

Countercyclical Risks and Portfolio Choice over the Life Cycle: Evidence and Theory

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Abstract

I show that countercyclical earnings risk alone can generate moderate stock holdings for young households, while the standard lifecycle models struggle to predict such a realistic age profile of risky share. Moreover, countercyclical earnings risk has quantitatively important effects on saving and portfolio choice decisions over the business cycle. During expansions when expected future earnings growth is high, households save less and also invest a higher share of their financial wealth in the stock market. The opposite holds during recessions. Further negative skewness in the earnings process during recessions additionally reduces households' stock market exposure and consumption. These quantitative predictions are consistent with microeconomic evidence from the Panel Study of Income Dynamics and macroeconomic evidence from the Flow of Funds. Counterfactual simulations using the calibrated model generate wealth inequality dynamics similar to their empirical counterparts.

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1 Introduction

How does uninsurable labor income affect household portfolio choice? Numerous papers study this question theoretically, and a common implication of these models is that young households should invest exclusively in stocks¹. This is at odds with the empirical facts: for example, the Survey of Consumer Finances reports that on average, young households only invest almost half of their financial wealth to the stocks. Realistically modelling portfolio choice for younger households remains a challenge².

One possible concern is that the vast majority of this portfolio choice literature has focused on analyzing labor income shocks that follow a log-normal distribution, based on early microeconomic evidence on labor income³. However, more recent work from Guvenen, Ozkan, and Song (2014) shows that this may not be reasonable in the data: skewness in earnings shocks is strongly countercyclical⁴. This countercyclical skewness is so strong that even the expected growth rate is countercyclical. Alongside this recent empirical evidence on labor income dynamics, recent work by Constantinides and Ghosh (2017) shows that household consumption risk is also countercyclical and more importantly, drives asset prices.

¹For instance, Heaton and Lucas (1997), Koo (1998), Viceira (2001), Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005) and Polkovnichenko (2007) are typical examples showing how labor income can be viewed as an implicit riskless asset, abundant early in life, and therefore inducing higher stock market exposure in that period.

²An active literature addresses this discrepancy between the prediction of the model and the data in various settings. Chang, Hong and Karabarbounis (2018) use three different age-dependent labor income uncertainty to moderate stock holdings over the life cycle. Polkovnichenko (2007) considers additive habit utility to generate more conservative portfolios for younger households. Wachter and Yogo (2010) show that introducing nonhomothetic utility implies a much flatter age profile in the portfolio share.

³The studies of Abowd and Card (1989), Deaton (1991), Carroll (1997), Carroll and Samwick (1997) and Blundell, Pistaferri and Preston (2008) are seminal examples estimating or relying on this labor income process to draw out implications for consumption. A variant of this process also allows for serial correlation in the second moment (Meghir and Pistaferri (2004)) and/or countercyclical variance (Storesletten, Telmer and Yaron (2004), Lynch and Tan (2011)).

⁴Guvenen, Ozkan, and Song (2014) use a very large and confidential database from U.S. Social Security Administration and study business cycle variation in labor income process. Contrary to previous research, they find that variance in earnings shocks is not countercyclical, instead it is skewness in earnings shocks that is countercyclical.

In this paper, I revisit the implications of background labor income risk on portfolio choice by changing the exogenous specification of labor income risk to allow for countercyclical risks consistent with the data. Specifically, I use a mixture normal distribution to construct any desired moment in labor income shocks. My definition of countercyclical earnings risk has two primary differences from traditional models: (i) in a recession, households expect a lower mean growth rate in their labor income and (ii) they expect to draw labor income from a distribution that exhibits negative skewness. These assumptions are not only consistent with the findings of Guvenen, Ozkan and Song (2014), but also are arguably more appealing than assuming that in a recession households merely draw from the same distribution with higher variance: it is difficult to imagine that workers face symmetric risks in a recession and some workers may even receive more labor income in a recession compared with an expansion.

The model is calibrated to match average wealth accumulation and average risky asset holdings for different age groups over the life cycle. To account for limited market participation, I allow preference heterogeneity between stockholders and non-stockholders. Moreover, when I take into account a rare disaster event in the stock market (benchmark 2), the model generates similar implications but more realistic risk aversion (6.3 instead of 6.8). With this specification, I show that the model can do a reasonable job in matching the cross-sectional wealth and portfolio choices observed in the 1989 SCF Survey.

To show how the addition of these risks provides insight into the impact of countercyclical earnings risk on saving and portfolio choice over the life cycle, I compare my results relative to a model with log-normal earnings risk. I find that negative skewness reduces the share of wealth in stocks but does not substantially influence the saving decision. Moreover, the large observed change in expected growth of labor income when moving from an expansion

to a recession has an effect on both the saving and portfolio choice decision. A high expected mean growth in labor income makes the household feel richer and increases consumption. The reduction in saving, coming at the same time as higher positive skewness, allows the household to take on more financial risk, increasing the share of wealth in stocks. Lower saving from higher productivity is consistent with the results of Carroll (1997); an important point to note here is that this intuition also has intuitive portfolio choice implications: in a boom, households save less and invest a higher proportion of their financial wealth in the stock market. I show that the countercyclical labor income specification is key to delivering these results. Moreover, the model is able to match one very important stylised fact: moderate risky asset holdings for stockholders, especially for young stockholders.

To answer a more important question, that is, to what extent the model is able to explain the features of the data, I conduct three empirical analyses. First, I test the model using the individual level data. I show that in the Panel Study of Income Dynamics, negative skewness has statistically and economically significant effects on portfolio choices. In general, the model provides evidence that is consistent with its empirical counterparts. The model generates significantly negative effects of changes in the negative skewness of earnings shocks on changes in risky asset shares, and the inclusion of rare events in the stock market amplifies this effect. Specifically, one standard deviation increase in the negative skewness of earnings shocks is associated with a 1.5% decrease in risky asset shares for benchmark 1 and 1.8% for benchmark 2. Both magnitudes are close to their empirical counterparts: in the data, one standard deviation increase in the negative skewness of earnings shocks decreases risky asset shares by 2.0% to 2.5%, depending on the definition of financial wealth being used in the analysis.

Second, I identify the importance of skewness by running regressions at the aggregate level. I use the aggregate Flow of Funds data and show that the share of wealth in the stock market goes down during recessions, a feature that is also present in the model. Additionally, the model is capable of predicting upward trends in the data.

Lastly, I investigate the macroeconomic implications of the model with regard to wealth inequality. What would the model have predicted for the evolution of wealth inequality from 1989 to 2013? The model matches the observed degree of wealth inequality in the U.S. for nonstockholders, accounting for the wealth Gini index and share of wealth held by the top 10%. It effectively replicates the Gini index and share of wealth held by the richest top 10% for stockholders, while inclusion of rare events seems to underestimate these two measures. Overall, both variants of the model explain the wealth inequality well for all households.

The paper draws on several strands of the literature. First, it relates to the literature on the dynamics in individual earnings risk, motivated by the recent work of Guvenen, Ozkan and Song (2014), who document countercyclical skewness in individual earnings risk using a very large data set from the US Social Security Administration. Earlier research argues that idiosyncratic earnings risk has countercyclical variance (e.g., Storesletten, Telmer, and Yaron (2004)), and investigates the asset pricing implications of this kind of risk (e.g., Constantinides and Duffie (1996), Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008)). However, recent studies show that higher job displacement risk in recessions gives rise to the countercyclical skewness of earnings shocks and the cost of job losses can be very large, especially when it happens during a recession (e.g., Krebs (2007), Davis and von Wachter (2011)). This paper links this countercyclical skewness in earnings shocks to the life-cycle consumption decision and portfolio choices, and displays the importance of this

uninsured and unforeseen earnings risk.

I also build on a large body of recent literature, which studies the role of non-diversifiable labor income risk on life-cycle consumption and portfolio choice. Research in this literature usually focuses on analysing labor income shocks that follow a log-normal distribution (e.g., Deaton (1991), Carroll (1997), Carroll and Samwick (1997), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Polkovnichenko (2007), Fagereng, Gottlieb and Guiso (2017), among other papers). In contrast to these models, my model allows higher moments in labor income shocks, and is able to generate similar results to those in the PSID dataset. The analysis provides new insights into the determinants and dynamics of the portfolio allocation over the life-cycle and views the labor income uncertainty from a new angle.

Relating more closely to my work, Galvez (2017) uses quantile regression methods to study earnings risk and its effect on stock market participation and portfolio choice. Catherine (2017) sets up a life-cycle model with CRRA preferences and stock market participation costs to study the implications of uninsurable earnings risk on portfolio choices. More recent work from Chang, Hong and Karabarbounis (2018) has assessed age-dependent labor market uncertainty. In contrast to these papers, I disentangle the RRA coefficient from the EIS with recursive preferences, and examine the implications of differential expected growth rate in earnings shocks on life-cycle profiles, besides skewness in earnings shocks. In addition, my model illustrates the importance of preference heterogeneity between stockholders and nonstockholders, and investigates the empirical implications of earnings risk on portfolios and consumption with the PSID dataset. Moreover, the implications from my models ties in well with the evidence on household asset allocation and consumption in Chang, Hong and Karabarbounis (2018).

Finally, my findings are related to the literature on wealth inequality. Wealth distribution and its determinants have important implications for capital accumulation and growth, and the design of optimal taxation schemes and their welfare consequences⁵. The pioneering study of Aiyagari (1994, 1995) shows that uninsurable earnings risk influences the determination of wealth inequality. I develop a model with both countercyclical skewness in earnings shocks and preference heterogeneity in order to quantify their effects on wealth inequality. The results show that this model can match several features of the wealth distribution well.

The paper is organised as follows. Section 2 presents the model and Section 3 calibrates the parameters for the model with the 1989 SCF data. Section 4 compares the model's implications for consumption and portfolio choice relative to the log-normal earnings process. Section 5 compares the implications of counterfactual simulations of the model with their empirical counterparts with regards to exposure in the stock market over the business cycles and wealth inequality. The paper concludes with Section 6.

2 The Model

2.1 Preferences

I solve an annual frequency model and follow households from age 20 until their death. Death happens by age 100 at the latest, but could happen earlier as households are faced with an age-specific survival rate. Households start working at age 20 and receive uncertain labor income exogenously. They retire at age 65.

⁵These issues have been studied, for example, by Krusell and Smith (1998), Imrohroglu (1998), Ventura (1999), Castañeda, Díaz-Giménez, and Ríos-Rull (2003), Heathcote (2005), and Conesa, Kitao and Krueger (2009).

Households have Epstein-Zin (1989) preferences defined recursively over consumption C_{it} and the elasticity of intertemporal substitution is separated from the relative risk aversion,

$$(1) \quad V_{it} = \left\{ (1 - \beta)C_{it}^{1-1/\psi} + \beta(E_t(p_{t+1}V_{i,t+1}^{1-\gamma} + b(1 - p_{t+1})X_{i,t+1}^{1-\gamma}))^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$$

where β is the discount factor, b is the strength of bequest motive, γ is the coefficient of relative risk aversion and ψ is the elasticity of intertemporal substitution. p_{t+1} is the probability that the household is alive at date $t + 1$ conditional on being alive at date t .

2.2 Labor Income Process

Households work for the first K (46) periods out of T (81) periods. During the working period, household i 's labor income at age t (Y_{it}) is given in logs ($y_{it} = \log Y_{it}$), by

$$(2) \quad y_{it} = f(t, Z_{it}) + v_{it} + \varepsilon_{it} \text{ for } t \leq K$$

where $f(t, Z_{it})$ is a deterministic function of age t and a vector of other individual characteristics Z_{it} , ε_{it} is temporary shock to labor income, which is normally distributed with mean $-\sigma_\varepsilon^2/2$, variance σ_ε^2 , and the permanent component v_{it} is given by

$$(3) \quad v_{it} = v_{i,t-1} + u_{it}$$

where u_{it} is permanent shock, uncorrelated with ε_{it} . For simplicity, income during retirement is assumed to be exogenous and deterministic. Income is specified as a constant fraction λ

of the permanent component of labor income in the last working period,

$$(4) \quad y_{it} = \log(\lambda) + f(K, Z_{iK}) + v_{iK} \text{ for } t > K$$

where $K = 46$, corresponding to the retirement age 65.

A key variation relative to the prior literature on life cycle portfolio choice is allowing countercyclical earnings risks. To be consistent with the empirical findings in Guvenen, Ozkan and Song (2014). I not only allow skewness to depend on the business cycle, but also on expected growth rates. In what follows subscript $s(t)$ indicates whether year t is a boom or recession. Countercyclical earnings risks are captured by assuming u_{it} is a mixture of normal distributions, so that conditional on the state of the economy $s(t)$ with probability p_1 the u_{it} draw is from one distribution and with probability $(1 - p_1)$ from a second distribution:

$$(5) \quad u_{it} = \begin{cases} u_{it}^1 \sim N(\mu_{1s(t)}, \sigma_{1s(t)}^2) & \text{with prob. } p_1 \\ u_{it}^2 \sim N(\mu_{2s(t)}, \sigma_{2s(t)}^2) & \text{with prob. } 1 - p_1 \end{cases}$$

One of the key contributions of the paper is to understand how these countercyclical earnings risks affect saving/consumption and portfolio choices. Therefore, I also report results from a model where the permanent income shock u_{it} is distributed as $N(-\sigma_u^2/2, \sigma_u^2)$, which is a common setting, for example, from Deaton (1991), Hubbard et. al. (1995), Carroll (1997), Carroll and Samwick (1997) and Gourinchas and Parker (2002) for consumption-saving problems and, for instance, from Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005), Polkovnichenko (2007) and Guiso, Fagereng and Gottlieb (2017) for portfolio choice problems.

2.3 Financial Asset Returns

I assume there are only two assets in the market where households can invest, one riskless and one risky. The riskless asset has a constant gross return r_f , and the excess return of the risky asset is μ . The gross return of the risky asset is r_{t+1} and given by

$$(6) \quad r_{t+1} = r_f + \mu + \eta_{t+1}$$

where η_{t+1} is the innovation to returns, and independently and identically distributed as $N(0, \sigma_\eta^2)$.

I also introduce a variant of this model that allows a rare disaster in the stock market. In this case I change the stock return structure and households may lose τ_{tail} of their returns invested in the stock market with probability p_{tail} during recessions:

$$(7) \quad r_{t+1} = \begin{cases} (1 - \tau_{tail})(r_f + \mu + \eta_{t+1}) & \text{with prob. } p_{tail} \\ r_f + \mu + \eta_{t+1} & \text{with prob. } 1 - p_{tail} \end{cases}$$

I also allow for positive correlation between innovations to excess stock returns and permanent income shocks, $\rho_{u,\eta}$.

2.4 Wealth Accumulation

At each period t , households start with accumulated financial wealth W_t and receive labor income Y_t , which are available for consumption and saving. I denote it as cash on hand.

$$(8) \quad X_{it} = W_{it} + Y_{it}$$

Households decide to consume C_t , allocate α_t share of wealth to risky assets and save the rest of cash on hand. Hence, the next period cash on hand can be re-written as

$$(9) \quad X_{i,t+1} = (X_{it} - C_{it})r_{i,t+1}^p + Y_{i,t+1}$$

where $r_{i,t+1}^p$ is the portfolio return and given by

$$(10) \quad r_{i,t+1}^p = \alpha_{it}r_{t+1} + (1 - \alpha_{it})r_f$$

Stocks are not allowed to be sold short and the allocation to stocks can not be levered up. Hence, the fraction of wealth invested in stocks cannot be negative or larger than one:

$$(11) \quad 0 \leq \alpha_{it} \leq 1$$

Borrowing against future income is not allowed either. Hence, consumption can not exceed the contemporaneous cash on hand:

$$(12) \quad 0 < C_{it} \leq X_{it}$$

2.5 Household Optimisation Problem

Households face an optimisation problem to maximise their lifetime recursive value function subject to liquidity constraints and three sources of uncertainty, the labor income shocks ϵ_{it} and u_{it} and the stock return shock η_t . This optimisation problem can be stated as:

$$(13) \quad \max_{\{\alpha_{it}\}_{t=1}^T, \{C_{it}\}_{t=1}^T} E(V_0)$$

where V_0 is given by equation (8) and is subject to the constraints given by equations (9) to (19).

The state variables in this problem are time t , cash on hand X_{it} , the permanent component of labor income v_{it} and the business cycle indicator $s(t)$. At each time period t , depending on different states, households control their consumption $\{C_{it}^*\}_{t=1}^T$ and allocation to the stocks $\{\alpha_{it}^*\}_{t=1}^T$ to maximise the value function. Because of the unit-root process assumption for the labor income process, the state space can be reduced to two variables by standardising the entire problem by the permanent component of labor income $e^{v_{it}}$, which is denoted by P_{it} for simplicity.

Let $x_{it} = \frac{X_{it}}{P_{it}}$ and $c_{it} = \frac{C_{it}}{P_{it}}$ be the normalised cash-on-hand and consumption, then the normalised value function can be given by

$$(14) \quad V_{it}(x_{it}, s(t)) = \max_{\{\alpha_{it}\}_{t=1}^T, \{C_{it}\}_{t=1}^T} \left\{ (1 - \beta)c_{it}^{1-\frac{1}{\psi}} + \beta(E_t((\frac{P_{i,t+1}}{P_{it}})^{1-\gamma} p_{t+1} V_{it+1}(x_{i,t+1}, s(t+1)))^{1-\gamma} \right. \\ \left. + b(\frac{P_{i,t+1}}{P_{it}})^{1-\gamma} (1 - p_{t+1}) x_{i,t+1}^{1-\gamma})^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$$

subject to

$$(15) \quad x_{i,t+1} = (x_{it} - c_{it}(x_{it}, s(t)))r_{i,t+1}^p \frac{P_{it}}{P_{i,t+1}} + e^{\varepsilon_{i,t+1}} \quad \text{for } t \leq K$$

$$(16) \quad x_{i,t+1} = (x_{it} - c_{it}(x_{it}, s(t)))r_{i,t+1}^p \frac{P_{it}}{P_{i,t+1}} + \lambda \quad \text{for } t > K$$

Appendix A presents the details of the numerical solution method and Appendix B details the approximation accuracy of continuous distributions of mixture normals. I follow the techniques implemented by Zoia (2009) and Faliva, Poti, and Zoia (2016) that allow the numerical approximation of mixture normal distributions without using too many grid points. An online appendix provides accuracy tests that justify this choice.

3 Baseline Calibration

3.1 Financial Asset Returns

Table 1 presents the benchmark parameters that I take from the relative literature. Panel A describes the choices for asset returns. The risk-free rate (r_f) is set to 2% per year and the equity premium (μ) is equal to 4% per year, which is a common choice (for example, Campbell et al. (2001) to reflect transaction costs). I set the correlation between innovations to stocks and permanent income shocks ($\rho_{u,\eta}$) to 0.15, consistent with the estimates in Campbell et al. (2001), while the correlation between innovations and transitory income shocks ($\rho_{\varepsilon,\eta}$) is zero, taken from Cocco et al. (2005). I also use a second specification of stock returns that follows Rietz (1988) and Barro (2006) and assume a rare disaster event in the stock market. Barro and Ursúa (2009) use long-term data for 30 countries up to 2006 reveal stock

market crashes and macroeconomics depression. Market crashes are defined as cumulative real returns of -25% or worse. During recessions, households who participate in the stock market can experience around 2 to 3 market crashes over their life cycle and lose on average 55% of investments in the stock market. Hence, I set the probability of rare disaster (p_{tail}) to 3% and the size of loss (τ_{tail}) to 55% . I recognise there is disagreement on this choice (see the discussion in Constantinides and Ghosh (2017)) but this framework allows me to explicitly compare the implications of a model with, and a model without, a rare stock market disaster event and compare the implications with the literature.

3.2 Labor Income Process

Panel B discusses the labor income process calibration. The replacement ratio during the retirement (λ) is set to 0.68 and the deterministic component of the labor income process is set to be the same as that in Cocco, Gomes and Maenhout (2005). I use 0.1^2 for the transitory variance (σ_ϵ^2), which is similar to the one in Gourinchas and Parker (2002). For the permanent income shocks I rely on the estimates in Guvenen, Ozkan and Song (2014) who estimate a quantitative labor income model using a large and confidential US data set. The moments of permanent income shocks can be easily calculated based on these estimates and therefore the parameters with respect to the mixture of normal distribution during expansions and recessions can be calibrated. I slightly deviate from the data in Guvenen, Ozkan and Song (2014) by assuming the same variance and kurtosis during expansions and recessions because I would like to isolate the effects coming from changes in the mean and skewness of labor income shocks over the business cycle. I therefore fix the variance and kurtosis to be the same during expansions and recessions: the variance is 0.05 and the kurtosis is 3.0 , both

slightly lower than the Guvenen et. al. (2014) estimates. The probability of the mixture normal distribution ($p_1 = 0.49$) is the same as in Guvenen et. al. (2014). I then estimate the remaining eight moments to match the first four moments during expansions and the first four moments during recessions, yielding similar estimates to Guvenen et. al. (2014). The estimated moments imply a substantially higher mean growth in booms (20.7%) rather than in recessions (-17.3%) in one of the two normal distributions, and a negative mean growth in booms (-11.0%) rather than in recessions (16.2%) in the other normal distribution.

If the NBER peak of the previous expansion takes place in the first half of a given year, that year is classified as the first year of the new recession. If the peak is in the second half, the recession starts in the subsequent year. The ending date is defined as the next year after the start year of the expansion announced by the NBER, since the unemployment rate is a lagging variable and does not fall immediately after NBER troughs. According to this definition, recessions are 1991-1992, 2001-2002 and 2008-2010.

3.3 Preference and Bequest Motive

I calibrate the preference parameters and the bequest motive with the 1989 Survey of Consumer Finances (SCF) for the model with skewed permanent income shocks (benchmark 1) and the model with skewed permanent income shocks and rare events in the stock market (benchmark 2). I assume both stockholders and nonstockholders have Epstein-Zin preferences.

I calibrate preference parameters to best match the average normalised wealth and average risky asset share for different age groups at different points in the life cycle. Specifically, for stockholders I calibrate the discount factor (β) to match the average normalised wealth

during the working phase and the bequest motive (b) to match the average normalised wealth during retirement. The relative risk aversion coefficient (γ) determines the average risky asset share over the life cycle. For nonstockholders, I assume the relative risk aversion coefficient (γ) is the same as that of stockholders and calibrate the discount factor (β) to match the lower normalised wealth over working life and the bequest motive (b) to match the average normalised wealth during retirement.

Table 2 shows the main findings for benchmark 1. For stockholders, the preference parameters are $\beta = 0.98$ and $\gamma = 6.8$, and the strength of the bequest motive is $b = 2.0$, which are within the range of existing empirical evidence and calibrations. Nonstockholders are more impatient compared with stockholders, with the discount factor 0.92. Stockholders are wealthier and have a balanced portfolio between bonds and stocks.

Table 3 shows what happens in benchmark 2 (adding a small probability of a big loss in the stock market in recessions). Compared with benchmark 1, benchmark 2 generates a more moderate coefficient of risk aversion (relative risk aversion drops from 6.8 to 6.3). Wealth accumulation decreases slightly relative to the previous model at each stage of the life cycle. Nevertheless, the remaining parameters are not affected: the discount factor and bequest motive generate substantial wealth accumulation during the work phase and even higher wealth accumulation during retirement. As for nonstockholders, the discount factor decreases from 0.92 to 0.90 in order to keep the risk aversion the same as stockholders.

3.4 Life-cycle Profiles

Figure 1 compares the life-cycle profiles of average normalised wealth and risky asset share implied by benchmark 1, benchmark 2 and the equivalent profiles in the data. Graph A shows

mean normalised wealth accumulation over the life cycle for stockholders and shows that benchmark 1 and benchmark 2 match exactly the wealth accumulation during retirement. During working life, both models slightly overshoot normalized wealth accumulation in the data but overall, these models can generate predictions close to the data. Graph B compares the share of wealth in stocks and shows that the models are able to generate a low share of wealth in stocks that can match the data even for younger ages. Graph C illustrates that both benchmark 1 and benchmark 2 match the wealth accumulation well for the nonstockholders.

4 Understanding Model Predictions

To better understand the implications of countercyclical earnings shocks and rare events in the stock market compared with the log-normal earnings model, I present results with the calibrated preference parameters and bequest motive from benchmark 1: the discount factor (β) is equal to 0.98, the coefficient of relative risk aversion (γ) is set to 6.8, the elasticity of intertemporal substitution (ψ) is 0.5 and the bequest motive (b) is 2.

4.1 Understanding the Model: Policy Functions

4.1.1 Share of Wealth in Stocks

The most interesting question is how the presence of countercyclical earnings shocks affects portfolio choice behavior relative to the log-normal earnings shocks model. Figure 2 answers this question by presenting the policy functions for the share of wealth in stocks for four models: the model with normal permanent income shocks (log-normal earnings model), the model with normal permanent income shocks and different expected growth rate during

booms and recessions (log-normal model with business cycle), the model with skewed permanent income shocks (benchmark 1) and the model with skewed permanent income shocks and rare events in the stock market (benchmark 2).

Looking at Graph A and Graph B reveals that adding countercyclical skewness in the labor income process reduces the share of wealth in stocks by a large extent: portfolios drop from 100% to around 50% (Graph A). Including rare disasters lowers the optimal portfolio rule further, and has a stronger effect during recessions (Graphs D, E and F). This asymmetry is driven by the fact that a rare event might happen in recessions but not in expansions.

The model also has differential behavioural implications over the business cycle. I first compare portfolio policy functions over the business cycle. Figure 3 shows the optimal portfolio choice policy as a function of normalised cash on hand at age 25, 55 and 75. The left graphs (Graph A, Graph B and Graph C) plot two versions of the model: (i) the model with all risks conditional on being in a boom, and (ii) the same model conditional on being in a recession. The model with the rare disaster (benchmark 2) is shown in the right hand side graphs (Graph D, Graph E and Graph F).

Two common patterns emerge from these six graphs. First, the optimal portfolio rule is decreasing in age (and cash on hand), a standard result in the literature arising from the nature of labor income. Non-tradable labor income is an implicit riskless asset, and younger (and financially poorer) households have more of this asset as a share of their existing financial wealth, resulting in a greater incentive to diversify their portfolio profile and invest more in stocks when young (financially wealth).

Moreover, business cycles have three distinct effects. First, recessions discourage households from holding risky assets, leading to less aggressive investments compared with booms.

During recessions, households are faced with a slightly negative expected growth rate and negative cross-sectional skewness in the labor income process, both making human wealth riskier and less valuable. Households therefore tend to reduce their holdings of risky assets relative to expansions. Second, for a given level of cash on hand, the business cycle effect has a stronger effect on the optimal risky share of younger households relative to older households. Young households have a relatively higher human wealth to financial wealth ratio compared with older households, and thus they have more to lose and respond more vigorously. Third, during retirement, the business cycle effect disappears because households' income does not depend on the business cycle by assumption. However, rare events could still happen in recessions. As a result, the business cycle effect is still prominent during retirement (Graph F).

In sum, from the policy functions, I show that countercyclical skewness in the earnings process and a rare disaster in the stock market lower the optimal share of wealth in stocks.

4.1.2 Consumption

In this section, I study the behavior of the normalised consumption functions. Figure 4 plots consumption policy functions at age 25, 55 and 75 for the same four models as Figure 2. The left graphs show consumption policy functions conditional on being in a boom and the right graphs show consumption policy functions conditional on being in a recession.

The following observations can be made. First, during the working phase (Graphs A, B, D and E), the differential expected earnings growth overall encourages households to consume more, because it generates an average higher expected growth rate compared with the log-normal earnings model. As households expect to receive more labor income, they are

more willing to consume. Second, during the working phase (Graphs A, B, D and E), countercyclical skewness in earnings shocks generate an average negative skewness compared with log-normal earnings model with business cycle during booms, which leads to less consumption. More negative skewness during recessions leads to a further reduction in consumption. This is actually consistent with my empirical finding with the PSID in Section 2: skewness in earnings risk has a positive effect on consumption. Third, adding the rare events in the stock market lowers consumption, as households need to bear more risk in stock returns. Last but not least, during retirement (Graph C and F), households start receiving constant labor income and all earnings risks disappear. As a result, the log-normal earnings model, the log-normal earnings model with business cycle and benchmark 1 all share the same consumption level. However, risk in stock returns still exists because rare events could happen in the stock market. Hence, benchmark 2 generates lower consumption.

Figure 5 plots consumption policy functions at age 25, 55, and 75 conditioning in a boom and conditioning in a recession. Looking at the left-hand side graphs (A, B, C) reveals that adding negative skewness in the labor income process reduces consumption. The right-hand side graphs (Graph D, E and F) show that adding the rare disaster in the stock market (benchmark 2) lowers the consumption policy rule further. The distance between booms and recessions is much larger than before since a rare event might happen in recessions but not in expansions. As a result, the business cycle effect is still prominent during retirement as well.

Overall, policy functions show that differential expected earnings growth and positive skewness in the labor income process raise the normalized consumption, while negative skewness in the labor income process, business cycles and a rare disaster in the stock market lower

the normalised consumption.

4.2 Understanding the Model: Simulation Results

To highlight the importance of countercyclical earnings risk, I simulate the wealth accumulation and portfolio profiles of 10,000 agents over the life cycle and present the average profiles in Figure 6. To better understand the role of expansions and recessions, I also report results where I assume that a whole life cycle in each model can be spent entirely in a boom or entirely in a recession.

Figure 6 presents the life-cycle profile of mean wealth, the share of wealth in stocks, and consumption with bequest motive, simulated from four models. First, I solve the standard life-cycle model with normal permanent income shocks and no differential expected growth in labor income between booms and recessions. I compare it with a model with normal permanent income shocks, but differential expected growth in labor income between booms and recessions. Overall, the model with differential mean has an average higher expected growth rate compared with the standard normal life-cycle model. Higher expected growth rate in labor income accumulates less wealth (Graph A) and increases the share of wealth in stocks (Graph B). In the beginning of the life cycle, all households start with similar wealth accumulation. Households with a higher expected growth rate consume more because of their lower saving rates. When households approach their middle age, those with a lower expected growth rate accumulate so much wealth that even with a higher savings rate, they are still able to consume more than households with a higher expected growth rate (Graph C).

Next, I add the mixture normal specification to the model with differential expected

growth in labor income during booms and recessions (benchmark 1), which can capture countercyclical left skewness in permanent shocks. This introduction of a higher moment decreases the share of wealth in stocks to a large extent, but leads to only a very tiny reduction in mean wealth and mean consumption. The existence of higher moments in the labor income process indicates that large downward movements are more likely, which makes labor income more uncertain and undermines the nature of income serving as a riskless asset. Moreover, adding stock market crashes (benchmark 2) lowers mean wealth, the share of wealth in stocks, and mean consumption further. As stock becomes much riskier, households choose to consume less, save more and rebalance their portfolio towards cash.

Figure 7 reports the separate profiles, assuming all booms and recessions. Business cycle variation in earning shocks comes from differential expected earnings growth during booms and recessions, and drop in skewness during recessions. Households save much less and invest more aggressively in stocks during booms and do the opposite during recessions. In the beginning of the life cycle, households are faced with similar initial wealth and consume more during booms. Around age 40, much more wealth accumulation during booms leads to more consumption even with higher savings rates. The difference in mean wealth, risky asset shares and consumption between booms and recessions is non-negligible, suggesting that business cycle variation has large impact on life-cycle profiles. The rare disaster in stock returns amplifies this business cycle effect over the life cycle.

4.3 Understanding the Model: Sensitivity Analysis

I perform sensitivity analysis to see whether my results are robust with respect to different positive correlations between permanent income shocks and stock returns. In particular, I

am concerned with correlations during recessions. There is evidence that correlations between asset returns conditional on downside and upside moves display asymmetric patterns⁶: correlations tend to be greater for downside moves than for upside moves. It is important to bear in mind that a labor income stream can be considered as the implicit holding of an asset. Thus correlation between earnings shocks and stock return should also be greater during recessions, as more downward movements in both processes during recessions are more likely. However, the empirical evidence on the correlation between permanent shocks and stock returns is mixed, not to mention this correlation during recessions.

Huggett and Kaplan (2016) argue the correlation should be quantitatively small. Davis and Willen (2000) obtain estimates between 0.1 and 0.3. Heaton and Lucas (1999) report positive correlation for entrepreneurs. I explore the implications of greater correlations during recessions in Figure 8. Benchmark correlation is 0.15 and the same over the business cycle. Keeping correlations during booms the same, Figure 8 illustrates the effects of correlation during recessions equal to 0.3 and 0.4. As simulated profiles of wealth and consumption are similar, they are omitted here. Yet the portfolio effects are significant early in life. I document two important findings. First, the increased correlation makes stocks significantly less attractive and induces households to hold more riskless assets. For a correlation of 0.4, young households hold almost zero risky assets. Secondly, both models predict a non-negative relationship between age and mean share of wealth in stocks before retirement. This prediction is consistent with the data. Both Bertaut and Starr-McCluer (2002), and Ameriks and Zeldes (2004) show that risky asset shares do not decrease with age during the working period, conditional on participation. Overall, changing correlation over the business

⁶For instance, Ang and Chen (2002), Hong, Tu, and Zhou (2007), Dahlquist, Farago, and Tédongap (2016).

cycle is a useful step towards understanding the nature of the portfolio specialisation puzzle and the correlation between age and stock holdings.

5 Comparison between Model and Data

I estimate the preference parameters and the bequest parameter to match the mean wealth accumulation and risky asset share over the life cycle in the sample of the 1989 Survey of Consumer Finances (SCF). If I only read the parameter values one by one, I may lose some important implication about the overall quantitative performance of the models. Therefore, I take the performance of the models to another level, by conducting counterfactual simulations to evaluate to what extent models are able to capture the features of the real world.

In this section, I conduct three counterfactual exercises: (i) regression analysis concerning how the change in the risky asset share responds to the change in the skewness of permanent income shocks by using Panel Study of Income Dynamics (PSID), which test the models at the individual level data, (ii) correlation between risky asset share and business cycles by using Financial Accounts of the U.S., which tests the models at the aggregate level data, and (iii) wealth inequality generated by the models, where I focus on two measures: the Gini index and the share of wealth held by the richest top 10%.

5.1 Simulation Method

For the cohorts in the sample of the 1989 SCF, I observe many of the state variables, such as age, wealth level, and stock market participation status. Using this information and the calibration in the previous section, I simulate optimal stock holdings, labor income,

consumption, and wealth accumulation for the repeated cross-sections of cohorts from 1989 to 2013, and calculate consumption risk over time.

In order to simulate portfolio choices and consumption decisions over time, I make certain assumptions when simulating the model forward from 1989 to 2013. There are two main sources of risk in the model: (i) aggregate stock returns, and (ii) idiosyncratic labor income shocks. When simulating forward, all stockholders are assumed to face the same realised annual equity return taken from the Center for Research in Security Prices (CRSP). Although the stock returns here are exogenous, I acknowledge the importance of endogenising stock returns in a production economic world to build a general equilibrium model. I follow the advice in Heaton and Lucas (2000) who argue that matching complicated models in partial equilibrium is a first necessary step before endogeneising stock returns. As for idiosyncratic labor income shocks, I simulate them from the model.

From 1989 to 2013, there are three NBER-dated recessions. In a similar spirit with realised stock returns, I assume that certain years in the annual simulation belong to an expansion and certain years belong to a recession based on the NBER dating methodology. Households know this information and make decisions conditional on the distributions they expect to face in those years⁷. Households die at 100 and once they die, they are dropped from the simulation. New twenty-year old households enter the labor market every year and are randomly assigned an initial wealth based on the wealth distribution with head aged 20 or less from the 1989 SCF.

I need to take into account the fact that stock market participation has increased from around 30% in 1989 to around 50% in 2013. Moreover, the sampling weights of the SCF

⁷Interesting future work will be whether they know recessions or not, such as Deaton (1991) and Sims (2003)

change over time. Therefore, starting from the initial wealth distribution in the data in 1989, I can use our two benchmark models to follow what would happen to the two different population groups (stockholders and non-stockholders). I use a zero-one indicator variable based on NBER-dated recessions to denote recessions and expansions. Given an initial wealth distribution, I can then track each group separately from 1989 onwards. I combine these two groups by the realised participation rate. As I do not assume population growth, I adjust the weights for each household in order to match the increasing participation rate. Two steps are taken. First, I match the participation rate. I fix the weights for the non-stockholders, and only adjust the weights for the stockholders by simply multiplying the ratio of the number of the stockholders I want to the actual number of stockholders in our simulation. Although the participation rate is matched, the total population changes because of the adjusted weight. Next, I adjust both the weights of the stockholders and the non-stockholders. I multiple both weights by the ratio of the population in 1989 to the population in our simulation to keep the population the same from 1989 to 2013.

5.2 Portfolio Choices and Earnings Risk

Using the PSID, I investigate the microeconomic implications of the model with regard to portfolios. The PSID was an annual survey from 1968 to 1997 and a biennial survey after 1997. It contains a variety of information, including the labor market, risky asset holdings, consumption and demographic variables, such as age, education, household composition and household marital status. The detailed information enabled me to explore the empirical link between labor income and portfolios.

I have used data collected in 1999, 2001, 2003, 2005, 2007 and 2009. I define the variables

as Brunnermeier and Nagel (2008) do. Financial assets are defined as the sum of stocks and mutual funds plus riskless assets. Subtracting other debts from financial assets yields financial wealth. Total wealth is defined as the sum of financial wealth, home equity and equity in private business. To be included in the sample, household marital status is required to remain the same in two consecutive survey years and no assets move in or move out due to a family member moving into or out of a family unit. If a household head retires in the current survey year, I delete all the information about this household. Stock market participants are defined as those whose risky assets shares are larger than zero. Table 4 shows summary statistics for stockholders.

5.2.1 An Empirical Model of the Labor Income Process

First, I need to calculate skewness in earnings shock in the data. To do so, I follow the literature on earning regressions in adopting a log-linear specification. The process is similar as that in my model setup, except I do not assume any distribution of earnings shocks. This particular labor income process decomposes income shocks in permanent and transitory components, and following Pistaferri (2001) and Attanasio, Kovacs, and Molnar (2017), I can identify permanent shocks and transitory shocks separately with the data. Assuming $f(t, Z_{it})$ is already estimated and known, I can write the one-period ahead expected income as follows:

$$(17) \quad E_{i,t-1}(y_{it}) = f(t, Z_{it}) + v_{i,t-1}$$

$$(18) \quad E_{it}(y_{i,t+1}) = f(t+1, Z_{i,t+1}) + v_{it}$$

Subtracting (3) from (4) I obtain

$$(19) \quad E_{it}(y_{i,t+1}) - E_{i,t-1}(y_{it}) = f(t+1, Z_{i,t+1}) + v_{it} - f(t, Z_{it}) - v_{i,t-1}$$

Using labor income process in (1) and (2), permanent income shocks can be easily derived

$$(20) \quad u_{it} = E_{it}(y_{i,t+1}) - E_{i,t-1}(y_{it}) - f(t+1, Z_{i,t+1}) + f(t, Z_{it})$$

Permanent income shocks are identified by the change in the expectations of income, once one removes predictable life-cycle effect. Next, note that the expectational error in income can be written as the sum of the temporary and permanent income shocks:

$$(21) \quad y_{it} - E_{i,t-1}(y_{it}) = u_{it} + \varepsilon_{it}$$

Therefore, transitory income shocks are calculated as

$$(22) \quad \varepsilon_{it} = y_{it} - E_{it}(y_{i,t+1}) + f(t+1, Z_{i,t+1}) - f(t, Z_{it})$$

that is, the income innovation between time t and $t+1$ given the information available at time t and a factor that governs predictable life-cycle income.

Therefore both permanent income shocks and transitory income shocks can be identified by combining observed and expected income data at hand. In this paper, I only focus on permanent earnings shocks and its skewness, as it has been shown that transitory income shocks can be easily smoothed out by households both empirically and theoretically. As

I study the cross-sectional distribution of permanent earnings shocks, the moments of the cross-sectional distribution are the same for all households in each year, which results in one observation of skewness in each year.

Therefore, in order to increase the variability of moments, I exploit the region information in the dataset by calculating the skewness in earnings shocks based on the region where the household lives. I consider four regions: Northeast, North Central, South and West. I would like to be able to consider the state where the household lives, but that would lead to a decrease in sample size and to an increase in measurement error. In order to be consistent with the existing literature, I also investigate the implications of the variance. To do so, I calculate the variance in the same way as the skewness and include the variance into the regression.

5.2.2 An Empirical Model of Risky Asset Shares

Various empirical literature considers the following empirical model for the portfolio share in risky assets ($\Delta_k \alpha_{it}$):

$$(23) \quad \Delta_k \alpha_{it} = \beta q_{i,t-k} + \gamma \Delta_k h_{it} + \psi \Delta_k w_{it} + \lambda \Delta_k L_{it} + \epsilon_{it}$$

where life-cycle controls ($q_{i,t-k}$) include the variables related to the life cycle, background and financial situation of the household at $t - k$, and preference shifters ($\Delta_k h_{it}$) are the variables related to the changes in the household between $t - k$ and t . L_{it} measures uninsurable earnings risk. I omit region subscripts for all variables to reduce clutter.

Theory predicts $\lambda < 0$, because households choose to reduce their overall risk exposure

by lowering risky assets with the presence of unavoidable earnings risk⁸. Motivated by these theoretical predictions, an early empirical literature initiated by Guiso, Jappelli and Terlizzese (1996) uses variance in earnings shock to measure earnings risk and points out the temperance effect of labor income uncertainty on portfolios. I follow the same empirical strategy, but I also define earnings risk as skewness in earnings shocks, instead of only variance in earnings shocks, and investigate whether a link exists between this earnings risk and portfolios.

I calculate two risky asset shares: stocks and mutual funds divided by financial assets (α_1) and a second measure - the sum of stocks, home equity and equity in a private business, divided by total wealth (α_2). L_{it} is measured by skewness in earnings shocks ($l_{skew,t}$). In order to be compatible with the existing literature, I also control for the variance in earnings shocks and a broad set of household characteristics as follows:

$$(24) \quad \Delta_k \alpha_{it} = \beta q_{i,t-k} + \gamma \Delta_k h_{it} + \psi \Delta_k w_{it} + \rho \Delta_k l_{skew,t} + \kappa \Delta_k l_{var,t} + \epsilon_{it}$$

Table 5 reports the main results for $\Delta\alpha_1$ (Column 1 – 3) and $\Delta\alpha_2$ (Column 4 – 6). The table shows that the point estimates for the two definitions of risky asset shares are positive and statistically significant. The coefficient of skewness for $\Delta\alpha_1$ is 0.008 and for $\Delta\alpha_2$, it is 0.010. During recessions, when more downward movements in the labor income process are more likely (skewness becomes more negative), households reduce their holdings of risky assets, which is consistent with the positive coefficients of the change in skewness. For $\Delta\alpha_1$, the coefficient implies that one standard deviation increases in the negative skewness leads

⁸Such behaviour has been termed 'temperance' in a number of important theoretical contributions. For instance, Pratt and Zeckhauser (1987), Kimball (1991), Gollier and Pratt (1996), Heaton and Lucas (2000)

to a decrease in the risky asset share by 2%. Meanwhile, the coefficient for $\Delta\alpha_2$ implies that one standard deviation increases in the negative skewness leads to a decrease in the risky asset share decreases by 2.5%.

No matter how the risky asset share is defined, the coefficients of the variance are both negative. For $\Delta\alpha_1$, the coefficient of the variance is -0.174 , significantly different from 0. For $\Delta\alpha_2$, the coefficient of the variance is quite similar to that for $\Delta\alpha_1$. The estimate is of the same order of magnitude and significance. These results imply that the risky asset share of households with higher variance in the labor income process is much less than that of households with lower variance, other things being equal, which is consistent with Guiso, Jappelli and Terlizzese (1996). Compared with skewness, variance is less important for risky asset shares: a one standard deviation increase in variance of earnings shocks decreases risky asset shares by 1.6%.

These empirical findings show that the background risk decreases households' willingness to bear other avoidable risks. When households face negative shocks to the cross-sectional skewness, their uninsurable labor risk increases and they choose to reduce their holdings of risky assets. The regression analysis in this section confirms that the presence of negative skewness is crucial to the portfolio choice problem.

Then, I investigate what the model would have predicted for the same regressions as those with the PSID data. My models do not make a distinction between α_1 and α_2 , and actually it is much closer to α_1 . Table 6 shows the results for benchmark 1 (Columns 1 – 3) and benchmark 2 (Columns 4 – 6). The model is able to capture the significantly positive effect of changes in skewness in earnings shocks on risky asset shares. The point estimates are 0.006 for benchmark 1 and 0.009 for benchmark 2, indicating that the inclusion of rare

events in the stock market amplifies the effect of skewness in earnings shock on portfolios. One standard deviation increase in the skewness of earnings shocks is associated with 1.5% increase in risky asset shares for benchmark 1 and 1.8% for benchmark 2, which generates the prediction close to the data: one standard deviation increase in the skewness of earnings shocks increases risky asset shares by 2.0% with financial assets and 2.5% with total wealth. Meanwhile, both models produce a negative effect of variance in earnings shock, but not as significantly as the data indicate. This is largely because of the assumption made in the model: no business cycle variation exists in variance in earnings shock.

5.3 Portfolio Choices over the Business Cycles

In this section, I test the models with the Financial Accounts of the U.S.. This data set includes data on the flow of funds and levels of financial assets and liabilities, by sector and financial instrument, and thus allows me to construct aggregate version of the wealth accumulation and the share of wealth invested in the stock market. The main tables I use are B.101 (Balance Sheet of Households and Nonprofit Organizations) and F.101 (Households and Nonprofit Organizations) from 1989 to 2013.

I define the variables as those in the previous section. Although my models do not make a distinction between α_1 and α_2 , I still report the α_2 as a reference when I calculate the correlation in Flow of Funds. Table 7 presents the correlations between the share of wealth in stocks and the business cycle. I introduce a dummy variable for business cycle, taking value 1 if this year is in boom and value 0 if this year is in recession. The larger the correlation, the stronger the countercyclicality. Correlations between risky asset shares and dummy for booms are all positive, no matter it is implied by the models or estimated from

the data, meaning aggregate risky asset share suffers drastic drops during the recession.

Specifically, looking at first column, I find that the correlations are both significantly different from zero in the data. However, the log-earnings model only generates very weak correlation. With the addition of differential expected growth rate in labor income, the point estimate increases, and it is weakly significant. If I switch on the countercyclical skewness in income shocks in the model (benchmark 1), the correlation between risky asset shares and skewness in income shocks increases significantly. The inclusion in the model of a rare disaster in the stock market to the model (benchmark 2) does not have significant effect on the estimates. It only leads to a slightly larger correlation compared with Benchmark 1, meaning households reduce more risky asset share when they have the chances to lose most of their returns in stocks. Considering the relatively large standard error, I can not conclude that the difference between benchmark 1 and benchmark 2 is significant, but evidently the addition of countercyclical skewness in income shocks results in a significantly strong correlation between risky asset shares and business cycle.

To visualize the variation of the asset allocation over the business cycle more intuitively, I contrasts the model-generated aggregate share of wealth in stocks and the data-estimated aggregate share of wealth in stocks in Figure 10. Aggregate risky asset share estimated from the data drops in the recessions and has a very strong and clear upward trend. Both models are capable of reproducing the drops in the recessions and generating the upward trend in the aggregate risky asset share, which are in line with the simulated life-cycle profiles in Section 3.2 that households reduce their holdings in stocks during the recession. In a word, the business cycle effect is still valid at the aggregate level. More importantly, comparing Benchmark 1 and Benchmark 2 clearly shows introducing rare disaster in stock market

during recession makes households react more intensely to the recessions, as expected in the simulated life-cycle profiles too. On the other hand, the models without countercyclical earnings shocks can not capture the large drops during recessions and seem to overestimate the aggregate share of wealth in stocks compared with the data.

5.4 Evolution of Wealth Inequality

How much do observable preference heterogeneity in risk aversion and countercyclical earnings risks account for U.S. wealth inequality? In this section I conduct counterfactual simulations to answer this question and to understand how countercyclical earnings risks contribute to the evolution of wealth inequality.

It is well known that wealth is highly concentrated in the United States. The Gini index rises from 0.78% in 1989 to 0.82% in 2013. It is skewed to the right, and displays a thick, right tail: the top 1% of the richest households hold over 30% of wealth. Redistribution of wealth is a central issue in the discussion of economic policy. Because of its importance, a large body of literature has focused on the mechanisms behind increasing wealth inequality, and it has related wealth distribution to income distribution. Specifically, it focuses on the uninsurable labor income risk. However, the existing literature has not considered countercyclical moments in earnings shocks. In previous sections, I construct a life-cycle model with business cycle variation in earnings shocks, and show that countercyclical first and third moments have a quantitative impact on households' consumption/saving decisions and portfolio choices. Based on these optimal choices of households, I ask whether this model is able to capture the evolution of wealth inequality in the data.

Moreover, my model allows bequest motive and preference heterogeneity across stock-

holders and nonstockholders, which have not been fully explored in accounting for wealth inequality. As shown, households' bequest motive and preference towards risk are important ingredients in shaping their consumption/saving decisions and portfolio choices. Bequest motive ensures enough savings during retirement to account for wealth inequality. This favorable feature overcomes the problems in previous literature, such as Domeij and Klein (2000). They set up an overlapping generations model with old households consuming most of their wealth before they died and they fail to explain wealth inequality. Meanwhile, preference heterogeneity accounts for the dispersion in wealth accumulation, and contributes to wealth concentration.

I set out my main results in Figure 12, and focus on two measures of inequality: the Gini index (Graphs A, C, and E) and the share of wealth held by the richest top 10% of households (Graphs B, D, and F). I report the behavior of my benchmark models, which I have calibrated to the wealth accumulation and risky asset shares in Section 5 above. Moreover, I also report the results of the log-normal earnings model and the log-normal earnings model with business cycle. The comparisons among these models illustrate the importance of countercyclical moments in earnings shocks to wealth inequality.

Figure 12 contrasts the model-implied evolution of wealth inequality and data-simulated evolution of wealth inequality from 1989 to 2013 for stockholders, non-stockholders and all households, respectively. Looking at Graph A and Graph B, I find that for stockholders, the Gini index increases during recessions and drops during succeeding booms. Overall, the Gini index shows a clear upward trend from 1989 to 2013, reflecting the fact that wealth inequality in the United States has become pronounced. At the same time, the share of wealth held by the richest top 10% of stockholders is similar to the Gini index, both pointing

to the high concentration of wealth.

A glance at the model-implied evolution of wealth inequality for stockholders (Graphs A and B) shows that the log-normal earnings model fails to account both for the rise in the Gini index and top tails of the wealth distributions: both measures from the model are almost flat over time, contrary to the data. I then include a differential growth rate in the model (log-normal earnings model with business cycle). The Gini index and the share of wealth owned by the richest 10% of the stockholders rise gradually, and both are higher than those in the log-normal earnings model. This countercyclical first moment allows the model to do a fairly good job of accounting for the increasing wealth inequality.

Next, I allow for countercyclical skewness in earnings shocks (benchmark 1). A comparison between the model-implied evolution of wealth inequality and data-simulated evolution of wealth inequality shows that countercyclical skewness plays an important role in explaining the observed wealth inequality. The Gini index and the top 10% wealth share resemble the data well. For instance, the Gini index peaks in 2010 with 0.71 in the model and 0.72 in the data, and the share of wealth in the top 10% is 0.54 in the model and 0.59 in the data. By incorporating business cycle variation into the earnings process, labor income in benchmark 1 becomes more risky and households have more incentive to accumulate large amounts of wealth. This procedure allows benchmark 1 to account for wealth inequality much better than the log-normal earnings model.

I also explore the role played by rare disasters in the stock market (benchmark 2), and find that rare disasters in the stock market have a quantitatively significant impact on wealth inequality, although they make the model worse by substantially underestimating both inequality measures, especially the Gini index. In benchmark 2, stockholders suffer a

small probability of big losses in stocks during recessions and react more drastically to the recessions by significantly reducing their holdings of risky asset shares. This leads to the large reduction in financial wealth, and reduces the difference between the rich and the poor, which explains why benchmark 2 generates less wealth inequality than benchmark 1.

Figure 12, Graphs C and D show the evolution of wealth inequality for nonstockholders. Benchmark 1 and benchmark 2 generate the same results, as stock market crashes do not affect nonstockholders. As for the log-normal earnings model, the Gini index and the top 10% wealth share do not show any upward trend, and are irrelevant to business cycles. Moreover, differential expected growth rate in the labor income process alone cannot fully account for the wealth inequality for nonstockholders. This highlights the importance of countercyclical skewness in the labor income process to wealth inequality again.

Figure 12, Graphs E and F show the evolution of wealth inequality for all households. With the rise in the stock market participation rate, the gap between benchmark 1 and benchmark 2 widens. The effect of rare disasters in the stock market becomes stronger. Again, business cycle variation in the labor income process accounts for the observed wealth inequality.

Overall, using SCF data, I find that wealth inequality in the United States is increasing. Furthermore, both the Gini index and the top 10% wealth share show a business cycle pattern: rises during recessions and drops during the subsequent expansions. I empirically assess the role played by countercyclical labor income risks on wealth inequality. I find that benchmark 1 is able to capture this pattern, and does a good job of accounting for the wealth inequality, while the inclusion of rare disasters in the stock market (benchmark 2) underestimates the level of wealth concentration.

6 Conclusion

In this paper, I build a life-cycle model, which allows business cycle variation in labor income shocks. This model shows the clear implications of countercyclical earnings risk on the portfolio choice decisions over the life cycle and the business cycle. I find that countercyclical earnings risk is a key ingredient of balanced portfolio choices over the life cycle, especially for young households. Moreover, I find that negative skewness in labor income shocks lowers households' consumption and reduces the share of wealth in stocks. Meanwhile, positive expected growth rate encourages households to consume and hold more risky asset shares.

To better understand to what extent models are able to capture the features of the real world, I conduct three counterfactual exercises. First, using the PSID data, I show that skewness in earnings shocks is statistically positively correlated with risky asset shares for stockholders. The implicit risk-free asset holdings in the form of labor income lose importance as negative skewness in earnings shock increases. All other things being equal, when stockholders are exposed to more downward movement in their labor income process, they reduce their risky asset shares. Therefore, earnings risk crowds out risky asset holdings.

Secondly, I test the models with the Financial Accounts of the U.S.. Countercyclical earnings risk is able to capture significant drops in aggregate risky asset shares, which accords well with the data. Summing up, my model implications are analogous to the empirical findings using both micro-level and macro-level data.

Furthermore, with preference heterogeneity and countercyclical earnings shocks, the model replicates wealth distribution relatively well. The Gini index and share of wealth held by the richest top 10% in the model and in the data are very close to each other, and

indicate increasing wealth inequality from 1989 to 2013. The effect of countercyclical earnings shocks on wealth accumulation and portfolio choice generates highly unequal wealth distribution, and causes wealth concentration. These results show the mechanisms behind the transition from uninsurable earnings risk towards increasing inequality, and provide evidence that earnings distribution can map onto wealth distribution.

A Appendix

A.1 Supplementary Data

A.1.1 Panel Study of Income Dynamics

The PSID is the longest longitudinal household survey. Started in 1968, the PSID was an annual survey through 1997 and a biennial survey afterwards. PSID provides quite rich information on household socioeconomic characteristics, labor market experiences, income, wealth, health status, and family structure. Total family labor income contains the labor income of the head of the household and labor income of the wife. Labor income is the sum of wages and salaries, bonuses, overtime, tips, commissions, professional practice or trade, market gardening, additional job income, and miscellaneous labor income. Riskless assets comprise cash (checking and savings accounts, money market funds, certificates of deposits, savings bonds, and treasury bills) plus bonds and life insurance (bonds, bond funds, cash value in a life insurance, valuable collection for investment purposes, and rights in a trust or estate). Risky financial assets are defined as the amount reported in the PSID survey question asking for the combined value of the shares of stock in publicly held corporations, mutual funds, and investment trusts.

A.1.2 SCF Data

The SCF has been conducted by the Federal Reserve Board every three years to provide detailed information on the finances of US households. The survey deliberately over-samples relatively wealthy households to produce more accurate statistics; in my analysis I then use the sampling weights provided by the SCF to obtain unbiased statistics for the US population.

The SCF also handles the survey nonrespondents by using weighting adjustments. These weights are used to calculate the values reported in the tables and graphs. I use data from the 1989 to 2013 wave. Variables are constructed using the codebook and macro-variable definitions from the Federal Reserve website.

Wealth is made up of checking accounts, savings accounts, certificates of deposit, saving bonds, money market accounts, cash/call money accounts, trusts, life insurance, thrift plans, IRAs, future pensions, total directly held mutual funds, stocks, bonds, savings bonds, other managed assets and other financial assets. Household income refers to the household's cash income, before taxes, for the full calendar year preceding the survey. The components of income are the sum of wages and salaries, unemployment insurance, worker's compensation, Social Security income, other pension income, annuities, and other disability or retirement programmes. Wealth invested in risky assets is the sum of directly held stock, stock mutual funds, and amounts of stock in retirement accounts. Stock market participants are those who have the full value of stocks greater than zero. Risky assets share is constructed as the ratio of wealth invested in the risky assets to wealth, which are defined above.

A.1.3 U.S. Financial Accounts

The U.S. Financial Accounts are the key component of the Bureau of Economic Analysis' international transactions accounts. They include data on the flow of funds and levels of financial assets and liabilities, by sector and financial instrument. Sectors are compiled into three categories: households and nonprofit organizations, nonfinancial corporate businesses, and nonfinancial noncorporate businesses. In this paper, I focus on the households, but the U.S. Financial Accounts report households and nonprofit organizations together. Therefore,

the main tables I use are B.101 (Balance Sheet of Households and Nonprofit Organizations) and F.101 (Households and Nonprofit Organizations) from 1989 to 2013.

Riskless assets are defined as the sum of deposits (private foreign deposits, checkable deposits and currency, total time and savings deposits), debt securities and loans. Financial assets are constructed as the sum of stocks (corporate equities), mutual funds (mutual funds shares) and riskless assets. α_1 , the financial risky asset share, is defined as the ratio of stocks (corporate equities) and mutual funds (mutual funds shares) to financial assets.

A.2 Numerical Solution

The model does not have an analytical solution but can be solved with backward induction numerically. The policy functions and value functions are functions of the state variables: time t , business cycle indicator $s(t)$, and cash on hand relative to the permanent labor income, which is continuous and thus needs to be discretised appropriately. In the last period, the policy functions are determined by the bequest motive and the value function corresponds to the bequest function. I use grid search to optimise the value function. I compute the value associated with each level of consumption and the share of wealth invested in stocks. Then I choose the level of consumption and the share of wealth invested in stocks achieving the maximum value, which are saved as the policy rules for the previous period. For every time t prior to T , and for each point in the state space, this procedure is iterated backwards.

To approximate the distributions of innovations to the permanent labor income shocks, I use numerical integrations. My density function for permanent income shock can be rewritten as a sum of Hermite polynomials with Gaussian Kernel so that I can use Gaussian quadrature points with some adjusted weights to approximate numerical integrations. For points that do

not lie on the state space grid, I evaluate the value function using a cubic spline interpolation. I use cubic spline interpolation for value function evaluation of the chosen grids. As for the transition matrix between expansion and recession, I assume that the probability of the current state staying the same in the next period is 0.75 and the probability of the current state changing to the other state in the next period is 0.25. During recession, there is a small probability 3% of losing 55% of stock returns.

After the optimal policy rules are derived, I start simulating life-cycle profile for each household in 1989 SCF until 2013. Following the NBER dating methodology specified in the previous section, I have three recessions from the 1989 SCF to 2013 SCF: 1992, 2001 and 2010. To make the results comparable, I use the 1989 to 2013 waves for the U.S. Financial Accounts as well. All households face the same annual stock returns from CRSP and choose the income distribution based on the business cycle status. Once households die at age 100, they are dropped from the simulation. New twenty-year old households enter the labor market every year with initial wealth distribution of aged 20 or less from the 1989 SCF.

A.3 Continuous Distributions Approximation Experiments

I now provide experimentation with the orthogonal polynomials approximation method in Zoia (2009) and Faliva, Poti and Zoia (2016). To test the accuracy of the approximation method, I use two different methods. The first method is based on simulation. I simulate based on the discretization for a given number of grid points and then perform a Monte Carlo analysis to investigate how close the estimated parameters are to the actual parameters used to generate the discrete approximation. I generate 100000 simulation paths, and report the means, variance, skewness and kurtosis of each variable and the distance between the

estimations and true values. The second method uses the nodes and weights used in the numerical solution to compute the first four moments of the variables. These values should be close to the simulations.

I test the orthogonal polynomials approximation method for three different situations: (i) a variable distributed normally, (ii) a variable distributed as a mixture of normal distribution with negative skewness and excess kurtosis, and (iii) two correlated variables.

Experiment 1: Assume a variable follows a normal distribution $N(0,0.1)$. I report the first four moments of this variable and change the number of grid points (N) to check if the accuracy can be improved by increasing the number of grid points. The first four moments and the average distance by simulation are:

N	Mean	Variance	Skewness	Kurtosis	Avg. Distance
5	$-3.3043e - 05$	0.0101	-0.0085	3.0044	$9.2008e - 05$
10	$2.0576e - 04$	0.0100	$-1.1483e - 4$	2.9963	$1.4082e - 05$
15	$-3.4842e - 06$	0.0100	-0.0027	3.0018	$1.0594e - 05$
20	$2.4127e - 17$	0.0100	$-1.1044e - 15$	3.0000	$8.1986e - 06$

The first four moments computed using the numerical integration method are:

N	Mean	Variance	Skewness	Kurtosis
5	$2.0817e - 17$	0.0100	$3.5128e - 16$	3.0000
10	$-3.1113e - 17$	0.0100	$3.0709e - 16$	3.0000
15	$-5.5311e - 17$	0.0100	$2.1441e - 16$	3.0000
20	$-1.3772e - 17$	0.0100	$-9.8642e - 17$	3.0000

From these two tables, I can find that the orthogonal polynomials approximation method can produce accurate first four moments for the normal variable with only five grid points. Increasing the number of grid points does not improve the accuracy too much. Considering the computation speed and accuracy, I use five grid points for the numerical approximation.

Now, an interesting question is whether this orthogonal polynomials approximation method can also be applied to the non-normal variables, which leads to experiment 2.

Experiment 2: Assume a variable follows a mixture of normal distributions with mean 0, standard deviation 0.1, skewness -0.5 and kurtosis 5. I report the first four moments of this variable and change the number of grid points (N) to check if the accuracy can be improved by increasing the number of grid points. The first four moments and the average distance by simulation are:

N	Mean	Variance	Skewness	Kurtosis	Avg. Distance
5	$2.9796e - 04$	0.0100	-0.4944	4.9972	$3.9133e - 05$
10	$-9.0775e - 05$	0.0100	-0.4995	5.0047	$2.2194e - 05$
15	$-4.6554e - 05$	0.0100	-0.4996	4.9988	$1.5714e - 06$
20	$-2.4788e - 05$	0.0100	-0.5001	5.0001	$8.1986e - 07$

The first four moments computed using the numerical integration method are:

N	Mean	Variance	Skewness	Kurtosis
5	$3.4694e - 17$	0.0100	-0.5000	5.0000
10	$-4.8843e - 17$	0.0100	-0.5000	5.0000
15	$-2.6057e - 17$	0.0100	-0.5000	5.0000
20	$-2.3259e - 17$	0.0100	-0.5000	5.0000

From these two tables, I find that the orthogonal polynomials approximation method can produce accurate first four moments for the variable with non-zero skewness and excess kurtosis with only five grid points. Increasing the number of grid points does not improve the accuracy too much. Considering the computation speed and accuracy, I use five grid points for the numerical approximation.

Experiment 3: Assume there are two correlated variables with correlation 0.15: one (v_1) follows a normal distribution $N(0, 0.1)$, and the other one (v_2) follows a mixture of

normal distributions with mean 0, standard deviation 0.1, skewness -0.5 and kurtosis 5. I report the correlation and the first four moments of each variable and change the number of grid points (N) to check if the accuracy can be improved by increasing the number of grid points. For each N , I report the correlation, the first four moments (v_1 on the first row and v_2 on the second row), and the average distance by simulation:

N	Correlation	Mean	Variance	Skewness	Kurtosis	Avg. Distance
5	0.1674	$-1.3310e - 04$	0.0100	-0.0024	2.9907	$9.3236e - 05$
		$-2.7394e - 05$	0.0100	-0.4981	4.9951	$2.7826e - 05$
10	0.1662	$5.1730e - 05$	0.0100	-0.0036	3.0016	$1.5511e - 05$
		$-1.8385e - 05$	0.0100	-0.5015	5.0025	$1.3567e - 05$
15	0.1571	$1.3257e - 05$	0.0100	$6.4496e - 04$	2.9979	$4.8294e - 06$
		$1.6052e - 05$	0.0100	-0.5005	5.0019	$6.3069e - 06$
20	0.1533	$5.9415e - 06$	0.0100	$-3.0275e - 04$	3.0015	$2.2885e - 06$
		$-1.7116e - 06$	0.0100	-0.4998	4.9994	$3.1394e - 06$

The correlation and the first four moments (v_1 on the first row and v_2 on the second row) computed using the numerical integration method are:

N	Correlation	Mean	Variance	Skewness	Kurtosis
5	0.1500	$2.7322e - 17$	0.0100	$1.0192e - 16$	3.0000
		$3.8164e - 17$	0.0100	-0.5000	5.0000
10	0.1500	$-2.7566e - 17$	0.0100	$3.2543e - 16$	3.0000
		$-4.8843e - 17$	0.0100	-0.5000	5.0000
15	0.1500	$-6.1494e - 17$	0.0100	$3.7788e - 16$	3.0000
		$-2.0095e - 17$	0.0100	-0.5000	5.0000
20	0.1500	$2.4127e - 17$	0.0100	$1.1044e - 16$	3.0000
		$1.7961e - 17$	0.0100	-0.5000	5.0000

From these two tables, I find that the orthogonal polynomials approximation method can produce accurate correlation and the first four moments for the correlated processes with only five grid points. Increasing the number of grid points does not improve the accuracy

too much. Considering the computation speed and accuracy, I use five grid points for the numerical approximation.

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Table 1
Baseline Calibration Parameters

Table 1 reports calibration parameters for the baseline annual frequency life-cycle model. Panel A shows the parameters for stock returns. For stock returns, I consider two cases sequentially: stock returns without a rare disaster and stock returns with a rare disaster. The risk-free rate (r_f) and the excess return on stocks (μ) are common choices in Campbell et al. (2001). The parameters related to the rare disasters are calibrated by the empirical evidence in Barro and Ursúa (2009). Panel B shows the parameters for the labor income process. The replacement ratio (λ) is taken from Cocco, Gomes and Maenhout (2005) and the standard deviation of transitory shocks (ε) is set following Gourinchas and Parker (2002). The rest of the income parameters are calculated based on the first four moments from Guvenen, Ozkan and Song (2014).

Description	Parameter value
<i>Panel A. Asset returns</i>	
Risk-free rate (r_f)	0.02
Equity premium (μ)	0.04
Standard deviation of stock return (σ_η)	0.157
Probability of big loss during recessions (p_{tail})	0.03
Big loss during recessions (τ_{tail})	0.55
Correlation between innovations and permanent shocks ($\rho_{u,\eta}$)	0.15
<i>Panel B. Labor income process</i>	
Replacement ratio (λ)	0.68
Standard deviation of transitory shocks (σ_ε)	0.1
Probability of mixture normal distribution (p_1)	0.49
Normal distribution 1 mean during booms (μ_{1b})	0.207
Normal distribution 2 mean during booms (μ_{2b})	-0.110
Normal distribution 1 standard deviation during booms (σ_{1b})	0.212
Normal distribution 2 standard deviation during booms (σ_{2b})	0.076
Normal distribution 1 mean during recessions (μ_{1r})	-0.173
Normal distribution 2 mean during recessions (μ_{2r})	0.162
Normal distribution 1 standard deviation during recessions (σ_{1r})	0.212
Normal distribution 2 standard deviation during recessions ($\sigma_{2,r}$)	0.003

Table 2
Baseline Results

Table 2 presents the preference parameters of the model with skewed permanent shocks but without rare event (Benchmark 1) calibrated to the 1989 Survey of Consumer Finances (SCF) and it compares the data with the model for different age groups. Both stockholders and non-stockholders have Epstein-Zin preferences. I calibrate discount factor (β) to match average ratio of financial wealth to labor income during the working time, strength of bequest motive (b) to match average ratio of financial wealth to labor income during the retirement phase, coefficient of relative risk aversion (γ) to match the optimal risky asset shares, and elasticity of intertemporal substitution (ψ) is set at 0.5. For nonstockholders, coefficient of relative risk aversion (γ) is the same as stockholders, discount factor (β) matches the average ratio of financial wealth to labor income during the working time and strength of bequest motive (b) matches the average ratio of financial wealth to labor income during the retirement phase.

	Stockholders		Non-stockholders	
	Data	Model	Data	Model
Benchmark 1				
Discount factor (β)	(mean W/Y work phase)	0.98		0.92
	20 – 34	1.772	1.315	0.270
	35 – 44	1.907	3.289	0.912
	45 – 54	2.653	4.989	1.611
	55 – 64	5.078	6.933	2.364
Strength of bequest motive (b)	(mean W/Y retirement)	2.0		1.5
	65 – 74	8.785	8.819	2.955
	75 – 64	9.934	10.002	3.282
Coefficient of relative risk aversion (γ)	(mean α)	6.8		6.8
	20 – 34	0.300	0.403	
	35 – 44	0.322	0.293	
	45 – 54	0.286	0.248	
	55 – 64	0.262	0.211	
	65 – 74	0.340	0.326	
	75 – 100	0.324	0.315	
Elasticity of intertemporal substitution (ψ)		0.5		0.17

Table 3
Baseline Results

Table 3 presents the preference parameters of the model with skewed permanent shocks and rare event (Benchmark 2) calibrated to the 1989 Survey of Consumer Finances (SCF) and it compares the data with the model for different age groups. Both stockholders and non-stockholders have Epstein-Zin preferences. I calibrate discount factor (β) to match average ratio of financial wealth to labor income during the working time, strength of bequest motive (b) to match average ratio of financial wealth to labor income during the retirement phase, coefficient of relative risk aversion (γ) to match the optimal risky asset shares, and elasticity of intertemporal substitution (ψ) is set at 0.5. For nonstockholders, coefficient of relative risk aversion (γ) is the same as stockholders, discount factor (β) matches the average ratio of financial wealth to labor income during the working time and strength of bequest motive (b) matches the average ratio of financial wealth to labor income during the retirement phase.

	Stockholders		Non-stockholders	
	Data	Model	Data	Model
Benchmark 2				
Discount factor (β)	(mean W/Y work phase)	0.98		0.90
	20 – 34	1.772	1.245	0.386
	35 – 44	1.907	3.078	0.611
	45 – 54	2.653	4.750	0.980
	55 – 64	5.078	6.485	1.882
Strength of bequest motive (b)	(mean W/Y retirement)	2.0		1.5
	65 – 74	8.785	8.357	2.786
	75 – 64	9.934	8.832	3.099
Coefficient of relative risk aversion (γ)	(mean α)	6.3		6.3
	20 – 34	0.300	0.416	
	35 – 44	0.322	0.307	
	45 – 54	0.286	0.245	
	55 – 64	0.262	0.219	
	65 – 74	0.340	0.304	
	75 – 100	0.324	0.279	
Elasticity of intertemporal substitution (ψ)		0.5		0.17

Table 4
Summary Statistics

Table 4 presents summary statistics for the 1999-2009 stockholders sample ($k = 2$). Financial asset is defined as the sum of stocks and mutual funds plus riskless assets. Subtracting other debts from financial assets yields financial wealth. Total wealth is defined as the sum of financial wealth, home equity and equity in private business. $\Delta_k \log$ financial assets (total wealth) is the change in financial asset (total wealth) between $t - k$ and t , α_1 is the sum of stocks and mutual funds held divided by financial assets, α_2 is defined as the sum of stocks, home equity and equity in a private business, divided by total wealth. Income is total family labor income, and $\Delta_k \log$ income is the change in total family labor income.

Variable	Mean	Std.Dev.	Min	Max
Financial assets	316097	948635	11155	6795220
$\Delta_k \log$ financial assets	0.187	1.795	-5.228	5.577
α_1	0.579	0.320	0.015	1.000
$\Delta_k \alpha_1$	0.085	0.370	-0.939	0.956
Total wealth	503121	1179555	36475	9715000
$\Delta_k \log$ total wealth	0.096	1.426	-6.477	6.718
α_2	0.815	0.229	0.021	1.000
$\Delta_k \alpha_2$	0.051	0.262	-0.966	0.938
Income	149050	126648	14216	1218500
$\Delta_k \log$ income	0.055	0.528	-2.616	2.670

Table 5
Regressions on Changes in Skewness of Labor Income Shocks

Table 5 presents the results for the 1999-2009 sample ($k = 2$) with financial assets (Columns 1 – 3) and total wealth (Columns 4 – 6), how changes in risky asset shares ($\Delta_k \alpha$) respond to changes in the skewness of earnings shocks ($\Delta_k l_{skew}$) conditional on changes in the variance of earnings shocks ($\Delta_k l_{var}$) and a vector of the variables that may cause common movements. Financial asset is defined as the sum of stocks and mutual funds plus riskless assets. Subtracting other debts from financial assets yields financial wealth. Total wealth is defined as the sum of financial wealth, home equity and equity in private business. Regressions control for preference shifters and life-cycle controls (not reported). Preference shifters include changes in household characteristics. Life-cycle controls are related to the life cycle, background and financial situation of the household. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Explanatory variable	All households	Stockholders	Nonstockholders	All households	Stockholders	Nonstockholders
	<i>Column 1 – 3: Financial assets</i>			<i>Column 4 – 6: Total Wealth</i>		
	<i>Dependent variable: Changes in risky assets shares ($\Delta_k \alpha$)</i>					
$\Delta_k l_{skew}$	0.008**			0.010***		
	(0.004)			(0.002)		
$\Delta_k l_{var}$	-0.174**			-0.120**		
	(0.078)			(0.061)		

Table 6
Regressions on Changes in Skewness of Labor Income Shocks (Model)

Table 6 presents the results for the 1999-2009 sample ($k = 2$) with benchmark 1 (Columns 1 – 3) and benchmark 2 (Columns 4 – 6), how changes in the risky assets shares ($\Delta_k \alpha$) respond to changes in the skewness of earnings shocks ($\Delta_k l_{skew}$) conditional on changes in the variance of earnings shocks ($\Delta_k l_{var}$) and a vector of the variables that may cause common movements. Financial asset is defined as the sum of stocks and mutual funds plus riskless assets. Subtracting other debts from financial assets yields financial wealth. Total wealth is defined as the sum of financial wealth, home equity and equity in private business. Regressions control for life-cycle controls (not reported). Life-cycle controls are related to the life cycle, background and financial situation of the household. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Explanatory variable	All households	Stockholders	Nonstockholders	All households	Stockholders	Nonstockholders
	<i>Column 1 – 3: Benchmark 1</i>			<i>Column 4 – 6: Benchmark 2</i>		
	<i>Dependent variable: Risky assets shares ($\Delta_k \alpha$)</i>					
$\Delta_k l_{skew}$	0.006***			0.009***		
	(0.001)			(0.001)		
$\Delta_k l_{var}$	-0.011			-0.012		
	(0.007)			(0.008)		

Table 7
Correlation between Risky Asset Share and Business Cycle

Table 7 presents the correlations between the share of wealth in stocks and the dummy variable for boom, and between different wealth to income ratios and the dummy variable for boom. All variables are defined as those in Brunnermeier and Nagel (2008), α_1 is the sum of stocks and mutual funds held divided by financial assets (the financial risky asset share) and α_2 is the sum of stocks and mutual funds, home equity, and equity in a private business, divided by total wealth (the total risky asset share). The models do not make a distinction between α_1 and α_2 , and actually are much closer to α_1 . The table shows the results for the model with normal permanent income shocks (Lognormal Earnings Model), the model with normal permanent shocks but different growth rate during booms and recessions (Lognormal Earnings Model with Business Cycle), the model with skewed permanent shocks (Benchmark 1) and the model with skewed permanent shocks and rare events in the stock market (Benchmark 2). *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Variable	Flow of Funds	Log-normal earnings model	Log-normal earnings model with business cycle	Benchmark 1	Benchmark 2
α_1	0.368* (0.213)	0.208 (0.184)	0.304* (0.162)	0.759*** (0.136)	0.771*** (0.133)
α_2	0.340** (0.172)				

Figure 1

Life-cycle Profiles for Mean Wealth and Share of Wealth in Stocks

Figure 1 presents the mean wealth and mean share of wealth in stocks for different age groups. Graph A and B plot the life-cycle profile for stockholders and Graph C plots the life-cycle profile for nonstockholders.

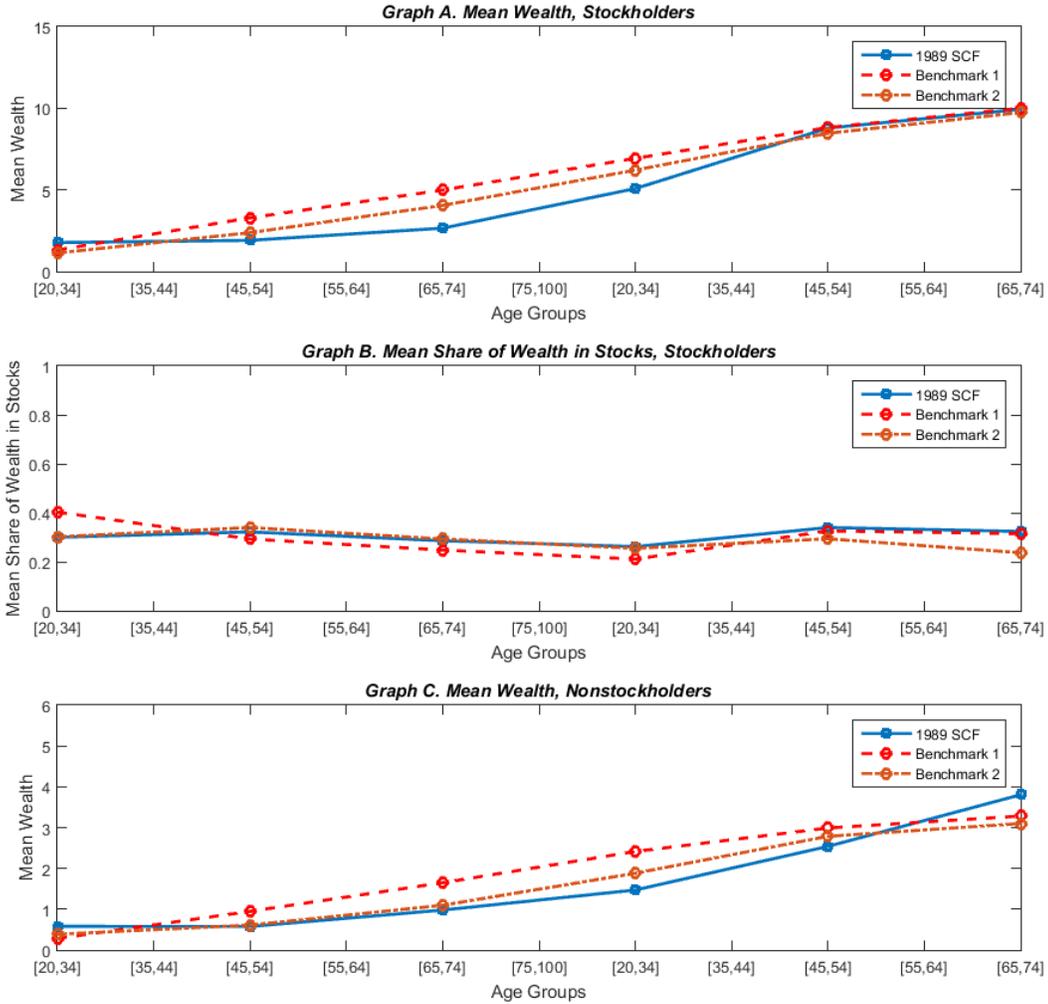


Figure 2

Share of Wealth in Stocks Policy Function

Figure 2 presents policy functions for share of wealth in stocks and provides a comparison between different models.

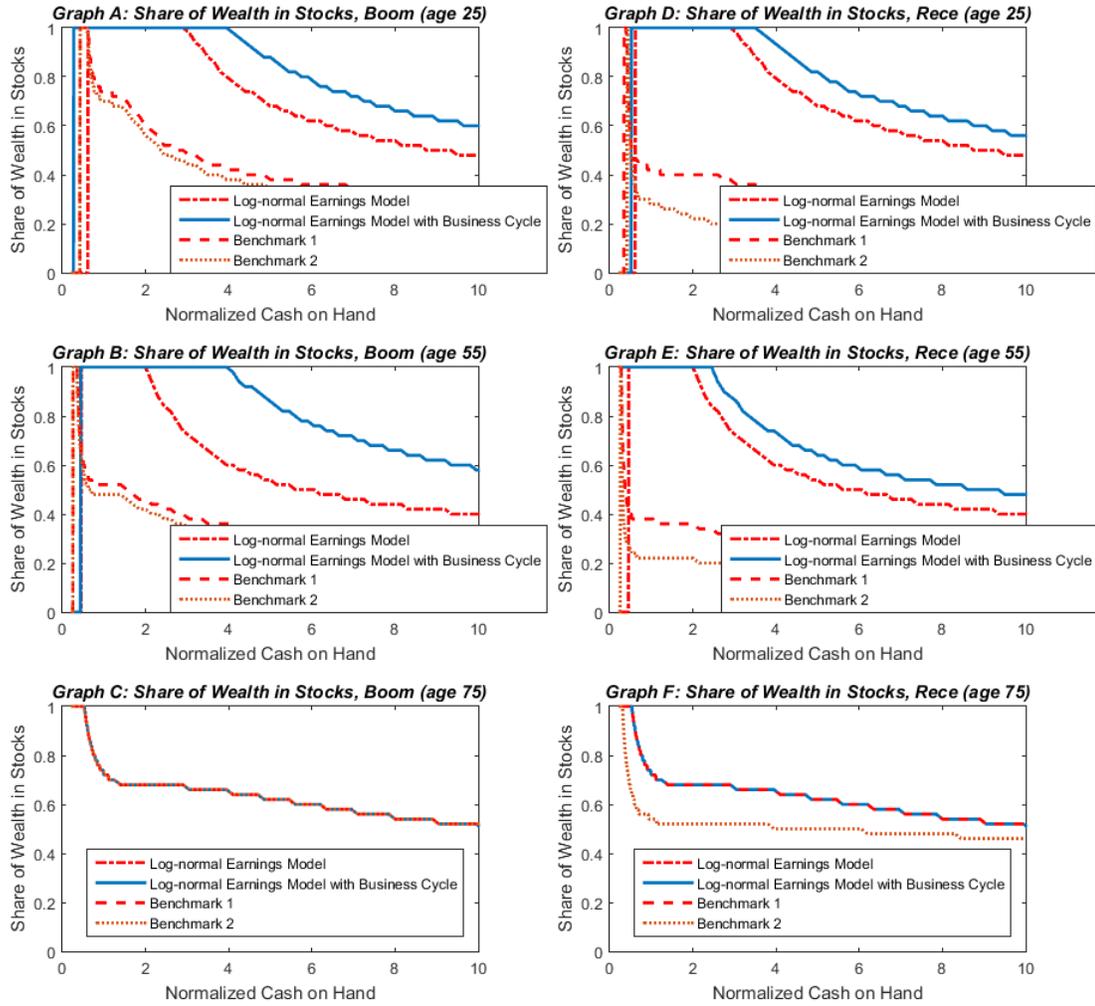


Figure 3

Share of Wealth in Stocks Policy Function

Figure 3 presents policy functions for share of wealth in stocks and provides a comparison between booms and recessions.

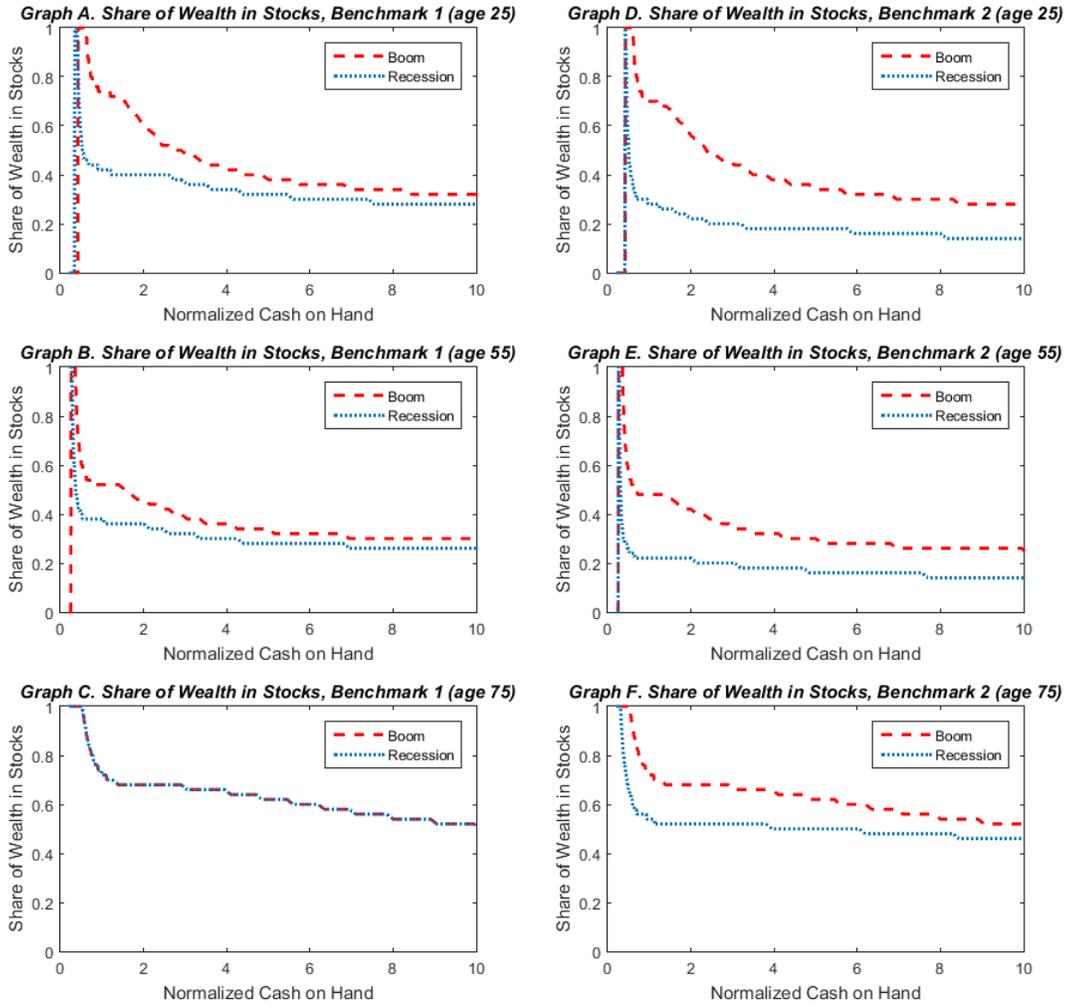


Figure 4
Consumption Policy Function

Figure 4 presents policy functions for consumption and provides a comparison between different models.

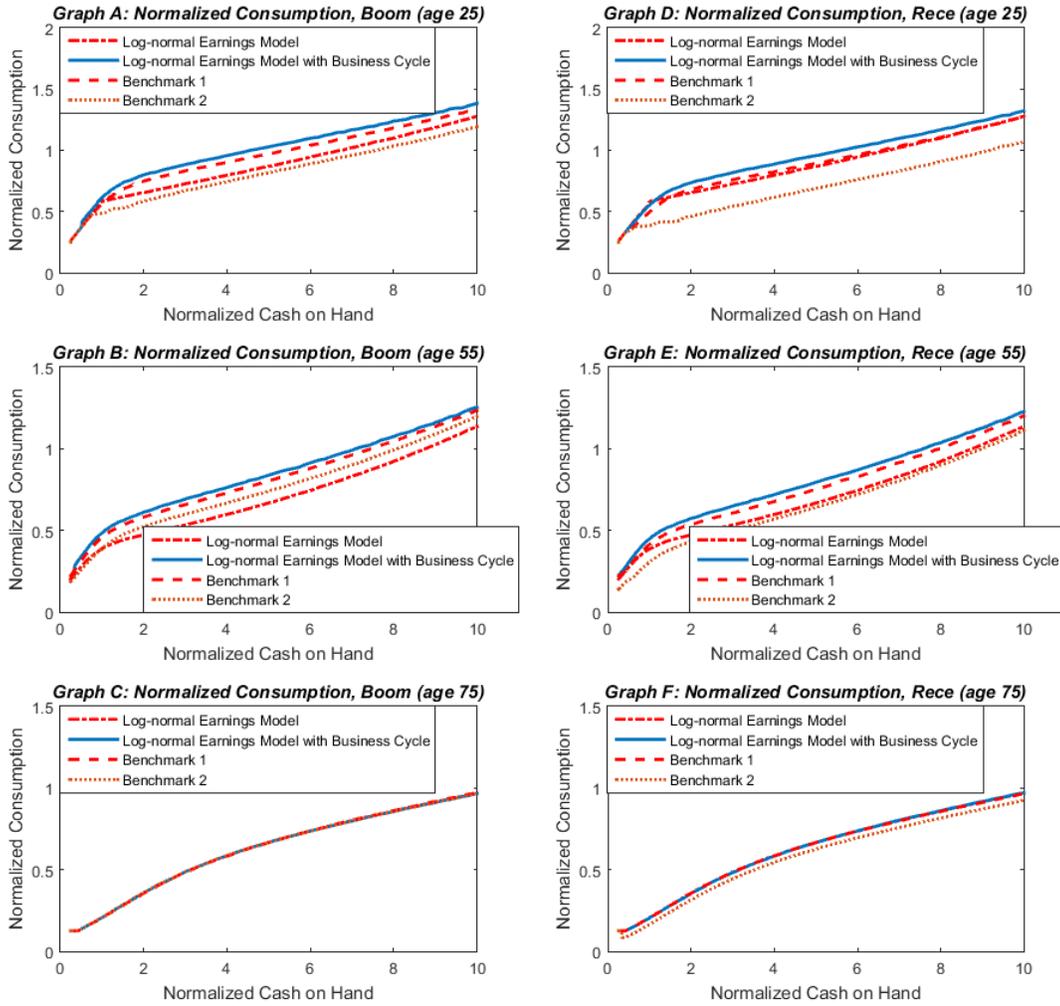


Figure 5

Consumption Policy Function

Figure 5 presents policy functions for consumption and provides a comparison between booms and recessions.

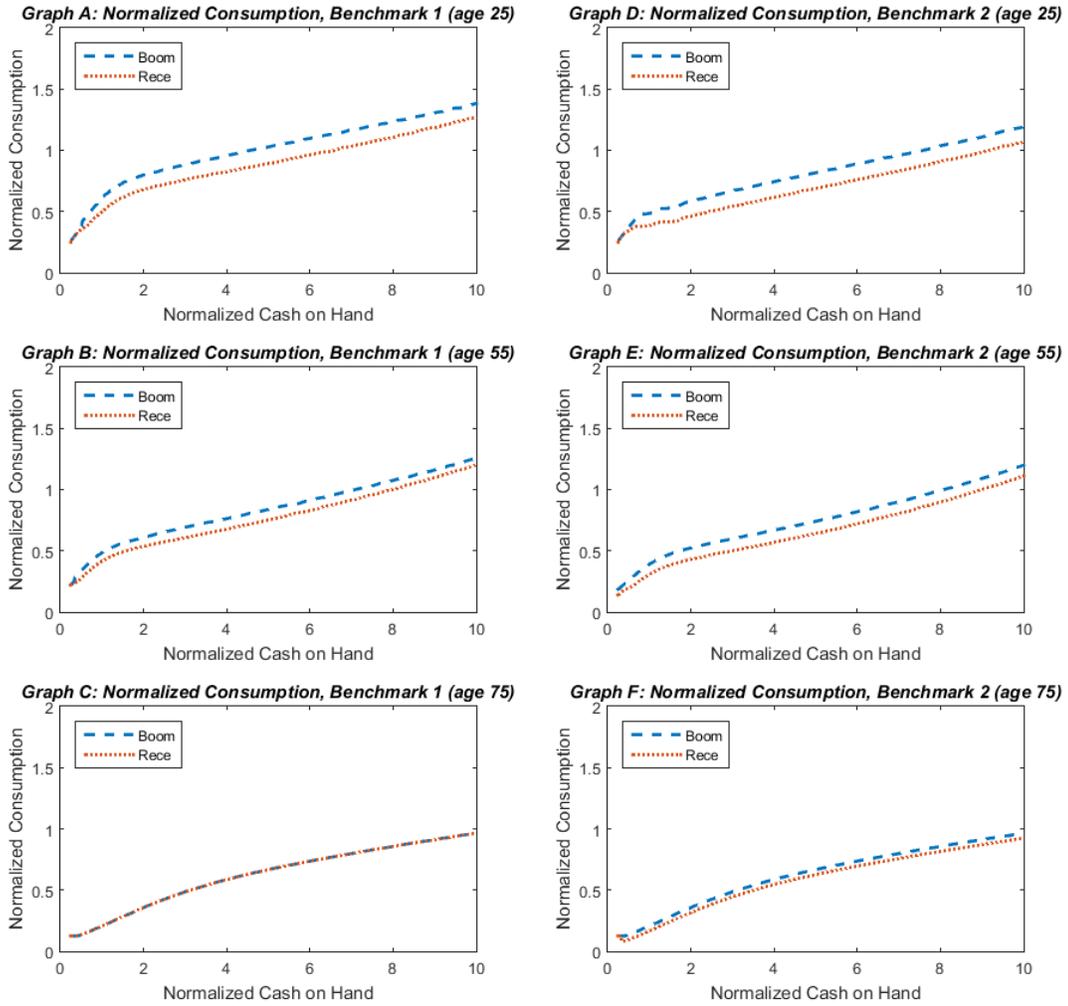


Figure 6

Life-Cycle Profile

Figure 6 presents the life-cycle profile comparison between the model with normal permanent income shocks (Log-normal Earnings Model), the model with normal permanent shocks but different growth rate during booms and recessions (Log-normal Earnings Model with Business Cycle), the model with skewed permanent shocks (Benchmark 1) and the model with skewed permanent shocks and rare events in the stock market (Benchmark 2).

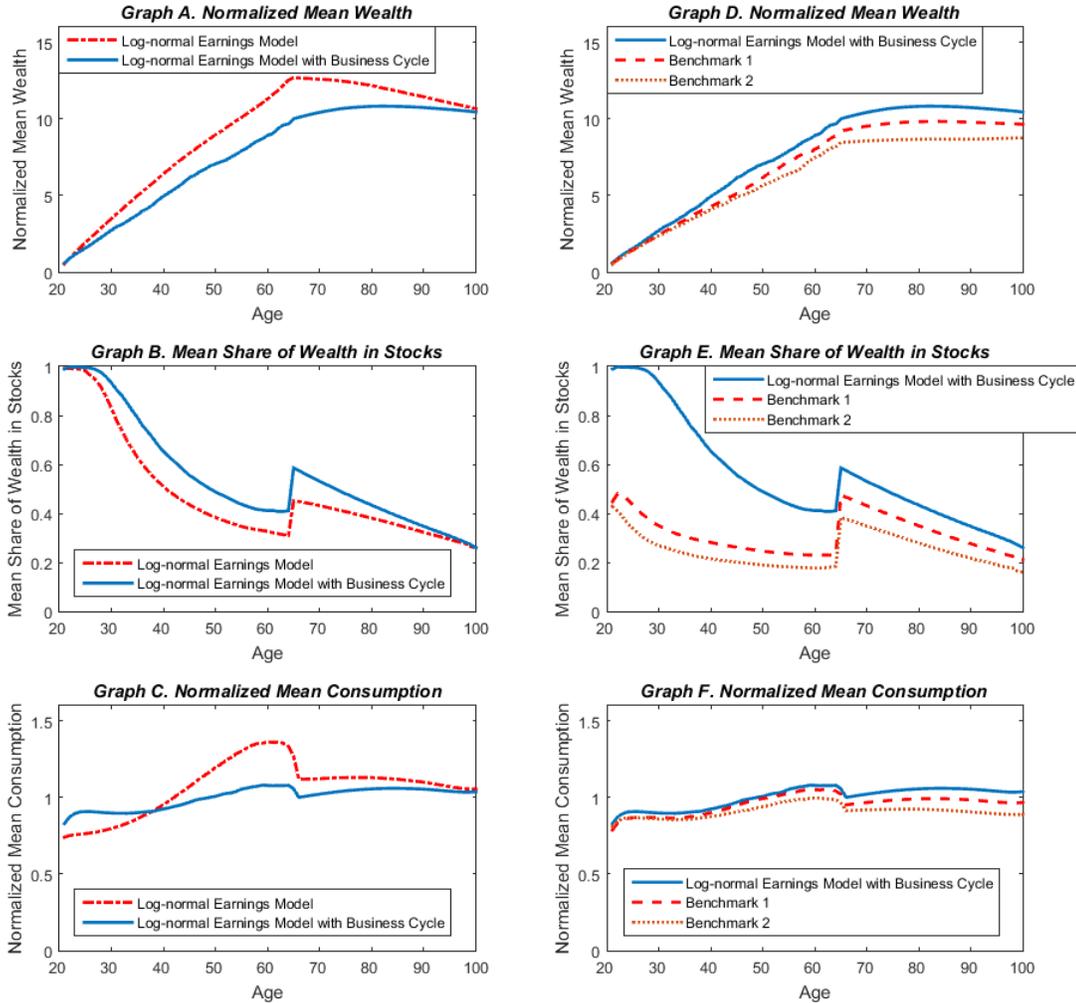


Figure 7

Business Cycle Variation in Life-Cycle Profile

Figure 7 presents the business cycle variation in life-cycle profiles. To show effect clearly, I assume a recession in all life cycle or a boom in all life cycle. The left graphs plot the model with skewed permanent shocks (Benchmark 1) under the circumstance of a boom in all life cycle, and the model with skewed permanent shocks (Benchmark 1) under the circumstance of a recession in all life cycle. The right graphs plot the model with skewed permanent shocks and rare events in the stock market (Benchmark 2) under the circumstance of a boom in all life cycle, and the model with skewed permanent shocks and rare events in stock market (Benchmark 2) under the circumstance of a recession in all life cycle.

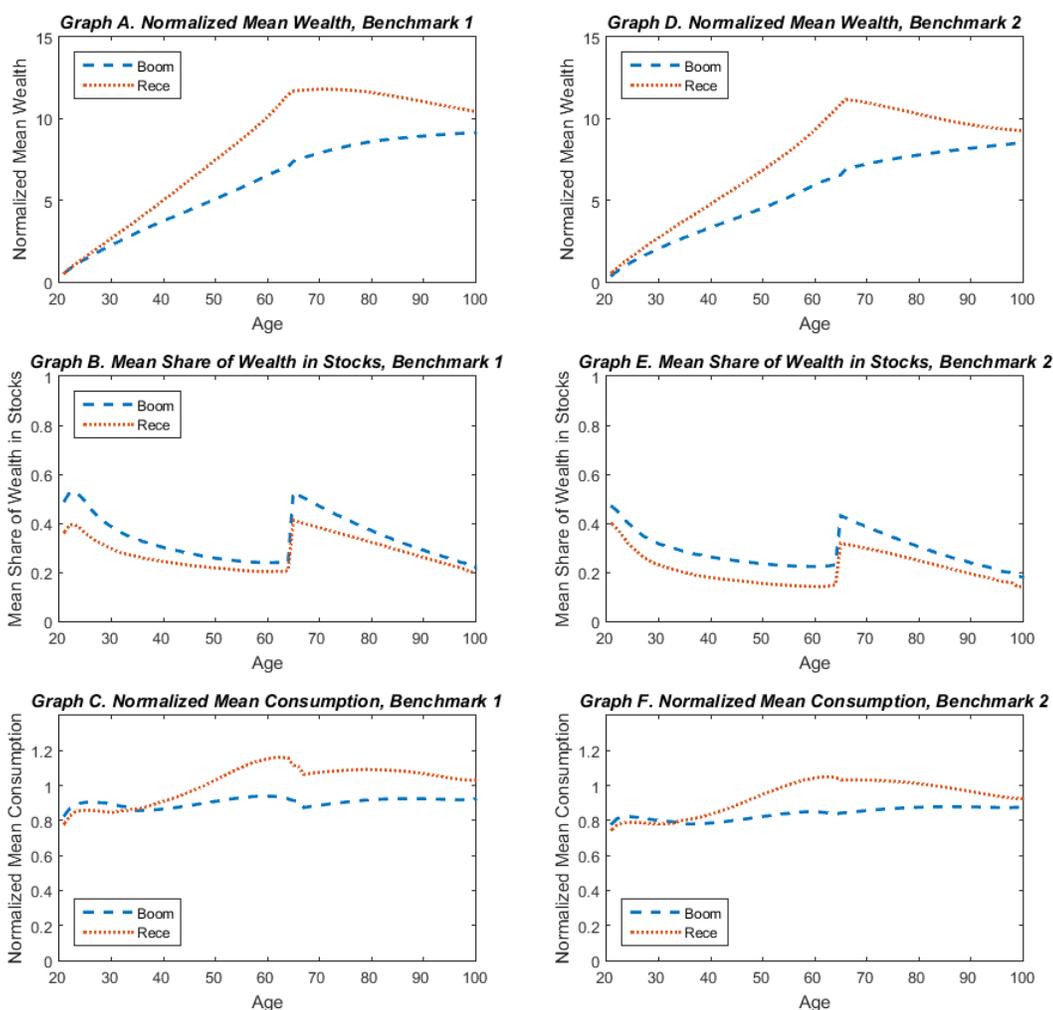


Figure 8

Mean Share of Wealth in Stocks

Figure 8 presents the mean share of wealth in stocks comparison among different correlations between permanent earnings shocks and stock returns innovation during recessions. Graph A plots the model with skewed permanent shocks (Benchmark 1), and Graph B plots the model with skewed permanent shocks and rare events in the stock market (Benchmark 2).

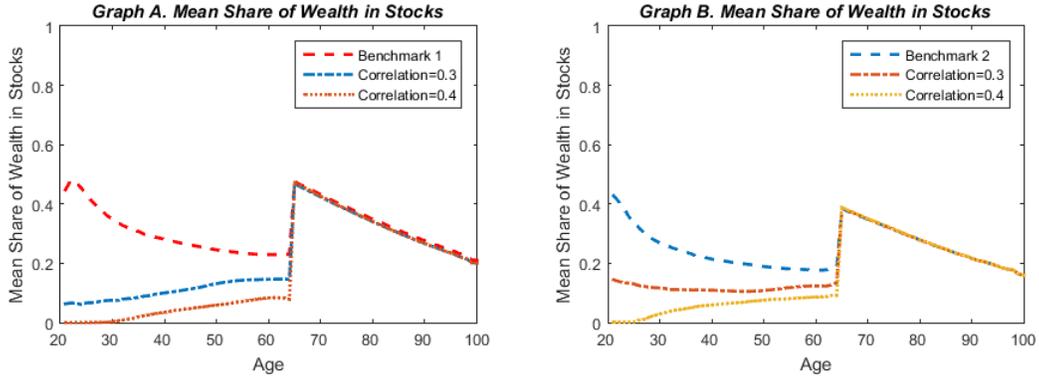


Figure 9

Aggregate Share of Wealth in Stocks and Business Cycle

Figure 9 presents the aggregate share of wealth in stocks from 1989 to 2013, and provides comparison among different models and the Flow of Funds. The grey shadow indicates that the year is in a recession.

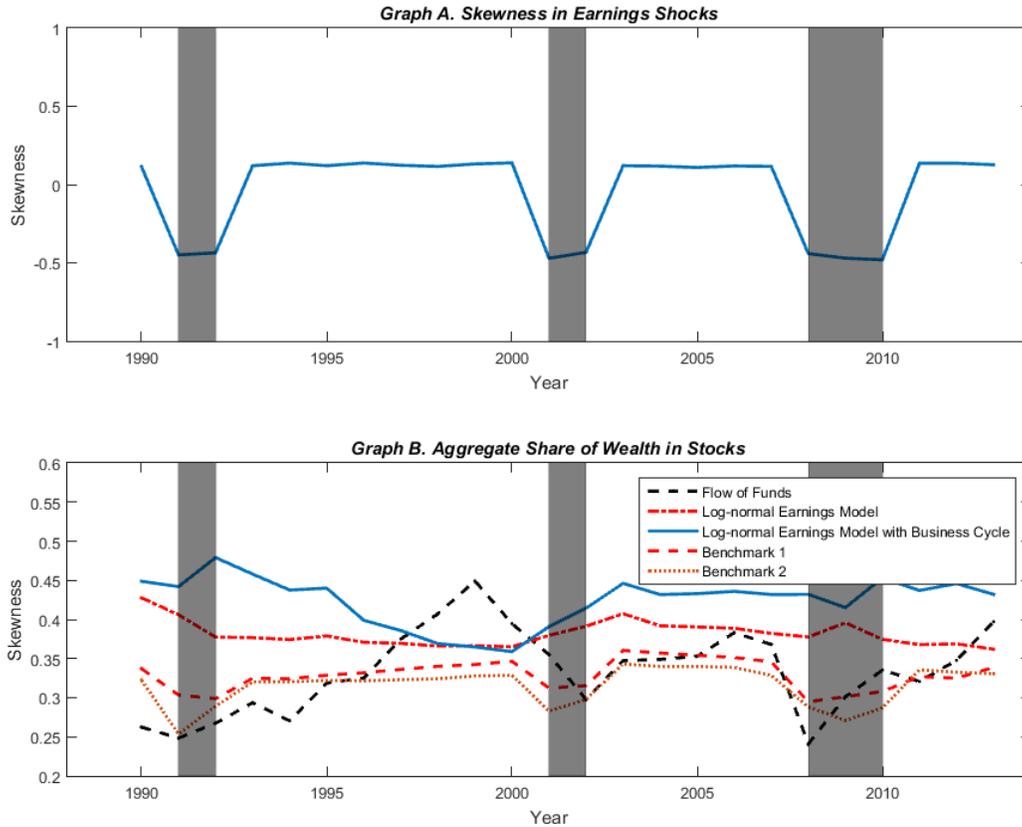


Figure 10
Wealth Inequality

Figure 10 presents the evolution of wealth inequality from 1989 to 2013 for stockholders, non-stockholders and total population. The left graphs plot Gini Index and the right graphs plot share of wealth held by the richest top 10%. Each graph contrasts the wealth inequality implied by the models with the sample of the SCF.

