The Impact of Equity Tail Risk on Bond Risk Premia: Evidence of Flight-to-Safety in the U.S. Term Structure*

Dario Ruzzi†

University of Bristol

December 12, 2018

Abstract

This paper quantifies the effects of equity tail risk on the term structure of U.S. government securities. We combine the downside jump intensity factors of international stock market indices into a single measure of equity tail risk and show that it is strongly priced in an affine term structure model for the U.S. interest rates. Consistent with the theory of flight-to-safety, we find that the response of Treasury bond yields and future excess returns to a contemporaneous shock to the equity tail factor is negative and opposite to what happens in the stock market. The significance of these results decreases with the maturity of the bonds, suggesting that the short end of the U.S. yield curve is more strongly affected by flight-to-safety than the long end.

JEL classification: C52, C58, G12, E43.

Keywords: Flight to safety, bond risk premium, equity tail risk, affine term structure model.

*This paper benefits from suggestions from conference participants at the Financial Econometrics Conference: Market Microstructure, Limit Order Books and Derivative Markets at Lancaster University Management School. Ruzzi gratefully acknowledges the valuable material and advice on option pricing received from Torben Andersen, Nicola Fusari and Viktor Todorov at conferences and at the Society for Financial Econometrics (SoFiE) Summer School 2017. The author also thanks Mirco Rubin for helpful discussions of preliminary results.

†Corresponding author: School of Economics, Finance and Management, University of Bristol, Bristol, BS8 1TU, UK. Email: dario.ruzzi@bristol.ac.uk.

1 Introduction

In times of financial distress, the disengagement from risky assets, such as stocks, and the simultaneous demand for a safe-haven, such as top-tier government bonds, generate a flight-to-safety (FTS) event in the capital markets. A large body of literature examines the linkages between the stock and bond markets during crisis periods and their implications for asset pricing, see Vayanos (2004), Hartmann et al. (2004), Chordia et al. (2005), Adrian et al. (2015) and Ghysels et al. (2016), among others. We add to this literature by estimating a model of the term structure of interest rates that incorporates the effect of extreme events happening in the stock market. If Treasury bonds are a major beneficiary of the FTS flows occurring when the stock market is hit by heavy losses, then we expect the downside tail risk of equity to affect bond risk premia and determine both stock and bond prices during distress periods. We investigate this conjecture by considering a Gaussian affine term structure model (ATSM) for U.S. interest rates where the pricing factors are the principal components of the yield curve and an equity left tail factor derived from options on international stock market indices. This provides, to the best of our knowledge, the first evidence for the importance of equity tail risk in bond pricing.

Understanding the dynamics of bond yields is particularly useful for forecasting financial and macro variables, for making debt and monetary policy decisions and for derivative pricing. Most of these applications require the decomposition of yields into expectations of future short rates (averaged over the lifetime of the bond) and term premia, i.e. the additional returns required by investors for bearing the risk of long-term commitment. Gaussian affine term structure models have long been used for this purpose, see, e.g., Duffee (2002), Kim and Wright (2005) and Abrahams et al. (2016). In the setup of a Gaussian ATSM, a number of pricing factors that affect bond yields are selected and assumed to evolve according to a vector autoregressive (VAR) process of order one. The yields of different maturities are all expressed as linear functions of the factors with restrictions on the coefficients that prevent arbitrage opportunities, implying that long-term yields are merely risk-adjusted expectations of future short rates.

The selection of pricing factors typically starts by extracting from the cross-section of bond yields a given number of principal components (PCs), which are linear combinations of the rates themselves. The first three PCs are prime candidates as they generally explain over 99% of the variability in the term structure and, due to their loadings on yields, may be interpreted as the level, slope and curvature factor. However, it is well established in the literature that additional factors are needed to explain the cross-section of bond returns. For this reason, the first five principal components of the U.S. Treasury yield curve are used as pricing factors in Adrian et al. (2013), while Malik and Meldrum (2016) adopt a four-factor specification for U.K. government bond yields. Furthermore, several studies suggest that a great deal of information about bond risk premia can be found in factors that are not principal components of the yield curve. Cochrane and Piazzesi (2005) discover a new linear combination of forward rates which is a strong predictor of future excess bond returns and, based on this evidence, Cochrane and Piazzesi (2008) use it in an ATSM along with the classical level, slope and curvature factors. More recently, Duffee (2011) and Joslin et al. (2014) show that valuable information about bond premia is located outside of the yield curve and contained, for example, in macro variables that have little or no impact on current yields but strong predictive power for future bond returns.

This paper explores the use of factors, other than combinations of yields, to drive the curve of U.S. Treasury rates and explain bond returns. In contrast to earlier work, however, we draw on the literature that deals with comovements in the equity and bond markets and we consider the possibility that pricing factors of Treasury bonds originate also in the stock market. The findings of Connolly et al. (2005) and Baele et al. (2010) suggest that measures linked to stock market uncertainty explain time variation in the stock-bond return relation and have important cross-market pricing effects.[1] Therefore, we select a risk measure which is known to

[1]Connolly et al. (2005) find that when the implied volatility from equity index options, measured by the VIX, increases to a considerable extent, bond returns tend to be higher than stock returns (flight-to-quality) and the correlation between the two assets over the next month is lower. Baele et al. (2010) show that the time-varying and sometimes negative stock-bond return correlations cannot be explained by macro variables but instead by

predict equity returns and we examine its role in an ATSM. The existing literature suggests that the variance risk premium (VRP) forecasts stock returns at shorter horizons than other predictors like dividend yields or price-to-earning ratios, see Bollerslev et al. (2009), Bollerslev et al. (2014) and Bekaert and Hoerova (2014), among others. In view of recent studies showing that the predictive power of the VRP for future equity returns stems from a jump tail risk component (see, for example, Bollerslev et al. (2015) and Andersen et al. (2017a,b)), we opt for the jump intensity factor extracted from the Andersen et al. (2015b) model to assess the impact of equity tail events on U.S. Treasury bonds. Hence, our main contribution is to integrate the downside tail risk of the stock market into the study of Treasury bond risk premia.

Caballero et al. (2017) describe U.S. government securities as global stores of value in what they call "safe asset shortage". They illustrate how U.S. government debt rose between 2007 and 2011 in response to an increased demand for safe assets. During the same years, however, the supply of safe assets contracted since debt from fiscally weak sovereigns, such as Italy and Spain, stopped being perceived as safe. The observed surge in cross-border purchases of safe assets indicate that Treasury bonds may react to tail events that originate in the stock market of countries other than the U.S. Therefore, in order to analyze the effect of the international stock market, we follow the methodology of Bollerslev et al. (2014) and define the equity tail risk measure of this study as the market-capitalization weighted average of the downside jump intensity factors extracted from U.S., U.K. and Euro-zone equity-index options.

Our empirical analysis relies on monthly data for the U.S. zero-coupon yield curve and the S&P 500, FTSE 100 and EURO STOXX 50 index options over the period 2007 to 2016. We obtain the equity tail factor from option data, which are well known to embed rich information about the pricing of extreme events, and then we estimate an ATSM as in Adrian et al. (2013) but including also the equity tail factor. Overall, the results show that equity jump tail risk

liquidity factors and the variance risk premium, which represents the compensation demanded by investors for bearing variance risk and is defined as the difference between the risk-neutral and statistical expectations of the future return variation. Although the variance risk premium is a major contributor to the stock-bond return correlation dynamics, Baele et al. (2010) find significant exposures to it only for stock but not for bond returns.

is strongly priced in the term structure model. This risk premium in the Treasury market is consistent with the evidence in Krishnamurthy and Vissing-Jorgensen (2012), who document the existence of a significant price for the safety attribute of Treasuries. Further, we find that bond prices, which move inversely to yields, increase and future expected excess returns shrink in response to a contemporaneous shock to the equity tail factor. These observations confirm the role of U.S. Treasuries as a safe haven and, when combined with the previously documented positive relationship between jump tail risk and future equity returns, indicate the presence of a common predictor across the two asset classes. This remark is in line with the findings reported by Adrian et al. (2015), who show that the same nonlinear function of the VIX can forecast both stock and bond returns, but in opposite directions as predicted by the theory of FTS. Finally, we observe that the predictive power of the equity tail factor for lower bond returns is statistically significant only at the short end of the U.S. yield curve. Based on this evidence, we claim that short-term bonds are more sensitive to flight-to-safety than are long-term bonds.

This study is related to the work of Kaminska and Roberts-Sklar (2015), who document the importance of global market sentiment for the term structure of U.K. government bonds. The authors observe that future excess returns on U.K. bonds load positively on a VRP-based proxy of risk aversion. Their results are consistent with the findings of Bekaert et al. (2010), who show that both equity and bond premia increase with risk aversion, but contrast with the negative relationship between U.S. bond returns and our measure of downside equity tail risk.

Our paper is structured as follows. In section 2 we review the methodology used to identify a left tail factor for the stock market. Section 3 outlines the term structure modeling approach. Section 4 covers the empirical application of equity tail risk in an ATSM. Section 5 concludes.

2 Equity Left Tail Factor

This section summarizes the estimation of the equity tail risk measure whose impact on U.S. Treasuries is discussed later in the paper. This measure, which we denote by $\widetilde{U}^{Equity}$, is obtained as the market capitalization weighted average of downside jump intensity factors driving the returns of international stock market indices. To identify each of these index-specific factors, we rely on the Three-Factor Double Exponential Model proposed for option pricing by Andersen et al. (2015b).[2] The authors specify a parametric model for the risk-neutral dynamics of equity-index returns that includes two volatility factors, V_1 and V_2, plus a separate jump intensity factor, U, which is capable of detecting the priced downside risk in the option surface.[3] The model features two separate jump components: one captures co-jumps in the level of the index, X, the first volatility factor, V_1, and the tail factor, U, and one captures jumps that affect U only. The distribution for the size of return jumps is assumed to be double exponential with two distinct parameters governing the decay of left and right tail. Although the time variation in positive and negative jumps is not the same, both intensities are affine functions of the state vector (V_1, V_2, U). This procedure allows for "cross self-exciting" jumps: a shock to one factor can increase the jump intensity, which in turns increases the probability of future jumps in that and all other factors. The Three-Factor Double Exponential Model is represented by the following equations:

$$
\begin{aligned}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)dt + \sqrt{V_{1,t}}\ dW_{1,t}^{\mathbb{Q}} + \sqrt{V_{2,t}}\ dW_{2,t}^{\mathbb{Q}} + \eta\sqrt{U_t}\ dW_{3,t}^{\mathbb{Q}} + \int_{\mathbb{R}^2} (e^x - 1)\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy), \\
dV_{1,t} &= \kappa_1(\bar{v}_1 - V_{1,t})dt + \sigma_1\sqrt{V_{1,t}}\ dB_{1,t}^{\mathbb{Q}} + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}}\mu(dt, dx, dy)\ , \\
dV_{2,t} &= \kappa_2(\bar{v}_2 - V_{2,t})dt + \sigma_2\sqrt{V_{2,t}}\ dB_{2,t}^{\mathbb{Q}}\ , \\
dU_t &= -\kappa_u U_t dt + \mu_u \int_{\mathbb{R}^2} [(1-\rho_u)x^2 1_{\{x<0\}} + \rho_u y^2]\mu(dt, dx, dy)\ .
\end{aligned}
\tag{1}
$$

where r_t is the risk-free rate, δ_t is the dividend yield on the index, and $(W_{1,t}^{\mathbb{Q}}, W_{2,t}^{\mathbb{Q}}, W_{3,t}^{\mathbb{Q}}, B_{1,t}^{\mathbb{Q}}, B_{2,t}^{\mathbb{Q}})$ is a five-dimensional Brownian motion with $\mathrm{corr}\left(W_{1,t}^{\mathbb{Q}}, B_{1,t}^{\mathbb{Q}}\right) = \rho_1$, $\mathrm{corr}\left(W_{2,t}^{\mathbb{Q}}, B_{2,t}^{\mathbb{Q}}\right) = \rho_2$, and

[2] The interested reader is directed to Andersen et al. (2015b) for an in-depth description of the formulation since here we limit ourselves to highlighting the distinctive features.

[3] Nonparametric and seminonparametric approaches to estimating a tail risk measure from option data are also available, see, e.g., Aït-sahalia and Lo (2000), Bollerslev and Todorov (2011), Bollerslev et al. (2015) and Andersen et al. (2017a). For tail risk measures that are computed from the cross-section of returns without relying on option price information see, e.g., Kelly and Jiang (2014) and Almeida et al. (2017)

mutual independence for the remaining Brownian motions. In addition, μ is the jump counting measure with instantaneous intensity, under the risk-neutral measure, given by $dt \otimes \nu_t^{\mathbb{Q}}(dx, dy)$. The difference $\widetilde{\mu}^{\mathbb{Q}}(dt, dx, dy) = \mu(dt, dx, dy) - dt\nu_t^{\mathbb{Q}}(dx, dy)$ constitutes the associated martingale measure. The contemporaneous co-jumps in X, V_1 and, if $\rho_u < 1$, also in U are captured by x, while y represents the independent shocks to the U factor. The jump component x is distributed according to a double exponential density function with separate tail decay parameters, λ_- and λ_+, for negative and positive jumps, respectively. The jump component y is distributed identically to the negative price jumps. Moreover, $c^-(t)$ and $c^+(t)$ define the time-varying intensities of, respectively, negative and positive jumps as follows,

$$c^-(t) = c_0^- + c_1^- V_{1,t} + c_2^- V_{2,t} + c_3^- U_t \quad , \qquad c^+(t) = c_0^+ + c_1^+ V_{1,t} + c_2^+ V_{2,t} + c_3^+ U_t \ . \tag{2}$$

Finally, the jump compensator characterizes the conditional jump distribution and is given by,

$$\frac{\nu_t^{\mathbb{Q}}(dx, dy)}{dxdy} = \begin{cases} (c^-(t) \cdot 1_{\{x<0\}} \lambda_- e^{-\lambda_- |x|} + c^+(t) \cdot 1_{\{x>0\}} \lambda_+ e^{-\lambda_+ x}) \ , & \text{if } y = 0 \\ c^-(t) \lambda_- e^{-\lambda_- |y|} \ , \quad \text{if } x = 0 \text{ and } y < 0 \end{cases}$$

Supported by the data, Andersen et al. (2015b) constrain the statistically insignificant parameters to zero and set c_3^- to unity for identification purposes.[4] The implication of this is that U becomes a left tail factor that affects the intensity of only negative jumps and does not contribute directly to the diffusive spot variance. Given these characteristics, we are motivated to formulate the equity tail risk measure of the present paper in terms of the U factor.

The period-by-period estimates of the state variables, $(V_{1,t}\ V_{2,t}\ U_t)$, together with values for the model parameters, are obtained by using the penalized nonlinear least squares estimator developed by Andersen et al. (2015a). The Andersen et al. (2015b) model is fitted to a panel of equity-index options by minimizing the weighted sum of squared deviations of the Black-

[4]The restrictions on the parameters are the same as those shown in Table 1, 2 and 3 of this paper.

Scholes implied volatilities generated by the model from the observed ones.[5] In solving this minimization problem, the estimator also penalizes for discrepancies between the model-implied spot volatilities and those estimated, in a nonparametric fashion, from high-frequency data on the underlying asset returns. Using the same notation as in Andersen et al. (2015b), we denote the parameter vector of the model by θ and the state vector at time t by $\mathbf{Z_t} = (V_{1,t}\ V_{2,t}\ U_t)$. Further, we use $\kappa(k, \tau, \mathbf{Z_t}, \theta)$ and $\bar{\kappa}(t, k, \tau)$ to denote, respectively, the model-implied Black-Scholes implied volatility (IV) and the observed Black-Scholes IV corresponding to the average of bid and ask quotes of the option with tenor τ and log-forward moneyness k at time t. As for the diffusive spot variance, we denote the model-implied measure by $V(\mathbf{Z_t}, \theta) = V_{1,t} + V_{2,t} + \eta^2 U_t$ and its nonparametric estimator constructed from intraday returns by $\widehat{V}_t$.[6] Finally, letting N_t denote the number of option contracts available on day t, the estimator takes the form,

$$\left(\{\hat{V}_{1,t}, \hat{V}_{2,t}, \hat{U}_t\}_{t=1,\ldots,T}, \hat{\theta}\right) = \underset{\{\mathbf{Z_t}\}_{t=1,\ldots T}, \theta \in \Theta}{\arg\min} \sum_{t=1}^{T} \left\{ \frac{\text{Option Fit}_t + \lambda \times \text{Vol Fit}_t}{V_t^{ATM}} \right\} \tag{3}$$

$$\text{Option Fit}_t = \frac{1}{N_t} \sum_{j=1}^{N_t} \left(\bar{\kappa}(t, k_j, \tau_j) - \kappa(k_j, \tau_j, \mathbf{Z_t}, \theta\right)^2, \qquad \text{Vol Fit}_t = \left(\sqrt{\widehat{V}_t} - V(\mathbf{Z_t}, \theta)\right)^2 \tag{4}$$

where λ is a calibration parameter that we set to 0.05 as in Andersen et al. (2017b), and V_t^{ATM} is the squared Black-Scholes IV obtained for the option closest to at-the-money with the shortest available maturity on day t.[7] The standardization by V_t^{ATM} in the estimator is such that days with high market volatility are underweighted because option pricing errors tend to be larger.

Throughout the rest of the paper, we use the residual of the regression of the U factor on the spot variance V to construct our equity left tail factor and study the effect of equity tail risk on bond risk premia. This choice is motivated by the work of Andersen et al. (2015b, 2017b)

[5] Andersen et al. (2015b) provide in their *Supplementary Appendix* the conditional characteristic function of log-returns needed to price options according to the Three-Factor Double Exponential Model. The obtained option prices are then expressed in Black-Scholes implied volatility units for estimation purposes.

[6] For jump-robust volatility estimators based on high-frequency data see, for instance, Andersen et al. (2012).

[7] In practice, the joint optimization over parameters and state vector realizations is performed by concentrating, or profiling, the state vector and optimizing over the model parameters. Indeed, given a candidate vector θ, it is easy to obtain estimates of $(V_{1,t}\ V_{2,t}\ U_t)$ with local optimization search methods. By contrast, the search of a global optimum is done for vector θ.

who have recently shown that the component of the left jump tail intensity factor unspanned by volatility, the so-called "pure tail" factor, has strong predictive power for future equity returns. Building on the significant stock-bond return comovements documented in times of elevated risk, we investigate the explanatory power of this equity "pure tail" factor for future bond returns. As mentioned earlier, due to the scarcity of safe asset producers other than the United States (Caballero et al., 2017), we treat U.S. Treasury bonds as global safe haven and examine their response to the downside tail risk of the international stock market. Therefore, if we denote by $\widetilde{U}^i$ the "pure tail" factor relating to the i-th stock market index, we construct the equity left tail factor of this paper as follows,

$$\widetilde{U}_t^{Equity} = \sum_{i=1}^{I} w_t^i \, \widetilde{U}_t^i \ , \tag{5}$$

where w_t^i is the time-t market capitalization of the i-th stock market index divided by the sum of the market capitalizations of the I indices at time t.[8]

3 Term Structure Modeling

We now introduce the term structure framework adopted in this paper and we present its estimation procedure. To set up the model, we rely on the approach suggested by Adrian et al. (2013), which has the advantage that the pricing factors of bonds are not restricted to linear combinations of yields. Factors can indeed also be of different origin, such as the equity tail measure defined in Section 2 and whose use in a model for U.S. interest rates is the main novelty of this paper. After deriving the data generating process of log excess bond returns from a dynamic asset pricing model with an exponentially affine pricing kernel, Adrian et al. (2013) propose a new regression-based estimation technique for the model parameters. The linear regressions of this simple estimator avoid the computational burden of maximum likelihood

[8] In results available upon request, we show that the option-implied "pure tail" factor of the U.S. stock market provides, within the model for the U.S. term structure, qualitatively identical performance to that of $\widetilde{U}_t^{Equity}$.

methods, which have previously been the standard approach to the pricing of interest rates.

The formulation and estimation of the Gaussian ATSM in Adrian et al. (2013) can be summarized as follows. A $K \times 1$ vector of pricing factors, $\mathbf{X}_t$, is assumed to evolve according to a VAR process of order one:

$$\mathbf{X}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\phi}\mathbf{X}_t + \mathbf{v}_{t+1} \ , \tag{6}$$

where the shocks $\mathbf{v}_{t+1} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$ are conditionally Gaussian with zero mean and variance-covariance matrix $\boldsymbol{\Sigma}$. Letting $P_t^{(n)}$ denote the price of a zero-coupon bond with maturity n at time t, the assumption of no-arbitrage implies the existence of a pricing kernel M_{t+1} such that,

$$P_t^{(n)} = \mathrm{E}_t\Big[M_{t+1}P_{t+1}^{(n-1)}\Big] \ . \tag{7}$$

The pricing kernel M_{t+1} is assumed to have the following exponential form:

$$M_{t+1} = \exp\Big(-r_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\Sigma}^{-1/2}\mathbf{v}_{t+1}\Big) \ , \tag{8}$$

where $r_t = -\ln P_t^{(1)}$ is the continuously compounded one-period risk-free rate and $\boldsymbol{\lambda}_t$ is the $K \times 1$ vector of market prices of risk, which are affine in the factors as in Duffee (2002):

$$\boldsymbol{\lambda}_t = \boldsymbol{\Sigma}^{-1/2}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\mathbf{X}_t) \ . \tag{9}$$

The log excess one-period return of a bond maturing in n periods is defined as follows,

$$rx_{t+1}^{(n-1)} = \ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} - r_t \ . \tag{10}$$

After assuming the joint normality of $\{rx_{t+1}^{(n-1)}, \mathbf{v}_{t+1}\}$, Adrian et al. (2013) derive the return generating process for log excess returns, which takes the form[9],

$$rx_{t+1}^{(n-1)} = \boldsymbol{\beta}^{(n-1)'}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 \mathbf{X}_t) - \frac{1}{2}(\boldsymbol{\beta}^{(n-1)'}\boldsymbol{\Sigma}\boldsymbol{\beta}^{(n-1)} + \sigma^2) + \boldsymbol{\beta}^{(n-1)'}\mathbf{v}_{t+1} + e_{t+1}^{(n-1)} \ , \tag{11}$$

where the return pricing errors $e_{t+1}^{(n-1)} \sim$ i.i.d. $(0, \sigma^2)$ are conditionally independently and identically distributed with zero mean and variance σ^2. Letting N be the number of bond maturities available and T be the number of time periods at which bond returns are observed, Adrian et al. (2013) rewrite equation (11) in the stacked form,

$$\mathbf{rx} = \boldsymbol{\beta}'(\boldsymbol{\lambda}_0 \boldsymbol{\iota}_T' + \boldsymbol{\lambda}_1 \mathbf{X}_\_) - \frac{1}{2}(\mathbf{B}^* \text{vec}(\boldsymbol{\Sigma}) + \sigma^2 \boldsymbol{\iota}_N)\boldsymbol{\iota}_T' + \boldsymbol{\beta}'\mathbf{V} + \mathbf{E} \ , \tag{12}$$

where $\mathbf{rx}$ is an $N \times T$ matrix of excess bond returns, $\boldsymbol{\beta} = \left[\boldsymbol{\beta}^{(1)} \ \boldsymbol{\beta}^{(2)} \ ... \ \boldsymbol{\beta}^{(N)}\right]$ is a $K \times N$ matrix of factor loadings, $\boldsymbol{\iota}_T$ and $\boldsymbol{\iota}_N$ are a $T \times 1$ and $N \times 1$ vector of ones, $\mathbf{X}_\_ = [\mathbf{X}_0 \ \mathbf{X}_1 \ ... \ \mathbf{X}_{T-1}]$ is a $K \times T$ matrix of lagged pricing factors, $\mathbf{B}^* = \left[\text{vec}(\boldsymbol{\beta}^{(1)}\boldsymbol{\beta}^{(1)'}) \ ... \ \text{vec}(\boldsymbol{\beta}^{(N)}\boldsymbol{\beta}^{(N)'})\right]'$ is an $N \times K^2$ matrix, $\mathbf{V}$ is a $K \times T$ matrix and $\mathbf{E}$ is an $N \times T$ matrix.

The main novelty of the approach taken by Adrian et al. (2013) to model the term structure of interest rates is the use of ordinary least squares to estimate the parameters of equation (12). In particular, the authors propose the following three-step procedure:

1. Estimate the coefficients of the VAR model in equation (6) by ordinary least squares.[10] Stack the estimates of the innovations $\hat{\mathbf{v}}_{t+1}$ into matrix $\hat{\mathbf{V}}$ and use this to construct an estimator of the variance-covariance matrix $\hat{\boldsymbol{\Sigma}} = \hat{\mathbf{V}}\hat{\mathbf{V}}'/T$.

2. From the excess return regression equation $\mathbf{rx} = \mathbf{a}\boldsymbol{\iota}_T' + \boldsymbol{\beta}'\hat{\mathbf{V}} + \mathbf{c}\mathbf{X}_\_ + \mathbf{E}$, obtain estimates of $\hat{\mathbf{a}}$, $\hat{\boldsymbol{\beta}}$ and $\hat{\mathbf{c}}$. Use $\hat{\boldsymbol{\beta}}$ to construct $\hat{\mathbf{B}}^*$. Stack the residuals of the regression into matrix $\hat{\mathbf{E}}$ and use this to construct an estimator of the variance $\hat{\sigma}^2 = \text{tr}(\hat{\mathbf{E}}\hat{\mathbf{E}}')/NT$.

3. Noting from equation (12) that $\mathbf{a} = \boldsymbol{\beta}'\boldsymbol{\lambda}_0 - \frac{1}{2}(\mathbf{B}^* \text{vec}(\boldsymbol{\Sigma}) + \sigma^2 \boldsymbol{\iota}_N)$ and $\mathbf{c} = \boldsymbol{\beta}'\boldsymbol{\lambda}_1$, estimate

[9] For the full derivation of the data generating process see Section 2.1 in Adrian et al. (2013).
[10] For estimation purposes, Adrian et al. (2013) advise to set $\boldsymbol{\mu} = 0$ in case of zero-mean pricing factors.

the price of risk parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$ via cross-sectional regressions,

$$\hat{\boldsymbol{\lambda}}_0 = (\hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}^{'})^{-1}\hat{\boldsymbol{\beta}}\Big(\hat{\mathbf{a}} + \frac{1}{2}(\hat{\mathbf{B}}^{*}\text{vec}(\hat{\boldsymbol{\Sigma}}) + \hat{\sigma}^2\boldsymbol{\iota}_N)\Big) \ , \tag{13}$$

$$\hat{\boldsymbol{\lambda}}_1 = (\hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}^{'})^{-1}\hat{\boldsymbol{\beta}}\hat{\mathbf{c}} \ . \tag{14}$$

The analytical expressions of the asymptotic variance and covariance of $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\Lambda}} = [\hat{\boldsymbol{\lambda}}_0 \ \hat{\boldsymbol{\lambda}}_1]$, which we do not report here to save space, are provided in Appendix A.1 of Adrian et al. (2013). From the estimated model parameters, Adrian et al. (2013) show how to generate a yield curve. Indeed, within the proposed framework, bond prices are exponentially affine in the pricing factors. Consequently, the yield of a zero-coupon bond with maturity n at time t, $y_t^{(n)}$, can be expressed as follows,

$$y_t^{(n)} = -\frac{1}{n}[a_n + \mathbf{b}_n^{'}\mathbf{X}_t] + u_t^{(n)} \ , \tag{15}$$

where the coefficients a_n and $\mathbf{b}_n$ are obtained from the following no-arbitrage recursions,

$$a_n = a_{n-1} + \mathbf{b}_{n-1}^{'}(\boldsymbol{\mu} - \boldsymbol{\lambda}_0) + \frac{1}{2}(\mathbf{b}_{n-1}^{'}\boldsymbol{\Sigma}\mathbf{b}_{n-1} + \sigma^2) - \delta_0 \ , \tag{16}$$

$$\mathbf{b}_n^{'} = \mathbf{b}_{n-1}^{'}(\boldsymbol{\phi} - \boldsymbol{\lambda}_1) - \boldsymbol{\delta}_1^{'} \ , \tag{17}$$

subject to the initial conditions $a_0 = 0$, $\mathbf{b}_n = \mathbf{0}$, $a_1 = -\delta_0$ and $\mathbf{b}_1 = -\boldsymbol{\delta}_1$. The parameters δ_0 and $\boldsymbol{\delta}_1$ are estimated by regressing the short rate, $r_t = -\ln P_t^{(1)}$, on a constant and contemporaneous pricing factors according to,

$$r_t = \delta_0 + \boldsymbol{\delta}_1\mathbf{X}_t + \epsilon_t \ , \quad \epsilon_t \sim i.i.d. \ (0, \sigma_\epsilon^2) \ . \tag{18}$$

By setting the price of risk parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$ to zero in equation (16) and (17), Adrian et al. (2013) obtain a_n^{RN} and $\mathbf{b}_n^{\text{RN}}$, which they use to generate the risk-neutral yields, $y_t^{(n)\ \text{RN}}$.

These yields reflect the average expected short rate over the current and the subsequent $(n-1)$ periods and are computed as follows,

$$y_t^{(n)\ \mathrm{RN}} = \frac{1}{n}\sum_{i=0}^{n-1} \mathrm{E}_t[r_{t+i}] = -\frac{1}{n}[a_n^{\mathrm{RN}} + \mathbf{b}_n^{\mathrm{RN}\prime}\mathbf{X}_t] \ . \tag{19}$$

Given equation (15) and (19), the term premium $TP_t^{(n)}$, which is the additional compensation required for investing in long-term bonds relative to rolling over a series of short-term bonds, can be calculated as follows,

$$TP_t^{(n)} = y_t^{(n)} - y_t^{(n)\ \mathrm{RN}} \ . \tag{20}$$

Starting from the expressions for the zero-coupon bond yields, it is possible to show that also forward rates are affine functions of the pricing factors. In particular, we calculate $f_t^{m,n}$, which denotes the forward rate at time t for an investment that starts m periods after time t and terminates n periods after time t, as follows,

$$f_t^{(m,n)} = \frac{1}{n-m}\Big[(a_m - a_n) + (\mathbf{b}_m^{'} - \mathbf{b}_n^{'})\mathbf{X}_t\Big] \ . \tag{21}$$

By replacing a_m, a_n, $\mathbf{b}_m$ and $\mathbf{b}_n$ in equation (21) with their risk-neutral counterparts a_m^{RN}, a_n^{RN}, $\mathbf{b}_m^{\mathrm{RN}}$ and $\mathbf{b}_n^{\mathrm{RN}}$, we obtain the risk-neutral forward rates $f_t^{(m,n)\ \mathrm{RN}}$ which we use to calculate the forward term premium $FTP_t^{(m,n)}$ according to,

$$FTP_t^{(m,n)} = f_t^{(m,n)} - f_t^{(m,n)\ \mathrm{RN}} \ . \tag{22}$$

In the next section we specify and estimate a term structure model for U.S. interest rates following the procedure outlined above. The main difference between the Gaussian ATSM in Adrian et al. (2013) and ours is that we use a different set of pricing factors. Indeed, we include in $\mathbf{X}_t$ not only factors of bond-market origin (principal components of the yield curve) but also the left jump tail risk measure extracted from equity-index options and described in Section 2.

4 Empirical Application

We provide in this section an application to data of a bond pricing model featuring equity tail risk. We present empirical results using bond data from the U.S. market and equity option data from the U.S., U.K. and Euro-zone markets. We start by examining the role of the equity left tail factor in predicting bond returns for horizons up to one year. We then estimate a Gaussian ATSM that uses the equity left tail factor, along with the first five principal components of Treasury yields, to explain the cross-section of one-month excess bond returns. We report the estimation results for the full sample and we claim that equity jump tail risk is strongly priced within the model and is a significant predictor of lower expected returns on short-term bonds. Finally, we discuss how equity tail risk has influenced the Treasury term structure over time.

4.1 Data

All data considered here are sampled at the end of each month, or the previous trading day if the month-end value is missing, for the period from January 2007 through November 2016. In this study on bond premia, the start date of the sample is chosen in accordance with Andersen et al. (2017b), who analyze the impact of market tail risk on the equity risk premium instead.[11]

To construct the equity left tail factor, we use the closing bid and ask prices reported by OptionMetrics IvyDB US for the European style S&P 500 equity-index (SPX) options, and the last prices reported by OptionMetrics IvyDB Europe for the European style FTSE 100 (FTSE) and EURO STOXX 50 (ESTOXX) equity-index options. We apply the following standard filters to our dataset. We discard options with a tenor of less than seven days or more than one year. We discard options with zero bid prices and options with non-positive open interest. We only use options with non-negative bid-ask spread and options with an ask-to-bid ratio smaller than five. We retain only options whose prices are at least threefold the minimum tick size. For each

[11] Since 2007, the available maturity and strike coverage of the option data of this study is broad enough to estimate the Three-Factor Double Exponential Model discussed in Section 2 instead of the simplified two-factor model used by Andersen et al. (2017b).

day in the sample, we retain only option tenors for which we have at least five pairs of call and put contracts with the same strike price. We exploit these cross sections to derive, via put-call parity, the risk-free rate and the underlying asset price adjusted for the dividend yield that apply to a given option tenor on a given day. Finally, we discard all in-the-money options and we use only out-of-the-money options whose volatility-adjusted log-forward moneyness is between -15 and 5. The option data so obtained are supplemented by the time series of the three indices' Bipower Variation (5-min) provided by the OxfordMan Institute's "realised library". This is the nonparametric estimator of variance, constructed from high-frequency returns, that we use in equation (4) to compute Vol Fit_t. The estimator belongs to the class of jump-robust measures of volatility and was introduced by Barndorff-Nielsen and Shephard (2004).

The term structure model of this paper is estimated using the Gürkaynak et al. (2007) zero-coupon bond yields derived from U.S. Treasuries.[12] In line with the range of maturities in Adrian et al. (2013) and Abrahams et al. (2016), for our analysis we consider bonds maturing in less than or equal to ten years. More specifically, we extract the principal components, which we then use as pricing factors in the ATSM, from yields of maturities $n = 3, 6, ..., 120$ months. Furthermore, setting the risk-free short rate equal to the $n = 1$ month yield, we calculate the one-month excess returns for Treasury bonds with maturities $n = 6, 12, ..., 120$ months.

4.2 Equity Tail Risk and Bond Pricing

The estimated parameters of the Three-Factor Double Exponential Model applied to S&P 500, FTSE 100 and EURO STOXX 50 equity-index option data are listed, respectively, in Table 1, 2 and 3. We note that our estimates for the S&P 500 index are very close to those reported by Andersen et al. (2015b) in their Table 4.

Insert Table 1, 2 and 3 here

[12]These yield data are available at a daily frequency for annually spaced maturities ranging from 1 to 30 years from the Federal Reserve website https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html. The parameters used to calculate the yields of any desired maturity are also available.

Using the index-specific estimated parameter vector $\hat{\theta}$, we recover the month-by-month realizations of the state variables for the S&P 500, FTSE 100 and EURO STOXX 50 equity-index returns, which are displayed in Figure 1. The top panel shows the model-implied diffusive spot variance, which we denote by V and is given by the sum of the two volatility factors, V_1 and V_2. The middle panel displays the downside jump intensity factor, U. The bottom panel presents the pure tail factor, $\widetilde{U}$, which corresponds to the residual obtained from the linear regression of U on V, and then normalized to have mean zero and unit variance. High values of $\widetilde{U}$ reflect a high perception, or fear, of future negative return jumps in the stock market. The equity left tail risk factor that we use in the term structure model of this paper is given by the market-capitalization weighted average of the $\widetilde{U}$ factor of the three stock market indices.[13]

Insert Figure 1 here

Inspection of Figure 1 immediately reveals that, as documented in Andersen et al. (2015b, 2017b), the negative jump intensity factor is far more persistent than diffusive volatility in the years following a crisis. In those studies, the component of the left jump intensity factor unspanned by volatility, i.e. $\widetilde{U}$, is shown to be a strong predictor of future excess equity-index returns, with predictive power superior to that of the VRP. Starting from this result and inspired by the theory of flight-to-safety, we explore the relationship of $\widetilde{U}$ with the U.S. government bond market. We start by considering the role of S&P 500 option-implied measures in explaining Treasury risk premia. To this end, we regress the future excess returns of one-, two-, three-, four-, five-, seven- and ten-year Treasury bonds ($n = 12, 24, 36, 48, 60, 84, 120$ months, respectively) on the left jump intensity factor (orthogonal to spot variance) of S&P 500 equity-index returns, $\widetilde{U}^{\text{SPX}}$, and the variance risk premium orthogonal to $\widetilde{U}^{\text{SPX}}$, which we denote by $VRP^{\perp}$. We calculate the variance risk premium (on a monthly basis) as the difference between the

[13] All the code has been written in Python to benefit from its computational speed. Also, since the estimation of state variables in the option pricing exercise is inherently independent from one day to another, we relied on BlueCrystal, the High Performance Computing (HPC) machine provided by the Advanced Computing Research Centre at University of Bristol, to exploit the power of multiple CPUs at the same time. Finally, it is a pleasure to acknowledge the invaluable help and advice received from Nicola Fusari in support of this part of the work.

square of the VIX index (obtained from the Chicago Board Options Exchange (CBOE)) and the summation of current and previous 20 trading days' daily realized variances and overnight squared returns of the S&P 500 (obtained from the OxfordMan Institute's "realised library").[14] As is pointed out in Andersen et al. (2017b), there is a strong correlation between the tail factor and the variance risk premium since the former is part of the risk-neutral measure of expected return variation used to calculate the latter. For this reason, the auxiliary explanatory power of the VRP is assessed in the following bivariate regressions,

$$rx_{t+h}^{(n-h)} = c_{0,h} + c_{u,h} \cdot \widetilde{U}_t^{\mathrm{SPX}} + c_{p,h} \cdot VRP_t^{\perp} + \xi_{t+h} \ , \tag{23}$$

where h is the holding period (in months) and $rx_{t+h}^{(n-h)} = \ln P_{t+h}^{(n-h)} - \ln P_t^{(n)} - r_t$ is the h-month excess log-return on a bond with maturity n (in months) at time t. The risk-free rate used in the calculation of the excess returns is the yield of a zero-coupon bond with maturity h at time t. We run the predictive regressions using the full sample of monthly data over forecast horizons $h = 1, 2, ..., 12$ months. We compute the robust Newey-West standard errors using a window of twice as many lags as the number of months within the holding period horizon.[15] The significance of the regression coefficients and the degree of explanatory power provided by $\widetilde{U}^{\mathrm{SPX}}$ and $VRP^{\perp}$ are presented in Figure 2.

Insert Figure 2 here

The immediate point that stands out is that Treasury risk premia load negatively on the S&P 500 option-implied tail factor, $\widetilde{U}^{\mathrm{SPX}}$. Hence, as reflecting flight-to-safety, a higher fear of abrupt negative return shocks to the U.S. equity market forces a contraction in the risk premia that investors require for holding U.S. government bonds. For bonds with maturity less than

[14] This is the same methodology used by Bollerslev et al. (2009) and Bollerslev et al. (2014) to approximate the variance risk premium. This proxy of VRP has the advantage of being directly observable and completely model-free as the risk-neutral expectation of the future return variation is measured by the CBOE VIX squared, while the statistical expectation is quantified by the total realized variation over the previous month.

[15] The regression standard errors so constructed should also account for the estimation error in the projection generating the pure tail factor, see Andersen et al. (2015b, 2017b).

five years, we find that the excess returns are significantly related to $\widetilde{U}^{\text{SPX}}$, for most of the forecast horizons considered. For longer-term bonds, the explanatory power of $\widetilde{U}^{\text{SPX}}$ is never statistically significant at the 10% level. Examining the coefficient of $VRP^{\perp}$, we observe that the component of the variance risk premium orthogonal to $\widetilde{U}^{\text{SPX}}$ is highly significant for the longer return horizons of short-term bonds and for the shorter horizons of long-term bonds. The sign of the coefficient is positive in the former case and negative in the latter. Therefore, since the variance risk premium captures both jump and diffusive spot volatility, the risk premium associated with the diffusive part tends to predict lower bond returns at short horizons and higher returns at long horizons. Thus we can conclude that the component of the variance risk premium unrelated to the tail factor may have some predictive power for the Treasury risk premia. However, since the relevant return horizon in this paper is one month and $VRP^{\perp}$ does not seem to have (at this horizon) predictive power over-and-above $\widetilde{U}^{\text{SPX}}$ for most of the excess bond returns, it seems reasonable to us to consider the equity tail index, rather than the variance risk premium, as a potentially relevant pricing factor for the bond market.

Motivated by the global safe haven status of U.S. government bonds (Caballero et al., 2017) and the commonalities across countries in stock return predictability (Bollerslev et al., 2014; Andersen et al., 2017b), we now extend our analysis to the explanatory power of the U.K. and Euro-zone equity tail risk measures.[16] We explore predictive regressions for Treasury risk premia including the option-implied left jump intensity factor (orthogonal to spot variance) of FTSE 100 equity-index returns, $\widetilde{U}^{\text{FTSE}}$, and the option-implied left jump intensity factor (orthogonal to spot variance) of EURO STOXX 50 equity-index returns, $\widetilde{U}^{\text{ESTOXX}}$, as single explanatory variables. Hoping to uncover stronger predictability than the one provided by the individual country measures, we also use as predictor $\widetilde{U}^{Equity}$, which is the market-capitalization weighted

[16] In their multi-country study, Bollerslev et al. (2014) and Andersen et al. (2017b) find, respectively, that a country's equity "pure tail" factor and variance risk premium forecast the future equity market returns of that country. The predictability pattern that holds true for a number of countries in addition to the United States leads Bollerslev et al. (2014) to define a "global" variance risk premium, which, they find, is a highly significant predictor for all of the different country returns.

average of the option-implied pure tail factor of S&P 500, FTSE 100 and EURO STOXX 50 equity-index returns, calculated from equation (5). The univariate regressions take the form,

$$rx_{t+h}^{(n-h)} = d_{0,h} + d_{y,h} \cdot Y_t + \xi_{t+h}^{Y} \ , \tag{24}$$

where Y_t denotes one of the possible predictors, and the quantities on the left-hand side are the same log-returns as those used for equation (23). The results are presented in Figure 3 and 4.[17]

Insert Figure 3 and 4 here

Inspection of Figure 3 reveals that excess Treasury returns are linked to the left jump tail intensity factor driving stock index returns in the U.K. and Euro-area markets, for horizons beyond four-five months and with a peak at around six months. Importantly, the same holds true for bonds with maturity of five years or longer, for which we failed to find significant exposures to $\widetilde{U}^{\mathrm{SPX}}$ in Figure 2. Furthermore, examining the short return horizons, we note that the explanatory power of $Y_t = \widetilde{U}_t^{\mathrm{ESTOXX}}$ is statistically insignificant. On the contrary, the option-implied tail factor of the U.K. stock market, $Y_t = \widetilde{U}_t^{\mathrm{FTSE}}$, contains predictive power for the one-month excess returns on short-term bonds. A potential explanation for the observed inverse relationship between risk premia in the U.S. government bond market and downside tail risk in the U.K. equity market can be the role of safe asset producer that the United States play for many economies, including the United Kingdom. For instance, as reported by the Economist (2015), the world aggregate demand for safe assets shifted away from the U.K. supply toward the U.S. one in 1920-45, when Britain ceased to be the world's pre-eminent power and passed the safe asset baton to the United States.[18] Figure 4 shows qualitative features that are similar to those of Figure 2 and 3. That is, Treasury risk premia, computed over horizons up to one year, load negatively on a measure of downside equity tail risk, this time associated with the

[17]The standard errors are computed in the same way as those for the regression coefficients in equation (23).

[18]See Chiţu et al. (2014) for a thorough examination of how and why the dollar overtook sterling as the dominant currency of denomination for international bonds.

international stock market, $Y_t = \widetilde{U}_t^{Equity}$. Despite the negative sign of the coefficient, which is in agreement with the theory of flight-to-safety, we note, however, that the results tend to be statistically significant at the 10% level only for short-maturity bonds.

On the basis of the significant interactions observed between excess Treasury returns and the $\widetilde{U}$ factor of all three equity market indices considered, we deem it best to evaluate the overall effect of the international stock market on US government securities. Hence, the equity left tail factor that we use throughout the rest of the paper is $\widetilde{U}^{Equity}$, which we interpret as a measure of international fear of future abrupt negative return shocks to the equity market.

The limited statistical power of the preliminary analysis presented thus far may be justified by the misspecification of equation (23) and (24) that should logically include additional explanatory variables of bond risk premia. To go further in the analysis, we now estimate a Gaussian ATSM for U.S. interest rates that includes as pricing factors the first five principal components of Treasury yields and the equity left tail factor, $\widetilde{U}^{Equity}$. This is a richer framework that allows us to explore in detail the effect of equity tail risk on contemporaneous bond yields and future excess bond returns. The first five principal components of the U.S. yield curve have proven to be remarkably effective in fitting the cross-section of bond yields and returns in Adrian et al. (2013). Based on this evidence, we let these PCs drive the interest rates of our model as well, but with a slight modification of the methodology. Indeed, in order to have pricing factors that are uncorrelated with each other, we follow Cochrane and Piazzesi (2008) and extract the principal components not from the conventional yields, but instead from the yields orthogonalized to the extra factor, which in our study is $\widetilde{U}^{Equity}$. By doing so, we obtain yield curve factors that are unrelated to the pricing of tail risk in the stock market, which is entirely ascribed to the $\widetilde{U}^{Equity}$ factor. The choice of those state variables for our model is supported by the following observations. First, we note that the equity left tail factor is poorly spanned by the first five PCs extracted from the non-orthogonalized yields. Indeed, a regression of $\widetilde{U}^{Equity}$ on the traditional level, slope and curvature factors augmented with the fourth and fifth principal components results in an R^2 of only 28%. We find no significant relationships between the equity left tail factor and these PCs, as the largest correlation coefficient is -0.35 with the level factor. Therefore, if we want to capture the effect of equity tail risk on bond risk premia we must include $\widetilde{U}^{Equity}$ separately in the vector of pricing factors, $\mathbf{X}_t$, and orthogonalize for convenience the remaining factors. The second observation that we make about the choice of the state variables in $\mathbf{X}_t$ is that we cannot exclude the fourth and fifth principal components of the yield curve. The regressions of $PC4$ and $PC5$ on the equity left tail factor yield an R^2 of, respectively, 6% and 2%. These results imply that $\widetilde{U}^{Equity}$ does not subsume the predictive ability of the fourth and fifth principal components of the yield curve, which are, therefore, needed to explain the cross-section of bond returns as well as the model of Adrian et al. (2013). In view of these considerations, we employ the following set of pricing factors in our Gaussian ATSM,

$$\mathbf{X}_t = \left[\widetilde{U}_t^{Equity},\ PC1_t,\ PC2_t,\ PC3_t,\ PC4_t,\ PC5_t\right]', \tag{25}$$

where $\widetilde{U}^{Equity}$ is the equity left tail factor from equation (5) and $PC1$–$PC5$ are the first five principal components estimated from an eigenvalue decomposition of the covariance matrix of zero-coupon bond yields of maturities $n = 3, 6, ..., 120$ months, orthogonal to $\widetilde{U}^{Equity}$. All factors have mean zero and unit variance, and they are plotted in Figure 5. The panels of $PC1$–$PC5$ also present the principal components of the conventional non-orthogonalized bonds yields. We find that estimates of the factors extracted using the two yield curves track each other quite closely, with the largest differences occurring for $PC1$ at the onset of the financial crisis. Therefore, the orthogonalization of the rates with respect to $\widetilde{U}^{Equity}$ does not appear to significantly alter the interpretation and role of the principal components in describing the characteristics of the U.S. Treasury yield curve.

Insert Figure 5 here

Given the vector of state variables in (25), we estimate our Gaussian ATSM using the method put forward by Adrian et al. (2013) and discussed in Section 3. In particular, we use a total of $N = 20$ one-month excess returns for Treasury bonds with maturities $n = 6, 12, ..., 120$ months to fit the cross-section of yields. The estimation approach by Adrian et al. (2013) allows for direct testing of the presence of unspanned factors, i.e. factors that do not help explain variation in Treasury returns. The specification test is implemented as a Wald test of the null hypothesis that the exposures of bond returns to a given model factor are jointly equal to zero. Letting $\boldsymbol{\beta}_i$ be the i-th column of $\boldsymbol{\beta}'$, the Wald statistic, under the null $H_0 : \boldsymbol{\beta}_i = \mathbf{0}_{N\times 1}$, is defined as follows,

$$W_{\beta_i} = \hat{\boldsymbol{\beta}}_i' \hat{\mathcal{V}}_{\beta_i}^{-1} \hat{\boldsymbol{\beta}}_i \overset{\alpha}{\sim} \chi^2(N) \ , \tag{26}$$

where $\hat{\mathcal{V}}_{\beta_i}$ is an $N \times N$ diagonal matrix that contains the estimated variances of the $\hat{\boldsymbol{\beta}}_i$ coefficient estimates.[19] The results of the Wald test on the pricing factors of both the proposed ATSM with equity tail risk and a benchmark model based on only the first five PCs of the yield curve are shown in Table 4. As we can see, we strongly reject the hypothesis of unspanned factor for each of our state variables. This means that the data support the use of the equity left tail factor $\widetilde{U}^{Equity}$, together with the yield curve factors indicated by Adrian et al. (2013), for pricing bonds in the U.S. market over the period 2007 – 2016.

Insert Table 4 here

The summary statistics of the pricing errors implied by our term structure model, which accounts for equity tail risk, and the benchmark PC-only specification are provided in Table 5. Overall the results indicate a good fit between the data and the proposed model with equity tail risk. Indeed, both the mean and the standard deviation of our yield pricing errors remain well below half of a basis point for all maturities and they never exceed, in absolute value, those of the benchmark. As for the return pricing errors, we can see that including our equity tail

[19]See Appendix A.1 in Adrian et al. (2013) for the analytical expressions of the asymptotic variance of the estimators.

risk measure explicitly in the Gaussian ATSM improves the fit especially to the short end of the U.S. yield curve. Moreover, consistent with the way Adrian et al. (2013) construct their framework for the term structure of interest rates, we observe a strong autocorrelation in the yield pricing errors and a negligible one in the return pricing errors. The success of our model in fitting the yield curve is shown graphically in the left panels of Figure 6. In these plots, the solid black lines of observed yields are visually indistinguishable from the dashed gray lines of model-implied yields. Similarly, the right panels of Figure 6 display the tight fit between actual and fitted excess Treasury returns. The dashed red lines plot the model-implied dynamics of bond term premia in the left panels and of the expected component of excess returns in the right panels.

Insert Table 5 and Figure 6 here

We now examine whether the risk factors that we use in our Gaussian ATSM are priced in the cross-section of Treasury returns. To this end, we follow Adrian et al. (2013) and perform a Wald test of the null hypothesis that the market price of risk parameters associated with a given model factor are jointly equal to zero. Letting $\boldsymbol{\lambda}_i^{\prime}$ be the i-th row of $\boldsymbol{\Lambda} = [\boldsymbol{\lambda}_0 \ \boldsymbol{\lambda}_1]$, the Wald statistic, under the null $H_0 : \boldsymbol{\lambda}_i^{\prime} = \mathbf{0}_{1\times(K+1)}$, is defined as follows,

$$W_{\Lambda_i} = \hat{\boldsymbol{\lambda}}_i^{\prime} \hat{\mathcal{V}}_{\lambda_i}^{-1} \hat{\boldsymbol{\lambda}}_i \overset{\alpha}{\sim} \chi^2(K+1) \ , \tag{27}$$

where $\hat{\mathcal{V}}_{\lambda_i}$ is a square matrix of order $(K+1)$ that contains the estimated variances of the $\hat{\boldsymbol{\lambda}}_i$ coefficient estimates.[20] In addition, in order to test whether the market prices of risk are time-varying, Adrian et al. (2013) propose the following Wald test which focuses on $\boldsymbol{\lambda}_1$ and excludes the contribution of $\boldsymbol{\lambda}_0$. Letting $\boldsymbol{\lambda}_{1_i}^{\prime}$ be the i-th row of $\boldsymbol{\lambda}_1$, the Wald statistic of this

[20]See Appendix A.1 in Adrian et al. (2013) for the analytical expressions of the asymptotic variance of the estimators.

second test, under the null $H_0 : \boldsymbol{\lambda}_{1_i}^{'} = \mathbf{0}_{1\times(K)}$, is defined as follows,

$$W_{\lambda_{1_i}} = \hat{\boldsymbol{\lambda}}_{1_i}^{'} \hat{\mathcal{V}}_{\lambda_{1_i}}^{-1} \hat{\boldsymbol{\lambda}}_{1_i} \overset{\alpha}{\sim} \chi^2(K) \ . \tag{28}$$

In Table 6, we report the estimates and t-statistics for the market price of risk parameters in the proposed Gaussian ATSM, together with the Wald statistics and p-values for the two tests just described. Examining the first row of the table, we note that equity tail risk, as measured by exposure to $\widetilde{U}^{Equity}$, is strongly priced in our term structure model with a p-value of 5.7%. We detect statistically significant time variation in the market price of equity tail risk, which are mostly explained by the equity left tail factor itself. Furthermore, we find that nearly all the coefficients in the second column of the table are statistically significant at the 1% level. These results suggest that $\widetilde{U}^{Equity}$ is an important driver of the market price of risk related to the factors that explain the yield curve movements. The only exception is in the risk associated with $PC4$, which, however, in accordance with the findings of Adrian et al. (2013), does not seem to be strongly priced in the bond market. Finally, we observe that the second principal component carries a significant price of risk in our term structure model. This result, together with the fact that Adrian et al. (2013) find a significant market price of slope risk only after adding an unspanned real activity factor to their framework, corroborates the hypothesis that valuable information about bond premia is located outside of the yield curve.

Insert Table 6 here

We now discuss the impact of the state variables of our Gaussian ATSM on the pricing of U.S. Treasury bonds. The loadings of the yields on all model factors are reported in Figure 7, whereas the loadings of the expected one-month excess returns are displayed in Figure 8. From an examination of the state variables that are in common with the work of Adrian et al. (2013), we can see that our results are broadly consistent with the well-established role of these factors. Indeed, given the sign of the yield loadings on $PC1$, $PC2$ and $PC3$, we can argue that the first three principal components of yields preserve in our study the interpretation of, respectively, level, slope and curvature of the term structure. Moreover, the yield loadings on $PC4$ and $PC5$ are both quite small, reflecting the modest variability of bond rates explained by these factors. As can be seen from Figure 8, however, all the principal components, including the higher order ones, are important to explain variation in Treasury returns. Specifically, in line with previous findings concerning the predictability of bond returns with yield spreads, our evidence suggests that an increase in the slope factor forecasts higher expected excess returns on bonds of all maturities. Now turning to the new pricing factor that we propose in this paper, we observe from the top left panel of Figure 7 that the yield loadings on $\widetilde{U}^{Equity}$ are negative across all maturities. These results imply that bond prices, which move inversely to yields, rise in response to a contemporaneous shock to the equity left tail factor. And since, by construction, $\widetilde{U}^{Equity}$ is associated with a downturn in the international stock market, we confirm the hypothesis that U.S. Treasury bonds benefit from flight-to-safety flows during periods of turmoil. Further, it is worth noting that, according to the size of the loadings, the contemporaneous effect of the equity left tail factor on the yield curve is not negligible compared to that of the first three principal components. Additional evidence of flight-to-safety is provided in the top left panel of Figure 8 where the expected excess return loadings on $\widetilde{U}^{Equity}$ are displayed. The coefficients are negative and tend to decrease with the maturity of the bond. Therefore, calculation of the second term in equation (11) suggests that the risk premium required by investors for holding U.S. Treasury securities for one month shrinks in response to a contemporaneous shock to the equity left tail factor. In particular, we find that a one standard deviation increase in the $\widetilde{U}^{Equity}$ factor reduces the annualized expected excess return by up to about 2% for short-maturity and medium-maturity bonds. These observations about Treasury returns, combined with the previously documented positive relationship between the "pure tail" factor and future equity returns (Andersen et al., 2015b, 2017b), indicate a common predictor across the two asset classes, whose existence can be justified by the safe haven potential of U.S. Treasuries.

Insert Figure 7 and 8 here

In order to assess the significance of our results, we report in Table 7 the estimates and t-statistics of the expected excess return loadings associated with the $N = 20$ Treasury maturities used to fit the cross-section of yields.[21] To ease visual interpretation of the results, Figure 9 plots the absolute value of the t-statistics against the critical value of 1.64 for the 10% significance level. From an examination of the loadings on $\widetilde{U}^{Equity}$, it seems that, although the equity left tail factor predicts lower future returns across the whole yield curve, the significance of the results decreases with the maturity of the bonds. Indeed, we find that the $\widetilde{U}^{Equity}$ factor has highly significant explanatory power for future returns only on Treasuries with maturities ranging from one to four years. Based on this evidence, we argue that when the equity market tumbles, the short end of the U.S. yield curve is more strongly affected by flight-to-safety than the long end. When looking at the return loadings on the remaining pricing factors of the Gaussian ATSM, we note a remarkably strong predictive ability of $PC1$ and $PC2$ over a wide range of maturities. By contrast, the higher order principal components have significant forecast power for future returns on Treasuries with either only short maturity or only long maturity.

Insert Table 7 and Figure 9 here

The analysis presented thus far can be related to the work of Kaminska and Roberts-Sklar (2015), who assess the importance of global market sentiment for the term structure of U.K. government bonds. The authors use the variance risk premium of U.S., U.K. and Euro-area equity markets to construct a proxy of global risk aversion, which then they introduce explicitly as a pricing factor into a Gaussian ATSM.[22] However, by studying the impact of risk aversion

[21] We use a delta method approach to estimate the standard errors of the expected excess return loadings on the pricing factors of the model. The coefficients are calculated as $\boldsymbol{\beta}^{(n)'}\boldsymbol{\lambda}_1^{(i)}$ and represent the response of the expected one-month excess return on the n-month bond to a contemporaneous shock to the i-th factor. The use of $\boldsymbol{\beta}^{(n)'}$ in place of $\mathbf{b}_n^{'}$ is allowed since the derivation of log bond prices in the ATSM is exact conditional on the equivalence of the two measures. Standard errors are calculated using the analytical expressions for the asymptotic variance and covariance of $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\Lambda}}$ provided in Appendix A.1 of Adrian et al. (2013).

[22] Kaminska and Roberts-Sklar (2015) obtain the global measure of risk aversion either as the market capitalization weighted average or as the first principal component of the individual VRPs. As the authors claim, the results are not sensitive to the choice of the aggregation method.

on U.K. bond data, Kaminska and Roberts-Sklar (2015) reach a different conclusion from ours: future excess bond returns for all maturities load positively on the equity market factor. We can interpret this as evidence of "weak" FTS affecting the U.K. term structure if we believe that their VRP-based measure and our equity left tail factor capture similar attributes of the stock market. Alternatively or concomitantly, the different conclusions drawn can be traced to differences in the equity factor used in the Gaussian ATSM of the two studies.

We conclude this subsection by discussing how equity tail risk has affected bond term premia over the course of time. To conduct the analysis, we use forward rates because, as suggested by Abrahams et al. (2016), their variation may be ascribed more to changes in risk premia than to changes in the expected future short rate. The left panels of Figure 10 show the dynamics of the 2-3y, 2-5y and 5-10y forward Treasury rates and their components, whereas the right panels illustrate the effect of the equity left tail factor on the term premia of those forward rates. We determine the contribution of $\widetilde{U}^{Equity}$ to FTP in equation (22) as the difference between the component of fitted forward rates and the component of their risk-neutral counterparts that the model attributes to the equity left tail factor. The following remarks can be made by observing Figure 10. As anticipated, we confirm that the expectations of future short spot rates embedded in forward yields (and represented by the risk-neutral forward rates) remain stable throughout time, especially in the case of far in the future forwards. Therefore it follows that oscillations in forward rates reflect, in large part, adjustments in the required term premia. We note that the outburst of the 2008-09 financial crisis marks the beginning of a long period of declining rates which was interrupted only briefly by the Federal Reserve's "taper tantrum" in 2013. Although the same pattern is observed for all yields presented in Figure 10, it is interesting to see how the $\widetilde{U}^{Equity}$ factor influenced the downward trend of term premia differently depending on the maturity. Indeed, from the right panels of Figure 10, it appears that the term premium of short-maturity forward rates was strongly affected by equity tail risk, whereas the response of far in the future forward rates was consistently very small. This further corroborates our previous conclusion that short-term bonds provide a more effective shelter against equity market losses than long-term bonds do. For the 2-3y and 2-5y forward Treasury rates, we measure the impact of $\widetilde{U}^{Equity}$ on FTP to be as large as -100 and -75 basis points, respectively, at the peak of the crisis. The forward term premia show strong downward oscillations also in the first half of 2010 and second half of 2011, when the equity left tail factor increased in response to the intensification of the European sovereign debt crisis. In both these instances, the extent of the reduction in bond term premia that can be credited to equity tail risk is approximately 50 basis points. In conclusion, we can state that equity jump tail risk has played a central role in shaping the short end of the U.S. Treasury yield curve since the outburst of the recent financial crisis.

Insert Figure 10 here

5 Conclusion

In this paper, we study the response of U.S. Treasury bonds to extreme events happening in the international stock market. We propose an affine term structure model in which the main drivers of interest rates are the principal components of the zero-coupon yield curve and a downside jump intensity factor extracted from S&P 500, FTSE 100 and EURO STOXX 50 equity-index options. While earlier approaches to pricing bonds with factors other than combinations of yields have proven useful when macro variables are considered, we focus here on the safe haven potential of U.S. Treasuries and use a factor that originates in the equity option markets of developed countries.

The results of our main application to U.S. bond market and international stock market data are summarized as follows. First, equity jump tail risk is strongly priced and exhibits significant time variation within the term structure model. Second, consistent with the theory of flight-to-safety, bond prices increase and future expected excess returns shrink in response to a contemporaneous shock to the equity left tail factor. Third, the equity left tail factor has significant explanatory power for future returns on Treasuries with maturities ranging from one to four years. Finally, large drops in term premia at the short end of the U.S. yield curve are attributable to equity tail risk since the outburst of the recent financial crisis.

A natural direction for future research is to apply the methodology outlined in this paper to the yield curve of a wide range of developed countries. In particular, the model would allow to explore in detail the effect of equity tail risk and, therefore, uncover evidence of FTS in the government bond market of countries other than the U.S.

Given our findings with a downside jump intensity factor related to the international stock market, it would also be worth assessing the impact on the yield curve of a tail factor implied by Treasury options. For instance, it would be interesting to see whether the downside tail risk of the bond market receives compensation in a term structure model and how its pricing differs from that of equity jump tail risk. This would contribute to the recent literature on the auxiliary role of Treasury variance risk premium in predicting higher expected bond returns (Mueller et al., 2016). We leave investigation of such possibilities to future research.

References

Abrahams, M., T. Adrian, R. K. Crump, E. Moench, and R. Yu (2016): "Decomposing real and nominal yield curves," *Journal of Monetary Economics*, 84, 182–200.

Adrian, T., R. K. Crump, and E. Moench (2013): "Pricing the term structure with linear regressions," *Journal of Financial Economics*, 110, 110–138.

Adrian, T., R. K. Crump, and E. Vogt (2015): "Nonlinearity and Flight to Safety in the Risk-Return Trade-Off for Stocks and Bonds," *Federal Reserve Bank of New York Staff Report*.

Aït-sahalia, Y. and A. Lo (2000): "Nonparametric risk management and implied risk aversion," *Journal of Econometrics*, 94, 9–51.

Almeida, C., K. Ardison, R. Garcia, and J. Vicente (2017): "Nonparametric Tail Risk, Stock Returns, and the Macroeconomy," *Journal of Financial Econometrics*, 15, 333–376.

Andersen, T. G., D. Dobrev, and E. Schaumburg (2012): "Jump-robust volatility estimation using nearest neighbor truncation," *Journal of Econometrics*, 169, 75–93.

Andersen, T. G., N. Fusari, and V. Todorov (2015a): "Parametric Inference and Dynamic State Recovery from Option Panels," *Econometrica*, 83, 1081–1145.

——— (2015b): "The risk premia embedded in index options," *Journal of Financial Economics*, 117, 558–584.

——— (2017a): "Short-Term Market Risks Implied by Weekly Options," *Journal of Finance*, 72, 1335–1386.

——— (2017b): "The Pricing of Tail Risk and the Equity Premium: Evidence from International Option Markets," *Working Paper*.

Baele, L., G. Bekaert, and K. Inghelbrecht (2010): "The determinants of stock and bond return comovements," *Review of Financial Studies*, 23, 2374–2428.

Barndorff-Nielsen, O. E. and N. Shephard (2004): "Power and Bipower Variation with Stochastic Volatility and Jumps," *Journal of Financial Econometrics*, 2, 1–37.

Bekaert, G., E. Engstrom, and S. R. Grenadier (2010): "Stock and bond returns with Moody Investors," *Journal of Empirical Finance*, 17, 867–894.

Bekaert, G. and M. Hoerova (2014): "The VIX, the variance premium and stock market volatility," *Journal of Econometrics*, 183, 181–190.

Bollerslev, T., J. Marrone, L. Xu, and H. Zhou (2014): "Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence," *Journal of Financial and Quantitative Analysis*, 49, 633–661.

Bollerslev, T., G. Tauchen, and H. Zhou (2009): "Expected stock returns and variance risk premia," *Review of Financial Studies*, 22, 4463–4492.

Bollerslev, T. and V. Todorov (2011): "Tails, Fears, and Risk Premia," *Journal of Finance*, 66, 2165–2211.

Bollerslev, T., V. Todorov, and L. Xu (2015): "Tail risk premia and return predictability," *Journal of Financial Economics*, 118, 113–134.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2017): "The Safe Assets Shortage Conundrum," *Journal of Economic Perspectives*, 31, 29–46.

Chiţu, L., B. Eichengreen, and A. Mehl (2014): "When did the dollar overtake sterling as the leading international currency? Evidence from the bond markets," *Journal of Development Economics*, 111, 225–245.

Chordia, T., A. Sarkar, and A. Subrahmanyam (2005): "An empirical analysis of stock and bond market liquidity," *Review of Financial Studies*, 18, 85–129.

Cochrane, J. H. and M. Piazzesi (2005): "Bond Risk Premia," *The American Economic Review*, 95, 138–160.

——— (2008): "Decomposing the Yield Curve," *Working Paper*.

Connolly, R., C. Stivers, and L. Sun (2005): "Stock Market Uncertainty and the Stock-Bond Return Relation," *Journal of Financial and Quantitative Analysis*, 40, 161–194.

Duffee, G. R. (2002): "Term premia and interest rate forecasts in affine models," *Journal of Finance*, 57, 405–443.

——— (2011): "Information in (and not in) the Term Structure," *The Review of Financial Studies*, 24, 2895–2934.

Economist (2015): "Dominant and dangerous," Available at https://www.economist.com/leaders/2015/10/03/dominant-and-dangerous.

Ghysels, E., A. Plazzi, and R. Valkanov (2016): "The Risk-Return Relationship and Financial Crises," *Working Paper*.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007): "The U.S. Treasury yield curve: 1961 to the present," *Journal of Monetary Economics*, 54, 2291–2304.

Hartmann, P., S. Straetmans, and C. G. de Vries (2004): "Asset Market Linkages in Crisis Periods," *Review of Economics and Statistics*, 86, 313–326.

Joslin, S., M. Priebsch, and K. J. Singleton (2014): "Risk premiums in dynamic term structure models with unspanned macro risks," *Journal of Finance*, 69, 1197–1233.

Kaminska, I. and M. Roberts-Sklar (2015): "A global factor in variance risk premia and local bond pricing," *Bank of England Staff Working Paper*.

Kelly, B. and H. Jiang (2014): "Tail Risk and Asset Prices," *Review of Financial Studies*, 27, 2841–2871.

Kim, D. H. and J. H. Wright (2005): "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates," *Federal Reserve Board Finance and Economics Discussion Series.*

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

Malik, S. and A. Meldrum (2016): "Evaluating the robustness of UK term structure decompositions using linear regression methods," *Journal of Banking and Finance*, 67, 85–102.

Mueller, P., P. Sabtchevsky, A. Vedolin, and P. Whelan (2016): "Variance risk premia on stocks and bonds," *Working Paper.*

Vayanos, D. (2004): "Flight to quality, flight to liquidity, and the pricing of risk," *NBER Working Paper*, 10327.

Table 1 – **SPX** – Three-Factor Double Exponential Model - Estimation Results

Parameter	Estimate	Constrained	Parameter	Estimate	Constrained
ρ_1	-0.961	-	ρ_u	0.530	-
$\bar{v}_1$	0.003	-	c_0^-	0.000	✓
κ_1	11.425	-	c_0^+	0.353	-
σ_1	0.580	-	c_1^-	115.137	-
μ_1	12.962	-	c_1^+	24.943	-
ρ_2	-0.978	-	c_2^-	0.000	✓
$\bar{v}_2$	0.010	-	c_2^+	84.677	-
κ_2	1.881	-	c_3^-	1.000	✓
σ_2	0.192	-	c_3^+	0.000	✓
η	0.000	✓	λ_-	26.016	-
μ_u	7.492	-	λ_+	37.235	-
κ_u	0.096	-			

Notes: This table provides the in-sample estimates of parameter vector θ of the Three-Factor Double Exponential Model discussed in Section 2 and applied to S&P 500 equity-index options. All parameters are expressed in annualized terms. A ✓ in the "Constrained" column means that the corresponding parameter is not freely estimated, but instead is set to the value reported in the "Estimate" column. Model is estimated using data sampled at the end of each month over the period from January 2007 through November 2016.

Table 2 – **FTSE** – Three-Factor Double Exponential Model - Estimation Results

Parameter	Estimate	Constrained	Parameter	Estimate	Constrained
ρ_1	-0.956	-	ρ_u	0.505	-
$\bar{v}_1$	0.004	-	c_0^-	0.000	✓
κ_1	14.282	-	c_0^+	0.286	-
σ_1	0.425	-	c_1^-	189.405	-
μ_1	11.220	-	c_1^+	20.163	-
ρ_2	-0.984	-	c_2^-	0.000	✓
$\bar{v}_2$	0.005	-	c_2^+	83.979	-
κ_2	1.540	-	c_3^-	1.000	✓
σ_2	0.247	-	c_3^+	0.000	✓
η	0.000	✓	λ_-	23.894	-
μ_u	5.865	-	λ_+	44.372	-
κ_u	0.093	-			

Notes: This table provides the in-sample estimates of parameter vector θ of the Three-Factor Double Exponential Model discussed in Section 2 and applied to FTSE 100 equity-index options. All parameters are expressed in annualized terms. A ✓ in the "Constrained" column means that the corresponding parameter is not freely estimated, but instead is set to the value reported in the "Estimate" column. Model is estimated using data sampled at the end of each month over the period from January 2007 through November 2016.

Table 3 – **ESTOXX** – Three-Factor Double Exponential Model - Estimation Results

Parameter	Estimate	Constrained	Parameter	Estimate	Constrained
ρ_1	-0.921	-	ρ_u	0.786	-
$\bar{v}_1$	0.009	-	c_0^-	0.000	✓
κ_1	13.173	-	c_0^+	0.581	-
σ_1	0.557	-	c_1^-	118.055	-
μ_1	14.306	-	c_1^+	19.145	-
ρ_2	-0.024	-	c_2^-	0.000	✓
$\bar{v}_2$	0.034	-	c_2^+	52.345	-
κ_2	0.000	-	c_3^-	1.000	✓
σ_2	0.385	-	c_3^+	0.000	✓
η	0.000	✓	λ_-	24.926	-
μ_u	11.636	-	λ_+	40.905	-
κ_u	0.654	-			

Notes: This table provides the in-sample estimates of parameter vector θ of the Three-Factor Double Exponential Model discussed in Section 2 and applied to EURO STOXX 50 equity-index options. All parameters are expressed in annualized terms. A ✓ in the "Constrained" column means that the corresponding parameter is not freely estimated, but instead is set to the value reported in the "Estimate" column. Model is estimated using data sampled at the end of each month over the period from January 2007 through November 2016.

Table 4 – Gaussian ATSM - Factor Risk Exposures

	Equity Tail Risk ATSM		**PC-only ATSM**	
Factor	W_{β_i}	p-value	W_{β_i}	p-value
$\widetilde{U}^{Equity}$	25905227.061	0.000	-	-
$PC1$	87227968.158	0.000	74933420.309	0.000
$PC2$	19231616.842	0.000	19299290.011	0.000
$PC3$	3162887.185	0.000	2823467.516	0.000
$PC4$	370962.032	0.000	359581.671	0.000
$PC5$	32999.643	0.000	31014.780	0.000

Notes: This table provides the Wald statistics and corresponding p-values for the Wald test of whether the exposures of bond returns to a given model factor are jointly zero. Under the null $H_0 : \boldsymbol{\beta}_i = \mathbf{0}_{N\times 1}$ the i-th pricing factor is unspanned, i.e. Treasury returns are not exposed to that factor. The p-values of the statistics are obtained from a chi-squared distribution with $N = 20$ degrees of freedom. The test is conducted on the pricing factors of both the proposed ATSM specified with equity tail risk and a benchmark PC-only model specification.

Table 5 – Gaussian ATSM - Fit Diagnostics

Panel A: Equity Tail Risk ATSM

	$n = 12$	$n = 24$	$n = 36$	$n = 60$	$n = 84$	$n = 120$
Panel A1: Yield Pricing Errors						
Mean	−0.001	0.000	0.000	0.000	0.000	0.000
Standard Deviation	0.002	0.001	0.001	0.001	0.001	0.002
Skewness	−0.915	1.959	1.772	−0.771	1.311	−1.001
Kurtosis	5.639	9.615	7.100	5.510	6.819	6.021
$\rho(1)$	0.751	0.741	0.822	0.756	0.751	0.775
$\rho(6)$	0.184	0.195	0.286	0.137	0.137	0.053
Panel A2: Return Pricing Errors						
Mean	−0.001	0.001	0.001	−0.002	0.001	−0.004
Standard Deviation	0.023	0.019	0.025	0.058	0.035	0.191
Skewness	−0.232	−0.876	−1.208	0.180	−0.555	0.110
Kurtosis	5.355	13.607	9.911	5.072	11.396	4.474
$\rho(1)$	−0.089	−0.207	−0.067	−0.122	−0.174	−0.033
$\rho(6)$	0.021	0.194	0.178	0.032	0.194	0.023

Panel B: PC-only ATSM

	$n = 12$	$n = 24$	$n = 36$	$n = 60$	$n = 84$	$n = 120$
Panel B1: Yield Pricing Errors						
Mean	0.002	0.000	0.001	0.000	0.000	0.000
Standard Deviation	0.004	0.002	0.001	0.002	0.001	0.002
Skewness	−1.179	2.066	2.584	−0.925	0.290	−1.229
Kurtosis	3.845	8.830	10.829	2.977	2.488	5.495
$\rho(1)$	0.893	0.836	0.856	0.856	0.898	0.779
$\rho(6)$	0.520	0.381	0.325	0.461	0.519	0.121
Panel B2: Return Pricing Errors						
Mean	0.000	−0.004	0.000	0.000	−0.007	0.006
Standard Deviation	0.028	0.020	0.029	0.057	0.041	0.178
Skewness	−0.247	−0.067	−1.884	−0.220	−0.090	−0.195
Kurtosis	14.254	13.450	16.218	11.378	6.291	5.735
$\rho(1)$	−0.106	−0.177	−0.110	−0.189	−0.097	−0.078
$\rho(6)$	0.123	0.267	0.244	0.120	0.158	0.067

Notes: This table contains the summary statistics of the pricing errors implied by the Gaussian ATSM that includes equity tail risk (Panel A) and by the benchmark model that only uses the first five PCs of the yield curve (Panel B). Models are estimated over the period 2007 to 2016. Reported are the sample mean, standard deviation, skewness, kurtosis and the autocorrelation coefficients of order one and six. Panels A1 and B1: properties of the yield pricing errors $\hat{u}$. Panels A2 and B2: properties of the return pricing errors $\hat{e}$. n denotes the maturity of the bonds in months.

Table 6 – Gaussian ATSM - Market Prices of Risk

Factor	λ_0	$\lambda_{1,1}$	$\lambda_{1,2}$	$\lambda_{1,3}$	$\lambda_{1,4}$	$\lambda_{1,5}$	$\lambda_{1,6}$	W_{Λ_i}	$W_{\lambda_{1_i}}$
$\widetilde{U}^{Equity}$	0.112	0.776	0.031	−0.140	0.207	−0.075	−0.217	13.675	13.464
	(0.697)	(3.548)	(0.193)	(−0.885)	(1.269)	(−0.459)	(−1.343)	(0.057)	(0.036)
$PC1$	0.007	0.325	−0.037	−0.104	0.067	0.009	−0.047	16.401	16.400
	(0.107)	(3.817)	(−0.589)	(−1.658)	(1.035)	(0.134)	(−0.730)	(0.022)	(0.012)
$PC2$	−0.065	−0.264	0.009	−0.015	−0.072	0.087	0.075	15.748	14.955
	(−1.058)	(−3.457)	(0.148)	(−0.241)	(−1.165)	(1.414)	(1.218)	(0.028)	(0.021)
$PC3$	0.106	−0.180	0.040	0.071	−0.131	0.054	0.010	25.002	20.674
	(2.081)	(−3.356)	(0.790)	(1.398)	(−2.558)	(1.069)	(0.200)	(0.001)	(0.002)
$PC4$	−0.172	−0.035	0.035	−0.045	0.055	−0.168	0.035	11.931	6.704
	(−2.308)	(−0.399)	(0.480)	(−0.614)	(0.739)	(−2.255)	(0.464)	(0.103)	(0.349)
$PC5$	0.047	0.223	−0.017	−0.026	0.123	−0.177	−0.118	39.019	38.271
	(0.905)	(4.116)	(−0.331)	(−0.507)	(2.364)	(−3.423)	(−2.260)	(0.000)	(0.000)

Notes: This table provides the estimates of the market price of risk parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$ in equation (9) for the Gaussian ATSM specified with equity tail risk. Estimated t-statistics are reported in parentheses. Wald statistics for tests of the rows of $\boldsymbol{\Lambda}$ and of $\boldsymbol{\lambda}_1$ being different from zero are reported along each row, with the corresponding p-values in parentheses below. The null hypothesis underlying W_{Λ_i} is that the risk related to a given factor is not priced in the term structure model. The null hypothesis underlying $W_{\lambda_{1_i}}$ is that the price of risk associated with a given factor does not vary over time.

Table 7 – Gaussian ATSM - Expected Excess Return Loadings

Maturity	$\widetilde{U}^{Equity}$	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
$6m$	−0.005	0.023	0.002	0.015	−0.010	−0.014
	(−0.902)	(3.909)	(0.400)	(2.544)	(−1.684)	(−2.302)
$12m$	−0.029	0.050	0.016	0.021	−0.012	−0.030
	(−2.108)	(3.723)	(1.173)	(1.525)	(−0.901)	(−2.227)
$18m$	−0.058	0.078	0.036	0.018	−0.012	−0.047
	(−2.537)	(3.400)	(1.596)	(0.775)	(−0.509)	(−2.022)
$24m$	−0.087	0.106	0.062	0.011	−0.014	−0.063
	(−2.588)	(3.140)	(1.835)	(0.336)	(−0.426)	(−1.862)
$30m$	−0.112	0.133	0.090	0.005	−0.024	−0.080
	(−2.470)	(2.939)	(1.986)	(0.107)	(−0.520)	(−1.757)
$36m$	−0.132	0.160	0.121	0.001	−0.040	−0.098
	(−2.279)	(2.774)	(2.093)	(0.010)	(−0.703)	(−1.692)
$42m$	−0.146	0.185	0.153	0.000	−0.065	−0.117
	(−2.060)	(2.628)	(2.173)	(−0.002)	(−0.924)	(−1.654)
$48m$	−0.154	0.208	0.187	0.003	−0.096	−0.137
	(−1.836)	(2.494)	(2.238)	(0.038)	(−1.155)	(−1.631)
$54m$	−0.157	0.229	0.222	0.011	−0.133	−0.157
	(−1.620)	(2.367)	(2.291)	(0.110)	(−1.376)	(−1.615)
$60m$	−0.157	0.247	0.257	0.022	−0.173	−0.178
	(−1.419)	(2.244)	(2.335)	(0.199)	(−1.577)	(−1.602)
$66m$	−0.153	0.262	0.293	0.037	−0.216	−0.197
	(−1.236)	(2.124)	(2.371)	(0.296)	(−1.752)	(−1.588)
$72m$	−0.148	0.275	0.329	0.054	−0.260	−0.217
	(−1.074)	(2.007)	(2.401)	(0.392)	(−1.898)	(−1.571)
$78m$	−0.141	0.285	0.365	0.074	−0.304	−0.235
	(−0.934)	(1.894)	(2.425)	(0.485)	(−2.016)	(−1.549)
$84m$	−0.135	0.294	0.402	0.094	−0.346	−0.252
	(−0.815)	(1.786)	(2.444)	(0.569)	(−2.107)	(−1.522)
$90m$	−0.128	0.300	0.439	0.116	−0.387	−0.268
	(−0.717)	(1.682)	(2.459)	(0.644)	(−2.172)	(−1.491)
$96m$	−0.123	0.304	0.475	0.137	−0.425	−0.282
	(−0.640)	(1.584)	(2.470)	(0.709)	(−2.213)	(−1.456)
$102m$	−0.120	0.308	0.511	0.159	−0.460	−0.294
	(−0.580)	(1.492)	(2.479)	(0.763)	(−2.235)	(−1.417)
$108m$	−0.119	0.309	0.547	0.179	−0.493	−0.305
	(−0.538)	(1.407)	(2.486)	(0.807)	(−2.240)	(−1.376)
$114m$	−0.120	0.310	0.583	0.198	−0.521	−0.314
	(−0.511)	(1.327)	(2.491)	(0.840)	(−2.231)	(−1.332)
$120m$	−0.124	0.310	0.618	0.216	−0.546	−0.321
	(−0.498)	(1.255)	(2.496)	(0.865)	(−2.210)	(−1.287)

Notes: This table provides the estimates and t-statistics (in parentheses) of the expected excess return loadings on the factors of the proposed ATSM with equity tail risk. These coefficients are calculated as $\boldsymbol{\beta}^{(n)'}\boldsymbol{\lambda}_1^{(i)}$ and can be interpreted as the response of the expected one-month excess return on the n-month bond to a contemporaneous shock to the i-th pricing factor. Results are provided for the $N = 20$ Treasury returns used for model estimation.

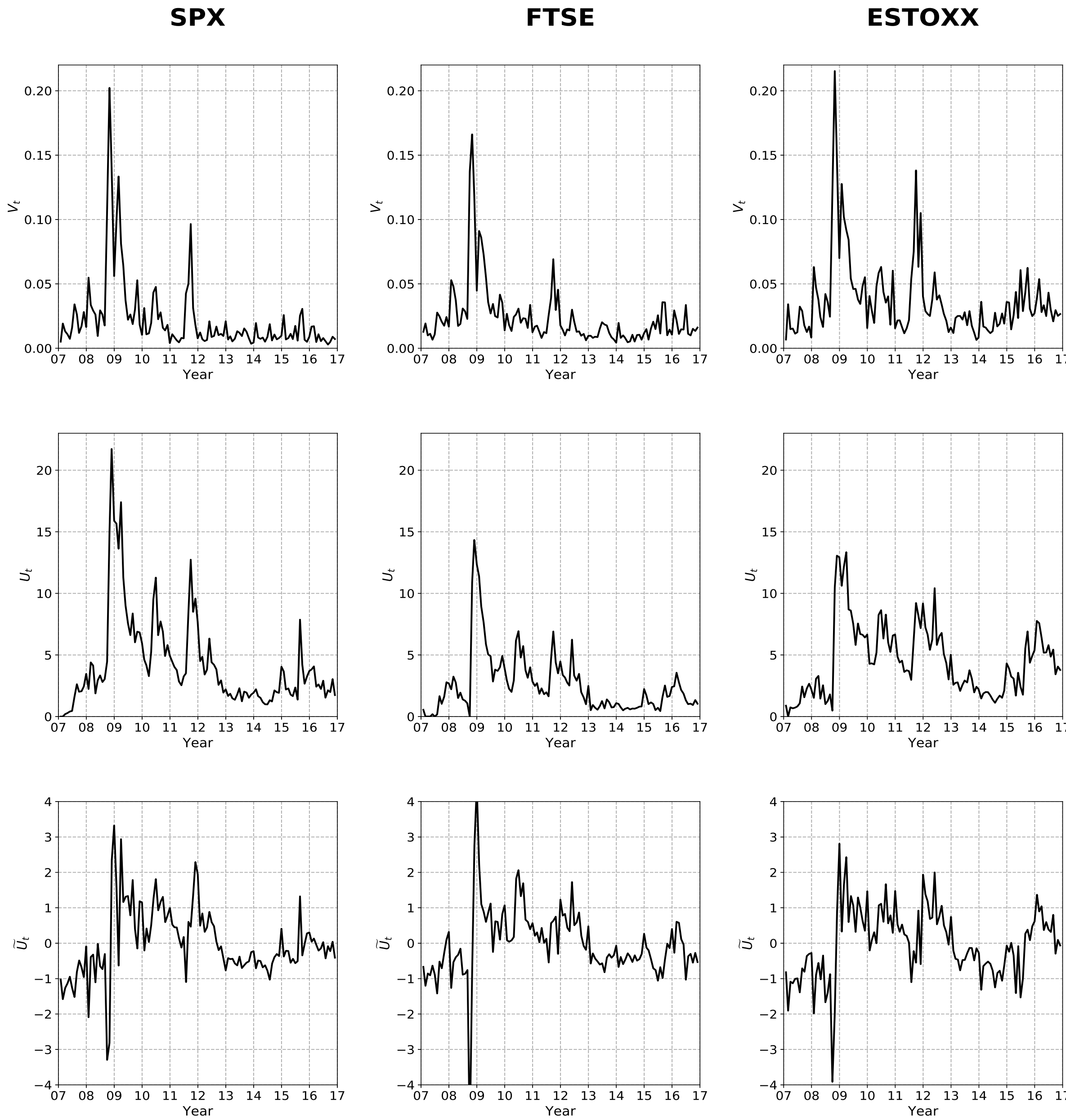

Figure 1 – Monthly option-implied state variables for the S&P 500, FTSE 100 and EURO STOXX 50 equity-index returns. Estimates are obtained using the model parameter values from Table 1, 2 and 3. Top panel: annualized spot variance. Middle panel: annualized negative jump intensity factor. Bottom panel: component of the negative jump intensity factor orthogonal to spot variance and normalized to have mean zero and unit variance. The equity left tail risk factor that we use in the Gaussian ATSM for U.S. interest rates is obtained as the market-capitalization weighted average of the $\widetilde{U}$ factor of the three stock market indices.

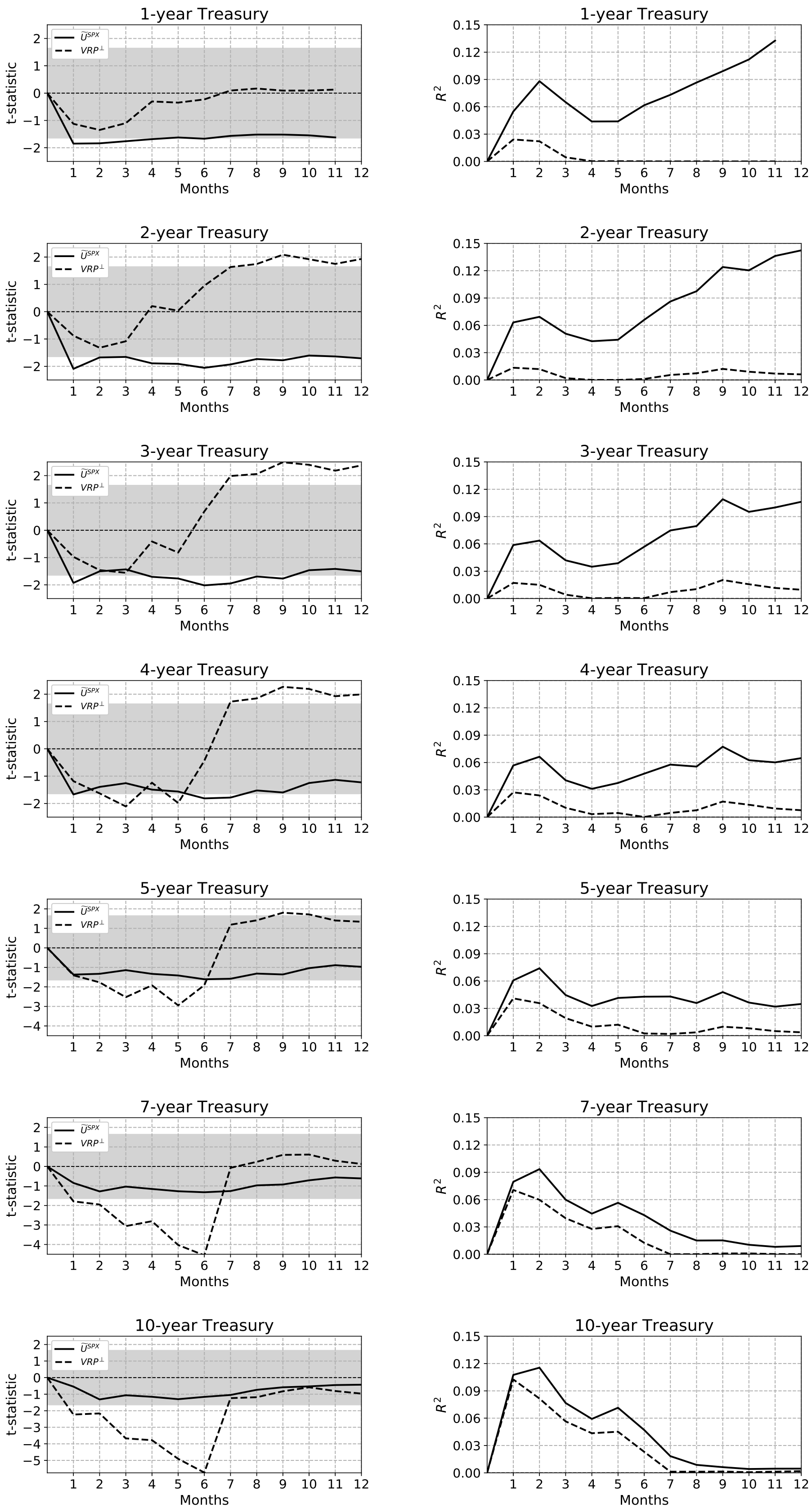

Figure 2 – Results of the regressions of excess returns of U.S. Treasury bonds with 1-, 2-, 3-, 4-, 5-, 7- and 10-year maturities on the option-implied left jump intensity factor (orthogonal to spot variance) of S&P 500 equity-index returns, $\widetilde{U}^{\mathrm{SPX}}$, and the variance risk premium orthogonal to $\widetilde{U}^{\mathrm{SPX}}$, $VRP^{\perp}$. Regressions are run for holding periods from 1 to 12 months using the full sample of data from 2007 to 2016. Left panels: Newey-West t-statistics for the regression slopes, along with regions of statistical insignificance at the 10% level (shaded area). Right panels: regression R^2 obtained using just $VRP^{\perp}$ (dashed line) and both $\widetilde{U}^{\mathrm{SPX}}$ and $VRP^{\perp}$ (solid line).

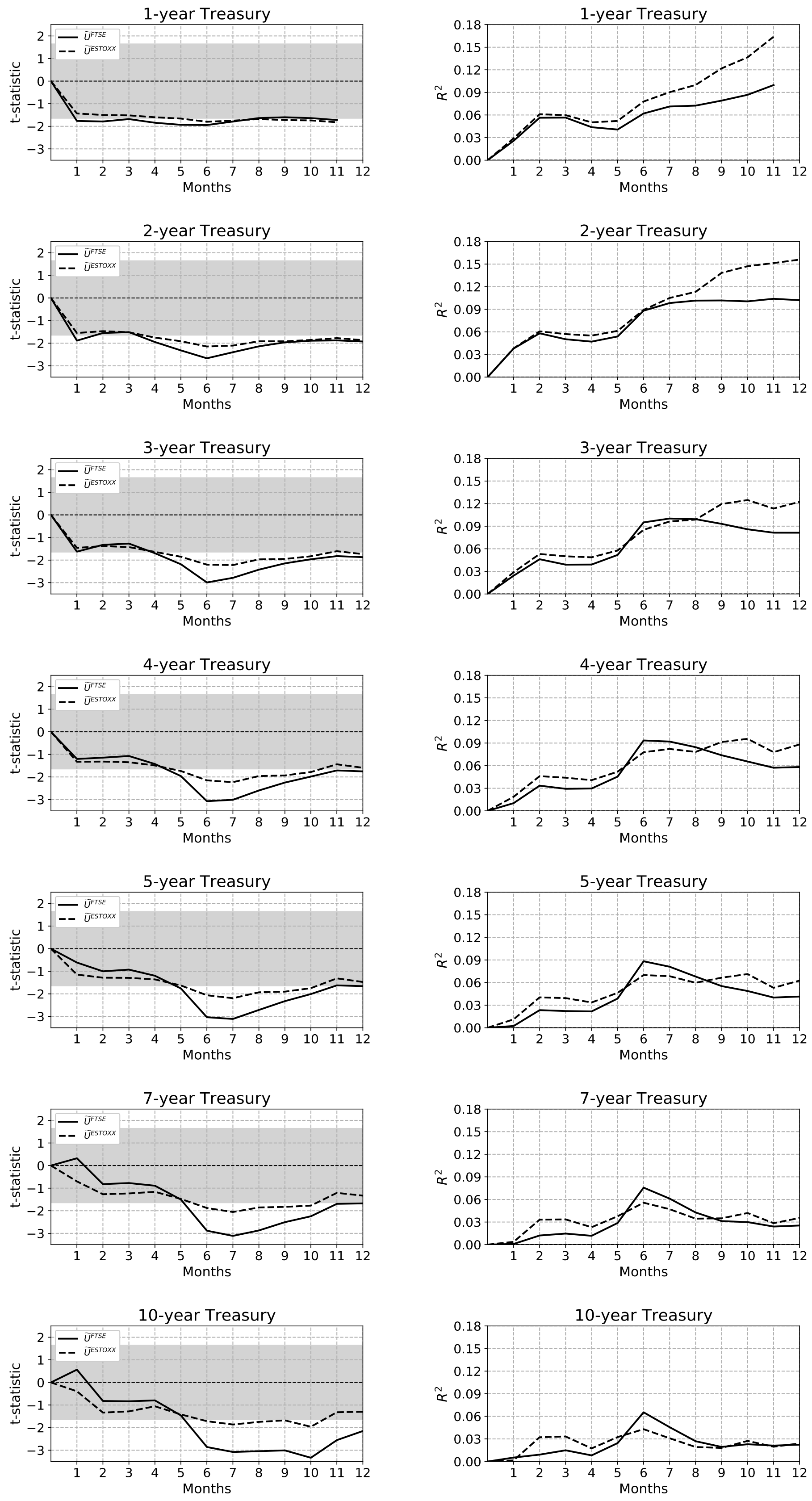

Figure 3 – Results of the regressions of excess returns of U.S. Treasury bonds with 1-, 2-, 3-, 4-, 5-, 7- and 10-year maturities on the option-implied left jump intensity factor (orthogonal to spot variance) of FTSE 100, $\widetilde{U}^{\mathrm{FTSE}}$, and of EURO STOXX 50 equity-index returns, $\widetilde{U}^{\mathrm{ESTOXX}}$. Regressions are run separately on each intensity factor, for holding periods from 1 to 12 months using the full sample of data from 2007 to 2016. Left panels: Newey-West t-statistics for the regression slopes, along with regions of statistical insignificance at the 10% level (shaded area). Right panels: R^2 of the regressions on $\widetilde{U}^{\mathrm{FTSE}}$ (solid line) and on $\widetilde{U}^{\mathrm{ESTOXX}}$ (dashed line).

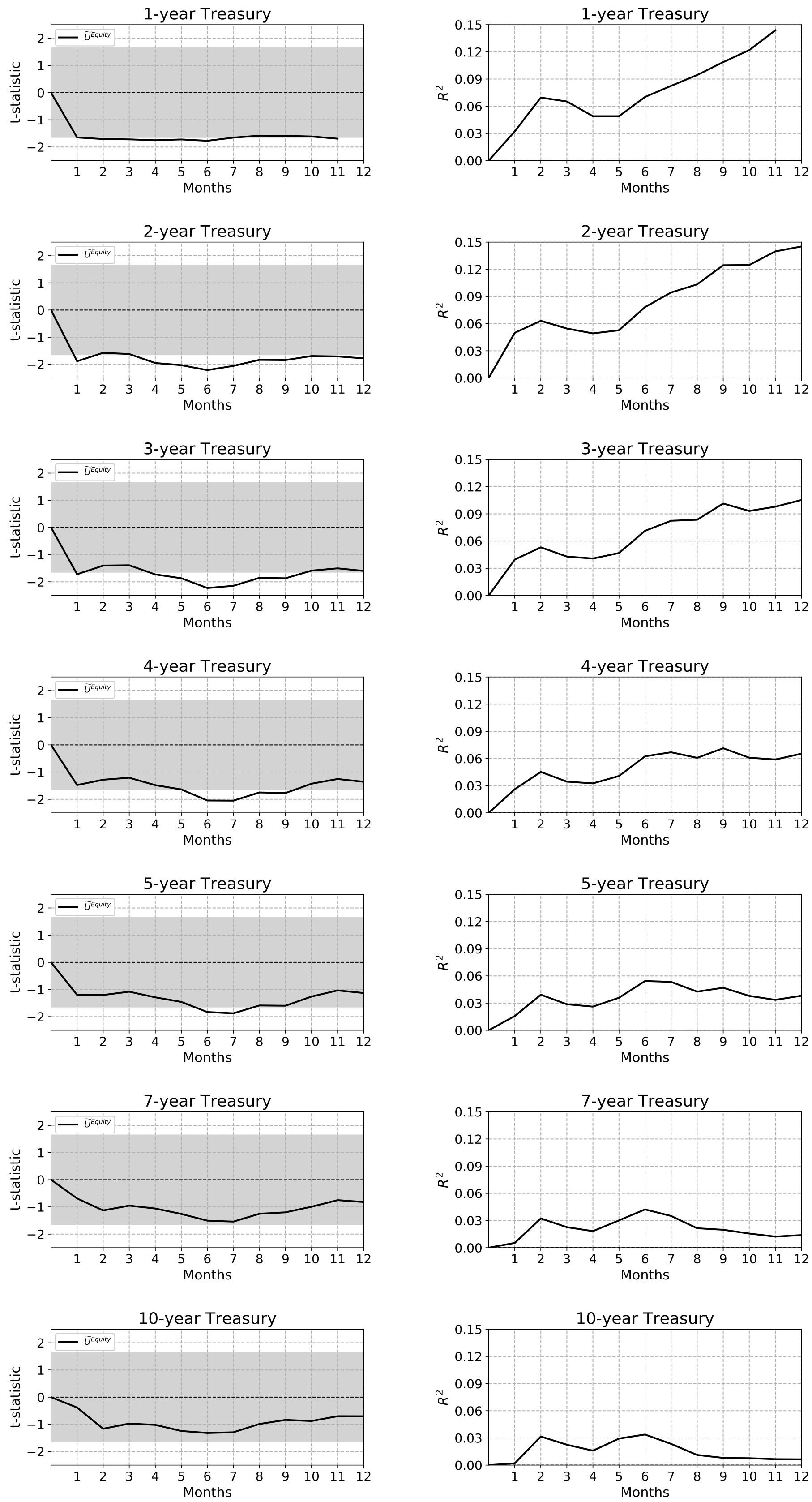

Figure 4 – Results of the regressions of excess returns of U.S. Treasury bonds with 1-, 2-, 3-, 4-, 5-, 7- and 10-year maturities on a constant and $\widetilde{U}^{Equity}$, which is the market-capitalization weighted average of the option-implied left jump intensity factor (orthogonal to spot variance) of S&P 500, FTSE 100 and EURO STOXX 50 equity-index returns. Regressions are run for holding periods from 1 to 12 months using the full sample of data from 2007 to 2016. Left panels: Newey-West t-statistics for the coefficient of $\widetilde{U}^{Equity}$, along with regions of statistical insignificance at the 10% level (shaded area). Right panels: R^2 of the regressions.

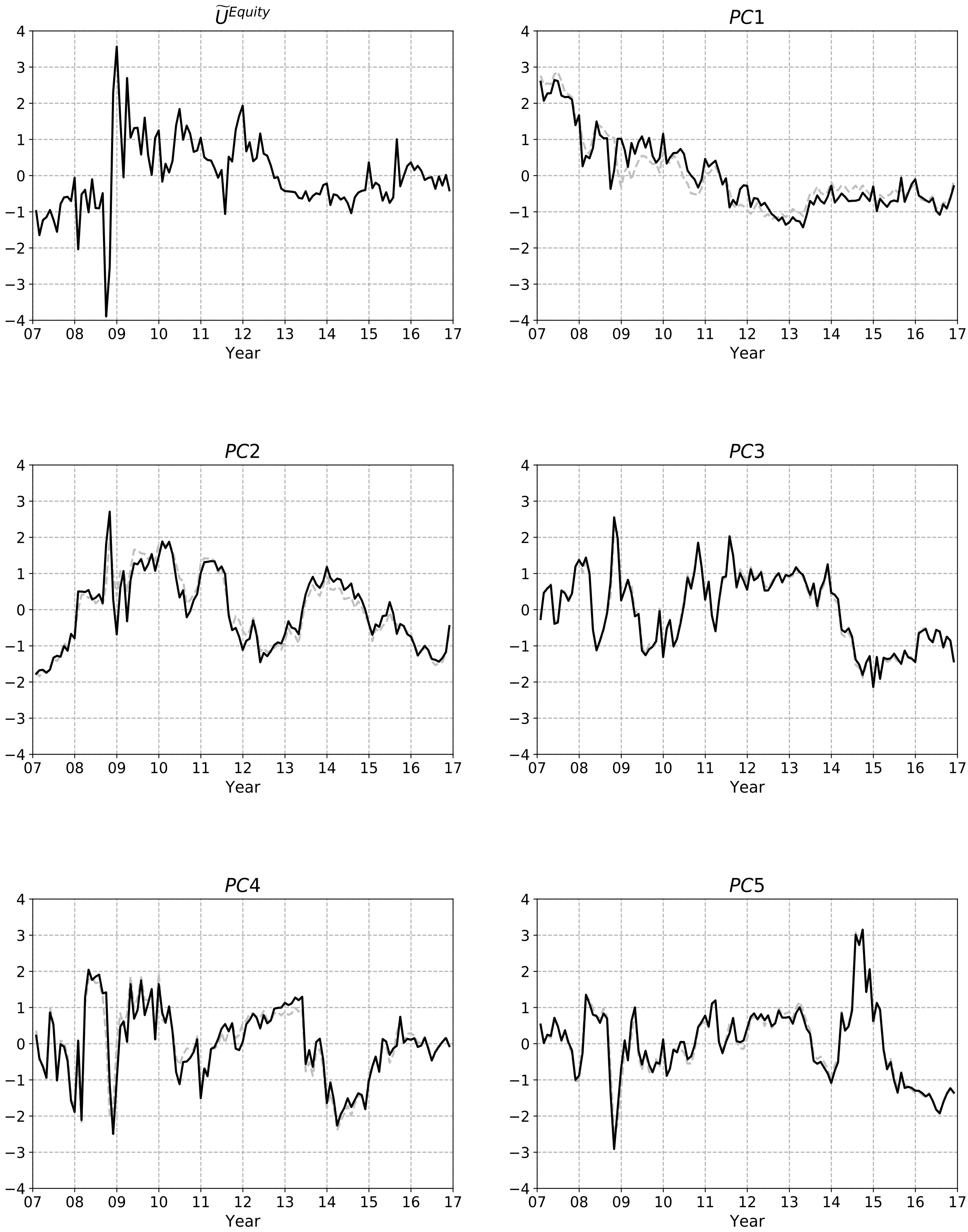

Figure 5 – Monthly time series of the pricing factors of our Gaussian ATSM. The top-left panel shows the equity left tail factor associated with the S&P 500, FTSE 100 and EURO STOXX 50 index returns, calculated from equation (5) and then normalized to have mean zero and unit variance. The remaining panels show the first five standardized principal components extracted from the U.S. Treasury yields of maturities $n = 3, 6, ..., 120$ months, orthogonal to the $\widetilde{U}^{Equity}$ factor. The light-colored dashed lines show the principal components extracted from non-orthogonalized yields, which, however, are not used as pricing factors in our model.

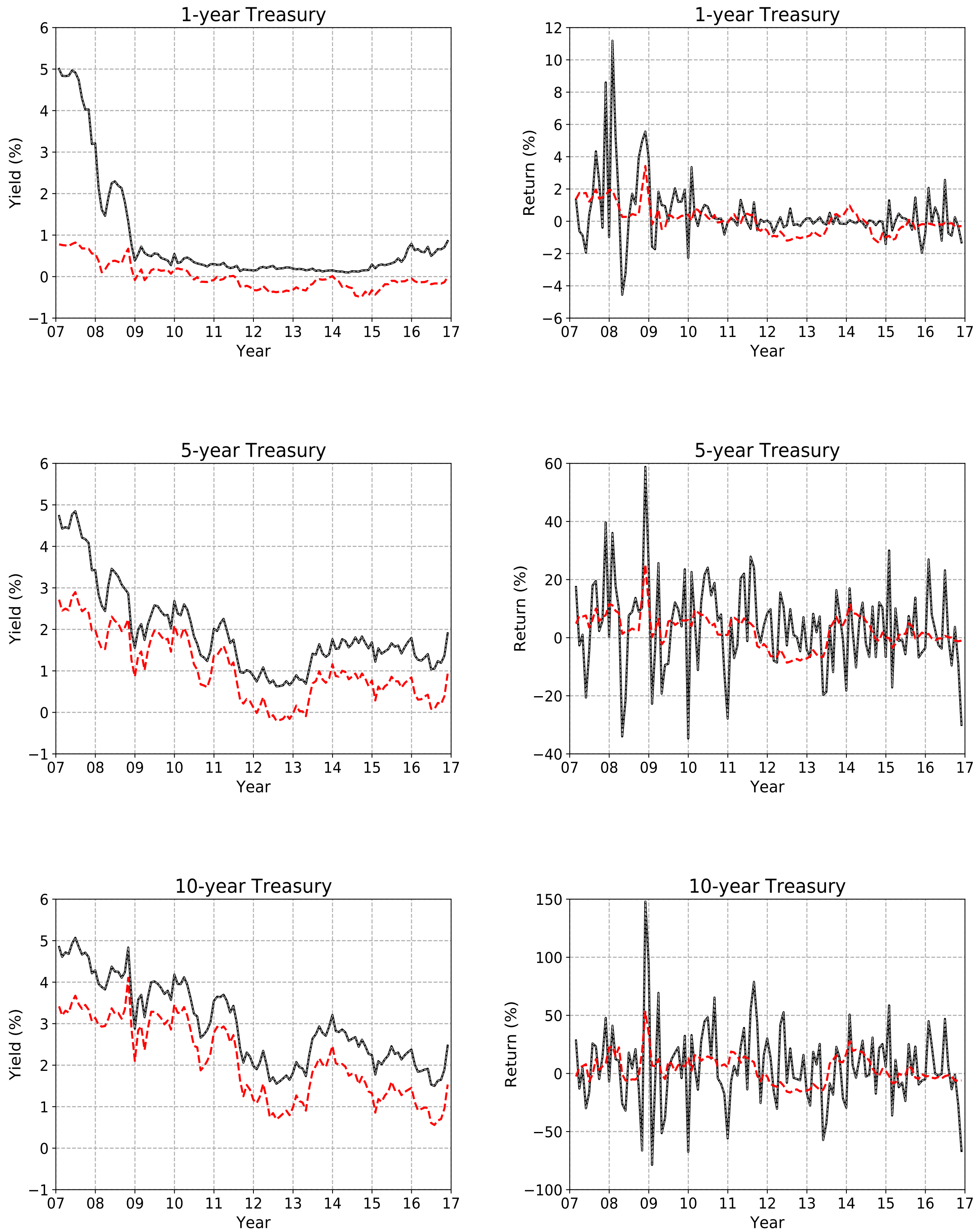

Figure 6 – Observed and model-implied time series of yields and one-month excess returns on U.S. Treasury bonds with 1-, 5- and 10-year maturities. In the left panels, the solid black lines show the observed yields, the dashed gray lines plot the model-implied yields, while the dashed red lines indicate the model-implied term premia. In the right panels, the solid black lines show the observed excess returns, the dashed gray lines plot the model-implied excess returns, while the dashed red lines indicate the model-implied expected excess returns.

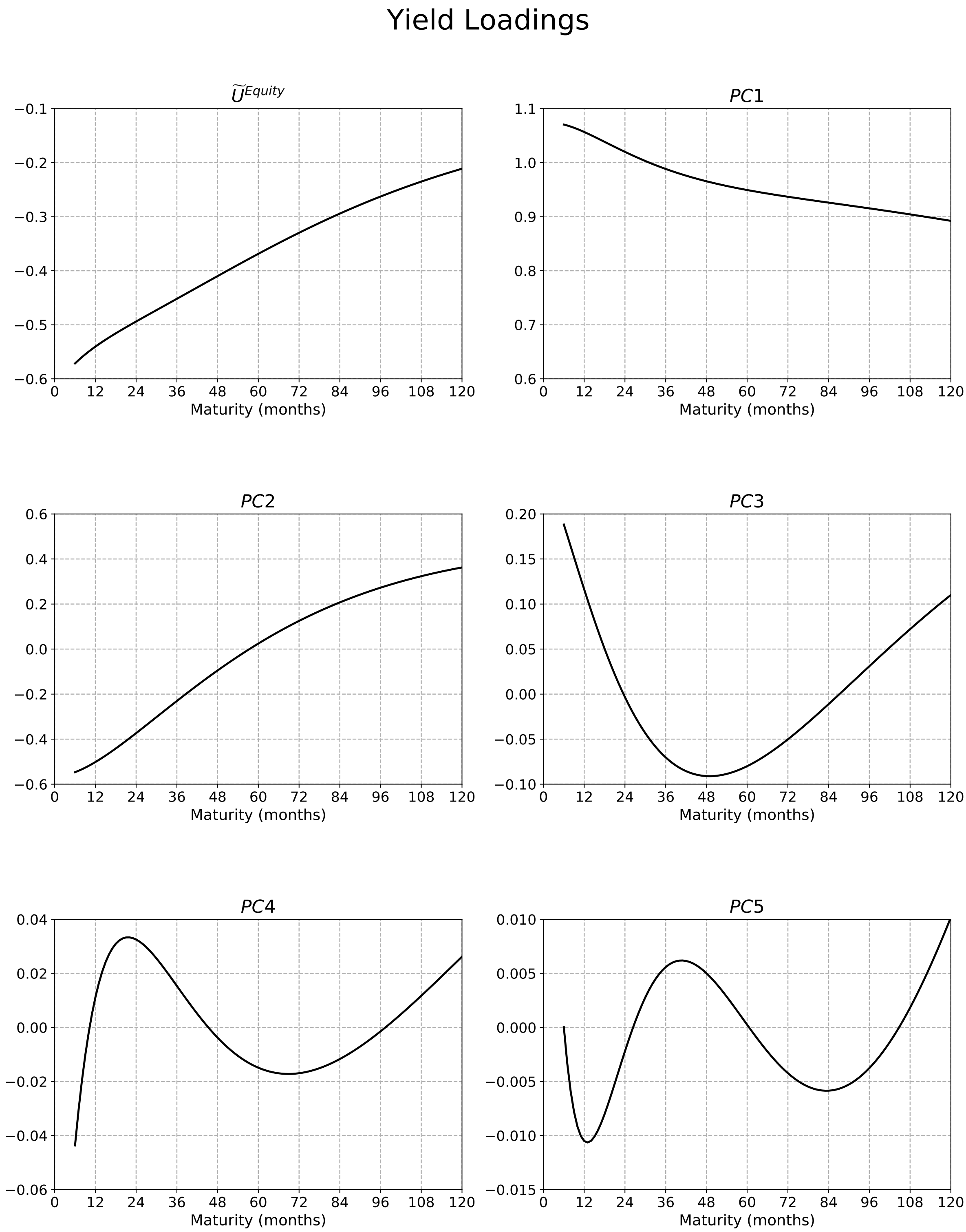

Figure 7 – Model-implied yield loadings on the pricing factors of the proposed ATSM with equity tail risk. These coefficients are calculated as $-(1/n)\mathbf{b}_n$ and can be interpreted as the response of the n-month yield to a contemporaneous shock to the respective factor. $\widetilde{U}^{Equity}$ represents the equity left tail factor associated with the S&P 500, FTSE 100 and EURO STOXX 50 index returns, calculated from equation (5) and then standardized. $PC1$ – $PC5$ denote the first five standardized principal components extracted from the U.S. Treasury yields orthogonal with respect to the $\widetilde{U}^{Equity}$ factor.

Expected Excess Return Loadings

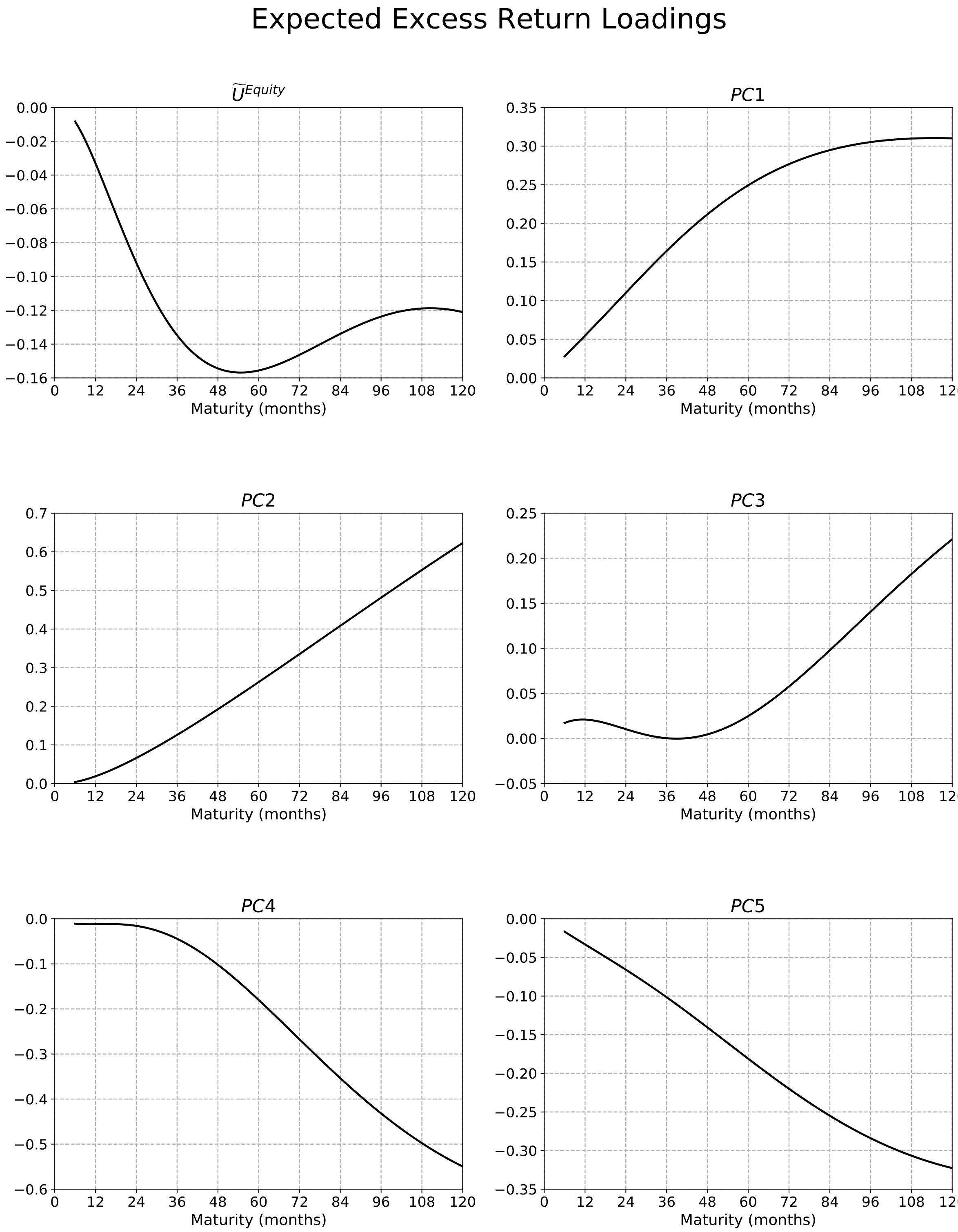

Figure 8 – Model-implied expected excess return loadings on the pricing factors of the proposed ATSM with equity tail risk. These coefficients are calculated as $\mathbf{b}_n^{'}\boldsymbol{\lambda}_1$ and can be interpreted as the response of the expected one-month excess return on the n-month bond to a contemporaneous shock to the respective factor. $\widetilde{U}^{Equity}$ represents the equity left tail factor associated with the S&P 500, FTSE 100 and EURO STOXX 50 index returns, calculated from equation (5) and then standardized. $PC1$ – $PC5$ denote the first five standardized principal components extracted from the U.S. Treasury yields orthogonal with respect to the $\widetilde{U}^{Equity}$ factor.

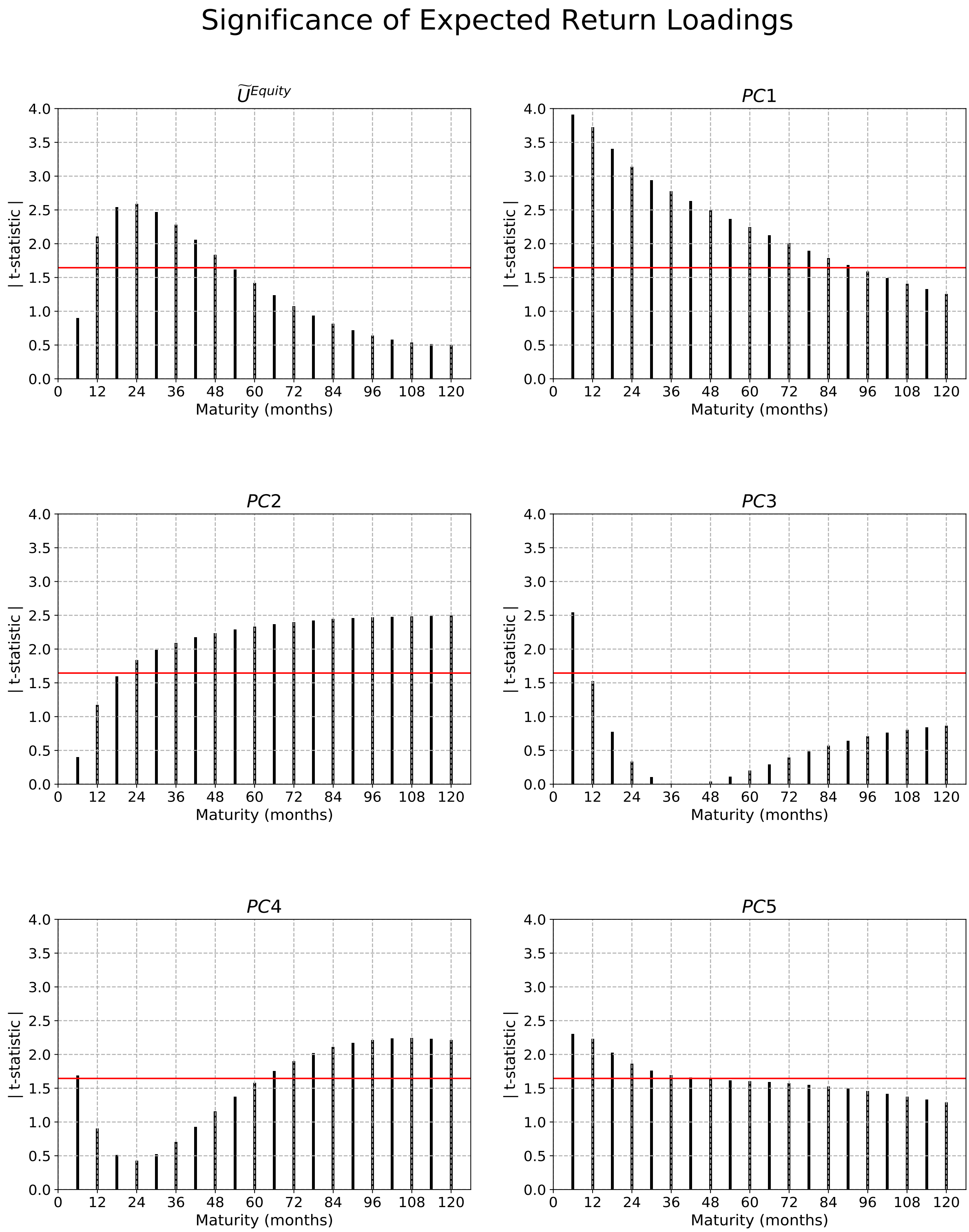

Figure 9 – Significance of expected return loadings on the pricing factors of the proposed ATSM with equity tail risk. The absolute value of the t-statistic is reported for the $N = 20$ one-month excess Treasury returns used to fit the cross-section of yields. The solid red lines depict the critical value of the statistics for the significance level of 10%. $\widetilde{U}^{Equity}$ represents the equity left tail factor associated with the S&P 500, FTSE 100 and EURO STOXX 50 index returns, calculated from equation (5) and then standardized. $PC1$ – $PC5$ denote the first five standardized principal components extracted from the U.S. Treasury yields orthogonal with respect to $\widetilde{U}^{Equity}$.

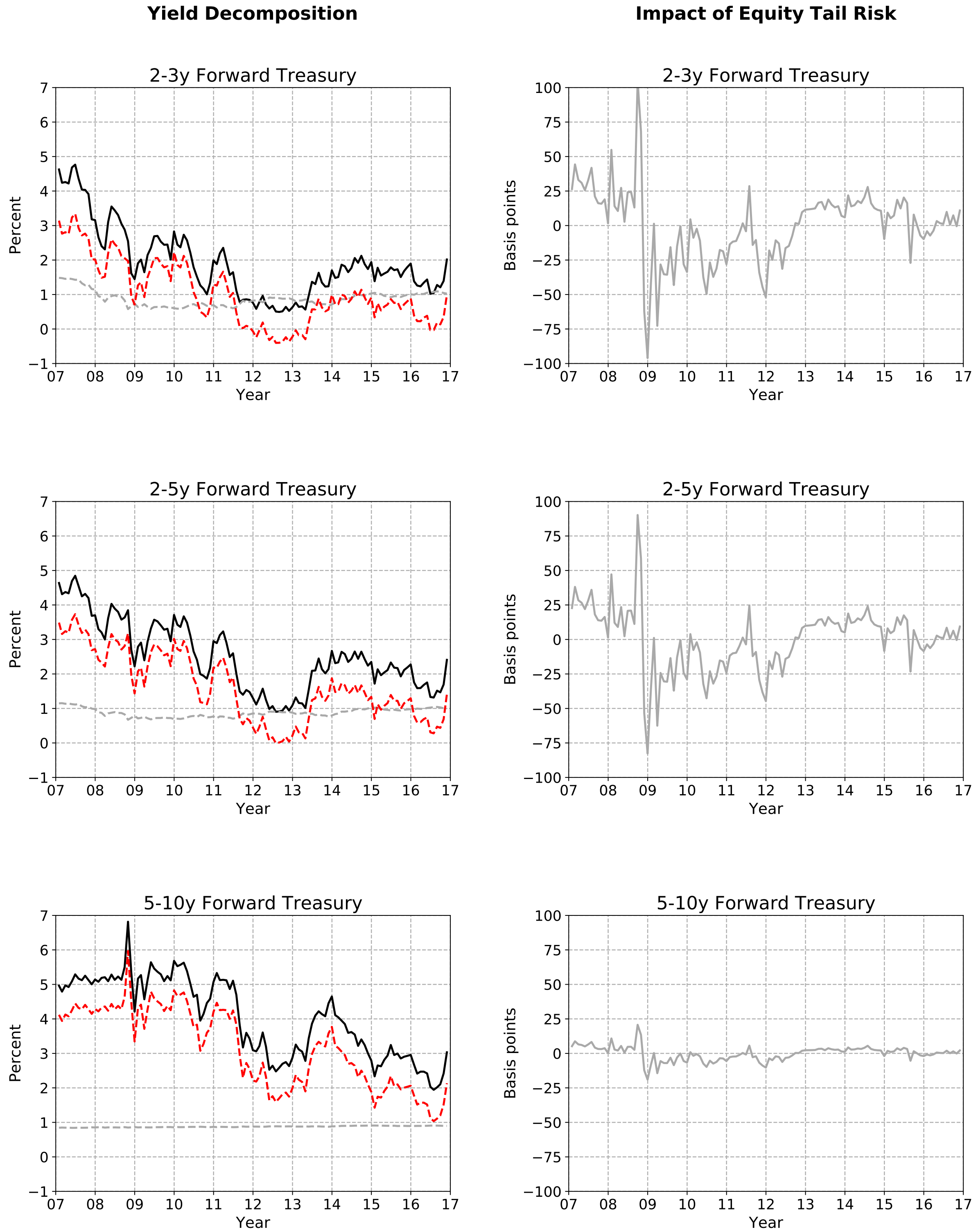

Figure 10 – Contribution of changes in equity tail risk to the term premia embedded in the 2-3y, 2-5y and 5-10y forward Treasury rates. In the left panels, the solid black lines show the model-implied m-n forward rates, the dashed gray lines plot the risk-neutral m-n forward rates (the average expectation of the short rates over the next m to n periods), while the dashed red lines indicate the model-implied term premia embedded in the m-n forward rates. In the right panels, the solid dark gray lines show the impact over time of the equity left tail factor $\widetilde{U}^{Equity}$ on the model-implied term premium of the m-n forward Treasury rates.