# Modeling Non-Normal Corporate Bond Yield Spreads by Copula


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## Abstract


This research focuses on modeling for how corporate bond yield spreads are affected by explanatory variables such as equity volatility, interest rate volatility, *r*, slope, rating, liquidity, coupon rate, and maturity.

Therefore, we use the Gaussian copula regression method with Weibull marginal distributions and also employ several copula functions to test for the tail dependence between yield spreads and other explanatory variables. We show that our regression model is a better-fitting model than the one based on the lower AIC value.

We find that (1) coupon rates increase noncallable bond yield spreads, while coupon rates do not affect callable bond yield spreads in the traditional regression and (2) stronger tail dependence in the joint upper tail for the relation between equity volatility and noncallable yield spreads, among others.


## Introduction

The existing literature on how corporate bond yield spreads are determined assumes normality and linearity. However, our data analysis shows that (1) yield spreads and some explanatory variables do not follow the normal distribution and (2) relations between yield spreads and some explanatory variables are not necessarily linear.

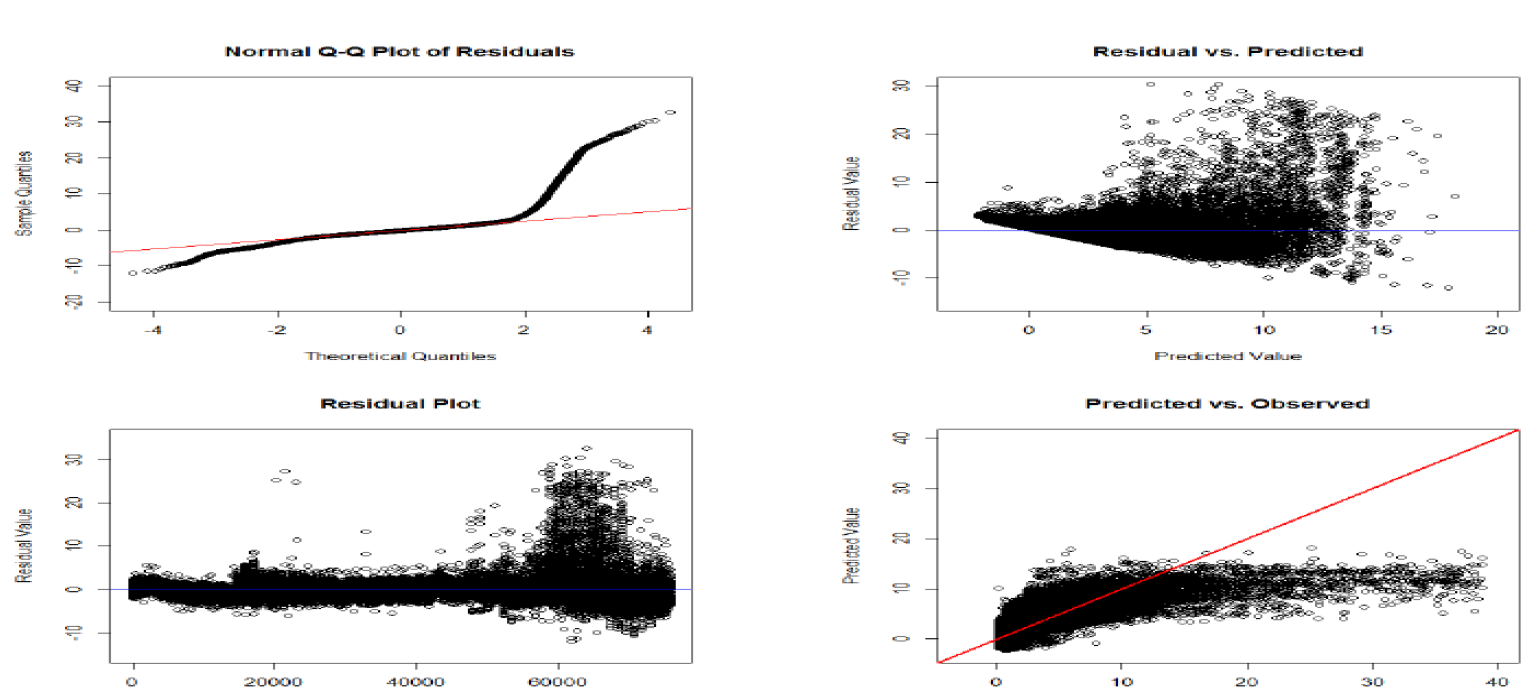


Figure 1: Checking linear model assumptions

To relax normality and linearity assumptions, copula-based methods have been explicitly recognized and employed in the finance literature.

Copula has been increasingly popular as a flexible tool for modeling the dependence of multivariate data in many fields of application, such as biostatistics, medical research, econometrics, finance, actuarial science, and hydrology.

Our finding should be useful to practitioners, such as investors. By relying on better-fitting, more meaningful statistical models, this paper contributes to the extant literature on how corporate bond yield spreads are determined.

## Economic Models

Sklar (1973) shows that any bivariate distribution function, $F_{XY}$ (x; y), can be represented by the marginal distributions, $F_X$(x) and $F_Y$ (y), by using C determined in $[0; 1]^n$.

1) GCMR model

$$Spread_t = \beta_0 + \beta_1 Equity\ volatility_t + \beta_2 Interest\ rate\ volatility_t + \beta_3 r_t + \beta_4 Slope_t + \beta_5 Rating_t + \beta_6 Liquidity_t + \beta_7 Coupon\ rate_t + \beta_8 Maturity_t + u_t,$$

where $u_t = \phi u_{t-1} + \omega_t \sim iid\ N(0, \sigma_\omega^2)$.

2) Copula tail dependence

The coefficients of lower (left) and upper (right) tail dependence of (X; Y ) are defined in terms of copula by Nelsen (2007) as:

$$\tau^L = \lim_{\varepsilon \to 0} P\left[u \le \varepsilon | v \le \varepsilon\right] = \lim_{\varepsilon \to 0} P\left[v \le \varepsilon | u \le \varepsilon\right]$$
$$\tau^U = \lim_{\delta \to 1} P\left[u > \delta | v > \delta\right] = \lim_{\delta \to 1} P\left[v > \delta | u > \delta\right]$$

## Empirical Results (Noncallable Case)

**Table 3:** Estimation results: noncallable case

| | Linear Regression | Gaussian Copula Regression |
|---|---|---|
| Equity volatility | 100.300*** (0.601) | 21.699*** (0.166) |
| Interest rate volatility | 1.480*** (0.032) | 0.788*** (0.008) |
| $r$ | -1.282*** (0.020) | -0.320*** (0.005) |
| Slope | -1.702*** (0.030) | -0.321*** (0.007) |
| Rating | 0.323*** (0.003) | 0.104*** (0.001) |
| Liquidity | -0.371*** (0.025) | -0.349*** (0.006) |
| Coupon rate | -0.005 (0.006) | 0.021*** (0.001) |
| Maturity | -0.000 (0.001) | 0.007*** (0.000) |
| Constant | 3.295*** (0.105) | 0.357*** (0.025) |
| Log likelihood | 170,690 | 93,316 |
| AIC | 341,405 | 186,652 |

**Table 4:** Statistics for copulas between standardized uniform pairs: noncallable case

| Copula type | Equity volatility-Spread | | Interest rate volatility-Spread | | $r$-Spread | | Slope-Spread | |
|---|---|---|---|---|---|---|---|---|
| | $\tau^L$ | $\tau^U$ | $\tau^L$ | $\tau^U$ | $\tau^L$ | $\tau^U$ | $\tau^L$ | $\tau^U$ |
| Gaussian | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| **Clayton** | **0.437** | **0.000** | **0.087** | **0.000** | **0.000** | **0.000** | **0.179** | **0.000** |
| **Rotated Clayton** | **0.000** | **0.628** | **0.000** | **0.262** | **0.000** | **0.000** | **0.000** | **0.240** |
| Plackett | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Frank | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| **Gumbel** | **0.000** | **0.558** | **0.000** | **0.290** | **0.000** | **0.122** | **0.000** | **0.291** |
| **Rotated Gumbel** | **0.495** | **0.000** | **0.234** | **0.000** | **0.122** | **0.000** | **0.277** | **0.000** |
| Student's $t$ | 0.022 | 0.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| **SJC** | **0.009** | **0.637** | **0.000** | **0.305** | **0.000** | **0.000** | **0.080** | **0.232** |

| Copula type | Rating-Spread | | Liquidity-Spread | | Coupon rate-Spread | | Maturity-Spread | |
|---|---|---|---|---|---|---|---|---|
| | $\tau^L$ | $\tau^U$ | $\tau^L$ | $\tau^U$ | $\tau^L$ | $\tau^U$ | $\tau^L$ | $\tau^U$ |
| Gaussian | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| **Clayton** | **0.169** | **0.000** | **0.000** | **0.000** | **0.134** | **0.000** | **0.126** | **0.000** |
| **Rotated Clayton** | **0.000** | **0.381** | **0.000** | **0.000** | **0.000** | **0.041** | **0.000** | **0.033** |
| Plackett | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Frank | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| **Gumbel** | **0.000** | **0.354** | **0.000** | **0.122** | **0.000** | **0.169** | **0.000** | **0.159** |
| **Rotated Gumbel** | **0.296** | **0.000** | **0.122** | **0.000** | **0.203** | **0.000** | **0.199** | **0.000** |
| Student's $t$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| **SJC** | **0.001** | **0.411** | **0.000** | **0.000** | **0.180** | **0.010** | **0.177** | **0.002** |

## Discussion

Since a model with the smallest AIC value among competing models is the best fitting model, the GCMR model is a better- fitting model than the traditional linear regression model.

Coupon rates increase noncallable bond yield spreads, while coupon rates do not affect callable bond yield spreads in the traditional regression.

The relative magnitude between the lower and upper tail dependence coefficients indicates the possibility of asymmetry in the tail dependence.

We find evidence that greater upper tail dependence than low tail dependence for the spread and equity volatility pair.

## Conclusions

In this paper, our corporate bond data do not follow normal distribution and have outliers. Therefore, we employ the copula methods to examine the relationship between yield spreads and other explanatory variables.

We find a positive effect of equity volatility on yield spreads in the upper tail is greater than that in the low tail.

Our findings have important implications for practitioners such as hedgers and policymakers in the corporate bond market.

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