Option-Implied Correlations, Factor Models, and Market Risk

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Motivation

- Correlations are changing, and increase during market downturns.
- Correlation risk negatively affects investor welfare by making diversification more difficult.
- The estimation of the correlations and factor models are typically performed using historical data.

Major Goals and Contributions

1. Construct option-implied covariances (COV) without historical data.
2. Use options on sectors to infer correlations in and between sectors.
3. Identify and estimate an option-implied linear factor model.
4. Find the risk channel through which implied correlation (IC) predicts market returns.

Summary of the Major Results

- Correlations and variances (+premiums) vary across economic sectors.
- Implied correlation (IC) between sectors contains enough information to predict market returns and systematic risk.
- IC predicts not just (RC), but also the lower bound of non-diversifiable market risk—$\sigma^2(\beta_M)$.

  A high IC predicts a lower cross sectional dispersion of betas $\rightarrow$ β_M more clustered around the mean $\rightarrow$ less diversification benefits.
- Fully option-implied COV from sector data results in factors explaining more of stock dynamics than historical or hybrid approaches.

Literature Review

From many option-based variables two stand out in predicting market returns and risk:

- VRP performs best at the quarterly horizon - Bollerslev, Tauchen, and Zhou (2009)
- IC works at horizons up to a year - Driessen, Maenhout, and Vilkov (2005)
- Both variance and correlations contribute to the market variance risk.
- Pricing of the Index variance depends on the pricing of the individual variance and the correlation risk.

Input Correlation Matrix - inferred from option prices

Two alternatives are so far available in the literature:

1. **Homogenous IC** - option-implied - Equicorrelations
   Driessen, Maenhout, and Vilkov (2005), Skinzi and Refenes (2005).
2. **Heterogeneous IC** - historical correlations adjusted by a parametric correlation risk premium - Buss and Vilkov (2012).

NEW: Sector-based implied correlations: heterogenous correlation matrix built exclusively from options.

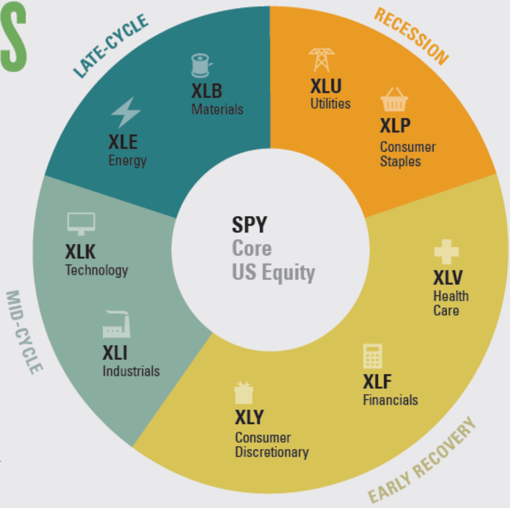
Data and Preparation of Variables - Data Availability

- **Major Indices**: S&P500, S&P100, DJ Industrial Average (DJ30).
- **Sector Indices**: ETFs for nine economic sectors of the S&P500.
- **Individual Level**: All constituents

The data on options are available until April 2016.

ECONOMIC CYCLES & SECTORS

The economy moves in cycles. Specific sectors may outperform or underperform during different phases, driven by cyclical factors such as corporate earnings, interest rates and inflation. For a sector rotation strategy around a core US equity exposure, investors can use ETFs to increase their allocation to sectors expected to outperform because of cyclical trends, and decrease their allocation to sectors that are expected to underperform.



Source: http://blog.spdrs.com

Data and Preparation of Variables - Three Databases

- Index composition from Compustat (GVKEY and IID) $\rightarrow$ merged with return data and market cap from CRSP (PERMNO).
- Matching CRSP/Compustat with Option Data through historical CUSIP link provided by Option Metrics.
- Options on SPDR ETFs serve as proxy for nine economic sectors.
- Group stocks corresponding to the composition of the respective indices and the nine Select Sector SPDR ETFs.

PERMNO is used as the main identified in our merged database.

Option-Implied Variables - Moments

Time horizon: $30, 91, 365$ days.

- For computing the option-based variables we rely on the **Surface Data** from **Option Metrics**.
- Option-implied variance (σ^2) are computed as Simple Variance Swaps (SMFIV) - Martin (2013).
- SMFIV is the risk-neutral expected quadratic variation of the underlying (robust to jumps).
- For realized variances we use daily returns (window = time horizon).
- VRP is computed in an ex ante version: $SMFIV_t - RV_{t-\Delta t,t.}$.

How is the Implied Correlation calculated?

Option-Implied Variables - Implied Correlations

ICs (for each day) are constructed using several methods:

Fully option-implied:

1. **Equicorrelations** - pairwise correlations are equal.
2. **Sector-based correlations** - equal correlations for stocks in the same sector, and between any two stocks in different sectors.

Hybrid:

3. **Heterogeneous correlations** Buss and Vilkov (2012)
   - $\rho_{ij}^{Q}(t) = \rho_{ij}^{P}(t) - \alpha^{Q}(t)(1 - \rho_{ij}^{P}(t))$

Option-Implied Variables - Main Identifying Restriction

Main Identifying Restriction (MIR): The variance of an index is equal to the variance of the portfolio, which the index represents:

$$
\begin{aligned}
\sigma_I^2(t) &= \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_i(t)\sigma_j(t)\rho_{ij}(t). \\
&= \sum_{i=1}^{N} w_i^2\sigma_i^2(t) + \sum_{i=1}^{N}\sum_{j\neq i}^{N} w_i w_j \sigma_i(t)\sigma_j(t)\rho_{ij}(t).
\end{aligned}
$$

Option-Implied Variables - Equicorrelations

Equicorrelations: use $\rho_{ij}(t) = \rho(t)$ and solve for $\rho(t)$:

$$\rho\left(t\right) = \frac{\sigma_I^2\left(t\right) - \sum_{i=1}^{N} w_i^2 \sigma_i^2\left(t\right)}{\sum_{i=1}^{N} \sum_{j \neq i} w_i w_j \sigma_i\left(t\right) \sigma_j\left(t\right)},$$

For example: *Reduced Sector-Based* Correlations for the S&P500

- Consider only the nine sector ETFs (as assets).

Hence:

$$\sigma_I^2(t) = \sum_{i=1}^{N=9} \sum_{j=1}^{N=9} w_i w_j \sigma_i(t) \sigma_j(t) \rho(t).$$

Option-Implied Variables - Block Diagonal COV

Full Sector-Based Correlation Matrix:

1. Estimate the equicorrelations ρ_{sect} using the MIR for each sector.

2. Determine the remaining correlations $\rho_{off-diag}(t)$ between stocks in different sectors using the identifying restriction:

$$\sigma_I^2(t) = \sum_{sect=1}^{Nsect} \sum_{i \in sect} \sum_{j \in sect} w_i w_j \sigma_i(t) \sigma_j(t) \rho_{sect}(t) + \sum_{i=1}^{N} \sum_{j:sect(i) \neq sect(j)} w_i w_j \sigma_i(t) \sigma_j(t) \rho_{off-diag}(t).$$

Option-Implied Variables - Block Diagonal COV

For **one** sector the option implied correlation matrix looks as follows:

$$\Omega_{mat}^{Q} = \begin{pmatrix} 1 & \rho_{mat} & \cdots & \rho_{mat} \\ \rho_{mat} & 1 & \cdots & \rho_{mat} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{mat} & \rho_{mat} & \cdots & 1 \end{pmatrix}$$

For the S&P500 (i.e for the nine sectors), the full sector-based block-diagonal correlation matrix (at a specific date t) looks as follows:

$$\Omega_{FSB}^{Q} = \begin{pmatrix} \Omega_{mat}^{Q} & \rho_{off-diag} & \cdots & \rho_{off-diag} \\ \rho_{off-diag} & \Omega_{hea}^{Q} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{off-diag} & \cdots & \cdots & \Omega_{utl}^{Q} \end{pmatrix}$$

The Price of Variance and Correlation Risks

- Heterogeneity in the average IC & CRP among economic indices.

- Within the S&P500 the correlations in the sectors are linked less than perfectly.

Table 1: (Some) Sector ICs and CRPs: Summary Statistics

	IC			*CRP = IC-RC*		
	30	91	365	30	91	365
Sector: Materials						
Mean	0.520	0.520	0.549	0.038	0.041	0.080
p-val	0.000	0.000	0.000	0.000	0.000	0.000
Sector: Health Care						
Mean	0.415	0.397	0.433	0.048	0.035	0.075
p-val	0.000	0.000	0.000	0.000	0.007	0.000
Sector: Energy						
Mean	0.702	0.715	0.717	0.009	0.022	0.024
p-val	0.000	0.000	0.000	0.351	0.077	0.164
Sector: Finance						
Mean	0.628	0.643	0.680	0.078	0.092	0.130
p-val	0.000	0.000	0.000	0.000	0.000	0.000
Sector: Utilities						
Mean	0.487	0.548	0.649	-0.049	0.016	0.111
p-val	0.000	0.000	0.000	0.000	0.131	0.000

Insample Predictability of Returns via IC

Approach: Predict market returns over $30, 91, 365$ days by RC, IC, VRP.

Result:

- ICs extracted from nine S&P500 ETF sectors are sufficient for predicting market returns.

Hence: Correlation between different sectors matters and not just the correlation between all stocks.

- IC predicts better than VRP for longer horizons, always significant, R^2 from $21\% - 33\%$.

Table 2: Market Return Predictability: Correlations and VRP

	Market ret, 30 days			
SP500 Sample (Equicorrelations)				
RC	0.030	-	-	-
	0.111	-	-	-
IC	-	0.067	-	0.072
	-	0.000	-	0.000
VRP	-	-	0.210	0.228
	-	-	0.003	0.001
R^2	0.008	0.030	0.023	0.057
SP500 Sample (Reduced Sector Based)				
RC	0.049	-	-	-
	0.000	-	-	-
IC	-	0.048	-	0.047
	-	0.000	-	0.000
VRP	-	-	0.205	0.205
	-	-	0.005	0.004
R^2	0.034	0.035	0.024	0.059

Table 3: Market Return Predictability: Correlations and VRP

	Market ret, 365 days			
SP500 Sample (Equicorrelations)				
RC	0.403	-	-	-
	0.093	-	-	-
IC	-	0.851	-	0.849
	-	0.000	-	0.000
VRP	-	-	-0.738	-0.699
	-	-	0.231	0.186
R^2	0.064	0.216	0.012	0.227
SP500 Sample (Reduced Sector Based)				
RC	0.700	-	-	-
	0.000	-	-	-
IC	-	0.642	-	0.634
	-	0.000	-	0.000
VRP	-	-	-1.550	-1.446
	-	-	0.015	0.027
R^2	0.307	0.291	0.058	0.342

Predictability of Risks via IC

Through which channel does IC predict the market risk premium?

Hypothesis: IC predicts diversification (RC) in the economy.

- With increasing horizon the lagged RC works better in predicting RC.
- **But:** IC beats RC in predicting the cross-sectional dispersion of market betas - $\sigma^2(\beta_M)$.
- Stronger effect for longer horizons.

Thus: IC predicts the level of non-diversifiable market risk - higher IC indicates closer clustering of market betas around the mean.

Table 4: Risk Predictability: Cross Sectional Dispersion and Realized Correlations

SP500 Sample: 30-day horizon

	$\sigma^2(\beta_M)$		RC	
RC	-0.512	-	0.510	-
	0.000	-	0.000	-
IC	-	-0.774	-	0.688
	-	0.000	-	0.000
R^2	0.063	0.108	0.261	0.357

Table 5: Risk Predictability: Cross Sectional Dispersion and Realized Correlations

SP500 Sample: 365-day horizon

	$\sigma^2(\beta_M)$		RC	
RC	-0.243	-	0.519	-
	0.000	-	0.000	-
IC	-	-0.626	-	0.430
	-	0.000	-	0.000
R^2	0.047	0.224	0.295	0.149

The Linear Factor Model - Motivation and Reasoning

In a linear factor model with K factors the return for asset i follows:

$$r_{i,t+1} = \mu_{i,t} + \sum_{k=1}^{K} \beta_{ik,t} F_{k,t+1} + \varepsilon_{i,t+1},$$

The COV derived from a factor model is given via:

$$\Sigma = B \Sigma^F B' + D.$$

- B is the $N \times K$ matrix of K factor betas for N stocks, Σ^F is the COV of factors, D is the diagonal matrix of residual variances.

Factor Identification via Principal Component Analysis

But we are confronted with the inverse problem:

Task: Find the factor betas and factor variances from the COV.

Solution: Apply PCA to extract statistical factors at the end of a month.

Findings:

- The first factor is highly correlated with the market returns ($> 85\%$).
- Option-implied information improves factor explanatory power.
- Fully implied sector-based correlations produce the best factors.

Implied Factors and Factor Exposures - S&P500

Approach:

- At the end of each month construct three COVs $(\Sigma^P, \Sigma^Q_{BV}, \Sigma^Q_{FSB})$
- Extract the five leading principal components (eigenvectors) and normalize each to obtain factor weights.
- Calculate the daily factor return for each factor for the next month.
- Regress each stock returns on the set of factor returns - daily return frequency for each date (EoM) (reported are the mean coefficients).
- Do this exercise for two set of factors - unrotated and rotated.

Implied Factors and Factor Exposures - S&P500

Table 6: One Factor Models: Individual Stocks

Factors	β_{mkt}	R^2				
Economic factors						
mkt	0.997	0.208	-	-	-	-

	30-day		91-day		365-day	
Factors	β_{PC1}	R^2	β_{PC1}	R^2	β_{PC1}	R^2
Covariance matrix: Σ^P						
PC1	0.844	0.231	0.844	0.230	0.849	0.235
Covariance matrix: Σ^Q_{BV}						
PC1	0.883	0.232	0.883	0.232	0.907	0.237
Covariance matrix: Σ^Q_{FSB}						
PC1	0.878	0.247	0.875	0.247	0.910	0.260

$\Rightarrow$ R^2 for FSB Model is higher than for others.

Implied Factors and Factor Exposures - S&P500

Table 7: 3 Factor Models: Individual Stocks

Factors	β_{mkt}	R^2				
Economic factors						
mkt + smb + hml	1.068	0.236	-	-	-	-

	30-day		91-day		365-day	
Factors	β_{PC1}	R^2	β_{PC1}	R^2	β_{PC1}	R^2
Covariance matrix: Σ^P						
PC1-3	0.827	0.279	0.828	0.279	0.838	0.284
Covariance matrix: Σ^Q_{BV}						
PC1-3	0.884	0.277	0.885	0.279	0.905	0.286
Covariance matrix: Σ^Q_{FSB}						
PC1-3	0.875	0.287	0.870	0.288	0.917	0.305

$\Rightarrow$ R^2 for FSB Model is higher than for others.

PCA - Factor Rotation - S&P500

Approach: Least Squares Rotation (of A) to a Partially Specified Target Matrix ($W * B$)

- For every month t search the Rotation Matrix - Λ such that the 5 extracted factors A are rotated towards the target B.
- The Rotation Matrix $\Lambda = A(T')^{-1}$, where T is a Transformation Matrix s.th $diag(T'T) = I$
- W is specified such that $w_{ij} = 1$ if $b_{ij} \in B$ is specified.
- Obtain $\Lambda(A)$ by solving the optimization problem:

$$\min_{\Lambda} ||W * \Lambda - W * B||^2$$

In our case:

- A consists of the 5 extracted factors, the first column of B are the S&P500 market weights, the other 4 columns are 0.
- After rotation the first factor is correlated with the market by $> 93\%$

Implied Rotated Factors and Factor Exposures - S&P500

Table 8: One Factor Models: Individual Stocks

Factors	β_{mkt}	R^2				
Economic factors						
mkt	0.997	0.208	-	-	-	-

	30-day		91-day		365-day	
Factors	β_{PC1}	R^2	β_{PC1}	R^2	β_{PC1}	R^2
Covariance matrix: Σ^P						
PC1	0.933	0.238	0.933	0.238	0.933	0.238
Covariance matrix: Σ^Q_{BV}						
PC1	0.941	0.238	0.943	0.238	0.951	0.238
Covariance matrix: Σ^Q_{FSB}						
PC1	0.939	0.260	0.936	0.261	0.942	0.261

$\Rightarrow \approx 5\%$ higher R^2 than with just the market.

Conclusion

- Correlation between sectors matters—not just between assets.
- IC based on nine sectors efficiently predicts market returns and risks.
- High IC $\Rightarrow$ lower dispersion in β_M $\Rightarrow$ less diversification benefits.
- Economic sectors bear different variance and correlation risks.
- Option-implied Variables explain returns better than historical ones.

Thank you!

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