

Market Power and Price Informativeness*

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Abstract

Large institutional investors dominate asset ownership in financial markets. We develop a general equilibrium model to study the effects of ownership patterns for price informativeness when some investors have price impact, and can learn about assets. We decompose the effects into: (i) a covariance channel, (ii) a concentration channel, and (iii) an information passthrough channel. We find that price informativeness is non-monotonic in institutional sector's size, monotonically decreasing in institutional sector's concentration, and monotonically decreasing in the size of the passive sector both due to decrease in aggregate information capacity and an additional amplification through endogenous information allocation response.

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1 Introduction

Institutional investors constitute an important part of financial landscape due to their economic size and amount of information they bring in. As of 2018, the institutional ownership of an average stock in the U.S. equals around 60%. Within the institutional sector, investors who actively produce information play a dominant role but passive investors who do not trade for information reasons also hold an economically significant 25% of shares outstanding. The ownership structure is heavily skewed, with ten largest investors (some active and some passive) holding an average 35% of total shares outstanding. The economic importance of large institutional investors, and their impact on market stability has drawn considerable attention from market participants, policy makers, and academics.

One major consideration is the implication of changing market structure on asset prices, specifically the effect of *large* active and passive investors on price discovery and, more broadly, on the efficiency of capital allocation in the economy. On the one hand, large active investors have greater capacity to conduct fundamental research, which would increase the amount of information revealed in their trading. On the other hand, they also recognize the price impact of their trades, which makes them trade less on any information they have. The questions that interest us in this paper are threefold: First, what happens to the informational content of prices as the *size* of large investors changes? Second, what happens to the informational content of prices as the *concentration* of large investors changes? Third, what happens to the informational content of prices as the *share of passively managed funds* changes?

To address the issues, a new, generalized theory is necessary. This paper

builds such a theory by combining elements of some well-established literatures. Our framework has a novel combination of two distinct elements: (i) investors have different sizes, and therefore different degrees of price impact;¹ (ii) investors have different abilities to acquire information, and make endogenous learning and portfolio decisions across several assets.² The interaction of these two elements allows us to study the impact of a particular market structure not only on trading decisions, but also on the ex-ante learning decisions of investors. We are therefore able to increase the breadth of analysis on factors that would impact price informativeness.

We define the *informational content of a price as the covariance of the price with the fundamental, normalized by the volatility of the price*. This measure is becoming prevalent in the literature,³ and is suited to our analysis for several reasons: (i) the measure is intuitive, as it can be expressed as the product of the correlation between the price and the fundamental and the volatility of the fundamental. Such a measure is higher when correlation is higher, which is intuitive, but also increases when the fundamental is more volatile, because higher correlation is more meaningful when the unobserved variable is more volatile; (ii) price informativeness gives the reduction in posterior beliefs if agents used price as a signal about fundamentals. That is, our definition is exactly the object that enters an agent's beliefs when she learns from the price, which would appear to be a good basis for a price-informativeness measure; (iii) as was shown by Bai, Philippon, and Savov (2016), the measure can be derived as a welfare measure using Q-theory. The definition we use is not essential for our comparative static results, but for the reasons discussed here,

¹See the literature started by Kyle (1985).

²See, for example, Van Nieuwerburgh and Veldkamp (2009, 2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016)

³See Bai, Philippon, and Savov (2016) and Farboodi, Matray, and Veldkamp (2018)

is the one that best fits our framework.

Given the formulation of price informativeness described above, we can analytically decompose the impact of any market structure on the informational content of prices into three channels. The first is the *covariance channel*, which captures how well large players' learning makes the price load more on the fundamental shock. This channel is determined by the linear combination of oligopolists' learning, weighted by their ownership shares of an asset. It hence favors ownership structures that concentrate ownership on the oligopolist that invests the most information capacity in that asset. The second channel is the *information passthrough channel*, which captures how responsive investors' trades are to their private signals about asset payoffs. We show that the information passthrough for each oligopolist is a hump-shaped function of their size. As an oligopolist grows in size of assets under management, initially it implies larger responses of their trades to their signals. However, above a certain size, the price impact of the oligopolist becomes too big, and actually the response of their trades to signals begins to shrink. This channel favors intermediate size of each oligopolist and the sector as a whole, and it is responsible for non-monotonic responses of price informativeness to size. The third channel impacting price informativeness is the *concentration channel*, expressed by a learning-weighted HHI index, which captures how concentration in investors' holdings leads to errors in signals being compounded into the price. The more concentrated the ownership of an asset, the larger is the impact on the price of errors in signals of the large holders of that asset, leading to larger price variability orthogonal to the fundamental.

Two additional results combine with the channels above to determine the responses of the model to different market structures. One is that, unlike in a perfectly competitive model, here *size introduces a concavity in investor's*

learning decision. Because of price impact, there are decreasing returns to learning about any one asset, and those returns decrease faster, the bigger is the investor. In equilibrium, it means that as an investor increases in size, they diversify their learning into more assets. The second result is that *price informativeness is concave in any large investor's learning decision.* Therefore, spreading oligopolists' learning helps average price informativeness through this result, and also through the concentration channel and potentially the information passthrough channel above, while hurting each individual asset's price informativeness through the covariance channel. The combination of all these forces is what drives our results, and we provide detailed decompositions of each of our findings.

We conduct a set of numerical exercises for typical parameterization of the model, in order to study the effects of different market structures on price informativeness, as well as to determine the relative importance of the channels we identify. Our first result is that *increases in the size of the large investor sector have a non-monotonic effect on average price informativeness.* This is true in the aggregate and for individual assets and is driven by the hump-shaped nature of the information passthrough channel, which quantitatively dominates this result. Our second result is that *increases in the concentration of the large investor sector, have a monotonically negative effect on average price informativeness.* On the asset level, this is driven by passthrough channel and the concentration channel. In terms of information passthrough, when one investor gets big at the expense of others, both groups hit the diminishing information passthrough region. In terms of aggregation, these asset level effects get amplified by the fact that when the sector size is more evenly distributed, agents learn roughly evenly about all the assets, which increases the aggregate price informativeness due to its concavity. Our third result is that *moving*

assets from being actively managed to passively managed reduces price informativeness through two channels. The first is the quantity channel—less smart money reduces price informativeness. The second is the learning channel—a smaller active sector is relatively more likely to specialize in its learning, which has a negative impact on aggregate price informativeness. An additional implication is that as active investors specialize more, some assets actually exhibit growth in their price informativeness and some less desirable assets exhibit a decrease in price informativeness. A heterogenous cross-sectional response to a growing passive sector has been documented in Farboodi, Matray, and Veldkamp (2018), which is consistent with this result. Finally, we contrast all of the results from our experiments with those from a model with exogenous information, as in prior literature. We find that the conclusions are very different in a model with exogenous information, which highlights the importance of modeling information choice when studying price informativeness.

We present a set of additional results relating to price informativeness in the cross section of assets as well. We find that large investors prefer to learn about volatile assets, which, in turn means that price informativeness is highest for more volatile assets, and less so for less volatile assets. Further, we find that assets that have high levels of concentration in holdings tend to have somewhat lower levels of price informativeness, as the errors in an agents learning pollute the price. The effects for concentration of holdings are smaller in magnitude than they are for size of holdings. Additionally, we solve special cases of the model to find closed-form solutions for price informativeness and learning, as well as for the thresholds of size at which investors decide to start spreading their learning across many assets. We are able to compare these thresholds to those at which it would be optimal *when maximizing price informativeness*.

Finally, we present an extension of our framework in which fringe investors

are allowed to learn from the price signal at no capacity cost.⁴ This extension introduces significantly more complexity to the equilibrium, and as a result only numerical solution is possible. In our simulations of this extension of the model, however, we find the same set of conclusions as in the benchmark model in terms of the shape and monotonicity of price informativeness relative to market structures.

1.1 Related Literature

Our paper spans several research themes. The literature on informed trading with market power dates back to Kyle (1985) and Grinblatt and Ross (1985) whose setup is one strategic trader, and Holden and Subrahmanyam (1992), which extends the model of Kyle into an oligopolistic framework. Lambert, Ostrovsky, and Panov (2018) extend the Kyle’s model to study the relation between the number of strategic traders and information content of prices.⁵ In all these studies, information is an exogenous process, which is a key dimension along which our model works. Also, these studies do not examine the role of concentration and active/passive traders, both being the central focus of our study. Kyle, Ou-Yang, and Wei (2011) allow for endogenous information acquisition but their mechanism depends on differences in risk aversion. Also, they focus on the contracting features of delegation and only consider one risky asset. In turn, our framework utilizes heterogeneity in information capacity and multi-asset economy.

⁴If the price signal is costly in terms of information capacity, agents would never find it optimal to use it. For a detailed discussion of this result, see Kacperczyk, Nosal, and Stevens (2017). Our baseline results all come in a setting in which any learning uses up capacity, which allows for analytical characterization of the price informativeness and the decomposition.

⁵Models in which traders condition on others’ decisions also include Foster and Viswanathan (1996) and Back, Cao, and Willard (2000).

Our general equilibrium model is anchored in the literature on the endogenous information choice, in the spirit of Sims (1998, 2003). More closely related to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), and Kacperczyk, Nosal, and Stevens (2017). Ours is the first theoretical study to introduce market power into a model with endogenous information acquisition. This novel aspect allows us to study strategic responses of oligopolistic traders in terms of their demand and information choices.

We also contribute to the literature on information production and asset prices. Bond, Edmans, and Goldstein (2012) survey the literature on information production in financial markets, emphasizing the differences between new information produced in markets (revelatory price efficiency: RPE) and what is already known and merely reflected in prices (forecasting price efficiency: FPE). Our focus is solely on RPE and is largely dictated by the modeling framework we use.⁶ Stein (2009) develops a model of market efficiency and sophisticated (arbitrage) capital in the presence of capital constraints. Garleanu and Pedersen (2015) examine the role of search frictions in asset management for price efficiency. Breugem and Buss (2018) study the impact of benchmarking on price informativeness in a costly information acquisition competitive equilibrium model. Davila and Parlato (2017) explore the equilibrium relation between price informativeness and price volatility, and characterize the conditions under which volatility and price informativeness co-move.

On an empirical front, Bai, Philippon, and Savov (2016) show that price

⁶Theoretical work on asset prices and real efficiency also includes Dow and Gorton (1997), Subrahmanyam and Titman (1999), Kurlat and Veldkamp (2015), and Edmans, Goldstein, and Jiang (2015).

informativeness is greater for stocks with greater institutional ownership. We confirm their findings for the range of the ownership values. However, we show that beyond certain levels (not observed in their data) ownership may in fact reduce price informativeness. Separately, we also investigate the role of ownership concentration and provide a micro-founded general equilibrium model that allows us to study the underlying economic mechanism in more depth. In a contemporaneous work, Farboodi, Matray, and Veldkamp (2018) examine differences in price informativeness between companies included and not included in the S&P 500 index. They show that the indexed companies exhibit larger efficiency, which they attribute to composition effect of these companies, being older and larger. Their focus, however, is not on market power and changes in market structure.

Finally, we add to a growing empirical literature that studies the impact of market structure in asset management on various economic outcomes. Following the diseconomies of scale argument of Chen et al. (2004), Pástor, Stambaugh, and Taylor (2015) show significant diseconomies of scale at the industry level. Using a merger between BlackRock and BGI as a shock to market power, Massa, Schumacher, and Yan (2016) study the asset allocation responses of their competitors. They find that competitors scale down positions which overlap with those held by the merged entity. Our work complements these studies by studying theoretically the effect of ownership structure on price informativeness.

2 Motivating Facts

In this section, we present the three empirical facts that motivate our study. First, institutional stock ownership is economically important, averaging about

60% over the last 35 years. Second, the ownership structure is skewed towards the largest owners. Third, the ownership mix includes significant shares of both active and passive investors. Except for the recent paper by Bai, Philippon, and Savov (2016) which emphasizes the first fact, no other study has exploited the implications of these facts for longer-horizon price informativeness.⁷

To provide more details related to the above facts, we collect data on institutional stock ownership from Thomson Reuters. The data span the period 1980–2015. Even though the formal requirements to report holdings allow for smaller companies to not report, the coverage of institutions in the data is more than 98% in value-weighted terms. We calculate the stock-level institutional ownership by taking the ratio of the number of stocks held by financial institutions at the end of a given year to the corresponding number of shares outstanding. Next, we aggregate the measures across stocks by taking a simple average across all stocks in our sample. Using equal weighting, rather than value weighting, gives a more conservative metric of the trends in the data. Subsequently, we calculate a similar measure, but only taking into account the holdings of the top-10 largest holders for a given stock. We present the time-series evolution of the two quantities in Figure 1.

Both series indicate a clear pattern underlying the recent policy discussions: Institutional ownership has grown and the increase has been mostly fueled by the growing concentration of ownership. The magnitudes of the growth are economically large: Over the period of over 35 years, each ownership statistic has more than doubled. While we focus here on the average effects in the data, even stronger results can be observed in the cross section of stocks with

⁷A parallel microstructure literature (Boehmer and Kelley (2009)) examines empirically the relation between institutional ownership and price efficiency due to trading intensity. Efficiency there is measured using variance ratios and pricing errors. Their conclusions are akin to ours.

different characteristics.

In our model, a more natural way to measure concentration is the Herfindahl-Hirshman Index (HHI), defined as the sum of squared shares of all institutional owners of a given stock. However, the problem with using the index is its mechanical correlation with the number of investors. To the extent that the number of institutions has been growing steadily over the same period the unadjusted index would reflect two effects going in opposite direction. To filter out this mechanical sorting, we take out the predicted component in the HHI accounted by the number of investors. We plot the filtered series in [Figure 2](#).

The results indicate that the concentration levels have been generally going up over time. This pattern has been particularly visible since the early 1990s. The magnitude of the growth is economically large and the large values of concentration, especially in the last few years, reflect the concerns policy makers have voiced with regard to this phenomenon.

To illustrate the effects on ownership mix we define active investors as those engaged in information acquisition process and passive investors as those who strictly invest in pre-defined index portfolios. The latter group includes both index mutual funds and ETFs. Because identifying passive funds in the institutional investors data is not trivial, we use the evidence from the Investment Company Institute Fact Book. We show the time-series evolution of the percentage of passively managed equity mutual funds in the U.S. in [Figure 3](#). The results indicate a significant increase in passive ownership in the period 2001–2016. While in early 2000’s passive funds accounted for less than 10% of total equity fund market in the U.S., this share has increased to 25% by 2016.

To conclude, we note that while the motivating facts we present relate to

institutional investors, the model we present next is a general theory of asset allocation and information acquisition by investors with market power. We believe institutions are natural candidates for this type of investors. Moreover, even though here we present the time-series evidence on our motivating facts, our study is motivated more by the mere economic size and interesting features of the distribution of institutional holders. In particular, since our model is static we do not aim to explain any of the dynamic effects in the data and rather take the trends as given.

3 Model

This section presents a noisy rational expectations portfolio choice model in which investors are constrained in their capacity to process information about assets payoffs. The setup departs from standard information choice models (e.g., Van Nieuwerburgh and Veldkamp (2010) or Kacperczyk et al. (2017)) by introducing market power for some investors. In the model, we solve for price informativeness of the aggregate economy and individual assets differentiated by their volatility.

3.1 Setup

The model features a finite continuum of traders, divided into $L + 1$ many segments, represented by λ_j , $j \in \{0, \dots, L\}$. The λ s represent the size of the investor and map monotonically to assets under management.⁸ Mass λ_0 of traders, indexed by h , are atomistic. These traders act as a *competitive fringe*, in that they pay attention to innovations in asset prices, but do not have

⁸Because of this monotonic relationship, we use assets under management as also denoting λ s.

any price impact (market power). Masses $\{\lambda_1, \lambda_2, \dots, \lambda_L\}$ of investors act as *oligopolists*, indexed by j . Each oligopolist collects information and trades, as the fringe does, but the each oligopolist realizes the impact of their learning and trading decisions on prices and returns, and hence acts strategically in information and trading strategy choice. Investors of both types maximize a mean-variance utility function, with common risk aversion ρ .

The market comprises one risk-free asset in unlimited supply, with a price normalized to one and a net payout of r , and $n > 1$ risky assets, indexed by i , with prices p_i and independent payoffs $z_i = \bar{z} + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_i^2)$. Each risky asset has a stochastic supply with mean \bar{x} and variance $\{\sigma_{xi}\}$ - we can think of these as noisy supply shocks.

Each investor is endowed with information capacity, K_j for oligopolist j and K_h for each member of the fringe. They can use that capacity to obtain information about innovations to the payoff for some or all of the risky assets. In particular, every member of the fringe and every oligopolist observes signals about innovations in z_i . The vector of signals for oligopolist j and assets 1 through n is $s_j = (s_{1j} \dots s_{nj})$, and the vector of signals for a member of the fringe h is $s_h = (s_{1h} \dots s_{nh})$. Signal choice is modeled using entropy reduction as in Sims (2003), and is governed by an information capacity constraint where for each investor the vector of signals is subject to an information capacity constraint based on Shannon's (1948) mutual information measure: $I(z; s_j) \leq K_j$ for each oligopolist j and $I(z; s_h) \leq K_h$ for each member of the fringe. After observing the signals and updating their beliefs using Bayes' rule, all investors choose quantities traded given the observed price.

We denote an agent j 's posterior variance on asset i as $\hat{\sigma}_{ji}^2 \leq \sigma_i^2$. We

conjecture and later verify the following price structure:

$$p_i = a_i + b_i \varepsilon_i - c_i \nu_i - \sum_{j=1}^L d_{ji} \zeta_{ji} \quad (1)$$

where ε_i and ν_i are the innovations in the payoff and noisy supply shocks, respectively. The last term is new relative to the literature and it captures the noise in oligopolists signals. It is given by

$$\zeta_{ji} \equiv \delta_{ji} - \frac{1}{\alpha_{ji}} \varepsilon_i$$

where $\alpha_{ji} = \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2}$ and δ_{ji} as the data loss of oligopolist j : $\delta_{ji} \equiv z_i - s_{ji}$. Then, $p_i \sim \mathcal{N}(a_i, \sigma_{pi}^2)$, where σ_{pi} is given by:⁹

$$\sigma_{pi}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}}\right) \hat{\sigma}_{ji}^2 \quad (2)$$

Before solving the oligopolists' problem, we first analyze the problem faced by the competitive fringe.

3.1.1 Competitive Fringe

Portfolio Problem Given posterior beliefs and equilibrium prices, each competitive investor h solves the following problem:

$$U_h = \max_{\{q_{hi}\}_{i=1}^n} E_h(W_h) - \frac{\rho}{2} V_h(W_h) \quad s.t. \quad W_h = r \left(W_{0h} - \sum_{i=1}^n q_{hi} p_i \right) + \sum_{i=1}^n q_{hi} z_i \quad (3)$$

⁹See the Appendix for the derivations.

where E_h and V_h are the perceived mean and variance of investor h conditional on her information set, and W_{0h} is her initial wealth. Optimal portfolio holdings are given by:

$$q_{hi} = \frac{\hat{\mu}_{hi} - rp_i}{\rho \hat{\sigma}_{hi}^2} \quad (4)$$

where $\hat{\mu}_{hi}$ and $\hat{\sigma}_{hi}^2$ are the mean and variance of investor h 's posterior beliefs about payoff z_i .

Information Problem Given the portfolio choice solution, ex-ante each member of the fringe faces the following information problem:

$$\max_{\{\hat{\sigma}_{hi}^2\}_{i=1}^n} U_{0h} \equiv \frac{1}{2\rho} \sum_{i=1}^n \frac{E_{0h} (\hat{\mu}_{hi} - rp_i)^2}{\hat{\sigma}_{hi}^2} \quad (5)$$

subject to the relative entropy constraint

$$\prod_{i=1}^n \frac{\sigma_i^2}{\hat{\sigma}_{hi}^2} \leq e^{2K_h}. \quad (6)$$

The information problem can be rewritten as:

$$U_{0h} = \sum_{i=1}^n G_i \frac{\sigma_i^2}{\hat{\sigma}_{hi}^2}, \quad (7)$$

and the optimum is a corner solution: each investor h learns about one asset $l_h \in \arg \max\{G_i\}$. The gain to the competitive investors from learning about asset i , derived in the Appendix, is:

$$G_i \equiv \frac{(\bar{z} - ra_i)^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2} + r^2 \left(\sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}} \right) \hat{\sigma}_{ji}^2 \right) - \frac{\hat{\sigma}_{hi}^2}{\sigma_i^2} (1 - 2rb_i)$$

The gain from learning about a particular asset is the same across all compet-

itive investors. However, this gain is a function of the learning by the fringe sector as a whole and by the oligopolists. The gains to learning about an asset's payoff are negatively related to the information about z_i reflected in the price already (second term) and positively related to the noise in price (third term and fourth term). The last term drops out of the objective function once multiplied by the reduction in uncertainty.

3.1.2 Oligopolists

Portfolio Problem Oligopolists have a similar trading problem as the fringe, and the quantity demanded by each oligopolist is:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i(q_{ji})}{\rho\widehat{\sigma}_{ji}^2 + r\frac{dp_i(q_{ji})}{dq_{ji}}}, \quad (8)$$

The price impact term in the denominator reflects the fact that oligopolists have market power. Each oligopolist internalizes the fact that their asset purchase decisions affect the equilibrium price. Using market clearing, we can solve for this derivative to get:

$$\frac{dp_i(q_{ji})}{dq_{ji}} = \frac{\lambda_j\rho\sigma_i^2}{\lambda_0r(1 + \Phi_{hi})} > 0, \quad (9)$$

where

$$\Phi_{hi} \equiv m_{hi}(e^{2K_h} - 1), \quad (10)$$

and m_{hi} is the mass of competitive investors learning about asset i . Hence, how sensitive the price is to an oligopolist's demand depends positively on oligopolist's size λ_j relative to the size of the fringe, λ_0 scaled by fringe learning about that asset.

The oligopolist's demand becomes:

$$q_{ji} = \frac{\hat{\mu}_{ji} - rp_i}{\rho \left(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2 \right)} \quad (11)$$

where $\hat{\lambda}_{ji} = \frac{\lambda_j}{\lambda_0(1+\Phi_{hi})}$ is again the relative size ratio. Given the expression for quantity demanded, we can calculate indirect utility, derived in the Appendix, as:

$$U_j = \frac{1}{2\rho} \sum_{i=1}^n (\hat{\mu}_{ji} - rp_i)^2 \left[\frac{\hat{\sigma}_{ji}^2 + 2\hat{\lambda}_{ji} \sigma_i^2}{\left(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2 \right)^2} \right]. \quad (12)$$

As with the fringe, oligopolists' expected utilities depend positively on the deviations of their personal estimates of payoffs from the equilibrium price (larger deviations mean larger quantities demanded). Further, the smaller is the oligopolists' posterior variance the larger is their utility. The larger is the oligopolists' market power (or conversely the smaller is the fringe, or the less informed the fringe), the larger is the oligopolists' price impact, and therefore the smaller their utility.

Information Problem The oligopolist's information problem is

$$\max_{\{\hat{\sigma}_{ji}^2\}_{i=1}^n} U_{0j} \quad s.t. \quad \prod_{i=1}^n \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \leq e^{2K_j}. \quad (13)$$

We can also write the constraint, defining the increase in precision of beliefs as $\alpha_{ji} \equiv \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2}$, as

$$\prod_{i=1}^n \alpha_{ji} \leq e^{2K_j} \Leftrightarrow \sum_{i=1}^n \ln \alpha_{ji} \leq 2K_j. \quad (14)$$

with

$$\ln \alpha_{ji} \geq 0. \quad (15)$$

The Lagrangean is (dropping the $1/2\rho$ term)

$$\mathcal{L} = \sum_{i=1}^n [u_i(\alpha_{ji}) - \mu \ln \alpha_{ji} + \eta_i \ln \alpha_{ji}] + n\gamma 2K_j, \quad (16)$$

and the optimality conditions are

$$u'_i(\alpha_{ji}) - \frac{\mu}{\alpha_{ji}} + \frac{\eta_i}{\alpha_{ji}} = 0, \quad \forall i = 1, \dots, n. \quad (17)$$

The capacity constraint is always binding, so $\sum_{i=1}^n \ln \alpha_{ji} = 2K_j$ and $\mu > 0$. Let P denote the set of assets that are learned about by the oligopolist. We have that

$$\alpha_{jp} > 1 \quad \text{and} \quad \eta_p = 0 \quad \text{and} \quad \mu = \alpha_{jp} u'_p(\alpha_{jp}) \quad \forall p \in P \quad (18)$$

and

$$\sum_{p \in P} \ln \alpha_{jp} = 2K_j. \quad (19)$$

For assets $i \notin P$,

$$\alpha_{jp} = 1 \quad \text{and} \quad \eta_p = \mu - u'_i(1) \geq 0 \quad \Leftrightarrow \quad \alpha_{ji} u'_i(\alpha_{ji}) \geq u'_i(1) \quad \forall p \in P. \quad (20)$$

These conditions yield the oligopolist's allocation of attention across assets, $\{\alpha_{ji}\}$, as a function of the equilibrium price coefficients, a_i, b_i, c_i, d_i , and the share of competitive investors' learning about each asset, m_{hi} . Given the oligopolist's choice of the set $\{\alpha_{ji}\}$, variance of the posterior belief of the oligopolist is σ_i^2/α_{ji} and the corresponding mean is just the signal s_{ji} . The

signal is distributed, conditional on the realizations $z_i = \bar{z} + \varepsilon_i$, as

$$E(s_{ji}|z_i) = \bar{z} + \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right) \varepsilon_i = \bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right) \varepsilon_i,$$

$$Var(s_{ji}|z_i) = \sigma_i^2 \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right) \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2} = \left(1 - \frac{1}{\alpha_{ji}}\right) \frac{1}{\alpha_{ji}} \sigma_i^2.$$

3.2 Equilibrium

We solve for the coefficients of equation (1): a_i, b_i, c_i, d_{ki} , and d_{ji} (derivation in the Appendix):

$$a_i = \frac{\bar{z}}{r} - \frac{\bar{x}}{r} \frac{N_i \rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} \quad (21)$$

$$b_i = N_i \left(\sum_{j=1}^L \frac{M_{ji} (\alpha_{ji} - 1)}{r \alpha_{ji}} + \frac{\Phi_{hi}}{r (1 + \Phi_{hi})} \right) \quad (22)$$

$$c_i = \frac{N_i \rho \sigma_i^2}{r \lambda_0 (1 + \Phi_{hi})} \quad (23)$$

$$d_{ji} = \frac{N_i M_{ji}}{r} \quad (24)$$

where $M_{ji} \equiv \frac{\hat{\lambda}_{ji} \sigma_i^2}{(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)}$ and $N_i \equiv \frac{1}{1 + \sum_{j=1}^L M_{ji}}$.

The fundamental component of the price, a_i , depends positively on \bar{z} . An increase in supply also decreases a_i , as do increased risk aversion and fundamental volatility. As the fringe's size or attentional capacity increase, their demand increases, and thus prices increase. As the oligopolists' size increases, or as their attention to asset i increases, demand goes up, M_{ji} increases, and N_i decreases, again driving up the price.

The coefficient b_i depends almost exclusively on the information choices of the fringe and oligopolists. If the fringe does not pay attention, then Φ drops

to zero, and so does the second term of the expression. If the oligopolists does not pay attention, each α_{ji} goes to one. b_i is increasing in Φ_{hi} and α_{ji} , because increased attention increases investors' ability to predict the fundamental, and therefore their information will be better reflected in prices. The same reasons for demand's fluctuation in a_i apply to c_i , as c_i corresponds to the random component, while a_i corresponds to the mean component. We next show the existence of an equilibrium.

Proposition 1. *An equilibrium is defined as: A set of posterior beliefs of the fringe $N(\widehat{\mu}_{hi}, \widehat{\sigma}_{hi}^2)$ and of the oligopolists $N(\widehat{\mu}_{ij}, \widehat{\sigma}_{ij}^2)$, a set of learning allocations $\{m_i\}$, and $\{\alpha_{ij}\}$, and a set of quantity allocations $\{q_{hi}\}$ and $\{q_{ij}\}$ such that the asset market clears, and such that oligopolists solve equations (12) and (13) and the fringe solves equations (3) and (5). Such an equilibrium exists.*

All proofs are in the Appendix.

3.3 Price Informativeness and its Determinants

Price informativeness in the model is given by the covariance of the price with the fundamental shock, normalized by the standard deviation of the price:¹⁰

$$PI_i = \frac{b_i \sigma_i}{\sqrt{b_i^2 + c_i^2 \sigma_{xi}^2 / \sigma_i^2 + \sum_j d_{ji}^2 \frac{\alpha_{ji} - 1}{\alpha_{ji}^2}}},$$

where a_i, b_i, c_i , and d_{ji} are the coefficients of the equilibrium price function. In the expression for PI_i , b_i parameterizes the covariance of the price with the shock z_i ; the second term in the denominator captures the noise in the price coming from the noise trader demand shock, and the third term in the

¹⁰Equivalently, it is the correlation of the price with the fundamental, multiplied by the asset's price variance.

denominator captures the noise in the price coming from the noise in the oligopolists' private signals.

We can use the equilibrium expressions for the price coefficients to express PI_i as

$$PI_i = \frac{\sigma_i \sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}} + \Phi_{hi}}{\sqrt{\left(\sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}} + \Phi_{hi}\right)^2 + \frac{1}{(\sum_j \Omega_{ji})^2} \frac{\sigma_{xi}^2}{\sigma_i^2} + \sum_j \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \quad (25)$$

where ω_{ji} is the average share of asset i held by oligopolist j , given by

$$\omega_{ji} \equiv \frac{Q_{ji}}{\sum_k Q_{ki}},$$

Q_{ji} is the average quantity of asset i held by oligopolist j , and Ω_{ji} is the responsiveness of the quantity traded of asset i by oligopolist j to the private signal of oligopolist j . We call Ω the *information passthrough*, and calculate it as,

$$\Omega_{ji} = \frac{\partial \lambda_j q_{ji}}{\partial \hat{\mu}_{ji}} = \frac{\lambda_j \alpha_{ji}}{\rho \sigma_i^2 (1 + \hat{\lambda}_{ji} \alpha_{ji})}. \quad (26)$$

And Φ_{hi} is the informational contribution of the fringe.

Expression (25) allows us to express price informativeness of any asset as determined by three channels. First is the *covariance channel*, defined by the term $\sum_j \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}$, which is the ownership share-weighted average of the reduction in uncertainty¹¹ about asset i 's payoffs due to learning by oligopolist j . PI_i is increasing in the covariance channel, which in turn is maximized when all the ownership weight (i.e. $\omega = 1$) is placed on the oligopolist with the largest reduction in uncertainty. Hence, with heterogeneous learning, the covariance channel favors specialized learning. Second is the *information passthrough*

¹¹Note that $(\alpha_{ji} - 1)/\alpha_{ji} = (\sigma_i^2 - \hat{\sigma}_{ji}^2)/\sigma_i^2$

channel. By (26), the passthrough of oligopolist's j signals to quantities is hump-shaped in their size λ_j . Hence, as λ_j increases, initially the information passthrough goes up, but above a threshold level of size λ_j further increases in size result in less information pass-through, decreasing PI_i . This channel implies that very unequal sizes - as in a high concentration of ownership case - will mean that both the small and the large oligopolists will be on the lower end of the information passthrough curve, implying low PI_i . Also, at very low and very high overall sizes of oligopolists, PI_i will be low as well. Finally, the third channel is the *concentration channel* expressed by the term $\sum_j \omega_{ji}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$, which is given by the weighted sum of the noise in private signals, with weights given by the square ownership shares of each oligopolist. This term can be thought of as a learning-weighted HHI index. To see that this expression is related to ownership concentration, notice that, in a symmetric case of $\alpha_{ji} = \alpha_i$, it simplifies to $\frac{\alpha_i-1}{\alpha_i^2} HHI_i$, where HHI_i is the Herfindahl index for asset i . Therefore, if the noise in oligopolists' signals is equally volatile, high concentration hurts PI_i through this channel. In the general case, the effect of this channel depends on learning choices as well. However, in our numerical experiments, we find that it always favors lower concentration.

The expression in (25) highlights the importance of modeling the choice of information for price informativeness. For an exogenously fixed learning structure (that is, fixed $\{\alpha_{ji}\}_{j=1,\dots,L,i=1,\dots,n}$), putting high weight on the highest- α oligopolist is beneficial as it always increases PI_i through the covariance channel. However, working through the third term in the denominator, high concentration of ownership could be detrimental (e.g., for equal α), or beneficial (e.g., for very unequal distribution of α). Hence, the information structure one assumes in an exogenous information model dictates the conclusion on the benefits of concentration.

We next present two results that together with the three channels identified above drive our results. The first one is that as oligopolists get bigger (their λ increases), they prefer to spread their information acquisition across a larger number of assets. This follows directly from the shape of the indirect utility function of the oligopolists which, unlike the fringe, is concave in own learning:

Proposition 2. *An oligopolist's utility function is concave in her own learning.*

The result follows from the proof of the existence of an equilibrium. We provide detailed analysis of the expansion of learning with size in Section 5.

The second result is that for each asset, the PI_i function is concave in each oligopolist's learning, which means that spreading learning of each oligopolist across a larger number of assets is beneficial for the aggregate measure of PI:

Proposition 3. *Price Informativeness of asset i is concave in α_{ji} , $\forall i$. That is, $\frac{\partial^2 PI_i}{\partial \alpha_{ji}^2} < 0$.*

The concavity of the aggregate PI function interacts with the three economic channels identified by us to drive the numerical results of the next section.

4 Numerical Analysis

In this section, we provide a set of quantitative results from the solution to the equilibrium of the model.¹² We select parameter values for the return distribution \bar{z} and $\{\sigma_i\}_{i=1}^n$, the liquidity distribution \bar{x} and $\{\sigma_{xi}\}_{i=1}^n$, the risk-free return r , risk aversion ρ , fringe and oligopolists' learning capacities K_h and

¹²This involves solving a fixed point of the best responses of the oligopolists to each other's learning and trading policies.

$\{K_j\}_{j=1}^L$, and their respective sizes λ_0 and $\{\lambda_j\}_{j=1}^L$. The simulation generates equilibrium levels of price informativeness, oligopoly holdings, and oligopoly concentration for each asset.

In our simulations, we choose the parameters with two goals in mind: they have to be in an empirically relevant region of the parameter space and the solution needs to involve some degree of learning. Specifically, we consider parameters such that the benchmark model exhibits: (i) learning about all assets, (ii) aggregate oligopoly holdings of between 60% and 70% (which corresponds to the data in Figure 1), (iii) market excess real return of around 7% (which corresponds to the average over 1980-2015). For the results reported below, we set the number of assets to $n = 10$ and the number of oligopolists to $L = 6$.¹³ We report parameter values in Table 1.

Table 1: Parameter values

Parameter	Symbol	Value
Mean payoff, supply	\bar{z}_i, \bar{x}_i	10, 5 for all i
Number of assets	n, L	10, 6
Risk-free rate	$r - 1$	2.5%
Vol. of noise shocks	σ_{xi}	0.41 for all i
Vol. of asset payoffs	σ_i	$\in [1, 1.5]$, linear distribution
Risk aversion	ρ	1.3
Information capacities	$K_h, \{K_j\}$	0, 4.5, constant
Investor masses	$\lambda_0, \lambda_l/\lambda_1$	0.45, 4 λ_j s linearly distributed

4.1 Market Structure Experiments

We begin by analyzing the effect of different market structures on the aggregate price informativeness. In our exercise, each point corresponds to

¹³The choice is largely dictated by the computational speed. Experiments with larger values of each parameter do not change the conclusions from this exercise.

one solution of the model. The experiments are useful as a way to isolate the relative effects of levels and concentration of oligopoly holdings on price informativeness.

Size of the Oligopoly Sector In our first experiment, we look at how average price informativeness across assets changes in response to different levels of λ_0 . Holding the relative distribution of λ_j fixed, we look at simulations of the model by varying λ_0 from 0.05 to 0.95. The type of change we test here could be thought of as a limit on entry, or a limit on a per-agent size in a given market, which would then affect the *composition* of ownership in the market between the fringe and the oligopolists, keeping the total mass of all investors constant.

Figure 4 shows the relation between the size of the oligopoly sector (parameterized by $1 - \lambda_0$) and endogenous variables of interest. The price informativeness in Panel (a) shows a hump-shaped relation with the parameterized size, on average and also for each asset (as indicated by interquartile 10-90 range), and also with the actual realized ownership which is monotonically increasing (Panel (b)). The model's results point to an interior solution for optimal oligopoly sector size. The forces that drive price informativeness up as the size of the oligopoly sector increases are: more efficient (that is, diversified) learning due to convexity of aggregate PI (Panel (d), Propositions 2 and 3), and the positive effect of decreased concentration due to the concentration channel (Panel (c)). The forces that push PI down as the size of the sector increases are the covariance channel and—above a certain size—the information pass-through channel (Section 3.3).

Figure 5 decomposes the overall average PI response into the three channels identified in Section 3.3. The black line represents the total effect on PI, and

each of the color lines represents PI as in equation (25), where the value of one of the channels has been fixed at its $\lambda_0 = 0.05$ level. Several patterns emerge from the decomposition. First, the information passthrough is responsible for the hump shape in aggregate PI—the blue line fixes information pass-through at the initial level, which PI response monotonically decreasing. Second, the covariance channel—as discussed earlier—favors a small, specialized oligopoly sector, as can be seen from the red line that fixes the covariance channel. Third, the concentration channel works to increase PI as the size of the sector increases. This is because we are keeping the distribution of λ_j s intact, and hence growing the size of the sector results in less concentration (also see Figure 4, Panel c).

The Concentration of the Oligopoly In our second experiment, we consider the effects of a change that affects the concentration of actively trading oligopolists. Holding λ_0 constant, we vary the size distribution of $\{\lambda_j\}$ in order to measure an impact on the concentration measure. Specifically, we vary λ_1 relative to the rest of the oligopoly sector in order to vary the share of assets owned by the largest oligopolist (indexed $j = 1$) from 26% to 78%. In doing so, we keep the sum of all λ_j s equal to $1 - \lambda_0$ to isolate the effect of concentration on endogenous variables. Figure 6 presents the aggregate PI response as well as the decomposition where we keep each key channel fixed at the level corresponding to the initial point.

Overall, price informativeness is downward sloping in the level of concentration of oligopolist ownership. The results of the decomposition are consistent with the intuition we build in Section 3.3. Fixing the covariance channel at the initial level – corresponding to high diversification – lowers the PI curve across the experiment. On the other hand, fixing the concentration channel

at the initial level raises the PI curve across all cases. The quantitatively dominant force, just as in the size experiment, turns out to be the information passthrough channel, which in this case is responsible for the downward sloping nature of the PI response. Fixing the information passthrough channel at the initial level, the covariance and concentration channels cancel each other out, resulting in a flat PI response. The information passthrough channel works towards generating a downward sloping PI response as when ownership shares become more unequal, both the increasingly small and the increasingly big oligopolists are going *down* the hump-shaped information passthrough curve.

Active/Passive Effects The importance of oligopoly traders results from two sources: their informational advantage and their size. While all oligopolists exert price impact, not all of them are necessarily equally informed. In particular, passive investors do not directly participate in the market for information. In this section, we explore the predictions of the model with respect to the size of such passive investors. In this exercise, we split oligopolists half-half into passive (zero information capacity) and active (positive information capacity). We then vary the size of the passive versus active sector. The results are presented in Figure 7. Overall, PI decreases as we increase the size of the passive sector, and the main channel at play is the covariance channel. This is due to the fact that (i) the active investors are getting smaller, and (ii) they adjust their information choices in order to specialize more, hence decreasing the aggregate PI via the aggregate concavity property.

More specialization among the active investors also means a heterogeneous PI response for individual assets. Those assets that are being specialized in will increase in PI while those that are being 'dropped' will decrease in PI. Such heterogeneity in asset PI responses over time has been documented in

Farboodi, Matray, and Veldkamp (2018). Our framework links such responses to the rise of passive investing, which can be one of the channels driving the data trends.

4.2 Cross-sectional Patterns

We next analyze the cross section of equilibrium output variables across assets for the benchmark parameter values in Table 1. Figure 8 presents the relation between equilibrium price informativeness per asset (on the y -axis) and equilibrium oligopoly holdings per asset (on the x axis). We find that price informativeness is increasing in underlying volatility, and so are total oligopoly holdings.¹⁴

Figure 9 presents the relation between equilibrium price informativeness and equilibrium oligopoly concentration. The larger an oligopolist's presence in a particular asset's market, the more likely she is to internalize the price effect of her trade. As such, she would like to be less informed than she would be if she had a small presence. As a result, concentration in a particular asset is associated with lower levels of price informativeness.

4.3 The Role of Endogenous Learning Choice

In this section, we present a comparison of the predictions of our benchmark model with endogenous learning choice with a model in which the information structure is exogenously given. The model with a fixed information structure is similar in spirit to that presented in Kyle (1985), in that the effect of market power in the absence of endogenous reoptimization of information choices

¹⁴Volatility is the only asset heterogeneity we model. More generally, these patterns will hold across any type of heterogeneity that affects the return from learning, for example size (\bar{x}_i) or the volatility of the noise/liquidity shocks (σ_{ix}).

depends entirely on how the quantities adjust.

Figure 10 presents the interaction of oligopoly ownership and price informativeness in the two models, where the different points are generated by varying λ_0 , i.e. the size experiment of the previous section. The black dots represent the benchmark case in which we allow both quantities and information choices to adjust, while the red crosses correspond to a case with a fixed learning structure. For the fixed learning cases, the information choice is either fixed at the benchmark value (i.e., $\lambda_0 = 0.45$, Panel (a)), or at values such that information structures are optimal at $\lambda_0 = 0.95$ (small oligopolists, Panel (b)) and $\lambda_0 = 0.05$ (large oligopolists, Panel (c)). In all the fixed-information cases, the level of price informativeness is below that of the benchmark model for which capacity choice adjusts optimally. The gains in price informativeness from optimal learning can be quite large. For example, for the benchmark specification of fixed alphas, price informativeness is reduced by up to 40%. More important, fixing the learning choices leads to very different conclusions about the optimal size of the oligopoly sector. Depending on what values of learning one exactly fixes, the optimal size lies either below or above the actual optimum derived when all the choices are endogenous. This finding underscores the importance of modeling the information choice margin when making normative statements about the size of the oligopoly sector.

Next, we evaluate the *concentration of oligopoly* exercise of Section 4.1, in which we hold λ_0 fixed but vary the distribution of λ s. Figure 11 presents the relation between concentration of ownership and price informativeness for the benchmark model with endogenous information choice (black dots), as well as three cases of fixing the information choice at the benchmark values (largest oligopolist owns 64%, Panel (a)), as well as at values that are optimal at two extremes of the size distribution of the oligopolists, (largest oligopolist owns

26% (Panel (b)) or 78% (Panel (c)). For all three cases, the exogenous and endogenous information models give remarkably different predictions in terms of the relation of concentration and price informativeness. In particular, for the benchmark model, lower concentration always increases price informativeness. In contrast, models with fixed information structure exhibit a hump-shaped relation between concentration and price informativeness. Similar to the previous exercise, the two models give very different recommendations regarding the level of concentration that maximizes price informativeness. The exogenous information models optimally imply an intermediate level of concentration, while at the same time the fully endogenous model prescribes a concentration level that is at the lower bound of the potential values.

Overall, we conclude that the predictions resulting from a model with endogenous learning choices are not a simple extension of the model where information choices are fixed. The differences are not only quantitatively important but also qualitatively relevant from the perspective of policy making.

5 Additional Results and Extensions

In this section, we discuss an extension of the model in which we allow the fringe to learn from the price observation for ‘free’, i.e. without using information capacity. This extension significantly increases the complexity of the equilibrium of the model, making only numerical solutions possible. In simulations, we show that the expanded model gives qualitatively the same predictions in terms of the relationship of market structure and price informativeness. Then, we provide a discussion of a number of additional results that shed more light on the importance of oligopolists’ size and asset prices for learning process in the benchmark model.

5.1 Learning from Prices

Our benchmark model studies the effects of oligopolists' trades, and resulting price movements, on price informativeness. In the model, the more informed oligopolists are about an asset in equilibrium, and the more assets under management they have, the bigger is the price impact of their trades, which diminishes their ability to generate rents from their superior information. In this section, we extend the benchmark model to study the effects of an additional force—that price movements may additionally *directly* reveal the signal that an oligopolist received. Such an extension unfortunately reduces the tractability of the model quite substantially and precludes analytical characterization. Hence, we present numerical results from the solution to the model only.

Specifically, we allow the fringe to learn from the price realization about the shock z . For simplicity, we assume that the fringe has zero information capacity, $K_h = 0$, but that they update their beliefs about the shock after observing the price, without using any capacity.¹⁵ The details for the rest of the model remain the same.

Formally, each competitive investor demands

$$q_{hi} = \frac{y_i - rp_i}{\rho\sigma_{y_i}^2}, \quad (27)$$

where y_i and $\sigma_{y_i}^2$ are the mean and variance of the fringe investor's posterior beliefs about payoff z_i after observing the price signal: $p_i = a_i + b_i\varepsilon_i - c_i\nu_i - \sum_{j=1}^L d_{ji}\zeta_{ji}$.

¹⁵If we imposed the same constraint on reduction in entropy using the price signal as for the private signals, no member of the fringe would ever choose to use the price signal, as it is strictly dominated by the strategy of using private signals. For detailed arguments, see Kacperczyk, Nosal, and Stevens (2017).

$y_i \sim N(\bar{y}_i, \sigma_{y_i}^2)$ is generated using Bayes' rule given the price signal, that is:

$$\sigma_{y_i}^2 = \sigma_i^2 - \frac{\text{cov}^2(z_i, p_i)}{\sigma_{p_i}^2} = \sigma_i^2 - PI_i^2$$

$$\bar{y}_i = \bar{z}_i + \frac{\text{cov}(z_i, p_i)}{\sigma_{p_i}^2} (p_i - \bar{p}_i)$$

where

$$\bar{p}_i = a_i$$

$$\text{cov}(z_i, p_i) = b_i \sigma_i^2$$

$$\sigma_{p_i}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{x_i}^2 + \sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}}\right) \hat{\sigma}_{j_i}^2.$$

Note that since now the demand of the fringe depends on the posterior belief which in turn depends on the price functional, then the equilibrium solution becomes a fixed point on the coefficients of the price: given $\{a_i, b_i, c_i, d_{ji}\}$'s, fringe Bayes' rule and implied fringe demand and the optimal oligopolists demand must give market clearing consistent with those coefficients. Hence, there is no closed-form solution for the price coefficients and we must resort to numerical solutions to the model. Also, note that in the framework with learning from prices, our price informativeness measure, PI_i , enters directly the updating process as the reduction in noise of the volatility parameter. This feature provides a strong economic rationale for our informativeness measure.

The main result coming from this extension of the model is that the incentives to diversify learning are the same as in the benchmark model and the results on the effects of overall institutional ownership and concentration on PI remain the same. Figures 12 and 13 below present the case for two

oligopolists. As in the benchmark model, PI is hump-shaped in the aggregate institutional ownership, holding concentration of assets under management constant (Figure 12). Similarly, when we increase the concentration of ownership among oligopolists, holding the aggregate institutional size constant, PI monotonically decreases, just like in the benchmark model (Figure 13).

5.2 Thresholds

In this section, we derive closed-form solutions for the size thresholds at which investors find it privately optimal to specialize in their learning, and at which it would be optimal for them to specialize in their learning from a price informativeness standpoint. We relate these thresholds to changes in the monopolist's capacity and differences in volatilities between most volatile assets. We find that the optimal thresholds vary depending on what objective function a monopolist maximizes and also depending on the assumptions about her information capacity and underlying asset volatilities. These results suggest that the objective function plays an important role to establish the optimality conditions in learning behavior.

Our model shows that oligopolists try to spread their learning across assets whenever possible to mitigate their price impact. In particular, the point at which the oligopolist stops specializing in her learning might be different from the point at which she would stop specializing if her objective was to maximize aggregate PI . For analytical tractability, we consider a special case of a monopolist, of size λ_1 , who has a positive K and a fringe that is uninformed.

From our earlier discussion, we know that, for sufficiently small levels of λ_1 , the monopolist will choose to specialize in her learning, and learn only about the most volatile asset. This specialization result arises from the fact that the

monopolist's returns to learning are diminishing when she has positive size. We can characterize this threshold implicitly using the following expression:

$$\begin{aligned} & \frac{e^{2K_j}}{\left(1 + 2\frac{T}{\lambda_0}e^{2K_j}\right)^2} \left(\left(\frac{\rho}{\lambda_0}\right)^2 (\bar{x}^2 + \sigma_x^2)\sigma_1^2 + 1 + 2\frac{T}{\lambda_0} \right) \\ &= \frac{1}{\left(1 + 2\frac{T}{\lambda_0}\right)^2} \left(\left(\frac{\rho}{\lambda_0}\right)^2 (\bar{x}^2 + \sigma_x^2)\sigma_2^2 + 1 + 2\frac{T}{\lambda_0} \right) \end{aligned}$$

where σ_1 is the volatility of the most volatile asset, and σ_2 is the volatility of the second-most volatile asset. If the monopolist specializes, she learns only about asset 1. When the marginal benefit of additional learning about asset 1 when the agent specializes is equal to the marginal benefit of starting to learn about asset 2, an increase in the agent's size will make her not specialize.

Next, we characterize the size threshold between specialization and diversification from the perspective of a monopolist maximizing aggregate price informativeness. This threshold is given by comparing the derivative of price informativeness for the most volatile asset with respect to the monopolist's learning (assuming specialization) to the derivative of price informativeness of the second-most volatile asset with respect to the monopolist's learning (assuming no learning):

$$\begin{aligned} \frac{\partial PI_i}{\partial \alpha} = mP(\alpha, \lambda_1) &\equiv \frac{\sigma_i^2(((2\lambda_1^2 + 2\lambda_1\lambda_0)\rho^2\sigma_x^2 + \lambda_0^2 + \lambda_1^2)\alpha_i + (2\lambda_0\lambda_1 + 2\lambda_1^2)\rho^2\sigma_x^2 - \lambda_0^2 - \lambda_1^2)}{2(\lambda_0^2 + \lambda_1^2) \left((\alpha_i - 1)^2\sigma_i^2 + (\alpha_i - 1)\sigma_i^2 + \frac{(\lambda_0 + \lambda_1\alpha_i)^2\rho^2\sigma_i^4\sigma_x^2}{\lambda_0^2\lambda_1^2} \right)^{3/2}} \\ mP(e^{2K}, T) &= mP(1, T) \end{aligned}$$

We can analyze the sensitivity of the threshold level with respect to the monopolist's capacity and differences in volatilities between asset 1 and 2. To

provide a numerical solution to the above equations, for the analysis based on changes in capacity, we choose the following parameter values: $K_h = 0$, $\sigma_1 = 2$, $\sigma_2 = 1$, $\bar{x} = 5$, $\rho = 1.3$, and $\sigma_x = 0.41$, consistent with our calibration exercise. We plot the results in Figure 14 below.

The two size thresholds vary with the parameters as follows: First, they are both decreasing in K . As the monopolist has greater capacity to learn, specializing in learning means better ability to trade that asset, and higher price impact. Therefore, a monopolist wants to diversify her learning at smaller sizes. Similarly, the more information a monopolist can collect, the more quickly a planner might want her to spread her wealth (learning-wise) to other assets. For lower values of K , the monopolist wants to specialize later than the planner. For higher values of K , the monopolist wants to specialize sooner.

For the analysis based on differences in volatilities, we set $K_h = 0$, $K_j = 2$, $\bar{x} = 5$, $\rho = 1.3$, and $\sigma_x = 0.41$. We report the results from this analysis in Figure 15.

As is evident from the graph, the larger the gap in volatilities, the more a monopolist wants to specialize. However, the opposite is true for the planner. As the gap in volatilities grows, the monopolist would diversify sooner to increase PI . Notably, we only analyze the threshold conditions for two sets of parameter values, because increases in ρ , \bar{x} , and σ_x all increase the size threshold for a monopolist by economically small margins.

6 Concluding Remarks

The last few decades have witnessed important changes in equity ownership structure, with significant consequences for financial stability and social welfare. These trends have triggered an active discussion among financial reg-

ulators and finance industry. While several participants in the debate have raised important reasons for or against regulatory changes, the ultimate verdict is difficult to reach in the absence of a well-specified economic model. This paper aims to take one step in this direction by offering a general equilibrium model in which asymmetric information, market power, and heterogeneity of assets play an important role. We think this setting is a good way to characterize the world of equity ownership. Our goal is to rank various equilibria by comparing their implications for average price informativeness.

We show that for the level of ownership equal to the currently observed levels in the U.S. (roughly 60%), the average price efficiency is positively related to the levels of large ownership and negatively related to its concentration. This cross-sectional result is strongly supported by the data. Further, we show that the average price informativeness across assets is maximized for the values of ownership and concentration that are strictly within the range of admissible outcomes. This result suggests an interesting role for policy makers to enforce optimal structure of equity ownership.

Our model can be flexibly applied to settings with rich cross-section of assets, differences in information asymmetry across agents, and differences in market power. Hence, it can generate interesting policy implications at the aggregate and cross-sectional dimensions. It can also be a good tool to evaluate asset pricing implications in the presence of market power and information asymmetry. We leave these questions for future research. At the same time, while our research can inform the debate for the role of large owners for price informativeness and learning in the economy, it naturally abstracts from other important dimensions relevant for policy makers, such as investment costs or flows of funds in and out of the sector, as we take the size distribution as an input in our analysis.

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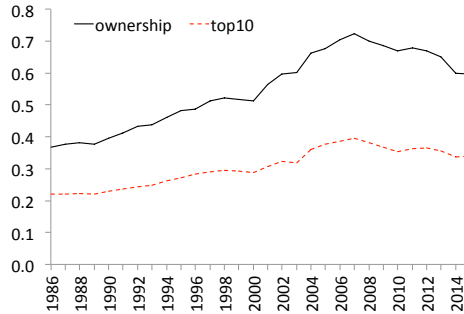


Figure 1: The black line plots the fraction of S&P 500 equities that are held by firms that have at least \$100 million in assets. The red, dashed line plots the fraction held by the 10 largest such holders of any stock.

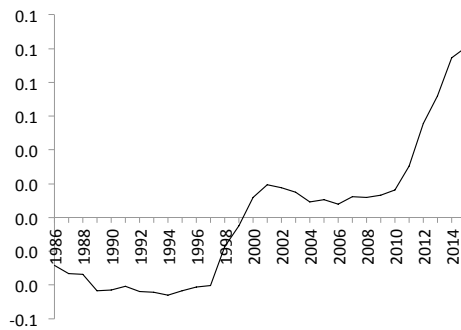


Figure 2: Adjusted Ownership Concentration. The line plots the residual of a regression of the HHI of the institutional holdings of each stock in the S&P 500 equities on the number of institutional investors in that stock.

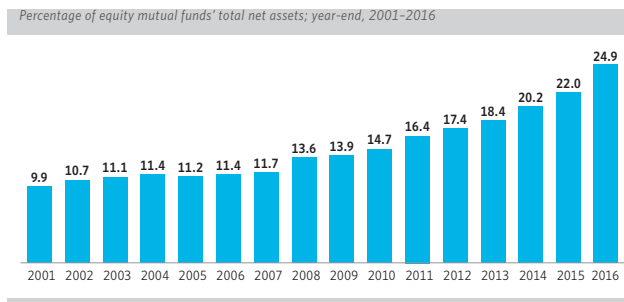


Figure 3: Passive Equity Fund Share. This figure is taken from the ICI FactBook 2017, and shows the fraction of institutional equity holdings that are invested by passive funds.

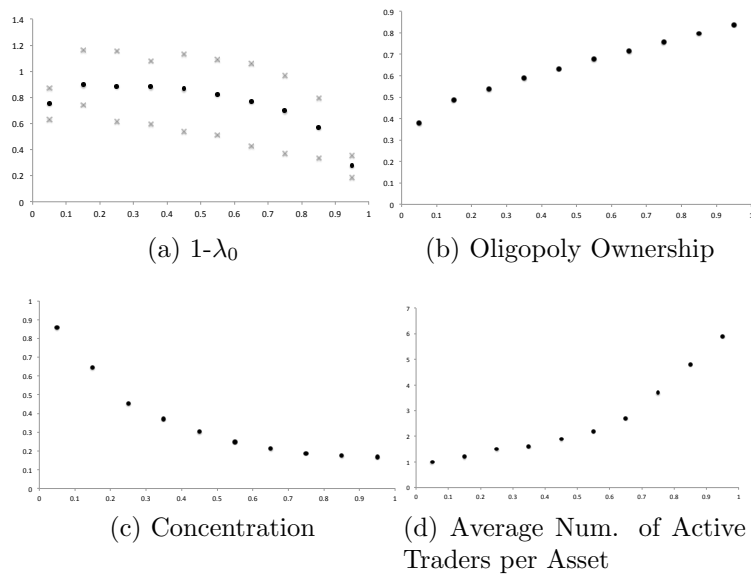


Figure 4: Response of equilibrium allocations to increasing the size of oligopoly sector ($1 - \lambda_0$).

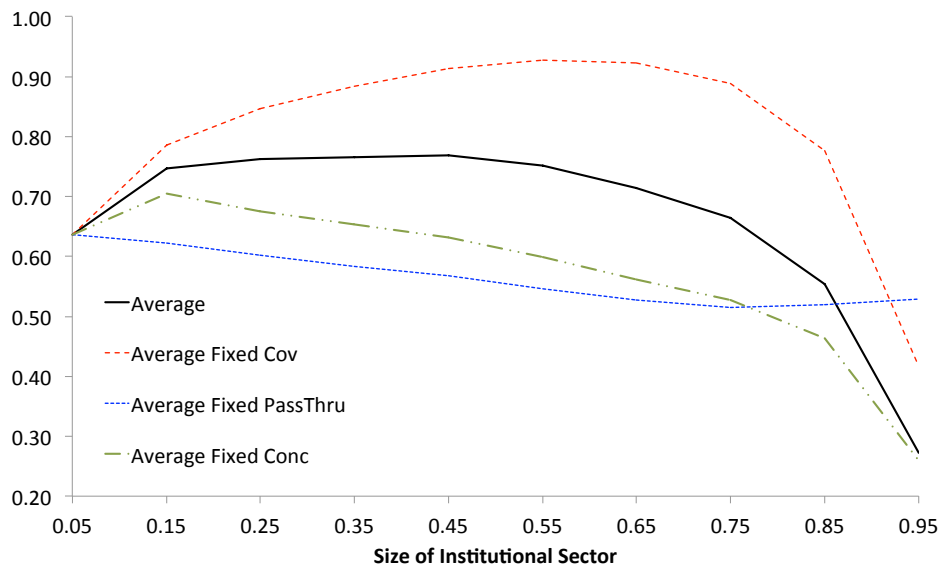


Figure 5: Price Informativeness decomposed into the relative contribution of the three channels as the size of the oligopoly sector varies from 5% to 95% of assets under management.

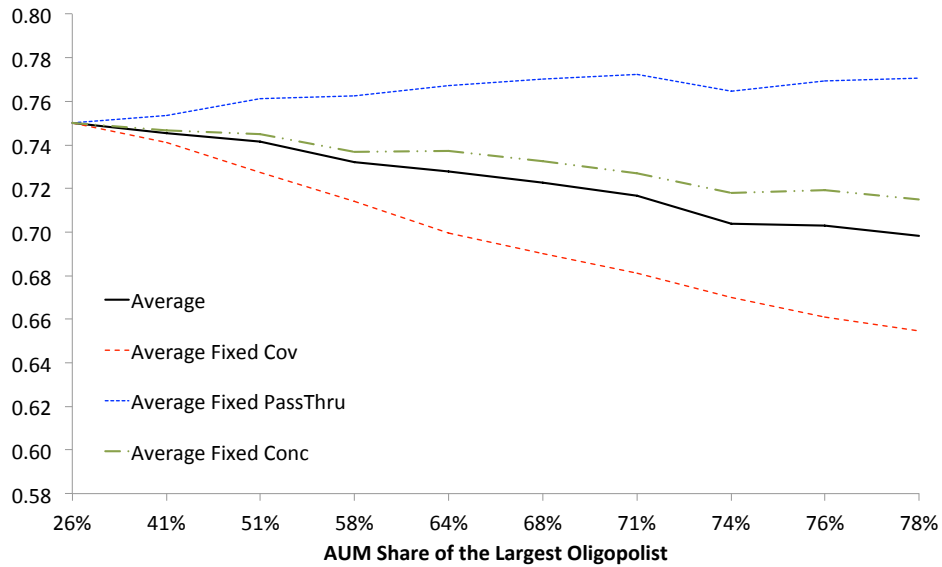


Figure 6: Price Informativeness decomposed into the relative contribution of the three channels as the size of the largest oligopolist is increased from 26% of oligopoly holdings to 78%.

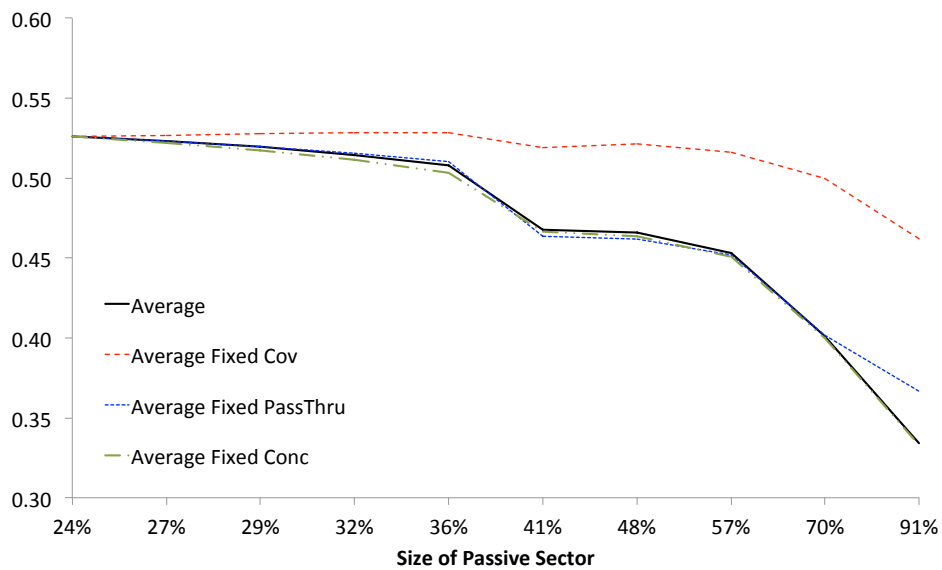


Figure 7: Price Informativeness decomposed into the relative contribution of the three channels as the assets under management of the three passive oligopolists increase from 24% to 91% of all oligopoly AUM.

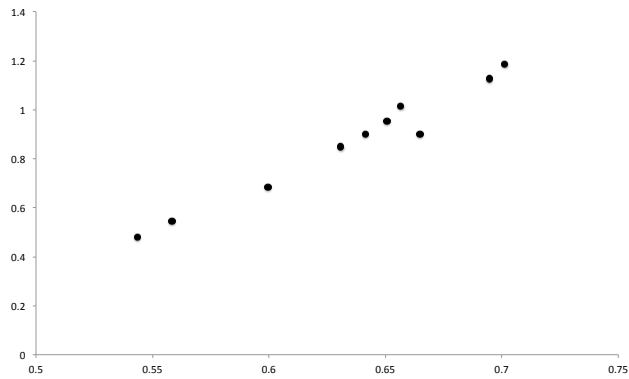


Figure 8: A cross-sectional relationship between oligopoly holdings and PI. Assets with higher levels of oligopoly holdings exhibit higher levels of PI.

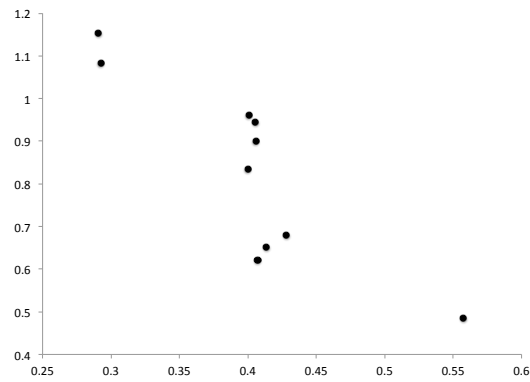


Figure 9: A cross-sectional relationship between oligopoly concentration and PI. Assets with higher levels of oligopoly concentration in holdings exhibit lower levels of PI.

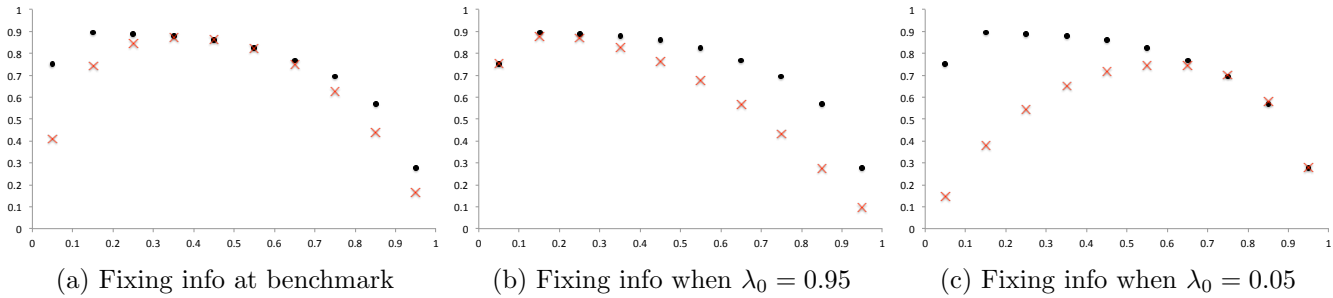


Figure 10: Aggregate price informativeness and oligopoly ownership with varying λ_0

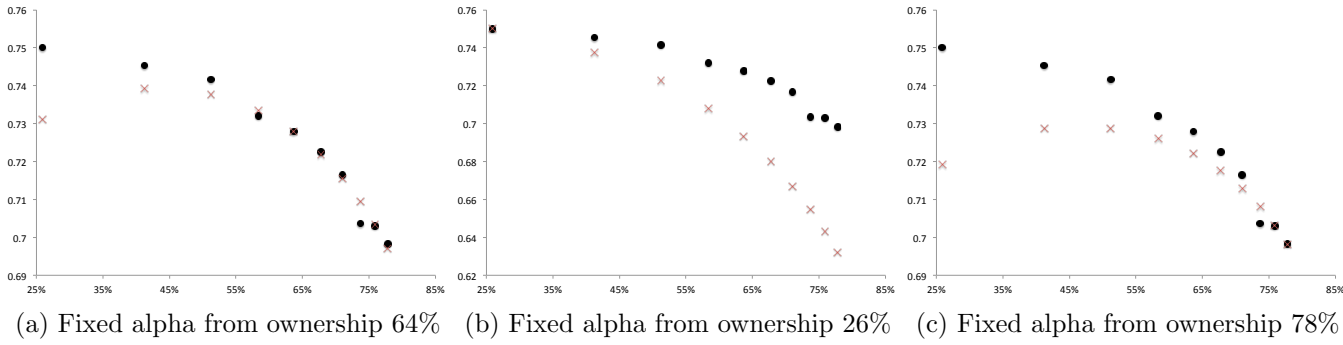


Figure 11: Aggregate price informativeness and concentration of oligopoly ownership with varying λ_l/λ_1

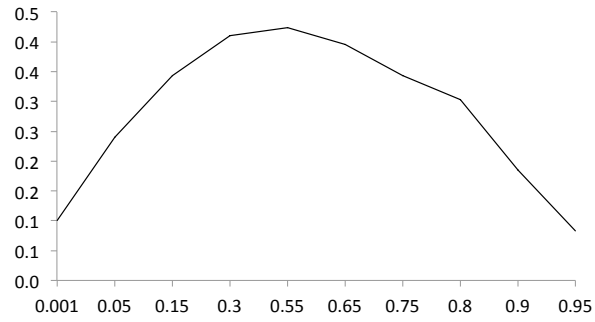


Figure 12: Price informativeness and oligopoly sector size

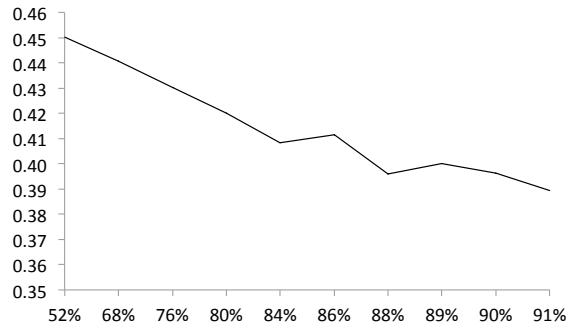


Figure 13: Price informativeness and the share of AUM of the largest oligopolist

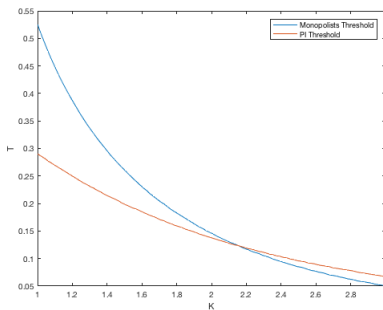


Figure 14: Optimal size thresholds for the monopolist as a function of her capacity.

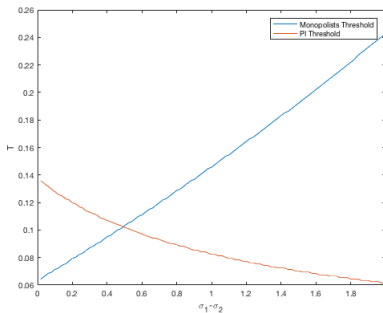


Figure 15: Optimal size thresholds for the monopolist as a function of differences in asset volatilities.

7 Appendix: Proofs

7.0.1 Derivation of Equation 2

$$\begin{aligned}
\sigma_{pi}^2 &= b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L \left(\widehat{\sigma}_{ji}^2 + \frac{\widehat{\sigma}_{ji}^4}{\sigma_i^4} \sigma_i^2 - \frac{2}{\alpha_{ji}} \text{Cov}(\epsilon_i, \delta_{ji}) - 2b_i d_{ji} \text{Cov}(\epsilon_i, \zeta_{ji}) - 2c_i d_{ji} \text{Cov}(\nu_i, \delta_{ji}) \right) \\
&\quad + \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \text{Cov}(\zeta_{ji}, \zeta_{ki}) \\
&= b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L \left(d_{ji}^2 \left(\widehat{\sigma}_{ji}^2 + \frac{\widehat{\sigma}_{ji}^4}{\sigma_i^2} - \frac{2\widehat{\sigma}_{ji}^2 \widehat{\sigma}_{ji}^2}{\sigma_i^2} \right) - 2b_i d_{ji} \left(\widehat{\sigma}_{ji}^2 - \frac{\sigma_i^2}{\alpha_{ji}} \right) \right) \\
&\quad + \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \text{Cov} \left(\delta_{ji} - \frac{1}{\alpha_{ji}} \epsilon_i, \delta_{ki} - \frac{1}{\alpha_{ki}} \epsilon_i \right) \\
&= b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}} \right) \widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \left(\frac{1}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \widehat{\sigma}_{ki}^2 - \widehat{\sigma}_{ji}^2 + \sigma_i^2 - \frac{1}{\alpha_{ki}} \widehat{\sigma}_{ki}^2 - \frac{1}{\alpha_{ji}} \widehat{\sigma}_{ji}^2 \right) \\
&= b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}} \right) \widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \left(\frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki}}{\alpha_{ki}} \widehat{\sigma}_{ki}^2 - \frac{1 + \alpha_{ji}}{\alpha_{ji}} \widehat{\sigma}_{ji}^2 \right)
\end{aligned}$$

7.0.2 Derivation of Equation 8

The objective is $U_{0h} = \frac{1}{2\rho} \sum_{i=1}^n \frac{E_{0h}(\widehat{\mu}_{hi} - rp_i)^2}{\widehat{\sigma}_{hi}^2} = \frac{1}{2\rho} \sum_{i=1}^n \frac{\widehat{R}_i^2 + \widehat{V}_{hi}}{\widehat{\sigma}_{hi}^2}$, where

$$\widehat{R}_i \equiv E_{0h}(\widehat{\mu}_{hi} - rp_i) = \bar{z} - r\bar{p}_i = \bar{z} - ra_i, \quad \widehat{V}_{hi} \equiv V_{0h}(\widehat{\mu}_{hi} - rp_i) = \text{Var}(\widehat{\mu}_{hi}) + r^2 \sigma_{pi}^2 - 2r \text{Cov}(\widehat{\mu}_{hi}, p_i).$$

$$\begin{aligned}
\text{Var}(\widehat{\mu}_{hi}) &= \sigma_i^2 - \widehat{\sigma}_{hi}^2. \quad \sigma_{pi}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}} \right) \widehat{\sigma}_{ji}^2 + \\
&\quad \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \left(\frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki}}{\alpha_{ki}} \widehat{\sigma}_{ki}^2 - \frac{1 + \alpha_{ji}}{\alpha_{ji}} \widehat{\sigma}_{ji}^2 \right).
\end{aligned}$$

Posterior beliefs and prices are conditionally independent given payoffs.

$$\begin{aligned}
Cov(\widehat{\mu}_{hi}, p_i) &= \frac{1}{\sigma_i^2} Cov(\widehat{\mu}_{hi}, z_i) Cov(z_i, p_i) \\
&= \frac{1}{\sigma_i^2} (\sigma_i^2 - \widehat{\sigma}_{hi}^2) \left(Cov(\varepsilon_i, b_i \varepsilon_i) - \sum_{j=1}^L Cov(\varepsilon_i, d_{ji} \zeta_{ji}) \right) \\
&= \frac{1}{\sigma_i^2} (\sigma_i^2 - \widehat{\sigma}_{hi}^2) \left(b_i \sigma_i^2 - \sum_{j=1}^L d_{ji} Cov\left(\varepsilon_i, \delta_{ji} - \frac{1}{\alpha_{ji}} \varepsilon_i\right) \right) \\
&= \frac{1}{\sigma_i^2} (\sigma_i^2 - \widehat{\sigma}_{hi}^2) \left(\left(b_i + \sum_{j=1}^L \frac{d_{ji}}{\alpha_{ji}} \right) \sigma_i^2 - \sum_{j=1}^L d_{ji} \widehat{\sigma}_{ji}^2 \right) \\
&= (\sigma_i^2 - \widehat{\sigma}_{hi}^2) b_i
\end{aligned}$$

Hence

$$\begin{aligned}
\widehat{V}_{hi} &= \sigma_i^2 - \widehat{\sigma}_{hi}^2 + r^2 \left(b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 + \sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}} \right) \widehat{\sigma}_{ji}^2 + \right. \\
&\quad \left. \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \left(\frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki}}{\alpha_{ki}} \widehat{\sigma}_{ki}^2 - \frac{1 + \alpha_{ji}}{\alpha_{ji}} \widehat{\sigma}_{ji}^2 \right) \right) - 2r (\sigma_i^2 - \widehat{\sigma}_{hi}^2) b_i
\end{aligned}$$

Expected utility becomes Hence $U_{0h} = \frac{1}{2\rho} \sum_{i=1}^n G_i \frac{\sigma_i^2}{\widehat{\sigma}_{hi}^2} - \frac{1}{2\rho} \sum_{i=1}^n (1 - 2rb_i)$, where

$$G_i \equiv G^{KNS} + r^2 \left(\sum_{j=1}^L d_{ji}^2 \left(1 - \frac{1}{\alpha_{ji}} \right) \widehat{\sigma}_{ji}^2 + \sum_{j=1}^L \sum_{k \neq j} 2d_{ji} d_{ki} \left(\frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki}}{\alpha_{ki}} \widehat{\sigma}_{ki}^2 - \frac{1 + \alpha_{ji}}{\alpha_{ji}} \widehat{\sigma}_{ji}^2 \right) \right).$$

Note: $G_i^{KNS} \equiv \frac{\widehat{R}_i^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2}$.

7.0.3 Derivation of Equation 12

Market clearing for each asset i is

$$\begin{aligned}
x_i &= \sum_{j=1}^L \lambda_j q_{ji} + \int_{H_i} q_{hi} dh \\
&= \sum_{j=1}^L \lambda_j q_{ji} + \int_{H_i} \frac{\hat{\mu}_{hi} - rp_i}{\rho \hat{\sigma}_{hi}^2} dh \\
&= \sum_{j=1}^L \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} \left[e^{2K_h} \int_{H_i} \hat{\mu}_{hi} dh - m_{hi} e^{2K_h} rp_i + (1 - m_{hi})(\bar{z} - rp_i) \right]
\end{aligned}$$

where H_i is the mass of competitive investors learning about asset i , of measure m_{hi} .

$$\text{Using } E[s_{hi}|z_i] = \begin{cases} \bar{z} + (1 - e^{-2K_h}) \varepsilon_i & \text{if } i = l_h \\ \bar{z} & \text{if } i \neq l_h, \end{cases}$$

$$\int_{H_i} \hat{\mu}_{hi} dh = m_{hi} [\bar{z} + (1 - e^{-2K_h}) \varepsilon_i].$$

Then market clearing becomes

$$x_i = \sum_{j=1}^L \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} [(1 - m_{hi} + e^{2K_h} m_{hi}) \bar{z} + (e^{2K_h} - 1) \varepsilon_i m_{hi} - (1 - m_{hi} + e^{2K_h} m_{hi}) rp_i]$$

Defining $\Phi_{hi} \equiv m_{hi}(e^{2K_h} - 1)$,

$$x_i = \sum_{j=1}^L \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma_i^2} [\bar{z}(1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - rp_i(1 + \Phi_i)]$$

which becomes

$$\frac{\rho \sigma_i^2}{\lambda_0} x_i = \frac{\rho \sigma_i^2}{\lambda_0} \sum_{j=1}^L \lambda_j q_{ji} + \bar{z}(1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - rp_i(1 + \Phi_i)$$

and then

$$rp_i = \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \sum_{j=1}^L \lambda_j q_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i$$

Hence,

$$\frac{dp_i(q_{ji})}{dq_{ji}} = \frac{\lambda_j \rho \sigma_i^2}{\lambda_0 r (1 + \Phi_{hi})} > 0$$

$$\text{Let } \hat{\lambda}_{ji} \equiv \frac{\lambda_j}{\lambda_0(1 + \Phi_{hi})}.$$

Then $q_{ji} = \frac{\hat{\mu}_{ji} - rp_i}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)}$, and similarly for k .

Plugging in the expression for q_{ji} :

$$rp_i = \sum_{j=1}^L \hat{\lambda}_{ji} \rho \sigma_i^2 \frac{\hat{\mu}_{ji} - rp_i}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i$$

which becomes

$$rp_i \left(1 + \sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \right) = \sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \hat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i$$

dividing through gives

$$rp_i = \frac{\sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \hat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i}{\left(1 + \sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \right)}$$

The indirect utility function $U_j = \sum_{i=1}^n q_{ji} (\hat{\mu}_{ji} - rp_i) - \frac{\rho}{2} \sum_{i=1}^n q_{ji}^2 \hat{\sigma}_{ji}^2$ becomes

$$U_j = \sum_{i=1}^n \left[q_{ji}^2 \rho \left(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2 \right) - \frac{\rho}{2} q_{ji}^2 \hat{\sigma}_{ji}^2 \right]$$

$$U_j = \sum_{i=1}^n \left[\rho q_{ji}^2 \left(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2 - \frac{1}{2} \hat{\sigma}_{ji}^2 \right) \right]$$

$$U_j = \sum_{i=1}^n \left\{ \frac{(\hat{\mu}_{ji} - rp_i)^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)^2} \left(\frac{1}{2} \hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2 \right) \right\}$$

$$U_j = \frac{1}{2\rho} \sum_{i=1}^n \left\{ (\hat{\mu}_{ji} - rp_i)^2 \left[\frac{\hat{\sigma}_{ji}^2 + 2\hat{\lambda}_{ji} \sigma_i^2}{(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)^2} \right] \right\}.$$

More detailed expression for U : We can rewrite $E_{0j}(\hat{\mu}_{ji} - rp_i)^2$ as $\hat{R}_i^2 + \hat{V}_{ji}$, where \hat{R}_i and \hat{V}_{ji} denote the ex-ante mean and variance of expected excess returns, which

$$\text{means: } \hat{R}_i \equiv E_{0j}(\hat{\mu}_{ji} - rp_i) = \frac{\frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})}}{\left(1 + \sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \right)} \bar{x} \text{ Define}$$

$$M_{ji} \equiv \frac{\hat{\lambda}_{ji} \sigma_i^2}{(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \quad N_i \equiv \frac{1}{1 + \sum_{j=1}^L M_{ji}}$$

$$\hat{V}_{ji} \equiv V_{0j}(\hat{\mu}_{ji} - rp_i)$$

$$= V_{0j} \left(\hat{\mu}_{ji} - N_i \sum_{k=1}^n M_{ki} \hat{\mu}_{ki} - N_i \bar{z} - N_i \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i + N_i \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i \right)$$

$$= V_{0j} \left(N_i (\hat{\mu}_{ji} + \sum_{k=1}^n M_{ki} (\hat{\mu}_{ji} - \hat{\mu}_{ki})) - N_i \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i + N_i \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i \right)$$

$$= N_i^2 V_{0j} \left(\hat{\mu}_{ji} + \sum_{k \neq j}^n M_{ki} (\hat{\mu}_{ji} - \hat{\mu}_{ki}) - \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i + \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i \right)$$

$$= N_i^2 \left(1 + \sum_{k \neq j}^n M_{ki} \right)^2 (\sigma_i^2 - \hat{\sigma}_{ji}^2) + \left(\frac{N_i \Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \sigma_i^2 + \left(\frac{N_i \rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 \sigma_{x_i}^2 + \sum_{k \neq j} M_{ki}^2 (\sigma_i^2 - \hat{\sigma}_{ki}^2) - 2 \left(\frac{N_i^2 \Phi_{hi} (1 + \sum_{k \neq j}^n M_{ki})}{1 + \Phi_{hi}} \right) (\sigma_i^2 - \hat{\sigma}_{ji}^2) + 2 \sum_{k \neq j} N_i^2 \frac{\Phi_{hi}}{1 + \Phi_{hi}} M_{ki} (\sigma_i^2 - \hat{\sigma}_{ki}^2)$$

$$U_{0j} = \frac{1}{2\rho} \sum_i \frac{N_i^2 (\hat{\sigma}_{ji}^2 + 2\hat{\lambda}_{ji} \sigma_i^2)}{(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)^2} \left[\left(\frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 (\bar{x}^2 + \sigma_{ix}^2) + \left(\frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \sigma_i^2 + \sum_{k \neq j} M_{ki}^2 (\sigma_i^2 - \hat{\sigma}_{ki}^2) \right. \\ \left. + \left(\frac{(1 + \Phi_{hi}) \left(1 + \sum_{k \neq j}^n M_{ki} \right) - 2\Phi_{hi}}{1 + \Phi_{hi}} \right) (\sigma_i^2 - \hat{\sigma}_{ji}^2) \left(1 + \sum_{k \neq j}^n M_{ki} \right) + 2 \sum_{k \neq j} \frac{\Phi_{hi}}{1 + \Phi_{hi}} M_{ki} (\sigma_i^2 - \hat{\sigma}_{ki}^2) \right]$$

7.0.4 Derivation of Equations 21

The market clearing condition is

$$rp_i = \frac{\sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)} \hat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i}{\left(1 + \sum_{j=1}^L \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho(\hat{\sigma}_{ji}^2 + \hat{\lambda}_{ji} \sigma_i^2)}\right)} \quad (28)$$

From here we identify the price coefficients as a function of the monopolist learning and the competitive fringe learning. Now, conditionally on z_i , we have

$$\hat{\mu}_{ji} = s_{ji}$$

and s_{ji} is normally distributed with mean $\bar{z} + (1 - \frac{1}{\alpha_{ji}}) \varepsilon_i$ and variance $(1 - \frac{1}{\alpha_{ji}}) \frac{1}{\alpha_{ji}} \sigma_i^2$. What we want is to express the posterior mean in terms of delta as in $z_i = s_i + \delta_i$. Given that,

$$\delta_{ji} = z_i - s_{ji} = -\frac{1}{\alpha_{ji}} \varepsilon_i + \text{noise}$$

$$rp_i = N_i \sum_{j=1}^L M_{ji} \left(\bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right) \varepsilon_i - \zeta_{ji} \right) + N_i \left[\bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} x_i \right] \quad (29)$$

$$\begin{aligned} rp_i &= \bar{z} - \bar{x} \frac{N_i \rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} + \varepsilon_i N_i \left(\sum_{j=1}^L \frac{M_{ji}(\alpha_{ji} - 1)}{\alpha_{ji}} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right) \\ &\quad - \frac{N_i \rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \nu_i - N_i \sum_{j=1}^L M_{ji} \zeta_{ji} \end{aligned}$$

7.0.5 Derivation of Proposition 1

Proof. In order to apply Kakutani's Fixed Point Theorem, we need to define a few terms. Agents select α_i . First, define A_i ($\{\alpha_{-j}\}$) to be the best response correspondence for oligopolist j . Next define $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_L\}$, and let \aleph define the set of all possible α . Then the best response correspondence can be defined as $A : \aleph \Rightarrow \aleph$ such that for all $\alpha \in \aleph$, we have that $A(\alpha) = [A_j(\alpha_{-j})]_{j \in L}$. This best response function takes into account the associated demand schedule for every oligopolist, as well as the learning and demand decisions for the fringe. Now we need to check whether there is a fixed point to A .

- \aleph is compact and convex. Each α_j must satisfy the capacity constraint. Therefore each α_j is convex, closed, and bounded, and therefore compact. Therefore \aleph is as well.
- A is non-empty. This is trivially true if an interior solution exists. If an interior solution does not exist, then the solutions are corners, so A is always non-empty.
- A has a closed graph. The first order conditions of the oligopolist are continuous, so this is trivial. (see above).
- A is convex-valued. A is convex iff A_i are all convex. The oligopolist's objective function is weakly more concave than the fringe's due to size. We show here that the second derivative is negative.

We just need to show that the utility function is concave in α . To show this, we can write the utility function down:

$$\begin{aligned}
U_{0j} = & \frac{1}{2\rho} \sum_i \frac{N_i^2 (\hat{\sigma}_{ji}^2 + 2\hat{\lambda}_i \sigma_i^2)}{(\hat{\sigma}_{ji}^2 + \hat{\lambda} \sigma_i^2)^2} \left[\left(\frac{\rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} \right)^2 (\bar{x}^2 + \sigma_{ix}^2) + \left(\frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \sigma_i^2 + \sum_{k \neq j} M_{ki}^2 (\sigma_i^2 - \hat{\sigma}_{ki}^2) \right. \\
& \left. + \left(\frac{(1 + \Phi_{hi}) \left(1 + \sum_{k \neq j}^n M_{ki} \right) - 2\Phi_{hi}}{1 + \Phi_{hi}} \right) (\sigma_i^2 - \hat{\sigma}_{ji}^2) \left(1 + \sum_{k \neq j}^n M_{ki} \right) + 2 \sum_{k \neq j} \frac{\Phi_{hi}}{1 + \Phi_{hi}} M_{ki} (\sigma_i^2 - \hat{\sigma}_{ki}^2) \right]
\end{aligned}$$

And we can rewrite this as:

$$U_{0j} = \frac{N_i (1 + \hat{\lambda}_{ji} \alpha_{ji})}{(1 + \hat{\lambda}_{ji} \alpha_{ji})^2} \frac{\alpha_{ji}}{\sigma_i^2} \left[X + Y \left(\frac{\alpha_{ji} - 1}{\alpha_{ji}} \right) \right]$$

where X and Y are positive. This in turn can be rewritten as:

$$U_{0j} = \frac{\frac{1}{1 + \sum M_{ji}} (1 + \hat{\lambda}_{ji} \alpha_{ji})}{(1 + \hat{\lambda}_{ji} \alpha_{ji})^2} \frac{\alpha_{ji}}{\sigma_i^2} \left[X + Y \left(\frac{\alpha_{ji} - 1}{\alpha_{ji}} \right) \right]$$

And rewriting yet again:

$$U_{0j} = \frac{1}{Z + \frac{\hat{\lambda}_{ji} \alpha_{ji}}{1 + \hat{\lambda}_{ji} \alpha_{ji}}} \frac{1 + 2\hat{\lambda}_{ji} \alpha_{ji}}{(1 + \hat{\lambda}_{ji} \alpha_{ji})^2} \hat{\lambda}_{ji} \alpha_{ji} \left[X + Y \left(\frac{\alpha_{ji} - 1}{\alpha_{ji}} \right) \right]$$

Where $Z > 1$. To finally get to:

$$U_{0j} = X \frac{\hat{\lambda}_{ji} \alpha_{ji} + 2\hat{\lambda}_{ji}^2 \alpha_{ji}^2}{Z + \hat{\lambda}_{ji} \alpha_{ji} (1 + 2Z) + \hat{\lambda}_{ji}^2 \alpha_{ji}^2 (1 + Z)} + Y \frac{\hat{\lambda}_{ji} \alpha_{ji}^2 + 2\hat{\lambda}_{ji}^2 \alpha_{ji}^3 - \hat{\lambda}_{ji} \alpha_{ji} - 2\hat{\lambda}_{ji}^2 \alpha_{ji}^2}{Z \alpha_{ji} + \hat{\lambda}_{ji} \alpha_{ji}^2 (1 + 2Z) + \hat{\lambda}_{ji}^2 \alpha_{ji}^3 (1 + Z)}$$

Both terms are concave in α , and the sum of concave functions is concave, so the proof is completed. \square

7.0.6 Derivation of Proposition 3

Proof.

$$\begin{aligned}
PI'_i &= \frac{b'_i \sigma}{\sqrt{b_i^2 + c_i^2 \frac{\sigma_x^2}{\sigma_i^2} + \sum_j d_j^2 \frac{\alpha_{ji}^{-1}}{\alpha_{ji}^2}}} \\
&+ \frac{b_i \sigma_i}{\left(\sqrt{b_i^2 + c_i^2 \frac{\sigma_x^2}{\sigma_i^2} + \sum_j d_j^2 \frac{\alpha_{ji}^{-1}}{\alpha_{ji}^2}}\right)^3} \left(2b_i b'_i + \frac{2c_i c'_i \sigma_x^2}{\sigma_i^2} - \frac{d_{ji}^2 (\alpha_{ji} - 2)}{\alpha_{ji}^3} + \frac{2d_{ji} d'_{ji} (\alpha_{ji} - 1)}{\alpha_{ji}^2} \right) \\
PI''_i &\propto \underbrace{\frac{b''_i}{<0}}_{<0} \underbrace{\left(b_i^2 + c_i^2 \frac{\sigma_x^2}{\sigma_i^2} + \sum_j d_j^2 \frac{\alpha_{ji}^{-1}}{\alpha_{ji}^2} \right)}_{>0} \underbrace{\left(2b_i b'_i + \frac{2c_i c'_i \sigma_x^2}{\sigma_i^2} - \frac{d_{ji}^2 (\alpha_{ji} - 2)}{\alpha_{ji}^3} + \frac{2d_{ji} d'_{ji} (\alpha_{ji} - 1)}{\alpha_{ji}^2} \right)}_{>0} \underbrace{\left(-2b'_i - \frac{3b_i}{\left(b_i^2 + c_i^2 \frac{\sigma_x^2}{\sigma_i^2} + \sum_j d_j^2 \frac{\alpha_{ji}^{-1}}{\alpha_{ji}^2} \right)} \right)}_{<0} \\
&+ \frac{b_i \left(2b_i b'_i + \frac{2c_i c'_i \sigma_x^2}{\sigma_i^2} - \frac{d_{ji}^2 (\alpha_{ji} - 2)}{\alpha_{ji}^3} + \frac{2d_{ji} d'_{ji} (\alpha_{ji} - 1)}{\alpha_{ji}^2} \right)'}{\underbrace{\left(2b_i b'_i + \frac{2c_i c'_i \sigma_x^2}{\sigma_i^2} - \frac{d_{ji}^2 (\alpha_{ji} - 2)}{\alpha_{ji}^3} + \frac{2d_{ji} d'_{ji} (\alpha_{ji} - 1)}{\alpha_{ji}^2} \right)}_{<0}} \\
&PI''_i < 0
\end{aligned}$$

\square

7.0.7 Derivation of Proposition 2

Proof. The sign of $\frac{\partial^2 U}{\partial \alpha_{ji}^2}$ was shown in the proof of Proposition 1.

\square