Present Bias

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December 18, 2017

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Extremely thirsty subjects (McClure et al, 2007)

“Yesterday is history, tomorrow is a mystery, but *today* is a gift. That is why it is called *the present*.”

- Master Oogway, *Kung Fu Panda* movie

Extremely thirsty subjects (McClure et al, 2007)

- Subjects choose between:

Juice now	*vs*	2x juice in 5 minutes
(60%)		(40%)
	AND	
Juice in 20 minutes	*vs*	2x juice in 25 minutes
(30%)		(70%)

Present Bias in money tasks

A. \$100 today	B. \$110 in a week
C. \$100 in 4 weeks	D. \$110 in 5 weeks

- People sometimes choose A over B, and D over C. (Present bias)
- Stationarity or Exponential Discounting: If A over B, then C over D. Vice-versa. Only temporal difference between the prizes matter. (violated)

Model(s) of present bias?

Model	Author(s)	Discount Function$\Delta(t)$	Present Bias
Exponential	Samuelson (1937)	$(1+g)^{-t}, g > 0$	No
Quasi-hyperbolic	Phelps, Pollak (1968)	$(\beta + (1-\beta)_{t=0})(1+g)^{-t}$	Yes
Proportional	Herrnstein (1981)	$(1+gt)^{-1}, g > 0$	Yes
Power	Harvey (1986)	$(1+t)^{-\alpha}, \alpha > 0$	Yes
Hyperbolic	Loewenstein, Prelec (1992)	$(1+gt)^{-\alpha/\gamma}, \alpha > 0, g > 0$	Yes
Constant sensitivity	Ebert, Prelec (2007)	$\exp[-(at)^b], a > 0, 1 > b > 0$	Yes

Not models for present bias per se

- They are all models of present bias + **additional temporal behavior idiosyncratic to the models.** For example...
- $\beta - \delta$: $\Delta(0) = 1, \Delta(t) = \beta\delta^t$
- Constant discounting $\frac{\Delta(t+1)}{\Delta(t)} = \delta$ in the future (from $t > 0$). Is it intuitive? Empirically sound?
- Hyperbolic discounting: $\Delta(t) = (1 + gt)^{-\alpha/\gamma}$
- $\frac{\Delta(t+1)}{\Delta(t)}$ increasing with t. (increasing patience in the future)
- Can we do away with such extraneous assumptions, and provide a general class of utility functions that would nest the aforementioned models?

What we will do

- We give Present Bias a precise definition, and impose it on the decision maker.
- We will axiomatize an general class of utility functions, given basic tenets of behavior alongside Present Bias.
- What insights would the axiomatization provide us about behavior?
- What additional empirical bite would the generalization provide us?

Additional Anomalies

- Anomalies that existing models cannot account for.

1. Stake dependent Present Bias: Cognitive optimization can result in non-existent present bias at high stakes.
2. Magnitude effect: Empirically estimated discount factors are higher for higher stakes.
3. Risk-time relations: Present Bias disappears in the presence of risk.

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- Applications to a dynamic decision-making game provides novel implications.

Placing this work in the literature

- Axiomatic theory: Linking testable/ observable conditions on behavior and utility theory.
- Behavioral Economics: Providing an alternative representation to Exponential Discounting or QHD, that adheres to laboratory and field evidence.

Outline for the talk

Theory

- Main Theorem
- Major take aways

Anomalies

- Anomaly 1: Stake dependence
- Anomaly 2: Risk-Time relations

Conclusion and possible extensions

Roadmap

- Theory
  - Main Theorem
  - Major take aways
- Anomalies
  - Anomaly 1: Stake dependence
  - Anomaly 2: Risk-Time relations
- Conclusion and possible extensions

An axiom for Weak Present Bias

Consider a present biased subject who chooses B over A.

B. \$110 in 1 week	$\succsim$	A. \$100 today
"Size of prize effect"	$\geq$	"present premium" AND "early factor"
(110>100)		(A is in the present) (A comes earlier)

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Moving both prizes equally into the future

D. \$110 in 5 weeks	?	C. \$100 in 4 weeks
"Size of prize effect"	$\geq$	~~"present premium"~~ AND "early factor"
D. \$110 in 5 weeks	$\succsim$	C. \$100 in 4 weeks

- $B \succsim A \implies D \succsim C$ for any DM with present-premium≥ 0

A novel Weakening of Stationarity

- $\mathbb{X}$ =$[0, M]$, $\mathbb{T} = \mathbb{N}_0$ or $[0, \infty)$. $\succsim$ on $\mathbb{X} \times \mathbb{T}$
- Objects of choice: Prize $x \in \mathbb{X}$ received at time $t \in \mathbb{T}$.
- *Weak Present Bias (WPB):* $(y, t) \succsim (x, 0) \implies (y, t + t_1) \succsim (x, t_1)$ for all $x, y \in X$ and $t, t_1 \in \mathbb{T}$.

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- *Stationarity:* $(y, t) \succsim (x, 0) \iff (y, t + t_1) \succsim (x, t_1)$ for all $x, y \in X$ and $t, t_1 \in \mathbb{T}$.

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Present biased choice reversal does not violate WPB, such choices vacuously satisfy the axiom.

A. \$100 today	$\succ$	B. \$110 in a week
C. \$100 in 4 weeks	$\prec$	D. \$110 in 5 weeks

Starting with preferences

- **A0:** $\succsim$ is complete and transitive.
- Ok and Masatlioglu [2007], Rubinstein [2003] consider temporal preferences without transitivity, and such preferences are outside the scope of our paper.
- **A1: CONTINUITY:** $\succsim$ is continuous.

Starting with preferences

- **A2: DISCOUNTING:**
- i) For $t, s \in \mathbb{T}$, if $t > s$ then $(x, s) \succ (x, t)$ for $x > 0$ and $(x, s) \backsim (x, t)$ for $x = 0$.
- ii) For $y > x > 0$, there exists $t \in \mathbb{T}$ such that, $(x, 0) \succsim (y, t)$.

Starting with preferences

- **A3: MONOTONICITY:** For all $t \in \mathbb{T}$ $(x, t) \succ (y, t)$ if $x > y$.

Starting with preferences

- **A3: MONOTONICITY:** For all $t \in \mathbb{T}$ $(x, t) \succ (y, t)$ if $x > y$.

- **A4: WEAK PRESENT BIAS:** If $(y, t) \succsim (x, 0)$ then, $(y, t + t_1) \succsim (x, t_1)$ for all $x, y \in X$ and $t, t_1 \in \mathbb{T}$.

Comparison with [Fishburn and Rubinstein, 1982]

$$\begin{aligned}\textit{A0-A3, Stationarity} \quad &\iff \textit{For any } \delta \in (0,1) \textit{ there exists } u_\delta \textit{ such that} \\ &\qquad G(x,t) \equiv \delta^t u_\delta(x) \\ &\iff \textit{For any } \delta \in (0,1) \textit{ there exists } u_\delta \textit{ such that} \\ &\qquad G(x,t) \equiv u_\delta^{-1}(\delta^t u_\delta(x))\end{aligned}$$

- $u_\delta^{-1}(\delta^t u_\delta(x))$ is the present equivalent of (x, t) w.r.t function u_δ and exponential discounting with discount factor δ.

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My result:

A0-A3, WPB $\iff$ *For any* $\delta \in (0,1)$ *there exists a set of utility functions* $\mathcal{U}_\delta$ *such that* $F(x,t) \equiv \min_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^t u(x)))$.

- $|\mathcal{U}| = 1 \implies$ Stationarity.

Comparison with [Fishburn and Rubinstein, 1982]

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- $|\mathcal{U}| = 1 \implies$ Stationarity.
- DM picks the most conservative (minimum) present equivalent under WPB.

Starting with preferences

Theorem

The following statements are equivalent:

i) $\succsim$ *satisfies Axioms A0-A4*

ii) For $\delta \in (0,1)$*, there exists a set* $\mathcal{U}_\delta$ *of monotonically increasing continuous functions such that*

$$F(x,t) \equiv \min_{u \in \mathcal{U}_\delta} (u^{-1}(\delta^t u(x)))$$

represents $\succsim$*.* $F(x,t)$ *is continuous. The set* $\mathcal{U}_\delta$ *has the following properties:* $u(0) = 0$ *and* $u(M) = 1$ *for all* $u \in \mathcal{U}$*.*

Intuition

- Intuition of Present Bias in the representation:
- $F(x, 0) = \min_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^0 u(x))) = \min_{u \in \mathcal{U}_\delta} x = x.$

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- Cerreia-Vioglio et al. [2015]
- $F(L) = \inf_{u\in\mathcal{U}}(u^{-1}(\sum_i p_i u(x_i)))$
- Bias for certainty, with similar intuition.

Minimum function

- $F(x,t) = \min_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^t u(x)))$.
- Subjective uncertainty about future tastes (Kreps, 1979), and max-min representation.
- Do you want coffee right now? : You can answer confidently.
- Do you want coffee in 379 days, 5 hours and 6 minutes? You might be uncertain about your answer, and might want to resolve uncertainty prudently.

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- Non-uniqueness of δ implies that a researcher cannot estimate the discount factor of the DM even if he observes the DM making infinite choices in this domain. Similar result in Fishburn and Rubinstein [1982] Non-uniqueness
- Uniqueness of δ will be obtained in an extension.

Major take aways from the theorem

- Minimum representation implies WPB.
- Any representation which calculates the minimum of present equivalents from possible future tastes must belong to a DM who has Weak Present Bias.

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- Minimum representation implies WPB.
- Any representation which calculates the minimum of present equivalents from possible future tastes must belong to a DM who has Weak Present Bias.

- WPB implies minimum representation.
- Result holds irrespective of $\mathbb{T} = \mathbb{N}_0$ or $[0, \infty)$.
- We start with just testable, intuitive conditions on behavior, and show that behavior is logically equivalent to a story of prudence under uncertainty of future tastes.
- β-δ, hyperbolic discounting and other popular utility functions can be interpreted as that of a prudent decision maker unsure about his/ her future tastes.

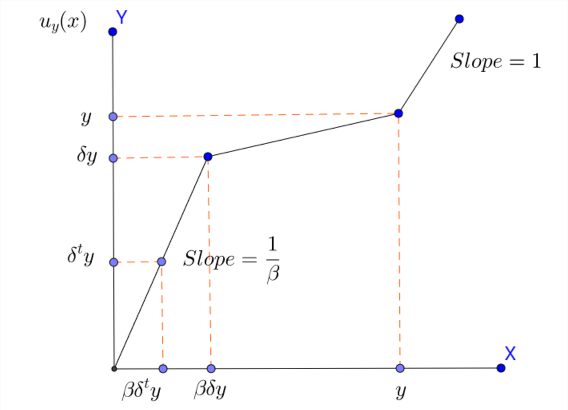
Constructing $\beta - \delta$

- $\beta - \delta: \ V(x,t) = \begin{cases} x & \text{for } t = 0 \\ \beta\delta^t x & \text{for } t > 0 \end{cases}$

$$\begin{aligned} u_y(x) &= \frac{x}{\beta} \text{ for } x \le \beta\delta y \\ &= \delta y + (x - \beta\delta y)\frac{1-\delta}{1-\beta\delta} \text{ for } \beta\delta y < x \le y \\ &= x \text{ for } x > y \end{aligned}$$

$V(x,t) = \min_{y \in \mathbb{R}_+} u_y^{-1}(\delta^t u_y(x))$. Proof for beta-delta case

Constructing $\beta - \delta$ (typical $u \in \mathcal{U}$)



Side-note: Future Bias

- $\mathbb{X} =$[0, M], $\mathbb{T}_0 = [0, \infty)$. $\succsim$ on $\mathbb{X} \times \mathbb{T}$
- Objects of choice: Prize $x \in \mathbb{X}$ received at time $t \in \mathbb{T}$.
- *Weak Future Bias (WFB):* $(x, 0) \succsim (y, t) \implies (x, t_1) \succsim (y, t + t_1)$ for all $x, y \in X$ and $t, t_1 \in \mathbb{T}$.
- The complimentary axiom that together with WPB implies stationarity.
- $F(x, t) = \max_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^t u(x)))$.
- Attitude towards uncertainty of future tastes determines bias for present or future.

Representation $\Longrightarrow$ WPB

$$(y, t) \succsim (x, 0)$$

$$\Longrightarrow \; (y, t + t_1) \succsim (x, t_1)$$

Representation $\implies$ WPB

$$(y, t) \succsim (x, 0)$$

$$\implies \min_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^t u(y))) \geq \min_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^0 u(x)))$$

$$\implies (y, t + t_1) \succsim (x, t_1)$$

Skip to anomalies section Uniqueness results

Representation $\implies$ WPB

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Uniqueness results

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$$\implies u^{-1}(\delta^t u(y)) \geq x \qquad \forall u \in \mathcal{U}_\delta$$

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\implies & \delta^t u(y) \geq u(x) \qquad \forall u \in \mathcal{U}_\delta
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\implies & \delta^t u(y) \geq u(x) \qquad \forall u \in \mathcal{U}_\delta \\
\implies & \delta^{t+t_1} u(y) \geq \delta^{t_1} u(x) \qquad \forall u \in \mathcal{U}_\delta
\end{aligned}
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\implies (y, t+t_1) \succsim (x, t_1)
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Representation $\implies$ WPB

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\implies u^{-1}(\delta^t u(y)) &\geq x && \forall u \in \mathcal{U}_\delta \\
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\implies & (y, t+t_1) \succsim (x, t_1)
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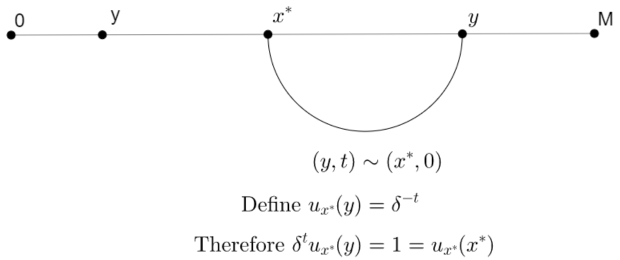
$$

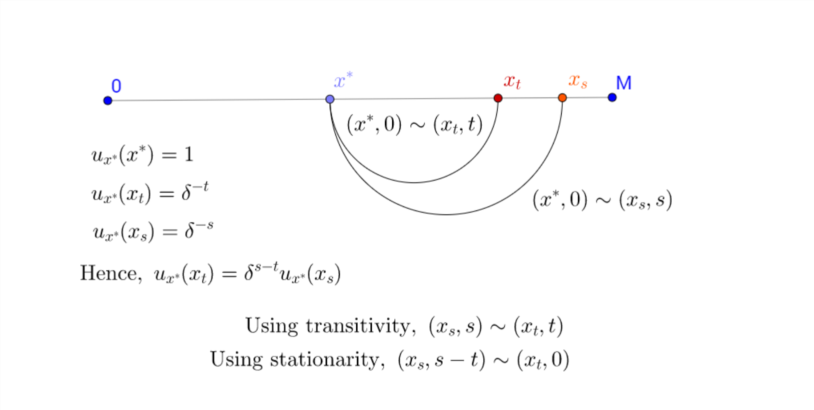
Skip to anomalies section
Uniqueness results

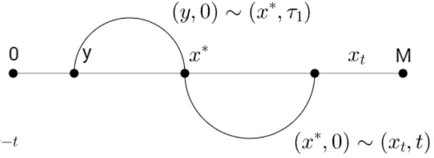
Construction under Stationarity

Fix $u_{x^*}(x^*) = 1$, $u_{x^*}(0) = 0$.



Construction under Stationarity





$u(x_t) = \delta^{-t}$

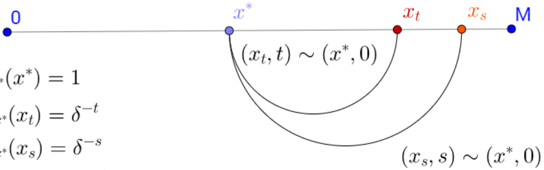
$u(y) = \delta^{\tau_1}$

$\delta^{t+\tau_1} u(x_t) = u(y)$

Under Stationarity $(x_t, t + \tau_1) \sim (x^*, \tau_1) \sim (y, 0)$

Hence, $\delta^{t+\tau_1} u(x_t) = u(y)$ works perfectly

Construction under WPB



$u_{x^*}(x^*) = 1$

$u_{x^*}(x_t) = \delta^{-t}$

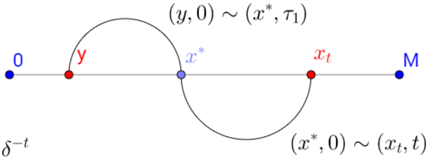
$u_{x^*}(x_s) = \delta^{-s}$

$u_{x^*}(x_t) = \delta^{s-t} u_{x^*}(x_s)$

$(x_s, s) \sim (x_t, t)$, hence, by WPB $(x_t, 0) \succsim (x_s, s-t)$

Hence u_{x^*} assigns a higher present equivalent to $(x_s, s-t)$

Construction under WPB



$$u_{x^*}(x_t) = \delta^{-t}$$

$$u_{x^*}(y) = \delta^{\tau_1}$$

$$\delta^{t+\tau_1} u_{x^*}(x_t) = u(y)$$

$$(x_t, t+\tau_1) \succsim (x^*, \tau_1) \sim_(y, 0)$$

WPB

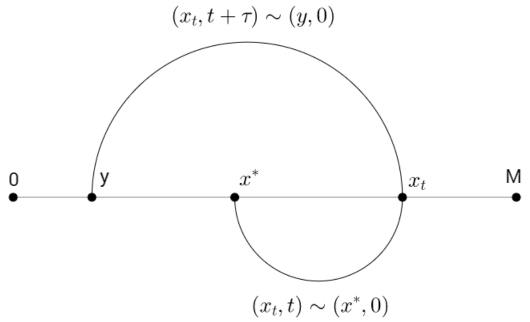
The present equivalent assigned by $u_{x^*}()$ to $(x_t, t+\tau_1)$

is y which is lower than its actual one according to $\sim$

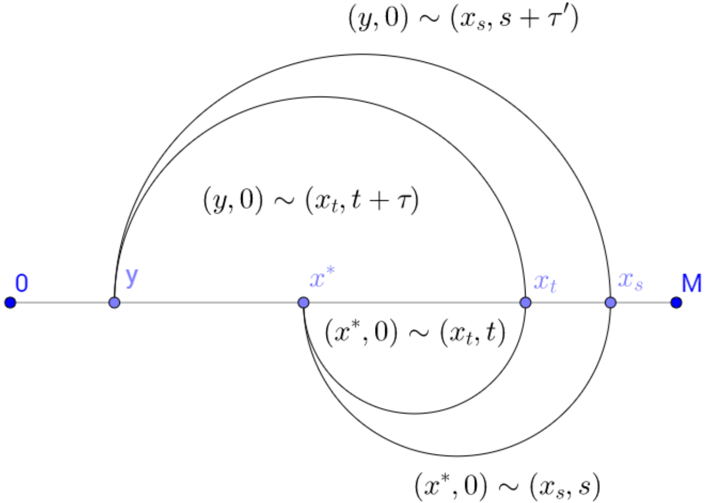
Solution

Same construction on the right of x^* as before.

$\delta^t u_{x^*}(x_t) = u_{x^*}(x^*)$ for all $(x_t, t) \sim (x^*, 0)$. Fix y.



Solution



Construction of $\mathcal{U}_\delta$

Now, for $y \in (0, x^*)$, define

$$u_{x^*}(y) = \min\{\delta^\tau : \text{ There exists } t \text{ such that } (x_t, t+\tau) \sim (y, 0)\}$$

- Minimum exists.

Construction of $\mathcal{U}_\delta$

- Constructed $u_{x^*}()$ is an increasing utility function on $[0, M]$ which has $\delta^\tau u_{x^*}(x) \geq u_{x^*}(y)$ if $(x, \tau) \sim (y, 0)$. Additionally it would also have $\delta^t u_{x^*}(x_t) = u_{x^*}(x^*)$ for all $(x_t, t) \sim (x^*, 0)$.
- Choose $\mathcal{U}_\delta = \{u_{x^*}(.) : \ x^* \in (0, M]\}$ to complete the proof.
- All utility functions in $\mathcal{U}_\delta$ assign either greater or exact present equivalents, and by construction there is atleast one function u_z that assigns exact present equivalent z for any $(x, t) \sim (z, 0)$.
- Hence the minimum of present equivalents represents the relation.
- Skip to anomalies section

Uniqueness of set of utilities

- Any set of utilities $\mathcal{U}$ and its convex hull have the same minimum representation: Only extreme tastes matter when extreme caution is practised.
- Any $\mathcal{U}$ and its closure have the same representation: The representation is continuous in the set of functions.
- If the two sets $\mathcal{U}, \mathcal{U}'$ have the same convex closure and there is a minimum representation for both of those sets, then, $\min_{u \in \mathcal{U}} u^{-1}(\delta^t u(x)) = \min_{u \in \mathcal{U}'} u^{-1}(\delta^t u(x))$.

Uniqueness of set of utilities

Definition

f is concave relative to g if $f \circ g^{-1}$ is concave.

Alternatively, $\frac{f''(x)}{f'(x)} \geq \frac{g''(x)}{g'(x)}$ or, $\frac{xf''(x)}{f'(x)} \geq \frac{xg''(x)}{g'(x)}$.

- If $u_1, u_2 \in \mathcal{U}_\delta$ and u_1 is concave relative to u_2, then, $\min_{u \in \mathcal{U}_\delta}(u^{-1}(\delta^t u(x))) = \min_{u \in \mathcal{U}_\delta \setminus u_2}(u^{-1}(\delta^t u(x)))$.

Details on Uniqueness Comparative Present Premium

Roadmap

Theory

- Main Theorem
- Major take aways

Anomalies

- Anomaly 1: Stake dependence
- Anomaly 2: Risk-Time relations

Conclusion and possible extensions

Anomaly 1: Stake dependence Example

$$
\begin{aligned}
\$100 \text{ today} &\sim \$110 \text{ in a week} \\
\$100 \text{ in 4 weeks} &\sim \$110 \text{ in 5 weeks} \\
\$10 \text{ today} &\succ \$11 \text{ in a week} \\
\$11 \text{ in 5 weeks} &\succ \$10 \text{ in 4 weeks}
\end{aligned}
$$

- Both pairs of DM's choices are consistent with Weak Present Bias (hence the choices can be supported by a minimum representation), but there is a classical choice reversal (or a local violation of Stationarity) only in the last pair.
- Evidence of such behavior in Halevy [2015]. Inconsistent with all existing models of Present Bias.
- Cognitive Optimization: If Present Bias is a cognitive phenomenon, people might be able to fight it off better when larger stakes are involved.

Anomaly 2: Risk-Time relations

- For the preference reversal $(100, 0) \succ (110, 4)$ and $(110, 30) \succ (100, 26)$, a $\beta - \delta$ model would suggest the equations

$$\begin{aligned} \beta\delta^4 u(110) &< u(100) \\ \beta\delta^{30} u(110) &> \beta\delta^{26} u(100) \end{aligned}$$

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- What would happen if all the choices now come with only probability .5?
- When coupled with Expected Utility, multiplication on both sides with the same probability, keeps the inequalities unchanged, suggesting the same reversal behavior as above. We get clear testable predictions.

$$\begin{aligned} .5\beta\delta^4 u(100) &< .5u(100) \\ .5\beta\delta^{30} u(110) &> .5\beta\delta^{26} u(100) \end{aligned}$$

Anomaly 2: No present bias without certainty

- In absence of certainty, present bias often disappears/ diminishes. Violations of separability
- The evidence is inconsistent with models like β-δ but consistent with the following justification:

Anomaly 2: No present bias without certainty

- In absence of certainty, present bias often disappears/ diminishes. Violations of separability
- The evidence is inconsistent with models like β-δ but consistent with the following justification:

- The future is inherently uncertain. Bias for the present is driven by the certainty of the present.
- But, this is really close in concept to the minimal functional written on the domain (x, p, t): $F(x, p, t) \equiv \min_{u \in \mathcal{U}}(u^{-1}(p\delta^t u(x)))$.
- The functional would favorably evaluate when all the present-certainty equivalents are equal, i.e, when $t = 0$ and $p = 1$.

Roadmap

Theory

- Main Theorem
- Major take aways

Anomalies

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Conclusion and possible extensions

Extension to streams

- Representation 1:

$$F(x_0, x_1, .., x_{T-1}) = \min_{u \in \mathcal{U}_\delta} u^{-1}\big(\sum_{0}^{T-1} \delta^t u(x_t))\big)$$

- This would tie present bias with violation of additivity (habit formation?), and potentially "resolve" taste uncertainty right away after the current period.
- Alternative Representation:

$$F(x_0, x_1, .., x_{T-1}) = x + \sum_{1}^{T-1} \min_{u \in \mathcal{U}_\delta} u^{-1}(\delta^t u(x_t))$$

Theorem

DM's preferences $\succsim$ are defined over $[0,\infty)^T$, the set of all consumption streams of finite length $T > 1$.

For any $\delta \in (0,1)$, there exists a set $\mathcal{U}_\delta$ of monotonically increasing continuous functions such that

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represents the binary relation $\succsim$.

Impose axioms that would imply the previous axioms on the sub-relation over streams which are positive only over a single-period. More Details

D5: STRONG ADDITIVITY: For any pair of orthogonal consumption bundles $(x_0, x_1, ..x_{T-1})$, $(y_0, y_1, ..y_{T-1}) \in [0,\infty)^T$, if, $(x_0, x_1, ..x_{T-1}) \sim (z_0, 0, .., 0)$ and $(y_0, y_1, ..y_{T-1}) \sim (z_0', 0, .., 0)$, then, $(x_0 + y_0, x_1 + y_1, ..x_{T-1} + y_{T-1}) \sim (z_0 + z_0', 0, .., 0)$.

Conclusion

- We introduce a novel axiom for Weak Present Bias.
- We provide the most general class of utilities that is consistent with present-biased behavior, and does not impose any extraneous behavior on the decision maker.

Conclusion

- We introduce a novel axiom for Weak Present Bias.
- We provide the most general class of utilities that is consistent with present-biased behavior, and does not impose any extraneous behavior on the decision maker.

- Anomalies that our model can explain that existing models cannot.
- Stake dependent Present Bias, Time-risk relations
- Non-standard implications in terms of policy.

Thank you

Movie tickets

- DM gets a coupon to watch a free movie, over the next four Saturdays.
- Theater is showing a mediocre movie on week 1, a good movie on week 2, a great movie on week 3 and Forrest Gump on week 4.
- DM perceives the quality of these movies as $30, 40, 60$ and 90 on a scale of $0-100$.

Dynamic decision-making problem

- He has to redeem the coupon an hour before the movie starts.
- His free ticket is issued subject to availability of tickets, and if there are no available tickets, the coupon is wasted.
- The DM can make a decision maximum 4 times, at $\tau = 1, 2, 3, 4$ (weeks).

Time inconsistency with time-risk preferences

Utility at calendar time τ from watching a movie of quality x with probability p at calendar time $t+\tau$(in weeks):

$$U^{\tau}(x,p,\tau+t) \quad = \quad \begin{cases} p^{100}(.36)^t x & \text{for } p^{100}(.36)^t \geq (.36)^{\frac{1}{2}} \\ \left(\frac{.36}{.99}\right)^{\frac{1}{2}} p(.99)^t x & \text{for } p^{100}(.36)^t < (.36)^{\frac{1}{2}} \end{cases}$$

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- Long run weekly discount factor $\beta = .99$ after a delay of half a week, or, $p < (.36)^{1/200} = (.99)^{\frac{1}{2}}$.
- Short run weekly discount factor $\alpha = (.99)^{100} \approx .36$.

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- These preferences fall under my representation and have the time-risk relation feature from Keren and Roelofsma [1995].
- Back to Welfare implications

Time inconsistency

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Time inconsistency

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- Short run weekly discount factor $\alpha = .36$.

- Quality of movies on weeks $1:4$ are $30,\ 40,\ 60$ and 90 on a scale of $0-100$.
- Optimal decision from a long run perspective (Period 0): To wait.

Time Inconsistency

- We will study the game under 2 conditions, 1) when demand of tickets are low ($p = 1$), and 2) when demand for tickets are high. ($p = .99$)

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- The outcome of the dynamic game would depend on the beliefs the subjects have about their future preferences.

Time Inconsistency

- We will study the game under 2 conditions, 1) when demand of tickets are low ($p = 1$), and 2) when demand for tickets are high. ($p = .99$)
- The outcome of the dynamic game would depend on the beliefs the subjects have about their future preferences.
- One could be aware of his time inconsistency of future preferences (**sophistication**).

Equilibrium notion for sophisticates

- **A Perception Perfect Strategy for sophisticates** is a strategy $s^s = (s_1^s, s_2^s, s_3^s, s_4^s)$, such that such that for all $t < 4$, $s_t^s = Y$ if and only if $U^t(t) \geq U^t(\tau')$ where $\tau' = \min_{\tau > t}\{s_\tau^s = Y\}$.
- Sophisticates care about the earliest period in which they would cash the coupon if they do not cash it right now.

Huge inefficiency from long run perspective for $p = 1$

		t				s_τ^s
		1	2	3	4	
τ	4				90	Y
	3			60	54.2	Y
	2		40	36.1	53.6	Y
	1	30	24	35.8	53	Y

$$p = 1$$

$$U^0(30, 1, 1) = 18, \quad U^0(90, 1, 4) = 53$$

Higher efficiency when $p = .99$

		t				s^s_τ
		1	2	3	4	
τ	4				54.2	Y
	3			36.1	53.6	N
	2		24	35.8	53	N
	1	18	24	35.8	52.57	N

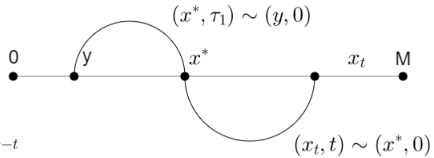
$p = .99$

$U^0(30, 1, 1) = 18 < U^0(90, .99, 4) = 52$ Second best

$U^0(90, 1, 4) = 53$ Global best

- Back to Welfare implications

Construction Question



$$u(x_t) = \delta^{-t}$$

$$u(y) = \delta^{\tau_1}$$

$$\delta^{t+\tau_1} u(x_t) = u(y)$$

$$(x_t, t+\tau_1) \succsim (x^*, \tau_1) \sim (y, 0)$$

Therefore, if the $\succsim$ is actually $\succ$, then, there would exist $y' > y$ such that $(x_t, t+\tau_1) \sim (y', 0)$ and $\delta^{t+\tau_1} u(x_t) < u(y')$

- Back to construction

Non-uniqueness of δ

- Consider the famous Rubinstein-Stahl Bargaining game with infinite horizon. When agents have utility function $u(x,t) = \delta^t x$, the model predicts an SPNE with immediate agreement over the split $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$.
- Utility functions are unique upto increasing transformations, hence, it would be equivalent to imagine the same game with agents having preferences $u(x,t) = (\sqrt{\delta})^t \sqrt{x}$.
- δ is not uniquely identified in this case too.
- Back to Minimum fn

Uniqueness of discount function

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- Consider a general discounting function $\delta(t) \neq \delta^t$.
- Could we have an alternative representation $\min_{u \in \mathcal{U}} u^{-1}(\delta(t)u(x))$, where $\delta(t)$ is decreasing in t, $\delta(0) = 1$.

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- Does treat the present and future differently.

Uniqueness of discount function

Theorem

Given the axioms A0-4, the representation form is unique in the discounting function $\delta(t) = \delta^t$ *inside the present equivalent function in* $\min_{u \in \mathcal{U}} u^{-1}(\delta(t)u(x))$.

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Given the axioms A0-4, the representation form is unique in the discounting function $\delta(t) = \delta^t$ inside the present equivalent function in $\min_{u \in \mathcal{U}} u^{-1}(\delta(t)u(x))$.

- Stationarity is a special case of the Weak Present Bias Axiom, and it is embedded in it.
- Back to Uniqueness

Comparative present premium

- For any discount factor δ , we can find a set of functions $\mathcal{U}_\delta$.
- For $\alpha, \delta \in (0,1)$, if $(\delta, \mathcal{U}_\delta)$ is a representation of $\succsim$,then so is $(\alpha, \mathcal{F}_\alpha)$,where $v \in \mathcal{F}_\alpha$ for $v = u^{\frac{\log \beta}{\log \delta}}$ for some $u \in \mathcal{U}$.

Comparative present premium

- Goal: Define comparative present premium in a model-free or context-free way.

Comparative present premium

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Definition

$\succsim_1$ allows a higher premium to the present than $\succsim_2$ if for all $x, y \in \mathbb{X}$ and $t \in \mathbb{T}$

$$(x, t) \succsim_1 (y, 0) \implies (x, t) \succsim_2 (y, 0)$$

Comparative present premium

Theorem

Let $\succsim_1$ and $\succsim_2$ be two binary relations which allow for minimum representation with respect to sets $\mathcal{U}_{\delta,1}$ and $\mathcal{U}_{\delta,2}$ respectively. The following two statements are equivalent:

i) $\succsim_1$ allows a higher premium to the present than $\succsim_2$.

ii) Both $\mathcal{U}_{\delta,1}$ and $\mathcal{U}_{\delta,1} \cup \mathcal{U}_{\delta,2}$ provide minimum representations for $\succsim_1$.

- Back to Uniqueness

Axioms $\Longrightarrow$ Representation

- Consider $\mathbb{T} = \mathbb{R}_+$. Now, we will outline the direction from Axioms to the representation.

Axioms $\implies$ Representation

- Consider $\mathbb{T} = \mathbb{R}_+$. Now, we will outline the direction from Axioms to the representation.

- For any (z, τ), there are immediate prizes that are weakly better and weakly worse: $(z, 0) \succsim (z, \tau) \succsim (0, 0)$.

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- Continuity: There exists a unique $x \in [0, M]$ such that $(z, \tau) \sim (x, 0)$.

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- Define $V : \mathbb{X} \times \mathbb{T} \to \mathbb{R}_+$ as, $V(z, \tau) = x$, if $(z, \tau) \sim (x, 0)$. (Present-equivalence representation)

Axioms $\implies$ Representation

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- Define $V : \mathbb{X} \times \mathbb{T} \to \mathbb{R}_+$ as, $V(z, \tau) = x$, if $(z, \tau) \sim (x, 0)$. (Present-equivalence representation)
- We will show that there exists a set of utilities such that the previously defined function can be rewritten as

$$V(z, \tau) = x \quad = \quad \min_{u \in \mathcal{U}_\delta} u^{-1}(\delta^\tau u(z))$$

Construction of $\mathcal{U}_\delta$

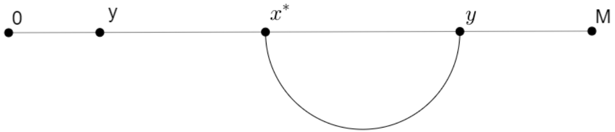
- For $(z,\tau) \sim (x,0)$, we need $\min_{u\in\mathcal{U}_\delta} u^{-1}(\delta^\tau u(z)) = x$, that is,

$$
\begin{aligned}
(z,\tau) \sim (x,0) \quad &\iff \quad \min_{u\in\mathcal{U}_\delta} u^{-1}(\delta^\tau u(z)) = x \\
&\iff \quad u^{-1}(\delta^\tau u(z)) \geq x \ \forall u \in \mathcal{U}_\delta \\
&\qquad \text{and } u_x^{-1}(\delta^\tau u_x(z)) = x \text{ for some } u_x \in \mathcal{U}_\delta
\end{aligned}
$$

- This is what is required of the constructed set of utility functions.
- We are going to provide an algorithm of constructing such functions. For arbitrary $x^* \in (0, M]$, we will construct a $u_{x^*}(.)$, which will have $u(x^*) = \delta^t u(y)$ for all $(y,t) \sim (x^*, 0)$ and the property above.

Construction on the right of x^*

Fix $u_{x^*}(x^*) = 1$, $u_{x^*}(0) = 0$.



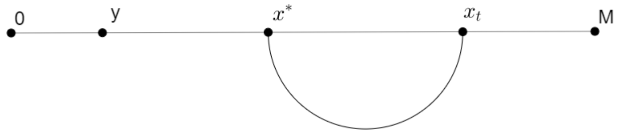
$$(y, t) \sim (x^*, 0)$$

$$\text{Define } u_{x^*}(y) = \delta^{-t}$$

$$\text{Therefore } \delta^t u_{x^*}(y) = 1 = u_{x^*}(x^*)$$

Construction on the right of x^*

Any point y to the right of x^* can be re-labelled as x_t for some t, such that $(x_t, t) \sim (x^*, 0)$.



$$(x_t, t) \sim (x^*, 0)$$

Define $u_{x^*}(x_t) = \delta^{-t}$

Therefore $\delta^t u_{x^*}(x_t) = 1 = u_{x^*}(x^*)$

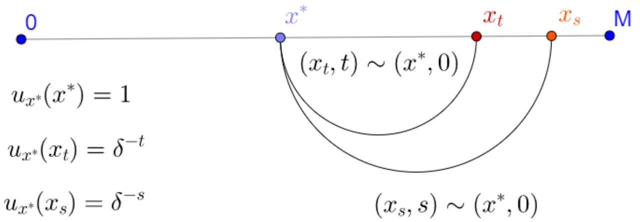
Go back

Construction on the right of x^*

For all prizes (y,τ) which have a present equivalent of $(x^*,0)$,

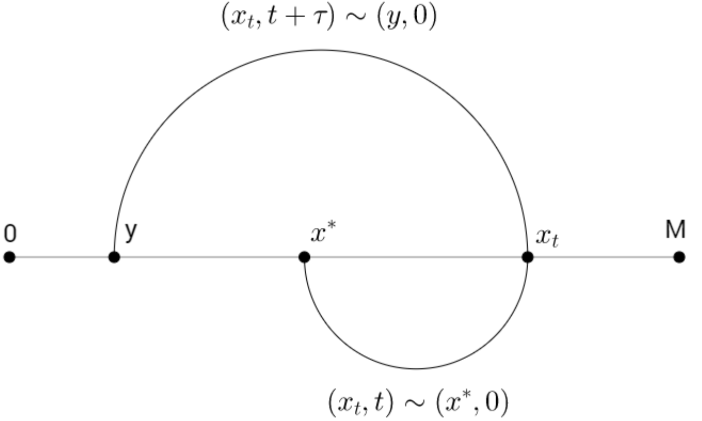
$$\delta^\tau u_{x^*}(y) = u_{x*}(x^*), \text{ or, } u_{x^*}^{-1}(\delta^\tau u_{x^*}(y)) = x^*.$$

$$u_{x^*}(x) = \{\delta^{-t(x)} : (x, t(x)) \sim (x^*, 0)\} \text{ for } x > x^*$$

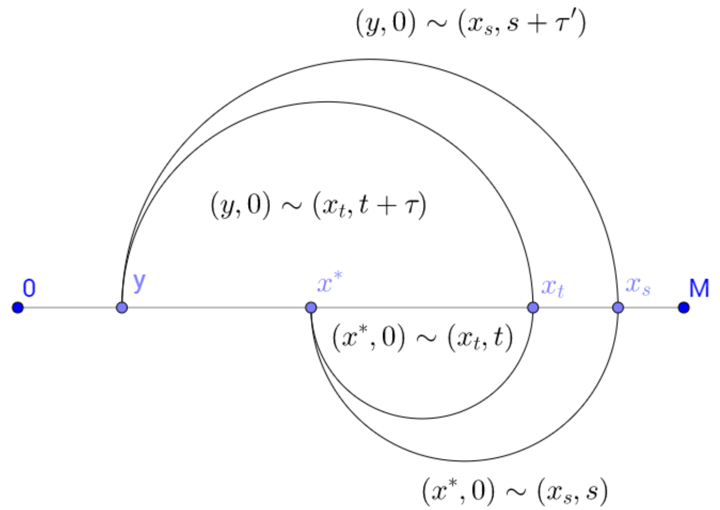


Construction on the left of x^*

Fix a point y to the left of x^*.



Construction on the left of x^*



Construction on the left of x^*

Now, for $y \in (0, x^*)$, define

$$u_{x^*}(y) = \min\{\delta^\tau : \text{ There exists } t \text{ such that } (x_t, t+\tau) \sim (y, 0)\}$$

Questions about Asymmetric Construction

Construction

- We additionally need to show that for any $(x, \tau) \sim (y, 0)$, we have $\delta^\tau u_{x^*}(x) \geq u_{x^*}(y)$.
  There are three cases depending on the relative postions of x and y with respect to x^*.
- The first case $x > y > x^*$ means that both x, y are to the right of x^*.
- We will show this case, the other cases follow similarly.

Construction

Let $x > y > x^*$ and $(x, \tau) \sim (y, 0)$. Show diagram

Need to show, $\delta^\tau u_{x^*}(x) \geq u_{x^*}(y)$.

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Need to show, $\delta^\tau u_{x^*}(x) \geq u_{x^*}(y)$.

Let, $(y, t_1) \sim (x^*, 0)$ and consequently $u(y) = \delta^{-t_1}$.

Applying WPB on $(x, \tau) \sim (y, 0)$ with delay of t_1 yields

$$(x, \tau + t_1) \precsim (y, t_1) \sim (x^*, 0)$$

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Hence, x must have to be delayed further than $\tau + t_1$ to make it indifferent to $(x^*, 0)$.

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Hence, x must have to be delayed further than $\tau + t_1$ to make it indifferent to $(x^*, 0)$.

Let, $(x, t_2) \sim (x^*, 0)$, and consequently, $u_{x^*}(x) = \delta^{-t_2}$

$$\begin{aligned}
\tau + t_1 &\leq t_2 \\
\iff \tau - t_2 &\leq -t_1 \\
\iff \delta^\tau . \delta^{-t_2} &\geq \delta^{-t_1} \\
\iff \delta^\tau u_{x^*}(x) &\geq \delta^{-t_1} = u_{x^*}(y)
\end{aligned}$$

Construction of $\mathcal{U}_\delta$

- We constructed an increasing utility function u_{x^*} on $[0, M]$ which would have $\delta^\tau u_{x^*}(x) \geq u_{x^*}(y)$ if $(x, \tau) \sim (y, 0)$. Additionally it would also have $\delta^t u_{x^*}(x_t) = u_{x^*}(x^*)$ for all $(x_t, t) \sim (x^*, 0)$.
- Choose $\mathcal{U}_\delta = \{u_{x^*}(.) : x^* \in (0, M]\}$ to complete the proof.
- Cerreia-Vioglio study

Present Bias, Allais paradox

- Risk and time create similar effects
- Reversals caused by loss of certainty/ present premium

Back to Anomaly2

	Prospect A	Prospect B	% chosing A	% chosing B	N
1	(100,1,0)	(110,1,4)	82%	18%	60
2	(100,1,26)	(110,1,30)	37%	63%	60
3	(100,.5,0)	(110,.5,4)	39%	61%	100
4	(100,.5,26)	(110,.5,30)	33%	67%	100

More evidence against risk time separability

- Andreoni and Sprenger [2012] find evidence against existing temporal models that are separable in time and risk.
- Baucells and Heukamp [2010]

	Prospect A	Prospect B	% chosing A	% chosing B	N
1	(9,1,0)	(12,.8,0)	58%	42%	142
2	(9,1,3)	(12,.8,3)	43%	57%	221

- Back to slides

Accounting for Anomaly 2

- Identification relation for δ: $(x, p^*, 0) \sim (x, 1, 1) \Longrightarrow \delta = p^*$.
- (B4) WEAK PRESENT BIAS: If $(y, 1, t) \succsim (x, 1, 0)$ then, $(y, 1, t + t_1) \succsim (x, 1, t_1)$

PTT

- B5: PROBABILITY-TIME TRADEOFF: For all $x, y \in \mathbb{X}$, $p \in (0,1]$, and $t, s \in \mathbb{T}$, $(x, p\theta, t) \succsim (x, p, t+\Delta) \implies (y, q\theta, s) \succsim (y, q, s+\Delta)$.
- Time and Risk have a similar and uniform effect on behavior.
  Evidence
- Also proposes the following estimation method for discount factor: $(x, 0, 1) \sim (x, \delta, 0)$.

Back to Anomaly2

Representation II

Theorem

The following statements are equivalent:

i) $\succsim$ *is complete, transitive, satisfies continuity, monotonicity, WPB, B5.*

ii) There exists a ***unique*** $\delta \in (0,1)$ *and a set* $\mathcal{U}$ *of monotinically increasing continuous functions such that* $F(x,p,t) \equiv \min_{u\in\mathcal{U}}(u^{-1}(p\delta^t u(x)))$. $F(x,p,t)$ *is continuous. Additionally,* $u(0) = 0$, $u(M) = 1$.

Example

Consider $\mathcal{U}_\delta = \{u_1, u_2\}$, where, $a = .99$, $b = .00021$, $\delta = .91$.

$$\begin{aligned} u_1(x) &= x^a \text{ for } a > 0 \\ u_2(x) &= 1 - \exp(-bx) \text{ for } b > 0 \end{aligned}$$

$V(x, p, t) = \min_{u \in \mathcal{U}} u^{-1}(p\delta^t u(x))$

- It is not difficult to find a subset of $\mathcal{U}$ from simple parametric families to fit choice data.

Allais Paradox and risk-time relations

$$
\begin{aligned}
V(100, 1, 0) &> V(110, 1, 1) \\
V(100, 1, 4) &< V(110, 1, 5)
\end{aligned}
$$

$$
\begin{aligned}
V(100, .5, 0) &< V(110, .5, 1) \\
V(100, .5, 4) &< V(110, .5, 5)
\end{aligned}
$$

- Rows 1 and 2 Present Bias, 1 and 3 Allais Paradox, 1-2 vs 3-4 time-risk relations

Proof

For all $x \in \mathbb{R}_+$, and for any $y \in \mathbb{R}_+$, $x \leq u_y(x) \leq \dfrac{x}{\beta}$. As u_y is an increasing function, it must be that $x \geq u_y^{-1}(x) \geq \beta x$. Since, $u_y(x) \geq x$, we get $\delta^t u_y(x) \geq \delta^t x$, which implies,

$$u_y^{-1}(\delta^t u_y(x)) \geq u_y^{-1}(\delta^t x) \geq \beta\delta^t x$$

Finally, for $x = y$, $\delta^t u_y(x) = \delta^t x < \delta x$ and, hence, $u_y(\delta^t u_y(x)) = \beta\delta^t x$.

Therefore, $V(x, t) = \min_{y \in \mathbb{R}_+} u_y^{-1}(\delta^t u_y(x))$

Back to beta delta slide

Axioms

DM's preferences $\succsim$ are defined over $[0,\infty)^T$, the set of all consumption streams of finite length $T > 1$.

- **D0:** $\succsim$ is complete and transitive.
- **D1: CONTINUITY:** $\succsim$ is continuous, that is the strict upper and lower contour sets of each consumption stream are open w.r.t the product topology.

Axioms

D2: DISCOUNTING:

If $0 \leq s < t \leq T-1$, then

$$(0,..\underbrace{y}_{\text{in period } s},..,0) \succsim (0,..\underbrace{y}_{\text{in period } t},..,0)$$

for $y \geq 0$ with the relation being strict if and only if $y > 0$.

Further, for $y_0 > x > 0$, and for any sequences $(y^1, y^2, y^3, ..y^m)$ and $(n^1, n^2, .., n^m)$, where,

$$(0,..0,\underbrace{y^{i-1}}_{\text{in period } n^i},0..,0) \succsim (y^i, 0, .., 0)\ \forall i \in \{1, 2, ..., m\} ,$$

$0 < n^i \leq T-1$ and $\sum_1^m n^i = t$,

there exists $t \in \mathbb{N}$ such that, $y_m \leq x$.

Axioms

D3: MONOTONICITY:

For any $(x_0, x_1, ..x_{T-1}),\ (y_0, y_1, ..y_{T-1}) \in [0, \infty)^T$,
$(x_0, x_1, ..x_{T-1}) \succsim (y_0, y_1, ..y_{T-1})$ if $x_t \geq y_t$ for all $0 \leq t \leq T-1$.
The preference is strict if at least one of the inequalities is strict.

D4: WEAK PRESENT BIAS:

If $(0, .. \underbrace{y}_{\text{in period } t}, .., 0) \succsim (x, 0, .., 0)$ then,

$(0, .. \underbrace{y}_{\text{in period } t+t_1}, .., 0) \succsim (0, . \underbrace{x}_{\text{in period } t_1} ., 0)$ for all $x, y \in \mathbb{X}$ and $t, t_1 \in \mathbb{T}$.

Axioms

D5: STRONG ADDITIVITY: For any pair of orthogonal consumption bundles $(x_0, x_1, ..x_{T-1})$, $(y_0, y_1, ..y_{T-1}) \in [0, \infty)^T$, if, $(x_0, x_1, ..x_{T-1}) \sim (z_0, 0, .., 0)$ and $(y_0, y_1, ..y_{T-1}) \sim (z_0', 0, .., 0)$, then, $(x_0 + y_0, x_1 + y_1, ..x_{T-1} + y_{T-1}) \sim (z_0 + z_0', 0, .., 0)$.

Theorem

i) The relation $\succsim$ *on* $[0,\infty)^T$ *satisfies properties D0-D5.*

ii) For any $\delta \in (0,1)$*, there exists a set* $\mathcal{U}_\delta$ *of monotonically increasing continuous functions such that*

$$F(x_0, x_1, .., x_{T-1}) = x + \sum_{1}^{T-1} \min_{u \in \mathcal{U}_\delta} u^{-1}(\delta^t u(x_t))$$

represents the binary relation $\succsim$*. The set* $\mathcal{U}_\delta$ *has the following properties:* $u(0) = 0$ *and* $u(M) = 1$ *for all* $u \in \mathcal{U}_\delta$*.* $F(.)$ *is continuous.*

James Andreoni and Charles Sprenger. Risk preferences are not time preferences. *American Economic Review*, 102(7): 3357–3376, 2012.

Manel Baucells and Franz H Heukamp. Common ratio using delay. *Theory and Decision*, 68(1-2):149–158, 2010.

Simone Cerreia-Vioglio, David Dillenberger, and Pietro Ortoleva. Cautious expected utility and the certainty effect. *Econometrica*, 83(2):693–728, 2015.

Peter C. Fishburn and Ariel Rubinstein. Time preference. *International Economic Review*, 23(3):677–694, 1982. ISSN 00206598, 14682354. URL `http://www.jstor.org/stable/2526382`.

Yoram Halevy. Time consistency: Stationarity and time invariance. *Econometrica*, 83(1):335–352, 2015.

Gideon Keren and Peter Roelofsma. Immediacy and certainty in intertemporal choice. *Organizational Behavior and Human Decision Processes*, 63(3):287–297, 1995.

Efe A Ok and Yusufcan Masatlioglu. A theory of (relative) discounting. *Journal of Economic Theory*, 137(1):214–245, 2007.

Ariel Rubinstein. Economics and psychology? the case of hyperbolic discounting. *International Economic Review*, 44(4): 1207–1216, 2003. ISSN 1468-2354. doi: 10.1111/1468-2354.t01-1-00106. URL `http://dx.doi.org/10.1111/1468-2354.t01-1-00106`.