Influenced Preferences: Consumption under Uncertainty

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Abstract

This work studies the impact of uncertainty on an agent's decision-making process in the interdependent preference context. Mainly, it analyzes the influence of society on the agent's consumption level when the agent is uncertain about the consumption level of the society. This issue is modeled in a hypothetical economy with two consumers and one good, where one of the consumers is the decision maker (DM), and the other is a peer. The primary result of this research proves that under uncertainty about the peer's consumption level, the inequity averse DM increases her consumption level.

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Men do not desire merely to be rich, but to be richer than other men.

John Stuart Mill

1 Introduction

Sometimes we do things which are entirely different from what we have planned to do. We choose an alternative which does not represent our taste. Why? There are different motives for such instability. Here I am going to focus on one of them, the influence of society on our life. In fact, the role of the social environment is significant (sometimes primary) in our life and has a great impact on our decisions. We choose things not only because we prefer what we have chosen, but also because our neighbors have chosen them. In other words, our well-being depends on our peers' well-being as well, and the dependence could be negative or positive. Moreover, when it is negative then it is conducive to envy, and when it is positive we are motivated to be altruistic.

The idea that the well-being of the DM depends not only on her material consumption but also on her peer's consumption level dates back to Veblen (1899) [10]. He argued that " ...it is extremely gratifying to possess more than others...". People frequently compare themselves with others and try to be at least as "good" as theirs peers. In the economics literature, this phenomenon is well known as the relative income hypothesis (Duesenberry, 1949 [1]) and "Keeping up with the Joneses" (Gali, 1994 [3]). However, sometimes, the DM has to make a decision without knowing the peer's choices. In other words, the DM tries to keep up with the Joneses without knowing the "social level" of the Joneses. The existence of uncertainty is novel in the interdependent preference literature and will be the cornerstone of this work.

As far as I know, in the economics literature, there is no paper which addresses the role of motivation to "keep up with the Joneses" under conditions of uncertainty. However, most of the research related to social preferences and interdependent preferences is still very relevant to this study. A few related papers are: Pollak (1976) [7], Maccheroni, Marinacci and Rustichini (2012) [5], and Ok and Kockesen (2000) [6]. The models explored in these papers examine the impact of a peer's choices on the DM's preferences when the choices are known.

In contrast to this literature, this paper discusses how the DM's preferences are influenced by a peer's choices in a context of uncertainty. That is, the DM does not know her peer's choice and the only information that she has is the set of alternatives available for the peer. The objective of this work is to show that when an inequity averse DM is uncertain about the peer's consumption level, then she will increase her consumption level. In the next sections, I will state formal conditions under which the DM will increase her material consumption. It should be noted that the additional consumption does not increase the DM's utility and actually aims to compensate for the possible disutility which is the result of uncertainty. There is good evidence in the book "Nudge" (Thaler & Sunstein, 2008 [8]) related to the extra consumption under uncertainty in this context. According to Thaler and Sunstein (2008), the exaggerated (which is the consequence of uncertainty) perception of classmates' alcohol consumption level generates alcohol abuse among college students. In other words, because of uncertainty, students are consuming more.

Let me present a few other examples which will help to elucidate this phenomenon better.

Example 1: Consider two university students who are preparing for an examination, and assume that the result of the exam depends only on the duration they study. They do not want to spend much time on the training, but it is essential for each of them to get a mark which is not lower than the classmate's score. In this situation, if one knows the duration that the other is going to spend on studying, then she will practice as much as her classmate. However, what will happen if one does not know how much time the other is going to allocate to studying? In this case, everything depends on her thoughts about the other. If she has an inflated opinion about the classmate, then she will study more than she would have done under certainty.

In short, in this example, the students (DMs) are increasing their consumption (knowledge) under uncertainty. The basic explanation for such behavior is the pessimistic and risk-averse nature of human beings. Since the majority of people are uncertainty averse, then this makes sense in the large.

Example 2: Consider a small village in which citizens are going to donate money for some project. Assume that they care about that project equally and also think that at the donation moment they do not know about the others' contributions and will only learn of them after the donation process. In this example, people who care about their rating, which depends on the donated amount, will give more if they have believe others will donate more than is actually the case. So, in such a situation, donation under uncertainty may raise the total amount of donated money.

Example 3: John, who is invited to an important ceremony, is deciding which type of car to rent to go there. He is going to be there first, and does not want to go there in a lower-status car than his colleagues have. Assume that John does not know what kinds of cars his colleagues have. On one hand he does not want to spend much money for the car, but on the other hand, it is paramount for him to be there in a car which is not of lower quality than the others' cars. So, what should John do; which type of car should he rent? As in the previous examples, it depends on his beliefs. If he is pessimistic, he will rent an expensive car; otherwise he will rent an inexpensive one.

These hypothetical examples shed light on the plausibility of the notion that lack of information can be the reason for excess consumption. Moreover, in such an environment the additional consumption does not increase the agent's utility, and it is only aimed at preventing possible disutility related to the peer's consumption. In addition, the explanation of the possible disutility is the negative interdependence of the agent's utility on the peer's consumption. So, the agent needs to increase her consumption, to balance the peer's (possible) excessive consumption.

In all these examples the DM is worse off when she consumes less than her peer, but she is worse off also when her consumption exceeds the peer's consumption level. This statement seems a bit controversial, since consuming more than the peer seems more pleasurable. However, even though the higher level of consumption could be more pleasurable, in this work I will assume that the DM is worse off when her consumption exceeds the peer's consumption. There are two sufficient grounds for this statement. First, consumption is costly itself, and if the DM's primary objective is to consume approximately the same amount as the peer, then the cost of the "excess" consumption could be higher than the benefit. Second, if the DM is altruistic, when her consumption exceeds the peer's consumption level, then she will prefer to consume less than her peer.

In the economics literature, this phenomenon is known as inequity aversion (For example: Fehr and Schmidt (1999) [2], Tricomi, Rangel, Camerer, and O'Doherty (2010) [9]). I will borrow this terminology to make use of this statement, which is one the most crucial assumptions in this work. In the next section, I will state a list of assumptions to describe some specific inequity averse preferences.

The principal objective of this work is to describe sufficient conditions to show that the DM will increase her consumption when she has uncertainty about the peer's consumption level. In order to show this, I will set up a simple economy consisting of two agents and one good. I will take one of the agents to be the DM and the other to be the peer, and the material good to be infinitely divisible.

The novelty of this work is the analysis of uncertainty in the influenced preferences context. The main result of this research shows that the agent is increasing consumption, without receiving additional utility, when she manifests inequity aversion and has uncertainty (risk or ambiguity) about the peer's consumption level.

The paper has the following structure: In the next section, I am going to discuss an economy consisting of one material good and two agents. Here I will state a few assumptions regarding the DM's utility function which depends on her and the peer's level of consumption. I will define two types of utility functions. In section 3 I will assume that the DM has an uncertainty (risk) about the peer's choice. Since the DM's utility also depends on the peer's choice, then the uncertainty (in this case risk) will affect her choice (consumption level). In particular, I will prove that under some sufficient conditions, the DM is consuming more than under certainty. Section 4 is similar to section 3, but here instead of risk, I will assume that there is ambiguity about the peer's choice. In the next section, I will discuss some policies that aim to decrease society's consumption without lowering the agents' well-being. The last section is the conclusion.

2 Influenced preferences in the one good economy context

Consider an economy consisting of two consumers and one type of material good. The real number X denotes the maximum amount of the material good available for each consumer. I.e., each consumer chooses a real number from $[0, X]$ interval. I will abuse the notation and denote by X the set of all real numbers in the $[0, X]$ interval. The ordinary elements of the set X are denoted by $x, y, s, t....$ In other words, X is a set of alternatives for both consumers.

Let one of the consumers be the DM and the other be the peer. Assume that the DM's preferences within set X depend on the peer?s choice as well. That is, she has different preference relations corresponding to each consumption level of the peer. So, if I extend the set of alternatives of the DM to be $[(x, s) \in]X \times X$, where x and s correspond to the DM's and the peer's choices respectively, then I can deal with only one preference relation over the set $X \times X$. To avoid further misunderstanding, I will call $X \times X$ a set of supporting alternatives.

Assume that the DM's preference over $X \times X$ is representable by the real-valued, cardinal, noncontinuous utility function $v : X \times X \to \mathbb{R}^+$, where the first argument corresponds to her choice and the second argument to the peer's choice. So, the DM calibrates her utility trough $v(\cdot, \cdot)$ function based on her and her peer's choices. Note that function v is a state dependent utility function per se, where the state is the peer's choice. In this specific framework, the state space coincides with the set of alternatives.

The following assumptions are intended to adjust the function v to make it fit for this research.

Assumption 1. *For all $x \in X, v(x, \cdot)$ is a continuous, linear and strictly increasing function on $[0, x]$ and is a continuous, linear and strictly decreasing function on $(x, X]$.*

This assumption restricts function $v(x, \cdot)$, depending on the peer's choice, to be a linear function within the given intervals. Also, it assumes that when the peer's consumption is less than her consumption, then $v(x, \cdot)$ is a strictly increasing linear function, and when the peer's consumption exceeds her consumption, then $v(x, \cdot)$ is a strictly decreasing linear function. In other words, this implies that the DM is not purely selfish and dislikes inequity. Note that Assumption 1 rules out the possibility of the DM preferring to consume more than the peer. I agree that there are some people who like to possess more than their peers. However, the existence of such a preference has no impact on the objective of this study.

Assumption 1*. *For all $x \in X, v(x, \cdot)$ is a continuous and increasing function on $[0, x]$ and is a continuous and decreasing function on $(x, X]$.*

This assumption is the weaker version of Assumption 1. As you can see Assumption 1* does not require linearity and does not say "strictly".

Assumption 2. *For all $x, y \in X$, if $y > x$ then*

$$v(y, y) \geq v(x, x).$$

Assumption 2 is almost the same as weak monotonicity. It assumes that the DM weakly prefer a higher level of consumption when equity is preserved. In other words, it affirms that function v is a non-decreasing function on the diagonal.

Assumption 3. *For all $x \in X$ and for all feasible*[1] $s, t > 0$,

$$v(x, x + s) - v(x, x + s + t) > v(x, x - s) - v(x, x - s - t).$$

I will give the intuition of this assumption in the context of Assumption 1. Remember that Assumption 1 states that agents dislike inequity. So, agents are worse off when their consumption deviates from the peers' consumption level. However, the agents do worry about the direction of the deviation. According to this assumption, agents suffer more from inequality when they have relative consumption disadvantage than from the inequity when they have relative consumption advantage.

[1] By feasibility I mean that the arguments of function $v(\cdot, \cdot)$ are in $[0, X]$ interval.

Assumption 4. *For all $x, y \in X$ and*
a) *for all feasible $s, t \geq 0$ if $y > x$ then*

$$v(x, x-s) - v(x, x-s-t) > v(y, y-s) - v(y, y-s-t),$$

b) *for all feasible $s, t > 0$ if $y > x$ then*

$$v(x, x+s) - v(x, x+s+t) > v(y, y+s) - v(y, y+s+t).$$

This assumption states that the more the DM increases her consumption the less she "cares" about the peer's consumption. That is, the slope of the utility function becoming less steep, along with the increasing consumption level of the DM.

Assumption 5. *For all $x \in X$ and for all feasible $s \geq 0, t > 0$*

$$v(x, x-s) > v(x, x+t).$$

According to Assumption 5 the DM strongly prefers to consume more than or the same as the peer over consuming less than the peer. Note that it does not matter how much more or less she consumes compared to the peer.

Assumption 6. *For all $x, y \in X$ if $y > x$ then*

$$v^o(y, y) \geq v^o(x, x)^2.$$

This assumption is similar to Assumption 2. The necessity of this assumption arises because function v is not a continuous function. The meaning of Assumption 6 is the following. $v(x, x+\epsilon)$ is a non-decreasing function on x when ϵ goes to zero. Instead, Assumption 6* states that $v(x, x+\epsilon)$ is an increasing function on x when ϵ goes to zero.

Assumption 6*. *For all $x, y \in X$ if $y > x$ then*

$$v^o(y, y) > v^o(x, x).$$

Thus I established eight assumptions which restrict utility function v in different ways. Relying on these assumptions I will define two different types of utility functions. The following definition is based on assumptions 1-6.

Definition 1. The real valued function $v : X \times X \to \mathbb{R}^+$ is called inequity averse (IA) cardinal utility if and only if it satisfies assumptions 1-6.

One of the key features of this function is that the DM maximizes her utility on the diagonal, that is when her consumption is equal to the peer's consumption. The next crucial point of the inequity averse utility function is the competitiveness property, Assumption 5. From another point of view, this characteristic of function v states that in the $\{(x, s) : x \geq s\}$ domain DM's utility is greater than in the rest of the domain, $\{(x, s) : x < s\}$. Figure 1 is the graphical interpretation of the inequity avers cardinal utility function.

[2] For all $x \in X$ by $v^o(x, x)$ denotes the $\lim_{\epsilon \to 0} v(x, x+\epsilon)$ for $\epsilon > 0$.

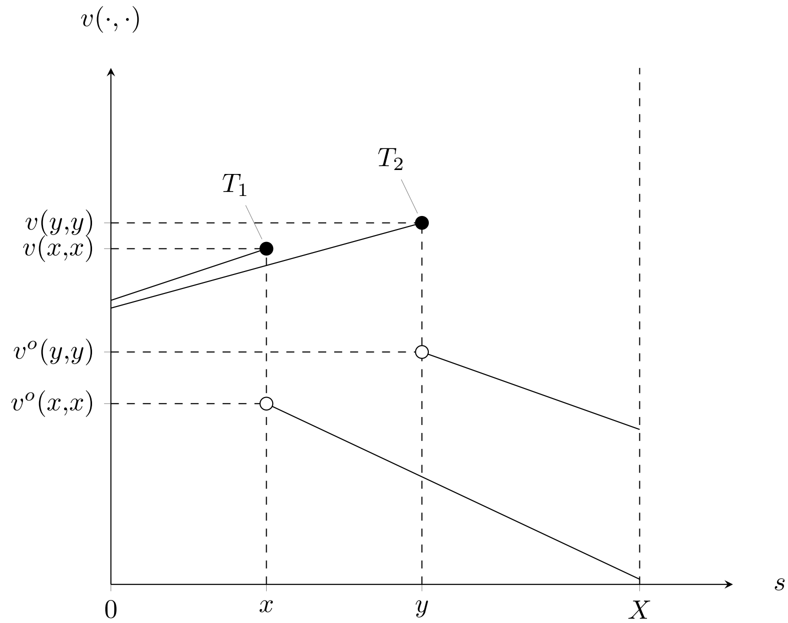

Figure 1: The projection of the DM's utility function onto the $s \times v(\cdot,\cdot)$-plane, corresponding to x and y consumption levels of the DM. T_1 and T_2 are projections of the points where the DM's consumption equal to the peer's consumption.

Definition 2. The real valued function $v : X \times X \to \mathbb{R}^+$ is called weak inequity averse (WIA) cardinal utility if and only if it satisfies assumptions 1^*, 5 and 6^*.

Definition 2 is the weaker version of Definition 1. It relaxes the assumptions of Definition 2 in several aspects. Some of them are the followings. The WIA utility function does not need to be linear. It is increasing and decreasing over the given intervals but not necessarily strictly, and so it can reach the maximum on more than one point. The other relaxation of the WIA utility function is that it does not assume increasing monotonicity on the diagonal (equity line).

3 Consumption vs Risk

Let we have an economy introduced in the previous section, and assume that the DM's preferences over $X \times X$ are given by the inequity averse cardinal utility function. Moreover, the DM does not know the peer's choice. Assume that the DM's belief about the peer's choice is given by the cumulative probability distribution, given by the following definition.

Definition 3. For each $\bar{s} \in X$, the probability distribution p, on X, is called an exaggerated belief with respect to $\bar{s}$ ($EB_{\bar{s}}$) if and only if the median of p is greater or equal than $\bar{s}$, i.e.

$$p \text{ is a } EB_{\bar{s}} \text{ iff } \int_{\bar{s}}^{X} dp(s) \geq \int_{0}^{\bar{s}} dp(s).$$

Since the DM's information about the peer's choice is presented by the probability distribution, p, over set X, then by $u_p : X \to \mathbb{R}$ I will denote the DM's utility function on the set of alternatives, X. In this section, I will assume that the function $u_p(\cdot)$ is given by (1).

Definition 4. The function $u : X \to \mathbb{R}$ called choice influenced additive expected utility (CIAEU) if and only if it is given by the following form

$$u_p(x) = \int_{s \in X} v(x, s) dp(s),^{3} \tag{1}$$

where $v(\cdot, \cdot)$ is a state (choice) dependent utility function and $p(\cdot)$ is a probability distribution on X.

The following theorem is one of the two main results of this work. It shows that if the DM's preferences on X is given by the CIAEU function, where the choice dependent function, $v(x, s)$, is IA cardinal utility, and there is risk about the peer's consumption level which is continuous and $EB_{\bar{s}}$, then she consumes strictly more than her consumption under certainty.

Theorem 1. *Let the peer's choice be $\bar{s}$. If the DM's preferences over X is given by the function* (1)*, where v is a IA cardinal utility function and p is a continuous probability distribution on X and $EB_{\bar{s}}$, then the DM's consumption, x, is strictly greater than his consumption under certainty, $\bar{s}$.*

Here I will provide only the intuition of the proof. The formal proof of Theorem 1 is discussed in the Appendix.

Under certainty, the DM's consumption level is $\bar{s}$. The theorem states that if the peer's consumption level is then the consumption level which provides maximum utility for the DM is strictly greater than $\bar{s}$. That is, for each level of consumption less than or equal to $\bar{s}$, there is a strictly greater level of consumption, which provides strictly greater utility than the other. First, I will consider the difference between the function $v(\bar{s}, \cdot)$ corresponding to the DM's consumption equal to $\bar{s}$ and the function $v(x, \cdot)$ corresponding to any consumption of the DM strictly lower than $\bar{s}$, and I will show that the minimum of this difference depending on the peer's consumption on $[\bar{s}, X]$ interval is strictly greater than the opposite difference depending on the peer's choice on $[0, \bar{s}]$ interval. Having this, the final result follows from the property of the probability distribution.

Second, I will show the same for consumption of the DM strictly greater than $\bar{s}$ and for consumption equal to $\bar{s}$. Again, the final result will follow from the property of the probability distribution.

[3]Sometimes, when it is clear from the context, I will skip to mention index p, and instead of writing $u_p(\cdot)$ I will use $u(\cdot)$.

Thus Theorem 1 describes sufficient conditions under which the DM consumes more. This result can be useful to study issues concerning welfare and consumption in the interdependent preference context. For instance, based on this theorem and the theorem provided in the next section, I will propose a reduction mechanism for the aggregate consumption of society, without worsening any member's welfare.

4 Consumption vs Ambiguity

In the previous section, I showed that under the sufficient requirements, the DM raises her consumption when there is a risk of consuming less than the peer. Alternatively, in this section, my target is to state conditions and show that under those conditions, the DM will increase her consumption when there is an ambiguity about the peer's consumption. Hence, here I am going to extend the uncertainty about the peer's consumption and show that the lack of information causes the excess consumption.

As in section 3, in this section as well, I will use the framework proposed in section 2, that is, an economy consisting of one good and two consumers. To show the negative effect of ambiguity on the DM's consumption level, I need to specify the utility function depending on the peer's choice, the DM's expected utility function over the set X and the DM's knowledge about the peer's choice, that is the set of priors. I will assume that the DM's utility function over X is given by the following definition.

Definition 5. The function $u : X \to \mathbb{R}$ called ambiguity averse choice influenced additive expected utility (AACIAEU) if and only if it is given with the following form

$$u_\Pi(x) = \min_{p \in \Pi} \int_{s \in X} v(x, s) dp(s) \tag{2}$$

where $v(\cdot, \cdot)$ is a state (choice) dependent utility function and Π is a closed, convex and nonempty set of probability distributions on X.

The utility function, given by (2), has the same structure as the Maxmin expected utility function (Gilboa and Schmeidler, 1989 [4]). The only difference is that every state corresponds to the peer's choice, and the set of the states is X itself.

In Theorem 2 I will assume that the DM has a set of priors of the peer's consumption level, which at least one probability distribution must satisfy (3). That is, there is a "scenario" which ensures that the peer's consumption level is strictly greater than her real consumption level (in this case $\bar{s}$).

Definition 6. For each $\bar{s} \in X$, the probability distributions p, on X, is called strong exaggerated belief with respect to $\bar{s}$ ($SEB_{\bar{s}}$) if and only if

$$\int_{\bar{s}}^{X} dp(s) = 1. \tag{3}$$

The objective of the following theorem is the same as Theorem 1. That is, the goal is to show that in the presence of some sufficient (some of them are also necessary) conditions, the DM's consumption under ambiguity exceeds her consumption under certainty, in the interdependent preference context.

Theorem 2. *Let the peer's choice be $\bar{s}$. If the DM's preferences over X is representable by the function* (2)*, where v is a WIA utility function and Π is a closed, convex and nonempty set of continuous probability distributions on X with at least one $SEB_{\bar{s}}$, then the DM's consumption is strictly greater than his consumption under the certainty, $\bar{s}$.*

Furthermore, the existence of at least one $SEB_{\bar{s}}$ probability distribution in the set of priors, is a necessary condition.

The proof of the theorem is given in the appendix.

When the DM knows the peer's consumption level, then the optimal consumption is equal to the peer's consumption level. But when she does not know the peer's consumption level then her optimal consumption depends on the peer's expected consumption. Theorem 2 proves that if the DM's utility on the set of alternatives is an AACIAEU, choice dependent utility on the set of supporting alternatives is a WIA and the set of priors contains at least one SEB with respect to the peer's real consumption, then her consumption increases.

5 Nudging the society to consume less

As we saw in the last two sections, when the agent is inequity averse and has exaggerated beliefs about the peer's consumption level, then she increases her consumption level. However, what is remarkable is that the aim of the agent's increased consumption is not to get more utility, but to cover disutilities generated by the peer's possible high level of consumption. So, the agent, whose utility depends on the peer's consumption level in a defined manner (given by the definitions 1 and 2), will be better off if she has complete information about the peer's consumption. In this section, I will recommend a method which will allow the social planner to reduce the aggregate consumption level, without detriment to each agent's well-being.

Consider a society consisting of N agents, where everyone should decide her consumption level regarding some particular product. Each agent's utility depends on her consumption and the mean consumption of the society. Assume that each agent has uncertainty about the social consumption level, and so does not know the average consumption of the society. If the agents satisfy the conditions defined in Theorem 1 or Theorem 2, then they will consume more than the mean, and so the average consumption will increase. Hence, under Theorem 1 or Theorem 2, the aggregate consumption will increase when there is uncertainty about the general consumption level in the society. In this case, the social planner can nudge the agents to consume less (or more) by providing information about the social average consumption. Note that this policy will be useful to regulate the aggregate consumption level without affecting the agents' well-being.

6 Conclusion

In this paper, I discussed issues concerning the decision-making process when the agent's utility depends on her peer's choice. I studied it under the assumption of uncertainty; that is, the agent does not know the choice of her peer and only has some given type of beliefs about it.

What I have shown is that when an agent does not know her peer's consumption level, then she increases her consumption level to compensate for the disutility from the possible inequity. The inequity is possible since there is uncertainty about the peer's choice. Hence, by providing information to the agents, it is possible to influence agents to consume less, without harming the agents' well-being.

Appendix

Two Lemmata

Before starting the proof of Theorem 1, I will state and prove two corner lemmata.

Lemma 3. *Let $v : X \times X \to \mathbb{R}^+$ be a IA cardinal utility function and x and y be arbitrary elements in $[0, X)$, if $y > x$ then*

$$\min_{s\in[y,X]}(v(y, s) - v(x, s)) > \max_{s\in[0,y]}(v(x, s) - v(y, s)). \tag{4}$$

Proof. Let $d^x_y(s)$ be the difference between the functions $v(x, \cdot)$ and $v(y, \cdot)$ at the point $s \in X$, i.e.

$$d^x_y(s) = v(x, s) - v(y, s). \tag{5}$$

So given this notation, I need to prove that for all $x, y \in [0, X)$ if $y > x$ then

$$\min_{s\in[y,X]} d^y_x(s) > \max_{s\in[0,y]} d^x_y(s). \tag{6}$$

I will show this in the following steps.

a) $d^x_y(\cdot)$ is an increasing function on $[0, x]$.

From assumption 1 $v(y, \cdot)$ is a linear function on $[0, y]$ interval, so assumption 4.a could be written the following way:

$$v(x, x-s) - v(x, x-s-t) > v(y, x-s) - v(y, x-s-t), \tag{7}$$

or equivalently

$$v(x, x-s) - v(y, x-s) > v(x, x-s-t) - v(y, x-s-t). \tag{8}$$

From (5) and (8) we have

$$d_y^x(x-s) > d_y^x(x-s-t) \tag{9}$$

From (9) it is straightforward that $d_y^x(\cdot)$ is an increasing function on $[0, x]$, because $s, t \geq 0$.

b) $d_y^x(s) < 0$ when $s \in (x, y]$.

From assumption 5 it follows that for all $y \in [0, X]$, $r \in [0, y]$ and $t \in [0, X-y]$,

$$v(y, y-r) > v(y, y+t),$$

so for $r = y$ it will be

$$v(y, 0) \geq \lim_{t \to 0} v(y, y+t) = v^o(y, y), \tag{10}$$

Since $y > 0$, then for all $r \in [0, y]$ we have that $y - r > 0$. So

$$v(y, y-r) \geq v(y, 0). \tag{11}$$

because $v(y, \cdot)$ is an increasing function on $r \in [0, y]$ (assumption 1).

Let $s = y - r$, then from (11)

$$v(y, s) \geq v(y, 0), \tag{12}$$

for all $s \in [0, y]$, because $s = y - r$ and (11) holds for all $r \in [0, y]$.

Hence from (10) and (12), for all $y \in [0, X]$ and $s \in [0, y]$

$$v(y, s) \geq v^o(y, y). \tag{13}$$

On the other hand from the assumption 6 and the fact that $v(x, \cdot)$ is strictly decreasing on $(x, X]$ we have

$$v^o(y, y) > v(x, s) \tag{14}$$

for all $s \in (x, X]$.

Hence from (13) and (14) it follows that

$$v(x, s) - v(y, s) < 0. \tag{15}$$

for all $s \in (x, y]$.

So (b) is straightforward.

c)$d_x^y(\cdot)$ is an increasing function on $(y, X]$.

The proof is the same as step (a) just instead of assumption 4.a use assumption 4.b.

d) $v^o(y,y) - v(x,y) > d^x_y(x)$.

From the linearity of the functions $v(x,\cdot)$ and $v(y,\cdot)$ on the intervals $(x,y]$ and $(y,X]$ follows that the slop of each function is the same on each interval. So from the assumptions 3 and 4.b it is straightforward that

$$v^o(x,x) - v(x,y) > v(y,y) - v(y,x) \tag{16}$$

From equation (16) and assumptions 2 and 6 it follows

$$v^o(y,y) - v(x,y) > v(x,x) - v(y,x) \tag{17}$$

Which is the same as (d).

e) $d^y_x(y) > v^o(y,y) - v(x,y) > 0$.

From (13) it follows

$$v(y,y) > v^o(y,y). \tag{18}$$

Hence it is obvious that

$$v(y,y) - v(x,y) > v^o(y,y) - v(x,y). \tag{19}$$

From (14) it follows

$$v^o(y,y) - v(x,y) > 0 \tag{20}$$

Now let me summarize all these steps. If there is $s \in [0,y]$ such that $d^x_y(s) > 0$ then from steps (a) and (b), we have that

$$\max_{s\in[0,y]} d^x_y(s) = d^x_y(x). \tag{21}$$

From steps (c) and (e), we have also that

$$\min_{s\in[y,X]} d^y_x(s) = v^o(y,y) - v(x,y). \tag{22}$$

Hence from (21), (22) and step (d) the result is straightforward.

If $d^x_y(s) \leq 0$ for all $s \in [0,y]$ then the result follows from (22) and steps (d) and (e).

□

Lemma 4. *Let $v : X \times X \to \mathbb{R}^+$ be a IA cardinal utility function, then for all $x \in (0,X)$ there is $y \in (x,X]$ such that*

$$v(y,x) - v^o(x,x) > v(x,x) - v(y,x). \tag{23}$$

Proof. I will prove this lemma by contradiction. Suppose there is $\bar{x} \in (0, X)$ such that for all $y \in (\bar{x}, X]$

$$v(y, \bar{x}) - v^o(\bar{x}, \bar{x}) \leq v(\bar{x}, \bar{x}) - v(y, \bar{x}). \tag{24}$$

So I need to show that there is some $\bar{y} \in (\bar{x}, X]$ such that (24) doesn't hold. Let $\bar{y} \in (\bar{x}, \frac{3}{2}\bar{x}]$[4], if $\frac{3}{2}\bar{x} > X$ then I will take $\bar{y} \in (\bar{x}, X]$. Figure 2 corresponds to the case when $\bar{y} = \frac{3}{2}\bar{x}$.

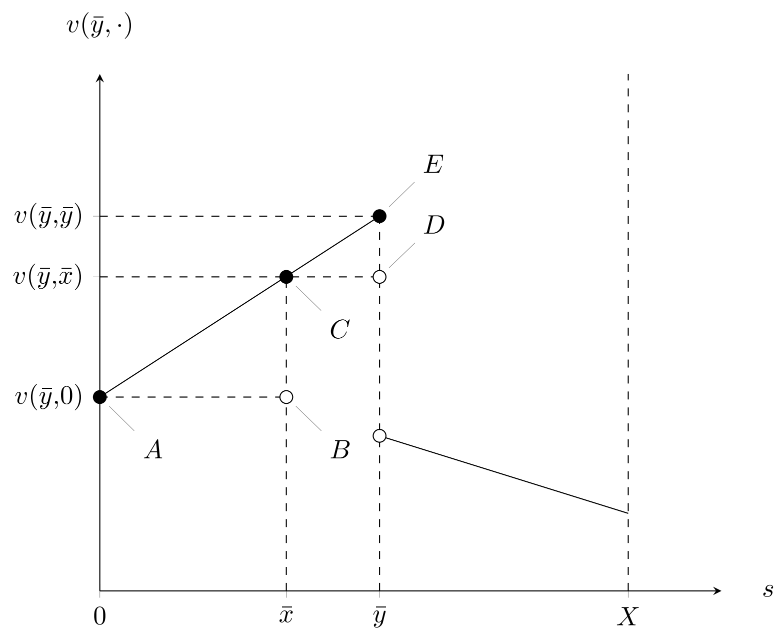

Figure 2: The DM's utility function depending only on the peer's choice, in the case in which the DM's consumption is fixed to the $\bar{y}$ level. The points $A; B; C; D; E$ respectively correspond to $(\bar{y}, 0, v(\bar{y}, 0)); (\bar{y}, \bar{x}, v(\bar{y}, 0)); (\bar{y}, \bar{x}, v(\bar{y}, \bar{x})); (\bar{y}, \bar{y}, v(\bar{y}, \bar{x})); (\bar{y}, \bar{y}, v(\bar{y}, \bar{y}))$

From selection of $\bar{y}$ it is obvious that the segment (A, B) is longer than (C, D). Since function $v(\bar{y}, \cdot)$ is a linear and strictly increasing on $[0, \bar{y}]$ interval, we have that $\triangle ABC$ and $\triangle CDE$ are similar triangles. Hence interval (B, C) is longer than (D, E) interval, which is equivalent to the following:

$$v(\bar{y}, \bar{x}) - v(\bar{y}, 0) > v(\bar{y}, \bar{y}) - v(\bar{y}, \bar{x}). \tag{25}$$

From assumptions 5 and 6 it follows

$$v(\bar{y}, 0) > v^o(\bar{x}, \bar{x}), \tag{26}$$

[4]The interval is not empty because $\bar{x} > 0$.

or equivalently

$$-v^o(\bar{x},\bar{x}) > -v(\bar{y},0). \tag{27}$$

From (26), (27) and assumption 2 it is straightforward, that

$$v(\bar{y},\bar{x}) - v^o(\bar{x},\bar{x}) > v(\bar{x},\bar{x}) - v(\bar{y},\bar{x}). \tag{28}$$

A contradiction.

□

Proof of Theorem 1

Proof. First, note that if $\bar{s} = X$, then there is no continuous probability distribution, p, which is $EB_{\bar{s}}$. So I will prove the theorem for $\bar{s} \in [0, X)$.

If the DM knows that the peer's choice is $\bar{s}$ then her optimal consumption is $\bar{s}$ as well, because of the shape of function $v(\cdot,\cdot)$.

So I need to show that for each given $\bar{s} \in [0, X)$,

$$\operatorname*{argmax}_{x\in X} \int_0^X v(x,s)dp(s) > \bar{s}.^5 \tag{29}$$

I will distinguish two cases: $\bar{s} = 0$ and $\bar{s} > 0$.

Let $\bar{s} = 0$. From assumptions 4.b, 5 and 6 it follows that for all $y \in (0, X]$ and for all $s \in (0, X]$

$$v(y,s) > v(0,s). \tag{30}$$

So from (30) and definition 3 we have (29), because at the only point, where $v(0,s) > v(y,s)$ could be possible, the probability of that point is zero, $p(0) = 0$, from continuity of the probability distribution.

Now suppose $\bar{s} \in (0, X)$. It is easy to see that equation (29) can equivalently be rewritten as follows:

For all $x \leq \bar{s}$ there is $y > \bar{s}$ such that

$$\int_0^X v(y,s)dp(s) > \int_0^X v(x,s)dp(s) \tag{31}$$

First I will show that if $y = \bar{s}$ then for all $x < \bar{s}$ (31) holds. And then I will show that for all $x = \bar{s}$ there is $y > x$ such that (31) holds as well.

Let x be an arbitrary element from $(0, \bar{s})$ and $y = \bar{s}$. From lemma 3 we have that for all $x, y \in X$ if $y > x$ then

$$\min_{s\in[y,X]}(v(y,s) - v(x,s)) > \max_{s\in[0,y]}(v(x,s) - v(y,s)). \tag{32}$$

Because of p is $EB_{\bar{s}}$ and $y = \bar{s}$ we have that for all $x < \bar{s}$

[5] The maximum of $\int_0^X v(x,s)dp(s)$ integral exists, because $v(x,\cdot)$ is a bounded function on the compact set [0,X].

$$\int_{\bar{s}}^{X} \min_{s\in[\bar{s},X]}(v(\bar{s},s)-v(x,s))dp(s) > \int_{0}^{\bar{s}} \max_{s\in[0,\bar{s}]}(v(x,s)-v(\bar{s},s))dp(s). \quad (33)$$

Easy to see that for all $x < \bar{s}$ the following two inequalities hold:

$$\int_{\bar{s}}^{X} (v(\bar{s},s)-v(x,s))dp(s) > \int_{\bar{s}}^{X} \min_{s\in[\bar{s},X]}(v(\bar{s},s)-v(x,s))dp(s), \quad (34)$$

and

$$\int_{0}^{\bar{s}} \max_{s\in[0,\bar{s}]}(v(x,s)-v(\bar{s},s))dp(s) > \int_{0}^{\bar{s}} (v(x,s)-v(\bar{s},s))dp(s). \quad (35)$$

Hence from (33), (34) and (35) it follows

$$\int_{\bar{s}}^{X} (v(y,s)-v(x,s))dp(s) > \int_{0}^{\bar{s}} (v(x,s)-v(y,s))dp(s). \quad (36)$$

Which is the same as

$$\int_{0}^{X} (v(y,s)-v(x,s))dp(s) > 0, \quad (37)$$

or

$$\int_{0}^{X} v(y,s)dp(s) > \int_{0}^{X} v(x,s)dp(s). \quad (38)$$

Now let me prove that for $x=\bar{s}$ there is $y^* > \bar{s}$ such that (31) holds. From lemma 4 we have that for all $x \in (0,X)$ (i.e., also for $x=\bar{s}$, because $\bar{s} \in (0,X)$) there is $y' \in (x,X)$ (in this case $y' \in (\bar{s},X)$) such that

$$v(y',\bar{s}) - v^{o}(\bar{s},\bar{s}) > v(\bar{s},\bar{s}) - v(y',\bar{s}). \quad (39)$$

Let $y^* = y'$. From assumption 1 (function $v(y',\cdot)$ is strictly increasing on $[\bar{s},y']$ and function $v(\bar{s},\cdot)$ is strictly decreasing on $[\bar{s},y']$) and (39) we have that for all $s \in [\bar{s},y']$

$$v(y',s) - v(\bar{s},s) > v(y',\bar{s}) - v^{o}(\bar{s},\bar{s}). \quad (40)$$

And from lemma 3 step (a) and (40) we have

$$\min_{s\in(\bar{s},y')}(v(\bar{s},s)-v(y',s)) > \max_{s\in[0,\bar{s}]}(v(y',s)-v(\bar{s},s)). \quad (41)$$

From lemma 3 we have that (4) holds for all $y > \bar{s}$ so it will be true also for given $y' > \bar{s}$. Hence from this and (41) we can write

$$\min_{s\in(\bar{s},X)}(v(\bar{s},s)-v(y',s)) > \max_{s\in[0,\bar{s}]}(v(y',s)-v(\bar{s},s)). \quad (42)$$

Since the probability distribution is continuous (i.e., $prob(\bar{s}) = 0$) then it doesn't matter (42) holds also for $s = \bar{s}$ or not. So the analog way as above we will get

$$\int_0^X v(y^*, s)dp(s) > \int_0^X v(\bar{s}, s)dp(s). \tag{43}$$

for $y^* = y'$.

Hence from (38) and (44) we have that for all $x \leq \bar{s}$ there is $y^* > \bar{s}$ such that

$$\int_0^X v(y^*, s)dp(s) > \int_0^X v(x, s)dp(s). \tag{44}$$

□

Proof of Theorem 2

Proof. If $\bar{s} = X$, then there is no continuous probability distribution, p, which is a $SEB_{\bar{s}}$[6]. So I will prove the theorem for $\bar{s} \in [0, X)$.

Note that if there is no uncertainty, i.e., the DM knows that the peer's consumption is $\bar{s}$, then her consumption will be

$$\operatorname*{argmax}_{x \in X} v(x, \bar{s}) = \bar{s}. \tag{45}$$

So the theorem will be proven if I show that for each given $\bar{s} \in [0, X)$,

$$\operatorname*{argmax}_{x \in X} \min_{p \in \Pi_{\bar{s}}} \int_0^X v(x, s)dp(s) > \bar{s}. \tag{46}$$

From assumption 1* function $v(\bar{s}, \cdot)$ is decreasing on $(\bar{s}, X]$, so for all $s > \bar{s}$

$$v^o(\bar{s}, \bar{s}) \geq v(\bar{s}, s). \tag{47}$$

So for all continuous probability distributions in $\Pi_{\bar{s}}$

$$v^o(\bar{s}, \bar{s}) \geq \int_{\bar{s}}^X v(\bar{s}, s)dp(s). \tag{48}$$

But $\Pi_{\bar{s}}$ includes at least one continuous $SEB_{\bar{s}}$, $\bar{p}$, such that

$$\int_0^{\bar{s}} v(\bar{s}, s)d\bar{p}(s) = 0. \tag{49}$$

Hence from (48) and (49) we have

$$v^o(\bar{s}, \bar{s}) \geq \int_0^{\bar{s}} v(\bar{s}, s)d\bar{p}(s) + \int_{\bar{s}}^X v(\bar{s}, s)d\bar{p}(s) = \int_0^X v(\bar{s}, s)d\bar{p}(s). \tag{50}$$

[6] All probability distributions in Π must be continuous and at least one must be $SEB_{\bar{s}}$.

Since $\bar{p} \in \Pi_{\bar{s}}$ then

$$v^o(\bar{s},\bar{s}) \geq \int_0^X v(\bar{s},s)d\bar{p}(s) \geq \min_{p\in\Pi_{\bar{s}}} \int_0^X v(\bar{s},s)dp(s). \tag{51}$$

Now I will show that for all $x < \bar{s}$

$$v^o(\bar{s},\bar{s}) > \min_{p\in\Pi_{\bar{s}}} \int_0^X v(x,s)dp(s). \tag{52}$$

From assumption 1^* for all $x \in X$ the function $v(x,\cdot)$ is decreasing on $(x, X]$, so for all $x < \bar{s}$ and for all $s > \bar{s}$

$$v^o(x,x) \geq v(x,s). \tag{53}$$

Applying assumption 6^*, from equation (53) we have that for all $x < \bar{s}$ and for all $s > \bar{s}$

$$v^o(\bar{s},\bar{s}) > v(x,s). \tag{54}$$

Having (54) the proof of the equation (52) is the analog of the non strict case (equation (51)), we just need to replace (47) with (54) and keep the strict inequality.

Easy to see that the proof of the theorem will be complete if I show that there is $y > \bar{s}$ such that

$$\min_{p\in\Pi_{\bar{s}}} \int_0^X v(y,s)dp(s) > v^o(\bar{s},\bar{s}), \tag{55}$$

Let y' be an arbitrary element from $(\bar{s}, X)$ interval and denote by $\delta = v^o(y',y') - v^o(\bar{s},\bar{s})$. From assumption 6^* it is obvious that $\delta > 0$.

From assumptions 5 and 6^* it is easy to see that for all $y \geq y'$ and for all $s \leq y$

$$v(y,s) > v^o(y',y') = v^o(\bar{s},\bar{s}) + \delta. \tag{56}$$

Let us denote by $k_p(\epsilon)$ the following

$$k_p(\epsilon) = \int_0^{X-\epsilon} dp(s). \tag{57}$$

Since each $p \in \Pi_{\bar{s}}$ is a continuous function then

$$\lim_{\epsilon\to 0} k_p(\epsilon) = 1. \tag{58}$$

From (56) we have, that for all $y \in (y', X)$ if $s \in [0, y]$

$$v(y,s) - v^o(\bar{s},\bar{s}) > \delta. \tag{59}$$

And obvious that for all $y \in (y', X)$ and for all $s \in [0, X]$

$$v^o(\bar{s},\bar{s}) - v(y,s) < v^o(\bar{s},\bar{s}), \tag{60}$$

because $v(y,s) \geq 0$ for all $y \in (y', X)$ and for all $s \in [0, X]$.

Let ϵ be chosen such that

$$k_{p_y}(\epsilon) > \frac{v^o(\bar{s},\bar{s})}{v^o(\bar{s},\bar{s}) + \delta} \tag{61}$$

for $p_y = \text{argmin}_{p \in \Pi_{\bar{s}}} \int_0^X v(y,s)dp(s)$.

Such ϵ exists because the right-hand side of (61) is less then one (because $\delta > 0$) and from (58) we have that $k_{p_y}(\epsilon) \to 0$ when $\epsilon \to 0$.

Let $y = X - \epsilon$. The equation (61) is equivalent to the following

$$\delta k_{p_y}(\epsilon) > (1 - k_{p_y}(\epsilon)) v^o(\bar{s},\bar{s}), \tag{62}$$

or

$$\int_0^y \delta dp_y(s) > \int_y^X v^o(\bar{s},\bar{s}) dp_y(s). \tag{63}$$

because δ and $v^o(\bar{s},\bar{s})$ are constants.

From (59) we have that

$$\int_0^y (v(y,s) - v^o(\bar{s},\bar{s})) dp_y(s) > \int_0^y \delta dp_y(s). \tag{64}$$

And from (60) it follows that

$$\int_y^X v^o(\bar{s},\bar{s}) p_y(s) > \int_y^X (v^o(\bar{s},\bar{s}) - v(y,s)) dp_y(s). \tag{65}$$

Hence from (63), (64) and (65) we have

$$\int_0^y (v(y,s) - v^o(\bar{s},\bar{s}) dp_y(s) > \int_y^X (v^o(\bar{s},\bar{s}) - v(y,s)) dp_y(s). \tag{66}$$

Equivalently

$$\int_0^y v(y,s) dp_y(s) - \int_0^y v^o(\bar{s},\bar{s}) dp_y(s) > \int_y^X v^o(\bar{s},\bar{s}) dp_y(s) - \int_y^X v(y,s)) dp_y(s). \tag{67}$$

Since $v^o(\bar{s},\bar{s}) = const$, then

$$\int_0^y v(y,s) dp_y(s) + \int_y^X v(y,s) dp_y(s) > v^o(\bar{s},\bar{s}), \tag{68}$$

or

$$\int_0^X v(y,s)dp_y(s) > v^o(\bar{s},\bar{s}), \tag{69}$$

Because of $p_y = \text{argmin}_{p\in\Pi_{\bar{s}}} \int_0^X v(y,s)dp(s)$, we have that

$$\min_{p\in\Pi_{\bar{s}}} \int_0^X v(y,s)dp(s) = \int_0^X v(y,s)dp_y(s) > v^o(\bar{s},\bar{s}), \tag{70}$$

Which is the same as (55).

The necessity of at least one $SEB_{\bar{s}}$, I will prove by contradiction. Suppose it is not necessary, that is, for any given $\bar{s}$ there is no $SEB\bar{s}$ in the set of priors. I.e., for all $p \in \Pi_{\bar{s}}$

$$\int_{\bar{s}}^X dp(s) < 1. \tag{71}$$

I will prove that in this case generally speaking theorem 2 doesn't hold. For that I will show that if (71) holds then there is a function $\tilde{v}(\cdot,\cdot)$ which satisfies assumptions 1*, 5 and 6*, but theorem 2 doesn't hold. I.e., for the given $\bar{s}$ there is $\tilde{v}(\cdot,\cdot)$ such that

$$\underset{x\in X}{\text{argmax}} \min_{p\in\Pi_{\bar{s}}} \int_0^X \tilde{v}(x,s)dp(s) \leq \bar{s}. \tag{72}$$

In other words I need to find $\tilde{v}(\cdot,\cdot)$ such that for all $y \in (\bar{s}, X]$

$$\min_{p\in\Pi_{\bar{s}}} \int_0^X \tilde{v}(\bar{s},s)dp(s) > \min_{p\in\Pi_{\bar{s}}} \int_0^X \tilde{v}(y,s)dp(s). \tag{73}$$

So it is left to find such a $\tilde{v}(\cdot,\cdot)$ function.

Without loss of generality I can choose $\tilde{v}(\cdot,\cdot)$ such as $\tilde{v}(\bar{s},s) = const$ for all $s \in (\bar{s}, X]$[7]. So, function $\tilde{v}(\bar{s},s) = \tilde{v}^o(\bar{s},\bar{s})$ if $s \in (\bar{s}, X]$ and from assumption 5 we have that $\tilde{v}(\bar{s},s) > \tilde{v}^o(\bar{s},\bar{s})$ if $s \in [0,\bar{s}]$. Hence from (71) I can write that

$$\min_{p\in\Pi_{\bar{s}}} \int_0^X \tilde{v}(\bar{s},s)dp(s) > \tilde{v}^o(\bar{s},\bar{s}). \tag{74}$$

Let us denote by e_v the following difference

$$e_v = \min_{p\in\Pi_{\bar{s}}} \int_0^X v(\bar{s},s)dp(s) - v^o(\bar{s},\bar{s}). \tag{75}$$

From (74) it follows that $e_{\tilde{v}} > 0$. Now let us restrict function $\tilde{v}(\cdot,\cdot)$ a bit more and assume that for all $y \in (\bar{s}, X]$

$$\tilde{v}^o(y,y) - \tilde{v}^o(\bar{s},\bar{s}) < \frac{e_{\tilde{v}}}{2}. \tag{76}$$

[7]Easy to see that function $\tilde{v}(\cdot,\cdot)$ chosen like this doesn't contradict any assumption in theorem 2.

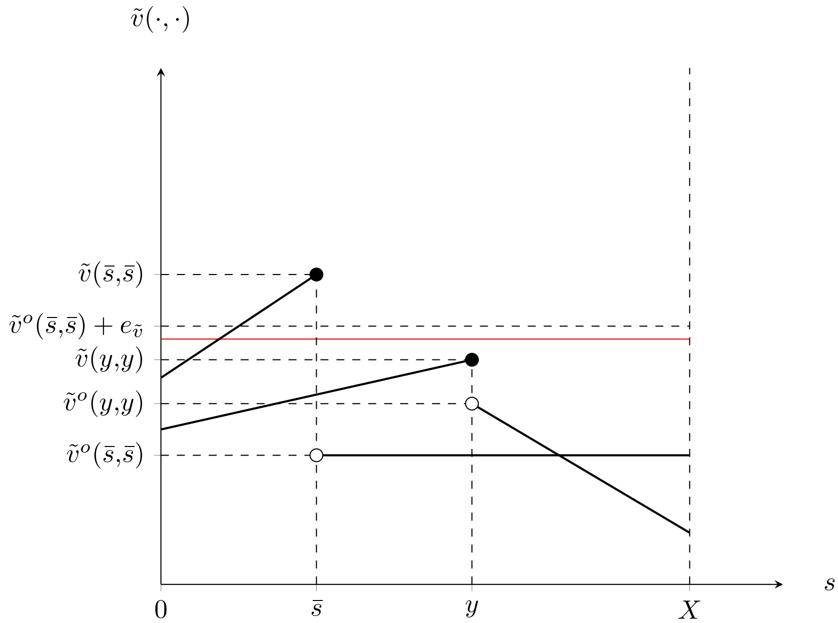

Figure 3: The projection of the function $\tilde{v}(\cdot,\cdot)$ onto the $s \times \tilde{v}(\cdot,\cdot)$-plane, corresponding to $\bar{s}$ and y consumption levels of the DM. The red line corresponds to $\tilde{v}^o(\bar{s},\bar{s}) + \frac{e_{\tilde{v}}}{2}$ level of $\tilde{v}$ function.

Such $\tilde{v}(\cdot,\cdot)$ exists because $e_{\tilde{v}} > 0$, so, it does't contradict assumption 6^* (and any other assumptions in Theorem 2).

Since $e_{\tilde{v}} > 0$, then without loss of generality (it doesn't contradict assumption 5) I can assume that for all $y \in (\bar{s}, X]$ and for all $s \in [0, y]$[8]

$$\tilde{v}(y,s) < \tilde{v}^o(y,y) + \frac{e_{\tilde{v}}}{2}. \tag{77}$$

From (76) and (77) we have that for all $y \in (\bar{s}, X]$ and for all $s \in [0, y]$

$$\tilde{v}(y,s) < \tilde{v}^o(\bar{s},\bar{s}) + e_{\tilde{v}}. \tag{78}$$

And from (76) and assumption 1^* we have that (78) holds for all $y \in (\bar{s}, X]$ and $s \in (y, X]$ as well. So (78) holds for all $y \in (\bar{s}, X]$ and $s \in [0, X]$.

Hence it is straightforward that for all $y \in (\bar{s}, X]$ and for all $p \in \Pi_{\bar{s}}$

$$\int_0^X \tilde{v}(y,s)dp(s) < \int_0^X (\tilde{v}^o(\bar{s},\bar{s}) + e_{\tilde{v}})dp(s). \tag{79}$$

[8]Graphically speaking, (for all $y \in (\bar{s}, X]$) on the interval $[0, y]$, the graph corresponding to the function $\tilde{v}(y,\cdot)$ is at the lower place than the horizontal line corresponding to $v(x,s) = \tilde{v}^o(\bar{s},\bar{s}) + \frac{e_{\tilde{v}}}{2}$ level (in figure 3, it corresponds to the red line).

Since $\tilde{v}^o(\bar{s}, \bar{s}) + e_{\tilde{v}} = const$, $\int_0^X dp_y(s) = 1$ and (79) holds for all $p \in \Pi_{\bar{s}}$ then

$$\min_{p \in \Pi_{\bar{s}}} \int_0^X \tilde{v}(y, s) dp(s) < \tilde{v}^o(\bar{s}, \bar{s}) + e_{\tilde{v}}. \tag{80}$$

But from (75) we have that

$$\min_{p \in \Pi_{\bar{s}}} \int_0^X \tilde{v}(\bar{s}, s) dp(s) = \tilde{v}^o(\bar{s}, \bar{s}) + e_{\tilde{v}}. \tag{81}$$

Hence, from (80) and (81) we have that for such a $\tilde{v}(\cdot, \cdot)$ (73) holds.

$\square$

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