Purchasing Seats for High School Admission in China

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Abstract

For more than 15 years, many Chinese cities gave students the option of paying higher tuition to acquire seats in their preferred schools. Yet real-world matching mechanisms that include an option to purchase seats may yield inefficient and unstable matching outcomes. This paper combines high school admission and survey data from China to estimate students' preferences regarding schools and tuition. The counterfactual experiments indicate that when the number of seats for sell is limited, the change from the deferred acceptance mechanism to the existing matching mechanism (with the seat-purchasing option) may have benefited moderately or poor performing students while reducing the welfare of top students. Meanwhile, the upper-tier schools may benefit from the increase in tuition collection with a minimal change in student quality, but middle-tier schools face a large uncertainty in student quality when they collect more tuition; and the seat-purchasing policy has a mixed effect on low-tier schools.

1 Introduction

The analysis of centralized school choice mechanisms has become a key focus of research in market design (Abdulkadiroglu and Sönmez (2003)). In extant literature on the school choice problem, the influence of monetary transfers between students and schools is seldom considered because public schools are either free or have fixed (and low) tuition. Yet unlike school choice systems in the United States and most other counties, since the 1990s there were

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many Chinese cities that offered students the option to pay higher tuition and thereby gain admission to public schools.[1] This procedure is referred to as the *Ze Xiao* (ZX) policy.[2] This policy was controversial because it was considered unfair to students whose families could not afford the extra costs (Shen and Wu (2006)). The controversy lasted for more than a decade and was somewhat defused in 2012, when the Ministry of Education announced restrictions on the ZX policy and requested that public high schools cease using it within three years. Many cities did abandon this policy for high school admissions, including Shanghai (which ceased using it in 2012), Beijing (in 2014), and Shenzhen and Tianjin (in 2015).

The ZX policy was a practical application of the *matching with contracts* model (Hatfield and Milgrom (2005); Hatfield and Kojima (2008, 2010)), which connects the matching market and auction behaviors. Analyzing student responses to a "price menu" for an individual good in matching markets may shed light on broader applications, such as considering the influence of financial aid in the school choice problem. However, when the Chinese government decided to discontinue its ZX policy in the face of widespread objections, it had never been rigorously analyzed by policy makers or researchers. Yet the questions that naturally arose still merit examination. For instance: Does this policy reduce (as often claimed) or enhance student welfare? Does this policy have the same influence on all students? If the ZX policy led to welfare losses for students, did that loss stem from the option to purchase admissions—or was the matching mechanism itself flawed?

This paper addresses these questions by exploiting a new data set covering high school admissions for the period 2012–2014 in a large Chinese city.[3] We combine these admission records with data from a 2014 survey to explore the strategic behaviors evident in student applications; in addition, we estimate student preferences regarding schools and tuition. We then use those estimated preferences to evaluate how student welfare might have been affected by the ZX policy. Furthermore, we address the trade-off faced by schools—given that selling seats to students may increase school profits but reduce the quality of admitted students.

The high school admission in our focal city is a centralized matching process based on standardized tests. The City Education Bureau adopted a typical ZX policy for its admission procedure until that policy was contraindicated in 2014 (see Appendix I for details of the ZX

[1] Zhu Kaixuan, then chairman of the state education commission, publicly addressed the seat-purchasing problem in public schools. In 1995 he argued, in the *People's Daily*, against charging high tuition to purchase admission to compulsory education.

[2] Ze Xiao in Chinese means school selection

[3] Confidentiality restrictions prevent this city from being identified by name.

policy and its corresponding admission mechanisms in different Chinese cities). Candidates were allowed to rank as many as three schools when applying; also, each student indicated whether or not he was willing to pay higher tuition in order to gain admission to each school on his preferred list—that is, if her admission would otherwise be denied. Students and schools were "price takers" in the admission procedure, while both the basic and the higher tuition levels were set by the local government. Admitted students who paid only the basic tuition were referred to as *normal* students; those whose admission depended on paying extra tuition were the *ZX students*.

From 2008 to 2013, the centralized matching mechanism used to assign students to schools mainly followed the *Chinese parallel* (CP) mechanism (Chen and Kesten (2017)). However, an additional stage was involved with each choice so as to match ZX students with their preferred school; the resulting system is known as the *Chinese parallel purchasing seats* (CPPS) mechanism. Suppose, for example, that a student's first and second choices are (respectively) school A and school B, and suppose she also indicates a willingness to pay extra tuition for the privilege of attending school A. Under the CPPS mechanism, this student will be considered for admission in school A first as a normal student; if she is rejected, then she will be considered for admission to that school as a ZX student before being considered for school B. Prior to 2008, the matching algorithm usually followed the so-called Boston mechanism by adding a similar additional stage to match the ZX students with their respective choices; this is referred to as the *Boston mechanism with purchasing seats option* (BMPS).

In contrast to the CP and Boston mechanisms, the CPPS and BMPS mechanisms are not *direct* mechanisms (see Section 2). An equilibrium outcome can be inefficient under these latter two mechanisms and, moreover, can be unstable under the CPPS mechanism. One way to overcome these undesirable features—while retaining the option to purchase admission—is to adopt the *student optimal purchasing seats* (SOPS) mechanism, a direct mechanism first proposed by Sönmez and Switzer (2013).[4] The SOPS mechanism is an extension of the *student-proposing deferred acceptance* (DA) mechanism (Gale and Shapley (1962)) and shares its favorable properties, which include being both stable and strategy-proof (see Section 2 for the definition of these terms).

One difficulty with empirical analysis of the school choice problem is estimating student preferences when only the submitted applications can be observed. The reason is that, if

[4] In Sönmez and Switzer (2013), this mechanism is called the *cadet optimal stable mechanism* (COSM). For the purposes of this paper, COSM was changed to SOPS to fit the school choice problem.

the adopted mechanism is *not* strategy-proof, then students have an incentive to manipulate their true preferences when submitting their rank-ordered lists (ROLs). After the CP mechanism replaced the CPPS mechanism in 2014, we conducted a survey that aimed to uncover students' true preferences and thereby counter, to some extent, the problems associated with assessing those preferences in the presence of strategic behavior. A comparison of survey responses and the ROLs actually submitted indicates that students seldom revealed their true preferences when applying for admission. Only 1.4% of the students reported their true preferences on their respective ROLs. Students also sought to increase their chances of being admitted by strategically maintaining sufficient gaps between their ROL choices. That is, the admission cutoffs of the second choices were (on average) more than 20 points below the admission cutoffs of the first choices—and most students listed a "leftover" school (see Section 4.3) as their third choice.

We estimate students' true preferences in two steps. In the first step, the survey results are used to estimate student preferences over schools without considering strategic behavior in ROLs. Given that the ZX policy ceased after 2013, the survey data cannot be used to identify any ZX policy–related parameters (e.g., tuition). So in the second step, the ROLs submitted in 2012 and 2013 are used to estimate other parameters. In this step, we assume that students have homogeneous beliefs about the likelihood of being admitted to each school. These assumptions reflect the year-to-year stability of schools' admission cutoffs and the dearth of naïve students. Our estimated results indicate if a school's quality increases by 1 unit, high-scoring students would prefer to pay an additional 226 yuan to attend this school, and medium and low-scoring students are only willing to pay 93 and 78 yuan respectively.

We use the estimated student preferences to conduct counterfactual experiments that enable assessment of how the different matching mechanisms perform. When strategy-proof mechanisms are adopted, retaining the option to purchase admission ends up reducing the welfare of students. On average, the change from the DA mechanism to the SOPS mechanism reduces student welfare by the equivalent of a 71-yuan tuition increase ($\approx$ 11.6 USD) when 10% of the seats are reserved for ZX students.[5] This welfare loss increases to 380 and 856 yuan when the quota for ZX students is increased to 30% and 50%, respectively. If an indirect mechanism is adopted to replace the DA mechanism, then students will experience a neglected welfare loss (on average) under the CPPS mechanism, and will benefit under the BMPS mechanism when the quota for ZX students is 10%; however, when the quota

[5]The exchange rate as of June 1, 2013, was $1 to 6.1 yuan.

increases to 30%, the welfare change becomes negative. However, the ZX policy's effect varies across student groups. The top 10% of students experience a welfare loss, relative to the DA scenario, under all mechanisms that involve seat purchases. Students who score between the 70th percentile and 90th percentile on their exams tend to gain in welfare under both the CPPS mechanism with a small ZX quota. The low-scoring group (below the 70th percentile) may benefit under the BMPS mechanism.

Our study allows for investigating the welfare of schools as well, a topic that has long been overlooked in studies of the school choice problem. From a policy maker's perspective, the purpose of designing a matching mechanism is to benefit students in public education. However, the competition among schools to recruit the best students exists across all education levels, especially after compulsory education levels. We measure the welfare of high schools in terms of (a) the quality of admitted students and (b) the profit derived from collecting student tuition. For the upper-tier high schools, the tuition collected increases—after replacement of the DA mechanism—in proportion to the ZX quota across all mechanisms that have seat-purchasing options; meanwhile, the quality of their students (measured by percentage grade) declines by no more than 1%. Although an increase of the tuition collection are observed with respect (w.r.t.) to the so-called middle-tier (medium-quality) high schools. However, the seat-purchasing option involve substantial variations in student quality for these schools. On the one hand, some schools' tuition receipts increase by more than 70% when the quota is 50%; on the other hand, student quality may decline by 5%. Although the low-tier (leftover) schools do not admit ZX students, the ZX policy still increases their tuition receipts as compared with the DA mechanism. It is because the students who are not willing to pay extra costs to retain their seats in better school may attend the low-tier schools. Meanwhile, the changes of student quality in these schools are uncertain.

This paper is closely related to the work of Sönmez (2013) and Sönmez and Switzer (2013), who investigated cadet–branch matching in the US military—whereby cadets may choose a longer term of enlistment in exchange for being assigned to their desired branch of service. These papers provided the first theoretical analysis of a practical application of the matching-with-contracts model, and we complement that research by offering the first empirical analysis. Our work is also directly related to the extensive theoretical literature addressing the centralized school choice problem (e.g., Balinski and Sönmez (1999); Abdulkadiroglu and Sönmez (2003); Ergin and Sönmez (2006); Pathak and Sönmez (2008); Abdulkadiroğlu et al. (2009); Haeringer and Klijn (2009); Kesten (2010); Pathak and Sönmez

(2013)).[6]

The research undertaken here contributes to growing body of empirical work on the school choice mechanism. One thread of that literature uses the preferences reported under non–strategy-proof mechanisms to estimate student preferences (Agarwal and Somaini (2018); Hwang (2015); He (2016); Calsamiglia et al. (2017)). Other papers focus on strategy-proof mechanisms. Abdulkadiroğlu et al. (2015) treated preferences reported under the DA mechanism as students' true preferences and then used them to analyze the demand for particular schools in New York City. Fack et al. (2015) proposed an approach for estimating preferences that did not require truth telling to be the unique equilibrium under a DA mechanism. Also, scholars have increasingly begun to use survey data when exploring strategic behavior under a matching mechanism. Budish and Cantillon (2010) conducted a survey on student preferences for offered courses to study the course allocation mechanism at Harvard Business School. Burgess et al. (2014) used survey data to assess directly the preferences of students over schools. Surveys were used also by De Haan et al. (2015) to analyze shortcomings of the Boston mechanism and by Kapor et al. (2017) to study heterogeneous beliefs in the school choice problem.

Finally, our paper is related to previous research on the school choice mechanism in China. With the largest centralized school choice systems in the world, the Chinese government has continually implemented admission policy reforms at all school levels over the past decades.[7] These reforms offer opportunities for research into the various types of mechanisms of interest to economists. He (2016) studied middle school choice in Beijing, and Chen and Kesten (2016, 2017) focused on college admission.

The rest of this paper proceeds as follows. In Section 2 we present school choice mechanisms that incorporate seat-purchasing options and develop the theoretical properties of those mechanisms. Section 3 provides details on the local ZX policy's background, and Section 4 describes our data and analyzes students' strategic behavior in the applications. We present our empirical model and estimates of student preferences in Section 5, and Section 6 discusses model fit. In Section 7 we conduct counterfactual experiments across mechanisms. Section 8 concludes with a summary of our findings.

[6]See Pathak (2011) for a survey on the school choice problem from the perspective of market design.

[7]Since China's Ministry of Education adopted (in 1952) a centralized matching mechanism for its college admission procedure, the centralized school choice system has been implemented throughout all levels of education—from elementary schools to colleges. There were 9.2 million high school seniors who took college entrance exams in 2015, a modest reduction from the peak of 10.5 million students in 2008.

2 School Choice with a Purchasing Seats Option

The components of a school choice problem—when purchasing seats is an option—can be listed as follows:

1. a finite set of students, $I = \{i_1, \dots, i_n\}$;
2. a finite set of schools, $J = \{j_1, \dots, j_m\} \cup \emptyset$, where $\emptyset$ denotes null school;
3. a finite set of tuition options, $C = \{c_0, c_1\}$ with $c_0 < c_1$;
4. two vectors of school quotas, $q^a = \{q_1^a, \dots, q_m^a\}$ and $q^z = \{q_1^z, \dots, q_m^z\}$ with $\sum_{j \in J}(q_j^a + q_j^z) \geq n$;
5. a list of strict student preferences, $\pi = (\pi_{i_2}, \pi_{i_2}, \dots, \pi_{i_n})$ over schools and tuition, $H \times C$; and
6. a school's normal priority $\succ$ over students.

The tuition amount c_0 is the basic tuition paid by normal students, and c_1 is the tuition paid by ZX students.[8] For any school j, the terms q_j^a and q_j^z denote its quotas for normal and ZX students respectively. For any student i, we use π_i to denote the strict preference relation over $J \times C$, where $(j, c_0)\pi_i(j, c_1)$ means that student i strictly prefers paying basic tuition for a seat in school j to paying extra tuition for a seat in the same school. We make the reasonable assumption that no student prefers high tuition to low tuition for attending a given school.[9] The term Π denotes the set of all preferences over $J \times C$, and $\widetilde{\Pi}$ denotes the set of all preferences over schools J only. Because high school admission in China is merit based (by entrance exam scores), we simply assume that all schools have the same priority ranking of students.

According to a *contract* $x = (i, j, c) \in I \times J \times C$, student i is assigned a seat in school j by paying tuition c; hence (j, c) is student i's *assignment*. A *matching* X is a set of contracts such that (a) each student appears in only one contract and (b) no school appears in more

[8] The model can easily be extended to accommodate multiple levels of tuition, see Sönmez and Switzer (2013).

[9] Sönmez and Switzer (2013) described a more general structure that forgoes this assumption about student preferences. If a student prefers (j, c_1) to (j, c_0), then a stable matching may not be fair (see Example 2 in their paper for details). Under our assumption, a stable matching is equivalent to a fair matching.

contracts than its total quota, $q^a + q^z$, of students. Let $\mathcal{X}$ denote the set of all matching outcomes.

A *mechanism* is a strategy space A_i for each student i along with an outcome function, $\psi\colon (A_{i_1} \times A_{i_2} \times \cdots \times A_{i_n}) \to \mathcal{X}$, that selects a matching outcome for each strategy vector $\mathbf{a} = (a_{i_1} \times a_{i_2} \times \cdots \times a_{i_n}) \in (A_{i_1} \times A_{i_2} \times \cdots \times A_{i_n})$. A *direct* mechanism is one for which the strategy space is simply the set of preferences Π for each student i. It follows that a direct mechanism is simply a function $\psi\colon \Pi \to \mathcal{X}$ that selects a matching outcome for each preference profile.

A matching is *stable* if: (i) there is no unselected contract (i, j, c) such that student i prefers assignment (j, c) to her current assignment and also i has sufficiently high priority that she is selected by j after paying cost c; and (ii) no student prefers a pair (j, c) with an unfilled quota to her current assignment. In turn, a mechanism ψ is stable if it always selects a stable matching. A mechanism ψ is *strategy-proof* if, for each student, it is at least a weakly dominant strategy to report her true preference.

2.1 Chinese Parallel Purchasing Seats (CPPS) Mechanism

In recent years, the Chinese parallel mechanism has replaced the Boston mechanism in Chinese provinces and cities (Chen and Kesten (2017)).[10] Mechanisms in the CP family are characterized by a vector $\mathbf{e}$=$(e_1, e_2, ...)$ for $e \in \mathbb{N}$, where e is the *permanency-execution period.* In each matching round j, a total of e_j subchoices are considered. Within each round, the algorithm implements a deferred acceptance procedure in which applications are tentatively held until no new proposals are made. Assignments are finalized after all e_j choices have been considered. The CPPS mechanism is an extension of the CP mechanism that simply adds a stage to each choice for the purpose of matching ZX students after the normal students are tentatively assigned. Unlike the CP mechanism, the CPPS mechanism used to assign students is not a direct mechanism. In particular, each student is asked

(i) to rank her school preferences; and

(ii) to indicate, for each ranked school, whether she wants the *ZX option* (i.e., would pay extra tuition) to attend that school if she is not assigned a seat there as a normal student.

[10]See Chen and Kesten (2017) for additional details of the college admission reform in China.

Under the CPPS mechanism, the strategy space of each student is $\widetilde{\mathcal{P}} \times 2^J$. For each school j, students who choose the ZX option receive higher priority for its q_j^z ZX seats than do any other students. More precisely, the *ZX priority* $\succ^+$ is constructed as follows: (i) if student i chooses the ZX option w.r.t. j but student i' does not, then $i \succ^+ i'$; (ii) if both i and i' choose the ZX option w.r.t. j (or neither does), then $i \succ^+ i'$ if and only if $i \succ i'$. Hence the relative priority of two students does not change under $\succ^+$ unless one of the students chooses the ZX option w.r.t. school j and the other student does not.

The CPPS mechanism selects the matching outcome as described next

Round 1:

• Each student applies to her first choice. Each school j tentatively holds the top q_j^a applicants in the normal pool, based on the normal priority. Among the remaining applicants, it tentatively holds the top q_j^z applicants in its ZX pool based on the ZX priority. All other applicants are rejected.

In general,

• Each rejected student i who has not yet applied to her (e_1)th-choice school applies to her next choice school. A student who has been rejected by all her first e_1 choices does not apply to any other schools until the next round. Each school j reviews the new applicants, along with those currently held in the normal pool, then tentatively holds the top q_j^a applicants in its normal pool based on the normal priority. Then j considers all remaining applicants, along with those currently held in its ZX pool, and tentatively holds the top q_j^z applicants based on the ZX priority. The other applicants are rejected.

• The round terminates whenever each student either is held in a school's pool or has been rejected by all her first e_1 choices. At this point, all tentative assignments become final. For each school, the quotas q_j^a and q_j^z are reduced by the number of students permanently assigned to those respective pools in this round. The resulting new quotas are denoted $q_{j,2}^a$ and $q_{j,2}^z$.

In general,

Round $\boldsymbol{k > 1}$:

• Each student applies to her $\sum_{j=1}^{k-1} e_j + 1$-th choice school. Then, much as in Round 1, each school j tentatively holds the top $q_{j,k}^a$ applicants in the normal pool, based on the normal priority. Among the remaining applicants, it tentatively holds the top $q_{j,k}^z$ applicants based on the ZX priority. All other applications are rejected.

In general,

• Each rejected student i, who has not yet applied to her $\sum_{j=1}^{k} e_j$-th choice, applies to the next choice school. A student who has been rejected by all her first $\sum_{j=1}^{k} e_j$ choices does not apply to any other schools until the next round. Each school j reviews the new applicants, along with those currently held in the normal pool, then tentatively holding the top $q_{j,k}^{a}$ applicants in its normal pool based on the normal priority. Then j considers all remaining applicants, along with those currently held in its ZX pool, and tentatively holds the top $q_{j,k}^{z}$ applicants based on the ZX priority. The other applicants are rejected.

• The algorithm terminates when each student is admitted to a school and all the tentative assignments are final. Each student who receives a normal seat pays tuition c_0. Each student who receives a ZX seat after choosing the ZX option for those schools pays the higher tuition c_1, while each student who receives a ZX seat but did not choose the ZX option pays the basic tuition c_0.

The Boston mechanism is a special case of the Chinese parallel mechanism (Chen and Kesten (2017)). Similarly, a special case of the CPPS mechanism—when $e_j = 1$ for all j—is known as the "Boston mechanism with purchasing seats option"(BMPS); in this mechanism, the assignments made after each choice are final. When the Boston mechanism was being used by most Chinese provinces for college admissions, the BMPS mechanism was adopted as the secondary school admission procedure throughout the country. (A formal description of the BMPS matching algorithm is given in Appendix A.)[11]

Neither the Chinese parallel mechanism nor the Boston mechanism is strategy-proof, and Sönmez and Switzer (2013) also indicated that it is not a weakly dominant strategy for cadets to reveal their true preferences over preferred service branches under the United States Military Academy mechanism. We extend the latter result to the CPPS mechanism as follows.

Proposition 1. *Revealing the truth preference over schools under the CPPS (or BMPS) mechanism may not be a weakly dominant strategy.*

(The example and proof can be found in Appendix B)

Before we show the details of the local school admissions, we first illustrate the ZX policy's potential influence on students and schools by way of an example. Suppose there are three schools of different quality: an upper-tier school, a middle-tier school, and an lower-tier (leftover) school. Each school has its own quota for admissions. Exam scores are the sole

[11] The CPPS and BMPS algorithms described here are the ones actually used, although some cities and provinces adopt "adjusted" versions of these mechanisms.

criterion used to admit students under a centralized matching mechanism without the option to purchase seats. Now suppose a ZX policy is implemented that replaces the old mechanism with a new mechanism with the seat-purchasing option, and let λ denote the proportion of a school's capacity that is devoted to ZX seats.

For the upper-tier school, the top $1 - \lambda$ of students admitted under the old mechanism is unaffected by adoption of the new mechanism. However, some of the next λ of students, who won't pay extra tuition to attend this school, are assigned to other schools, after which lower-scoring students take those seats and pay extra tuition. The rest of these λ students are still admitted by the upper-tier school, but they pay extra tuition. As a result, the dispersion of quality students increases and the average quality of admitted students declines.

For the middle-tier school, some of the top students who would normally be assigned there will now attend the upper-tier school because they are willing to pay extra tuition. Furthermore, this middle-tier school admits some students who—under the old mechanism—would have been assigned to the upper-tier school. Of the remaining students, those who were just barely admitted under the old mechanism must now pay extra tuition in order to keep their seats; otherwise, they will be assigned to the lower-tier school and their seats will be taken by lower-scoring students who do pay the extra tuition. Here the dispersion of student quality increases but the average quality of students is indeterminate. Finally, some of the lower-tier school's students will, under a ZX policy, gain admission to better-quality schools by paying the extra tuition. The lower-tier school then admits some students who were assigned to a better school under the old mechanism. Hence we will see an increase not only in the dispersion of quality students but also in average student quality at the lower-tier school.

This example illustrates that some high-scoring students may suffer a welfare loss when the option to purchase seats is offered. They must now compete for the limited number of basic tuition seats, which means that they must either pay extra tuition to attend the good schools or matriculate at a lower-quality school. Yet at the same time, low-scoring students can take advantage of this opportunity to gain admission into better schools by paying additional tuition. The influence of the ZX policy is determined by three factors: the quota of ZX seats, student preferences (demand for schools), and matching mechanisms.

2.2 Student Optimal Purchasing Seats (SOPS) Mechanism

Next we analyze the Student Optimal Purchasing Seats mechanism, which was first described by Sönmez and Switzer (2013) as the "cadet optimal stable mechanism". Recall that the SOPS mechanism is a simple extension of the DA mechanism. Each student's strategy space is Π under the SOPS mechanism, which makes it a direct mechanism. Here $\widetilde{\succ}$, the ZX priority of school j, is adjusted as follows. Suppose school j is considering two applicants, i and i', for ZX seats: (i) if i's application is (j, c_1) and i''s application is (j, c_0), then the school prefers i to i' (i.e., $i \widetilde{\succ}_j i'$); (ii) if both applicants choose (j, c_0) or (j, c_1), then $i \widetilde{\succ}_j i'$ if and only if $i \succ_j i'$.

Given the submitted preference lists, the SOPS mechanism selects the outcome as follows.

Round 1: Each student applies to her first choice. Each school j holds the top q_j^a students with their contracts whose first choices are (j, c_0) based on the normal priority $(\succ)$ in its normal pool tentatively. Among the remaining applicants, it holds the top q_j^z students with their contracts whose first choices are (j, c_1) or (j, c_0) based on the ZX priority $(\widetilde{\succ}_j)$ in its ZX pool tentatively. The other applicants are rejected.

Round k$>$ 1: Each rejected student applies to her next choice. Each school j considers the new applicants whose choices are (j, c_0) along with those who are held in the normal pool with their contracts from the last round, then hold up the top q_j^a applicants with their contracts in the normal pool based on the normal priority tentatively. Among the remaining applicants, j considers the new applicants whose choice is (j, c_1) or (j, c_0) along with those who are held in its ZX pool with their holding contracts from the last round, then holds the top q_j^z applicants based on the ZX priority. The other applicants are rejected.

The algorithm terminates when each students is tentatively held by a school, and the tentative assignments are final. Student i who is assigned a seat in j pays the tuition c_0 if her assigned contract is (i, j, c_0), and c_1 if the assigned contract is (i, j, c_1).

Some properties of the SOPS mechanism are similar to those of the DA mechanism. Sönmez and Switzer (2013) indicated that the SOPS mechanism is both strategy-proof and stable; moreover, the matching outcome under SOPS is weakly preferred by all students to any stable matching.[12]

The SOPS mechanism rules out the possibility of a student manipulating her preferences to "game" the system. In contrast, from Proposition 1 it follows that—under the CPPS

[12] Sönmez and Switzer (2013) proved additional theoretical properties of the SOPS mechanism.

or BMPS mechanism—students can achieve better outcomes by misreporting their preferences. Those two mechanisms also share some shortcomings, as revealed in our next two propositions.

Proposition 2. *Nash equilibrium outcomes under the CPPS mechanism with $e_1 > 1$ can be unstable and also Pareto inferior to outcomes of the SOPS mechanism.*

Proposition 2 shows that, even in a Nash equilibrium, the matching outcome under the CPPS mechanism may still exhibit undesired properties (e.g., instability) and may be Pareto-dominated by the SOPS mechanism. Our next result indicates that, although the BMPS equilibrium outcome can be stable, like the Boston mechansim (Ergin and Sönmez (2006)), it is still Pareto inferior to the outcome under the SOPS mechanism.

Proposition 3. *1. The set of Nash equilibrium outcomes under the BMPS mechanism is equal to the set of stable matchings.*
2. Every Nash equilibrium outcome of the BMPS mechanism is Pareto dominated by the outcome of the SOPS mechanism.

3 Background on High School Admissions

The schools in our focal city can be categorized into several types based on their educational goals after students graduate from middle school. There are general high schools that prepare students for colleges and universities in China, foreign language schools (or classes) for foreign colleges or universities, fine arts schools for the fine arts colleges in China, and vocational schools for the labor market. General high schools can also be categorized into public and private high schools.

The City Education Bureau requires that all schools, regardless of type or ownership, join the centralized admission system with regard to middle school graduates. In addition, each student who undergoes this admission procedure must register at the school to which she is assigned by the system. Hence, no outside option is available for students if they intend to continue their education in this city.[13]

[13] To avoid an unacceptable assignment, a student may either forgo the admission procedure or leave the application blank. Another way to avoid an undesirable assignment is to register at—but not actually attend—the assigned school. By paying additional costs, such students can instead attend schools in other cities.

At the end of March in each year, the City Education Bureau presents an admission plan that includes the quota of students that can be allocated to each school.[14] The quota for each public high school j comprises three parts: quotas q_j^e for early admission students, quotas q_j^a for normal admission students, and quotas q_j^z for ZX students. In mid-May, students submit their rank-ordered lists of schools. Thereafter, all students take the centralized high school entrance exam in early June. During 2012–2014, the full mark (i.e., the highest possible score) on this the exam was 665.[15] After the exams are graded, students are assigned to the schools by a centralized matching mechanism. The exam score is the only criterion for determining which students are admitted to a school.

Each student can list at most three schools on her ROL; students also select (or not) the ZX option w.r.t. those schools. Finally, every student must indicate whether she will accept a random assignment in the event she is rejected by her three preferred schools.

Local public high schools play a dominant role in preparing students for college. Thus, gaining entry into a public high school is the only hope for most students in China who want to attend college. However, high school education in this country now involves more than compulsory education, and local public high schools can accommodate fewer than half of all middle school graduates. After receiving the students' ROLs and exam scores, the education bureau provides a threshold based on the score distribution and total available seats. Only students whose scores are above that threshold will be considered for a seat in public high schools. The threshold is meant to guarantee that the number of qualified students does not exceed the total number of available seats in public high schools.

Because each students' rank-order list contains no more than three schools, the matching mechanism used by the City Education Bureau differs slightly from the model described in Section 2; the bureau uses what we refer to as the *constrained* mechanism (Haeringer and Klijn (2009)), as the matching algorithm terminates after each student's three choices have been considered. Unmatched students who have indicated acceptance of a random assignment are then randomly assigned to public high schools that still have available seats; the rest may find their own paths either to continue their schooling or join the labor market. Before 2008, the (constrained) BMPS mechanism was employed to assign students. Since then, the (constrained) CPPS mechanism—with permanency- execution periods $(2, 1)$—has been used. This new mechanism's matching algorithm lasts two rounds. The first and second choices in students' ROLs are considered in the first round, and their third choices are

[14]The admission quotas for private and vocational schools are announced at the same time.

[15]Prior to 2012, the highest possible score was 650; after 2014, it was 780.

considered in the second round. Without loss of generality, hereafter we shall reference the CPPS mechanism when describing the specific mechanism used in this city (i.e., without indicating its permanency-execution periods).

The tuition structure of public high schools is also different from our baseline model. Given that the exam score is the unique admission criterion, each school establishes a cutoff for its normally admitted students. The annual basic tuition paid by normal students is 1,600 yuan (about $260 in 2013), so a public high school education is relatively inexpensive.[16]

Three levels of the additional tuition paid by ZX students are based on their exam scores. A ZX student pays 3,333.3 yuan per year if her score is within 10 points of the school's cutoff, 5,000 yuan per year if it is within 11–20 points, and 6,000 yuan per year if it is within 21–30 points.[17] No school is allowed to admit a ZX student whose exam score is more than 30 points below its cutoff. Note that students submit their ROLs prior to taking the exam, and they can indicate only "yes" or "no" to the ZX option—that is, without making any stipulations about tuition levels. In accordance with instructions from the Ministry of Education, the local education bureau discontinued the ZX option after its 2013 admission process was completed.

4 Data Description

4.1 Data Source and Sample Selection

Since we are analyzing the ZX policy, which is designed specifically for public high schools, we focus on the students qualified for admission to those schools.

The data set we use consists of two parts, administrative data and survey data. The former comprise admission records from 2012 through 2014. Those records include the three choices listed on students' ROLs, exam scores, final assignments, whether a student was admitted as a normal student or ZX student, and each student's middle school and home address. We also have some data on school characteristics: admission quotas, tuition, and dormitory accommodations. To assess the quality of public high schools, our proxy is the quality of students that they admitted in previous years. More specifically, each school's

[16]In 2013, a local urban household's annual disposable income was 35,227 yuan. Hence the basic tuition is equivalent to 4.5% of that income.

[17]Unlike normal students, who pay tuition annually, ZX students must make a lump-sum payment for all three years of their high school education.

quality is measured as the average high school entrance exam scores (percentage grade) of students (in the 10th to 90th percentile of those scores) admitted over the previous three years.[18] We do not take a separate approach to estimate the schools' added value to measure school quality. It is because when students and their family evaluate school quality in a school choice problem, they seldom consider the schools' added value to students, instead, they use some relative simple indexes, such as the rank of school, college admission record, or admission cutoff. Since we try to mimic the students' strategies to estimate their preferences, it is not necessary to consider a relative complicated approach to estimate the schools' "true quality". In addition, we conducted a supplement survey that involved about 60 middle school teachers (the details of this survey can be find in Appendix C part two). We asked the teachers to what degree did they agree that a high school's education quality can be represented by the incoming students' test scores, and the most teachers answered "Strongly Agree" or "Agree", a few of them answered "Neutral" and no one answered "disagree".

In early May 2014, we conducted a survey of middle school graduates that asked each student to list five high schools she might attend and to rank them based on her preferences. The surveyed students were asked explicitly to report their genuine preferences, and there was no compelling reason for them not to honor this request. And given that the survey was conducted just two weeks before students submitted their ROLs, it seems unlikely that their preferences would change within that short period (The details of this survey can be found in Appendix C part one).

Unlike most surveys that seek to discover students' true preferences, we did not ask them to simply rank their favorite schools. Instead, respondents were asked to rank those schools they think that they may attend based on their true preferences (the reliability of our survey data is discussed in Section 4.3). Recall that the exam score is the only admission criterion, and note that the highest admission cutoff may exceed the lowest cutoff by more than 80 points. Our survey design aims to avoid instances of a low-scoring student ranking schools at which she has no chance of being admitted, meanwhile, such a student could list three schools with low cutoffs in the ROL. That possibility could lead to top schools being overreported in the survey, which would complicate attempts to compare the survey

[18]Our school quality measure is highly correlated (0.96) with the schools' college admission rate, which is valued by most students and their parents. We did not use the college admission rate itself to measure school quality because (a) that information was missing for some of the schools and (b) the college admission rate is not publicly available to students. However, both the high school admission cutoff and the score distribution on the entrance exam are public information.

responses and reported ROLs of low-scoring students.

In the administrative data, a total of 41,939 students were included in the 2012–2014 admission records. We first exclude students who were admitted by schools with special quotas, which did not affect the normal admission procedure. Students excluded for this reason included those who were admitted early or by fine arts schools as well as those on sports or art scholarships.[19] Second, we exclude students whose exam scores were below the threshold, since they were not qualified for admission to public high school. Finally, we exclude the students whose assignment outcomes were inconsistent with official rules.[20] After these exclusions, our final sample size from the administrative data was 11,217.[21]

We surveyed 6,980 students in 2014, or nearly half (49.17%) of the middle school graduates in that year's admission records. After we matched these students with the final administrative data sample just described—and deleted the invalid observations (e.g., students who ranked no school or only one school in the survey)—we were left with 1,447 survey observations for the subsequent analysis. Thus our survey covers 43.74% of the selected sample in 2014.

4.2 School Characteristics

In the administrative data, all nonpublic high schools were coded with a single number; we therefore treated all these schools as a whole without distinctions. Table 1 summarizes the characteristics of public high schools over the study period. A total of 13 public high schools were identified, with three special classes in 2012. Special classes are designed to admit gifted students and are independently operated. Moreover, these special classes have their own admission quotas in the matching mechanism. In the table's last row, the changes in total number of public high schools are due to the addition of special classes in some years.

The first row of Table 1 summarizes school quality for each school (as defined in Sec-

[19]An early admission decision is one that is made before students submit their ROLs. A student who is admitted early is still required to take the exam and to list the pre-admitting school as her first choice. Students admitted to fine arts schools must take an additional (art) exam; their admission process is handled separately from other students.

[20]For example, a few students were assigned to schools at which the cutoff was higher than their actual exam scores.

[21]48.6% of students are dropped out of the sample because their scores were below the thresholds. Another 13.3% of students are excluded for special quotas. The rest drop out is due to inconsistency with official rules or missing address.

tion 4.1). The average is approximately 80, with a standard deviation of 12.[22] School quality is stable across years, reflecting the stability both of admission cutoffs and of students' perceptions of the schools' relative ranks. There is considerable variation in the normal admission quotas. The largest school can admit 600 students; a small, "special class" school admits 40 students each year. The decrease in the average normal admission quota across years can be attributed to the newly established special classes and the increased number of early admissions. The average quota for ZX students ranges between 95 and 100 across years, with a standard deviation of about 35. Special classes and also four public high schools do not admit ZX students.[23] The table's fourth row indicates that the number of schools providing dormitories increases from nine in 2012 to thirteen in 2014.

4.3 Student Characteristics and Behaviors

Exam score distributions are summarized in Table 2. The first data column gives the percentile benchmarks, and the next three columns report the corresponding absolute scores across years. Exam scores are slightly lower in 2013 than in 2012 and 2014, but the variation in absolute scores of the same percentile level never exceeds 1.7% of the full mark. This finding confirms that exam scores were stable across years.

Our analysis focuses on students who were qualified to be assigned to public high schools. Approximately 94.3% of these students, whose scores were above the threshold, received seats in public high schools in 2012—as compared with, and this number was 95.1% and 90.3% in 2013 and 2014, respectively. These values indicate that most students who qualified for admission to take seats in the public high schools end up going there rather than entering other types of schools.

The first part of Table 3 summarizes the number of schools on the submitted ROLs. More than 93% of the students submit full (three-school) lists, approximately 5% of them list two schools, and fewer than 1% of all students list only one school.[24]

The second part of this table shows the assignment results, which exhibited similar patterns in 2012 and 2013. About 30% (resp. 37%) of students were assigned to their first (resp.

[22]School quality is defined by the percentage grade) in Section 4.1. To scale the measurement in the estimate, we increase the percentage grade by 100 times. For example if the school quality is 80%, then we record it as 80 instead of 0.8.

[23]Special classes and one public high school are not allowed to admit ZX students. The other three public high schools are the admission procedure's leftover schools. These schools admit students with scores above the threshold and then, if any unassigned seats remain, ZX students with scores below the threshold.

[24]Schools that are listed twice in the same ROL are treated as a single school.

second) choice, and approximately 11%–13% of students were rejected by all three of their preferred schools. Some 13%–15% (resp. 5%–6%) of students were assigned to their first (resp. second) choice as ZX students. No ZX student was assigned to the third choice. After cancellation of the ZX policy in 2014, fewer students (26%) were assigned to their first choice and more students (17%) were rejected by all three choices.

Because the Chinese parallel mechanism is not strategy-proof, it is difficult to assess—while referring only to submitted ROLs—the extent to which students misrepresent their true preferences. Our survey data provide an opportunity for direct comparisons between each student's true ordinal preferences and her strategic behavior. More than 60% of the surveyed middle school graduates ranked five schools, 17% of them ranked four schools, and approximately 21% of them ranked fewer than four schools (Table 4).

The admission cutoffs of schools reflect their popularity among students. We define a *popular* school as one whose first-round cutoff is higher than the threshold; that is, the demand for admission to these schools is greater than the number of available seats.[25] At the other extreme, schools whose cutoffs are equal to the threshold are referred to as *leftover* schools.

Figure 1 shows the average admission cutoffs of schools chosen by students in the survey and the ROLs.[26] Students are grouped into four categories according to their score percentiles. In the survey, the top 10% students' exam score school cutoffs average 606.1 and 599.4 for (respectively) their first and second choices; the average cutoff for third choices (593.2) is another 6 points lower. The gaps between the third and fourth choices and the fourth and fifth choices in the survey are 5 and 9 points, respectively. The choices of students in the other three groups follow a similar pattern. Within a group, the average cutoff gap between consecutive choices is approximately 6 points and never more than 10 points. Between groups, the average cutoff for the first choice of the 80th–90th percentile students is 6 points lower than that for the highest decile of students, and this average cutoff decreases by another 9 points (to 591) for the 70th–80th percentile students. The average first-choice cutoff of students below the 70th percentile of exam scores is 585. For each additional choice, average cutoffs are similarly decreasing (at a rate of 4–10 points) in exam scores.

The decline in average cutoff of students' first choice when their scores decrease indicates that the surveyed students answered our questions truthfully by listing and ranking schools

[25] Since no school's second-round cutoff was higher than the threshold when its first-round cutoff was equal to the threshold, no confusion can arise if we base popularity solely on the first-round cutoffs.

[26] The corresponding table can be found in Appendix D.

into which they might actually be admitted. The gaps between consecutive choices within groups in the survey indicate that student preferences w.r.t. schools was decreasing in the popularity of those schools; in 2014, the consecutive cutoff gaps for two popular schools were between 3 and 9 points. Also, the small cutoff gaps (4–10 points) between consecutive choices within each group implies that the preferences reported in the survey are reliable enough to be considered the students' true preferences.

In the ROLs, the average cutoffs for the first choices of students whose exam scores were above the 70th percentile nearly coincide with the corresponding parts in the survey, although the average cutoffs for the first choices of low-scoring students (i.e., with exam scores below the 70th percentile) are 6 points lower than in the survey. However, the gap between the first and second choices increases significantly with declining exam scores. The gap in the average cutoffs between the first and second choices for the top 10% students is almost the same as that in the survey, but this gap increases to 19 points for the 80th–90th percentile students and to about 25 points for the two groups of low-scoring students. Finally, the average cutoffs for third choices are consistently close to the threshold (of 535) for all groups in the ROLs.

When compared with the survey data, the large gaps between consecutive ROL choices reveal students' strategic behavior in their submitted preferences: maintaining a sufficiently large gap between choices toward the end of increasing their chances of being admitted to some school.[27] The coincidence between the first choices in the survey and the ROLs indicates that students prefer applying to their favorite attainable schools. This coincidence, and the small cutoff gaps among choices reported in the survey, provide further evidence that the surveyed students accurately reported their five favorite attainable schools. Yet students—especially those who were not in the top-scoring group—strategically manipulated their true preferences in the ROLs so as to increase their overall likelihood of being admitted should they be rejected by their first choices. Thus the second choices in the ROLs of 80th–90th percentile (resp., 70th–80th percentile) students are close to their fourth (resp., fifth) choices in the survey. Moreover, most students (across all four groups) chose a leftover school as their third choice because the ROL is restricted to only three choices.

One drawback of the non-strategy-proof mechanism is that students who submit strategically modified ROLs may take advantage of the naïve students who reveal their true prefer-

[27] This finding is consistent with the literature that indicates the students' strategic behavior in non-strategy-proof mechanisms (Abdulkadiroğlu et al. (2005); Chen and Sönmez (2006); Abdulkadiroglu et al. (2017)), and among many others.

ences (Pathak and Sönmez (2008)). Calsamiglia et al. (2017) indicated that, in Barcelona's local school choice setting, the proportion of such naïve students is less than 4%. We can estimate the proportion of naïve players in our data set by directly comparing the schools listed in the survey and in the ROLs. Only 20 students (1.38% of all observations) submitted ROLs that matched their lists in the survey. In fact, there may be even fewer naïve students because reporting true preferences could be a weakly dominant strategy for some students (e.g., those in the top-scoring groups). Our findings here accord with previous research in suggesting that few students submit an ROL without any strategic considerations—especially when a strict criterion is used to assign students. Unlike the assignment of students via coarser criteria (e.g., walking zones or siblings), high school admission in our context offers no safe choice for students before their exam scores are known; it follows that estimating this score is a student's first strategic move. Hence one must anticipate an extremely low percentage of naïve students among those who engage in admission procedures such as those described here.

5 Empirical Model and Preference Estimate

We simply adjust the structure of tuition fees in the school choice problem from Section 2 based on the local admission rule: a set of tuition fees $C = \{c_0, c_1, ..., c_4\}$, where c_0 is the basic tuition for normal students and the other amounts are the additional tuition levels paid by ZX students; here $c_{t'} < c_t$ for $t < t'$.

We assume that student i's (indirect) utility from being assigned to public high school j with tuition c_{ij} is

$$u_{i,j,c} = \sum_l \beta_l y_j^l + \sum_w \beta_w x_i^w y_j^w + \beta_D f(d_{ij}, Y_j) + \sum_k \alpha_k (c_{ij} - c_0) x_i^k + \varepsilon_{ij} \tag{1}$$

and that the utility from being assigned to nonpublic high school o is

$$u_{i,o} = F_o + \varepsilon_{io}. \tag{2}$$

Here $Y_j \equiv \{y_j\}$ is a vector of school j's observed characteristics *other than* its ZX quota (e.g., school quality and its quota for normal students); $X_i \equiv \{x_i\}$ is a vector of student i's observed characteristics (e.g., her exam score); d_{ij} is the distance, by road, between student i's home and school j; F_o is the fixed effect of non-public high schools; and ε_{ij} and ε_{io} are i's idiosyncratic tastes for, respectively, public high school j and nonpublic high schools.[28] In the estimate, we follow a common approach in other school choice literature (Agarwal and Somaini (2018) and many others) by assuming the home–school distance is additively separable and independent of unobserved student preference, moreover normalize the coefficient for the home–school distance (d_{ij}) to be -1.[29]

We do not adopt the random coefficient model to estimate students' heterogeneous taste to observed school characteristics (e.g. Abdulkadiroğlu et al. (2015); Agarwal and Somaini (2018)) because of the limited variation in the data. Except school quality, other observed school characterises are indicator variables. It is because general high schools have the unique education goal to prepare students for the college entrance exams, schools' observed characteristics exhibit homogeneity, such as facility and dormitory. To examine our empirical model, we present an alternative random coefficient model in Appendix **??** to compare the estimated results, which exhibit worse performance in the within and out of sample test.

Assumption 1. *The terms ε_{ij} and ε_{io} are independent of the explanatory variable, X_i, Y_j, d_{ij}, C, and F_o. Both ε_{ij} and ε_{io} are independent and identically distributed (i.i.d.) and exhibit a type I extreme value distribution with cumulative distribution function (CDF) $F(\varepsilon)$.*

We follow Abdulkadiroğlu et al. (2015) in not explicitly modeling an outside option. The first reason for this choice is that, as mentioned in Section 3, no outside option can be observed in the current admission record. Second, the commonly used model of an outside option—which implies that an unranked school is not acceptable—would require us to assume that students who did not list three schools when submitting their respective ROLs actually prefer the outside option to attending school in the city. However that assumption cannot rule out the possibility of students, especially those with high exam scores, ranking fewer than three schools simply because they were confident about being admitted by their first (or second) choice.

We use both the administrative data and our survey data to estimate student preferences. The advantage of survey data is that our estimates can proceed without having to account for the students strategic behaviors in the submitted ROLs. However, our survey data cannot reveal student preferences w.r.t. ZX options because the ZX policy was discontinued after 2013; thus, in 2014, all students paid the same basic tuition for all public high

[28] The road distance d_{ij} is calculated via Google Maps by inputting the focal school's address and the student's home address.

[29] Unlike the admissions in the elementary schools and middles schools, the high school admission procedure does not consider the home locations and school districts. Therefore, we do not think school choice mechanism in this city directly affects the local house price and residential decisions.

schools. As a result, neither α can be identified by the survey data. We therefore divide our estimation procedure into two steps. First, the survey data from 2014 are used to estimate the parameters *unrelated* to the ZX option. Second, the parameters related to the ZX policy are estimated from the student ROLs submitted prior to 2014. We use $\Theta \equiv [\boldsymbol{\beta}, \boldsymbol{\alpha}]$ to denote the vector of model parameters, where $\boldsymbol{\beta} = \{\beta\}$ is the vector of parameters unrelated to the ZX policy and $\boldsymbol{\alpha}$ is the vector of parameters that are related to that policy.

5.1 Step One: Estimating the Non-ZX-Related Parameters $\boldsymbol{\beta}$

In this step, we focus on the survey data without considering students' strategic behavior when submitting their ROLs. Each surveyed student ranked five schools that she believed herself capable of attending. This procedure implies that the student first selects the schools for which admission is a distinct possibility and then, after identifying those schools, ranks them. That process complicates our constructing a model of how these middle school graduates select schools in the first place. For example, if a school with a high admission cutoff does not make the surveyed student's list, then it is difficult to distinguish between (a) her preferring the listed schools to the focal school and (b) her thinking that admission to the high-cutoff school is not possible. From the evidence presented in Section 4.3, we conclude that the survey responses reflect students' true preferences—that is, conditional on their belief in the possibility of admission. To simplify the estimation process, we focus on the listed schools' ranks in the survey (i.e., without considering the unlisted schools). In other words, we do not attempt to infer the relative ranks of listed and unlisted schools. This approach renders the estimate less efficient, but it is consistent when surveyed students report their true rankings.

Given that $c_{ij} = c_0$ in this step, we can rewrite student i's utility function as

$$u_{i,j} = \sum_{l} \beta_l y_j^l + \sum_{w} \beta_w x_i^w y_j^w + \beta_D f(d_{ij}, Y_j) + \varepsilon_{ij}, \tag{3}$$

$$u_{i,o} = \varepsilon_{io}. \tag{4}$$

While referring to the survey data, we use the rank-ordered logit model (Beggs et al. (1981)) to estimate $\boldsymbol{\beta}$. Given a surveyed student i's ranked school list $(j_1, ..., j_{l_i})_i$ of length $l_i \leq 5$, we conclude that j_1 is her favorite school among all the l_i schools on her list, that j_2 is her second-favorite school, and so on. The joint probability of these choices is $\Pr(u_{i,j_1} > u_{i,j_1} > \cdots > u_{i,j_{l_i}})$. Recall that we assume the ε to be i.i.d. with type I extreme value distribution; it follows that, for an observed ordinal ranking of the choices,

$$\Pr(u_{i,j_1} > u_{i,j_1} > \cdots > u_{i,j_{l_i}}) = \prod_{k=1}^{l_i-1} \frac{e^{\mu_{i,j_k}}}{e^{\mu_{i,j_k}} + e^{\mu_{i,j_{k+1}}} + \cdots + e^{\mu_{i,j_{l_i}}}}, \tag{5}$$

where $\mu_{i,j}$ is the deterministic component of $u_{i,j}$ or $u_{i,o}$.[30] Then the log-likelihood function can be written as

$$\log L_1(\boldsymbol{\beta}) = \sum_{i=1}^{n} \sum_{k=1}^{l_j-1} \mu_{i,j_k} - \sum_{i=1}^{n} \sum_{k=1}^{l_i-1} \log\left(\sum_{s=k}^{l_i} e^{\mu_{i,j_s}} \right). \tag{6}$$

Now we can estimate $\boldsymbol{\beta}$ via maximum likelihood estimation.[31]

5.2 Step Two: Estimating the ZX-Related Parameters $\boldsymbol{\alpha}$

In the second step, we estimate $\boldsymbol{\alpha}$ while considering students' strategic behavior in the admission procedure. After plugging the estimated $\widehat{\boldsymbol{\beta}}$ into equations (1) and (2), we can rewrite student i's utility function as

$$u_{i,j,c} = \hat{u}_{i,j} + \sum_{k} \alpha_k (c_{ij} - c_0) x_i^k + \varepsilon_{ij}, \tag{7}$$

$$u_{i,o} = \hat{F}_o + \varepsilon_{io}, \tag{8}$$

where $\hat{u}_{i,j} = \sum_l \hat{\beta}_l y_j^l + \sum_w \hat{\beta}_w x_i^w y_j^w + \hat{\beta}_D f(d_{ij}, Y_j)$.

Given the evidence (from Section 4.3) that very few students report their true preferences when submitting ROLs, we model the students' strategic behaviors by assuming that students submit ROLs that are optimal given a set of beliefs about the admission probability of schools.

Assumption 2. *Students are fully informed about their own preferences and maximize their expected utility.*

[30] More precisely, $\mu_{i,j} = \sum_l \beta_l y_j^l + \sum_w \beta_w x_i^w y_j^w + \beta_D f(d_{ij}, Y_j)$ when j is a public high school and $\mu_{i,j} = F_o$ when j is not a public high school.

[31] Because we assume the utility function has an additively separable form, it is easy to show that $\log L_1$ is globally concave in $\boldsymbol{\beta}$—from which it follows that there exists a unique maximum of the likelihood function.

5.2.1 Beliefs

Students evaluate their likelihood of being admitted to each school before submitting their ROLs. Admission requires that the student's score be no less than the school's admission cutoff. Those cutoffs are announced to the public after the annual admission season.

Figure 2 shows the fluctuation of popular schools' cutoffs in terms of percentage grades (on the entrance exam) across years. Compared with the previous year's admission cutoffs, none of the popular schools' cutoffs (with one exception) between 2011 and 2013 increased by more than 0.04 or decreased by more than 0.02. For example, the admission cutoff of School 183, which has the highest cutoff, was 97.5% in 2011 and increased by half a percentage point (to 98%) in 2012. Furthermore, the list of popular and leftover schools did not change across years. We therefore assume that students form the correct beliefs about admission cutoffs in the current year—that is, given the stability of those cutoffs (Figure 2) and of the exam score distribution (Table 2). The probability of admission is calculated in Section 5.2.3. An alternative assumption is that students use the previous year's admission cutoffs to form their beliefs (i.e., that students exhibit "adaptive expectations"). Estimated results based on this assumption are reported in the robustness check.

5.2.2 Students' Decision Problem

At the start, each student submits an ROL $a_i = \{(j_i^1, v_i^1), (j_i^2, v_i^2), (j_i^3, v_i^3), t_i\}$; here $v_i^k \in \{0, 1\}$ indicates whether student i selects the ZX option for her kth choice j_i^k, and $t_i \in \{0, 1\}$ indicates whether i accepts a random assignment if she is rejected by all three of her chosen schools. Thereafter, all students take the entrance exams and receive the corresponding scores $\{s_i\}_{i=1}^n$.

After collecting the ROLs ($\{a_i\}_{i=1}^n$) and exam scores ($\{s_i\}_{i=1}^n$), the City Education Bureau runs the algorithm that assigns students to schools based on the CPPS mechanism. Because the exam scores and ZX options are used to determine the assignment of students, the matching algorithm generates an admission cutoff $\bar{S}_j^k$ for each school j when j is listed as the kth choice. Thus $\bar{S}_j^k$ equals the lowest exam score of the student who is assigned to j based on her kth choice as a normal student—or the threshold S^* if unassigned normal seats are still available after the current matching round. We put $\bar{S}_j^k = \infty$ if school j has no normal seat available for that kth choice.[32] Similarly, $\hat{S}_j^k$ denotes school j's cutoff for the ZX student

[32] Since students' first two choices are considered in the matching algorithm's first round, it follows that $\bar{S}_j^1 = \bar{S}_j^2$ for all schools. If j is a popular school, then $\bar{S}_j^k$ is higher than the threshold when $k < 3$ and

as the kth choice.[33]

Finally, a student who is assigned to school j based on her kth choice as a normal student will pay the basic tuition c_0. If she is assigned as a ZX student then the tuition she pays depends on her exam score, as detailed in Section 3.

Given this model setup, student i's decision problem is to select the a_i that maximizes the expected payoff. Formally, we have

$$\max_{a_i \in A_i} \sum_{j \in \{j_i^k\}} \left[I_{hj} \left(\sum_{c \in C} P_{i,j,c} u_{i,j,c} \right) + I_{oj} P_{i,o} u_{i,o} \right] + \widetilde{P}_{a_i} \tilde{u}_i. \tag{9}$$

Here A_i is the set of all possible choices for student i; the terms I_{hj} and I_{oj} are indicators for whether school j is (respectively) a public school or a nonpublic school; $P_{i,j,c}$ represents the probability that student i is assigned to school j with tuition c; and $P_{i,o}$ is the probability that student i is assigned to a nonpublic high school. Finally, $\widetilde{P}_{a_i}$ and $\tilde{u}_i$ represent, respectively, the probability and payoff of student i being rejected by all three of her chosen schools.

A student's choice consists of the combination of her three choices and her ZX options. Therefore, the size $|A_i|$ of student i's choice set is equal to $|(|J| \times 2)^3 \times 2|$; this amounts to more than 50,000 alternatives. To simplify calculations, we rule out a few weakly dominant strategies and thereby limit the choice set to $A_i' \subset A_i$, as described next.

First, if a student lists a leftover school as her first or second choice, then the rest of her choices should be blank. Hence no student will be admitted by a school that is listed after a leftover school. Second, if a student lists a popular school as her first or second choice, then her subsequent choice should not be blank. Third, no student's ROL can select a particular school more than once.

Fourth, no student selects the ZX option for her third choice. According to the admission records, students are admitted by their third choice only when those choices are leftover schools, which admit all students as normal students. That is, selecting the ZX option for one's third choice does not affect the admission result. In the data set, we do not observe any student who was admitted as a ZX student for her third choice.

Fifth, a student accepts the randomly assigned school if she is rejected by all of her listed schools. So if a student in those circumstances does not accept the randomly assigned

$\bar{S}_j^3 = \infty$. If j is a leftover school then $\bar{S}_j^1 = \bar{S}_j^2 = \bar{S}_j^3$, which is equal to the threshold. There is no school (in the admission records) for which $\bar{S}_j^1 = \bar{S}_j^2$ is equal to the threshold and $\bar{S}_j^3$ is higher than the threshold.

[33]Recall from Section 3 that $\hat{S}_j^k$ must satisfy an additional condition: it cannot be more than 30 points below $\bar{S}_j^k$.

school, then her only option is to attend a nonpublic high school as assigned by the matching procedure for students whose scores are below the threshold. In the admission records, all nonpublic high schools have admission cutoffs that are below the threshold; in other words, their admission probability is equivalent to that of leftover high schools. The implication is that, if a student would rather attend a nonpublic high school than be randomly assigned to a leftover school, then she should list that nonpublic school as one of her three choices. In the admission records, 1.21% of the students did not accept the random assignment after being rejected by their preferred schools.

After excluding all these weakly dominated strategies, we can simplify our expression for student i's submitted ROL as follows: $a_i = \{(j_i^1, v_i^1), (j_i^2, v_i^2), j_i^3\}$. Hence the choice set A_i' includes alternatives and so is significantly smaller than its parent set A_i.

5.2.3 Admission Probability

Given student i's ROL a_i, the probability of i being admitted (as a normal student) by her first choice is $\Pr\left(\bar{S}^1_{j_i^1} \leq s_i\right)$; that is, her score is no less than the school's cutoff. The probability of i being admitted by this school as a ZX student with tuition c_t is $v_i^1 \Pr\left(\max\left\{\hat{S}^1_{j_i^1}, \bar{S}^1_{j_i^1} - 10t\right\} \leq s_i < \bar{S}^1_{j_i^1} - 10(t-1)\right)$, which means that her score is too low to receive the seat with ZX tuition c_{t-1} but is high enough to receive a seat with tuition c_t. In general, the probability that student i is assigned to school j_i^k as a normal student can be written as

$$P_{i,j_i^k,c_0} = \Pr\left(\bar{S}^k_{j_i^k} \leq s_i < (\bar{S}^{k-1}_{j_i^{k-1}})^{(1-v_i^{k-1})}\left(\hat{S}^{k-1}_{j_i^{k-1}}\right)^{v_i^{k-1}}\right), \tag{10}$$

with $\bar{S}_j^{k-1} = \infty$ and $v_i^{k-1} = 0$ when $k = 1$. The probability of being admitted as a ZX student with tuition c_t is

$$P_{i,j_i^k,c_t} = v_i^k \Pr\left(\max\{\hat{S}^1_{j_i^k}, \bar{S}^1_{j_i^k} - 10t\} \leq s_i < \bar{S}^1_{j_i^k} - 10(t-1)\right). \tag{11}$$

When school j_i^k is a nonpublic school, the admission probability $P_{i,o}$ is a special case of equation (10) in which $\bar{S}^k_{j_i^k}$ equals the threshold S^*.

Uncertainty regarding the exam scores is evident because students submit their ROLs prior to taking the exam. Taking the perspective of student i, we assume that her exam score will be $\tilde{s}_i = m_i + \eta_i$; here m_i represents either i's mock exam score or her true ability (by which she estimates her exam score) and η_i is the uncertainty. We assume that η is i.i.d. and distributed normally as $N(0, \delta)$. Note that m_i cannot be directly observed from the data.

Instead, we use the student's actual exam score s_i as the estimate of m_i. To simplify our estimation process, we also set $\delta = 20$, which is 3% of the full mark.[34]

After we replace s_i with $s_i + \eta$ in equations (10) and (11), the admission probabilities can be rewritten as

$$P_{i,j_i^k,c_0} = \max\left\{0, \left(\Phi\left(\left(\bar{S}^{k-1}_{j_i^{k-1}} - s_i\right)/\eta\right)\right)^{(1-v_i^{k-1})}\left(\Phi\left(\left(\hat{S}^{k-1}_{j_i^{k-1}} - s_i\right)/\eta\right)\right)^{v_i^{k-1}} - \Phi\left(\left(\bar{S}^{k}_{j_i^{k}} - s_i\right)/\eta\right)\right\}, \tag{12}$$

where Φ is the CDF of the standard normal distribution, and

$$P_{i,j_i^k,c_t} = v_i^k \max\left\{0, \Phi\left(\left(\bar{S}^{k}_{j_i^{k}} - 10(t-1) - s_i\right)/\eta\right) - \max\left\{\Phi\left(\left(\bar{S}^{k}_{j_i^{k}} - 10t - s_i\right)/\eta\right), \Phi\left(\left(\hat{S}^{k}_{j_i^{k}} - s_i\right)/\eta\right)\right\}\right\}. \tag{13}$$

Finally, the probability of the student being randomly assigned to a leftover school can be calculated as the probability of her being (a) rejected by all three of her chosen schools yet (b) qualified to be assigned a seat in public high school. That probability is

$$\widetilde{P}_{a_i} = \max\left\{0, \Phi\left(\left(\min\left[\left\{\bar{S}^{k}_{j_i^{k}}\right\}_k, \left\{\hat{S}^{k}_{j_i^{k}}\right\}_k\right] - s_i\right)/\eta\right) - \Phi((S^* - s_i)/\eta)\right\}. \tag{14}$$

5.2.4 Likelihood Function and Identification

Although we can observe the students' three choices in ROLs, their choices w.r.t. the ZX option cannot be observed from the admission records. All we know in that regard is whether a student is assigned to a school as a normal or a ZX student. Therefore, students' ZX options can be partially (or sometimes fully) inferred from their assignment results. Suppose, for example, that a student is assigned to her first choice as a ZX student; then we know she must have selected the ZX option for that choice. If she is assigned to the second choice but was qualified for admission by the first choice as a ZX student, then we can infer that she did not select the ZX option for the first choice. Hence observations can be categorized into three groups. The first group (G_1) includes only students whose ZX options in ROLs can be unambiguously inferred from the admission records data. For these students, we shall continue using a_i to denote their choices. The second group (G_2) comprises students whose decisions w.r.t. ZX options can be observed or inferred for their first choice but not for their second choice; we use a_i^2 to denote the choices of these students, although information concerning the ZX option for the second choice is insufficient (i.e., v_i^2 is unobserved). The

[34] The estimated results when $\delta = 13.3$ (2% of the full mark), when $\delta = 26.6$ (4% of the full mark), and when $\delta = 33.35$ (5% of the full mark) are reported in the robustness check.

third group (G_3) consists of students whose decisions w.r.t. ZX options can be observed or inferred for their second choice but not for their first choice. For students in this group, we use a_i^3 to denote i's choice when v_i^1 is unobserved.

For an observation in G_1, we write the probability of observing an ROL a_i as $\Pr(a_i \in A_i^*)$; here $A_i^* \subset A_i'$ is the optimal solution set of the student's problem in (9). For student i in G_2, we can neither observe nor infer whether i selected the ZX option for her second choice; however, we do know that she either (a) selected the ZX option for her second choice, $a_i^{2+} = \{(j_i^1, v_i^1), (j_i^2, 1), (j_i^3)\}$, or (b) did not select that option, $a_i^{2-} = \{(j_i^1, v_i^1), (j_i^2, 0), (j_i^3)\}$. Hence the probability of observing a_i^2 is $\Pr(a_i^{2+} \in A_i^*) + \Pr(a_i^{2-} \in A_i^*)$. Similarly, the probability of observing a_i^3 in group G_3 is written as $\Pr(a_i^{3+} \in A_i^*) + \Pr(a_i^{3-} \in A_i^*)$, where $a_i^{3+} = \{(j_i^1, 1), (j_i^2, v^2), (j_i^3)\}$ and $a_i^{3-} = \{(j_i^1, 0), (j_i^2, v^2), (j_i^3)\}$.

The total log-likelihood function for the entire sample is then provided as follows:

$$\log L_2(\boldsymbol{\alpha}) = \sum_{i \in G_1} \log(\Pr(a_i \in A_i^*)) + \sum_{g=2}^{3} \sum_{i \in G_g} \log[\Pr(a_i^{g+} \in A_i^*) + \Pr(a_i^{g-} \in A_i^*)]. \tag{15}$$

Identification of the model's $\boldsymbol{\alpha}$ parameters with the utility function given in equations (7) and (8) is similar to that for a multinomial discrete choice model, which has been established based on general conditions (Matzkin (1993)). Since all students are assumed to be sophisticated—that is, to behave strategically—our model differs only in that each student considers the admission probabilities $P_{i,j,c}$ of schools and then chooses the option with the highest expected payoff. The identification power comes from the observed variation in choices by students who were rejected, as normal students, by either their first or second choices. When students submit their ROLs, they must decide whether or not to pay higher tuition in order to increase the probability of being admitted by their first or second choices. In a simplified example with only two schools (A and B), suppose that student i prefers school A to school B; then she should list school A as her first choice and school B as her second choice. If she chooses the ZX option for school A, this implies that she would rather attend school A as a ZX student (and pay extra tuition) than attend school B as a normal student (assuming that school A rejects her as a normal student); otherwise, she should not choose the ZX option for school A (See Appendix E for a proof of the identification in this example). The same intuition regarding identification can be applied to the case of students who are rejected by their second choices.

There is no closed-form solution to equation (15) because the distribution of the summation of a type I extreme distribution does not itself follow a type I extreme distribution.

We therefore estimate parameters using the maximal simulated likelihood estimate with the logit-smoothed accept–reject simulator. See Train (2009, ch. 5) for the algorithmic procedure; details are given in Appendix F to our paper.

5.3 Estimation Results

Table 5 presents estimated coefficients for the utility function, including our estimates of both $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$. The coefficients for the tuition are estimated from the administrative data; the other coefficients are estimated from our survey data.

Column 3 of Table 5 represents the full model with the school fixed effect. Rows 2-4 of Column 3 show the preferences regarding school quality. Students are classified into three groups based on their exam scores: *high-scoring students*, whose scores are above the 90th percentile; *medium-scoring students*, whose scores are between the 70th and 90th percentile; and *low-scoring students*, whose scores are below the 70th percentile yet above the threshold. The top students are much more sensitive to school quality than the other two student groups. For example, if school quality increases by 1 unit then high-scoring girls are willing to travel an additional distance nearly 0.54 km, 0.2 km for the medium-scoring girls and 0.18 km for the lowing-scoring girls. In the same situation, high-scoring boys are willing to travel 2.75 km more, 1.03 km for the moderate-scoring boys and 0.92 for the lowing-scoring boys.

Different groups of students exhibit similar sensitivities to paying extra tuition fees (normalize to 1,000 yuan) to purchase seats in public high schools, when preferences is normalized by the home-school distance. When school tuition decreases by 1000 yuan, girls are willing to travel an additional 2.1 to 2.4 km, and boys could travel more. However, if we compare the trade-off between school quality and tuition cost, different groups exhibit various attitude to purchase seats. If school quality increases by 1 unit, high-scoring students would prefer to pay an additional 226 yuan, and medium and low-scoring students are only willing to pay 93 and 78 yuan respectively.

The students' valuation of the school capacity which we normalize to 100 seats, varies across student groups. Although all students group prefer small schools when other variables are fixed, medium-scoring students dislike large schools the most. If the school capacity decreases by 100 seats, then medium-scoring students are willing to travel an additional 1.54 km, but high-scoring students only prefer to travel 0.94 km more and 1.19 km for low-scoring students.

Table 5 also reports our estimates for other parameters. Rows 6–8 of column 3 indicate that the high-scoring students have a somewhat unfavorable attitude toward special classes, whereas these classes are viewed positively by the other two student groups. Row 9-10 shows that a student's utility from attending a school increases when her exam score is close to—that is, within 15% of—the average for other students admitted there. This effect reflects peer pressure in schools. Row 15-16 indicates, providing dorm can decrease the students' negative concerns about the travel distance.

6 Model Fit and Robustness Check

6.1 Model Fit

This section examines how well our preference estimates match the data. We conduct within-sample and out-of-sample tests to check the aggregate-level matching patterns. Table 6 compares the actual and predicted admission cutoffs of each high school.[35]

For the within-sample test, column 3 gives schools' predicted cutoffs for 2013. With only two exceptions, the gaps between the actual and predicted cutoff are less than 1% of the full mark (665). Column 6 reports the schools' predicted cutoffs for 2012. The gaps between predicted and actual cutoffs are less than 5.2 points in 12 out of 13 schools, and the other school are about 2.9% of the full mark. The predicted results also correctly identify all the leftover schools, whose cutoff is equal to 530.

For the out-of-sample test, we estimate our parameters for preferences using the procedure described in Section 5 but while excluding the 2012 data. Then, using the newly estimated parameters, we simulate the behavior of students based on their 2012 preference profiles. The schools' predicted cutoffs are reported in column 9 of Table 6. The results are quite similar as column 6. Except one school, the largest gap between predicted and actual cutoff is 5.8 point, which is about 0.87% of the full mark.

Table 7 explores the aggregate-level matching patterns for the first two choices. In the data, 30.7% (resp. 29.7%) of the students were admitted by their first choices in 2013 (resp. 2012); our prediction is (respectively) 30.5% and 30.4%. More specifically, for 2013 we predicted that 18.8% of students are admitted by their first choice as normal students and 11.7% as ZX students; these predictions are close to the actual respective values of 15.6%

[35]The reported results are the admission cutoff for the first round. The actual second-round cutoffs of all popular schools are infinity, and those of all leftover schools are equal to the threshold. Given that our predicted results correctly identify all popular and leftover schools, we report results only for the first-round cutoffs and ignore those for the second round.

and 15%. For 2012, our model predicts that 19.6% of students are admitted by their first choice as normal students and 13.8% as ZX students, where the actual values were 15.5% and 13.8%, respectively. As for the second choices, 38.9% of students in 2013 were admitted by their second choice and 32% were admitted as normal students; the model predicts respective values of 32.3% and 28%. The data for 2012 show that 36.7% of students were admitted by their second choice and that 31% were admitted as normal students; our corresponding predictions are 28.5% and 25%.

In the out-of-sample test, our predicted results for 2012 indicate that 30.4% of students were admitted by their first choice and 19.1% were admitted as normal students; and 29.7% of students were admitted by their second choice and 25.7% were admitted as normal students. In sum, our estimated results predict the admission pattern well for students' first choices but underestimate the percentages for their second choices—especially in the case of normal students, whose admissions the model underestimates by about 4-6%.

6.2 Other Robustness Check

The estimated results in Section 5.3 are based on the rational expectation that students can predict the correct admission probabilities of schools. Because students must submit their ROLs before taking the entrance exam, the primary source of information for students is reasonably assumed to be based on preview year information (adaptive expectations). Column (1) in Table 8 reports the estimated ZX-related parameters when students use the admission cutoffs from the prior year to establish the admission probability.[36] The reported results are fairly similar to those under the rational expectation. These results may result from the stability of admission cutoffs across years.

Students may exhibit different strategic behaviors when the uncertainties of exams faced by them vary. Columns (2)-(4) in Table 8 indicate the estimated ZX-related parameters when the standard deviation of the exam score (δ) is 13.3 (2% of the full mark), 26.6 (4% of the full mark), and 33.35 (5% of the full mark), respectively. These results affirm similarity in the pattern of the results in Table 1: middle-level students are more willing to pay extra tuition (than students in the other groups) to purchase seats.

[36]The estimate of the non-ZX-related parameters does not rely on any assumption about students' beliefs.

7 Counterfactual Analysis

Because of its controversial nature, the ZX policy was cancelled in 2014. To evaluate this policy, we do not directly compare the variation of welfare before and after the policy change. Because, after cancellation of the ZX policy, the City Education Bureau did not simply add the original ZX quota to the normal quota; instead, they considerably increased the quota for early admission.[37] Therefore, we conducted experiments to compare how different assignment mechanisms affect the welfare of students and schools, especially as regards discontinuance of the option to purchase seats.

Using the estimated preferences, we simulate the application list of the students according to their true preferences. In the simulation, we used the profiles of students and schools from the 2014 administrative data. Since there were no ZX students in that year, we treated the normal admission quota as the corresponding school's total capacity. We also excluded the special classes from the choice set of the students, as such classes were not allowed to admit ZX students.[38]

The ratios of ZX quotas to normal quotas in 2012 and 2013 varied from 0.5 to 1.0 across the schools that admitted ZX students. Because we assume that each school's total capacity is equal to its true normal quota in 2014, we simplify the counterfactual analysis by running our experiments under three different setups—namely, with the ZX quota accounting for 10%, 30%, and 50% of the total quota when the focal mechanism includes the option to purchase seats.

Under the DA and SOPS mechanisms, we assumed that students' ROLs report their true preferences. Under other mechanisms, we created ROLs that reflected each student's best response in equilibrium (details are provided in Appendix G). We used 500 simulations in which each student experienced a different vector of random utility shocks.

7.1 Students' Welfare

Our baseline is the welfare of students under the deferred acceptance mechanism. For each tested mechanism, we use an intuitive metric to measure the welfare change of students: the welfare-equalizing tuition adjustment Δ yuan, defined as the amount of tuition that a

[37]The ratios of early admission quotas to normal quotas increased from 0.48 in 2013 to 0.64 in 2014.

[38]The total capacity of special classes accounted for 5% of the total normal admission quota. After excluding them, the total available seats still exceeded (by far) the number of the students who qualified for assignment to public high schools.

student must pay (or be credited) under the DA mechanism to reach the utility level achieved under the new mechanism being tested (Figure 3 and Figure 4).[39]

When the DA mechanism is replaced with the SOPS mechanism, the average welfare of students falls as the ZX quota rises. The red curve in Figure 3 indicates that, overall, students under the DA mechanism need to pay additional 71 yuan (on average) to achieve the same utility level as under the SOPS mechanism when the ZX quota is 10% of the total quota. This loss due to increased tuition becomes 380 yuan (resp., 856 yuan) when the ZX quota rises to 30% (resp., 50%). Different student groups experience a similar welfare loss that grows larger as the ZX quota increases(Figure **??**(c)). In particular, the medium-scoring student group suffers a tuition increase by 127 yuan when the ZX quota is 10% case, and this welfare loss increases to 1209 yuan when the ZX quota is 50% of the total quota.

When the DA mechanism is replaced with the CPPS mechanism, average student welfare decreases slightly; this decreased welfare is equivalent to a 4.5-yuan increase in their tuition (the blue curve in Figure 3). When the ZX quota increases to 30% and 50%, the average welfare of students declines, that is, equivalent to tuition increments of 256.5 and 620 yuan, respectively.

The welfare changes are not constant across the student groups (Figure 4(b)). The top students suffer a welfare loss under the CPPS mechanism. When the ZX quota is 10% of the total, top students would pay 53 yuan more in tuition than those under the DA mechanism—an amount that increases to 303 yuan in the 30% ZX quota case and to 802 yuan in the 50% ZX quota case. Yet medium-scoring students enjoy a welfare gain under the CPPS mechanism: the equivalent of a 41-yuan reduction in tuition with a 10% ZX quota. However, this gain becomes a welfare loss of 376 yuan when the ZX quota increases to 30% and 822 yuan when it increases to 50%. The change in welfare of low-scoring students is similar to that among the high-scoring students. Specifically their tuition increases by 8 yuan under the 10% ZX quota, by 103 yuan under the 30% ZX quota and by 275 yuan under the 50% ZX quota.

When the BMPS mechanism is adopted to replace the DA mechanism (Figure 4(c)), high and medium-scoring students still experience a welfare loss that grows larger as the ZX quota increases. In contrast, the welfare of low-scoring students is better under the

[39]We fix the other parameters (except for tuition). Formally, let $u_{ij} = U(c_{ij})$ be i's utility derived from admittance to school j when paying tuition c_{ij} under the DA mechanism. If that mechanism is replaced by the focal new mechanism and so student i is assigned to school j'—and achieves utility $u_{ij'}$—then the welfare-equalizing tuition adjustment (Δyuan) is the solution to $U(c_{ij} + \Delta\text{yuan}) = u_{ij'}$.

BMPS mechanism: their tuition would be reduced by 195 yuan when the ZX quota is 10% of the total quota or by 171 yuan and 166 yuan when that quota is (respectively) 30% and 50%.

These results confirm that, students' welfare declines somewhat on average and the welfare loss occurs across all student groups when the DA mechanism is changed into the SOPS mechanism. However, replacing the DA mechanism with the CPPS mechanism may benefit medium-scoring students when the ZX quota is low. As that quota increases, their benefit may be reduced—or even become a loss. Adoption of the BMPS mechanism improves the welfare of the low-scoring group, but the other two student groups invariably suffer a welfare loss when purchasing seats is an option.

Table 9 and Table 10 identify (respectively) the percentage of "winners" and "losers" when the DA mechanism is replaced by a mechanism under which seats may be purchased. All cases have fewer winners (who enjoy increases in welfare) than losers (who suffer decreases) among high- and medium-score students. The proportion of winners among the high-scoring students is only 3.02% when the ZX quota is 10% under the SOPS mechanism, This amount never exceeds 6.5% in all other cases—and the number of losers is at least double the number of winners in this student group. When the ZX quota is 50%, nearly half of the high-scoring students are worse-off. Low-scoring students have more losers than winners when the DA mechanism is replaced with the SOPS or CPPS mechanism, but the winners outnumber the losers when the BMPS mechanism is adopted.[40]

7.2 Schools' Welfare

There are several reasons why the changes in schools' welfare have been ignored in the school choice literature. First, no mechanism can be optimal for both sides of a matching mechanism (Gale and Shapley (1962)). Second, improved matching results for students (e.g., increasing their average welfare) is the primary goal of most studies addressing the school choice problem; this generalization holds in particular for public school systems. Third, the welfare of public schools—especially at the elementary and secondary level—is difficult to measure when their admission criteria are "coarse", for instance, walking distance and/or whether siblings are in the same school are used as the admission criteria.

Nevertheless, it is important to analyze schools' welfare. There is intense competition among schools with regard to admissions, and even more so following compulsory education.

[40]The amounts of the gains and losses in welfare are given by, respectively, Table 14 and Table 15 in Appendix H.

Although schools cannot make strategic moves in a centralized admission system, they still prefer to admit students of high quality. It follows that schools, which suffer a welfare loss when admission mechanisms undergo certain types of reform, have a strong incentive to block such reforms. When exam scores are used as the criterion for admission, it is easy to compare how different admission mechanisms affect schools' welfare.

The purpose of allowing schools to sell seats is to increase profit. The ZX policy offers two ways of analyzing school welfare: the quality of admitted students and the tuition collected by schools. As illustrated by the example in Section 2, the ZX policy may impose a trade-off. On the one hand, allowing students to buy seats will likely increase the income of schools; on the other hand, seat purchasing has the effect of high-quality students being dispersed more widely among different schools.

Table 11 presents the change in the average scores of admitted students and tuition collection across different mechanisms for all schools. Figure 5 plots the changes in tuition and quality for a upper-tier school (#183), a middle-tier school (#185), and a lower-tier school (#142). For the upper-tier school (Figure 5(a)), the collected fees increase in proportion to the ZX quota when the DA mechanism is replaced by the SOPS mechanism. This school collects approximately 10% more tuition when the ZX quota is 10% as well as 30% when the ZX quota is increased to 30%. When the ZX quota is 50%, the tuition collection increases to more than 60% under SOPS and BMPS, but only around 30% under CPPS.

When the DA mechanism is replaced by the SOPS mechanism in the 10% ZX quota case, student quality declines by only 0.06% (measured by the percentage grades), although it declines by an additional 0.51% (resp. 1.2%) when the ZX quota is 30% (resp. 50%). When the CPPS or the BMPS mechanism is adopted to replace the DA mechanism, the decrease in student quality does not exceeds 1.05%. In view of our findings for the second good school listed in Table 11, the demand for upper-tier schools is clearly such that they can profit significantly by selling seats yet without lowering the quality of admitted students.

Figure 5(b) illustrates the case of a middle-tier school. When the ZX quota is 10%, the collected tuition increases about 10% under all three mechanisms that include the option to purchase seats. If the ZX quota is increased to 50% then, under the SOPS and BMPS mechanisms, we observe a 43-48% increase in tuition collected compared with the baseline case and an increase of more than 60% under the CPPS mechanism. As for student quality, the change from the DA mechanism to the SOPS mechanism does not alter the quality of admitted students by more than 1% when the ZX quota is 30% or 50%. However, when the ZX quota increases to 50%, the student quality increases by 3.6%.

If the DA mechanism is replaced by the CPPS mechanism, then student quality declines by more than 1% and decreases even further as the ZX quota increases. Under the BMPS mechanism, student quality is reduced by about 4-5%. Combined with our findings for the other middle-tier schools (i.e., the third through the seventh schools listed in Table 11), the results indicate that the purchasing seat option may generate significant profits for most middle-tier schools, but several schools may still experience profit under specific cases. Meanwhile, middle-tier schools may experience large variations in the quality of admitted students in positive and negative directions.

Figure 5(c) plots our findings for a lower-tier school. When the ZX quota is 10%, the school collects approximately 13% extra tuition when the DA mechanism is replaced by the SOPS mechanism. When the ZX quota increases to 30% (resp. 50%), this school collects 60% (resp. 237%) extra tuition. At the same time, student quality experiences a slight decrease with the increasing ZX quota. The large change in tuition collection indicates that a number of students switch from middle-tier schools to low-tier schools to fill their empty seats when the ZX quota increases.

When the CPPS mechanism is used to replace the DA mechanism, tuition collection experiences a similar trend as that under the SOPS mechanism but at a reduced magnitude. Under this mechanism, student quality also decreases slightly when the ZX quota increases. When the DA mechanism is replaced by the BMPS mechanism, this school collects increased tuition (3.38%) when the ZX quota is 10%, but this profit increase becomes negative when the ZX quota increases to 30% and 50%. Meanwhile the student quality of this school experience a slight decreases.

Table 12 presents the standard deviation (S.D.) of student quality for each school across the various mechanisms. The S.D. of student quality for the best school (#183) under the DA mechanism is 0.015, and this value increases as school quality declines for middle-tier schools but decreases for lower-tier schools. Under the SOPS mechanism, the S.D. of student quality for most schools increases and does not vary significantly when the ZX quota grows. Under the CPPS mechanism, S.D. gradually increases with the ZX quota—a trend that is magnified under the BMPS mechanism.

In summary, the seat-purchasing option significantly increases the tuition collection for upper-tier schools with a minimal influence on student quality. This condition implies a strong demand of the most popular schools. However, middle-tier schools can also benefit from the tuition increase from the ZX policy, but experience a large uncertainty in the change of student quality. For lower-tier schools, the influence of the ZX policy is uncertain on both tuition collection and student quality.

8 Conclusion

This paper investigates a controversial but long-ignored Chinese school choice policy, *Ze Xiao*. This policy allowed students to "purchase" seats in their desired schools by paying higher tuition. Our theoretical results indicate that the associated matching mechanisms likely resulted in unstable and/or inefficient matching outcomes for students.

We combine high school admission records with survey data to estimate student preferences over schools and tuition. In the estimation, we first use the survey data to estimate student preferences on schools without considering students' strategic behavior when submitting their rank-ordered lists of preferred schools. Next, we use information from those ROLs to estimate the parameters of student preferences vis-à-vis the ZX policy. Our results indicate that students give considerable weight to the quality of potential high schools but largely ignore the difficulty caused by travel distance. We also find that students who receive low scores on the entrance exam are more willing than other students to pay an extra cost (higher tuition) to secure seats in their preferred schools.

Using the estimated preferences, we conduct several counterfactual experiments to evaluate the welfare consequences, for both students and the schools, under different assignment mechanisms. Our results indicate that when the strategy-proof SOPS mechanism replaces the deferred acceptance mechanism, students' welfare decreases overall—and this effect is consistent across different student groups. However, replacing the DA mechanism with a non–strategy-proof mechanism (e.g., CPPS or BMPS) may affect students differently depending on the group (high-, medium-, or low-scoring) to which they belong. The high-scoring students in this scenario almost always experience a welfare loss, especially when a large proportion of seats are assigned to ZX students. In contrast, the medium-scoring students may benefit from the option to purchase seats under the CPPS mechanism when the ZX quato is low. The low-scoring student experience a welfare gain under the BMPS mechanism across all ZX quota setup.

From the school's perspective, using the upper-tier schools generally results in a considerable increase in collected tuition and with only a limited decline (relative to the DA mechanism) in the quality of their admitted students. However, middle-tier school may collect significant amount tuition under the seat-purchasing option on the one hand, on the other hand, they could also face a large uncertainty of student quality change.

Our findings should encourage scholars to apply a matching-with-contracts approach when addressing the school choice problem. An extension that merits exploring would be to model the effect of financial aid on the school choice problem when students at the same school may pay different tuition amounts. Another research avenue worth investigating is the integration of private schools into the centralized school choice system when private schools have the flexibility to adjust their tuition.

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Table 1: School Characteristics

	2012			2013			2014		
	mean(s.d)	max	min	mean(s.d)	max	min	mean(s.d)	max	min
Quality	80(12)	97	64	81(12)	97	64	83 (11)	97	66
Normal Quota	215.9 (182.6)	600	40	197 (183.8)	600	40	186.4 (164.5)	600	40
ZX Quota	94.8(37.9)	146	22	101.3(33.1)	142	37			
♯ of schools with dorms	9			11			13		
♯ of schools	16			18			19		

Notes: Schools and special classes that did not admit ZX students are excluded when the ZX quota is calculated.

Table 2: Distributions of High School Entrance Exam Scores

Percentile	2012	2013	2014
90th	597	590.5	598
80th	579.5	572	578
70th	562	553	557.5
60th	542	531	532.5
Threshold	535	530	535

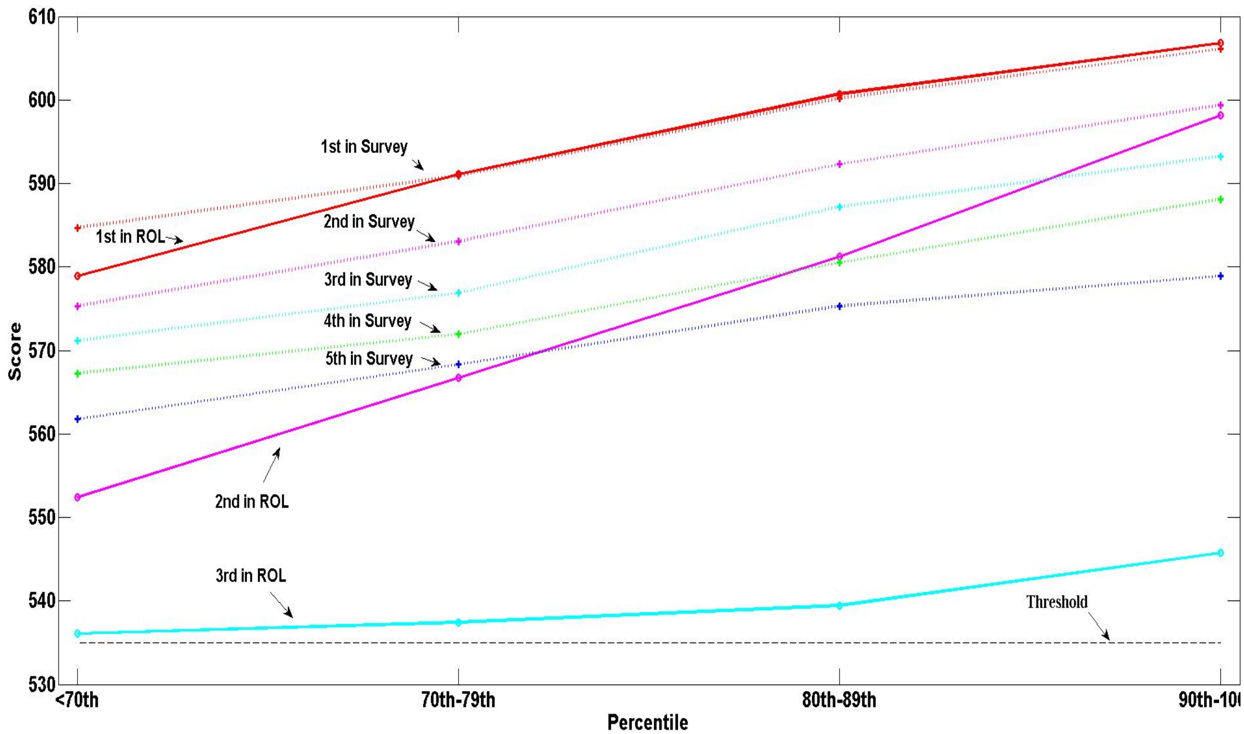
Table 3: School Lists and Assignments

	2012		2013		2014	
	Freq.	Percent	Freq.	Percent	Freq.	Percent
Rank Ordered Lists						
3 Schools	3696	94.33%	3793	95.04%	3100	93.71%
2 Schools	191	4.87%	167	4.18%	189	5.71%
1 Schools	31	0.79%	31	0.78%	19	0.57%
Assignment Results						
1st Choice	1153	29.43%	1227	30.74%	875	26.45%
ZX students	542	13.83%	599	15.01%		
2nd Choice	1441	36.78%	1545	38.71%	1290	39%
ZX students	217	5.54%	262	6.56%		
3rd Choice	803	20.50%	751	18.82%	565	17.08%
ZX students	0	0	0	0		
Rejected by all 3	521	13.30%	460	11.53%	578	17.47%
Total observations	3918		3991		3308	

Table 4: Survey Length

	Freq.	Percent
5 schools	900	62.20%
4 schools	242	16.72%
3 schools	130	8.98%
2 schools	175	12.09%
Total	1447	100%

Figure 1: Average Admission Cutoffs of Schools: Survey versus ROLs



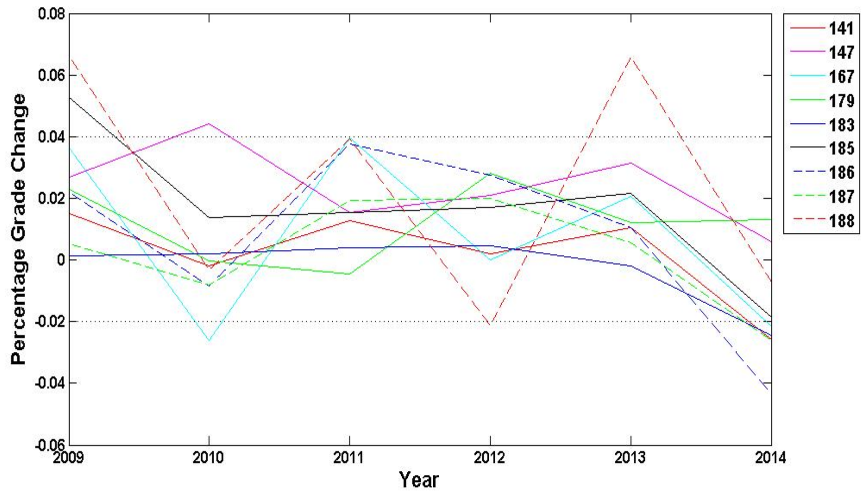
Notes: This figure indicates the admission cutoffs of schools chosen by students in their ROLs and survey. The y-axis represents absolute scores, and the x-axis represents the students' exam scores in percentile. Students are separated into four groups according to their scores: above 90%, 80-90%, 70-80% and below 70%. The threshold for public high school admission is 535 (60.95 percentile) in 2014.

Table 5: Preference Parameters

	No student interactions		With student interactions	
	(1)	(2)	(3)	(4)
Quality	0.835***	0.296***		
	(0.011)	(0.020)		
Quality × H			0.539***	0.688***
			(0.155)	0.032
Quality × M			0.201***	0.376 ***
			(0.038)	(0.012)
Quality × L			0.181***	0.361***
			(0.030)	(0.014)
Special class	-1.006***	-2.121**		
	(0.325)	(1.015)		
Special class × H			-6.675***	-2.657***
			(1.972)	(0.560)
Special class × M			0.602	1.204***
			(1.592)	(0.342)
Special class × L			6.504	5.300***
			(5.591)	(1.193)
Score range			0.597	0.216
			(0.430)	(0.183)
Score range × Male			0.315	0.898***
			(0.550)	(0.220)
Same district			-1.896***	-2.401***
			(0.247)	(0.107)
Same district × Male			1.739***	2.586***
			(0.309)	(0.143)
Distance	-1	-1	-1	-1
Distance × Male			0.804***	0.933***
			(0.034)	(0.010)
Dorm	-3.924***	4.253***	4.445***	-0.907***
	(0.119)	(0.967)	(1.095)	(0.137)
Dorm × Male			0.684*	0.756***
			(0.307)	(0.164)
Capacity	-0.011	-1.969***		
	(0.055)	(0.136)		
Capacity × H			-0.941	0.217
			(0.835)	(0.318)
Capacity × M			-1.542***	-0.632***
			(0.291)	(0.081)
Capacity × L			-1.190***	-0.540***
			(0.237)	(0.062)
Cost	-2.878***	-2.370***		
	(0.001)	(0.002)		
Cost × H			-2.388***	-1.676***
			(0.008)	(0.004)
Cost × M			-2.156***	-1.910***
			(0.003)	(0.003)
Cost × L			-2.309***	-2.422***
			(0.007)	(0.004)
Non-public high school	43.909***	2.005 *	1.347*	13.364***
	(0.946)	(0.799)	(0.653)	(1.115)
School Fixed Effect		Y	Y	

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Distance is measured by kilometre. Both normal and ZX quotas are normalized to 100 seats. Tuition is normalized to 1000 Yuan.

Figure 2: Fluctuation of Admission Cutoffs



Notes: This figure indicates the fluctuation of admission cutoffs of schools as measured by percentage grade. The y-axis represents the change in school cutoff from the previous year, and the x-axis represents the year.

Table 6: Admission Cutoffs

	Within Sample						Out of Sample		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
School	True 2013	Predicted	Diff.	True 2012	Predicted	Diff.	True 2012	Predicted	Diff.
141	604.0	597.8	6.2	607.0	603.0	4.0	607.0	602.8	4.2
142*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
147	552.5	562.6	-10.1	555.5	560.6	-5.1	555.5	559.8	-4.3
167	590.0	588.6	1.4	592.5	592.9	-0.4	592.5	592.1	0.4
173*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
179	565.0	572.8	-7.8	571.5	570.9	0.6	571.5	570.7	0.79
181*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
183	611.0	605.1	5.9	617.0	612.2	4.9	617.0	612.2	4.8
184*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
185	580.0	576.1	3.9	583.0	580.6	2.4	583.0	580.5	2.5
186	578.0	574.0	4	583.0	577.3	5.7	583.0	577.2	5.8
187	594.5	595.1	-0.6	599.5	600.1	-0.6	599.5	599.7	-0.2
188	575.0	580.1	-5.1	571.5	591.1	-19.6	571.5	590.5	-19

Notes: This table indicates the within- and out-of-sample tests for the schools' cutoffs. The full mark is 665. The threshold is 535 in 2012 and 530 in 2013. * indicates the leftover schools with cutoff equal to the threshold.

Table 7: Admission Patterns (%)

	Within Sample						Out of Sample		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Data 2013	Predeted	Diff.	Data 2012	Predeted	Diff.	Data 2013	Predeted	Diff.
Total 1st Choice	30.7	30.5	0.3	29.4	30.8	-1.3	29.4	30.4	-1.0
Normal 1st	15.6	18.8	-3.2	15.5	19.6	-4.1	15.5	19.1	-3.6
Top	10.6	12.4	-1.7	10.0	11.8	-1.9	10.0	11.8	-1.9
Middle	3.7	6.3	-2.7	3.9	6.6	-2.7	3.9	6.3	-2.4
Low	1.3	0.1	1.2	1.7	1.2	0.5	1.7	1.0	0.6
ZX 1st	15.0	11.7	3.4	13.8	11.2	2.7	13.8	11.4	2.4
Top	5.0	4.4	0.6	4.9	5.1	-0.2	4.9	5.5	-0.7
Middle	8.2	6.7	1.5	7.5	5.7	1.8	7.5	5.5	2.0
Low	1.9	0.6	1.3	1.5	0.4	1.1	1.5	0.4	1.1
Total 2nd choice	38.9	32.4	6.4	36.7	28.5	8.2	36.7	29.7	7.0
Normal 2nd	32.0	28.0	4.0	31.0	25.0	6.0	31.0	25.7	5.3
Top	6.2	4.1	2.1	6.3	3.5	2.8	6.3	3.1	3.2
Middle	18.1	21.6	-3.4	14.7	17.5	-2.8	14.7	18.3	-3.6
Low	7.6	2.3	5.3	10.0	3.9	6.0	10.0	4.4	5.6
ZX 2nd	6.6	4.4	2.2	5.5	3.6	2.0	5.5	4.0	1.6
Top	0.2	0.5	-0.3	0.2	0.1	0.1	0.2	0.1	0.1
Middle	4.6	3.3	1.3	4.0	3.2	0.9	4.0	3.5	0.5
Low	1.8	0.6	1.3	1.3	0.3	1.0	1.3	0.3	1.0

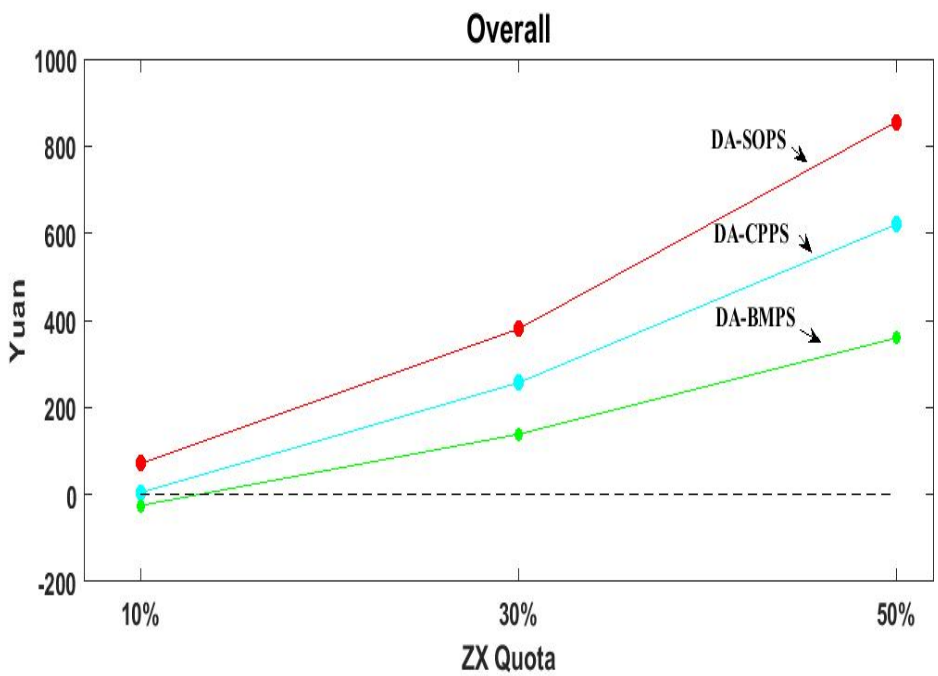
Notes: This table indicates the within- and out-of-sample test of the matching patterns for the 1st and 2nd choices in the ROLs.

Table 8: Robustness Check

	(1)	(2)	(3)	(4)
Extra Tuition*Top_Stu	-2.27***	-2.41***	-2.58***	-2.75***
	(0.007)	(0.005)	(0.018)	(0.011)
Extra Tuition*Mid_Stu	-2.01***	-2.03***	-2.27***	-2.50***
	(0.001)	(0.003)	(0.001)	(0.007)
Extra Tuition*Low_Stu	-2.12***	-2.39***	-2.41***	-2.62***
	(0.007)	(0.004)	(0.009)	(0.01)

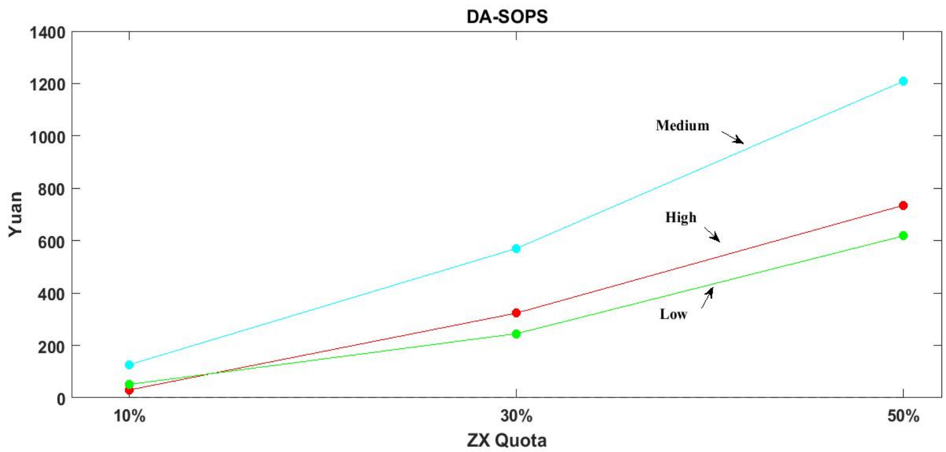
Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Column (1) reports the estimated coefficients under adaptive expectation. Columns (2)-(4) report the estimated coefficients when δ is 13.3, 26.6 and 33.25 respectively.

Figure 3: Change in Students' Welfare

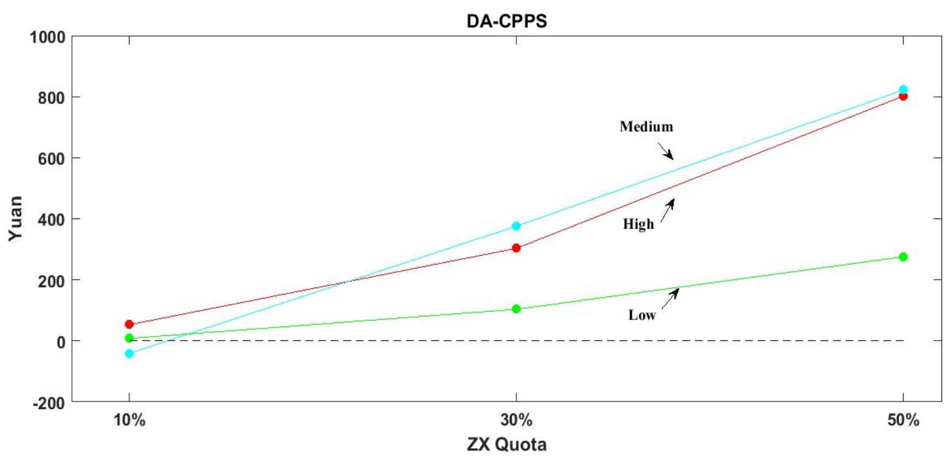


Notes: This figure indicates the change in students' welfare when the DA mechanism is replaced by the SOPS, CPPS, and BMPS mechanisms. The circles above zero represent welfare loss that the students need to pay more tuition under the DA mechanism to achieve the same level of utility under the new mechanism, and the circles below zero represent welfare gain.

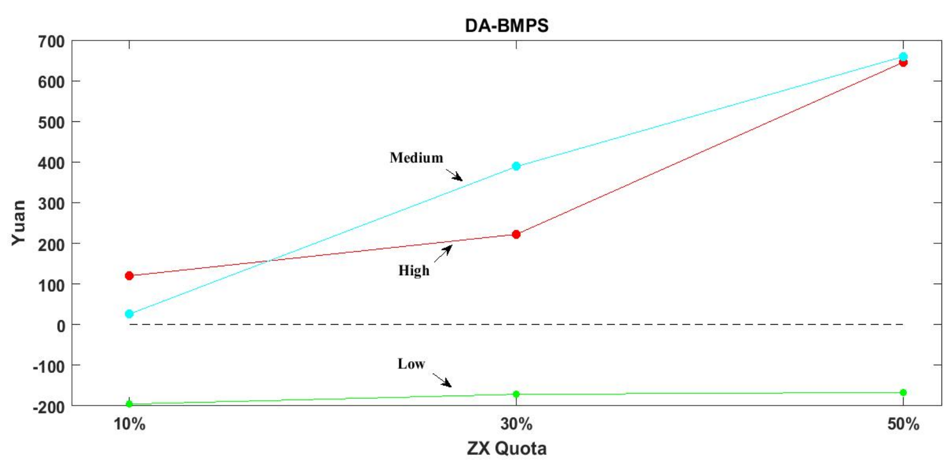
Figure 4: Change in Students' Welfare by Groups



(a)



(b)



(c)

Table 9: Number of Winners (%)

	DA-SOPS			DA-CPPS			DA-BMPS		
	10%	30%	50%	10%	30%	50%	10%	30%	50%
Overall	3.23	3.27	1.73	6.64	4.84	3.04	15.43	12.30	11.45
High	3.02	3.79	2.41	3.33	3.43	2.50	6.13	5.34	3.16
Medium	5.11	5.11	2.74	12.55	8.42	5.25	20.93	12.92	12.23
Low	1.59	1.07	0.18	3.71	2.59	1.36	17.90	17.51	17.61

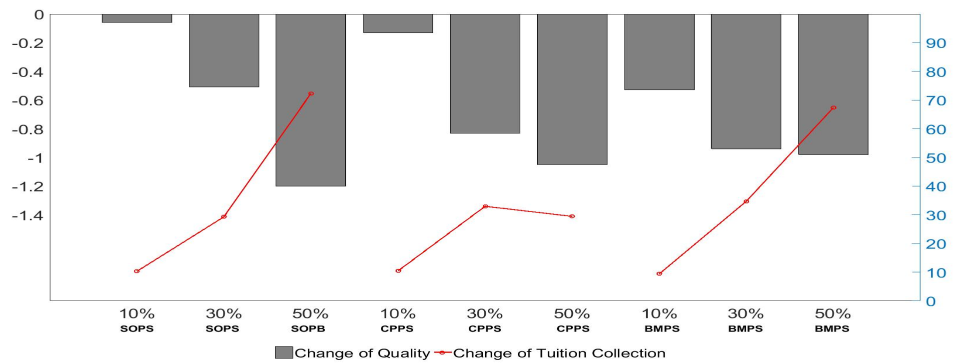
Notes: This table indicates the percentage change in the number of students whose utilities decrease when the DA mechanism is replaced by the SOPS, CPPS, and BMPS mechanisms. For each mechanism change, utility changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. "Top" represents students whose scores are above the 90th percentile, "Middle" represents students whose scores are between the 70th and 90th percentiles, and "Low" represents students whose scores are below the 70th percentile and above the threshold.

Table 10: Number of Losers (%)

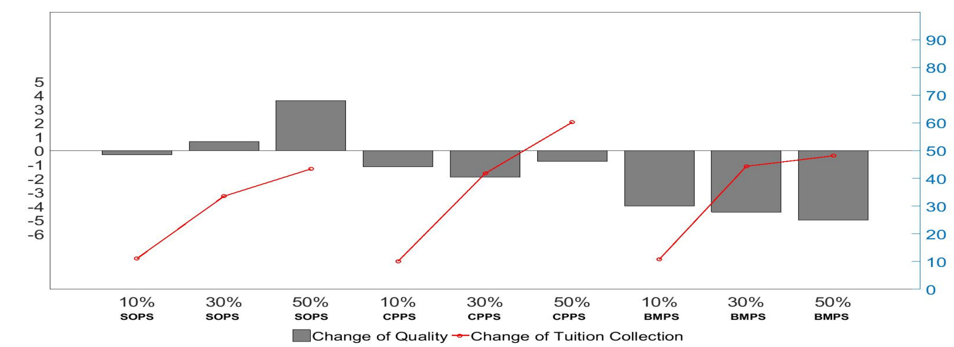
	DA-SOPS			DA-CPPS			DA-BMPS		
	10%	30%	50%	10%	30%	50%	10%	30%	50%
Overall	10.25	37.27	64.21	11.17	26.06	40.04	16.16	24.10	33.91
High	5.32	29.57	54.00	9.40	28.66	53.36	15.82	28.18	49.40
Medium	18.26	54.18	79.98	20.01	43.69	57.28	28.18	41.30	49.70
Low	6.68	27.48	57.60	4.16	7.01	12.45	4.93	4.23	5.87

Notes: This table indicates the percentage change in the number of students whose utilities decrease when the DA mechanism is replace by the SOPS, CPPS, and BMPS mechanisms. For each mechanism change, utility changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. "Top" represents students whose scores are above the 90th percentile, "Middle" represents students whose scores are between the 70th and 90th percentiles, and "Low" represents students whose scores are below the 70th percentile and above the threshold.

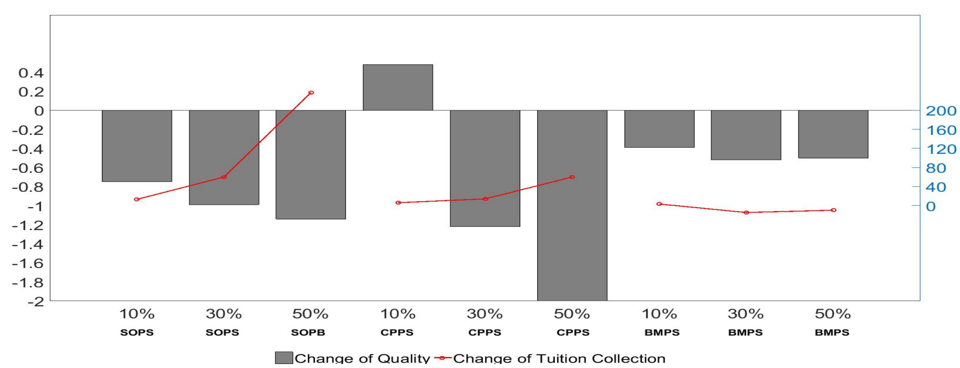
Figure 5: Quality-Tuition Tradeoff



(a) 183



(b) 185



(c) 142

Notes: These figures indicate the change of tuition collection and the students quality when the DA mechanism is replaced by the SOPS, CPPS and BMPS mechanisms for three schools, 183, 186 and 184. Left y-axis represents the percentage change of tuition collection, right y-axis represents the percentage change of the students quality. For each mechanism change, the utility changes are measured in three cases in which the ZX quotas are 10%, 30%, and 50% of the total quotas.

Table 11: Tuition Collection vs Student Quality (%)

School		DA-SOPS			DA-CPPS			DA-BMPS		
		10%	20%	30%	10%	20%	30%	10%	20%	30%
183	Δ Tuition	10.27	29.28	72.32	10.43	32.95	29.44	9.37	34.71	67.34
	Δ Qual.	-0.06	-0.51	-1.20	-0.13	-0.83	-1.05	-0.53	-0.94	-0.98
141	Δ Tuition	11.35	39.77	43.33	12.26	43.91	71.26	10.83	38.54	73.40
	Δ Qual.	-0.07	0.38	0.92	-0.76	-0.57	0.49	-0.46	-0.06	0.00
187	Δ Tuition	-1.04	-16.30	-25.45	1.76	-11.32	-14.85	5.66	18.35	46.71
	Δ Qual.	0.35	1.67	2.32	-0.46	1.17	1.20	-0.42	-0.03	0.32
167	Δ Tuition	10.61	34.92	3.75	11.69	44.30	74.62	9.77	38.50	73.21
	Δ Qual.	0.59	2.12	5.48	1.04	2.77	4.81	0.42	1.59	1.55
185	Δ Tuition	11.05	33.56	43.41	10.09	41.76	60.23	10.77	44.35	48.14
	Δ Qual.	-0.31	0.66	3.61	-1.17	-1.92	-0.77	-4.00	-4.43	-5.02
186	Δ Tuition	10.01	5.65	-33.15	6.51	16.27	33.77	9.00	8.45	16.27
	Δ Qual.	0.74	3.90	12.06	1.25	3.95	7.00	-2.64	-1.87	-1.78
179	Δ Tuition	12.10	18.55	-3.87	9.38	15.36	22.87	10.29	11.15	9.12
	Δ Qual.	0.40	3.50	9.23	0.99	2.92	4.08	-3.56	-2.71	-3.20
184	Δ Tuition	-3.87	-0.79	11.23	-3.61	4.25	38.77	-4.25	-3.04	0.51
	Δ Qual.	-0.51	-0.59	-0.17	-1.13	-2.34	-3.20	-0.19	-0.49	0.05
147	Δ Tuition	-4.86	-15.49	-34.29	1.13	3.84	4.84	2.55	1.59	-1.26
	Δ Qual.	0.81	4.04	10.25	0.55	1.51	2.89	4.45	3.60	3.71
181	Δ Tuition	22.18	102.13	340.05	0.28	6.93	23.46	-5.39	-1.81	0.27
	Δ Qual.	-0.91	-1.33	-1.11	0.24	0.28	0.38	0.69	1.80	1.57
173	Δ Tuition	17.54	82.14	153.26	4.85	15.39	26.69	-0.85	5.92	12.91
	Δ Qual.	-0.80	-1.03	2.45	0.85	1.43	2.20	3.26	3.76	4.10
142	Δ Tuition	13.12	59.95	237.27	6.11	14.13	60.03	3.38	-14.48	-9.60
	Δ Qual.	-0.75	-0.99	-1.14	0.48	-1.22	-2.00	-0.39	-0.52	-0.50

Notes: This table indicates the percentage change in the tuition collection and the student quality when the DA mechanism is replaced by the SOPS, CPPS, and BMPS mechanisms. For each mechanism change, utility changes are measured in three cases in which the ZX quotas are 10%, 30%, and 50% of the total quotas.

Table 12: Standard Deviation of Student Quality

School	DA	SOPS			CPPS			BMPS		
		10%	20%	30%	10%	20%	30%	10%	20%	30%
School	DA									
		10%	20%	30%	10%	20%	30%	10%	20%	30%
183	0.015	0.016	0.027	0.031	0.017	0.033	0.041	0.020	0.025	0.025
141	0.026	0.028	0.032	0.035	0.037	0.044	0.035	0.040	0.040	0.039
187	0.035	0.035	0.025	0.023	0.050	0.037	0.048	0.050	0.045	0.039
167	0.044	0.043	0.048	0.038	0.052	0.049	0.039	0.055	0.053	0.053
185	0.042	0.042	0.050	0.056	0.051	0.067	0.069	0.056	0.068	0.066
186	0.049	0.053	0.057	0.047	0.062	0.064	0.069	0.069	0.070	0.069
179	0.040	0.040	0.047	0.043	0.052	0.057	0.057	0.055	0.056	0.056
184	0.035	0.032	0.033	0.037	0.030	0.035	0.035	0.046	0.045	0.048
147	0.037	0.037	0.038	0.039	0.042	0.041	0.046	0.077	0.067	0.067
181	0.032	0.035	0.035	0.039	0.033	0.037	0.041	0.048	0.053	0.055
173	0.030	0.032	0.033	0.039	0.036	0.040	0.047	0.058	0.059	0.061
142	0.028	0.029	0.028	0.033	0.028	0.031	0.030	0.033	0.037	0.039

Notes: This table indicates the standard deviation (s.d.) of the admitted students quality under different mechanisms. Except the DA mechanism, the standard deviations are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas.

Appendices

A The Boston Mechanism with Purchasing Seats Option (BMPS)

Formally the BMPS selects the matching outcome as follows:

Round 1: Each student applies to her first choice. Each school j admits the top q_j^a students based on the normal priority ($\succ$). Among the remaining applicants, j admits the top q_j^z students based on the ZX priority ($\succ^+$). The rest are rejected. All assignments are final and the quotas of each school, q_j^a and q_j^z, are reduced by the number of the students permanently assigned to it in this round as respectively. The new quotas are denoted as $q_{j,2}^a$ and $q_{j,2}^z$.

Round $k > 1$: Each rejected student applies to her kth choice. Each school j admits the top $q_{j,k}^a$ students based on the the normal priority. Among the remaining applicants, its admits the top $q_{j,k}^z$ students based on the ZX priority. The rest are rejected. All assignments are final and the quotas of each school $q_{j,k}^a$ and $q_{j,k}^z$ are reduced by the number of students permanently assigned to it in this round respectively. The new quotas are denoted as $q_{j,k+1}^a$ and $q_{j,k+1}^z$.

The algorithm terminates when there are no students left. The students who receive the normal seats pay tuition c_0. The student who receive ZX seats and choosing ZX option pay tuition c_1, while those who receive ZX seats but not choose ZX option pay basic tuition c_0.

In summary, the BMPS simply extends the Boston Mechanism buy adding a stage to assign ZX seats to students in each round. In this newly added stage, the students choosing ZX options have higher priority to receive the seats for sell than those who do not. Similar as the BM, choosing the first choice school is very important for the students. If a student put school j in her second choice, then she loses the priority in j than those who put j in the first choice regardless whether choose the ZX option.

B Mathematical Proofs

Proof of Proposition 1. For the CPPS mechanism with permanency-execution period $(e_1, e_2, ...)$ with $e_1 \geq 1$. There are three students i_1, i_2, i_3 and three high schools j_1, j_2, j_3 with one ZX seat each school and no normal seat. Students are ordered as $i_1 \succ i_2 \succ i_3$ by schools under the normal priority. Suppose that the true preference of student i_3 over schools and tuitions are as follows:

$$(j_1, c_0)\pi_{i_3}(j_2, c_0)\pi_{i_3}(j_1, c_1)\pi_{i_3}(j_2, c_1)\pi_{i_3}(j_3, c_0).$$

So student i_3's true preference over schools is $j_1\widetilde{\pi}_{i_3}j_2\widetilde{\pi}_{i_3}j_3$.

We need to show that no truthful strategy weakly dominates all other strategies.

Case 1: i_3 chooses the ZX option for j_1.

Given i_2 and i_3 choose the same strategy as $\{(j_1, 0), (j_3, 0), (j_2, 0)\}$, where 1 represents choosing the ZX option for the school and 0 otherwise.

If i_3 chooses the strategy as $\{(j_1, 1), (j_2, 0), (j_3, 0)\}$, then i_3 will receive the assignment (j_1, c_1). If i_3 switches to the strategy $\{(j_2, 0), (j_1, 1), (j_3, 0)\}$, she gets better off by receiving the assignment (j_2, c_0).

Case 2: i_3 does not choose the ZX option for j_1.

Given i_1's strategy as $\{(j_1, 0), (j_2, 0), (j_3, 0)\}$, and i_2's strategy as $\{(j_2, 0), (j_1, 0), (j_3, 0)\}$.

Subcase 2.1: $e_1 > 1$.

i_3 cannot receive an assignment better than (j_2, c_1) if she put j_1 as the first choice and does not choose the ZX option for it, because her normal priority is lower than i_1 and i_2. In this situation, if i_3 switches to the strategy $\{(j_2, 0), (j_1, 1), (j_3, 0)\}$, she gets better off by receiving the allocation (j_1, c_1).

Subcase 2.2: $e_1 = 1$.

In this mechanism, i_3 will be assigned to j_3 if she put j_1 as the first choice. If she switches to the strategy $\{(j_2, 1), (j_1, 0), (j_3, 0)\}$, then she gets better off by receiving the allocation (j_2, c_1).

Therefore, revealing the true preference over schools may not be a dominant strategy for i_3. □

Proof of Proposition 2. There are four students i_1, i_2, i_3, i_4 and four schools j_1, j_2, j_3, j_4 with one ZX seat each and no normal seat. Schools order the students in the same way as $i_1 \succ i_2 \succ i_3 \succ i_4$. Students' preferences are as follows:

π_{i_1}: $(j_1, c_0)\pi_{i_1}(j_2, c_0)\pi_{i_1}(j_1, c_1)\pi_{i_1}(j_3, c_0)\cdots$

π_{i_2}: $(j_1, c_0)\pi_{i_2}(j_1, c_1)\pi_{i_2}(j_2, c_0)\pi_{i_2}(j_2, c_1)\pi_{i_2}(j_4, c_0)\pi_{i_2}(j_4, c_1)\pi_{i_2}(j_3, c_0)\pi_{i_2}(j_3, c_1)$.

π_{i_3}: $(j_1, c_0)\pi_{i_3}(j_3, c_0)\pi_{i_3}(j_1, c_1)\pi_{i_3}(j_2, c_0)\pi_{i_3}(j_3, c_1)\pi_{i_3}(j_2, c_1)\pi_{i_3}(j_4, c_0)\pi_{i_3}(j_4, c_1)$.

π_{i_4}: $(j_4, c_0)\pi_{i_4}(j_2, c_0)\pi_{i_4}(j_4, c_1)\pi_{i_4}(j_2, c_1)\cdots$

Consider the following strategy profile under the CPPS mechanism:

$a_{i_1} = \{(j_1, 1), (j_2, 0), (j_3, 0), (j_4, 0)\}$,

$a_{i_2} = \{(j_1, 0), (j_2, 0), (j_4, 1), (j_3, 0)\}$,

$a_{i_3} = \{(j_1, 1), (j_3, 0), (j_2, 0), (j_4, 0)\}$,

$a_{i_4} = \{(j_4, 0), (j_2, 1), (j_1, 0), (j_3, 0)\}$.

Then the matching outcome is

$\{(i_1, j_1, c_1), (i_2, j_2, c_0), (i_3, j_3, c_0), (i_4, j_4, c_0)\}$.

This strategy profile is a Nash equilibrium but not stable. Because i_1 prefer (j_2, c_0) to her assignment (j_1, c_1), and under the normal priority j_2 prefers i_1 to i_2. Furthermore, this outcome is Pareto dominated by the outcome of the SOPS mechanism

$\{(i_1, j_2, c_0), (i_2, j_1, c_1), (i_3, j_3, c_0), (i_4, j_4, c_0)\}$. □

Proof of Proposition 3. Part 1: For any Nash equilibrium strategy profile $(a_1, ..., a_n)$ and matching outcome τ of the BMPS mechanism, suppose τ is not stable under the true preference. Then there is an contract (i, j, c) such that student i prefers assignment (j, c) to her assignment in τ and either school j has an empty seat for tuition c or i has higher priority at school j than another student who receives a seat with tuition c. In the first case, the unstable matching implies i does not put j as the first choice if $c = c_0$, then i can move school j to the first choice and receives the assignment (j, c). In the second case, if $c = c_1$, the unstable matching implies either i does not choose j as the first choice and choose the ZX option for it. Then i can put j as the first choice and choose the ZX option for it, and i can receive the assignment (j, c). In either case, student i has the incentive to deviate, so the matching result is not an equilibrium.

For a stable matching outcome τ, student i's assignment is (j, c). Then consider a strategy profile A as follow, if $c = c_0$, then student i put j as the first choice, if $c = c_1$, then student i put j as the first choice and choose the ZX option for it. Under this profile, every student receives the assignment in the first round and receives the same assignment as in τ. For student i, if she prefer an assignment (j', c') to the current assignment (j, c), since τ is stable, it implies the seats with tuition c' in school j' have assigned to other students who have higher priority to receive the seats. When $c' = c_1$, since the students who receive assignment (j', c') must put j' as the first choices and choose the ZX options for j', therefore even if student i put j as the first choice and choose the ZX option for j', she still cannot receive the assignment (j', c'). When $c' = c_0$, similarly, putting j' as the first choice cannot help i to receive (j', c_0). Therefore, student i has no way to deviate to get better assignment, and the strategy profile A is a Nash equilibrium.

Part 2 is straightforward, because Proposition 3 in Sönmez and Switzer (2013) have proven that students prefer the outcome under the SOPS mechanism to any stable outcomes.

$\square$

C Survey Details

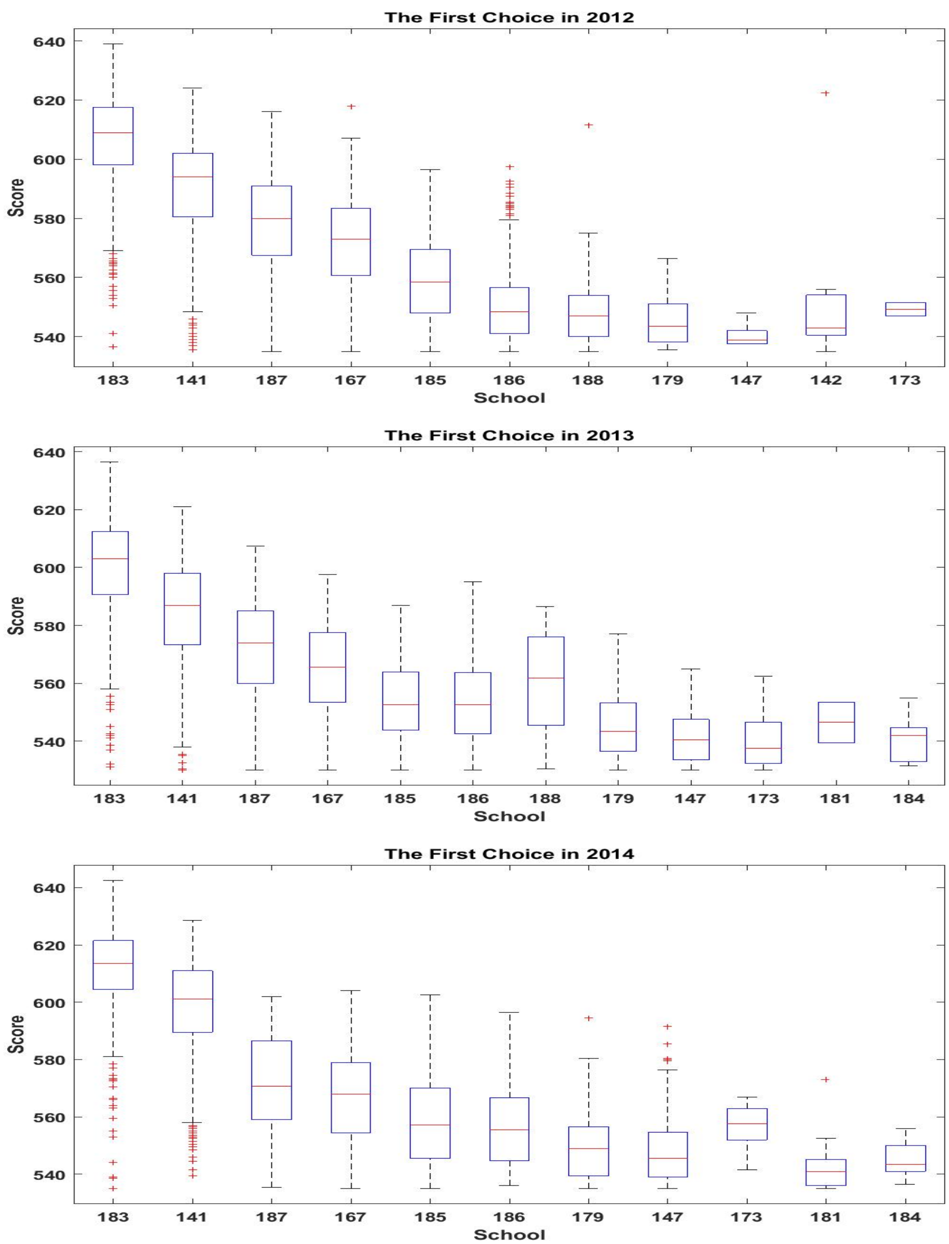
D More Results of Summary Statistics

Table 13: Distance Distribution (km)

	Overall	2014	2013	2012
1st Choice	8.87	8.68	8.91	9.01
	(6.04)	(6.18)	(6.06)	(5.90)
2nd Choice	7.56	7.63	7.42	7.60
	(5.73)	(5.92)	(5.44)	(5.78)
3rd Choice	6.46	6.74	6.41	6.26
	(5.01)	(5.27)	(4.94)	(4.84)

Notes: This table indicates the distribution of the home-school distance in the ROLs. The standard deviations are reported in the parenthesis.

Figure 6: Score Distribution for the First Choice



Notes: These figures are the box plots of the first choice score distribution for each school that has been chosen by students as their first choices The x-axis represents the schools chosen as the first choices in the ROLs. The y-axis represents the exam score.

E Identification of Parameters α

In this section, we provide a simplified version for the identification of parameters Θ_2. The general case can be conducted from the simple version. Since Θ_2 contains the parameters related with the ZX policy, therefore, we consider only three schools, school 1, and 2. Students' ROLs contain only two schools, and the order of schools are fixed, school 1 must be in the first place and school 2 in the second position. The students can only decide whether choose the ZX option for the first schools in the ROLs, but not the second school. Also we assume there is only one ZX tuition level c_1 and the student's payoff of being rejected by both schools in the ROLs is zero. The students can be divided into two groups.

This simplified version emphasizes the identification of the ZX policy related parameters, so it only gives students a binary choice that whether choose the ZX option for their first listed schools.

If student i chooses the ZX option for school 1, then her expected payoff of action $a_i = \{(1,1),(2,0)\}$ is

$$P^{a_i}_{i,1,c_0}(\hat{u}_{i,1} + \gamma q^z_1 + \epsilon_{i1}) + P^{a_i}_{i,1,c_1}(\hat{u}_{i,1} + \gamma q^z_1 - \alpha(c_1 - c_0) + \epsilon_{i1}) + P^{a_i}_{i,2,c_0}(\hat{u}_{i,1} + \epsilon_{i2}),$$

and the payoff of not choosing ZX option for school 1 with action $a'_i = \{(1,0),(2,0)\}$ is

$$P^{a'_i}_{i,1,c_0}(\hat{u}_{i,1} + \gamma q^z_1 + \epsilon_{i1}) + P^{a'_i}_{i,2,c_0}(\hat{u}_{i,2} + \epsilon_{i2}).$$

Therefore, student i chooses the ZX option for school 1, i.e. $v^1_i = 1$ if and only if

$$U_{i,12} + \gamma Q - \alpha C > \tilde{\epsilon},$$

where $U_{i,12} = (P^{a_i}_{i,1,c_0} + P^{a_i}_{i,1,c_1} - P^{a'_i}_{i,1,c_0})\hat{u}_{i,1} + (P^{a_i}_{i,2,c_0} + P^{a'_i}_{i,2,c_0})\hat{u_{i,2}}$, $Q = (P^{a_i}_{i,1,c_0} + P^{a_i}_{i,1,c_1})q^z_1$, $C = P^{a_i}_{i,1,c_0}(c_1 - c_0)$, and $\tilde{\epsilon} = (P^{a'_i}_{i,2,c_0} + P^{a_i}_{i,2,c_0})\epsilon_{i2} - (P^{a_i}_{i,1,c_0} + P^{a_i}_{i,1,c_1} - P^{a'_i}_{i,1,c_0})\epsilon_{i1}$.

Therefore the probability of observing a decision $v^1_i = 1$ is $F(U_{i,12} + \gamma Q - \alpha C)$ where F is the cdf of $\tilde{\epsilon}$. Then the log-likelihood of an observation a is

$$L_1(\Theta_2) = vln(F) + (1 - v)ln(1 - F).$$

The score function is

$$\frac{\partial L_1}{\partial \Theta_2} = \frac{y - F}{F(1 - F)} \frac{\partial F}{\partial \Theta_2}.$$

Furthermore

$$\begin{aligned} E\big[\frac{\partial L_1}{\partial \Theta_2}\frac{\partial L_1}{\partial \Theta_2'}\big] &= \frac{1}{F(1-F)}\frac{\partial F}{\partial \Theta_2}\frac{\partial F}{\partial \Theta_2'} \\ &= \frac{f^2}{F(1-F)}\left(\begin{array}{cc} Q^2 & -CQ \\ -CQ & C^2 \end{array}\right), \end{aligned}$$

where f is the pdf of $\tilde{\epsilon}$. Then it is easy to show that the information matrix $E\big[\frac{\partial L_1}{\partial \Theta_2}\frac{\partial L_1}{\partial \Theta_2'}\big]$ is positive definite. This result is equivalent to indicate that parameters γ and α are locally identified from the observed decisions.

F Maximum Simulated Likelihood Estimate

This appendix describes the algorithm used in the maximum simulated likelihood estimate to estimate the ZX related parameters with logit smoothed accept reject simulator. The procedure is implemented in the following steps, similar to the steps in Chapter 5 of Train (2009).

Step 1. Draw a value of J dimensional vector of errors, ϵ_i from type I extreme value distribution. Label the draw ϵ_i^r with $r = 1$ and the elements of the draw as $\epsilon_{i1}^r, ..., \epsilon_{iJ}^r$.

Step 2. Calculate the utility for each alternative. That is, $u_{i,j,c}^r = \tilde{u}_{i,j,c} + \epsilon_{ij}^r$, where $\tilde{u}_{i,j,c}$ is the deterministic part of the utility when student i enters school j and pay tuition c, and $u_{i,o}^r = \tilde{F}_o + \epsilon_{ij}^r$ that is denoted the utility when student i gets into a non-public high school.

Step 3. Given the beliefs and thus the admission probabilities, calculate the expected utility, $EU_i^r(a)$ of submitting a ROL $a = \{(j^1, v^1), (j^2, v^2), j^3\}$

In this step, the utility that the student i gets into one of her chosen school is $u_{i,j,c}^r$ obtained from step 2. The utility of being randomly assigned into a leftover school is $(\sum_{k=1,...,n_e}^{n_e} u_{i,j_{lo}^k,c_0}^r)/n_e$ where n_e is the number of leftover schools in year e, u_{i,j_{lo}^k,c_0}^r is the utility of i getting into the leftover school j_{lo}^k by paying the basic tuition c_0.

Given the student i's ROL $a = \{(j^1, v^1), (j^2, v^2), j^3\}$ and exam score s_i, the probability of i being admitted by school j^k as a normal student or by a non-public school can be calculated as follows:

$$P_{i,j_i^k,c_0} \text{or} P_{i,o} = \max\{0, P_i^{k-1} - \Phi((\bar{S}_{j_i^k}^k - s_i)/\eta)\},$$

where Φ is the cdf of the standard normal, $P_i^{k-1} = 1$ if $k = 1$, $P_i^{k-1} = \Phi((\bar{S}_{j_i^{k-1}}^{k-1} - s_i)/\eta)$ if $v^{k-1} = 0$, and $P_i^{k-1} = \Phi((\hat{S}_{j_i^{k-1}}^{k-1} - s_i)/\eta)$ if $v^{k-1} = 1$. The probability of being admitted by school j_i^k as a ZX student with tuition c is

$$P_{i,j_i^k,c} = \sum_{t=1}^{4} I(c_t = c)[\max\{0, \Phi((\bar{S}_{j_i^k}^k - 10(t-1) - s_i)/\eta) - \max\{\Phi((\bar{S}_{j_i^k}^k - 10t - s_i)/\eta), \Phi((\hat{S}_{j_i^k}^k - s_i)/\eta)\}\}],$$

Finally, the probability of being randomly assigned to a leftover school can be calculated as one minus the probability of being rejected by all three choices.

Step 4. For any student i in group 1, put these expected utilities into the logit formula, i.e.,

$$S_i^r = \frac{exp(Eu_i^r(a_i)/\lambda)}{\sum_{i'} exp(Eu_i^r(a_{i'})/\lambda)}, \tag{16}$$

where a_i is the student i's observed choice, $a_{i'}$ is her alternatives including a_i, and $\lambda > 0$ is a scale factor ($\lambda = 0.01$ in the reported results, the experimental results with other λs are available upon request).

For any student i in group 2, calculate $S_i^{r,2+}$ and $S_i^{r,2-}$ by using $a_i^{2+} = \{(j_i^1, v^1), (j_i^2, 1), j^3\}$ and $a_i^{2-} = \{(j_i^1, v^1), (j_i^2, 0), j^3\}$ to replace a_i in equation (16) respectively. Similarly, for any student i in group 3, calculate $S_i^{r,3+}$ and $S_i^{r,3-}$ by using $a_i^{3+} = \{(j_i^1, 1), (j_i^2, v^2), j^3\}$ and $a_i^{3-} = \{(j_i^1, 0), (j_i^2, v^2), j^3\}$ to replace a_i in equation (16) respectively.

Step 5. Repeat step 1-4 for R times, so that r takes the value from 1 to R.

Step 6. The simulated probability of student i in group 1 choosing the observed ROL a_i is the average of the values of the logit formula: $\hat{P}(a_i \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R} S_i^r$. For the students in group 2, the simulated probability of observing a_i^2 is $\hat{P}(a_i^{2+} \in A_i^*) + \hat{P}(a_i^{2-} \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R}(S_i^{r,2+} + S_i^{r,2-})$. Similarly, for the students in group 3, the the simulated probability of observing a_i^3 is $\hat{P}(a_i^{3+} \in A_i^*) + \hat{P}(a_i^{3-} \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R}(S_i^{r,3+} + S_i^{r,3-})$.

Finally, the log-likelihood function can be calculated in the following equation.

$$\begin{aligned} LogL_2 \quad = \quad & \sum_{i \in G_1} log(P(a_i \in A_i^*)) \\ & + \sum_{i \in G_2} log[P(a_i^{2+} \in A_i^*) + P(a_i^{2-} \in A_i^*)] + \sum_{i \in G_3} log[P(a_i^{3+} \in A_i^*) + P(a_i^{3-} \in A_i^*)]. \end{aligned}$$

G Simulations in Counterfactual Analysis

The section describes the simulation procedure used to analyze in the welfare comparison. We use the students' profiles from 2014. To simplified the calculation, the special classes and non-public schools are excluded. To calculate the equilibrium of the outcomes for different mechanism, the procedure is described as follows:

Step 1. For each student i, draw a value of J dimensional vector of errors, ϵ_i from type I extreme value distribution. Label the draw ϵ_i^r with $r = 1$ and the elements of the draw as $\epsilon_{i1}^r, ..., \epsilon_{iJ}^r$.

Step 2. Calculate the utility function as, $u_{i,j,c}^r = \tilde{u}_{i,j,c} + \epsilon_{ij}^r$, where $\tilde{u}_{i,j,c}$ is the deterministic part of the utility. The parameters used to calculate it come from table 5.

Step 3. The DA mechanism and SOPS mechanism are strategy-proof. For the DA mechanism, we treat students' true preferences across all schools as their reported ROLs. For the SOPS, there are three tuition levels for each school. We treat students' true preferences across school-tuition pairs as their ROLs. Then we run the use serial dictatorship algorithm to match students and schools.

Step 4. The CP, CPPS, BM, BMPS mechanisms are not strategy-proof. We describe the calculation of the equilibrium outcomes as follows:

Step 4.1. For each of these non-strategy-proof mechanism, use the admission cutoffs generated by the DA mechanism as the first prior beliefs for all students.

Step 4.2. Use the prior beliefs to calculate the optimal choice for each student. When there are more than one choice as the optimal choices, then randomly choose one of them. Then each student reports the calculated optimal choice as the ROL.

Step 4.3. Given the submitted ROL, run the matching algorithm base on the definition of the mechanism to match students to schools. Then rank all N students by exam scores.

Step 4.4. The matching outcome from the last step generates new admission cutoffs for schools. Then use the these cutoffs as the new prior beliefs.

Start from the first student and let $k = 1$.

Step 4.5. Calculate the k-th student's best response to the prior beliefs. If there exists at least one choice of this student making him/her strictly better off, then jump to step 4.6. If there does not exist any choice of this student making him/her strictly better off, then let $k = k + 1$. If $k = N$, then jump to step 5. If $k < N$, then repeat step 4.5.

Step 4.6. Choose the k-th student's best response to replace his/her old choice in the submitted ROL. When there is more than one best response, then randomly choose one of them. Thereafter, repeat step 4.3.

Step 5. The current ROLs are the equilibrium strategies of the students.

After calculate one equilibrium outcome for each mechanism, repeat the step 1 to 5 for R times ($R = 100$ in the reported results).

H More results for the welfare comparison

Table 14: Welfare Gain (%)

	DA-SOPS			DA-CPPS			DA-BMPS		
	10%	30%	50%	10%	30%	50%	10%	30%	50%
Overall	939.4	1079.7	1118.5	1494.8	1318.6	1185.6	1541.9	1281.0	1253.4
	(904.9)	(990.9)	(946.3)	(1164.9)	(1017.3)	(943.4)	(1024.0)	(839.2)	(725.0)
Hihg	934.4	977.4	967.5	1208.0	1140.5	1095.3	1605.2	954.8	918.3
	(775.0)	(813.0)	(830.0)	(802.8)	(799.2)	(804.6)	(1025.6)	(862.6)	(787.2)
Medium	1070.7	1270.6	1280.7	1636.7	1532.4	1265.0	1627.6	1341.6	1218.6
	(1025.8)	(1114.8)	(1013.1)	(1325.0)	(1109.8)	(967.4)	(1223.2)	(1007.2)	(779.5)
Low	544.0	508.3	432.6	1249.3	849.6	1030.2	1427.8	1320.9	1326.6
	(486.5)	(460.1)	(421.9)	(641.3)	(680.0)	(1015.9)	(723.8)	(666.0)	(658.8)

Notes: This table indicates the average gain of welfare of the winners when the DA mechanism is replace by the SOPS, CPPS and BMPS mechanisms. For each mechanism change, the welfare changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. Top represents the students are the students whose score is above 90%, Middle represents the students whose scores are between 90% and 70%, and Low represents the students whose scores are below 70% and above the threshold. The standard deviations are in the parenthesis.

Table 15: Welfare Loss (%)

	DA-SOPS			DA-CPPS			DA-BMPS		
	10%	30%	50%	10%	30%	50%	10%	30%	50%
Overall	-988.4	-1114.7	-1362.8	-928.7	-1229.4	-1638.3	-1315.7	-1227.8	-1483.4
	(577.4)	(584.2)	(779.4)	(904.0)	(917.0)	(1145.8)	(1509.6)	(1055.6)	(1010.8)
High	-1099.7	-1219.3	-1404.3	-991.8	-1194.1	-1554.3	-1380.7	-969.6	-1365.5
	(558.6)	(539.4)	(673.8)	(869.8)	(623.6)	(759.1)	(2235.9)	(1042.1)	(901.4)
Medium	-994.1	-1172.6	-1555.1	-820.9	-1155.5	-1550.7	-1301.5	-1361.0	-1627.1
	(588.1)	(596.5)	(810.4)	(757.6)	(772.1)	(1080.4)	(1031.6)	(920.9)	(999.2)
Low	-899.7	-911.5	-1074.7	-1306.9	-1791.0	-2324.6	-1219.8	-1413.8	-1143.3
	(546.7)	(549.1)	(726.8)	(1359.7)	(1868.2)	(2013.4)	(1281.1)	(1786.0)	(1515.3)

Notes: This table indicates the average loss of welfare of the winners when the DA mechanism is replace by the SOPS, CPPS and BMPS mechanisms. For each mechanism change, the welfare changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. Top represents the students are the students whose score is above 90%, Middle represents the students whose scores are between 90% and 70%, and Low represents the students whose scores are below 70% and above the threshold. The standard deviations are in the parenthesis.

I ZX policy in other cities in China

In this appendix, we describe the implementation of ZX policy in three direct-controlled municipalities of China, i.e., Beijing, Shanghai, and Tianjin.

Beijing integrated the ZX policy into its centralized high school admission system in 2005. After the Ministry of Education announced the cancellation of the ZX policy in 2012, the percentage of ZX students of each school decreased from 18% to 15% and further to 10% in 2013. The ZX policy was fully terminated in 2014. The basic tuition of public high schools was 1,600 Yuan/year for a normal student in 2011, whereas that for a ZX student cannot exceed 10,000 Yuan/year.

The admission mechanism of the ZX policy applied in Beijing was an adjusted constrained DA mechanism with purchasing seat options. In this process, no more than eight schools can be selected in the ROL. Each student can select no more than two options from each specific school choice. The options of a school include normal, ZX, special class, and dorm. This mechanism is a special case of CPPS mechanism, wherein the matching algorithm follows the CPPS mechanism with permanency-execution period (8, 0, 0,...).

Shanghai is one of the cities that discontinued the ZX policy immediately after the announcement from the Ministry of Education in 2012. The total percentage of ZX students was restricted within 15% for each school in 2011, which is the percentage for ZX policy in the previous year. The ZX tuition in Shanghai was charged according to the type of school. In district-level key high schools, the basic tuition for students was 2,400 Yuan/year, whereas the ZX tuition was 6,000 Yuan/year before 2011 and 4,266 Yuan/year in 2011. For the city-level key high schools, the basic tuition was 3,000 Yuan/year, whereas the ZX tuition was 10,000 Yuan/year before 2011 and 7,000 Yuan/year in 2011. For the boarding schools, the basic tuition was 4,000 Yuan/year, whereas the ZX tuition was 13,333 Yuan/year before 2011 and 9,333 Yuan/year in 2011. The admission mechanism adopted in Shanghai was the constrained SOPS mechanism where no more than 15 schools can be selected from the ROL.

Tianjin cancelled its ZX policy in 2015. Before 2015, the ZX tuition was standardized across all general high schools at 8,000 Yuan/year, which was a fourfold increase in the basic tuition (2,000 Yuan/year). The matching algorithm used in Tianjin was a constrained CPPS mechanism with permanency-execution period (2, 8). The students can select two key high schools in the first round and eight ordinary high schools in the second round.