A Semiparametric Network Formation Model with Multiple Linear Fixed Effects

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- Unrestricted dependence.
- No distributional assumptions on the unobserved components.

Introduction

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Why are they important?

1. **Peer effects:** network endogeneity.
    - Goldsmith-Pinkham and Imbens 2013.
2. **Policy:** social programs.
    - Banerjee et al. 2013.
3. **Social meaning:** homophily.
    - McPherson et al. 2001.

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Implications:

- Biased and inconsistent results if these attributes are omitted.

Friendship network

Definition (Network)

A network is an ordered pair, $(\mathcal{N}_n, D^n)$, where $\mathcal{N}_n = \{1, \cdots, n\}$ is a set of nodes and $D^n = (D^n_{ij})$ is a $n \times n$ adjacency matrix.

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Assume the network is

- **Undirected:** $D^n_{ij} = D^n_{ji}$ for any $i, j \in \mathcal{N}_n$.
- **Unweighted:** $D^n_{ij} \in \{0, 1\}$ for any $i, j \in \mathcal{N}_n$.

Normalize $D^n_{ii} = 0$ for any $i \in \mathcal{N}_n$.

Example: Friendship network

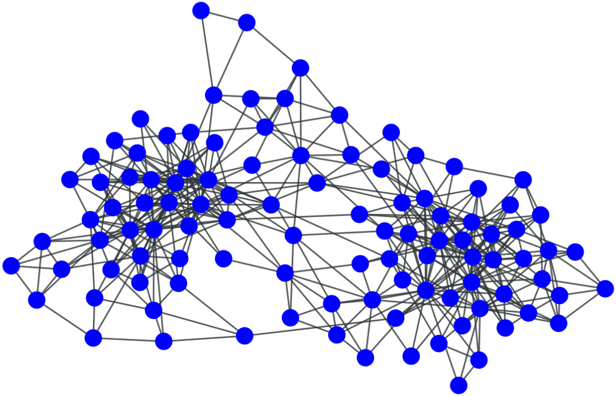

Figure: Undirected Network

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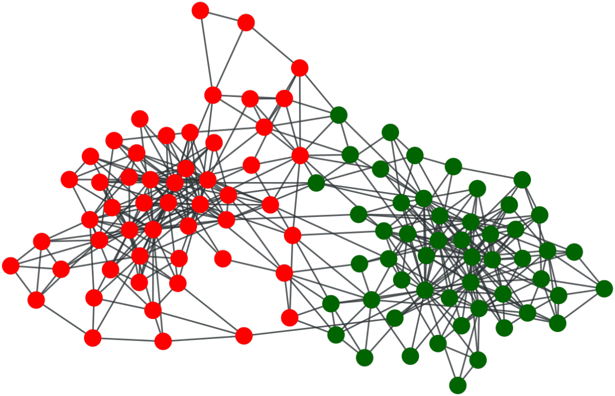

Figure: Homophily on Observed Characteristics

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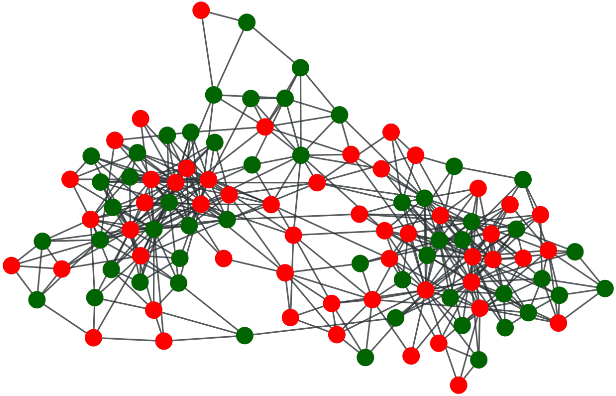

Figure: Unobserved Agent-Specific Heterogeneity

Model of Interest

Agents $i, j \in \mathcal{N}_n$ form an undirected link according to the equation:

$$D_{ij}^n = \mathbf{1}\left[X_{ij}^{n'}\beta_0 + \mu_i + \mu_j - \varepsilon_{ij}^n \geq 0\right], \tag{NF}$$

for $i \neq j$.

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- ε_{ij}^n: pair-specific exogenous factor.
- $\beta_0 \in \mathbb{R}^K$: unknown parameter.

Overview

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$$D_{ij}^n = \mathbf{1}\left[X_{ij}^{n\prime}\beta_0 + \mu_i + \mu_j - \varepsilon_{ij}^n \geq 0\right].$$

with two main features:

1. Multiple and unobserved agent-specific fixed effects:

$$\mu_i + \mu_j.$$

2. Semiparametric approach:

$$F_{\varepsilon_{ij}^n|\mathbf{x},\mu} \quad \text{and} \quad F_{\mu|\mathbf{x}} \quad \text{are unrestricted.}$$

Objective: Identification and estimation of β_0.

Main Results

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Main Results

1. New identification strategy.
   - Point identification of β_0.
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2. Semiparametric pairwise estimator.
   - Computationally tractable.
   - Asymptotics: growing number of agents.
3. Empirical application.
   - Friendship network: Add Health dataset.
   - Evidence for homophily on age, Hispanic, and father's education.

Literature Review

I. Network Formation.

- **Observed Heterogeneity:** Brock and Durlauf (2005), Christakis, Fowler, Imbens, and Kalyanaraman (2010), Sheng (2012), Boucher and Mourifié (2013), Chandrasekhar and Jackson (2014), Souza (2014), Leung (2015a,b, 2016), Menzel (2015), Ridder and Sheng (2015), Hsieh and Lee (2016), de Paula, Richards-Shubik, and Tamer (2017), and Mele (2017).
- **Unobserved Heterogeneity:** Goldsmith-Pinkham and Imbens (2013), Charbonneau (2014), Auerbach (2016), Dzemski (2017), Graham (2017) and Jochmans (2017).

II. Semiparametric Methods.

- **Identification:** Andersen (1973), Manski (1985, 1987) and Vytlacil and Yildiz (2007).
- **Maximum Rank:** Han (1987) and Abrevaya (1999).
- **Inference:** Andrews and Schafgans (1998), Newey (1990), Chamberlain (2010), Khan and Tamer (2010) and Khan and Nekipelov (2015).

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1. Network Formation Model

2. Identification

3. Inference

4. Simulations

5. Application

6. Conclusions and Extensions

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   - Cardinality: $N \equiv \#\mathcal{N}_n^{(2)} = O(n^2)$.

   - Each $(i, j) \in \mathcal{N}_n^{(2)}$ is endowed with X_{ij}^n, and let

$$\mathbf{X}^n \equiv \left(X_{12}^n, \cdots, X_{n-1,n}^n\right).$$

Model - Preferences

Agent i's latent marginal benefit of adding the link $\{ij\}$ to D^n is

$$V_{ij}(\mathbf{X}^n, \eta_{ij}; \beta_0) = u_{ij}(\mathbf{X}^n; \beta_0) + \eta_{ij}.$$

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Remarks:

Rules out:

- Network externalities: $u_{ij}(\mathbf{X}^n, D^n; \beta_0)$.
- Unobserved complementarity: $g(\mu_i, \mu_j)$ as in Candelaria (2016).

Model - Stability Condition

A network D^n is stable with transfers if for each $(i, j) \in \mathcal{N}_n^{(2)}$:

1. for all $D_{ij} = 1$, $V_{ij}(\mathbf{X}^n, \eta_{ij}; \beta_0) + V_{ji}(\mathbf{X}^n, \eta_{ji}; \beta_0) \geq 0$;
2. for all $D_{ij} = 0$, $V_{ij}(\mathbf{X}^n, \eta_{ij}; \beta_0) + V_{ji}(\mathbf{X}^n, \eta_{ji}; \beta_0) < 0$.

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2. for all $D_{ij} = 0$, $V_{ij}(\mathbf{X}^n, \eta_{ij}; \beta_0) + V_{ji}(\mathbf{X}^n, \eta_{ji}; \beta_0) < 0$.

Equivalently, the network D^n is stable with transfers if:

$$D_{ij} = \mathbf{1}\left[X_{ij}'\beta_0 + \mu_i + \mu_j - \varepsilon_{ij} \geq 0\right], \quad \forall (i, j) \in \mathcal{N}_n^{(2)}. \qquad \text{(NF)}$$

Model-Assumptions

Assumption (A1)

The following hold for any n.

1. *For any distinct* $(i,k),(j,l) \in \mathcal{N}_n^{(2)}$*:*

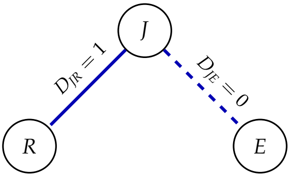
$$\varepsilon_{ik} \perp\!\!\!\perp \varepsilon_{jl} \mid \mathbf{X^n} = \mathbf{x}, \mu^n = \mu, \text{ and } F_{\varepsilon_{ik}|\mathbf{x},\mu} = F_{\varepsilon_{jl}|\mathbf{x},\mu}.$$

2. *The pdf* $f_{\varepsilon_{i1}|\mathbf{x},\mu}$ *is positive everywhere for all* $(\mathbf{x}, \mu)$*.*

- A1 used in Graham (2017) and Menzel (2015).
- Agnostic about $F_{\varepsilon_{i1}|\mathbf{x},\mu}$ and $F_{\mu|\mathbf{x}}$.

Identification Strategy

Consider the subnetwork given by $J, R, E \in \mathcal{N}_n$.



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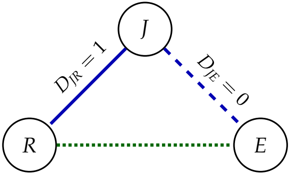

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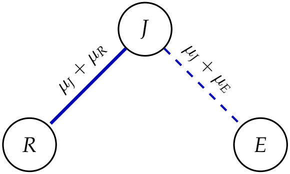
Conditional on $\{\mathbf{X}^n = \mathbf{x}, \mu^n = \mu\}$:



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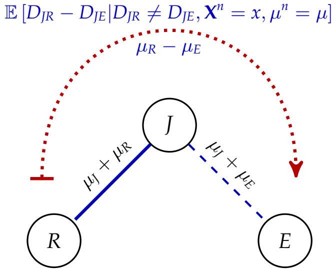
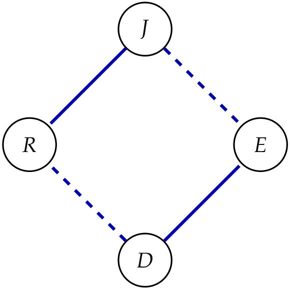


Identification Strategy

Consider the tetrad given by $\{J, R, E, D\}$.



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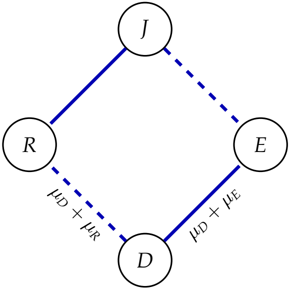

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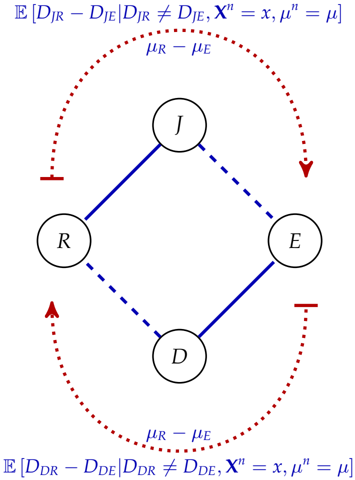

Assumptions

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Assumption (A2)

The following hold for any n, and any distinct $(i,k),(i,l) \in \mathcal{N}_n^{(2)}$.

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2. *Exists* $\Delta_{kl} X_i^{(1)}$ *with* $\beta_0^{(1)} \neq 0$ *s.t. the cond. density of* $\Delta_{kl} X_i^{(1)}$ *is positive everywhere for any* $\Delta_{kl} x_i^{(-1)}$.

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- Sign of $\beta_0^{(1)}$ is identified, and scale is normalized: $\mid \beta_0^{(1)} \mid = 1$.
- A2 used in Manski(1985,1987), Han (1987) and Abrevaya (1999).

Assumption (A3)

For any $i \in \mathcal{N}_n$*,*

$$\text{supp}(\mu_i \mid \mathbf{X}^n = \mathbf{x}) \subseteq [-B, B],$$

for any $\mathbf{x} \in \text{supp}(\mathbf{X}^n)$*, and some* $B < \infty$*.*

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- Allows for continuous or a discrete fixed effects.
- Intuitively:

$$\text{supp}(\mu_k - \mu_l \mid \mathbf{X}^n = \mathbf{x}) \subset \text{supp}(\Delta_{kl} X_i' \beta_0)$$

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- Let:

$$\mathcal{X}_B = \{\mathbf{x} \in \mathbf{X}^n : \text{ for any } \; i, j, k, l \in \mathcal{N}_n, \mid \Delta_{kl} x_i \beta_0 \mid \geq 2B, \text{ and } \\ \text{sign}\{\Delta_{kl} x_i \beta_0\} \neq \text{sign}\{\Delta_{kl} x_j \beta_0\}\}$$

Point Identification

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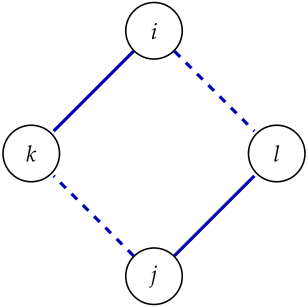

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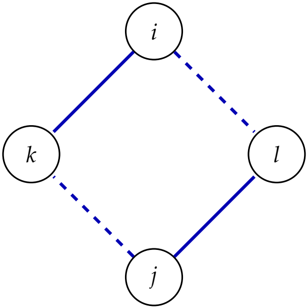
$$\begin{aligned} Y_{kl}^{(s)} &\equiv (D_{sk} - D_{sl}) \quad \text{for} \quad s = i, j, \\ \Omega(ijlk) &\equiv \{ \;\; D_{ik} \neq D_{il}, \;\; D_{jl} \neq D_{jk} \;\; \underbrace{D_{ik} \neq D_{jk}}_{\text{Across-inds}} \;\; \} \end{aligned}$$



Point Identification

Theorem

1. *Let assumptions 1-3 hold. Then, for any n, and any* $i, j, k, l \in \mathcal{N}_n$*:*

$$\text{Med}\left[Y_{kl}^{(i)} - Y_{kl}^{(j)} | \mathbf{X}^n = \mathbf{x}, \Omega(ijlk)\right] = 2 \times \text{sign}\left\{\left[\Delta_{kl} x_i - \Delta_{kl} x_j\right]' \beta_0\right\}, \tag{MC}$$

   where $\mathbf{x} \in \mathcal{X}_B$*.*

2. *Let assumptions 1-3 hold. Then* β_0 *is point identified.*

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$\Omega(ijlk)$ contains the subnetworks with enough variation to identify β_0.

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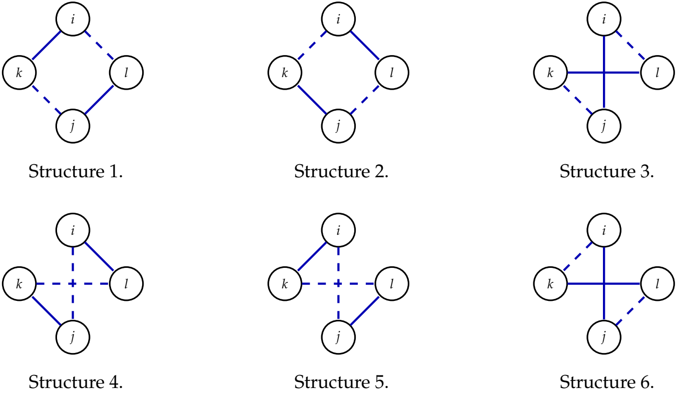

Figure: Subnetwork by the tetrad (i, j, k, l) in $\Omega(ijlk)$.

Identification Failure

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Under Ass. 1-3. If the class Ω_n has probability zero, then:

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Under Ass. 1-3. If the class Ω_n has probability zero, then:

$\text{Med}\left\{ Y_{kl}^{(i)} - Y_{kl}^{(j)} \,\middle|\, \mathbf{X}^n = \mathbf{x} \right\}$ *does not have identification power.*

Identification Failure

I. Thin Set

Let

$$\Omega_n \equiv \{\Omega(ijlk) : \text{ for any distinct } i, j, k, l \in \mathcal{N}_n\}.$$

Theorem (Thin Set)

Under Ass. 1-3. If the class Ω_n has probability zero, then:

$\text{Med}\left\{ Y_{kl}^{(i)} - Y_{kl}^{(j)} \middle| \mathbf{X}^n = \mathbf{x} \right\}$ *does not have identification power.*

- In the empirical application: $P(\Omega_n) = 2.24\%$.
- "Thin set identification" as in Khan and Tamer (2010).

Thin Set

Lemma (Sufficient Conditions)

For any n, the class Ω_n has probability zero if for any $(i,j) \in \mathcal{N}_n^{(2)}$:

1. *D^n is empty, i.e.,*

$$\operatorname{supp}\left(X_{ij}'\beta_0 \mid \mu^n = \mu, \varepsilon_{ij} = e\right) = \left(-\infty,\ \mu_i + \mu_j - e\right]$$

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2. *D^n is dense, i.e.,*

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3. *D^n is homogeneous, i.e.,*

$$\text{supp}\left(\mu_i + \mu_j \mid X_{ij} = x, \varepsilon_{ij} = e\right) = \left[\, e - x'\beta_0,\ \infty\right)$$

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   $\Rightarrow \beta_0$ is still identified

2. A2″: they are all discrete variables.

   $\Rightarrow$ Bounds for each element of β_0 are obtained.

Outline

1. Network Formation Model
2. Identification
3. Inference
4. Simulations
5. Application
6. Conclusions and Extensions

Inference

The identification condition in (MC) suggests an estimator for β_0.

Limiting objective function:

$$Q(b) \equiv 2\mathbb{E}\left[S(\mathcal{X}_B) \times \text{sign}\left\{\left[\Delta_{kl}X_i - \Delta_{kl}X_j\right]' b\right\} \times \left(Y_{kl}^{(i)} - Y_{kl}^{(j)}\right) \mid \Omega(ijlk)\right],$$

where, $S(\mathcal{X}_B) = 1$ if $\mathbf{x} \in \mathcal{X}_B$, and 0 otherwise.

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- $Q(b)$ is uniquely maximized at $b = \beta_0$.

The semiparametric pairwise difference estimator is

$$\hat{\beta}_n = \arg\max_{b \in \tilde{\mathcal{B}}} Q_n(b)$$

Inference

Given a random sample of n agents, let $\left\{z_{ij}^{n}\right\}_{(i,j)\in\mathcal{N}_n^{(2)}} = \left\{D_{ij}^{n}, x_{ij}\right\}_{(i,j)\in\mathcal{N}_n^{(2)}}$.

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The sample analog of $Q(b)$:

$$Q_n(b) \equiv \binom{n}{4}^{-1} \sum_{C_{n,4}} h(z_{i_{1,3}}, z_{i_{1,4}}, z_{i_{2,3}}, z_{i_{2,4}}, b), \qquad \text{(Qn)}$$

where $C_{n,4}$ indexes all the unique tetrads in $\{1, 2, \cdots, n\}$.

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where $C_{n,4}$ indexes all the unique tetrads in $\{1, 2, \cdots, n\}$.

Kernel function:

$$h(z_{i_{1,3}}, z_{i_{1,4}}, z_{i_{2,3}}, z_{i_{2,4}}, b) \equiv \frac{2}{4!} \sum_{P_4} \{\text{sign}\left\{[\Delta_{3,4}x_1 - \Delta_{3,4}x_2]' \, b\right\}$$
$$\times \, (y_{3,4}^{(1)} - y_{3,4}^{(2)}) \times \mathbf{1}\left\{|y_{3,4}^{(1)} - y_{3,4}^{(2)}| = 2\right\} \times S(x_{i_{1,3}}, x_{i_{1,4}}, x_{i_{2,3}}, x_{i_{2,4}}, B)\right\},$$

where P_4 denotes the 4! permutations of $\{i_{1,3}, i_{1,4}, i_{2,3}, i_{2,4}\}$.

Choice of B

1. If B is known

$$\mathcal{X}_B = \{\mathbf{x} \in \mathbf{X}^n : \text{ for any } \ i, j, k, l \in \mathcal{N}_n, \ | \Delta_{kl} x_i b | \geq 2B, \text{ and } \\ \text{sign}\{\Delta_{kl} x_i b\} \neq \text{sign}\{\Delta_{kl} x_j b\}\}$$

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2. Trimming

$$\mathcal{X}_B(\gamma_n) = \{\mathbf{x} \in \mathbf{X}^n : \text{ for any } i,j,k,l \in \mathcal{N}_n, \mid \Delta_{kl} x_i b \mid \geq \gamma_n, \text{ and } \\ \text{sign}\{\Delta_{kl} x_i b\} \neq \text{sign}\{\Delta_{kl} x_j b\}\},$$

with $\gamma_n \to \infty$.

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with $\gamma_n \to \infty$.

$$\sup_{\gamma_n \in \Gamma} \sup_{b \in \tilde{\mathcal{B}}} ||Q_n(b; \gamma_n) - \mathbb{E}[Q_n(b; \gamma_n)]|| \xrightarrow{p} 0.$$

Assumptions

Assumption (B1)

The researcher observes a random sample of n agents. For each dyad in $\mathcal{N}_n^{(2)}$, the researcher observes the link status and dyad-level attributes.

$$\left\{D_{ij}, \mathbf{x}_{ij}\right\}_{(i,j)\in\mathcal{N}_n^{(2)}} .$$

- Used in Graham (2017), Leung (2015b) and Menzel (2015).

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- Used in Graham (2017), Leung (2015b) and Menzel (2015).

Assumption (B2)

The parameter space $\tilde{\mathcal{B}}$ is compact and β_0 is an interior point of $\tilde{\mathcal{B}}$.

- Used in Han (1987), Sherman (1993, 1994) and Abrevaya (1999).

Assumptions

Assumption (B3)

Let $p_n \equiv \mathbb{P}(\Omega_n)$, *where*

1. $p_n \to p_0 \geq 0$, *as* $n \to \infty$.
2. $\sqrt{N} p_n \to \infty$, *as* $n \to \infty$.

- The probability p_n is allowed to decay as $n \to \infty$.

Consistency

Theorem (Consistency)

Let assumptions A1, A2, B1-B3 hold. Then,

$$\hat{\beta}_n - \beta_0 \xrightarrow{p} 0$$

as $n \to \infty$.

Theorem (Asymptotic Normality)

If assumptions A1, A2, B1-B4. hold, then:

$$p_n \sqrt{N}(\hat{\beta}_n - \beta_0) \xrightarrow{d} \mathcal{N}(0, V^{-1} \Delta V^{-1}) \tag{AN}$$

with

$$\begin{aligned} 4V &= \mathbb{E}\left[\nabla_2 \tau_2(\cdot, \beta_0) \mid \Omega_n\right], \\ \Delta &= \mathbb{E}\left[\nabla_1 \tau(\cdot, \beta_0)\right]\left[\nabla_1 \tau(\cdot, \beta_0)\right]'. \end{aligned}$$

Recall that $N = O(n^2)$.

Convergence Rate

The convergence rate depends on the limit of $p_n \equiv \mathbb{P}(\Omega_n)$.

1. Regular Estimator: $p_n \to \bar{p} > 0$, as $n \to \infty$.

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1. Regular Estimator: $p_n \to \bar{p} > 0$, as $n \to \infty$.
2. Irregular Estimator: $p_n \to 0$, as as $n \to \infty$.
   (Newey 1990, Andrews and Schafgans 1998 and Khan and Tamer 2010).

Theorem (Information bound)

In model given by equation (NF), *under assumptions A1, A2, B1-B4.*

*If $p_n \to 0$, then the information bound for β_0 is **zero**.*

Adaptive Rate Inference

Consider the "studentized" estimator, as in Andrews and Schafgans (1998) and Khan and Tamer (2010),

$$\hat{\Sigma}_n^{-1/2}\sqrt{N}(\hat{\beta}_n - \beta_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, I), \quad \text{as } n \to \infty$$

where $\hat{\Sigma}_n$,

$$\hat{\Sigma}_n = \hat{S}_n / \hat{p}_n^2,$$

and $\hat{S}_n$ is the Bootstrap estimate of

$$S = V^{-1} \Delta V^{-1}.$$

Subbotin (2007): Bootstrap validity for Maximum Rank estimators.

Outline

Computation

The objective function $Q_n(b)$ is a 4th order U-statistic.

- $O(n^4)$ operations.

Proposition

The estimator $\hat{\beta}_n$ can be equivalently computed from:

$$\tilde{Q}_n(b) \equiv \frac{1}{n(n-1)(n-2)} \sum_{i \neq j \neq k} S(B) Rank_{j,k} \left[(x_{ik} - x_{il})' b \right] y_{k,l}^{(i)}$$

- *$\tilde{Q}_n(b)$ can be computed in $O(n^3 log(n))$ operations.*

Simulations

Consider the following model:

$$D_{ij} = \mathbf{1}\left[X_{ij}'\beta_0 + \mu_i + \mu_j - \varepsilon_{ij} \geq 0\right], \quad \text{for } (i,j) \in \mathcal{N}_n^{(2)}.$$

1. Dyad-specific attributes, X_{ij} for $(i,j) \in \mathcal{N}_n^{(2)}$:

$$X_{ij} = \left[z_{i1}z_{j1},\ z_{i2}z_{j2},\ z_{i3}z_{j3}\right].$$

   where the individual-specific attributes are drawn as:

$$\begin{aligned} z_{i1} &\sim \text{Normal}(0,3), \\ z_{i2} &\sim \text{Uniform}\{-1,0,1\} \text{ with } p_k = 1/3, \\ z_{i3} &\sim \text{Uniform}(-2,2). \end{aligned}$$

Simulations

2. Fixed effects:

$$\alpha_i = \lambda \left(z_{i1} + z_{i2} + z_{i3} \right) /3 + (1 - \lambda)\text{Normal}(0, 1),$$

where $\lambda \in \{1/4, 1/2, 3/4\}$ measures the degree of dependence.

$$\mu_i = \begin{cases} -B & \text{if} \quad \alpha_i < -B \\ \alpha_i & \text{if} \quad -B \leq \alpha_i \leq B \\ B & \text{if} \quad B < \alpha_i \end{cases},$$

with $B = 1$.

3. Link-specific disturbance term: $\varepsilon_{ij}^{(2)} \sim \text{Normal}(0, 2)$.

True DGP: $\beta_0 = [1,\ 1.5,\ -1.5]'$

MC Simulations: Normal(0,2)

	Pairwise Difference				Graham (2015)				$P(\Omega_n)$
	Median	Mean	Bias(%)	RMSE	Median	Mean	Bias(%)	RMSE	
$N = 100$									7.914%
$\beta_2/\beta_1 = 1.5$	1.630	1.585	5.715	0.727	1.651	1.665	7.454	0.437	
$\beta_3/\beta_1 = -1.5$	-1.734	-1.702	13.613	1.836	-1.735	-1.763	15.712	0.438	
$N = 250$									7.376%
$\beta_2/\beta_1 = 1.5$	1.567	1.551	5.061	0.686	1.524	1.512	4.133	0.325	
$\beta_3/\beta_1 = -1.5$	-1.677	-1.632	7.245	1.074	-1.691	-1.674	13.128	0.325	

M=500, $\lambda = 0.5$

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$N = 500$									7.148%
$\beta_2/\beta_1 = 1.5$	1.529	1.542	4.761	0.591					
$\beta_3/\beta_1 = -1.5$	-1.572	-1.553	5.281	0.801					

M=500, $\lambda = 0.5$

Simulation: Discrete and Bounded Support:

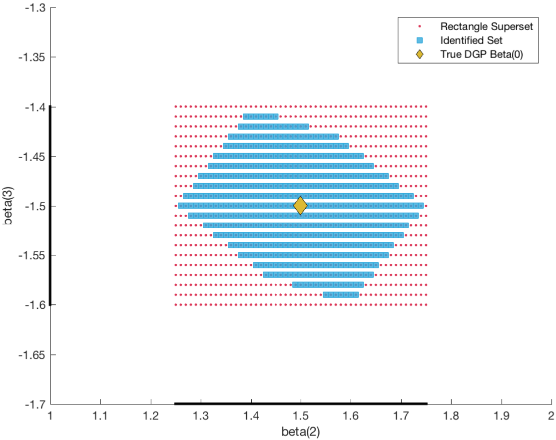
Consider the next specification for the observed covariates:

- $X_{ij}^{(1)}$ takes the values $\{0, 1, 2, 3, 4\}$.
- $X_{ij}^{(2)}$ takes the values $\{-1, 0, 1\}$.
- $X_{ij}^{(3)}$ takes the values $\{-1, 0, 1, 2\}$.

Thus, the support of X_{ij} contains 60 points.

Discrete and Bounded Support: Sharp Bounds

Figure: Bounds and Rectangular Superset



Outline

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- Observed network: availability of respondents' friendship network.
- Saturated high schools: each student nominates at most 5 male and 5 female friends.
- Wave I In-home interview: One high school with 319 students.

Exogenous Covariates

Table: Descriptive Statistics

Variable	Mean	Std. Dev.	Min	Max
Household Income	51.405	29.68	4	200
Age	15.707	1.183	14	19
Female	0.441	0.497	0	1
Grade	10.255	1.085	9	12
Hispanic	0.025	0.150	0	1
White	0.942	0.233	0	1
Black	0.006	0.079	0	1
Asian	0.014	0.121	0	1
Indian	0.029	0.170	0	1
Other races	0.036	0.187	0	1
Overall GPA	2.346	0.956	0	4
Mother's Education	4.240	2.419	0	9
Father's Education	4.147	2.794	0	9
Sample size = 469.				

Estimation Results

	Logistic	Pairwise Difference	Graham (2015)
Age	−1.245***	−0.826	−1.088
Female	−1.875***	0.635**	0.032
Grade	0.764***	1.264*	0.553*
Hispanic	0.772	1.322***	1.100***
White	−3.758***	1.661**	1.544***
Black		0.382	0.085
Asian		−1.172**	−1.491**
Indian	−0.597	−0.318	−0.742
Other races	−0.461	−0.553*	−1.061
Overall GPA	−0.102***	2.436**	2.350**
Mother Education	0.276***	−0.352*	−0.615*
Father Education	0.240***	1.549***	0.748

$P(\Omega) = 2.24\%$
Average Degree = 3.62.
Number of Students = 319.
Number of dyads = 50,721.

*,**,*** represents the significant at 10%, 5%, and 1% level.

Conclusions

1. Semiparametric network model with unobserved heterogeneity.

2. Point identification and sharp bounds for each component of β_0.

3. Semiparametric pairwise difference estimator.

4. Empirical application considers a friendship network.

Thanks!

Appendix

Covariates with Bounded Support

I. At Least One Continuous Covariate

Assumption (A2′)

The following hold for any n, and any $i, l, k \in \mathcal{N}_n$, with $l \neq k$.

1. *The random vector $\Delta_{kl} X_i$ has a bounded support on $\mathbb{R}^K$.*
2. *For some $\delta > 0$, there exists an interval $I_\delta = [-\delta, \delta]$ and a set $N_\delta \in \mathbb{R}^{K-1}$ such that*
    - *N_δ is not contained in any proper linear subspace of $\mathbb{R}^{K-1}$.*
    - $\mathbb{P}\left(\Delta_{kl}\tilde{X}_i \in N_\delta\right) > 0.$
    - *For almost every $\Delta_{kl}\tilde{x} \in N_\delta$, the distribution of $\Delta_{kl}X_i'\beta_0$ conditional on $\Delta_{kl}\tilde{X}_i = \Delta_{kl}\tilde{x}_i$ has a probability density that is everywhere positive on I_δ.*

Proposition

Let Assumptions A1, A2′, and A3 hold; then β_0 is point identified.

Covariates with Bounded Support

II. Discrete Support

I obtain sharp bounds for each component in β_0 using Komarova (2013).

Assumption (A2″)

For any n, and any $i, k, l \in \mathcal{N}_n$, with $k \neq l$.

1. *The support of $F_{X_{ik}}$ is not contained in any proper linear space of $\mathbb{R}^K$.*
2. *The profile vector of observed attributes $\mathbf{X}^n \equiv (X_{12}, \cdots, X_{n-1,n})$ has a discrete support given by*

$$\text{supp}(\mathbf{X}^n) = \left\{\mathbf{x}^1, \cdots, \mathbf{x}^D\right\},$$

for a finite D.

Thin Set

Table: Stochastic Dominance and Sparsity

	Empty		Sparse		Dense	
	E [Degree]	$P\left[\Omega_n\right]$ (%)	E [Degree]	$P\left[\Omega_n\right]$ (%)	E [Degree]	$P\left[\Omega_n\right]$ (%)
$\lambda = 0.25$						
Log	20.30	4.32	49.53	16.71	97.15	0.06
LnN	9.34	1.01	36.98	13.73	95.88	0.11
N	19.47	3.84	49.52	18.11	98.56	0.00
Gam	19.54	3.87	49.36	19.63	87.12	1.56
T	28.59	8.30	49.45	18.25	90.54	1.03
$\lambda = 0.5$						
Log	23.56	5.71	49.44	16.95	95.48	0.21
LnN	10.58	1.28	36.62	13.72	92.34	0.47
N	22.44	5.03	49.39	18.58	98.13	0.01
Gam	23.11	5.41	49.32	21.04	76.73	4.72
T	33.90	11.29	49.30	18.84	84.53	2.71
$\lambda = 0.75$						
Log	27.81	7.88	49.30	17.14	91.75	0.86
LnN	12.38	1.74	36.06	13.64	80.39	3.52
N	26.38	6.92	49.21	18.82	96.75	0.07
Gam	27.08	7.34	49.20	22.42	54.40	11.08
T	40.51	15.00	49.26	19.29	72.11	7.27

Notes: N=100, M=500.

Thin Set

Table: Thin Set Simulations: Homogeneous Network

		$\mu = 10 * \text{Bernoulli(p)} + (-5) * (1 - \text{Bernoulli(p)})$		
N=100	E [Degree]	$P[\Omega(ijkl)]$ (%) (%)	Jaccard SI (Mean) (Mean)	Cosine SI (Mean) (Mean)
$p = 0.2$				
Log	37.66	0.38	0.55	0.70
LnN	20.52	0.83	0.35	0.53
N	36.66	0.31	0.60	0.73
Gam	31.14	0.42	0.56	0.70
T	27.30	0.34	0.57	0.70
$p = 0.8$				
Log	92.56	0.12	0.87	0.93
LnN	83.46	1.16	0.74	0.85
N	95.10	0.01	0.91	0.95
Gam	94.42	0.05	0.90	0.94
T	93.26	0.10	0.88	0.93

Notes: M=500.

Identification Failure

Proposition

1. *Let assumption 1 hold; then, for any n, and any $i, l, k \in \mathcal{N}_n$.*

$$\begin{aligned} &\text{Med}(D_{ik} - D_{il} | \mathbf{X}^n = x, D_{il} + D_{ik} = 1) \\ &\qquad = \text{sign}\left[(x_{ik} - x_{il})'\beta_0 + (\mu_k - \mu_l)\right] \end{aligned} \tag{MS}$$

2. *Let Assumptions 1 and 2 hold; then, the equation* (MS) *does not have identification power.*

References I

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