Contagious Bank Runs and Dealer of Last Resort*

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Abstract

In a global-games framework, we show how a dealer-of-last-resort policy can promote financial stability while traditional lender-of-last-resort policies are informationally constrained: Central banks and private investors can be uncertain whether banks selling assets to fend off runs are insolvent or illiquid. Such uncertainty leads to asset price collapses and runs and restricts central banks' role as a lender of last resort. In the presence of aggregate uncertainty, contagion and price volatility emerge as a multiple-equilibria phenomenon despite the global-games refinement. A dealer-of-last-resort policy that requires no information on individual banks' solvency can contain contagion and stabilize prices at zero-expected costs.

Keywords: Dealer of last resort, Global games, Bank runs, Asset price volatility

JEL Classification: G01, G11, G21

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1 Introduction

In the recent financial crises, central banks have been creative in providing facilities for liquidity injection, and certain policy interventions have deviated from the classic lender-of-last-resort (LoLR) policy formulated by Bagehot (1873). As observed by Mehrling (2010) and Mehrling (2012), the Fed has increasingly become a dealer of last resort (DoLR) in its crisis management: Instead of lending to banks directly, the Fed boosted the market liquidity of banks' assets which in turn increased banks' capability to raise funding.[1] The liquidity injection of European Central Bank features a similar practice: By Long-term Refinancing Operation (LTRO) and Outright Monetary Transactions (OMT), the central bank either swapped illiquid securities of European banks with cash or pledged to purchase assets that are otherwise illiquid in markets. It is fair to say that in providing liquidity support, the central banks increasingly focus on boosting the market liquidity of assets rather than directly lending to financial institutions.[2] Despite the significance of this transition, we still lack a formal theory that defines the attributes of DoLR policies, and there is little formal theoretical discussion on why a DoLR can outperform a traditional LoLR. In this paper, we aim to formalize the concept of DoLR policies—to highlight their defining attributes, discuss their key differences from classic LoLRs, and answer the question why such policies can be more effective.

We provide a micro-founded exposition why central banks should boost the market liquidity of bank assets rather than directly lend to 'solvent-but-illiquid' banks: In crises, it can be difficult—if not impossible—to distinguish illiquid banks from insolvent ones.[3] Thus, informationally constrained central banks will not be able to lend only to the solvent-but-illiquid banks as suggested by Bagehot.

The lack of information also applies to private institutions and can generate a vicious cycle between bank runs and declining asset prices.[4] When private asset buyers cannot distinguish

[1]For example, on top of the usual LoLR policies such as open market operations (OMOs) and the discount window, the Fed also launched novel emergency liquidity assistance such as Primary Dealer Credit Facility (PDCF), Term Securities Lending Facility (TSLF), and Term Asset-Backed Securities Loan Facility (TALF). Via these programs, the central bank substituted illiquid assets of private institutions with liquid assets, by accepting a wide range of collateral.

[2]This trend also continues after the crisis. In 2017, The Reserve Bank of Australia announced its new Committed Liquidity Facility, by which the central bank can commit to entering repo transactions with qualified deposit-taking institutions.

[3]The information constraint is widely recognized both in practice and in the academic literature. It is considered a main challenge for central banks to act as lenders of last resort. See, e.g., Freixas et al. (2004).

[4]Indeed, the recent banking crisis highlights the two-way feedback: As asset market liquidity evaporated, asset prices dropped sharply. At the same time, funding liquidity dried up, and even well-capitalized banks found it difficult to roll over their short-term debt.

assets sold by illiquid banks from those sold by insolvent banks, their offered price will only reflect an average asset quality. As a result, an illiquid bank cannot recoup a fair value for its assets on sale. In a global-games framework, we study the impact of such information friction on a bank's funding liquidity risk. We show that the expectation for low asset prices can deprive the bank of its short-term funding: Anticipating the liquidation loss caused by other creditors' early withdrawals, a creditor has incentives to join the run. However, it is the run and the forced liquidation, which pools the illiquid bank with insolvent ones, that lead to the decline of asset price in the first place. In a sense, the information friction that creates the financial fragility also limits the effectiveness of traditional LoLR policies.

Price volatility and contagious bank runs emerge once we introduce aggregate uncertainty: That is, distinct equilibrium outcomes (i.e., different asset prices and different numbers of bank runs) can arise for the same realization of bank fundamentals. This is because economic agents can now coordinate on their beliefs about the aggregate state. In particular, when asset buyers observe more bank runs, they revise their beliefs about the aggregate state downwards. Their deteriorating beliefs reduce the asset price that they are willing to pay. The depressed asset price, in turn, precipitates runs at more banks. Therefore, pessimistic expectations can realize and justify themselves, leading to a vicious cycle between collapsing asset prices and contagious bank runs. Formally, this is captured by the multiple equilibria of the model.[5]

The existence of multiple equilibria leaves scope for policy intervention, and a dealer of last resort can break the vicious cycle at zero expected cost, without demanding information on individual banks' solvency. We suggest that A DoLR should not concern herself with the (in)solvency of individual institutions but should focus on maintaining a 'fair price' of banks' assets. This reduces the amount of information required. We highlight one essential difference between an equilibrium market price and the price offered by a DoLR: To an extent, the market price is belief-driven. Whenever private asset buyers observe more bank runs, they will price in the observation and lower their bids. However, it is their reduced willingness to pay that makes short-term debt-holders panic and leads to the observed bank runs in the first place. The price offered by a DoLR, on the other hand, can be based on the long-run fundamentals.[6] Furthermore, regulatory authorities such as central banks can hold greater commitment power

[5]The belief-driven runs in our model are different from those in papers such as Diamond and Dybvig (1983) because the beliefs about the aggregate state have to be rationalized by the observed number of runs, which in the global-games framework depend on fundamentals.

[6]Underlying the pricing strategy is the assumption of mean reversion of asset performance in the long run, which can be valid for assets such as real estate and underlies models like Allen and Gale (1997).

than private institutions do. Thus, when acting as a DoLR, a regulator can set the price according to the long-run fundamental of bank assets and commit to not revoking the offer when runs actually happen. We suggest that an effective DoLR policy should be pre-emptive, in the sense that the regulator makes a stand-by offer that serves as a backstop at asset prices. While such a price can be irrelevant in normal times as it can be below the prevailing market price, it prevents belief-driven contagious runs associated with extremely low asset prices. In our model, the DoLR policy will not eliminate inefficient liquidation of individual banks but will disengage the two-way feedback between market and funding illiquidity and thereby mitigate systemic crises.

We believe DoLR policies can be more effective than LoLR for two reasons: First, from an operational point of view, a bank that fails because of illiquidity cannot be distinguished from one that fails because of fundamental insolvency, which leaves regulators informationally constrained. Whereas in practicing DoLR policies, instead of assessing the solvency of a bank, regulators only need to estimate the value of assets to be purchased, which can be much less informationally demanding. Second, lending only to banks that are solvent but illiquid, a LoLR sets a policy object that is both ambitious and limited. It is ambitious because the policy aims to eliminate inefficient liquidation completely. But at the same, the policy aims only at individual banks and does not pay enough attention to the systemic stability. DoLR policies, on the other hand, can focus on preventing contagion and systemic crises by maintaining the stability of strategic bank assets.[7] To make a clear contrast, we compare DoLR and classic LoLR policies as suggested by Bagehot side by side in Table 1.[8]

Table 1: Comparison between DoLR policies and classic LoLR policies

	Lender of Last Resort (LoLR)	**Dealer of Last Resort (DoLR)**
Direct target	Individual financial institutions	(Strategic) bank assets
Policy channels	Funding liquidity	Market liquidity
Eligible collateral	'Good' collateral	A wide range of collateral
Duration	Term of loan typically overnight, up to a few weeks	Up to years, indefinitive in the case of asset purchase
Information required	Info on individual FIs' solvency	Valuation of securities to purchase
Timing	Ex-ante/ex-post intervention	Ex-ante intervention
Policy objective	Avoiding inefficient liquidation of individual FIs	Preventing systemic meltdowns

In a broader sense, our paper presents an attempt to answer the question: How should emergency liquidity assistance (ELA) programs be designed in the presence of information

[7]Examples of strategic assets would include sovereign bonds in Europe and mortgage-related assets in the US.

[8]Regarding the timing of intervention, Bagehot (1873) does suggest that a LoLR should make clear in advance her readiness to lend to troubled institutions that fulfill solvency and collateral conditions, but the lending is conditional on the actual occurrence of runs.

constraints? We show that conditioning policy interventions on more information does not necessarily improve the financial stability better than the pre-emptive policy that is based on a minimum amount of information.[9] As an extension, we also discuss the effectiveness of the DoLR policy in light of other market imperfections, such as moral hazard of private institutions.

Our paper contributes to the literature of central bank liquidity intervention and global-game based bank run models.

Central bank liquidity injection in a global-games framework is first studied in Rochet and Vives (2004). The authors considered a single-bank setup and derived a unique threshold equilibrium that features solvent-but-illiquid banks as in Bagehot (1873). The authors further assume commercial banks' fundamental to be perfectly observable to the central bank and suggest that the central bank can act as a LoLR by lending directly and only to solvent banks.[10] In a multiple-bank setup, we generalize Rochet and Vives (2004) by introducing information constraints, endogenous liquidation value, and aggregate uncertainty. We focus on systemic crises instead of runs on individual banks and show that the uniqueness achieved by global-games refinement does not survive the introduction of aggregate uncertainty. We emphasize that emergency liquidity assistance programs need to take into account information constraints faced by central banks, and provide a formal and micro-founded rationalization of central bank policies during the recent crisis.[11]

In terms of predicting feedback between market liquidity and funding liquidity, our model is most related to Liu (2016) and Brunnermeier and Pedersen (2009). In the context of banking, the two-way feedback of our model is most related to Liu (2016), who studies the interaction between bank runs and rising interbank market rates. While the driving mechanism in Liu (2016) is limited participation in the interbank market, we show that in the presence of asymmetric information, multiple equilibria emerge even if the supply of liquidity is perfectly elastic. More importantly, the main purpose of our paper is to provide a tractable model to rationalize all the features of DoLR policies as summarized in Table 1.[12] In a non-bank setup that

[9]To an extent, the result provides a justification for policy designs such as the repo facility pre-committed by Reserve Bank of Australia.

[10]In an alternative framework, Vives (2010) studies open-market operations and central banks' response to the liquidity crisis in a uniform price auction model, focusing on the design of liquidity auction when banks' marginal values of liquidity are idiosyncratic, assessed imperfectly, and yet correlated with each others'.

[11]On the empirical side, Acharya et al. (2017) argue that the asset purchase program of ECB stabilized markets better than its lending facilities did. And Veronesi and Zingales (2010) show that evaluated from an ex-ante perspective, "Paulson's Plan" yielded a net benefit between \$86 and \$109 bn.

[12]Liu (2016) also discusses a policy intervention which is modeled as an ex-post net transfer from the central bank to private institutions (in the form of helicopter money), conditional on central bank's observation of bad state. In contrast, we emphasize that the intervention should be pre-emptive: DoLR policies should be announced

has no coordination failures, Brunnermeier and Pedersen (2009) emphasize a margin constraint on a speculator who supplies liquidity to a financial market with limited participation. In their model, asset prices are volatile because selling and buying of assets are not synchronized. In comparison, we emphasize the funding liquidity risk caused by the equilibrium bank runs and how asymmetric information on asset qualities causes asset illiquidity.[13]

Our model's policy suggestion on central bank directly holding risky assets is related to the recent contribution of Koulischer and Struyven (2014) and Choi et al. (2017). Both papers show that it can be efficient for central banks to accept a broad range of collateral in their liquidity injection.[14] This view contrasts the classic view of Bagehot's that central banks' liquidity injection should avoid credit risks by accepting high-quality collateral only. Compared to the existing papers, we relax their assumption that central banks can perfectly observe collateral qualities and analyze the design of liquidity assistance programs under information constraint. In a global-games framework, our analysis also takes a general equilibrium approach by analyzing the interaction between creditors' run decisions in banks' short-term funding market and asset buyers' pricing decisions in a secondary asset market. Regarding policy implementations, we consider direct asset purchase with price support which is also recommended by Bolton et al. (2009, 2011). While in the two papers the public price support helps to maintain efficient origination and distribution of risky assets, we emphasize the role of the price support in disengaging the vicious feedback between systemic bank runs and falling asset prices.

Eisenbach (2017) also studies financial fragility caused by aggregate uncertainty using global games. Assuming observable aggregate states, the author suggests using contingent liabilities to maintain both financial stability and the disciplinary power of runs. Our model, by contrast, emphasizes that incomplete information on the aggregate state leads to multiple equilibria and financial fragility. We also show that the information about the aggregate state does not necessarily help in central banks' liquidity injection.

The paper relates to the broader literature that use global-games refinement to study bank runs, e.g., Morris and Shin (2000), Rochet and Vives (2004), Goldstein and Pauzner (2005),

before the realization of the aggregate uncertainty and can still be effective even if the central bank does not observe the aggregate state.

[13]Using historical data, Fohlin et al. (2016) empirically document the feedback between market and funding illiquidity, providing evidence that information asymmetry on asset qualities contributes to the vicious cycle.

[14]Koulischer and Struyven (2014) show that the central bank can improve welfare by lending against low-quality collateral after all high-quality collateral is exhausted. Choi et al. (2017) furthers the argument and identify a tradeoff between shielding central bank from counterparty risks and imposing negative externalities on private funding market when only high-quality collateral is accepted.

and Morris and Shin (2016),[15] but we relax the common simplifying assumption of exogenous liquidation losses.[16] From a modeling point of view, the simplifying assumption implicitly excludes the possibility for bank runs to affect asset prices, despite that bank failures often put downward pressure on asset prices in reality and that the mechanism is central to theories such as Allen and Gale (1998) and Gromb and Vayanos (2002). We introduce the impact of runs on asset prices in the framework of global games. We emphasize that buyers' lack of knowledge about the aggregate state and the inability to distinguish illiquid banks from insolvent ones result in a downward spiral between runs and declining asset prices. In the broader literature of global games, Ozdenoren and Yuan (2008) and Angeletos and Werning (2006) also introduce endogenous asset prices and predict the co-existence of price volatility and multiple equilibria. In generating multiple equilibria, both papers emphasize the impact of endogenous market price on the precision of public signals. In contrast, we study a case where asset prices directly affect players' payoffs in coordination games. In this context, we show that even if the price is endogenous, one can still have a unique equilibrium, which disappears only upon the introduction of aggregate uncertainty. Also, our focus is not the multiplicity *per se*, but the policy intervention that reduces financial fragility and price volatility.

In line with the literature of information contagion, e.g., Acharya and Thakor (2011) and Oh (2012), contagious bank runs in our model are caused not only by the actual realization of the common risk but also by its perception: An extra bank failure casts shadow on the perception of the common risk factor, and the negative informational externalities affect all other banks. Such informational contagion is introduced into global-games setups in Ahnert and Bertsch (2017) and in Chen and Suen (2016). Yet, neither paper allows the outcome of later stages to affect coordination games in earlier stages. As a result, the papers predict sequences of unique equilibrium that are path-dependent. In contrast, we show that once creditors in the bank run game are *forward-looking*, the later stage will feedback to the bank run decision, and multiple equilibria will emerge.

The paper proceeds as follows. Section 2 lays out the model. Section 3 presents the equilibrium of the model using a progressive approach: We start with a baseline case with no

[15]The literature refines the multiple equilibria in Diamond and Dybvig (1983) and emphasizes the role of liquidation loss in causing bank runs. That is, to prevent runs, an extra buffer of cash flow is needed against the liquidation loss. A bank that fails to provide the extra buffer will become "solvent but illiquid"—being able to repay its debt in full if no run happens, but will fail in equilibrium as its creditors do not roll over their debt. This bridges the panic view and the fundamental view of bank runs.

[16]For example, Rochet and Vives (2004) assume an exogenous fire-sale discount; Goldstein and Pauzner (2005) assumes unit liquidation value; and Morris and Shin (2016) assume an exogenous haircut of 100%.

systematic risk and derive its unique equilibrium. We then introduce aggregate uncertainty, showing that price volatility and contagion emerge as a multiple-equilibria phenomenon. Section 4 carries out the equilibrium analysis with DoLR policies. We show that even if a regulator is no better informed than private market participants, a DoLR policy can still improve financial stability at zero expected cost. Section 5 concludes.

2 Model setup

We consider a three-date ($t = 0, 1, 2$) economy with N banks ($i = 1, 2, ..., N$). There are three groups of active players: a continuum of wholesale creditors to banks, a large number of secondary-market asset buyers, and a regulator. All players are risk neutral.

2.1 Banks

Banks are identical at $t = 0$. Each of them holds a unit portfolio of long-term assets and finances the portfolio with equity E, retail deposits F, and short-term wholesale debt $1 - E - F$. We consider banks as contractual arrangements among claim holders, designed to fulfill the function of liquidity transformation (Diamond and Dybvig (1983)). Therefore, banks in our model are passive, with given loan portfolios and liability structures.

A Bank i's assets generate a random cash flow $\tilde{\theta}^i \sim U\left(\underline{\theta}_s, \overline{\theta}\right)$. The realization of the cash flow is not only affected by the idiosyncratic risk of the bank, but also by a systematic risk factor s. The systematic risk, as indicated by the subscript of the lower bound, determines the distribution of all banks' cash flows. There are two possible aggregate states, $s = G$ and $s = B$. With $\underline{\theta}_G \geq \underline{\theta}_B$, State G is assumed to be more favorable. All players hold a prior belief that State G and B occur with probabilities α and $1 - \alpha$ respectively.[17] Note that the upper bound of banks' cash flows is assumed to be the same across the two aggregate states. This reflects the fact that banks hold mostly debt claims whose highest payoffs are capped by their face values. Once the systematic risk factor s realizes, the N banks' cash flows are determined by their idiosyncratic risks and are assumed to be independently and identically distributed.[18] The fundamental of the banking sector can be represented by a vector $\boldsymbol{\theta} \equiv \left(\theta^1, \theta^2, \ldots, \theta^N\right)$.

[17]Probabilities α and $1 - \alpha$ have a frequentist interpretation. One can consider them derived from historical observations and corresponding to the long-run frequencies of economic booms and recessions respectively.

[18]For instance, when State G realizes, all banks' cash flows are independently drawn from a uniform distribution with support $\left[\underline{\theta}_G, \overline{\theta}\right]$.

On the liability side, we assume that retail deposits are fully protected by deposit insurance, and this financial safety net is provided to banks free of charge. Therefore, retail depositors will hold their claims passively to maturity and demand only a gross risk-free rate which we normalize to 1. On the other hand, banks' wholesale debt is risky, demandable, and raised from a continuum of creditors of mass 1. Provided that a bank does not fail, a wholesale debt contract promises a gross interest rate $r_D > 1$ if a wholesale creditor waits till $t = 2$, and qr_D if the wholesale creditor withdraws early at $t = 1$. Here $q < 1$ reflects the penalty for the early withdrawal. A bank run occurs if a positive mass of wholesale creditors withdraws funds from their bank at $t = 1$. For the ease of future presentation, we denote by D_1 the total amount of debt a bank needs to repay at $t = 1$ if *all* wholesale creditors withdraw early, and by D_2 the total amount of debt a bank needs to repay at $t = 2$ if *no* wholesale creditor withdraws early.

$$D_1 \equiv (1 - E - F)qr_D$$

$$D_2 \equiv (1 - E - F)r_D + F$$

We further make the following three parametric assumptions.

$$D_2 > \underline{\theta}_s \tag{1}$$

$$F > D_1 \tag{2}$$

$$q > \frac{1}{2} + \frac{\underline{\theta}_G}{2D_2} \tag{3}$$

Inequality (1) states that banks are not risk-free, and there is a positive probability of bankruptcy even in State G. Inequality (2) suggests that banks' retail debt exceeds their short-term wholesale debt,[19] which is a realistic scenario and helps to simplify the analysis of bank run games.[20] Finally, inequality (3) states that the penalty for early withdrawal is only moderate, which is in line with banks' role as liquidity providers (Diamond and Dybvig (1983)).[21] While we do not endogenize banks' capital structure (therefore taking q, D_1, and D_2 as given), as long as the optimal capital structure satisfies the aforementioned conditions, all of our results will apply.

[19]Note that for $q < 1$, inequality (2) also implies $D_2 > 2D_1$, because $D_2 = D_1/q + F > D_1 + F > 2D_1$.

[20]The condition is more than a technical assumption. It is realistic in the sense that despite the rapid growth of wholesale funding, most commercial banks and bank holding companies are still financed more by retail deposits than wholesale debt. For example, Cornett et al. (2011) document that the median core deposit to asset ratio for US commercial banks was 67.88% over the period from 2006 to 2009.

[21]For example, when $\underline{\theta}_G = \underline{\theta}_B = 0$, the condition states that $q > 1/2$. That is, by withdrawing early, a wholesale creditor will not lose more than a half of the face value of his claim.

We assume that banks' long-term assets cannot be physically liquidated at $t = 1$. If a wholesale run happens, a bank has to financially liquidate its assets in a secondary market and sell them to outside asset buyers. As early liquidation is costly in this model, we assume that a bank will sell its assets if and only if it faces a bank run.[22]

2.2 The bank run game

A bank run game of complete information can have two strict equilibria: All short-term debt holders withdraw from the bank, and no one withdraws. To refine the multiplicity, we take the global-games approach pioneered by Carlsson and Van Damme (1993) and assume that creditors observe noisy signals of banks' cash flows. At the beginning of $t = 1$, both systematic risk (State s) and banks' idiosyncratic risks (cash flow $\tilde{\theta}^i$) have realized, but the information is not fully revealed to players. We assume that wholesale creditors hold claims in *all N banks* and observe independent noisy signals for the banks' cash flows.[23] Specifically, a representative Creditor j privately observes a vector of noisy signals $\boldsymbol{x}_j = (x_j^1, x_j^2, \ldots, x_j^N)$, where $x_j^i = \theta^i + \epsilon_j^i$ is his signal on Bank i's realized cash flow θ^i. Noise ϵ_j^i is drawn from a uniform distribution with support $[-\epsilon, \epsilon]$. For simplicity, we assume that noises are independent across banks as well as across creditors. We also focus on a limiting case where ϵ approaches zero.[24]

After receiving his signals $\boldsymbol{x}_j$, Creditor j has two possible actions at each bank: to wait till $t = 2$ or to withdraw early at $t = 1$. We assume that creditors play a bank run game with each other in all banks simultaneously, and focus on *threshold strategies that are symmetric across all creditors for all banks*.[25,26] That is, any creditor j withdraws from any bank i if and only if $x_j^i < x^*$. As a result, an equilibrium bank run will happen if and only if the bank's cash flow $\theta^i < \theta^*$. We show in the limiting case where $\epsilon \to 0$, the critical cash flow θ^* converges to x^*.[27]

[22] Diamond and Rajan (2011) also provide an exposition why banks protected by the limited liability prefer not to sell their asset until runs happen, in which case the sale is too late and causes bank failures.

[23] It is not uncommon for institutional investors to hold demandable debt claims in multiple banks. A similar setup is analyzed by in Goldstein and Pauzner (2004).

[24] We show that in the limiting case where the noise of creditors' private signals converges to 0, the critical cash flow below which bank runs occur will coincide with the threshold signal that triggers creditor withdrawals. As a result, a bank has either no early withdrawal or all of its creditors withdrawing.

[25] In the finance application of global games, the threshold equilibrium is of primary interest. For example, see Morris and Shin (2004) and Liu (2016). Following Vives (2014) and Angeletos and Lian (2016), we also show in Appendix A that the restriction to threshold strategies is without loss of generality.

[26] Once State s realizes, all N banks' cash flows are independently and identically distributed. As creditors are ex-ante homogenous and banks are also assumed to have the same capital structure, there is no loss of generality to focus on symmetric strategies.

[27] Due to its tractability, it is common to study the limiting case in the literature. For example, see Liu and Mello (2011). In our model, the limiting case also allows for a clear-cut definition for a bank run. For a given

As in standard global-games models, creditors formulate posterior beliefs about banks' fundamentals $\boldsymbol{\theta}$ and the fraction of creditors who will withdraw early in each bank. A novelty of our model is that the creditors also need to formulate rational beliefs about the number of bank runs and anticipate its impact on the equilibrium asset price in the secondary market.

A wholesale creditor's payoff from a bank depends both on his withdrawal decision and on the bank's solvency. The creditor will receive D_1 if he withdraws early and the bank does not fail at $t = 1$; he will receive D_1/q, if he waits and the bank stays solvent at $t = 2$. In the case of failure, a bank incurs a bankruptcy cost C, which is a constant and can be interpreted as the legal cost of bankruptcy. We further assume C to be sufficiently high such that if the wholesale creditor waits and the bank fails at either $t = 1$ or $t = 2$, the wholesale creditors will receive a zero payoff and a senior deposit insurance company obtains the residual value of the bank.[28] Finally, we assume that the creditor can obtain an arbitrarily small reputational benefit by running on a bank that fails at $t = 1$.[29,30]

2.3 Secondary asset market

When facing withdrawals, banks have to liquidate their long-term assets in a secondary asset market. We assume that a large number of identical, deep-pocketed buyers participate in the market and that they are called into action only when a run happens. When no bank run occurs, the asset buyers will not have the opportunity to move, and the game between wholesale creditors and asset buyers ends. The buyers observe neither the aggregate state s nor any signals about banks' cash flows. Thus, they cannot determine the exact quality of assets on sale. They can, however, observe the outcome of creditors' bank run game, i.e., the number of banks that are forced into liquidation and can infer the quality of assets on sale from the observation.

When buyers observe any positive number of runs, $M \in \{1, 2, ..., N\}$, they compete in prices to purchase banks' assets on sale. A strategy for an asset buyer is a complete price schedule $\boldsymbol{P} = (P_1, P_2, \dots, P_N)$ that specifies a price offer P_M in the contingency that M bank runs are observed. Given the homogeneity of asset buyers, their equilibrium strategy will be symmetric.

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equilibrium and a realized cash flow θ^i, either all creditors withdraw or no one withdraws from the bank. That a fraction of creditors withdraw becomes a zero probability event.

[28]As it will be clear from the analysis, this case is off equilibrium. We then also drive a critical value for C.

[29]As we will show later, wholesale creditors receiving this small reputational payoff is also off equilibrium.

[30]The reputational benefit may come from the fact that the creditor makes a "right decision". More detailed discussion on this assumption is provided in Rochet and Vives (2004). The authors argue that the vast majority of wholesale deposits are held by collective investment funds, whose managers are compensated if they build a good reputation, and penalized otherwise.

In fact, the equilibrium strategy $\boldsymbol{P}^* = (P_1^*, P_2^*, \dots, P_N^*)$ can be viewed as the market demand for bank assets.

The price schedule $\boldsymbol{P}$ offered by an asset buyer will aggregate all information available to her. First, the buyer understands the creditors' bank run game and knows that the quality of assets on sale must be below an equilibrium threshold θ^*. Second, the buyer updates her beliefs about the aggregate state s: After the aggregate state realizes, all N bank's cash flows are i.i.d., so that more bank runs (i.e., more cases where $\theta^i < \theta^*$) suggest State B more likely.[31]

Finally, asset buyers make competitive price offers based on their beliefs. Therefore, upon the contingency where M runs have occurred, P_M^* in the equilibrium strategy profile must leave the buyers breaking even in expectation.

2.4 Dealer of Last Resort policy

As an alternative to market price, a regulator can intervene by making a standby offer to purchase bank assets for a price P_A—before the realization of the systematic risk s and idiosyncratic risks $\boldsymbol{\theta}$. Such a policy intervention, in spite of its simple form, captures the main features of DoLR policies as we summarized in Table 1. Notice that the policy intervention is not targeted at any particular bank but directly at banks' assets. Instead of lending to banks against safe collateral for a short period, the regular would swap banks' risky asset with cash via asset purchase and hold the asset indefinitely. The intervention is ex-ante in the sense that the regulator commits to the standby offer before the observation of any actual bank runs.[32] Therefore, the intervention requires neither information on the (in)solvency of individual banks nor knowing the aggregate state of the economy. We assume that the regulator possesses full commitment power and will not revoke her offer ex post. We will evaluate the proposed DoLR policy in terms of its effect of containing contagious bank runs and will compare it to alternative policies that are conditional on more granular information.

The DoLR policy in our model does not exclude the private asset market. In case that bank runs happen and the prevailing market price is higher than P_A, the private buyers will acquire banks' asset. In this sense, the regulator only provides a backstop. We will show that the DoLR policy only takes effect when the market features pessimistic beliefs and there is a severe risk of financial contagion. While the DoLR policy does not eliminate inefficient liquidation of

[31]Note that buyers' belief about s is endogenous to creditors' strategy.

[32]Notice that, instead of providing a price schedule, the regulator commits to a price P_A which is a scalar and does not depend on the number of bank runs.

individual banks, it would contain the risk of systemic meltdowns. Last but not least, the DoLR can break even from an ex-ante point of view. The intervention, therefore, is different from a public bailout, as the regulator does not incur any expected loss and makes no net transfers to banks' creditors.

2.5 Timing

The timing of the game, with and without the policy intervention, is summarized in Panel (a) and (b) of Figure 1, respectively. Events at $t = 1$ take place sequentially.

Figure 1: Timing of the game

t = 0	t = 1	t = 2
Banks are established, with their portfolios and liability structures as given.	1. s and $\boldsymbol{\theta}$ realize sequentially. 2. Creditors receive noisy private signals about $\boldsymbol{\theta}$ and simultaneously decide whether to run on each of the banks. 3. If any bank run occurs, buyers bid for and acquire assets on sale according to the number of runs observed.	1. Bank assets pay off. 2. Remaining obligations are settled.

(a) Timing of the game in a laissez-faire market

t = 0	t = 1	t = 2
1. Banks are established, with their portfolios and liability structures as given. 2. *A regulator announces her commitment to buy bank assets for a unified price P_A in case any bank run happens.*	1. s and $\boldsymbol{\theta}$ realize sequentially. 2. Creditors receive noisy private signals about $\boldsymbol{\theta}$ and simultaneously decide whether to run on each of the banks. 3. If any bank run occurs, buyers bid for assets on sale according to the number of runs observed and *acquire the asset only if their bids are higher than P_A.*	1. Bank assets pay off. 2. Remaining obligations are settled.

(b) Timing of the game under the DoLR policy

3 Equilibrium analysis in a laissez-faire market

To solve this dynamic game with incomplete information, we apply the concept of Perfect Bayesian Equilibrium.

Definition. *A PBE of our dynamic game is characterized by an equilibrium strategy profile $(x^*, \boldsymbol{P}^*)$ and a system of beliefs: (i) Each creditor plays a threshold strategy: Withdraw from a bank if and only if his private signal about the bank's cash flow is lower than the threshold x^*.*

Asset buyers purchase banks' assets on sale according to a price schedule $\boldsymbol{P}^* = (P_1^*, P_2^*, ..., P_N^*)$, *where* P_M^* *is the asset price given creditors' threshold* x^* *and the observation of M bank runs. (ii) Each creditor forms beliefs about the realized cash flows* $\boldsymbol{\theta}$ *for all N banks, and calculates the ex-post distribution of early withdrawals in each bank, conditional on his private signals and other players' equilibrium strategies. Based on the observed number of bank runs, the buyers then form beliefs about the qualities of banks' assets on sale and beliefs about the realized aggregate state, conditional on the creditors' equilibrium strategy. (iii) The strategy profile described in (i) is sequential rational given the beliefs described in (ii).*

For an equilibrium $(x^*, \boldsymbol{P}^*)$ and a fundamental $\boldsymbol{\theta}$, *an equilibrium outcome* in a laissez-faire market can feature no bank runs and no asset liquidation (which we denote by *No Run*), or be summarized by a duplex $\left(M, P_M^*\right)$, where M is the number of bank runs and P_M^* is the prevailing market price for banks' asset. When multiple equilibria exist, different equilibrium outcomes can emerge for the same fundamental, which captures the concept of financial fragility.

It takes three steps to establish an equilibrium $(x^*, \boldsymbol{P}^*)$. We start with asset buyers who move last and characterize their beliefs and optimal actions—taking creditors' equilibrium strategy as given (section 3.1). As buyers bid competitively and make zero profits in expectation, we determine the competitive asset price in the case of M runs as a response to creditors' equilibrium threshold strategy. As buyers understand the bank run game and know $x^* = \theta^*$ in the limit, we denote the competitive price as $\mathbb{P}_M(\theta^*)$.

We then solve the global games played by wholesale creditors who move first (section 3.2). We construct a representative creditor j's posterior beliefs about $\boldsymbol{\theta}$ and the other creditors' withdrawal decisions. Furthermore, forward-looking creditors foresee the equilibrium outcome in the secondary asset market and expect the price to be $\mathbb{P}_M(\theta^*)$ when anticipating M runs, $M \in \{1, 2, ..., N\}$. Based on the expected asset price and Creditor j's beliefs, we calculate his best response to the strategy x^*. For symmetric equilibria, we derive a condition that an equilibrium critical cash flow θ^* must satisfy.

Finally, we establish the existence of the equilibrium by solving for θ^* as a fixed point, which, in turn, pins down the threshold strategy x^* and the equilibrium price schedule $\boldsymbol{P^*}$, where $P_M^* = \mathbb{P}_M(\theta^*)$, $M \in \{1, 2, ..., N\}$. For illustrative purpose, we analyze two contrasting cases. When there is no aggregate uncertainty, ($\underline{\theta}_B = \underline{\theta}_G$), the model has a unique equilibrium in closed form (section 3.3). With aggregate uncertainty ($\underline{\theta}_B < \underline{\theta}_G$), multiple equilibria can emerge: For an intermediate range of fundamentals, while the equilibrium price schedule is unique, creditors' equilibrium switching strategy can take multiple thresholds. As a result, the equilibrium outcome cannot be determined, which carries the natural interpretation of contagion and price volatility (section 3.4).

3.1 Competitive bidding in the secondary asset market

In this section, we solve asset buyers' bidding game: That is, given creditors' strategy, what would be the secondary-market asset prices?

Asset buyers observe neither cash flows $\boldsymbol{\theta}$ nor State s, nevertheless form rational beliefs about the quality of assets on sale. In a Perfect Bayesian Equilibrium, buyers who believe creditors using a symmetric switching threshold x^* understand that a bank run happens if and only if the bank's cash flow is lower than θ^*. Asset buyers also Bayesian update their beliefs about State s. Given their beliefs of creditors' equilibrium strategy and the observation of M bank runs, we can calculate buyers' posterior beliefs of State s as follows:[33]

$$\omega_M^B(\theta^*) \equiv Prob(s = B|\theta < \theta^*, M) = \frac{\left(\theta^* - \underline{\theta}_B\right)^M}{\left(\theta^* - \underline{\theta}_B\right)^M + \kappa\left(\theta^* - \underline{\theta}_G\right)^M} \tag{4}$$

$$\omega_M^G(\theta^*) \equiv Prob(s = G|\theta < \theta^*, M) = \frac{\kappa\left(\theta^* - \underline{\theta}_G\right)^M}{\left(\theta^* - \underline{\theta}_B\right)^M + \kappa\left(\theta^* - \underline{\theta}_G\right)^M}, \tag{5}$$

where κ is defined as

$$\kappa \equiv \frac{\alpha}{1-\alpha}\left(\frac{\overline{\theta} - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G}\right)^N.$$

When buyers bid competitively for banks' assets on sale, the equilibrium of the secondary market entails buyers' bid to be equal to the expected asset quality. Otherwise, undercutting would happen among the buyers. Specifically, when M bank runs occur, the competitive price offered by the homogeneous buyers can be written as follows.

$$\mathbb{P}_M(\theta^*) = E\left[\theta\middle|\theta < \theta^*, M\right] = \omega_M^B(\theta^*)\, E_B\left[\theta\middle|\theta < \theta^*\right] + \omega_M^G(\theta^*)\, E_G\left[\theta\middle|\theta < \theta^*\right]$$

For a given aggregate state s, the buyers perceive the average quality of asset on sale to be

$$E_s\left[\theta\middle|\theta < \theta^*\right] = \int_{\underline{\theta}_s}^{\theta^*} \theta \cdot \frac{1}{\theta^* - \underline{\theta}_s} d\theta = \frac{\underline{\theta}_s + \theta^*}{2}.$$

[33]The detailed calculation can be found in Appendix B.1.

Therefore, the competitive asset price can be written explicitly as the following.

$$\mathbb{P}_M(\theta^*) = \omega_M^B(\theta^*)\frac{\underline{\theta}_B + \theta^*}{2} + \omega_M^G(\theta^*)\frac{\underline{\theta}_G + \theta^*}{2} = \frac{E\left(\underline{\theta}_s|M\right) + \theta^*}{2}. \tag{6}$$

Expression $E\left(\underline{\theta}_s|M\right) = \omega_M^B(\theta^*) \cdot \underline{\theta}_B + \omega_M^G(\theta^*) \cdot \underline{\theta}_G$ represents the expected lower bound of θ, based on the observation of M runs.

It is worth noticing that creditors' strategy affects the secondary market price in two ways. First, x^* and the associated θ^* directly determine the types of assets on sale, with asset quality following a uniform distribution on $[\underline{\theta}_s, \theta^*]$. Second, θ^* affects buyers' perception of the aggregate state. For a given number of runs, a more optimistic strategy on the creditors' side (i.e., a lower x^*) is associated with a more pessimistic perception of State s (i.e., a higher ω_M^B). Via both channels, higher x^* and θ^* are associated with higher asset prices.

Finally, it should be pointed out that a candidate equilibrium price must belong to $[\underline{P}, qD_2)$, where $\underline{P} = \left(\underline{\theta}_B + D_2\right)/2$. The result is intuitive: If the price is greater than qD_2, early liquidation will not hurt a bank's solvency so that its creditors would not run in the first place.[34] On the other hand, as all fundamentally insolvent banks will be liquidated, the average asset quality is guaranteed to be no lower than $\left(\underline{\theta}_B + D_2\right)/2$. This restricts the set of candidate equilibria and will facilitate the solution of bank run games in the next section.

Lemma 1. *When asset buyers believe that creditors follow a switching threshold x^* and that a bank fails if and only if $\theta < \theta^*$, an equilibrium asset price is characterized by equation (6), given an observation of $M \in \{1, 2, ..., N\}$ bank runs. The price cannot be greater than or equal to qD_2, nor can it be smaller than $\underline{P}$.*

Proof. See Appendix B.2. □

A corollary of $\mathbb{P}_M(\theta^*) \geq \underline{P}$ is that banks do not fail at $t = 1$. This is because $\mathbb{P}_M(\theta^*) \geq \underline{P} > D_1$ so that banks can always repay their $t = 1$ liabilities. Runs on the intermediate date, however, do accelerate bank failures because liquidation losses will lead to a higher probability of $t = 2$ bankruptcy. Specifically, while a partial liquidation can generate sufficient cash to pay early withdrawals and the bank does immediate failure at $t = 1$, the cash flow from the residual portfolio will insufficient to cover the remaining liabilities at $t = 2$, make a bank that

[34]For an asset price equal to qD_2, one can show that any run will reduce a bank's asset and liabilities by the same amount, resulting in a neutral impact on the solvency of the bank.

is otherwise solvent fail at $t = 2$.[35] It is also worth noticing that parametric assumption (3) guarantees $qD_2 > \underline{P}$, so that the set of candidate equilibrium prices is non-empty.

3.2 Bank run game

We now turn to creditors' bank run game and derive the condition that an equilibrium critical cash flow θ^* needs to satisfy. We do so by examining a representative creditor j's best response—running a bank if and only if his private signal is below $\hat{x}$—to other players' equilibrium strategy $(x^*, \boldsymbol{P}^*)$.

For a realized fundamental $\boldsymbol{\theta}$, an equilibrium switching threshold x^* results in runs for all banks with $\theta^i < \theta^*$. As creditor j receives sufficiently accurate signals of all banks' fundamentals, he perfectly foresees the number of runs.[36] Accordingly, creditor j rationally expects the asset price to be $\mathbb{P}_M(\theta^*)$ when he foresees M runs, $M \in \{1, 2, ..., N\}$. As we assume creditors take symmetric strategies across all N banks, the analysis of creditor j's withdrawal decisions in any of the N banks would stay the same. As a result, we suppress the index i of banks and focus our discussion on a representative bank.

We start with establishing the existence of two dominance regions. First, there exists θ^L such that a bank of $\theta \in \left[\underline{\theta}_s, \theta^L\right)$ will always fail at $t = 2$, independently of the fraction of runs. So it is a dominant strategy for creditor j to withdraw. Similarly, provided $\overline{\theta} > F/(1 - D_1/\underline{P})$, there exists θ^U such that a bank of $\theta \in \left(\theta^U\left(\mathbb{P}_M(\theta^*)\right), \overline{\theta}\right]$ will always survive at $t = 2$, independently of the fraction of runs. It is, therefore, a dominant strategy for creditor j to wait. We show in Appendix A that $\theta^L = D_2$ and that θ^U has an upper bound $F/(1 - D_1/\underline{P})$.

To solve for the best response of creditor j in the intermediate range $\left[\theta^L, \theta^U(\mathbb{P}_M(\theta^*))\right]$, we derive his payoffs for action 'wait' and 'withdraw' as functions of the fraction of other creditors who withdraw early. When L fraction of creditors withdraw early, a bank will face a liquidity demand of LD_1, $L \in [0, 1]$ and need to liquidate a λ fraction of its assets at a price $\mathbb{P}_M(\theta^*)$.

$$\lambda(M, \theta^*) = \frac{LD_1}{\mathbb{P}_M(\theta^*)} \in [0, 1)$$

[35] As pointed out by Morris and Shin (2016), even if a bank survives $t = 1$ runs, it would be doomed to fail at $t = 2$. The funding liquidity risk is captured by higher ex-ante probability of bank failure and the fact the survival threshold is higher than the solvency threshold. (See section 3.1 for their discussion on impairment function and FireSale risk.) The similar feature of no interim date failure also emerges in Ahnert et al. (2018), in their analysis of rollover risk without asset encumbrance.

[36] Recall that we define a run in a bank when a positive mass of creditors who made withdrawals in that bank. Consequently, creditor j's decision alone has no impact on the bank run outcome.

Note that λ is between 0 and 1, since we have established in Lemma 1 that $\mathbb{P}_M$ must be higher than $\underline{P}$, which is higher than D_1. After liquidating a fraction λ of its assets, the bank will fail at $t = 2$ if and only if the value of its remaining assets is lower than its remaining liabilities.

$$[1 - \lambda(M, \theta^*)] \cdot \theta < F + (1 - L)(1 - E - F)r_D \tag{7}$$

In other words, a bank will fail at $t = 2$ if and only if the fraction of creditors' withdrawal exceeds a critical value L^c.

$$L > \frac{\mathbb{P}_M(\theta^*) \cdot [\theta - F - (1 - E - F)r_D]}{[q\theta - \mathbb{P}_M(\theta^*)]\,(1 - E - F)r_D} = \frac{\mathbb{P}_M(\theta^*) \cdot (\theta - D_2)}{D_1 \cdot [\theta - \mathbb{P}_M(\theta^*)/q]} \equiv L^c(\theta, \theta^*) \in [0, 1] \tag{8}$$

Creditor j's payoff, therefore, depends on the actions of other creditors, in particular, the fraction of runs L. Depending on L, creditor j's payoffs of playing 'withdraw' or 'wait' are tabulated as follows.

	$L \in [0, L^c]$	$L \in (L^c, 1]$
withdraw	D_1	D_1
wait	D_1/q	0

Note that if the creditor withdraws, his payoff will always be $W_{run}(L) = D_1$. Instead, if he waits, his payoff depends on the action of other creditors.

$$W_{wait}(L) = \begin{cases} D_1/q & L \in [0, L^c] \\ 0 & L \in (L^c, 1] \end{cases}$$

Defining the difference between the creditor's payoffs of "wait" and "withdraw" as $DW(L) \equiv W_{wait}(L) - W_{run}(L)$, we have the following expression.

$$DW(L) = \begin{cases} (1 - q)D_1/q & L \in [0, L^c] \\ -D_1 & L \in (L^c, 1] \end{cases}$$

The strong strategic complementarity in creditors' game is clear. In a perfect information benchmark, creditor j strictly prefers 'wait' ('withdraw') if L is marginally lower (higher) than L^c, so that the slope of his best-response function tends to infinity when L approaches to L^c. In fact, the bank run game can have two equilibria in which either all creditors withdraw or all creditors wait. We refine the multiplicity using the technique of global games.

By adding noise about the banks' fundamentals, the analysis resembles the standard global-games procedure except for the endogenous asset price. We outline the analysis here, and interested readers can refer to Appendix A for full details. As a first step, we have established the existence of lower and upper dominance regions. Second, we characterize creditor j's posterior belief about L when the bank's fundamental is out of the dominance regions. Finally, we show that creditor j's best response to the other creditors' threshold strategy is a threshold strategy too. And a symmetric equilibrium implies that the critical cash flow θ^* must satisfy the following condition.

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/\mathbb{P}_M(\theta^*)} \tag{9}$$

We summarize these results in Lemma 2.

Lemma 2. *When creditors receive noisy signals* $\mathbf{x}$ *and expect asset price to be* $\mathbb{P}_M(\theta^*)$*, the only equilibium of the bank run game that survives iterated elimination of dominated strategies is a threshold equilibrium characterized by a critical signal* x^**. The corresponding bank critical cash flow is given by equation* (9)*. In the limiting case* $\epsilon \to 0$*,* $x^* = \theta^*$*.*

Proof. See Appendix A. □

A few comments are due. First, a $t = 2$ failure happens when $L > L^c$ because the partial liquidation at $t = 1$ incurs a liquidation loss. When a sufficiently large number of creditors withdraw, the bank will be forced to liquidate prematurely a significant share of it assets, and the remaining assets will not generate sufficient cash flows to meet the remaining liabilities. The creditors who withdraw therefore can impose negative externalities on creditors who wait.

Second, our model differs from standard global-games based bank run models because creditors in our model are forward-looking and understand the impact of their decisions to run on asset prices. While the critical fraction L^c would only depend on the bank's fundamental and liquidation value in models with exogenous asset prices, L^c in our model is also a function of the threshold θ^*, because θ^* affects the endogenous asset price $\mathbb{P}_M(\theta^*)$.

Finally, equation (9) characterizes a symmetric threshold equilibrium if it exits. One still needs to establish the existence of the equilibrium given the endogenous asset price, which is the focus of the next sections.

3.3 Unique equilibrium without aggregate uncertainty

We start with a baseline case where $\underline{\theta}_B = \underline{\theta}_G = \underline{\theta}$. With no aggregate uncertainty, banks are only exposed to the idiosyncratic risks of their cash flows. With cash flows independently distributed, the failure of one bank carries no information for the other banks' fundamentals. The asset buyers, therefore, will offer a unified price P independent of the number of runs observed. In other words, their strategy features a price schedule $\boldsymbol{P} = (P_1, P_2, ..., P_N) = (P, P, ..., P)$ so that the demand for banks asset is perfectly elastic.

As discussed in Section 3.1, a candidate equilibrium price P^* must satisfy zero-profit condition (6). Without aggregate uncertainty, $E(\underline{\theta}_s|M)$ degenerates to $\underline{\theta}$ and the condition becomes

$$P^* = \mathbb{P}(\theta^*) = \frac{\theta^* + \underline{\theta}}{2}, \tag{10}$$

which is no longer a function of M. As creditors anticipate the unified asset price P^*, equation (9) that defines the critical fundamental θ^* becomes

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/\mathbb{P}(\theta^*)}. \tag{11}$$

An equilibrium of the game, if exists, is a solution of the system of two equations (10) and (11). In the absence of aggregate uncertainty, we show that there exist a unique pair of critical fundamental $\theta^* \in \left[\theta^L, \theta^U(P^*)\right]$ and asset price $P^* \in [\underline{P}, qD_2)$, which are independent of the number of runs observed and jointly solve the system of equations (10) and (11). We present the closed-form solution in Appendix B.3 and summarize the results in Proposition 1.

Proposition 1. *Without aggregate uncertainty, there exists a unique PBE characterized by* $(x^*, \boldsymbol{P}^*)$*, with* $\boldsymbol{P}^* = (P^*, P^*, ..., P^*)$ *and* $x^* = \theta^*$ *when noise* $\epsilon \to 0$*. A bank run happens if and only if the bank's fundamental is below* θ^**, and the bank's asset will be sold for price* P^**.*

Proof. See Appendix B.3. □

The equilibrium is unique and stable despite the two-way feedback between bank runs and asset prices. Intuitively, if creditors take a strategy more optimistic than the equilibrium one, they will rationally anticipate the asset buyers to bid a lower price P^* according to equilibrium condition (10). The lower P^*, however, implies aggravated coordination problem by equilibrium condition (11). This, in turn, restores the equilibrium threshold strategy.[37]

A few comments are due regarding the unique equilibrium. Regarding the bank run game, in the absence of aggregate uncertainty, the equilibrium outcome is qualitatively similar to the classic global-games based bank run models such as Rochet and Vives (2004) and Vives (2014). That is, a bank with $\theta \in [D_2, \theta^*)$ can fully repay its debt if no bank run happens, but will fail in equilibrium because of premature asset liquidation caused by runs. The uniqueness, however, obtains only because we examined a special case without aggregate uncertainty. With a single aggregate state, creditors' strategy cannot affect buyers' perception of the aggregate state, which makes equation (6) reduce to (10) and makes only one asset price rationalizable. The introduction of aggregate uncertainty will open the possibility for players to coordinate on different beliefs about the aggregate state, generating both price volatility and financial contagion in the form of multiple equilibria.

Regarding the asset market, it is worth noticing that our model does not feature asset fire sales. As the buyers pay the expected payoff of the asset given their information set, no welfare loss emerges due to the change of ownership of the asset.[38] The information asymmetry has only redistributional effects. The welfare loss in our model solely stems from the bankruptcy cost. To see so, one can refer to Table 2 for the distribution of surplus among different parties. Since running on fundamentally insolvent banks is a dominant-strategy equilibrium, the insolvent banks will incur the bankruptcy cost inevitably. Yet, the efficiency can be improved by saving solvent banks from runs. We will evaluate the effectiveness of different policies by this measure.

Table 2: Payoffs to different parties

	$\theta \in [\theta^*, \overline{\theta}]$	$\theta \in [P^*, \theta^*]$	$\theta \in [\underline{\theta}, P^*]$
Retail creditors	F	F	F
Wholesale creditors	$D_2 - F$	D_1	D_1
Shareholders	$\theta - D_2$	0	0
Asset buyers	0	$(\theta - P^*)\, D_1/P^* > 0$	$(\theta - P^*)\, D_1/P^* < 0$
Deposit insurance	0	$(1 - D_1/P^*)\,\theta - C - F < 0$	$(1 - D_1/P^*)\,\theta - C - F < 0$
Surplus	θ	$\theta - C$	$\theta - C$

[37]It is worth noticing that the information asymmetry in our model does not generate standard adverse selection problems where lower prices are associated with lower average qualities. Since banks in our model are forced into asset liquidation rather than strategically choose to do so, a lower asset price in our model is associated with a *higher* average quality.

[38]This differs from classic views of asset fire sale, such as in Shleifer and Vishny (1992). When the seller of the asset can make better use of the capital as compared to the potential buyers, the asset sale itself generates negative impact on social welfare.

Finally, while the classic LoLR policies suggest targeting only at solvent but illiquid banks, such policies will be infeasible when regulators do not possess accurate information on individual banks' solvency. It should also be noted that in this baseline case, the proposed DoLR policy cannot improve financial stability either. An uninformed regulator cannot offer a better price than the private market participants—at least without incurring expected losses. The proposed DoLR policy is effective when there are aggregate uncertainty and the risk of contagion, which are the focus of our analysis in the next two sections.

3.4 Multiple equilibria with aggregate uncertainty

In this section, we characterize the equilibrium of the fully-fledged model with both idiosyncratic and aggregate risks. The complication as compared to the last section is that now players form beliefs about the aggregate state s, and the beliefs have to be rationalized by the number of bank runs observed, given the players' equilibrium strategies.[39]

To construct an equilibrium, we first derive a critical cash flow θ^*_M and the corresponding asset price P^*_M, for any given number of runs, $M \in \{1, 2, ..., N\}$.[40] Upon the observation of M runs, asset buyers update their belief about the quality of asset on sale, and their competitive bidding leads to an asset price $P^*_M = \mathbb{P}_M(\theta^*_M)$ as specified by equation (6). On the other hand, expecting M runs and an asset price P^*_M, creditors follow their threshold strategy, which implies a critical cash flow θ^*_M formulated by equation (9). Creditors' strategy and belief need to be consistent in the sense that exactly M bank runs should happen according to the critical cash flow θ^*_M. We establish in in Lemma 3 that the pair $\left(\theta^*_M, P^*_M\right)$ is unique for any $M \in \{1, 2, ..., N\}$.

Lemma 3. *In the presence of aggregate uncertainty, for any given* $M \in \{1, 2, ..., N\}$*, there exists a unique pair of an asset price* $P^*_M \in [\underline{P}, qD_2)$ *and a critical cash flow* $\theta^*_M \in \left[\theta^L, \theta^U(P^*_M)\right]$ *that jointly solve the system of equations* (6) *and* (9)*.*

Proof. See Appendix B.4. □

The critical cash flows θ^*_M and secondary asset prices P^*_M exhibit monotonicity with respect to M. Intuitively, asset buyers form more pessimistic beliefs about State s when they observe

[39] The rationalizable beliefs are given by equation (4) and (5). By comparison, the case without aggregate uncertainty can be seen as a degenerated case where $\omega_s = 1$.

[40] Since the creditors' strategy needs to be consistent with their beliefs about the number of runs and can, in principle, changes with M, we index the corresponding critical cash flow θ^* by M.

more bank runs. As a result, they will offer a lower asset price, since the expected asset quality is lower in State B. This, in turn, pushes up the threshold cash flow for a bank to survive a run.

Lemma 4. *When more runs happen, asset buyers bid less, and a bank needs to meet a higher critical cash flow to survive the runs. That is,* $P^*_{M+1} < P^*_M$ *and* $\theta^*_{M+1} > \theta^*_M$, $\forall M = 1, 2, ..., N-1$.

Proof. See Appendix B.5. □

To better understand the monotonicity in Lemma 4, one may consider a hypothetical case where the aggregate state is observable. Following the same reasoning of Proposition 1, one can derive critical cash flow θ^*_s and price P^*_s, associated with probability 1 that State s has realized, $s \in \{G, B\}$. However, the aggregate state is *not observable*, and asset buyers have to form posterior beliefs about s. In fact, each $\left(\theta^*_M, P^*_M\right)$ pair is associated a unique posterior belief $\omega^B_M \in (0, 1)$ that can be rationalized by the observation of M bank runs. As M increases, the posterior belief worsens, generating the monotonicity.

Lemma 5. *When State s is perceived to be G or B with probability* 1*, there exists a unique PBE characterized by* $(\theta^*_s, \boldsymbol{P}^*_s)$*, with* $\boldsymbol{P}^*_s = (P^*_s, P^*_s, ..., P^*_s)$, $s \in \{G, B\}$*. It holds that* $\theta^*_G < \theta^*_1 < ... < \theta^*_N < \theta^*_B$ *and* $P^*_G > P^*_1 > ... > P^*_N > P^*_B$.

Proof. See Appendix B.7. □

Lemma 3 and 4 do not fully characterize the equilibrium of the model, because M is part of the equilibrium outcome and cannot be taken as given. Different from classic bank run models à la Rochet and Vives,[41] the equilibrium of our model can vary with banks' fundamentals. In particular, if $(\theta^*_M, \boldsymbol{P^*})$ can sustain as an equilibrium for fundamental $\boldsymbol{\theta}$, the fundamental and the equilibrium strategy must imply exactly M bank runs. In fact, for certain fundamentals, creditors can form distinct rational beliefs about the number of runs and choose among multiple thresholds.[42] As a result, multiple equilibria can emerge.

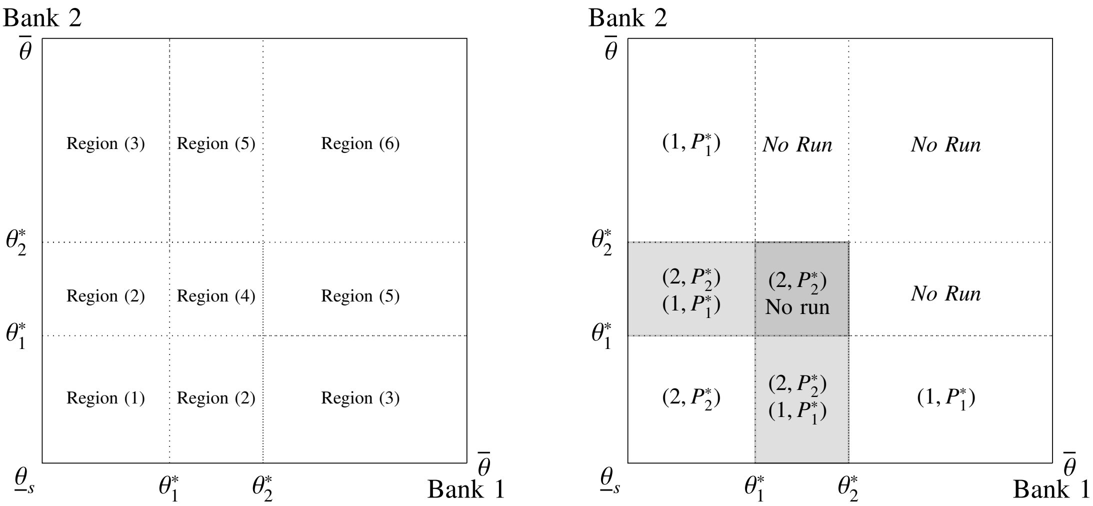
To illustrate the multiple equilibria, we start with a complete characterization of the equilibrium for a two-bank case ($N = 2$). As Lemma 3 and 4 imply $\theta^*_1 < \theta^*_2$, the critical cash flows make a unique partition of the set of bank fundamentals as shown in Panel (a) of Figure 2. For the ease of exposition, we dub creditors' threshold strategy "to run a bank if and only if the

[41] In the classic models, the equilibrium is independent of a bank's fundamental, even though the equilibrium outcome can change with the fundamental.

[42] Asset buyers' equilibrium strategy, on the other hand, remains the same price schedule $\boldsymbol{P^*} = \left(P^*_1, P^*_2, ..., P^*_N\right)$, where $P^*_M = \mathbb{P}_M(\theta^*_M)$, $M = 1, 2, ..., N$.

private signal about the bank's fundamental is below θ_1^* (θ_2^*)" the optimistic (pessimistic) strategy. Noticing that buyers' equilibrium price schedule $\boldsymbol{P}^* = \left(P_1^*, P_2^*\right)$ does not change across the different regions, we characterize the equilibrium of the game as follows.

Figure 2: Equilibrium outcomes for given regions of fundamentals



(a) Partition of bank fundamentals

(b) Equilibrium outcomes ($N = 2$)

The equilibrium outcome for given fundamentals is indicated in Panel (b). Shaded areas indicate regions where multiple equilibrium outcome can emerge with the same fundamental.

- In Region (1) where both banks' cash flow is below θ_1^*. The only *PBE* is that creditors play the pessimistic strategy. Consequently, two bank runs happen, and the asset buyers purchase bank assets for price P_2^*.
- In Region (2) where one bank's fundamental is between θ_1^* and θ_2^*, and the other bank's below θ_1^*, two equilibria can emerge. If creditors take the optimistic strategy, only the weaker bank will fail, which leads to an asset price of P_1^*. Whereas if creditors take the pessimistic strategy, both banks will fail, and the asset price will be P_2^*.
- In Region (3) where one bank's fundamental is below θ_1^* and the other's greater than θ_2^*, we have a unique equilibrium that creditors play the optimistic strategy. As a result, only the weaker bank fails, resulting an asset price P_1^*.
- In Region (4) where both banks' cash flow is between θ_1^* and θ_2^*, two equilibria can emerge. If all creditors take the pessimistic strategy, the banking sector will experience two bank runs, and the asset buyers purchase banks' asset for P_2^*. Whereas if creditors play the optimistic strategy, no bank will fail and no asset liquidation will occur.

- In Region (5) where one bank's fundamental is between θ_1^* and θ_2^* and the other's greater than θ_2^*, we have a unique equilibrium that creditors play the optimistic strategy. Only the weak bank fails, resulting an asset price P_1^*.
- In Region (6) where both banks' fundamental exceeds θ_2^*, two equilibria can sustain: The creditors can justify either the optimistic or the pessimistic strategy. But the equilibrium outcome is unique with no bank runs and no asset sale.

For an intermediate range of fundamentals (i.e., Region of (2) and (4)), multiple equilibria can emerge for the same fundamental, so that price volatility and financial fragility arise endogenously. This contrasts the full determinacy (conditional on the fundamental) of classic global-games based bank run models. The multiple equilibria are driven by players' belief about the aggregate state. When creditors hold a pessimistic belief that $s = B$ is likely, they will expect the low asset price P_2^* and set their threshold at θ_2^*. Their pessimistic strategy will generate two bank runs, justifying their pessimistic belief in the first place. We summarize the results in Proposition 2 and indicate the equilibrium outcomes in Panel (b) of Figure 2.

Proposition 2. *In the two-bank case, depending on the fundamental, the equilibrium outcome of the game is as follows:* $(2, P_2^*)$ *in Region (1);* $(1, P_1^*)$ *or* $(2, P_2^*)$ *in Region (2);* $(1, P_1^*)$ *in Region (3); No Run or* $(2, P_2^*)$ *in Region (4); and No Run in Region (5) and (6).*

We show in the following corollary that multiple equilibrium outcomes can be ranked according to the social cost. This creates the scope for policy intervention.

Corollary 1. *When the same fundamental allows for multiple equilibrium outcomes, the outcomes can be ranked according to efficiency. In the two-bank case, No Run* $\succ \left(1, P_1^*\right) \succ \left(2, P_2^*\right)$. *The three outcomes have social costs of 0, C and* $2C$*, respectively.*

To conclude this section, we discuss the equilibrium of the fully-fledged model in the general N-bank setup. While higher dimensionality makes it less practical to give a complete characterization of the equilibrium, we show that regions of multiple equilibria always exist.

Proposition 3. *For any realization of banks' fundamental, a PBE exists. When the fundamental can generate M runs, a PBE is characterized by* $(\theta_M^*, \boldsymbol{P}^*)$*, with* $\boldsymbol{P}^* = (P_1^*, P_2^*, ..., P_N^*)$ *and* $P_M^* = \mathbb{P}_M(\theta_M^*)$*,* $M = 1, 2, ..., N$*. Multiple equilibria exist when all banks' fundamentals belong to* (θ_1^*, θ_N^*)*, allowing for volatile asset prices and contagious bank runs for the same fundamental.*

Proof. Appendix B.6 □

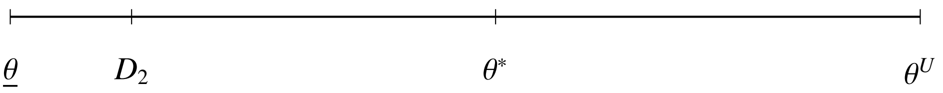
An implication of Proposition 3 is that extreme financial fragility can emerge in a laissez-faire market. Even if the banking sector's fundamental is strong, e.g., all banks' fundamentals only marginally below the highest critical value θ^*_N, runs can happen to all banks if the creditors play a pessimistic strategy that sets the switching threshold to θ^*_N.

A few comments are due regarding the multiple equilibria. First, note that we do not give a full characterization of the equilibrium for the N-bank case, but instead emphasize the existence of financial fragility. Price volatility and financial fragility also exist out of the region described in Proposition 3. For example, suppose that one bank's fundamental is between D_2 and θ^*_1, and one only one bank's fundamental is between θ^*_M and θ^*_{M+1}, $M = 1, 2, ..., N-1$. One can show that N distinct equilibria exist, and that any θ^*_M can make a PBE.

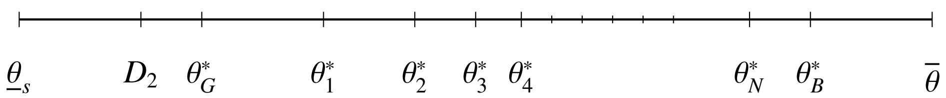
Second, our model can be seen as a hybrid model of fundamental and belief-driven runs. While beliefs about the aggregate state are, to some extent, self-fulfilling in our model, they differ from those in models such as Diamond and Dybvig (1983), as in the latter, the beliefs are entirely independent of the fundamental. In contrast, in our model the rationalizable beliefs and equilibrium outcome are always based on fundamentals. The link between fundamentals and the beliefs differ from classic global-games based bank run models too, because creditors cannot coordinate on their beliefs in classic models such as Rochet and Vives (2004) and Vives (2014), implying a unique equilibrium and full determinacy in the equilibrium outcome.

Our model generalizes the classic models by introducing endogenous asset price, aggregate states and multiple banks. The generalization may be best understood from Figure 3. In the classic models, the refinement generates a unique cut-off value, as shown in Panel (a). In contrast, the introduction of aggregate states G and B creates two corresponding cutoff values θ^*_G and θ^*_B. The difference between θ^*_G and θ^*_B creates a range for possible multiple equilibria. The multiple equilibria associated with $\theta^*_1, \theta^*_2, ..., \theta^*_N$ are ranked according to the associated beliefs about the aggregate state, which have to be rationalized by the number of observed runs.

Figure 3: The comparison between our model and classic global-games models



(a) The critical fundamental in classic models



(b) Critical fundamentals in our model

4 Dealer-of-last-resort policies

We now show that a dealer of last resort with commitment power can promote financial stability without incurring any expected loss, even if she is not better informed than private asset buyers. We then go beyond the proposed DoLR policy and consider in general the design of emergency liquidity assistance (ELA) programs in light of information constraints. We conclude the section by evaluating the proposed DoLR policy in the presence of frictions other than information constraints.

4.1 Equilibrium analysis with the DoLR policy

To examine the stability effect of the DoLR policy as proposed in section 2.4, we start with deriving a price P_A^* that allows the regulator to break even in expectation. We then analyze the equilibrium outcomes when the regulator makes such a stand-by offer of P_A^*.

Suppose that the regulator commits to purchase banks' assets for a price P_A and that creditors expect banks to sell their assets to the regulator for that price. The equilibrium of the bank run game entails a critical cash flow

$$\theta_A^*(P_A) = \frac{D_2 - D_1}{1 - qD_1/P_A}. \tag{12}$$

As the regulator announces P_A at $t = 0$, she holds the prior belief $Prob(s = G) = \alpha$ and $Prob(s = B) = 1 - \alpha$. From an ex-ante perspective, the regulator's expected payoff is

$$\alpha \cdot \frac{\underline{\theta}_G + \theta_A^*(P_A)}{2} + (1 - \alpha) \cdot \frac{\underline{\theta}_B + \theta_A^*(P_A)}{2} - P_A. \tag{13}$$

The regulator's break-even price P_A^* obtains as expression (13) equals zero. We prove the uniqueness of P_A^* and compare it to the market prices in Proposition 4.

Proposition 4. *When a regulator commits to purchase banks' assets before the realization of states, there exists a unique price P_A^* that allows the regulator to break even ex ante. Compared to the market prices, $P_A^* > P_N^*$ always holds. And $P_A^* > P_1^*$ is true if the number of banks is sufficiently small, $N < \underline{N}$, and (or) the probability $s = G$ is sufficiently high, $\alpha > \hat{\alpha}$.*

Proof. See Appendix B.8. □

The intuition of the uniqueness follows directly that of Proposition 1. Given the unique P_A^*, there exists a unique $\theta_A^* \in \left(D_2, \theta^U(P_A^*)\right)$ defined by Equation (12). As $\theta_A^* > D_2$, the proposed intervention only mitigates banks' funding liquidity and has no impact on the pure insolvency risk.[43] We have $P_A^* > P_N^*$ always holds because observing all banks failing leads to a belief that is definitely worse than the prior, i.e., $\omega_N^B > 1 - \alpha, \forall \theta^* < \overline{\theta}$. On the other hand, whether P_A^* is smaller or greater than P_1^* depends on whether observing one and only one bank run is good or bad news as compared to the prior. Intuitively, when only one bank fails out of a large number of banks, the observation leads to a posterior belief about s that is more optimistic than the prior. Whereas if the prior is very optimistic (high α), observing one bank run can lead to a dramatic downward revision of the prior beliefs.

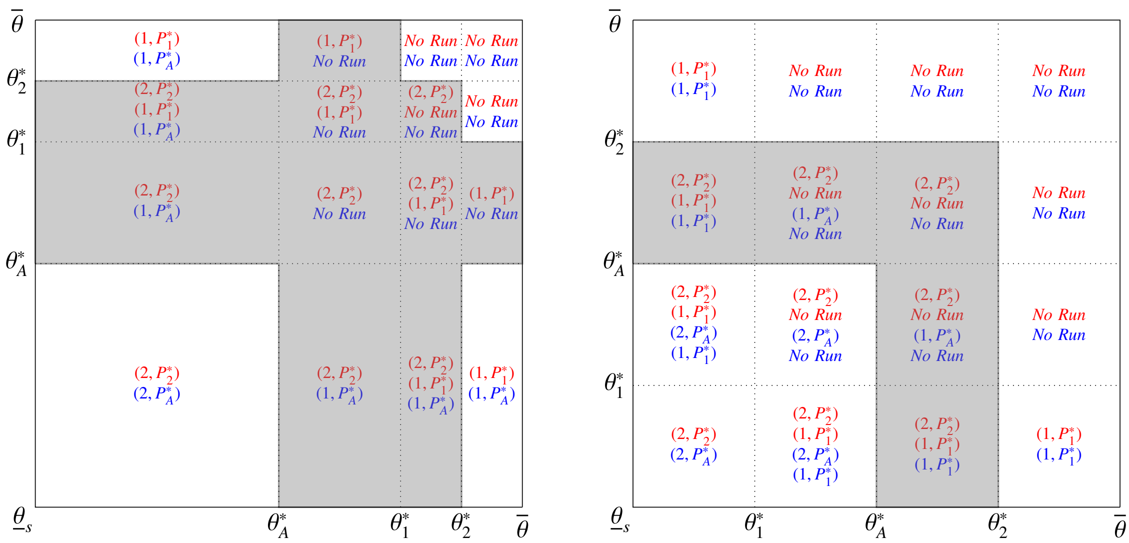
To appreciate the stability effect, one may consider the extreme fragility discussed in the last section. That is, all banks' cash flows only marginally below θ_N^*. We know that one possible equilibrium outcome in a laissez-faire market is that all N banks fail and the prevailing asset price drops to P_N^*, while the other market equilibrium entails no bank runs at all. Once the regulator pre-commits to the price $P_A^* > P_N^*$, the bad equilibrium can no longer sustain. As creditors expect price not to go below P_A^*, the switching threshold θ_N^* can no longer be rationalized. As a result, no bank run happens under the DoLR policy. This example also illustrates that promoting financial fragility does not always involve the regulator purchasing banks' assets. A credible commitment to price P_A^* is sufficient to eliminate the inefficient equilibrium that is belief-driven. This feature resembles the consequences of ECB's outright monetary transactions (OMT) program, in which case the market stabilized even if no actual transaction happen under the program.

More generally, being based on the prior belief, P_A^* exceeds all prices associated with beliefs that are more pessimistic than the prior. The DoLR, therefore, eliminates all market equilibria driven by worse-than-prior beliefs by providing a backstop in asset prices. To see so, one may again consider the example where one bank's fundamental is below θ_1^*, and for all other banks, one and only one bank's fundamental falls in interval $\left[\theta_M^*, \theta_{M+1}^*\right)$, $M = 1, 2, ..., N-1$. In a laissez-faire market, the equilibrium outcome can feature any price P_M^* with the corresponding critical cash flow θ_M^*. Whereas in the presence of P_A^*, those 'bad' equilibria with $\theta_M^* > \theta_A^*$ can no longer sustain. This example also makes a clear illustration that market equilibria can co-exist with the regulatory intervention: As the those equilibria associated with better-than-prior beliefs remain,

[43] Insolvent banks with $\theta < D_2$ will not be saved by the proposed DoLR policy. In this sense, the suggested public guarantee is only limited and does not go as far as that in the recent Irish banking crisis.

the equilibrium outcome may feature only a small number of runs and a critical cash flow lower than θ_A^*. The price offered by DoLR only limits the severity of contagion. We summarize the result in Proposition 5. To see the stability effect in an even more intuitive way, readers may also refer to Figure 4 and Appendix B.9, where we provide a complete characterization of the equilibrium outcome under the DoLR policy for the two-bank case.

Figure 4: The stability effects of the DoLR policy



The equilibrium outcome of the laissez-faire market is indicated in red, while the equilibrium outcome under the DoLR intervention is indicated in blue. The grey areas indicate regions of fundamentals where the DoLR policy promotes financial stability. Detailed derivation of can be found in Appendix B.9.

Proposition 5. *When the regulator pre-commits to price P_A^*, any market equilibrium associated with a belief $\omega_M^G(\theta_M^*) < \alpha$ cannot sustain. Whereas market equilibria associated with better-than-prior beliefs can co-exist with the regulatory intervention.*

4.2 Commitment power and DoLR

The regulator differs from private asset buyers because she holds commitment power.

Private buyers without commitment power must make no expected losses given any realized number of bank runs. In other words, they are constrained by ex-post break-even conditions. In fact, if an asset buyer offers the same price P_A^*, she will revoke the offer when a sufficiently large number of bank runs happen, because she will then form a posterior belief $\omega_M^B > 1-\alpha$ and consider herself making a loss. To break even from this ex-post perspective, the asset buyer has to lower her offered price, so as to decrease the loss from purchasing assets with $\theta \in \left[\underline{\theta}_s, P^*_M\right)$, and to increase the profit from purchasing assets with $\theta \in \left[P^*_M, \theta^*_M\right)$. The lack of commitment power, therefore, leads to a lower asset price, which in turn results in more bank runs and justifies the pessimistic belief in the first place.

By contrast, the regulator with the commitment power can choose not to react to the outcome of bank run games and stick to a unified asset price. To break even ex ante, the regulator does not require to break even for each observed number of bank runs. This allows the regulator to break the vicious cycle between bank runs and asset fire sales that is fuelled by pessimistic beliefs in market. As the regulator only needs to break even ex ante given her prior belief about State s, she can use the profit from State G to compensate the loss in State B.

The key for the regulator to achieve financial fragility is, therefore, her commitment to the price based on priors.[44] To the extent that no market participants learn the true state of the world, both ex-ante and ex-post beliefs can be justified. In a sense, the difference between asset prices in a laissez-faire market and the price offered by a DoLR reflects the difference in the concept of probability by Bayesian and frequentist schools.

We believe that regulators such as central banks can, and more importantly, be perceived to, hold the commitment power to offer price P^*_A for at least three reasons. First, central banks have different objective functions as compared to private parties. While we did not consider any possible negative externalities other than the contagion, negative externalities from bank failures (such as the threat to the payment system, the loss of soft information on borrowers) are major concerns in reality. In the presence of such externalities, P^*_A can be ex-post optimal for the central bank even if the observed number of runs suggests expected losses under Bayesian updating. Second, central banks are not subject to stark bankruptcy constraints as private institutions and can sit on temporary losses caused by short-term price fluctuations. Third, a central bank can employ commitment devices such as establishing financial stability funds to signal its commitment to support the asset price.[45] Finally, even if it is challenging to achieve full commitment, a central bank can still be in a much better position to disengage the feedback

[44] The role of commitment power in improving allocation is central to classic models such as Holmstrom and Tirole (1998), where banks' commitment to credit line contracts based on prior beliefs of financing need allows the borrower to move spare pledgeable income from favorable states to unfavorable ones and thereby mitigates credit rationing.

[45] A similar observation can be made about deposit insurance. While a small number of countries use ex-post schemes, most countries require banks to pay deposit insurance premium ex ante into a deposit insurance fund, which, we believe, adds to the credibility of deposit insurance schemes.

between falling asset prices and contagious bank runs, as long as it holds greater commitment power than private market participants.

4.3 Information constraints and emergency liquidity assistance

In the last section, we discussed the stability effect of one particular liquidity assistance program that we interpret as a DoLR policy. We now take a broader view and try to answer the question: How should the information constraint affect the design of liquidity assistance programs in general? We do so by examining two dimensions of information constraints: the granularity of available information, and whether the information can be costlessly and credibly communicated to the market participants.

Note that in our model, the policy intervention can in principle be based on four information sets, ranked according to the level of granularity: (1) precise information on banks' idiosyncratic cash flows, i.e., the realization of $\boldsymbol{\theta}$, (2) the range of idiosyncratic cash flows as reflected in the number of bank runs M,[46] (3) the aggregate state, i.e., the realization of s,[47] and (4) no information on any realized states. Having discussed the DoLR intervention that requires no information on the realized states, we now analyze the other three scenarios in turn.

Let's start with a benchmark case where the regulator knows θ. In this case, it is possible to lend directly to the solvent-but-illiquid banks. But such public intervention would not be necessary if the regulator can communicate the information to the market. This is because once the private buyers learn banks' fundamentals, the asset price will be $P = \theta$ and no inefficient liquidation will occur. In other words, the regulator can eliminate market inefficiency by eradicating its root in information friction. A more interesting case arises when the regulator observes θ but cannot perfectly communicate the information to private market participants. As pointed out by Angeletos et al. (2006), if an informed regulator aims to defend a regime (e.g., saving a solvent-but-illiquid bank) by sending a costly signal of θ, the ex-post intervention itself can create fragility in the form of multiple equilibria. Therefore, a more efficient solution would be lending directly to the solvent-but-illiquid institutions.

What if the intervention is based on the number of observed bank runs? In this case, the regulator cannot outperform the market—at least not without incurring expected losses. This

[46] We know that the M banks experiencing runs have cash flow below an equilibrium threshold, whereas the $N - M$ banks facing no runs must have cash flows greater than the equilibrium threshold.

[47] We consider observing M represents a more granular information set than observing s, because based on the number of runs, players can update their beliefs about the aggregate state. But the opposite is not true.

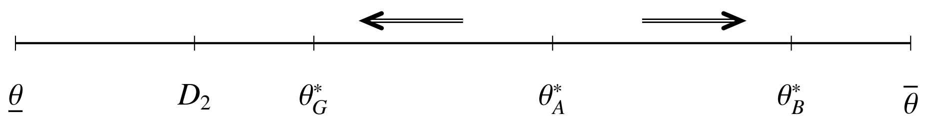
is because the only friction in our model is the information friction. As private buyers also observe M, the efficiency of the policy intervention will be bounded by the market outcome.

We consider it particularly relevant to assess intervention conditional on the aggregate state s. For if s is interpreted as a macro-economic variable, it is not unreasonable to assume the central bank being better informed than the rest of the economy. If a central bank purchases bank assets after learning s, it will need to condition the price on the aggregate state to avoid expected losses (i.e., offering P_B^* and P_G^* in state B and G respectively). And the offered price will generate thresholds θ_B^* and θ_G^*. Compared with the ex-ante intervention, the ex-post intervention creates a trade-off: When $s = G$, the policy intervention boosts asset prices and saves banks with $\theta \in \left(\theta_G^*, \theta_A^*\right)$ from illiquidity. However, when $s = B$, the policy intervention exacerbates liquidity problems by pushing the critical cash flow all the way up to θ_B^*.[48] We depict the comparison in Figure 5. And it should be clear that ex-post intervention conditional on s not necessarily dominates the DoLR policy. We summarize the result in Proposition 6.

Proposition 6. *Suppose that the regulator knows the aggregate state s. Ex-post intervention, such as disclosing s or committing to purchase bank asset for P_s^*, $s \in \{G, B\}$, is dominated by pre-committing to price P_A^* if and only if $\overline{\theta} > \overline{\theta}_c$.*

Proof. See Appendix B.10. □

Figure 5: Intervention conditional on the aggregate state s



The effectiveness of ex-post interventions will be further compromised when the knowledge of s cannot be costlessly communicated to private market participants. Indeed, if the regulator can communicate s in a separating equilibrium, the efficiency will be lower than the case where the communication is costless, since the regulator will need to pay the cost of signaling. On the other hand, if separating equilibrium is impossible and the regulator does not directly purchase banks' asset, the allocation will be the same as in a laissez-faire market.

[48]When the information on s can be perfectly communicated to the private market participants, the intervention is equivalent to disclosing the State s, as private buyers will offer the same price P_s^*, $s \in \{G, B\}$. The disclosure faces the same trade-off: If the state is good, the reassuring disclosure can calm down the market and save banks from illiquidity; but if the state is unfavorable, acknowledging a crisis will result in even more runs by pushing asset prices further down.

In conclusion, we believe that if the only market friction is the information constraint to learn banks' fundamental θ, the pre-emptive DoLR policy can be more effective than ex-post interventions. We summarize the policy options for different information sets in Table 3.

Table 3: The design ELA policies in light of information constraints

	Perfect communication	Imperfect communication
θ	Traditional LoLR is feasible. Disclosing θ is equally efficient.	Traditional LoLR is feasible. Disclosing θ can create a policy trap.
M	Interventions cannot to improve over market outcome.	
s	Purchasing asset for P_s^* in State s. Disclosing s is equally efficient.	Purchasing asset for P_s^* in State s Disclosure in a separating equilibrium: efficiency bounded by the costless disclosure Disclosure in a pooling equilibrium: equivalent to intervention based on M
No Info	DoLR is the only feasible intervention and can be more efficient than ex-post interventions conditional on s	

4.4 Further policy discussion

The key message of our paper is that regulators have to take into account information constraints when designing ELA policies. Admittedly, the information constraints are not the only challenge faced by regulators. In reality, successful policy designs must take into account additional constraints such as incentive compatible constraints of market participants (e.g., moral hazard problem from banks that can receive support), budget constraints of regulators, constraints due to limited market participation, etc. In this section, we discuss briefly the proposed DoLR policy in light of constraints other than the informational one.

One of the major concerns for ELA policies is that the policy intervention can weaken market discipline and fuel risk-taking. As banks understand that they will receive the contingent subsidy from the regulators in the case of runs, the incentive to manage funding liquidity risk can decrease. Indeed, this is the main concern as highlighted by papers such as Goodhart (1999), Goodhart and Huang (2005), Repullo (2005), and Freixas et al. (2004). In fact, the moral hazard problem and information constraints intertwine: It is regulators' inability to distinguish an illiquid bank from the insolvent that raises the concern that that blinded intervention will benefit insolvent banks and compromise market discipline, whereas taking no action risks letting solvent banks fail. To an extent, certain degree of moral hazard problem appears inevitable given the existence of information constraints.

We believe that compared to traditional LoLR policies as suggested by Bagehot, the DoLR policy we propose may suffer less from banks' moral hazard actions. As the regulator in our model does not aim to avoid inefficient liquidation of individual banks, the DoLR policy still allows banks with $\theta \in (D_2, \theta_A^*)$ to fail. In other words, those banks still pay a price for their mismanagement of risk, which would reduce the incentive of risk-taking. Whereas the traditional LoLR policy that aims to save all banks with $\theta > D_2$ is likely to be associated with greater liquidity support and provides stronger incentives for banks to take on the liquidity risk. So, our model suggests a possibility that via limited intervention, a regulator subject to information constraints may still improve financial without causing excessive risk-taking.[49]

Another prominent market friction is the limited market participation. As emphasized by Allen and Gale (1994, 2005), the supply and demand of liquidity can be inelastic in the short run, so that small aggregate uncertainty can generate large volatility in asset prices.[50] This calls for ex-post intervention contingent on the aggregate state. Liu (2016) highlights this mechanism in a global-games framework and suggest that central banks should provide liquidity when there is an aggregate shortage.[51] In our opinion, as different market imperfections can co-exist, policy interventions targeting at different market imperfections may not mutually exclude each other. While the limited market participation can be addressed by ex-post aggregate liquidity injections, information asymmetry may call for ex-ante intervention such as the DoLR policy.

Indeed, the Fed's intervention during the recent crisis provides an interesting example in this regard. As observed by Mehrling (2010), the intervention of the Fed went through three stages. After the initial cuts of policy rates, the Fed first sold its treasury holdings and lent the proceeds to banks. As the crisis deepened, the Fed eventually expanded its balance sheet by purchasing bank assets directly. While the lending stage is more conventional and can be motivated by the friction of limited market participation, the less conventional policies in the later stage made the Fed more a DoLR and may be justified by the information constraints.

5 Concluding remarks

We have presented a theory to rationalize the recent shifts in central bank emergency liquidity assistance. In particular, our model formalizes the concept of dealer of last resort (DoLR) by highlighting its defining features. In a global-games based bank run model à la Rochet and Vives, we show that the lack of granular information on individual banks' solvency creates

[49]In this sense, we agree with Goodhart and Huang (2005) that in the design of ELA policies, the regulator should focus more on the systemic risk than on individual banks.

[50]By contrast, our model shows that even if the supply of liquidity is perfectly elastic, the information constraints and aggregate uncertainty can still generate volatile asset prices and financial fragility.

[51]The policy suggestion is consistent with Kaufman (1991) in the sense that OMO, as form of aggregate liquidity injection, is preferred to discount windows that target at individual banks.

two-way feedback between bank runs and falling asset prices. In the presence of aggregate uncertainty, multiple equilibria with financial contagion and volatile asset prices emerge despite global-games refinement. We emphasize that the same information constraint also restrict central banks' policy options: Without granular information on individual banks' solvency, it is infeasible to target only at solvent-but-illiquid banks as suggested by Bagehot. We show that pre-emptive price support from central banks can require a minimum amount of information while promoting financial stability at a zero expected cost. We emphasize the main difference between the price support provided by the central bank and the price in a laissez-faire market is private parties' lack of commitment power. Whenever a crisis escalates, the private parties will need to price in the negative observation and reduce the price offered on banks' asset. However, it is the expectation of falling asset prices that precipitates runs and contagion in the first place. Public authorities such as a central bank can intervene preemptively. The commitment to the price support alone can promote financial stability by disengaging the two-way feedback between contagious runs and falling asset prices.

We also took a broader view to examine the design of emergency liquidity assistance programs in light of information constraints. That is, how effective different ELA programs can be, given the lack of granular information and the challenges for central banks to communicate the information to private market participants. We show that the DoLR policy in the model is least informationally demanding, and yet can outperform alternative programs. In particular, pre-emptive intervention based on a minimum amount of information does not necessarily underperform policies that are conditional on the knowledge of the realized aggregate states.

In a more general sense, our model provides an illustration that market imperfections such as information asymmetry both create financial fragility (and therefore, the need for policy intervention) and restrict the set of feasible policy tools. While we focus on the information friction, other forms of market imperfection, such as moral hazard and market incompleteness, also need to be taken into account in the design of ELA policies. Though our intuition suggests that the proposed DoLR policy is unlikely to perform poorly or to contradict policies designed for the other frictions, more careful analysis may lead to fruitful future research.

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Appendix A Equilibrium condition of the bank run game

In this appendix, we show that a representative creditor j's best response to other creditors' symmetric threshold strategy 'to withdraw if the private signal lower than x^*' is also a threshold strategy. We establish a condition that a symmetric equilibrium must satisfy. Given creditors' belief about the asset price, this symmetric equilibrium is the only one that survives iterated elimination of dominated strategies. In the analysis, we integrate forward-looking asset prices into a standard bank run game with incomplete information.[52] Appendix A.1-A.3 elaborate the three steps outlined in Section 3.2.

Appendix A.1 Lower and upper dominance regions

First, denoting the lower dominance region by $\left[\underline{\theta}, \theta^L\right)$, we prove its existence by construction. By definition, when a bank's cash flow belongs to this region, a representative creditor j would be better off to withdraw even if all other creditors wait. This is the case if and only if inequality (7) holds for $L = 0$ so that $\theta < F + (1 - E - F)r_D = D_2$. In other words, the bank is fundamentally insolvent and will fail at $t = 2$ even if no premature liquidation takes place at $t = 1$. In this case, a creditor, if chooses to wait, will receive a zero payoff because of the bank failure, but will receive qr_D if he withdraws early.[53] Therefore, if creditor j is sure that $\theta < D_2$ given his signal, his best action is to withdraw, independently of his belief about the other creditors' actions. As a result, we establish a dominance region $\left[\underline{\theta}, \theta^L\right)$ with $\theta^L = D_2$.

Second, denoting $\theta^U(\mathbb{P}_M(\theta^*)) \equiv F/\left(1 - D_1/\mathbb{P}_M(\theta^*)\right)$, we show that for an expected asset price $\mathbb{P}_M(\theta^*)$, the upper dominance region is $\left(\theta^U(\mathbb{P}_M(\theta^*)), \overline{\theta}\right]$. Suppose all other creditors withdraw early (i.e., $L = 1$). The bank will survive if and only if its cash flow $\theta \geq \theta^U(\mathbb{P}_M(\theta^*))$. To see that it is creditor j's dominant strategy to wait, notice that creditor j will receive qr_D if he withdraws, and r_D if he waits. Again, when the creditor's signal is accurate, he will be sure that the bank's cash flow is in the upper dominance region, and will choose to wait independently of his belief about other creditors' actions. Note that $\theta^U(\mathbb{P}_M(\theta^*))$ decreases in $\mathbb{P}_M(\theta^*)$, and it has been established in Lemma 1 that the asset price cannot be lower than $\underline{P}$. Therefore, $\theta^U(\mathbb{P}_M(\theta^*))$ has an upper bound $F/\left(1 - D_1/\underline{P}\right)$. Provided that $\overline{\theta}$ is sufficiently high, $\overline{\theta} > F/\left(1 - D_1/\underline{P}\right)$, the

[52]Our assumption that the noise of private signals diminishes ($\epsilon \to 0$) also guarantees the creditors to anticipate the same equilibrium outcome, in particular, the same number of runs M and the corresponding asset price $\mathbb{P}_M(\theta^*)$.

[53]Recall Lemma 1 that banks will not fail at $t = 1$ as an equilibrium asset price must satisfy $\mathbb{P}_M(\theta^*) > \underline{P} > D_1$.

upper dominance region exists. Different from standard global-games models, θ^U is a function of $\mathbb{P}_M(\theta^*)$, reflecting creditors' rational expectation of asset price.

Appendix A.2 Creditors' beliefs outside the dominance regions

When a bank's realized cash flow is in the intermediate region $\left[\theta^L, \theta^U(\mathbb{P}_M(\theta^*))\right]$, creditor j's optimal action depends on his beliefs about other creditors' actions. In this subsection, we characterize such beliefs.

Note first that the fraction of creditors who withdraw early is a function of a bank's fundamental θ and the threshold signal x^*. We denote this fraction by $L = L(\theta, x^*)$ and determine the functional form of $L(\theta, x^*)$. For a realized θ, we have three cases. (1) When $\theta + \epsilon < x^*$, even the highest possible signal is below the threshold x^*. By the definition of the threshold strategy, all creditors will withdraw and $L(\theta, x^*) = 1$. (2) When $\theta - \epsilon > x^*$, even the lowest possible signal exceeds the threshold x^*. All creditors will wait and $L(\theta, x^*) = 0$. (3) When θ falls into the intermediate range $[x^* - \epsilon, x^* + \epsilon]$, the fraction of creditors who withdraw at $t = 1$ is as follows, where x_k denotes the private signal of an arbitrary creditor k other than creditor j.

$$L(\theta, x^*) = Prob\left(x_k < x^* \middle| \theta\right) = Prob\left(\epsilon_k < x^* - \theta \middle| \theta\right) = \frac{x^* - \theta - (-\epsilon)}{2\epsilon} = \frac{x^* - \theta + \epsilon}{2\epsilon} \tag{A.14}$$

$L(\theta, x^*) \in (0, 1)$ would look uncertain from the perspective of creditor j, as the creditor only receives a noisy signal x_j and perceives θ with uncertainty. In particular, creditor j has a posterior belief $\theta \sim U(x_j - \epsilon, x_j + \epsilon)$ conditional his private signal x_j. Depends on the value of x_j, we have five cases.

Case (1) $x_j > x^* + 2\epsilon$: In this case, creditor j is certain that all other creditors must have received signals higher than x^*. As all other creditors choose to wait, creditor j has a posterior belief $Prob\left(L(\theta, x^*) = 0 \middle| x_j\right) = 1$ when observing $x_j > x^* + 2\epsilon$.

Case (2) $x^* < x_j \leq x^* + 2\epsilon$: Recall that $\theta \sim U(x_j - \epsilon, x_j + \epsilon)$ conditional x_j. As $x_j > x^*$, it follows $x_j + \epsilon > x^* + \epsilon$, so that we can divide the support of θ into two intervals: $\left[x_j - \epsilon, x^* + \epsilon\right]$ and $\left(x^* + \epsilon, x_j + \epsilon\right]$. When θ lies in the second interval, all other creditors receive signals higher than x^* and chosen to wait, so that $L = 0$. Creditor j, therefore, assigns the event $L(\theta, x^*) = 0$ with the following posterior probability:

$$Prob\left(L(\theta, x^*) = 0 \middle| x_j\right) = Prob\left(x^* + \epsilon < \theta < x_j + \epsilon \middle| x_j\right) = \frac{x_j + \epsilon - (x^* + \epsilon)}{(x_j + \epsilon) - (x_j - \epsilon)} = \frac{x_j - x^*}{2\epsilon} \in (0, 1]$$

On the other hand, the first interval $\left[x_j - \epsilon, x^* + \epsilon\right]$ is a subset of $[x^* - \epsilon, x^* + \epsilon]$ so that $L(\theta, x^*)$ is given by expression (A.14). Creditor j's posterior belief of $L(\theta, x^*)$ can be calculated as

$$Prob\left(L(\theta, x^*) \leq \hat{L} \middle| x_j\right) = Prob\left(\frac{x^* - \theta + \epsilon}{2\epsilon} \leq \hat{L} \middle| x_j\right) = Prob\left(\theta \geq x^* + \epsilon - 2\epsilon\hat{L} \middle| x_j\right),$$

where $\hat{L} \in (0, 1)$. Since $\theta \sim U(x_j - \epsilon, x_j + \epsilon)$ conditional on x_j, we know that θ is still uniformly distributed on $[x_j - \epsilon, x^* + \epsilon]$. And the probability above can be written explicitly as

$$Prob\left(L(\theta, x^*) \leq \hat{L} \middle| x_j\right) = \frac{(x^* + \epsilon) - (x^* + \epsilon - 2\epsilon\hat{L})}{(x^* + \epsilon) - (x_j - \epsilon)} = \frac{2\epsilon\hat{L}}{2\epsilon - (x_j - x^*)} = \frac{\hat{L}}{1 - (x_j - x^*)/2\epsilon}.$$

Therefore, for a signal $x_j \in (x^*, x^* + 2\epsilon]$, creditor j perceives $L(\theta, x^*)$ having a mixed distribution, with a positive probability mass $(x_j - x^*)/2\epsilon$ at $L = 0$ and being uniformly distributed on $\left(0, 1 - \left(x_j - x^*\right)/2\epsilon\right]$ with density 1.

Case (3) $x_j = x^*$: Creditor j still perceive $L(\theta, x^*)$ as given by expression (A.14) as $\theta \sim U(x^* - \epsilon, x^* + \epsilon)$ conditional on $x_j = x^*$. Creditor j calculates the posterior distribution of $L(\theta, x^*)$ as follows.

$$Prob\left(L(\theta, x^*) \leq \hat{L} \middle| x_j = x^*\right) = Prob\left(\frac{x^* - \theta + \epsilon}{2\epsilon} \leq \hat{L} \middle| x_j = x^*\right)$$
$$= Prob\left(\theta \geq x^* + \epsilon - 2\epsilon\hat{L} \middle| x_j = x^*\right) = \frac{(x^* + \epsilon) - (x^* + \epsilon - 2\epsilon\hat{L})}{(x^* + \epsilon) - (x^* - \epsilon)} = \hat{L}.$$

Therefore, creditor j holds a posterior belief that $L(\theta, x^*) \sim U(0, 1)$, when observing $x_j = x^*$.

Case (4) $x^* - 2\epsilon \leq x_j < x^*$: This case can be analyzed in the same way as Case (2). When observing a signal $x_j \in [x^* - 2\epsilon, x^*)$, creditor j perceives $L(\theta, x^*)$ having a mixed distribution, with a positive probability mass $(x^* - x_j)/2\epsilon$ at $L = 1$ and being uniformly distributed on $\left[(x^* - x_j)/2\epsilon, 1\right)$ with density 1.

Case (5) $x_j < x^* - 2\epsilon$: Similar to Case (1), when observing a signal $x_j < x^* - 2\epsilon$, creditor j is certain that all other creditors must have received signals lower than x^*, and therefore, has a posterior belief $Prob\left(L(\theta, x^*) = 1 \middle| x_j\right) = 1$.

It worth noticing that creditor j becomes more pessimistic about the proportion of early withdrawals when observing a lower signal. That is, the distribution of $L(\theta, x^*)$ associated with a lower x_j first-order-stochastic dominates one associated with a higher x_j.

Appendix A.3 Threshold equilibrium of the bank run game

Creditor j's expected payoff difference between action 'wait' and 'withdraw' conditional on his signal x_j dictates his action. We denote this expected payoff difference by $E[DW(L)|x_j]$ and calculate it explicitly using the posterior distribution of $L(\theta, x^*)$.

By the definition of θ^*, the following equality must hold.

$$\left(1 - \frac{LD_1}{\mathbb{P}_M(\theta^*)}\right)\theta^* = F + (1 - L)(1 - E - F)r_D \tag{A.15}$$

That is, a bank is on the verge of failure if its fundamental equals θ^*. The critical fundamental θ^* then implies a critical run proportion $L^c(\theta^*)$.

$$L^c(\theta^*) = \frac{\mathbb{P}_M(\theta^*)(\theta^* - D_2)}{D_1\left[\theta^* - \mathbb{P}_M(\theta^*)/q\right]} \tag{A.16}$$

As we know $\mathbb{P}_M(\theta^*) \in [\underline{P}, qD_2)$ and focus on $\theta \in \left[\theta^L, \theta^U\right]$, it holds that $L^c(\theta^*) \in (0, 1)$.

In Case (1) $x_j > x^* + 2\epsilon$: Creditor j perceives $L(\theta, x^*) = 0$ with probability 1. As $DW(L) = (1 - q)D_1/q$ when $L = 0$, we have

$$E[DW(L)|x_j] = (1 - q)\frac{D_1}{q}, \text{ for } x_j > x^* + 2\epsilon.$$

In Case (2) $x^* < x_j \leq x^* + 2\epsilon$: Creditor j believes L has a mixed distribution. Notice that $1 - (x_j - x^*)/2\epsilon$ decreases in x_j, with $1 - (x_j - x^*)/2\epsilon = 1$ when $x_j = x^*$ and $1 - (x_j - x^*)/2\epsilon = 0$ when $x_j = x^* + 2\epsilon$. So there exists a $\overline{x} \in (x^*, x^* + 2\epsilon]$, such that $L^c(\theta^*) = 1 - (x_j - x^*)/2\epsilon$. When $x_j \in \left(\overline{x}, x^* + 2\epsilon\right]$, we have $1 - (x_j - x^*)/2\epsilon < L^c(\theta^*)$ and the expected payoff difference

$$E[DW(L)|x_j] = \frac{x_j - x^*}{2\epsilon}(1 - q)\frac{D_1}{q} + \int_0^{1-\frac{x_j - x^*}{2\epsilon}} (1 - q)\frac{D_1}{q}dL = (1 - q)\frac{D_1}{q}.$$

When $x_j \in \left(x^*, \overline{x}\right]$, we have $1 - (x_j - x^*)/2\epsilon > L^c(\theta^*)$ and the expected payoff difference

$$\begin{aligned} E[DW(L)|x_j] &= \frac{x_j - x^*}{2\epsilon}(1 - q)\frac{D_1}{q} + \int_0^{L^c(\theta^*)} (1 - q)\frac{D_1}{q}dL + \int_{L^c(\theta^*)}^{1-\frac{x_j - x^*}{2\epsilon}} (-D_1)dL \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*) - q] + \frac{D_1}{q}\frac{x_j - x^*}{2\epsilon}. \end{aligned}$$

In Case (3) $x_j = x^*$: Creditor j believes $L \sim U(0,1)$. We have

$$E[DW(L)|x_j] = \int_0^{L^c(\theta^*)} (1-q)\frac{D_1}{q}dL + \int_{L^c(\theta^*)}^1 (-D_1)dL = \frac{D_1}{q} \cdot [L^c(\theta^*) - q]$$

In Case (4) $x^* - 2\epsilon \leq x_j < x^*$: The analysis mirrors that of Case (2). There exists a $\underline{x} \in [x^* - 2\epsilon, x^*)$, such that $\left(x^* - \underline{x}\right)/2\epsilon = L^c(\theta^*)$. When $x_j \in \left(\underline{x}, x^*\right)$, we have $\left(x^* - x_j\right)/2\epsilon < L^c(\theta^*)$. The expected payoff difference can be written as

$$\begin{aligned} E[DW(L)|x_j] &= \int_{\frac{x^*-x_j}{2\epsilon}}^{L^c(\theta^*)} (1-q)\frac{D_1}{q}dL + \int_{L^c(\theta^*)}^1 (-D_1)dL + \frac{x^* - x_j}{2\epsilon}(-D_1) \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*) - q] - \frac{D_1}{q}\frac{x^* - x_j}{2\epsilon}. \end{aligned}$$

When $x_j \in \left[\theta^* - 2\epsilon, \underline{x}\right]$, we have $\left(x^* - x_j\right)/2\epsilon > L^c(\theta^*)$ and

$$E[DW(L)|x_j] = \int_{\frac{x^*-x_j}{2\epsilon}}^1 (-D_1)dL + \frac{x^* - x_j}{2\epsilon}(-D_1) = -D_1.$$

Lastly, in Case (5) $x_j < x^* - 2\epsilon$: Creditor j perceives $L(\theta, x^*) = 1$ with probability 1. As $DW(L) = -D_1$ when $L = 1$, we have

$$E[DW(L)|x_j] = -D_1, \text{ for } x_j < x^* - 2\epsilon.$$

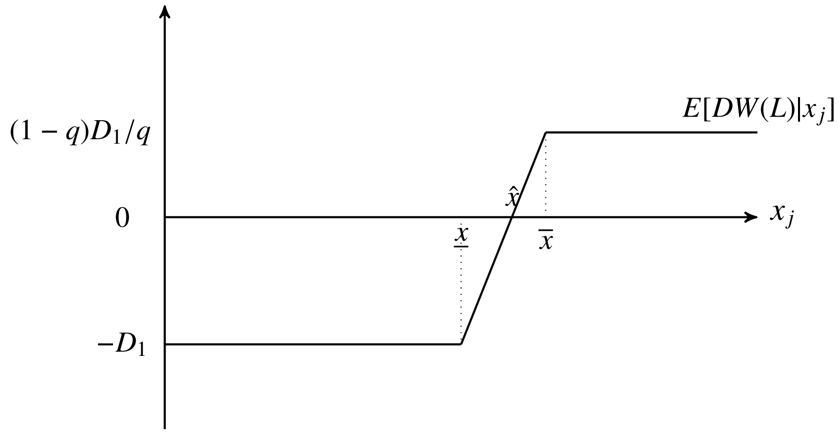
We plot $E[DW(L)|x_j]$ in Figure 6. It is straightforward to see that when $x_j \in \left(\underline{x}, \overline{x}\right)$, $E[DW(L)|x_j]$ strictly increases in x_j with a slope $r_D/2\epsilon$. As a result, there must exist a unique $\hat{x}$ such that $E[DW(L)|x_j = \hat{x}] = 0$. Therefore, creditor j's best response to the other creditors' threshold strategy is a threshold strategy: to withdraw if $x_j < \hat{x}$ and to wait if $x_j > \hat{x}$. One can derive explicitly the best response of $\hat{x}$ to x^*.

$$\hat{x} = x^* - 2\epsilon \left[L^c\left(\theta^*\right) - q\right] \tag{A.17}$$

In a symmetric equilibrium, it must hold that $\hat{x} = x^*$. Therefore, a condition for a symmetric equilibrium of the bank run game to exist is $L^c(\theta^*) = q$. Using (A.16), we can derive the condition explicitly as

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/\mathbb{P}_M(\theta^*)}. \tag{A.18}$$

Figure 6: Payoff differences and the decision to withdraw



Provided that θ^* exists, the fraction of early withdrawal when the bank's fundamental θ happens to be θ^* is given by

$$L(x^*, \theta^*) = \frac{x^* - \theta^* + \epsilon}{2\epsilon}.$$

By the definition of θ^*, the bank will be exactly on the verge of failure when $L(x^*, \theta^*)$ fraction of creditors withdraw. This in turn suggests $L(x^*, \theta^*)$ must exactly equal to $L^c(\theta^*)$, the critical run proportion derived in (A.16). Thus, the equilibrium threshold signal x^*, if exist, must satisfy

$$\frac{x^* - \theta^* + \epsilon}{2\epsilon} = \frac{\mathbb{P}_M(\theta^*)(\theta^* - D_2)}{D_1\left[\theta^* - \mathbb{P}_M(\theta^*)/q\right]} \tag{A.19}$$

In sum, when creditors expecting the asset price to be $\mathbb{P}_M(\theta^*)$, the equilibrium threshold signal x^* and the critical cash flow θ^* jointly satisfy equation (A.18) and (A.19).

It should be noted that θ^*, if exists, must belong to the interval $[x^* - 2\epsilon, x^* + 2\epsilon]$. Otherwise, we would have contradictions. For example, suppose $\theta^* > x^* + 2\epsilon$. When a bank's fundamental happens to be $\theta^* - \epsilon/2$, the bank should fail by the definition of θ^*. Yet, all creditors will receive signals greater than $x^* + \epsilon/2$ and should not withdraw from the bank according to their equilibrium strategy. Therefore θ^* must be no higher than $x^* + 2\epsilon$. Similarly, one can argue that θ^* must be no lower than $x^* - 2\epsilon$. In the limiting case where noise ϵ approaches to zero, θ^* and x^* converge.

Finally, we show that the symmetric equilibrium, if exists in the interval $[\theta^L, \theta^U(\mathbb{P}_M(\theta^*))]$, is the only one that survives iterated elimination of strictly dominated strategies.

We construct a sequence $\{\underline{x}_j\}_{j=0}^{\infty}$ starting by $\underline{x}_0 = \underline{\theta}_s$, $s = G$ or B. We let

$$\underline{x}_{j+1} = \underline{x}_j - 2\epsilon \left[L^c \left(\underline{\theta}_j, \theta^* \right) - q \right] \tag{A.20}$$

where $L^c(\underline{\theta}_j, \theta^*) = \frac{\mathbb{P}_M(\theta^*)(\underline{\theta}_j - D_2)}{D_1 \left[\underline{\theta}_j - \mathbb{P}_M(\theta^*)/q \right]}$. Note that θ^* enters into the expression of (A.20) because creditors have to form belief that asset price to be $\mathbb{P}_M(\theta^*)$ before they play the global games. And it can be checked that $\frac{\partial L^c(\underline{\theta}_j, \theta^*)}{\partial \underline{\theta}_j} > 0$ when $\mathbb{P}_M(\theta^*) \in [\underline{P}, qD_2)$. We further let $\underline{\theta}_j$, $j \geq 0$ satisfies the equation $\frac{\underline{x}_j - \underline{\theta}_j + \epsilon}{2\epsilon} = L^c(\underline{\theta}_j, \theta^*)$. Or equivalently,

$$\underline{\theta}_j + 2\epsilon L^c(\underline{\theta}_j, \theta^*) = \underline{x}_j + \epsilon \tag{A.21}$$

It is easily seen that $\underline{\theta}_j$ increases as $\underline{x}_j$ increases. Given $\underline{x}_0 = \underline{\theta}_s$, $\underline{\theta}_0$ solves the equation $\underline{\theta}_0 + 2\epsilon L^c(\underline{\theta}_0, \theta^*) = \underline{x}_0 + \epsilon$. By the monotonicity of L^c with respect to $\underline{\theta}_j$, there exists a unique solution of $\underline{\theta}_0$. The solution is sufficiently close to $\underline{x}_0 = \underline{\theta}_s$ because ϵ is sufficiently small. Recall that we claimed $\theta^* \geq D_2$, if exists, solves $L^c(\theta^*) = L^c(\theta^*, \theta^*) = q$. By $\underline{\theta}_0 < D_2$ and $\frac{\partial L^c(\underline{\theta}_j, \theta^*)}{\partial \underline{\theta}_j} > 0$, we have $L^c(\underline{\theta}_0, \theta^*) < q$. Consequently, we obtain

$$\underline{x}_1 = \underline{x}_0 - 2\epsilon \left[L^c \left(\underline{\theta}_0, \theta^* \right) - q \right] > \underline{x}_0$$

This in turn means $\underline{\theta}_1 > \underline{\theta}_0$. We can iterate this process and claim both sequences $\{\underline{x}_j\}_{j=0}^{N}$ and $\{\underline{\theta}_j\}_{j=0}^{N}$ are increasing for finite number N.

On the other hand, it is easily seen that $\{\underline{\theta}_j\}_{j=0}^{\infty}$ has a upper limit of θ^*. To see why, because each incremental value from $\underline{\theta}_j$ to $\underline{\theta}_{j+1}$ is small enough, there exists a value of $\underline{\theta}_k$ such that $\underline{\theta}_k = \theta^*$. Then by (A.20), we have $\underline{x}_{k+1} = \underline{x}_k$ as θ^* makes $L^c(\theta^*, \theta^*) = q$. Consequently, we have $\underline{\theta}_{k+1} = \underline{\theta}_k = \theta^*$ by (A.21). By iteration, we have $\underline{\theta}_k = \underline{\theta}_{k+1} = \underline{\theta}_{k+2} = ... = \theta^*$, θ^* is the upper bound of the sequence. We then know that both $\{\underline{x}_j\}_{j=0}^{\infty}$ and $\{\underline{\theta}_j\}_{j=0}^{\infty}$ are increasing. By (A.21), $\{\underline{x}_j\}_{j=0}^{\infty}$ has a upper limit of $\theta^* - \epsilon$.

Similarly, we can construct sequences $\{\overline{x}_j\}_{j=0}^{\infty}$ and $\{\overline{\theta}_j\}_{j=0}^{\infty}$ starting by $\overline{x}_0 = \overline{\theta}$. It takes the same procedure to show both sequences are decreasing, and with lower bounds $\theta^* - \epsilon$ for the former and θ^* for the latter.

To summarize, $x^* = \theta^*$ when ϵ approaches to zero is the only strategy that survives iterated elimination of strictly dominated strategies.

Appendix B Proofs to lemmas and propositions

Appendix B.1 Asset buyers' posterior beliefs

Proof. We derive the form of $\omega_M^B(\theta^*)$ for an illustration. Asset buyers evaluate the posterior probability of $s = B$ as $\omega_M^B(\theta^*)$ conditional on their belief that the equilibrium critical cash flow of the bank run game to be θ^* and their observation of M bank runs.

By Bayes' rule, we have

$$\begin{aligned}\omega_M^B(\theta^*) \equiv Prob\left(s = B \middle| \theta < \theta^*, M\right) &= \frac{Prob\,(s = B, \theta < \theta^*, M)}{Prob\,(\theta < \theta^*, M)} \\ &= \frac{Prob(s = B) \cdot Prob(\theta < \theta^*, M | s = B)}{Prob(s = B) \cdot Prob(\theta < \theta^*, M | s = B) + Prob(s = G) \cdot Prob(\theta < \theta^*, M | s = G)},\end{aligned}$$

Here, $Prob(\theta < \theta^*, M | s = B)$ calculates the probability that M banks' cash flows be lower than θ^* and rest $N - M$ banks' cash flows be higher than θ^*, conditional the distribution of banks' assets to be $U(\underline{\theta}_B, \overline{\theta})$. We can write $\omega_M^B(\theta^*)$ in details as

$$\begin{aligned}\omega_M^B(\theta^*) &= \frac{(1-\alpha)\left(\frac{\theta^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right)^M \left(\frac{\overline{\theta} - \theta^*}{\overline{\theta} - \underline{\theta}_B}\right)^{N-M}}{(1-\alpha)\left(\frac{\theta^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right)^M \left(\frac{\overline{\theta} - \theta^*}{\overline{\theta} - \underline{\theta}_B}\right)^{N-M} + \alpha \left(\frac{\theta^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G}\right)^M \left(\frac{\overline{\theta} - \theta^*}{\overline{\theta} - \underline{\theta}_G}\right)^{N-M}} \\ &= \frac{(1-\alpha)\left(\frac{\theta^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right)^M}{(1-\alpha)\left(\frac{\theta^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right)^M + \alpha \left(\frac{\theta^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G}\right)^M \left(\frac{\overline{\theta} - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G}\right)^{N-M}} = \frac{\left(\theta^* - \underline{\theta}_B\right)^M}{\left(\theta^* - \underline{\theta}_B\right)^M + \kappa \left(\theta^* - \underline{\theta}_G\right)^M}.\end{aligned}$$

For simplicity, we define $\kappa \equiv \frac{\alpha}{1-\alpha}\left(\frac{\overline{\theta} - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G}\right)^N$ as an exogenous parameter, which does not depend on the endogenous threshold θ^*.

□

Appendix B.2 Proof of Lemma 1

Proof. Since buyers' equilibrium bid cannot be negative, an equilibrium price $\mathbb{P}_M(\theta^*)$, if exits, must be in one of the three regions: $[0, \underline{P})$, $[\underline{P}, qD_2)$, or $[qD_2, +\infty)$. We show that it cannot be greater than or equal to qD_2, nor can it be lower than $\underline{P}$.

Suppose $\mathbb{P}_M(\theta^*) \geq qD_2$ for any runs $M \in \{1, 2, ..., N\}$ observed, then it is not sequentially rational for the wholesale creditors to withdraw from a solvent bank, i.e., $\theta > D_2$. To see so, one can take the perspective of a representative creditor j. Even when all other creditors withdraw, the bank needs to liquidate no more than D_1/qD_2 fraction of its asset, for $\mathbb{P}_M(\theta^*) \geq qD_2$. While the bank's $t = 2$ liability drops to F, its residual cash flow is $\left(1 - \frac{D_1}{\mathbb{P}_M(\theta^*)}\right)\theta \geq \left(1 - \frac{D_1}{qD_2}\right)D_2 \geq F$ as $\theta \geq D_2$. As a result, by running on the bank, creditor j will only incur a penalty for early withdrawal. This implies that whenever a run happens when $\mathbb{P}_M(\theta^*) \geq qD_2$, the bank must be fundamentally insolvent with $\theta < D_2$. Therefore, buyers must expect asset quality to be lower than $(D_2 + \underline{\theta}_G)/2$, which is in turn lower than qD_2 given our parametric assumption (3). Buyers would make a loss by offering $\mathbb{P}_M(\theta^*) \geq qD_2$, a contradiction.

An equilibrium price $\mathbb{P}_M(\theta^*)$ cannot be smaller than $\underline{P}$ either. Note that when a bank is fundamentally insolvent with cash flow $\theta < D_2$, it is a dominant strategy for its wholesale creditors to run, independently of the asset price. To see so, notice that if $\mathbb{P}_M(\theta^*) \geq D_1$ and the bank does not fail at $t = 1$, a creditor is better off to run and receive qr_D than to wait and receive 0.[54] On the other hand, if $\mathbb{P}_M(\theta^*) < D_1$, a creditor will receive a zero payoff for his claim whether he runs or not, but can still obtain an arbitrarily small reputational benefit by running on a bank that is doomed to fail. This implies that runs must happen to those banks with $\theta < D_2$, and the expected quality of assets on sale is at least $\left(\underline{\theta}_B + D_2\right)/2 = \underline{P}$. As asset buyers break even with their competitive bidding, the price they offer must be greater than or equal to $\underline{P}$.

□

Appendix B.3 Proof of Proposition 1

Proof. In this proof, we characterize the equilibrium solution to the baseline case without aggregate uncertainty.

We insert (10) into (11) and obtain a quadratic form

$$(\theta^*)^2 - \left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right]\theta^* - (D_2 - D_1)\,\underline{\theta} = 0, \tag{B.22}$$

The solution(s) of this quadratic function is given by

$$\theta^*_{+,-} = \frac{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right] \pm \sqrt{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right]^2 + 4\,(D_2 - D_1)\,\underline{\theta}}}{2}$$

[54]Note that asset sale will never increase an insolvent bank's solvency as we just proved that $\mathbb{P}_M(\theta^*) \geq qD_2$ could never happen.

Consequently, we have

$$P^*_{+,-} = \frac{\theta^*_{+,-} + \underline{\theta}}{2} = \frac{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right] \pm \sqrt{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right]^2 + 4\,(D_2 - D_1)\,\underline{\theta} + 2\underline{\theta}}}{4} \tag{B.23}$$

P^* and θ^* can be futher rearranged into

$$P^*_{+,-} = \frac{\Psi \pm \sqrt{\Psi^2 - 8qD_1\underline{\theta}}}{4}, \quad \theta^*_{+,-} = \frac{\Psi \pm \sqrt{\Psi^2 - 8qD_1\underline{\theta}} - 2\underline{\theta}}{2}$$

For simplicity, we denote $\Psi \equiv (D_2 - D_1) + 2qD_1 + \underline{\theta}$ as an exogenous parameter.

We check that only the positive root

$$P^* = \frac{\Psi + \sqrt{\Psi^2 - 8qD_1\underline{\theta}}}{4} = P^*_+ \tag{B.24}$$

belongs to the interval $[\underline{P}, qD_2)$ and can be the equilibrium price.

We first verify that both roots monotonically increase in q. In particular, it can be checked that the cross derivative $\partial^2 P^*_-/(\partial q \partial \underline{\theta}) > 0$, and $\partial P^*_-/\partial q = 0$ when $\underline{\theta} = 0$.[55] Therefore, we obtain that $\partial P^*_-/\partial q > 0$ for all $\underline{\theta} > 0$. Direct calculation shows that P^*_- is smaller than D_1 when $q = 1$. As $D_1 < \underline{P}$, we conclude that P^*_- is less than $\underline{P}$ for all $q \in (0, 1)$ and cannot be part of the equilibrium of our model by Lemma 1.

For P^*_+, we can also check that $\partial^2 P^*_+/(\partial q \partial \underline{\theta}) < 0$, and $\partial P^*_+/\partial q > 0$ when $\underline{\theta} = (2q - 1)D_2$. Therefore, we obtain that $\partial P^*_+/\partial q > 0$ for all $\underline{\theta} < (2q - 1)D_2$.[56] Furthermore, one can verify that $P^*_+ = \underline{P}$ when $q = 0$,[57] so that $P^*_+ > \underline{P}$ for any $q > 0$ and $\underline{\theta} < (2q - 1)D_2$. On the other hand, we can directly check that $P^*_+ < qD_2$ when parametric assumption (3) holds. Consequently, we retain only P^*_+ and obtain expression (B.24) as the unique equilibrium asset price.

The (only) equilibrium critical cash flow, if exits, is

$$\theta^* = \frac{\Psi + \sqrt{\Psi^2 - 8qD_1\underline{\theta}} - 2\underline{\theta}}{2} = \theta^*_+ \tag{B.25}$$

We then show that the corresponding this θ^* indeed belongs to $[\theta^L, \theta^U(P^*)]$.

[55] The detailed calculation of the derivatives in this proof can be provided upon request.

[56] Note that this is actually parametric assumption (3) in the baseline case. The assumption degenerates to $q > \frac{1}{2} + \frac{\underline{\theta}}{2D_2}$ as $\underline{\theta}_G = \underline{\theta}_B = \underline{\theta}$.

[57] Note that in this baseline case, $\underline{P} = (\underline{\theta} + D_2)/2$

To show $\theta^* > \theta^L = D_2$, note that the inequality can be explicitly written as $\sqrt{\Psi^2 - 8qD_1\underline{\theta}} > 2D_2 + 2\underline{\theta} - \Psi$. To see it indeed holds, we can square both sides and rearrange the terms to get $(D_2 + \underline{\theta})\Psi - (D_2 + \underline{\theta})^2 - 2qD_1\underline{\theta} > 0$. By the definition of Ψ, the inequality can be simplified to $2qD_2 - D_2 - \underline{\theta} > 0$, which is guaranteed again by assumption (3).

For the part $\theta^* < \theta^U(P^*)$, we rewrite the equilibrium condition in the secondary asset market as $\theta^* = 2P^* - \underline{\theta}$. Therefore, $\theta^* < \theta^U(P^*)$ is equivalent to

$$\frac{F}{1 - D_1/P^*} - (2P^* - \underline{\theta}) > 0,$$

or $2(P^*)^2 - (2D_1 + \underline{\theta} + F)P^* + D_1\underline{\theta} < 0$. Using the closed-form solution of P^*, we can rewrite the inequality as follows.

$$(1-q)D_1 \left[\frac{1-2q}{q} \frac{\Psi + \sqrt{\Psi^2 - 8qD_1\underline{\theta}}}{4} + \underline{\theta} \right] < 0 \tag{B.26}$$

It can be verified that the term inside the square bracket of (B.26) increases in $\underline{\theta}$, and it equals zero when $\underline{\theta} = (2q-1)D_2$. Use assumption (3), this term must be negative for $0 < \underline{\theta} < (2q-1)D_2$, and this proves $\theta^* < \theta^U(P^*)$.

To conclude, we prove the pair θ^* and P^* characterized by (B.25) and (B.24) is unique and satisfy our equilibrium requirements. There exists a unique PBE $(\theta^*, \boldsymbol{P}^*)$ for this baseline case.

□

Appendix B.4 Proof of Lemma 3

Proof. Suppose all other creditors take a θ^* as the critical threshold of their switching strategy. A creditor j rationally anticipates that M runs will occur after receiving signals of all N banks' cash flows. He understands asset buyers' bidding game and expects the equilibrium asset price to satisfy (6). His best response to all other player' strategies is then derived in Appendix A. For θ^* to be a symmetric equilibrium of the bank run game, it must satisfy (9).

To ease discussion, we transfer (6) and (9) to the following system of two equations.

$$P^*_M = \omega^B_M(\theta(P^*_M)) \frac{\underline{\theta}_B + \theta(P^*_M)}{2} + \omega^G_M(\theta(P^*_M)) \frac{\underline{\theta}_G + \theta(P^*_M)}{2}, \tag{B.27}$$

and

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/P_M^*} \equiv \theta(P_M^*). \tag{B.28}$$

To prove this lemma, it is equivalent to prove the existence and uniqueness a pair of θ^* and P_M^* as the solution of (B.27) and (B.28). It will take two steps.

Step 1: We prove there exists a unique $P_M^* \in (\underline{P}, qD_2]$ satisfying (B.27).

We can define an asset buyers' expected profit as

$$\Pi_M(P_M) \equiv \omega_M^B(\theta(P_M))\left(\frac{\underline{\theta}_B + \theta(P_M)}{2} - P_M\right) + \omega_M^G(\theta(P_M))\left(\frac{\underline{\theta}_G + \theta(P_M)}{2} - P_M\right). \tag{B.29}$$

Then P_M^* must satisfy the zero-profit condition $\Pi_M(P_M^*) = 0$. We further have

$$\Pi_M(P_M) = \frac{\theta(P_M) + \underline{\theta}_G}{2} - \frac{\underline{\theta}_G - \underline{\theta}_B}{2}\omega_M^B(\theta(P_M)) - P_M$$

as $\omega_M^B(\theta(P_M)) + \omega_M^G(\theta(P_M)) = 1$. Take the first order derivative with respect to P_M, we obtain

$$\frac{d\Pi_M(P_M)}{dP_M} = \frac{1}{2}\frac{d\theta(P_M)}{dP_M} - \frac{\underline{\theta}_G - \underline{\theta}_B}{2}\frac{d\omega_M^B(\theta(P_M))}{dP_M} - 1. \tag{B.30}$$

From (B.28), it is straightforward to check that $d\theta(P_M)/dP_M < 0$. It can be verified that

$$\frac{d\omega_M^B(\theta(P_M))}{dP_M} = -\frac{\partial\theta(P_M)}{\partial P_M}\frac{M\cdot\kappa\cdot\left(\theta(P_M) - \underline{\theta}_B\right)^{M-1}\cdot\left(\theta(P_M) - \underline{\theta}_B\right)^{M-1}\cdot\left(\underline{\theta}_G - \underline{\theta}_B\right)}{\left[\left(\theta(P_M) - \underline{\theta}_B\right)^M + \kappa\left(\theta(P_M) - \underline{\theta}_G\right)^M\right]^2} > 0.$$

Consequently, $\Pi_M(P_M)$ monotonically decreases in P_M.

Notice that the asset market zero-profit condition can be rewritten as the following.

$$\Pi_M(P_M^*) = \omega_M^B(\theta(P_M^*))\,\pi^B(\theta(P_M^*)) + \omega_M^G(\theta(P_M^*))\,\pi^G(\theta(P_M^*)) = 0. \tag{B.31}$$

$\pi^s(P_M) \equiv \left(\underline{\theta}_s + \theta(P_M)\right)/2 - P_M$ denotes a buyer's profit in State s when her offered price is P_M.

Independently of State s, a buyer makes a profit if $P_M = \underline{P}$. To see so, a sufficient condition for $\pi^s(\underline{P}) > 0$ is $\theta(\underline{P}) > D_2$, which is implied by assumption (3).

$$\pi^s(\underline{P}) = \frac{\theta(\underline{P}) + \underline{\theta}_s}{2} - \underline{P} = \frac{\theta(\underline{P}) + \underline{\theta}_s}{2} - \frac{D_2 + \underline{\theta}_B}{2} > 0$$

It can also be showed that a buyer makes a loss if $P_M = qD_2$. Note that $\theta(qD_2) = D_2$ and

$$\pi^s(qD_2) = \frac{\theta(qD_2) + \underline{\theta}_s}{2} - D_2 = \frac{D_2 + \underline{\theta}_s}{2} - D_2 < 0.$$

With the posterior beliefs $\omega_M^B(\theta(P_M))$ and $\omega_M^G(\theta(P_M))$ positive and smaller than one,[58] we have $\Pi_M(\underline{P}) > 0$ and negative when $\Pi_M(qD_2) < 0$. As $\Pi_M(P_M)$ monotonically decreases in P_M, there exists a unique $P_M^* \in [\underline{P}, qD_2)$ such that $\Pi_M(P_M^*) = 0$.

Step 2: The corresponding $\theta^* = \theta(P_M^*)$ defined in (B.28) is indeed between θ^L and $\theta^U(P_M^*)$. This step is a direct replication of the second part of Appendix B.3, thus we suppress it for simplicity. The detailed derivation can be provided upon request.

Finally, it should be pointed out that the integer M in (B.27) and (B.28) defines a unique system of equations and a unique pair of θ^* and P_M^*. And it varies from 1 to N. For any $M \in \{1, 2, ..., N\}$ runs, there always exists a P_M^* being the unique equilibrium asset price P_M^*; there always exists a θ^* being the equilibrium critical threshold when creditors anticipating P_M^*. To avoid ambiguity, we label this θ^* as θ_M^*, that is, the critical threshold associated with M runs.

□

Appendix B.5 Proof of Lemma 4

Proof. For any value $\theta \in [D_2, \overline{\theta}]$, the proof hinges on the monotonicity of two ratios.

$$\frac{\omega_M^B(\theta)}{\omega_M^G(\theta)} = \frac{(\theta - \underline{\theta}_B)^M}{\kappa(\theta - \underline{\theta}_G)^M} \qquad \text{and} \qquad \frac{\pi^G(\theta)}{\pi^B(\theta)} = \frac{(\theta + \underline{\theta}_G)/2 - P(\theta)}{(\theta + \underline{\theta}_B)/2 - P(\theta)}$$

The former is a ratio of posterior beliefs about state when observing M runs, and the latter is a ratio of buyers' profit across two states defined in Appendix B.4. Note that $P(\theta) = qD_1\theta/\left[\theta - (D_2 - D_1)\right]$ from (9).

It can be checked easily that $\partial P(\theta)/\partial\theta < 0$. Then, both ratios strictly monotonically decrease in θ for $\theta > D_2 > \underline{\theta}_s$ as

$$\frac{d}{d\theta}\left(\frac{\omega_M^B(\theta)}{\omega_M^G(\theta)}\right) = -\frac{1}{\kappa} \cdot \frac{M \cdot (\theta - \underline{\theta}_B)^{M-1} \cdot (\underline{\theta}_G - \underline{\theta}_B)}{\left(\theta - \underline{\theta}_G\right)^{M+1}} < 0$$

$$\frac{d}{d\theta}\left(\frac{\pi^G(\theta)}{\pi^B(\theta)}\right) = -\frac{\left[1/2 - \partial P(\theta)/\partial\theta\right](\underline{\theta}_G - \underline{\theta}_B)}{2\left[\frac{\theta + \underline{\theta}_B}{2} - P(\theta)\right]^2} < 0$$

[58]This result is guaranteed by $\theta(P_M) > D_2 > \underline{\theta}_G > \underline{\theta}_B$, which can be calculated directly.

Furthermore, notice that $(\theta - \underline{\theta}_B)/(\theta - \underline{\theta}_G) > 1$ for $\theta > D_2$, therefore

$$\kappa \cdot \frac{\omega_M^B(\theta)}{\omega_M^G(\theta)} = \left(\frac{\theta - \underline{\theta}_B}{\theta - \underline{\theta}_G}\right)^M < \left(\frac{\theta - \underline{\theta}_B}{\theta - \underline{\theta}_G}\right)^{M+1} = \kappa \cdot \frac{\omega_{M+1}^B(\theta)}{\omega_{M+1}^G(\theta)}. \tag{B.32}$$

As a result, for a given value $\theta > D_2$, $\frac{\omega_M^B(\theta)}{\omega_M^G(\theta)} < \frac{\omega_{M+1}^B(\theta)}{\omega_{M+1}^G(\theta)}$ holds for any $M = 1, 2, ..., N-1$.

We then prove the result by contradiction. Suppose the equilibrium critical cash flows are such that $\theta_M^* = \theta(P_M^*) \geq \theta_{M+1}^* = \theta(P_{M+1}^*)$.[59] By the monotonicity of $\pi^G(\theta)/\pi^B(\theta)$, we have

$$\frac{\pi^G(\theta_M^*)}{\pi^B(\theta_M^*)} \leq \frac{\pi^G(\theta_{M+1}^*)}{\pi^B(\theta_{M+1}^*)}. \tag{B.33}$$

By (B.31) from Appendix B.4, we have

$$\frac{\pi^G(\theta_M^*)}{\pi^B(\theta_M^*)} = -\frac{\omega_M^B(\theta_M^*)}{\omega_M^G(\theta_M^*)} \quad \text{and} \quad \frac{\pi^G(\theta_{M+1}^*)}{\pi^B(\theta_{M+1}^*)} = -\frac{\omega_{M+1}^B(\theta_{M+1}^*)}{\omega_{M+1}^G(\theta_{M+1}^*)}.$$

Together with (B.33), the two equations above imply that

$$\frac{\omega_{M+1}^B(\theta_{M+1}^*)}{\omega_{M+1}^G(\theta_{M+1}^*)} \leq \frac{\omega_M^B(\theta_M^*)}{\omega_M^G(\theta_M^*)}.$$

By inequality (B.32), we know

$$\frac{\omega_{M+1}^B(\theta_{M+1}^*)}{\omega_{M+1}^G(\theta_{M+1}^*)} \leq \frac{\omega_M^B(\theta_M^*)}{\omega_M^G(\theta_M^*)} < \frac{\omega_{M+1}^B(\theta_M^*)}{\omega_{M+1}^G(\theta_M^*)}.$$

Then by the monotonicity of $\omega_{M+1}^B(\theta)/\omega_{M+1}^G(\theta)$, we have $\theta_{M+1}^* > \theta_M^*$, a contraction.

Therefore, $\theta_{M+1}^* > \theta_M^*$, for any $M = 1, 2, 3, ..., N-1$. Then, it follows directly $P_{M+1}^* < P_M^*$, for any $M = 1, 2, 3, ..., N-1$ from the monotonicity of $P(\theta)$. □

Appendix B.6 Proof of Proposition 3

Proof. Following Lemma 3 and 4, we show that for any realization of fundamental $\boldsymbol{\theta}$, a Perfect Bayesian Equilibrium could be found: a rational expectation about the number of runs of creditors', i.e., $M \in \{1, .., N\}$ runs will occur; a strategy profile $\left(\theta_M^*, \boldsymbol{P}^*\right)$ where $\boldsymbol{P}^* = \left(P_1^*, ..., P_M^*, ..., P_N^*\right)$.[60]

[59] Note that we prove in Lemma 3, those critical cash flows exist and are unique in $[D_2, \theta^U] \subset [D_2, \overline{\theta}]$.

[60] We suppress creditors' beliefs about the proportion of withdrawals and buyers' belief about the quality of assets being sold, as we have already pinned them down in Section 3.1 and 3.2.

For a $\boldsymbol{\theta} = \left(\theta^1, ..., \theta^N\right)$,[61] we denote $\theta^{(1)} = \max_{i\in\{1,...,N\}} \theta^i$ as the largest realized fundamental. If $\theta^{(1)} \leq \theta_N^*$, we immediately find a PBE: strategy $\left(\theta_N^*, \boldsymbol{P}^*\right)$ along with creditors' expectation that runs will occur to all N banks. Note that the expectation N runs will happen is rational, as creditors indeed run all N banks following threshold strategy θ_N^*. Then, P_N^* is the equilibrium price of the asset market upon N runs, and θ_N^* is the BNE of bank run game when creditors anticipating P_N^*, the strategy is sequentially rational.

If $\theta^{(1)} > \theta_N^*$, we denote $\theta^{(2)} = \max_{i\in\{1,...,N\}/\{(1)\}} \theta^i$ as the second largest value. Then if $\theta^{(2)} \leq \theta_{N-1}^*$, by the same argument, we find a PBE: a strategy $\left(\theta_{N-1}^*, \boldsymbol{P}^*\right)$ along with creditors' expectation that runs will occur to all $N-1$ banks except for the bank with the highest fundamental $\theta^{(1)}$. Observing sufficiently accurate signals, creditors' expectation of $N-1$ runs is confirmed by the equilibrium bank run outcome, and playing θ_{N-1}^* is sequentially rational when the equilibrium asset price is P_{N-1}^*.

If $\theta^{(2)} > \theta_{N-1}^*$, we can further denote $\theta^{(3)} = \max_{i\in\{1,...,N\}/\{(1),(2)\}} \theta^i$ and iterate the process.

Finally, if we iterate this process for $N-1$ times and still find no PBE, banks' fundamentals must be $\theta^{(N-1)} > \theta_2^*$, $\theta^{(N-2)} > \theta_3^*$, ... , $\theta^{(1)} > \theta_N^*$. Then we denote $\theta^{(N)} = \min_{i\in\{1,...,N\}} \theta^i$ as the smallest value. If $\theta^{(N)} \leq \theta_1^*$, we find a PBE $(\theta_1^*, \boldsymbol{P}^*)$ along with creditors' expectation that only 1 run will occur to the bank with fundamental $\theta^{(N)}$. If instead $\theta^{(N)} > \theta_1^*$, the aforementioned strategy profile and creditors' expectation that there will be no run consist a PBE. As creditors will not run any bank following threshold strategy θ_1^*, the whole game ends immediately. Then they expect the secondary market asset price to be any one in the price schedule $\boldsymbol{P}^*$, i.e., P_1^*. The price in turn rationalizes θ_1^* as their threshold strategy. To summarize, we have shown that there always exists a PBE for any realized fundamentals.

Next, we show that there exist multiple PBEs when all N banks' fundamentals belong to (θ_1^*, θ_N^*). By the same token established above, it is easy to see that one PBE is $\left(\theta_1^*, \boldsymbol{P}^*\right)$ along with creditors' expectation of no run will ever occur. While, a second PBE is $\left(\theta_N^*, \boldsymbol{P}^*\right)$ along with creditors' expectation that runs occur to all N banks.

□

[61]We only consider the case where banks' fundamentals are distinct with each other. The cases where any two or more than two banks' fundamentals being equal are zero-possibility events. Thus, we drop off the discussion for all those cases.

Appendix B.7 Proof of Lemma 5

Proof. When $s = G$ or B with probability 1, then the secondary market equilibrium is characterized by

$$P_s^* = \frac{\underline{\theta}_s + \theta_s^*}{2}$$

Rationally anticipating the equilibrium price, the bank run equilibrium solves

$$\theta_s^* = \frac{D_2 - D_1}{1 - qD_1/P_s^*}$$

where $s = G$ (or B) in the above equations. Solving this system of equations, we obtain the equilibrium critical cash flow θ_s^* and the equilibrium asset price P_s^*, where $s = G$ or $s = B$.

We can directly follow the proof of Appendix B.3 to show $P_s^* \in [\underline{P}, qD_2)$ and $\theta_s^* \in [\theta^L, \theta^U(P_s^*)]$ for both $s = G$ and B. The corresponding PBE when $s = G$ or B with certainty is as being characterized in Lemma 5.

Moreover, we can also follow directly the proof of Appendix B.5 to show $\theta_G^* < \theta_1^* < ... < \theta_N^* < \theta_B^*$.

□

Appendix B.8 Proof of Proposition 4

Proof. We first characterize the existence and uniqueness of such a pair θ_A^* and P_A^*.

Inserting the expression (12) into the regulator's break-even condition (13), one can obtain the following equation of P_A^*.

$$2(P_A^*)^2 - \overline{\Psi}P_A^* + qD_1E(\underline{\theta}_s) = 0 \tag{B.34}$$

where $\overline{\Psi} = (D_2 - D_1) + 2qD_1 + E(\underline{\theta}_s)$, and $E(\underline{\theta}_s) = \alpha\underline{\theta}_G + (1-\alpha)\underline{\theta}_B$. Note that $\overline{\Psi}$ parallels to Ψ in the baseline case. So the solution resembles to Appendix B.3, we can verify

$$P_A^* = \frac{\overline{\Psi} + \sqrt{\overline{\Psi}^2 - 8qD_1E(\underline{\theta})}}{4}$$

is the unique solution of (B.34). And it belongs to $[\underline{P}, qD_2)$. Similarly, we can obtain $\theta_A^* = \theta(P_A^*)$ by inserting P_A^* back to (12) and verify that it belongs to $\left[\theta^L, \theta^U(P_A^*)\right]$.

We then show $P_A^* > P_N^*$, and derive two sufficient conditions to guarantee $P_A^* > P_1^*$.

Notice that $P_A^* > P_N^*$ is equivalent to $\theta_A^* < \theta_N^*$. It is convenient to compare the two ratios of posterior beliefs about states ω_A^B/ω_A^G and $\omega_N^B(\theta_N^*)/\omega_N^G(\theta_N^*)$. Note that

$$\frac{\omega_A^B}{\omega_A^G} = \frac{1-\alpha}{\alpha} < \frac{1-\alpha}{\alpha} \cdot \left(\frac{\overline{\theta} - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_B} \cdot \frac{\theta_N^* - \underline{\theta}_B}{\theta_N^* - \underline{\theta}_G} \right)^N = \frac{(\theta_N^* - \underline{\theta}_B)^N}{\kappa(\theta_N^* - \underline{\theta}_G)^N} = \frac{\omega_N^B(\theta_N^*)}{\omega_N^G(\theta_N^*)}.$$

As $\theta_N^* < \overline{\theta}$ for any $N \geq 1$, the term inside the above parenthesis is always larger than 1. Asset buyers must hold a more pessimistic belief about the state, i.e., $s = B$ more likely, than the prior after observing runs to all N banks. By the asset market equilibrium conditions, we have

$$\frac{\pi^G(\theta_A^*)}{\pi^B(\theta_A^*)} = -\frac{\omega_A^B}{\omega_A^G} > -\frac{\omega_N^B(\theta_N^*)}{\omega_N^G(\theta_N^*)} = \frac{\pi^G(\theta_N^*)}{\pi^B(\theta_N^*)}$$

Recall that we have showed $\pi^G(\theta)/\pi^B(\theta)$ is decreasing for $\theta \in [D_2, \overline{\theta}]$. By this monotonicity, $\theta_A^* < \theta_N^*$ always holds.

Next, $P_A^* > P_1^*$ is equivalent to $\theta_A^* < \theta_1^*$. Then it holds if the ratios of belief about the state satisfy

$$\frac{\omega_A^B}{\omega_A^G} = \frac{1-\alpha}{\alpha} < \frac{(\theta_1^* - \underline{\theta}_B)}{\kappa(\theta_1^* - \underline{\theta}_G)} = \frac{\omega_1^B(\theta_1^*)}{\omega_1^G(\theta_1^*)}, \tag{B.35}$$

Recall that $\kappa = \frac{\alpha}{1-\alpha}\left(\frac{\overline{\theta}-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_G}\right)^N$. The inequality (B.31) turns into

$$\frac{1-\alpha}{\alpha} < \frac{1-\alpha}{\alpha} \left(\frac{\overline{\theta} - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_B} \right)^N \left(\frac{\theta_1^* - \underline{\theta}_B}{\theta_1^* - \underline{\theta}_G} \right).$$

$\theta_A^* < \theta_1^*$ holds if

$$\left(\frac{\overline{\theta} - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G} \right)^N < \left(\frac{\theta_1^* - \underline{\theta}_B}{\theta_1^* - \underline{\theta}_G} \right). \tag{B.36}$$

First, we characterize a condition on N, i.e., the number of banks, to ensure (B.36) holds. Recall θ_G^* and θ_B^* are the equilibrium thresholds of the bank run game when $s = G$ and B with certainty. We have

$$1 < \frac{\overline{\theta} - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G} < \frac{\theta_B^* - \underline{\theta}_B}{\theta_B^* - \underline{\theta}_G} < \frac{\theta_1^* - \underline{\theta}_B}{\theta_1^* - \underline{\theta}_G} < \frac{\theta_G^* - \underline{\theta}_B}{\theta_G^* - \underline{\theta}_G}.$$

From this inequality, we can find an integer $\underline{N} \equiv \left\lfloor log_{\left(\frac{\overline{\theta}-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_G}\right)} \left(\frac{\theta_B^*-\underline{\theta}_B}{\theta_B^*-\underline{\theta}_G}\right) \right\rfloor \geq 1$[62] such that the inequality $\left(\frac{\overline{\theta}-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_G}\right)^N < \left(\frac{\theta_B^*-\underline{\theta}_B}{\theta_B^*-\underline{\theta}_G}\right) < \frac{\theta_1^*-\underline{\theta}_B}{\theta_1^*-\underline{\theta}_G}$ holds when $N \leq \underline{N}$. As a result, $\theta_A^* < \theta_1^*$ when $N \leq \underline{N}$.

[62] Note that $\lfloor x \rfloor$ takes the largest integer that is smaller than the value x.

Similarly, we can also find an integer $\overline{N} \equiv \left\lceil log_{\left(\frac{\bar{\theta}-\theta_B}{\bar{\theta}-\underline{\theta}_G}\right)}\left(\frac{\theta_G^*-\theta_B}{\theta_G^*-\underline{\theta}_G}\right)\right\rceil > \underline{N} > 1$[63] such that $\left(\frac{\bar{\theta}-\theta_B}{\bar{\theta}-\underline{\theta}_G}\right)^N > \left(\frac{\theta_G^*-\theta_B}{\theta_G^*-\underline{\theta}_G}\right) > \frac{\theta_1^*-\theta_B}{\theta_1^*-\underline{\theta}_G}$ when $N \geq \overline{N}$, $\theta_A^* > \theta_1^*$ when $N \geq \overline{N}$.

Second, we characterize a condition on α, i.e., the prior probability of $s = G$, to ensure (B.36) holds. To make this characterization non trivial, we focus on the cases where $\frac{\theta_B^*-\theta_B}{\theta_B^*-\underline{\theta}_G} < \left(\frac{\bar{\theta}-\theta_B}{\bar{\theta}-\underline{\theta}_G}\right)^N < \frac{\theta_G^*-\theta_B}{\theta_G^*-\underline{\theta}_G}$, i.e., $N \in \left(\underline{N}, \overline{N}\right)$.

Notice that θ_1^* is continuous in α when $\alpha \in [0, 1]$. It is also decreasing in α. Intuitively, when the prior that banks' fundamental has a better distribution increases, a creditor has less incentive to withdraw. This reduces the equilibrium threshold. Consequently, we have $\frac{\partial}{\partial\alpha}\left(\frac{\theta_1^*-\theta_B}{\theta_1^*-\underline{\theta}_G}\right) > 0$. Additionally, we have $\lim_{\alpha\to 0}\theta_1^* = \theta_B^*$ and $\lim_{\alpha\to 1}\theta_1^* = \theta_G^*$. So, there must exist a critical $\hat{\alpha}$, such that $\left(\frac{\bar{\theta}-\theta_B}{\bar{\theta}-\underline{\theta}_G}\right)^N > \frac{\theta_1^*-\theta_B}{\theta_1^*-\underline{\theta}_G}$ when $\alpha < \hat{\alpha}$, and $\left(\frac{\bar{\theta}-\theta_B}{\bar{\theta}-\underline{\theta}_G}\right)^N \leq \frac{\theta_1^*-\theta_B}{\theta_1^*-\underline{\theta}_G}$ when $\alpha \geq \hat{\alpha}$.

□

Appendix B.9 DoLR Policy in the two-bank case

Proof. To present the stability effect in a more intuitive way, we refer again to the two-bank example. Figure 4 provides a complete characterization of the equilibrium outcome under the DoLR policy. We start with the extreme case where $P_1^* < P_A^*$, for it is simple to analyze. Given $P_A^* > P_1^* > P_2^*$, rational creditors expect that banks to sell their asset for P_A^*, no matter one or two bank runs occur. Therefore, the only rationalizable strategy for creditors is 'to run a bank if its fundamental is below θ_A^*'. As a result, no run will happen if both banks' cash flows exceed θ_A^*, whereas two bank runs will occur if both banks' fundamentals fall below the threshold. And one bank run occurs if one bank's $\theta < \theta_A^*$ and the other one's $\theta > \theta_A^*$. The equilibrium outcomes are illustrated in Panel (a) of Figure 4.

We now turn to the more interesting case $P_A^* < P_1^*$ as demonstrated in Panel (b). In this case, note that P_A^* would only affect the equilibrium outcome when both banks' fundamental is below θ_2^*. This is because when at least one bank's fundamental is higher than θ_2^*, rational creditors would never expect two bank runs. As a result, that the lowest possible equilibrium price offered by private buyers would be P_1^*. Given $P_A^* < P_1^*$, the price offered by the DoLR would not be relevant. Therefore, for all regions where at least one bank's fundamental is greater than θ_2^*, the presence of DoLR intervention does not change the equilibrium outcome.

[63] Note that $\lceil x \rceil$ takes the smallest integer that is larger than the value x.

We now turn to regions where both banks' fundamental is below θ_2^*. With $P_2^* < P_A^*$, creditors understand the lowest price the would face is P_A^*, even if two bank runs occur. Therefore, the creditors would have two equilibrium strategies: an optimistic one that is 'to run if and only if the observed signal is below θ_1^*', and a pessimistic one 'to run if and only if the observed signal is below θ_A^*'.

(i) When both banks' cash flow is below θ_1^*, the creditors understand that both banks will fail even facing the highest possible price P_1^*. Therefore, the only possible equilibrium is to run both banks according to threshold θ_A^*. As a result, runs will happen to both banks, and the DoLR will purchase both banks' asset for price P_A^*.

(ii) When both banks' cash flow is between θ_1^* and θ_A^*, no bank run will happen if creditors take the optimistic strategy, but two runs will happen if creditors run according to the threshold θ_A^*.

(iii) When both banks' cash flow is between θ_A^* and θ_2^*, no bank run will happen no matter which strategy the creditors take.

(iv) When one bank's fundamental is below θ_1^* and the other bank's falls in $\left[\theta_1^*, \theta_A^*\right)$, both the optimistic and pessimistic equilibrium can sustain. In the former case, runs happen only to the bank with $\theta < \theta_1^*$; whereas in the latter case, both banks be forced to sell their asset to the DoLR.

(v) When one bank's fundamental is below θ_1^* and the other bank's falls in $\left[\theta_A^*, \theta_2^*\right)$, only one bank run will be observed, because the bank with $\theta > \theta_A^*$ will experience no run even if creditors take the pessimistic strategy. As a result, creditors expect private buyers to offer $P_1^* > P_A^*$ and run only the bank with $\theta < \theta_1^*$.

(vi) When one bank's fundamental falls in $\left[\theta_1^*, \theta_A^*\right)$ and the other bank's in $\left[\theta_A^*, \theta_2^*\right)$, no run will happen if creditors take the optimistic strategy; whereas runs will only happen to the bank whose $\theta \in \left[\theta_1^*, \theta_A^*\right)$, forcing the bank to sell its asset to DoLR.

The introduction of DoLR policy promotes financial stability on the intermediate range as indicated by the shaded area. It is worth noting that improvements in financial stability do not always involve DoLR actually purchasing banks' asset, as evident from case (iii) and (v). □

Appendix B.10 Proof of Proposition 6

Proof. We derive a condition to guarantee the expected social costs under DoLR policy is lower than that under regulatory disclosure policy.

To proceed, we assume regulatory disclosure takes the following form: when s realizes, the regulator observe it and perfectly communicates this information to the players. Accordingly, the equilibrium under regulatory disclosure is the same as the one characterized in Appendix B.7. Banks sell their assets to asset buyers for a price P_s^* and run occurs when a bank's fundamental is less than θ_s^* depends on the disclosure to be $s = G$ or B. Remember that we must have $\theta_G^* < \theta_A^* < \theta_B^*$.

For DoLR, we focus on the case where it takes over the market, i.e., $P_A^* > P_1^* > P_2^*$. Then banks sell their assets only to the regulators. We analyze this case because it can be showed that the expected social cost when DoLR and asset market coexist is strictly lower.

In our model, when a successful bank run occurs, the bank fails at $t = 2$ because its remaining asset value is insufficient to repay F. In case of failure, a bankruptcy cost C is incurred. We denote the expected social losses under DoLR as SC_{DoLR} and under regulatory disclosure as SC_{RD}. Those costs can be calculated as

$$SC_{DoLR} = N\left[\alpha\frac{\theta_A^* - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_A^* - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right]C, \quad SC_{RD} = N\left[\alpha\frac{\theta_G^* - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_B^* - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right]C$$

To see the two expressions are true, consider the case of DoLR. We can explicitly write the expected social losses under DoLR policy as

$$SC_{DoLR} = \sum_{s=G,B}\left\{\sum_{M=1}^{N} C(N, M)[Pr(\theta \le \theta_A^*|s)]^M [Pr(\theta > \theta_A^*|s)]^{N-M} M \cdot C\right\}$$

To understand, note that any bank with fundamental less than θ_A^* will encounter a run. When the number of runs is M, it must be the case that M out of N banks' fundamentals are less than θ_A^*. The term $C(N, M)[Pr(\theta \le \theta_A^*|s)]^M [Pr(\theta > \theta_A^*|s)]^{N-M}$ is the ex ante probability that M runs happen under DoLR policy. $C(N, M)$ calculates the M-run combinations of an N-bank system. The term $M \cdot C$ is the social costs due to the M bankruptcies. The term inside the parenthesis then sums up the expected social costs according to the number of runs, given an aggregate state. Use the mathematical definition of combination, we can obtain

$$\sum_{M=1}^{N} C(N,M)[Pr(\theta \leq \theta_A^*|s)]^M[Pr(\theta > \theta_A^*|s)]^{N-M} M \cdot C = NPr(\theta \leq \theta_A^*|s)C = N\frac{\theta_A^* - \underline{\theta}_s}{\overline{\theta} - \underline{\theta}_s}C$$

As a result, we obtain the expression of SC_{DoLR}. SC_{RD} can be derived in the same token.

Lastly, we derive a sufficent condition under which $SC_{DoLR} < SC_{RD}$. Note that it is equivalent to the inequality

$$N\left[\alpha\frac{\theta_A^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_A^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right]C < N\left[\alpha\frac{\theta_G^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_B^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right]C$$

This inequality can be further rearranged into

$$\frac{\underline{\theta}_G - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G} < \frac{\alpha\theta_G^* + (1-\alpha)\theta_B^* - \theta_A^*}{\alpha(\theta_A^* - \theta_G^*)} \tag{B.37}$$

As a result, from (B.37), we can find a $\overline{\theta}^c$ such that when

$$\overline{\theta} > \underline{\theta}_G + \frac{\alpha(\theta_A^* - \theta_G^*)}{\alpha\theta_G^* + (1-\alpha)\theta_B^* - \theta_A^*}(\underline{\theta}_G - \underline{\theta}_B) = \overline{\theta}^c. \tag{B.38}$$

the expected social costs under DoLR is lower than that under regulatory disclosure. □