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Abstract

We study the short-run macroeconomic effects of trade policies that are equivalent in a frictionless economy, namely a uniform increase in import tariffs and export subsidies (IX), an increase in value-added taxes accompanied by a payroll tax reduction (VP), and a border adjustment of corporate profit taxes (BAT). Using a dynamic New Keynesian open-economy framework, we summarize conditions for exact neutrality and equivalence of these policies. Neutrality requires the real exchange rate to appreciate enough to fully offset the effects of the policies on net exports. We argue that a combination of higher import tariffs and export subsidies is likely to trigger only a partial exchange rate offset and thus boosts net exports and output (with the output stimulus largely due to the subsidies). Under full pass-through of taxes, IX and BAT are equivalent but VP is not. We show that a temporary VP can increase intertemporal prices enough to depress aggregate demand and output, even when wages are sticky. These contractionary effects are especially pronounced under fixed exchange rates.

JEL classification: E32, F30, H22

Keywords: Trade Policy, Fiscal Policy, Exchange Rates, Fiscal Devaluation

1 Introduction

There is a longstanding debate about how trade policies can stimulate the macroeconomy. In considering different ways of alleviating a deep economic recession within the confines of the gold standard, Keynes (1931) argued that the U.K. could derive a similar degree of stimulus from raising import tariffs and providing export subsidies as through devaluing the pound against gold.[1] More recently,

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[1] Eichengreen (1981) provides a detailed account of the contentious political debate that preceded the United Kingdom's shift towards protectionist trade policies in the early 1930s.

there has been renewed interest in the question of how countries constrained by membership in a currency union can implement tax policies with economic effects akin to a currency depreciation (e.g. Calmfors 1998 and Farhi, Gopinath, and Itskhoki 2014). The approach of cutting payroll taxes to lower domestic relative to foreign prices, and thus boost external competitiveness, has strong intuitive appeal. In this vein, a number of countries have reduced payroll taxes and financed these cuts with VAT increases.

However, even if these policies can provide stimulus under fixed exchange rates, it is unclear to what extent they would do so under flexible exchange rates. Mundell (1961) questioned whether the mercantilist prescription of higher import tariffs and export subsidies would stimulate demand in economies with floating exchange rates, as "equilibrium in the balance of payments is automatically maintained by variations in the price of foreign exchange". Similarly, Feldstein (2017) and Auerbach, Devereux, Keen, and Vella (2017) have argued that a border adjustment of corporate taxation − that in effect taxes imports and subsidies exports − would not affect the trade balance provided that the nominal exchange rate was free to adjust.

In this paper, we examine the short-run macroeconomic effects of three alternative policies that are equivalent in a frictionless economy, namely a uniform increase in import tariffs and export subsidies, a reduction in employer payroll taxes financed by an increase in VAT rates, and a border adjustment of corporate profit taxation. To do so, we use a New Keynesian open-economy framework that builds on contributions by Galì and Monacelli (2005) and Corsetti, Dedola, and Leduc (2010). We analyze the extent to which these three policies elicit equivalent macroeconomic effects − under both flexible and fixed exchange rates − as well as their efficacy in providing cyclical stimulus.

The first key finding of our analysis is that the combination of import tariffs and export subsidies (IX henceforth) induces expenditure-switching towards domestic goods that tend to boost domestic output and inflation *even under flexible exchange rates.* While IX policies clearly stimulate demand under fixed exchange rates – as hypothesized by Keynes and corroborated by Farhi, Gopinath, and Itskhoki (2014) – our finding that these policies are stimulative under flexible exchange rates contrasts sharply with the conventional view, in which the exchange rate appreciates enough to fully offset any allocative effects of import tariffs and export subsidies on the domestic economy.[2]

We lay out the conditions under which the conventional view holds and IX policies are "neutral,"

[2] See, for instance, the orginal contribution by Lerner (1936) and, more recently, Costinot and Werning (2017).

i.e., have no allocative effects, and argue that these conditions appear extremely restrictive and hence unlikely to hold in practice. Crucially, the neutrality of IX policies hinges on the expectation that the real exchange rate will appreciate permanently, reflecting the public's belief that trade actions will remain in place forever and not induce foreign retaliation (even in the long run). However, historical experience suggests that trade policy actions are often reversed or spur retaliation. These reversals may occur because the trade policies are implemented as cyclical measures to boost the economy or as a negotiating tool in foreign policy;[3] alternatively, they may result from an electoral shift towards a political party more supportive of free trade.[4] Moreover, although some trade policy legislation has been enacted with the expectation that it will remain in effect for a long time − as in the Great Depression − the tariff wars that ensued during the 1930s (especially in response to Smoot-Hawley) serve to underscore the high likelihood of foreign retaliation under such circumstances.

Given this motivation, we use a Markov-switching framework to consider two mechanisms that cause the exchange rate to revert to its initial level in the long-run: first, an eventual abandonment of the policy; and second, retaliation by foreign countries.[5] In both cases, we find that the policy boosts output so long as the unilateral actions remain in effect (that is, before the foreign retaliation occurs). Intuitively, when the exchange rate is expected to eventually revert to its pre-shock level, the immediate appreciation of the currency falls short of completely offsetting the expenditure-switching effects of the policy on imports and exports. While the expectation that the policy will be reversed raises the relative price of current consumption − since tariffs are expected to decline − the resulting fall in consumption due to this intertemporal substitution channel is swamped by the boost to net exports, so that output expands. As a matter of fact, a key insight of our paper is that the output stimulus of unexpected IX policies is largely driven by the export subsidy whereas tariffs, depending

[3] In this vein, Irwin (2013) discusses how President Nixon favored the imposition of a 10 percent across-the-board tariff in 1971 partly to enhance his electoral prospects in the 1972 election, as well as to put pressure on foreign trading partners to revalue their exchange rates. As it turned out, the tariffs were lifted fairly quickly when the foreign policy objectives were viewed as largely achieved, as well as from pressure coming even from some members of the Administration.

[4] For example, in the U.S. experience, President Wilson, a free-trade Democrat, strongly supported the passage of the Underwood Tariff Act of 1913 which scaled back the high tariffs that had prevailed under previous Republican Administrations (see Irwin 2017).

[5] In standard DSGE models, expectations about how trade policy will be set in the distant future – by affecting the exchange rate that must prevail in the long-run to satisfy intertemporal trade balance – can exert powerful effects on the exchange rate today. However, such implications rest on the doubtful premise that agents have a high degree of confidence about the stance of policy far in the future.

on parameter values, have a negligible or even contractionary effect on output.[6,7]

We then turn our attention to the analysis of a reduction in payroll taxes financed by an increase in VAT (VP policy), an alternative tax policy that is often considered either equivalent or a close substitute to IX. Some European governments have attempted to provide macroeconomic stimulus by implementing such "fiscal devaluations", including the governments of Denmark in 1988, Sweden in 1993, Germany in 2007, and Portugal in the context of the 2011-2014 EU-IMF Economic Stabilization Program.[8]

Our second key finding is that, in general, the effects of IX policies diverge markedly from VP, even qualitatively. To illustrate the different general equilibrium response to these policies it is helpful to consider the same conditions under which IX is neutral − namely, when policies are implemented permanently and exchange rates are flexible. In this case, neutrality of IX occurs through an immediate jump in the exchange rate which ensures that the price of imported goods remains unchanged relative to domestically-produced goods; no change in factor prices, including the wage, is required. While VP also turns out to be neutral – at least if wages are flexible – a striking difference is that no adjustment in the exchange rate is required to keep the relative price of traded goods constant, reflecting that the VAT applies to both imported and domestic goods. Moreover, the wage must jump under VP, both to offset the competitiveness-enhancing effect of the subsidy on firm marginal cost, and to induce households to keep their labor supply unchanged. A direct consequence of these different relative price responses is that any departure from the specific conditions that deliver allocative equivalence (and neutrality) will result in markedly different macroeconomic effects. For instance, in the special case in which wages are flexible but exchange rates are fixed, IX has expansionary effects while VP remains neutral.

An important open question remains whether a temporary implementation of VP can provide stimulus in the event of cyclical downturns. We find that a VP policy can easily have contractionary effects on aggregate demand and inflation, even when wages are sticky. While the payroll subsidy to employers increases competitivenes by reducing marginal costs, the temporary increase in VAT rates

[6] Our emphasis on intertemporal substitution channels is in the spirit of earlier work by Svensson and Razin (1983), who use a two-period model to illustrate how a temporary tariff affects the current account through intertemporal-substitution effects on consumption.

[7] Barattieri, Cacciatore, and Ghironi (2017) incorporate additional supply-side channels, including endogenous entry and exit of firms, that amplify the negative effects of tariffs. These authors, however, focus exclusively on the effects of tariffs rather than the combination of import tariffs and export subsidies (IX policies), as we do here.

[8] A number of quantitative and empirical papers have tried to gauge the effects of fiscal devaluations on trade, including Lipińska and Von Thadden (2012), de Mooij and Keen (2012), Franco (2013), and Gomes et al. (2016).

raises the price of current consumption relative to future consumption, thus depressing aggregate demand. The latter intertemporal substitution effect exerts a strong contractionary impetus unless monetary policy cuts interest rates sufficiently. Hence, VP tends to be sharply contractionary under fixed exchange rates and may well cause output to fall even under flexible exchange rates.

These results may seem surprising in light of the existing literature, including the seminal work by Farhi et al. (2014), which shows that, under fixed exchange rates, VP provides equivalent stimulus to output and inflation as IX or an exchange rate devaluation. A critical assumption responsible for the contractionary effects of VP is that, in our framework, pre-tax prices are sticky, so that VAT increases are immediately passed through to consumer prices. Given the centrality of this assumption about VAT pass-through for our theoretical results, we discuss some empirical evidence in support of our specification that shows that consumer prices tend to increase quickly in response to VAT increases. A particularly applicable case was the implementation of the German fiscal devaluation in 2007 − a rare case of a VAT increase accompanied by an equally-sized payroll subsidy − in which the pass-through of VAT increases was large and immediate.

While our analysis focuses heavily on IX and VP policies, we also study the effects of a border adjustment of corporate taxes (BAT). Several authors, including Feldstein (2017) and Auerbach et al. (2017), have recently argued that a border adjustment of corporate taxation, that amounts to taxing imports and subsidizing exports, would not affect the trade balance provided that the nominal exchange rate was free to adjust. A key theoretical insight of our analysis is that, with nominal rigidities and full pass-through of taxes, the BAT is equivalent to IX. Consequently, the BAT would provide macroeconomic stimulus exactly like IX policies and have no allocative effects only under fairly extreme assumptions.

A few authors, including Barbiero, Gopinath, Farhi, and Itshoki (2018) and Linde and Pescatori (2018), have recently provided quantitative assessments of a possible adoption of a BAT by the United States. These papers mainly consider a unilateral permanent implementation of the BAT which implies a large jump in the exchange rate. They focus on incomplete pass-through of exchange rate changes to import prices as a key source of non-neutrality, and show how, under these conditions, the BAT can have large effects on both trade prices and volumes. While our analysis has clear complementarities with this research, we focus on features – such as the possibility of retaliation or reversal as captured by our Markov-switching framework – that greatly diminish the scope for trade policies to exert large effects on the long-run level of the exchange rate. As emphasized, these features cause trade policies to have allocative effects by damping the near-term movement in the exchange rate. Although we retain a standard infinite-horizon general equilibrium framework, our use of a Markov-switching framework to limit the role of long-run expectations in determining the economic effects of trade policies is in the same spirit as a recent literature that attempts to damp the role of beliefs about future policies on current outcomes.[9]

The paper is organized as follows. Section 2 describes the model. Section 3 develops some intuition about the effects of IX, VP, and BAT policies by discussing their partial equilibrium effects. Section 4 discusses the macroeconomic effects of IX policies, including conditions for neutrality. Section 5 investigates the relation between IX, VP, and BAT policies. Section 6 concludes.

2 Model

The economy consists of a home (H) country and a foreign (F) country that are isomorphic in structure. Foreign variables are denoted with an asterisk. Agents in each economy include households, retailers, producers of intermediate goods, and the government. For ease of exposition, the next sections describe the optimization problems solved by each type of agent under the assumptions of producer currency pricing (PCP), fully flexible wages, and a simple financial market structure in which only a foreign currency bond is traded internationally. Appendix A presents a more general model that allows for alternative assumptions about price and wage setting and financial market structure, as well as for differences in country size; all of the theoretical results are derived within the context of this general framework.

2.1 Households

Households in the home country derive utility from a final good consumption (C_t) and disutility from labor (N_t). They maximize expected lifetime utility

$$\mathbb{E}_0 \Sigma_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{1}$$

[9] See, for instance, research on the "forward guidance puzzle" by McKay, Nakamura, and Steinsson (2014), Fahri and Werning (2018), and Angeletos and Lian (2017), as well as related analysis of the effects of finite planning horizons by Woodford (2018).

subject to the budget constraint

$$P_tC_t + B_{Ht} + \varepsilon_t\left[B_{Ft} + \frac{\chi}{2}\left(B_{Ft} - \bar{B}_F\right)^2\right] = R_{t-1}B_{Ht-1} + \varepsilon_t R^*_{t-1}B_{Ft-1} + W_tN_t + \widetilde{\Pi}_t + T_t \tag{2}$$

where P_t is the consumer price index, B_{Ht} are noncontingent nominal bond holdings denominated in domestic currency, B_{Ft} are noncontingent nominal bond holdings denominated in foreign currency, R^*_{t-1} is the foreign nominal interest rate, ε_t is the nominal exchange rate (defined as the price of one unit of foreign currency in terms of units of home currency), W_t is the wage rate, $\widetilde{\Pi}_t$ is the aggregate profit of the home firms assumed to be owned by the home consumers, T_t is a lump-sum transfer from the government. The parameter $\chi \geq 0$ allows for the possibility that home households face quadratic costs of adjusting their holdings of foreing bonds.[10] In our baseline calibration we focus on the case, often considered in the literature, in which foreign households cannot invest in the domestic bond so that only the foreign bond is traded internationally.[11]

We assume that the period utility function takes the form

$$U(C,N) = \frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{\eta+1}N_t^{1+\eta} \tag{3}$$

Optimality requires

$$N_t^{\eta}C_t^{\sigma} = \frac{W_t}{P_t} \tag{4}$$

$$1 = \beta\mathbb{E}_t\left[\Lambda_{t,t+1}\frac{P_t}{P_{t+1}}R_t\right] \tag{5}$$

$$1 = \beta\mathbb{E}_t\left[\Lambda_{t,t+1}\frac{P_t}{P_{t+1}}\frac{\varepsilon_{t+1}}{\varepsilon_t}R^*_t\right] \tag{6}$$

where $\Lambda_{t,t+1} = \left(\frac{C_t}{C_{t+1}}\right)^{\sigma}$ is the real stochastic discount factor of the home household. The corresponding optimality condition for foreign household holdings of the foreign bond is

$$1 = \beta\mathbb{E}_t\left[\Lambda^*_{t,t+1}\frac{P^*_t}{P^*_{t+1}}R^*_t\right] \tag{7}$$

Combining the optimality conditions for bond holdings (6) and (7), one obtains the risk-sharing

[10] All of our theoretical results go through irrespective of the value of χ provided that $\chi \geqq 0$. For simplicity, the first order conditions we report in the text assume $\chi = 0$. In our simulations we introduce very small costs of adjustment to ensure stability of a first order approximation. See Neumeyer and Perri (2001) and Schmitt-Grohe and Uribe (2003).

[11] That is, the budget constraint for foreign households is given by
$P^*_tC^*_t + B^*_{Ft} + \frac{1}{\varepsilon_t}\left[B^*_{Ht} + \frac{\chi^*}{2}\left(B^*_{Ht} - \bar{B}_H\right)^2\right] = R^*_{t-1}B^*_{Ft-1} + \frac{1}{\varepsilon_t}R_{t-1}B^*_{Ht-1} + W^*_tN^*_t + \widetilde{\Pi}^*_t + T^*_t$
In our baseline analysis we set $\chi^* = \infty$ so that only foreign currency bonds are traded internationally. We consider relaxing this assumption in Section 4.4.

condition

$$\mathbb{E}_t \left\{ \left[\Lambda_{t,t+1} \frac{Q_{t+1}}{Q_t} - \Lambda^*_{t+1} \right] \frac{P^*_t}{P^*_{t+1}} \right\} = 0 \tag{8}$$

where Q_t is the real exchange rate expressed as the price of the foreign consumption bundle in home currency relative to the price of the domestic consumption bundle, that is

$$Q_t = \varepsilon_t \frac{P^*_t}{P_t} \tag{9}$$

2.2 Retailers

Competitive home retailers combine home and foreign intermediate goods to produce the final consumption good according to the constant-elasticity-of-substitution (CES) aggregator

$$C_t = \left[\omega_H^{\frac{1}{\theta}} Y_{Ht}^{\frac{\theta-1}{\theta}} + (1-\omega_H)^{\frac{1}{\theta}} Y_{Ft}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{10}$$

where $\theta \geq 0$ determines the elasticity of substitution between home and foreign intermediate goods and $\omega_H \in [0.5, 1]$ governs home bias. The home good (Y_{Ht}) and the foreign good (Y_{Ft}) consist of CES aggregators over home and foreign varieties

$$Y_{Ht} = \left[\int_0^1 Y_{Ht}(i)^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}} \tag{11}$$

$$Y_{Ft} = \left[\int_0^1 Y_{Ft}(i)^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}} \tag{12}$$

where $\gamma \geq 0$ determines the elasticity of substitution across varieties.

Profit for the home retailers are

$$\Pi^R_t = (1-\tau^v_t)(1-\tau^\pi_t) \left[P_t C_t - P_{Ht} Y_{Ht} - \frac{P_{Ft}}{(1-\tau^\pi_t BAT_t)} Y_{Ft} \right] \tag{13}$$

where P_{Ht} and P_{Ft} are the price indexes of the home and foreign goods, τ^v_t is the value-added tax rate, τ^π_t is the tax rate on profits, and $BAT_t \in \{0, 1\}$ indicates whether profit taxes are adjusted at the border or not. The border adjustment implies that the cost of imported goods (Y_{Ft}) cannot be deducted from profits. Prices are inclusive of value-added taxes and, in the case of imported goods, are also inclusive of home tariffs (τ^m_t).

Given the CES structure of these aggregators, the home and foreign good demand functions are characterized by

$$Y_{Ht} = \omega \left[\frac{P_{Ht}}{P_t} \right]^{-\theta} C_t \tag{14}$$

$$Y_{Ft} = (1-\omega) \left[\frac{P_{Ft}}{(1-\tau_t^{\pi} BAT_t)\, P_t} \right]^{-\theta} C_t \tag{15}$$

$$Y_{Ht}(i) = \left[\frac{P_{Ht}(i)}{P_{Ht}} \right]^{-\gamma} Y_{Ht} \tag{16}$$

$$Y_{Ft}(i) = \left[\frac{P_{Ft}(i)}{P_{Ht}} \right]^{-\gamma} Y_{Ht} \tag{17}$$

The zero profit conditions for home retailers imply that price indexes satisfy:

$$P_t = \left[\omega P_{Ht}^{1-\theta} + (1-\omega) \left(\frac{P_{Ft}}{1-\tau_t^{\pi} BAT_t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{18}$$

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\gamma}\, di \right]^{\frac{1}{1-\gamma}} \tag{19}$$

$$P_{Ft} = \left[\int_0^1 P_{Ft}(i)^{1-\gamma}\, di \right]^{\frac{1}{1-\gamma}} \tag{20}$$

2.3 Producers

Each country features a continuum $i \in [0,1]$ of monopolistically-competitive firms that produce different varieties of intermediate goods. Producers use the technology

$$Y_{Ht}(i) + Y_{Ht}^{*}(i) = A_t N_t^{\alpha}(i) \tag{21}$$

where $Y_{Ht}(i)$ and $Y_{Ht}^{*}(i)$ are firm i's sales in the domestic and foreign market, respectively, A_t is the aggregate level of technology, and $\alpha \in (0,1)$ controls the curvature of the production function.

In our benchmark specification we assume producer currency pricing (PCP), that is, producers set prices in the domestic currency while letting prices in the foreign market adjust to ensure that unit revenues are equalized across markets. We can then write firm $i's$ profits as

$$\Pi_t^{P}(i) = (1-\tau_t^{\pi}) \left\{ P_{Pt}(i) \left[Y_{Ht}(i) + Y_{Ht}^{*}(i) \right] - (1-\varsigma_t^{v})\, W_t N_t(i) \right\} \tag{22}$$

where $P_{Pt}(i)$ denotes the unit revenue from domestic sales of the home variety.

The presence of value-added taxes introduces a wedge between unit revenues $P_{Pt}(i)$ and the price paid by domestic retailers for $P_{Ht}(i)$:

$$P_{Pt}(i) = (1-\tau_t^v)\, P_{Ht}(i) \tag{23}$$

Similarly, import tariffs, export subsidies, and the deductability of export sales from the corporate profit tax when the border adjustment is in place (i.e. $BAT = 1$) create a wedge between the foreign currency price paid by foreign retailers, $P_{Ht}^*(i)$, and firm $i's$ foreign currency unit revenue from exports, $\frac{P_{Pt}(i)}{\varepsilon_t}$:

$$P_{Ht}^*(i) = \frac{(1-\tau_t^\pi BAT_t)(1+\tau_t^{m*})}{(1+\varsigma_t^x)} \frac{P_{Pt}(i)}{\varepsilon_t} \tag{24}$$

Producers set prices in staggered contracts following a Calvo-style timing assumption and with full pass-through of value added taxes. That is, a domestic firm that adjusts its price at time t sets the unit revenues $P_{Pt}(i)$ and, absent any price adjustment until time $s > t$, changes in value-added taxes are fully reflected in retailer's costs of purchasing the home variety

$$P_{Hs}(i) = \frac{P_{Pt}(i)}{(1-\tau_s^v)}. \tag{25}$$

Each firm that reoptimizes at time t will then choose $\bar{P}_{Pt,}$ to solve

$$\max E_t \sum_{s\geq t} \zeta_P^{s-t} \Lambda_{t,s} (1-\tau_s^\pi) \left\{ \frac{\bar{P}_{Pt}(i)\,[Y_{Hs}(i) + Y_{Hs}^*(i)] - (1-\varsigma_s^v)\, W_s N_s(i)}{P_s} \right\} \tag{26}$$

where ζ_P is the probability that the firm won't be able to adjust its price in any given period, labor demand satisfies (21), and domestic and foreign sales are determined by retailers' demand schedules in both the home and foreign market (i.e., equation (16) and its foreign analogue, respectively). The reset price $\overline{P}_{Pt}(i)$ satisfies the usual optimality condition:

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \left[Y_{Hs}(i) + Y_{Hs}^*(i)\right] (1-\tau_s^\pi) \frac{1}{P_s} \left[\overline{P}_{Pt}(i) - \frac{\gamma}{\gamma-1} \frac{(1-\varsigma_s^v)\, W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0 \tag{27}$$

Equation (27) indicates that the contract price $\overline{P}_{Pt}(i)$ is set as a fixed markup over the appropriately discounted measure of firm marginal costs that takes into account the expected duration that the contract price will remain in effect. We let the producer price index P_{Pt} be defined in a way that mimics the consumer price index in (19) :

$$P_{Pt} = \left[\int P_{Pt}(i)^{1-\gamma}\, di \right]^{\frac{1}{1-\gamma}}, \tag{28}$$

our Calvo-style pricing assumption then implies that producer price inflation is given by

$$\pi_{Pt} = \left[\zeta_P + (1-\zeta_P) \left(\frac{\bar{P}_{P,t}}{P_{P,t-1}} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{29}$$

Expression (29) indicates that producer price inflation depends on future marginal costs through the optimal reset price $\bar{P}_{P,t}$, which is identical across all firms that reset at time t. Combining equations (27) and (29) one obtains the familiar New Keynesian Phillips Curve linking domestic price inflation to current and future marginal costs.

Similarly, foreign firm j sells its good in the foreign country at a price of $P^*_{Ft}(j)$ and in the home country according to the PCP condition[12]

$$P_{Ft}(j) = \frac{(1+\tau^m_t)}{(1+\varsigma^{x*}_t)(1-\tau^v_t)} \varepsilon_t P^*_{Pt}(j) \tag{30}$$

Foreign firms that are allowed to reset their price choose their contract price $\overline{P}^*_{Ft}(j)$ so that

$$\mathbb{E}_t \Sigma^{\infty}_{s=t} \zeta^{*s-t}_P \Lambda^*_{s,t} \left[Y^*_{Fs}(j) + Y_{Fs}(j) \right] \left[\overline{P}^*_{Pt}(j) - \frac{\gamma}{\gamma-1} \frac{W^*_s}{\alpha A^*_s Z^*_s(i)^* N^*_s(j)^{\alpha-1}} \right] = 0 \tag{31}$$

2.4 Government Policy

Fiscal policy in the home and in the foreign country is characterized by a vector of fiscal instruments

$$s_t = (\tau^m_t, \varsigma^x_t, \tau^v_t, \varsigma^v_t, \tau^\pi_t, BAT_t, \tau^{m*}_t, \varsigma^{x*}_t) \tag{32}$$

that are assumed to follow a Markov chain as described below. The home government balances its budget in every period through levying lump-sum taxes T_t

$$\left[\frac{\tau^m_t + \tau^v_t}{1+\tau^m_t} \right] P_{Ft} Y_{Ft} + \tau^v_t P_{Ht} Y_{Ht} - \varsigma^x_t \frac{\varepsilon_t P^*_{Ht}}{1+\tau^{m*}_t} Y^*_{Ht} + \frac{\tau^\pi_t}{1-\tau^\pi_t} \widetilde{\Pi}^\pi_t - \varsigma^v_t W_t N_t + T^I_t = T_t \tag{33}$$

where T^I_t are net international transfers.

Monetary policy follows a Taylor-style interest rate rule:

$$R_t = \frac{1}{\beta} (\pi_{Pt})^{\varphi_\pi} (\tilde{y}_t)^{\varphi_y} (\widetilde{\varepsilon}_t)^{\varphi_\varepsilon} \tag{34}$$

where φ_π is the weight on producer price inflation (π_{Pt}), φ_y the weight on the output gap $(\tilde{y}_t)$, and φ_ε determines how policy rates respond to deviations of the nominal exchange rate from an exchange

[12] As we specify later, we assume that the foreign governments does not make use of VP and BAT policies. Hence, unit revenues to producers equal retailers cost in the foreign country (i.e. $P^*_{F,t} = P^*_{P,t}$).

rate target $\left(\text{i.e. } \widetilde{\varepsilon}_t = \frac{\varepsilon_t}{\bar{\varepsilon}}\right)$.[13] When $\varphi_\varepsilon = 0$, the home interest rate responds exclusively to fluctuations in output gaps and domestic inflation. This specification implies that the central bank looks through changes in inflation due to the direct effects of tariffs and value-added taxes. When $\varphi_\varepsilon = M$, with M large, the interest rate is set so that the country pegs its exchange rate to a predetermined target ($\bar{\varepsilon}$).

2.5 Market Clearing and Equilibrium

Labor market clearing equates household supply of labor with aggregate firms' demand

$$N_t = \int N_t(i)\, di. \tag{35}$$

Bond market clearing requires

$$B_{Ft} + B_{Ft}^* = 0 \tag{36}$$

$$B_{Ht} + B_{Ht}^* = 0 \tag{37}$$

Combining home and foreign households budget constraints and using the bond market clearing conditions we get a balance of payment equilibrium equation

$$\varepsilon_t B_{Ft} = \varepsilon_t B_{Ft-1} R_{t-1}^* + NX_t \tag{38}$$

which requires that home households increase their holdings of foreign bonds to meet the total amount of new borrowing demand from abroad, given by home net exports:

$$NX_t = \frac{\varepsilon_t P_{Ht}^*}{(1+\tau_t^{m*})} \left(Y_{Ht}^* - S_t Y_{Ft}\right) \tag{39}$$

where S_t denotes the terms of trade

$$S_t = \frac{(1+\tau_t^{m*})}{(1+\tau_t^{m})} \frac{(1-\tau_t^{v})\, P_{Ft}}{\varepsilon_t P_{Ht}^*} \tag{40}$$

which is the key relative price determining the behavior of the trade balance.

Let the initial condition for home holdings of bonds and individual producer prices in the home and foreign market be:

$$x_0 = \left[B_{F-1} R_{-1}^*, P_{H-1}(i), P_{F-1}^*(i)\right]$$

Definition. *Given an initial state x_0, a stochastic process for fiscal policy $\{s_t\}$ and international transfers $\left\{T_t^I\right\}$, an equilibrium consists of (i) an allocation at home, $\Xi = \{C_t, B_F, N_t, Y_{Ht}, Y_{Ft}, Y_{Ht}(i),$*

[13] See Benigno *et al.* (2007) for a discussion of interest rate rules that maintain a fixed exchange rate.

$Y_{Ft}(i)\}_{t\geq 0}$, *and abroad* Ξ^*; (ii) *firm-level prices and production decisions at home,* $\Phi = \{\bar{P}_{Pt}(i), N_t(i), P_{Ht}(i), P^*_{Ht}(i)\}_{t\geq 0}$ *,and abroad* Φ^**;* (iii) *aggregate prices at home* $\Gamma = \{P_t, P_{Ht}, P_{Ft}, P_{Pt}, \pi^P_t, W_t, R_t\}_{t\geq 0}$ *and abroad* Γ^*; (iv) $($*domestic*$)$ *bond holdings, net exports, currency exchange rates and terms of trade* $\{B_{Ht}, B^*_{Ft}, NX_t, \varepsilon_t, S_t\}$ *such that:*

1. *The allocation* Ξ *satisfies households and retail firms optimality conditions* $(4)-(6)$ *and* $(14)-(17)$ *as well as the analogous conditions in the foreign country;*

2. *Individual producer prices and production decisions* Φ *maximize firm profits, i.e. they satisfy conditions* (21),(23), (24) and (27) *as well as the analogous conditions in the foreign country;*

3. *Prices* Γ *clear all markets. That is, price indexes,* $\{P_t, P_{Ht}, P_{Ft}, P_{Pt}, \pi^P_t\}_{t\geq 0}$*, satisfy* $(18)-(20)$, $(28)-(29)$*; wages clear the labor market, i.e.* (35) *is satisfied; and nominal interest rates are determined according to* (34)*. Analogous conditions pin down* Γ^*.

4. *The bond market clears, i.e. equations* $(36)-(40)$ are satisfied.

2.6 Calibration

In our discussion of the transmission of trade policies, we calibrate the model using fairly standard values in the literature.[14] Table 1 shows our baseline parameter values.

Table 1. Calibration

		Parameter	Value
Households	Discount factor	β	0.99
	Risk aversion	σ	1.00
	Frisch elasticity of labor supply	η^{-1}	1.00
Producers	Labor share	α	0.36
	Price stickiness	ζ_P	0.90
	Trade elasticity	θ	1.25
	Import share	ω_H	0.15
Monetary Policy	Output gap weight in the rule	φ_y	0.125
	Inflation weight in the rule	φ_π	1.50

[14] See, for instance, Galì (2008).

3 Partial equilibrium effects of trade and fiscal policies

The trade and fiscal tax instruments considered in this paper directly affect three key margins determining the equilibrium allocation in our model economy, namely relative demand for home and foreign varieties, for consumption and leisure, and for consumption at different dates. In this section, we focus on these margins to show how the effects of IX and BAT differ markedly from VP in partial equilibrium - i.e. holding fixed nominal interest rates, exchange rates, wages, and producer prices. This analysis will prove helpful in understanding the different general equilibrium effects of these policies under various assumptions about nominal rigidities and monetary policy.

Starting with the relative demand for home and foreign varieties, the PCP conditions (30) and (25) allow us to express the relative price of imports as

$$\frac{P_{Ft}}{P_{Ht}} = \frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})}\frac{\varepsilon_t P_{Pt}^*}{P_{Pt}} = \frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})}Q_{Pt} \tag{41}$$

where

$$Q_{Pt} = \frac{\varepsilon_t P_{Pt}^*}{P_{Pt}} \tag{42}$$

is the home relative to the foreign level of producer prices (henceforth, the *producer real exchange rate*). Note that this relative price is not directly affected by changes in taxes.

Equations (14), (15), and (41) imply that the relative demand for imported goods in the home country can be rewritten as

$$\frac{Y_{Ft}}{Y_{Ht}} = \left[\frac{P_{Ft}}{P_{Ht}\left(1-\tau_t^\pi BAT_t\right)}\right]^{-\theta} = \left[\frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})\left(1-\tau_t^\pi BAT_t\right)}Q_{Pt}\right]^{-\theta} \tag{43}$$

This equation shows that the imposition of a tariff raises the relative price of imports for any given level of the producer real exchange rate, and hence shifts demand away from imports towards domestically-produced goods. Similarly, the non-deductibility of imports from corporate profits under the BAT operates like a tariff by raising the effective price of imports. The foreign subsidy to exports (ς_t^{x*}) has a countervailing effect, by allowing foreign exporters to decrease their price and hence stimulate the home demand for imports. In contrast, changes in the VAT have no effect on relative import prices provided that the producer real exchange rate is unchanged, reflecting that the VAT has equal-sized effects on both the price of imported as well as domestically-produced goods (see e.g. Feldstein and Krugman (1982) for a similar argument).

Foreign relative demand can be expressed in a symmetric way as

$$\frac{Y_{Ht}^*}{Y_{Ft}^*} = \left[\frac{(1+\tau_t^{m*})}{(1+\varsigma_t^x)}\frac{1}{Q_{Pt}}\right]^{-\theta} \tag{44}$$

which shows that the imposition of an export subsidy in the home country (ς_t^x) boosts foreign demand for home exports, while higher tariffs abroad reduce them. Once again, the VAT has no direct effect on this margin: foreign demand for home exports is unaffected by changes in VAT rates (holding the producer real exchange rate constant).

Turning to the consumption-leisure margin, we can rewrite the labor supply schedule (4) in terms of a *product real wage*, defined as the nominal wage in terms of producer prices $\left(\frac{W_t}{P_{pt}}\right)$, as follows:

$$\frac{W_t}{P_{pt}} = \frac{P_t}{P_{pt}} C_t^\sigma N_t^\eta \tag{45}$$

where

$$\frac{P_t}{P_{pt}} = \frac{1}{1-\tau_t^v}\left\{\omega + (1-\omega)\left[\frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})(1-\tau_t^\pi BAT_t)}Q_{Pt}\right]^{1-\theta}\right\}^{\frac{1}{1-\theta}} \tag{46}$$

Equations (45) and (46) show that the imposition of import tariffs, the BAT adjustment, and an increase of VAT rates all induce a contraction in labor supply by causing an increase in the relative price of the consumption bundle (i.e., a rise in $\frac{P_t}{P_{pt}}$) holding fixed the producer real exchange rate. As consumption becomes more expensive, households' willingness to supply labor, at any given product real wage, falls. Notably, this effect is larger in the case of VAT increases as these taxes raise the price of both domestic and foreign goods whereas import tariffs and the BAT raise prices only for the share $(1-\omega)$ of imported goods. In the case of VP policies, payroll subsidies (ς_t^v) also affect the labor market equilibrium by increasing firms' labor demand at any given level of the product real wage, as implied by equation (27).

Finally, to see how these policies affect the intertemporal margin, we can rewrite the optimal consumption-saving condition (5) by using (41) and (46) to express consumer price inflation π_{t+1} in terms of producer price inflation (π_{pt}), the producer real exchange rate (Q_{Pt}), and the tax instruments as:

$$\beta E_t\left\{\frac{C_t^\sigma}{C_{t+1}^\sigma}\frac{R_t}{\pi_{pt+1}}\frac{1-\tau_{t+1}^v}{1-\tau_t^v}\left[\frac{\omega+(1-\omega)\left(\frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})(1-\tau_t^\pi BAT_t)}Q_{Pt}\right)^{1-\theta}}{\omega+(1-\omega)\left(\frac{\left(1+\tau_{t+1}^m\right)}{\left(1+\varsigma_{t+1}^{x*}\right)\left(1-\tau_{t+1}^\pi BAT_{t+1}\right)}Q_{Pt+1}\right)^{1-\theta}}\right]^{\frac{1}{1-\theta}}\right\} = 1 \tag{47}$$

Equation (47) shows that consumption depends crucially on agents' expectations about the evolution of these policies. For instance, when agents expect these policies to be temporary, the price of the consumption bundle in the future will be lower than the current price, making savings more attractive. Once again, while IX and BAT policies generate inflationary pressure through only higher prices of imported goods, VP policies make the entire consumption bundle more expensive. The implication of this difference is that expected dynamics of VAT changes have stronger effects on internal demand than the dynamics of IX and BAT tax changes.

4 Macroeconomic Effects of IX Policy

In this section, we analyze the macroeconomic effects of an increase in import tariffs and export subsidies in the home country (IX policy) assuming that other tax policies (BAT, VP) remain unchanged. We begin by briefly describing the specification of trade policy regimes. We then provide conditions under which IX policies are neutral, and explore how various departures from these conditions, especially in terms of agents' beliefs about the persistence of tax changes and the risk of retaliation, can induce IX policies to exert sizable allocative effects.

4.1 Trade Policy Shocks: Retaliation and Policy Reversal

In our benchmark model, we assume that trade policy actions $s_t \in S$ follow a finite state Markov chain, which provides a convenient way of capturing the possibility that foreign economies may retaliate, or that the policies may be reversed. Specifically, the trade policy regime can be categorized as belonging to one of three different states $s_t \in S^R = \left\{s^{NT}, s^{IX}, s^{TW}\right\}$. In the first state $\left(s^{NT}\right)$, no country levies any taxes, tariffs, or provides export subsidies ("No Tax" state). In the second state $\left(s^{IX}\right)$, the home country unilaterally adopts an IX policy that raises import tariffs and export subsidies by the same amount δ. In the third state $\left(s^{TW}\right)$, the foreign country retaliates in a symmetric way by raising its own tariffs and subsidies by the same amount as the home country, that is, $\tau_t^m = \varsigma_t^x = \tau_t^{m*} = \varsigma_t^{x*} = \delta$ ("Trade War" state).[15] All other taxes, including the VAT, the payroll subsidy, and the corporate tax rate, are held unchanged (we consider shocks to these tax rates in Section 5).

[15] Although we restrict our analysis to symmetric retaliatory actions by the foreign government, we also experimented with departures from this assumption (e.g., the foreign government retaliates by only imposing a tariff).

The transition probability matrix Ω can be expressed:

$$\Omega^R = \begin{bmatrix} 1-a & a & 0 \\ (1-\pi)(1-\rho) & \rho & \pi(1-\rho) \\ (1-\varphi) & 0 & \varphi \end{bmatrix} \tag{48}$$

with element $\Omega_{i,j}$ indicating the probability of moving from state i to state j. The first row of matrix Ω^R implies that the transition from the no-tax state s^{NT} to the s^{IX} state - where the home country implements the IX policy unilaterally - is anticipated with probability a. The second row indicates that, given an implementation of IX, the economy remains in the state s^{IX} with probability ρ, returns to the no-tax state with probability $(1-\pi)(1-\rho)$, and transitions to the retaliation state s^{TW} with probability $\pi(1-\rho)$. Once the foreign country retaliates, the economy returns to a no-tax state with probability $1-\varphi$, while with probability φ it remains in the trade war regime. In this specification, the foreign country does not abandon its retaliatory policies unilaterally, so that a trade war can only end through a coordinated policy reversal by both countries.[16]

This general specification for the trade policy regime is helpful for considering a wide range of policy configurations and dynamics as special cases, including permanent unilateral tariffs, trade (or tax) policies that are expected to eventually be reversed, and foreign retaliation. Moreover, the Markov structure will prove very useful in analyzing how uncertainty about prospective trade policies affects current macroeconomic outcomes.[17]

4.2 Neutrality of IX Policies

A large literature has analyzed the Lerner Symmetry Theorem (Lerner, 1936) establishing conditions under which changes in import tariffs and export subsidies do not have allocative effects on the equilibrium allocation.[18] While most of the literature has focused on tax changes within static models of international trade, here we enumerate conditions that generalize Lerner's result in the context of our dynamic monetary framework.[19]

Proposition 1. *In an economy with flexible exchange rates ($\varphi_\varepsilon = 0$), a unilateral implementation*

[16] In our calibration the exact value of φ does not have material effects on outcomes (see the discussion in Section 4.3.2). Thus, in our experiments, we set φ equal to ρ.

[17] Ossa (2014, 2016) present estimates about the effects of cooperative and noncooperative commercial policies in multi-country and multi-industry general equilibrium models of international trade. Although Ossa is able to characterize the optimal trade policies, his analysis abstracts from dynamic considerations which are the focus of this paper.

[18] See, for instance, McKinnon (1966) and, more recently, Costinot and Werning (2017). The Lerner's Symmetry Theorem is also a relevant result for the neutrality of border tax adjustments, as in Meade (1974), Grossman (1980), and Auerbach et al. (2017), Lindé and Pescatori (2017), and Barbiero et al. (2018).

[19] Eichengreen (2018) provides an intuitive discussion of these neutrality conditions.

of IX of size δ (i.e. $\tau_t^m = \varsigma_t^x = \delta$ and $\tau_t^{m*} = \varsigma_t^{x*} = 0$) *causes a* $\delta-$*percentage appreciation of the exchange rate and has no allocative effect if*

1. *It is permanent, unanticipated, and there is no probability of retaliation (*$a = \pi = 0$, and $\rho = 1$*);*

2. *Foreign holdings of home currency-denominated bonds are always zero (*$\chi^* = \infty$*);*

3. *Export prices are set in the producer's currency (PCP) or prices are flexible.*

The formal proof of proposition 1 is in Appendix B.[20] The main intuition behind this proposition, however, can be summarized by the observartion that, under conditions *1.- 3.,* a permanent and immediate jump in the real exchange rate is sufficient to insulate international relative prices and quantities, and thus the entire allocation, from the effects of IX policies.

Recall from section (3) that IX policies affect directly the relative demand of domestic and foreign varieties in the home and foreign country through the relative prices

$$\frac{P_{Ft}}{P_{Ht}} = (1 + \tau_t^m)\, Q_{Pt} \tag{49}$$

$$\frac{P_{Ht}^*}{P_{Ft}^*} = \frac{1}{(1 + \varsigma_t^x)\, Q_{Pt}} \tag{50}$$

where Q_{Pt} is the producer real exchange rate as defined in equation (42). These relative prices remain unchanged if a δ-percentage increase in both import tariffs and export subsidies causes an exchange rate appreciation of the same exact size. In other words, under PCP the exchange rate appreciation lowers the cost of imports in the home country just enough to offset the increase in tariffs and lowers the revenues from sales of domestic varieties in the foreign country by as much as the higher export subsidy.

In a flexible exchange rate regime *(*$\varphi_\varepsilon = 0$*)*, an appreciation of the nominal exchange rate delivers neutrality. That is, in response to an IX policy of size δ implemented at time 0, the nominal exchange rate $\varepsilon_t(\delta)$ satisfies

$$\frac{\varepsilon_t(\delta)}{\varepsilon_t(0)} = \frac{1}{1+\delta}, \tag{51}$$

and *all other prices and quantities are unaffected.*[21] To give a sketch of the proof, note first that expressions (49) and (50) imply that this permanent appreciation of the exchange rate leaves relative

[20] While we do not prove that these conditions are necessary, we illustrate in sections 4.3 - 4.5 that they are *tight* in the sense that relaxing any one of them breaks the neutrality of IX.

[21] Note that Proposition 1 does not impose any restriction on wage setting, as neutrality of IX does not require any adjustment in wages. As we discuss later, this result is in stark contrast with the case of VP.

prices $\left(\frac{P_{Ft}}{P_{Ht}}\right)$ and $\left(\frac{P^*_{Ht}}{P^*_{Ft}}\right)$ unchanged. It is then immediate to verify that the three margins discussed in section (3) – the relative demand for home and foreign varieties (equations 43 and 44), for consumption and leisure (equation 45), and for consumption at different points in time (equation 47) – are also unaffected by the policy change. In addition, the two optimality conditions for holdings of foreign currency denominated bonds, (equations 6 and 7), are unaffected as these holdings depend on *future* exchange rate changes while the IX policy causes a permanent appreciation of the exchange rate.[22] Finally, after substituting equations (39) and (40) , the expression for the balance of payment

$$B_{Ft} = B_{Ft-1}R^*_{t-1} + P^*_{Ht}\left[Y^*_{Ht} - S_t Y_{Ft}\right] \tag{52}$$

shows that the external balance is also unaffected as the permanent appreciation of the exchange rate insulates the terms of trade

$$S_t = \frac{1}{(1+\tau^m_t)\,\varepsilon_t(\delta)} \frac{P_{Ft}}{P^*_{Ht}} \tag{53}$$

4.3 Policy Reversal, Retaliation, and Anticipation: The Role of Exchange Rate Dynamics

The neutrality of IX policies in our dynamic framework requires that the real exchange rate jumps to a new long-run value, reflecting the public's belief that trade actions will be permanent and not induce foreign retaliation, even in the future. However, as argued in the introduction, these assumptions seem at odds with historical experience. Given these considerations, we next apply our benchmark model to study the effects of IX policies that have no long-run effect on the real exchange rate. Through the lens of the Markov structure presented earlier, the effects on the exchange rate may prove temporary because the policy action is reversed, or alternatively, because the home country's implementation of IX policies prompts the foreign government to retaliate by adopting similar policies. As the implications of either type of policy turn out to be nearly identical, for expositional simplicity

[22] This discussion implies that a permanent IX policy would not be neutral under complete markets, absent the use of additional tax instruments (such as appropriately chosen consumption taxes). The risk-sharing condition would impose a tight link between the ratio of (marginal utility of) consumption across countries and the current real exchange rate. Thus, any movement in the current real exchange rate would have allocative effects. Lindé and Pescatori (2017) explore a similar insight in the context of their analysis of the effects of BAT policies.

we begin by focusing on the case in which a unilateral IX policy is expected to be reversed ($1-\rho > 0$, $\pi = 0$) and later discuss differences between policy reversal and retaliation ($\pi > 0$).

4.3.1 Reversal of IX policies

In our benchmark framework, a unilateral IX policy of size δ that is expected to be reversed with probability $1-\rho > 0$ exerts allocative effects by boosting real net exports, as the associated exchange rate appreciation only partially insulates international relative prices. In order to explain the intuition behind this result, it is helpful to proceed by way of contradiction. Assume that the allocation is unaffected and that the exchange rate appreciates by δ – as in (51) – for as long as the policy remains in effect. This exchange rate movement suffices to completely offset the effects of IX on relative prices (49) and (50). However, the expectation that the IX policy will eventually be reversed implies that the exchange rate depreciates in the future, which in turn requires an increase in the real interest rate to satisfy the uncovered interest parity condition

$$\mathbb{E}_t \left[\frac{C_t^\sigma}{C_{t+1}^\sigma} \frac{P_t}{P_{t+1}} \left(R_t - \frac{\varepsilon_{t+1}}{\varepsilon_t} R_t^* \right) \right] = 0 \tag{54}$$

As a rise in the interest rate clearly has allocative effects, this contradicts the hypothesis of neutrality of a transitory IX. The increase in the demand for foreign bonds associated with the expected depreciation of the currency turns out to lead to a smaller simultaneous appreciation of the exchange rate and an expansion of net exports.[23] Notably, transitory IX policies are non-neutral both under flexible prices as well as under sticky prices, although specific assumptions about the form of nominal rigidities and the monetary policy rule are of course key to determining how the stimulus to net exports is transmitted to the rest of the economy.

In this vein, the solid lines in Figure 1 show the expected paths of key variables after the home country adopts a unilateral IX policy in our benchmark model with sticky prices. The IX policy consists of a 10 percentage points increase in import tariffs and export subsidies that is expected to be reversed with probability $(1-\rho) = 0.05$ by the following quarter. The policy causes a small appreciation of the exchange rate that does not fully insulate relative prices and, as a consequence, imports fall and exports rise. In our general equilibrium model, monetary policy reacts to the stronger external demand by raising interest rates, which reduces home consumption and contributes to the appreciation

[23] The use of appropriately targeted capital controls, i.e. designed so that equation (54) holds without requiring an adjustment in the interest rate, restores neutrality. We thank our discussant Emmanuel Farhi for this observation.

of the real exchange rate, thus dampening some of the stimulus to net exports. Because the stimulus to domestic output occurs through expenditure-switching channels, it has negative spillovers to the foreign economy, so that both foreign GDP and inflation decline.

To clarify the transmission channels through which IX policies that are expected to be reversed affect the economy – and highlight the distinction between import tariffs and export subsidies in violating Lerner's symmetry – it is helpful to consider the following log-linearized equations of the model:

$$\tilde{y}_t \equiv \omega \tilde{y}_{ht} + (1-\omega)\,\tilde{y}^*_{ht} = \tilde{c}_t + (1-\omega)\widetilde{nx}^q_t \tag{55}$$

$$\widetilde{nx}^q_t = \tilde{y}^*_{ht} - \tilde{y}_{ft} = \theta\omega[(\tau^m_t + \sigma^x_t) + 2\tilde{q}_{pt}] + (\tilde{c}^*_t - \tilde{c}_t) \tag{56}$$

$$\Delta\tilde{q}_{pt+1} = (\tilde{r}_t - \tilde{\pi}_{pt+1}) - (\tilde{r}^*_t - \tilde{\pi}^*_{pt+1}) + \phi_b b_t \tag{57}$$

$$\tilde{c}_t = \tilde{c}_{t+1} - \frac{1}{\sigma}\left[\tilde{r}_t - \tilde{\pi}_{pt+1} - (1-\omega)\left(\Delta\tau^m_{t+1} + \Delta\tilde{q}_{pt+1}\right)\right] \tag{58}$$

where a 'tilde' denotes the percent or percentage point deviation of a variable from its steady state level. Equation (55) specifies that output may be allocated to consumption $\widetilde{c}_t$ or to real net exports $\widetilde{nx}^q_t$, where the latter is the difference between exports and imports (evaluated at steady state prices). Equation (56) states that real net exports rise in response to higher import tariffs (τ^m_t) or export subsidies (σ^x_t), a depreciation of the producer real exchange rate (i.e., a rise in $\widetilde{q}_{pt}$), or to a rise in foreign relative to home consumption. The direct effects of the trade policy shocks on aggregate demand are stronger if technology allows higher substitution between domestic and foreign varieties (i.e. higher trade elasticity θ) and if the country is more open (i.e. higher ω). Equation (57) is the uncovered interest parity condition, linking the evolution of the real exchange rate to the difference between real interest rates between countries, with the final term $\phi_b\widetilde{b}_t$ capturing that investors are willing to accept a relatively lower real return on home bonds if net foreign assets $\widetilde{b}_t$ are positive. Finally, using the Euler equation (47), expression (58) links consumption changes to the producer real interest rate ($\tilde{r}_t - \tilde{\pi}_{pt+1}$) as well as to changes in import tariffs and in the producer real exchange rate (with the strength of the latter changes controlled by the import share). This expression reveals that IX policies operate not only through trade channels, but also through intertemporal channels, as an increase in import tariffs – that is expected to be reversed – raises the cost of current consumption relative to future consumption. These dynamic effects of tariffs contrast sharply with the effects of export subsidies, which do not affect the consumer real interest rate directly (but only through the strength of the monetary policy response).

This intertemporal substitution effect of import tariffs that pushes down consumption is consequential. Returning to Figure 1, the dashed lines show the effects of import tariffs only: An increase in import tariffs has essentially no effect on output under our baseline calibration ($\sigma = 1; \theta = 1.25$), so that all of the output stimulus from IX policies comes from the increase in export subsidies (i.e., the distance between the blue and red lines). The quasi-invariance of output to the tariff increase reflects that the expenditure-switching effect, which pushes up the desired share of consumption spent on home goods, is offset by the intertemporal-substitution effect, that pushes down overall consumption. Stepping beyond our specific calibration, the output effects of higher import tariffs depend on the relative strength of these two effects. If the intertemporal elasticity of substitution is low relative to the trade price elasticity, higher tariffs would tend to boost output (as the expenditure-switching effect dominates), whereas the tariff would reduce output if the intertemporal elasticity is relatively high relative to the trade elasticity. Even so, under reasonable assumptions about intertemporal substitution and trade elasticities, a combination of import tariffs and export subsidies that is expected to be reversed increases output in the near term.

The magnitude of the stimulus from temporary IX policies depends on the response of monetary policy as well. For instance, a larger interest rate response to producer price inflation (higher φ_π in the policy rule) and, consequently, to the external demand stimulus would imply smaller output effects. By contrast, when monetary policy gives high weight to stabilization of the exchange rate (high φ_ε in the policy rule), the output stimulus is larger, with a fixed exchange rate regime an interesting limiting case. In this spirit, Figure 2 shows how the IX policies play out in our baseline model in which the home exchange rate is fixed to that of the foreign economy (solid lines). Home output rises significantly more in this case than under flexible exchange rates. This larger output expansion largely reflects that consumption expands robustly – rather than contracts – as the home policy rate declines in lockstep with the foreign policy rate. The rise in output is also reinforced by a smaller appreciation of the real exchange rate, which boosts net exports relatively more.

4.3.2 Reversal of IX policies and retaliation

We have asserted that the IX policy with reversal considered so far has very similar effects to an IX policy subject to possible retaliation, meaning in the latter case that agents expect that the foreign government may retaliate in kind sometime in the future. As shown by the lemma below, the difference between the equilibrium allocation under retaliation and the equilibrium allocation with policy reversal is attributable to wealth effects that can be offset by appropriate international transfers.

Lemma 1 *If prices are flexible or set in the producer's currency, a unilateral implementation of IX with policy reversal implements the same equilibrium allocation as a unilateral implementation of IX that triggers retaliation coupled with international transfers that satisfy:*

$$T_{t_1}^I = -\frac{\delta}{1+\delta}\left[B_{F,t_1-1}R_{t_1-1}^*\varepsilon_{t_1} + B_{H,t_1-1}R_{t_1-1}\right]$$

$$T_{t_2}^I = \delta\left[B_{F,t_2-1}R_{t_2-1}^*\varepsilon_{t_2} + B_{H,t_2-1}\frac{R_{t_2-1}}{\pi_{t_2}}\right]$$

where t_1 is the first time the economy transits to the retaliation state s^{TW} and $t_2 > t_1$ is the first time it leaves the retaliation state s^{TW}.

Proof. See Appendix C.

The intuition of this lemma can be easily understood by considering the special case of a permanent transition to a trade war regime starting from balanced trade. In this case, $T_{t_1}^I = 0$ and $T_{t_2}^I$ never occurs so that Lemma 1 implies that the effects of starting a trade war are identical to the effects of abolishing all tariffs and subsidies in both countries. The reason can be easily understood by inspecting equation (43) ,where export subsidies in the foreign country exactly offset import tariffs in the home country, and, symmetrically, equation (44) .

When the home country has a positive net foreign asset position, however, a transition to a trade war regime will not be equivalent to a transition to a state with no taxes. Given that a positive net foreign asset position implies that the home country is expected to run trade deficits in the future, import tariff revenues will exceed export subsidy expenditures, implying a positive wealth effect and an associated appreciation of the home currency. Symmetrically, the foreign economy will suffer wealth losses from its implementation of IX. Consequently, a transfer of resources that corrects this international wealth redistribution is needed to implement the same allocation under policy reversal and retaliation. Under our assumption of balanced trade in the long run, however, the economic effects of these transfers are of second order.

4.3.3 Anticipation Effects of IX

While we have shown that IX policies may boost output if their implementation is a surprise, the anticipation that such policies may be implemented sometime in the future can have immediate contractionary effects. The importance of anticipation effects was recognized by Krugman (1982) in a setting in which agents were certain about the future implementation date, but is useful to revisit in our Markov-switching framework given that it provides a convenient way of capturing uncertainty about the implementation date. In this vein, Figure 3 shows the response of the economy when agents learn that IX policies will be introduced in the future, but are unsure about the timing. Specifically, as long as IX policies are not implemented, agents believe that there is a 10 percent chance that IX policies will be implemented in the subsequent period (i.e., $a = 0.10$), and that – once implemented – the policies will not be reversed ($\rho = 1.0$). The figure plots a realized simulation path in which IX policies are not actually implemented in the 5-year period shown, even though agents continue to see a substantial likelihood that they will be put into effect in the near term.

The anticipation effects of IX policies work through an exchange rate channel: The expectation that the exchange rate must appreciate in the long-run causes the exchange rate to appreciate in the near-term, when agents first come to believe that IX policies will eventually be implemented (first panel). The stronger currency leads to a decline in competitiveness for domestic firms, a drop in exports, and an output contraction.

4.4 Trade in home currency bonds

The neutrality result presented in Proposition 1 requires the strong condition that asset market incompleteness takes the form of no international trade in home currency denominated bonds. To understand the role of this restriction, note that the implementation of IX induces changes in two different components of households wealth. First, the IX policy generates fiscal revenues whenever the home country has a trade deficit since in this case revenues from tariffs exceed subsidies to exporters. The wealth increase associated with a permanent IX policy of size δ, $G_t^F(\delta)$, is then given by the present discounted value of the fiscal revenues it generates

$$
\begin{aligned}
G_t^F(\delta) &= E_t \sum_{i\geq 0} \left(\prod_{j=1}^{i} \frac{\pi_{t,t+j}^*}{R_{t+j}^*} \right) \frac{\delta}{1+\delta} \left(\frac{P_{Ft+i}}{P_{t+j}} Y_{Ft+i} - Q_{t+i}(0) \frac{P_{Ht+i}^*}{P_{t+j}^*} Y_{Ht+i}^* \right) \\
&= \frac{\delta}{1+\delta} \left[Q_t(0) \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} - \frac{B_{Ht-1}^*}{P_{t-1}} \frac{R_{t-1}}{\pi_t} \right] \qquad (59)
\end{aligned}
$$

where the second equality uses the fact that in equilibrium the present discounted value of future trade deficits is equal to the net foreign asset position of the home country, that is, the difference between home country holdings of foreign bonds $\left[Q_t(0) \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*}\right]$ and foreign country holdings of home bonds $\left[\frac{B_{Ht-1}^*}{P_{t-1}} \frac{R_{t-1}}{\pi_t}\right]$.

Second, the exchange rate appreciation decreases the value of home holdings of foreign bonds. Denote with $L_t^B(\delta)$ the losses on foreign bond holdings under an appreciation of size δ, then

$$
L_t^B(\delta) = \left[Q_t(\delta) - Q_t(0)\right] \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} = -\frac{\delta}{1+\delta} Q_t(0) \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} \qquad (60)
$$

Equations (59) and (60) imply:

$$
L_t^B(\delta) = G_t^F(\delta) + \frac{\delta}{1+\delta} \frac{B_{Ht-1}^*}{P_{t-1}} \frac{R_{t-1}}{\pi_t}. \qquad (61)
$$

Expression (61) summarizes the wealth effects associated with IX policies. When there is no international trading of bonds denominated in home currency ($B_{Ht}^* = 0$), as required in Proposition 1, wealth gains through higher fiscal revenues $G_t^F(\delta)$ are exactly offset by the wealth losses induced by lower valuations of foreign holdings $L_t^B(\delta)$, thus preserving neutrality of IX policies. In contrast, when the home country borrows in home currency bonds $\left(B_{Ht-1}^* > 0\right)$ and invests in foreign currency bonds $(B_{Ft-1} > 0)$, it acquires a leveraged exposure to foreign exchange variations and the sensitivity of wealth in the home country to an exchange rate appreciation is bigger than its net foreign asset position. Consequently, given an unchanged path for future trade deficits, an exchange rate appreciation of the same size of the policy reduces wealth in the home country as the increase in fiscal revenues is not large enough to offset the capital losses on foreign bonds holdings implied by equation (61). These wealth losses induce households to reduce their savings and, in equilibrium, the exchange rate appreciates less while the trade balance increases.

Figure 4 shows the response of the economy to a permanent unilateral IX policy when the home country has a leveraged exposure to exchange rate fluctuations. In particular, this experiment assumes that in the initial state international trade is balanced but countries hold offsetting positions in domestic and foreign currency denominated bonds $\left(\text{i.e. } B_{F_{-1}} = B^*_{H-1} > 0\right)$ scaled to be twice as large as the value of annual GDP. As anticipated in our previous discussion, the implementation of a permanent IX when foreign holdings of home currency denominated bonds are positive lowers households wealth and reduces savings, thus dampening the appreciation of the exchange rate (solid lines). As a result, the home country runs a permanently positive trade balance to pay interest on its negative net foreign asset position. For comparison, we also plot the response of the baseline economy when there is no international trade in domestic currency bonds, as required in Proposition 1, and a permanent IX policy is neutral (dashed lines).

4.5 Departing from Producer's Currency Pricing

We conclude this section with a brief discussion on the requirement of producer's currency pricing (PCP) in Proposition 1 to deliver neutrality of IX policies. We follow the literature and compare the transmission of policies under PCP and local currency pricing (LCP).[24] Under LCP, producers in each country set a price for the home market in domestic currency and a (pre-tariff) price for the foreign market in foreign currency, with both prices adjusted infrequently because of the Calvo friction.[25]

Figure 5 compares the effects of an IX policy under PCP (dashed lines) and under LCP (solid lines), assuming that all other conditions in Proposition 1 are satisfied. As discussed before, under PCP international relative prices are insulated by the immediate appreciation of the exchange rate and the allocation is unaffected. Under LCP, in contrast, the IX policy has allocative effects: Imports contract, inflation jumps, while exports and output experience a very small boost.

The source of non-neutrality is the asymmetric pass-through of tariff changes and exchange rate movements to import prices. As shown by the expression for the price of imported goods in the home country

$$P_{Ft} = (1 + \tau^m_t)\, P_{X^*_t} \tag{62}$$

changes in import tariffs are fully passed through to import prices (P_{Ft}) whereas movements in the exchange rate only pass-through gradually as foreign exporters adjust their prices in the home market $\left(P_{X^*_t}\right)$ infrequently under our Calvo pricing assumption. Hence, the rise in import prices reduces the

[24] For a discussion of transmission under PCP and LCP see, for instance, Devereux and Engel (2002). See section 5.4 for a discussion of alternative pricing assumptions that also considers the "Dominant Currency Paradigm" (DCP). If the dominant currency is the currency of the country that implements IX, IX would still not be neutral for the same reasons described in this section.

[25] Appendix A presents a full derivation of the optimization conditions of producers under the LCP assumption.

demand for imported varieties and boosts output through import-substitution channels.

5 Can Alternative Tax Policies Substitute for Trade Policy Actions?

In this section, we compare IX policies to tax policies that are often considered as having similar macroeconomic effects. These tax policies include "fiscal devaluation" – implemented through a rise in the VAT that is coupled with a payroll subsidy to employers (VP) – as well as a border-adjustment of corporate profit taxes (BAT).

Our main contribution is to provide conditions under which these three policies are equivalent and to investigate their transmission when they are not. To that end, we first show that under the (quite stringent) conditions of Proposition 1, all three policies in our baseline framework are equivalent and neutral. We next present a general result stating that while IX and BAT remain equivalent when the conditions required by Proposition 1 are relaxed, IX and VP are generically not equivalent. Our analysis thus emphasizes that the equivalence of these policies often ascribed in the existing literature and among many policymakers relies crucially on a very specific assumption about how VAT changes are assumed to be passed through to consumer prices. In particular, under our preferred specification of full and immediate passthrough of VAT changes to consumer prices – for which we provide some empirical evidence at the end of this section – we show that the effects of IX policies often diverge markedly from VP, even qualitatively. Given these differences, we conclude that alternative assumptions about tax passthrough are highly consequential for the macroeconomic effects of VP, a result reminiscent of Poterba et al. (1986).

5.1 A Special Case: Equivalence and Neutrality

To understand the differences between IX (and BAT) and VP policies, it is helpful to first consider a special case in which the effects of these policies look beguilingly similar – namely, the case in which the policies are implemented permanently, exchange rates are flexible, and monetary policy follows the benchmark Taylor-style rule that responds only to producer price inflation and the output gap:

$$R_t = \frac{1}{\beta} \left(\pi_{Pt}\right)^{\varphi_\pi} \left(\tilde{y}_t\right)^{\varphi_y} \tag{63}$$

Under these conditions, VP, IX, and BAT policies have no allocative effects – meaning no effects on output, employment, or inflation – and hence may be regarded as equivalent and neutral. These results are formalized in the following proposition.

Proposition 2. *If monetary policy is described by (63), wages are flexible, and conditions 1.-3. of Proposition 1 are satisfied, the following policies are equivalent, neutral, and induce the real exchange rate to appreciate by δ:*

1. *A permanent unexpected IX policy of size δ;*

2. *A permanent unexpected BAT policy with corporate taxes $\tau^{\pi} = \frac{\delta}{1+\delta}$;*

3. *A permanent unexpected VP policy of size $\frac{\delta}{1+\delta}$;*

The formal proof of this proposition is relegated to Appendix D. It is immediate to conclude why the IX policy is neutral as the conditions of Proposition 1 apply. Given that IX and BAT are always equivalent in our environment, as we show later in Section (5.2), here we focus on the relation between IX and VP.[26] In particular, we provide intuition for why VP is neutral in this special case and underscore key differences in transmission with respect to the neutrality of IX. Figure 6 compares the economic adjustment that delivers neutrality under IX (solid blue lines) and VP policies (dashed red lines).

The three partial-equilibrium margins discussed in section (3) are a useful starting point to understand why VP has no allocative effects in this special case. Inspection of equations (43) and (44) reveals that VP has no direct effect on relative demand of home and foreign varieties, as VAT changes affect equally the price of imported and domestically-produced goods. Consequently, under VP no general equilibrium adjustment of the product real exchange rate (Q_{Pt}) is required to insulate relative prices from the effects of the policy. This transmission of VP contrasts sharply with IX where, as shown in Figure 6, the nominal exchange rate has to jump to insulate relative prices.

Turning to the labor market equilibrium, the increase in the payroll subsidy and the VAT hike have offsetting effects on labor demand and labor supply. Under our assumption of full pass-through of taxes, at fixed producer prices, a VAT hike induces consumer prices (P_t) to jump by δ percent (see

[26] In independent work developed around the same time as our paper, Barbiero et al. (2018) provide similar conditions required for the BAT to be neutral. Their neutrality result, however, is slightly different from ours because of a different assumption about the response of import prices to the adoption of the BAT, which allows them to relax condition 3 in the proposition. In particular, these authors assume that under LCP, the actual cost of imports, inclusive of the effect of the non-deductibility from corporate profits under BAT $\left(i.e. \frac{P_{Ft}}{(1-\tau_t^{\pi})} \right)$, remains fixed absent price adjustment by foreign exporters (whereas this cost jumps in our environment as the price P_{Ft} remains fixed). This assumption implies that neutrality goes through under LCP as well in their setup.

equation 46). In order for the household labor supply to remain unchanged, equation (45) requires an adjustment in the "product" real wage $\left(\frac{W_t}{P_{Pt}}\right)$ of the same exact percentage of the VAT hike. That is, in response to a VP policy of size $\frac{\delta}{1+\delta}$, the nominal wage $W_t(\delta)$ satisfies

$$\frac{W_t(\delta)}{W_t(0)} = 1+\delta. \tag{64}$$

while producer prices are unaffected. In addition, as evident from the optimal pricing decision of producers (27), the commensurate increase in payroll subsidies (ς_t^v) ensures that firms are willing to pay this higher wage. As shown in Figure 6, the increase in consumer prices and in the nominal wage under VP is also in sharp contrast to IX, where the nominal exchange rate is the only price that neutralizes the effects of the policy.

Finally, the intertemporal optimality condition in (47) clarifies why the consumption profile is unaffected by VP, as long as the implementation of the policy is unexpected and permanent. We have already noted that under VP both the producer real exchange rate (Q_{Pt}) and the producer price inflation (π_{Pt}) do not need to adjust. Given that VAT rates jump permanently to a higher level (i.e., $\tau_{t+1}^v = \tau_t^v$), a constant nominal interest rate keeps consumption growth constant. This result hinges critically on the assumption that the monetary policy rule responds to producer price inflation and thus "sees through" rising consumer prices.

In sum, IX and VP are both equivalent and neutral under the restrictive conditions of Proposition 2. That said, while IX policies achieve allocative neutrality through an adjustment in the nominal exchange rate, VP policies achieve neutrality through a jump in consumer prices and nominal wages without requiring any nominal exchange rate adjustment. This observation turns out to be the basis for the main result of this section, namely that IX and VP policies generically implement different allocations.

5.2 The General Case

We begin this section with a more precise definition of the the three policies that we consider.

Definition. An IX , a VP, and a BAT policy of size δ and probability of reversal ρ are described by stochastic processes $\left\{\widetilde{S}^{IX}, \widetilde{\Omega}^{IX}\right\}, \left\{\widetilde{S}^{VP}, \widetilde{\Omega}^{VP}\right\}, \left\{\widetilde{S}^{BAT}, \widetilde{\Omega}^{BAT}\right\}$ respectively, with $\widetilde{S}^{IX} = \left\{s^{NT}, s^{IX}\right\}$ as defined above; $\widetilde{S}^{VP} = \left\{s^{NT}, s^{VP}\right\}$ where in state s^{VP} all taxes are zero apart from value-added taxes and payroll subsidies given by $\tau^v = \varsigma^v = \frac{\delta}{1+\delta}$; $\widetilde{S}^{BAT} = \left\{s^{NBAT}, s^{BAT}\right\}$ where in state s^{NBAT} all taxes are zero apart from corporate profit taxes given by $\tau^\pi = \frac{\delta}{1+\delta}$ and in state

s^{BAT} corporate profit taxes are adjusted at the border (i.e. $BAT = 1$ and $\tau^{\pi} = \frac{\delta}{1+\delta}$). The transition matrix is given by

$$\widetilde{\Omega}^{i} = \begin{bmatrix} 1 & 0 \\ 1-\rho & \rho \end{bmatrix} \tag{65}$$

where $i = IX, VP, BAT$.

Our main result, summarized in Proposition 3, is that while IX and BAT always implement the same allocation, VP does not (as long as prices are sticky).

Proposition 3. *Under full pass-through of taxes, an unexpected IX policy of size δ and an unexpected BAT policy of size δ implement the same allocation. Generically, a VP policy of size $\frac{\delta}{1+\delta}$ does not implement the same allocation as IX or BAT. Equivalence of the three policies requires that policies are permanent, i.e. $\rho = 1$, or that prices are flexible.*

Appendix E provides a formal proof showing that BAT and IX policies have identical allocative effects if there is full pass-through of tariffs, irrespective of the specific assumptions about price and wage settings. Intuitively, recall from Section (3) that the non-deductibility of imports from firm profits under the BAT is similar to an import tariff, whereas the exemption of export sales acts like an export subsidy. Hence, a BAT policy of size δ exerts the same effects on trade prices and, more generally, on the allocation as an IX policy of the same size.

The equivalence of BAT and IX policies relies importantly on the assumption of full pass-through of tariffs. For instance, it is immediate to show that if import tariffs were not fully passed through, then BAT and IX policies would be no longer equivalent when foreign exporters price in local currency.

Having established the equivalence between the BAT and IX policies in our baseline model, in the remainder of this section we study the relation between VP and IX (or BAT) by discussing cases that make differences in transmission more evident.

5.2.1 Fixed Exchange Rates

Proposition 4. *In a fixed exchange rate regime $(\varphi_{\varepsilon} = \infty)$, under assumptions 1.- 3. of Proposition 1, an IX policy of size δ and a BAT policy of size $\frac{\delta}{1+\delta}$ have the same allocative effects as a once-and-for-all unexpected currency devaluation of size δ. A VP policy of the same size $\frac{\delta}{1+\delta}$ has no effect on the allocation but causes the real exchange rate to appreciate by δ.*

Appendix E contains a formal proof of Proposition 4. The equivalence between IX and a currency devaluation in this framework confirms Keynes' original argument and has already been formally established in Farhi et al (2014). Under our assumption of full pass-through of import tariffs, however, this result is less general as it requires that changes in the exchange rates are also fully passed through: Condition 3. of Proposition 1 ensures that this is the case by assuming producer currency pricing.

Here we make use of Figure 7 to compare the effects of VP (dashed red lins) and IX (solid blue lines) under fixed exchange rates and provide intuition for the different effects. The neutrality of VP is a direct consequence of the neutrality result discussed in Proposition 2: Given that VP does not cause changes in relative demand and thus does not require any adjustment in the producer real exchange rate (Q_{Pt}), the fixed exchange rate regime does not pose any constraint to achieving the same outcome as under flexible exchange rates that was described above. This outcome contrasts sharply with the large output stimulus provided by IX policies under fixed exchange rates. As shown in Figure 7, absent an appreciation of the nominal exchange rate, the IX policy boosts net exports and, with nominal interest rates unchanged, domestic consumption expands as well.

The lack of equivalence between IX and VP policies under fixed exchange rates appears in contrast with recent results in the fiscal devaluation literature, such as Farhi et al. (2014). The key difference between the two frameworks is that our analysis assumes that pre-tax prices are sticky and taxes are fully passed through, so that consumer prices jump immediately after the implementation of a VP policy. In the fiscal devaluation literature, in contrast, prices are typically assumed to be sticky *inclusive* of taxes (and hence pre-tax prices are free to adjust). Therefore, a permanent VP in this environment exactly replicates the effects of an IX policy as the sluggish adjustment in consumer prices is associated with a decline in real interest rates that boosts consumption. While we consider our contribution as largely theoretical, we later discuss some evidence pointing to fairly rapid passthrough of VAT changes to consumer prices, in line with our benchmark specification. Nevertheless, it is plausible that this policy could have some delayed effects on consumer prices so that the contrast between IX and VP would not be quite as stark as in our baseline model.

5.2.2 Alternative wage settings

We next examine the implications of departing from the assumption of flexible wages for the macroeconomic consequences of IX and VP policies. A large macroeconomic literature assumes that households set nominal wages in Calvo-style staggered contracts that are similar in form to the price contracts outlined in Section 2.[27] We choose the parameter controlling the degree of wage stickiness to imply

[27] See, for instance, Erceg et al. (2000).

that wages are adjusted with the same frequency as prices.

Figure 8 shows the response of the economy to unexpected and permanent VP (dashed red lines) and IX (solid blue lines) policies under flexible exchange rates. Recall from our discussion of Proposition 2 that IX policies do not rely on adjustment in wages to achieve neutrality. Hence, IX policies continue to have no allocative effects when wages are sticky, as shown by the blue lines in Figure 8. Given that VP policies required a jump in the product real wage to have no allocative effects, it follows that when wages are sticky VP policies are not neutral. The nominal wage rises only gradually under VP, and the product real wage follows a similar pattern, thus preventing labor supply from falling. However, the higher payroll subsidy persistently reduces producers' marginal costs, thus pushing producer price inflation down and increasing labor demand. As monetary policy cuts the policy rate in response to below-target producer price inflation, consumption expands. The increase in consumption and the rise in (net) exports – due to the induced decline in the terms of trade – contribute to an expansion in home country output.

An equally important macroeconomic literature considers formulations of downward wage rigidities of the form

$$W_t \geqslant \xi W_{t-1}, \quad \xi > 0 \tag{66}$$

where ξ governs the degree of downward wage rigidity.[28] Given that both IX and VP policies either do not affect nominal wages or increase them, it is immediate to conclude that our non-equivalence results would go through when wages are downwardly rigid.

5.2.3 IX and VP with Reversal

We conclude our analysis of VP and IX policies with a comparison of their effects when agents perceive that these policies will be reversed according to the transition matrix in equation (41). When agents expect policies to be reversed, changes in the price of the consumption bundle over time can have strong intertemporal substitution effects on consumption, as shown by the linearized version of equation (47)

$$\widetilde{c}_t = \widetilde{c}_{t+1} - \frac{1}{\sigma}\left[\tilde{r}_t - \tilde{\pi}_{pt+1} - \Delta\tau^v_{t+1} - (1-\omega)\left(\Delta\tilde{q}_{pt+1} + \Delta\tau^m_{t+1}\right)\right] \tag{67}$$

Equation (67) shows that this intertemporal substitution effect is much stronger in the case of temporary VP as this policy, under full pass-through of taxes, induces direct inflationary pressure on

[28] See, for instance, Schmitt-Grohé and Uribe (2016).

the entire consumption bundle. IX policies, in contrast, generate inflationary pressure through only higher prices of imported goods.

Figure 9 compares VP (dashed red lines) and IX (solid blue lines) policies of size $\delta = 10$ percent that are reversed with probability $(1 - \rho) = 0.05$ by the following quarter. For simplicity, we focus here on the case in which wages are sticky and exchange rates are flexible. The higher consumption real interest rate under VP depresses consumption markedly, causing a *contraction* in output. As output declines, the central bank lowers policy rates, which depreciates the exchange rate and boosts net exports. The contractionary effects of VP contrast substantially with the *expansion* of output under IX.

While the two policies have similar effects of trade quantities, this outcome reflects very different channels. The IX policy has direct "competitiveness-enhancing" effects on relative trade prices which raise exports and cause imports to contract. This stimulus is only partially counterbalanced by a tightening of policy rates and an appreciation of the home currency. In contrast, the stimulus to net exports from the VP policy is mainly due to the depreciation of the exchange rate induced by the fall in policy rates in the face of lower aggregate demand.

The disparity between VP and IX policies under full pass-through of taxes is not affected by different assumptions about wage settings and becomes even greater under fixed exchange rates (since the interest rate cut under VP shown in the figure would be precluded, as would the interest rate hike under IX).

Taken together, our results underscore that VP and IX operate through substantially different transmission channels. Given the importance of intertemporal substitution channels in shaping the macroeconomic effects of VP, the imposition of VP on a temporary basis runs the risk of providing a contractionary impetus to output, especially if the policy interest rate and exchange rates cannot adjust much.

5.3 Survey of Empirical Evidence on the Pass-through of Tax Changes

Our finding that an increase in VAT accompanied by a commensurate increase in payroll subsidies has significantly different macroeconomic effects relative to a increase in import tariffs and export subsidies appears in sharp contrast with the conventional wisdom established in the fiscal devaluation literature. The key insight of our analysis is that assumptions about the pass-through of tax changes critically determine the equivalence of these policies as nominal rigidities alter the incidence of these taxes. While the theoretical literature typically assumes that prices are rigid *inclusive* of taxes, our analysis has assumed that pre-tax prices are rigid and VAT increases are fully and immediately passed through to consumer prices.

In this section, we briefly review some evidence in support of the assumption of full pass-through of taxes. Nevertheless, rather than viewing this evidence as dispositive in favor of our specification of price-setting, we view our main contribution as highlighting the sensitivity of the equivalence results of IX and VP policies to assumptions about tax pass-through when firm prices cannot be immediately adjusted.

Several studies present estimates of large pass-through from the analysis of country-specific episodes of VAT changes. Carbonnier (2006) reports that two large reductions in French VAT rates for car and housing services were associated with pass-through of between 60 and 75 percent, with most of the price adjustment completed within two months. Cashin and Unayama (2016) use the 1997 VAT increase in Japan to derive estimates of the intertemporal elasticity of substitution using information about the response of very detailed consumption expenditure categories. They find that the pass-through of the VAT increase into consumer prices was full and that prices adjusted within one month of the tax change. Karadi and Reiff (2016) investigate the response of prices to two increases and one decrease in VAT rates that were implemented in Hungary between 2004 and 2006 across different categories of goods. In their data, positive increases in VAT rates elicited immediate adjustments of prices (within one month) with very high pass-through (between 75 and 99 percent). The response to reduction in VAT rates, in contrast, was significantly smaller, pointing to an asymmetric response of prices to positive and negative VAT changes.

A few cross-country studies also present evidence of large and quick pass-through of VAT changes. Benedek et al. (2015) analyze the response of prices to VAT changes that took place in euro-area countries over the years 1999-2013 and find complete pass-through for changes in standard VAT rates. Exploiting a different identification strategy on similar data, Benzarti and Carloni (2017) find that the pass-through of standard VAT rate changes to prices is about 39 percent for VAT increases and only 11 percent for VAT decreases. Despite this asymmetry, in both cases the passthrough happens in the first month after the implentation of the VAT change.[29]

[29] The tax shift of sale taxes across U.S. states also appears to be very high and immediate, as discussed in Besley and Rosen (1999), although it bears noting that the nature of the sales tax base and incidence of the tax differ somewhat

Direct evidence on fiscal devaluations also point to large VAT pass-throughs. For instance, in 2007 the German government raised the standard VAT rate by 3 percentage points and lowered the marginal rate for social security contributions by a similar amount. Figure 10 shows the evolution of German core inflation and motor vehicle inflation (blue lines in the top panels) around the implementation of these tax changes. For comparison, the figure also presents the behavior of inflation in other euro-area countries where no changes in VAT rates or payroll contributions were implemented (red lines). Both panels clearly show a discrete jump in German inflation in the month January 2007, when the tax changes went into effect, whereas inflation in other euro-area countries does not reveal any outsized change throughout the 2007 year. This observation suggests that the spike in German inflation is likely to reflect the increase in value-added taxes. The immediate pass-through to core inflation is around 50 percent, a large number if one considers that the good categories affected by the VAT change accounted for just half of the consumption basket.[30] Consequently, the VAT pass-through must have been very high for a large number of consumption items, as exemplified by the evolution of car prices.

The bottom panels of Figure 10 provide some evidence on the macroeconomic effects of the 2007 fiscal devaluation in Germany. In particular, data on various measures of nominal wages (e.g. negotiatied, compensation per hour) show a substantial pick up in labor costs in the aftermath of VAT changes, possibly reflecting workers' requests for stability in after-tax real wages. Finally, private consumption in Germany underperformed consumption in other euro-area countries, suggesting, at least qualitatively, that VAT hikes may have adverse effects on aggregate demand even when accompanied by commensurate reductions in payroll taxes.

All told, evidence from country-specific episodes as well as cross-sectional analysis suggests that VAT changes tend to be passed through to consumer prices quickly and, in many cases, nearly in full. Within the context of theoretical frameworks where producer prices are adjusted infrequently, this empirical observation appears easier to reconcile with the assumption that pre-tax prices are sticky and valued added tax changes are fully passed through.

from value-added taxes.

[30] The 2007 VAT hike did not apply to goods taxed at a reduced rate (such as food, entertainement, and books).

5.4 Alternative Pricing Assumptions

For ease of exposition, we described the transmission of the IX (and BAT) and VP policies under the assumption of producer currency pricing (PCP) and, thus, full pass-through of exchange rate changes to traded good prices. In this section we study the effects of these policies under alternative pricing assumptions. In particular, we present simulations of transitory IX and VP policies under local currency pricing (or LCP, when exporters set prices in the destination currency) and under dominant currency pricing (or DCP, when exporters both in the home and the foreign country set prices in the home currency).[31,32] There are two key takeaways from these experiments. First, given that IX policies directly affect trade prices, different assumptions about pass-through change their effects on export prices and hence affect transmission on imports and exports. Nonetheless, compared to the PCP case, the overall macroeconomic effects of IX remain qualitatively similar. Second, the transmission of VP policies is very little changed under different assumptions on exchange rate pass-through. This observation should come as no surprise given that, as we argued above, VP policies do not operate by directly affecting relative trade prices.

Figure 11 shows the response of the economy to a transitory implementation of IX under PCP (solid lines), DCP (crossed lines), and LCP (dashed lines). While the response of exports is virtually identical under DCP as under PCP, DCP implies somewhat greater expenditure switching away from imports, reflecting that foreign exporters under DCP are slow to adjust their ''at the dock" prices ($P_{X_t^*}$ in equation 62) so that home (cum-tariff) import prices rise by more. Accordingly, output rises a tad more under DCP than PCP. LCP is similar to DCP in inducing larger expenditure-switching away from imports than PCP. However, under LCP domestic exporters do not immediately reduce the prices at which they sell the domestic good in the foreign country (P_{Ht}^*) in response to the export subsidy, leading to a much more limited boost to domestic exports under LCP than PCP (or DCP). The smaller export boost under LCP translates into a smaller output rise than under PCP or DCP, though the qualitative responses are clearly aligned.

Figure 12 shows the response of the economy to a transitory implementation of VP under PCP (solid lines), LCP (dashed lines), and DCP (crossed lines). In all cases, monetary policy cuts interest

[31] We focus on policies that are eventually reversed given our interest in assessing the scope for IX and VP policies to provide macroeconomic stimulus. Our simulations allow wages to be sticky: As discussed before, when wage are sticky VP induces the competitive-enhancing effects which typically motivate the use of such policies (see, for example, Calmfors 1998).

[32] Casas, Diez, Gopinath, and Gourinchas (2016) study the implications of DCP in a standard New Keynesian model.

rates in response to weaker domestic demand, thus contributing to a depreciation of the exchange rate. Compared to PCP, the slower responsiveness of trade prices to such depreciation under LCP results in a smaller boost to net exports, so that output initially contracts even more. The drop in output, in contrast, is very similar to PCP under DCP as exporters benefit from the depreciation of the home currency. Notwithstanding these minor differences, the overall transmission of VP is very similar across pricing assumptions as these policies do not directly alter trade prices.

6 Conclusions

Our paper has presented a systematic analysis of the short-run effects of trade policies that are equivalent in a frictionless economy, namely a uniform increase in import tariffs and export subsidies (IX), an increase in value-added taxes accompanied by a payroll tax deduction (VP), and a border adjustment of corporate taxation (BAT). Using a New Keynesian dynamic general equilibrium model, we have studied the transmission of these policies and summarized conditions for their equivalence and neutrality.

Our paper has shown that the conventional view that trade policies do not stimulate output under flexible exchange rates – on the grounds that long-run balance of payment equilibrium should induce an immediate currency appreciation large enough to insulate trade prices – rests on fragile assumptions. Specifically, we argue that an increase in import tariffs and export subsidies is likely to elicit a much smaller response of the exchange rate than required for "full insulation" to hold, so that expenditure-switching effects show through to higher output. This output stimulus is largely driven by the export subsidy whereas tariffs tend to have a negligible or even contractionary effect on output.

We have also shown how VP can differ substantially from either IX or the BAT under full pass-through of taxes. VP tends to have contractionary effects on output if implemented on a temporary basis, reflecting that the competitiveness-enhancing effects of producer subsidies – the typical emphasis in the fiscal devaluation literature – are outweighed by the drag on consumption from the implied rise in the real interest rate (since the VAT is expected to decrease through time).

We caution that our results should not be interpreted as implying that IX policies are likely to provide a reliable and effective way of providing macroeconomic stimulus relative to other fiscal measures. Indeed, given that the stimulus is due to expenditure-switching, it is contingent on foreign economies refraining from retaliation, which seems unlikely if the IX policies are put in place for very long. Conversely, our analysis of VP should not be interpreted as dismissive of the possibility of using these policies to boost the economy; rather, in doing so, the program should be designed to pay keen attention to intertemporal substitution effects in addition to the competitiveness effects emphasized in the literature.

While we have used a workhorse open-economy model to facilitate comparison between the policies considered, trade policies in practice have widely divergent effects across different industries and economic sectors, with political economy considerations often playing a paramount role in determining the structure of tariffs or subsidies. In future work, it would be of interest to use a medium-scale model with a broader array of frictions and more sectoral differentiation to revisit some of the questions we have considered here.

References

[1] Angeletos, G.M. and C. Lian, (2017), "Dampening GE: from Micro to Macro", working paper.

[2] Auerbach, A., Devereux, M.P., Keen, M. and Vella, J. (2017), "Destination-Based Cash Flow Taxation", *Oxford University Center for Business Taxation*.

[3] Barattieri, A., Cacciatore, M., and F. Ghironi (2018), "Protectionism over the Business Cycle", NBER Working Paper No. 24353.

[4] Barbiero, O., Farhi, E., Gopinath, G., and Itskhoki, O. (2018), "The Economics of Border Adjustment", working paper.

[5] Benigno, G., Benigno, P. and Ghironi, F. (2007), "Interest Rate Rules for Fixed Exchange Rate Regimes", *Journal of Economic Dynamics and Control*, **31**, 2196-2211.

[6] Besley, T. J. and Rosen, H. S. (1999), "Sales Taxes and Prices: An Empirical Analysis", *National Tax Journal*, **52**, 157-178.

[7] Calvo, G. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, **12**, 383-398.

[8] Calmfors, L. (1998), "Macroeconomic Policy, Wage Setting, and Employment - What Difference Does The EMU Make?", *Oxford Review of Economic Policy*, pp. 125-151.

[9] Carbonnier, C. (2007), "Who Pays Sales Taxes? Evidence from French VAT Reforms, 1987-1999", *Journal of Public Economics*, **91**, 1219-1229.

[10] Casas, C.F., Diez, F., Gopinath, G., and P.O. Gourinchas, (2018), "Dominant Currency Paradigm", working paper.

[11] Cashin, D. and T. Unayama, (2016), "Measuring Intertemporal Substitution in Consumption: Evidence from a VAT Increase in Japan", Review of Economics and Statistics, **98**, 285-297.

[12] Corsetti, G., Dedola, L., and S. Leduc, (2010), "Optimal Monetary Policy in Open Economies", *Handbook of Monetary Economics*, Chapter 16, pp. 861-933.

[13] Costinot, A. and Werning, I. (2017), "The Lerner Symmetry Theorem: Generalizations and Qualifications", NBER Working Paper 23427.

[14] Devereux, M.B. and C. Engel, (2002), "Exchange Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect", *Journal of International Economics*, **49**, 913-940.

[15] de Mooij, R. and M. Keen, (2012), "Fiscal Devaluation and Fiscal Consolidation: The VAT in Troubled Times", IMF Working Paper 85.

[16] Eichengreen, B. J. (1981), "Sterling and the Tariff, 1929-32", Princeton Studies in International Finance, No. 48.

[17] Eichengreen, B.J. (2017), "Trade Policy and the Macroeconomy", IMF Mundell-Fleming Lecture.

[18] Erceg, C. J., Dale, W. H., and A. T. Levin, (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts", *Journal of Monetary Economics*, **46**, pp. 281-314.

[19] Farhi, E., Gopinath, G. and O. Itskhoki, (2014), "Fiscal Devaluations", *Review of Economic Studies*, **81**, 725-760.

[20] Farhi, E. and I. Werning, (2018), "Monetary Policy, Bounded rationality, and Incomplete Markets", working paper.

[21] Feldstein, M., (2017), "The Shape of US Tax Reform", *Project Syndicate*.

[22] Feldstein, M. S. and Krugman, P. R. (1990), "International Trade Effects of Value-Added Taxation", *Taxation in the Global Economy*, NBER Chapters (National Bureau of Economic Researcxh, Inc.) 263-282.

[23] Franco, F. (2013), "External Rebalancing in the EMU: The Case of Portugal", Working Paper.

[24] Galì, J. (2008), *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework,* Princeton, NJ: Princeton University Press.

[25] Galì, J. and Monacelli, T. (2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies*, **72**, 707-734.

[26] Gomes, S., Jacquinot, P. and Pisani, M. (2016), "Fiscal Devaluation in the Euro Area: A Model-Based Analysis", *Economic Modelling*, **52**, 58-70.

[27] Grossmann, G. M. (1980), "Border Tax Adjustment", *Journal of International Economics*, **10**, 117-128.

[28] Harberger, A. C. (1950), "Currency Depreciation, Income, and the Balance of Trade", *Journal of Political Economy*, **58**, 47-60.

[29] Irwin, D. A. (2013), "The Nixon Shock after 40 Years: The Import Surcharge Revisited", *World Trade Review*, **12**, pp. 29-66.

[30] Irwin, D. A. (2017), "Clashing Over Commerce: A History of U.S. Trade Policy", University of Chicago Press.

[31] Keynes, J.M., (1931), "Committee on Finance and Industry Report", Addendum, *Macmillan Report*.

[32] Krugman, P. (1982), "The Macroeconomics of Protection with a Floating Exchange Rate", Carnegie-Rochester Conference Series in Public Policy 16, pp. 141-182.

[33] Laursen, S. and Metzler, L. (1950), "Flexible Exchange Rate and the Theory of Employment", *Review of Economic and Statistics*, **32**, 281-99.

[34] Lerner, A. P. (1936), "The Symmetry Between Import and Export Taxes", Economics 3(11), 306-313.

[35] Linde', J. and Pescatori, A. (2017), "The Macroeconomic Effects of Trade Tariffs: Revisiting the Lerner Symmetry Result", IMF working paper.

[36] Lipińska, A. and Von Thadden, L. (2012), "On the (In)effectiveness of Fiscal Devaluations in a Monetary Union", Finance and Economics Discussion Series, **2012-71**.

[37] Maier, P. (2008), "A Wave of Protectionism? An Analysis of Economic and Political Considerations", Bank of Canada Working Paper 2008-2.

[38] McKay, A., Nakamura, E., and J. Steinsson (2016), "The Power of Forward Guidance Revisited", American Economic Review, 106(10), 3133-3158.

[39] McKinnon, Ronald I., (1966), "Intermediate Products and Differential Tariffs: a Generalization of Lerner's Symmetry Theorem", *Quarterly Journal of Economics*, **80**, 584-615.

[40] Meade, James E., (1974), "A Note on Border Tax Adjustments", *Journal of Political Economy*, **82,** 1013-1015.

[41] Mundell, R. (1961), "Flexible Exchange Rate and Employment Policy", *The Canadian Journal of Economics and Political Science*, **27**, 509-517.

[42] Neumeyer P. and F. Perri, (2001), " Business Cycles in Emerging Economies: The Role of Interest Rates", *Journal of Monetary Economics,* 52/2, 345-380.

[43] Ossa, R. (2014), "Trade Wars and Trade Talks with Data", *American Economic Review*, **104**(12), 4104-4146.

[44] Ossa, R. (2016), "Quantitative Models of Commercial Policy", *Handbook of Commercial Policy*, Ch. 4, 207-259.

[45] Poterba, J. M., Rotemberg, J.J., and L. H. Summers, (1986), "A Tax-Based Test for Nominal Rigidities", *American Economic Review*, **76**, 659-75.

[46] Schmitt-Grohé S. and M. Uribe, (2003), "Closing Small Open Economies", *Journal of International Economics,* **61**, 163-185.

[47] Schmitt-Grohé S. and M. Uribe, (2016), "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment", *Journal of Political Economy,* **124**, 1466-1514.

[48] Svensson L. and A. Razin, (1983), "The Terms of Trade and the Current Account: The Harberger-Laursen-Metzler Effect", *Journal of Political Economy, vol.* 91, issue 1, pp. 97-125.

[49] Woodford, M., (2018), "Monetary Policy Analysis When Planning Horizons Are Finite", prepared for the NBER Macroeconomics Annual Conference.

7 Appendix A. Model Equations

7.1 Households

Household $i \in H = [0,1]$ chooses $\{\bar{w}_t(i), w_t(i), n_t(i), c_t(i), a_{t,t+1}(i), B_{Ht}(i), B_{Ft}(i)\}$ to maximize

$$\max \mathbb{E}_0 \Sigma_{t=0}^{\infty} \beta^t \left[\frac{[c_t(i)]^{1-\sigma}}{1-\sigma} - \frac{[n_t(i)]^{1+\eta}}{1+\eta} \right] \tag{68}$$

$$s.t$$

$$\begin{array}{c} P_t c_t(i) + \Sigma_{t+1} q_{t,t+1} a_{t,t+1}(i) + B_{Ht}(i) + \varepsilon_t \left[B_{Ft}(i) + \frac{\chi}{2} \left(B_{Ft}(i) - \bar{B}_F \right)^2 \right] = \\ R_{t-1} B_{Ht-1}(i) + \varepsilon_t R_{t-1}^* B_{Ft-1}(i) + P_t a_{t-1,t}(i) + w_t(i) n_t(i) + \widetilde{\Pi}_t + T_t \end{array} \tag{69}$$

$$w_t(i) = \left\{ \begin{array}{cc} w_{t-1}(i) & w.p.\ \zeta_W \\ \bar{w}_t(i) & w.p.\ 1-\zeta_W \end{array} \right. \tag{70}$$

$$n_t(i) = \left[\frac{w_t(i)}{W_t} \right]^{-\gamma_n} N_t \tag{71}$$

where W_t is a wage index (descrobed below) and $q_{t,t+1}$ is the price of a state contingent Arrow security paying one unit of consumption in a specific state at time $t+1$. We assume that a complete set of Arrow securities is traded domestically so that perfect risk sharing within each country allows for simple aggregation. Equation (70) states that housheolds can only adjust their wage with probability ζ_W. Equation (71) is the firms' demand schedule for labor variety i, derived below.

Optimality conditions are

$$1 = \beta \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} R_t \right] \tag{72}$$

$$1 + \chi \left(B_{Ft}(i) - \bar{B}_F \right) = \beta \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} R_t^* \right] \tag{73}$$

$$E_t \zeta_W^{s-t} \sum C_s^{-\sigma} \left\{ \frac{[n_s(i)]^{\eta}}{C_s^{-\sigma}} \frac{\gamma_n}{(\gamma_n - 1)} - \frac{\bar{W}_t}{P_s} \right\} n_s(i) = 0 \tag{74}$$

7.2 Retailers

The problem of retailers is as described in the main text.

7.3 Producers

7.3.1 PCP pricing

Producer $i \in F = [0,1]$ chooses an optimal reset price $P_{Pt}(i)$, export prices $\{P^*_{Hs}(i)\}_{s\geq t}$ quantities $\{Y_{Hs}(i), Y^*_{Hs}(i)\}_{s\geq t}$ and employment $\left\{N_s(i), \{n_s(j;i)\}_j\right\}_{s\geq t}$ to maximize

$$\max E_t \sum_{s\geq t} \zeta_P^{s-t} \Lambda_{t,s} (1-\tau_s^\pi) \left\{ \frac{\bar{P}_{Pt}(i)\left[Y_{Hs}(i) + \frac{s^*}{s} Y^*_{Hs}(i)\right] - (1-\varsigma_s^v)\int w_s(j)\, n_s(j;i)\, dj}{P_s} \right\} \tag{75}$$

$$s.t.$$

$$Y_{Hs}(i) + \frac{s^*}{s} Y^*_{Hs}(i) = A_s N_s^\alpha(i) \tag{76}$$

$$N_s(i) = \left\{ \int [n_s(j;i)]^{\frac{\gamma_n - 1}{\gamma_n}} dj \right\}^{\frac{\gamma_n}{\gamma_n - 1}} \tag{77}$$

$$Y_{Hs}(i) = \left[\frac{\bar{P}_{Pt}(i)}{P_{Ps}}\right]^{-\gamma} Y_{Hs} \tag{78}$$

$$Y^*_{Ht}(i) = \left[\frac{P^*_{Hs}(i)}{P^*_{Hs}}\right]^{-\gamma} Y^*_{Ht} \tag{79}$$

$$P^*_{Hs}(i) = \frac{(1-\tau_s^\pi BAT_s)(1+\tau_s^{m*})}{(1+\varsigma_t^x)} \frac{\bar{P}_{Pt}(i)}{\varepsilon_s} \tag{80}$$

where s^* and s are the size of the foreign and home country respectively.

The optimality conditions for this problem are constraints $(76)-(80)$ as well as an optimal pricing condition as in the text:

$$\mathbb{E}_t \Sigma_{s=t}^\infty \zeta_P^{s-t} \Lambda_{t,s} \left[Y_{Hs}(i) + \frac{s^*}{s} Y^*_{Hs}(i)\right] (1-\tau_s^\pi) \frac{1}{P_s} \left[\overline{P}_{Pt}(i) - \frac{\gamma}{\gamma-1} \frac{(1-\varsigma_s^v) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0 \tag{81}$$

where W_s is the wage index

$$W_s = \left[\int [w_s(j)]^{1-\gamma_n} dj\right]^{\frac{1}{1-\gamma_n}} \tag{82}$$

7.3.2 LCP pricing

Producer i chooses optimal reset prices $\bar{P}_{Pt}(i)$ and $\bar{P}^*_{Xt}(i)$, where $\bar{P}^*_{Xt}(i)$ is the foreign currency price of domestic export net of tariffs, export prices $\{P^*_{Hs}(i)\}_{s\geq t}$, quantities $\{Y_{Hs}(i), Y^*_{Hs}(i)\}_{s\geq t}$ and employment $\left\{N_s(i), \{n_s(j;i)\}_j\right\}_{s\geq t}$ to maximize

$$\max E_t \sum_{s\geq t} \zeta_P^{s-t} \Lambda_{t,s} (1-\tau_s^\pi) \left\{ \frac{\bar{P}_{Pt}(i) Y_{Hs}(i) + \varepsilon_s P^*_{Xt}(i) \frac{(1+\varsigma_s^x)}{(1-\tau_s^\pi BAT_s)} \frac{s^*}{s} Y^*_{Hs}(i) - (1-\varsigma_s^v)\int w_s(j)\, n_s(j;i)\, dj}{P_s} \right\} \tag{83}$$

$$s.t.$$

$$Y_{Hs}(i) + \frac{s^*}{s} Y^*_{Hs}(i) = A_s N^\alpha_s (i) \tag{84}$$

$$N_s(i) = \left\{ \int [n_s(j;i)]^{\frac{\gamma_n - 1}{\gamma_n}} dj \right\}^{\frac{\gamma_n}{\gamma_n - 1}} \tag{85}$$

$$Y_{Hs}(i) = \left[\frac{\bar{P}_{Pt}(i)}{P_{Ps}} \right]^{-\gamma} Y_{Hs} \tag{86}$$

$$Y^*_{Ht}(i) = \left[\frac{P^*_{Hs}(i)}{P^*_{Hs}} \right]^{-\gamma} Y^*_{Ht} \tag{87}$$

$$P^*_{Hs}(i) = (1 + \tau^{m*}_s) P^*_{Xt}(i) \tag{88}$$

The optimality conditions for this problem are constraints (84) $-$ (88) and optimal pricing conditions for domestic and foreign markets:

$$\mathbb{E}_t \Sigma^\infty_{s=t} \zeta^{s-t}_P \Lambda_{t,s} (1 - \tau^\pi_s) \frac{Y_{Hs}(i)}{P_s} \left[\bar{P}_{Pt}(i) - \frac{\gamma}{\gamma - 1} \frac{(1 - \varsigma^v_s) W_s}{\alpha A_s N_s(i)^{\alpha - 1}} \right] = 0 \tag{89}$$

$$\mathbb{E}_t \Sigma^\infty_{s=t} \zeta^{s-t}_P \Lambda_{t,s} (1 - \tau^\pi_s) \frac{Y^*_{Hs}(i)}{P_s} \left[\varepsilon_s \frac{(1 + \varsigma^x_s)}{(1 - \tau^\pi_s BAT_s)} P^*_{Xt}(i) - \frac{\gamma}{\gamma - 1} \frac{(1 - \varsigma^v_s) W_s}{\alpha A_s N_s(i)^{\alpha - 1}} \right] = 0 \tag{90}$$

where W_s is the wage index

$$W_s = \left\{ \int [w_s(j)]^{1 - \gamma_n} dj \right\}^{\frac{1}{1 - \gamma_n}} \tag{91}$$

An analogous problem for the foreign producers yield

$$\mathbb{E}_t \Sigma^\infty_{s=t} \zeta^{s-t}_P \Lambda_{t,s} \frac{Y^*_{Fs}(i)}{P_s} \left[\bar{P}^*_{Pt}(i) - \frac{\gamma}{\gamma - 1} \frac{W^*_s}{\alpha A^*_s N^*_s(i)^{\alpha - 1}} \right] = 0 \tag{92}$$

$$\mathbb{E}_t \Sigma^\infty_{s=t} \zeta^{s-t}_P \Lambda_{t,s} \frac{Y_{Fs}(i)}{P_s} \left[\frac{1}{\varepsilon_s} (1 + \varsigma^{x*}_s) \bar{P}_{X^*t}(i) - \frac{\gamma}{\gamma - 1} \frac{W^*_s}{\alpha A^*_s N^*_s(i)^{\alpha - 1}} \right] = 0 \tag{93}$$

where

$$P_{Fs}(i) = \frac{(1 + \tau^m_s) \bar{P}_{X^*t}(i)}{(1 - \tau^v_s)}$$

8 Equilibrium equations

Equations (94)$-$(125) determine the equilibrium process $\{\Psi(s^t)\}_{s^t \in (S)^t, t \geq 0}$. For ease of exposition we we group elements of Ψ into variables that we associate with households optimality conditions,Ψ_{HH} and Ψ^*_{HH} abroad, retailers optimality conditions, Ψ_{RE} and Ψ^*_{RE}, firms optimality conditions, Ψ_{FI}

and Ψ^*_{FI}, price indexes, Ψ_{PI} and Ψ^*_{PI}, and market clearing conditions, Ψ_{MC}. We have that $\Psi = \{\Psi_{HH}, \Psi^*_{HH}, \Psi_{RE}, \Psi^*_{RE}, \Psi_{FI}, \Psi^*_{FI}, \Psi_P, \Psi^*_P, \Psi_{MC}\}$

Households optimality

$\Psi_{HH} = \left\{w_t(i), \bar{W}_t, n_t(i), C_t, B_{Ht}\right\}$ (leaving out budget constraint and B_{Ft})

$$w_{t+1}(i) = \begin{cases} w_t(i) & w.p.\ \zeta_W \\ \bar{W}_{t+1} & w.p.\ 1-\zeta_W \end{cases} \tag{94}$$

$$E_t \zeta_W^{s-t} \sum C_s^{-\sigma} \left[\frac{[n_s(i)]^{\eta}}{C_s^{-\sigma}} \frac{\gamma_n}{(\gamma_n - 1)} - \frac{\bar{W}_t}{P_s} \right] n_s(i) = 0 \tag{95}$$

$$n_t(i) = \left(\frac{w_t(i)}{W_t} \right)^{-\gamma_n} N_t \tag{96}$$

$$1 = \beta \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} R_t \right] \tag{97}$$

$$1 + \chi\left(B_{Ft}(i) - \bar{B}_F\right) = \beta \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} R_t^* \right] \tag{98}$$

and symmetric conditions for $\Psi^*_{HH} = \left\{w^*_t(i), \bar{W}^*_t, n^*_t(i), C^*_t, B^*_{Ft}\right\}$ abroad

Retailers optimality

$\Psi_{RE} = \{Y_{Ht}, Y_{Ft}, Y_{Ht}(i), Y_{Ft}(i)\}$

$$Y_{Ht} = \omega \left[\frac{P_{Ht}}{P_t} \right]^{-\theta} C_t \tag{99}$$

$$Y_{Ft} = (1-\omega) \left[\frac{P_{Ft}}{(1 - \tau_t^{\pi} BAT_t) P_t} \right]^{-\theta} C_t \tag{100}$$

$$Y_{Ht}(i) = \left(\frac{P_{Pt}(i)}{P_{Pt}} \right)^{-\gamma} Y_{Hs} \tag{101}$$

$$Y_F(i) = \left(\frac{P_{Ft}(i)}{P_{Ft}} \right)^{-\gamma} Y_{Ft} \tag{102}$$

and symmetric conditions for $\Psi_{RE} = \{Y^*_{Ft}, Y^*_{Ht}, Y^*_{Ft}(i), Y^*_{Ht}(i)\}$

Firms optimality

$\Psi_{FI} = \left\{ P^*_{Hs}(i), P_{Ft}(i), P_{Pt}(i), P^*_{Pt}(i), \overline{P}_{Pt}(i), \overline{P}^*_{Pt}(i), \bar{P}^*_{Xt}(i), \bar{P}^*_{X^*t}(i), P^*_{Xt}(i), P_{X^*t}(i) \right\}$

$$P^*_{Ht}(i) = (1 + \tau_t^{m*}) P^*_{Xt}(i) \tag{103}$$

$$P_{Ft}(i) = \frac{1 + \tau_t^m}{1 - \tau_t^v} P_{X^*t}(i) \tag{104}$$

$$P_{Pt+1}(i) = \begin{cases} P_{Pt}(i) & w.p.\ \zeta_p \\ \bar{P}_{Pt+1}(i) & w.p.\ 1-\zeta_p \end{cases} \tag{105}$$

$$P^*_{Pt+1}(i) = \begin{cases} P^*_{Pt}(i) & w.p.\ \zeta_p \\ \bar{P}^*_{Pt+1}(i) & w.p.\ 1-\zeta_p \end{cases} \tag{106}$$

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \left[Y_{Hs}(i) + \tfrac{s^*}{s} Y^*_{Hs}(i) \right] \tfrac{1}{P_s} \left[\overline{P}_{Pt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{(1-\varsigma_s^v) W_s}{\alpha A_s N_s(i)^{\alpha-1}} \right] = 0 \quad PCP \tag{107a}$$

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \tfrac{Y_{Hs}(i)}{P_s} \left[\bar{P}_{Pt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{(1-\varsigma_s^v) W_s}{\alpha A_s N_s(i)^{\alpha-1}} \right] = 0 \quad LCP \tag{107b}$$

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda^*_{t,s} \left[\tfrac{s}{s^*} Y_{Fs}(i) + Y^*_{Fs}(i) \right] \tfrac{1}{P^*_s} \left[\overline{P}^*_{Pt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{W^*_s}{\alpha A_s N^*_s(i)^{\alpha-1}} \right] = 0 \quad PCP \tag{108a}$$

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda^*_{t,s} \tfrac{Y^*_{Fs}(i)}{P^*_s} \left[\overline{P}^*_{Pt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{W^*_s}{\alpha A_s N^*_s(i)^{\alpha-1}} \right] = 0 \quad LCP \tag{108b}$$

$$P^*_{Ht}(i) = \tfrac{(1-\tau^{\pi} BAT_t)(1+\tau_t^{m*})}{(1+\varsigma_t^x)} \tfrac{P_{Pt}(i)}{\varepsilon_t} \quad PCP \tag{109a}$$

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \tfrac{Y^*_{Hs}(i)}{P_s} \left[\varepsilon_s \tfrac{(1+\varsigma_s^x)}{(1-\tau^{\pi} BAT_s)} \bar{P}^*_{Xt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{(1-\varsigma_s^v) W_s}{\alpha A_s N_s(i)^{\alpha-1}} \right] = 0 \quad LCP \tag{109b}$$

$$P_{Ft}(i) = \tfrac{1+\tau_t^m}{1-\tau_t^v} \tfrac{P^*_{Pt}(i)\varepsilon_t}{(1+\varsigma_t^{x*})} \quad PCP \tag{110a}$$

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda^*_{t,s} \tfrac{Y^*_{Hs}(i)}{P_s} \left[\tfrac{(1+\varsigma_s^{x*})}{\varepsilon_s} \bar{P}_{X^*t}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{W^*_s}{\alpha A_s N^*_s(i)^{\alpha-1}} \right] = 0 \quad LCP \tag{110b}$$

$$P_{X^*t}(i) = \bar{P}_{X^*t}(i) \quad PCP \tag{111a}$$

$$P_{X^*t+1}(i) = \begin{cases} P_{X^*t}(i) & w.p.\ \zeta_W \\ \bar{P}_{X^*t+1}(i) & w.p.\ 1-\zeta_W \end{cases} \quad LCP \tag{111b}$$

and symmetric conditions for $\Psi^*_{FI} = \left\{ N^*_t(i), P_{Ft}(i), P^*_{Pt}(i), \overline{P}^*_{Pt}(i), \bar{P}_{X^*t}(i), P_{X^*t}(i) \right\}$

Price indexes

$\Psi_{PI} = \{P_t, P_{Ht}, P_{Pt}, P_{Ft}, W_t\}$

$$P_t = \left[\omega P_{Ht}^{1-\theta} + (1-\omega) \left(\frac{P_{Ft}}{1-\tau^{\pi}_t BAT_t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{112}$$

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}} \tag{113}$$

$$P_{Pt} = P_{Ht}(1-\tau^v_t) \tag{114}$$

$$P_{Ft} = \left[\int_0^1 P_{Ft}(i)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}} \tag{115}$$

$$W_s = \left[\int [w_s(j)]^{1-\gamma_n} dj \right]^{\frac{1}{1-\gamma_n}} \tag{116}$$

and symmetric conditions for $\Psi^*_{PI} = \{P^*_t, P^*_{Ft}, P^*_{Pt}, P^*_{Ht}, W^*_t\}$

Market Clearing

$\Psi_{MC} = \{N_t(i), N^*_t(i), N_t, N^*_t, B_{Ft}, B^*_{Ht}, \varepsilon_t, R_t, R^*_t\}$

$$Y_{Ht}(i) + \frac{s^*}{s} Y^*_{Ht}(i) = A_t N^{\alpha}_t(i) \tag{117}$$

$$Y^*_{Ft}(i) + \frac{s}{s^*} Y_{Ft}(i) = A_t N^{*\alpha}_t(i) \tag{118}$$

$$N_t = \int_{j \in F} N_t(j)\, dj \tag{119}$$

$$N^*_t = \int_{j \in F} N^*_t(j)\, dj \tag{120}$$

$$B_{Ft} + B_{Ft}^{*} = 0 \tag{121}$$

$$B_{Ht} + B_{Ht}^{*} = 0 \tag{122}$$

$$\varepsilon_t B_{Ft} + B_{Ht} = \varepsilon_t B_{Ft-1} R_{t-1}^{*} + B_{Ht-1} R_{t-1} + \frac{s^*}{s} \varepsilon_t \frac{P_{Ht}^{*}}{1+\tau_t^{m*}} Y_{Ht}^{*} - \frac{\left(1-\tau_t^{v}\right) P_{Ft}}{\left(1+\tau_t^{m}\right)} Y_{Ft} \tag{123}$$

$$R_t^{*} = \frac{1}{\beta} \left(\frac{P_{pt}^{*}}{P_{pt-1}^{*}} \right)^{\varphi_\pi} \left(\frac{Y_{Ft} + Y_{Ft}^{*}}{Y_{Ft}^{flex} + Y_{Ft}^{*flex}} \right)^{\varphi_y} \left(\frac{\varepsilon_t}{\bar{\varepsilon}_t} \right)^{\varphi_\varepsilon^{*}} \tag{124}$$

$$R_t = \frac{1}{\beta} \left(\frac{P_{pt}}{P_{pt-1}} \right)^{\varphi_\pi} \left(\frac{Y_{Ht} + Y_{Ht}^{*}}{Y_{Ht}^{flex} + Y_{Ht}^{*flex}} \right)^{\varphi_y} \left(\frac{\varepsilon_t}{\bar{\varepsilon}_t} \right)^{\varphi_\varepsilon} \tag{125}$$

9 Appendix B. Proof of Proposition 1

We start by giving a formal definition of neutrality of a policy and equivalence between policies.

Definition 1. *Let the evolution of the policy regime s_t before the implementation of a given policy be determined by the stochastic process $\{S, \Omega\}$ and let $\left\{\tilde{S}, \tilde{\Omega}\right\}$ describe the process for policy after the implementation of the policy. We say that the implementation of the policy has no allocative effects or that it is neutral if for any equilibrium process $\{\Psi\left(s^t\right)\}_{s^t \in (S)^t, t \geq 0}$ before the implementation of the policy there is an equilibrium process $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t \in \left(\tilde{S}\right)^t, t \geq 0}$ under which the real allocation*

$\tilde{\Xi} = \left\{ \tilde{C}\left(s^t\right), \tilde{C}^{*}\left(s^t\right), \left\{\tilde{n}\left(i, s^t\right)\right\}, \left\{\tilde{n}^{*}\left(i, s^t\right)\right\}, \left\{\tilde{Y}_H\left(i, s^t\right)\right\}, \left\{\tilde{Y}_F\left(i, s^t\right)\right\}, \left\{\tilde{Y}_H^{*}\left(i, s^t\right)\right\}, \left\{\tilde{Y}_F^{*}\left(i, s^t\right)\right\} \right\}_{s^t \in \left(\tilde{S}\right)^t, t \geq 0}$

is unaffacted: that is there exists a sequence of functions $\mu_t : \left(\tilde{S}\right)^t \to (S)^t$ such that for each element $\tilde{\varkappa}$ in $\tilde{\Xi}$

$$\tilde{\varkappa}\left(s^t\right) = \varkappa\left(\mu_t\left(s^t\right)\right) \text{ for any } s^t \in \left(\tilde{S}\right)^t, t \geq 0$$

where $\varkappa$ is the corresponding element in the real allocation Ξ which is part of equilibrium process Ψ. We also say that the two policies described by $\{S, \Omega\}$ and $\left\{\tilde{S}, \tilde{\Omega}\right\}$ are equivalent.

Definition 2 gives a definition of a unilateral IX policy that generalizes the example used in Lemma 1.

Definition 2. *Let the evolution of trade policy at home and abroad $s_t = \left(\tau_t^{m}, \varsigma_t^{x}, \tau_t^{m*}, \varsigma_t^{x*}\right)$ before the IX implementation be determined by the stochastic process $\{S, \Omega\}$. A unilateral implementation of IX of size δ that is anticipated w.p. π^{IX} and is reversed w.p. ρ, is described by a new stochastic process $\left\{GP^{IX}, \Omega^{IX}\right\}$ such that $GP^{IX} = S \cup S^{IX}$ where the set of states is given by*

$$S^{IX} = \sigma_\delta^{IX}\left([S]\right) \tag{126}$$

where $\forall\left(\tau_t^m, \varsigma_t^x, \tau_t^{m*}, \tau_t^{x*}\right) \in S$ *the function* $\sigma_\delta^{IX}\left(\tau_t^m, \varsigma_t^x, \tau_t^{m*}, \tau_t^{x*}\right) = \left(\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^{m*}, \tau_t^{x*}\right)$ *with*

$$\frac{1+\tilde{\tau}_t^m}{1+\tau_t^m} = \frac{1+\tilde{\varsigma}_t^x}{1+\varsigma_t^x} = 1+\delta \tag{127}$$

and

$$\Omega^{IX} = \begin{bmatrix} \left(1-\pi^{IX}\right)\Omega & \pi^{IX}\Omega \\ (1-\rho)\,\Omega & \rho\Omega \end{bmatrix} \tag{128}$$

where the ordering of states in the matrix Ω^{IX} *is the obvious one.*

Notice that the process $\left\{GP^{IX}, \Omega^{IX}\right\}$ does not encompass the possibility of retaliation.

Proposition 1. *In an economy with flexible exchange rates, a unilateral implementation of IX of size* δ *has no allocative effect if*

1. *It is unanticipated, permanent, and there is no probability of retaliation;*
2. *Foreign holdings of home currency are always zero*;
3. *Export prices are set in producer currency (PCP) or prices are flexible*

The only effect of the policy is to cause a δ *percent appreciation of the currency* ε_t*.*

Proof. Condition 1 implies that $\pi^{IX} = 0$ and $\rho = 1$. In this case the transition matrix is simply

$$\Omega^{IX} = \begin{bmatrix} \Omega & 0 \\ 0 & \Omega \end{bmatrix} \tag{129}$$

Let $\left\{\Psi\left(s^t\right)\right\}_{s^t \in (S^T)^t, t\geq 0}$ denote an equilibrium process before the IX implementation, i.e. when s_t is governed $\{S, \Omega\}$.

Consider now a process $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t \in \left(\tilde{S}\right)^t, t\geq 0}$ with an unanticipated permanent IX such that, for each element $\tilde{\varkappa}$ of $\tilde{\Psi}$, *other than the nominal exchange rate,* $\tilde{\varepsilon}_t$*, and home currency producer prices of foreign exporters,* $\tilde{P}_{X^*t}(i)$, we have

$$\tilde{\varkappa}\left(\tilde{s}^t\right) = \varkappa\left(\mu_t\left(\tilde{s}^t\right)\right) \qquad \forall \tilde{s}^t \in \left(GP^{IX}\right)^t, \quad \forall t \geq 0 \tag{130}$$

where $\varkappa$ is the corresponding element of the equilibrium process Ψ without IX and function μ_t maps all histories in which IX is implemented into a history in which IX is not implemented: i.e. $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$, $\mu_t\left(\tilde{s}^t\right) = s^t = (s_1, ..., s_t) \in (S)^t$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_i) & if\ \tilde{s}_i \in GP^{IX} \end{cases}$$

For ease of notation in what follows, for any $\tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$, we let $\tilde{\varkappa}_t = \tilde{\varkappa}(\tilde{s}^t)$ and $\varkappa_t = \varkappa\left(\mu_t^{IX}(\tilde{s}^t)\right)$.

The nominal exhange rate and the home currency producer prices of foreign exporters are $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$

$$\tilde{\varepsilon}_t = \begin{cases} \varepsilon_t & if\ \tilde{s}_t \in S \\ \frac{\varepsilon_t}{1+\delta} & if\ \tilde{s}_t \in S^{IX} \end{cases} \tag{131}$$

$$\tilde{P}_{X^*t}(i) = \begin{cases} P_{X^*t}(i) & if\ s_t \neq s^{TW} \\ \\ \frac{1}{1+\delta} P_{X^*t}(i) & if\ s_t = s^{TW} \end{cases} \tag{132}$$

We want to show that $\left\{\tilde{\Psi}(s^t)\right\}_{s^t \in (GP^{IX})^t, t \geq 0}$ is an equilibrium.

We first show that $\tilde{\Psi}(s^t)$ satisfies all of the equations directly affected by the tariffs and export subsidy change when $\tilde{s}_t = (\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^{m*}, \tau_t^{x*}) \in S^{IX}$. These equations are the laws of one price $(109a)-(110a)$, the tax pass-through equations $(104)-(103)$, and the balance of payment equilibrium (123) Considering the law of one price for domestic goods at an history $\tilde{s}^t$ such that $\tilde{s}_t = (\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^{m*}, \tau_t^{x*}) \in S^{IX}$ and letting $\left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_t) = (\tau_t^m, \tau_t^x, \tau_t^{m*}, \tau_t^{x*}) \in S$ we see that

$$\begin{aligned} \tilde{P}^*_{H,t}(i) &= P^*_{H,t}(i) = P_{H,t}(i) \frac{1+\tau_t^{m*}}{1+\sigma_t^x} \frac{1}{\varepsilon_t} && (133) \\ &= \tilde{P}_{H,t}(i) \frac{1+\tau_t^{m*}}{(1+\tilde{\sigma}_t^x)} \frac{1}{\tilde{\varepsilon}_t} && (134) \end{aligned}$$

where the first and third equalities follow from (130), (131) and (127) and the second from the fact that Ψ is an equilibrium. An analogous arguemt holds for $(110a)$ and (104).

Consider now the balance of payment equilibrium which, under condition 2 $B^*_{H-1} = 0 = \bar{B}$ and $\chi = \infty$, is

$$\tilde{B}_{Ft} = \tilde{B}_{Ft-1}\tilde{R}^*_{t-1} + \frac{\tilde{P}^*_{Ht}}{1+\tau_t^{m*}}\tilde{Y}^*_{Ht} - \frac{\tilde{P}_{Ft}}{(1+\tilde{\tau}_t^m)\tilde{\varepsilon}_t}\tilde{Y}_{Ft}$$

to see that this is satisfied, let again $\left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_t) = (\tau_t^m, \tau_t^x, \tau_t^{m*}, \tau_t^{x*}) \in S$ to get

$$\begin{aligned} \tilde{B}_{Ft} &= B_{Ft} = B_{Ft-1}R^*_{t-1} + \frac{P^*_{Ht}}{1+\tau_t^{m*}}Y^*_{Ht} - \frac{P_{Ft}}{(1+\tau_t^m)\varepsilon_t}Y_{Ft} \\ &= \tilde{B}_{Ft-1}\tilde{R}^*_{t-1} + \frac{\tilde{P}^*_{Ht}}{1+\tau_t^{m*}}\tilde{Y}^*_{Ht} - \tilde{P}_{Ft}\frac{\tilde{y}_{Ft}}{(1+\tilde{\tau}_t^m)\tilde{\varepsilon}_t} \end{aligned}$$

where the first and third equality follow from (130) (131) and (127) and the second from the fact that Ψ is an equilibrium .

We then need to check that the adjustment of the nominal exchange rate and local currency producer prices of exports in $(131)-(132)$ does not induce violations in other equilibrium equations.

Under PCP $\tilde{P}^*_{Xt}(i)$ and $\tilde{P}_{X^*t}(i)$ only affect (104) and (103), i.e. they are definitions.The exchange rate ε_t affects optimal holdings of foreign bonds (98) and an anlogous condition abroad. As long as $\pi^{IX} = 0$ and $\rho = 1$ we have that $\forall s^t \in \left(GP^{IX}\right)^t$, if $s^{t+1} \in \left(GP^{IX}\right)^t$ has positive probability, $\Pr\left\{s^{t+1}\,|s^t\right\} > 0$, the appreciation is identical across equilbria:

$$\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t} = \frac{\varepsilon_{t+1}}{\varepsilon_t}$$

and since these conditions only depend on exchange rate appreciation they are satisfied.

10 Appendix C. Proof of Lemma 1

Consider an IX policy subject to policy reversal and characterized by $\left\{S^T, \Omega^T\right\}$ where $S^T = \left\{s^{NT}, s^{IX}\right\}$. In state $\left(s^{NT}\right)$ no country levies any taxes and in the second state $\left(s^{IX}\right)$ the home country unilaterally raises import tariffs and export subsidies by the same amount δ. The transition matrix is

$$\Omega^T = \begin{bmatrix} 1 & 0 \\ 1-\rho & \rho \end{bmatrix} \tag{135}$$

Consider also an IX policy that triggers retaliation and characterized by $\left\{S^R, \Omega^R\right\}$, where $S^R = \left\{S^T, s^{TW}\right\}$. S^T includes the same two states as described above but in s^{TW} the foreign country retaliates with a symmetric policy (i.e. $\tau^m_t = \varsigma^x_t = \tau^{m*}_t = \varsigma^{x*}_t = \delta$). In this case the transition probability matrix is:

$$\Omega^R = \begin{bmatrix} 1 & 0 & 0 \\ (1-\pi)(1-\rho) & \rho & \pi(1-\rho) \\ 1-\varphi & 0 & \varphi \end{bmatrix} \tag{136}$$

Lemma 1 *If export prices are set in producer currency, a unilateral implementation of IX with policy reversal, i.e.* s_t *governed by* $\left\{S^T, \Omega^T\right\}$, *implements the same equilibrium allocation as a unilateral implementation of IX that triggers retaliation,* i.e. s_t *governed by* $\left\{S^R, \Omega^R\right\}$, *coupled with international transfers that satisfy:*

$$T^I_{t_1} = -\frac{\delta}{1+\delta}\left[B_{F,t_1-1}R^*_{t_1-1}\varepsilon_{t_1} + B_{H,t_1-1}R_{t_1-1}\right]$$

$$T^I_{t_2} = \delta\left[B_{F,t_2-1}R^*_{t_2-1}\varepsilon_{t_2} + B_{H,t_2-1}\frac{R_{t_2-1}}{\pi_{t_2}}\right]$$

where t_1 *is the first time the economy transits to the retaliation state* s^{TW} *and* $t_2 > t_1$ *is the first time it leaves the retaliation state* s^{TW}.

Proof. Let $\{\Psi(s^t)\}_{s^t\in(S^T)^t,t\geq 0}$ be an equilibrium with no international transfers and no retaliation, i.e. $T(s^t)=0\ \forall s^t\in\left(S^T\right)^t$.

Consider now the process $\left\{\tilde{\Psi}(s^t)\right\}_{s^t\in(S^R)^t,t\geq 0}$ such that, for each element $\tilde{\varkappa}$ of $\tilde{\Psi}$, *other than bond holdings and local currency producer prices of exports*, we have

$$\tilde{\varkappa}\left(s^t\right)=\varkappa\left(\mu_t\left(s^t\right)\right)\qquad \forall s^t\in\left(S^R\right)^t,\ \ \forall t\geq 0 \tag{137}$$

where $\varkappa$ is the corresponding element of the equilibrium process Ψ without trade wars and function μ_t maps all histories in which a trade war occurs into a history in which no taxes are levied: that is $\forall s^t=(s_1,...,s_t)\in\left(S^R\right)^t$, $\mu_t\left(s^t\right)=\tilde{s}^t=(\tilde{s}_1,...,\tilde{s}_t)\in\left(S^T\right)^t$ where $\forall i\geq 1$

$$\tilde{s}_i=\begin{cases} s_i & if\ s_i\neq s^{TW} \\ s^{NT} & if\ s_i=s^{TW}\end{cases}.$$

For ease of notation in what follows, for any $s^t=(s_1,....,s_t)\in\left(S^R\right)^t$, we let $\tilde{\varkappa}_t=\tilde{\varkappa}(s^t)$ and $\varkappa_t=\varkappa\left(\mu\left(s^t\right)\right).$

Bond holdings and local currency producer prices of exports satisfy $\forall s^t=(s_1,....,s_t)\in\left(S^R\right)^t$

$$\frac{\tilde{B}_{F,t}}{B_{F,t}}=\frac{\tilde{B}_{H,t}}{B_{H,t}}=\begin{cases} 1 & if\ s_t\neq s^{TW} \\ \frac{1}{1+\delta} & if\ s_t=s^{TW}\end{cases} \tag{138}$$

$$\frac{\tilde{P}_{X^*t}}{P_{X^*t}}=\frac{\tilde{P}^*_{Xt}}{P^*_{Xt}}=\begin{cases} 1 & if\ s_t\neq s^{TW} \\ \frac{1}{1+\delta} & if\ s_t=s^{TW}\end{cases} \tag{139}$$

We want to show that $\left\{\tilde{\Psi}(s^t)\right\}_{s^t\in(S^R)^t,t\geq 0}$ is an equilibrium when international transfers satisfy

$$\tilde{T}\left(s^t\right)=\begin{cases} 0 & if\ s_{t-1}\neq s^{TW}\text{ and }s_t\neq s^{TW} \\ -\frac{\delta}{1+\delta}\left[\tilde{B}_{F,t-1}\tilde{R}^*_{t-1}\tilde{\varepsilon}_t+\tilde{B}_{H,t-1}\tilde{R}_{t-1}\right] & if\ s_{t-1}\neq s^{TW}\text{ and }s_t=s^{TW} \\ \frac{\delta}{1+\delta}\left[\tilde{B}_{F,t-1}\tilde{R}^*_{t-1}\tilde{\varepsilon}_t+\tilde{B}_{H,t-1}\tilde{R}_{t-1}\right] & if\ s_{t-1}=s^{TW}\text{ and }s_t\neq s^{TW}\end{cases}. \tag{140}$$

It is straightforward to check that if Ψ_t is an equilibrium then $\tilde{\Psi}_t$ satisfies all equilibrium equations other than (123). When $s_t=s^{TW}$ the only conditions that need to be checked are the laws of one price $(109a)-(110a)$ and the tax pass-through equations $(104)-(103)$ which are satisfied under (139). All the other equations are clearly satisfied by construction of $\tilde{\Psi}$, and by the fact that the probability of leaving the unilateral IX state is the same in (135) and (136).

Consider now the balance of payment equilibrium (123) which we rewrite as follows

$$\tilde{A}_t = \tilde{A}_{t-1}\tilde{r}^a_t + N\tilde{X}_t + \tilde{T}_t$$

where

$$\tilde{A}_{t-1} = \tilde{B}_{F,t-1}\tilde{\varepsilon}_{t-1} + \tilde{B}_{ht-1}$$

$$r^a_t = \frac{\left[\tilde{B}_{F,t-1}\tilde{R}^*_{t-1}\tilde{\varepsilon}_t + \tilde{B}_{ht-1}\tilde{R}_{t-1}\right]}{\tilde{A}_{t-1}}$$

$$N\tilde{X}_t = \varepsilon_t \frac{P^*_{Ht}}{1+\tau^{m*}_t}\frac{s^*}{s}Y^*_{Ht} - \frac{(1-\tau^v_t)\,P_{Ft}}{(1+\tau^m_t)}Y_{Ft}$$

Take any history $\tilde{s}^\infty = (\tilde{s}_1, ..., \tilde{s}_t, ...) \in \left(S^R\right)^\infty$ such that $s_i = s^{TW}$ $\exists i$. Let t_1 and t_2 satisfy $s_{t_1} = s^{TW}$, $s_{t_1-1} \neq s^{TW}$, $s_{t_2} \neq s^{TW}$, $s_{t_2-1} = s^{TW}$. At t_1 we have

$$\begin{aligned} \tilde{A}_{t_1} &= \frac{A_{t_1}}{1+\delta} \\ &= \frac{A_{t_1-1}r^a_{t_1} + NX_{t_1}}{1+\delta} \\ &= A_{t_1-1}r^a_{t_1} + \frac{NX_{t_1}}{1+\delta} - \frac{\delta}{1+\delta}A_{t_1-1}r^a_{t_1} \\ &= \tilde{A}_{t_1-1}\tilde{r}^a_{t_1} + N\tilde{X}_{t_1} + \tilde{T}_{t_1} \end{aligned} \tag{141}$$

where, the first follows from (138) given $s_{t_1} = s^{TW}$; the second from the fact that Ψ is an equilibrium; and the last follows from the fact that (138) imply $A_{t_1-1}r^a_{t_1} = \tilde{A}_{t_1-1}\tilde{r}^a_{t_1}$ given $s_{t_1-1} \neq s^{TW}$ together with the fact that $s_{t_1} = s^{TW}$ implies $N\tilde{X}_{t_1} = \frac{NX_{t_1}}{1+\delta}$ and that $\tilde{T}_{t_1}$ is given by (140).

As long as the trade war is in place (138) readily imply that $\forall s$ and $t_1 < s < t_2$

$$\begin{aligned} \tilde{A}_s &= \frac{A_s}{1+\delta} \\ &= \tilde{A}_{s-1}\tilde{r}^a_s + N\tilde{X}_s \end{aligned} \tag{142}$$

And when it ends, at t_2, we have

$$\begin{aligned} \tilde{A}_{t_2} &= A_{t_2} \\ &= A_{t_2-1}r^a_{t_2} + NX_{t_2} \\ &= \frac{A_{t_2-1}r^a_{t_2}}{1+\delta} + NX_{t_2} + \frac{\delta}{1+\delta}A_{t_2-1}r^a_{t_2} \\ &= \tilde{A}_{t_2-1}\tilde{r}^a_{t_2} + N\tilde{X}_{t_2} + \tilde{T}_{t_2} \end{aligned} \tag{143}$$

where we are using again (138) as in (141).

11 Appendix D. Proof of Proposition 2

We start by giving a definition of a permanent unexpected implemnation of BAT and VP.

Definition 2. *Let the evolution of trade policy at home and abroad*

$$s_t = (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, BAT_t, \tau_t^{m*}, \varsigma_t^{x*})$$

be determined by the stochastic process $\{S, \Omega\}$ *that satisfies* $BAT = 0\ \forall s \in S$. *A unilateral implementation of BAT and an implementation VP of size* $\frac{\delta}{1+\delta}$ *are described by stochastic processes* $\{GP^{BAT}, \Omega^{BAT}\}$ *and* $\{GP^{VP}, \Omega^{VP}\}$ *respectively such that* $GP^{BAT} = S \cup S^{BAT}$ *and* $GP^{VP} = S \cup S^{VP}$ where $S^{BAT} = \sigma_\delta^{BAT}([S])$ and $S^{VP} = \sigma_\delta^{VP}([S])$. The functions σ_δ^{BAT} and σ_δ^{VP} satsify $\forall (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}) \in S$

$$\sigma_\delta^{BAT}(\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}) = (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*})$$

$$\sigma_\delta^{VP}(\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}) = (\tau_t^m, \varsigma_t^x, \tilde{\tau}_t^v, \tilde{\varsigma}_t^v, , \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*})$$

with

$$\frac{1-\tilde{\tau}_t^v}{1-\tau_t^v} = \frac{1-\tilde{\varsigma}_t^v}{1-\varsigma_t^v} = \frac{1}{1+\delta}$$

The transition matrices are given by

$$\Omega^{BAT} = \begin{bmatrix} \left(1-\pi^{BAT}\right)\Omega & \pi^{BAT}\Omega \\ \left(1-\rho^{BAT}\right)\Omega & \rho^{BAT}\Omega \end{bmatrix} \tag{144}$$

$$\Omega^{VP} = \begin{bmatrix} \left(1-\pi^{VP}\right)\Omega & \pi^{VP}\Omega \\ \left(1-\rho^{VP}\right)\Omega & \rho^{VP}\Omega \end{bmatrix} \tag{145}$$

where the ordering of states in the transition probability matrices is the obvious one.

Proposition 2. *If monetary policy is described by* (63), *wages are flexible, and the three conditions of Proposition 1 are satisfied, the following policies are equivalent and neutral:*

1. *A permanent unexpected IX policy of size* δ;

2. *A permanent unexpected BAT policy when corporate taxes are* $\tau^\pi = \frac{\delta}{1+\delta}$;

3. *A permanent unexpected VP policy of size* $\frac{\delta}{1+\delta}$;

These three policies have no effect on the real allocation and induce the real exchange rate to appreciate by δ.

Proof.

Neutrality of IX under the assumptions of Proposition 1 was already proved. Therefore, we just need to prove that a BAT and a VP implementations of size $\frac{\delta}{1+\delta}$ are neutral and generate a real exchange rate appreciation of size δ.

Let $\{\Psi(s^t)\}_{s^t\in(S)^t,t\geq 0}$ denote an equilibrium process before the implementation of any policy, i.e. when s_t is governed $\{S,\Omega\}$.

Consider now the process $\left\{\tilde{\Psi}^{BAT}(s^t)\right\}_{s^t\in(GP^{BAT})^t,t\geq 0}$ with an unanticipated permanent BAT such that, for each element $\tilde{\varkappa}$ of $\tilde{\Psi}^{BAT}$, *other than retailers and producers import prices*, $\tilde{P}^{BAT}_{Ft}(i)$ and $\tilde{P}_{X^*t}(i)$, *and the nominal exchange rate*, ε_t, we have

$$\tilde{\varkappa}^{BAT}(\tilde{s}^t) = \varkappa\left(\mu^{BAT}_t(\tilde{s}^t)\right) \qquad \forall\tilde{s}^t \in \left(GP^{BAT}\right)^t, \quad \forall t\geq 0 \tag{146}$$

where $\varkappa$ is the corresponding element of the equilibrium process Ψ and $\forall\tilde{s}^t = (\tilde{s}_1,....,\tilde{s}_t) \in \left(GP^{BAT}\right)^t$ the function $\mu_t(\tilde{s}^t) = s^t = (s_1,...,s_t,...) \in (S)^t$ where $\forall i\geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \left(\sigma^{BAT}_\delta\right)^{-1}(\tilde{s}_i) & if\ \tilde{s}_i \in S^{BAT} \end{cases}.$$

. Import prices and the exchange rate satisfy $\forall\tilde{s}^t = (\tilde{s}_1,....,\tilde{s}_t) \in \left(GP^{BAT}\right)^t$

$$\frac{\tilde{P}^{BAT}_{F,t}(i)}{P_{F,t}(i)} = \frac{\tilde{P}^{BAT}_{F,t}}{P_{F,t}} = \frac{\tilde{P}^{BAT}_{X^*,t}(i)}{P_{X^*,t}(i)} = \begin{cases} 1 & if\ \tilde{s}_t\in S \\ (1-\tau^\pi_t) & if\ \tilde{s}_t \in S^{BAT} \end{cases} \tag{147}$$

$$\frac{\tilde{\varepsilon}^{BAT}_t}{\varepsilon_t} = \begin{cases} 1 & if\ \tilde{s}_t\in S \\ (1-\tau^\pi_t) & if\ \tilde{s}_t \in S^{BAT} \end{cases} \tag{148}$$

We want to show that $\left\{\tilde{\Psi}^{BAT}(s^t)\right\}_{s^t\in(GP^{BAT})^t,t\geq 0}$ is an equilibrium which, given (148) and the fact that P_t and P^*_t are unaffected also implies that the real echange rate appreciates by δ.

The conditions that are directly affected by BAT are the law of one price for domestic exporters, $(109a)$, retailers optimal demand of imports, (100), and the domestic price index, (112). Fix an history $\tilde{s}^t = (\tau^m_t, \varsigma^x_t, \tau^v_t, \varsigma^v_t, \tau^\pi_t, 1, \tau^{m*}_t, \varsigma^{x*}_t\tau^{m*}_t, \varsigma^{x*}_t) \in \left(GP^{BAT}\right)^t$. Equation $(109a)$ is satisfied since:

$$\begin{aligned} \tilde{P}^{*BAT}_{Ht} &= P^*_{Ht} = \frac{(1+\tau^{m*}_t)(1-\tau^v_t)}{(1+\varsigma^x_t)}\frac{P_{Ht}(i)}{\varepsilon_t} \\ &= \frac{(1+\tau^{m*}_t)(1-\tau^v_t)(1-\tau^\pi)}{(1+\varsigma^x_t)}\frac{\tilde{P}^{BAT}_{Ht}(i)}{\tilde{\varepsilon}^{BAT}_t} \end{aligned} \tag{149}$$

the last equality from (148) and $\tau^\pi_t = \frac{\delta}{1+\delta}$.

Retailers optimal demand of imports is satisfied since

$$\begin{aligned}\tilde{Y}_{Ft}^{BAT} &= Y_{Ft} = (1-\omega)\left[\frac{P_{Ft}}{P_t}\right]^{-\theta} C_t \\ &= (1-\omega)\left[\frac{\tilde{P}_{Ft}^{BAT}}{\tilde{P}_t^{BAT}}\frac{1}{1-\tau_t^{\pi}}\right]^{-\theta}\tilde{C}_t^{BAT}\end{aligned} \tag{150}$$

where the last equality follows from (147). And analogously for the price index:

$$\begin{aligned}\tilde{P}_t^{BAT} &= P_t = \left[\omega P_{Ht}^{1-\theta} + (1-\omega)(P_{Ft})^{1-\theta}\right]^{\frac{1}{1-\theta}} \\ &= \left[\omega\left(\tilde{P}_{Ht}^{BAT}\right)^{1-\theta} + (1-\omega)\left(\frac{\tilde{P}_{Ft}^{BAT}}{1-\tau_t^{\pi}}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}\end{aligned} \tag{151}$$

Moreover to make sure that the adjustment in import prices is still consistent with firms optimality conditions we also need to check the law of one price for foreign exporters:

$$\begin{aligned}\tilde{P}_{Ft}^{BAT} &= P_{Ft}(1-\tau_t^{\pi}) = \frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})(1-\tau_t^v)}P_{Ft}^*\varepsilon_t(1-\tau_t^{\pi}) \\ &= \frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})(1-\tau_t^v)}\tilde{P}_{Ft}^{*BAT}\tilde{\varepsilon}_t^{BAT}\end{aligned}$$

Finally we need to check the balance of payment equilibrium

$$\begin{aligned}\tilde{B}_{Ft}^{BAT} &= B_{Ft} = B_{Ft-1}R_{t-1}^* + \frac{P_{Ht}^*}{1+\tau_t^{m*}}\frac{s^*}{s}Y_{Ht}^* - \frac{(1-\tau_t^v)P_{Ft}}{(1+\tau_t^m)}\frac{Y_{Ft}}{\varepsilon_t} \\ &= \tilde{B}_{Ft-1}^{BAT}\tilde{R}_{t-1}^{*BAT} + \frac{\tilde{P}_{Ht}^{*BAT}}{1+\tau_t^{m*BAT}}\frac{s^*}{s}\tilde{Y}_{Ht}^{*BAT} - \frac{(1-\tau_t^v)\tilde{P}_{Ft}^{BAT}}{(1+\tau_t^m)}\frac{\tilde{y}_{Ft}^{BAT}}{\tilde{\varepsilon}_t^{BAT}}\end{aligned}$$

where the third equality uses $\frac{\tilde{P}_{Ft}^{BAT}}{(1-\tau^{\pi})} = P_{Ft}$, $\tau^{\pi} = \frac{\delta}{1+\delta}$ and $\tilde{\varepsilon}_t^{BAT}(1+\delta) = \varepsilon_t$.

Inspecting all of the other dynamic equations we observe that since BAT adjustemnts and import prices do not enter any of those equations and the exchange rate only enters through the future appreciation which by (148) and under conditions of Proposition 1 is identical across allocations with probability one, all equations will be satisfied by $\tilde{\Psi}^{BAT}$ since they are satisfied by Ψ.

Let's now turn to equivalence with VP.

Consider the process $\left\{\tilde{\Psi}^{VP}(s^t)\right\}_{s^t\in(GP^{VP})^t, t\geq 0}$ with an unanticipated permanent VP implementation such that, for each element $\tilde{\varkappa}^{VP}$ of $\tilde{\Psi}^{VP}$, *other than domestic prices* $\left(\tilde{P}_{Ht}^{VP}(i), \tilde{P}_{Ft}^{VP}(i), \tilde{P}_t^{VP}(i)\right)$ *and wages* $\left(\tilde{\tilde{w}}_t^{VP}(i), \tilde{w}_t^{VP}(i), \tilde{W}_t^{VP}\right)$ *and the associated price indexes* $\left(\tilde{P}_{Ht}^{VP}, \tilde{P}_{Ft}^{VP}, \tilde{P}_t^{VP}\right)$,

$$\tilde{\varkappa}^{VP}(\tilde{s}^t) = \varkappa\left(\mu_t^{VP}(\tilde{s}^t)\right) \qquad \forall \tilde{s}^t \in \left(\tilde{S}^{VP}\right)^t, \quad \forall t \geq 0 \tag{152}$$

where $\varkappa$ is the corresponding element of the equilibrium process $\tilde{\Psi}$ with IX and $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t, ...) \in \left(GP^{VP}\right)^t$, $\mu_t(\tilde{s}^t) = s^t = (s_1, ..., s_t, ...) \in (S)^t$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if \ \tilde{s}_i \in S \\ \left(\sigma_\delta^{VP}\right)^{-1}(\tilde{s}_i) & if \ \tilde{s}_i \in S^{IX} \end{cases}$$

Prices and wages satisfy $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{VP}\right)^t$

$$\frac{\tilde{P}_{H,t}^{VP}(i)}{P_{H,t}(i)} = \frac{\tilde{P}_{F,t}^{VP}(i)}{P_{F,t}(i)} = \frac{\tilde{P}_t^{VP}(i)}{P_t(i)} = \begin{cases} 1 & if \ \tilde{s}_t \in S \\ \\ (1+\delta) & if \ \tilde{s}_t \in S^{VP} \end{cases} \tag{153}$$

$$\frac{\tilde{\overline{w}}_t^{VP}(i)}{\overline{w}_t(i)} = \frac{\tilde{w}_t^{VP}(i)}{\tilde{w}_t(i)} = \frac{\tilde{W}_t^{VP}}{W_t} = \begin{cases} 1 & if \ \tilde{s}_t \in S \\ \\ (1+\delta) & if \ \tilde{s}_t \in S^{VP} \end{cases} \tag{154}$$

We want to show that $\left\{\tilde{\Psi}^{VP}(s^t)\right\}_{s^t \in (GP^{VP})^t, t \geq 0}$ is an equilibrium, which given (153) and the fact that ε_t is unaffected also implies that the real echange rate appreciates by δ.

The two laws of one price and the balance of payment are again straightforward: fix an history $\tilde{s}^t \in \left(GP^{VP}\right)^t$ such that $\tilde{s}_t = (\tau_t^m, \varsigma_t^x, \tilde{\tau}_t^v, \tilde{\varsigma}_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}) \in S^{VP}$. We have

$$\begin{aligned} \tilde{P}_{Ht}^{*VP}(i) &= P_{Ht}^*(i) = \frac{(1+\tau_t^{m*})(1-\tau_t^v)}{(1+\varsigma_t^x)} \frac{P_{Ht}(i)}{\varepsilon_t} && (155) \\ &= \frac{(1+\tau_t^{m*})(1-\tilde{\tau}_t^v)}{(1+\varsigma_t^x)} \frac{\tilde{P}_{Ht}^{VP}(i)}{\tilde{\varepsilon}_t^{VP}} && (156) \end{aligned}$$

where we are making use of (153) and $\frac{1-\tilde{\tau}_t^v}{1-\tau_t^v} = \frac{1}{1+\delta}$.And similarly for foreign producers. For the balance of payment equilibrium condition we use the same argument as well:

$$\begin{aligned} \tilde{B}_{Ft}^{VP} &= B_{Ft} = B_{Ft-1}R_{t-1}^* + \frac{P_{Ht}^*}{1+\tau_t^{m*}} \frac{s^*}{s} Y_{Ht}^* - \frac{(1-\tau_t^v) P_{Ft}}{(1+\tau_t^m)\varepsilon_t} Y_{Ft} \\ & \tilde{B}_{Ft-1}^{VP} \tilde{R}_{t-1}^{*VP} + \frac{\tilde{P}_{Ht}^{*VP}}{1+\tau_t^{m*}} \frac{s^*}{s} \tilde{Y}_{Ht}^{VP*} - \frac{(1-\tilde{\tau}_t^v)\tilde{P}_{Ft}^{VP}}{(1+\tau_t^m)\tilde{\varepsilon}_t} \tilde{Y}_{Ft}^{VP} \end{aligned}$$

Now consider the optimality condition for the price of the domestic good at home an history $\tilde{s}^t \in \left(GP^{VP}\right)^t$ such that $\tilde{s}_t = (\tau_t^m, \varsigma_t^x, \tilde{\tau}_t^v, \tilde{\varsigma}_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}) \in S^{VP}$:

$$\begin{aligned} \tilde{\mathbb{E}}_t^{VP} \Sigma_{s=t}^\infty \zeta_P^{s-t} \tilde{\Lambda}_{t,s}^{VP} \left[\tilde{Y}_{Hs}^{VP}(i) + \frac{s^*}{s}\tilde{Y}_{Hs}^{*VP}(i)\right] \frac{1}{\tilde{P}_s^{VP}} \left[\tilde{\overline{P}}_{Pt}^{VP}(i) - \frac{\gamma}{\gamma-1} \frac{(1-\tilde{\varsigma}_s^v)\tilde{W}_s^{VP}}{\alpha A_s \tilde{N}_s^{VP}(i)^{\alpha-1}}\right] &= && (157) \\ \mathbb{E}_t \Sigma_{s=t}^\infty \zeta_P^{s-t} \Lambda_{t,s} \left[Y_{Hs}(i) + \frac{s^*}{s} Y_{Hs}^*(i)\right] \frac{1+\delta}{P_s} \left[\overline{P}_{Pt}^{VP}(i) - \frac{\gamma}{\gamma-1} \frac{(1-\varsigma_s^v) W_s^{VP}}{\alpha A_s \tilde{N}_s^{VP}(i)^{\alpha-1}}\right] &= 0 && (158) \end{aligned}$$

where the first equality follows from $\frac{\tilde{P}_s^{VP}}{P_s} = \frac{\tilde{W}_s^{VP}}{W_s} = \frac{(1-\varsigma_s^v)}{(1-\tilde{\varsigma}_s^v)} = 1+\delta$ w.p. 1.

With flexible wages, optimal labor supply is also satisfied since real wages are unaffected:

$$\frac{\left[\tilde{n}_t^{VP}(i)\right]^{\eta}}{\tilde{C}_t^{VP-\sigma}}\frac{\gamma_n}{(\gamma_n-1)}-\frac{\tilde{\bar{w}}_t^{VP}(i)}{\tilde{P}_t^{VP}}=\frac{[n_t(i)]^{\eta}}{C_t^{-\sigma}}\frac{\gamma_n}{(\gamma_n-1)}-\frac{\bar{w}_t(i)}{P_t}=0$$

Morevoer, since the transition from $s_{t-1}\in S$ to $s_t\in S^{VP}$ is unanticipated, the different inflation dynamic ex post does not affect optimal bond holdings ex ante. On the other hand since the policy is permanent, future inflation is unaffected by its implementation as is clear from (153).

12 Appendix E. Proof of Proposition 3

We start by giving a definition of a permanent unexpected appreciation of the nominal exchange rate.

Definition 3. *Let the evolution of trade policy and monetary policy at home and abroad*

$$s_t=(\tau_t^m,\varsigma_t^x,\tau_t^v,\varsigma_t^v,\tau_t^\pi,BAT_t,\tau_t^{m*},\varsigma_t^{x*},\bar{\varepsilon}_t)$$

be determined by the stochastic process $\{S,\Omega\}$ *that satisfies* $BAT=0\ \forall s\in S$. *A currency devaluation of size* δ *starting from* $\{S,\Omega\}$ *is described by a stochastic processes* $\{GP^\varepsilon,\Omega^\varepsilon\}$ *such that* $GP^\varepsilon=S\cup S^\varepsilon$ where

$$S^\varepsilon=\sigma_\delta^\varepsilon([S]) \tag{159}$$

$$\sigma_\delta^{BAT}(\tau_t^m,\varsigma_t^x,\tau_t^v,\varsigma_t^v,\tau_t^\pi,0,\tau_t^{m*},\varsigma_t^{x*},\bar{\varepsilon}_t)=(\tau_t^m,\varsigma_t^x,\tau_t^v,\varsigma_t^v,\tau_t^\pi,0,\tau_t^{m*},\varsigma_t^{x*},\bar{\varepsilon}_t(1+\delta))$$

and the transitiong matrix is

$$\Omega^\varepsilon=\begin{bmatrix}(1-\pi^\varepsilon)\Omega & \pi^\varepsilon\Omega\\ (1-\rho^\varepsilon)\Omega & \rho^\varepsilon\Omega\end{bmatrix} \tag{160}$$

where the ordering of states in the transition probability matrices is the obvious one.

Proposition 3. *If monetary policy is described by (34) in a fixed exchange rate regime* ($\varphi_\varepsilon=\infty$) *and assumptions 1.-3. of Proposition 1 hold, an unexpected IX policy of size* δ *and an expected BAT policy of size* $\frac{\delta}{1+\delta}$ *have the same allocative effects of a once and for all unexpected currency devaluation of size* δ. *An unexpected VP policy of the same size* $\frac{\delta}{1+\delta}$ *has no effect on the real allocation but causes the real exchange rate to appreciate by* δ.

Proof.

Let $\{\Psi^\varepsilon(s^t)\}_{s^t\in(S^T)^t,t\geq 0}$ denote an equilibrium process under $\{GP^\varepsilon,\Omega^\varepsilon\}$.

Consider now the process $\left\{\tilde{\Psi}^{IX}(s^t)\right\}_{s^t\in(GP^{IX})^t,t\geq 0}$ with an unanticipated permanent IX such that, for each element $\tilde{\varkappa}^{IX}$ of $\tilde{\Psi}^{IX}$, *apart from the nominal exchange rate*, we have

$$\tilde{\varkappa}^{IX}(\tilde{s}^t)=\varkappa^\varepsilon\left(\mu_t^\varepsilon(\tilde{s}^t)\right)\qquad\forall\tilde{s}^t\in\left(GP^{IX}\right)^t,\ \ \forall t\geq 0 \tag{161}$$

where $\varkappa^{\varepsilon}$ is the corresponding element of the equilibrium process Ψ^{ε} and $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t, ...) \in \left(GP^{IX}\right)^t$, $\mu_t^{\varepsilon}(\tilde{s}^t) = s^t = (s_1, ..., s_t, ...) \in (GP^{\varepsilon})^t$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \sigma_\delta^{\varepsilon}\left(\left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_i)\right) & if\ \tilde{s}_i \in S^{IX} \end{cases}.$$

For ease of notation in what follows, for any $\tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$, we let $\tilde{\varkappa}_t^{IX} = \tilde{\varkappa}^{IX}(s^t)$ and $\varkappa_t^{\varepsilon} = \varkappa\left(\mu_t^{\varepsilon}(s^t)\right)$.

The exchange rate satisfies $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$

$$\tilde{\varepsilon}_t^{IX} = \begin{cases} \varepsilon_t^{\varepsilon} & if\ \tilde{s}_t \in S \\ \frac{\varepsilon_t^{\varepsilon}}{1+\delta} & if\ \tilde{s}_t \in S^{IX} \end{cases} \tag{162}$$

To show that $\left\{\tilde{\Psi}^{IX}(s^t)\right\}_{s^t \in (GP^{IX})^t, t \geq 0}$ is an equilibrium we can follow the same steps as in the proof Proposition 1.

At $\tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$ such that $\tilde{s}_t \in S^{IX}$, the laws of one price and the balance of payment equilibrium equations are satisfied since

$$\frac{\tilde{\varepsilon}_t^{IX}}{\tilde{\varepsilon}_t^{IX}} = \frac{(1+\tilde{\sigma}_t^x)}{(1+\sigma_t^x)} = \frac{(1+\tilde{\tau}_t^m)}{(1+\tau_t^m)}$$

and the only other equations in which the exchange rate appears only depend on its expected appreciation which is the same in the two processes.

The argument for the BAT policy is analogous but in that case optimal import demands and the price index need to be checked as well just as in the proof of Proposition 2.[33]

Finally, the fact that VP is still neutral even under fixed exchange rates is a straightforward consequence of the proof of Proposition 2. Since under flexible exchange rates VP is neutral and the nominal exchang rate is unaffected by its implementation, it follows that even if monetary policy targets a given fixed exchange rate the policy still remains neutral.

13 Appendix F. Proof of Proposition 4

Proposition 4. *Under full pass-through of taxes, an unexpected IX policy of size δ and an unexpected BAT policy of size δ implement the same allocation. Generically, a VP policy of size $\frac{\delta}{1+\delta}$ does not implement the same allocation as IX or BAT. Equivalence of the three policies requires that policies are permanent, i.e. $\rho = 1$, or that prices are flexible.*

[33] The equivalence result between BAT and IX in Proposition 4 also delivers taht BAT is equivalent to IX in this context and hence it is equivalent to an exchange rate devaluation.

Proof.

Let $\left\{\Psi^{IX}\left(s^t\right)\right\}_{s^t\in(GP^{IX})^t,t\geq 0}$ denote an equilibrium process with IX.

Consider the process with BAT $\left\{GP^{BAT},\Omega^{BAT}\right\}$ and define a sequence of functions $\mu_t^{BAT,IX}$: $\left(GP^{BAT}\right)^t \to (S)^t$ as follows: $\forall\tilde{s}^t = (\tilde{s}_1,...,\tilde{s}_t,....) \in \left(GP^{BAT}\right)^t$, $\mu_t^{BAT,IX}\left(\tilde{s}^t\right) = s^t = (s_1,...,s_t,...) \in (S)^t$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \sigma_\delta^{IX}\left(\left(\sigma_\delta^{BAT}\right)^{-1}(\tilde{s}_i)\right) & if\ \tilde{s}_i \in S^{BAT} \end{cases}$$

that is function $\mu_t^{BAT,IX}$ maps all histories in which BAT has occurred into histories in which IX has occurred instead.

Consider now a process $\left\{\tilde{\Psi}^{BAT}\left(s^t\right)\right\}_{s^t\in(GP^{BAT})^t,t\geq 0}$ with an unanticipated permanent BAT such that, for each element $\tilde{\varkappa}^{BAT}$ of $\tilde{\Psi}^{BAT}$, *other than import prices*, $\left(\tilde{P}_{Ft}^{BAT}(i), \tilde{P}_{Ft}^{BAT}\right)$, we have

$$\tilde{\varkappa}^{BAT}\left(\tilde{s}^t\right) = \varkappa^{IX}\left(\mu_t^{BAT,IX}\left(\tilde{s}^t\right)\right) \qquad \forall\tilde{s}^t \in \left(GP^{BAT}\right)^t,\ \ \forall t \geq 0 \tag{163}$$

where $\varkappa^{IX}$ is the corresponding element of the equilibrium process Ψ^{IX} and $\forall\tilde{s}^t = (\tilde{s}_1,...,\tilde{s}_t) \in \left(GP^{BAT}\right)^t$, $\mu_t^{BAT,IX}\left(\tilde{s}^t\right) = s^t = (s_1,...,s_t) \in (S)^t$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \sigma_\delta^{IX}\left(\left(\sigma_\delta^{BAT}\right)^{-1}(\tilde{s}_i)\right) & if\ \tilde{s}_i \in S^{BAT} \end{cases}.$$

Import prices satisfy $\forall\tilde{s}^t = (\tilde{s}_1,....,\tilde{s}_t) \in \left(GP^{BAT}\right)^t$

$$\frac{\tilde{P}_{F,t}^{BAT}(i)}{P_{F,t}(i)} = \frac{\tilde{P}_{F,t}^{BAT}}{P_{F,t}} = \begin{cases} 1 & if\ \tilde{s}_t \in S \\ (1-\tau_t^\pi) & if\ \tilde{s}_t \in S^{BAT} \end{cases} \tag{164}$$

We want to show that $\left\{\tilde{\Psi}^{BAT}\left(s^t\right)\right\}_{s^t\in(GP^{BAT})^t,t\geq 0}$ is an equilibrium.

Under PCP the equilibrium equations that are directly affected by BAT and IX are the laws of one price for exporters, $(109a)$ and $(110a)$; the tax passthrough equation, (104);retailers optimal demand of imports, equation (100); the price index (115); and the balance of paymet equilibrium (**??**) Fix an history $\tilde{s}^t \in \left(GP^{BAT}\right)^t$ with $\tilde{s}_t = (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}) \in S^{BAT}$ and let $s^t = \mu_t^{BAT,IX}\left(\tilde{s}^t\right)$ where $s_t = (\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*})$ has IX in place instead of BAT.

Under LCP instead of $(109a)$ and $(110a)$ we only need to check equation $(109b)$.

The laws of one price $(109a)$ and $(110a)$ are satisfied since:

$$\begin{aligned} \tilde{P}_{Ht}^{*BAT} &= P_{Ht}^{*IX} = \frac{(1+\tau_t^{m*})(1-\tau_t^v)}{(1+\tilde{\varsigma}_t^x)}\frac{P_{Ht}^{IX}}{\varepsilon_t^{IX}} \\ &= \frac{(1+\tau_t^{m*})(1-\tau_t^v)(1-\tau_t^\pi)}{(1+\varsigma_t^x)}\frac{\tilde{P}_{Ht}^{BAT}}{\tilde{\varepsilon}_t^{BAT}} \end{aligned} \tag{165}$$

$$\tilde{P}_{Ft}^{BAT}(i) = (1-\tau^{\pi}) P_{Ft}^{IX}(i) = \frac{(1-\tau^{\pi})(1+\tilde{\tau}_t^m)\varepsilon_t^{IX}}{(1+\varsigma_t^{x*})(1-\tau_t^v)} P_{Ft}^{*IX}(i)$$
$$= \frac{(1+\tau_t^m)\tilde{\varepsilon}_t^{BAT}}{(1+\varsigma_t^{x*})(1-\tau_t^v)} \tilde{P}_{Ft}^{*BAT}(i)$$

Under LCP, equation (109b) is satsified since:

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \frac{Y_{Hs}^*(i)}{P_s} \left[\varepsilon_s (1+\tilde{\varsigma}_s^x) \bar{P}_{Xt}^*(i) - \frac{\gamma}{\gamma-1} \frac{(1-\varsigma_s^v) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] =$$
$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \frac{Y_{Hs}^*(i)}{P_s} \left[\varepsilon_s \frac{(1+\varsigma_s^x)}{(1-\tau^{\pi} BAT_s)} \bar{P}_{Xt}^*(i) - \frac{\gamma}{\gamma-1} \frac{(1-\varsigma_s^v) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0$$

and similarly for equations and (104)

$$\tilde{P}_{Ft}^{BAT} = (1-\tau^{\pi}) P_{Ft}^{IX} = \frac{(1-\tau^{\pi})(1+\tilde{\tau}_t^m)}{(1-\tau_t^v)} P_{X_t^*}^{IX}(i) = \frac{(1+\tau_t^m)}{(1-\tau_t^v)} \tilde{P}_{X_t^*}^{BAT}(i)$$

Retailers optimal demand of imports is satisfied since

$$\tilde{y}_{Ft}^{BAT} = y_{Ft}^{IX} = (1-\omega)\left[\frac{P_{Ft}^{IX}}{P_t^{IX}}\right]^{-\theta} C_t^{IX} \tag{166}$$
$$= (1-\omega)\left[\frac{\tilde{P}_{Ft}^{BAT}}{\tilde{P}_t^{BAT}} \frac{1}{1-\tau_t^{\pi}}\right]^{-\theta} \tilde{C}_t^{BAT}$$

And analogously for the price index:

$$\tilde{P}_t^{BAT} = P_t^{IX} = \left[\omega \left(P_{Ht}^{IX}\right)^{1-\theta} + (1-\omega)\left(P_{Ft}^{IX}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{167}$$
$$= \left[\omega \left(\tilde{P}_{Ht}^{BAT}\right)^{1-\theta} + (1-\omega)\left(\frac{\tilde{P}_{Ft}^{BAT}}{1-\tau_t^{\pi}}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{168}$$

Finally we need to check the balance of payment equilibrium:

$$\tilde{B}_{Ft}^{BAT} = B_{Ft}^{IX} = B_{Ft-1}^{IX} R_{t-1}^{*IX} + \frac{P_{Ht}^{*IX}}{1+\tau_t^{m*}} \frac{s^*}{s} Y_{Ht}^{*IX} - \frac{(1-\tau_t^v) P_{Ft}^{IX}}{(1+\tilde{\tau}_t^m)\varepsilon_t^{IX}} Y_{Ft}^{IX}$$
$$= \tilde{B}_{Ft-1}^{BAT} \tilde{R}_{t-1}^{*BAT} + \frac{\tilde{P}_{Ht}^{*BAT}}{1+\tau_t^{m*}} \frac{s^*}{s} \tilde{Y}_{Ht}^{*BAT} - \frac{(1-\tau_t^v)\tilde{P}_{Ft}^{BAT}}{(1+\tau_t^m)\tilde{\varepsilon}_t^{BAT}} \tilde{Y}_{Ft}^{BAT}$$

Given that import prices, BAT adjustments and IX policies don't enter any other equilibrium equations and Ψ^{IX} is an equilibrium, than Ψ^{BAT} is an equilibrium as well.

To see that VP will generically implement a different allocation notice that the first order condition for optimal reset price of domestic firms can be rewritten as

$$\frac{\overline{P}_{Ht}(i)}{P_t} = \frac{\gamma}{\gamma-1} \frac{\mathbb{E}_t \Sigma_{s=t}^{\infty} \frac{(1-\varsigma_s^v)}{(1-\tau_t^v)} \zeta_P^{s-t} \Lambda_{t,s} Y_{Hs}(i) \frac{1}{\alpha A_s Z_s(i) N_s(i)^{\alpha-1}} \frac{W_s}{P_s}}{\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \frac{\Lambda_{t,s}}{\pi_{t,s}} Y_{Hs}(i)} \tag{169}$$

which implies that as long as there are price rigidities, i.e. $\varsigma_P \in (0, 1)$, and policies are not permanent, i.e. $\frac{(1-\varsigma_s^v)}{(1-\tau_t^v)}$ is not equal to unity w.p. 1, the introduction of VAT taxes and payroll subsidies will induce a different optimal level of relative prices and hence production by domestic firms. So within our setup equivalence can only old if $\rho = 1$ or when prices are flexible.

14 Appendix G. Data Sources

We report below details on the data used in Figure 10. In the case of price series, the "Euro-area excluding Germany" aggregate refers to the weighted average of data for Belgium, France, Italy, the Netherlands, and Spain (these countries account for nearly 80 percent of the actual euro-area aggregate excluding Germany). All series were obtained from Haver Analytics and corresponding mnemonics are reported in parenthesis.

Core Inflation. Inflation series in the first panel refer to seasonally-adjusted core HICP data (i.e. excluding energy, food, alcohol, and tobacco) for Germany (EUDATA'H134HOEF), Belgium (EUDATA'H124HOEF), France (EUDATA'H132HOEF), Italy (EUDATA'H136HOEF), the Netherlands (EUDATA'H138HOEF), and Spain (EUDATA'H184HOEF). Data are plotted in 12-month percent changes.

Motor Vehicle Inflation. The price series in the second panel refer to seasonally-adjusted new and used automobiles prices for Germany (EUDATA'H134H711), Belgium (EUDATA'H124H711), France (EUDATA'H132H711), Italy (EUDATA'H136H711), the Netherlands (EUDATA'H138H711), and Spain (EUDATA'H184H711).

Wage Inflation. The wage series for Germany in the third panel refer to seasonally-adjusted negotiated hourly wages (GERMANY-DENEDBS), wages in the production and service sector (GERMANY-DENE6I), total labor costs in all sectors excluding agriculture (GERMANY'DESLTXA), and gross wages in all sectors excluding agriculture (GERMANY'DESLEXA). Data are presented in 4-quarter percent changes.

Real Consumption. The consumption series in the fourth panel refer to Households and Nonprofit Final Consumption Expenditures (SWDA, millions of chained 2010 euros) for Germany (EUDATA'J134PCT) and the euro area (EUDATA'J025PCT).

Figure 1. Macroeconomic Effects of IX with Retaliation

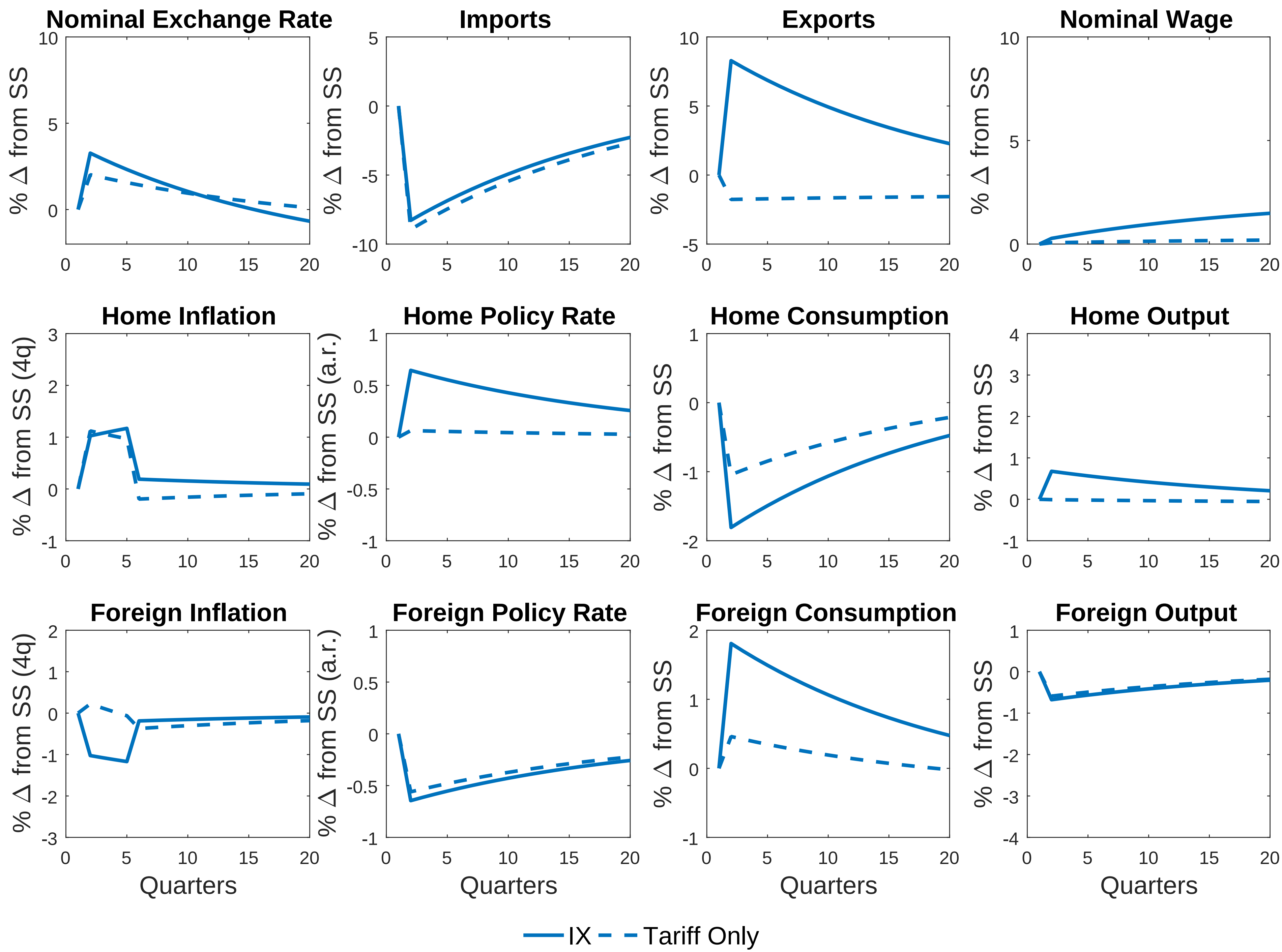

Figure 2. Macroeconomic Effects of IX with Retaliation: Fixed Exchange Rates

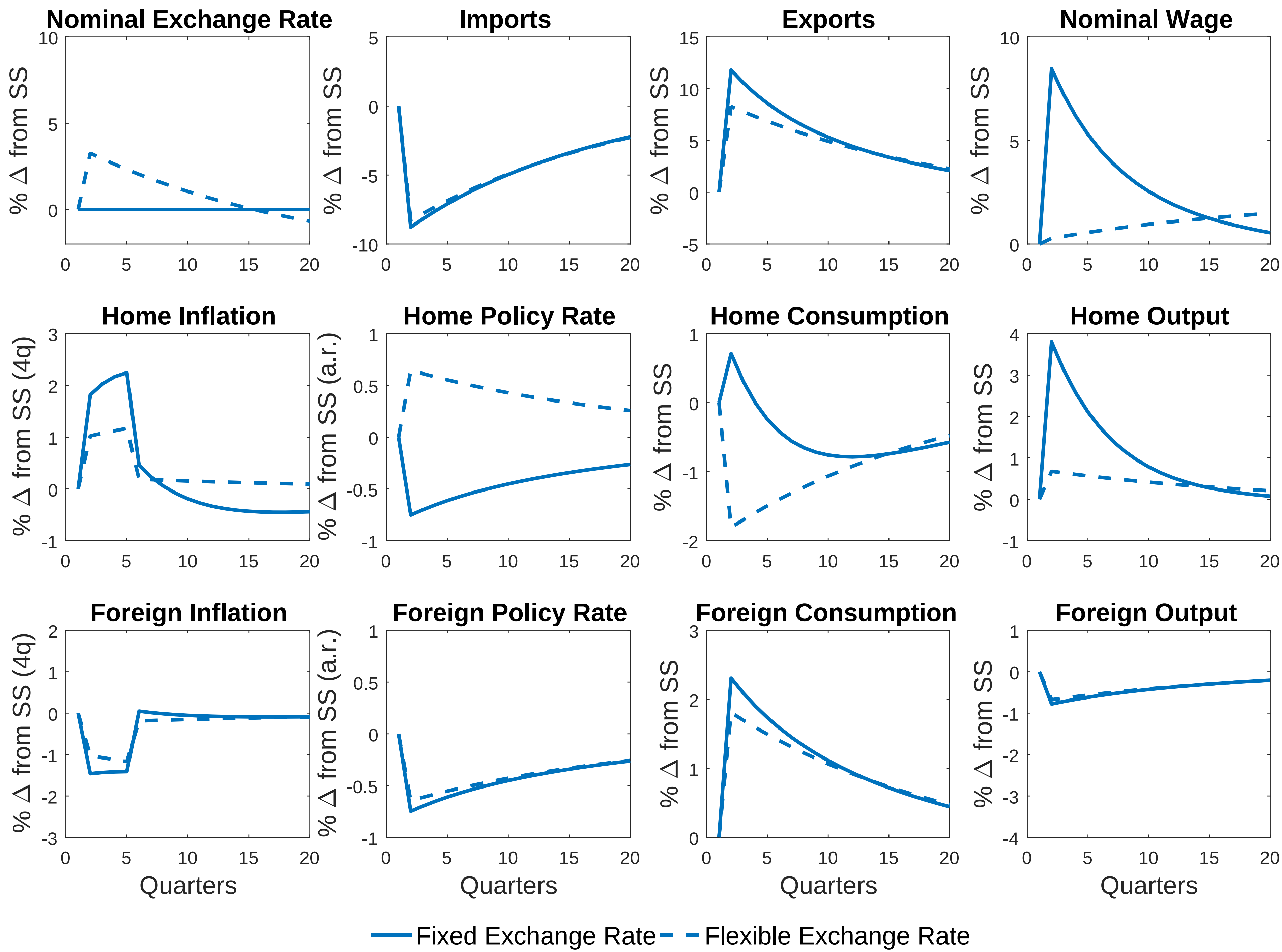

Figure 3. Anticipation Effects of Permanent IX

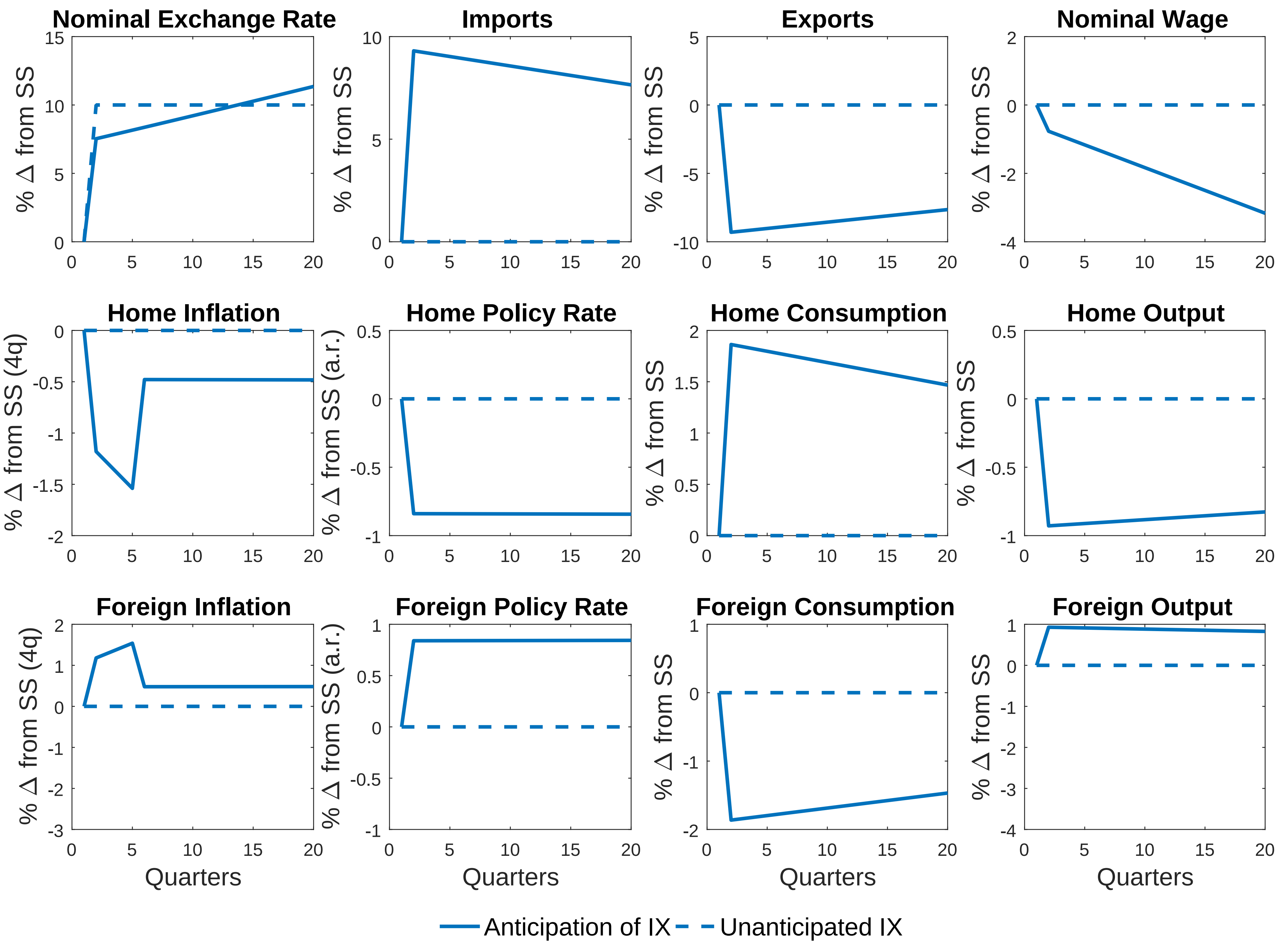

Figure 4. Permanent IX With and Without Foreign Holdings of Home Currency Bonds

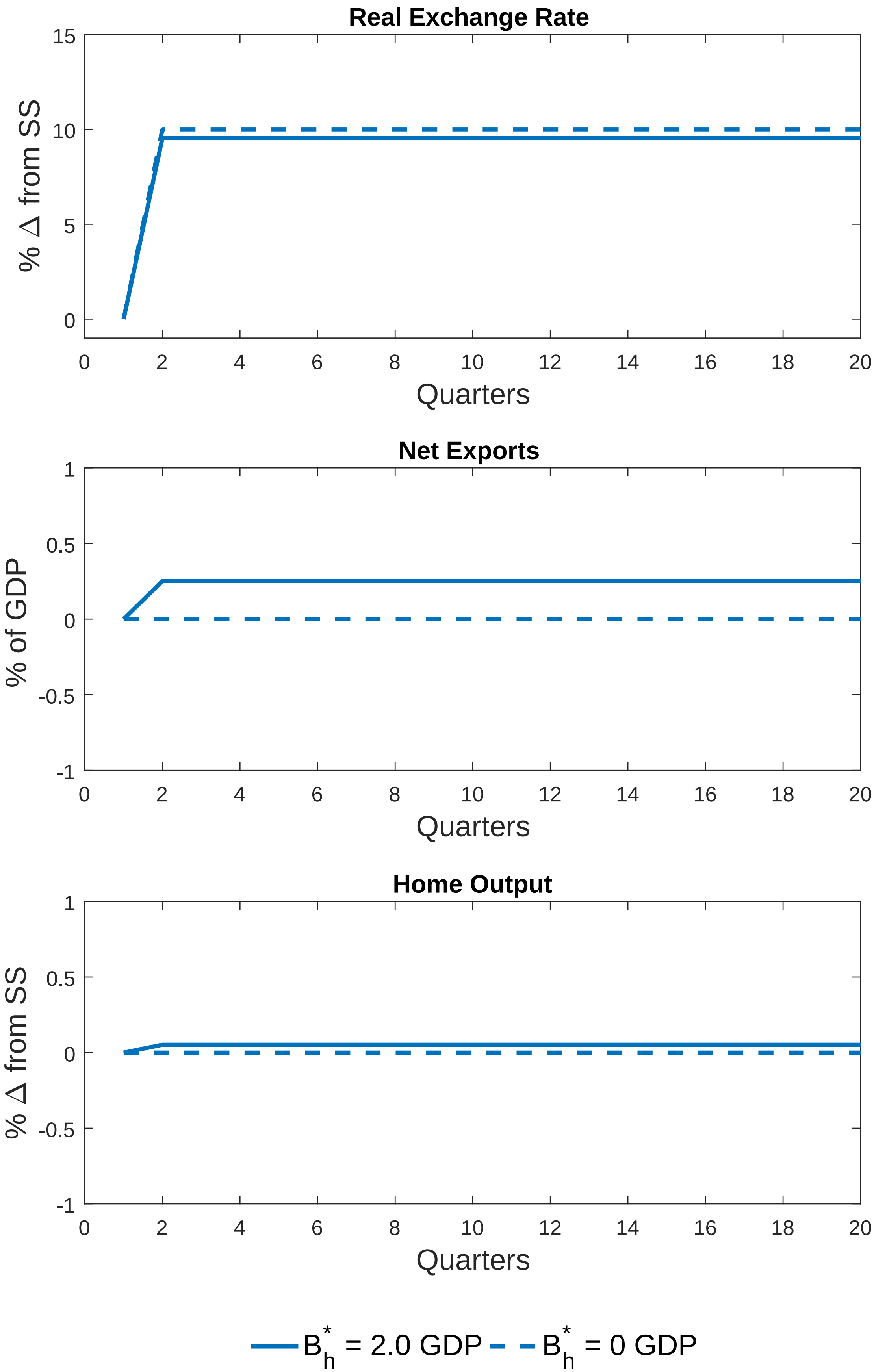

Figure 5. Permanent IX: LCP vs. PCP

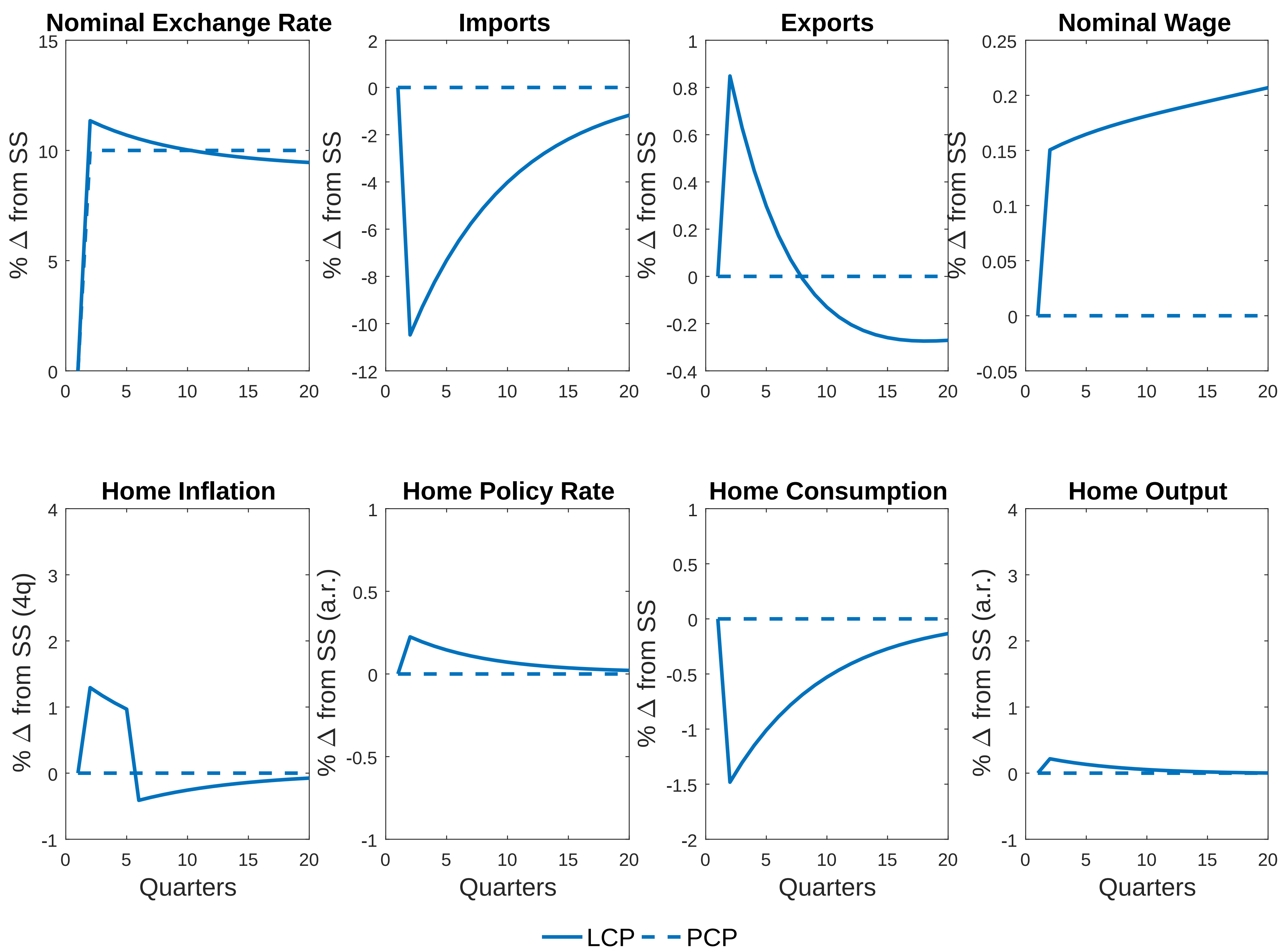

Figure 6. Permanent IX vs. VP: Neutrality

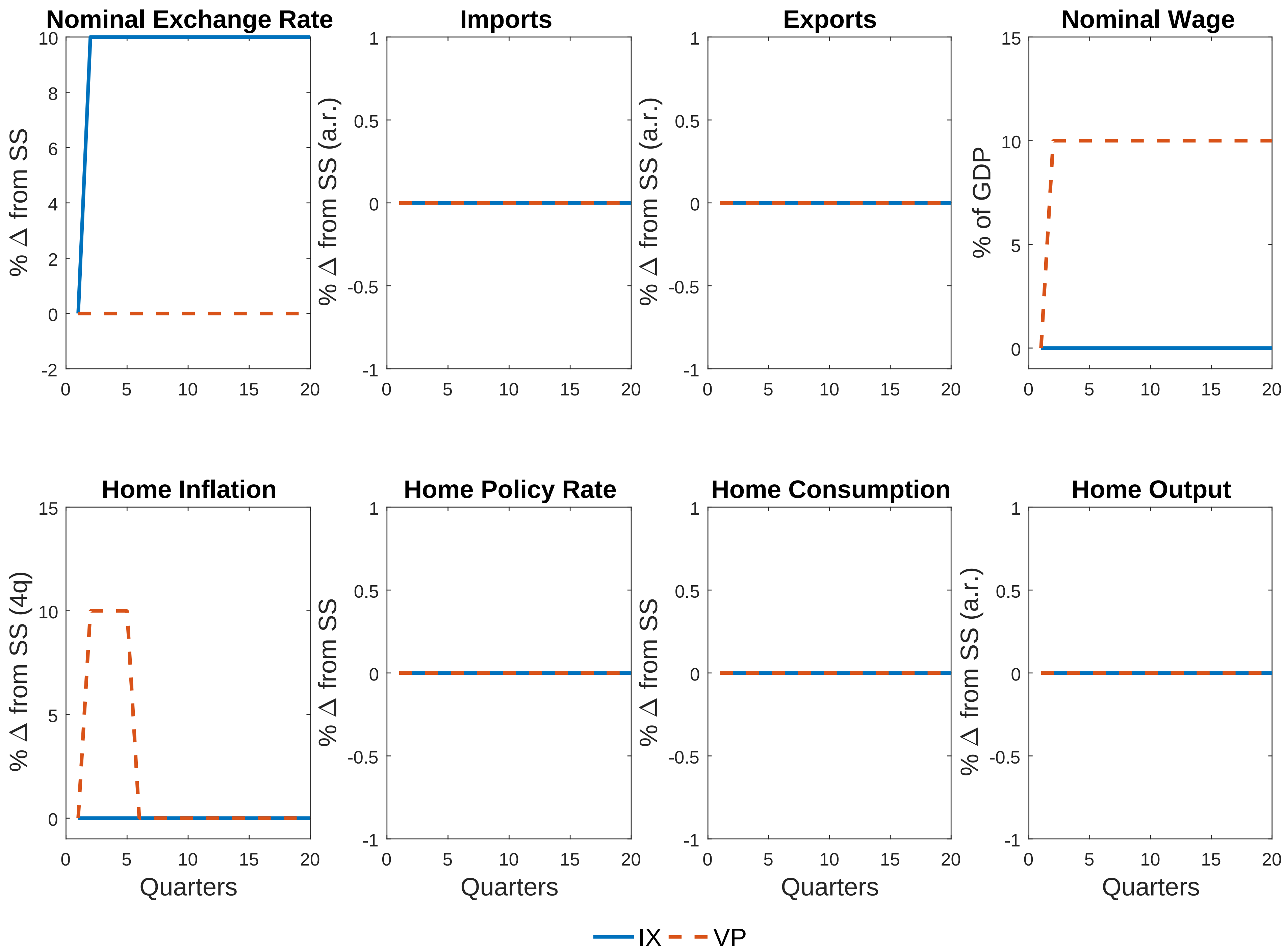

Figure 7. Permanent IX and VP: Fixed Exchange Rates

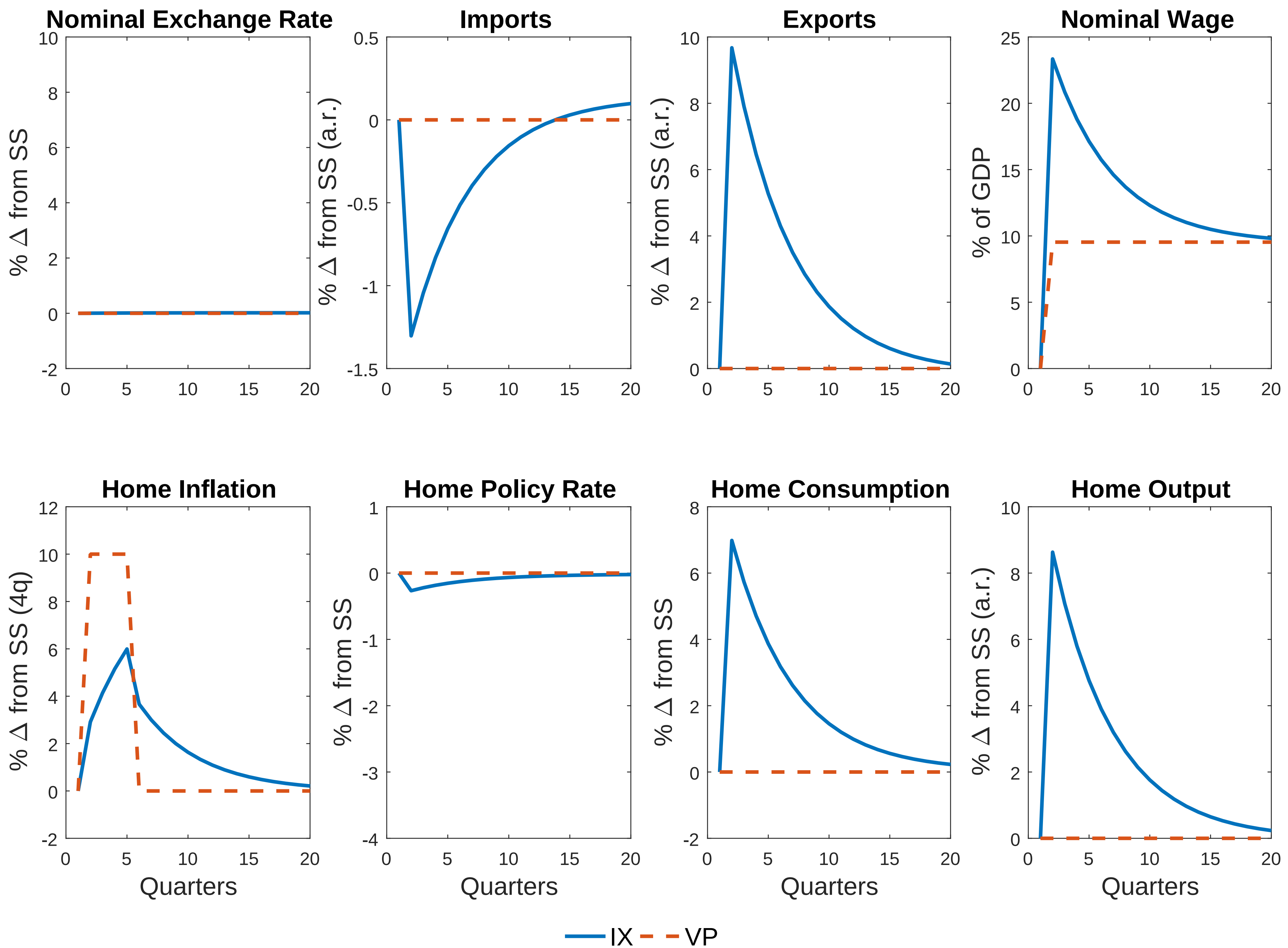

Figure 8. Permanent IX and VP: Sticky Wages

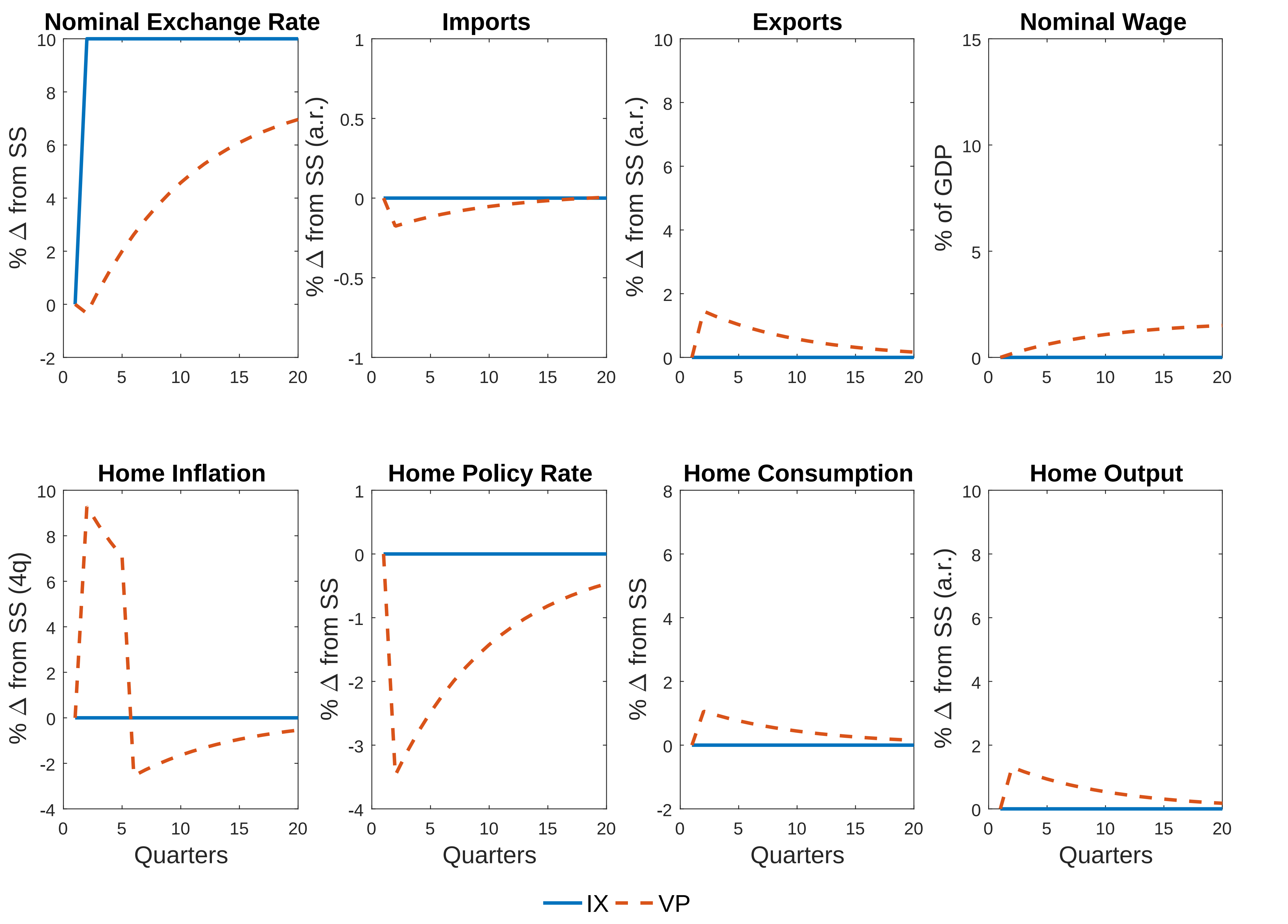

Figure 9. Temporary IX vs. VP: Stick Wages (PCP)

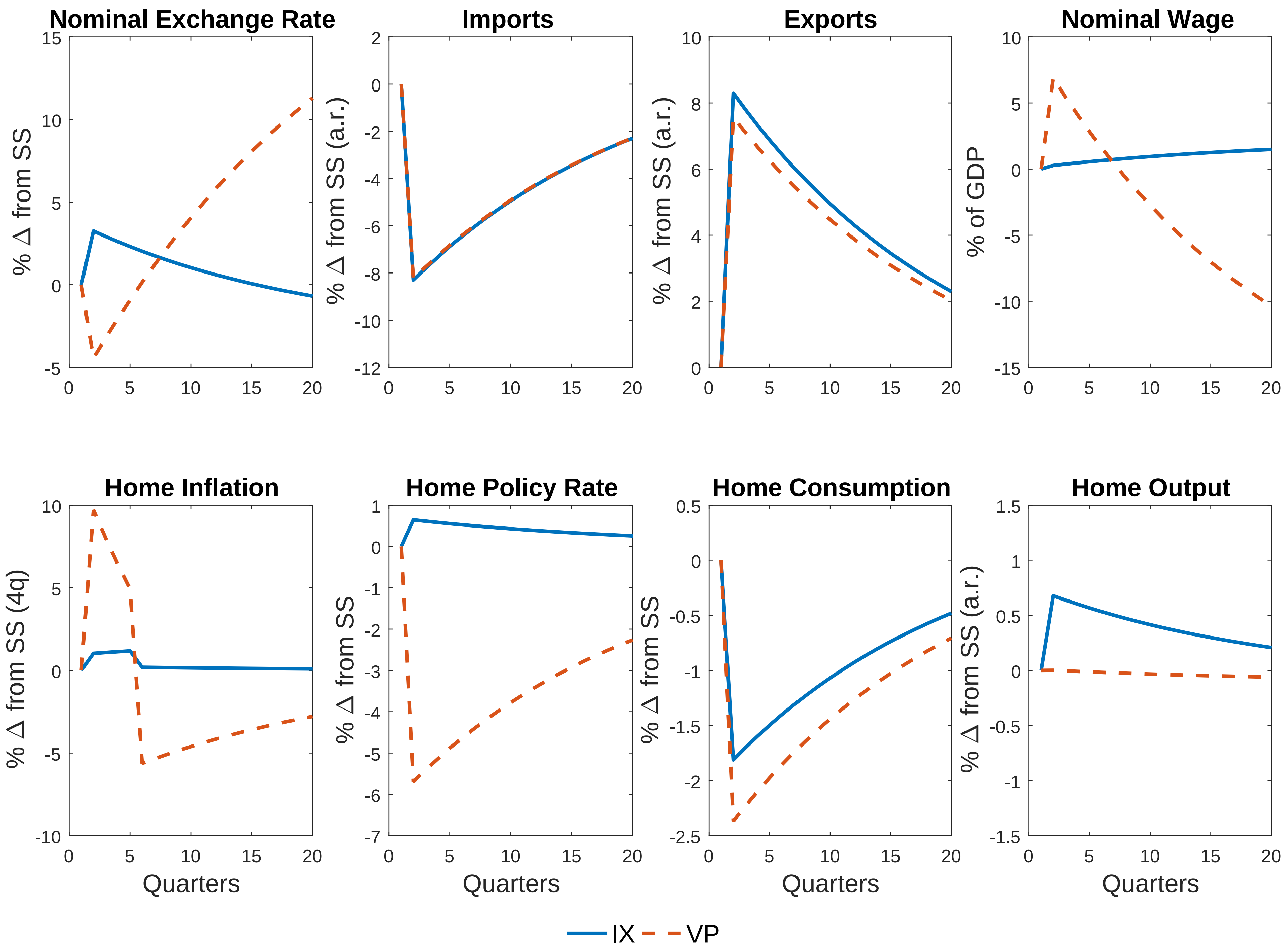

Figure 10. Fiscal Devaluation in Germany (2007)

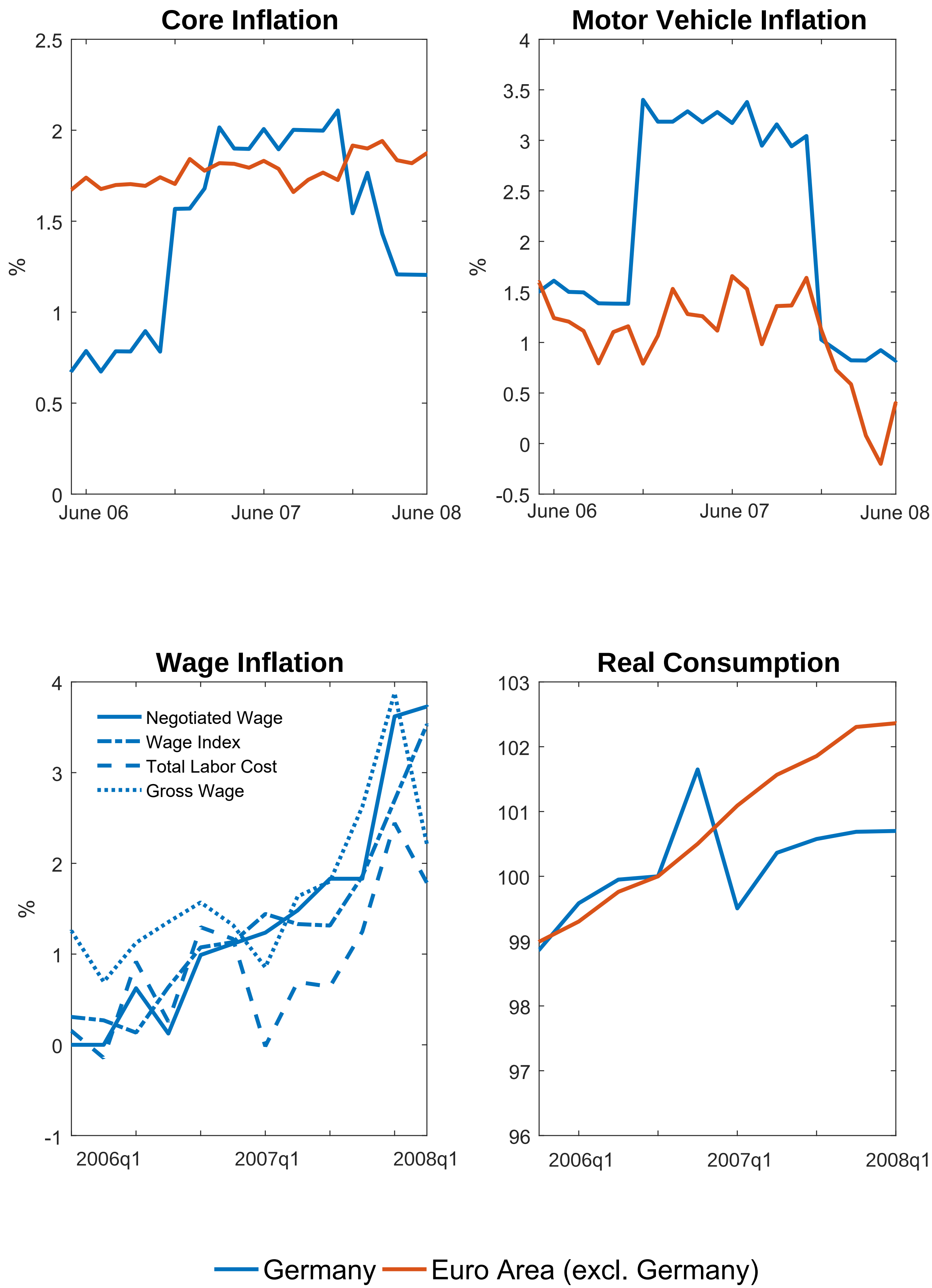

Figure 11. Temporary IX with Sticky Wages: PCP vs LCP vs DCP

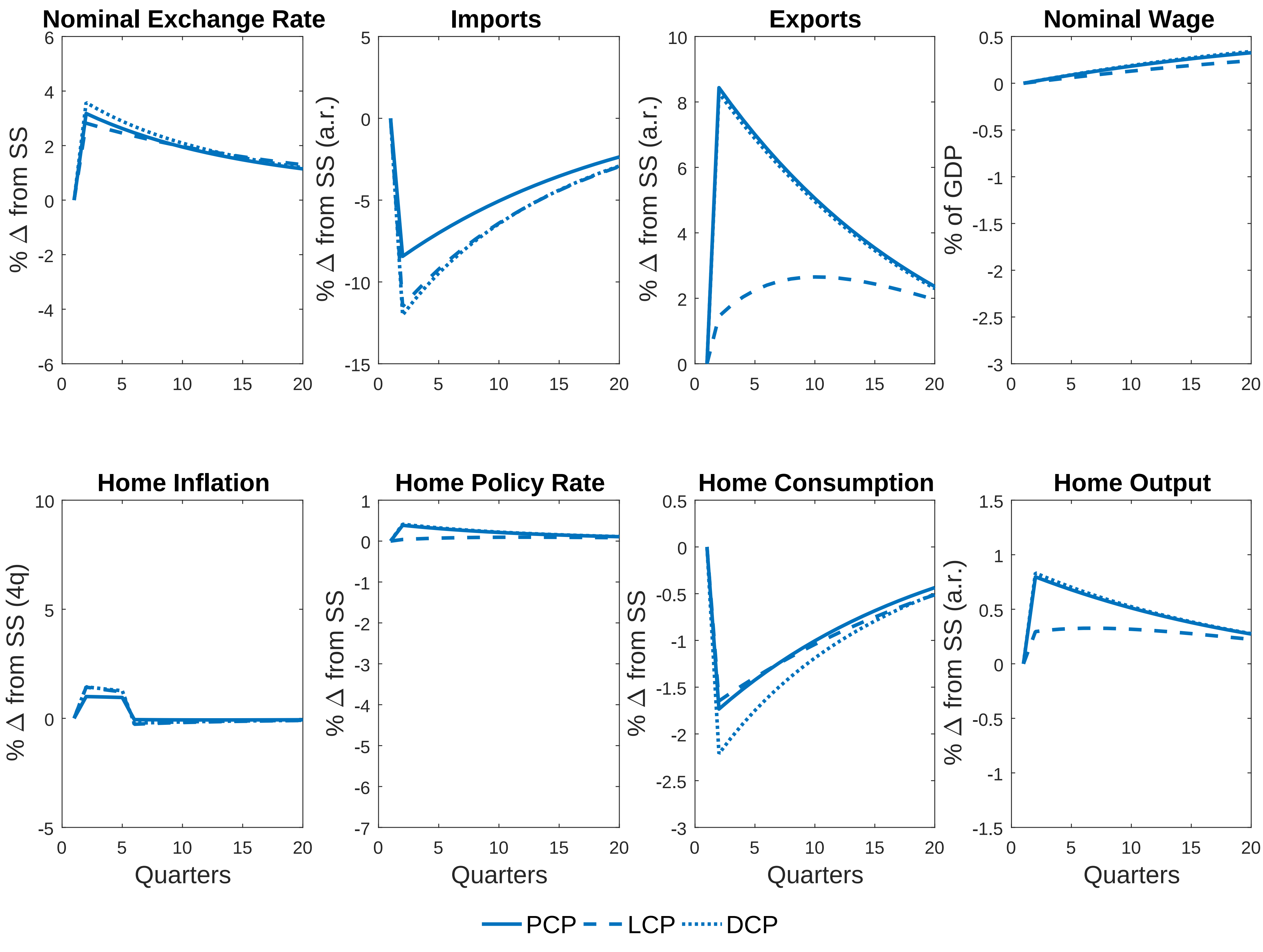

Figure 12. Temporary VP with Sticky Wages: PCP vs LCP vs DCP