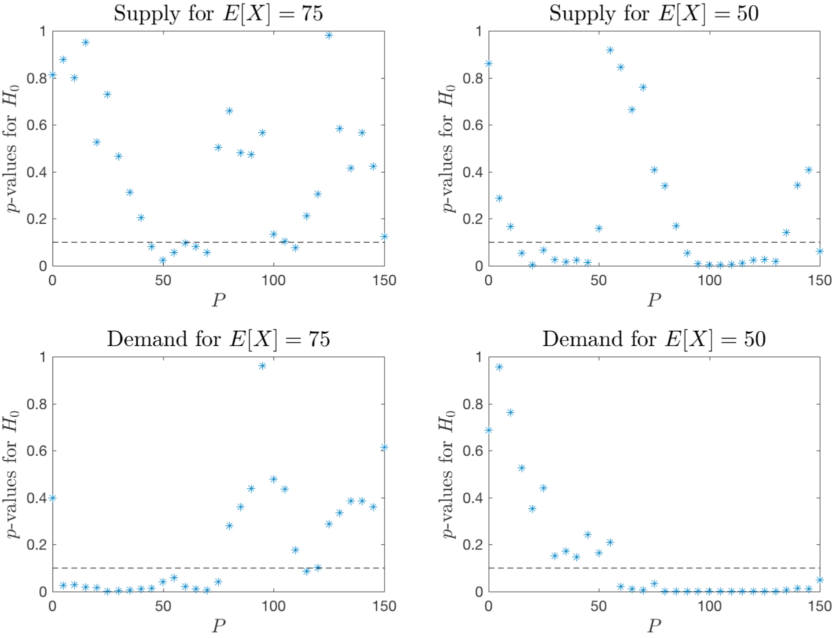
Aggregate Market Outcomes

Simple vs. Complex Risks

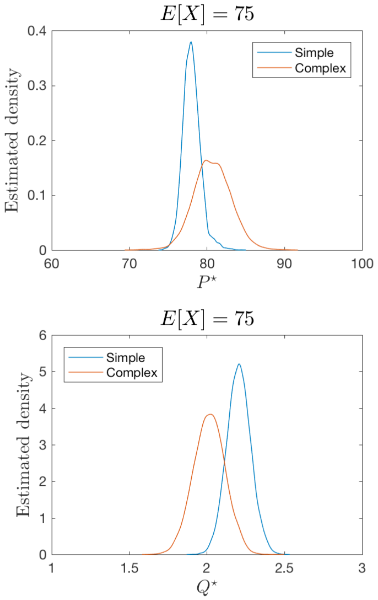
▸ price-taking?



Simple vs. Complex Risks (cont'd): Wilcoxon Signed-Rank Test



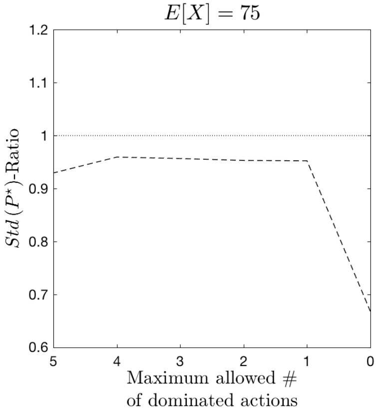
Bootstrapped Equilibrium Distribution (resampling size: 10k)

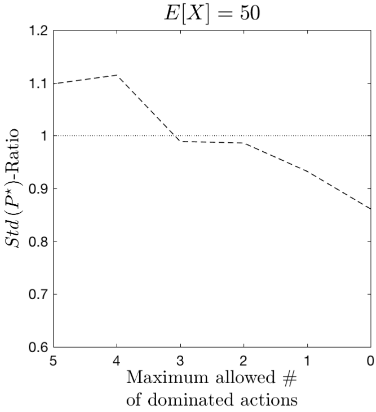


Relative Variability of Market-clearing Prices

I propose the following measure to assess markets' information aggregation efficiency:

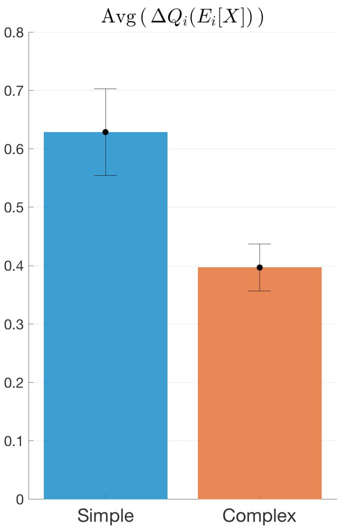
$$Std(P^{\star})\text{-Ratio} = \sqrt{\frac{Var(P^{\star}_c)}{Var\left(P^{\star}_s + E^{\star}_c[X]\right)}}.$$

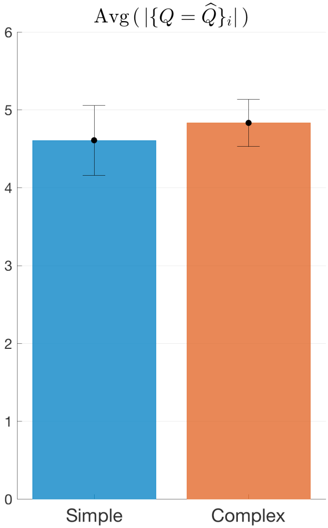




Individual Behavior

Inconclusive Results





Reconciling Individual and Aggregate Behavior

- What about complexity induced errors/noise in decision making?
- More severe bounds on rationality than in Biais et al. (2017)?
- Random choices in the spirit of McKelvey and Palfrey (1995, 98)'s quantal response model:

$$\mathbb{P}_i(Q_j|P) = \frac{\psi_i\left(E_i[U_i(Q_j|P)]\right)}{\Sigma_k\, \psi_i\left(E_i[U_i(Q_k|P)]\right)}$$

- Implications:
  1. $P = E_i[X]$: distribution of Qs symmetric around $\widehat{Q}$
  2. $P < E_i[X]$: Distribution of Qs asymmetric around $\widehat{Q}$ and decreasing above (below) $\widehat{Q}$ for sellers (buyers)
  3. $P > E_i[X]$: Distribution of Qs asymmetric around $\widehat{Q}$ and decreasing below (above) $\widehat{Q}$ for sellers (buyers)

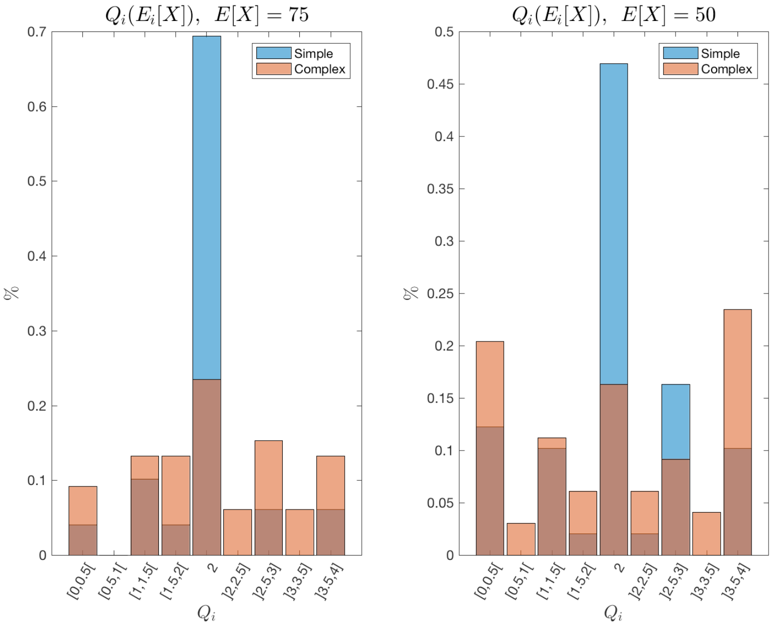
Reconciling Individual and Aggregate Behavior (cont'd)

- What about complexity induced errors/noise in decision making?
- More severe bounds on rationality than in Biais et al. (2017)?
- Random choices in the spirit of McKelvey and Palfrey (1995, 98)'s quantal response model:

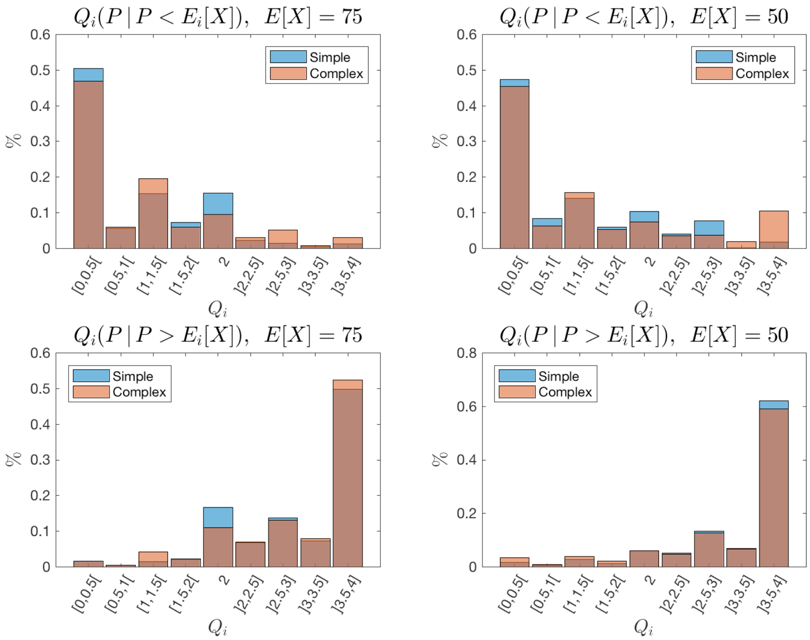
$$\mathbb{P}_i(Q_j|P) = \frac{\boldsymbol{\psi_i}\left(E_i[U_i(Q_j|P)]\right)}{\Sigma_k\, \boldsymbol{\psi_i}\left(E_i[U_i(Q_k|P)]\right)}$$

- Hypotheses:
    1. $\boldsymbol{\psi_i}$ likely to depend on complexity: $\overline{\boldsymbol{\psi}}_{\boldsymbol{i}}$ vs. $\underline{\boldsymbol{\psi}}_{\boldsymbol{i}}$
    2. $\overline{\boldsymbol{\psi}}_{\boldsymbol{i}}(x) > \underline{\boldsymbol{\psi}}_{\boldsymbol{i}}(x)$ and $\overline{\boldsymbol{\psi}}_{\boldsymbol{i}}'(x) > \underline{\boldsymbol{\psi}}_{\boldsymbol{i}}'(x)$

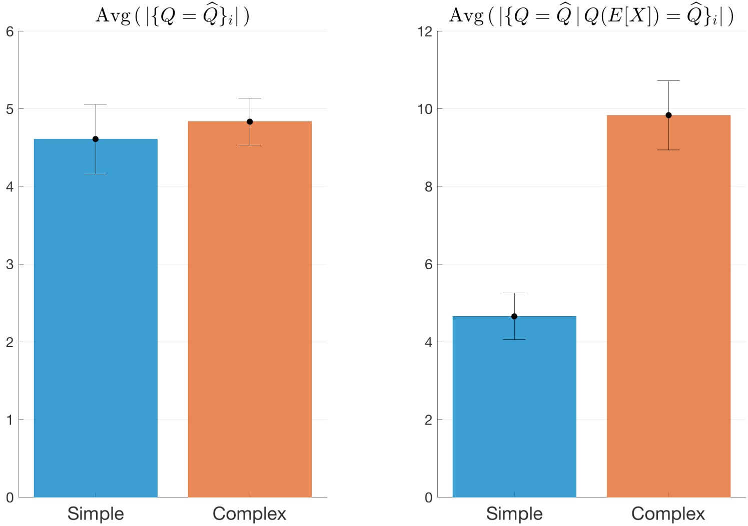
Reconciling Individual and Aggregate Behavior: Sellers



Reconciling Individual and Aggregate Behavior: Sellers (cont'd)



From Unconditional to Conditional Individual Behavior



What do we learn?

- Consistent with decision theory under ambiguity, subjects' demand and supply curves are less price sensitive for complex relative to simple risks.
- In the presence of complex risks, equilibrium prices are more sensitive whereas risk allocations are less sensitive to subjects' incorrect beliefs.
- Markets' effectiveness in aggregating beliefs about complex risks is determined by the trade-off between reduced price sensitivity and reinforced bounded rationality.

Appendix

Solution to Complexity Treatment

- Now, what is the probability of receiving a dividend equal to 150?
- We start with the SDE of the GBM

$$dS_t = 10\% S_t \, dt + 32\% S_t \, dW_t.$$

- Applying Itô to $f := \ln(S_t)$, we get

$$S_2 = \exp\left\{ \left(10\% - \frac{32\%^2}{2} \right) + 32\%(W_2 - W_1) \right\}.$$

- Hence,

$$\mathbb{P}(S_2 \geq 1.05) = \mathbb{P}\left(W_2 - W_1 \leq \underbrace{\left(\ln(1.05) - 10\% + \frac{32\%^2}{2} \right) \frac{1}{32\%}}_{\approx 0} \right).$$

- Given the distribution of $W_2 - W_1$ (known), we find $\mathbb{P}(S_2 \geq 1.05) = \frac{1}{2}$.

back

Expected Utility Theory: Individual Behavior and Aggregate Risk

Agent i's expected utility from consumption is given by

$$E\Big[U_i(C_i(\omega))\Big] = \pi\, U_i\left(\mu_i + \sqrt{\frac{1-\pi}{\pi}}\sigma_i\right) + (1-\pi)\, U_i\left(\mu_i - \sqrt{\frac{\pi}{1-\pi}}\sigma_i\right),$$

where $\mu_i \equiv \pi C_i(u) + (1-\pi)C_i(d)$ and $\sigma_i^2 \equiv \pi(1-\pi)\left(C_i(u) - C_i(d)\right)^2$.

No Aggregate Risk

If there is no aggregate risk, i.e., there exists a tradeable quantity $\widehat{Q}$ at which every seller and buyer is perfectly hedged, i.e., $\sigma_i = 0 \;\; \forall\, i \in I$, then:

For any family of concave utility functions $(U_i)_{i\in I}$, seller i's supply and buyer j's demand curve have the unique intersection point $(E[X], \widehat{Q}) \;\; \forall \{i,j\} \subset I$.

▸ back

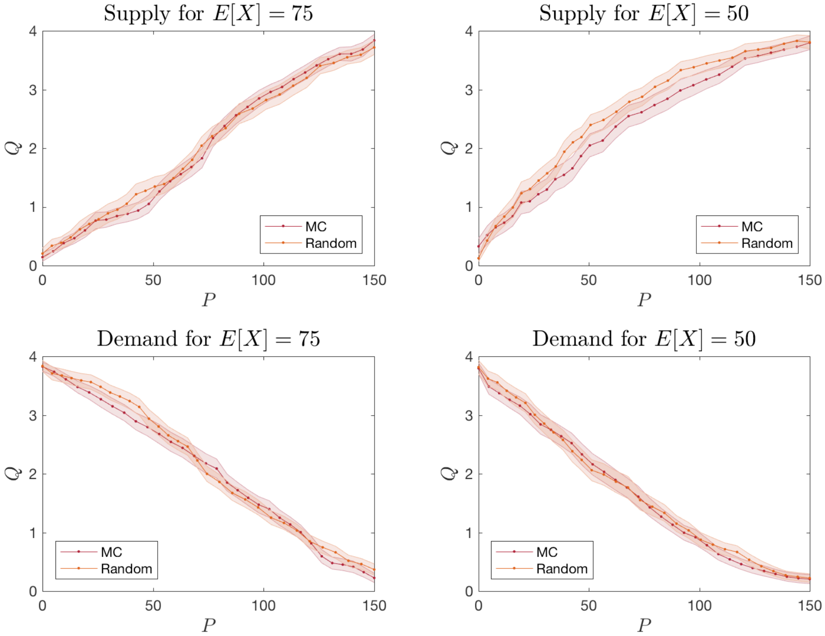
Overview of Experiment

	Session 1 (#16)			Session 2 (#18)			Session 3 (#16)		
Round	π	Type	Pricing	π	Type	Pricing	π	Type	Pricing
1	1	C (P)	MC	1	C (P)	MC	1	C (P)	MC
2	high	C (P)	random	high	C (P)	random	high	C (P)	random
3	low	C (P)	MC	low	C (P)	MC	low	C (P)	MC
4	1/2	C	MC	1/3	C	random	1/3	C	MC
5	1/3	C	MC	1/2	C	random	1/3	C	random
6	1/2	C	random	1/3	C	MC	1/2	C	MC
7	1/3	C	random	1/2	C	MC	1/2	C	random
8	1/2	R	MC	1/2	R	random	1/2	R	MC
9	1/3	R	random	1/3	R	MC	1/3	R	random
10	ambig	A	MC	ambig	A	random	ambig	A	MC

	Session 4 (#16)			Session 5 (#16)			Session 6 (#16)		
Round	π	Type	Pricing	π	Type	Pricing	π	Type	Pricing
1	1/2	R (P)	MC	1/2	R (P)	MC	1/2	R (P)	MC
2	9/10	R (P)	random	9/10	R (P)	random	9/10	R (P)	random
3	1/2	R	MC	1/2	R	random	1/2	R	MC
4	1/3	R	random	1/3	R	MC	1/3	R	random
5	high	C (P)	MC	high	C (P)	MC	high	C (P)	MC
6	1/2	C	MC	1/3	C	random	1/3	C	MC
7	1/3	C	MC	1/2	C	random	1/3	C	random
8	1/2	C	random	1/3	C	MC	1/2	C	MC
9	1/3	C	random	1/2	C	MC	1/2	C	random
10	ambig	A	MC	ambig	A	random	ambig	A	MC

back

Price-taking Behavior under Complex Risks?



Price-taking Behavior (cont'd): Wilcoxon Signed-Rank Test

back