

# Turning Alphas into Betas: Arbitrage and the Cross-section of Risk

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## Abstract

What determines the cross-section of *betas* with respect to a risk factor? The act of arbitrage plays an important role. If the capital of arbitrageurs loads on a systematic factor, the assets traded by the arbitrageurs gain different sensitivities to that factor, depending on the asset positions taken by the arbitrageurs. I develop predictions about such “arbitrage-driven” betas in a model of constrained arbitrage and test them in the cross-section of equity anomalies. The arbitrage channel accounts for a substantial part of the cross-sectional variation in equity anomalies’ betas in intermediary-based and multifactor asset pricing models.

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# 1 Introduction

What generates the cross-section of betas, the sensitivities of asset returns to systematic risk factors? Since betas measure the risk of a financial asset, what determines the cross-section of betas is a central question in finance.<sup>1</sup> The traditional explanation relies on the fundamental cash-flow channel: a stock has a high beta if the company’s expected cash flow covaries strongly with the risk factor.<sup>2</sup> But what is an important non-fundamental channel for the cross-section of betas?

The channel for the cross-section of betas I explore is the act of arbitrage by institutional arbitrageurs. Growing evidence suggests that shocks to institutional arbitrageurs’ capital propagate to the assets they trade (e.g., Coval and Stafford, 2007; Mitchell, Pedersen, and Pulvino, 2007; Krishnamurthy, 2010; Gârleanu and Pedersen, 2011; Greenwood and Vayanos, 2014; Du, Tepper, and Verdelhan, 2018). Hence, if arbitrageurs’ capital is exposed to systematic factors, the assets traded by the arbitrageurs could inherit these factor exposures.

To see this, consider assets  $A$  and  $B$  with positive and negative abnormal returns, respectively, but no prior factor exposure. Hence, arbitrageurs go long on  $A$  and short on  $B$ , but their capital is limited and—for whatever reason—loads positively on systematic factor  $F$ . Then, a positive- $F$  shock that enables the arbitrageurs to increase their long/short positions on  $A$  and  $B$  would raise  $P_A$  (price of  $A$ ) and lower  $P_B$  (price of  $B$ ). On the other hand, a negative- $F$  shock that causes the arbitrageurs to unwind their positions would lower  $P_A$  back down and raise  $P_B$  back up. In this way, through the act of arbitrage, assets  $A$  and  $B$  with no prior factor exposure can covary positively and negatively with  $F$ , respectively.

Several questions arise. To what kinds of systematic factors does the arbitrage channel for betas apply? What is an important right-hand variable that explains the cross-sectional variation in betas generated by the act of arbitrage? What distinct patterns do these betas feature?

To answer these questions, this paper develops a model in which the cross-section of betas of

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<sup>1</sup>Cochrane (2011).

<sup>2</sup>E.g., Fama and French (1992, 1993) attribute “value” betas of high book-to-market firms to their common distress risk. Theoretical explanations of cross-sectional anomalies, such as the value premium, tend to model firms with different (conditional) cash-flow betas with a systematic factor (e.g., Chan and Chen, 1991; Gomes, Kogan, Zhang, 2003; Carlson, Fisher, and Giammarino, 2004; Bansal, Dittmar, and Lundblad, 2005; Zhang, 2005; Cooper, 2006; Ozdagli, 2012; Choi, 2013; Kogan and Papanikolaou, 2013, 2014).

assets arises through the act of arbitrage. Testing the model’s predictions using equity anomaly portfolios (“anomalies”), I show empirically that the arbitrage channel explains a substantial part of the cross-sectional variation in the factor betas of anomalies.

My three-period model features a continuum of assets with i.i.d. dividends and hence no aggregate cash-flow risk. Behavioral investors form a downward-sloping demand curve for each asset and also create negative distortions in asset demands. Crucially, the magnitude of this distortion is constant over time but differs across assets, generating a cross-section of different “pre-arbitrage” alphas. A representative arbitrageur trades these assets using capital that loads on a systematic factor and may be constrained.

In this model, the act of arbitrage causes asset prices to covary with factors that arbitrage capital loads on, generating “arbitrage-driven” betas. Importantly, in the cross-section of assets, the magnitude of the demand distortion in the asset determines the magnitude of the arbitrage-driven beta. A larger demand distortion means that the arbitrageur plays a larger price-correcting role in that asset in equilibrium. This also means, however, that the asset’s price is more sensitive to the variation in the arbitrage capital and therefore to factors that the capital loads on. It follows that an asset’s arbitrage-driven beta depends positively on the *arbitrage position* on the asset as a fraction of its market capitalization (measuring the arbitrageur’s equilibrium price-correcting role) and more intrinsically on the asset’s *pre-arbitrage alpha* (measuring its demand distortion) ([Propositions 1 and 2](#)).

Additional predictions arise. Since arbitrage capital is the sum of arbitrageur wealth and external funding, arbitrage-driven betas arise with factors that either arbitrageur funding shocks (e.g., funding liquidity) or wealth shocks (e.g., common factors in the arbitrated assets) load on ([Lemma 4](#)). Arbitrage-driven betas arise when the arbitrageur is capital constrained but disappear when the arbitrageur has a “deep pocket” ([Proposition 3](#)). Arbitrage-driven betas are “discount-rate” betas arising from the asset’s market valuation—as opposed to expected cash flow—covarying with the factor, so a high arbitrage-driven beta means more discount-rate shocks and greater return predictability of the asset in the time series ([Proposition 4](#)). Finally, asset return response to a sharp decline in arbitrage capital reveals the cross-section of arbitrage-driven betas ([Proposition 5](#)).

Testing these predictions in a cross-section of 40 equity anomalies, I show that their betas with

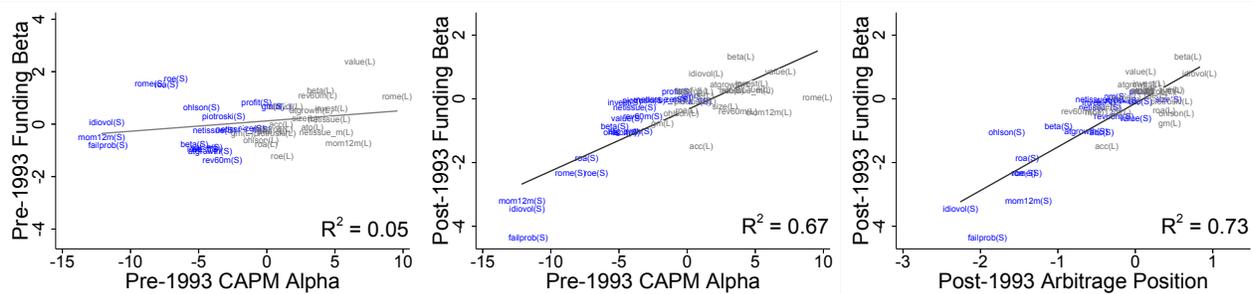


Figure 1: **Explaining the Cross-Section of Funding-Liquidity Betas**

The first figure shows that the funding-liquidity betas of anomalies in the pre-1993 period (1974–1993) cluster around zero, irrespective of their CAPM alphas. The next two figures show that the funding-liquidity betas of anomalies in the post-1993 period (1994–2016) are explained by their pre-arbitrage alphas (pre-1993 CAPM alpha) and post-1993 arbitrage position (abnormal short interest ratio). Long-side and short-side anomalies are denoted in gray and blue, respectively.

respect to systematic factors feature patterns consistent with the model. I show this in the context of the intermediary-based model of Adrian, Etula, and Muir (2014), interpreting their aggregate funding-liquidity factor as a factor that arbitrage loads on. This interpretation is plausible since anomaly arbitrageurs, such as quantitative long/short equity hedge funds, are levered and rely on funding liquidity provided by their prime brokers. I measure the arbitrage position based on abnormally high or low shorting on the anomaly, and I measure the pre-arbitrage alpha using the anomaly’s factor alpha (e.g., CAPM alpha) in the pre-1993 period (1974–1993) when institutional arbitrageurs (e.g., quantitative long/short equity hedge funds) were small. I look for evidence of arbitrage-driven betas primarily in the post-1993 period (1994–2016) with more arbitrage on the anomalies.<sup>3</sup>

I find strong evidence that the funding-liquidity (“funding”) betas of anomalies are arbitrage-driven betas. In the pre-1993 period with less arbitrage on the anomalies, funding betas cluster around zero for both the anomalies with positive CAPM alpha and the anomalies with negative CAPM alpha (Figure 1a). In the post-1993 period with more arbitrage, however, the anomalies attain either positive or negative funding betas depending on whether their pre-1993 alphas were positive or negative (Figure 1b), consistent with the model’s prediction (Proposition 2). Furthermore, actual arbitrage position explains the magnitude and the direction of the post-1993 funding betas (Figure 1c), consistent with Proposition 1. I find similar patterns using panel regressions that allow the funding betas to vary more freely over time: funding beta increases with arbitrage posi-

<sup>3</sup>See Schwert (2003), Chordia, Roll, and Subrahmanyam (2008, 2011), and Chordia, Subrahmanyam, and Tong (2014) for evidence of increased arbitrage activity on anomalies following early 1990s.

tion, with the academic publication of the anomaly, and with the post-1993 dummy. In contrast, the anomalies' fundamental characteristics (e.g., size or book-to-market ratio) are not strongly related to funding betas.

The funding betas of anomalies display additional patterns expected from arbitrage-driven betas. Funding betas strengthen in periods when arbitrageurs are likely to be constrained and weaken when arbitrageurs are likely to be unconstrained, consistent with [Proposition 3](#). This evidence addresses the concern that the funding factor proxies for an arbitrageur wealth portfolio rather than for aggregate funding shocks, in which case anomalies' betas with the factor should remain, if not strengthen, during unconstrained periods when arbitrageurs hold more anomalies. Furthermore, in the cross-section of different anomalies, the time-series return predictability increases in the funding beta in the post-1993 period but not in the pre-1993 period. This is consistent with post-1993 funding betas measuring discount-rate shocks generated by the act of arbitrage as opposed to fundamental cash-flow shocks that happen to covary with funding-liquidity shocks ([Proposition 4](#)).

Next, I argue—more aggressively—that arbitrage-driven betas are pervasive, arising in conventional multifactor models. Factors in these models are long-short portfolios with positive mean returns, so arbitrageurs who seek positive market-adjusted returns would load positively on these factors. Hence, these factors generate shocks to arbitrageur wealth that can propagate to other assets traded by the arbitrageurs, thus generating arbitrage-driven betas with respect to the factors: a large negative (positive) factor shock increases (decreases) the wealth of arbitrageurs and leads them to unwind (increase) their position on all anomalies, causing anomalies with no prior exposure to the factor to attain arbitrage-driven betas with the factor.

I use the five-factor model of Fama-French (FF) (2015) to support this claim. I find that, among the cross-sectional factors, *profitability (RMW)* and *investment (CMA)* are the factors that the capital of anomaly arbitrageurs loads strongly on. Since long-side (short-side) anomalies consistently load positively (negatively) on *RMW* and *CMA*, long/short arbitrageurs who trade the anomalies trade *RMW* and *CMA*, deliberately or inadvertently. Indeed, my proxy for a quantitative long/short equity hedge fund portfolio loads strongly on *RMW* and *CMA* but not on *SMB* and *HML*—the other cross-sectional factors—once I control for *RMW/CMA*. Consistent with this observation, in the post-1993 period, an arbitrage-related variable (i.e., the arbitrage position or the pre-arbitrage alpha) explains around 30% of the cross-sectional variation in the *RMW/CMA*

betas of anomalies but does not explain the cross-section of *SMB/HML* betas. I find consistent evidence in panel regressions and in my tests of other predictions of the model.

Finally, I use the “quant” crisis of August 2007 to test the cross-sectional relationship between arbitrage-driven betas and their determinants without relying on a factor model. Over the three-day period of the crisis, seemingly distinct equity anomalies commonly underperformed, while hedge fund strategies in other asset classes remained unaffected. Khandani and Lo (2007, 2011) and Pedersen (2009) attribute this unusual covariance event to a rapid decline in the capital of quantitative long/short equity hedge funds that trade these anomalies.

My model predicts the cross-section of anomaly returns during the quant crisis. Since anomaly returns during a crash of arbitrage capital reveal their sensitivity to arbitrage-capital shocks, an anomaly’s quant-crisis return should *decrease* in its arbitrage position and pre-arbitrage alpha, which determine the cross-section of arbitrage-driven betas ([Proposition 5](#)). Evidence strongly supports the prediction: anomalies with a greater arbitrage position or pre-arbitrage alpha experienced a sharper crash over the three-day crash period (August 7–9, 2007). These anomalies in turn experienced a sharper recovery following the crash (August 10–14), consistent with the quant-crisis returns being discount-rate movements generated by a decline in arbitrage capital.

Taken together, my results suggest that arbitrage-driven betas predicted by the model arise in equity anomalies and help determine the cross-section of their factor betas. Although alternative explanations may exist for each individual result, the arbitrage channel in my model offers a unifying explanation for the joint occurrence of my empirical results.

To my knowledge, this paper is the first to study the arbitrage channel for the cross-section of factor betas. Previous work on betas studies whether firm characteristics determine the stocks’ market betas (e.g., Rosenberg, McKibben, and March, 1973; Lev, 1974; Thompson, 1976; Thurnbull, 1977; Bowman, 1979) and whether factor betas are driven by the discount-rate or cash-flow part of factor returns (Campbell and Mei, 1993; Campbell, Polk, and Vuolteenaho, 2009). Others have shown that institutional arbitrageur trading affects the second moments of the arbitrated assets as a *whole* (e.g., Shleifer and Vishny, 1997; Barberis and Shleifer, 2003; Barberis, Shleifer, and Wurgler, 2005; Anton and Polk, 2010; Greenwood and Thesmar, 2011; Lou and Polk, 2013; Liu, Lu, Sun, and Yan, 2015; McLean and Pontiff, 2016; Huang, Lou, and Polk, 2018). My contribution is to show how this insight can be applied to understand the *cross-section* of risk.

In a simple framework, my model shows that future research can use arbitrage position or pre-arbitrage alpha as the cross-sectional determinant of arbitrage-driven risk. Retrospectively, the empirical work of Brunnermeier, Nagel, and Pedersen (2009) uses both arbitrage position and “alpha” (interest rate differential) to explain the cross-section of crash risks in currency carry trades. My model provides a theoretical ground for their approach and shows that the alpha in this type of study should ideally be measured in periods with little arbitrage. My model shares similarities to both Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2017), but I focus on differences in the assets’ demand distortion rather than fundamental volatility and provide new predictions about arbitrage-driven betas. Kozak, Nagel, and Santosh (2018) show that, in the presence of arbitrageurs, asset returns in a “behavioral” model with mispricings feature a factor structure and therefore are not easily distinguishable from those in a fully rational model. My work suggests that factor models that arise in this manner *are* distinguishable, as factor *betas* in such models feature patterns expected from arbitrage-driven betas.

This paper offers an explanation as to why, despite arbitrage, anomaly returns persist.<sup>4</sup> They persist because of the very fact that many arbitrageurs are attempting to exploit them, which generates arbitrage-driven betas in the anomalies. This allows the initial abnormal returns of anomalies to persist in the form of risk premia associated with the arbitrage-driven betas.<sup>5</sup> Hence, my paper relates to Drechsler and Drechsler (2016), who find that anomaly arbitrageurs face risk concentrated in negative-alpha stocks and require a return premium for bearing this risk. I find that the role of arbitrageurs extends beyond recognizing such risk, as they propagate this type of risk to other assets that they trade.

Finally, this paper contributes to the growing intermediary-based asset pricing literature. This literature has shown that financial intermediaries generate asset price movements in the short-to-medium horizon (see the references in Paragraph 2) and help determine assets’ expected returns over a longer horizon (Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2012, 2013, 2018; Adrian, Etula, and Muir, 2014; Brunnermeier and Sannikov, 2014; He, Kelly, and Manela, 2017; Avdjiev, Du, Koch, and Shin, 2017; Haddad and Muir, 2017). Arbitrageurs in my model can be interpreted more broadly as financial intermediaries with preferences that differ from the preferences

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<sup>4</sup>McLean and Pontiff (2016) document a 32% decline in the returns of 97 anomalies after their publication. Chordia, Subrahmanyam, and Tong (2014) also find that anomaly returns have not completely disappeared.

<sup>5</sup>This argument can also be attributed to the “adaptive market hypothesis” of Lo (2004), who first coined the term “alphas becoming betas.”

of households. Hence, my results suggest that financial intermediaries also shape the cross-section of factor betas and that betas used in intermediary asset pricing may be generated by the financial intermediaries themselves.

The rest of the paper is organized as follows. [Section 2](#) lays out the theory, [Section 3](#) describes the data and methodology, [Section 4](#) tests [Propositions 1-4](#) on funding-liquidity betas, [Section 5](#) tests the propositions on Fama-French (2015) betas, [Section 6](#) tests [Proposition 5](#) on the quant crisis, and [Section 7](#) concludes. [Appendix A](#) contains a detailed derivation of the model and all proofs, and an [online appendix](#) to this paper provides additional results and robustness checks.

## 2 A Model of Assets with Arbitrage-Driven Betas

The economy has three periods ( $t = 1, 2, 3$ ) and two types of security: a risk-free bond and a continuum of anomaly assets  $i \in [0, 1]$ . The risk-free bond is supplied elastically at a zero interest rate; hence an asset's excess return equals its return. An anomaly asset ("asset") is a claim to a stream of cash flows  $\{\delta_{i,2}, v + \delta_{i,3}\}$  over  $t \in \{2, 3\}$  and has a zero net supply. The dividends  $\{\delta_{i,t}\}$  are conditionally i.i.d. across assets, and  $v > 0$  is a constant; hence there is zero aggregate cash-flow risk. I assume for convenience that the dividends have a zero conditional mean.

There are two types of investors: behavioral investors and a representative arbitrageur. Behavioral investors generate negative distortions in asset demands that push asset prices downward.<sup>6</sup> Importantly, these distortions are constant over time but *increasing* in magnitude in  $i$ . I model this as a distortion  $-\phi i$  in the aggregate behavioral investor demand for asset  $i$  (in units of wealth) at time  $t \in \{1, 2\}$ :

$$B_{i,t} = E_t [r_{i,t+1}^e] - \phi i, \quad (1)$$

with  $E_t [r_{i,t+1}^e]$  denoting the *objective* conditional expected (excess) return and  $\phi > 0$ . Besides the distortion, the demand function has three additional features:

- Demand falls as price rises (since price is inversely related to expected return).

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<sup>6</sup>The direction of the distortions is chosen for convenience and does not affect the model's predictions. Furthermore, I do not specify the reason for this distortion, which can be behavioral (e.g., sentiment) or rational (e.g., local risk to behavioral investors that arbitrageurs are willing to share).

- “Narrow framing” in that covariances do not matter (Barberis, Huang, and Thaler, 2006). This feature simplifies the model solution.
- Equal “size” or “liquidity” of all assets. That is, a marginal increase in the arbitrage position lowers the equilibrium expected return by an amount that is constant across all assets.<sup>7</sup>

A representative, risk-neutral arbitrageur with mass  $\mu$  trades to maximize the expected wealth at time 3 but faces a capital constraint. Specifically, the arbitrageur is not short-sale constrained but faces a margin rate of one in all positions, which prevents the arbitrageur from raising cash by shorting an asset.<sup>8</sup> The arbitrageur can borrow up to an exogenous stochastic funding constraint  $f_t \in [0, \infty)$  and additionally faces exogenous shocks to its wealth  $w_t$ , both of which are independent of dividends  $\{\delta_{i,t}\}$  and generate shocks to the level of deployable capital of the unit arbitrageur (“arbitrage capital”):<sup>9</sup>

$$k_t = w_t + f_t. \quad (2)$$

The presence of these shocks and the possibility of a binding capital constraint make the risk-neutral arbitrageur behave in a risk-averse manner through the intertemporal speculative motive (Merton, 1973). Finally, the arbitrageur faces a non-negativity constraint: a negative realized wealth forces the arbitrageur to exit the market immediately and pay an interest cost  $c \geq \phi$  on the negative wealth in all future periods.<sup>10</sup>

To summarize, the arbitrageur’s objective at time  $t$  is to choose asset positions  $\mathbf{x}_t$  to maximize  $E_t[w_3]$  s.t.

$$w_{t+1} = \begin{cases} w_t + \int_0^1 r_{i,t+1} x_{i,t} di + \tilde{w}_{t+1} & \text{if } w_t > 0 \\ (1+c)w_t & \text{if } w_t \leq 0 \end{cases} \quad (3)$$

$$\int_0^1 |x_{i,t}| di \leq \mathbf{1}(w_t > 0) k_t$$

$$k_t = w_t + \mathbf{1}(w_t > 0) f_t,$$

where  $\mathbf{x}_t$  is the unit arbitrageur’s sequence of dollar positions on all assets over all remaining

<sup>7</sup>Suppose  $\mu x_{i,t}$  is the aggregate arbitrageur demand for asset  $i$  at time  $t$ . Since market clearing implies  $\mu x_{i,t} = -B_{i,t}$ ,  $\partial E_t[r_{i,t+1}^e] / \partial (\mu x_{i,t}) = -\partial E_t[r_{i,t+1}^e] / \partial B_{i,t} = -1$  for all assets.

<sup>8</sup>This is analogous to how actual arbitrageurs, such as hedge funds, are not short-sale constrained but face a nonzero margin requirement. However, I hold the margin rate fixed rather than make it a function of asset volatility, as in Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2017), to emphasize that arbitrage-driven betas can arise without differences in fundamental volatility.

<sup>9</sup>As the reader will see, the funding channel and the wealth channel play an identical role in the model, but I keep both channels for a tighter link to my empirical results.

<sup>10</sup>This allows me to obtain the arbitrageur’s marginal value of wealth in the negative-wealth region.

trading periods,  $\tilde{w}_t$  is the wealth shock,  $r_{i,t}$  is the asset return, and  $\mathbf{1}(\cdot)$  is an indicator function.

I look for a competitive equilibrium in which (i) the aggregate behavioral investor demand  $\{B_{i,1}\}$  and  $\{B_{i,2}\}$  satisfy eq. (1) given prices, (ii) the arbitrageur's chosen positions  $\{x_{i,1}\}$  and  $\{x_{i,2}\}$  solve problem (3) given prices, and (iii) all asset markets clear s.t.  $\mu x_{i,t} + B_{i,t} = 0 \forall i, t$ .

I analyze the three-period equilibrium under two different assumptions about the arbitrageur's mass  $\mu$ : the trivial “pre-arbitrage” equilibrium with  $\mu = 0$  and the more interesting “post-arbitrage” equilibrium with  $\mu = \frac{1}{2}\phi$ . These two equilibria respectively capture sample periods before and after the growth of arbitrage on the assets.

## 2.1 The pre-arbitrage equilibrium

In the “pre-arbitrage” economy with a negligible mass of arbitrageurs ( $\mu = 0$ ), the assets have different alphas but no systematic risk. (All proofs and derivations are in [Appendix A](#).)

**Lemma 1. (Asset returns in the pre-arbitrage economy).** *If  $\mu = 0$ , excess return on asset  $i$  is*

$$r_{i,t}^e = \phi i + \epsilon_{i,t} \quad (4)$$

$\forall t$  where  $\epsilon_{i,t}$  is a mean-zero idiosyncratic return and the “pre-arbitrage alpha,”

$$\alpha_i^{pre} \equiv \phi i, \quad (5)$$

*increases monotonically from asset  $i = 0$  to asset  $i = 1$ .*

Hence the unobserved demand distortion  $\phi i$  is revealed in the abnormal return or “alpha” in the pre-arbitrage economy,  $\alpha_i^{pre}$ , which continues to proxy for demand distortions latent in the post-arbitrage economy. This identification of the post-arbitrage-economy demand distortion using the pre-arbitrage alpha is valid up to the cross-sectional ordering if the relative ordering of the distortion is invariant over the two economies.<sup>11</sup>

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<sup>11</sup>I maintain this assumption in my empirical tests using pre-arbitrage alphas. This is likely to be true despite the growth of institutional capital in the stock market if mutual fund managers exhibit behavioral patterns similar to those of retail investors (Frazzini, 2006; Frazzini and Lamont, 2008).

## 2.2 The post-arbitrage equilibrium

Next, consider the “post-arbitrage” economy in which the arbitrageur has a non-negligible mass of  $\mu = \frac{1}{2}\phi$ . If the arbitrageur is always unconstrained with sufficient capital ( $k_1, k_2 \geq 1$ ), all alphas are arbitrated away and no endogenous arbitrage-driven risk arises.

**Lemma 2. (Asset returns with unconstrained arbitrageurs).** *Suppose  $\mu = \frac{1}{2}\phi$  and  $k_1, k_2 \geq 1$  with certainty so that the arbitrageur is always unconstrained. Then, excess return on asset  $i$  is*

$$r_{i,t}^e = \epsilon_{i,t} \quad (6)$$

$\forall i, t$  where  $\epsilon_{i,t}$  is a mean-zero idiosyncratic return.

Hence, with the frictionless “textbook” arbitrage, assets subject to different degrees of demand distortion become effectively identical riskless assets. Comparing eq. (6) with eq. (4), the pre-arbitrage alpha has disappeared completely with no emergence of endogenous risk.

However, the more realistic case is if, during the arbitrage, the level of arbitrage capital may fall below the value required to counteract all demand distortions, an assumption I maintain from hereon:<sup>12</sup>

**Assumption 1.**  $\mu = \frac{1}{2}\phi$  so that the arbitrageur is large and  $k_2$  is in  $[0, 1]$  with positive (conditional) conditional probability so that the arbitrageur may be constrained during arbitrage.

In this case, asset returns from time 1 to time 2 follow a factor structure with endogenous “arbitrage-driven” betas with respect to  $k_2$ :<sup>13</sup>

**Lemma 3. (Asset returns with constrained arbitrageurs).** *Under Assumption 1, the expected excess return on asset  $i$  from time 1 to time 2 approximately follows*

$$E_1 r_{i,2}^e = \alpha_{i,2} + \lambda_k \beta_{i,k}, \quad (7)$$

<sup>12</sup>The case in which  $k_1$  may be below 1 but  $k_2 \geq 1$  is not considered explicitly, since this case is analogous to the “perfect arbitrage” case except for a positive return from time 1 to time 2.

<sup>13</sup>Since the level of arbitrage capital  $k$  is the state variable in the model, the stochastic discount factor in this economy is a nonlinear function of  $k$ . I therefore state an approximate factor model with respect to  $k$  rather than the exact model with respect to the stochastic discount factor for better intuition. An analogous result for the exact factor model is available in [Appendix A](#).

where  $\beta_{i,k}$  is an “arbitrage-driven” beta with respect to  $k_2$ ,  $\beta_{i,k} > 0 \forall i \in (0, 1]$ ,  $\lambda_k > 0$ , and  $\alpha_{i,2} \geq 0$ . Since mispricing disappears at time 3 with certainty, the expected excess return on asset  $i$  from time 2 to time 3 is

$$E_2 r_{i,3}^e = \alpha_{i,3}, \quad (8)$$

with  $\alpha_{i,3} \geq 0$  and no arbitrage-driven beta.

**Lemma 3** is intuitive. Equation (7) essentially restates the classic “limits of arbitrage” result of Shleifer and Vishny (1997) within a beta pricing framework. Arbitrage that requires capital is endogenously risky since the price of the arbitrated asset comoves with the level of arbitrage capital during arbitrage; i.e., since  $\beta_{i,k}$  is positive for the arbitrated assets.<sup>14</sup> Although the arbitrageur has a risk-neutral preference, the arbitrageur perceives this beta as risk due to the intertemporal speculative motive: asset return tends to drop precisely when arbitrage capital drops and investment opportunity improves. However, this result does not depend on the risk preference, since a low- $k$  state remains a high-marginal-value-of-wealth state under other risk preferences, as I explain further at the end of the section.

Comparing **Lemma 3** with **Lemma 2** shows that the entire cross-section of betas in **Lemma 3** arises from frictions in the arbitrage, justifying the name “arbitrage-driven” betas. Furthermore, equation (8) shows that no arbitrage-driven beta arises when demand distortion is about to disappear and asset prices are about to converge to the fundamental value, since the return on the asset would not comove with arbitrage capital in the next period. That is, arbitrage-driven betas do not arise in assets or portfolios with a short mispricing horizon (Gromb and Vayanos, 2017).

In both equations (7) and (8), the abnormal return  $\alpha_{i,t}$  can differ across assets when the arbitrageur is constrained, violating the law of one price (e.g., Gârleanu and Pedersen, 2011; Geanakoplos and Zame, 2014). A positive margin rate means that when the capital constraint binds, the arbitrageur would not equalize all abnormal returns if doing so through a long-short trade makes less money than other trades the arbitrageur currently engages in. For example, if the arbitrageur’s shadow cost of capital is 3% and the margin rate is 50%, the arbitrageur would not engage in a long-short trade on two portfolios with 1% and  $-1\%$  abnormal returns and identical factor exposures to earn a 2% return.

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<sup>14</sup>Also see Gromb and Vayanos (2017) and Kondor and Vayanos (2017).

Since wealth  $w$  and funding conditions  $f$  determine the level of arbitrage capital, the beta pricing model in equation (7) can be restated in terms of wealth and funding betas:

**Lemma 4. (Decomposing the arbitrage-capital beta).** Eq. (7) in Lemma 3 can be restated as

$$E_1 r_{i,2}^e = \alpha_{i,0} + \lambda_w \beta_{i,w} + \lambda_f \beta_{i,f} \quad (9)$$

where  $\beta_{i,w}$  and  $\beta_{i,f}$  are betas with respect to  $w_2 = w_1 + \int_0^1 r_{i,2} x_{i,2} di + \tilde{w}_2$  and  $f_2$ , respectively.

Hence, arbitrage-driven betas can arise with respect to two kinds of systematic factors. First, they arise with systematic shocks to the arbitrageur's wealth  $w_2$  coming from the assets being arbitrated ( $\int_0^1 r_{i,2} x_{i,2} di$ ) and from more exogenous shocks such as fund flows to institutional arbitrageurs ( $\tilde{w}_2$ ). Hence an arbitrated asset with no prior factor exposure can attain betas with factors that other arbitrated assets are exposed to. Second, they arise with systematic funding shocks  $f_2$ . An arbitrated asset with no prior factor exposure can attain betas with factors that determine arbitrageur funding conditions. Restrictions on  $\beta_{i,k}$  derived below apply analogously to both  $\beta_{i,w}$  and  $\beta_{i,f}$ , but not necessarily to other systematic factors in the market.

### 2.3 The cross-section of arbitrage-driven betas

Previous results show that arbitrated assets obtain endogenous betas with respect to arbitrage-capital shocks (dubbed “arbitrage-driven” betas), but which asset obtains a *larger* arbitrage-driven beta? The first proposition shows that arbitrage-driven beta increases in the arbitrageur's total position on the asset:

**Proposition 1. (Arbitrage position determines the cross-section of arbitrage-driven betas).** Arbitrage-driven beta increases in the expected arbitrage position on the asset:  $\frac{\partial \beta_{i,k}}{\partial (E_1[\mu x_{i,2}])} > 0$ .

Proposition 1 is intuitive. If a larger fraction of the market capitalization of the asset is owned by the arbitrageur, the price of the asset is more sensitive to the variation in the aggregate arbitrage capital. Hence, such an asset has a higher arbitrage-driven beta than other assets.

Although intuitive, Proposition 1 is not completely satisfactory since arbitrage position—the right-hand variable determining the level of arbitrage-driven beta—is itself an endogenous quantity

determined in equilibrium. The next proposition shows that it is ultimately the asset’s demand distortion that determines its arbitrage-driven beta. A larger demand distortion from the arbitrageur’s perspective means that the arbitrageur plays a larger price-correcting role in the asset in equilibrium through a larger arbitrage position, which results in a higher arbitrage-driven beta. This demand distortion may be unobserved by the econometrician but is revealed by the pre-arbitrage alpha. The next proposition restates [Proposition 1](#) using this “instrument” for the arbitrage position.

**Proposition 2.** *(Pre-arbitrage alpha predicts the cross-section of arbitrage-driven betas). Arbitrage-driven beta increases in the magnitude of the demand distortion in the asset proxied by the pre-arbitrage alpha:  $\frac{\partial \beta_{i,k}}{\partial \alpha_i^e} > 0$ . That is, “alphas turn into betas.”*

I illustrate [Proposition 2](#) using an example. Consider assets  $A$  and  $B$ , which are claims to some deterministic payoff of \$10 in present value. Suppose also that, absent arbitrage capital, demand distortions in behavioral investors drive down the prices of the assets to  $P_A = \$5$  and  $P_B = \$8$ , creating “pre-arbitrage” alphas of  $\alpha_A = 100\%$  and  $\alpha_B = 25\%$ . Now suppose that arbitrageurs begin trading these assets but their capital loads positively on some factor  $k$ . Then in “normal” arbitrage times, the arbitrageurs would drive up  $P_A$  and  $P_B$  to nearly \$10. However, if during the arbitrage, a large negative- $k$  shock depletes the arbitrage capital completely,  $P_A$  and  $P_B$  would drop 50% (\$10 to \$5) and 20% (\$10 to \$8) respectively, assuming that the behavioral investors’ demand distortion stays. Hence, precisely because  $A$  has a larger pre-arbitrage alpha and arbitrageurs play a larger price-correcting role in the asset in normal times,  $A$  has a larger endogenous sensitivity to (i.e., higher beta with) factor  $k$  than  $B$ .<sup>15</sup>

[Figure 2](#) illustrates [Lemma 1](#) as well as propositions [1](#) and [2](#) to show that the model generates patterns observed in the data ([Figure 1](#)). In the pre-arbitrage economy, the assets have zero betas with respect to  $k_2$  irrespective of their pre-arbitrage alphas, since arbitrageurs are too small to generate price pressure on the assets. However, in the post-arbitrage economy, the assets obtain a cross-section of different betas with  $k_2$  that line up with their pre-arbitrage alpha or expected arbitrage position.

Next, a useful restriction on arbitrage-driven betas is that the cross-section of different arbitrage-driven betas comes from the constrained states of time 2. Put differently, an arbitrageur does not

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<sup>15</sup>And this endogenous risk means that  $P_A$  and  $P_B$  would actually be lower than \$10 even with sufficiently large arbitrage capital, except in the period immediately before the deterministic payoff.

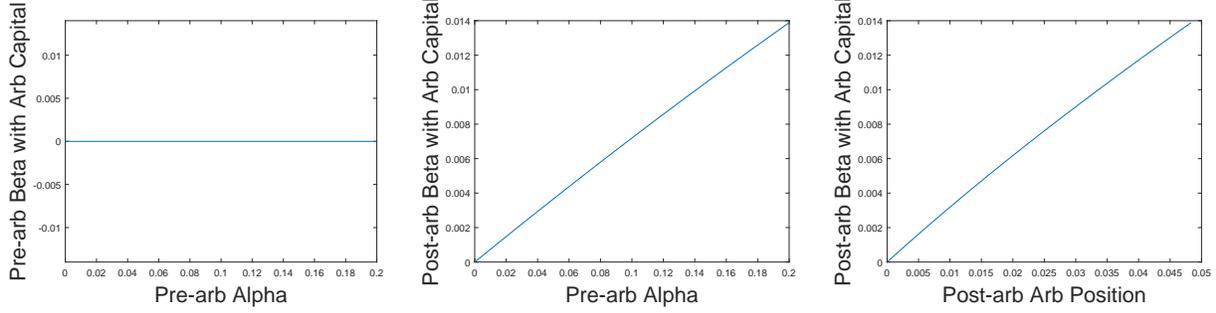


Figure 2: “Turning Alphas into Betas” in the Model

The first figure shows that assets’ betas with respect to arbitrage capital in the pre-arbitrage economy cluster around zero. The next two figures show that the assets’ arbitrage-capital betas in the post-arbitrage economy are explained by their pre-arbitrage alpha and expected arbitrage position. Parameter values used:  $\phi = 0.2$ ,  $\mu = \phi/2$ ,  $k_2 \sim U[-10, 10]$ ,  $k_1 \geq 1$ ,  $c = 0.5$ , and  $\delta_{i,t}/v \sim N(0, 0.1)$ .

generate endogenous  $\beta$ s in the assets when he has a “deep pocket,” which was the case in [Lemma 2](#):

**Proposition 3.** (*Arbitrage-driven betas arise when the arbitrageur is constrained*). *Arbitrage-driven betas arise only when the arbitrageur is constrained. That is,*

$$\begin{aligned} \beta_{i,k} | (k_2 \geq 1) &= 0 \\ \beta_{i,k} | (k_2 < 1) &> 0 \end{aligned} \tag{10}$$

for all  $i \in (0, 1]$ . For this reason, if  $k_t$  follows a process such that  $k_1, k_2 \geq 1$  almost surely, then neither beta nor abnormal return arises:

$$\beta_{i,k} = 0 \text{ and } E_1[r_{i,2}] = 0 \text{ for all } i \in [0, 1]. \tag{11}$$

Although intuitive, the exact statement of [Proposition 3](#) relies on the assumption that the arbitrageur is risk-neutral and dividends are i.i.d. However, a weaker version of the proposition would hold under risk aversion and undiversifiable dividends (in which case arbitrageurs would not correct asset prices completely despite high  $k_2$ ): the arbitrage-driven beta is lower if  $k_2$  is expected to be higher. Intuitively, the arbitrageur’s optimization implies that  $p_{i,2}$ , a non-decreasing function of  $k_2$ , is capped at  $v$ . Hence  $\partial p_{i,2}/\partial k_2$  should approach zero as  $k_2$  increases, which implies a decreasing price sensitivity to  $k_2$  for higher values of  $k_2$ .

Testing [Proposition 3](#) requires empirically identifying constrained vs. unconstrained periods.

In this model of time-varying arbitrage capacity, abnormal returns ( $\alpha_{i,2}$  in Lemma 3) emerge only when the arbitrageur is constrained, providing one approach to identifying constrained periods for the arbitrageur.

Next, arbitrage-driven betas are “discount-rate” betas. They arise because a positive arbitrage-capital shock increases the *valuation*—as opposed to expected cash flows—of an underpriced asset while a negative arbitrage-capital shock lowers it. It follows that return predictability in the post-arbitrage economy increases in the magnitude of  $\beta_{i,k}$  and its determinants.

**Proposition 4. (Cross-section of time-series return predictability).** *Arbitrage-driven betas are discount-rate betas. Hence return predictability measured by the  $R^2$  increases in the absolute value of the arbitrage-driven beta:*

$$\frac{\partial R_i^2}{\partial |\beta_{i,k}|} > 0,$$

where  $R_i^2 = \text{Var}_1(E_2 r_{i,3}^e) / \text{Var}_1(r_{i,3}^e)$ . Hence it also increases in the absolute value of the arbitrage position or the pre-arbitrage alpha.

Intuitively, an asset with a larger arbitrage-driven beta has larger discount-rate variation generated by arbitrage capital. Hence, if return volatility unassociated with arbitrage activity is constant or similar across the assets—which is the case in this model—the explained part of the asset return increases in arbitrage-driven beta. It is important to note that the  $R^2$  increases in the *absolute value* of the arbitrage-driven beta or its determinants. In a return predictive regression, the conditioning information can be either the level of arbitrage capital  $k_2$  or past return  $r_{i,2}$ . I use the latter in my empirical tests.

Except for one part of Proposition 4 that relates predictability to the arbitrage position and the pre-arbitrage alpha, testing the previous propositions requires computing betas with respect to factors that arbitrage capital loads on. This means having to take a stance on which factor generates systematic shocks to arbitrage capital. However, one can circumvent this problem by observing asset returns during a severe crash of arbitrage capital, which reveals the assets’ betas with respect to arbitrage capital shocks. Therefore, if arbitrage-capital beta increases in the asset’s demand distortion (Propositions 1 and 2), the asset return response to the crash should also line up cross-sectionally with the arbitrage position and the pre-arbitrage alpha that proxy the distortion. The next proposition formalizes this idea, focusing on the case in which the arbitrageur is unconstrained

at time 1 to state an analytical result.<sup>16</sup>

**Proposition 5. (Cross-section of asset returns during a crash in arbitrage capital).** *Asset return during a crash of arbitrage capital decreases in the asset's demand distortion. Specifically, if  $k_1$  is sufficiently large, a crash in  $k_2$  that leads to a complete unwinding of arbitrage positions on assets  $[0, i_2^*]$  generates negative returns on these assets that increase in magnitude in  $\phi_i$ :*

$$r_{i,2}^e < 0 \text{ and } \frac{\partial r_{i,2}^e}{\partial (\phi_i)} < 0 \quad \forall i \in [0, i_2^*].$$

Furthermore, since these asset returns are discount-rate shocks, asset returns going forward display the opposite pattern:

$$\frac{\partial E_2 [r_{i,3}^e]}{\partial (\phi_i)} > 0 \quad \forall i \in [0, i_2^*].$$

Proposition 5 implies that a large enough crash in arbitrage capital generates negative returns in almost all arbitrated assets and that the magnitude of the return response is greater in assets with larger demand distortions since the arbitrageur plays a larger price-correcting role in the assets. This allows me to test the predicted relationship between measures of demand distortion and arbitrage-capital beta without having to identify an arbitrage-capital factor.

The [online appendix](#) states an additional prediction that in the presence of arbitrage-driven betas, asset pricing tests can produce biased price of risk estimates if the econometrician does not distinguish between pre- and post-arbitrage economies. Simply put, the bias arises because the arbitrage channel causes factor betas to be cross-sectionally correlated with the alphas, the disturbance term in a cross-sectional asset pricing regression.

## 2.4 Discussion

Although the model makes a few simplifying assumptions to deliver a simple framework, these assumptions are relatively innocuous in that the predictions I draw from the model are likely to survive various model extensions.

First, as in Shleifer and Vishny (1997), Brunnermeier and Pedersen (2009), and Brunnermeier

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<sup>16</sup>See [Lemma 7](#) in [Appendix A](#) for the exact condition.

and Sannikov (2014), the arbitrageur in my model is risk-neutral but perceives the arbitrated assets to be endogenously risky since shocks to arbitrage capital at time 2 makes the arbitrageur's marginal value of wealth (MVW) stochastic at time 2 and covary negatively with returns on the arbitrated assets. Adding risk aversion would not change the endogenous negative relationship between the MVW and returns on the arbitrated assets that generates the arbitrage-driven betas. In the presence of a margin constraint, a large negative arbitrage-capital shock during arbitrage would still force the arbitrageur to unwind its positions in the assets, generating negative asset returns. And low capital would still mean high marginal value of wealth.

Second, the cross-sectional relationship between demand distortion and arbitrage-driven beta would also remain with risk aversion, even if I reduce the number of assets to be finite so that the arbitrageur cannot disregard idiosyncratic risks. Despite idiosyncratic risk, the arbitrageur would take a larger position on the higher-distortion asset in equilibrium, since equal arbitrage position on two assets that have different demand distortions means that the arbitrageur should marginally increase his position on the higher-distortion asset. Then, the same example as above implies that the price of the higher-distortion asset with a larger arbitrage position would drop more in response to the arbitrage-capital shock.

Finally, arbitrageur wealth shocks in my model are “exogenous” in that they do not come from cash-flow shocks to the arbitrated assets, which are assumed i.i.d. to deliver analytical results. In reality, arbitrage-driven betas can arise with respect to “endogenous” wealth shocks coming from cash-flow shocks to arbitrated assets. Introducing cash-flow shocks (i.e., correlated dividends) to the model would not change the analytical result, since dividends are part of the arbitrageur portfolio return that determines arbitrageur wealth, and [Lemma 4](#) already shows that arbitrage-driven betas arise with respect to arbitrageur wealth shocks. However, in the pre-arbitrage economy, correlated dividend shocks combined with arbitrageur risk aversion would mean that the pre-arbitrage alpha from the arbitrageur's perspective would need to be computed with respect to the common cash-flow factor.

### 3 Application to Equity Anomalies: Data and Methodology

I test the model’s predictions in the cross-section of betas of equity “anomalies,” trading strategies known to generate abnormal returns. Quantitative long/short equity hedge funds—the primary arbitrageurs of anomalies—dedicate around \$300-400 billion to trading these anomalies (as of 2007), suggesting that their trading may expose anomalies to factors that their capital is exposed to and generate arbitrage-driven betas.<sup>17</sup>

Although my predictions apply to other trading strategies, I use equity anomalies as a laboratory for my tests for three reasons. First, they are a subject of a large body of research, being increasingly used as test assets in the cross-sectional asset pricing literature, so understanding how their factor exposures arise seems particularly important. Second, they offer a reasonably rich cross-section (Green, Hand, and Zhang, 2016), allowing me to test the model’s cross-sectional predictions. Third, previous research documents approximately when institutional arbitrageurs began trading the anomalies: around early 1990s (1993 in particular) due to improved market liquidity and increased hedge fund capital (Schwert, 2003; Chordia, Roll, and Subrahmanyam, 2008, 2011; Stein, 2009; Chordia, Subrahmanyam, and Tong, 2014) and around academic publication due to increased publicity (McLean and Pontiff, 2016). This allows me to look for signs of arbitrage-driven betas in a disciplined manner.

#### 3.1 The anomalies

I use 40 equity anomalies that are “long” and “short” portfolios (top and bottom deciles) of 20 anomaly characteristics (see the list in [Table 1](#)).<sup>18</sup> I compute each anomaly’s monthly value-weighted returns over 1974m1–2016m12 based on all domestic common stocks from the three major exchanges (NYSE, AMEX, and NASDAQ) that belong to the extreme decile portfolios

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<sup>17</sup>These numbers are from Pedersen (2009).

<sup>18</sup>This list of 20 characteristics represents a standard set of low-turnover anomaly characteristics. One can arrive at this set by taking the 32 characteristics surveyed by Novy-Marx and Velikov (2016) and excluding the 5 redundant (e.g., “high-frequency combo”) and 7 highest-turnover (e.g., short-term reversal) characteristics. I exclude the high-turnover anomalies since arbitrage-driven beta should *not* arise in anomalies with a short mispricing horizon, as discussed in [section 2](#). I use long and short portfolios as separate test assets rather than forming long-short portfolios for two reasons: (i) actual arbitrageurs typically do not form a long-short portfolio based on single anomaly characteristic but consider multiple anomaly characteristics of stocks; (ii) it ensures a large cross-sectional variation in the right-hand variable (e.g., arbitrage position and pre-arbitrage alpha), which increases the power of the test.

formed at the end of the previous month.<sup>19</sup> I use cumulative quarterly returns over 1974q1–2016q4 for analysis using quarterly factors. To construct standard errors, I use bootstrapping to account for cross-anomaly covariances.<sup>20</sup>

### 3.2 Factor models of anomalies

I study the anomalies’ betas in the contexts of two factor models that summarize the cross-section of risk of anomalies reasonably well: the intermediary-based model of Adrian, Etula, and Muir (2014) and the five-factor model of Fama and French (FF) (2015). I do this in the separate context of each model rather than combine the two models in an arbitrary manner. Within each factor model, the model predictions apply to factors that anomaly-arbitrageur capital loads on, a point I come back to.

The intermediary-based model of Adrian et al. studies the cross-section of asset risk from the perspective of levered financial intermediaries such as banks and hedge funds. To do so, it proposes shocks to the book leverage of security broker-dealers as a factor capturing aggregate funding-liquidity shocks.<sup>21</sup> Adrian et al. find that the funding factor explains both value and momentum anomalies; i.e., value and momentum stocks have high betas with aggregate funding-liquidity shocks, making them risky from the perspective of levered financial intermediaries. Furthermore, I find that the factor helps explain anomalies beyond value and momentum—the cross-section of 40 anomaly returns tends to line up with their funding-liquidity exposures (Figure 3a).

Since anomaly arbitrageurs such as quantitative long/short equity hedge funds rely on leverage and hence funding liquidity, I treat the funding-liquidity factor as a factor that arbitrage capital

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<sup>19</sup>My data are from CRSP and Compustat. Similarly to Novy-Marx and Velikov (2016), I do not use data from before the early 1970s because of the poor quality of quarterly accounting data. See the [online appendix](#) to this paper as well as Novy-Marx and Velikov (2016) for more information on the anomaly construction.

<sup>20</sup>See [online appendix](#) for the bootstrapping procedure. My bootstrapped standard errors tend to reduce my  $t$ -statistics by 50-70%. However, these bootstrapped standard errors likely *overstate* the estimated coefficient’s standard deviation since part of the cross-anomaly covariances would come from arbitrage-capital shocks that my factors do not account for.

<sup>21</sup>Specifically,  $f_t = \ln(\text{leverage}_t^{BD}) - \ln(\text{leverage}_{t-1}^{BD})$ , where *leverage* is the aggregate leverage of the entire broker-dealer sector obtained from the Federal Reserve Board’s flow of funds data. The book leverage is adjusted for seasonality before taking the growth rate. Adrian et al. refer to the factor as a “leverage” factor, but I follow a related paper by Asl and Etula (2012) in calling it a funding factor. Another intermediary-based factor is the “capital ratio” factor of He, Kelly, and Manela (2017). However, this factor is aimed at explaining the cross-section of expected returns in multiple asset classes rather than different anomalies within equity and does not explain the equity momentum anomaly. Hence for my analysis on equity anomalies, I use the funding factor of Adrian et al.

loads on.<sup>22</sup> This seems plausible given how the factor is defined since security broker-dealers provide funding to hedge funds as their prime brokers. Therefore, I test to what extent the anomalies' funding-liquidity betas are arbitrage-driven betas. To do so, I extend the quarterly funding factor to 2016q4 and standardize it over my sample period 1974q1-2016q4.<sup>23</sup> I compute funding betas of anomalies in a two-factor model that includes the market factor, which Adrian et al. show is more robust to subsample analysis.<sup>24</sup>

I also study betas in the context of the five-factor model of FF, which augments the three-factor model comprising the market, size, and value factors (labeled *MKT*, *SMB*, and *HML*) with the new profitability and investment factors (labeled *RMW* and *CMA*) to summarize the cross-sectional covariances of stocks.<sup>25</sup> FF (2016) find that the various equity anomalies' "abnormal returns" are explained by their large loadings on *RMW* and *CMA*. Consistent with this claim, [Figure 3b](#) shows that the five-factor model captures almost 80% of the cross-section variation in 40 anomalies' returns over 1974m1-2016m12. For my analysis in [section 5](#), I use multivariate betas estimated in monthly time-series regressions using all five factors, which I download from Kenneth French's data library. Unlike the intermediary-based model, it is less obvious which Fama-French factors are shocks to anomaly arbitrageurs. I show in [section 5](#) that *RMW* and *CMA* are the likely candidates.

### 3.3 Arbitrage position

My model shows that total arbitrage position in an anomaly is a key cross-sectional determinant of arbitrage-driven betas. Following previous studies (Ben-David, Frazoni, and Moussawi, 2012; Boehmer, Jones, and Zhang, 2013; Hanson and Sunderam, 2014; Hwang, Liu, and Xu, 2018), I infer arbitrage positions on anomalies from an abnormal level of shorting (short interest).<sup>26</sup> Since

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<sup>22</sup>Brunnermeier and Pedersen (2009), Aragon and Strahan (2012), and Mitchell and Pulvino (2012) are selective works that document funding-liquidity exposures of hedge funds.

<sup>23</sup>See Adrian et al. and the online appendix to this paper for detailed instructions on constructing the series.

<sup>24</sup>The market factor is downloaded from Kenneth French's data library.

<sup>25</sup>*RMW* stands for "robust minus weak" profitability and *CMA* stands for "conservative minus aggressive" investment. The logic behind the two new factors is that the profitability and investment of a company are natural predictors of future returns: between two companies with the same market and book values and future growth in book equity, the one with a higher growth in earnings (i.e., more profitable) has a higher expected return; similarly, between two companies with the same market and book values and future growth in earnings, the one with a higher expected growth in equity (e.g., more investment) has a lower expected return (FF 2015).

<sup>26</sup>Ben-David, Frazoni, and Moussawi (2012) and Boehmer, Jones, and Zhang (2013) use short interest to infer hedge fund positions in stocks. My approach of inferring both short and long positions of hedge funds based on short

most short positions are held by hedge funds (approximately 85%, according to Goldman Sachs, 2008), abnormally high (low) short interest on an anomaly signals a net short (long) arbitrage position taken by the arbitrageurs.

Specifically, in each month, I measure arbitrage position in each stock as the *negative* ( $-1 \times 100$  to express in %) of the abnormal short interest defined as the deviation in the short interest ratio (shares shorted  $\div$  shares outstanding) from the level predicted by its size and liquidity deciles.<sup>27</sup> Then, I compute the arbitrage position in an anomaly as the value-weighted average of the abnormal short interests in the underlying stocks.

Figure 4 plots the equal-weighted average of anomaly positions on long- and short-side anomalies over 1974m1–2016m12. The figure shows that the gap in the arbitrage on short vs. long anomalies widens around the early 1990s, after which the short interest ratio is 30–50 basis points lower (hence an arbitrage position of 0.3–0.5%) for long-side anomalies and 1–2 percentage points (%p) higher (hence arbitrage position of  $-1\%$  to  $-2\%$ ) for short-side anomalies.

### 3.4 Pre-arbitrage alpha

The model shows that the abnormal return in the absence of arbitrageurs (i.e. the pre-arbitrage alpha) is an alternative right-hand variable that explains arbitrage-driven betas in periods with more arbitrage activity. I use CAPM alpha in the pre-1993 period as a baseline proxy for the pre-arbitrage alpha; however, using alternative factor alphas generates similar results. I use the pre-1993 period as a proxy for the pre-arbitrage economy in the model, since this period features less arbitrage on the anomalies as observed in Schwert (2003), Chordia, Roll, and Subrahmanyam (2008, 2011), Chordia, Subrahmanyam, and Tong (2014), and Stein (2009).

Table 2 shows that pre-1993 alphas inferred from CAPM and other factor models are a strong predictor of arbitrage activity in the post-1993 period, suggesting that pre-1993 alpha proxies

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interests is closest to Hanson and Sunderam (2014) and Hwang, Liu, and Xu (2018). On the other hand, Chen, Da, and Huang (2018) infer arbitrage positions from both short interest and 13F data. I do not use hedge fund long positions inferred from 13F filings since the holdings data are available only at the holding company level, which makes it difficult to identify quantitative long/short equity hedge funds that trade anomalies. Consistent with this, I find that aggregate hedge fund holdings in the 13F do not have a strong relation to past alphas, contrary to aggregate short interest. (I thank Chen, Da, and Huang for generously allowing me to check this using their data.) The short interest data come from Compustat.

<sup>27</sup>I do this in a cross-sectional OLS regression using size and liquidity deciles as dummy variables, but the exact method I use to obtain abnormal short interest does not affect my results.

for the amount of demand distortion arbitrageurs perceive in an anomaly (see the coefficient on  $\alpha^{pre} \times \text{Post-1993}$ ): arbitrage positions on positive- $\alpha$  anomalies become more positive and those on negative- $\alpha$  anomalies become more negative around 1993.<sup>28</sup> The table shows that academic publication is also important, consistent with McLean and Pontiff (2016). Interestingly, under some specifications, academic publication triggers increased arbitrage activity *only* in the post-1993 period, when hedge funds have sufficient capital to generate an observable effect on short interests in response to an academic publication.

### 3.5 Post-arbitrage beta

I use the post-1993 period as a proxy for the post-arbitrage economy in the model. Hence, my cross-sectional tests on arbitrage-driven betas use betas estimated in the time series within the post-1993 period as the left-hand variable. However, the year 1993 does not represent a structural break in arbitrage activity, and using alternative cutoffs in the early 1990s does not affect my results. To further alleviate concerns about using the 1993 cutoff, I also study betas in a panel regression and find consistent results.

### 3.6 Other determinants of beta

Although my model in [section 2](#) shuts down the fundamental cash-flow channel for betas, in practice, the cash-flow channel also explains cross-anomaly covariances (Campbell, Polk, and Vuolteenaho, 2009; Lochstoer and Tetlock, 2016). Therefore, I use the anomalies' size, book-to-market ratio, profitability, and investment characteristics as the fundamental determinants of factor betas.<sup>29</sup> An anomaly's characteristic is defined as the value-weighted average characteristic decile of the underlying stocks, where the deciles are determined only by the NYSE stocks. An alternative way to control for the non-arbitrage determinants of betas is to use betas from the pre-1993 period with less arbitrage on the anomalies.

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<sup>28</sup>My finding on the 1993 cutoff is somewhat at odds with the finding that no return decay is observed in the anomalies following 1993 (McLean and Pontiff, 2016). The main reason for this difference is that short interest measures the arbitrage activity by a group of sophisticated arbitrageurs, whereas return decay reflects investment by all types of investors. Another contributing factor is that I use the year in which the anomaly was first published, not when it was first well publicized. For example, the academic publication of the value anomaly is Rosenberg, Reid, and Lanstein (1985) in my data, but it is Fama and French (1992) in McLean and Pontiff.

<sup>29</sup>See the [online appendix](#) to this paper for more information about how I construct these characteristics for each stock.

### 3.7 Testing Proposition 5

To test [Proposition 5](#), I use the crash of quantitative hedge fund capital during the quant crisis of 2007. See [section 6](#) for more explanations for this choice.

## 4 Funding-Liquidity Betas as Arbitrage-Driven Betas

Equity anomalies display a large cross-sectional variation in their funding-liquidity betas ([Figure 3](#)). To what extent does this variation arise because levered arbitrageurs such as hedge funds are exposed to aggregate funding-liquidity shocks in the first place and then transmit these shocks to the anomalies they trade? [Figure 1](#) summarizes the answer given in this section: the cross-section of funding-liquidity betas arises almost entirely through the arbitrage channel. The analysis in this section supplements this observation using formal tests.

### 4.1 Explaining the cross-section of funding betas

Applying [Propositions 1](#) and [2](#), I formally test whether arbitrage position and pre-arbitrage alpha explain the cross-section of post-1993 funding betas. [Table 3](#) shows that anomalies with greater arbitrage position have higher funding betas, controlling for other potential determinants of the beta using pre-1993 funding betas or post-1993 fundamental characteristics of anomalies (columns (1)-(2)). A 1%p rise in arbitrage position (1%p fall in the short interest ratio) raises the anomaly's funding beta by 1.40–1.60, which means that the anomaly return responds 1.40–1.60%p more to a one-standard-deviation shock in funding liquidity. The  $R^2$  is close to 80%, highlighting the economic importance of arbitrage positions in determining funding-liquidity exposures. Therefore, consistent with [Proposition 1](#), anomalies in which arbitrage capital plays a larger price-correcting role respond more to the variation in arbitrage capital due to funding-liquidity shocks.

However, using arbitrage position as the right-hand variable raises reverse-causality concerns: arbitrageurs may take larger positions on stocks with larger funding betas to earn extra risk premium (Jurek and Stafford, 2015).<sup>30</sup> A remedy is to use the pre-arbitrage alpha as a right-hand

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<sup>30</sup>Relatedly, Yang (2017) finds that mutual funds with lower skills increase factor exposures.

variable, since it measures the demand distortion in the anomaly that ultimately determines the equilibrium arbitrage position and since an alpha—when correctly measured—captures the part of expected return unrelated to factor exposures ([Proposition 2](#)). I use pre-1993 CAPM alpha and pre-1993 “unexplained” mean return as alternative proxies for the pre-arbitrage alpha. The unexplained mean return is defined as the mean excess return net of market risk premium measured by the multivariate (two-factor) market beta times the realized market premium:  $\overline{r}_i^e - \hat{\beta}_{i,m} \overline{r}_m^e$ . The coefficient on the pre-1993 unexplained mean return can be interpreted as the coefficient on the pre-1993 alpha net of the funding-liquidity premium when the pre-1993 funding beta is included as an additional regressor since it absorbs part of the unexplained return due to funding-liquidity exposure.

Columns (3)-(6) show that pre-arbitrage alpha strongly explains the cross-section of funding betas both as an OLS regressor and as a 2SLS instrument.<sup>31</sup> The coefficients on the two measures of pre-arbitrage alpha are almost identical since the anomalies’ pre-1993 funding betas are close to zero.

Controlling for the arbitrage-related regressors, fundamental characteristics do not explain the anomalies’ funding betas in the post-1993 period. Furthermore, arbitrage position does not explain the cross-section of funding betas in the pre-1993 period, consistent with my interpretation of the pre-1993 period as the pre-arbitrage period in which arbitrage-driven betas do not arise. Instead, fundamental characteristics explain around 3/4 of the cross-sectional variation in funding betas, suggesting that anomalies’ fundamental characteristics determine their funding-liquidity exposures in the pre-1993 period.<sup>32</sup>

## 4.2 Panel of funding betas

An alternative to the cross-sectional analysis is to study a panel (anomalies×time) of betas using time-varying arbitrage positions and pre-1993 alphas interacted with predictors of arbitrage

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<sup>31</sup>I thank Yao Zeng for the instrumental-variable interpretation of the regression using pre-arbitrage alpha.

<sup>32</sup>In particular, firms with value characteristics (i.e., high book-to-market ratios) are associated with high funding-liquidity exposures in the pre-1993 period, which may reflect that institutional arbitrage on value anomalies began earlier than 1993 due to Rosenberg, Reid, and Lanstein (1985), who introduced the value anomaly to the academic literature. Not controlling for arbitrage-related regressors, both profitability and investment characteristics are positively associated with funding betas because stocks with high pre-1993 alphas tend to be stocks with high profitability and investment characteristics. (Here, firms with *high* investment characteristics are those with low investment.)

activity (post-1993 and post-publication dummies) as the right-hand variables. On the left-hand side, I use betas estimated in a window of 29 quarters (7 years) surrounding each quarter for each anomaly, which allows the betas to vary slowly over time.<sup>33</sup> For additional controls, I include (i) anomaly fixed effects to control for unobserved mean differences among anomalies that may be correlated with funding betas, (ii) time-varying fundamental characteristics to control for changes in the characteristics that may affect the betas, and (iii) quadratic time trends to control for average trends in the betas.

Table 4 shows results consistent with the cross-sectional result. A time-series increase in the arbitrage position leads to an increase in the funding beta (column (1)). Furthermore, the increase in arbitrage positions on high- $\alpha^{pre}$  anomalies around 1993 and around academic publication have a combined effect of increasing the funding beta by around 0.28 for each  $\alpha^{pre}$  of 1%p (columns (2)-(5) and (7)-(9)). This magnitude is similar to the effect of  $\alpha^{pre}$  estimated in the cross-section (0.20) but slightly larger, suggesting that exploiting both the post-1993 and the post-publication increase in arbitrage activity may have led to a sharper identification. Comparing the  $R^2$ s in columns (4) and (6) shows that changes in arbitrage position around 1993 and academic publication explain around 30% of the time-series variation in funding betas.<sup>34</sup>

It is interesting to relate my panel regression result to the finding that academic publication increases the anomaly's correlation with other published anomalies (McLean and Pontiff, 2016). My result shows that the increased correlation arises partly from an increased exposure to the funding factor and that the post-publication increase in the beta (correlation) has a cross-sectional pattern consistent with my model: the increase is larger for an anomaly with a larger pre-arbitrage alpha.

### 4.3 Funding betas during constrained vs. unconstrained periods

What is an alternative explanation for the previous results? Suppose that the funding factor actually proxies for the arbitrageur wealth portfolio rather than aggregate funding-liquidity shocks. In this

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<sup>33</sup>The online appendix shows that using alternative windows (5-year or 9-year) does not affect my results.

<sup>34</sup>Profitability and investment characteristics, which were not strongly associated with post-1993 betas in the cross-sectional approach, are now more strongly associated with funding betas. This is because anomalies whose profitability and investment characteristics increase over time would also see their CAPM alphas increase (Fama and French, 2015), attracting more arbitrage capital and attaining higher funding-liquidity exposures. The cross-sectional approach may not detect this effect if changes in fundamental characteristics tend to mean-revert over the post-1993 period.

case, even if arbitrageurs were too small to affect the covariances of anomalies, an anomaly with a larger arbitrage position or pre-arbitrage alpha would mechanically have a higher beta with the factor since the anomaly would be a larger part of the arbitrageur wealth portfolio.<sup>35</sup>

[Proposition 3](#) is useful in this regard. If funding betas were arbitrage-driven betas, they would arise primarily when arbitrageurs are constrained such that shocks to their capital that relax or tighten their constraint generate variation in arbitrage positions in the anomalies. In contrast, if funding betas were mechanical wealth portfolio betas, they would strengthen when arbitrageurs are *unconstrained* and can hold more anomalies in their portfolio.

To test the proposition, I define constrained vs. unconstrained periods in two ways. First, I follow Nagel (2012) to proxy constrained (unconstrained) times for institutional arbitrageurs as quarters in which the moving average of the VIX is above (below) the sample median.<sup>36</sup> Second, since abnormal returns are competed away during unconstrained times ([Proposition 3](#)), I use years in which the anomalies' alphas re-emerge (disappear) as the constrained (unconstrained) times. Specifically, I use years in which CAPM alphas estimated from daily data have a cross-sectional  $R^2$  with pre-1993 CAPM alphas above the median.<sup>37</sup> [Figure 5](#) plots the constrained vs. unconstrained post-1993 quarters (or years) defined by the two methods. Despite some differences, they commonly identify the dot-com crash and the financial crisis of 2008-2009 as constrained periods.

[Table 5](#) strongly favors the arbitrage interpretation over the wealth-portfolio interpretation of my previous results. Funding betas are large and cross-sectionally explained by both arbitrage position and pre-arbitrage alpha during constrained times, but they tend to disappear during unconstrained times. Furthermore, although both correlation and volatility can affect beta, my finding is driven by changes in the anomaly *correlation* with the funding factor rather than changes in anomaly volatility. [Figure 6](#) shows that anomaly return correlations with the funding factor feature the same patterns as betas.

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<sup>35</sup>Note that this argument differs from the claim in [Lemma 4](#) that an arbitrated asset with no prior beta can attain an endogenous beta with a factor that is an undiversifiable *component* of the wealth portfolio. In the next section, I will interpret the *RMW* and *CMA* factors as components of the arbitrageur wealth portfolio.

<sup>36</sup>I use the exponential-weighted moving average with a smoothing factor 0.3. However, since quarterly VIX tends to be persistent, using the original quarterly VIX series delivers similar results.

<sup>37</sup>Theoretically, the correct alpha to use here should additionally control for the risk premium associated with arbitrage-driven beta. Yearly alphas that additionally account for exposure to the mimicking portfolio of the funding factor does not change leads to similar classification of constrained times, so I prefer using yearly CAPM alphas for simplicity.

## 4.4 Funding betas as discount-rate betas: Cross-section of time-series return predictability

Next, since arbitrage-driven betas are discount-rate betas, high funding-beta anomalies should feature greater booms and busts induced by arbitrage capital, which implies greater return predictability in the time series.<sup>38</sup> I test this in a two-stage regression. The first stage is a time-series predictive regression by anomaly. I predict each anomaly's 1-, 2-, and 3-year future returns using its own long-run past returns in overlapping monthly data, obtaining the  $R^2$  as the measure of how predictable the anomaly return is in the time series:<sup>39</sup>

$$r_{i,t \rightarrow t+s}^e = \theta_0 + \theta_1 r_{i,t-L \rightarrow t}^e + \epsilon_{i,t \rightarrow t+s}. \quad (12)$$

The second stage is a cross-sectional regression that explains first-stage  $R^2$ s using the absolute values of the funding betas, arbitrage positions, and the pre-arbitrage alphas of anomalies.<sup>40</sup> I do this for the post-1993 period, where I expect to find predictability lining up with the absolute value of the funding beta and its cross-sectional determinants, as well as for the pre-1993 period, which should *not* feature the same pattern unless return predictability, for whatever reason, is intrinsically correlated with the absolute value of funding beta and its determinants.

I use the anomaly's past 3- or 5-year cumulative excess return as the predictor of future return in the 1st stage (DeBondt and Thaler, 1985; Moskowitz, Ooi, and Pedersen, 2012). Past return predicts future return in my model since high (low) past return means that arbitrageurs have driven up (down) the price of the anomaly at the expense of a lower (higher) expected return going forward.<sup>41</sup> Empirically, long-run returns can proxy for valuation ratios such as the book-to-market ratio, often used in return predictability studies, when accounting data are unavailable or subject to seasonality issues, as is the case in my predictability regressions with monthly data (Fama and

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<sup>38</sup>An alternative way to check that a factor beta is a discount-rate beta is to decompose stock returns into discount-rate vs. cash-flow shocks using VAR, as in Campbell and Vuolteenaho (2004) and Campbell, Polk, and Vuolteenaho (2009) (CPV). However, as explained in CPV, this approach works most naturally for decomposing yearly returns and is not suitable for my paper with a relatively short sample period.

<sup>39</sup>Past and future returns I use as the left-hand and right-hand variables in the 1st-stage regression are based on the same stocks that belong to the anomaly portfolio as of  $t$ . Using past and future returns on rebalanced portfolios would be an incorrect approach.

<sup>40</sup>I take an absolute value since predictability increases in the magnitude of the discount-rate beta, regardless of its sign.

<sup>41</sup>In contrast, valuation ratios are not well-defined in my model with mean-zero dividends.

French, 1996; Gerakos and Linnainmaa, 2012; Asness, Moskowitz, and Pedersen, 2013).<sup>42</sup>

Figure 7 summarizes my finding on the cross-section of time-series predictability. Return predictability increases in the absolute value of the anomaly's funding beta in the post-1993 period but not in the pre-1993 period, consistent with post-1993 funding betas being discount-rate betas arising from arbitrage trades. Table 6 shows that this cross-sectional pattern holds with respect to the absolute value of the arbitrage position and the pre-arbitrage alpha as well as the funding beta (columns (1)-(3)). That is, an anomaly with a larger demand distortion attracts greater arbitrage position and suffers greater booms and busts due to variation in aggregate arbitrage capital.

The economic magnitude is large. In the baseline case, an increase in the absolute value of the funding beta by 1 (pre-arbitrage alpha by 1%p) increases the 1st-stage  $R^2$  of the predictability regression by 0.04–0.06 (0.01–0.02), depending on the return horizon. The large  $R^2$  of the 2nd-stage cross-sectional regression reported in the brackets shows that the arbitrage variables explain as much as 56% of the cross-sectional variation in predictability. The pre-1993 period does not display a cross-sectional relationship between predictability and funding beta, suggesting that the large discount-rate variation in high-funding-beta anomalies is unique to the post-1993 period with increased arbitrage activity.

By showing that the cross-section of different return predictabilities of anomalies is an equilibrium outcome of arbitrage trades, my results shed new light on the growing literature on time-series predictability of anomaly returns. My finding is consistent with Lou and Polk (2013), Frazzini and Pedersen (2014), and Huang, Lou, and Polk (2018), who find that the act of arbitrage generates predictable time-series patterns in momentum and low-beta stocks. My finding also suggests that the strong predictability of anomalies found in Haddad, Kozak, and Santosh (2018) may be unique to the recent post-1993 sample period with greater arbitrage-driven discount-rate shocks to the anomalies.

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<sup>42</sup>In my data,  $-1$  times the cumulative 3-year excess return and the book-to-market ratio taken in each June have a median annual time-series correlation of 0.66 (with 0.29 and 0.83 being the bottom and top 5% values) among the 40 anomalies. In the appendix, I show that return predictability using the book-to-market ratio also lines up with the absolute value of funding beta, albeit with a lower  $R^2$ .

## 4.5 Discussion

The results in this section suggest that the cross-section of funding-liquidity betas of anomalies arise endogenously through the act of arbitrage on the anomalies. Recall from [Section 3](#) that the intermediary-based asset pricing model of Adrian, Etula, and Muir (2014) prices the cross-section of anomaly returns using these betas as the right-hand variable. In this sense, my results suggest that intermediary-based asset pricing may be a cross-sectional manifestation of limits to arbitrage. Asset portfolios that are mispriced from intermediaries' perspective get traded by them and obtain a cross-section of endogenous sensitivities to factors that the intermediaries are exposed to. If intermediaries recognize this endogenous risk, the equilibrium expected returns on those portfolios should line up cross-sectionally with the betas that intermediaries themselves generate, which leads to the "intermediary-based asset pricing" result.

Next, my results provide one explanation for what equity anomalies are. They are, at least from arbitrageurs' point of view, mispricings turned into endogenous risks. This explanation is consistent with both the extensive trading of anomalies by institutional arbitrageurs and the partial persistence in anomaly returns.

However, the interpretation of my results does not depend heavily on whether equity anomalies actually represent hidden risk, mispricing, or measurement error. What matters is that institutional arbitrageurs trade anomalies, regardless of the debate. In fact, even if anomalies represent rational compensation for risk, arbitrage-driven betas would arise through risk sharing. Intuitively, anomalies with larger arbitrageur positions rely more heavily on the risk-sharing role of arbitrageurs and hence become more sensitive to factors that arbitrage capital load on. Also, even if some anomalies were measurement errors, a high past alpha that occurs by chance would still attract arbitrage capital and give rise to arbitrage-driven beta. In this case, however, arbitrage-driven beta would eventually disappear after arbitrageurs realize that an anomaly was a measurement error and stop trading it.

Relatedly, one may suggest that once arbitrage has driven down the anomaly alphas, arbitrage-driven betas should no longer arise. This is not the case. If the original source of the demand distortion remains, alphas can remain low only in the presence of arbitrage trades that generate the arbitrage-driven betas. In other words, in the presence of systematic shocks to arbitrage capital,

low alphas and nonzero arbitrage-driven betas should coexist in equilibrium.

## 4.6 Robustness checks

My findings are robust to alternative choices I could have made in my empirical analysis.<sup>43</sup> First, I use the sample cutoff of 1993 in some of my analysis, but my results are robust to using years 1991, 1992, 1994, and 1995 as the end of the pre-arbitrage period. To illustrate, I repeat the main cross-sectional regression in [Table 3](#) using the alternative cutoffs and find similar results.

Second, controlling for additional market characteristics of the anomalies, such as volatility and market liquidity, does not affect the ability of the arbitrage position and the pre-arbitrage alpha to explain the cross-section of funding betas. Although not statistically significant and somewhat sensitive to specification, assets with a larger pre-1993 volatility (which may proxy its fundamental volatility) tend to feature a greater funding-liquidity exposure in the post-1993 period, consistent with the predictions of Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2017). The low statistical significance suggests that liquidity spirals may be less pronounced in the stock market than in other markets.

Third, the funding-liquidity beta I estimate is a multivariate beta from a two-factor model that includes the market factor. How does including additional pricing factors change my result? This exercise is useful since it helps address the concern that the cross-sectional relationship between funding beta and arbitrage variable in the post-1993 period is a spurious result that arises from the funding factor—for whatever reason—becoming increasingly more correlated with a factor that has always featured the cross-sectional relationship. On the other hand, if the additional factors are portfolio factors that summarize the cross-section of returns well, including additional factors may eventually drive out the part of the funding-liquidity beta that is priced since the part of the stochastic discount factor that matters is the projection to the return space. Regardless, I study how the core cross-sectional result changes as I add an increasing number of return-based factors to the model, including the market liquidity factor that may have become more correlated with funding liquidity in recent years. Interestingly, the relationship stays relatively intact. Still, the result in this section does rely on the funding factor representing arbitrage-capital shock. [Section 6](#) alleviates this concern by providing evidence that does not rely on a factor model.

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<sup>43</sup>See the [online appendix](#) for relevant tables and additional robustness checks.

The online appendix also considers alternative specifications of time-varying betas used in the panel regressions and reports robustness checks on results in the next section.

## 5 Arbitrage-Driven Betas in the Fama-French 5-Factor Model

Next, I argue that arbitrage-driven betas arise in conventional multifactor models. Since factors in these models are long-short portfolios with positive mean returns, arbitrageurs that seek positive market-adjusted returns would load positively on these factors. Since portfolio shock is a component of arbitrage capital shock, arbitrated assets with no prior exposures to these factors would attain arbitrage-driven betas with respect to the factors (Lemma 4). Furthermore, even the anomalies with pre-arbitrage exposures to the factors would see their betas rise or fall depending on the direction of the arbitrageur's position in the anomaly; i.e., *controlling for* the anomaly's pre-arbitrage beta or fundamental characteristic, an anomaly with a larger arbitrage position or a higher pre-arbitrage alpha should have a higher beta with these factors.

In particular, I present evidence of arbitrage-driven betas in the five-factor model of Fama and French (FF) (2015). The first task is to infer which of the five factors the portfolio of anomaly arbitrageurs loads on. Columns (1)-(2) of Table 7 show that the equal-weighted long-short portfolio of 40 anomalies has economically large exposures to *RMW* and *CMA*, implying that arbitrageurs trading these anomalies would find it difficult to neutralize their *RMW* and *CMA* exposures. Consistent with this, columns (3)-(8) show that the actual portfolio of anomaly arbitrageurs proxied by quantitative long/short equity hedge funds have relatively large positive loadings on *RMW* and *CMA*.<sup>44</sup> Hence, I examine if anomalies' post-1993 *RMW* and *CMA* betas are partly arbitrage-driven betas.

### 5.1 Explaining the cross-section of Fama-French betas

Table 8 explains the cross-section of post-1993 Fama-French betas using both the arbitrage and non-arbitrage determinants of betas as right-hand variables. Starting with *SMB* and *HML*, the two

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<sup>44</sup>The finding that the quant equity hedge funds do not extensively trade *MKT* and *SMB* may not be surprising, but it may surprise the reader that they do trade *HML*. In the context of the original three-factor model of Fama and French (1993), these hedge funds *do* trade *HML*. However, since the new *RMW* and *CMA* factors meant to subsume *HML*, these hedge funds load on *RMW* and *CMA* but not to the residual *HML*.

traditional factors of Fama and French, I find that anomalies' *SMB* and *HML* betas are largely explained by the anomalies' fundamental characteristics (Panel A). The size (value) characteristic alone explains 79% (51%) of the cross-sectional variation in *SMB* (*HML*) betas, and adding arbitrage-related regressors does not seem to matter either statistically or economically.

On the other hand, both arbitrage position and two different proxies for the pre-arbitrage alpha (CAPM  $\alpha$  and FF5  $\alpha$ ) account for a substantial part of the cross-sectional variation in *RMW* and *CMA* betas (a marginal increase in the  $R^2$  of around 30%), consistent with [Proposition 2](#) (Panel B). In terms of magnitude, a 1%p rise in arbitrage position (1%p fall in the short interest ratio) raises the anomaly's *RMW* beta by 0.35–0.41 and *CMA* beta by 0.18–0.23. The statistical significance of the coefficients is stronger for *RMW* betas than *CMA* betas, perhaps due to anomaly-arbitrageur portfolios having a larger exposure to *RMW* than to *CMA* ([Table 7](#)). Fundamental characteristics do continue to matter. The profitability and investment characteristics are both highly significant and economically important in determining *RMW* and *CMA* betas, respectively.

## 5.2 Additional tests on *RMW* and *CMA* betas

Panel A of [Table 9](#) presents similar findings in the panel of *RMW* and *CMA* betas. An increase in the arbitrage position over time increases the anomaly's *RMW* and *CMA* betas. Furthermore, the betas show different projectiles from 1993 depending on their pre-arbitrage alphas proxied by the FF five-factor alpha.<sup>45</sup> Controlling for the post-1993 effect, however, anomalies' *RMW* and *CMA* do not change predictably around their academic publication, contrary to funding-liquidity betas. Next, *RMW* and *CMA* betas during unconstrained times exhibit reduced cross-sectional predictability based on arbitrage variables than those during constrained times, although the contrast is less pronounced for *RMW* (Panel B). Finally, the predictabilities of anomaly returns in the post-1993 period tend to line up with the absolute value of *RMW* and *CMA* betas as they do with funding betas, consistent with post-1993 *RMW/CMA* betas being discount-rate betas (Panel C).

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<sup>45</sup>The result is similar using CAPM alphas, but I use the five-factor alphas to be consistent with the multivariate nature of the betas. Although the five factors were not discovered until recently, to the extent that they represent principal components of anomalies, sophisticated arbitrageurs may have inadvertently accounted for exposures to *RMW* and *CMA* when making investment decisions.

This is true for all return horizons (1-3 years) for *RMW* and for the 1-year horizon for *CMA*.<sup>46</sup>

## 6 A Test Without A Factor Model: The Quant Crisis of 2007

Tests of Propositions 1-4 in the last two sections required choosing systematic factors that generate arbitrage-capital shocks. This section tests [Proposition 5](#), which instead focuses on the single event of a severe arbitrage-capital shock and predicts that the cross-section of returns during the event line up with the determinants of arbitrage-driven betas. To do this, I use the crash of quantitative long/short equity hedge fund capital in August 2007, which I describe briefly before proceeding to my tests.

### 6.1 Description of the crisis

Over a three-day period of August 7–9, 2007, seemingly distinct equity anomalies commonly underperformed. [Figure 9a](#) shows that long-side anomalies that arbitrageurs go long on commonly posted losses (in 17 out of 20 cases) and short-side anomalies that arbitrageurs go short on commonly posted gains (in 19 out of 20 cases). Naturally, hedge funds that took long-short positions on these anomalies also suffered severe losses. [Figure 9b](#) shows that these arbitrageurs suffered a cumulative loss of almost 6% over August 7–9, after which the return rebounded back over the following three days (August 10–14). Remarkably, the crash and the recovery were exclusive to equity anomalies; other arbitrage strategies remained unaffected.

The primary explanation for this unusual covariance event in the anomalies is a systematic drop in arbitrage capital. It is speculated that following a portfolio underperformance since early July 2007, one or more arbitrageurs rapidly unwound their arbitrage position on anomalies, possibly due to margin calls (Khandani and Lo 2007, 2011; Pedersen, 2009; Stein, 2009). This led to losses by other arbitrageurs that in turn triggered more margin calls until anomaly arbitrageurs commonly suffered capital losses.

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<sup>46</sup>Consistent with arbitrage-driven betas being endogenous, the relationship between predictability and the absolute value of the *RMW/CMA* beta in the pre-1993 period features lower  $R^2$ s.

## 6.2 The cross-section of anomaly returns during the crisis

Treating the three-day crash period of the crisis as the period in which the level of arbitrage capital dropped severely, I ask whether returns on different anomalies during the crash can be *cross-sectionally* explained by the differences in their arbitrage position or pre-arbitrage alpha (Proposition 5), analogous to my analysis on betas. Furthermore, since this drop in the asset price is a discount-rate (valuation) shock rather than cash-flow shock, I also ask whether the anomaly return during the three-day recovery period following the crisis can be cross-sectionally explained.

Figure 10 summarizes my finding. Cumulative raw and abnormal returns on anomalies during the crisis are cross-sectionally and strongly explained by their pre-1993 CAPM alphas. This is consistent with the key mechanism of the model that generates a cross-section of arbitrage-driven betas: an asset with a more positive (negative) pre-arbitrage alpha and hence positive (negative) arbitrage position drops (gains) more in response to a sharp decline in arbitrage capital. An opposite pattern holds during recovery, consistent with anomalies' quant-crisis returns being discount-rate movements.<sup>47</sup> Table 10 shows that this result is robust to using alternative measures of the arbitrage position and the pre-arbitrage alpha.

## 7 Conclusion

This paper uses a simple model to develop a set of predictions that help identify arbitrage-driven betas in a cross-section of asset portfolios. Testing these predictions on equity anomalies, I show that arbitrage-driven betas arise in the data and are an important part of the cross-section of betas that equity anomalies have with respect to funding-liquidity shocks as well as conventional cross-sectional factors.

Some qualifications are in order. First, my intention is not to claim that the arbitrage channel is the only possible explanation for the patterns I find in the betas of equity anomalies. Instead, my contribution is to show that the arbitrage channel in my model offers a unifying explanation for the joint occurrence of my empirical findings. Relatedly, my intention is to develop and test predictions

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<sup>47</sup>The cross-sectional pattern of long anomalies earning positive returns and short anomalies earning negative returns during the recovery is clearer in abnormal returns than in raw returns.

that use price and aggregate holdings data, which allows for a wide range of applications. Micro evidence based on detailed holdings data could complement my tests to provide further evidence for the arbitrage channel in the cross-section of betas, but I leave this to future work.

The framework in this paper can apply to other settings. My results suggest that the cross-section of asset betas in other asset classes may also be affected by the extent of arbitrage on the assets and that the predictions in my model can help detect such arbitrage-driven betas. My model may also shed light on the cross-sectional variation in the return correlations between the U.S. stock market and other equity markets. From a U.S. investor's perspective, the stock market of one country may appear riskier than another country's markets precisely because U.S. investors hold a larger fraction of the stock market, which causes the market to covary more with shocks to U.S. investor wealth.

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## Tables and Figures

Table 1: List of 40 Anomalies

This table describes the 40 anomalies used in this paper’s empirical sections.  $\alpha^{pre}$  is the CAPM alpha over the pre-1993 period of 1974m1–1993m12. (Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.) Mktcap Share is the time-series average (over 1974–2016) of the anomaly’s total market capitalization normalized by the total market capitalization of all domestic common U.S. stocks listed on the NYSE, AMEX, and NASDAQ.

Type	Academic Publication		Long (Top Decile)				Short (Bottom Decile)			
	Year	Sample	No	Label	$\alpha^{pre}$	Mktcap Share	No	Label	$\alpha^{pre}$	Mktcap Share
Beta arbitrage	1973	1926-1968	1	beta(L)	<b>3.9</b>	0.09	21	beta(S)	<b>-5.3</b>	0.09
Return on market equity	1977	1956-1971	2	rome(L)	<b>9.6</b>	0.05	22	rome(S)	<b>-8.6</b>	0.03
Ohlson’s O-score	1980	1970-1976	3	ohlson(L)	-0.4	0.29	23	ohlson(S)	<b>-4.8</b>	0.01
Size	1981	1926-1975	4	size(L)	2.8	0.02	24	size(S)	-1.1	0.58
Long-run reversals	1985	1926-1982	5	rev60m(L)	3.7	0.03	25	rev60m(S)	<b>-3.3</b>	0.13
Value	1985	1980-1990	6	value(L)	<b>6.8</b>	0.04	26	value(S)	<b>-4.4</b>	0.20
Momentum	1990	1964-1987	7	mom12m(L)	<b>6.0</b>	0.10	27	mom12m(S)	<b>-12.1</b>	0.04
Net issuance	1995	1980-1990	8	netissue(L)	<b>4.6</b>	0.11	28	netissue(S)	<b>-3.8</b>	0.08
Net issuance monthly	1995	1980-1990	9	netissue_m(L)	<b>4.4</b>	0.11	29	netissue_m(S)	-1.7	0.09
Accruals	1996	1962-1991	10	acc(L)	1.0	0.06	30	acc(S)	<b>-4.6</b>	0.05
Return on assets	1996	1979-1993	11	roa(L)	-0.0	0.17	31	roa(S)	<b>-7.4</b>	0.03
Return on book equity	1996	1979-1993	12	roe(L)	1.1	0.14	32	roe(S)	<b>-6.7</b>	0.04
Failure probability	1998	1981-1996	13	failprob(L)	0.5	0.16	33	failprob(S)	<b>-11.6</b>	0.02
Piotroski’s f-score	2000	1976-1997	14	piotroski(L)	0.6	0.21	34	piotroski(S)	<b>-3.2</b>	0.09
Investment	2004	1973-1996	15	invest(L)	<b>4.7</b>	0.03	35	invest(S)	<b>-4.6</b>	0.07
Idiosyncratic volatility	2006	1986-2000	16	idiovol(L)	1.4	0.25	36	idiovol(S)	<b>-11.7</b>	0.04
Asset growth	2008	1968-2003	17	atgrowth(L)	3.3	0.03	37	atgrowth(S)	<b>-4.2</b>	0.10
Asset turnover	2008	1984-2002	18	ato(L)	<b>3.4</b>	0.05	38	ato(S)	0.9	0.09
Gross margins	2008	1984-2002	19	gm(L)	-1.8	0.20	39	gm(S)	0.5	0.04
Gross profitability	2010	1976-2005	20	profit(L)	0.4	0.10	40	profit(S)	-0.8	0.07

**Table 2: Determinants of Arbitrage Position on Anomalies**

$$\text{Baseline: } Arb\ Position_{i,t} = b_0 + b_1\alpha_i^{pre} \times \mathbf{1}(t > 1993m12) + b_3\alpha_i^{pre} + b_4\mathbf{1}(t > 1993m12) + b_5t + b_6t^2 + u_i + \epsilon_{i,t}$$

This table shows that pre-1993 alpha predicts post-1993 and post-publication arbitrage position in a panel regression (40 anomalies  $\times$  1974q1–2016q4). The dependent variable measures arbitrage position on anomaly  $i$  in quarter  $t$  using the negative ( $\times -100$ ) of the “abnormal” short interest on the anomaly over the three months in the quarter. Abnormal short interest is defined as the value-weighted average of the residual short interest in a cross-sectional regression with 10 size deciles and 10 liquidity (Amihud) deciles as dummy variables, where the average is taken over all stocks that belong to the anomaly portfolio. I use short interests reported in mid-month and shares outstanding on the same day (if available) or the previous trading day. The post-1993 dummy is 0 for the pre-1993 period (1974q1–1993q4) and 1 for the post-1993 period (1994q1–2016q4). An anomaly’s “pre-arbitrage” alpha, denoted  $\alpha^{pre}$ , is measured by its pre-1993 alpha with respect to the factor model specified in the column heads. For failure probability,  $\alpha^{pre}$  is computed from 1981 onward to account for its sensitivity to sample period emphasized in Dichev (1998). Post-Publication, Post-Sample, Post-1993, and Post-1993  $\times$  Post-Pub (whenever appropriate) as well as quadratic time trends ( $t$  and  $t^2$ ) and a constant are included in the regression but not reported in the table. In the parentheses are  $t$ -statistics based on standard errors with clustering by anomaly and quarter. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	$\alpha^{pre} = \text{CAPM Alpha}$					$\alpha^{pre} = \text{FF3 Alpha}$			$\alpha^{pre} = \text{FF5 Alpha}$			Long vs. Short			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
$\alpha^{pre} \times \text{Post-1993}$	<b>0.087</b> (5.60)		<b>0.068</b> (5.17)	<b>0.060</b> (4.02)	<b>0.061</b> (6.57)	<b>0.069</b> (7.04)	<b>0.066</b> (5.30)	<b>0.054</b> (6.52)	<b>0.078</b> (6.92)	<b>0.073</b> (5.44)	<b>0.061</b> (5.97)				
$\alpha^{pre} \times \text{Post-Publication}$		<b>0.086</b> (4.90)	<b>0.034</b> (2.13)	0.020 (1.62)	0.028 (1.48)	<b>0.035</b> (2.72)	<b>0.030</b> (2.27)	0.010 (0.70)	<b>0.046</b> (3.06)	<b>0.037</b> (2.47)	0.021 (1.26)				
$\alpha^{pre} \times \text{Post-Sample}$				0.026 (1.56)			0.010 (0.80)			0.016 (1.16)					
$\alpha^{pre} \times \text{Post-1993} \times \text{Post-Pub}$					0.012 (0.65)			<b>0.040</b> (2.50)			<b>0.040</b> (2.27)				
Long $\times$ Post-1993												<b>0.743</b> (4.75)	<b>0.633</b> (4.84)	<b>0.610</b> (3.91)	
Long $\times$ Post-Publication													<b>0.674</b> (3.72)	0.197 (1.32)	0.160 (1.17)
Long $\times$ Post-Sample															0.071 (0.42)
Anomaly FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880	6,880
$R^2$	0.22	0.18	0.23	0.24	0.23	0.31	0.31	0.31	0.30	0.31	0.31	0.18	0.14	0.19	0.19

Table 3: **Explaining the Cross-Section of Funding-liquidity Betas**

$$\text{Baseline: } \beta_i^{post} = b_0 + b_1 \text{Arb Position}_i^{post} + b_2 \beta_i^{pre} + u_i$$

This table shows that the post-1993 funding-liquidity betas of 40 anomalies can be cross-sectionally explained by arbitrage position and the pre-arbitrage alpha, consistent with Proposition 1 and Proposition 2. Arbitrage position is inferred from abnormal short interest ratio as explained in Section 3. The pre-arbitrage alpha is proxied by the pre-1993 CAPM alpha. Funding-liquidity betas are betas with the funding-liquidity factor of Adrian, Etula, and Muir (2014) estimated in a two-factor model that includes the market factor. Characteristic ranks are the value-weighted decile rank of the underlying stocks' characteristics. Pre-1993 and post-1993 periods are 1974q1–1993q4 and 1994q1–2016q4, respectively. In the parentheses are  $t$ -statistics based on standard errors that account for cross-anomaly covariances through bootstrapping. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	LHS: Post-1993 Funding Beta							Pre-93 Funding Beta		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Post-1993 Arb Position	<b>1.40</b> (2.51)	<b>1.60</b> (2.66)			<b>1.69</b> (2.73)	<b>1.69</b> (2.73)				
Pre-1993 CAPM Alpha			<b>0.20</b> (2.58)							
Pre-1993 Unexplained Return <sup>†</sup>				<b>0.19</b> (2.57)						
Pre-1993 Arb Position								0.57 (0.31)		
Pre-1993 Funding Beta	0.28 (1.46)		-0.09 (-0.43)	-0.11 (-0.49)	0.30 (1.61)	0.30 (1.61)	0.16 (0.56)			
Size Rank		0.10 (0.83)						-0.41 (-1.47)	-0.02 (-0.17)	0.02 (0.10)
Value Rank		0.09 (0.47)						0.37 (1.86)	0.46 (1.73)	<b>0.48</b> (2.17)
Profitability Rank		-0.07 (-0.48)						0.19 (1.08)	0.07 (0.42)	0.09 (0.58)
Investment Rank		-0.04 (-0.32)						0.16 (1.15)	-0.07 (-0.66)	-0.09 (-0.97)
Constant	-0.15 (-0.83)	-0.13 (-0.07)	-0.33 (-1.36)	-0.32 (-1.32)	-0.07 (-0.36)	-0.07 (-0.37)	-0.56 (-1.70)	-2.71 (-1.22)	-2.74 (-1.17)	-2.89 (-1.44)
Observations	40	40	40	40	40	40	40	40	40	40
$R^2$	0.77	0.78	0.68	0.67			0.01	0.36	0.74	0.74
OLS/2SLS	OLS	OLS	OLS	OLS	2SLS	2SLS	OLS	OLS	OLS	OLS
<i>Instrumental variables</i>										
Pre-93 CAPM Alpha					✓					
Pre-1993 Unexplained Return <sup>†</sup>						✓				

<sup>†</sup>The pre-1993 unexplained return is defined as  $\overline{r_i^{pre}} - \hat{\beta}_{i,m}^{pre} \cdot \overline{r_m^e}$ , where  $\beta_{i,m}$  is a multivariate beta from a two-factor model with  $r_m^e$  and the funding liquidity factor.

**Table 4: Explaining the Panel of Funding-liquidity Betas**

$$\text{Baseline: } \beta_{i,t} = b_0 + b_1 \text{Arb Position}_{i,t} + b_3 t + b_4 t^2 + u_i + \epsilon_{i,t}$$

This table uses a panel regression to show that arbitrage position and pre-arbitrage alpha explain the panel of funding-liquidity betas of anomalies (40 anomalies  $\times$  1974q1–2016q4). Quarterly funding-liquidity betas are estimated in a window of 29 quarters (7 years) surrounding each quarter for each anomaly.  $\alpha^{pre}$  is the anomaly’s pre-1993 CAPM alpha.  $\alpha^{pre}$  interacted with Post-1993 and Post-Publication dummies are used as proxies or instruments for arbitrage position. Post-1993 and Post-Publication (whenever appropriate) as well as quadratic time trends ( $t$  and  $t^2$ ) and a constant are included in the regression but not reported in the table. In the parentheses are  $t$ -statistics based on standard errors that account for cross-anomaly covariances through clustering by quarter and serial correlation through Newey-West with a lag of 29 quarters. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	OLS						2SLS		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Arb Position	<b>0.75</b> (3.52)						<b>2.79</b> (4.99)	<b>3.50</b> (4.24)	<b>2.94</b> (4.95)
$\alpha^{pre} \times \text{Post-1993}$		<b>0.21</b> (3.72)		<b>0.12</b> (2.54)	<b>0.13</b> (2.67)				
$\alpha^{pre} \times \text{Post-Publication}$			<b>0.25</b> (5.30)	<b>0.16</b> (4.48)	<b>0.16</b> (4.24)				
Size Rank	-0.02 (-0.18)	-0.10 (-0.90)	-0.12 (-1.25)	-0.11 (-1.09)		-0.12 (-0.83)	0.25 (1.31)	0.35 (1.66)	0.27 (1.39)
Value Rank	0.15 (1.27)	0.14 (1.54)	0.11 (1.62)	0.14 (1.83)		0.14 (1.06)	0.18 (1.83)	0.19 (1.82)	0.18 (1.84)
Profitability Rank	0.19 (1.60)	<b>0.20</b> (2.08)	0.17 (1.94)	<b>0.19</b> (2.21)		0.21 (1.68)	0.14 (1.00)	0.12 (0.81)	0.13 (0.96)
Investment Rank	<b>0.21</b> (2.96)	<b>0.15</b> (2.84)	<b>0.23</b> (3.48)	<b>0.17</b> (3.53)		<b>0.23</b> (2.22)	<b>0.16</b> (2.23)	0.14 (1.21)	0.15 (1.94)
Anomaly FE	Yes								
Observations	5,760	5,760	5,760	5,760	5,760	5,760	5,760	5,760	5,760
$R^2$	0.18	0.34	0.35	0.39	0.38	0.10			
<i>Instrumental variables</i>									
$\alpha^{pre} \times \text{Post-1993}$							✓		✓
$\alpha^{pre} \times \text{Post-Pub}$								✓	✓

Table 5: **Funding-liquidity Betas Arise During Constrained Times**

$$\text{Baseline: } \beta_i^{post, constrained} = b_0 + b_1 Arb\ Position_i^{post} + b_2 \beta_i^{pre} + u_i$$

This table shows that post-1993 funding-liquidity betas of anomalies strengthen in periods when arbitrageurs are likely to be constrained and weaken when they are likely to be unconstrained, consistent with Proposition 3. I define constrained (unconstrained) times for institutional arbitrageurs as (1) quarters in which the moving average of the VIX is above (below) the sample median (“VIX”) and as (2) years in which the CAPM alphas estimated from daily data have a cross-sectional  $R^2$  with pre-1993 CAPM alphas above the median among the years in the post-1993 period (“Alphas”). In the parentheses are  $t$ -statistics based on standard errors that account for cross-anomaly covariances through bootstrapping. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	LHS: Post-1993 Constrained-time Funding Beta				Post-1993 Unconstrained-time Funding Beta			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Post-1993 Arb Position	<b>1.97</b> (2.41)		<b>2.18</b> (2.81)		0.22 (0.43)		0.26 (0.33)	
Pre-1993 CAPM Alpha		<b>0.29</b> (2.56)		<b>0.30</b> (2.85)		0.03 (0.49)		0.05 (0.47)
Pre-1993 Funding Beta	0.39 (1.19)	-0.14 (-0.48)	0.27 (0.97)	-0.29 (-1.01)	0.21 (0.98)	0.15 (0.55)	0.27 (1.05)	0.19 (0.50)
Constant	-0.11 (-0.36)	-0.35 (-0.88)	-0.24 (-0.82)	-0.52 (-1.35)	-0.15 (-0.83)	-0.18 (-0.79)	-0.02 (-0.09)	-0.04 (-0.14)
Constrained indicator	VIX		Alphas		VIX		Alphas	
Observations	40	40	40	40	40	40	40	40
$R^2$	0.77	0.74	0.84	0.74	0.27	0.27	0.25	0.30

**Table 6: Funding-liquidity Betas as Discount-Rate Betas: Evidence from Return Predictability**

$$\begin{aligned} \text{1st Stage: } r_{i,t \rightarrow t+s}^e &= \theta_0 + \theta_1 r_{i,t-L \rightarrow t}^e + \epsilon_{i,t \rightarrow t+s} \quad (t = 1, \dots, T) \\ \text{2nd Stage: } R_{1\text{st stage},i}^2 &= b_0 + b_1 |\beta_{funding,i}| + u_i \quad (i = 1, \dots, n) \end{aligned}$$

This table shows that high-funding-beta anomalies feature greater return predictability than other anomalies in the post-1993 period but not in the pre-1993 period, consistent with post-1993 funding betas being discount-rate betas arising from the act of arbitrage (Proposition 4). The regression has 2 stages. The 1st stage is a time-series return predictive regression by anomaly: I regress an anomaly's cumulative future excess returns (+1, +2, and +3 year returns denoted  $r_{t \rightarrow t+s}^e$ ) on its past 3- or 5-year cumulative excess return (denoted  $r_{t-L \rightarrow t}^e$ ). The 2nd stage is a single-variable cross-sectional regression in which I explain the predictabilities of 40 anomalies measured by the R-squared of the 1st-stage regression ( $R_{1\text{st stage}}^2$ ) using the absolute value of its funding beta, arbitrage position, or pre-arbitrage alpha ( $|\beta_f|$ ,  $|\text{Arb Position}|$ , or  $|\alpha^{pre}|$ ) that measures the the magnitude of discount-rate shocks in the anomaly generated by the act of arbitrage. The pre-arbitrage alpha is measured by pre-1993 CAPM alpha. In the parentheses are  $t$ -statistics based on standard errors that account for cross-anomaly covariances through bootstrapping. In the brackets are the  $R^2$ s of the 2nd-stage cross-sectional regressions. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

RHS Variable	LHS: $R^2$ from 1st-stage Predictive Regressions					
	+1 Year		+2 Years		+3 Years	
	-3yr Return	-5yr Return	-3yr Return	-5yr Return	-3yr Return	-5yr Return
1st-stage Predictor Variable:	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel A. Post-1993 period return predictability increases in <math> \beta_{funding} </math>, <math> \text{Arb Position} </math>, and <math> \alpha^{pre} </math></i>						
$ \beta_{funding} $	<b>0.06</b> (3.39) [0.55]	<b>0.05</b> (3.31) [0.55]	<b>0.06</b> (3.57) [0.48]	<b>0.06</b> (3.49) [0.51]	<b>0.04</b> (2.52) [0.19]	<b>0.04</b> (2.51) [0.21]
$ \text{Arb Position} $	<b>0.10</b> (3.37) [0.56]	<b>0.10</b> (3.18) [0.54]	<b>0.11</b> (3.47) [0.49]	<b>0.10</b> (3.25) [0.49]	<b>0.08</b> (2.58) [0.20]	<b>0.07</b> (2.45) [0.21]
$ \alpha^{pre} $	<b>0.02</b> (2.89) [0.47]	<b>0.02</b> (2.89) [0.49]	<b>0.02</b> (2.77) [0.39]	<b>0.02</b> (2.78) [0.42]	<b>0.01</b> (2.13) [0.17]	<b>0.01</b> (2.18) [0.20]
<i>Panel B. Pre-1993 period return predictability does not increase in <math> \beta_{funding} </math>, <math> \text{Arb Position} </math>, and <math> \alpha^{pre} </math></i>						
$ \beta_{funding} $	0.03 (1.23) [0.04]	0.04 (1.46) [0.06]	0.05 (1.61) [0.08]	0.05 (1.67) [0.10]	0.06 (1.95) [0.11]	<b>0.06</b> (2.06) [0.13]
$ \text{Arb Position} $	-0.05 (-0.49) [0.01]	-0.03 (-0.32) [0.00]	-0.00 (-0.03) [0.00]	0.01 (0.11) [0.00]	0.10 (1.04) [0.02]	0.13 (1.32) [0.03]
$ \alpha^{pre} $	0.00 (0.37) [0.01]	0.00 (0.67) [0.02]	0.01 (1.05) [0.04]	0.01 (1.36) [0.06]	0.01 (1.89) [0.10]	<b>0.01</b> (2.24) [0.14]

**Table 7: RMW and CMA Factors as Shocks to Long-Short Arbitrageurs of Anomalies**

$$\text{Baseline: } r_t^{LS} = \alpha^{LS} + \beta_m r_{m,t}^e + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{RMW} RMW_t + \beta_{CMA} CMA_t + \epsilon_t$$

This table regresses the time-series of equal-weighted (EW) long-short anomaly returns and proxies for quantitative long/short equity hedge fund returns on the five factors of Fama and French (2015) to show that arbitrageurs of equity anomalies are likely to be exposed to *RMW* and *CMA*. The EW long-short anomaly portfolio is the portfolio that gives equal positive weights to the 20 long-side portfolios and equal negative weights to the 20 short-side portfolios. The quantitative long/short equity hedge fund portfolio is proxied by a mix of the equity-market-neutral hedge fund index and the short-bias hedge fund index from the Hedge Fund Research (HFR). For instance, “90/10” means a combined portfolio that places 90% weight on equity market neutral hedge funds and 10% weight on equity short-bias hedge funds. I display results for 100/0 through 75/25 only since equity market-neutral hedge funds are about six times larger than equity short-bias hedge funds (TASS hedge fund data). The hedge fund returns are examined only in the post-1993 period. In the parentheses are *t*-statistics based on heteroskedasticity-robust OLS standard errors. Boldface indicates coefficient estimates greater than 1.96 standard errors in absolute value.

	EW Long-Short Anomaly Portfolio		Quantitative Long/Short Equity Hedge Fund Portfolio					
	Pre-1993 Period	Post-1993 Period	100/0	95/5	90/10	85/15	80/20	75/25
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
MKT	<b>-0.03</b> (-3.26)	<b>-0.10</b> (-5.71)	<b>0.08</b> (5.42)	<b>0.05</b> (3.04)	0.01 (0.49)	<b>-0.03</b> (-2.02)	<b>-0.07</b> (-4.35)	<b>-0.11</b> (-6.36)
SMB	<b>-0.06</b> (-4.00)	-0.03 (-1.45)	<b>0.04</b> (2.35)	0.02 (1.37)	0.00 (0.28)	-0.01 (-0.85)	-0.03 (-1.91)	<b>-0.05</b> (-2.84)
HML	0.00 (0.25)	-0.05 (-1.81)	-0.02 (-0.85)	-0.01 (-0.27)	0.01 (0.33)	0.02 (0.91)	0.04 (1.44)	0.05 (1.91)
RMW	<b>0.17</b> (7.13)	<b>0.22</b> (6.42)	<b>0.07</b> (2.48)	<b>0.08</b> (3.09)	<b>0.09</b> (3.72)	<b>0.10</b> (4.29)	<b>0.11</b> (4.69)	<b>0.11</b> (4.89)
CMA	<b>0.12</b> (4.01)	<b>0.20</b> (5.44)	0.06 (1.61)	<b>0.07</b> (1.99)	<b>0.08</b> (2.38)	<b>0.08</b> (2.74)	<b>0.09</b> (3.04)	<b>0.10</b> (3.24)
$\alpha^{LS}$	<b>2.61</b> (6.31)	<b>1.82</b> (2.83)	<b>0.15</b> (2.90)	<b>0.14</b> (2.95)	<b>0.14</b> (2.93)	<b>0.13</b> (2.82)	<b>0.13</b> (2.64)	<b>0.12</b> (2.41)
Observations	240	276	276	276	276	276	276	276
$R^2$	0.53	0.71	0.23	0.17	0.23	0.36	0.49	0.59

Table 8: Explaining the Cross-Section of Fama-French Betas

$$\text{Baseline: } \beta_i^{post} = b_0 + b_1 \text{Arb Position}_i^{post} + b_2 \beta_i^{pre} + u_i$$

This table shows that anomalies' *SMB* and *HML* betas are largely explained by the fundamental characteristics of anomalies (Panel A), whereas both the arbitrage position and the two different proxies for pre-arbitrage alpha (CAPM  $\alpha$  and FF5  $\alpha$ ) account for a substantial part of the cross-sectional variation in anomalies' *RMW* and *CMA* betas (Panel B). Each beta is a multivariate beta estimated in the time series regression over the post-1993 period (1994m1–2016m12). In the parentheses are *t*-statistics based on standard errors that account for cross-anomaly covariances through bootstrapping. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

**Panel A. Betas with SMB and HML**

	Post-1993 SMB Beta						Post-1993 HML Beta					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Post-1993 Arb Position	0.01 (0.13)	-0.04 (-0.71)					-0.09 (-1.12)	-0.08 (-1.00)				
Pre-1993 CAPM Alpha			0.00 (0.50)						-0.01 (-1.14)			
Pre-1993 FF5 Alpha				0.01 (0.57)							-0.01 (-0.91)	
Pre-1993 Beta	<b>0.60</b> (4.31)		<b>0.60</b> (6.81)	<b>0.63</b> (6.37)	<b>0.58</b> (5.80)		<b>0.64</b> (6.98)		<b>0.70</b> (5.72)	<b>0.62</b> (7.52)	<b>0.66</b> (6.40)	
Size Rank		<b>0.13</b> (9.36)				<b>0.14</b> (7.29)						
Value Rank								<b>0.12</b> (8.64)				<b>0.12</b> (7.65)
Constant	0.00 (0.08)	<b>-0.24</b> (-5.47)	0.00 (0.14)	-0.00 (-0.01)	0.00 (0.15)	<b>-0.27</b> (-5.06)	-0.01 (-0.28)	<b>-0.51</b> (-8.66)	0.01 (0.14)	-0.00 (-0.07)	0.02 (0.39)	<b>-0.51</b> (-7.98)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
$R^2$	0.75	0.80	0.76	0.76	0.75	0.79	0.61	0.55	0.61	0.60	0.56	0.51

**Panel B. Betas with RMW and CMA**

	Post-1993 RMW Beta						Post-1993 CMA Beta					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Post-1993 Arb Position	<b>0.35</b> (4.34)	<b>0.41</b> (5.19)					<b>0.23</b> (2.58)	0.18 (1.96)				
Pre-1993 CAPM Alpha			<b>0.03</b> (2.76)						<b>0.04</b> (2.87)			
Pre-1993 FF5 Alpha				<b>0.05</b> (4.82)							<b>0.03</b> (2.04)	
Pre-1993 Beta	<b>0.51</b> (4.52)		<b>0.72</b> (3.84)	<b>0.58</b> (5.06)	<b>0.98</b> (4.04)		<b>0.60</b> (5.04)		<b>0.46</b> (4.03)	<b>0.68</b> (4.91)	<b>0.63</b> (4.04)	
Profitability Rank		<b>0.06</b> (3.53)				<b>0.11</b> (3.75)						
Investment Rank								<b>0.15</b> (4.93)				<b>0.18</b> (5.29)
Constant	0.02 (0.50)	<b>-0.36</b> (-3.49)	-0.03 (-0.75)	0.01 (0.36)	-0.05 (-1.04)	<b>-0.76</b> (-3.70)	-0.02 (-0.35)	<b>-0.81</b> (-4.64)	-0.04 (-0.94)	-0.04 (-0.75)	-0.09 (-1.54)	<b>-0.99</b> (-5.09)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
$R^2$	0.81	0.78	0.65	0.87	0.49	0.25	0.66	0.71	0.67	0.58	0.37	0.54

**Table 9: Additional Tests on *RMW* and *CMA* Betas**

This table provides additional evidence that the post-1993 *RMW* and *CMA* betas of anomalies are partly arbitrage-driven betas. Panel A shows that an increase in the arbitrage position over time increases an anomaly's *RMW* and *CMA* betas, where the time-varying betas are estimated in an 85-month window (7 years) surrounding each month. Panel B shows that *RMW* and *CMA* betas shrink in unconstrained times and expand in constrained times. I use the VIX-implied constrained quarters as the constrained periods. Panel C shows that the post-1993 time-series return predictabilities of anomalies line up with the *RMW* and *CMA* betas, consistent with the post-1993 *RMW/CMA* betas being arbitrage-driven discount-rate betas. It employs the two-stage procedure as in Table 6. In the parentheses are *t*-statistics, and in the brackets in panel C are the  $R^2$ s of the 2nd-stage cross-sectional regressions. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

**Panel A. Panel of *RMW* and *CMA* betas explained by arbitrage variables**

	RMW							CMA							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	
Arb Position	<b>0.059</b> (2.77)				<b>0.339</b> (4.03)	<b>0.255</b> (3.70)	<b>0.319</b> (4.10)	<b>0.107</b> (4.17)					<b>0.384</b> (4.58)	<b>0.323</b> (3.76)	<b>0.366</b> (4.40)
$\alpha^{pre} \times$ Post-1993		<b>0.029</b> (3.94)		<b>0.030</b> (3.78)					<b>0.034</b> (3.30)		<b>0.033</b> (2.42)				
$\alpha^{pre} \times$ Post-Pub			<b>0.022</b> (3.19)	-0.001 (-0.15)							<b>0.028</b> (3.79)	0.002 (0.28)			
Profitability Rank	<b>0.039</b> (3.03)	<b>0.040</b> (3.41)	<b>0.041</b> (3.18)	<b>0.040</b> (3.44)	0.020 (1.12)	0.026 (1.68)	0.022 (1.24)								
Investment Rank								<b>0.051</b> (3.05)	<b>0.047</b> (3.14)	<b>0.052</b> (3.03)	<b>0.047</b> (3.21)	<b>0.048</b> (3.21)	<b>0.049</b> (3.46)	<b>0.048</b> (3.30)	
Anomaly FE	Yes														
Observations	17280	17280	17280	17280	17280	17280	17280	17280	17280	17280	17280	17280	17280	17280	17280
$R^2$	0.07	0.16	0.09	0.16				0.12	0.19	0.12	0.19				
OLS/2SLS	OLS	OLS	OLS	OLS	2SLS	2SLS	2SLS	OLS	OLS	OLS	OLS	2SLS	2SLS	2SLS	2SLS
<i>Instrumental variables</i>															
$\alpha^{pre} \times$ Post-1993					✓		✓						✓		✓
$\alpha^{pre} \times$ Post-Pub						✓	✓						✓		✓

**Panel B. Post-1993 *RMW* and *CMA* betas during constrained vs. unconstrained times**

	Post-1993 <i>RMW</i> Beta						Post-1993 <i>CMA</i> Beta								
	Constrained Times			Unconstrained Times			Constrained Times			Unconstrained Times					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)			
Post-1993 Arb Position	<b>0.368</b> (2.96)			<b>0.244</b> (2.43)			<b>0.261</b> (2.03)				0.097 (0.95)				
Pre-1993 CAPM Alpha		<b>0.038</b> (2.17)				0.017 (1.22)			<b>0.042</b> (2.37)				0.011 (0.68)		
Pre-1993 FF5 Alpha			<b>0.057</b> (3.25)			<b>0.036</b> (2.42)				0.037 (1.91)					-0.001 (-0.03)
Pre-1993 Beta	<b>0.518</b> (3.23)	<b>0.716</b> (3.21)	<b>0.582</b> (3.67)	<b>0.436</b> (2.88)	<b>0.628</b> (3.00)	<b>0.488</b> (3.28)	<b>0.620</b> (4.98)	<b>0.464</b> (3.91)	<b>0.718</b> (5.00)	<b>0.472</b> (2.02)	0.436 (1.95)	<b>0.485</b> (1.98)			
Constant	-0.008 (-0.18)	-0.060 (-1.04)	-0.014 (-0.31)	-0.001 (-0.03)	-0.039 (-0.82)	-0.006 (-0.15)	-0.023 (-0.45)	-0.051 (-0.91)	-0.038 (-0.71)	-0.025 (-0.41)	-0.041 (-0.61)	-0.055 (-0.81)			
Observations	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
$R^2$	0.84	0.71	0.89	0.65	0.51	0.67	0.68	0.71	0.66	0.20	0.18	0.16			

**Panel C. Post-1993 *RMW* and *CMA* betas are discount-rate betas: evidence from return predictability**

1st-stage Forecast Horizon:	LHS: $R^2$ from 1st-stage Predictive Regressions					
	+1 Year		+2 Years		+3 Years	
	-5yr Return		-3yr Return		-5yr Return	
	(1)	(2)	(3)	(4)	(5)	(6)
1st-stage Predictor Variable:						
$ \beta_{RMW} $	<b>0.18</b> (2.68) [0.39]	<b>0.17</b> (2.60) [0.39]	<b>0.18</b> (2.69) [0.34]	<b>0.18</b> (2.64) [0.36]	<b>0.16</b> (2.59) [0.20]	<b>0.16</b> (2.60) [0.22]
$ \beta_{CMA} $	<b>0.17</b> (2.18) [0.21]	<b>0.17</b> (2.16) [0.21]	0.16 (1.94) [0.14]	0.15 (1.88) [0.14]	0.06 (0.67) [0.01]	0.05 (0.64) [0.01]

Table 10: **Explaining the Cross-Section of Quant-Crisis Returns**

$$\text{Baseline: } r_i^{crash} = b_0 + b_1 \text{Arb Position}_i + u_i$$

This table shows that the anomalies' arbitrage-capital betas, as revealed by the negative of their return during the quant crash (August 7–9, 2007), are cross-sectionally explained by the anomalies' prior arbitrage position and pre-arbitrage alpha, consistent with Proposition 5. Returns during the recovery from the crash (August 10–14, 2007) display an opposite pattern, suggesting that the anomaly returns during the crash were discount-rate shocks. Cumulative abnormal return is defined as the excess return net of market exposure (market excess return times the beta estimated over the preceding 1 year using 3-day returns). The July 2007 arbitrageur (“arb”) position is defined as negative of  $(-1 \times 10^2)$  the “abnormal” short interest on the anomaly in mid-July 2007. “Post-93 pre-quant-crisis” arbitrageur position is the negative of the average abnormal short interest over the post-1993 period preceding the crisis (1994m1–2007m7). The post-1993 arbitrageur position is computed over the entire post-1993 period. Pre-1993 variables refer to variables measured over 1974m1–1993m12. In the parentheses are standard errors that account for cross-anomaly covariances through bootstrapping (based on the preceding year’s data). Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

**Panel A. Cumulative raw return**

	Quant-crisis Return						Quant-recovery Return					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Post-93 Pre-Quant-Crisis Arb Position	<b>-1.94</b> (-3.70)						<b>1.22</b> (2.63)					
Post-1993 Arb Position		<b>-1.71</b> (-3.88)						<b>1.16</b> (2.92)				
July 2007 Arb Position			<b>-1.22</b> (-2.99)						<b>0.87</b> (2.47)			
Pre-1993 CAPM Alpha				<b>-0.27</b> (-5.04)						<b>0.18</b> (3.98)		
Pre-1993 FF3 Alpha					<b>-0.25</b> (-5.35)						<b>0.16</b> (3.85)	
Pre-1993 FF5 Alpha						<b>-0.26</b> (-4.45)						<b>0.17</b> (3.35)
Constant	-0.15 (-0.12)	-0.30 (-0.25)	-0.25 (-0.21)	-0.08 (-0.06)	-0.31 (-0.25)	-0.24 (-0.20)	-1.93 (-1.57)	-1.81 (-1.50)	-1.82 (-1.52)	-1.95 (-1.57)	-1.81 (-1.47)	-1.85 (-1.51)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
$R^2$	0.46	0.54	0.37	0.63	0.71	0.58	0.41	0.57	0.43	0.68	0.70	0.58

**Panel B. Cumulative abnormal return (net of market exposure and the risk-free rate)**

	Quant-crisis Abnormal Return						Quant-recovery Abnormal Return					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Post-93 Pre-Quant-Crisis Arb Position	<b>-2.05</b> (-4.23)						<b>1.11</b> (2.64)					
Post-1993 Arb Position		<b>-1.81</b> (-4.53)						<b>1.06</b> (2.98)				
July 2007 Arb Position			<b>-1.31</b> (-3.55)						<b>0.78</b> (2.49)			
Pre-1993 CAPM Alpha				<b>-0.28</b> (-5.55)						<b>0.17</b> (4.08)		
Pre-1993 FF3 Alpha					<b>-0.25</b> (-5.79)						<b>0.15</b> (3.89)	
Pre-1993 FF5 Alpha						<b>-0.27</b> (-4.86)						<b>0.16</b> (3.37)
Constant	0.48 (1.84)	0.32 (1.30)	0.36 (1.28)	<b>0.56</b> (2.30)	0.33 (1.42)	0.39 (1.53)	<b>-1.30</b> (-6.12)	<b>-1.19</b> (-6.01)	<b>-1.21</b> (-5.65)	<b>-1.31</b> (-6.49)	<b>-1.18</b> (-5.98)	<b>-1.22</b> (-5.83)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
$R^2$	0.49	0.58	0.41	0.65	0.72	0.59	0.37	0.51	0.37	0.66	0.68	0.56

Figure 3a. Explaining Anomalies with Adrian et al.  
Funding-liquidity  $\beta$ s

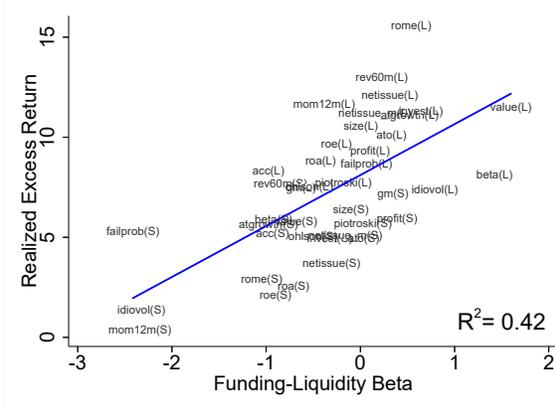


Figure 3b. Explaining Anomalies with FF5

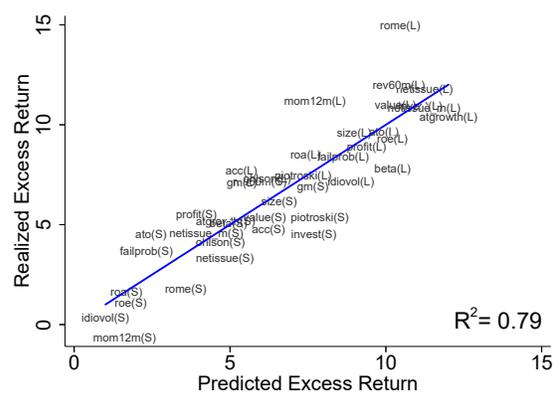


Figure 3: **Factor Models of Equity Anomalies**

The first figure the ability of the Adrian, Etula, and Muir (2014) funding-liquidity beta to explain the cross-section of mean excess returns of 40 anomalies over 1974q1–2016q4. The beta is a multivariate beta from a two-factor model that includes the market factor. The second figure shows the ability of the five Fama-French (2015) factors (*MKT*, *SMB*, *HML*, *RMW*, and *CMA*) to explain the cross-section of mean excess returns of 40 anomalies over 1974m1–2016m12. An intercept *is included* in both regressions.

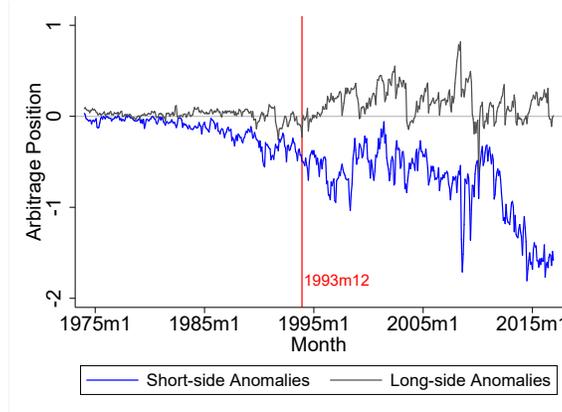


Figure 4: **Arbitrage Position Inferred from Short Interests**

The figure plots the equal-weighted (cross-sectional) average of arbitrage positions in long-side and short-side anomalies over 1974m1–2016m12. The arbitrage positions are inferred from short interest ratio (see Section 3).

Figure 5a. Constrained Quarters Implied by the VIX

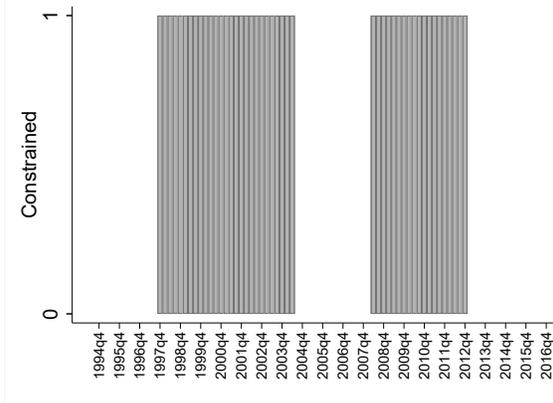


Figure 5b. Constrained Years Implied by Alpha

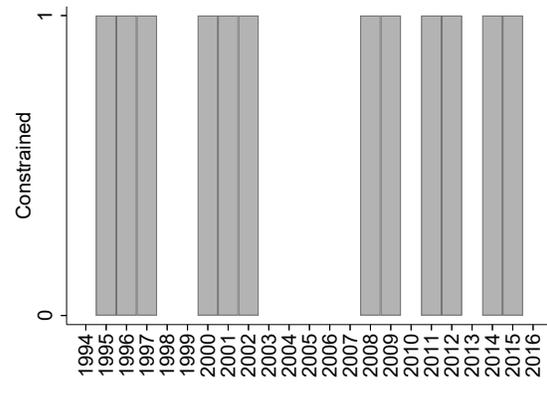


Figure 5: Proxies for Constrained vs. Unconstrained Periods

The first figure reports constrained (unconstrained) post-1993 quarters defined as quarters in which moving average of the VIX is above (below) the sample median. The second figure reports constrained (unconstrained) years defined as years in which the CAPM alphas estimated from daily data are cross-sectionally explained by pre-1993 alphas with a high  $R^2$  (above median among all year-specific cross-sectional  $R^2$ s).

Figure 6a. Constrained Post-1993 Periods

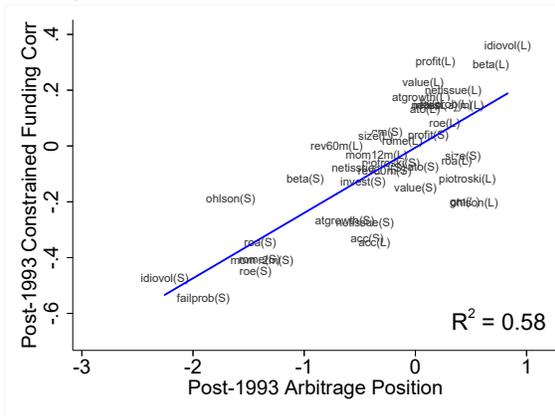


Figure 6b. Unconstrained Post-1993 Periods

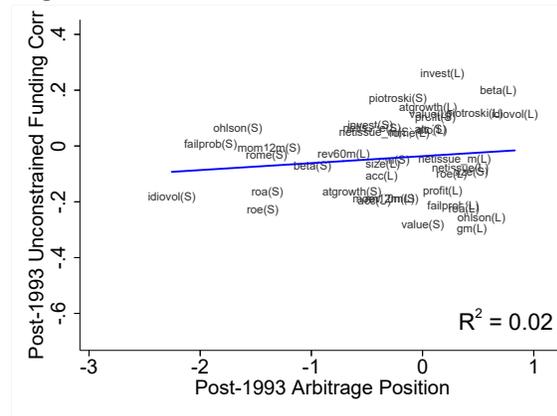
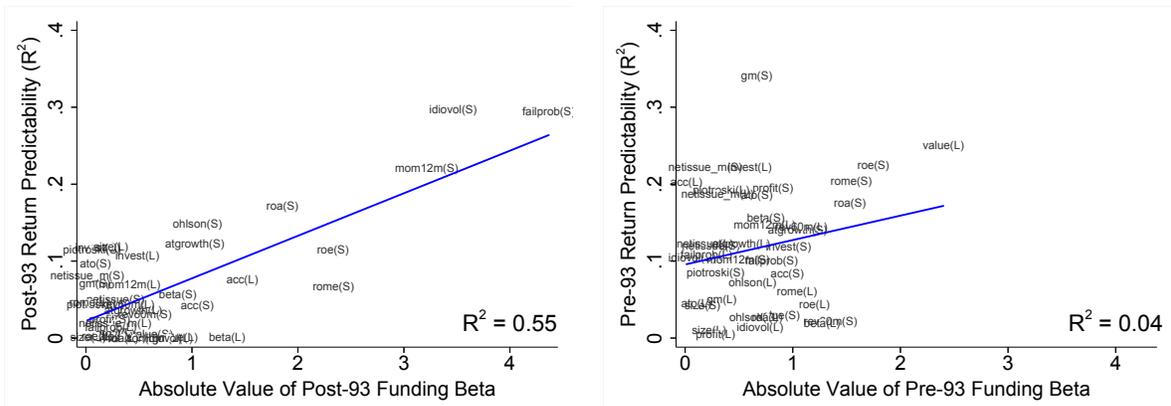


Figure 6: Funding-liquidity Correlations Arise During Constrained Times

The figures show that in the post-1993 period, anomalies' return correlation with the funding-liquidity factor lines up strongly with arbitrage position during constrained periods (left) but not during unconstrained periods (right). The result is similar if I use pre-1993 CAPM alpha (pre-arbitrage alpha) as the  $x$ -axis. To compute the correlations, I compute unexplained return as the realized return in excess of the risk-free rate and multivariate (2-factor) market beta times the excess market return. I then take the time-series correlation between the unexplained returns and the funding factor. I use constrained quarters implied by the VIX.



**Figure 7: Cross-Section of Time-Series Predictability: Post-1993 vs. Pre-1993**

The figures show that the return predictabilities of anomalies line up with their funding betas in the post-1993 period (left) but not strongly in the pre-1993 period (right). Return predictability of an anomaly is measured by the  $R^2$  of the time-series regression that explains future 1-year cumulative excess returns using the past 3-year cumulative excess returns.

Figure 8a. Cross-Section of *SMB* Betas

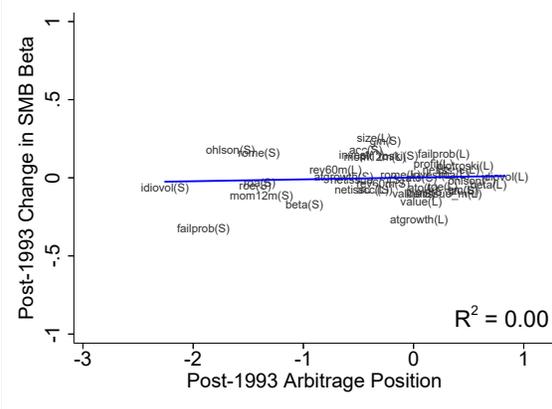


Figure 8b. Cross-Section of *HML* Betas

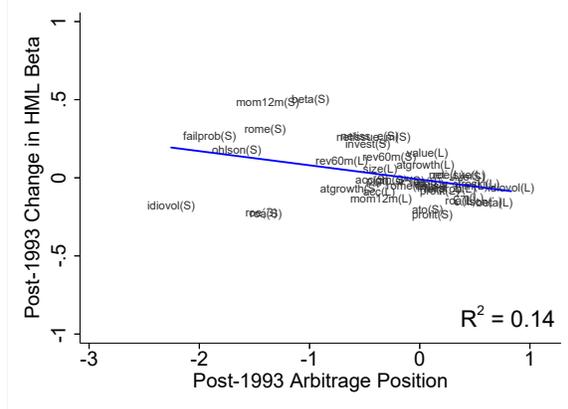


Figure 8c. Cross-Section of *RMW* Betas

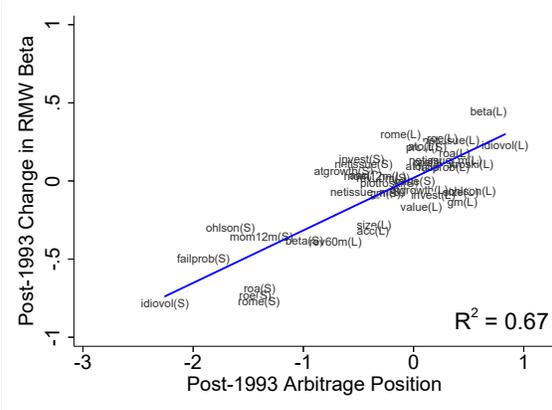
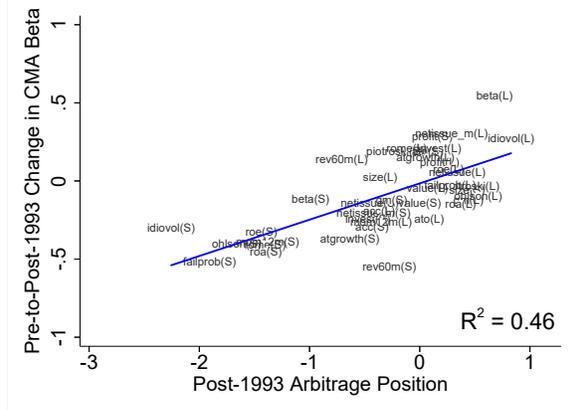


Figure 8d. Cross-Section of *CMA* Betas



**Figure 8: Explaining the Cross-Section of Fama-French Betas**

The figures show that arbitrage position in the post-1993 period explains the post-1993 change in the *RMW* and *CMA* betas of 40 anomalies, but not the change in *SMB* and *HML* betas, consistent with *RMW* and *CMA* being factors that generate shocks to the capital of anomaly arbitrageurs. Post-1993 arbitrage position is inferred from short interests. Post-1993 change in beta is defined as the post-1993 beta minus the pre-1993 beta multiplied by the approximate shrinkage factor of 0.6 implied by Table 8. The betas are multivariate betas based on the five-factor model of Fama and French (2015) and are estimated using monthly data over 1974m1–2016m12.

Figure 9a. Returns on Anomalies During Quant Crash

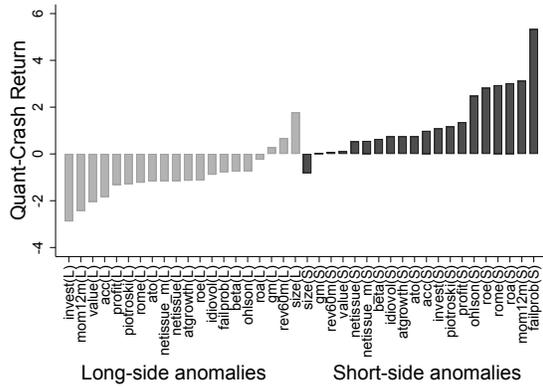


Figure 9b. Arbitrageur Portfolio Return

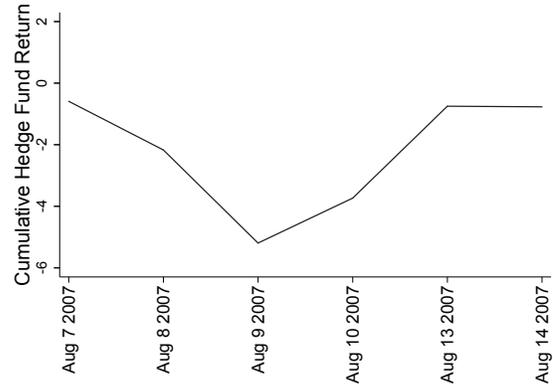


Figure 9: Description of the Quant Crisis of August 2007

The first figure plots the cumulative 3-day returns of long-side (first 20) and short-side (next 20) anomalies during the quant “crash” of August 7–9, 2007. The second figure plots the cumulative daily returns on equity market-neutral hedge funds during the entire quant crisis period, which includes both the crash (8/7–9) and recovery (8/10–14) periods. The hedge fund return data are from Hedge Fund Research.

Figure 10a. Cumulative Return During Quant Crash

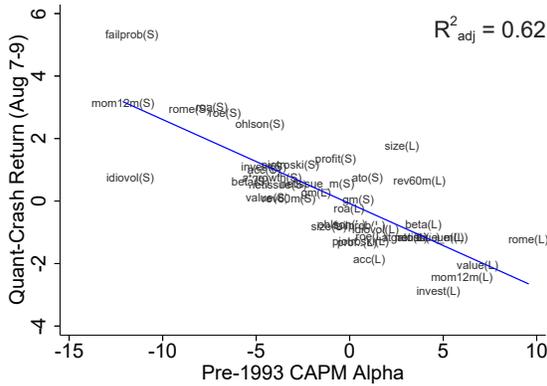


Figure 10b. Abnormal Return During Quant Crash

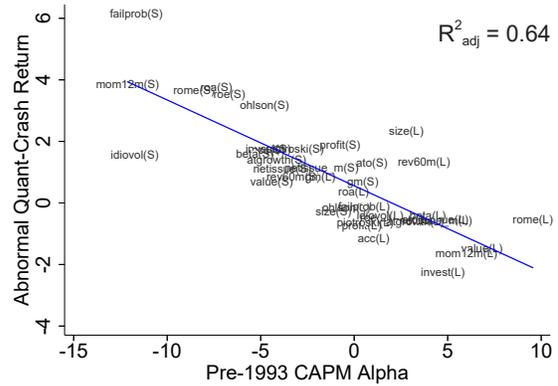


Figure 10c. Cumulative Return over Recovery

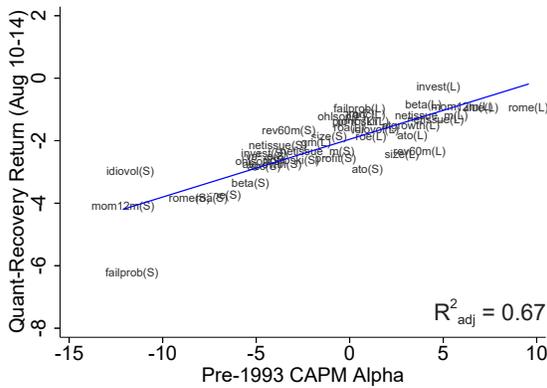
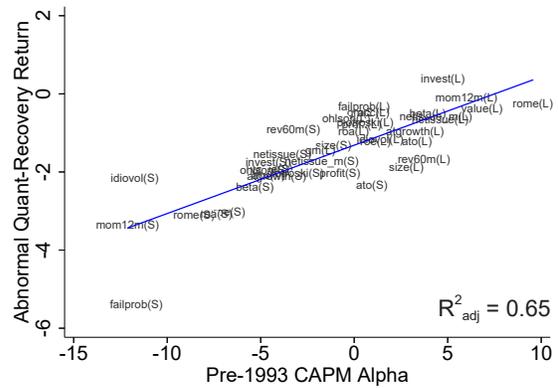


Figure 10d. Abnormal Return over Recovery



**Figure 10: Explaining the Cross-section of Anomaly Returns During Quant Crash and Recovery**

The figures show that the anomalies' pre-1993 CAPM alphas explain the cross-section of their returns during the quant "crash" (August 7--9, 2007; top two figures) and "recovery" (August 10--14; bottom two figures). Abnormal return is defined as the excess return net of market exposure (market excess return times the beta estimated over the preceding 1 year using 3-day returns).

# A Theory Appendix

## A.1 Solving the pre-arbitrage equilibrium

**Proof of Lemma 1 (Asset returns in the pre-arbitrage economy).** Since the behavioral investors alone clear the market, equation (1) implies  $B_{i,t} = 0 \implies \phi i = E_t [r_{i,t+1}^e]$ . Hence,  $r_{i,t}^e = E_{t-1} [r_{i,t}^e] + \epsilon_{i,t} = \phi i + \epsilon_{i,t}$  where  $\epsilon_{i,t}$  is a mean-zero idiosyncratic return by the i.i.d.-dividend assumption. Finally,  $\alpha_{i,t} = \phi i$  increases in  $i$  since  $\phi > 0$ .<sup>48</sup>

## A.2 Solving the post-arbitrage equilibrium

Before proving the rest of the lemmas and propositions, I first solve the post-arbitrage equilibrium, highlighting important steps as new lemmas.

The equilibrium in the post-arbitrage economy with  $\mu = \frac{\phi}{2}$  is solved backward from time 2, which represents the period immediately before mispricings disappear and asset prices converge to their fundamental value. Hence arbitrageurs at time 2 invest all available capital in the mispriced assets without worrying about asset returns covarying with the level of arbitrage capital in the future. Time 1 represents the earlier periods of arbitrage in which arbitrageurs do worry about asset returns covarying endogenously with their capital before the assets realize their fundamental value. The asset prices at time 1 therefore take this endogenous risk into account.

To find the equilibrium in each period, note first that the arbitrageur's objective function in (3) implies the following value function at  $t \in \{1, 2\}$ :

$$\begin{aligned} V_t(w_t, f_t) &= \max_{\{x_{i,t}\}} E_t [V_{t+1}(w_{t+1}, f_{t+1})] \\ &\text{s.t. } \int_0^1 |x_{i,t}| di \leq (w_t + f_t) \\ w_{t+1} &= w_t + \int_0^1 \left( \frac{p_{i,t+1} + \delta_{i,t+1}}{p_{i,t}} - 1 \right) x_{i,t} di + \tilde{w}_{t+1} \\ &V_3 = w_3 \end{aligned} \tag{13}$$

---

<sup>48</sup>To solve for prices, since the riskless rate is zero,  $\phi i = E_t [r_{i,t+1}^e] = E_t [r_{i,t+1}] \implies p_t = E_t \left[ \frac{1}{1+\phi i} (p_{i,t+1} + \delta_{i,t+1}) \right]$ . That is, price at time  $t$  is the price and dividend at time  $t+1$  discounted by the asset-specific constant discount factor  $\frac{1}{1+\phi i}$  imposed by behavioral investors.

in the non-default state ( $w_t > 0$ ), and

$$V_t = (1 + c)^{3-t} w_t \quad (14)$$

in the default state ( $w_t \leq 0$ ). Then, equilibrium prices at time 2 are given by the following lemma:

**Lemma 5. (Time-2 equilibrium prices).** *The equilibrium price of asset  $i$  at time 2 is*

$$p_{i,2} = m_{i,3} v \quad (15)$$

- s.t. (i)  $m_{i,3} = \frac{1}{1+\phi i_2^*}$  for the “exploited” assets  $i \in (i_2^*, 1]$ .  
(ii)  $m_{i,3} = \frac{1}{1+\phi i}$  for the “unexploited” assets  $i \in [0, i_2^*]$ .  
(iii)  $i_2^*$  is the marginal asset s.t.  $i_2^* = 1, 1 - \sqrt{k_2}$ , and 0 for  $k_2 \in (-\infty, 0], (0, 1)$ , and  $[1, \infty)$ , respectively.  
(iv) For completeness, the equilibrium arbitrage position is  $x_{i,2} = i - i_2^*$  for  $i \geq i_2^*$  and  $x_{i,2} = 0$  for  $i < i_2^*$ .

**Proof.** The arbitrageur’s value function at time 2 in the non-default state ( $w_2 > 0$ ) is

$$V_2 = w_2 + \max_{\{x_{i,2}\}} \left\{ \int_0^1 E_2 [r_{i,3}] x_{i,2} di + \psi_2 \left[ w_2 + f_2 - \int_0^1 |x_{i,2}| di \right] \right\} \quad (16)$$

where  $\psi_2$  is the shadow cost of capital at time 2 such that  $\psi_2 = 0$  ( $\psi_2 > 0$ ) if the arbitrageur is unconstrained (constrained). Since the arbitrageur does not take negative positions in equilibrium (doing so would generate a negative expected return due to the behavioral investor demand), the first order condition with respect to  $x_{i,2}$  within the value function implies

$$E_2 [r_{i,3}] \leq \psi_2,$$

which holds with equality if and only if  $x_{i,2} > 0$ . On the other hand, behavioral investor demand in (1) implies

$$E_2 [r_{i,3}] = \phi i - \frac{\phi}{2} x_{i,2}.$$

Hence  $x_{i,2} = 0$  for  $i \in \left[0, \frac{\psi_2}{\phi}\right]$  since  $E_2 [r_{i,3}] = \phi i \leq \psi_2$ ; i.e., these assets’ expected returns are already below or equal to the shadow cost of capital before arbitrageurs trade them. But  $x_{i,3} = 2 \left(i - \frac{E_2[r_{i,3}]}{\phi}\right) = 2 \left(i - \frac{\psi_2}{\phi}\right) > 0$  for  $i \in \left(\frac{\psi_2}{\phi}, 1\right]$ ; i.e., arbitrage positions in exploited assets ensure that the assets’ expected return equals  $\psi_2$ . So there is a marginal asset  $i_2^* = \frac{\psi_2}{\phi}$  s.t.  $x_{i,2} = 0$  for  $i \in [0, i_2^*]$  and  $x_{i,2} = 2(i - i_2^*)$  for  $i \in (i_2^*, 1]$ . If the arbitrageur’s capital

constraint binds, it must be that

$$k_2 = \int_0^1 x_{i,1} di = \int_{i_2^*}^1 x_{i,1} di = 2 \int_{i_2^*}^1 (i - i_2^*) di = (1 - i_2^*)^2 \implies i_2^* = 1 - \sqrt{k_2},$$

which is a solution when  $k_2 \in (0, 1)$ . If  $k_2 \in (-\infty, 0]$ , no asset is exploited so that  $i_2^* = 1$  and  $E_2[r_{i,3}] = \phi i$ . If  $k_2 \in [1, \infty)$ , all assets are fully exploited so that  $i_2^* = 0$  and  $E_2[r_{i,3}] = 0$ .

Intuitively, if  $k_2 \geq 1$  and thus  $i_2^* = 0$ , the arbitrageur has enough capital to restore all asset prices to the correct level  $v$ . If  $k_2 \leq 0$  and  $i_2^* = 1$ , all assets are priced by the behavioral investors. If  $k_2 \in (0, 1)$ , the arbitrageur trades some assets but faces a capital constraint. In this case, the risk-neutral arbitrageur equalizes the expected return on all exploited assets  $(i_2^*, 1]$  to  $\phi i_2^*$ , the arbitrageur's shadow cost of capital. The lower- $i$  assets  $[0, i_2^*]$  remain unexploited since their expected return is lower than  $\phi i_2^*$  even without arbitrage.

The equilibrium time-2 prices in [Lemma 5](#) offer a glimpse into why high- $i$  assets become endogenously riskier in this post-arbitrage equilibrium. It is because the prices of high- $i$  assets respond more to the variation in  $k_2$ ; as  $k_2$  ranges from 0 to 1, the price of asset  $i$  rises from  $\frac{v}{1+\phi i}$  to  $v$ , implying a  $\phi i$ -percent increase in its price. The intuition is that the an initially more-mispriced asset relies more heavily on the price-correcting role of arbitrage capital, which makes its price more sensitive to the variation in the level of arbitrage capital.

Next, to solve for equilibrium time-1 prices, I first show that the arbitrageur's marginal value of wealth at time 2 falls as  $k_2$  rises:

**Lemma 6. (Time-2 marginal value of wealth).** *The arbitrageur's value function at time 2 is*

$$V_2 = \Lambda_2 w_2 \tag{17}$$

where the marginal value of wealth in the non-default state ( $w_2 > 0$ ) is  $\Lambda_2 = 1 + \phi i_2^*$  and that in the default state ( $w_2 \leq 0$ ) is  $\Lambda_2 = 1 + c$ .

**Proof.** First, consider  $w_2 > 0$ . The derivative of the value function (16) with respect to  $w_2$  gives  $\Lambda_2 = 1 + \psi_2$ . For  $\psi_2$ , the derivative with respect to any exploited asset's  $x_{i,2}$  within the bracket implies  $\psi_2 = E_2[r_{i,3}] = \phi i_2^*$ , where the second equality follows from equation (15). Next,  $\Lambda_2$  for  $w_2 \leq 0$  follows from equation (14). Finally,  $V_2 = \Lambda_2 w_2$  since [Lemma 5](#) implies

that the marginal value of wealth  $\Lambda_2 = 1 + \psi_2 = 1 + \phi i_2^*$  is also the average return on wealth in the non-default state and  $w_3 = (1 + c) w_2$  in the non-default case.

**Lemma 6** implies that a low- $k_2$  state is a “bad” state in which the arbitrageur’s marginal value of wealth is high:  $\Lambda_2$  rises from 1 to  $1 + \phi$  and to  $1 + c$  as  $k_2$  decreases from  $\infty$  to  $0^+$  and to  $-\infty$ . This inverse relationship between  $\Lambda_2$  and  $k_2$  here is not driven by the preference for risk or intertemporal substitution, similarly to how the decreasing marginal utility of consumption does not rely on the curvature of the utility function. With risk-neutrality in particular, this happens because arbitrage capital  $k_2$  falls precisely when the investment opportunity  $\phi i_2^*$  improves.

Given **Lemma 6**, the equilibrium price at time 1 depends on the extent to which the asset’s return at time 2 covaries with the arbitrageur’s marginal value of wealth  $\Lambda_2$ :

**Lemma 7. (Time-1 equilibrium prices).** *The equilibrium price of asset  $i$  at time 1 is*

$$p_{i,1} = E_1 [m_{i,2} (p_{i,2} + \delta_{i,2})] \quad (18)$$

- s.t. (i)  $m_{i,2} = m_2^A \equiv \frac{\Lambda_2}{\Lambda_1}$  for the exploited assets  $i \in \mathbf{I}_1^*$  where  $\mathbf{I}_1^*$  is the set of exploited assets.  
(ii)  $m_{i,2} = m_{i,2}^B \equiv \frac{1}{1+\phi i}$  for the unexploited assets  $i \in \mathbf{I}_1^*$ .  
(iii)  $\Lambda_1$  is the time-1 marginal value of wealth s.t.  $\Lambda_1 = E_1 [\Lambda_2] + \psi_1$  where  $\psi_1 > 0$  if the arbitrageur is constrained and  $\psi_1 = 0$  if the arbitrageur is unconstrained.  
(iv) The arbitrageur is unconstrained if  $k_1$  is above some threshold  $k_1^* \leq 1$ .

**Proof.** By eq. (13) and **Lemma 6**, the arbitrageur’s value function at time 1 is

$$V_1 = E_1 [\Lambda_2] w_1 + \max_{\{x_{i,1}\}} \left\{ \int_0^1 E_1 [\Lambda_2 r_{i,2}] x_{i,1} di + \psi_1 \left[ w_1 + f_1 - \int_0^1 |x_{i,1}| di \right] \right\} \quad (19)$$

where  $\psi_1$  is the Lagrangian multiplier on the capital constraint  $\int_0^1 |x_{i,1}| di \leq (w_1 + f_1)$  s.t.  $\psi_1 = 0$  if the arbitrageur is unconstrained. First, I prove by contradiction that the arbitrageur does not take a negative position at time 1. Suppose  $x_{i,1} < 0$ . Then, first order condition within the maximization bracket implies  $E_1 [\Lambda_2 r_{i,2}] = -\psi_1 < 0$ . However, market clearing in eq. (1) implies  $E_1 [\Lambda_2 r_{i,2}] \geq 0$ , which is a contradiction. To see why, note that eq. (1) implies  $E_1 [r_{i,2}] = \phi \left( i - \frac{x_{i,1}}{2} \right) > \phi i \implies \frac{p_{i,2}}{p_{i,1}} > (1 + \phi i) \frac{p_{i,2}}{E_1 [p_{i,2}]} \geq 1$  for any possible realization of  $p_{i,2}$ , since  $p_{i,2} \in [v, (1 + \phi i) v]$  by **Lemma 7**. Since  $p_{i,2}/p_{i,1} \geq 1$  and  $\Lambda_2 > 0$ ,  $E_1 [\Lambda_2 r_{i,2}] \geq 0$ . Next, I prove the lemma. Since  $x_{i,1} \geq 0 \forall i$  in equilibrium, the first order

condition w.r.t.  $x_{i,1}$  within the maximization bracket implies  $E_1 [\Lambda_2 r_{i,2}] \leq \psi_1$  and hence

$$p_{i,1} \geq E_1 \left[ \frac{\Lambda_2}{E_1 [\Lambda_2] + \psi_1} (p_{i,2} + \delta_{i,2}) \right],$$

which holds with equality if and only if  $x_{i,2} > 0$  ( $i$  is exploited). To express this differently, the first order condition of both sides w.r.t.  $w_1$  implies

$$\Lambda_1 \equiv \frac{dV_1}{dw_1} = E_1 [\Lambda_2] + \psi_1,$$

which implies  $p_{i,1} = E_1 \left[ \frac{\Lambda_2}{\Lambda_1} (p_{i,2} + \delta_{i,2}) \right]$  for exploited assets. The unexploited assets are priced by the behavioral investors so that  $p_{i,1} = E_1 \left[ (1 + \phi i)^{-1} (p_{i,2} + \delta_{i,2}) \right]$ . To obtain  $k_1^*$ , assume that all assets are exploited and combine (1) and (18) to obtain

$$E_1 \left[ \frac{\Lambda_2}{E_1 [\Lambda_2]} (p_{i,2} + \delta_{i,2}) \right] = \frac{E_1 [p_{i,2} + \delta_{i,2}]}{1 + \phi \left( i - \frac{1}{2} x_{i,1} \right)},$$

which gives  $x_{i,1} = 2 \left( i - \frac{1}{\phi} \left[ \left( 1 + Cov_1 \left( \frac{\Lambda_2}{E_1 [\Lambda_2]}, \frac{p_{i,2} + \delta_{i,2}}{E_1 [p_{i,2}]} \right) \right)^{-1} - 1 \right] \right)$ . Rearranging and setting  $k_1^* = \int_0^1 x_{i,1} di$  gives

$$k_1^* = 1 - \frac{2}{\phi} \int_0^1 \left\{ \left( 1 + Cov_1 \left( \frac{\Lambda_2}{E_1 [\Lambda_2]}, \frac{p_{i,2} + \delta_{i,2}}{E_1 [p_{i,2}]} \right) \right)^{-1} - 1 \right\},$$

which is less than or equal to 1 since  $Cov(\Lambda_2, p_{i,2} + \delta_{i,2}) = Cov(1 + \phi i_2^*, p_{i,2} + \delta_2) \leq 0$   $\forall i$  since  $p_{i,2} = v / (1 + \phi i_1^*)$  or  $p_{i,2} = v / (1 + \phi i)$  and  $i_2^* = 1 - \sqrt{k_2}$  where  $k_2 = w_1 + \int_0^1 (p_{i,2} + \delta_2) x_{i,1} di$ .

**Lemma 8. (Asset returns using the SDF).** Under Assumption 1, the expected return on asset  $i$  at time 2 follows

$$E_1 r_{i,2}^e = \alpha_{i,0} + \lambda_m \beta_{i,m} \tag{20}$$

s.t. (i)  $\beta_{i,m}$  is the negative of the beta with respect to the arbitrageur's time-2 stochastic discount factor (SDF), which depends negatively on  $k_2$ .

(ii)  $\alpha_{i,0}$  is the asset-specific zero-beta rate that is also the abnormal return by the zero-risk-free-rate assumption.

(iii)  $\lambda_m > 0$  and  $\beta_{i,m} > 0$  for  $i > 0$ .

**Proof.** The expected return formula follows from an algebraic manipulation of [Lemma 7](#) where  $\lambda_m > 0$  since  $k_2$  is in  $[0, 1]$  with positive probability.  $\beta_{i,m} > 0$  is because

$$Cov_1(r_{i,2}, m_2^A) = Cov_1(p_{i,2} + \delta_{i,2}, m_2^A) = Cov_1\left(\frac{v}{m_{i,2}}, m_2^A\right),$$

where  $m_{i,2} = m_2^A$  when  $i > i_2^*$  and  $m_{i,2} = (1 + \phi i)^{-1}$  when  $i \leq i_2^*$ .

### A.3 Proof of lemmas 2-4 and propositions 1-5

Next, I prove the remaining lemmas and propositions in the main body of the paper.

**Proof of Lemma 2. (Equilibrium with unconstrained arbitrageurs).** Since  $k_2 \geq 1$  with certainty, item (iii) of [Lemma 5](#) implies that  $\Lambda_2 = 1$  and  $i_2^* = 1$  in all states. Hence item (i) of [Lemma 5](#) shows that all assets are completely exploited such that  $p_{i,2} = v$  and  $r_{i,3}^e = r_{i,3} = \epsilon_{i,3} \equiv \delta_{i,3}/v$ . Similarly, since  $k_1 \geq 1$ , item (iv) of [Lemma 7](#) implies that  $k_1$  is above the threshold value  $k_1^*$  that makes the arbitrageur unconstrained. Hence [Lemma 6](#) and [Lemma 7](#) imply that  $m_{i,2} = 1$  for all assets such that  $p_{i,1} = E_1[v + \delta_{i,2}] = v$  and  $r_{i,2}^e = r_{i,2} = \epsilon_{i,2} = \delta_{i,2}/v$ .

**Proof of Lemma 3. (Asset returns with constrained arbitrageurs).** Since  $m_2^A = (1 + \phi(1 - \sqrt{k_2})) / \Lambda_1$  at  $k \in (0, 1)$ , a first-order approximation around  $\bar{k}_2 \equiv (1 - \phi(E_1[\Lambda_2] - 1))^2$  is  $m_2^A \approx E_1[m_2^A] - \phi(2\Lambda_1\sqrt{\bar{k}_2})^{-1}(k_2 - \bar{k}_2)$ . Thus,

$$E[r_{i,2}] = \alpha_{i,0} + \lambda_m \beta_{i,m} \approx \alpha_{i,0} + \underbrace{\frac{\phi Var_1(k_2)}{2\Lambda_1 E_1[m_2^A] \sqrt{\bar{k}_2}}}_{\equiv \lambda_k} \underbrace{\frac{Cov_1(r_{i,2}, k_2)}{Var_1(k_2)}}_{\equiv \beta_{i,k}}. \quad (21)$$

To see that  $\beta_{i,k} > 0$  for  $i > 0$ , note that  $Cov_1(r_{i,2}, k_2) = p_{i,1}^{-1} Cov_1(p_{i,2} + \delta_{i,2}, k_2) = p_{i,1}^{-1} Cov_1\left(\frac{v}{m_{i,2}}, k_2\right)$ , where we know  $\frac{\partial m_{i,2}}{\partial k_2} \leq 0$  for  $i > 0$ . Also, for any random variable  $X$ , we know

$$\begin{aligned} Cov(X, f(X)) &= E[(X - E[X])(f(X) - E[f(X)])] \\ &= \underbrace{E[(X - E[X])(f(X) - f(E[X]))]}_{\geq 0} + \underbrace{E[(X - E[X])(f(E[X]) - E[f(X)])]}_{=0} \\ &\geq 0 \end{aligned}$$

if  $f'(X) \geq 0$ , which is the case when  $X$  is  $k_2$  and  $f(X)$  is  $m_{i,2}(k_2)$ .

**Proof of Lemma 4. (A factor model of asset returns).** Substituting  $k_2 = w_2 + f_2$  into eq. (21) gives

$$E[r_{i,2}] \approx \alpha_{i,0} + \underbrace{\frac{\phi \text{Var}_1(w_2)}{2\Lambda_1 E_1[m_2^A] \sqrt{k_2}}}_{\equiv \lambda_w} \beta_{i,w} + \underbrace{\frac{\phi \text{Var}_1(f_2)}{2\Lambda_1 E_1[m_2^A] \sqrt{k_2}}}_{\equiv \lambda_f} \beta_{i,f}.$$

**Proof of Proposition 1. (Arbitrage position determines the cross-section of arbitrage-driven**

**betas).** (a) First, I show that a higher expected arbitrage position on an asset means that the level of  $k_2$  at which the arbitrageur begins trading the asset is lower. Let  $k_2(i)$  denote the level of  $k_2$  that makes  $i$  the marginal asset. Then,  $E_1[\mu x_{i,2}] = \int_{k_2(i)}^1 (\sqrt{k_2} - \sqrt{k_2(i)}) dF(k_2) + \int_1^\infty (1 - \sqrt{k_2(i)}) dF(k_2) = \int_{k_2(i)}^\infty [\min\{\sqrt{k_2}, 1\} - \sqrt{k_2(i)}] \cdot dF(k_2)$ . Hence,  $E_1[\mu x_{i,2}]$  and  $k_2(i)$  are negatively related:

$$\frac{\partial k_2(i)}{\partial E_1[\mu x_{i,2}]} < 0.$$

(b) On the other hand, a lower  $k_2(i)$  means higher beta. To show this, I prove

$$\frac{\partial \text{Cov}_1(r_{i,2}, k_2)}{\partial k_2(i)} < 0.$$

This is because  $\frac{\partial p_{i,1}}{\partial k_2(i)} = \underbrace{\frac{\partial p_{i,1}}{\partial i}}_{\leq 0} \cdot \underbrace{\frac{\partial i}{\partial k_2(i)}}_{< 0} \geq 0$  and  $\frac{\partial \text{Cov}_1(p_{i,2}, k_2)}{\partial k_2(i)} = -v \int_{-\infty}^{k_2(i)} \frac{k_2}{(1+\phi i)^2} \cdot \frac{\partial i}{\partial k_2(i)} dF(k_2) +$

$v E_1[k_2] \int_{-\infty}^{k_2(i)} \frac{1}{(1+\phi i)^2} \cdot \frac{\partial i}{\partial k_2(i)} dF(k_2) = \underbrace{\frac{\partial \text{Cov}_1(p_{i,2}, k_1)}{\partial i}}_{> 0} \cdot \underbrace{\frac{\partial i}{\partial k_2(i)}}_{< 0} < 0$ . Combining (i) and (ii)

implies  $\frac{\partial \beta_{i,k}}{\partial E_1[\mu x_{i,2}]} > 0$ .

**Proof of Proposition 2. (Pre-arbitrage alpha predicts the cross-section of arbitrage-driven**

**betas).** The proof has two steps: first prove that the prices of high- $i$  assets respond more strongly to the variation in arbitrage capital and then prove that this implies that those assets have higher arbitrage capital betas. (a) For the first step, since  $\text{Cov}_1(p_{i,2}, k_2) =$

$$E_1 [p_{i,2}k_2] - E_1 [p_{i,2}] E_1 [k_2],$$

$$\begin{aligned} Cov_1 (p_{i,2}, k_2) &= v \int_{-\infty}^{k_2(i)} \frac{k_2}{1+\phi i} dF (k_2) + v \int_{k_2(i)}^{\infty} \frac{k_2}{1+\phi i_2^*} dF (k_2) \\ &\quad - v E_1 [k_2] \left( \int_{-\infty}^{k_2(i)} \frac{1}{1+\phi i} dF (k_2) + \int_{k_2(i)}^{\infty} \frac{1}{1+\phi i_2^*} dF (k_2) \right), \end{aligned}$$

where  $k_2(i)$  denotes the value of  $k_2$  that makes  $i$  the marginal asset and  $F$  is the conditional cumulative density function of  $k_2$ . The derivative of the covariance with respect to  $i$  gives

$$\frac{\partial Cov_1 (p_{i,2}, k_2)}{\partial i} = -v \int_{-\infty}^{k_2(i)} \frac{k_2}{(1+\phi i)^2} dF (k_2) + v E_1 [k_2] \int_{-\infty}^{k_2(i)} \frac{1}{(1+\phi i)^2} dF (k_2),$$

where the Leibniz terms cancel out by the fact that  $i_2^*(k_2(i)) = i$ . Rearranging the terms gives

$$\frac{\partial Cov_1 (p_{i,2}, k_2)}{\partial i} = \frac{v}{(1+\phi i)^2} (E_1 [k_2] - E_1 [k_2 | k_2 \leq k_2(i)]) F (k_2(i)) > 0.$$

(b) Next, to show how this monotonicity of the price covariance implies  $\partial Cov_1 (r_{i,2}, k_2) / \partial i > 0$ , it suffices to show that the equilibrium time-1 prices are non-increasing in  $i$ :

$$\frac{\partial p_{i,1}}{\partial i} \leq 0.$$

To see this, suppose for a contradiction that  $A < B$  but  $p_{A,1} < p_{B,1}$ . Suppose also that  $B$  is priced by the arbitrageur so that  $p_{B,1} = E_0 \left[ \frac{\Lambda_2}{\Lambda_1} p_{B,2} \right]$ . Since  $p_{A,2} \geq p_{B,2}$  in all states of  $t = 2$ , it must be that

$$p_{A,1} \geq E_1 \left[ \frac{\Lambda_1}{\Lambda_0} p_{A,2} \right] \geq E_1 \left[ \frac{\Lambda_1}{\Lambda_0} p_{B,2} \right],$$

which is a contradiction. Now suppose that  $B$  is priced by the behavioral investors so that  $p_{B,1} = \frac{1}{1+\phi B} E_1 [p_{B,2}]$ . Again, since  $p_{A,2} \geq p_{B,2}$  in all states of  $t = 2$ , it must be that

$$p_{A,1} \geq \frac{1}{1+\phi A} E_1 [p_{A,2}] \geq \frac{1}{1+\phi B} E_1 [p_{B,2}],$$

which is also a contradiction. Hence,  $p_{i,1}$  is non-increasing in  $i$ . Putting these together, we see that  $Cov_1 (r_{i,2}, k_2)$  is non-decreasing in  $i$ :

$$\frac{\partial Cov_1 (r_{i,2}, k_2)}{\partial i} > 0.$$

It follows that

$$\frac{\partial \beta_{i,k}}{\partial (\phi i)} = \frac{1}{\phi \text{Var}_1(k_2)} \times \frac{\partial \text{Cov}_1(r_{i,2}, k_2)}{\partial i} > 0.$$

Finally, since  $\alpha_i^{pre} = \phi i$ , this also implies  $\frac{\partial \beta_{i,k}}{\partial \alpha_i^{pre}} > 0$ .

**Proof of Proposition 3. (Arbitrage-driven betas arise when the arbitrageur is constrained).**

This follows trivially from the analysis in [Lemma 2](#) and from [Lemma 3](#).

**Proof of Proposition 4. (Cross-section of time-series return predictability).** First, I prove the statement that arbitrage-driven beta is a discount-rate beta:

$$\beta_{i,k}^{DR} \equiv \frac{\text{Cov}_1(E_2[r_{i,3}], k_2)}{\text{Var}_1(k_2)} < 0 \quad \forall i \in (0, 1].$$

Since the expected cash flow at time 3 is fixed,  $E_2[r_{i,3}] = \frac{v}{p_{i,2}} - 1 = m_{i,3} - 1$  where  $m_{i,3}$  is a non-increasing function of  $k_2$  (and equals one if  $i = 0$ ). Hence  $\beta_{i,k}^{DR} < 0$  for  $i > 0$ . Next, to see the cross-sectional relationship between  $R^2$  and  $\phi i$ , note [Lemma 5](#) implies

$$R_i^2 = \frac{\text{Var}_1(E_2 r_{i,3}^e)}{\text{Var}_1(r_{i,3}^e)} = \frac{\text{Var}_1(E_2 r_{i,3}^e)}{\text{Var}_1\left(\left(1 + \frac{\delta_{i,3}}{v}\right) E_2 r_{i,3}^e\right)} = \frac{\text{Var}_1(E_2 r_{i,3}^e)}{\text{Var}_1\left(1 + \frac{\delta_{i,3}}{v}\right) \left(\text{Var}_1(E_2 r_{i,3}^e) + \left(E_1[E_2 r_{i,3}^e]\right)^2\right)}$$

where the last equality follows from  $\delta_{i,e}$  and  $E_2 r_{i,3}^e$  being independent. Since  $\text{Var}(X) + (E[X])^2 = E[X^2]$  for any random variable  $X$ ,

$$R_i^2 = \frac{\text{Var}_1(E_2 r_{i,3}^e)}{\text{Var}_1\left(1 + \frac{\delta_{i,3}}{v}\right) E_1\left[\left(E_2 r_{i,3}^e\right)^2\right]}$$

Since  $\text{Var}_1\left(1 + \frac{\delta_{i,3}}{v}\right)$  is the same for all assets by the i.i.d. assumption on  $\delta_{i,3}$  and since  $\text{Var}_1(E_2 r_{i,3}^e) = E_1\left(\left(E_2 r_{i,3}^e\right)^2\right) - \left(E_1(E_2 r_{i,3}^e)\right)^2$ , it suffices to show that  $\left(E_1(E_2 r_{i,3}^e)\right)^2 / E_1\left[\left(E_2 r_{i,3}^e\right)^2\right]$  is decreasing in  $i$ , which in turns determines  $\beta_{i,k}$  by [Proposition 2](#) (I put absolute value in the proposition for empirical applications, but in the model both  $\beta_{i,k}$  and  $i$  are non-negative). Applying the formula for  $r_{i,3}^e$  from [Lemma 5](#), this is equivalent to proving that the function

$$S(i) \equiv \frac{\left(E[\min(Z, i)]\right)^2}{E[\min(Z, i)^2]}$$

decreases in  $i \in (0, 1]$  where  $Z = \max\left((1 - \sqrt{\max(k, 0)}, 0\right)$ . Then, function  $S(i)$  is differentiable outside at most countable set of points  $i \in (0, 1]$  and its derivative is equal to

$$S'(i) = \frac{2P(Z > i)E[\min(Z, i)]}{(E[\min(Z, i)]^2)^2} \left( E[\min(Z, i)]^2 - iE[\min(Z, i)] \right). \quad (22)$$

As  $\min(Z, i)^2 = \min(Z, i) \min(Z, i) \leq i \min(Z, i)$  with strict inequality at some values of  $i$ , it follows that  $S'(i) < 0$  outside at most countable set of points  $i \in (0, 1]$ . It remains to apply a well-known result from real analysis that if a continuous function on an interval has a negative derivative outside at most countable set of points, then it is decreasing (Dieudonné, 2006). The rest of the proof is to show (22). For a non-negative random variable  $\xi$ , the expectation can be written as  $E[\xi] = \int_0^\infty P(\xi > \omega) d\omega$ . It follows that

$$E[\min(Z, i)] = \int_0^i P(Z > \omega) d\omega \quad (23)$$

and it is a continuous function of  $i$  (a Lipschitz function). Similarly,

$$E[\min(Z, i)]^2 = E[\min(Z^2, i^2)] = \int_0^{i^2} P(Z^2 > \omega) d\omega$$

It follows that  $S(i)$  is continuous on  $(0, 1]$ . From (23) it follows that  $i \rightarrow E[\min(Z, i)]$  is differentiable at all points of continuity of the function  $i \rightarrow P(Z > i)$ , and the derivative is  $P(Z > i)$ . But the function  $i \rightarrow P(Z > i)$  is monotone and hence possess at most countable set of discontinuities. The same applies to the function  $i \rightarrow P(Z^2 > i)$ . So, outside at most countable set of points on  $(0, 1]$  the derivative of the numerator  $i \rightarrow (E[\min(Z, i)]^2)$  is  $2P(Z > i)E[\min(Z, i)]$  and the derivative of the denominator  $i \rightarrow E(\min(Z, i))^2$  is  $2iP(Z^2 > i^2) = 2iP(Z > i)$ . Hence (22).

### **Proof of Proposition 5. (Cross-section of asset returns during a crash of arbitrage capital).**

Suppose  $k_2 = k_2(i_2^*)$  for some  $i_2^* \in (0, 1)$  where  $k_2(i)$  denotes the level of  $k_2$  that makes  $i$  the marginal asset. I proceed in three steps. (a) First, returns are negative for assets  $[0, i_2^*]$ . Since  $p_{i,1} = E_1 \left[ \frac{\Lambda_2}{E_1[\Lambda_2]} (p_{i,2} + \delta_{i,2}) \right]$  (Lemma 7) and  $\delta_{i,2}$  and  $\Lambda_2$  are independent (since  $E[\Lambda_2 | \delta_{i,2}] = E[\Lambda_2]$ ),  $p_{i,1} = E_1 \left[ \frac{\Lambda_2}{E_1[\Lambda_2]} p_{i,2} \right] = \int_{-\infty}^0 \frac{1+c}{E_1[\Lambda_2]} \cdot \frac{v}{1+\phi i} dF(k_2) + \int_0^{k_2(i)} \frac{1+\phi(1-\sqrt{k_2})}{E_1[\Lambda_2]} \cdot \frac{v}{1+\phi i} dF(k_2) + \int_{k_2(i)}^\infty \frac{1+\phi(1-\sqrt{k_2})}{E_1[\Lambda_2]} \cdot \frac{v}{1+\phi(1-\sqrt{k_2})} dF(k_2)$ . Since  $p_{i,2} = \frac{v}{1+\phi i}$  (Lemma 5),  $\Delta p_{i,2} \equiv$

$p_{i,2} - p_{i,1} = \frac{v}{1+\phi i} \left[ 1 - \frac{E_1[\tilde{\Lambda}_{i,2}]}{E_1[\Lambda_2]} \right]$  where  $\tilde{\Lambda}_{i,2} \equiv 1 + c, 1 + \phi(1 - \sqrt{k_2})$ , and  $1 + \phi i$  for intervals  $k_2 \leq 0, k_2 \in (0, k_2(i)),$  and  $k_2 \geq k_2(i),$  respectively. Under Assumption 1,  $\tilde{\Lambda}_{i,2} > \Lambda_2 \implies \frac{E_1[\tilde{\Lambda}_{i,2}]}{E_1[\Lambda_2]} > 1 \implies \Delta p_{i,2} < 0 \implies r_{i,2}^e < 0 \forall i \leq i_2^*.$  (b) Next,  $\Delta p_{i,2}$  is decreasing in  $i \in [0, i_2^*].$  Let  $g(i) \equiv \frac{E_1[\Lambda_2] - E_1[\tilde{\Lambda}_{i,2}]}{1+\phi i} = \frac{\int_{k_2(i)}^{\infty} \phi(i^* - i) dF(k_2)}{1+\phi i}.$  Then,  $g'(i) = -\frac{\phi}{(1+\phi i)^2} \left( E_1[\Lambda_2] - E_1[\tilde{\Lambda}_{i,2}] \right) - \frac{\phi \int_{k_2(i)}^{\infty} dF(k_2)}{1+\phi i} = -\frac{\phi}{(1+\phi i)^2} (E_1[\Lambda_2] - E_1[\tilde{\Lambda}_{i,2}] + (1 + \phi i) Pr(k_2 > k_2(i))) = -\frac{\phi}{(1+\phi i)^2} \left( \phi \int_{k_2(i)}^{\infty} i^* dF(k_2) + Pr(k_2 > k_2(i)) \right) < 0.$  (c) Finally, since  $p_{i,1}$  is non-increasing in  $i$  (from the proof of Proposition 1) and  $\Delta p_{i,2} < 0$  (first part of this proof),  $r_{i,2}^e = \Delta p_{i,2}/p_{i,1}$  is decreasing in  $i \forall i \leq i_2^*.$