The Macroeconomic Announcement Premium*

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December 26, 2018

Abstract

Empirical studies demonstrate striking patterns in stock market returns in relation to scheduled macroeconomic announcements. First, a large proportion of the total equity premium is realized on days with macroeconomic announcements, despite the small number of such days. Second, the relation between market betas and expected returns is far stronger on announcement days as compared with non-announcement days. Finally, these results hold for fixed-income investments as well as for stocks. We present a model with rare events that jointly explains these phenomena. In our model, which is solved in closed form, agents learn about a latent disaster probability from scheduled announcements. We quantitatively account for the empirical findings, along with other facts about the market portfolio.

*We thank Frank Diebold, Winston Dou, Marco Grotteria, Nick Roussanov, Chaojun Wang, and seminar participants at the Stockholm School of Economics, at University of North Carolina, and at the Wharton School for helpful comments.

1 Introduction

Since the work of Sharpe (1964) and Lintner (1965), the Capital Asset Pricing Model (CAPM) has been the benchmark model of the cross-section of asset returns. While generalizations have proliferated, the CAPM, with its simple and compelling structure and tight empirical predictions, remains the major theoretical framework for understanding the relation between risk and return. Recently, Savor and Wilson (2014) document a striking fact about the fit of the CAPM. Despite its poor performance in explaining the cross section overall, the CAPM does quite well on a subset of trading days, namely those days in which the Federal Open Market Committee (FOMC) or the Bureau of Labor Statistics (BLS) releases macroeconomic news.

Figure 1 reproduces the main result of Savor and Wilson (2014) using updated data. We sort stocks into portfolios based on market beta (the covariance with the market divided by market variance) computed using rolling windows. We display the relation between portfolio beta and expected returns on announcement days and non-announcement days in the data. This relation is known as the security market line. On non-announcement days (the majority), the slope is indistinguishable from zero. That is, there appears to be no relation between beta and expected returns. This result holds unconditionally, and is responsible for the widely-held view of the poor performance of the CAPM. However, on announcement days, a strong positive relation between betas and expected returns appears. Moreover, portfolios line up well against the security market line, suggesting that the relation is not only strong, but that the total explanatory power is high. Finally, these results appear even stronger for fixed-income investments than for equities.

We summarize the facts as follows:

1. The equity premium is much higher on announcement days as opposed to non-announcement days
2. The slope of the security market line is higher on announcement days than on non-announcement days. The difference is economically and statistically significant.
3. The security market line is essentially flat on non-announcement days.
4. Results 1 and 2 hold for Treasury bonds as well as for stocks.

In this paper, we build a frictionless model with rational investors that explains these findings. Our model is relatively simple and solved in closed form, allowing us to clearly elucidate the elements of the theory that are necessary to explain these results. Nonetheless, the model is quantitatively realistic, in that we explain not only these findings above, but also the overall risk and return of the aggregate stock market.

One important aspect of our model is that, despite the lack of frictions, investors do not have complete information. Macroeconomic announcements matter for stock prices because they reveal information to investors. This only makes sense if investors do not have full information in the first place.[1] The information that is revealed matters greatly to investors, which is why a premium is required to hold stocks on announcement days (the first finding). In our model, the information concerns the likelihood of economic disaster similar to the Great Depression or what many countries suffered following the 2008 financial crisis.

We further assume that stocks have differential exposure to macroeconomic risk. We endogenously derive the exposure on stock returns from the exposure of the underlying cash flows. We also assume, plausibly, that there is some variability in the probability of disaster that is not revealed in the macroeconomic announcements. Stocks with greater exposure have endogenously higher betas, both on announcement and non-announcement days, than those with lower exposure. They have much higher returns, in line with the data, on announcement days, because that is when a disproportionate amount of information is revealed (the second finding). Finally, the presence of disasters and of time-varying disaster risk implies that a linear relation between expected returns and betas does not hold. Stocks can have high variances, and covariances with the market, driven by time-varying disaster risk, without exposure to the actual disasters rising in proportion. This explains the third finding.

An extension of the model to bonds us to explain the fourth finding. We assume that some information that is revealed on announcements is informative about expected inflation. Bonds are exposed to announcements to a greater extent than equities. In the model, as well as in the data, betas on bonds rise dramatically on announcement days (they are near zero on non-announcement days), while equity betas do not.

We find that the presence of rare events breaks the traditional relation between

[1] Another possibility is that macroeconomic announcements themselves create the risk perhaps because they reflect on the competence of the Federal Reserve. We do not consider that possibility here.

risk and return. This is important, because except for bonds, conventional measures of risk such as variance and covariance do not appear marketdly higher on announcement days. Our model is consistent with this finding, because of the asymmetric nature of the rare event. Most likely, investors will learn that the economy continues to be in good shape and the risk of disaster remains low. There is a small probability, however, that they will learn that the economy is in worse shape than believed. A sample could easily feature mainly events of the first type since the second type is rare.

While we focus on macroeconomic announcements, the tools we develop could be used to address other types of periodic information revelation. There is a vast empirical literature on announcement effects (La Porta et al., 1997; Fama, 1970), of which the literature on macro-announcements is a part. There is, at present, scant theoretical work (Ai and Bansal (2018) is an important recent exception). In this paper, we develop a set of theoretical tools to handle the fact that announcements occur at deterministic intervals, and that a finite amount of information is released over a vanishingly small period of time. Time just before and just after the announcement is connected through intertemporal optimization conditions. We show that these conditions form a set of boundary conditions for the dynamic evolution of prices in the interval between announcements. It is this insight that allows us to solve the model in closed form.

The rest of the paper proceeds as follows. Section 2 discusses the model. Section 3 discusses the fit of the model to the data, and Section 4 concludes.

2 A model of asset prices with macroeconomic announcements

In the section that follows, we describe the model. Section 2.1 gives the endowment and preferences, Section 2.2 the relation between cash flows and announcements, Section 2.3 describes state prices, Section 2.4 equity prices, and Section 2.5 risk premia. Finally Section 2.6 describes the pricing of nominal bonds. Unless otherwise stated, proofs are contained in the Appendices.

2.1 Endowment and preferences

We assume an endowment economy with an infinitely-lived representative agent. Aggregate consumption (the endowment) follows the stochastic process

$$\frac{dC_t}{C_{t^-}} = \mu_C dt + \sigma dB_{Ct} + \left(e^{Z_t} - 1\right) dN_t, \tag{1}$$

where B_{Ct} is a standard Brownian motion and where N_t is a Poisson process. The diffusion term $\mu_C dt + \sigma dB_{Ct}$ represents the behavior of consumption during normal times. The Poisson term $\left(e^{Z_t} - 1\right) dN_t$ represents rare disasters. The random variable Z_t represents the effect of a disaster on log consumption growth. We assume, for tractability, that Z_t has a time-invariant distribution, which we call ν; that is, Z_t is iid over time, and independent of all other shocks. We use the notation E_ν to denote expectations taken over ν.

We assume the representative agent has recursive utility with EIS equal to 1, which gives us closed-form solutions up to ordinary differential equations. We use the continuous-time characterization of Epstein and Zin (1989) derived by Duffie and Epstein (1992). The following recursion characterizes utility V_t:

$$V_t = \max E_t \int_t^\infty f(C_s, V_s) ds, \tag{2}$$

where

$$f(C_t, V_t) = \beta(1-\gamma)V_t \left(\log C_t - \frac{1}{1-\gamma} \log[(1-\gamma)V_t]\right). \tag{3}$$

Here β represents the rate of time preference, and γ represents relative risk aversion. The case of $\gamma = 1$ collapses to time-additive (log) utility. When $\gamma \neq 1$, preferences satisfy risk-sensitivity, the characteristic that Ai and Bansal (2017) show is a necessary condition for a positive announcement premium.

2.2 Scheduled announcements and the disaster probability

We assume that scheduled announcements convey information about the probability of a rare disaster (in what follows, we use the terminology probability and intensity interchangeably). The probability may also vary over time for exogenous reasons; this creates volatility in stock prices in periods that do not contain announcements.

To parsimoniously capture these features in the model, we assume the intensity of N_t is a sum of two processes, λ_{1t} and λ_{2t}.[2] We assume investors observe λ_{2t}, which follows

$$d\lambda_{2t} = -\kappa(\lambda_{2t} - \bar{\lambda}_2)dt + \sigma_\lambda \sqrt{\lambda_{2t}} dB_{\lambda t}, \tag{4}$$

with $B_{\lambda t}$ a Brownian motion independent of B_{Ct}. The process for λ_{2t} is the same as the one assumed for the disaster probability in Wachter (2013).

The intensity λ_{1t} follows a latent Markov switching process. Following Benzoni et al. (2011), we assume two states, λ^G (good) and λ^B (bad), with $0 \leq \lambda^G < \lambda^B$, and

$$\begin{aligned} P(\lambda_{1,t+dt} = \lambda^G | \lambda_{1t} = \lambda^B) &= \eta_{BG} dt \\ P(\lambda_{1,t+dt} = \lambda^B | \lambda_{1t} = \lambda^G) &= \eta_{GB} dt. \end{aligned} \tag{5}$$

Note that η_{BG} is the probability (per unit of time) of a switch from the bad to the good state and η_{GB} is similarly, the probability of a switch from the good to the bad state.

Announcements convey information about λ_{1t}. Let T be the length of time between announcements.[3] Define τ as the time elapsed since the most recent announcement:

$$\tau \equiv t \bmod T,$$

Furthermore, define

$$\begin{aligned} \mathcal{A} &\equiv \{t : t \bmod T = 0\}, \\ \mathcal{N} &\equiv \{t : t \bmod T \neq 0\}. \end{aligned} \tag{6}$$

That is, $\mathcal{A}$ is the set of announcement times, and $\mathcal{N}$ is the set of non-announcement times. Note that $\mathcal{N}$ is an open set, so we can take derivatives of functions evaluated at times $t \in \mathcal{N}$.

Let p_t denote the probability that the representative agent places on $\lambda_{1t} = \lambda^B$. For

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[2]Equivalently, decompose, N_t as

$$N_t = N_{1t} + N_{2t},$$

where N_{jt}, for $j = 1, 2$, has intensity λ_{jt}.

[3]In the data, announcements are periodic, but, depending on the type of announcement, the period length is not precisely the same. Our assumption of an equal period length is a convenient simplification that has little effect on our results.

$t \in \mathcal{N}$, assume

$$dp_t = (-p_t\eta_{BG} + (1 - p_t)\eta_{GB})\, dt = (-p_t(\eta_{GB} + \eta_{BG}) + \eta_{GB})\, dt. \tag{7}$$

This assumption implies that the agent learns only from announcements.[4] Outside of announcement periods, the agent updates based on (5). If the economy is in a good state, which it is with probability $1 - p_t$, the chance of a shift to the bad state over the next instant is $\eta_{GB}\, dt$. If the economy is in a bad state, which is with probability p_t, the chance of a shift to the good state over the next instant is $\eta_{BG}\, dt$. Define

$$\bar{\lambda}_1(p_t) \equiv p_t\lambda^B + (1 - p_t)\lambda^G,$$

as the agent's posterior value of λ_{1t}.

For simplicity, we assume announcements convey full information, that is, they perfectly reveal λ_{1t}.[5] We refer to announcements revealing λ_{1t} to be λ^G as positive and those revealing it to be λ^B as negative. The reason for this terminology is intuitive: an announcement revealing the disaster probability to be low should be good news. The following sections make this intuition precise.

It is useful to keep track of the content of the most recent announcement, because of the information it conveys about the evolution of the disaster probability. Let

$$p_{0t} \equiv p_{t-\tau}. \tag{9}$$

That is, p_{0t} is the revealed probability of a bad state at the most recent announcement.

[4] Bayesian learning implies

$$dp_t = p_{t^-}\left(\frac{\lambda^B - \bar{\lambda}_1(p_{t^-})}{\bar{\lambda}_1(p_{t^-})}\right) dN_{1t} + \big(-(p_{t^-})(\lambda^B - \bar{\lambda}_1(p_{t^-})) - (p_{t^-})\eta_{BG} + (1 - p_{t^-})\eta_{GB}\big)\; dt \tag{8}$$

(Liptser and Shiryaev, 2001). The first term multiplying N_{1t} corresponds to the actual effect of disasters. The term $-p(\lambda^B - \bar{\lambda}_1(p))$ in the drift corresponds to the effect of no disasters. We abstract from these effects in (8). Because disasters will be very unlikely, the term $-p(\lambda^B - \bar{\lambda}_1(p))$ is small (agents do not learn much from the fact that disasters do not occur). In what follows, we compare the data to simulations that do not contain disasters. Therefore ignoring the Poisson term can be understood as an implementation of realization utility, defined by Cogley and Sargent (2008). We allow agents to learn from disasters; however, they do not forecast that they will learn from disasters.

[5] In effect, we assume the government body issuing the announcement has better information, perhaps because of superior access to data. Stein and Sunderam (2017) model the strategic problem of the announcer and investors, and show that announcements might reveal more information than a naive interpretation would suggest.

By definition, $p_{0t} \in \{0, 1\}$. The process for p_t is right-continuous with left limits. In the instant just before the announcement it is governed by (7). On the announcement itself, it jumps to 0 or 1 depending on the true (latent) value of λ_{1t}.

Under these assumptions, p_t has an exact solution:

Lemma 1. *For $t \in \mathcal{N}$, the probability assigned to the bad state satisfies $p_t = p(\tau; p_{0t})$, where*

$$p(\tau; p_{0t}) = p_{0t} e^{-(\eta_{BG}+\eta_{GB})\tau} + \frac{\eta_{GB}}{\eta_{BG} + \eta_{GB}}(1 - e^{-(\eta_{BG}+\eta_{GB})\tau}). \tag{10}$$

Proof. Equation 7 implies that p_t is deterministic between announcements. Moreover, p_t is memoryless in that it contains no information prior to the most recent announcement. Because the information revealed at the most recent announcement is summarized in p_0, any solution for (7) takes the form $p_t = p(\tau; p_{0t})$, where $\tau = t \bmod T$ and $p_{0t} \in \{0, 1\}$. It follows directly from (7) that $p(\tau; p_{0t})$ satisfies

$$\frac{d}{d\tau} p(\tau; p_0) = -p(\tau; p_0)(\eta_{BG} + \eta_{GB}) + \eta_{GB}, \qquad \tau \in [0, T). \tag{11}$$

This has a general solution:

$$p(\tau; p_0) = K_{p_0} e^{-(\eta_{BG}+\eta_{GB})t} + \frac{\eta_{GB}}{\eta_{BG} + \eta_{GB}}, \tag{12}$$

where K_{p_0} is a constant that depends on p_0. The boundary condition $p(0; p_0) = p_0$ determines K_{p_0}. $\square$

Equation 10 shows that p_t is a weighted average of two probabilities. The first, p_{0t}, is the probability of the bad state, revealed in the most recent announcement. The second, $\frac{\eta_{GB}}{\eta_{GB}+\eta_{BG}}$, is the unconditional probability of the bad state. As τ, the time elapsed since the announcement, goes from 0 to 1, the agent's weight shifts from the former of these probabilities to the latter.

Agents forecast the outcome of the announcement based on p_t. The optimality conditions connecting the instant before the announcement to the instant after are crucial determinants of equilibrium. It is thus useful to define notation for p_t just before the announcement. Let

$$
\begin{aligned}
p^G &= \lim_{\tau \to T} p(\tau; 0) \\
p^B &= \lim_{\tau \to T} p(\tau; 1).
\end{aligned} \tag{13}
$$

Then p^G is the probability that the agent assigns to a negative announcement just before the announcement is realized, if the previous announcement was positive. If the previous announcement was negative, then the agent assigns probability p^B. The values of p^G and p^B, which are strictly between 0 and 1, follow from (10). Not surprisingly, $p^B > p^G$.

In what follows, all expectations should be understood to be taken with respect to the agent's posterior distribution, unless noted otherwise.

2.3 The state-price density

We will value claims to future cash flows using the state-price density π_t. This object is uniquely determined by the utility function and by the process for the endowment. Heuristically, we can think of π_t as the process for marginal utility.

Theorem 1. *For $t \in \mathcal{N}$, the evolution of the state price density π_t is characterized by*

$$
\begin{aligned}
\frac{d\pi_t}{\pi_{t^-}} &= -(r_t + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu \left[e^{-\gamma Z_t} - 1\right])dt \\
&\quad - \gamma \sigma dB_{Ct} + (1-\gamma) b_\lambda \sigma_\lambda \sqrt{\lambda_{2t}} dB_{\lambda t} + [e^{-\gamma Z_t} - 1] dN_t,
\end{aligned} \tag{14}
$$

where r_t is the riskless interest rate,

$$
r_t = \beta + \mu_C - \gamma \sigma^2 + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu \left[e^{-\gamma Z_t}(e^{Z_t} - 1)\right]. \tag{15}
$$

and where

$$
b_\lambda = \frac{1}{(1-\gamma)\sigma_\lambda^2} \left(\beta + \kappa - \sqrt{(\beta+\kappa)^2 - 2\sigma_\lambda^2 E_\nu [e^{(1-\gamma)Z_t} - 1]} \right).
$$

The instantaneous mean growth rate of the state-price density is (as usual) the riskfree rate r_t (to be characterized below).[6] The state-price density jumps upward in the case of a disaster, corresponding to the effect of a large decline in consumption on marginal utility. The state-price density also changes due to normal-time changes in consumption (this term will be small), and because of changes in the disaster probability not associated with announcements $(1-\gamma)b_\lambda \sigma_\lambda \sqrt{\lambda_{2t}} dB_{\lambda t}$. When $\gamma > 1$, $(1-\gamma)b_\lambda$ is positive and so marginal utility rises when the disaster probability rises. When $\gamma < 1$, marginal utility falls.[7]

Theorem 1 shows that there is no role for announcements for $t \in \mathcal{N}$. For a given intensity of N_t, π_t is the same as it would be without announcements (see, e.g. Tsai and Wachter (2015)). Announcements enter only indirectly, through the disaster intensity.[8] This is because announcements occur at pre-determined intervals. The announcement cycle does affect the level of the value function, but, because it is deterministic, it does not affect marginal utilities along the optimal consumption path.

Announcements do however affect the state-price density for $t \in \mathcal{A}$.

Theorem 2 (Announcement SDF)**.** *For $t \in \mathcal{A}$, with probability 1,*

$$\frac{\pi_t}{\pi_{t^-}} = \left(\frac{\exp\{\zeta_{p_{0t}} + b_p p_t\}}{\exp\{e^{\beta T}\zeta_{p_{0t^-}} + b_p p_{t^-}\}} \right)^{1-\gamma} \tag{16}$$

where

$$b_p = \frac{(\lambda^B - \lambda^G) E_\nu \left[e^{(1-\gamma)Z_t} - 1 \right]}{(1-\gamma)(\beta + \eta_{GB} + \eta_{BG})}, \tag{17}$$

and where $\zeta_{p_{0t}}, \zeta_{p_{0t^-}} \in \{\zeta_0, \zeta_1\}$ with

$$e^{(1-\gamma)\left(\zeta_0 e^{\beta T} + b_p p^G\right)} = p^G e^{(1-\gamma)(\zeta_1 + b_p)} + (1-p^G) e^{(1-\gamma)\zeta_0} \tag{18a}$$

$$e^{(1-\gamma)\left(\zeta_1 e^{\beta T} + b_p p^B\right)} = p^B e^{(1-\gamma)(\zeta_1 + b_p)} + (1-p^B) e^{(1-\gamma)\zeta_0}. \tag{18b}$$

The ratio of state-price densities just prior to and just after an announcement in

[6] For notational simplicity, the drift term in (14) is multiplied by time-t variables rather than time-t^- variables. Note that these variables are continuous for $t \in \mathcal{N}$ (they do not jump in the case of a disaster) and so this simplification is harmless.

[7] In a more general model, whether marginal utility falls or rises depends on γ relative to the inverse of the elasticity of intertemporal substitution. See Tsai and Wachter (2018).

[8] Note that the term in the drift involving p_t acts as compensation for the disaster, ensuring that r_t remains the instantaneous rate of change π_t.

(16) will play an important role in what follows. This ratio can be thought of as an *announcement stochastic discount factor*, or Announcement SDF, a concept defined in a discrete-time setting in Ai and Bansal (2018). Though the announcement is instantaneous, Theorem 2 shows that a finite amount of news is revealed: π_t undergoes a discrete change and thus the SDF is not trivially equal to one. This is what will produce a macroeconomic announcement premium in our model.

The Announcement SDF depends on p_{t^-} (the posterior probability just before the announcement) and on p_t (the posterior probability just after). It depends also on p_{0t^-} (the previously-announced probability of a bad state). Note that $p_{0t}, p_{0t^-} \in \{0, 1\}$ and, for announcement times t, $p_{t^-} \in \{p^G, p^B\}$ and $p_t = p_{0t}$. Equation 18 is thus simply the condition that, over an infinitesimal interval, agents' expectations of the SDF equal one, or, equivalently, that the state-price density must follow a martingale.[9]

That the posterior p_t should affect the SDF is intuitive. It follows from $\lambda^B > \lambda^G$ that $b_p < 0$. Thus for $\gamma > 1$, an increase in the posterior probability of being in a bad state increases marginal utility. Moreover, the increase in marginal utility is higher, the greater is the persistence of the probability (namely, the lower $\eta_{GB} + \eta_{BG}$), and the lower the discount factor β. In the numerator of this term is the instantaneous effect of a disaster on utility, multiplied by the incremental probability of disaster from being in a bad state.

However, the change in the state-price density is not only due to the change in the posterior probability. There is also an effect of the announcement itself. On the announcement, the state variable p_{0t}, representing the posterior on the most recent announcement, also jumps. Recall that this variable can either be 0 or 1, because the announcement perfectly reveals the state. The effect is thus characterized by a binary variable $\zeta_{p_{0t}}$, whose two values satisfy the system (18). When the agent receives news about λ_{1t} on the announcement, she changes her p_t, and incorporates the future predictable changes in p_t into the SDF (this is why mean reversion enters in Equation 17). The agent also incorporates forecasts of future announcements through (18).

Given the interpretation of (18) as the announcement SDF, we would expect it to

[9]Equation 16 holds "only" with probability 1. That is, there is a theoretical possibility that a disaster could coincide with an announcement. Because announcements are a set of measure zero the probability that a disaster and announcement coincide is zero, and so we can ignore the theoretical possibility when calculating expectations.

reflect our intuition about agent's marginal utilities. In fact it does, as Corollary 1 shows. The following technical result is helpful.

Lemma 2. *Let ζ_0, ζ_1, and b_p be defined as in Theorem 2. Then $b_p < 0$ and*

$$\zeta_0 > \zeta_1 + b_p. \tag{19}$$

Proof. See Appendix A. □

Indeed, given Lemma 2, the following corollary shows (assuming a preference for early resolution of uncertainty) that the announcement SDF exceeds 1 if the announcement is negative and is below 1 if it is positive:[10]

Corollary 1. *For $\gamma > 1$, the state-price density falls when the announcement is positive and rises when the announcement is negative.*

For $\gamma < 1$, the state-price density falls when the announcement is negative and rises when it is positive.

Proof. First assume that $\gamma > 1$. It follows from Lemma 2 that

$$e^{(1-\gamma)(\zeta_1+b_p)} > e^{(1-\gamma)\zeta_0}.$$

Consider the case where the previous announcement was positive. It follows from $p^G \in (0, 1)$ and (18a) that

$$e^{(1-\gamma)\zeta_0} < e^{(1-\gamma)\left(\zeta_0 e^{\beta T} + b_p p^G\right)} < e^{(1-\gamma)(\zeta_1+b_p)},$$

because the middle expression is a weighted average of the terms on either side, with weights strictly between 0 and 1. Therefore,

$$\left(\frac{e^{\zeta_0}}{e^{\zeta_0 e^{\beta T} + b_p p^G}}\right)^{1-\gamma} < 1 \tag{20}$$

$$\left(\frac{e^{\zeta_1+b_p}}{e^{\zeta_0 e^{\beta T} + b_p p^G}}\right)^{1-\gamma} > 1 \tag{21}$$

[10]This condition is consistent with risk-sensitivity, as defined by Ai and Bansal (2018). In their setting, as in ours, risk-sensitivity is a necessary condition for a nonzero announcement premium.

When the time-t announcement is positive, π_t/π_{t^-} equals the left-hand side of (20). When it is negative, π_t/π_{t^-} equals the left hand side of (21). This completes the proof.

The case in which the previous announcement was negative follows the same argument, using instead $p^B \in (0,1)$ and (18b). The proof for $\gamma < 1$ follows along the same lines. $\square$

Note that, because the SDF is greater than 1 for a negative announcement and less than 1 for a positive announcement (Corollary 1), the probability of a negative announcement is higher under the risk-neutral measure. Less obvious is the fact that the relation $p^G < p^B$ also holds for risk-neutral probabilities:

Theorem 3. *Let $\tilde{p}^B$ be the risk-neutral probability of a negative announcement, just prior to the announcement occurring, provided that the previous announcement was negative, and $\tilde{p}^G$ be the analogous quantity, provided that the previous announcement was positive. Then*

$$\tilde{p}^B > \tilde{p}^G.$$

In the next sections, we will use Theorem 3 to characterize announcement effects in the prices of bonds and stocks.

2.4 Equity prices

We consider a cross section of equities which differ in their sensitivity to disasters. For parsimony, we assume the claims are identical in all other respects. Let D_t^j equal the time-t dividend of claim j, for $j = 1, \ldots, J$. Assume

$$\frac{dD_t^j}{D_{t^-}^j} = \mu_D dt + \sigma dB_{Ct} + (e^{\phi_j Z_t} - 1)dN_t. \tag{22}$$

The parameter ϕ_j determines the sensitivity of the claim to disasters. Let S_t^j denote the time-t price of the jth claim (that is, the price of stock j). No-arbitrage then implies

$$S_t^j = E_t \int_t^\infty \frac{\pi_s}{\pi_t} D_s^j \, ds \tag{23}$$

In (23) and elsewhere in what follows, we take the expectation under the agents' subjective distribution.

Our model implies an analytical expression for (23) that, not surprisingly given the form of (23), takes the form of an integral over s. The expressions in this integral are equity strips, namely claims to a dividend payment at a single point in time.[11] To simplify the problem, we first give an analytical solution for these equity strips. We use superscript j to denote quantities that depend on ϕ_j and thus are asset specific.

Theorem 4. *Consider a claim to a dividend D^j_{s+t}, where the process for D_t solves (22). Let $H^j(D^j_t, p_t, \lambda_{2t}, \tau, s; p_{0t})$ denote the time-t price of this claim. That is,*

$$H^j(D^j_t, p_t, \lambda_{2t}, \tau, s; p_{0t}) = D^j_t E_t\left[\frac{\pi_{t+s}}{\pi_t}\frac{D^j_{t+s}}{D^j_t}\right]. \tag{24}$$

Then

$$H^j(D^j_t, p_t, \lambda_{2t}, \tau, s; p_{0t}) = D^j_t \exp\left\{a^j_\phi\big(\tau, s; p_{0t}\big) + b^j_{\phi p}(s)p_t + b^j_{\phi\lambda}(s)\lambda_{2t}\right\} \tag{25}$$

where

$$b^j_{\phi p}(s) = \frac{(\lambda^B - \lambda^G)E_\nu\left[e^{(\phi_j-\gamma)Z_t} - e^{(1-\gamma)Z_t}\right]}{\eta_{BG}+\eta_{GB}}\left(1 - e^{-(\eta_{BG}+\eta_{GB})s}\right), \quad s \geq 0, \tag{26}$$

where $b^j_{\phi\lambda}(s)$ solves

$$\frac{db^j_{\phi\lambda}(s)}{ds} = \frac{1}{2}\sigma^2_\lambda b^j_{\phi\lambda}(s)^2 + \left((1-\gamma)b_\lambda\sigma^2_\lambda - \kappa\right)b^j_{\phi\lambda}(s) + E_\nu\left[e^{(\phi_j-\gamma)Z_t} - e^{(1-\gamma)Z_t}\right], \tag{27}$$

with boundary condition $b^j_{\phi\lambda}(0) = 0$. Define the function a^j_ϕ such that

$$a^j_\phi\big(\tau, s; p_{0t}\big) = h^j\big(\tau + s; p_{0t}\big) + \\ \int_0^s \Big(-\beta - \mu_C + \mu_D + \lambda^G E_\nu\Big[e^{(\phi_j-\gamma)Z_t} - e^{(1-\gamma)Z_t}\Big] + \kappa\bar{\lambda}_2 b^j_{\phi\lambda}(u) + \eta_{GB}b_{\phi p}(u)\Big)\,du \tag{28}$$

for $\tau \in [0, T), s \geq 0, p_{0t} \in \{0, 1\}$. The function h^j uniquely solves

$$e^{h^j(u;p_{0t^-}) + b^j_{\phi p}(u-T)p_{t^-}} = E_{t^-}\left[\frac{e^{(1-\gamma)(\zeta_{p_{0t}}+b_p p_t)}}{e^{(1-\gamma)(e^{\beta T}\zeta_{p_{0t^-}}+b_p p_{t^-})}}e^{h^j(u-T;p_{0t})+b^j_{\phi p}(u-T)p_t}\right], \tag{29}$$

for $u \geq T$ and $h^j\big(u;\cdot\big) = 0$ for $u \in [0, T)$.

[11] See Lettau and Wachter (2007).

Equation 25 gives the price investors will pay today to receive the aggregate dividend s periods in the future. This price depends on s, the probability of the state p_t and the probability of observed disaster λ_{2t}. As is clear from both (26) and 27, the direction of this dependence varies according to whether ϕ_j is greater than or less than one, reflecting a tradeoff between the effect of the riskfree rate on the one hand and that of the the risk premium and cash flow expectation on the other.[12] The standard assumption is levered equity, with $\phi_j > 1$. In this case, the latter effect dominates, and equity prices fall when the probability of a disaster (as captured by either λ_{2t} or p_t) rises. This dependence is similar what one finds in previous work (Tsai and Wachter, 2015) and so we do not discuss it further here.

More novel is the dependence of the price on disasters, captured in the function $h(\tau + s; p_{0t})$. This function depends on the sum of the maturity and the time since the last disaster. When $\tau + s < T$, there are no announcements scheduled before the equity matures, and this term is zero. Now assume there is just one announcement left before maturity. The risk-neutral expectation of the discounted price just before the announcement has to match the price just after the announcement, pinning down $h(\tau + s; p_{0t})$ for $\tau + s \in [T, 2T)$. This is the content of condition (29).

Intuitively, a negative announcement should decrease prices. The following Corollary shows that this is true, provided that $\phi > 1$:

Corollary 2. *Assume that $\phi_j > 1$. Then the price of an equity strip with positive maturity on the announcement date increases when the announcement is positive and decreases when the announcement is negative. That is*

$$H(D, 1, \lambda_2, 0, s; 1) < \lim_{\tau \to T} H(D, p_{t^-}, \lambda_2, \tau, s; p_{0t^-}) < H(D, 0, \lambda_2, 0, s; 0)$$

for $s > 0$.

While a formal proof is in the Appendix, we give a heuristic proof here. When $\phi_j > 1$, a higher probability of disaster lowers the value of the dividend claim (this effect operates through $b_{\phi p}$). Consider first the claim with one announcement prior to maturity. Clearly, this claim will fall in price if the announcement is negative and rise if it is positive. Because $\tilde{p}^B > \tilde{p}^G$, the price of this claim will be lower prior

[12]Equivalently: the agent's desire to substitute across asset classes dominates the need to save more.

to announcement if the *previous* announcement is negative than if it positive (this is captured by (29), for $u \in [T, 2T)$). Thus a claim with two announcements before maturity will fall in price if the next announcement is negative, and so on.

To calibrate this model, we will consider a claim to a continuous stream of dividends. The price of the claim is an integral of prices of the form (25):

Corollary 3. *Let S_t^j be the time-t price of an asset paying the dividend process (22) with leverage parameter ϕ_j. Then*

$$S^j\big(D_t, p_t, \lambda_{2t}, \tau; p_{0t}\big) = \int_0^\infty H^j\big(D_t, p_t, \lambda_{2t}, \tau, s; p_{0t}\big) ds, \tag{30}$$

where H^j is given by (25).

Proof. The result follows directly from Theorem 4 and the no-arbitrage condition (23). $\square$

Using the characterization of the equity price in Corollary 3, we can sign the response to the announcement.

Corollary 4. *Assume that $\phi_j > 1$. Then $S^j\big(D_t, p_t, \lambda_{2t}, \tau; p_{0t}\big)$ increases when the announcement is positive and decreases when the announcement is negative. That is,*

$$S(D, 1, \lambda_2, 0; 1) < \lim_{\tau \to T} S(D, p_{t^-}, \lambda_2, \tau; p_{0t^-}) < S(D, 0, \lambda_2, 0; 0).$$

Proof. The result follows directly from Corollaries 2 and 3. $\square$

2.5 Risk premia

We first consider risk premiums when $t \in calN$. Let r_t^j denote the expected return on asset j per unit dt of time. Note that:

$$r_t^j = \mu_{Sjt} + \big(\bar{\lambda}_1(p_t) + \lambda_{2t}\big) E_\nu[e^{\phi_j Z_t} - 1] + \frac{D_t^j}{S_t^j}, \tag{31}$$

where μ_{Sjt} is the drift of S_t^j rate and where the expected jump size $E_\nu[e^{\phi_j Z_t} - 1]$ follows from the homogeneity of S_t^j in D_t^j.

Theorem 5. *For $t \in \mathcal{N}$, the risk premium on an asset with dividend stream (22) equals*

$$r_t^j - r_t = \gamma\sigma^2 - \lambda_{2t}(1-\gamma)b_\lambda \frac{1}{S^j}\frac{\partial S^j}{\partial \lambda_2}\sigma_\lambda^2 - \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu \left[\left(e^{-\gamma Z_t} - 1\right)\left(e^{\phi_j Z_t} - 1\right)\right]. \quad (32)$$

The theorem divides the premium into three components: the first is the standard consumption CAPM term (negligible in our calibration). The second term is the premium investors require for baring the risk of facing risk in λ_{2t}. Provided that the price falls when λ_{2t} rises, this is positive for $\gamma > 1$. See the discussion following Theorem 1 for further detail. The third term is the premium directly linked to the rare disasters. Note that the probability of the disaster outcome is the agent's posterior probability, $\bar{\lambda}_1(p_t) + \lambda_{2t}$. The disaster premium is positive provided that agents are risk averse and that asset has positive exposure to disasters $\phi_j > 0$.

We now consider the risk premium on announcement dates. On non-announcement dates, the risk premium earned on the asset is equal to $(r_t^j - r_t)\,dt$. Therefore the usual continuous-time result holds: the risk premium approaches zero for sufficiently small time periods. This is not true for announcements dates.

Intuitively, the announcement premium should be given by the covariance of returns with the stochastic discount factor. Both the SDF and the price process jump with the arrival of a discrete amount of information on the instant of the announcement. As the following theorem makes precise, this joint behavior creates an announcement premium.

Theorem 6. *For assets defined in Theorem 3, the announcement premium is given by*

$$E_{t^-}\left[\frac{S_t^j - S_{t^-}^j}{S_{t^-}^j}\right] = -E_{t^-}\left[\left(\frac{\pi_t - \pi_{t^-}}{\pi_{t^-}}\right)\left(\frac{S_t^j - S_{t^-}^j}{S_{t^-}^j}\right)\right] \quad (33)$$

for $t \in \mathcal{A}$.

Proof. Expanding the right-hand-side of (33) yields

$$E_{t^-}\left[\left(\frac{\pi_t - \pi_{t^-}}{\pi_{t^-}}\right)\left(\frac{S_t^j - S_{t^-}^j}{S_{t^-}^j}\right)\right] = E_{t^-}\left[\frac{\pi_t}{\pi_{t^-}}\frac{S_t^j}{S_{t^-}^j} - \frac{S_t^j}{S_{t^-}^j} - \left(\frac{\pi_t}{\pi_{t^-}} - 1\right)\right] \quad (34)$$

The process $\pi_t S_t^j$ follows a martingale, as follows from (23). Therefore

$$\pi_{t^-} S_{t^-}^j = E_{t^-}\left[\pi_t S_t^j\right].$$

Furthermore, the expected rate of change in π_t over any infinitesimal interval must equal r_t multiplied by the length of the interval. When the interval size length is zero, then

$$\pi_{t^-} = E_{t^-}\left[\pi_t\right].$$

The result then follows from (34). $\square$

Corollary 5. *Consider an asset paying dividends given by (22), such that the leverage parameter $\phi_j > 1$. The announcement premium is strictly positive if $\gamma > 1$ and strictly negative if $\gamma < 1$.*

Proof. Corollaries 1 and 4 show that changes in S^j and in π upon announcements have opposite signs when $\gamma > 1$ and the same sign when $\gamma < 1$. The result follows. $\square$

2.6 Nominal bonds

We now present a model of nominal bonds that incorporates a role for macroeconomic announcements.

2.6.1 Inflation process

The real return on nominal bonds depends on the inflation process. Following Barro (2006), Gabaix (2012) and Tsai (2016), we assume that bonds exhibit a loss in the event of disaster, and we assume, for simplicity that this loss is equal to the percent decline in consumption. Thus, the price level P_t follows

$$\frac{dP_t}{P_{t^-}} = q_t dt + \sigma_p dB_{Pt} + \left(e^{-Z_t} - 1\right) dN_t, \tag{35}$$

where q_t is the expected inflation process, and is given by

$$dq_t = \kappa_q(\bar{q}_t - q_t)dt + \sigma_q dB_{qt} \tag{36}$$

where B_{Pt} and B_{qt} are independent Brownian motion processes that are also independent of B_{Ct} and $B_{\lambda t}$.

Expected inflation $\bar{q}_t$ follows a Markov switching process, and, like λ_{1t}, is latent. Consistent with the data (Tsai, 2016; Dergunov et al., 2018), we assume that elevated disaster risk and elevated inflation co-occur. That is, $\bar{q}_t = \bar{q}^B$ when $\lambda_{1t} = \lambda^B$ and $\bar{q}_t = \bar{q}^G$ when $\lambda_{1t} = \lambda^G$, with $q^B > q^G$. This implies that the macro-announcements, which reveal the latent disaster-probability state, also reveal expected inflation. Given that macro-announcements are often ostensibly about inflation, this seems reasonable.[13]

2.6.2 The nominal state-price density and bond pricing

It is convenient to define a state-price density connecting nominal cash flows to nominal prices. As is well-known, the nominal state-price density equals

$$\pi_t^\$ = \frac{\pi_t}{P_t}. \tag{37}$$

Thus if $H^\$(p_t, q_t, \tau, s; p_{0t})$ denotes the price of a default-free nominal bond with s years to maturity and a face value of 1, no-arbitrage implies

$$H^\$(p_t, q_t, \tau, s; p_{0t}) = E_t\left[\frac{\pi_{t+s}^\$}{\pi_t^\$}\right]. \tag{38}$$

Given (37), the evolution of the nominal state-price density follows from Itô's Lemma (see Appendix C for more detail).

Theorem 7. *For $t \in \mathcal{N}$, the evolution of the nominal state price density $\pi_t^\$$ is characterized by*

$$\begin{aligned}\frac{d\pi_t^\$}{\pi_{t^-}^\$} = -(r_t^\$ + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu\left[e^{(1-\gamma)Z_t} - 1\right])dt \\ - \gamma\sigma dB_{Ct} + (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}dB_{\lambda t} - \sigma_P dB_{Pt} \\ + (e^{(1-\gamma)Z_t} - 1)dN_t,\end{aligned} \tag{39}$$

[13]We continue to assume that the agent infers the state only from announcements, and not from inflation observations. Because announcements are frequent and informative, this is reasonable.

where $r_t^\$$ is the nominal riskless interest rate,

$$r_t^\$ = r_t + q_t - \sigma_P^2 - (\bar{\lambda}_{1t} + \lambda_{2t}) E_\nu \left[e^{-\gamma Z_t} (e^{Z_t} - 1) \right], \tag{40}$$

and where b_λ is given by Theorem 1.[14]

Proof. By applying Itô's Lemma on (37) we get (39). □

Our specification implies that the announcement pertains to expected, not actual inflation. It follows that over the infinitesimal interval defined by the announcement, the nominal stochastic discount factor – namely the change in the state price density – is equal to the real stochastic discount factor.

Lemma 3 (Nominal announcement SDF)**.** *For $t \in \mathcal{A}$, with probability 1,*

$$\frac{\pi_t^\$}{\pi_{t^-}^\$} = \frac{\pi_t}{\pi_{t^-}} \tag{41}$$

Zero-coupon nominal bonds are priced in a manner analogous to equity strips in Theorem 4.[15]

Theorem 8. *The time-t price of a nominal zero-coupon bond maturing in s years is given by*

$$H^\$ \big(p_t, q_t, \tau, s; p_{0t} \big) = \exp \left\{ a^\$(\tau, s; p_{0t}) + b_p^\$(s) p_t + b_q^\$(s) q_t \right\} \tag{42}$$

where

$$b_q^\$(s) = \frac{1}{\kappa_q} (e^{-\kappa_q s} - 1), \tag{43}$$

and where $b_p^\$(s)$ solves

$$\frac{\partial b_p^\$(s)}{\partial s} = -(\eta_{BG} + \eta_{GB}) b_p^\$(s) + b_q^\$(s) \kappa_q \left(\bar{q}^B - \bar{q}^G \right) \tag{44}$$

[14] The nominal riskless interest rate (sometimes called the nominal riskfree rate) is the nominal return on the asset that is instantaneously riskfree when payoffs are expressed in nominal terms.

[15] In this specification, bond prices do not depend directly on the disaster probability. This is because the effect of the disaster probability on the nominal riskfree rate and on the risk premium cancels out. We make this assumption for simplicity: our results do not depend on it.

with boundary condition $b_p^\$(0) = 0$. *Define the function* $a^\$(\tau, s; p_{0t})$ *such that*

$$a^\$(\tau, s; p_{0t}) = h^\$(\tau + s; p_{0t}) + \int_0^s (-\beta - \mu_C + \gamma\sigma^2 + \sigma_p^2 + b_q^\$(u)\kappa_q \bar{q}^G) du, \tag{45}$$

for $\tau \in [0, T)$, $s \geq 0$, $p_{0t} \in \{0, 1\}$. *The function* $h^\$$ *uniquely solves*

$$e^{h^\$(u;p_{0t^-}) + b_p^\$(u-T)p_{t^-}} = E_{t^-}\left[\frac{e^{(1-\gamma)(\zeta_{p_{0t}} + b_p p_t)}}{e^{(1-\gamma)(e^{\beta T}\zeta_{p_{0t^-}} + b_p p_{t^-})}} e^{h^\$(u-T;p_{0t}) + b_p^\$(u-T)p_t}\right], \tag{46}$$

for $u \geq T$ *and* $h^\$\big(u; \cdot\big) = 0$ *for* $u \in [0, T)$.

Nominal bond prices fall when expected inflation rises ($b_q^\$(s) \leq 0$) and, when there is an increased probability of the bad state ($b_p^\$(s) \leq 0$, as shown in Appendix C). Because of the latter property, bond prices also fall upon a negative announcement:

Corollary 6. *The price of a zero-coupon bond with positive maturity on the announcement date increases when the announcement is positive and decreases when the announcement is negative. That is*

$$H^\$(1, \lambda_2, 0, s; 1) < \lim_{\tau \to T} H^\$(p_{t^-}, \lambda_2, \tau, s; p_{0t^-}) < H^\$(0, \lambda_2, 0, s; 0)$$

for $s > 0$.

The intuition is the same as for equities in Corollary 2.

2.6.3 Bond risk premia

Finally, we describe instantaneous covariances and risk premia on bonds. First note that the model implies that long-term nominal bonds have zero risk premia relative to short-term bonds on non-announcement days. That is, relative to the nominal riskfree rate (40), risk premia on bonds are zero. This is because bonds of all maturities are equally exposed to realized inflation.

However, long-term bonds have greater exposure than short-term bonds to expected inflation. Expected inflation is persistent, and thus the total loss, in real terms, on the nominal bond is greater. In principle, this could generate a risk premium during non-announcement periods if expected inflation were priced (i.e. if the Brownian motions

B_q and B_C were correlated). Given the low level of risk aversion we assume, however, this effect would be negligible.

What is not negligible, however, is the risk of changes in the mean of expected inflation due to announcements. The model predicts that the disaster probability is priced. When an announcement is negative, namely it reveals a high disaster probability, it simultaneously reveals that inflation is high. Marginal utilities rise on the news of the instability in the economy, exactly as bond prices fall because they will pay off less in real terms. This generates a positive premium on announcements that increases with maturity.

3 Quantitative results

We start by replicating the evidence of Savor and Wilson (2014) in an extended sample. Section 3.1 describes the data and Section 3.2 the empirical findings. We then simulate repeated samples from the model described in the previous section. Section 3.3 describes the calibration of our model and Section 3.4, the simulation results.

3.1 Data

We obtain daily stock returns from the Center for Research in Security Prices (CRSP). We consider individual stocks traded on NYSE, AMEX, NASDAQ and ARCA from January 1961 to September 2016. In addition, we also use the daily market excess returns and risk-free rate provided by Kenneth French. The scheduled announcement dates before 2010 are provided by Savor and Wilson (2014). Following their approach, we add target-rate announcements of the FOMC and inflation and employment announcements of the BLS for the remaining dates.

We define the daily excess return to be the daily (level) return of a stock in excess of the daily return on the 1-month Treasury bill. We estimate covariances on individual stock returns with the market return using daily data and 12-month rolling windows. We include stocks which are available for trading on 90% or more of the trading days. At the start of each trading month, we sort stocks by estimated betas, and create deciles. We then form value-weighted portfolios of the stocks in each deciles, and compute daily excess returns.

We obtain daily bond returns from CRSP. We use returns on the Fixed Term Indices. We subtract a daily riskfree rate, obtained from Kenneth french's website to obtain daily excess returns.

3.2 Empirical findings

Table 1 presents summary statistics on the ten beta-sorted portfolios. For each portfolio j, $j = 1, \ldots, 10$, we use the notation $E[RX^j]$ to denote the mean excess return, σ^j the volatility of the excess return, and β^j the covariance with the value-weighted market portfolio divided by the variance of the market portfolio. Table 1 shows statistics for daily returns computed over the full sample, over announcement days, and over non-announcement days.[16] There is a weak positive relation between full-sample returns and market betas. On non-announcement days, there is virtually no relation between betas and expected returns. However, on announcement days, there is a strong relation between beta and expected returns.

Figure 1 shows average daily excess returns in each of the ten portfolios, plotted against the betas on the portfolios for announcement days (diamonds) and non-announcement days (squares). Also shown is the fitted line on both days. This relation, known as the security market line, is strongly upward-sloping on announcement days, but virtually flat on non-announcement days.

Table 2 shows that Treasury bonds also feature much higher returns on announcement days. On non-announcement days, the beta on Treasury bond returns with respect to the market is negative, and there is no discernable relation between risk and return. However, this beta is strongly positive on announcement days, and a clear security market line emerges.

3.3 Calibration

We now describe the calibration of the model in Section 2. We choose preference parameters and normal-times consumption parameters to be the same as in Wachter

[16] Betas and volatilities are computed in the standard way, as central second moments. An announcement-day volatility therefore is computed as the mean squared difference between the announcement return and the mean announcement return. Announcement-day betas are computed analogously.

(2013). We also choose the same values for the mean reversion of the λ_2-process (κ) and the volatility parameter of this process, σ_λ. For simplicity, we assume that, when the economy is in the good state, the intensity λ_{1t} is zero, that is $\lambda_G = 0$. We choose η_{GB} so that the bad state of the economy is a rare event, and η_{BG} so that it is persistent. The unconditional probability of the bad state in our calibration is $\eta_{GB}/(\eta_{GB}+\eta_{BG}) = 23\%$. We then choose $\bar{\lambda}_2$ and λ_B with the restriction that the average disaster probability is 3.6%, as in Barro and Ursúa (2008). The values $\bar{\lambda}_2 = 2.1\%$ and $\lambda_B = 6.2\%$ satisfy that restriction. The disaster distribution is taken to be multinomial, as measured in the data by Barro and Ursua. See Wachter (2013) for further detail.

We choose the disaster sensitivity ϕ_j to match cash flow betas equal to the range of return betas given in the data, and so the average exposure is consistent with $\phi = 3$.[17] For simplicity, we assume that during normal times firm dividends grow at the same rate as each other and at the same rate as consumption $\mu_D = \mu_C$.

We take inflation parameters σ_q, σ_P, and κ_q from Tsai (2016), who chooses them to match the volatility and autocorrelation of inflation during normal times, as well as the volatility of the short-term interest rate. Expected inflation in each regime is chosen so that it equals normal-times expected inflation in the data. Table 3 reports parameter choices.

3.4 Simulation strategy

To evaluate the fit of the model, we simulate 500 artificial histories, each of length 50 years (240×50 days). We assume that announcements occur every 10 trading days. For each history, we simulate a burn-in period, so that we start the history from a draw from the stationary distribution of the state variables. We simulate the model using

[17] Very roughly, the beta during normal times is equal to

$$\beta^j \approx \frac{E_\nu\left[e^{(\phi^j-\gamma)Z_t} - e^{(1-\gamma)Z_t}\right]}{E_\nu\left[e^{(\bar{\phi}-\gamma)Z_t} - e^{(1-\gamma)Z_t}\right]}. \tag{47}$$

where $\bar{\phi}$ is the target market leverage. This follows because normal-times beta is driven by exposure to λ_{2t}. For assets of similar maturity structure and Brownian risk, the relative loadings on λ_{2t} are proportional to the numerator in (47), because it appears as the constant term in the differential equation for $b^j_{\phi\lambda}$ (see Equation 27). We solve for ϕ_j such that (47) gives us the range of betas observed in the data. The resultant values of ϕ_j range from 1.3 to 10.5. Our results are not at all sensitive to the precise values of the ϕ_j.

the true (as opposed to the agents') distribution. We report statistics for the full set of sample paths.

While time is continuous in our analytical model, it is necessarily discrete in our simulations. We simulate the model at a daily frequency to match the frequency of the data. We compute end-of-day prices, and assume the announcement occurs in the middle of a trading day.

Given a series of state variables and of shocks, we compute returns as follows. For each asset j, define the price-dividend ratio $G_t^j = F_t^j / D_t^j$. From (30), it follows that G_t^j is a function of the state variables alone. We approximate the daily return as

$$\begin{aligned} R^j_{t,t+\Delta t} &\approx \frac{F^j_{t+\Delta t} + D^j_{t+\Delta t}\Delta t}{F^j_t} \\ &= \frac{D^j_{t+\Delta t} G^j_{t+\Delta t} + D^j_{t+\Delta t}\Delta t}{D^j_t G^j_t} \\ &= \frac{D^j_{t+\Delta t}}{D^j_t} \frac{G^j_{t+\Delta t} + \Delta t}{G^j_t} \\ &\approx \exp\left\{\bar{\mu}_D \Delta t - \frac{1}{2}\sigma^2 \Delta t + \sigma(B_{C,t+\Delta t} - B_{C,t})\right\} \frac{G^j_{t+\Delta t} + \Delta t}{G^j_t}, \end{aligned} \tag{48}$$

where $\Delta t = 1/240$. The last line follows because we consider sample paths with no disasters. The risk free rate is approximated by

$$R_{ft} = \exp(r_t \Delta t). \tag{49}$$

The daily excess return of asset k is then

$$RX^j_{t,t+\Delta t} = R^j_{t,t+\Delta t} - R_{ft}. \tag{50}$$

We define the value-weighted market return just as in the data, namely we take a value-weighted portfolio of returns. We assume that the assets have the same value at the beginning of the sample. Because the assets all have the same loading on the Brownian shock and the same drift, and conditional on a history not containing rare events, the model implies a stationary distribution of portfolio weights. Given a time series of excess returns on firms (which, because we have no idiosyncratic risk, we take as analogous to portfolios), and a time series of excess returns on the market, we compute statistics exactly as in the data.

3.5 Stock returns on announcement and non-announcement days

Figure 2 displays our main result. We overlay the simulated statistics on the empirical statistics from Figure 1.[18] Each dot on the figure represents a statistic for one firm, for one simulated sample. Blue dots show pairs of average excess returns and betas on announcement days, while grey dots show pairs on non-announcement days. The figure shows that average returns on announcement days in the model are much higher than on non-announcement days. Furthermore, average returns vary with beta on announcement days in the model, whereas the do not on non-announcement days.

Figure 3 further clarifies the relation between the announcement and non-announcement days in the model by showing medians and interquartile ranges from the full set of simulated samples. Median returns closely match the data, whereas interquartile ranges show that the vast majority of samples with announcements can be clearly distinguished from those of non-announcements.

How is it that the model can explain these findings? Announcements convey important news about the distribution of future outcomes in the economy. On that day, it is possible that a bad state of the economy could be revealed. If the bad state is realized, not only will asset values be affected, but the marginal utility of economic agents will rise. Thus investors require a premium to hold assets over the risky announcement period.

In our model, some assets have cash flows that are more sensitive to others. The sensitivity parameter ϕ_j, while not the same as the beta, is closely related. Assets with high ϕ_j have a greater dividend response to disasters. Their prices thus move more with changes in the disaster probability, and in particular with λ_{1t} and λ_{2t}. The value-weighted market portfolio of course also moves with the disaster probability, and thus the higher is ϕ_j (over the relevant range), the higher is the return beta with the market, both on non-announcement days (which reveal information about λ_{2t}, and on announcement days, which reveal additional information about λ_{1t}.

[18]This figure reports simulated statistics from samples without disasters. As we show below, this does not affect inference from the model.

Thus the model predicts a relation between risk and return on both announcement and non-announcement days, but because the risk is so much greater on announcement days, the premium, and therefore the spread in expected returns between low and high-sensitivity assets, will also be much greater. A more subtle question is whether this slope is commensurate with the equity premium on announcement versus non-announcement days. We will see in fact that, while the equity premium on announcement days is low, the median slope is lower still, due to model mis-specification. This is discussed at the end of the next section.

In the discussion above, we focus on results for the cross section. The model also captures the time series result that most of the equity premium is realized on announcement days (Savor and Wilson, 2013; Lucca and Moench, 2015). Table 4 shows that the average market return is far higher on announcement days versus non-announcement days, both in the model and in the data. On the other hand, the increase in volatility is small. While the median increase in volatility is greater in the model than in the data, the data is well-within the 90 percent confidence intervals, reflecting the fact that a substantial fraction of the samples feature no increase in volatility on non-announcement days at all.

3.6 Bond returns on announcement and non-announcement days

A crucial difference between bonds and equities is that equities are, by virtue of their cash flows, exposed to aggregate stock market risk. For bonds, this need not be the case. Indeed, Table 2 shows that betas on bonds are close to zero on average. It is well-known that the covariances between Treasury bonds and stocks are unstable (Campbell et al., 2017), suggesting that the the beta does not reveal much about the risk in bonds. This makes it all the more striking that bonds exhibit positive betas on announcement days, and that these betas line up with the expected returns.

Table 5 compares the cross-section of bonds with that of equities. We run the regression

$$\hat{E}[RX_t^j \mid t \in i] = \delta_i \beta_i^j + \text{error}, \tag{51}$$

where $i = a$ (announcement days) or n (non-announcement days). The regression slope δ_i is the slope of the security market line. It is simulatenously a measure of risk and return, and a measure of the daily market risk premium. Table 5 shows an economically significant difference between the slope on announcement and non-announcement days for equities. The measure of the market risk premium is consistent with that found in Table 4 on both types of days. For bonds, the data reveal a slightly negative slope on non-announcement days. The slope on announcement days is strongly positive. To summarize: both equities and bonds exhibit a strong relation between risk and return on announcement days. Bonds, unlike equities exhibit no relation on non-announcement days. Furthermore, betas for bonds change substantially on announcement days versus non-announcement days.

What does the model have to say about these findings? Section 2 shows that, on non-announcement days, the true instantaneous covariance between bonds and stocks is equal to zero. This implies that the true security market line is undefined on non-announcement days. Thus the model is consistent both with negative observed betas on non-announcement days, and the fact that these betas exhibit no relation with expected returns. On the other hand, macro-announcements directly reveal news about bond cash flows, because they are informative about inflation. In our model, news of higher inflation is interpreted as indicating macroeconomic stability. Losses on bonds therefore coincide with losses on the stock market. Thus the model predicts both positive betas on bonds on announcement days, and a strong risk-return relation. Table 6 shows that, indeed, bonds have much higher betas on announcement days in simulated data. In contrast, equity betas can increase or decrease, with confidence intervals generally containing zero. Table 5 shows that the slope of the security market line in the model, while slightly negative on non-announcement days, is positive and large on announcement days.

Because betas on announcement days are higher in the model than in the data, the model does not succeed in capturing the full magnitude of the announcement-day slope. The model does succeed, however, in capturing the fact that bonds display market risk on announcement days, and no market risk on non-announcement days, and that this market risk is priced. In the model, news about disaster directly correlates with that of expected inflation. Stated differently, the announcements are concerned with inflation; investors perhaps infer that information concerning inflation also is informative about disasters. Moreover, because inflation tends to rise when the probability of a disaster rises, news about inflation is priced. The greater the bond maturity, the greater the impact of this news, and the greater is the expected return.

Finally, Table 5 shows that the slope of the security market line for equities predicted by the model is about half the size of the equity premium on non-announcement days. That is, while the model predicts that the premium is far lower on non-announcement days, as compared with announcement days, it also correctly predicts that the slope of the security market line is below even the premium on non-announcement days. The reason is that a the relation between normal-times covariance and expected returns is non-linear. Normal-times covariance is proportional to the premium for bearing risk of changes in the disaster probability, but not the disaster premium itself. In a sense, large betas are too large given their expected returns, and small betas are too small – both types of stocks have more similar exposure to disaster risk than one would think from their betas alone.

4 Conclusion

The Capital Asset Pricing Model has been a major focus of resesearch in financial economics, and the benchmark model in financial practice for over fifty years. Despite its pre-eminent status, years of empirical research has found little support for the CAPM. That is, until quite recently. The CAPM predicts a tight relation between market beta and expected return, known as the security market line. Recent research has shown that this security market line, seemingly absent on most days, appears on days with macro-economic announcements.

This paper builds a general equilibrium model to explain why the security market line appears on macroeconomic announcement days, but is hard to discern on others. The model derives the result from underlying economic principles in a frictionless environment. For this reason, we can explain why the relation between risk and return is not asset-class specific. It holds for both bonds and equities.

We explain the finding through a combination of two mechanisms. The first is a preference for early resolution of uncertainty, as described in Epstein and Zin (1989). The second is rare events. The risk that is realized on announcement days concerns the probability of a rare negative event to the economy. These mechanisms together imply that assets that are especially exposed to these events carry high premia, even though it is not necessary to observe the occurance of the event itself.

While our focus in this paper is on macro-announcements, our the methodology can be applied to scheduled announcements more generally, and understanding the rich array of empirical facts that the announcement literature has uncovered.

A Solving the representative agent's value function

Define the vector Brownian motion $dB_t = [dB_{Ct}, dB_{\lambda t}]^\top$. In what follows, we use E_t to denote expectations taken under the agents' subjective distribution. The notation E_ν denotes expectations taken with respect to the random variable Z.

A.1 Continuation Value

Lemma A.1. *In equilibrium, the representative agent's continuation value equals*

$$J(C_t, p_t, \lambda_{2t}, \tau; p_{0t}) = \frac{1}{1-\gamma} C_t^{1-\gamma} I(p_t, \lambda_{2t}, \tau; p_{0t})^{1-\gamma}, \tag{A.1}$$

with

$$I(p_t, \lambda_{2t}, \tau; p_{0t}) = \exp\{a\big(\tau; p_{0t}\big) + b_p p_t + b_\lambda \lambda_{2t}\}, \tag{A.2}$$

and

$$\begin{aligned} b_p &= \frac{(\lambda^B - \lambda^G) E_\nu \left[e^{(1-\gamma)Z_t} - 1\right]}{(1-\gamma)(\beta + \eta_{GB} + \eta_{BG})}, \\ b_\lambda &= \frac{1}{(1-\gamma)\sigma_\lambda^2} \left(\beta + \kappa - \sqrt{(\beta+\kappa)^2 - 2\sigma_\lambda^2 [E_\nu e^{(1-\gamma)Z_t} - 1]}\right), \end{aligned}$$

for $p_t \in [0,1]$, $\lambda_{2t} \in [0,\infty)$, $\tau \in [0,T)$, $p_{0t} \in \{0,1\}$ and where a defined as

$$a\big(\tau; p_{0t}\big) = \zeta_{p_{0t}} e^{\beta\tau} + \frac{1}{\beta}\left(\mu_C - \frac{1}{2}\gamma\sigma^2 + b_p \eta_{GB} + b_\lambda \kappa \bar{\lambda}_2 + \frac{\lambda^G}{1-\gamma}\left[E_\nu e^{(1-\gamma)Z_t} - 1\right]\right). \tag{A.3}$$

The constant terms ζ_0 and ζ_1 solve

$$\begin{aligned} e^{(1-\gamma)(\zeta_0 e^{\beta T} + b_p p^G)} &= p^G e^{(1-\gamma)(\zeta_1 + b_p)} + (1-p^G) e^{(1-\gamma)\zeta_0} \\ e^{(1-\gamma)(\zeta_1 e^{\beta T} + b_p p^B)} &= p^B e^{(1-\gamma)(\zeta_1 + b_p)} + (1-p^B) e^{(1-\gamma)\zeta_0}, \end{aligned} \tag{A.4}$$

with p^G and p^B defined by (13).

Proof. Conjecture that, at the optimum, the value function can be written as

$$V_t = J(C_t, p_t, \lambda_{2t}, \tau; p_{0t}). \tag{A.5}$$

Note that p_{0t} is a discrete variable that changes only for $t \in \mathcal{A}$. For $t \in \mathcal{N}$, the Hamilton-Jacobi-Bellman equation applies and

$$\begin{aligned} f(C_t, J_t) + \frac{\partial J}{\partial \tau} + \frac{\partial J}{\partial C} C_t \mu_C + \frac{\partial J}{\partial p}\left[-p_t(\eta_{GB} + \eta_{BG}) + \eta_{GB}\right] - \frac{\partial J}{\partial \lambda}\kappa(\lambda_{2t} - \bar{\lambda}_2) \\ + \frac{1}{2}\frac{\partial^2 J}{\partial C^2} C_t^2 \sigma^2 + \frac{1}{2}\frac{\partial^2 J}{\partial \lambda^2}\lambda_{2t}\sigma_\lambda^2 \\ + \left(p_t \lambda^B + (1 - p_t)\lambda^G + \lambda_{2t}\right) E_\nu \left[J(Ce^Z, \cdot) - J(C, \cdot)\right] = 0. \end{aligned} \tag{A.6}$$

Conjecture a solution of the form (A.1) and (A.2). The conjectured form (A.1) implies

$$\frac{1}{J}(J(Ce^Z, \cdot) - J(C, \cdot)) = E_\nu \left[e^{(1-\gamma)Z_t} - 1\right]. \tag{A.7}$$

Substituting (A.1) and (A.2) into (A.6) and dividing both sides by J, we obtain

$$\begin{aligned} & - \beta(1-\gamma)\Big[a\big(\tau; p_{0t}\big) + b_p p_t + b_\lambda \lambda_{2t}\Big] \\ & + (1-\gamma)\frac{da}{d\tau}(\tau; p_{0t}) + (1-\gamma)\mu_C + (1-\gamma) b_p \left[-p_t(\eta_{GB} + \eta_{BG}) + \eta_{GB}\right] - (1-\gamma) b_\lambda \kappa(\lambda_{2t} - \bar{\lambda}_2) \\ & - \frac{1}{2}\gamma(1-\gamma)\sigma^2 + \frac{1}{2}(1-\gamma)^2 b_\lambda^2 \sigma_\lambda^2 \lambda_{2t} \\ & + p_t(\lambda^B - \lambda^G) E_\nu \left[e^{(1-\gamma)Z_t} - 1\right] + \lambda^G E_\nu \left[e^{(1-\gamma)Z_t} - 1\right] + \lambda_{2t} E_\nu \left[e^{(1-\gamma)Z_t} - 1\right] = 0. \end{aligned} \tag{A.8}$$

Collecting coefficients on p_t and on λ_{2t}, we obtain

$$\begin{aligned} -\beta(1-\gamma) b_p - (1-\gamma) b_p (\eta_{GB} + \eta_{BG}) + (\lambda^B - \lambda^G) E_\nu \left[e^{(1-\gamma)Z_t} - 1\right] = 0 \\ -\beta(1-\gamma) b_\lambda - (1-\gamma) b_\lambda \kappa + \frac{1}{2}(1-\gamma)^2 b_\lambda^2 \sigma_\lambda^2 + E_\nu \left[e^{(1-\gamma)Z_t} - 1\right] = 0. \end{aligned} \tag{A.9}$$

The equation for b_p in the text follows.

We also have the following quadratic function of b_λ:

$$\frac{1}{2}(1-\gamma)\sigma_\lambda^2 b_\lambda^2 - (\beta + \kappa) b_\lambda + \frac{1}{1-\gamma} E_\nu \left[e^{(1-\gamma)Z_t} - 1\right] = 0, \tag{A.10}$$

which has solution:[19]

$$b_\lambda = \frac{1}{(1-\gamma)\sigma_\lambda^2}\left(\beta+\kappa-\sqrt{(\beta+\kappa)^2-2\sigma_\lambda^2 E_\nu\left[e^{(1-\gamma)Z_t}-1\right]}\right). \tag{A.11}$$

Finally we solve $a(\tau; p_{0t})$. Collecting constant terms gives the ODE

$$\begin{aligned}&-\beta(1-\gamma)a(\tau;p_{0t})+(1-\gamma)\frac{da}{d\tau}(\tau;p_{0t})\\ &+(1-\gamma)\mu_C+(1-\gamma)b_p\eta_{GB}+(1-\gamma)b_\lambda\kappa\bar{\lambda}_2-\frac{1}{2}\gamma(1-\gamma)\sigma^2+\lambda^G E_\nu\left[e^{(1-\gamma)Z_t}-1\right]=0,\end{aligned}$$

which is equivalent to

$$\frac{da}{d\tau}(\tau;p_{0t})=\beta a(\tau;p_{0t})-\mu_C+\frac{1}{2}\gamma\sigma^2-b_p\eta_{GB}-b_\lambda\kappa\bar{\lambda}_2-\frac{\lambda^G}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_t}-1\right]. \tag{A.12}$$

Equation A.12 implies a general form for $a(\tau; p_{0t})$:

$$\begin{aligned}&a(\tau;p_{0t})=\\ &\qquad\zeta_{p_{0t}}e^{\beta\tau}+\frac{1}{\beta}\left(\mu_C-\frac{1}{2}\gamma\sigma^2+b_p\eta_{GB}+b_\lambda\kappa\bar{\lambda}+\frac{\lambda^G}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_t}-1\right]\right),\end{aligned} \tag{A.13}$$

where $\zeta_{p_{0t}} \in \{\zeta_0, \zeta_1\}$ for as yet undetermined coefficients ζ_0 and ζ_1.

To obtain ζ_0 and ζ_1, we require boundary conditions for (A.12). We obtain these from the optimality condition at announcements. Along the optimal path, continuation value must satisfy

$$\begin{aligned}V_{t^-}&=E_{t^-}\left[\int_t^\infty f(C_s,V_s)ds\right]\\ &=E_{t^-}[V_t].\end{aligned} \tag{A.14}$$

Equation A.14 is trivial except for $t \in \mathcal{A}$. For $t \in \mathcal{A}$ however, (A.14) yields the required boundary conditions. First note that, by definition of $\mathcal{A}$ and of τ,

$$\lim_{\tau\to T} J(C_{t^-},p_{t^-},\lambda_{2,t^-},\tau;p_{0t^-})=E_{t^-}\left[J(C_t,p_t,\lambda_{2t},0;p_{0t})\right]. \tag{A.15}$$

That is, the value function the instant before the announcement must equal the expectation of its value just after the announcement. Furthermore, because C_t and λ_{2t} are

[19]See Tsai and Wachter (2015) for details about choosing the solution to b_λ.

continuous at t with probability 1,

$$\lim_{\tau \to T} J(C_t, p_{t^-}, \lambda_{2t}, \tau; p_{0t^-}) = E_{t^-}\left[J(C_t, p_t, \lambda_{2t}, 0; p_{0t})\right].$$

Equation A.15, together with the form of I and Lemma 1, restricts $a\big(\tau; p_{0t}\big)$. That is, because value must follow a martingale, and because consumption, and λ_{2t} will not change in the instant before and after the announcement, $a(0; p_{0t})$ must adjust to compensate the expected change in p_t. In technical terms,

$$\lim_{\tau \to T} e^{(1-\gamma)(a(\tau; p_{0t^-}) + b_p p_{t^-})} = E_{t^-}\left[e^{(1-\gamma)(a(0; p_{0t}) + b_p p_t}\right],$$

for $t \in \mathcal{A}$. Cancelling out the constant term in $a(\cdot, \cdot)$ implies

$$e^{(1-\gamma)(\zeta_{p_{0t^-}} e^{\beta T} + b_p p_{t^-})} = E_{t^-}\left[e^{(1-\gamma)\zeta_{p_{0t}} + b_p p_t}\right], \quad t \in \mathcal{A}.$$

Immediately following the announcement $p_t = p_{0t} \in \{0, 1\}$. Therefore,

$$e^{(1-\gamma)(\zeta_{p_{0t^-}} e^{\beta T} + b_p p_{t^-})} = (1 - p_{t^-}) e^{(1-\gamma)\zeta_0} + p_{t^-} e^{(1-\gamma)\zeta_1 + b_p} \tag{A.16}$$

Applying (A.16) at $p_{0t^-} = 0$ and $p_{0t^-} = 1$ implies (A.4), uniquely determining ζ_0 and ζ_1. □

A.2 The state price density

Lemma A.2. *The process π_t is characterized by*

$$\pi_t = \beta \exp\left\{\int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds\right\} C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; p_{0t})^{1-\gamma}, \tag{A.17}$$

where $I(p_t, p_{0t}, \lambda_{2t}, \tau)$ is defined by (A.2).

Proof. Duffie and Skiadas (1994) show that

$$\pi_t = \exp\left\{\int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds\right\} \frac{\partial}{\partial C} f(C_t, V_t). \tag{A.18}$$

The form of f implies

$$\begin{aligned}
\frac{\partial}{\partial C} f(C_t, V_t) &= \beta(1-\gamma)\frac{V_t}{C_t} \\
&= \beta(1-\gamma)\frac{(1-\gamma)^{-1}C_t^{1-\gamma}I(p_t, \lambda_{2t}, \tau; p_{0t})^{1-\gamma}}{C_t} \\
&= \beta C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; p_{0t})^{1-\gamma}.
\end{aligned} \tag{A.19}$$

Combining (A.18) and (A.19) implies

$$\pi_t = \beta \exp\left\{\int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds\right\} C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; p_{0t})^{1-\gamma}.$$

□

Proof of Theorem 1. For this proof, assume $t \in \mathcal{N}$. Ito's Lemma and Lemma A.2 imply

$$\frac{d\pi_t}{\pi_{t^-}} = \mu_{\pi t}\, dt + \sigma_{\pi t}\, dB_t + \frac{\pi_t - \pi_{t^-}}{\pi_{t^-}} dN_t, \tag{A.20}$$

for a scalar process $\mu_{\pi t}$ and a 1×2 vector process $\sigma_{\pi t}$.[20] It follows from (A.17) and Ito's Lemma that

$$\sigma_{\pi t} = [-\gamma\sigma, (1-\gamma) b_\lambda \sigma_\lambda \sqrt{\lambda_{2t}}], \tag{A.21}$$

and that, for $t_i = \inf\{t | N_t = i\}$

$$\frac{\pi_{t_i} - \pi_{t_i^-}}{\pi_{t_i^-}} = e^{-\gamma Z_{t_i}} - 1. \tag{A.22}$$

It follows from no-arbitrage that

$$E_{t^-}\left[\frac{d\pi_t}{\pi_{t^-}}\right] = -r_{t^-} dt.$$

It follows from the definition of an intensity that

$$E_{t^-}\left[\frac{d\pi_t}{\pi_{t^-}}\right] = \mu_{\pi t} + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu[e^{-\gamma Z_t} - 1],$$

[20] Lemma A.2 also implies the continuity of $\mu_{\pi t}$ and $\sigma_{\pi t}$ on non-announcement dates. This allows us to use t rather than t^- to subscript these variables in (A.20) and elsewhere.

implying

$$\mu_{\pi t} = -r_t - \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu[e^{-\gamma Z_t} - 1], \tag{A.23}$$

where $r_t = r_{t-}$ because $\mu_{\pi t}$, and λ_{2t} are continuous.

Finally, we show (15). Note taht

$$\begin{aligned}
\frac{\partial}{\partial V} f(C_t, V_t) &= \frac{\partial}{\partial V}\Big(\beta(1-\gamma)V_t \log C_t - \beta V_t \log[(1-\gamma)V_t]\Big) \\
&= \beta(1-\gamma)\log C_t - \beta \log[(1-\gamma)V_t] - \beta \\
&= -\beta\left\{1 + (1-\gamma)[a\big(\tau; p_{0t}\big) + b_p p + b_\lambda \lambda_{2t}]\right\}.
\end{aligned} \tag{A.24}$$

It follows from (A.17) and Ito's Lemma that

$$\begin{aligned}
\mu_{\pi t} = &\left\{-\beta\Big[1 + (1-\gamma)a\big(\tau; p_{0t}\big) + (1-\gamma)b_p p_t + (1-\gamma)b_\lambda \lambda_{2t}\Big] + (1-\gamma)\frac{\partial a}{\partial \tau}\right\} dt \\
&- \gamma \mu_C dt + (1-\gamma) b_p \left[-p_t \eta_{BG} + (1-p)\eta_{GB}\right] dt - (1-\gamma) b_\lambda \kappa(\lambda_{2t} - \bar{\lambda}_2) dt \\
&+ \frac{1}{2}\gamma(\gamma+1)\sigma^2 dt + \frac{1}{2}(1-\gamma)^2 b_\lambda^2 \sigma_\lambda^2 \lambda_{2t} dt.
\end{aligned}$$

Collecting terms and applying the equations for $a\big(\tau; p_{0t}\big)$, b_p and b_λ yields

$$\mu_{\pi t} = -\left(\beta + \mu_C - \gamma\sigma^2 + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right)\left[E_\nu e^{(1-\gamma)Z_t} - 1\right]\right) dt. \tag{A.25}$$

The result then follows from (A.23).

□

Proof of Theorem 2. Consider $t \in \mathcal{A}$, namely announcement times. With probability 1, a disaster does not coincide with an announcement. Therefore, it follows from (A.17) that

$$\frac{\pi_t}{\pi_{t-}} = \lim_{\tau \to T} \frac{I(p_t, \lambda_{2t}, 0; p_{0t})}{I(p_{t-}, \lambda_{2t}, \tau; p_{0t-})} = \lim_{\tau \to T} \frac{e^{(1-\gamma)(a(0;p_{0t}) + b_p p_t)}}{e^{(1-\gamma)(a(\tau; p_{0t-}) + b_p p_{t-})}}.$$

The first equality holds except on a set of outcomes of measure zero. We use the fact that λ_{2t} is continuous. The second inequality follows from the conjecture (A.2). The result then follows directly from the definition of $a(\cdot;\cdot)$ in (A.3). □

Proof of Lemma 2. Suppose by contradiction that

$$\zeta_0 \leq \zeta_1 + b_p. \tag{A.26}$$

Recall the following pair of equations which determine ζ_0 and ζ_1:

$$\begin{aligned} e^{(1-\gamma)\left(\zeta_0 e^{\beta T} + b_p p^G\right)} &= p^G e^{(1-\gamma)(\zeta_1 + b_p)} + (1 - p^G) e^{(1-\gamma)\zeta_0} \\ e^{(1-\gamma)\left(\zeta_1 e^{\beta T} + b_p p^B\right)} &= p^B e^{(1-\gamma)(\zeta_1 + b_p)} + (1 - p^B) e^{(1-\gamma)\zeta_0}, \end{aligned} \tag{A.27}$$

The expressions on the left hand side of (A.27) are weighted averages of $e^{(1-\gamma)(\zeta_1+b_p)}$ and $e^{(1-\gamma)\zeta_0}$ with weights between 0 and 1. Thus they must lie between these two terms. It follows that

$$\begin{aligned} \zeta_0 &\leq \zeta_0 e^{\beta T} + b_p p^G \\ \zeta_1 e^{\beta T} + b_p p^B &\leq \zeta_1 + b_p. \end{aligned} \tag{A.28}$$

However, (A.28) implies

$$\begin{aligned} \zeta_0(1 - e^{\beta T}) &\leq b_p p^G < 0 \\ \zeta_1(e^{\beta T} - 1) &\leq b_p(1 - p^B) < 0, \end{aligned}$$

because $b_p < 0$. Therefore $\zeta_0 > 0$ and $\zeta_1 < 0$, contradicting (A.26).

□

Proof of Theorem 3. It follows from (10), applied in the limit as $\tau \to T$, that $p^B > p^G$. This is intuitive: because states are persistent, if the previous announcement revealed a negative state, it is more likely that the next announcement will also reveal a negative state than if the previous announcement were positive.

Define the notation

$$\begin{aligned} \pi^G &= e^{(1-\gamma)\zeta_0} \\ \pi^B &= e^{(1-\gamma)(\zeta_1 + b_p)} \\ \bar{\pi}^G &= e^{(1-\gamma)\left(\zeta_0 e^{\beta T} + b_p p^G\right)} \\ \bar{\pi}^B &= e^{(1-\gamma)\left(\zeta_1 e^{\beta T} + b_p p^B\right)}. \end{aligned}$$

It follows from (16) that

$$\tilde{p}^G = p^G \frac{\pi^B}{\bar{\pi}^G}$$
$$\tilde{p}^B = p^B \frac{\pi^B}{\bar{\pi}^B}.$$

First consider the case of $\gamma > 1$. We want to show that

$$\frac{p_B}{p_G} > \frac{\bar{\pi}^B}{\bar{\pi}^G} = \frac{p^B + \frac{\pi^G}{\pi^B - \pi^G}}{p^G + \frac{\pi^G}{\pi^B - \pi^G}}$$

The second inequality follows from (18), or equivalently, $\pi_{t^-} = E_{t^-}\pi_t$. Moreover, Lemma 2 implies $\pi^B > \pi^G$. Because $\frac{p^B + x}{p^G + x}$ is a decreasing function of x, the result follows.

Now consider $\gamma < 1$. Because $\pi^G > \pi^B$, $\bar{\pi}^G > \bar{\pi}^B$. Thus

$$\frac{\bar{\pi}^B}{\bar{\pi}^G} < 1 < \frac{p^B}{p^G}.$$

□

B Pricing equity

Appendix B.1 derives result for equity strips (namely a claim to a dividend paid at a single point in time). Appendix B.2 uses these results to derive results for dividend streams. We suppress the j subscript when not essential for clarity.

B.1 Pricing equity strips

We first derive the no-arbitrage condition on intervals without announcements.

Lemma B.1. *Let H_t denote the time-t price of a dividend D_{t^*} with $t^* \geq t$, such that the distribution of D_{t^*}/D_t is determined by the state vector $p_t, p_{0t}, \lambda_{2t}$. Define $s = t^* - t$ and $\tau = t \bmod T$. Then*

$$H_t = H(D_t, p_t, \lambda_{2t}, \tau, s; p_{0t}) = D_t E_t \left[\frac{\pi_{t^*}}{\pi_t} \frac{D_{t^*}}{D_t} \right]. \tag{B.1}$$

Moreover, for $t \in \mathcal{N}$, H_t satisfies

$$\frac{dH_t}{H_{t^-}} = \mu_{Ht}dt + \sigma_{Ht}dB_t + (e^{\phi Z_t} - 1)dN_t, \tag{B.2}$$

with $\mu_{Ht} = \mu_H(p_t, \lambda_{2t}, \tau, s; p_{0t})$ and $\sigma_{Ht} = \sigma_H(p_t, \lambda_{2t}, \tau, s; p_{0t})$, satisfying

$$\mu_{Ht} + \mu_{\pi t} + \sigma_{Ht}\sigma_{\pi t}^\top + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu \left[e^{(\phi-\gamma)Z} - 1\right] = 0. \tag{B.3}$$

Proof. Equation B.1 follows from the absence of arbitrage and the Markov property for the dividend process D_t and the state-price density π_t. Given that H_t/D_t is a function of the state variables, (B.2) follows from Ito's Lemma and the fact that only D_t changes in a disaster.

Equation (B.1) implies that $\pi_t H_t$ is a martingale. Define

$$u_l = \inf\{t : N_t = l\}, \tag{B.4}$$

as the arrival time of the l^{th} Poisson arrival. Consider $t \in \mathcal{N}$ and chose Δt sufficiently small so that the interval $[t, t+\Delta t]$ does not contain an announcement. It follows from (B.2) that

$$H_{t+\Delta t}\pi_{t+\Delta t} = H_t\pi_t + \int_t^{t+\Delta t} \pi_u H_u(\mu_{Hu} + \mu_{\pi u} + \sigma_{Hu}\sigma_{\pi u}^\top)du + \int_t^{t+\Delta t} \pi_u H_u(\sigma_{Hu} + \sigma_{\pi u})dB_u \\ + \sum_{t<u_l\leq t+\Delta t} (\pi_{u_l} H_{u_l} - \pi_{u_{l^-}} H_{u_{l^-}}). \tag{B.5}$$

Rewriting, we have:

$$H_{t+\Delta t}\pi_{t+\Delta t} = H_t\pi_t + \underbrace{\int_t^{t+\Delta t} \pi_u H_u \left(\mu_{Hu} + \mu_{\pi u} + \sigma_{Hu}\sigma_{\pi u}^\top + \left(\bar{\lambda}_1(p_u) + \lambda_{2u}\right) E_\nu \left[e^{(\phi-\gamma)Z} - 1\right]\right) du}_{(1)}$$
$$+ \underbrace{\int_t^{t+\Delta t} \pi_u H_u(\sigma_{Hu} + \sigma_{\pi u}) dB_u}_{(2)}$$
$$+ \underbrace{\sum_{t<u_l\leq t+\Delta t} (\pi_{u_l} H_{u_l} - \pi_{u_{l}-} H_{u_{l}-}) - \int_t^{t+\Delta t} \pi_u H_u \left(\bar{\lambda}_1(p_u) + \lambda_{2u}\right) E_\nu \left[e^{(\phi-\gamma)Z} - 1\right] du}_{(3)}. \tag{B.6}$$

Since $H_t\pi_t$ is a martingale, the time-t expectation of $H_{t+\Delta t}\pi_{t+\Delta t}$ must be $H_t\pi_t$. In (B.6), (2) and (3) have zero expectation, so that the integrand in (1) must be zero. We obtain (B.3). □

Proof of Theorem 4. Define H_t as in Lemma B.1. Conjecture that H_t takes the form (25) for as-yet unspecified functions $a_\phi(\tau, s; p_0)$, $b_{\phi p}(s)$ and $b_{\phi\lambda}(s)$. No-arbitrage implies the following boundary condition for the zero-maturity claim:

$$H(D, p, \lambda_2, \tau, 0; p_0) = D.$$

Thus

$$a_\phi(\tau, 0; p_0) = b_{\phi p}(0) = b_{\phi\lambda}(0) = 0. \tag{B.7}$$

Consider $t \in \mathcal{N}$. Define μ_{Ht} and σ_{Ht} as in Lemma B.1. Applying Ito's Lemma to the conjecture (25) implies

$$\mu_{Ht} = \mu_D + \frac{\partial a_\phi}{\partial \tau} - \frac{\partial a_\phi}{\partial s} + b_{\phi p}(s)\eta_{GB} + b_{\phi\lambda}(s)\kappa\bar{\lambda}_2$$
$$+ \left(-\frac{\partial b_{\phi p}}{\partial s} - b_{\phi p}(s)(\eta_{BG} + \eta_{GB})\right) p_t + \left(-\frac{\partial b_{\phi\lambda}}{\partial s} + \frac{1}{2} b_{\phi\lambda}(s)^2\sigma_\lambda^2 + \kappa b_{\phi\lambda}(s)\right)\lambda_{2t}, \tag{B.8}$$

and

$$\sigma_{Ht} = \left[\sigma, b_{\phi\lambda}(s)\sigma_\lambda\sqrt{\lambda_{2t}}\right]. \tag{B.9}$$

Substituting (B.8), (B.9), (A.25), and (A.21) into Equation B.3 of Lemma B.1 and matching coefficients implies

$$-\frac{\partial b_{\phi p}(s)}{\partial s} - (\eta_{BG} + \eta_{GB}) b_{\phi p}(s) + (\lambda^B - \lambda^G) E_\nu \left[e^{(\phi-\gamma) Z_t} - e^{(1-\gamma) Z_t} \right] = 0 \tag{B.10}$$

$$-\frac{d b_{\phi\lambda}(s)}{ds} + \frac{1}{2}\sigma_\lambda^2 b_{\phi\lambda}(s)^2 + \left[(1-\gamma) b_\lambda \sigma_\lambda^2 - \kappa \right] b_{\phi\lambda}(s) + E_\nu \left[e^{(\phi-\gamma) Z_t} - e^{(1-\gamma) Z_t} \right] = 0, \tag{B.11}$$

and

$$\frac{\partial a_\phi}{\partial \tau} - \frac{\partial a_\phi}{\partial s} = \beta + \mu_C - \mu_D - \lambda^G E_\nu \left[e^{(\phi-\gamma) Z_t} - e^{(1-\gamma) Z_t} \right] - \kappa \bar{\lambda}_2 b_{\phi\lambda}(s) - \eta_{GB} b_{\phi p}(s). \tag{B.12}$$

Then (26) uniquely solves (B.10) together with the boundary condition (B.7). Moreover, (B.12) and (B.7) ensure that that a_ϕ takes the form (28).

Finally, we solve for the function h. Recall that $a(\tau, 0; p_0) = 0$, for all $\tau \in [0, T)$. Then, from (28), $h(\tau; p_0) = 0$ for all $\tau \in [0, T)$. However, h is only defined as a function of $\tau + s$. Therefore $h(u; p_0) = 0$, for $u \in [0, T)$.

For $u \geq T$, (29) iteratively determines $h(u; p_0)$. We now derive (29). Absence of arbitrage and the (almost sure) continuity of D_t, λ_{2t} and s imply, for $t \in \mathcal{A}$,

$$\lim_{\tau \to T} H(D_t, p_{t^-}, \lambda_{2t}, \tau, s; p_{0t^-}) = E_{t^-} \left[\frac{\pi_t}{\pi_{t^-}} H(D_t, p_t, \lambda_{2t}, 0, s; p_{0t}) \right]. \tag{B.13}$$

We use (25) to write (B.13) more explicitly as

$$\lim_{\tau \to T} \exp\{ a_\phi(\tau, s; p_{0t^-}) + b_{\phi p}(s) p_{t^-} \} = E_{t^-} \left[\frac{\pi_t}{\pi_{t^-}} \exp\{ a_\phi(0, s; p_{0t}) + b_{\phi p}(s) p_t \} \right] \tag{B.14}$$

Equation 28 and (B.14) then imply the following restriction on h:

$$\exp\{ h(T + s; p_{0t^-}) + b_{\phi p}(s) p_{t^-} \} = E_{t^-} \left[\frac{\pi_t}{\pi_{t^-}} \exp\{ h(s; p_{0t}) + b_{\phi p}(s) p_t \} \right]. \tag{B.15}$$

Defining $u = T + s$ and substituting in for the announcement SDF π_t / π_{t^-} from (16) gives us (29).

It remains to show that (29) uniquely characterizes h. The discussion above establishes $h(u, p_0) = 0$ is the unique solution for $u < T$. We show uniqueness by induction of the number of announcements prior to maturity of the asset. Define

$$n = \left\lfloor \frac{u}{T} \right\rfloor.$$

Assume by induction that h is unique for $u \in [(n-1)T, nT)$, $n \geq 1$. Then, for each $u \in [nT, (n+1)T)$, (29), applied at $p_{0t^-} = 0$ and $p_{0t^-} = 1$ is a system of two equations in two unknowns. It therefore uniquely pins down $h(u; p_0)$. □

We now prove the result on the effect of an announcement:

Proof of Corollary 2. We seek to determine the sign of $H_t - H_{t^-}$ for $t \in \mathcal{A}$.

Using (25), (28), and the almost-sure continuity of all variables around announcements, with the exception of p_t and p_{0t}, it suffices to show that

$$h(s; 0) > h(s; 1) + b_{\phi p}(s) \tag{B.16}$$

for $s > 0$. The reason is that (B.16) is equivalent to the result that H_t is lower for a negative announcement than for a positive announcement. Because H_{t^-} is a weighted average of these outcomes, it follows that $H_t < H_{t^-}$ when the announcement is positive and $H_t > H_{t^-}$ when the announcement is negative.

When $s < T$, (B.16) follows from $h(s; 1) = h(s; 0) = 0$ and $b_{\phi p}(s) < 0$ when $\phi > 1$. We now show (B.16) for general $s > T$ using induction on the number of announcements prior to maturity. Assume the condition holds for $s \in [(n-1)T, nT)$. Using (29) and the definition of the risk-neutral probabilities from Theorem 3 we have

$$\begin{aligned} e^{h(s;0)+b_{\phi p}(s-T)p^G} &= \tilde{p}^G e^{h(s-T;1)+b_{\phi p}(s-T)} + (1-\tilde{p}^G) e^{h(s-T;0)} \\ e^{h(s;1)+b_{\phi p}(s-T)p^B} &= \tilde{p}^B e^{h(s-T;1)+b_{\phi p}(s-T)} + (1-\tilde{p}^B) e^{h(s-T;0)}. \end{aligned}$$

Theorem 3 shows that $\tilde{p}^B > \tilde{p}^G$. Therefore, by the induction step

$$h(s; 0) + b_{\phi p}(s-T)p^G > h(s; 1) + b_{\phi p}(s-T)p^B.$$

Finally,

$$\begin{aligned} h(s;0) &> h(s;0) + b_{\phi p}(s-T)p^G \\ &> h(s;1) + b_{\phi p}(s-T)p^B \\ &> h(s;1) + b_{\phi p}(s-T) \\ &> h(s;1) + b_{\phi p}(s). \end{aligned}$$

The last inequality follows because $b_{\phi p}$ is a strictly decreasing function. Therefore (B.16) holds for all $s > 0$, which completes the proof. $\square$

B.2 Stock prices

The following Lemma extends Lemma B.1 to the case of an asset paying a stream of dividends. We drop the j superscript when not essential for clarity.

Lemma B.2. *Let $S_t = S(D_t, p_t, \lambda_{2t}, \tau; p_{0t})$ denote the time-t price of a future dividend stream $\{D_s\}_{s\in(t,\infty)}$ satisfying (22). Then*

$$S_t = E_t \int_t^\infty \frac{\pi_u}{\pi_t} D_u \, du \tag{B.17}$$

Moreover, for t such that $t \bmod T \neq 0$, there exist processes $\mu_{St} = \mu_S(p_t, \lambda_{2t}, \tau; p_{0t})$ and $\sigma_{St} = \sigma_S(p_t, \lambda_{2t}, \tau; p_{0t})$ such that

$$\frac{dS_t}{S_{t^-}} = \mu_{St}\, dt + \sigma_{St}\, dB_t + \frac{S_t - S_{t^-}}{S_{t^-}} dN_t \tag{B.18}$$

that satisfy the no-arbitrage restriction

$$\mu_{\pi t} + \mu_{St} + \frac{D_t}{S_t} + \sigma_{\pi t}\sigma_{St}^\top + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) E_\nu \left[e^{(\phi-\gamma)Z_t} - 1\right] = 0. \tag{B.19}$$

Proof. Applying (B.1) and interchanging the position of the integral and the expectation, we have

$$S_t = \int_0^\infty H(D_t, p_t, \lambda_{2t}, \tau, s; p_{0t})\, ds, \tag{B.20}$$

Equation B.18 then follows by Ito's Lemma and the homogeneity of H in D. Let $\mu_{H(s),t} = \mu_H(p_t, p_{0t}, \lambda_{2t}, \tau, s)$ and $\sigma_{H(s),t} = \sigma_H(p_t, p_{0t}, \lambda_{2t}, \tau, s)$, $s \in [0, \infty)$. It follows from Itô's lemma applied to both sides of (B.20) that

$$\begin{aligned} S_t \mu_{St} &= \int_0^\infty H_t(s) \mu_{H(s),t} ds - D_t \\ S_t \sigma_{St} &= \int_0^\infty H_t(s) \sigma_{H(s),t} ds \\ \pi_t S_t - \pi_{t^-} S_{t^-} &= (\pi_t D_t - \pi_{t^-} D_{t^-}) \int_0^\infty \frac{H_t(s)}{D_t} ds \end{aligned}$$

The last term expression from the continuity of $H_t(s)/D_t$ for $t \in \mathcal{N}$. Then (B.19) follows from (B.3). □

Proof of Theorem 5. For convenience, we drop the j superscript. The Theorem follows from the definition of the expected return (31), from Equation B.19 of Lemma B.2, and from the equation for the riskfree rate (15). We use Itô's Lemma to note that

$$\sigma_{St} = \left[\sigma, \frac{1}{S_t} \frac{\partial S_t}{\partial \lambda_2} \sigma_\lambda \sqrt{\lambda_{2t}} \right].$$

□

C Pricing nominal bonds

In this section we derive results for nominal zero-coupon bonds. We generalize the process in the main text. Assume the price level is given by

$$\frac{dP_t}{P_t} = q_t dt + \sigma_p dB_{Pt} + \left(e^{Z_{Pt}} - 1 \right) dN_t, \tag{C.1}$$

where q_t is the expected inflation process, and is given by

$$dq_t = \kappa_q (\bar{q}_t - q_t) dt + \sigma_q dB_{qt} + Z_{qt} dN_t. \tag{C.2}$$

so that a disaster is allowed to affect both realized and expected inflation. We recover the case in the main text by setting $Z_{Pt} = -Z_t$ and $Z_{qt} = 0$.

We first show the validity of the nominal stochastic discount factor.

Lemma C.1. *Let $H_t^\$$ be the time-t nominal price of a zero-coupon asset at time t.*

Then absence of arbitrage implies that there exists a nominal stochastic discount factor $\pi_t^\$ = \pi_t/P_t$*, such that*

$$\pi_t^\$ H_t^\$ = E_t\left[\pi_s^\$ H_s^\$\right], \forall s \geq t. \tag{C.3}$$

Proof. The time-t real price of the asset is given by $H_t^\$/P_t$. Absence of arbitrage implies that

$$\pi_t \frac{H_t^\$}{Pt} = E_t\left[\pi_s \frac{H_s^\$}{P_s}\right]. \tag{C.4}$$

Define $\pi_t^\$ = \pi_t/P_t$, then Equation C.4 is equivalent to (C.3), which implies that $\pi_t^\$$ can be used as a nominal stochastic discount factor process. □

The following corollary characterizes the dynamics of the nominal pricing kernel when there is a rare event.

Corollary C.1. *Conditioning on* $dN_t = 1$*, the dynamics of* $\pi_t^\$$ *is given by*

$$\frac{\pi_t^\$ - \pi_{t-}^\$}{\pi_{t-}^\$} = e^{-\gamma Z_t - Z_{Pt}} - 1. \tag{C.5}$$

The following lemma derives a no-arbitrage condition for nominal assets. The proof is very similar to that of Lemma B.1 and so we omit it.

Lemma C.2. *Now let* $B_t^\$$ *be the time-*t *price of unit nominal zero-coupon bond maturing at time* t^**,* $t^* > t$*. Define* $s = t^* - t$ *and* $\tau = t \pmod T$*. Then*

$$B_t^\$ = V^\$(p_t, q_t, \lambda_{2t}, \tau, s; p_{0t}) = E_t\left[\frac{\pi_{t^*}^\$}{\pi_t^\$}\right]. \tag{C.6}$$

Moreover, for $t \in \mathcal{N}$*,* $H_t^\$$ *satisfies*

$$\frac{dB_t^\$}{B_{t-}^\$} = \mu_{Bt}^\$ dt + \sigma_{Bt}^\$ dB_t^\$ + \frac{B_t^\$ - B_{t-}^\$}{B_{t-}^\$} dN_t, \tag{C.7}$$

and

$$\mu_{\pi t}^\$ + \mu_{Bt}^\$ + \sigma_{\pi t}^\$ {\sigma_{Bt}^\$}^\top + (\bar{\lambda}_1(p_t) + \lambda_{2t}) \frac{\mathcal{J}(\pi_t^\$ B_t^\$)}{\pi_{t-}^\$ B_{t-}^\$} = 0. \tag{C.8}$$

Here $\mu_{\pi t}^\$$ *and* $\sigma_{\pi t}^\$$ *are the local drift and (diffusion) volatility of* $\pi_t^\$$*, respectively.*

Proof of Theorem 8. Define $B_t^\$$ as the time-t price of asset in Lemma C.2. Conjecture that $B_t^\$$ takes the form (42) for as-yet unspecified functions $a^\$(\tau, s; p_0)$, $b_p^\$(s)$, $b_q^\$(s)$ and $b_\lambda^\$(s)$. No-arbitrage implies the following boundary condition for the zero-maturity claim:

$$\exp(a^\$(\tau, 0; p_0) + b_p^\$(0)p_t + b_q^\$(0)q_t + b_\lambda^\$(0)\lambda_{2t}) = 1.$$

Thus

$$a^\$(\tau, 0; p_0) = b_p^\$(0) = b_q^\$(0) = b_\lambda^\$(0) = 0. \tag{C.9}$$

Consider $t \in \mathcal{N}$. Define μ_{Ht} and σ_{Ht} as in Lemma C.2. Applying Ito's Lemma to the conjecture (42) implies

$$\begin{aligned}\mu_{Bt}^\$ = \frac{\partial a^\$}{\partial \tau} - \frac{\partial a^\$}{\partial s} - \frac{\partial b_p^\$}{\partial s}p_t - \frac{\partial b_q^\$}{\partial s}q_t - \frac{\partial b_\lambda^\$}{\partial s}\lambda_{2t} \\ + \frac{1}{2}b_\lambda^\$(s)^2\sigma_\lambda^2\lambda_{2t} + \frac{1}{2}b_q^\$(s)^2\sigma_q^2 \\ + b_p^\$(s)[-p_t\eta_{BG} + (1-p_t)\eta_{GB}] + b_q^\$(s)[-\kappa_q(q_t - (\bar{q}^G + p_t(\bar{q}^B - \bar{q}^G)))] + b_\lambda^\$(s)[-\kappa(\lambda_{2t} - \bar{\lambda}_2)] \\ = \frac{\partial a_\phi}{\partial \tau} - \frac{\partial a_\phi}{\partial s} + b_p^\$\eta_{GB} + b_q^\$(s)\kappa_q\bar{q}^G + b_\lambda^\$(s)\kappa\bar{\lambda}_2 \\ + \left(-\frac{\partial b_p^\$}{\partial s} - b_p^\$(\eta_{BG} + \eta_{GB})\right)p_t + \left(-\frac{\partial b_q^\$}{\partial s} - \kappa_q b_q^\$(s)\right)q_t + \left(-\frac{\partial b_\lambda^\$}{\partial s} + \frac{1}{2}b_\lambda^\$(s)^2\sigma_\lambda^2 + \kappa b_{\phi\lambda}(s)\right)\lambda_{2t},\end{aligned} \tag{C.10}$$

and

$$\sigma_{Bt}^\$ = \left[\sigma, b_\lambda^\$(s)\sigma_\lambda\sqrt{\lambda_{2t}}, 0, b_q^\$(s)\sigma_q\right]. \tag{C.11}$$

Moreover, by Itô's Lemman, $\pi_t^\$$ defined in Lemma C.1 has drift and diffusive volatility given by

$$\mu_{\pi t}^\$ = -\beta - \mu_C + \gamma\sigma^2 - q_t + \sigma_P^2 - (\lambda_1(p_t) + \lambda_{2t})\, E_\nu\left[e^{(1-\gamma)Z_t} - 1\right] \tag{C.12}$$

$$\sigma_{\pi t}^\$ = [-\gamma\sigma, (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}, -\sigma_P, 0]. \tag{C.13}$$

Finally, (42) and (C.5) imply

$$\frac{\bar{\mathcal{J}}(\pi_t^\$ B_t^\$)}{\pi_{t^-}^\$ B_{t^-}^\$} = E_\nu\left[e^{-\gamma Z_t - Z_{Pt} + b_q^\$(s)Z_{qt}} - 1\right]. \tag{C.14}$$

Combining results above and Lemma C.2, and then collecting the coefficients of the random variables yield

$$0 = \frac{\partial a^\$}{\partial \tau} - \frac{\partial a^\$}{\partial s} + b_p^\$(s)\eta_{GB} + b_\lambda^\$(s)\kappa\bar{\lambda}_2 + b_q^\$(s)\kappa_q \bar{q}^G + \frac{1}{2} b_q^\$(s)^2 \sigma_q^2 \tag{C.15}$$

$$- \beta - \mu_C + \gamma\sigma^2 + \sigma_p^2 + \lambda^G E_\nu \left[e^{-\gamma Z_t} \left(e^{b_q^\$(s) Z_{qt} - Z_{Pt}} - e^{Z_t} \right) \right] \tag{C.16}$$

$$0 = -\frac{\partial b_p^\$(s)}{\partial s} - (\eta_{BG} + \eta_{GB}) b_p^\$(s) + b_q^\$(s)\kappa_q \left(\bar{q}^B - \bar{q}^G \right) + (\lambda^B - \lambda^G) E_\nu \left[e^{-\gamma Z_t} \left(e^{b_q^\$(s) Z_{qt} - Z_{Pt}} - e^{Z_t} \right) \right] \tag{C.17}$$

$$0 = -\frac{\partial b_\lambda^\$(s)}{\partial s} - \kappa b_\lambda^\$(s) + \frac{1}{2} b_\lambda^\$(s)^2 \sigma_\lambda^2 + (1-\gamma) b_\lambda b_\lambda^\$(s) \sigma_\lambda^2 + E_\nu \left[e^{-\gamma Z_t} \left(e^{b_q^\$(s) Z_{qt} - Z_{Pt}} - e^{Z_t} \right) \right] \tag{C.18}$$

$$0 = -\frac{\partial b_q^\$(s)}{\partial s} - b_q^\$(s)\kappa_q - 1. \tag{C.19}$$

Then (43) uniquely solves (C.19) together with the boundary condition (C.9). Moreover, (C.15) and (C.9) ensure that that $a^\$$ takes the form (45).

Finally, we solve for the function $h^\$$. Recall that $a^\$(\tau, 0; p_0) = 0$, for all $\tau \in [0, T)$. Then, from (45), $h^\$(\tau; p_0) = 0$ for all $\tau \in [0, T)$. However, $h^\$$ is only defined as a function of $\tau + s$. Therefore $h(u; p_0) = 0$, for $u \in [0, T)$.

For $u \geq T$, (46) iteratively determines $h^\$(u; p_0)$. We now derive (46). Absence of arbitrage and the (almost sure) continuity of q_t, λ_{2t} and s imply, for $t \in \mathcal{A}$,

$$\lim_{\tau \to T} B^\$\big(p_{t^-}, q_t, \lambda_{2t}, \tau, s; p_{0t}\big) = E_{t^-}\left[\frac{\pi_t^\$}{\pi_{t^-}^\$} B^\$\big(p_t, q_t, \lambda_{2t}, \tau, s; p_{0t}\big) \right]. \tag{C.20}$$

We use (42) to write (C.20) more explicitly as

$$\lim_{\tau \to T} \exp\{a^\$(\tau, s; p_{0t^-}) + b^\$(s) p_{t^-}\} = E_{t^-}\left[\frac{\pi_t^\$}{\pi_{t^-}^\$} \exp\{a^\$(0, s; p_{0t}) + b^\$(s) p_t\} \right] \tag{C.21}$$

Equation 45 and (C.21) then imply the following restriction on h:

$$\exp\{h^\$(T + s; p_{0t^-}) + b^\$(s) p_{t^-}\} = E_{t^-}\left[\frac{\pi_t^\$}{\pi_{t^-}^\$} \exp\{h^\$(s; p_{0t}) + b^\$(s) p_t\} \right]. \tag{C.22}$$

Defining $u = T + s$ and substituting in for the nominal announcement SDF $\pi_t^\$/\pi_{t^-}^\$$ from (41) gives us (46).

It remains to show that (46)) uniquely characterizes $h^{\$}$. The discussion above establishes $h^{\$}(u, p_0) = 0$ is the unique solution for $u < T$. We show uniqueness by induction of the number of announcements prior to maturity of the asset. Define

$$n = \lfloor \frac{u}{T} \rfloor.$$

Assume by induction that $h^{\$}$ is unique for $u \in [(n-1)T, nT)$, $n \geq 1$. Then, for each $u \in [nT, (n+1)T)$, (46), applied at $p_{0t^-} = 0$ and $p_{0t^-} = 1$ is a system of two equations in two unknowns. It therefore uniquely pins down $h^{\$}(u; p_0)$. □

Corollary C.2. *When* $\bar{q}^B > \bar{q}^G$, $0 < \kappa_q < 1$, $0 < \eta_{BG} < 1$ *and* $0 < \eta_{GB} < 1$, $b_p^{\$}(s) \leq 0$.

Proof. We prove the corollary by contraditcion.

We know that

$$\frac{\partial b_p^{\$}(0)}{\partial s} = 0 \tag{C.23}$$

$$b_q^{\$}(s)\kappa_q(\bar{q}^B - \bar{q}^G) < 0, s > 0. \tag{C.24}$$

Then there is a sufficiently small but positive s_1, such that

$$b_p^{\$}(s_1) < 0$$

.

Suppose $\exists s_0$, such that $b_p^{\$}(s_2) > 0$. Then there must exists s^*, such that

$$b_p^{\$}(s^*) = 0 \tag{C.25}$$

$$\frac{\partial b_p^{\$}(s^*)}{\partial s} > 0. \tag{C.26}$$

However, as $b_p^{\$}(s^*) = 0$,

$$\frac{\partial b_p^{\$}(s^*)}{\partial s} = -0 + b_q^{\$}(s^*)\kappa_q(\bar{q}^B - \bar{q}^G) < 0,$$

which is a contradiction. □

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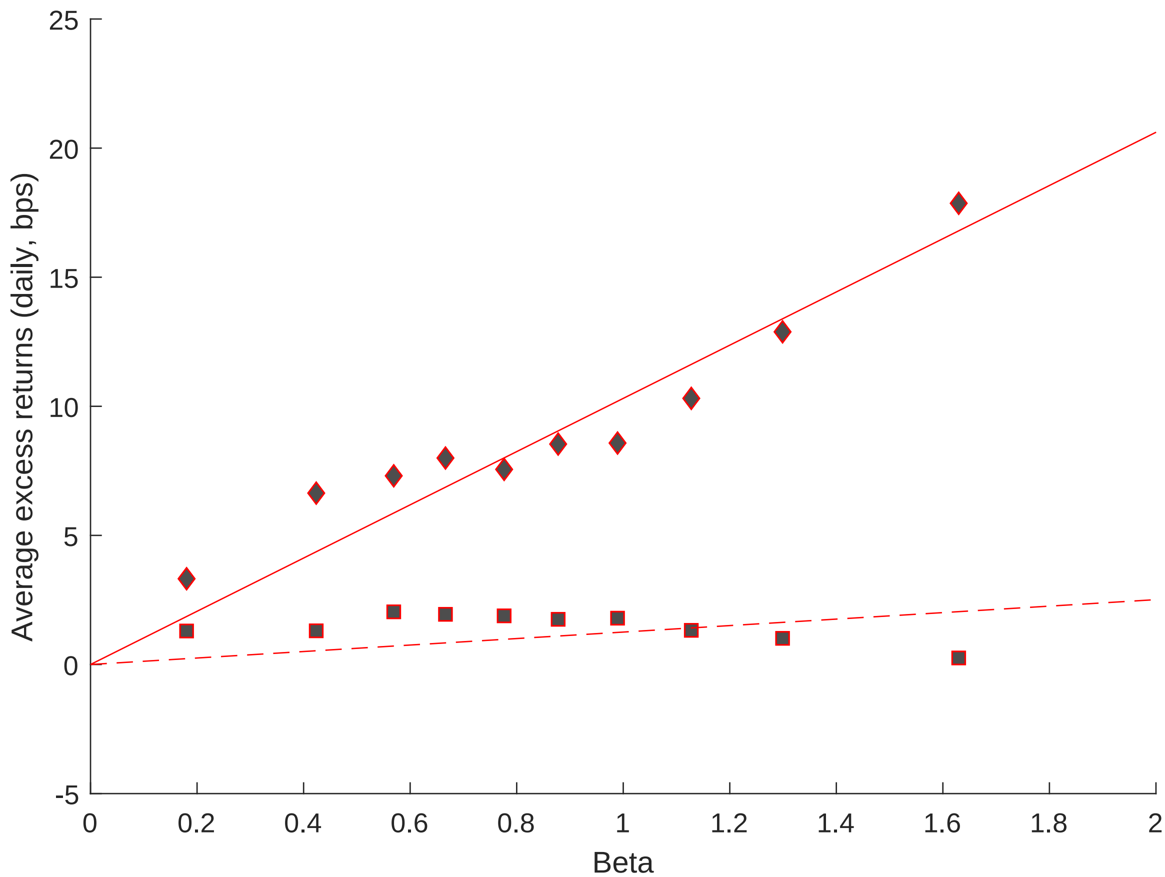
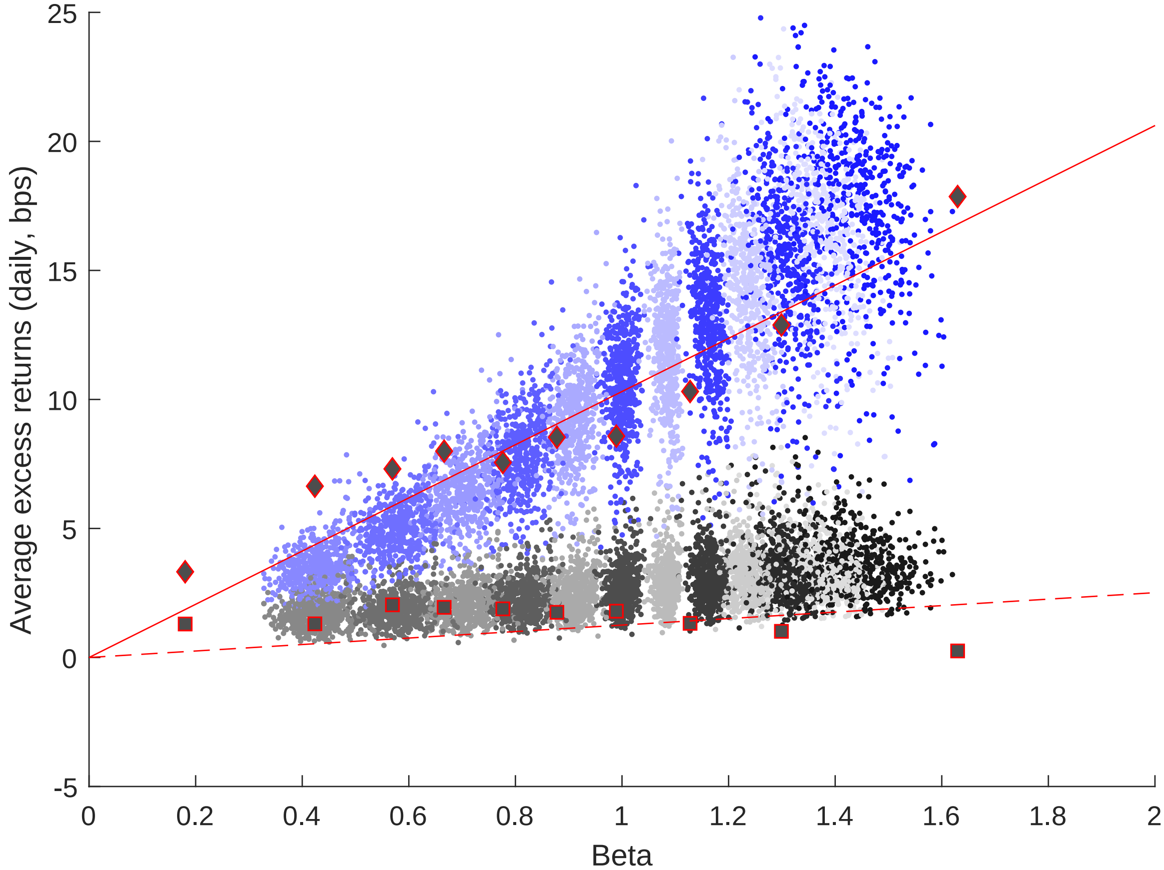
Figure 1: Portfolio excess returns against CAPM betas



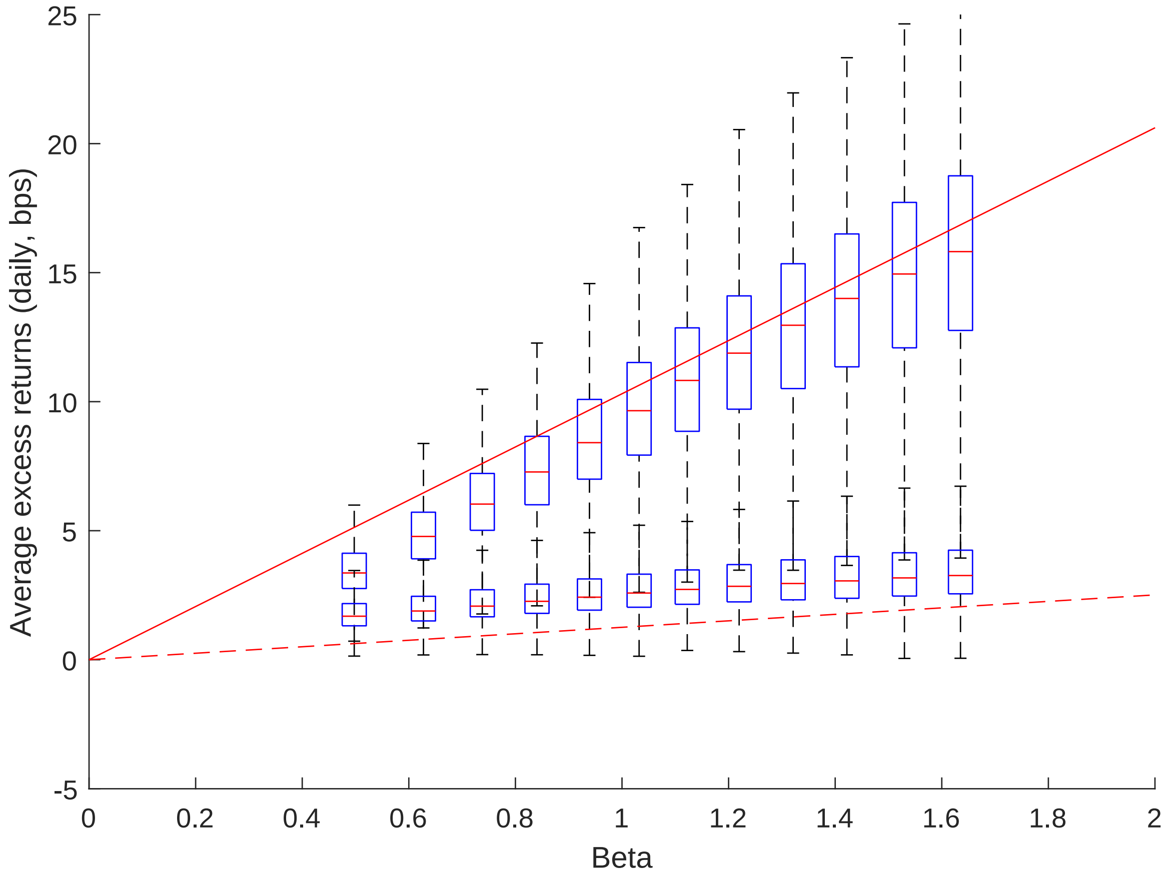
Notes: The figure shows average excess returns on announcement days (diamonds) and non-announcement days (squares) on beta-sorted portfolios in daily data from 1961.01-2016.09. On the horizontal axis is CAPM beta. Also shown are estimated regression lines for announcement day returns against beta (solid red) and non-announcement day returns against beta (dashed red).

Figure 2: Portfolio excess returns against CAPM betas on announcement and non-announcement days



Notes: The figure shows average excess returns on announcement days (diamonds) and non-announcement days (squares) on beta-sorted portfolios in daily data from 1961.01-2016.09 as a function of the CAPM beta. Also shown are estimated regression lines for announcement day returns against beta (solid red) and non-announcement day returns against beta (dashed red). We simulate 500 samples of artificial data from the model, each containing a cross-section of firms. We use samples that do not contain announcements. The blue and grey dots show average announcement day and non-announcement day returns for each sample as a function of beta, respectively.

Figure 3: Boxplots of simulated portfolio average excess returns on announcement and non-announcement days



Notes: We compute average excess returns on announcement and non-announcement days for a cross-section of assets in data simulated from the model. The red line shows the median for each portfolio across samples; the box corresponds to the interquartile range (IQR), and the whiskers correspond to the highest and lowest data value within $1.5 \times$ IQR of the highest and lowest quartile. Medians and interquartile ranges are computed using all samples (those with and without disasters). We plot returns against the median CAPM beta across samples for each portfolio. The red solid and dashed lines are the empirical regression lines of portfolio mean excess returns against market beta on announcement and non-announcement days, respectively.

Table 1: Statistics on excess returns of 10 beta-sorted portfolios

	Unconditional			Announcement day			Non-announcement day		
k	$E[RX^k]$	σ^k	β^k	$E[RX^k]$	σ^k	β^k	$E[RX^k]$	σ^k	β^k
1	1.53	53.1	0.20	3.32	52.8	0.18	1.30	53.2	0.20
2	1.91	59.2	0.44	6.64	58.8	0.42	1.30	59.2	0.44
3	2.64	69.2	0.57	7.31	70.8	0.57	2.04	69.0	0.58
4	2.63	77.4	0.69	8.00	77.1	0.67	1.94	77.4	0.69
5	2.53	87.9	0.81	7.56	87.6	0.78	1.88	87.9	0.81
6	2.52	96.2	0.90	8.54	96.7	0.88	1.75	96.1	0.91
7	2.56	105.4	1.00	8.58	107.5	0.99	1.79	105.1	1.00
8	2.34	118.9	1.14	10.31	121.8	1.13	1.32	118.5	1.14
9	2.36	136.5	1.31	12.88	139.1	1.30	1.01	136.2	1.31
10	2.25	176.2	1.67	17.86	176.9	1.63	0.25	176.0	1.67

Notes: Sample statistics for excess returns of ten beta-sorted portfolios. The sample period is 1961.01-2016.09. We show the sample mean excess returns ($E[RX^k]$), and CAPM beta (β^k). Each portfolio is labelled by k. Column 1-3 report estimates with all data available. Column 4-6 and column 7-9 use returns on announcement and non-announcement days, respectively. The unit is bps per day.

Table 2: Statistics on excess bond returns

Maturity	Unconditional		Announcement day		Non-announcement day	
k	$E[RX^k]$	β^k	$E[RX^k]$	β^k	$E[RX^k]$	β^k
1	0.363	0.000	−0.043	0.007	0.415	−0.001
5	0.855	−0.007	3.211	0.029	0.549	−0.013
10	0.779	−0.010	3.882	0.051	0.376	−0.019
20	1.122	−0.021	4.988	0.060	0.620	−0.033
30	0.986	−0.045	5.219	0.046	0.437	−0.058

Notes: Sample statistics for excess returns on Treasury bonds. The sample period is 1961.01-2016.09. We show the sample mean excess returns ($E[RX^k]$) and CAPM beta (β^k). Returns and betas are computed using the full sample (first two columns), announcement days (second two columns), and non-announcement days (last two columns). Maturity is in units of years; returns are in units of basis points per day.

Table 3: Parameter values for the simulated model

Panel A: Basic parameters	
Expected normal-times log growth in consumption $\bar{\mu}_C(\%)$	2.52
Expected normal-times growth in dividend $\bar{\mu}_D(\%)$	2.52
Volatility of consumption growth $\sigma(\%)$	2.00
Rate of time preference β	0.012
Relative risk aversion γ	3.00
Average leverage ϕ	3.00
Panel B: The process for λ_{1t}	
Probability of disaster in the good state λ^G	0
Probability of disaster in the bad state λ^B	0.062
Probability of switching to bad state η_{GB}	0.10
Probability of switching to good state η_{BG}	0.33
Panel C: The process for λ_{2t}	
Average probability of disaster $\bar{\lambda}_2$	0.021
Mean reversion in disaster probability κ	0.08
Volatility for disaster probability σ_λ	0.067
Panel D: Inflation	
Expected inflation in the good state $\bar{q}^G$	0.014
Expected inflation in the bad state $\bar{q}^G$	0.070
Mean reversion in expected inflation κ_q	0.09
Volatility for expected inflation σ_q	0.013
Volatility for realized inflation σ_P	0.008

Notes: Parameter values for the calibrated model, expressed in annual terms.

Table 4: The equity premium and volatility on announcement and non-announcement days

Statistic	Data	Simulation Median	90 % CI
$E[RX_t^{\text{mkt}} \mid \mathcal{A}]$	10.79	8.69	$[3.16, 12.73]$
$\text{std}[RX_t^{\text{mkt}} \mid \mathcal{A}]$	101.2	99.8	$[70.2, 168.4]$
$E[RX_t^{\text{mkt}} \mid \mathcal{N}]$	1.16	2.39	$[1.09, 4.47]$
$\text{std}[RX_t^{\text{mkt}} \mid \mathcal{N}]$	97.8	68.9	$[35.7, 104.8]$
$E[RX_t^{\text{mkt}} \mid \mathcal{A}] - E[RX_t^{\text{mkt}} \mid \mathcal{N}]$	9.63	6.21	$[0.45, 10.51]$
$\text{std}[RX_t^{\text{mkt}} \mid \mathcal{A}] - \text{std}[RX_t^{\text{mkt}} \mid \mathcal{N}]$	3.4	33.1	$[-22.6, 112.3]$

Notes: $E_a[RX_t^{\text{mkt}}]$ and $E_n[RX_t^{\text{mkt}}]$ denote the average excess return on the market portfolio on announcement days and non-announcement days respectively. $\text{std}_a[RX_t^{\text{mkt}}]$ and $\text{std}_n[RX_t^{\text{mkt}}]$ denote analogous statistics for the standard deviation. The first column reports the empirical estimate. The second column reports the median across samples simulated from the model. The third column reports the two-sided 90% confidence intervals from simulated samples.

Table 5: Cross-sectional regressions on announcement and non-announcement days

Panel A: Equity Portfolios			
Coefficient	Data	Simulation Median	90 % CI
δ_a	10.30	8.21	$[2.95, 12.88]$
δ_n	1.23	1.70	$[0.24, 4.36]$
$\delta_a - \delta_n$	9.07	6.40	$[0.59, 10.96]$
Panel B: Nominal Bonds			
Coefficient	Data	Simulation Median	90 % CI
δ_a	93.33	8.44	$[1.53, 32.69]$
δ_n	-0.51	-2.54	$[-643.09, 510.38]$
$\delta_a - \delta_n$	93.84	14.58	$[-494.50, 676.07]$

Notes: For each sample, the regression $E[RX_t^k \mid t \in i] = \delta_i \beta_i^k + \eta_i^k$ is estimated, where $i = \mathcal{A}, \mathcal{N}$ stands for sets of announcement and non-announcement days, respectively. These regressions are estimated for beta-sorted equity portfolios (Panel A) and for Treasury bonds (Panel B). The first column reports regression slopes in daily data from 1961.01-2016.09. The second column reports medians in simulated samples. The third column reports 90% confidence intervals computed using simulations.

Table 6: Difference in announcement and non-announcement day betas in simulated data

Panel A: Equity Portfolios						
Portfolio	1	2	3	4	5	6
Median	-0.18	-0.07	0.04	0.12	0.20	0.28
90% CI	$[-0.30, -0.04]$	$[-0.22, 0.10]$	$[-0.14, 0.27]$	$[-0.05, 0.45]$	$[-0.01, 0.61]$	$[-0.05, 0.82]$
Panel B: Bonds						
Maturity	1	3	5	7	10	
Median	0.01	0.22	0.49	0.81	0.95	
90% CI	$[0.00, 0.02]$	$[0.06, 0.29]$	$[0.14, 0.66]$	$[0.23, 1.11]$	$[0.27, 1.29]$	

Notes: In data simulated from the model, we compute betas on announcement days and non-announcement days. We do this for beta-sorted equity portfolios (Panel A) and for zero-coupon bonds (Panel B). The table reports the median difference and 90% confidence intervals for the difference.