Existence and uniqueness of solutions to dynamic models with occasionally binding constraints.

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Motivation

- Models with occasionally binding constraints (OBCs) are ubiquitous.
- Few general theoretical results on the behaviour of dynamic models featuring such constraints.
- Particularly relevant to the zero lower bound (ZLB) on nominal interest rates.
- Results on determinacy that justify the Taylor principle do not apply in models with OBCs.

This paper

- "Blanchard and Kahn (1980) with occasionally binding constraints."
- Existence and uniqueness results for linear models with occasionally binding constrains, **subject to terminal conditions**. (Chiefly perfect-foresight results.)
- In NK models: multiplicity is to be expected with a Taylor rule, but price-level targeting restores determinacy.
- All of the main existence and uniqueness conditions in the paper can be checked using my DynareOBC toolkit.

General intuition for multiplicity

- Since my results are conditional on a terminal condition, my multiplicity results are **not driven by multiple steady-states**.
- Suppose agents knew the bound would be escaped after one period, so expectations of tomorrow's outcomes are linear in today's.
- Substituting in these expectation rules give a set of equations in today's variables. Equations are non-linear (due to bound) so may have two solutions!

Representation result

- Let q be the perfect-foresight path of the bounded variable without OBCs.
- Let the T columns of M contain the IRFs of the bounded variable to news shocks to it at horizons $0, \dots, T-1$.
- Suppose y solves the linear complementarity problem (LCP):
$$y \geq 0, \qquad q + My \geq 0, \qquad y'(q + My) = 0$$
- Then $q + My$ is a perfect-foresight path of the bounded variable with OBCs.
- Any solution takes this form.

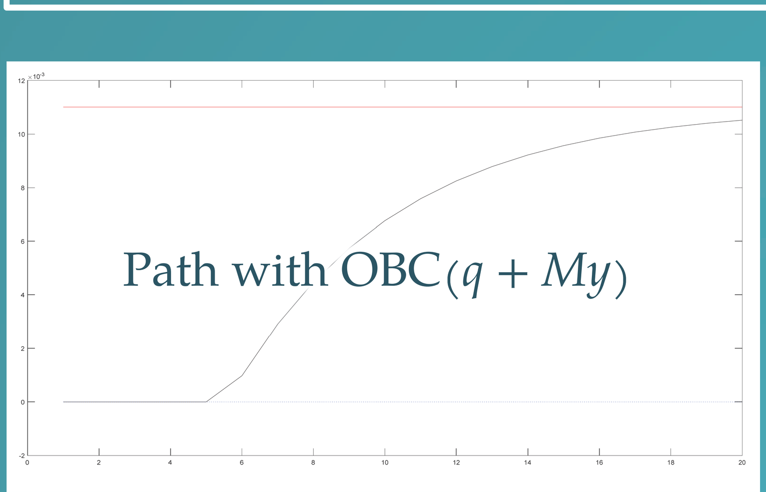
Uniqueness results

- Existence and uniqueness is determined by the properties of the M matrix.
- There is a unique solution for any q (path without OBCs) if and only if M is a P-matrix (i.e. all its principal minors are strictly positive).
- There is some q for which there are multiple solutions if M is not a P-matrix.
- The paper gives other more easily verified conditions for uniqueness.

Existence results

- The paper gives various conditions for existence of any solution, including some that do not impose any horizon by which the bound must be escaped.
- When there is no solution returning to the desired steady-state, if there is a solution at all, then it must tend to an alternative steady-state.
- If a solution does not exist in some states of the world, then agents should place positive probability on convergence to an alternative steady-state.

Graphical version of representation result



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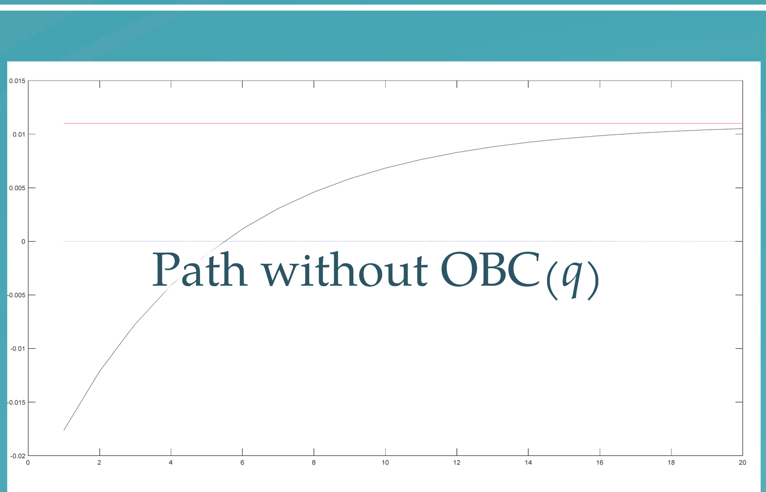

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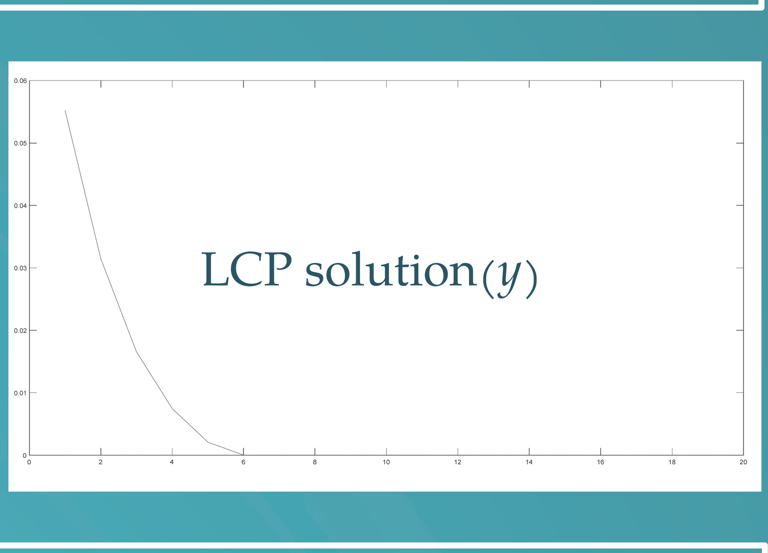
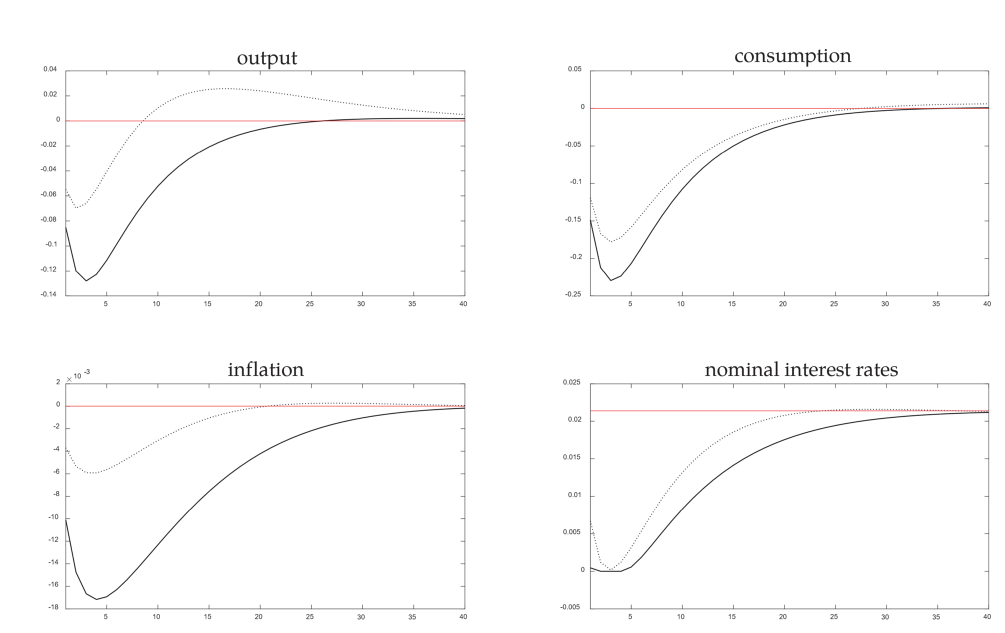


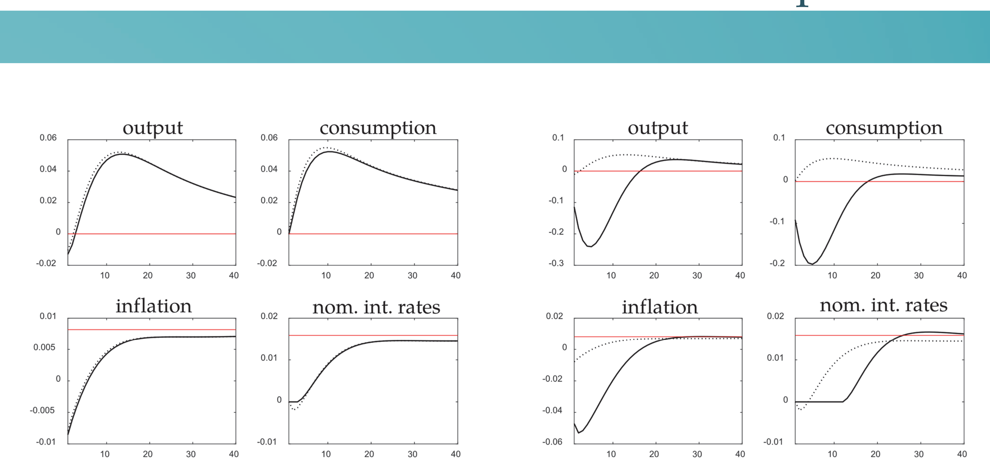
Multiplicity in New Keynesian models

- Without a response to the price-level in the monetary rule, multiplicity just requires a state variable in the model.
- E.g. consider the simple example of Brendon, Paustian & Yates (BPY) (2013):
$$i_t = \max\{0, 1 - \beta + \alpha_\Delta (y_t - y_{t-1}) + \alpha_\pi \pi_t\},$$
$$y_t = \mathbb{E}_t y_{t+1} - \sigma^{-1}(i_t + \beta - 1 - \mathbb{E}_t \pi_{t+1}), \qquad \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma y_t.$$
- Our results imply multiplicity if and only if $\alpha_\Delta > \sigma \alpha_\pi$. (Also shown by BPY.)
- Intuition: Self-fulfilling prophecy: If agents expect low inflation, and nominal rates cannot fall below zero, then real rates are high, so agents consume little. Thanks to the monetary rule, this implies low output and hence low inflation tomorrow, rationalising the initial expectations.
- Even the price-dispersion implied by a positive inflation target is enough to generate multiplicity though. Thus, it is really ubiquitous in NK models.
- Such NK models also have no perfect-foresight path returning to the positive inflation steady-state in some states of the world. Some probability must then be placed on getting stuck in the deflationary steady-state.
- Switching to a monetary rule containing an arbitrarily small term in the price-level is enough to solve these problems.

Examples of multiplicity in New Keynesian models



Two solutions after a preference shock in the Smets Wouters (2003) model.
The dotted solution does not hit the bound. Note the plausible amplification.



Two solutions after a combination of shocks in the Smets Wouters (2007) model.
Dotted lines do not impose the bound and are only for reference.