Knowledge Diffusion, Trade and Innovation across Countries and Sectors

Jie Cai, Nan Li and Ana Maria Santacreu*

CLICK HERE FOR LATEST VERSION
This version: December 28, 2018

Abstract

We provide a unified framework to quantify the cross-country and cross-sector interactions between trade, innovation and knowledge spillovers. We study the effect of trade liberalization in a multi-country and multi-sector endogenous growth model in which comparative advantage and the stock of knowledge are endogenously determined by innovation and knowledge spillovers. A reduction in trade costs induces a reallocation of innovation and comparative advantage across sectors, which translates into higher growth in the counterfactual balanced growth path (BGP). Welfare gains from trade are significantly larger than in static models of trade. Heterogeneous knowledge spillovers generate dispersion in comparative advantage, becoming additional sources of growth and welfare.

Keywords: Technology Diffusion; R&D; Patent Citations; International Trade

JEL Classification: F12, O33, O41, O47

*Cai, Shanghai University of Finance and Economics, cai.jie@mail.shufe.edu.cn; Li, International Monetary Fund, address: 700 19th Street NW, Washington DC 20431, email: nanli1@gmail.com; Santacreu, Federal Reserve Bank of St. Louis, email: am.santacreu@gmail.com. We are grateful to Lorenzo Caliendo, Jonathan Eaton, Sam Kortum, Andrei Levchenko, Fernando Parro, Esteban Rossi-Hansberg, Alan Taylor, and Dan Trefler for useful comments, as well as and participants of the seminars at Yale University, 2017 and 2016 Society for Economic Dynamics, the 2017 Econometric Society Summer Meeting, IMF, Federal Reserve Bank of St. Louis, Penn State University, RIDGE, Fudan University, and Tsinghua University for helpful comments. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of St. Louis or the Federal Reserve System. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

1 Introduction

The world has increasingly become a highly interconnected network of countries and sectors that not only trade goods and services but also exchange ideas. Recently, a growing strand of the trade literature has examined how the benefits of trade liberalization may spread across sectors through production input-output linkages (Eaton et al. (2016); Caliendo and Parro (2015); Costinot, Donaldson, and Komunjer (2012)). However, countries and sectors are also linked along another dimension—innovation and knowledge spillovers (Cai and Li (2018) and Acemoglu, Akcigit, and Kerr (2016)). Knowledge in one sector of a country can be used to enhance innovation in another country-sector, and much like production input-output linkages, knowledge linkages across countries and sectors are far from uniform. Therefore, in a world with multiple sectors, while trade liberalization affects countries' knowledge composition and diffusion, the reverse link is also prominent—productivity differences induced by innovation and diffusion also condition the patterns of trade and aggregate growth.[1]

This paper provides a novel unified framework to quantify the interactions between trade, innovation and knowledge spillovers in a multi-sector environment in which country-sectors are interconnected both in the product and knowledge space. Our framework is a multi-country and multi-sector endogenous growth model in which productivity evolves endogenously through innovation and knowledge diffusion. The model extends existing multi-sector models of trade with input-output linkages (see Caliendo and Parro (2015), and Costinot, Donaldson, and Komunjer (2012)) by adding dynamics through innovation and knowledge diffusion across countries and sectors. Production and trade are built upon the Ricardian model of trade with Bertrand competition (Bernard et al. (2003)). Innovation and knowledge diffusion are modeled in the spirit of Eaton and Kortum (1996, 1999), who analyze an endogenous growth model without trade.

Countries and sectors are heterogeneous in their efficiency of innovation and the strength of knowledge spillovers. Innovation is endogenous in that firms choose their research effort to create new ideas. Ideas diffuse across *all* sectors and countries, although the speed of diffusion may differ.[2] In our model, knowledge diffusion increases the stock of knowledge in two ways. First, it increases the stock of knowledge in the receiving country-sector. Second, the increase in the stock

[1] A vast empirical literature has documented the significant impact of trade on innovation (e.g. Aghion et al. (2018); Bloom, Draca, and Van Reenen (2016); Autor et al. (2016); Bustos (2011); Lileeva and Trefler (2010)) and its effect on knowledge diffusion (e.g. Keller (2010); MacGarvie (2006)). Regarding the reverse relationship, Santacreu and Zhu (2018) and Cameron, Proudman, and Redding (2000) find that innovation and knowledge diffusion help determine trade patterns. Hausmann, Hwang, and Rodrik (2007) and Hidalgo et al. (2007) argue that a country's knowledge composition conditions its income.

[2] This assumption is supported by evidence showing that a non-negligible amount of novel inventions are initiated outside the traditional frontier economies. This is in contrast with the technology adoption literature, which assumes only the best knowledge is adopted or diffused from the technologically frontier economies (Comin and Hobijn (2010)).

of knowledge enhances the innovation efficiency there, as new ideas are built on existing knowledge. While all new ideas contribute to the stock of knowledge, only ideas with the highest quality would be adopted for production. Different from recent papers in the literature (e.g. Buera and Oberfield (2016), Grossman and Helpman (1991)), diffusion in our model takes place independently from trade, reflecting that in practice other channels such as FDI, migration or direct communication outside trade, among others, also diffuse ideas across countries and sectors (see Fons-Rosen et al. (2017), Ramondo and Rodríguez-Clare (2013), Keller (1998)). Innovation and diffusion determine the distribution of knowledge stock across countries and sectors and economic growth. We solve for the BGP of the model in which all countries and sectors grow at a common and constant rate.

In a multi-sector model, trade liberalization induces an endogenous reallocation of research effort across countries and sectors, increasing aggregate innovation and long-run growth. This is in contrast with standard one-sector models, in which trade has a negligible effect on innovation and growth as the market size effect exactly offsets the competition effect (Eaton and Kortum (2006); Atkeson and Burstein (2010); Buera and Oberfield (2016)). In addition, as comparative advantage is endogenous, it results in welfare gains from trade beyond specialization effects present in static multi-sector models. In the presence of heterogeneous knowledge diffusion across country-sectors, the dispersion of comparative advantage resulting from research resource reallocation is even higher, further amplifying the specialization effect of trade on innovation and welfare.

Our model can be solved in two blocks. A 'trade block' determines the static equilibrium for the world economy, given the distribution of firm productivity together with trade barriers. A 'growth block' characterizes the dynamics of the economy. In particular, innovation and knowledge diffusion processes drive the endogenous evolution of comparative advantage and dynamic welfare gains from trade.

We calibrate the model to data on production, bilateral trade, R&D intensity, and patent citations at the country and sector level. Several parameters are estimated outside the model. In particular, we calibrate the speed of cross-country and cross-sector knowledge spillovers by fitting a citation function that includes both the rate of obsolescence and the diffusion lag. An advantage of this approach is that it allows for patents in different country-sectors to vary in terms of their obsolescence rates and their quality, in addition to diffusion speed. Moreover, we do not need to impose the assumption that citations are mapped into knowledge spillovers one-to-one. The diffusion speed parameters are thus estimated jointly with other parameters that also govern the citation process.[3] This procedure helps obtain a more accurate estimate of our parameter of interest—diffusion speed across country and sectors. The productivity parameters are calibrated

[3]Our method extends the approach proposed in Caballero and Jaffe (1993) into a multi-country multi-sector environment.

by running gravity regressions at the country-sector level based on the trade block of our model.[4] The rest of the parameters are calibrated by solving the two blocks of the model separately, using an algorithm based on excess demand iterations that solves for the 'trade block' and a fixed-point algorithm that solves for the 'growth block'. This algorithm helps us determine the exogenous efficiency of innovation as well as the stock of knowledge of the economy in the BGP.

We conduct a counterfactual exercise to study the effect of trade liberalization on innovation, comparative advantage and growth along the BGP. Changes in trade costs have a non-negligible effect on innovation in our model, as there is a reallocation of R&D toward sectors in which the country has comparative advantageAs a result, the economy grows at a higher rate in the counterfactual BGP. Knowledge spillovers amplify this effect, as countries and sectors have access to innovations that have been developed elsewhere. We calculate welfare gains from trade long the initial and counterfactual BGP, and we decompose them into static and dynamic gains.

Finally, we study the role of the different channels by considering the following three versions of our model: (i) homogeneous knowledge spillovers across counties and sectors, (ii) no knowledge spillovers across countries and sectors, and (iii) one-sector model. We recalibrate each version to match the same moments of the data. We find that welfare gains from trade are lower and less disperse in those cases, which exposes the importance of considering multi-sector models with heterogeneous knowledge spillovers in quantifying the effect of trade liberalization.

A few points merit mention regarding our calibration strategy for knowledge diffusion. Naturally, direct measures of technology spillovers do not exist. Patent citation data have been used extensively in a growing body of economic research as a way of tracking technological diffusion across time and geographic boundaries.[5] One patent application citing an earlier patent generally indicates that the applicant has benefited from the earlier patent. Although patent citations provide valuable rare insight into the knowledge spillovers, we first note as a caveat that they are subject to certain limitations. For example, they do not capture technology transfer or any types of learning that do not result in a patent, such as reverse-engineer, imitation or replication. Moreover, a substantial amount of inventions are not patented but are protected through trade secrets and other informal mechanisms. Although there are several considerations, all difficult to quantify, there is no pervasive evidence suggesting that we should expect nonpatented knowledge to diffuse at a systematically and significantly different speed than patented knowledge. Second and more

[4]Our method to estimate country-sector productivity is similar to the one used by Levchenko and Zhang (2016). Hanson, Lind, and Muendler (2015) also use gravity equations to estimate comparative advantage and they characterize the evolution of comparative advantage over time. However, different from our approach their estimation procedure relies only on bilateral trade data and their comparative advantage is calculated based on the estimate of technology adjusted by production cost, rather than technology per se.

[5]For example, see Li (2014); Jaffe, Trajtenberg, and Henderson (1993); Thompson and Fox-Kean (2005); Peri (2005); Griffith, Lee, and Van Reenen (2011).

importantly, our estimation procedure builds on the approach proposed by Caballero and Jaffe (1993) by extending it into a multi-country multi-sector environment. This approach is designed to incorporate, in addition to heterogeneous cross-country-sector diffusion speed, how variations in obsolescence rates, quality of patents and citations in different country-sectors affect citations. Controlling for these additional variations with a fairly rich structure of citation process helps to obtain a more accurate estimation of the diffusion speed. Third, consider the alternative regression approach in the literature which estimates how related domestic TFP in a certain sector is with foreign R&D capital stock in another sector and uses the estimated elasticity to proxy spillovers. Apparently such estimation requires data that are either not available (such as sectoral capital stock and R&D stock) or hard to measure (such as sectoral TFP) for most countries. In addition, using outcome-based measures may confound technology spillovers with other factors that lead to comovement between country-sectors.

Literature Review Our paper connects and extends existing theoretical literature on the relationship between trade and innovation (e.g. Sampson (2016); Atkeson and Burstein (2010); Rivera-Batiz and Romer (1991); Grossman and Helpman (1991)), between trade and diffusion (e.g. Perla, Tonetti, and Waugh (2015); Somale (2014)), and between innovation and diffusion (e.g. Eaton and Kortum (1999); Eaton and Kortum (1996)). Yet, rarely are trade, innovation and diffusion analyzed in one unified framework. Notable exceptions are Buera and Oberfield (2016), Santacreu (2015), Eaton and Kortum (2006) and Lind and Ramondo (2018). In both Buera and Oberfield (2016) and Santacreu (2015), trade is the only channel for cross-border exchange of ideas. We allow knowledge linkages and trade linkages to operate separately, even though trade liberalization would impact knowledge accumulation and the strength of diffusion as an endogenous outcome. In their survey paper, Lind and Ramondo (2018) consider multinational production as an alternative channel for diffusion (as in Ramondo and Rodríguez-Clare (2013)). Eaton and Kortum (2006)'s theoretical investigation analyze the effect of faster diffusion and lower trade barriers on the incentive to innovate. Our main departure from these papers is that we consider a multi-sector environment in which sectors are interconnected both through the input-output linkages and knowledge linkages. As thoroughly discussed in Eaton and Kortum (2006), in the absence of diffusion, the one-sector model predicts the same share of resources towards research regardless of trade barriers. Our multi-sector model, however, generates changes aggregate innovation and growth via the additional mechanism of research reallocation across sectors.

This paper joins forces on the growing literature quantifying dynamic gains from trade (Perla, Tonetti, and Waugh (2015), Buera and Oberfield (2016), Akcigit, Ates, and Impullitti (2018), Ramondo and Rodríguez-Clare (2013)). In a Melitz type of model, Perla, Tonetti, and Waugh (2015) find that lowering trade barrier induces faster technology adoption and growth as the relative profit gains between the average and marginal adopting firms become larger. However, they obtain lower gains from trade owing to a decrease in the number of varieties due to entry. Akcigit, Ates, and Impullitti (2018) focuses on the role of strategic interaction between firms in shaping their innovation responses to policy changes (such as tariffs and R&D subsidies) and the dynamic gains from trade. Ramondo and Rodríguez-Clare (2013) study the interaction between trade and multinational production. Although each has a different focus, these studies also show the gains from trade increase substantially compared to the static counterparts of those models, a result also found in our paper.

In analyzing multi-sector trade models of innovation with endogenous comparative advantage, our paper relates to two recent works by Somale (2014) and Sampson (2016). Somale (2014) studies the two-way relationship of trade and innovation in a multi-sector semi-endogenous model with only level effects of research in the BGP, while our model allows for growth effect as well. More importantly we analyze the three-way interactions between trade, innovation and knowledge spillovers, and allow for sectors to be interconnected. Our analysis shows that both considerations of knowledge spillovers and interconnections between country-sectors are important in understanding the endogenous evolution of comparative advantage and quantitatively contributes significantly to the welfare gains. Sampson (2016) develops a theoretical Armington framework of innovation and learning as sources of endogenous comparative advantage. Our emphasis is on the quantification of the model, which allows us to do counterfactuals.

The paper also contributes to a burgeoning strand of research that analyzes the implications of interconnections between different sectors in a closed economy (e.g. Carvalho (2014); Carvalho and Gabaix (2013); Acemoglu et al. (2012); Gabaix (2011)) or an open economy (Eaton et al. (2016); Caliendo and Parro (2015); Costinot, Donaldson, and Komunjer (2012)). Most of these papers focus on factor-demand linkages of production. In addition to the input-output linkages, this paper also simultaneously considers the intrinsic interconnections of technologies embodied in different sectors, which turns out to be significant and relevant when studying innovation and diffusion (Cai and Li (2016), Cai and Li (2018), Acemoglu, Akcigit, and Kerr (2016)). Most related, Cai and Li (2016) study knowledge spillovers across sectors within a country and how trade costs affect the distribution of endogenous knowledge accumulation across sectors. Different from our paper, however, cross-sector knowledge diffusion is not considered across countries and intermediate input-demand linkages across sectors are absent.

2 The Model

We develop a general equilibrium model of trade in intermediate goods, with sector heterogeneity and input-output linkages, in which technology evolves endogenously through innovation and knowledge diffusion. The model can be decomposed into two blocks: (i) a *trade block* which, given a distribution of technology and trade barriers, determines the static equilibrium, and (ii) a *growth block*, which determines the dynamics of technology through innovation and knowledge spillovers.

There are M countries and J sectors. Countries are denoted by i and n and sectors are denoted by j and k. Labor is the only factor of production, and we assume it to be mobile across sectors within a country but immobile across countries. There is trade in intermediate goods and trade is Ricardian.

2.1 Consumers

In each country there is a representative household with life-time utility

$$U_{nt} = \int_{t=0}^{\infty} \rho^t u\left(C_{nt}\right) dt, \tag{1}$$

where $\rho \in (0, 1)$ is the discount factor and C_{nt} represents consumption of country n at time t.

We assume that household's preferences are represented by a CRRA utility function

$$u\left(C_{nt}\right) = \frac{C_{nt}^{1-\gamma}}{1-\gamma}$$

with an intertemporal elasticity of substitution, $\gamma > 0$.

The household consumes and finances R&D activities of the entrepreneurs and owns all the firms. In return, she receives labor income and the profits of the entrepreneurs.

The budget constraint of the household is given by

$$P_{nt}C_{nt} + \dot{a}_{nt} = r_{nt}a_{nt} + \Pi_{nt}$$

where P_{nt} is the price of the final good, to be defined later, a_{nt} are household's holdings of firms shares, r_{nt} is the return on assets and Π_{nt} are profits of firms that household's obtain from financing firm's R&D activities.

2.2 Final Production

In each country n, a domestic final producer uses the composite output from each domestic sector j in country n at time t, Y_{nt}^{j}, to produce a non-traded final output Y_{nt} according to the following

Cobb-Douglas production function:

$$Y_{nt} = \prod_{j=1}^{J} \left(Y_{nt}^{j} \right)^{\alpha^{j}}, \tag{2}$$

with $\alpha^j \in (0,1)$ the share of sector production on total final output, and $\sum_{j=1}^{J} \alpha^j = 1$.

Final producers operate under perfect competition. Their profits are given by

$$\Pi_{nt} = P_{nt} Y_{nt} - \sum_{j=1}^{J} P_{nt}^{j} Y_{nt}^{j},$$

where P_{nt} is the price of the final product and P_{nt}^j is the price of the composite good produced in sector j from country n.

Under perfect competition, the price charged by the final producer to the consumers is equal to the marginal cost; that is

$$P_{nt} = \prod_{j=1}^{J} \left(\frac{P_{nt}^{j}}{\alpha^{j}} \right)^{\alpha^{j}}.$$

The demand by final producers for the sector composite good is given by

$$Y_{nt}^{j} = \alpha^{j} \frac{P_{nt}}{P_{nt}^{j}} Y_{nt}.$$

2.3 Intermediate Producers

In each sector j there is a continuum of intermediate producers indexed by $\omega \in [0,1]$ that use labor, $l_{nt}^j(\omega)$, and a composite intermediate good from every other sector k in the country, $m_{nt}^{jk}(\omega)$, to produce a variety ω according to the following constant returns to scale technology[6]:

$$q_{nt}^{j}(\omega) = z_{n}^{j}(\omega) [l_{nt}^{j}(\omega)]^{\gamma^{j}} \prod_{k=1}^{J} [m_{nt}^{jk}(\omega)]^{\gamma^{jk}}, \tag{3}$$

with $\gamma^j + \sum_{k=1}^{J} \gamma^{jk} = 1$. Here γ^{jk} is the share of materials from sector k used in the production of intermediate ω is sector j, and γ^j is the share of value added. Firms are heterogeneous in their productivity $z_n^j(\omega)$.

[6]The notation in the paper is such that every time there are two subscripts or two superscripts, the one on the right corresponds to the source country and the one on the left corresponds to the destination country.

The cost of producing each intermediate good ω is

$$c^j_{nt}(\omega) = \frac{c^j_{nt}}{z^j_{nt}(\omega)},$$

where c^j_n denotes the cost of the input bundle. With constant returns to scale,

$$c^j_{nt} = \Upsilon^j W_{nt}^{\gamma^j} \prod_{k=1}^{J} (P^k_{nt})^{\gamma^{jk}}, \tag{4}$$

with $\Upsilon^j = \prod_{k=1}^{J} (\gamma^{jk})^{-\gamma^{jk}} (\gamma^j)^{-\gamma^j}$ and W_{nt} the nominal wage rate.

2.4 Composite Intermediate Goods (Materials)

Each sector j produces a composite good combining domestic and foreign varieties from that sector. Composite producers operate under perfect competition and buy intermediate products ω from the minimum cost supplier.

The production for a composite good in sector j and country n is given by the Ethier (1982) CES function,

$$Q^j_{nt} = \left(\int e^j_{nt}(\omega)^{1-1/\sigma} d\omega \right)^{\sigma/(\sigma-1)}, \tag{5}$$

where $\sigma > 0$ is the elasticity of substitution across intermediate goods and $e^j_{nt}(\omega)$ is the demand of intermediate goods from the lowest cost supplier in sector j.

The demand for each intermediate good ω is given by

$$e^j_{nt}(\omega) = \left(\frac{p^j_{nt}(\omega)}{P^j_{nt}} \right)^{-\sigma} Q^j_{nt},$$

where

$$P^j_{nt} = \left(\int p^j_{nt}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}. \tag{6}$$

Composite intermediate goods are used as final goods in the final production and as materials for the production of the intermediate goods:

$$Q^j_{nt} = Y^j_{nt} + \sum_{k=1}^{J} \int m^{kj}_{nt}(\omega) d\omega.$$

2.5 International Trade

Trade in goods is costly. In particular, there are iceberg transport costs from shipping a good that is produced in sector j from country i to country n, $d^j_{ni} > 1$. We follow Bernard, Eaton, Jensen, and Kortum (2003) and assume Bertrand competition. With Bertrand competition, as with perfect competition, composite producers in each sector buy from the lowest cost supplier and the price charged by the producer will be the production cost of the second-lowest producer.

Ricardian motives for trade are introduced as in Eaton and Kortum (2002), since productivity is allowed to vary by country-sector. The productivity of producing intermediate good ω in country i and sector j is drawn from a Frechet distribution described by T^j_{it} and shape parameter θ. A higher T^j_{it} implies a higher average productivity of that country-sector, while a lower θ implies more dispersion of productivity across varieties:

$$F(z^j_i) = Pr\left[Z \leq z^j_i\right] = e^{-T^j_{it} z^{-\theta}},$$

and

Given these assumptions, the price index of goods in sector j in country n is

$$P^j_{nt} = B\left(\Phi^j_{nt}\right)^{-1/\theta}, \tag{7}$$

with $B = \left[\frac{1+\theta-\sigma+(\sigma-1)(\bar{m})^{-\theta}}{1+\theta-\sigma}\Gamma\left(\frac{2\theta+1-\sigma}{\theta}\right)\right]^{1/(1-\sigma)}$ and

$$\Phi^j_{nt} = \sum_{i=1}^{M} T^j_{it}(d^j_{ni} c^j_{it})^{-\theta}. \tag{8}$$

For prices to be well defined, we assume $\sigma < (1+\theta)$.[7]

Expenditure shares Given the distributional assumptions of productivity, the probability that country i is the lowest cost supplier of a good in sector j to be exported to country n is

$$\pi^j_{nit} = \frac{T^j_{it}\left(c^j_{it} d^j_{ni}\right)^{-\theta}}{\Phi^j_{nt}}, \tag{9}$$

where c^j_{it} is defined in equation (4). Because there is a continuum of intermediate goods, π^j_{nit} is also the fraction of goods that sector j in country i sells to any sector in country n. In particular,

[7] Details of these derivations can be found in Bernard, Eaton, Jensen, and Kortum (2003).

the share country n spends on sector j products from country i is

$$\pi^j_{nit} = \frac{X^j_{nit}}{X^j_{nt}}. \tag{10}$$

with X^j_{nit} the value of intermediate products from sector j that country n buys from country i and X^j_{nt} total expenditure of country n in sector j.

2.6 Total Expenditures and Balanced Trade

Total expenditures on goods from sector j and country n are given by the sum of what the composite producers from each sector k and country i buys and the spending from other final producers. Then, X^j_n is given by

$$X^j_n = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} X^k_i \pi^k_{in} + \alpha^j P_n Y_n. \tag{11}$$

We assume trade is balanced period by period. Total imports in country n are given by

$$IM_{nt} = \sum_{i=1 i\neq n}^{M} \sum_{k=1}^{J} X^k_{nit} = \sum_{k=1}^{J} X^k_{nt} \sum_{i=1 i\neq n}^{M} \pi^k_{nit}. \tag{12}$$

Total exports in country n are given by

$$EX_{nt} = \sum_{i=1 i\neq n}^{M} \sum_{k=1}^{J} X^k_{int} = \sum_{i=1 i\neq n}^{M} \sum_{k=1}^{J} \pi^k_{int} X^k_{it}.$$

Balanced trade implies

$$EX_{nt} = IM_{nt}.$$

2.7 Productivity and the Stock of Knowledge

So far, we have described the *trade block* of the model, which, given a distribution of technology, T^j_{it} and trade barriers, d^j_{ni}, determines the static equilibrium. Note that, different from static models of trade, T^j_{it} depends on t. Next we describe the *growth block* of the model, which determines the endogenous evolution of T^j_{it}.

We assume that the average productivity of each sector j in country i, T^j_{it}, is driven by two components: (i) The first is a time-varying component, A^j_{it}, which reflects the stock of knowledge of country i in sector j at time t. We refer to this component as "knowledge-based productivity," and it reflects the part of productivity that is driven by innovation and knowledge spillovers; (ii) The second is a time-invariant component, $T_{i,p}^{j}$, which captures the part of productivity that is not explained by innovation or knowledge spillovers. Factor endowments, institutions, geography, multinational production, or human capital could be factors embodied in this component.

Without loss of generality, we make the following assumption[8]:

$$T_{it}^{j} = A_{it}^{j} T_{p,i}^{j}. \tag{13}$$

Therefore, the dynamics of the average productivity T_{it}^{j} are driven by the dynamics of the knowledge-based productivity, A_{it}^{j}.[9]

The evolution of the stock of knowledge, A_{it}^{j} is determined by both innovation and knowledge spillovers. Next, we describe each channel in detail.

2.8 Endogenous Growth: Innovation and Knowledge Spillovers

Innovation and knowledge spillovers determine the endogenous evolution of the distribution of productivity. Innovation is conducted in a particular country and sector and requires effort. Knowledge spillovers across countries and sectors are costless. Firms in a country and sector learn about technologies that have been developed elsewhere. Both innvation ad knowledge spillovers increase the stock of knowledge of a particular country and sector.

Innovation In each sector j and country n, there is a continuum of entrepreneurs that invest final output, R_{nt}^{j}, to come up with a new idea. Ideas are blueprints used to produce an intermediate good with higher efficiency.[10] Research efforts are targeted at any of the continuum of intermediate goods in that sector. In each country n and sector j, ideas are drawn at a Poissson rate

$$\lambda_{n}^{j} A_{nt}^{j} \left(s_{nt}^{j} \right)^{\beta_r}, \tag{14}$$

with $s_{nt}^{j} = R_{nt}^{j}/Y_{nt}$ the fraction of final output invested into innovation; λ_{n}^{j} a country and sector specific parameter that determines the efficiency of innovation; A_{nt}^{j} the stock of knowledge in sector j and $\beta_r \in (0,1)$ is a parameter of diminishing returns to investing into R&D. country n. In this specification, $\lambda_{n}^{j} A_{nt}^{j}$ determines comparative advantage in innovation. In this specification, λ_{n}^{j} is the exogenous component whereas A_{nt}^{j} is the endogenous component. Everything else constant,

[8]This formulation is similar to the one introduced in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2013).

[9]As it will be clear in our quantitative exercise, $T_{p,i}^{j}$ is computed as a residual following development accounting. On the one hand, we will be able to identify A_{it}^{j} from innovation and diffusion data; on the other hand, we identify T_{it}^{j} from trade data. The part of T_{it}^{j} that cannot be explained by innovation and diffusion is $T_{i,p}^{j}$.

[10]We model the innovation process within each industry in a country as in Kortum (1997).

countries that have accumulated more knowledge over time (i.e., have a higher A_{nt}^j) are more productive at doing innovation. This process ensures that there is a balanced-growth path without scale effects (see Eaton and Kortum (1996, 1999) and Santacreu (2015)).

As it is standard in the quality-ladders literature, an idea is the realization of two random variables. One is the good ω to which the idea applies. An idea applies to only one good in the continuum. The good ω to which it is associated is drawn from the uniform distribution $[0,1]$. The other is the quality of the idea, which is drawn from the Pareto distribution. In equilibrium, only the best idea for each input is actually used to produce an intermediate good in any sector and country. In that case, the idea can be used to produce an intermediate product ω in sector j and country n with efficiency $z_n^j(\omega)$. Therefore, the efficient technology $z_n^j(\omega)$ for producing good ω in country n is the best idea for producing it yet discovered (see Eaton and Kortum (2006)).

The stock of ideas at time t in each sector j and country n is A_{nt}^j. Because there is a unit interval of intermediate goods, the number of ideas for producing a specific good is Poisson with parameter A_{nt}^j. This Poisson arrival implies that the quality distribution of successful ideas is $F(q) = e^{-A_{nt}^j q^{-\theta}}$, with q the quality of an idea. Therefore, the quality distribution of successful ideas inherits the distribution of productivity of the intermediate goods produced in a country.

Entrepreneurs finance R&D activities by issuing equity claims to households. These claims pay nothing if the entrepreneur is not successful in introducing a new technology in the market, and it pays the stream of future profits if the innovation succeeds. Because of the probabilistic distribution of productivity, entrepreneurs are indifferent to what product ω to devote their efforts, all products within a sector deliver the same expected profit. Innovators choose the amount of R&D investment, in terms of final output, R_{nt}^j that maximizes

$$\dot{A}_{nt}^j V_{nt}^j - P_{nt} R_{nt}^j$$

subject to equation (17). Here, V_{nt}^j is the value of an innovation created in sector j and country n, which is the expected flow of profits that will last until a new producer is able to produce the good at a lower cost. It is given by

$$V_{nt}^j = \int_t^\infty \left(\frac{P_{nt}^j}{P_{ns}^j} \right) e^{-\int_t^s r_{iu} du} \frac{\Pi_{ns}^j}{A_{ns}^j} ds. \tag{15}$$

with $1/A_{nt}^j$ being the probability of an idea being successful and Π_{nt}^j being profits, which are expressed as

$$\Pi_{nt}^j = \frac{\sum_{i=1}^M \pi_{in}^j X_i^j}{(1+\theta)}.$$

Th expression for V_{nt}^{j} introduces a competitive effect, by which the larger the stock of knowledge in a sector-country, the lower the probability that the new idea lowers the cost there. Furthermore, conditional on the idea being successful, expected profits of the innovator are determined by the probability that the intermediate good produced with her idea is produced at the lowest cost, which is determined by π_{in}^{j}. As we show later, changes in trade costs have an impact on the value of an innovation through their effect on Π_{in}^{j}.

The first-order condition for optimal R&D is

$$s_{nt}^{j} = \left(\beta_r \lambda_n^j A_{nt}^j \frac{V_{nt}^j}{P_{nt} Y_{nt}} \right)^{\frac{1}{1-\beta_r}}. \tag{16}$$

Knowledge Spillovers New ideas created in each sector j and country n increase its stock of knowledge, A_{nt}^{j}. Furthermore, ideas can diffuse exogenously across sectors and countries. Through diffusion, the stock of knowledge in each sector j and country n is composed of knowledge that has been developed in each sector k in country i.

Diffusion takes time. An idea discovered at time t in country i and sector k diffuses to country n and sector j at time $t + \tau_{ni}^{jk}$. We assume that the diffusion lag, τ_{ni}^{jk}, has an exponential distribution with parameter ε_{ni}^{jk} as the speed of diffusion, so that $Pr[\tau_{ni}^{jk} \leq x] = 1 - e^{-\varepsilon_{ni}^{jk} x}$. Therefore, the flow of ideas diffusing to country n and sector j is given by

$$\dot{A}_{nt}^{j} = \sum_{i=1}^{M} \sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \lambda_i^k A_{is}^k \left(s_{is}^k \right)^{\beta_r} ds. \tag{17}$$

The evolution of the stock of knowledge in sector j and country n at time t depends on the past research effort by each other sector k in each other country i up to time t and diffused at rate ε_{ni}^{jk}.

3 Endogenous Growth along the BGP

We define the BGP as an equilibrium in which all variables growth at a constant rate. Knowledge spillovers across countries and sectors guarantee that the stock of knowledge A_{nt}^{j} grows at a common rate, g_A, across all countries and sectors. We stationarize all the endogenous variables so that they are constant on the BGP and denote the normalized variables with a hat; therefore, we remove time subscripts in our derivation. Next, we describe how we normalize the variables.

The resource constraint equation is

$$Y_{nt} = C_{nt} + \sum_{j=1}^{J} s_{nt}^{j} Y_{nt}$$

From this expression, the fraction of final output that is invested into R&D, s_n^j, is constant on the BGP. This result, together with equation (16), implies that $\frac{V_n^j A_n^j}{P_n Y_n}$ is constant along the BGP. From equation (15)

$$\hat{V}_n^j = \left(\frac{1}{r - g_A/\theta + g_A} \right) \frac{\sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{(1+\theta)\hat{Y}_n},$$

with $\hat{V}_n^j = \frac{V_n^j A_n^j}{P_n Y_n}$. We impose $r - g_A/\theta + g_A > 0$ and we show in Appendix B that $\hat{X}_i^j = \frac{X_i^j}{W_M}$, and $\hat{Y}_n = \frac{P_n Y_n}{W_M}$, with W_M the nominal wage in the numeraire country M. From equation (10), π_{in}^j is constant along the BGP. From here, optimal R&D intensity can be expressed as

$$s_n^j = \left(\beta_r \lambda_n^j \frac{1}{(1+\theta)} \frac{1}{r - g_A/\theta + g_A} \frac{\sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \right)^{\frac{1}{1-\beta_r}}, \tag{18}$$

The growth rate of the stock of knowledge along the BGP is expressed as

$$g_A = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(s_i^k \right)^{\beta_r}, \tag{19}$$

Substituting equation (18) we obtain

$$g_A = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(\frac{1}{r - g_A/\theta + g_A} \beta_r \lambda_i^k \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_i} \right)^{\frac{\beta_r}{1-\beta_r}}.$$

The growth rate of the stock of knowledge on the BGP depends positively on the speed of diffusion, the expected profits, and the efficiency of innovation, and it depends negatively on the dispersion parameter. Following Eaton and Kortum (1999), the Frobenius theorem guarantees that there is a unique balanced-growth path in which all countries and sectors grow at the same rate g_A. The expression for the growth rate can be expressed in matrix form as

$$g_A A = \Delta(g_A) A.$$

If the matrix $\Delta(g_A)$ is definite positive, then there exists a unique positive balanced-growth rate of technology $g_A > 0$, given research intensities and diffusion parameters. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country-sector's relative level of knowledge along that balanced-growth path.

In Appendix C, we report the equations of the model after normalizing the endogenous variables.

4 The Mechanism

In this section we describe the mechanism through which a reduction of trade costs, d_{in}^{j}, has an impact on innovation, growth and comparative advantage. In multi-sector *static* models of trade, there is the well-known specialization effect: A decrease in d_{in}^{j} induces a reallocation of production towards those sectors in which the country has comparative advantage (Caliendo and Parro (2015)). The larger the dispersion in relative productivity, the stronger is comparative advantage, and hence the specialization effect.

In a multi-sector *dynamic* model, there are additional effects of trade liberalization that can potentially generate welfare gains. The first is the R&D reallocation effect. Through the specialization effect just described, profits increase in sectors with stronger comparative advantage due to the market size effect following the decline in trade costs. As a result, R&D resources reallocate towards sectors that experience a higher increase in production. Consider two sectors j and j' in country n. From equation (18), we can obtain an expression of the relative R&D expenditure between these two sectors as:

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j}{\lambda_n^{j'}} \frac{\sum_{i=1}^{M} \pi_{in}^{j} X_i^j}{\sum_{i=1}^{M} \pi_{in}^{j} X_i^{j'}}. \tag{20}$$

Everything else constant, lowering trade costs affects the production patterns in the economy and shifts R&D towards sectors that experience larger increase in profits (higher $\sum_{i=1}^{M} \pi_{in}^{j} X_i^j$). This reallocation effect changes the aggregate R&D intensity at the country level. The exact magnitude is a quantitative question and we will provide more details in the quantitative analysis.

The following two extreme cases further illuminates this R&D reallocation effect of changing trade barriers.

Case 1 (Autarky): Suppose all countries are closed from international trade in goods and services. That is, $d_{in}^{j} \to \infty$, $\forall i, n, j$. Equation (20) can be rewritten as:

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j X_{nn}^j}{\lambda_n^{j'} X_{nn}^{j'}}, \tag{21}$$

where X_{nn}^{j} is total domestic expenditure on sector j product. Thus, innovation efforts are distributed across sectors according to the exogenous component of innovation efficiency ($\frac{\lambda_n^j}{\lambda_n^{j'}}$) and the domestic market share ($\frac{X_{nn}^j}{X_{nn}^{j'}}$).

Case 2 (Free Trade): In the case of free trade, $d_{in}^j = 1$. Equation (20) then becomes

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j}{\lambda_n^{j'}} \frac{T_n^j (c_n^j)^{-\theta} / \sum_n T_n^j (c_n^j)^{-\theta}}{T_n^{j'} (c_n^{j'})^{-\theta} / \sum_n T_n^{j'} (c_n^{j'})^{-\theta}} \frac{X^j}{X^{j'}}, \tag{22}$$

where $X^j = \sum_n X_n^j$ denotes the world demand for sector-j good. This equation shows that under free trade, in addition to the sector-specific relative innovation efficiency, a country's R&D resources would be distributed also according to relative export capability (production comparative advantage) ($\frac{T_n^j (c_n^j)^{-\theta} / \sum_n T_n^j (c_n^j)^{-\theta}}{T_n^{j'} (c_n^{j'})^{-\theta} / \sum_n T_n^{j'} (c_n^{j})^{-\theta}}$), and the world expenditure share ($\frac{X^j}{X^{j'}}$). The latter captures the traditional market size effect of opening trade.

A comparison of these two cases shows that when a country opens up to trade, research efforts are directed more into sectors with production comparative advantage and higher world demand. Furthermore, all else equal, a higher share of R&D investment in a sector translate into higher relative productivity ($T_n^j / T_n^{j'}$). Production comparative advantage thus evolves with the distribution of innovation efforts over time, which in turn affects the R&D allocation as shown in Equation (22).

Cross-country cross-sector knowledge spillovers further add to the complexity of the interactions between innovation and production. Equation (19) implies that without cross-country cross-sector spillovers (i.e. $\varepsilon_{ni}^{jk} = 0$ for $nj \neq ik$), the evolution of the technology distribution ($T_n^j / T_n^{j'}$ or $A_n^j / A_n^{j'}$) eventually reflects the underlying specialization in innovation.[11]. In the presence of spillovers, the relative technology level is also determined by the relative amount of ideas diffused from elsewhere. The heterogeneous knowledge spillovers across country-sectors thus introduces another source of dispersion to the distribution of stock of knowledge and innovation specialization.

Note that this channel is absent in a one-sector model, in which changes in trade barriers have no effect on innovation or on aggregate growth. In a one-sector model, $\frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} = 1$ in equation (18), implying that the R&D intensity only depends on parameters that are not related to the trade cost (see Appendix E for a derivation of the one sector model).

The second effect of trade of trade liberalization on welfare is a growth effect. From equation (19), the reallocation of R&D across sectors induces changes in the growth rate and the relative stock of knowledge along the BGP. If R&D reallocates towards the sectors that are better at doing R&D (i.e. higher $\lambda_i^k \hat{A}_i^k$), the growth rate of the world increases. Knowledge spillovers reinforce this channel, as changes in R&D intensity across sectors will have a larger impact on growth along the BGP as countries and sectors can benefit from R&D done in other countries and sectors (see

[11] This result is similar to what Somale (2014) obtains in a semi-endogenous growth model without knowledge spillovers.

Equation (19).

Finally, heterogeneity in knowledge spillovers also play an important role in propagating the effect of trade liberalization. As discussed earlier, the reallocation of R&D intensity across sectors, together with heterogeneous knowledge spillovers, has an effect on the relative stock of knowledge. Rearranging terms in Equation (19), we have

$$g_A \hat{A}_n^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \hat{A}_i^k \left(s_i^k \right)^{\beta_r}, \tag{23}$$

Suppose the diffusion speed is common across all countries and sectors, the stock of knowledge ($\hat{A}_n^j$) would be the same everywhere. In contrast, when the diffusion speed is heterogeneous, the relative stock of knowledge is disperse. And the effect of the reallocation of R&D on the stock of knowledge will be stronger in those sectors and countries more connected to the sector and country that experiences larger changes in R&D intensity. The increase in the dispersion of relative stock of knowledge reinforces the specialization effect of trade, as larger relative productivity differences will translate into greater welfare gains from trade liberalization.

5 Quantitative Analysis

We calibrate our model to quantify the effect of a trade liberalization on innovation, comparative advantage and growth after a trade liberalization. We then study the role of innovation and knowledge spillovers in driving the results by simulating four versions of our model: (i) our baseline model with in innovation and cross-sector and cross-country knowledge linkages; (ii) a model with very low knowledge spillovers; (iii) a model with knowledge spillovers that are homogeneous across countries and sectors, and (iv) a one-sector model with knowledge flows across countries. In all cases, we recalibrate the parameters of the model to match the same moments of the data.

5.1 Calibration

We use data on bilateral trade flows, R&D intensity, production, and patent citations to calibrate the main parameters of the model. We assume that the world is on a BGP in 2005. We calibrate the model in two stages. In the first stage, we calibrate the production and knowledge diffusion parameters, as well as the average productivity $\hat{T}_i^j$ and trade barriers d_{in}^j, and solve for the static equilibrium of the model. In the second stage, we take as given the results from the static equilibrium and solve for the innovation parameters and the stock of knowledge, $\hat{A}_i^j$. Here we explain in more detail the calibration of the average productivity parameters $\hat{T}_i^j$, the diffusion parameters

ε_{in}^{jk}, and the parameters governing the innovation process—the elasticity of innovation, β_r, and the efficiency of innovation, λ_i^j. Details on the data used in the calibration are relegated to Appendix B, and the description of the calibration procedure to recover other parameters of interest is provided in Appendix C.

5.1.1 Estimation of T_i^j: Gravity Equation at the Sector Level

To estimate the technology parameters for tradable sectors, $j \leq J-1$, we follow the procedure in Levchenko and Zhang (2016) by estimating standard gravity equations for each sector in 2005. We start from the trade shares in equation (10):

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j \left(c_i^j d_{ni}^j\right)^{-\theta}}{\Phi_n^j}. \tag{24}$$

Dividing the trade shares by their domestic counterpart as in Eaton and Kortum (2002) and assuming $d_{nn}^j = 1$, we have

$$\frac{\pi_{ni}^j}{\pi_{nn}^j} = \frac{X_{ni}^j}{X_{nn}^j} = \frac{T_i^j \left(c_i^j d_{ni}^j\right)^{-\theta}}{T_n^j \left(c_n^j\right)^{-\theta}}. \tag{25}$$

Taking logs of both sides, we have

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right) - \theta \log(d_{ni}^j). \tag{26}$$

The log of the trade costs can be expressed as

$$\log(d_{ni}^j) = D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + ex_i^j + \nu_{ni}^j. \tag{27}$$

Following Eaton and Kortum (2002), $D_{ni,k}^j$ is the contribution to trade costs of the distance between country n and i falling into the k^{th} interval (in miles), defined as [0,350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, maximum). The other control variables include common border effect, B_{ni}, common currency effect CU_{ni}, and regional trade agreement RTA_{ni}, between country n and country i. We include an exporter fixed effect, ex_i^j, to fit the patterns in both country incomes and observed price levels as shown in Waugh (2010). ν_{ni}^j is the error term.

Substituting (27) back into (26) results in the following gravity equation at the sector level:

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \theta ex_i^j - \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right) - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \tag{28}$$

Define $\hat{F}_i^j = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \theta ex_i^j$ and $F_n^j = \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right)$. We then estimate the following equation using fixed effects and observables related to trade barriers, taking θ as known:

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \hat{F}_i^j - F_n^j - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \tag{29}$$

Using the estimates of equation (29), we can back out $\log(d_{ni}^j)$ based on equation (27). To obtain the exporter fixed effect in trade cost, ex_i^j, we use the importer and exporter fixed effects from the Gravity equation (29). That is, $ex_i^j = (F_i^j - \hat{F}_i^j)/\theta$. Figure 1 plots the distance parameters that we obtain from the sectoral gravity equations, d_{in}^j, against the trade share from the data that we use to estimate the gravity equations at the sector level, assuming $\theta = 8.28$.

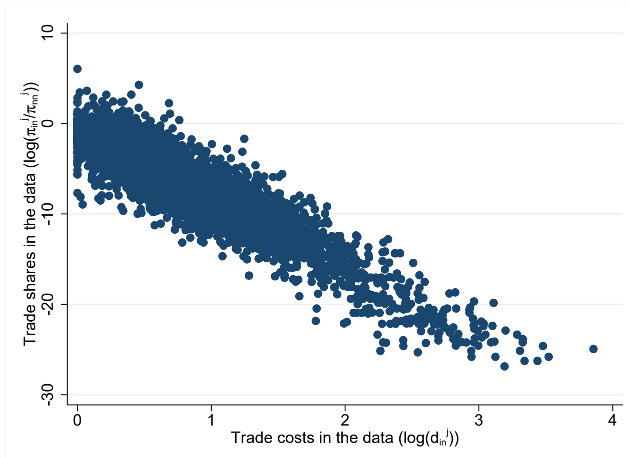


Figure 1: Trade shares and distance

The productivity of the tradable sector in country n relative to that in the United States, T_n^j/T_{US}^j, is then recovered from the estimated importer fixed effects as in

$$S_n^j = \frac{\exp(F_n^j)}{\exp(F_{US}^j)} = \frac{T_n^j}{T_{US}^j}\left(\frac{c_n^j}{c_{US}^j}\right)^{-\theta}, \tag{30}$$

in which the relative cost component can be computed by expressing (4) as

$$\frac{c_n^j}{c_{US}^j} = \left(\frac{W_n}{W_{US}}\right)^{\gamma^j} \prod_{k=1}^{J-1}\left(\frac{P_n^k}{P_{US}^k}\right)^{\gamma^{jk}} \left(\frac{P_n^J}{P_{US}^J}\right)^{\gamma^{jJ}}, \tag{31}$$

where J indicates the nontradable sector. Using data on wages (in USD), estimates of price levels in the tradable sector and the nontradable sector relative to the United States, we can back out the relative cost. The nontradable relative price is obtained using the detailed consumer price data collected by the International Comparison Program (ICP). To compute the relative price of the tradable sector, we follow the approach of Shikher (2012) by combining (7), (9), and (10) and get the following expression for relative prices of tradable goods:

$$\frac{P_n^j}{P_{US}^j} = \left(\frac{X_{nn}^j / X_n^j}{X_{US,US}^j / X_{US}^j} \frac{1}{S_n^j} \right)^{\frac{1}{\theta}}. \tag{32}$$

The right-hand side of this expression can be estimated using the observed expenditure shares of domestic product in country n and in the United States and the estimated importer fixed effects. Substituting the estimates for relative prices and wages in each country-sector and using the estimated S_n^j, we can construct the relative productivity T_n^j / T_{US}^j based on equation (30).

To compute the relative productivity in nontradable sectors, we combine (8), (7), and set the trade cost in nontradable sector d_{ni}^J to infinity for all i and n. This implies $\Phi_n^J = T_n^J \left(c_n^J\right)^{-\theta}$ based on equation (8). Substituting this expression into (7), we express the nontradable good price as

$$p_n^J = \frac{c_n^J}{\left(T_n^J\right)^{1/\theta}}. \tag{33}$$

The relative technology in nontradable sector can then be constructed based on

$$\frac{T_n^J}{T_{US}^J} = \left(\frac{c_n^J}{c_{US}^J} \frac{P_{US}^J}{P_n^J} \right)^{\theta}. \tag{34}$$

Again, the cost ratios are calculated following (31) and the price ratios for the non-tradable sectors are from the ICP database.

We now have estimated the relative productivity for all countries relative to the United States in every sector. To estimate the level of productivity, we need the U.S. productivity level. First, using OECD industry account data, we estimate the empirical sectoral productivity for each U.S. sector by the Solow residual (without capital in the production function):

$$\ln Z_{US}^j = \ln Y_{US}^j - \alpha^j \ln L_{US}^j - (1 - \alpha^j - \sum_{k=1}^{J} \alpha^{jk}) \ln K_{US}^j - \sum_{k=1}^{J} \alpha^{jk} \ln M_{US}^{jk}, j = 1, 2, ..., J, \tag{35}$$

where Z_{US}^j is measured U.S. productivity in sector j, Y_{US}^j is the output, L_{US}^j is the labor input, $K_U S^j$ is the capital input and M_{US}^{jk} is the intermediate input from sector k. Finicelli et al. (2013) show that trade and competition introduce selection in the productivity level, and the relationship between empirical productivity and the level of technology T_{US}^j in an open economy is given by

$$T_{US}^j = \left(Z_{US}^j\right)^\theta \left[1 + \sum_{i \neq US} S_i^j \left(d_{US,i}^j\right)^{-\theta}\right]^{-1}, \tag{36}$$

in which S_i^j and $d_{US,i}^j$ are estimated using (30) and (27) respectively. Lastly, we normalize the U.S. nontradable technology to 1, and express all T_{US}^j relative to T_{US}^J as

$$\hat{T}_{US}^j = \left(\frac{Z_{US}^j}{Z_{US}^J}\right)^\theta \left[1 + \sum_{i \neq US} S_i^j \left(d_{US,i}^j\right)^{-\theta}\right]^{-1}. \tag{37}$$

Throughout our analysis we assume that θ is common across countries and sectors and set it equal to 4.

We find that the cross-sector dispersion of the estimated relative productivity of country n relative to the United States is larger for countries that have a lower level of income per capita (see Figure 2). This result is consistent with Levchenko and Zhang (2016).

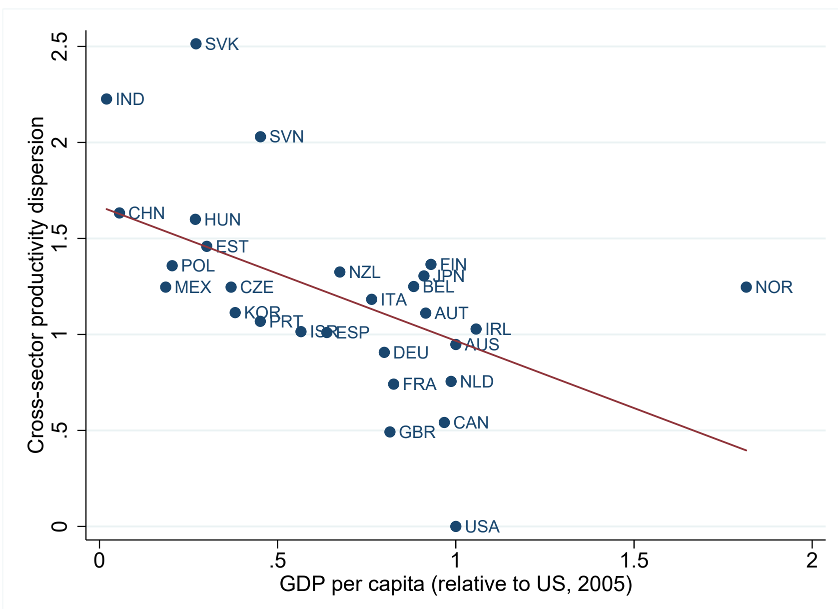


Figure 2: Dispersion of relative productivity and income per capita

5.1.2 The Speed of Knowledge Diffusion

Estimating the speed of knowledge diffusion is not a trivial task, as diffusion is conceptual and difficult to measure. The diffusion literature has typically found patent citations to represent a reasonable indicator of diffusion albeit with some degree of noise (Jaffe, Trajtenberg, and Fogarty

(2000); Bottazzi and Peri (2003))[12] When a patent is granted, its document identifies a list of citations made to previous patents upon which the current one builds. Thus, citations are informative of links between innovations. If a single technology is cited in numerous patents, it is apparently involved in many developmental efforts.

Using patent data, this section adapts the approach proposed in Caballero and Jaffe (1993) to estimate the diffusion speed parameters. To be consistent with our model, we extend their approach to a multi-sector multi-country environment. Similar to their paper, we use patents as an indicator of the creation of new ideas, and the citations as an indicator of use of existing ideas in the creation of new ideas.

A vast literature discusses the potential issues of using patent data to proxy ideas/knowledge and spillovers.[13] First, a considerable number of inventions or ideas are never patented but are protected by secrecy or other informal mechanism. Second, sectors differ in their propensity to patent and propensity to cite. Therefore, a relative abundant stock of patents in one sector may not necessarily imply a large accumulation of ideas. Third, individual patent varies in terms of its quality (the number of ideas embodied or the ability to generate spillovers). Lastly, not all citations necessarily represent spillovers as the decision to cite another patent sometimes rests with the patent examiner, who is supposed to be an expert in the area and able to identify relevant prior art that the applicant misses or conceals. This implies that the inventor may not be aware of the earlier work and the citation may not represent the true knowledge transmission.

A particular virtue of Caballero and Jaffe (1993) approach is that it is designed to deal with some of these issues by estimating these sector-specific factors—such as propensity to patent and to cite, the ability to generate spillovers and knowledge obsolescence rate, and the discrepancy between citations and spillovers—jointly with the diffusion speed parameters. Controlling for these additional sectoral variations with a fairly rich structure of citation process helps to obtain a more accurate estimation of the parameter of interest, the cross-country cross-sector speed of diffusion parameters, $\{\varepsilon_{ni}^{jk}\}_{MJ\times MJ}$.

In particular, we first specify a "citations" function which describes the usefulness of an idea generated at time s in country-sector ik for the production of new knowledge in country-sector nj at time t ($t \geq s$) $a_{ni}^{jk}(t,s)$. Let $P_{i,t}^{k}$ represents the number of patent applications by country i sector

[12] Although patent statistics have been widely used in studies of firm innovations, not all innovations are patented, especially process innovations, which are often protected in other ways such as copyright, trademarks and secrecy (see Levin et al. (1987)). Our measure implicitly assumes that for any sector, the nonpatented and patented knowledge utilizes knowledge (patented or nonpatented) from other sectors in the same manner, particularly with the same speed.

[13] See, for example, the survey by Griliches (1990).

k in period t. The citation function is written as below:

$$a_{ni}^{jk}(t,s) = \delta_{i,s}^{k} e^{-\sum_{\tau=s}^{t} \psi_{i,\tau}^{k} \tilde{P}_{i,\tau}^{k}} (1 - e^{-\varepsilon_{ni}^{jk}(t-s)}). \tag{38}$$

The first term, $\delta_{i,s}^{k}$, represents the strength of spillovers emanating from previous ideas in country-sector ik dated in period s.

The second term, $e^{-\sum_{\tau=s}^{t} \psi_{i,\tau}^{k} \tilde{P}_{i,\tau}^{k}}$, where $\tilde{P}_{i,\tau}^{k} = P_{i,\tau}^{k} / \sum_{\tau=s}^{t} P_{i,\tau}^{k}$ denotes the share of patent applications by ik in period τ among all applications by ik between period s to t. It can be interpreted as an index of knowledge obsolescence. It decreases the (normalized) size of inventions that take place between the recipient cohorts t and the source cohorts $\tau(\in [s,t])$, with a time-varying proportionality factor $\psi_{i,x}^{k}$. The idea is that old knowledge eventually is made obsolete by the emergence of superior new knowledge. Thus, the accumulation of new inventions (rather than simply the passage of time) that occur after the source cohorts increases the rate of which the source knowledge become obsolete.

The last term represents the probability of ideas in s having been seen by period t. Given the model assumption of the exponential distribution of diffusion lags, it follows that the probability of seeing an idea $(t-s)$ years old is given by $(1 - e^{-\varepsilon_{ni}^{jk}(t-s)})$, where ε_{ni}^{jk} is the constant diffusion speed from ik to nj. $\varepsilon_{ni}^{jk} \to \infty$ indicates instantaneous diffusion, whereas $\varepsilon_{ni}^{jk} = 0$ implies no diffusion. This is the parameter we are particularly interested in estimating.

Before bringing in the patent stock and and patent citation data, we need to make the assumption of the mapping between ideas and patents, and between spillovers and citations. Assume that the number of patents is proportional to the creation of ideas with the proportionality factor, $\psi_{n,t}^{j}$ and that citations are proportional to ideas used with a proportionality factor $\phi_{n,t}^{j}$, and we jointly estimate these parameters along with the other parameters in Equation (38). Let $C_{ni}^{jk}(t,s)$ be the observed citations from patents applied by country n sector j in year t to patents by country i sector k in year s. The left hand side of Equation (38) is then given by

$$a_{ni}^{jk}(t,s) = \frac{C_{ni}^{jk}(t,s)/\phi_{n,t}^{j}}{(P_{n,t}^{j}\psi_{n,t}^{j})(P_{i,s}^{k}\psi_{i,s}^{k})} \tag{39}$$

Define the empirical citation frequency as

$$\hat{a}_{ni}^{jk}(t,s) \equiv \frac{C_{ni}^{jk}(t,s)}{P_{n,t}^{j} P_{i,s}^{k}}. \tag{40}$$

Combining Equation (38) and (39) and rearranging terms, we obtain

$$\hat{a}^{jk}_{ni}(t,s) = \phi^j_{n,t}\psi^j_{n,t}\psi^k_{i,s}\delta^k_{i,s}e^{-\sum_{\tau=s}^{t}\psi^k_{i,\tau}\tilde{P}^k_{i,\tau}}(1-e^{-\varepsilon^{jk}_{ni}(t-s)}). \tag{41}$$

The empirical strategy is to use data on citation frequencies $\hat{a}^{jk}_{ni}(t,s)$ between patent cohorts (t,s) for each country-sector pairs (nj, ik) to estimate Equation (41) for many (t,s) observations.

We obtain patent and patent citation data across countries and sectors from the U.S. Patent and Trade Office (USPTO) for the period 2000-2010.[14] In the dataset, each patent is assigned to one of the IPC (International patent classification) categories. We use the probability mapping between IPC and ISIC Rev.3 provided by the World Intellectual Property Organization (WIPO) to assign patents into our 19 sectors. Our sample contains 1.15 million patents and over 13 million citations between the 28 countries and 19 sectors.

Equation (41) is estimated using Generalized Method of Moments (GMM) based on observations about $\hat{a}^{jk}_{ni}(t,s)$ with $t \in [2001, 2010]$, $s \in [2001, t]$, $j,k \in [1,J]$ and $n,i \in [1,M]$. Define $\Theta_{nj,ik}(t,s) = \{\varepsilon^{jk}_{ni}, \phi^j_{n,t}, \psi^j_{n,t}, \delta^j_{n,t}\}$ as the set of parameters to be estimated and $\Gamma(\Theta)$ the difference between data moments and the model-generated moments:

$$\Gamma(\Theta) = \frac{C^{jk}_{nt,is}}{P^j_{n,t}P^k_{i,s}} - \phi^j_{n,t}\psi^j_{n,t}\psi^k_{i,s}\delta^k_{i,s}e^{-\sum_{x=s}^{t}\psi^k_{i,x}\tilde{P}^k_{i,x}}(1-e^{-\varepsilon^{jk}_{ni}(t-s)}). \tag{42}$$

Our GMM estimators solve:

$$\Theta^* = \arg min_{\Theta} \sum_{n,i=1}^{M}\sum_{j,k=1}^{J}\sum_{t=2001}^{2010}\sum_{s=2001}^{t}\Gamma^2[\Theta_{nj,ik}(t,s)]. \tag{43}$$

The estimated ε^{jk}_{ni}s are then normalized following Eaton and Kortum (1999). Specifically, we fix the within-sector adoption speed in the U.S. to 2 years (taking the mid-point of the evidence reported by Pakes and Schankerman (1984)). The adoption lag in the model is given by $1/(\bar{\varepsilon}^{jj}_{USUS} + g) = 2$, which implies $\bar{\varepsilon}^{jj}_{USUS} = 0.38$ with $g = 0.12$ in our calibration. We then use this restriction to normalize all ε^{jk}_{ni}s.

[14] Note that patents applied in U.S. are not necessarily created by U.S. inventors. According to the territorial principle in U.S. patent laws, anyone intending to claim exclusive rights for inventions is required to file U.S. patents. In fact, about 50 percent of patents applied in the United States in the early 2000s were from foreign inventors. Given that the United States has been the largest technology consumption market in the world over the past few decades, it is reasonable to assume that most important innovations from other countries have been patented in the U.S. Therefore, the knowledge linkages uncovered in the U.S. patent data are reasonably representative of the deep fundamental relationship of technologies. All we really need is that statements of the following sort hold: If a patent that belongs to a German inventor in electronic components sector cites a Japanese patent in radio and television receiving equipment in the U.S. patent data, similar relationship also holds for German inventors filing a patent in Europe.

Figure 3: Contour mapping of ϵ_{ni}^{jk} - sectors

Several interesting findings emerge from estimating the citations function. First, there is a large heterogeneity in the diffusion speed across country and sectors (i.e. between $(nj,\, ik)$ cells), with a large number of country-sector pairs that diffuse knowledge very slowly to each other. The mean diffusion lag (i.e. $1/\varepsilon$) of about 5.5 years for cross-country-sector diffusion and a mean lag of less than 2 years for within country-sector diffusion. Second, although not reported here, a gravity-type regression shows that the diffusion speed significantly decreases with geographic distances, linguistic distances, and increases with being in the same currency or trade union, in the same continent, sharing a common colonizer or used to be a same country.

We present the heatmaps of the estimated ε_{ni}^{jk} in Figures 3 and 4. Figure 3, organizes the sub-blocks by sector and Figure 4 by country. Darker color means higher value. It is evident that there is large heterogeneity across country-sector-pairs.

Table 1 reports the average speed of diffusion by cited sector (i.e., a sector that diffuses knowledge) and citing sector (i.e, a sector that acquires knowledge). It shows that patents in the chemicals, agriculture, computer, electronic and medical instruments sectors have the highest diffusion

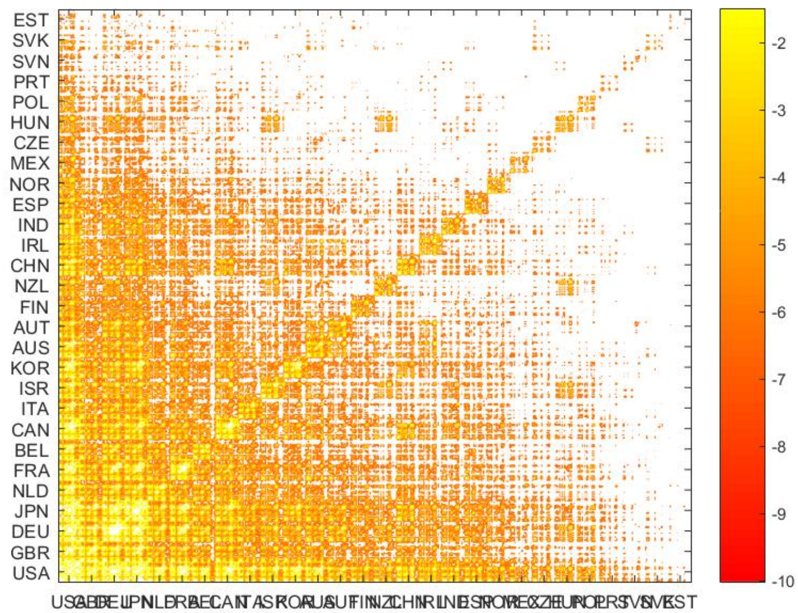


Figure 4: Contour mapping of ϵ_{ni}^{jk} - countries

speed, while patents in the wood products sector have the lowest diffusion speed. Across sectors, the citing speed (or speed of absorption) is highly correlated with the cited speed. Figure 5 shows the average speed of diffusion and absorption by country, with the U.S. observation normalized to 1. Unsurprisingly, new knowledge created in the United States, Japan, Germany, Canada and the United Kingdom diffuse the fastest. . Countries that diffuse knowledge (get cited) rapidly also tend to acquire new knowledge from other countries (citing others) fast. Emerging innovation powerhouse like China, India and Korea are faster at acquiring new knowledge than diffusing their own knowledge.

5.1.3 Parameters of Innovation

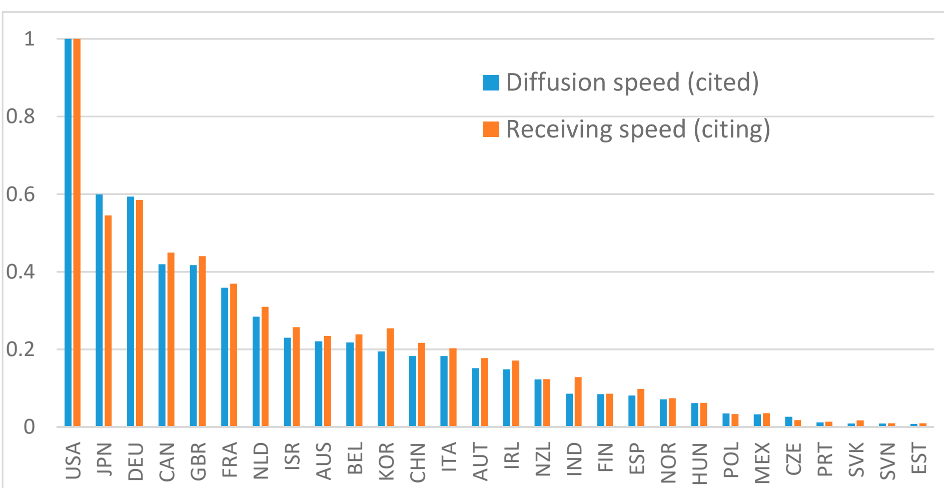
We calibrate the parameters of innovation $\{\beta_r, \lambda_n^j, \hat{A}_n^j\}$ in two steps. First, we solve for the static trade equilibrium taking as given the estimated sectoral productivity $\hat{T}_i^j$, the estimated trade barriers d_{in}^j, and production input-output linkages parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$ estimated using the U.S. input-output table for 2005. Our calibration strategy delivers relative wages and income that are broadly consistent with those observed in the data. The correlation between relative wages in our model and those in the data is around 0.8, and the correlation between GDP in our model and in the data is around 0.95.

Having computed wages and trade shares, in the second step we use the estimated parameters for

Table 1: Average diffusion speed by sectors

ISIC	Industry	Cited	Citing
C24	Chemicals and chemical products	0.055	0.053
C01T05	Agriculture, hunting, forestry and fishing	0.052	0.052
C30T33X	Computer, electronic and medical instruments	0.049	0.049
C29	Machinery and equipment, n.e.c.	0.046	0.048
C17T19	Textiles, textile products, leather and footwear	0.045	0.046
C10T14	Mining and quarrying	0.038	0.039
C28	Fabricated metal products, except machinery and equipment	0.035	0.038
C21T22	Pulp, paper, paper products, printing and publishing	0.032	0.028
C15T16	Food products, beverages and tobacco	0.025	0.025
C40T95	Nontradables	0.024	0.024
C25	Rubber and plastics products	0.019	0.020
C27	Basic metals	0.019	0.019
C23	Coke, refined petroleum products and nuclear fuel	0.019	0.018
C34	Motor vehicles, trailers and semi-trailers	0.012	0.011
C31	Electrical machinery and apparatus, n.e.c.	0.011	0.011
C26	Other non-metallic mineral products	0.010	0.010
C35	Other transport equipment	0.008	0.008
C36T37	Manufacturing n.e.c. and recycling	0.007	0.007
C20	Wood and products of wood and cork	0.002	0.002

Figure 5: Average speed of diffusion by country



Note: This figure presents the average diffusion speed and absorbing speed by country. Average diffusion (absorbing) speed is calculated as the average ϵ by cited (citing) country.

knowledge diffusion, ε_{in}^{kj}, data on R&D intensity at the country-sector level, s_n^j, and the expression for the growth rate of the economy on the BGP in equation (19) to calibrate the innovation parameters $\lambda_n^j, \beta_r, A_n^j$. We proceed as follows: First, we assume a growth of income per capita (productivity) on the BGP of $g_y = 2.8\%$. This corresponds to a growth rate for the stock of knowledge on the BGP of $g_A = \theta\left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j\right)^{-1} g_y = 0.12$. Because all countries and sectors' stock of knowledge grows at the same rate, all countries have the same productivity growth on the BGP (see Appendix B for details on the derivation). Second, we use the Frobenius theorem and equation (19) to obtain a value for the efficiency of innovation, λ_i^k, and the elasticity of innovation, β_r. Given data for s_n^j, the estimated values for ε_{ni}^{jk}, and g_A, we can use the Frobenius theorem and iterate on equation (19) to obtain β_r and λ_n^j. We obtain that $\beta_r = 0.67$ and λ_n^j ranges from $1.3 * 10^{-8}$ to 2.6, with mean 0.023 and standard deviation 0.13.

Figure 6 plots the calibrated values for λ_i^k against R&D intensity. As the figure shows, there is a positive relationship.

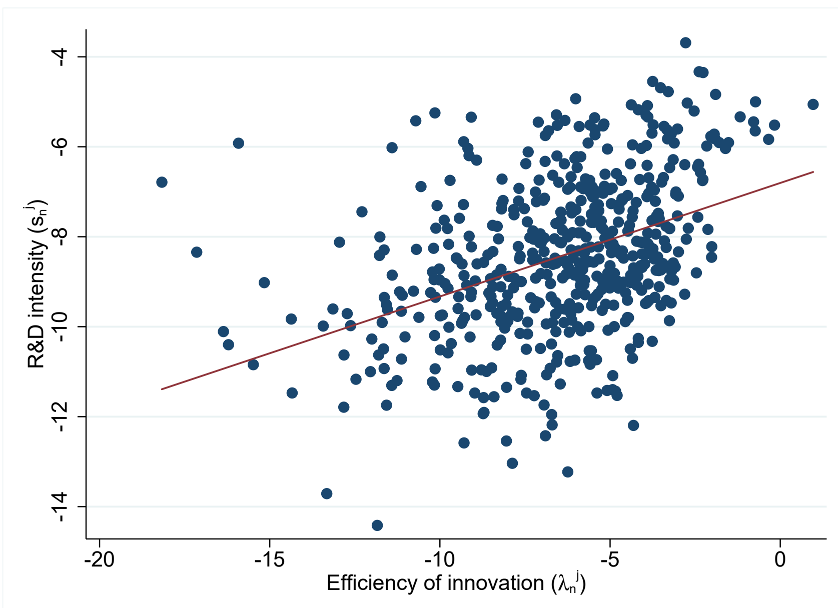


Figure 6: The exogenous efficiency of innovation and R&D intensity

The correlation between R&D intensity, s_n^j, and the parameter in the efficiency of innovation, λ_n^j, is around 0.45. However, the cross-sector correlations between these two variables are heterogeneous across countries. We find that lower-income countries, such as Slovakia, Slovenia and Estonia, do not allocate R&D across sectors according to the exogenous component of the efficiency of innovation, λ_n^j, and have a correlation below 0.2. In contrast, in countries such as United States and Japan or the United Kingdom, the correlation is above 0.5. Note that the efficiency of innovation that determines R&D intensity is actually a function of the parameter λ_n^j and the stock of knowledge $\hat{A}_n^j$. The stock of knowledge of a country-sector has two main components: (i) knowledge developed in that country-sector through the country's own innovation and (ii) knowledge developed somewhere else that has been diffused to that particular country-sector. In countries with a low correlation between R&D intensity and λ_n^j, R&D intensity is determined by the second component of the stock of knowledge. Diffusion is a key channel for promoting R&D in those countries and sectors. In what follows, we describe how we calibrate $\hat{A}_n^j$ and analyze its correlation with R&D intensity.

Given these parameter values, and using again the properties of the Frobenius theorem, the associated eigenvector to the growth rate of $g_A = 0.12$ corresponds to the normalized knowledge-related productivity $\hat{A}_n^j$. The correlation between $\hat{A}_n^j$ and $\hat{T}_n^j$ is .0.7. Moreover, $\hat{A}_n^j$ explains around three-fourths of the variability of $\hat{T}_n^j$. Figure 7 shows that there is a strong positive relation between the knowledge-related productivity, relative to the United States in sector J, $\hat{A}_i^j$ and R&D intensity.

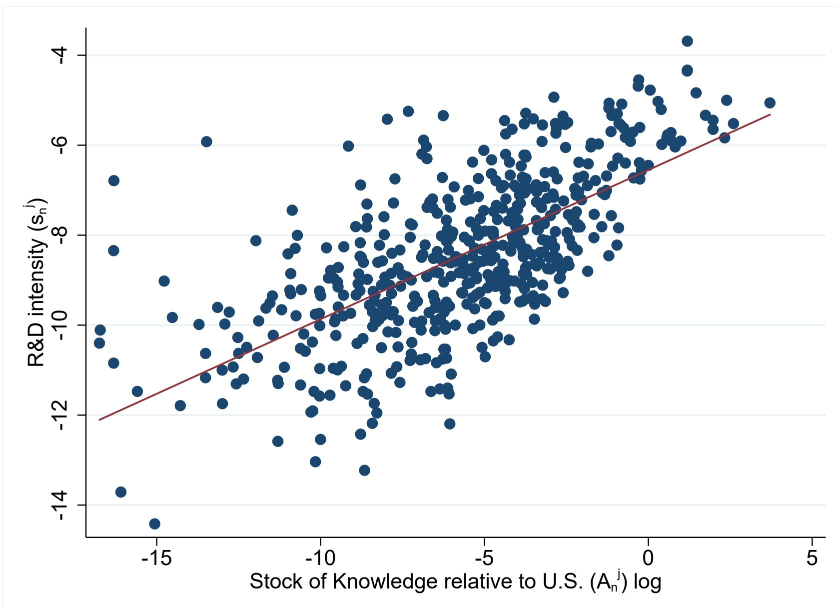


Figure 7: Research-related productivity and innovation

The correlation between A_n^j and s_n^j is around 0.7. The larger the stock of knowledge that the country-sector has accumulated, the larger the R&D intensity. This relation is heterogeneous across countries and depends on how much has been invested in R&D versus how much knowledge has been accumulated from other countries and sectors through diffusion. The relation also varies by sectors. The correlation is larger for the machinery and equipment, computer, electronic and optical equipment, and electrical machinery sectors.

5.1.4 The Algorithm

The calibration of the parameters of innovation, $\{\lambda_n^j, \beta_r, \hat{A}_n^j\}$ follows a recursive algorithm. First, knowing $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, \hat{T}_n^j, d_{in}^j\}$, we use the trade structure of the model to obtain wages, prices, expenditures, trade shares, and output, from equations (72), (74), (75), (76), (77), (79), (80), (81), and (82).

Then, given $\{\varepsilon_{in}^{jk}, g_A, s_n^j\}$, we iterate over equation (19) to obtain $\{\lambda_n^j, \beta_r\}$. We do given guessed values of λ_n^j and β_r, and we use R&D data, s_n^j, and keep iterating until $g_A = 0.25$. We use equation (83), (84) and the Frobenious theorem. The Frobenius theorem guarantees that there is a unique balanced-growth path in which all countries and sectors grow at the same rate g_A. The expression for the growth rate can be expressed in matrix form as

$$g_A A = \Delta(g_A) A.$$

If the matrix $\Delta(g_A)$ is definite positive, then there exists a unique positive balanced-growth rate of technology $g_A > 0$ given research intensities. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country and sector's relative level of knowledge along that balanced-growth path. We update β_r so that $g_A = 0.25$, and we update λ_n^j so that R&D intensity matches the data. Then we obtain $\hat{A}_n^j$ from the eigenvector associated to $\Delta(g_A = 0.25)$. Knowing $\hat{T}_n^j$ from the gravity regressions and $\hat{A}_n^j$ from the Frobenius theorem, we can obtain $T_{p,n}^j$ from equation (13).

5.2 Counterfactual Analysis

We perform a uniform and permanent reduction of trade barriers, d_{in}^j, of 10% for all country-pairs i, n and sector j. All other parameters are kept fixed at their calibrated values. We analyze the effect of this trade liberalization on innovation, long-run growth and comparative advantage. First, we describe briefly the algorithm that we develop to compute the counterfactual BGP. Different from the calibration algorithm, which could be solved in two stages—first characterizing the competitive equilibrium taking as given $\hat{T}_n^j$ (static equilibrium), and second solving for the innovation and diffusion parameters (dynamic equilibrium)—the algorithm to compute the transition is slightly more involved in that it requires us to solve for the static and dynamic parts of the model simultaneously. After having described the algorithm, we report our main results for our multi-country and multi-sector endogenous growth model featuring heterogeneous interlinkages in production and knowledge flows. First, we characterize welfare gains from trade in the baseline model and describe how changes in sectoral innovation and RCA across counterfactuals help shape gains from trade. Then we explore the role of the main two channels in our model: (i) the presence of knowledge spillovers and (ii) the multi-sector structure of the model.

5.2.1 The Algorithm

In our calibration, we took the average productivity, $\hat{T}_n^j$, as given by the estimated values from the gravity regressions. However, when there are changes in trade costs, $\hat{T}_n^j$ will change across counterfactuals to the extent that $\hat{A}_n^j$ also changes. In our model, there are changes in $\hat{A}_n^j$ that are induced by changes in the innovation intensity, s_n^j, and by knowledge diffusion. Our algorithm to solve for the counterfactual equilibrium uses the properties of the Frobenius theorem and allows $\hat{T}_n^j$ to evolve over time through changes in $\hat{A}_n^j$. First, we take $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, T_{p,n}^j, \hat{T}_n^j, \beta_r, \lambda_n^j\}$ as given and compute the static equilibrium that corresponds to the new trade barriers, d_{in}^j. With that equilibrium, we compute the new optimal R&D intensity s_n^j and use the Frobenius theorem to obtain the new g_A and associated eigenvector $\hat{A}_n^j$. We do this by iterating over equation (19) until $g_A(t-1) = g_A(t)$. The new $\hat{A}_n^j$ delivers a new $\hat{T}_n^j$ (we keep $T_{n,p}^j$ constant across counterfactuals). We then repeat the procedure until $\hat{T}_n^j$ converges.

5.2.2 Innovation, Growth and Comparative Advantage

In this section, we quantify the effect of trade liberalization on innovation, growth and comparative advantage according to the mechanisms exposed in Section 4. First, after trade liberalization, R&D reallocates towards sectors in which the country has increased its comparative advantage the most. In Figure 8, we examine the correlation between R&D intensity and comparative advantage in the baseline and counterfactual BGP. The figure plots a fitted line of the relation between the two variables in both equilibria. The line is steeper in the counterfactual, suggesting a reallocation effect towards sector sin which the country has increased its comparative advantage the most.

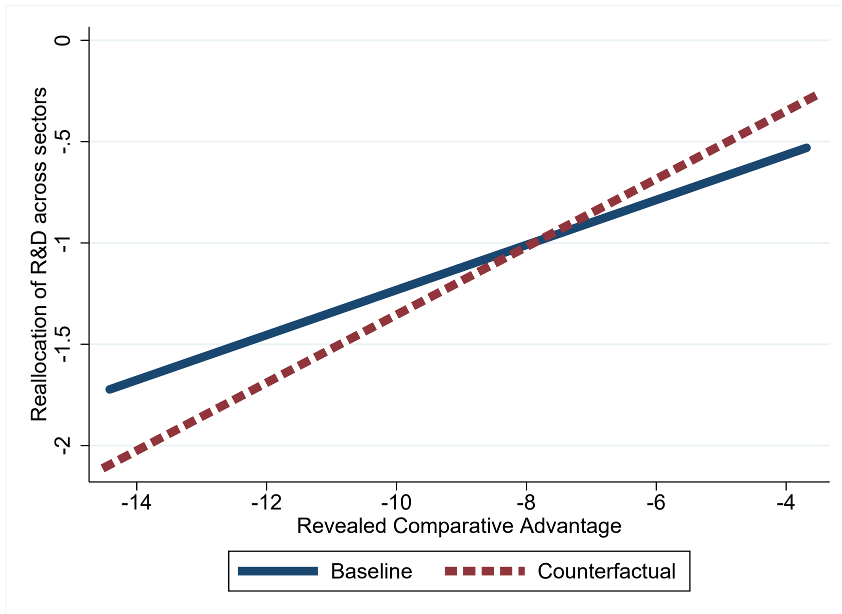


Figure 8: Reallocation of R&D and comparative advantage in innovation

As a result of the reallocation effect of R&D, growth jumps to a higher value in the new BGP. Growth of the stock of knowledge, g_T increases from 12% to 12.12%. The increase is exponential with the size of the trade liberalization, as Figure 9 exposes. After a 20% trade liberalization growth increases to 12.32%, whereas after a 90% trade liberalization it increases to 19.75%.

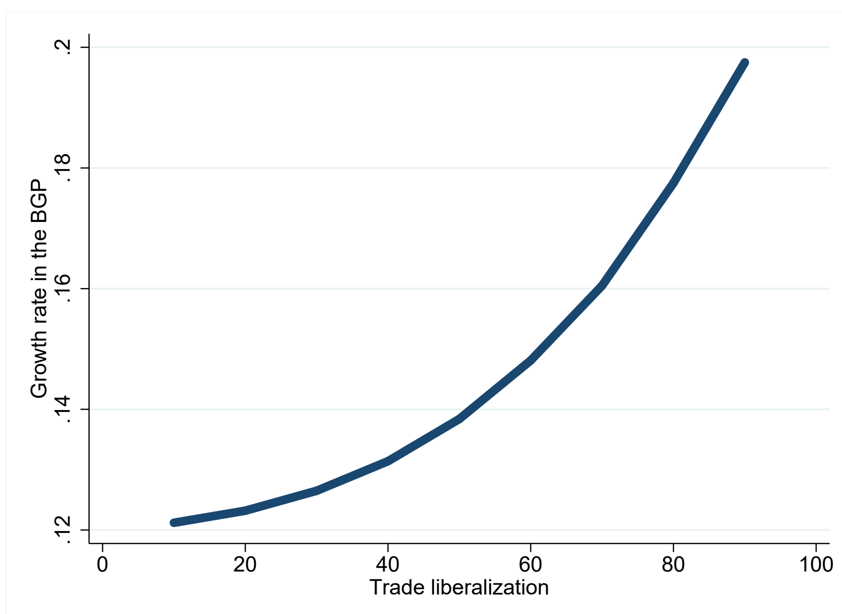


Figure 9: Effect of trade liberalization (% reduction on trade costs) of BGP growth

Comparative advantage is endogenous in our model. In the presence of heterogeneous knowledge spillovers, the reallocation of R&D effect the dispersion of relative productivity, reinforcing static forces of comparative advantage. In particular, the dispersion of relative productivity increases from 2 to 2.24. Within the sector, the average sector also experiences an increase in volatility, going from 0.6 to 0.7. Within each country, the dispersion of relative productivity increases from 0.95 to 1.1, suggesting that countries are also becoming more specialized towards particular sectors.

these results have consequences for welfare gains from trade, as we explore in Section 5.2.4.

5.2.3 Exploring the Mechanism

Knowledge spillovers We study the role of knowledge diffusion on welfare gains from trade. To do that, we recalibrate our baseline model in two ways. First, we consider the case of homogeneous diffusion, in which we set $\epsilon_{ni}^{jk} = \epsilon \ \forall i, n, j, k$, where ϵ is the speed of diffusion estimated in the data. Second, we consider the case of no diffusion by setting the diffusion parameters ε_{ni}^{jk} to a very small value of 0.0001, for all $i \neq n$ and $k \neq j$ (we set $\varepsilon_{nn}^{jj} \to \infty$; that is, we assume instantaneous diffusion within the same country-sector pair).[15] These recalibrations do not affect the first-stage calibration that solved for the competitive equilibrium of the model. However, we need to recalibrate the second-stage parameters, β_r and λ_n^j, by using the same input-output linkage parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$, estimated technology, T_n^j, R&D intensity, s_n^j, and growth rate, g_A, values than in the baseline model. We now obtain $\beta_r = 0.33$ in the case of homogeneous diffusion and $\beta_r = 0.13$ in the case of no diffusion. The effect of trade liberalization on growth rate is lower in the case of homogeneous diffusion or no diffusion. After trade liberalization, the growth rate increases to 0.1203 when there is homogeneous diffusion and to 0.1202 when there is no diffusion. Furthermore, the rate of increase of BGP growth with the size of trade liberalization is slower than in our baseline model with heterogeneous knowledge spillovers (see figure 10).

[15] The Frobenius theorem is only valid if there is at least some diffusion across all country-sector pairs. Setting ε_{ni}^{jk} to a very low number allows us to make use of the properties of the Frobenius theorem while allowing for very slow to virtually no diffusion.

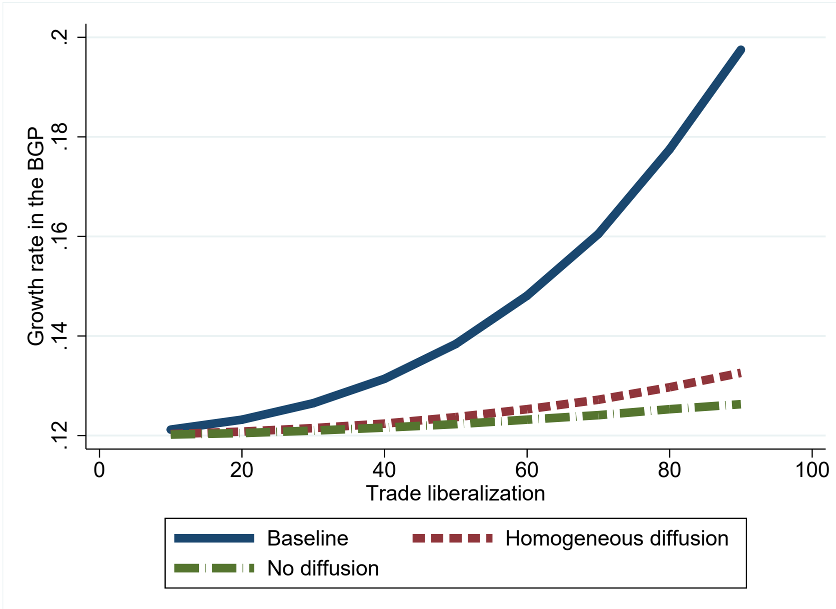


Figure 10: Effect of trade liberalization of BGP growth

The Role of Multiple Sectors In our baseline model, we have emphasized the importance of the role of multiple sectors in reallocating R&D intensity and RCA across sectors. This reallocation was important to understand the role of innovation and knowledge diffusion in generating dynamic gains from trade. We now recalibrate our baseline model to a one-sector model in which there are no production and knowledge diffusion interlinkages across sectors.[16]

We re-estimate the technology parameters, $\hat{T}_n$ by running gravity equations at the country level. Note that now there is not a sector j dimension in the model. The production and knowledge linkages parameters are also recalibrated at the country level. We set $\alpha^j = 1$, $\gamma^j = 1$ and $\gamma^{jk} = 0$ for all j and k. We obtain country-level data for R&D intensity, s_n. Then, assuming the same g_A as in the baseline model, we obtain a $\beta_r = 0.28$. In a one-sector model, trade liberalization does not have any effect on innovation or growth, even in the presence of knowledge spillovers. The reason is that the competition and market effect cancel out in a one sector model.

5.2.4 Welfare Gains from Trade

We compute welfare gains from trade after a trade liberalization between the baseline and the counterfactual BGP. Welfare in our model is defined in equivalent units of consumption. We ignore transitional dynamics in this analysis. We can use equation (1) to obtain the lifetime utility in the initial BGP as

[16]The one-sector model is equivalent to a special case of the multisector model in which all sectors are connected by symmetric IO and knowledge linkages.

$$\bar{U}_i^* = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{1-\gamma} e^{g^*(1-\gamma)t} dt = \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{\rho - g^*(1-\gamma)},$$

and in the counterfactual BGP as

$$\bar{U}_i^{**} = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{1-\gamma} e^{g^{**}(1-\gamma)t} dt = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}$$

with $*$ denoting the baseline BGP and $**$ denoting the counterfactual BGP.

Welfare gains are defined as the amount of consumption that the consumer is willing to give up in the counterfactual BGP to remain at the same level as in the initial BGP. We call this, λ_i, which is obtained as

$$\bar{U}_i^*(\lambda_i) = \bar{U}_i^{**}$$

$$\frac{\left(\hat{C}_i^* \lambda_i\right)^{1-\gamma}}{\rho - g^*(1-\gamma)} = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}.$$

From here,

$$\lambda_i = \frac{\hat{C}_i^{**}}{\hat{C}_i^*} \left(\frac{\rho - g^*(1-\gamma)}{\rho - g^{**}(1-\gamma)} \right)^{\frac{1}{1-\gamma}}. \tag{44}$$

Welfare gains depend on changes in normalized consumption between the BGPs and the change in growth rates. From equation (82), normalized consumption in the BGP is equal to income per capita net of R&D expenditures. That is,

$$\hat{C}_i = \hat{Y}_i - \sum_{k=1}^{J} s_i^k \hat{Y}_i = \left(1 - \sum_{k=1}^{J} s_i^k\right) \hat{Y}_i. \tag{45}$$

In static models or one-sector models of trade and innovation in which changes in trade costs do not have an effect on innovation, $g^* = g^{**}$ and $s_i^k = 0$. In that case, welfare gains from trade are computed as changes in the real wage. As in Caliendo and Parro (2015), we can obtain an expression for the real wage in country i as

$$\frac{W_i}{P_i} \propto \prod_{j=1}^{J} \left(\left(\frac{T_i^j}{\pi_{ii}^j} \right)^{\alpha^j/\theta} \prod_{k=1}^{J} \left(\frac{W_i}{P_i^k} \right)^{\alpha_i^j \gamma^{jk}} \right). \tag{46}$$

Note that this formula is the same as derived in Caliendo and Parro (2015) resembles the standard welfare formula in Arkolakis, Costinot, and Rodríguez-Clare (2012). In a one-sector version of our model, in which $j = 1$, γ^{jk}=0, and $\alpha^j = 1$, equation (46) becomes

$$\frac{W_i}{P_i} \propto \left(\frac{T_i}{\pi_{ii}}\right)^{1/\theta}. \tag{47}$$

This is the standard formula for welfare gains from trade that has been used in the literature and depends on aggregate productivity, the home trade shares and the trade elasticity.

Welfare Results We compute welfare gains from trade using equation (44). We find that welfare gains from trade are heterogeneous across countries, ranging from 4% to 28%, with a cross-country average gain of 19%. The gains are larger for smaller countries (see Figure 11), which is consistent with the findings in Waugh (2010).

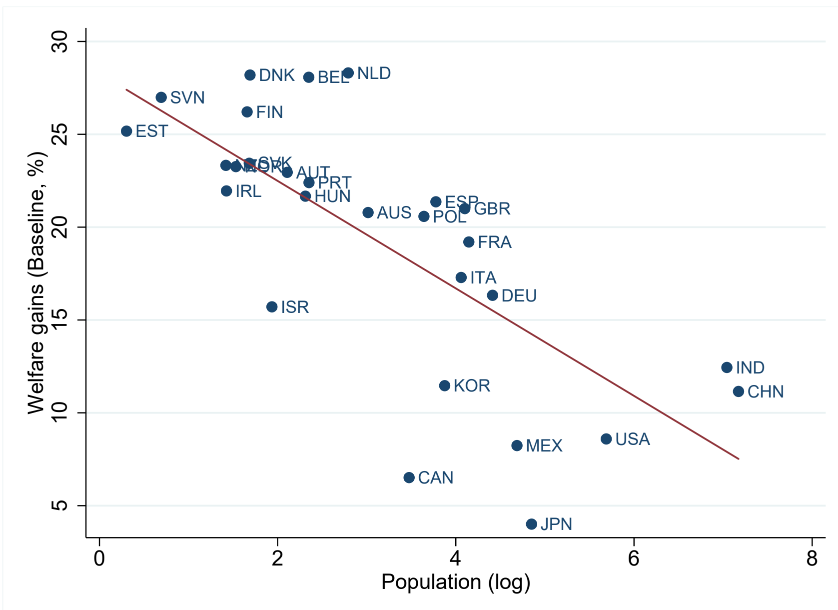


Figure 11: Welfare gains from trade and population

Welfare gains from trade can be divided into static and dynamic gains. Static gains correspond to those obtained in a model where the stock of knowledge, $\hat{A}^j_{nt}$, is not allowed to change over time. These are the gains that are obtained in standard static models of trade and are driven by increased specialization and comparative advantage. Dynamic gains take into account the effect of R&D and knowledge spillovers on the stock of knowledge. Both allow the stock of knowledge to increase over time. Higher innovation allows countries to increase their income per capita, which has an unambiguously positive effect on dynamic gains. Knowledge diffusion has two opposite effects on dynamic gains from trade. On the one hand, it increases the stock of knowledge of a country-sector as it can benefit from innovation created in other country-sectors. This has an additional effect on the efficiency of innovation, from equation (14), which reinforces the innovation channel. On the other hand, knowledge spillovers may generate convergence of comparative advantage over time, dampening the total welfare gains from trade that are driven by differences in comparative advantage.

To compute static welfare gains, we simulate our model keeping $\hat{T}_i^j$ and $\hat{A}_i^j$ constant across counterfactuals. Because we are analyzing only changes across BGPs, dynamic gains do not include the transition. We call them dynamic in that they reflect the gains that account for changes in the stock of knowledge across counterfactuals. Therefore, these gains are computed by letting $\hat{T}_i^j$ and $\hat{A}_i^j$ vary across counterfactuals. We then compare consumption in the initial and counterfactual BGPs.

Table 2 compares welfare gains from trade in our baseline model to those static gains in which the stock of technology is kept constant across counterfactuals. The difference between the two gains is a measure of dynamic gains from trade. The cross-country distribution of static gains is shifted to the left, which implies that dynamic gains are positive in every country.

Finally, we compare welfare gains from trade in our baseline model to those in a model with homogeneous diffusion, no diffusion and a one-sector model with heterogeneous diffusion. teh model that generates the lowest and least disperse gains from trade is the one-sector model. Trade liberalization has no effect on innovation and growth in this case, making it a static model. Furthermore, the lack of input-output linkages does not allow for additional gains from trade of multi-sector models. With respect to our baseline, the cases of homogeneous or no diffusion also deliver lower and less disperse gains from trade (see Table 2).

Model	Mean	Std. Dev.	Min	Max
Baseline	19.16	6.96	4.00	28.31
Static	9.61	3.39	2.58	17.97
Homogeneous diffusion	11.64	4.27	2.47	20.56
No diffusion	11.20	3.92	3.07	20.58
One sector	0.48	0.34	0.05	1.57

Table 2: Welfare gains from trade

6 Concluding Remarks

We develop a quantitative framework to study the effect of interlinkages among trade, knowledge flows and production on innovation, comparative advantage, growth and welfare. We distinguish between static gains from trade, which are driven by increased specialization, and dynamic gains from trade, which are driven by innovation and knowledge diffusion. Changes in trade barriers have a quantitatively important effect on innovation and welfare. After a trade liberalization, R&D reallocates toward sectors in which the country has a comparative advantage. Knowledge diffusion amplifies this effect, as comparative advantage is reallocated towards sectors with larger knowledge flows. Furthermore, knowledge spillovers allow sectors in a country to benefit for a larger pool of ideas, increasing dynamic welfare gains from trade. A one-sector version of our model delivers much smaller total gains in welfare and almost negligible, or even negative in some countries, dynamic welfare gains. This result reinforces the importance of modeling sectoral heterogeneity when studying the effect of trade liberalizations on innovation and welfare.

Our model can be extended to study other important issues in macroeconomics and international trade. If the production structure of the economy is assumed to be CES rather than Cobb-Douglas, a trade liberalization that changes technology and production costs will shift production shares across sectors, hence inducing structural change.

References

Acemoglu, Daron, Ufuk Akcigit, and William R Kerr. 2016. “Innovation network.” *Proceedings of the National Academy of Sciences* 113 (41):11483–11488.

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. “The network origins of aggregate fluctuations.” *Econometrica* 80 (5):1977–2016.

Aghion, Philippe, Antonin Bergeaud, Matthieu Lequien, and Marc J Melitz. 2018. “The impact of exports on innovation: Theory and evidence.” Tech. rep., National Bureau of Economic Research.

Akcigit, Ufuk, Sina T Ates, and Giammario Impullitti. 2018. “Innovation and Trade Policy in a Globalized World.” Tech. rep., National Bureau of Economic Research.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. “New trade models, same old gains?” *The American Economic Review* 102 (1):94–130.

Arkolakis, Costas, Natalia Ramondo, Andrés Rodríguez-Clare, and Stephen Yeaple. 2013. “Innovation and production in the global economy.” Tech. rep., National Bureau of Economic Research.

Atkeson, Andrew and Ariel Tomas Burstein. 2010. "Innovation, Firm Dynamics, and international Trade." *Journal of political economy* 118 (3):433–484.

Autor, David, David Dorn, Gordon H Hanson, Gary Pisano, and Pian Shu. 2016. "Foreign Competition and Domestic Innovation: Evidence from US Patents." Tech. rep., National Bureau of Economic Research, Inc.

Bernard, Jonathan Eaton, Bradford Jensen, and Samuel Kortum. 2003. "Plants and Productivity in International Trade." *The American economic review* 93 (4).

Bloom, Nicholas, Mirko Draca, and John Van Reenen. 2016. "Trade induced technical change? The impact of Chinese imports on innovation, IT and productivity." *The Review of Economic Studies* 83 (1):87–117.

Bottazzi, Laura and Giovanni Peri. 2003. "Innovation and spillovers in regions: Evidence from European patent data." *European economic review* 47 (4):687–710.

Buera, Francisco J and Ezra Oberfield. 2016. "The Global Diffusion of Ideas." Tech. rep., National Bureau of Economic Research.

Bustos, Paula. 2011. "Trade liberalization, exports, and technology upgrading: Evidence on the impact of MERCOSUR on Argentinian firms." *American economic review* 101 (1):304–40.

Caballero, Ricardo J and Adam B Jaffe. 1993. "How high are the giants' shoulders: An empirical assessment of knowledge spillovers and creative destruction in a model of economic growth." *NBER macroeconomics annual* 8:15–74.

Cai, Jie and Nan Li. 2016. "The Composition of Knowledge and Cross-Country Income Differences." .

———. 2018. "Growth through intersectoral knowledge linkages." *Review of Economic Studies* forthcoming.

Caliendo, Lorenzo and Fernando Parro. 2015. "Estimates of the Trade and Welfare Effects of NAFTA." *The Review of Economic Studies* 82 (1):1–44.

Cameron, Gavin, James Proudman, and Stephen Redding. 2000. "Productivity growth in an open economy: the experience of the UK." *Productivity, innovation, and economic performance* :149–176.

Carvalho, Vasco and Xavier Gabaix. 2013. "The great diversification and its undoing." *American Economic Review* 103 (5):1697–1727.

Carvalho, Vasco M. 2014. "From micro to macro via production networks." *Journal of Economic Perspectives* 28 (4):23–48.

Comin, Diego and Bart Hobijn. 2010. "An exploration of technology diffusion." *American Economic Review* 100 (5):2031–59.

Costinot, Arnaud, Dave Donaldson, and Ivana Komunjer. 2012. "What goods do countries trade? A quantitative exploration of Ricardo's ideas." *The Review of economic studies* 79:581–608.

Eaton, Jonathan and Samuel Kortum. 1996. "Trade in ideas: Productivity and patenting in the OECD." *Journal of International Economics* 40 (3-4):251–278.

———. 1999. "International technology diffusion: Theory and measurement." *International Economic Review* 40 (3):537–570.

———. 2002. "Technology, geography, and trade." *Econometrica* 70 (5):1741–1779.

———. 2006. "Innovation, diffusion, and trade." Tech. rep., National Bureau of Economic Research.

Eaton, Jonathan, Samuel Kortum, Brent Neiman, and John Romalis. 2016. "Trade and the global recession." *The American Economic Review* 106 (11):3401–3438.

Ethier, Wilfred J. 1982. "Dumping." *The Journal of Political Economy* :487–506.

Fons-Rosen, Christian, Sebnem Kalemli-Ozcan, Bent E Sorensen, Carolina Villegas-Sanchez, and Vadym Volosovych. 2017. "Foreign Investment and Domestic Productivity: Identifying knowledge spillovers and competition effects." Tech. rep., National Bureau of Economic Research.

Gabaix, Xavier. 2011. "The granular origins of aggregate fluctuations." *Econometrica* 79 (3):733–772.

Griffith, Rachel, Sokbae Lee, and John Van Reenen. 2011. "Is distance dying at last? Falling home bias in fixed-effects models of patent citations." *Quantitative economics* 2 (2):211–249.

Griliches, Zvi. 1990. "Patent Statistics as Economic Indicators: A Survey." *Journal of Economic Literature* 28 (4):1661–1707.

Grossman, Gene M and Elhanan Helpman. 1991. "Trade, knowledge spillovers, and growth." *European economic review* 35 (2-3):517–526.

Hanson, Gordon H, Nelson Lind, and Marc-Andreas Muendler. 2015. "The dynamics of comparative advantage." Tech. rep., National bureau of economic research.

Hausmann, Ricardo, Jason Hwang, and Dani Rodrik. 2007. "What you export matters." *Journal of economic growth* 12 (1):1–25.

Hidalgo, César A, Bailey Klinger, A-L Barabási, and Ricardo Hausmann. 2007. "The product space conditions the development of nations." *Science* 317 (5837):482–487.

Jaffe, Adam B, Manuel Trajtenberg, and Michael S Fogarty. 2000. "Knowledge spillovers and patent citations: Evidence from a survey of inventors." *American Economic Review* 90 (2):215–218.

Jaffe, Adam B, Manuel Trajtenberg, and Rebecca Henderson. 1993. "Geographic localization of knowledge spillovers as evidenced by patent citations." *the Quarterly journal of Economics* 108 (3):577–598.

Keller, Wolfgang. 1998. "Are international R&D spillovers trade-related?: Analyzing spillovers among randomly matched trade partners." *European Economic Review* 42 (8):1469–1481.

———. 2010. "International trade, foreign direct investment, and technology spillovers." In *Handbook of the Economics of Innovation*, vol. 2. Elsevier, 793–829.

Kortum, Samuel S. 1997. "Research, patenting, and technological change." *Econometrica* :1389–1419.

Levchenko, Andrei A and Jing Zhang. 2016. "The evolution of comparative advantage: Measurement and welfare implications." *Journal of Monetary Economics* 78:96–111.

Levin, Richard C, Alvin K Klevorick, Richard R Nelson, Sidney G Winter, Richard Gilbert, and Zvi Griliches. 1987. "Appropriating the returns from industrial research and development." *Brookings papers on economic activity* 1987 (3):783–831.

Li, Yao Amber. 2014. "Borders and distance in knowledge spillovers: Dying over time or dying with age?Evidence from patent citations." *European Economic Review* 71:152–172.

Lileeva, Alla and Daniel Trefler. 2010. "Improved access to foreign markets raises plant-level productivity for some plants." *The Quarterly journal of economics* 125 (3):1051–1099.

Lind, Nelson and Natalia Ramondo. 2018. "Trade with Correlation." Tech. rep., National Bureau of Economic Research.

MacGarvie, Megan. 2006. "Do firms learn from international trade?" *Review of Economics and Statistics* 88 (1):46–60.

Pakes, Ariel and Mark Schankerman. 1984. "The rate of obsolescence of patents, research gestation lags, and the private rate of return to research resources." In *R&D, patents, and productivity*. University of Chicago Press, 73–88.

Peri, Giovanni. 2005. "Determinants of knowledge flows and their effect on innovation." *Review of Economics and Statistics* 87 (2):308–322.

Perla, Jesse, Christopher Tonetti, and Michael E Waugh. 2015. "Equilibrium technology diffusion, trade, and growth." Tech. rep., National Bureau of Economic Research.

Ramondo, Natalia and Andrés Rodríguez-Clare. 2013. "Trade, multinational production, and the gains from openness." *Journal of Political Economy* 121 (2):273–322.

Rivera-Batiz, Luis A and Paul M Romer. 1991. "Economic integration and endogenous growth." *The Quarterly Journal of Economics* 106 (2):531–555.

Sampson, Thomas. 2016. "The Global Productivity Distribution and Ricardian Comparative Advantage." .

Santacreu, Ana Maria. 2015. "Innovation, diffusion, and trade: Theory and measurement." *Journal of Monetary Economics* 75:1–20.

Santacreu, Ana Maria and Heting Zhu. 2018. "Domestic Innovation and International Technology Diffusion as Sources of Comparative Advantage." 4.

Somale, Mariano. 2014. "Comparative advantage in innovation and production." Tech. rep., Mimeo, Princeton University.

Thompson, Peter and Melanie Fox-Kean. 2005. "Patent citations and the geography of knowledge spillovers: A reassessment." *American Economic Review* 95 (1):450–460.

Waugh, Michael. 2010. "International trade and income differences." *The American Economic Review* 100 (5):2093–2124.

Appendix

A Model Equations

We normalize $W_M = 1$. The endogenous variables are, for each $i = 1 \dots M$ and $n = 1 \dots M$

$$\{\pi_{in}^j, T_i^j, c_i^j, W_i, P_n^j, X_{ni}^j, X_n^j, P_n, Y_n, \Phi_n^j, C_n, s_n^j, V_n^j, \Pi_{nt}^j, A_n^j\}$$

The corresponding equations are as follows:

(1) Probability of Imports

$$\pi_{ni}^j = T_i^j \frac{\left(c_i^j d_{ni}^j\right)^{-\theta}}{\Phi_n^j}, \tag{48}$$

(2) Technology level

$$T_i^j = A_i^j T_{p,i}^j. \tag{49}$$

(3) Import shares

$$X_{ni}^j = \pi_{ni}^j X_n^j. \tag{50}$$

(4) Cost of production

$$c_n^j = \Upsilon^j W_{nt}^{\gamma^j} \prod_{k=1}^{J} (P_n^k)^{\gamma^{jk}}. \tag{51}$$

(5) Intermediate good prices in each sector

$$P_n^j = A^j \left(\Phi_n^j\right)^{-1/\theta}. \tag{52}$$

(6) Cost distribution

$$\Phi_n^j = \sum_{i=1}^{M} T_i^j \left(d_{ni}^j c_i^j\right)^{-\theta}. \tag{53}$$

(7) Price index

$$P_n = \prod_{j=1}^{J} \left(\frac{P_n^j}{\alpha^j}\right)^{\alpha^j}. \tag{54}$$

(8) Labor market clearing condition

$$W_nL_n = \sum_{j=1}^{J}\gamma^j\sum_{i=1}^{M}\pi_{in}^jX_i^j. \tag{55}$$

(9) Sector production

$$X_n^j = \sum_{k=1}^{J}\gamma^{kj}\sum_{i=1}^{M}X_i^k\pi_{in}^k + \alpha^jP_nY_n. \tag{56}$$

(10) Income

$$P_nC_n = W_nL_n + \frac{\sum_{j=1}^{J}\sum_{i=1}^{M}\pi_{in}^jX_i^j}{1+\theta}. \tag{57}$$

(11) Resource constraint

$$Y_n = C_n + \sum_{k=1}^{J}s_n^kY_n. \tag{58}$$

(12) Innovation

$$\dot{A}_{nt}^j = \sum_{i=1}^{M}\sum_{k=1}^{J}\varepsilon_{ni}^{jk}\int_{-\infty}^{t}e^{-\varepsilon_{ni}^{jk}(t-s)}\alpha_{is}^k\left(s_{is}^k\right)^{\beta^k}ds. \tag{59}$$

(13) R&D expenditures

$$\beta^j\lambda_{nt}^jV_{nt}^j\left(s_{nt}^j\right)^{\beta^j-1} = P_{nt}Y_{nt}. \tag{60}$$

(14) Value of an innovation

$$V_{nt}^j = \int_t^\infty\left(\frac{P_{nt}^j}{P_{ns}^j}\right)e^{-\int_t^s r_{nu}du}\frac{\Pi_{ns}^j}{A_{ns}^j}ds, \tag{61}$$

(15) Profits

$$\Pi_{nt}^j = \frac{1}{(1+\theta)}\sum_{i=1}^{M}X_{it}^j\pi_{int}^j. \tag{62}$$

(17) Trade balance

$$\sum_{k=1 k\neq j}^{J}X_{nt}^k\sum_{i=1 i\neq n}^{M}\pi_{nit}^k = \sum_{i=1 i\neq n}^{M}\sum_{k=1 k\neq j}^{J}\pi_{int}^kX_{it}^k. \tag{63}$$

B The Balanced-growth Path

Here, we derive an expression for the growth rate of the economy along the BGP. First, note that through technology diffusion, the level of knowledge-related productivity, A_n^j, grows at the same rate for every country n and sector j. Therefore, we can pick country M and sector J's technology level to normalize every A_n^j and T_n^j. Normalized variables are denoted with a hat. In particular, $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$.

From equation (60), we normalize the value of an innovation as $\hat{V}_n^j = \frac{V_n^j T_M^J}{W_M}$. Then, from equation (62), profits are normalized as $\hat{\Pi}_n^j = \frac{\Pi_n^j}{W_M}$, and from equation (55), X_i^j is normalized as $\hat{X}_i^j = \frac{X_i^j}{W_M}$ for all j. Hence, expenditures grow at a constant rate for all sectors, since π_{in}^j is constant in the BGP (see equations (48) and (53)). From equations (55) and (57), $P_n Y_n$ grow at the rate of W_M. Note that $g_{w_n} = g_w$ for all n.

To derive an expression for the BGP growth rate of the real output per capita, Y_n, we start from the fact that $\frac{W_n}{P_n Y_n}$ is constant in steady-state. Hence,

$$g_{Y_n} = g_w - g_{P_n}.$$

Using equation (54),

$$g_{P_n} = \sum_{j=1}^{J} \alpha^j g_{p_n^j}.$$

We then derive the expression for $g_{p_n^j}$ from equations (51), (52) and (53). First, we rewrite equation (51) as

$$\frac{c_n^j}{W_n} = \prod_{k=1}^{J} \left(\frac{p_n^k}{W_n} \right)^{\gamma_n^{jk}}.$$

In growth rates, it becomes

$$g_{\tilde{c}_n^j} = \sum_{k=1}^{J} \gamma_n^{jk} g_{\tilde{p}_n^k}, \tag{64}$$

where $\tilde{c}_n^j = \frac{c_n^j}{W_n}$ and $\tilde{p}_n^k = \frac{p_n^k}{W_n}$. From equation (53),

$$g_{\Phi_n^j} = g_T - \theta g_{c_n^j} = g_T - \theta g_{c_i^j}.$$

with $g_T = g_A$.

Hence, $g_{c_n^j} = g_{c^j}$ for all n. Normalizing by wages,

$$g_{\tilde{\Phi}_n^j} = g_T - \theta g_{\tilde{c}_n^j}, \tag{65}$$

where $\tilde{\Phi}_n^j = \frac{\Phi_n^j}{W_n^{-\theta}}$

Combining equation (52) and (65) implies that

$$g_{\tilde{p}_n^k} = -\frac{1}{\theta} g_T + g_{\tilde{c}^k}. \tag{66}$$

Substitution into (64) and using $\sum_{k=1}^{J} \gamma^{jk} = 1 - \gamma^j$, we get

$$g_{\tilde{c}^j} = -\frac{(1-\gamma^j)}{\theta} g_T + \sum_{k=1}^{J} \gamma^{jk} g_{\tilde{c}^k}. \tag{67}$$

We can express the previous expression in matrix form so that

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{1}{\theta} g_T \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} + \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} & \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} & \end{bmatrix} \begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} \tag{68}$$

From here

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{g_T}{\theta} (I-A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} \tag{69}$$

where

$$A = \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \vdots & \ddots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix}$$

Therefore, the cost of production c_n^j can be normalized as

$$\tilde{c}_n^j = \frac{c_n^j}{W_M (T_M^J)^{-\frac{1}{\theta}\Lambda_j}}, \tag{70}$$

where Λ_j is the jth entry of the vector $\Lambda = (I-A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix}$.

With this, we can obtain an expression for the growth rate of real output as

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j g_{p_n^j}.$$

From Equation (66), we have

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_{c^j} \right).$$

Based on Equation (70), the above equation becomes

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_w - \Lambda_j g_T \right).$$

Therefore,

$$g_{Y_n} = \frac{1}{\theta} \left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j \right) g_T = g_y, \forall n. \tag{71}$$

Note that in a one-sector economy in which $\gamma^{jk} = 0, \forall n, k$ and $\gamma^j = 1, \forall j$, the growth rate is

$$g_y = -\frac{1}{\theta} g_T.$$

as in Eaton and Kortum (1996, 1999). With multiple sectors, however, the growth rate of the economy is amplified by the input-output linkages.

C Model Equations (Normalized) along the BGP

In what follows, we report the equations of the model after normalizing the endogenous variables so that they are constant in the BGP. We follow the results obtained in Appendix B.

(1) Probability of imports

$$\pi_{ni}^j = \hat{T}_i^j \frac{\left(\hat{c}_i^j d_{ni}^j \right)^{-\theta}}{\hat{\Phi}_n^j}, \tag{72}$$

where $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$ and $\hat{\Phi}_n^j = \frac{1}{T_M^J} \frac{\Phi_n^j}{(W_M)^{-\theta} (T_M^J)^{\Lambda_j}}$ with Λ^j defined in Appendix B.

(2) Technology level

$$\hat{T}_i^j = \hat{A}_i^j T_{p,i}^j. \tag{73}$$

(3) Import shares

$$\hat{X}^j_{ni} = \pi^j_{ni}\hat{X}^j_n. \tag{74}$$

(4) Cost of production

$$\hat{c}^j_n = \Upsilon^j \hat{W}_n^{\gamma^j} \prod_{k=1}^{J} (\hat{P}^k_n)^{\gamma^{jk}}. \tag{75}$$

(5) Intermediate good prices in each sector

$$\hat{P}^j_n = B\left(\hat{\Phi}^j_n\right)^{-1/\theta}. \tag{76}$$

(6) Cost distribution

$$\hat{\Phi}^j_n = \sum_{i=1}^{M} \hat{T}^j_i \left(d^j_{ni}\hat{c}^j_i\right)^{-\theta}. \tag{77}$$

(7) Price index

$$\hat{P}_n = \prod_{j=1}^{J} \left(\frac{\hat{P}^j_n}{\alpha^j}\right)^{\alpha^j}. \tag{78}$$

(8) Labor market clearing condition

$$\hat{W}_n L_n = \sum_{j=1}^{J} \gamma^j \sum_{i=1}^{M} \pi^j_{in}\hat{X}^j_i. \tag{79}$$

(9) Sector production

$$\hat{X}^j_n = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} \pi^k_{in}\hat{X}^k_i + \alpha^j \hat{Y}_n. \tag{80}$$

where $\hat{Y}_n = \frac{P_n Y_n}{W_M}$.

(10)Income

$$\hat{C}_n = \hat{W}_n L_n + \frac{\sum_{j=1}^{J}\sum_{i=1}^{M}\pi^j_{in}\hat{X}^j_i}{1+\theta}. \tag{81}$$

(11) Resource constraint

$$\hat{Y}_n = \hat{C}_n + \sum_{k=1}^{J} s^k_n \hat{Y}_n. \tag{82}$$

(12) Innovation

$$g_A = \sum_{i=1}^{M}\sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(\frac{1}{r - g_y + g_A} \beta_r \lambda_i^k \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}}.$$

(13) R&D expenditures

$$\beta_r \lambda_n^j \hat{V}_n^j \left(s_n^j\right)^{\beta_r - 1} = \hat{Y}_n. \tag{83}$$

(14) Value of an innovation

$$\hat{V}_n^j = \left(\frac{1}{r - g_y + g_A} \right) \frac{\hat{\Pi}_n^j}{\hat{A}_n^j}, \tag{84}$$

(15) Profits

$$\hat{\Pi}_n^j = \frac{1}{(1+\theta)} \sum_{i=1}^{M} \hat{X}_i^j \pi_{in}^j. \tag{85}$$

(17) Trade balance

$$\sum_{k=1 k\neq j}^{J} \hat{X}_{nt}^k \sum_{i=1 i\neq n}^{M} \pi_{nit}^k = \sum_{i=1 i\neq n}^{M} \sum_{k=1 k\neq j}^{J} \pi_{int}^k \hat{X}_{it}^k. \tag{86}$$

D Data Description and Calculation

This appendix describes the data sources and the construction of various variables for the paper. Twenty-eight countries are included in our analysis based on data availability (mostly constrained by the availability of the R&D data): Australia, Austria, Belgium, Canada, China, Czech Republic, Estonia, Finland, France, Germany, Hungary, India, Israel, Italy, Japan, Korea, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovenia, Spain, Slovakia, Slovenia, the United Kingdom, and the United States. The model is calibrated for 2005. Eighteen tradable sectors and one aggregate nontradable sector are under consideration and reported in Table 3.

Bilateral trade flows at the sectoral level Bilateral trade data at the sectoral level (expenditure by country n of sector j goods imported from country i, X_{ni}^j) are obtained from the OECD STAN Bilateral Trade Dataset. Values are reported in thousands of U.S. dollars at current prices. Sectors are recorded at the ISIC (rev. 3) 2-3 digit level and are aggregated into the 19 sectors as listed in Table 3. We use the importer reported exports in each sector as the bilateral trade flows because it is generally considered to be more accurate than the exporter reported exports.

Value added and gross production Domestic sales in sector j, X_{nn}^j, are estimated based on the *domestic* input-output table provided by the OECD STAN database, which contains data at the ISIC 2-digit level that can be easily mapped into our 19 sectors. OECD provides separate IO tables for domestic output and imports. We sum up the values for a given row before the column "Direct purchases abroad by residents (imports)" to obtain X_{nn}^j. We compare this way of estimating the domestic expenditure on domestic product with an alternative calculation based on $X_{nn}^j = Y_n^j - \sum_{i\neq n}^M X_{in}^j$, where both gross production of country n in sector j, Y_n^j, and the total exports from n to i in sector j, $\sum_{i\neq n}^M X_{in}^j$, are from the OECD STAN Database for Structural Analysis. The first method proves to be superior, as the second generates a number of negative observations for some country-sectors. However, data are missing for India, for which we use the INDSTAT (2016 version) provided by United Nations Industrial Development Organization (UNIDO).

Trade barriers and gravity equation variables Data for variables related to trade costs used in gravity equations (such as geographic distance and common border dummies) at the country-pair level are obtained from the comprehensive geography database compiled by CEPII. The WTO's RTA database provides information on regional trade agreements. The currency union indicator is obtained from Rose (2004) and was updated to reflect Euro-area membership.

Wages Average annual wages are reported by the OECD labor statistics at current prices in local currency. They are transformed into U.S. dollars at the 2005 exchange rates to obtain the variable w_n in the model. However, wage data for China, India, and New Zealand are missing in this database, and are obtained from the International Labor Organization (ILO).

Factor shares and final consumption shares In our analysis, we used the U.S. factor shares in 2005 for all countries. Data on the share of materials from sector k used in the production in sector j, γ^{jk}, as well as the labor share of production in sector j, γ^j, come from the Input-Output Database maintained by OECD STAN. The I-O table gives the value of the intermediate input in row k required to produce one dollar of final output in column j. We then divide this value by the value of gross output of sector j to obtain γ^{jk}. Similarly, the labor share is calculated as the ratio of value added to gross output, as capital input does not exist in the model. In addition, the final consumption expenditure shares of each sector, α_n^j also come from the I-O matrix.

R&D data R&D expenditures at the country-sector level are obtained from the OECD database of Business Enterprise R&D expenditure by industry (ISIC Rev 3). Since sectoral R&D data for China, India and Sweden and several sectors in other countries are missing, we obtain estimates of these missing observations using the following approach. First, we run a regression using existing country-sector specific R&D and patent data from USPTO for 2005:

$$\log(R_n^j) = \beta_0 + \beta_1 \log\left(PS_n^j\right) + \mu_n + \gamma_j + \varepsilon_n^j, \tag{87}$$

where R_n^j is the R&D dollar expenditure of country i in sector j and PS_n^j is the patent stock of country i in sector j. μ_i and γ_j are country and sector fixed effects. This relation is built on the observations that (i) in the steady state, R&D expenditure should be a constant ratio of R&D stock and (ii) innovation input (R&D stock) is significantly positively related to innovation output (patent stock). In fact, the coefficient β_1 is large and significant at 99% and the R^2 is close to 0.90. Assuming that the relationship captured by equation (87) holds for China, India, and Sweden, we can obtain the fitted value of their sectoral level R&D expenditure:

$$\log(\widehat{R}_n^j) = \widehat{\beta}_0 + \widehat{\beta}_1 \log\left(PS_n^j\right) + \widehat{\mu}_n + \widehat{\gamma}_j.$$

For these three countries, we have information on all the right-hand-side variables except for the country fixed effects, $\widehat{\mu}_n$. This allows us to compute the *share* of R&D in a given sector for each country as

$$\widehat{r}_n^j = \frac{\widehat{R}_n^j}{\sum_j \widehat{R}_n^j} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}.$$

Second, we obtain the aggregate R&D expenditure as a percentage of GDP, $R\&D/GDP_n^{WB}$, for each country from the World Bank World Development Indicator database. The country-sector specific R&D can then be estimated as $s_n^j = \widehat{r}_n^j \times R\&D/GDP_n^{WB}$. For the countries with missing sectors, we estimate the fitted value using the same procedure. To maintain consistency across countries, we correct the OECD data-generated total R&D with the World Bank total R&D.

$$s_n^j = R\&D/GDP_n^{WB} \times \frac{R_n^{j,OECD}}{\sum_j R_n^{j,OECD}}$$

This estimated s_n^j is the R&D intensity used in our quantitative analysis.

Table 3: List of Industries

Sector	ISIC	Industry Description
1	C01T05	Agriculture, Hunting, Forestry and Fishing
2	C10T14	Mining and Quarrying
3	C15T16	Food products, beverages and tobacco
4	C17T19	Textiles, textile products, leather and footwear
5	C20	Wood and products of wood and cork
6	C21T22	Pulp, paper, paper products, printing and publishing
7	C23	Coke, refined petroleum products and nuclear fuel
8	C24	Chemicals and chemical products
9	C25	Rubber and plastics products
10	C26	Other non-metallic mineral products
11	C27	Basic metals
12	C28	Fabricated metal products, except machinery and equipment
13	C29	Machinery and equipment, nec
14	C30T33X	Computer, Electronic and optical equipment
15	C31	Electrical machinery and apparatus, n.e.c.
16	C34	Motor vehicles, trailers and semi-trailers
17	C35	Other transport equipment
18	C36T37	Manufacturing n.e.c. and recycling
19	C40T95	Nontradables

E One Sector Model

We show that in a one-sector version of our model, changes in trade barriers have no effect on the optimal R&D intensity, hence on growth rates along the BGP. In the one-sector model, $\gamma^j = 1$ and $\gamma^{jk} = 0$. The one-sector version of equations (79), (80) and (81) is

$$\hat{W}_n L_n = \sum_{i=1}^{M} \pi_{in} \hat{X}_i, \tag{88}$$

$$\hat{X}_n = \hat{Y}_n, \tag{89}$$

$$\hat{Y}_n = \hat{W}_n L_n + \frac{\sum_{i=1}^{M} \pi_{in} \hat{X}_i}{1+\theta}. \tag{90}$$

Using equations (88) and (90),

$$\hat{Y}_n = \frac{1+\theta}{\theta}\hat{W}_n L_n$$

and

$$\frac{\sum_{i=1}^{M}\pi_{in}\hat{X}_i}{1+\theta} = \frac{\hat{Y}_n}{2+\theta}.$$

From equations (83) and (84) in a one-sector model with royalties

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r\lambda_n\frac{\hat{V}_n}{\hat{Y}_n} = \beta_r\lambda_n\frac{1}{r-g_y+g_A}\frac{\sum_{i=1}^{M}\frac{\varepsilon_{in}}{g_A+\varepsilon_{in}}\frac{\sum_{m=1}^{M}\hat{X}_m\pi_{mi}}{1+\theta}}{\hat{Y}_n}. \tag{91}$$

Using the previous expression

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r\lambda_n\frac{\hat{V}_n^j}{\hat{Y}_n} = \beta_r\lambda_n^j\frac{1}{r-g_y+g_A}\frac{1}{2+\theta}\sum_{i=1}^{M}\frac{\varepsilon_{in}}{g_A+\varepsilon_{in}}\frac{\hat{Y}_i}{\hat{Y}_n}. \tag{92}$$

In this case, changes in trade costs have an effect on optimal R&D intensity to the extent that they have an effect on $\frac{\hat{Y}_i}{\hat{Y}_n}$.

If there are no royalties, the above expression becomes

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r\lambda_n^j\frac{1}{r-g_y+g_A}\frac{1}{2+\theta}. \tag{93}$$

In this case, changes in trade costs do not have an effect on optimal R&D intensity, hence on the growth rate along the BGP.