When is the Price of Dispersion Risk Positive?

Alexander David and Amel Farhat*

November 2018

Abstract

This paper documents that the price of analysts' dispersion risk in the cross-section of stock returns changes over time, and in particular turns positive in periods of high analyst dispersion. Our result holds using 100 test portfolios that are double-sorted on their betas and their coefficients on aggregate dispersion, as well as numerous test portfolios that have been used in the literature. We construct a general equilibrium model in the spirit of Merton's ICAPM, in which analysts of different types have heterogeneous beliefs and provide different forecasts of a macroeconomic factor (aggregate earnings growth). The consumer does not trust either analyst fully, and dynamically adjusts the weight given to each analyst, given the history of their past forecast performance. In equilibrium, each asset's risk premium depends on its exposure to three factors: (i) the market portfolio, (ii) the macroeconomic factor, and, (iii) a "flight-to-safety" factor, which is the variance of the market portfolio. The first term increases with dispersion, while the third term declines. The latter decline occurs because consumers shift into assets with lower cash flow betas during periods of high dispersion. The model provides a testable implication, that the changing sign of the price of risk is due to the flight-to-safety during periods of high dispersion. We find strong support for such a flight-to-safety in the data.

*Both authors are from the University of Calgary, Haskayne School of Business. We thank participants at the 2017 University of Alberta University of Calgary conference, the University of Manitoba, and the 2018 Essex Finance Center Conference for helpful comments. Address: University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4, Canada; E. Mail (David): adavid@ucalgary.ca. E. Mail(Farhat): afarhat@ucalgary.ca.

Introduction

Asset pricing implications of dispersion in investors' beliefs is among the most controversial issues in finance. Alternative theoretical and empirical frameworks have led to different signs on how dispersion affects stock returns.[1] The contribution of this paper is to show that the price of dispersion risk is time-varying in the cross-section, and potentially provides a resolution to the different views on the sign. Empirically, we find that in the cross-section of stock returns, aggregate dispersion is negatively priced in low dispersion months and positively priced in high dispersion months.[2] Our measure of aggregate dispersion is the value-weighted average of the standard deviation of analyst forecasts of the earning-per-share long-term growth rate at the firm level. This "bottom-up" measure has been used in a number of recent papers (e.g. Yu (2011) and Hong and Sraer (2016)) and incorporates the views of a vastly greater number of analysts than "top-down" measures, which are comprised of forecasts of the aggregate earnings growth rate.

We begin our empirical study by replicating the results of Yu (2011), who documents a negative relation between aggregate dispersion and market returns in the time series in the sample period from 1981 to 2005. Extending the sample to 2016, however, we find that the coefficient of aggregate dispersion is insignificant. Restricting the sample to 2006-2016, we see that the coefficient of aggregate dispersion is strongly positive. Looking at the time series of aggregate dispersion displayed in Figure 1, we see that the sample from 1981 to 2005 mainly had low dispersion months, while the sample from 2006 to 2016 mainly consisted of high dispersion months. This finding motivates our main finding, that the sign of the price of dispersion risk depends of the level of dispersion.

[1] On one hand, Varian (1985), Varian et al. (1989), Abel (1989), Qu, Starks, and Yan (2003), Doukas, Kim, and Pantzalis (2004), Anderson, Ghysels, and Juergens (2005), David (2008), Anderson, Ghysels, and Juergens (2009) and Carlin, Longstaff, and Matoba (2014) find a positive relation. On the other hand, Miller (1977) theorizes that the divergence of investor's beliefs in the presence of short sale constraints leads to overvaluation and lower returns. In support of this hypothesis, Diether, Malloy, and Scherbina (2002), Chen, Hong, and Stein (2002) , Park (2005), Sadka and Scherbina (2007), and Yu (2011) find a negative relation between dispersion and excess returns.

[2] We define low (high) dispersion months as those when one-year lagged aggregate dispersion is lower (higher) then the average aggregate dispersion at minus (plus) one standard deviation.

To study the effects of aggregate dispersion on the cross-section of stock returns, we start by estimating the market and aggregate dispersion pre-ranking risk loadings, β and δ, using one-year rolling regressions at the monthly frequency of individual stock returns on excess market returns as well as one-month lagged aggregate dispersion. These estimated risk loadings allow us to analyze the price of dispersion risk using two different methods described next.

First, we form ten portfolios using, δ, the estimated aggregate dispersion loadings. Figure 2 highlights the role played by aggregate dispersion in asset's returns using the full sample, low, medium and high dispersion months. Using the full sample, we find a U-shaped relation between dispersion and portfolio returns. Limiting our sample to low (high) dispersion months, we see a clear negative (positive) relation between dispersion in investor's beliefs and returns. During low (high) dispersion months, a portfolio of stocks in the highest decile of dispersion loading underperforms (outperforms) a portfolio of stocks in the lowest decile of dispersion loading. In medium dispersion months, the relationship is unclear.

Second, we control for the CAPM by forming 100 β-δ portfolios. Every month, we sort stocks into 10 β-deciles using the pre-ranking β. Then, for every β-decile, we sort stocks based on the pre-ranking δ into 10-deciles. Using a Fama and MacBeth (1973) style two-stage regression on the 100 β-δ portfolios, we show that the price of dispersion risk in low dispersion months is negative (-6.067, t-statistic $= -4.52$) and positive in high dispersion months (24.92, t-statistic $= 4.67$). The price of dispersion risk is statistically insignificant using the full sample period or medium-dispersion months. We next show that our finding is robust to the choice of test portfolios, as it holds using a variety of portfolios obtained from the website of Ken French.

To better understand the changing sign of the price of dispersion risk, we construct a production economy N-asset general equilibrium model in the spirit of Merton's ICAPM, in which analysts of different types have heterogeneous beliefs and provide different forecasts of a macroeconomic factor (aggregate earnings growth). The consumer does not trust either analyst fully, and dynamically adjusts the weight given to each analyst, given the history of their past forecast performance. The beliefs of each analyst and the weight the consumer assigns to each analyst's forecast, become state variables in the ICAPM framework whose shifts the consumer hedges against. We derive an equilibrium cross-sectional pricing equation for our model. There are three terms in each asset's risk premium, which we discuss next.

The first term, is the risk premium for covarying with the market portfolio, as in the CAPM; however, in the model, the exposure to the market is measured by the relative cash-flow beta of each asset, rather than the return beta. The second term, measures the risk premium for bearing the risk in the aggregate macroeconomic factor as well as any undiversified idiosyncratic risk. The last term, is the risk premium with respect to a "flight-to-safety" factor. This factor depends on the variance of the market portfolio and has a negative sign. The negative sign on the term implies that assets that provide higher returns in periods when variance is high, obtain a lower risk premium. The negative risk premium is consistent with the variance risk premium literature. The interesting aspect of this term is that it is time-varying and depends on the consumer's portfolio choices. As in Merton's ICAPM and Cox, Ingersoll, and Ross (1985), the representative consumer attempts to hedge against shifts in the state variables of the economy in addition to her diversification motive. The state variables in the economy are the beliefs of each analyst as well as well as a weight that the consumer places on the forecast of each analyst. The risk premia for shifts in these state variables, written as a function of the portfolio choices, collapses to the variance of the market portfolio. When analysts' dispersion becomes very high, consumers respond by allocating their investments to low cash flow beta assets, and this term declines. It is also interesting that the exposure to the flight-to-safety factor is again measured by the relative cash flow beta of each asset.

We estimate our model from 1971-2001 and out-of-sample beliefs of analysts are formulated until 2017. Using an unobserved regime shifting structure in the aggregate fundamental, which we take as aggregate S&P 500 operating earnings growth, we find two sets of parameters (one for each type of analyst), that maximizes the sum of the likelihoods of each agent type observing the fundamentals, and using the conditional expectations of each type of agent, we formulate the forecast of aggregate earnings growth, we formulate the 1-year ahead, aggregate earnings dispersion among the two types. The objective function also puts weight on matching the model dispersion to the dispersion of earnings growth from the Philadelphia Fed Survey of Professional Forecasters. The model provides a positive correlation between the dispersion in the analysts forecasts, and the price of dispersion risk and is higher in periods of high aggregate dispersion, as in the data.

A large literature in asset pricing has examined the risk-return trade-off. The Capital Asset Pricing Model of the Sharpe (1964), Lintner (1965) and Black (1972) states that the security excess return is proportional to the sensitivity of its return to the market return, denoted by

CAPM beta. Jensen, Black, and Scholes (1972) point out that ”high beta” assets earn lower returns on average. More recently, Frazzini and Pedersen (2014) document that a portfolio that holds low-beta assets and that shorts high-beta earns a positive average return. Hong and Sraer (2016) relax the CAPM homogeneous expectation assumption and show that when aggregate disagreement is low, expected return increases with beta due to risk-sharing confirming the CAPM prediction. But when it is large, expected return can decrease with beta through the ”short-sale constraint” channel. Our paper complements this finding by having aggregate dispersion as an explicitly priced factor with a generally negative price of risk, which turns positive in periods of high aggregate dispersion.

On the theoretical front, our paper contributes the growing literature on general equilibrium models with heterogeneous beliefs. starting with the seminal papers of Detemple and Murthy (1994) and Basak (2000). David (2008) studies the implications of heterogeneous beliefs for risk premium on the market index within this framework, while Dumas, Kurshev, and Uppal (2009) study the implications for “excess volatility” of the market index. Gallmeyer and Hollifield (2008) study the implications for asset prices in such a model with an added short-sales constraint, while Burashi, Trojani, and Vedolin (2014) extend the framework to multiple stocks in an exchange economy setting with multiple trees. Baker, Hollifield, and Osambela (2016) study investment in a single production technology where the representative agent has Epstein-Zin preferences. This paper is able to obtain a tractable equilibrium characterization by working with the Cox-Ingersoll-Ross (1985) framework with multiple firms, each having access to a linear production technology. In particular the scale of each firm is endogenous and we explicitly study the riskiness of the market portfolio with changing dispersion of beliefs. In addition, ours is the first model that makes a distinction between the beliefs of analysts, who we assume cannot trade in stocks, and the representative consumer, who does not directly follow firms’ fundamental, but dynamically weights the forecasts of the different analysts in the economy.

The rest of the paper is organized as follows. In Section 1, we describe the data used, define the aggregate dispersion measure and construct the β-δ portfolios. In Section 2, we present the results of the two-pass regressions. In Section 3, we provide a theoretical model that prices dispersion risk in the cross-section, and show that its pricing implications are in line with our empirical findings. Section 4 concludes. The results of the two-pass regression analysis for a wide range of test portfolios, and proofs of Propositions are in two appendices.

1 Data and Variables

The data in this study are the intersection of the Institutional Brokers Estimate System (I/B/E/S) and the Center of Research in Securities Prices database (CRSP) between December 1981 and September 2016. From I/B/E/S database, we obtain the analyst forecasts data. From CRSP, we obtain the monthly returns and stock characteristics (volume, shares outstanding, price...). We include all stocks listed in New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and The Nasdaq Stock Market with share code 10 or 11 (common stocks). In order to ensure that the illiquid stocks are not considered in our analysis, we exclude penny stocks (price $< \$5$) and micro-caps (stocks in the bottom 2 deciles of the monthly size distribution). A firm is kept in our sample if it has more then 12 consecutive monthly observations. We end up with a sample of 6428 firms.

1.1 Aggregate Dispersion

We start by displaying the main variable of interest, the aggregate dispersion. Similar to Yu (2011), we measure the aggregate dispersion as the value-weighted average of the stock-level dispersion, where stock-level dispersion is the standard deviation of analyst forecasts of the earning-per-share long-term growth rate (EPS LTG). Figure 1 plots the time series of the aggregate dispersion measure as well as the low and high dispersion thresholds shown by the two horizontal lines. We define low (high) dispersion month (t) as the month where aggregate dispersion is lower (higher) then the average aggregate dispersion at $(t-1)$ minus (plus) one standard deviation. Out of 418 months, 87 months are considered high dispersion months and only 55 months are considered low dispersion months. The remaining 276 months are considered medium dispersion months. Low dispersion months are concentrated prior to 2000. However, high dispersion months happen mostly after 2000. As can be seen in Figure 1, high levels of dispersion occur both during recession and growth periods. Table 1 presents the summary statistics of the aggregate dispersion measure. On average during the full sample period, the aggregate dispersion equals 3.51. The average ranges from 2.8 in low dispersion months to 4.48 in high dispersion months. Our main empirical findings also hold using the β-weighted measure of aggregate dispersion used in Hong and Sraer (2016).

We begin our study by replicating and expanding Yu (2011) results. Yu (2011) documents that for the market portfolio, market dispersion is negatively related to market returns at different horizons. In Table 2, we regress the excess market return from month t to $t + h$ ($h = 1, 6, 12, 24, 36$ months) on the aggregate dispersion at month t. In Panel A, we use Yu (2011)'s sample period covering observations from December 1981 to December 2005. Figure 1 shows that this period of time is composed mainly by low aggregate dispersion months. Results are consistent with Yu (2011)'s finding. The coefficient of aggregate dispersion is negative and significant for all return horizons and the explanatory power of aggregate dispersion is more pronounced for higher horizons. In Panel B, we repeat the same exercise using our full sample period including observations from December 1981 to September 2016, the coefficient of aggregate dispersion is negative but insignificant at all horizons. The effect of aggregate dispersion on market returns is less pronounced using the full sample compared to Yu (2011)'s sample. For example, at the two-year horizon, the coefficient of dispersion equals -9.27 (t-statistic = -1.17) using the full sample and equals -29.49 (t-statistic = -3.66) using data till December 2005. In Panel C, we only include the recent 10 years of data to run our regressions. The last 10 years of our monthly observations are mainly considered high dispersion months as shown in Figure 1. Results show that aggregate dispersion is positively related to market returns. The coefficient of dispersion is significant only at the 3-year return horizon with a t-statistic of 2.74. These results motivate us to look more carefully at the cross-sectional regression of stocks return on aggregate dispersion and examine the relation of aggregate dispersion to assets returns at low, medium and high dispersion months.

In this part, we report the contemporaneous correlation between aggregate dispersion, stock market excess returns and the standard pricing factors SMB, HML and UMD in Table 3. We note that aggregate dispersion is weekly correlated with other variables using the full sample, which may imply that, if the aggregate dispersion affects stock returns, then the reason may be different from those for other factors. Using low dispersion months or medium dispersion months, aggregate dispersion is positively correlated with the market excess return and the size factor and negatively correlated with the value factor and the momentum factor. And limiting our sample to the high dispersion months, aggregate dispersion is positively correlated with the market excess return, the size factor and the momentum factor and negatively correlated with the value factor. In the subsequent analysis, we show that aggregate dispersion remains significant after controlling for the standard pricing factors in various setting.

1.2 Pre-ranking Risk Loadings

To obtain time-varying risk loadings, we run rolling monthly time-series regressions of stock returns on excess market returns as well as one-month lagged aggregate dispersion.

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \delta_{i,t} \times Agg\_disp_{t-1} + \epsilon_{i,t} \tag{1}$$

For each security, every month, we estimate the pre-ranking β and δ as the slope coefficients from time series regression of individual securities on excess market return as well as lagged aggregate dispersion using the previous 12 months of monthly observations. The estimated risk loadings $\beta_{i,t}$ and $\delta_{i,t}$ allow us to analyze the cross-sectional characteristics of these parameters in two different ways. First, we form portfolios based on δ, the dispersion risk exposure, and we look at the average returns for these classes of stocks. Second, we use cross-sectional two-stage regressions to examine the role of aggregate dispersion on stock returns.

1.3 Univariate Portfolio Sorts

We use the estimated aggregate dispersion exposure δ, obtained from regression 1, to test if aggregate dispersion is priced in the cross-section of stocks returns using four sample periods: full sample, low, medium and high dispersion months. To do so, we assign stocks into portfolios using the aggregate dispersion loadings. Each calendar month, we sort stocks into 10 δ-deciles with the first decile having the lowest pre-ranking δ and the tenth decile having the highest pre-ranking δ. We calculate the monthly portfolio return, R_t^P, as the value weighted average of the returns of all stocks in the P^{th} δ-sorted portfolio. Figure 2 highlights the role played by aggregate dispersion in asset's returns. Panel A plots the aggregate portfolio returns using the full sample. We notice a U-shaped relation between dispersion and returns. Panel B uses only the low dispersion months, we clearly detect the negative relation between dispersion in investor's beliefs and returns. Panel C plots the sorting results using medium dispersion months, the graph does not show a clear relationship between dispersion and returns. Panel D reports the sorting results for high dispersion months, we now observe a strong positive relation between dispersion and portfolio returns.

2 The Price of Dispersion Risk in the Cross-Section

2.1 Two-stage Regression

In this section, we turn our attention to analyze whether the aggregate dispersion is a priced risk factor using our value weighted β-δ portfolios. To construct the β-δ portfolios, we use the estimated parameters, β and δ, obtained from equation (1). Every month, we sort stock into 10 β-deciles using the pre-ranking β. For every β-decile, we sort stocks based on the pre-ranking δ. We obtain 100 portfolios formed monthly. We calculate the value weighted monthly returns R_t^P on these 100 portfolios. We report the summary statistics in Table 4 for monthly β-δ portfolio return and pre-ranking risk loadings. The numbers reported represent time series averages of the monthly cross-sectional mean, standard deviation and the 10^{th} to 90^{th} percentile.

We use four regression models to estimate portfolio post-ranking β and post-ranking δ. As suggested by (Fama and MacBeth 1973), (Jensen, Black, and Scholes 1972) and (Cochrane 2009), we form portfolio to estimate the post-ranking risk loadings. Model one includes the market excess returns. Model two adds the one-month lagged aggregate dispersion measure. Model three includes the (Fama and French 1993) factors and the momentum factor ((Jegadeesh and Titman 1993)). And Model four, displayed in equation 2, includes all the variables listed above. We obtain monthly returns on the factors (R_t^m, SMB, HML, and UMD) from Ken Frenchs data web site. Every calendar month, post-ranking risk loadings are estimated using the previous one-year of monthly returns.

$$\begin{aligned} R_t^P = \alpha_{P,t} + \beta_{P,t}^{MKT} \times R_t^m + \delta_{P,t} \times Agg\_disp_{t-1} \\ + \beta_{P,t}^{SMB} \times SMB_t + \beta_{P,t}^{HML} \times HML_t + \beta_{P,t}^{UMD} \times UMD_t + \epsilon_{P,t}, \quad (2) \end{aligned}$$

where, $P = 1, ..., 100$, R_t^P is the value weighted monthly return of the P^{th} $\beta - \delta$-sorted portfolio at t, R_t^m is the market excess return at t, Agg_disp_{t-1} is the value weighted aggregate dispersion at month *(t-1)*, SMB_t is the average return on the three small portfolios minus the average return on the three big portfolios, HML_t is the average return on the two value portfolios minus the average return on the two growth portfolios and UMD_t is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios. We now define the portfolio post ranking β_P^{MKT} as the time series average of $\beta_{P,t}^{MKT}$ and the portfolio post ranking δ_P as the time series average of $\delta_{P,t}$. Second, as in (Fama and MacBeth 1973), each calendar month, we estimate the four cross-sectional regressions over our 100 β-δ portfolios using the specification

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P}, \quad (3)$$

where $P = 1, ..., 100$, β_P^{MKT} , δ_P, β_P^{SMB}, β_P^{HML} , β_P^{UMD} are the time series averages of the post-ranking portfolio loadings of the P^{th} $\beta - \delta$-sorted portfolio estimated using regression 2.

Table 5 reports the average month-by-month regression coefficients estimates as well as the *t* statistics, which are adjusted as proposed by (**?**). We also report the average month-by-month coefficients of determination R^2. The dependent variable in columns 1 - 4, R_t^P, is the return of the P^{th} $\beta - \delta$-sorted portfolio at t. The independent variable of interest is the time series average of dispersion risk loading δ_P. The first column, shows the simple CAPM results. In the second column, we include the market loading, β_P^{MKT} as well as the dispersion loading, δ_P, as independent variables in our regression. The price of dispersion risk is positive, 0.989% but insignificant with a t-statistic of 0.92. R^2 equals to 51% higher then the CAPM R^2 of 24%. In the fourth column, we extend our analysis to control for size, value and momentum effects. The price of dispersion risk is still positive (0.851%) and insignificant with a t-statistic of 0.76.

Evidence from both Fama MacBeth regressions and portfolio sorts suggests that the aggregate dispersion effect on stock returns varies across different sample periods. To test this hypothesis, we first retrieve a time series of the price of dispersion, ω_t. Figure 3 plots the time series of the price of dispersion risk estimated by the following two-factor regression:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \epsilon_{t,P},$$

where $P = 1, ..., 100$. We also plot in shaded area low and high dispersion months. We clearly notice that low dispersion months, shown in light grey, display negative values of price of dispersion risk. High dispersion months, shown in dark grey, display positive values of price of dispersion risk. These results strongly confirm that aggregate dispersion display a time varying risk premium.

In the next step, we run a two-stage regression to formally analyze the effect of aggregate dispersion in three different sub-samples: low, medium and high dispersion months. Table 6 column 2 reports the model two second-stage regression results, where the two independent variables are the market and dispersion loadings. Results suggest that, in periods of low beliefs dispersion (panel A), the coefficient of the exposure to aggregate dispersion is negative -6.07% and significant (t-statistic $= -4.52$) and in periods of high beliefs dispersion (Panel C), the coefficient of δ_P is positive 24.92% and significant (t-statistic $= 4.66$). However, in period of medium aggregate dispersion (Panel B), the price of dispersion risk is insignificant (t-statistic $= -0.20$) and very low (-0.24%). R^2s are around 50% in the three subsamples, higher then the CAPM R^2s equal to 22%, 24% and 23% in low, medium and high dispersion months respectively. These values of R^2 suggest that aggregate dispersion is an important state variable. Controlling for size, value and momentum factors generates equally good results. Column 4 shows that the aggregate dispersion risk loading is negatively related to portfolio returns in low dispersion months (Panel A), the coefficient of δ_P is equal to -6.32% with a t-statistic of -4.48 and it is positively related to portfolio returns in high dispersion months (Panel C) with a coefficient of 24.84% and a t-statistic of 4.74. Again, medium aggregate dispersion months (Panel C) display insignificant and low price of dispersion risk. The R^2s are around 54% using the three different sub-samples, again higher then the four factors model R^2s presented in column 3.

Overall, the results of Table 6 strongly suggest that the aggregate dispersion effect on portfolio returns is time varying. In particular, it is positive in high dispersion months and negative in low dispersion months. It is worth noting that the magnitude of the price of dispersion risk is more pronounced in high dispersion months (24.92% in column 2 and 24.84% in column 4) compared to the low dispersion months (-6.067% in column 2 and -6.325% in column 4) and almost equals to zero (-0.236% in column 2 and -0.414% in column 4) in medium dispersion months.

2.2 Additional Test Portfolios

In this section, we study whether the aggregate dispersion is a priced risk factor using test portfolios. In order to alleviate the critique of (Lewellen, Nagel, and Shanken 2010), ten sets of tests portfolios are used: 25 Value Size Portfolios, 25 Net Share Issues Size Portfolios, 10 Size Portfolios, 10 Value Portfolios, 25 Size Investment Portfolios, 100 Size Operating Prof

Portfolios, 30 Industry Portfolios, 10 E/P Portfolios, 10 Net Share Issues Portfolios, 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from Ken French's data web site.

First, every month, we estimate the risk loadings from a rolling time series regression of portfolio returns on excess market returns and one-month lagged aggregate dispersion using the previous 12 months. Similar to our analysis using the 100 β-δ portfolios, we also include (Fama and French 1993) factors and the momentum factor ((Jegadeesh and Titman 1993)) to control for size, value and momentum effects.

Second, we estimate the prices of risk by regressing the value weighted portfolio return on the first-pass risk loadings. Tables A.1 - A.4, in the appendix summarize the second pass regressions using the overall sample period for the 10 test portfolios listed above. Results show that the sign of the price of dispersion risk is undetermined. Table A.2, reports the results of the two-factor model with market excess return and aggregate dispersion being the two factors considered. The coefficient of aggregate dispersion risk loading is positive for six out of our ten portfolios. Only one of these six portfolios (25 Book-to-Market and Operating Profitability Portfolios) displays significant price of dispersion risk. The average month-by-month R^2 improves in this specification compared to the standard CAPM regression results in Table A.1. This is especially well pronounced for the 10 Size Portfolios. R^2 jumped from 32.3% in Table A.1 to 58% in Table A.2. Table A.4 includes the market, dispersion, size, value and momentum risk loadings in the regression. Only four portfolios out of ten display positive price of dispersion risk with t-statistics below conventional significance levels. The average month-by-month R^2 slightly increases in this specification compared to the standard four factors model shown in Table A.3.

At this stage of our analysis, we investigate the time varying characteristics of the aggregate dispersion using the ten test assets listed above. To do that, we repeat the steps of the two-stage regression mentioned above using the low, medium and high dispersion months. Tables A.5 - A.11 report the second-stage regressions. The price of dispersion is negative in low dispersion months and positive is high dispersion months for eight out of the ten test assets. These results hold using the two-factor model in Table A.7 and the full specification model in Table A.11. However, the *t* statistics are low. Adding the aggregate dispersion loading, δ_p, to the standard CAPM specification, shown in Table A.5 improves the average month-by-month R^2. Also, the magnitude of the price of risk is well pronounced in high dispersion months compared to the low dispersion months. In particular, the two-factor regression results using the 10 size portfolios show that the price of dispersion risk equals 41.97% (t statistic = 1.58), compared to -6.57% (t statistic = 0.86) in low dispersion months.

These empirical findings emphasize that the effect of aggregate dispersion on the portfolio returns varies with the respect to the level of disagreement on the market. Low dispersion periods are characterized with negative dispersion price and high dispersion periods are associated with positive dispersion price of risk.

3 The Model

In this section we provide a general equilibrium model that sheds light on why the price of dispersion risk can change sign over time depending on the level of aggregate dispersion. The model is based on the framework of Cox, Ingersoll, and Ross (1985) with two important specification choices. First, the state variables that we choose are the beliefs of each analyst type in the economy, and second, since we are modeling investment by a representative consumer, we restrict all portfolio choices to be non-negative.

3.1 The Macroeconomic Factor

A macroeconomic factor follows the process:

$$\frac{dY_t}{Y_t} = \nu_t dt + \sigma_Y d\tilde{W}_{Yt}. \tag{4}$$

The drift ν follows a 2-state Markov chain and it shifts between the states $\{\nu_1, \nu_2\}$ with generator matrix Λ.[3]

There are two types of analysts indexed by $m = 1, 2$. Each type m assumes that the process for ν has the correct specification, but differs in the estimates of the states as well as the generator. Let $\nu^{(m)}$ denote the estimated drift vector estimated by agent of type m, and let $\Lambda^{(m)}$ be their estimated generator matrix. Neither type of agent can observe the realizations of ν, although each does observe the entire history of Y. Based on their assumed models, analysts of type m form the posterior probability (see David (1997)) $\pi_{1t}^{(m)} = \text{prob}(\nu_t = \nu_1^{(m)} | \mathcal{F}_t^{(m)})$ of ν being in state 1 at time t. I denote conditional means with bars, for example,

[3] Over the infinitesimal time interval of length dt, $\Lambda_{ij} dt = \text{prob}\,(\nu_{t+dt} = \nu_j | \nu_t = \nu_i)$, for $i \neq j$, and $\Lambda_{ii} = -\Lambda_{ij}$. The transition matrix over any finite interval of time, s, is $\exp(\Lambda s)$.

$\overline{\nu}_t^{(m)} = \sum_{i=1}^{2} \pi_{it}^{(m)}\, \nu_i^{(m)}$. Given an initial belief $0 \le \pi_{10}^{(m)} \le 1$, the probabilities $\pi_{1t}^{(m)}$ follow the stochastic differential equations

$$d\pi_{1t}^{(m)} = \mu_{1t}^{(m)} dt + \sigma_{1t}^{(m)} d\tilde{W}_{Yt}^{(m)}, \tag{5}$$

where

$$\mu_{1t}^{(m)} = (\Lambda_{12}^{(m)} + \Lambda_{21}^{(m)})[\pi_1^{*(m)} - \pi_{1t}^{(m)}], \tag{6}$$

$$\sigma_{1t}^{(m)} = \pi_{1t}^{(m)}(1 - \pi_{1t}^{(m)})\frac{(\nu_1^{(m)} - \nu_2^{(m)})}{\sigma_Y}, \tag{7}$$

$$d\tilde{W}_{Yt}^{(m)} = \frac{1}{\sigma_Y}\left(\frac{dY_t}{Y_t} - E_t^{(m)}[\frac{dY_t}{Y_t}]\right) = \frac{(\nu_t - \overline{\nu}^{(m)})}{\sigma_Y} dt \, + \, d\tilde{W}_{Yt}. \tag{8}$$

In particular, $\pi_{1t}^{(m)}$ mean reverts to its unconditional mean, $\pi_1^{*(m)} = \Lambda_{21}^{(m)}/(\Lambda_{12}^{(m)} + \Lambda_{21}^{(m)})$, with a speed proportional to $(\Lambda_{12}^{(m)} + \Lambda_{21}^{(m)})$, and the volatility of an agent of type m's updating process is the product of his uncertainty, $\pi_{1t}^{(m)}(1 - \pi_{1t}^{(m)})$, and the signal-to-noise ratio, $\frac{(\nu_1^{(m)} - \nu_2^{(m)})}{\sigma_Y}$.

The two types of agents perceive the history of fundamentals differently. The process $\{\tilde{W}_{Yt}^{(m)}\}$ is the "innovations" process of analysts of type m, and is the shock process to the macroeconomic factor as perceived by agents of type m. According to analyst m, the macroeconomic factor dynamics are:

$$\frac{dY_t}{Y_t} = \bar{\nu}^{(m)} dt + \sigma_Y d\tilde{W}_{Yt}^{(m)}. \tag{9}$$

Taking the difference between the innovations of the two analysts we have

$$d\tilde{W}_{Yt}^{(2)} = d\tilde{W}_{Yt}^{(1)} + \sigma_{\eta t} dt, \tag{10}$$

where $\sigma_{\eta t} = \frac{(\bar{\nu_t}^{(2)} - \bar{\nu_t}^{(1)})}{\sigma_Y}$. Let $\mathcal{P}_t^{(m)}$ be analyst m's probability measure over the path of Y_s, $s \in [0, t]$. Again appealing to the results in David (2008) (see Corollary 1), the Radon-Nikodym derivative of $\mathcal{P}_t^{(2)}$ with respect to $\mathcal{P}_t^{(1)}$ is given by the process η_t, which follows;

$$\frac{d\eta_t}{\eta_t} = \sigma_{\eta t} d\tilde{W}_{Yt}^{(1)}, \tag{11}$$

which is a martingale with respect to $\mathcal{P}_t^{(1)}$. By relating the two innovations processes, we can write the beliefs of analyst 2 from the eyes of analyst 1 as

$$d\pi_{1t}^{(2)} = (\mu_{1t}^{(2)} + \sigma_{1t}^{(2)}\sigma_{\eta_t})dt + \sigma_{1t}^{(2)} d\tilde{W}_{Yt}^{(1)}, \tag{12}$$

and consequently solve the equilibrium of the model under analyst 1's probability measure.[4]

[4] Alternatively, we could also solve it under analyst 2's probability measure, or even the objective probability measure.

3.2 Speculation Among the Analysts

The analysts are able to speculate with each other on the realized value of the fundamental process Y.[5] Following Basak (2000), equilibrium among the analysts can be solved as the solution to the social planner's problem at each time t with weights $\lambda_{2t}/\lambda_{1t} = k\,\eta_t$, where η_t is the process in (11), and k is a constant that depends on the ratio of the initial wealths of the analysts at date 0. As discussed in David (2008), in equilibrium, $\eta_t = \xi_t^{(1)}/\xi_t^{(2)}$, which is the ratio of the analysts' state price densities (SPDs). Since the SPD is the state price per unit probability of that state, and the analysts agree on the state prices, η_t is the ratio of the probability of the state arising from the model of analyst 2, divided by the probability of the state arising from the model of analyst 1. Clearly, η_t belongs to the interval $[0, \infty]$. To obtain a bounded state variable set, we define

$$\varrho_t = \frac{1}{1+\eta_t}, \tag{13}$$

which is in $[0, 1]$, and in the competitive equilibrium, is the probability that the macro fundamental Y_t at date t arises from the model of analyst 2. Similarly $1 - \varrho_t$ is the probability that the data is generated by the model of analyst 1. By Ito's Lemma, $\{\varrho_t\}$ satisfies the process:

$$d\varrho_t = \varrho_t\,(1-\varrho_t)^2\,\sigma_{\eta t}\,dt - \varrho_t\,(1-\varrho_t)\,\sigma_{\eta t} dW_t^{(1)}. \tag{14}$$

3.3 Firms in the Economy

We model firms in a production economy with stochastically linear technologies as in Cox, Ingersoll, and Ross (1985). We assume that there are N production technologies, which we shall simply refer to as assets. The transformation of an investment of an amount X_i of the single good in the economy in the ith asset is given by the

$$\frac{dX_{it}}{X_{it}} = b_i \frac{dY_t}{Y_t} + \sigma_i d\tilde{W}_i. \tag{15}$$

Therefore the return on each technology is driven by the macroeconomic factor and an an idiosyncratic firm specific shock. The coefficient b_i is the "cash-flow beta" of the ith technology. We order firms in the economy with the ratio b_i/σ_i, so that

$$0 < \frac{b_1}{\sigma_1} < \frac{b_2}{\sigma_2} \cdots < \frac{b_N}{\sigma_N}. \tag{16}$$

[5] This could be done by derivative securities whose value mirrors that of Y.

Investment in the assets is made through competitive value maximizing firms. With free entry of firms and stochastic constant returns to scale, there is no incentive for firms to enter or leave industry i if and only if the returns on the shares of each firm are identical to the technologically determined physical returns on that asset. The equilibrium scale of each firm is determined by the supply of investment to that firm.

3.4 Consumers in the Economy

In contrast to models in the heterogeneous beliefs literature (for example Basak (2000) or David (2008) and the references in the introduction), we make a distinction between analysts and consumers. Analysts do not trade in stocks, but on derivatives on the macroeconomic factor. The representative consumer in the economy has the standard CRRA utility function

$$U(c) = E^{(m)}[\int_0^\infty \exp(-\rho s) \cdot u(c_s) ds], \tag{17}$$

with time discount factor ρ and felicity $u(c_t) = c_t^\gamma/\gamma$. The felicity function $u(.)$ has a constant coefficient of relative risk aversion $1-\gamma$, and satisfies the Inada conditions $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

We assume that the representative consumer does not fully trust any particular analyst in the economy, but behaves as a Bayes' Model Averager (BMA). However, she assesses analyst's relative performance and in particular uses the assessed probabilities that the data at any given date is generated by model of analyst m and given the analysts' equilibrium in Section 3.2, assign weights of ϱ_t and $1-\varrho_t$, to the beliefs of analysts 1 and 2, respectively. Given the asset return distributions in (15), the consumer's expected return for asset i is $\alpha_i = b_i((1-\varrho_t)\bar{\nu}^{(1)} + \varrho_t \bar{\nu}^{(2)})$. Let $w_i \geq 0$ be the proportion of the consumer's wealth invested in asset i for $i = 1, \cdots, N$, and w_0 be the proportion invested in the instantaneous riskless bond in the economy, which offers a rate of return r_t, and is in zero net supply. We will first solve the social planner's problem for the representative agent economy, which does not include investment in the riskless asset, and hence the portfolio weights satisfy $\sum_{i=1}^N w_i = 1$. Later, we will find the rate at which a choice of $w_0 = 0$ is optimal. Then, the consumer's wealth dynamics can be written as

$$dW_t = -C_t dt + W_t \left[\sum_{i=1}^N w_i \alpha_i dt + w_i b_i \sigma_Y d\tilde{W}_{Yt} + w_i \sigma_i d\tilde{W}_i \right], \tag{18}$$

where C_t is the rate of consumption.

Given the beliefs and the analysts' model probabilities, we can formulate the equilibrium in the production economy as in Cox, Ingersoll, and Ross (1985).[6] We start with the social planner's problem in this economy.

Proposition 1 *The value function of the representative consumer in the economy that maximizes utility takes the form:*

$$J(W_t, \pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t, t) = \exp(-\rho t)\frac{W_t^{\gamma}}{\gamma} I(\pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t), \tag{19}$$

in which the function $I(\pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t)$ satisfies the PDE:

$$\begin{aligned} 0 = \max_{w_i, s.t. w_i \geq 0, \sum_{i=1}^N w_i = 1} \Bigg[& \left(\frac{1}{\gamma} - 1\right) I^{\frac{\gamma}{\gamma-1}} - \frac{\rho}{\gamma} I \\ & + I\left(\sum_{i=1}^N w_i b_i ((1-\varrho)\bar{\nu}^{(1)} + \varrho\bar{\nu}^{(2)}) + \frac{1}{2}(\gamma-1)\sum_{i=1}^N w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2)\right) \\ & + I_{\pi_1^{(1)}}\left(\frac{\mu_1^{(1)}}{\gamma} + (\sum_{i=1}^N w_i b_i)\sigma_1^{(1)}\sigma_Y\right) + I_{\pi_1^{(2)}}\left(\frac{\mu_1^{(2)} + \sigma_2^{(2)}\sigma_\eta}{\gamma} + (\sum_{i=1}^N w_i b_i)\sigma_1^{(2)}\sigma_Y\right) \\ & + I_\varrho\left(\frac{\varrho(1-\varrho)^2\sigma_\eta}{\gamma} - (\sum_{i=1}^N w_i b_i)\varrho(1-\varrho)\sigma_\eta\right) \\ & \frac{1}{2} I_{\pi_1^{(1)}\pi_1^{(1)}}(\sigma_1^{(1)})^2 + \frac{1}{2} I_{\pi_1^{(2)}\pi_1^{(2)}}(\sigma_1^{(2)})^2 + \frac{1}{2} I_{\varrho\varrho}\varrho^2(1-\varrho)^2\sigma_\eta^2 \\ & - I_{\pi^{(1)}\varrho}\sigma_1^{(1)}\varrho(1-\varrho)\sigma_\eta - I_{\pi^{(2)}\varrho}\sigma_1^{(2)}\varrho(1-\varrho)\sigma_\eta + I_{\pi^{(1)}\pi^{(2)}}\sigma_1^{(1)}\sigma_1^{(2)}\Bigg]. \end{aligned} \tag{20}$$

The Kuhn-Tucker first-order conditions for the portfolio choices of the consumer are:

$$\begin{aligned} & b_i((1-\varrho)\bar{\nu}^{(1)} + \varrho\bar{\nu}^{(2)}) + (\gamma-1)w_i(b_i^2\sigma_Y^2 + \sigma_i^2) + \frac{I_{\pi_1^{(1)}}}{I} b_i \sigma_1^{(1)}\sigma_Y + \frac{I_{\pi_1^{(2)}}}{I} b_i \sigma_1^{(2)}\sigma_Y \\ & \qquad - \frac{I_\varrho}{I} b_i \varrho(1-\varrho)\sigma_\eta\sigma_Y + \kappa_i - \frac{\lambda^{(1)}}{I} \leq 0 \qquad \textit{for} \quad i = 1, \cdots, N \end{aligned} \tag{21}$$

$$\begin{aligned} w_i \Bigg[& b_i((1-\varrho)\bar{\nu}^{(1)} + \varrho\bar{\nu}^{(2)}) + (\gamma-1)w_i(b_i^2\sigma_Y^2 + \sigma_i^2) + \frac{I_{\pi_1^{(1)}}}{I} b_i \sigma_1^{(1)}\sigma_Y + \frac{I_{\pi_1^{(2)}}}{I} b_i \sigma_1^{(2)}\sigma_Y \\ & - \frac{I_\varrho}{I} b_i \varrho(1-\varrho)\sigma_\eta\sigma_Y + \kappa_i - \frac{\lambda^{(1)}}{I}\Bigg] = 0 \qquad \textit{for} \quad i = 1, \cdots, N \end{aligned} \tag{22}$$

[6] David (1997) extends the CIR model to the case with unobserved drifts of the production processes and learning.

$$\sum_{i=1}^{N} w_i = 1 \quad (23)$$

$$w_i \geq 0; i = 1, \cdots, N \quad (24)$$

$$w_i \kappa_i = 0; i = 1, \cdots, N, \quad (25)$$

where $\lambda^{(1)}$ *is the multiplier associated with the constraint* $\sum_{i=1}^{N} w_i = 1$*, and* κ_i *are the multipliers associated with the constraints* $w_i \geq 0$ *for* $i = 1, \cdots, N$.

The proof in Appendix 2.

We solve the PDE in (20) with Chebyshev polynomials using projection methods. A similar PDE has been solved in David (2008), and we follow the steps there in implementing this method. One difference though is that the PDE in David (2008) did not have the portfolio choices, which are needed here. We follow a recursive procedure to determine the portfolio choices and the solution to the PDE. Given the nth iteration of the solution, I^n, we solve the portfolio choices w^{n+1} using (21) – (25) using a standard equation solver at each point on the Chebshev grid. We then, use these portfolio choices in finding the projections to the polynomials, and hence find I^{n+1}. Using standard contraction mapping arguments, the recursive procedure converges.

3.5 The Cross Section of Equilibrium Risk Premia in the Economy

We now consider the cross section of equilibrium risk premia for the different stocks in the economy.

Proposition 2 *In equilibrium, the risk premium for stock* i *for any stock with* $w_i > 0$ *satisfies:*

$$\alpha_i - r = \frac{b_i}{b_m}(\alpha_m - r) + (1-\gamma)(b_i^2\sigma_Y^2 + \sigma_i^2) - \frac{b_i}{b_m}(1-\gamma)\sum_{i=1}^{N} w_i^2(b_i^2\sigma_Y^2 + \sigma_i^2), \quad (26)$$

in which $b_m = \sum_{i=1}^{N} w_i\, b_i$ *is the cash flow beta of the market portfolio, and* $\alpha_m = \sum_{i=1}^{N} w_i\, \alpha_i$ *is the expected return on the market portfolio. The riskless rate in the economy satisfies:*

$$r = \left(\sum_{i=1}^{N} w_i b_i\right)\left[((1-\varrho)\bar{\nu}^{(1)} + \varrho\bar{\nu}^{(2)}) + \frac{I_{\pi_1^{(1)}}}{I}\sigma_1^{(1)}\sigma_Y + \frac{I_{\pi_1^{(2)}}}{I}\sigma_1^{(2)}\sigma_Y - \frac{I_\varrho}{I}\varrho(1-\varrho)\sigma_\eta\sigma_Y\right] + (\gamma-1)\sum_{i=1}^{N} w_i^2\,(b_i^2\sigma_Y + \sigma_i^2) \quad (27)$$

The proof in Appendix 2.

We provide some comments on the equilibrium cross sectional pricing equation in (26). There are three terms in each asset's risk premium. The first term, is the risk premium for covarying with the market portfolio, as in the CAPM; however, in the model, the exposure to the market is measured by the relative cash-flow beta of each asset, rather than the return beta. The second term, measures the risk premium for bearing the risk in the aggregate macroeconomic factor as well as any undiversified idiosyncratic risk. The last term, is the risk premium with respect to a "flight-to-safety" factor. As can be seen, this factor depends on the variance of the market portfolio. The negative sign on the term implies that assets that provide higher returns in periods when variance is high, obtain a lower risk premium. The negative risk premium is consistent with the variance risk premium literature. The interesting aspect of this term is that it is time-varying and depends on the consumer's portfolio choices. As in Merton's ICAPM and Cox, Ingersoll, and Ross (1985), the representative consumer attempts to hedge against shifts in the state variables of the economy in addition to her diversification motive. The state variables in the economy are the beliefs of each analyst $\pi_t^{(m)}$ about the state of the macro fundamental, as well, as the variable ϱ_t, which is the probability at any time that the fundamentals are generated by the model of agent 1. The risk premia for shifts in these state variables, written as a function of the portfolio choices, collapses to the third term, which is the variance of the market portfolio. As we will see, when analysts' dispersion becomes very high, consumers respond by allocating their investments to low cash flow beta assets, and this term declines. It is also interesting that the exposure to the flight-to-safety factor is again measured by the relative cash flow beta of each asset.

As noted above, the most interesting aspect of our model is that the loading on aggregate dispersion depends on the portfolio choices, which are the market weights in equilibrium. To illustrate why our model implies that the price of dispersion risk depends on the level of dispersion, consider the following situations. Suppose there are only two assets in the economy, with $b_1 = 0.1$ and $b_2 = 2$. Suppose uncertainty about the growth rate of Y is very high, so that $w_1 = 0.9$, and $w_2 = 0.1$ Now the term $\sum_i w_i^2 b_i^2$, which is in the last term is just 0.0481. Alternatively, when uncertainty is low suppose the weights reverse, $w_1 = 0.1$ and $w_2 = 0.9$, this term equals 3.24. Since the last term enters with a negative sign in the risk premium (the consumer likes an asset that gives a positive return when variance is high), the model implies that the risk premium will be higher in periods of higher aggregate dispersion. We will formalize this intuition in the calibration section below.

To address the empirical results in our paper, we proceed to two-stage regressions on the market return, and the aggregate dispersion in the economy.

3.6 Belief Calibration and Model Two-Stage Regressions

We calibrate the model as in David (2008). A brief description of the calibration method is as follows. Using the unobserved regime shifting structure in the aggregate fundamental, which we take as aggregate S&P 500 operating earnings growth, we find two sets of parameters (one for each type of analyst), that maximizes the sum of the likelihoods of each agent type observing the fundamentals, and using the conditional expectations of each type of agent, we formulate the forecast of aggregate earnings growth, we formulate the 1-year ahead, aggregate earnings dispersion among the two types. The objective function also puts weight on matching the model dispersion to the dispersion of earnings growth from the Philadelphia Fed Survey of Professional Forecasters. The calibrated parameters are shown in Table 7. In Figure 4 we see the beliefs of each analyst type for the sample from 1971 to 2001, as well as their estimates of earnings growth for the period from 1971 to 2001. As seen, analyst of type 2 are more volatile. In addition, their expectations' differences are countercyclical, and the dispersion in their estimates is low in upturns and significantly higher in downturns.

Using the calibrated parameters, we simulate the model. The steps taken are as follows. We first extend the beliefs out-of-sample from 2002-2017 using the calibrated parameters and realized earnings growth. Using these beliefs we similarly extended $\{\varrho_t\}$ series, which is the conditional probability that the agents assign to the model of analyst 2. For the cross sectional specification, we use 10 assets, with cash flow betas that are $b_1 = 0.1$, $b_2 = 0.3$, ... $b_{10} = 0.2$. We use the same idiosyncratic volatility of 0.3 for each asset. Finally, for the consumer's preference, we use a time discount, ρ, of 2 percent, and a $\gamma = 0.5$. This low level of risk aversion (similar to that in David (2008), provides a low riskless rate. In addition, this choice implies an elasticity of intertemporal substitution, which is larger than 1, which has been documented in various studies. Using the beliefs and ϱ_t, we calculate the market portfolio weights as shown in Proposition 1. Then similar to the data, using rolling one-year lagged stock returns, we first run first-pass regressions of each asset's return on the market return and earnings dispersion. In the second-pass regression, at each date, we run the one-year lagged stock returns for each of the 10 assets on the average of beta and delta over the full sample.

The results of the out-of-sample calibration exercise are seen in Figures 6 to 8. The top panel Figure 6 shows the historical operating earnings growth of S&P 500 firms over the full sample from 1971 - 2017, while the bottom panel shows the conditional probability that the model of Analyst 2 generated the data at each time t, which is ϱ_t. As can be seen, the pattern of analysts' expectations post-2001 is very similar to that prior to 2001 (our calibration sample). In particular, the expectations of Analyst 1 are more volatile, overshooting those of Analyst 2 in each direction, and the gap between them is significantly wider in downturns. Figure 7 shows the expected growth over the next one year of each analyst type (top-left), and the dispersion of their forecasts (top-left). The bottom-left panel shows the time-series of the market's cash flow beta, which is obtained by using the beliefs of each type, ϱ_t, and the optimal portfolio choices in Proposition 1. Finally the bottom-right panel shows well the "flight-to-safety" pattern in the portfolio choices. Indeed, the cash flow beta of the market portfolio is nearly perfectly negatively correlated with the dispersion in analysts' forecasts. That is, when the consumer sees a large dispersion, she choose lower cash-flow assets in her portfolio.

The time-series of the model's prices of risk from the second-pass are shown in Figure 8. While not easily evident, the two prices of risk have a negative correlation of -0.26, and each fluctuates in sign. We focus or comments on the price of dispersion risk. As seen, the model's price or risk changes sign over time. Its maximum value is as high as 0.27, and as low as -0.28. The correlation between the dispersion price of risk and dispersion of analysts' forecasts is 0.2, so that in period of high dispersion, the price of dispersion risk tends to be higher, as in the data. Restricting our sample to periods when the dispersion is higher than its mean plus one standard deviation, the average price of dispersion risk is 0.06, while for the remaining sample, it is -0.03. This is consistent with our main empirical fact. The intuition for the positive correlation between dispersion and the price of dispersion risk is in our comments below Proposition 2. In periods of higher dispersion, the consumer invests in lower beta assets (as seen in Figure 7) and hence the loading on equilibrium loading on dispersion increases.

3.7 The Flight-to-Safety Effect in the Data

As discussed above, the change in the price of risk is related to the flight-to-safety behavior of consumers in the economy. In periods of high dispersion, consumers choose assets with lower cash flow betas. Here we verify that such a flight-to-safety occurs in the data. To do so, we first calculate a cash flow beta, b_i for each firm in the economy, which is estimated using 5-year rolling windows of firm level earnings growth on aggregate earnings growth. The weight, w_i is the market capitalization weight of each stock in the CRSP database. In Figure 9, we show the time series of aggregate dispersion, as well as the term $\sum_i b_i^2 \cdot w_i^2$, which is in the third term of the model risk premium in equation (26). It is interesting to see that during recessions and financial crises, this term declines. Such periods also happen to be in periods of high dispersion. Indeed, the two series have a correlation of -0.39, that is, there a portfolio reallocation towards lower cash flow beta assets in periods of higher dispersion, consistent with our model.

4 Conclusion

Understanding how dispersion in analyst's beliefs affects security prices and returns is one of the most fundamental issues in finance. This paper contributes by answering the following question: when is the price of dispersion risk positive? We show that aggregate dispersion effect on security's returns varies across different periods of time. Contrary to previous studies, we look at the price of dispersion risk at different sub-samples: full sample, low, medium and high dispersion months. We find, empirically, that the price of dispersion risk is significantly positive in high dispersion months and significantly negative in low dispersion months. The price of dispersion risk is insignificant and close to zero using the full sample and medium dispersion months. Also, the magnitude of the price of dispersion risk is more pronounced in high dispersion months compared to the low dispersion months.

We construct a general equilibrium model in which analysts of different types have heterogeneous beliefs and provide different forecasts of a macroeconomic factor (aggregate earnings growth). The consumer does not trust either analyst fully, and dynamically adjusts the weight given to each analyst, given the history of their past forecast performance. The model is estimated from 1971-2001, and out-of-sample beliefs of analysts are formulated until 2017. The model provides a positive correlation between the dispersion in the analysts forecasts, and the price of dispersion risk. A crucial part of the model's mechanism is that the market weights assigned to lower cash flow beta assets increases in periods of higher dispersion, leading to a large loading on the dispersion risk factor. We provide support for such a flight-to-safety phenomenon in the data.

Table 1: The table reports the summary statistics of aggregate dispersion from December 1981 to September 2016. Aggregate dispersion is the value-weighted average of the stock-level dispersion. Stock-level dispersion is measured as the standard deviation of analyst forecasts of the earning-per-share long-term growth rate (EPS LTG). We define low (high) dispersion months as the months where aggregate dispersion is lower (higher) then the average aggregate dispersion at $t-1$ minus (plus) one standard deviation.

	Aggregate Dispersion							
	obs	mean	sd	p10	p25	Median	p75	p90
Full Sample	418	3.51	0.63	2.84	2.98	3.33	3.93	4.53
Low Dispersion Months	55	2.80	0.09	2.70	2.73	2.79	2.87	2.92
Medium Dispersion Months	276	3.35	0.38	2.92	3.03	3.28	3.58	3.87
High Dispersion Months	86	4.48	0.32	4.10	4.32	4.50	4.68	4.81

Table 2: The table reports the regressions results of ex-post market excess returns on the market on aggregate dispersion. The t-statistics are adjusted for auto-correlation and heteroskedasticity using Newey and West (1987) with the number of lags equal to the return horizons. Panel A, the sample period is December 1981 to December 2005. Panel B, the sample period is December 1981 to September 2016 and Panel C, the sample period is January 2006 to September 2016.

Panel A: 12/1981 - 12/2005					
Return horizon (in months)	1	6	12	24	36
Aggregate dispersion	-0.622	-6.517**	-16.17***	-29.45***	-36.83*
	(-1.20)	(-2.85)	(-4.24)	(-3.66)	(-2.59)
Constant	2.732	25.94***	62.69***	115.8***	151.6***
	(1.62)	(3.61)	(5.12)	(4.41)	(3.36)
Observations	288	283	277	265	253
Adjusted R^2	0.15 %	8.66 %	25.24 %	40.85 %	33.59 %
Panel B: 12/1981 - 09/2016					
Return horizon (in months)	1	6	12	24	36
Aggregate dispersion	-0.200	-2.906	-6.559	-9.277	-10.18
	(-0.67)	(-1.78)	(-1.91)	(-1.17)	(-0.83)
Constant	1.365	14.62**	32.15**	50.85	65.00
	(1.27)	(2.68)	(2.83)	(1.94)	(1.58)
Observations	417	412	406	394	382
Adjusted R^2	-0.14 %	2.64 %	6.70 %	6 %	4.10 %
Panel C: 01/2006 - 09/2016					
Return horizon (in months)	1	6	12	24	36
Aggregate dispersion	0.333	-0.473	2.380	18.38	31.48**
	(0.53)	(-0.20)	(0.53)	(1.84)	(2.74)
Constant	-0.852	5.745	-2.728	-62.19	-107.9
	(-0.31)	(0.55)	(-0.12)	(-1.27)	(-1.87)
Observations	128	123	117	105	93
Adjusted R^2	-0.59 %	-0.78 %	-0.26 %	14.23 %	32.92 %

Table 3: The table reports the contemporaneous correlation between aggregate dispersion, stock market excess returns and the standard pricing factors SMB, HML and UMD using the full sample (Panel A), low dispersion months (Panel B), medium dispersion months (Panel C) and the high dispersion months (Panel D).

Panel A: Full sample

	Aggregate dispersion	$R_m - R_f$	SMB	HML	UMD
Aggregate dispersion	1				
$R_m - R_f$	0.0354	1			
SMB	0.0952	0.205	1		
HML	-0.0319	-0.261	-0.304	1	
UMD	-0.0458	-0.187	0.0643	-0.176	1

Panel B: Low dispersion months

	Aggregate dispersion	$R_m - R_f$	SMB	HML	UMD
Aggregate dispersion	1				
$R_m - R_f$	0.335	1			
SMB	0.186	0.409	1		
HML	-0.133	-0.455	-0.446	1	
UMD	-0.180	-0.467	-0.357	0.286	1

Panel C: Medium dispersion months

	Aggregate dispersion	$R_m - R_f$	SMB	HML	UMD
Aggregate dispersion	1				
$R_m - R_f$	0.0622	1			
SMB	0.149	0.158	1		
HML	-0.158	-0.266	-0.129	1	
UMD	-0.0834	-0.246	-0.153	-0.0567	1

Panel D: High dispersion months

	Aggregate dispersion	$R_m - R_f$	SMB	HML	UMD
Aggregate dispersion	1				
$R_m - R_f$	0.398	1			
SMB	0.0759	0.265	1		
HML	-0.130	-0.220	-0.493	1	
UMD	0.129	0.0371	0.542	-0.477	1

Table 4: Summary statistics: Sample: December 1981 - September 2016. The table reports the summary statistics of pre-ranking and post-ranking β and δ, and the average returns on the β-δ portfolios using the full sample (Panel A), low dispersion months (Panel B), medium dispersion (Panel C) and high dispersion months (Panel D). Pre-ranking β and δ are obtained from one-year rolling regression of individual stock return on the market excess return and lagged aggregate dispersion:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times \boldsymbol{R}_{m,t} + \delta_{i,t} \times \mathit{Agg\_disp}_{t-1} + \epsilon_{i,t}$$

Every month, we construct the β-δ Portfolios. We sort stock into 10 β-deciles using the pre-ranking β. For every β-decile, we sort stocks based on the pre-ranking δ. Post-ranking β and δ are the time series average of the estimated risk exposure obtained from one-year rolling regressions of portfolio return on the market excess return and lagged aggregate dispersion:

$$R_{P,t} = \alpha_{P,t} + \beta_{P,t} \times \boldsymbol{R}_{m,t} + \delta_{P,t} \times \mathit{Agg\_disp}_{t-1} + \epsilon_{P,t}.$$

The β-δ portfolios return R_t^P is the value weighted monthly portfolio return. The numbers represent the time series averages of the cross-sectional mean, standard deviation (sd), the 10^{th} to 90^{th} percentiles (p10 to p90) of each variables.

	mean	sd	p10	p25	Median	p75	p90
Panel A: Full Sample							
pre-ranking β	1.09	0.94	0.04	0.49	1.01	1.60	2.27
pre-ranking δ	-0.01	0.24	-0.29	-0.14	-0.01	0.12	0.20
post-ranking β	1.01	0.76	0.06	0.49	0.95	1.47	2.05
post-ranking δ	-0.00	0.19	-0.24	-0.12	-0.01	0.11	0.24
β-δ Portfolio return ($\times$ 100): R_t^P	1.50	5.32	-4.82	-1.76	1.30	4.58	8.03
Panel B: Low dispersion months							
pre-ranking β	1.12	1.04	-0.03	0.45	1.03	1.69	2.43
pre-ranking δ	0.00	0.33	-0.37	-0.18	0.00	0.19	0.38
post-ranking β	1.03	0.78	0.06	0.47	0.94	1.52	2.13
post-ranking δ	0.01	0.28	-0.33	-0.17	-0.00	0.17	0.37
β-δ Portfolio return ($\times$ 100): R_t^P	1.85	4.87	-3.91	-1.12	1.62	4.62	7.82
Panel C: Medium dispersion months							
pre-ranking β	1.10	0.95	0.04	0.49	1.01	1.61	2.29
pre-ranking δ	-0.02	0.26	-0.32	-0.16	-0.01	0.13	0.28
post-ranking β	1.01	0.77	0.06	0.50	0.96	1.46	2.05
post-ranking δ	-0.01	0.20	-0.26	-0.13	-0.01	0.11	0.25
β-δ Portfolio return ($\times$ 100): R_t^P	1.35	5.32	-4.97	-1.91	1.13	4.42	7.91
Panel D: High dispersion months							
pre-ranking β	1.06	0.84	0.09	0.50	0.98	1.52	2.12
pre-ranking δ	-0.00	0.11	-0.13	-0.06	0.00	0.06	0.12
post-ranking β	0.99	0.75	0.07	0.47	0.93	1.44	2.03
post-ranking δ	0.00	0.09	-0.11	-0.05	0.01	0.06	0.12
β-δ Portfolio return ($\times$ 100): R_t^P	1.80	5.64	-4.91	-1.69	1.63	5.13	8.60

Table 5: The table reports results from second-stage regression:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P}.$$

R_t^P is the value weighted returns of stocks in the P^{th} $\beta - \delta$-sorted portfolio at t, β_P^{MKT}, δ_P, β^{SMB}, β^{HML} and β^{UMD} the time series averages of the post-ranking portfolio loadings. Sample: December 1981 - September 2016. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992).

	(1)	(2)	(3)	(4)
$\bar{\pi}$	0.00157	0.00149	-0.00189	-0.00204
	(0.67)	(0.65)	(-0.73)	(-0.84)
$\bar{\omega}$		0.00989		0.00851
		(0.92)		(0.76)
$\bar{\phi}^{SMB}$			0.0102**	0.00824***
			(2.86)	(3.98)
$\bar{\phi}^{HML}$			-0.000568	-0.00534**
			(-0.10)	(-2.59)
$\bar{\phi}^{UMD}$			-0.0105**	-0.00683**
			(-2.76)	(-3.08)
Constant	0.00998***	0.0101***	0.0118***	0.0127***
	(6.84)	(7.02)	(6.13)	(8.43)
Num. portfolios	100	100	100	100
Num. time period	396	396	396	396
R^2	0.236	0.511	0.459	0.541
adjusted-R^2	0.228	0.501	0.436	0.517

Table 6: The table reports results from second-stage regression:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P},$$

R_t^P is the value weighted returns of stocks in the P^{th} $\beta - \delta$-sorted portfolio at t, β_P^{MKT}, δ_P, β^{SMB}, β^{HML} and β^{UMD} are the post-ranking portfolio risk loadings. Sample: Low (Panel A), medium (Panel B) and high (Panel C) aggregate dispersion months. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992).

Panel A: Low Dispersion Months				
	(1)	(2)	(3)	(4)
$\bar{\pi}$	0.00642	0.00861	0.00600	0.00862
	(1.32)	(1.80)	(1.11)	(1.73)
$\bar{\omega}$		-0.0607***		-0.0632***
		(-4.52)		(-4.48)
$\bar{\phi}^{SMB}$			-0.0101*	0.00164
			(-2.36)	(0.50)
$\bar{\phi}^{HML}$			-0.0198***	-0.00615*
			(-4.31)	(-2.14)
$\bar{\phi}^{UMD}$			0.00381	0.00443
			(1.20)	(1.68)
Constant	0.0106**	0.00869*	0.0121**	0.00890*
	(3.08)	(2.44)	(3.38)	(2.51)
Num. time period	51	51	51	51
R^2	0.223	0.506	0.486	0.540
adjusted-R^2	0.215	0.496	0.464	0.516
Panel B: Medium Dispersion Months				
$\bar{\pi}$	0.00196	0.00188	-0.00216	-0.00224
	(0.69)	(0.67)	(-0.74)	(-0.79)
$\bar{\omega}$		-0.00236		-0.00414
		(-0.20)		(-0.34)
$\bar{\phi}^{SMB}$			0.00730**	0.00852***
			(3.04)	(4.19)
$\bar{\phi}^{HML}$			-0.00504*	-0.00414
			(-2.02)	(-1.77)
$\bar{\phi}^{UMD}$			-0.00728	-0.00823***
			(-1.37)	(-3.41)
Constant	0.00859***	0.00867***	0.0119***	0.0119***
	(5.16)	(5.29)	(6.63)	(7.02)
Num. time period	267	267	267	267
R^2	0.240	0.514	0.303	0.546
adjusted-R^2	0.232	0.504	0.273	0.522
Panel C: High Dispersion Months				
$\bar{\pi}$	-0.0000816	-0.000618	-0.00631	-0.00340
	(-0.01)	(-0.10)	(-0.97)	(-0.51)
$\bar{\omega}$		0.249***		0.248***
		(4.66)		(4.74)
$\bar{\phi}^{SMB}$			0.0116*	0.00650
			(2.35)	(1.40)
$\bar{\phi}^{HML}$			0.0276***	0.00186
			(4.48)	(0.53)
$\bar{\phi}^{UMD}$			0.00190	-0.000195
			(0.38)	(-0.04)
Constant	0.0114**	0.0111*	0.0156**	0.0129**
	(2.69)	(2.62)	(3.42)	(2.74)
Num. time period	78	78	78	78
R^2	0.234	0.507	0.488	0.531
adjusted-R^2	0.226	0.496	0.466	0.506

Table 7: Two-State Heterogeneity Model Calibration Top Panel: GMM estimates of the following (discretized) model for real consumption, x_t, and real earnings, q_t:

$$x_{t+1} = x_t \cdot e^{(\kappa_t^{(m)} - \frac{1}{2}\sigma_x\sigma_x')\Delta t + \sigma_x \varepsilon_{t+1}} \; ; \; q_{t+1} = q_t \cdot e^{(\theta_t^{(m)} - \frac{1}{2}\sigma_q\sigma_q')\Delta t + \sigma_q \varepsilon_{t+1}}.$$

where $\sigma_q = (\sigma_{q1}, \sigma_{q2})$, $\sigma_x = (0, \sigma_{x,2})$, and $(\theta_t^{(m)}, \kappa_t^{(m)})$ jointly follows a 2-state regime-switching model. The estimates of the quarterly transition probability matrix are shown. The implied generator is $\Lambda^{(m)} = \sum_{i=1}^{\infty}(-1)^{i+1} \cdot \left((P^{(m)}(0.25))^4 - I\right)^i / i$, whose value is estimated using a series approximation of length 10 (see Israel, Rosenthal, and Wei (2001)). The GMM errors include the scores of the likelihood function of each type of agent and the difference in model-implied and historical dispersion in forecasts of Professional Forecasters as described in Appendix D. The $\chi^2(4)$ statistic for the specification test of the model is 7.6341, which has a *p*-value of 0.1059. Bottom Panel: Standard errors of parameter estimates are in parentheses. Units of measurement are quarterly and in percentage points. *T*-statistics are in parentheses. All *t*-statistics are adjusted for heteroskedasticity and autocorrelation using the methodology of Newey and West (1987). Figure 3 (top panel) shows the belief processes of the two agents. The top panel shows the actual and model-implied four-quarter-ahead dispersions of earnings growth, which are in the third regression.

Series Used: Real Earnings, Real Consumption,
and Dispersion of Earnings Growth Forecasts
Time Span (Quarterly): 1971-2001

	Analyst 1				Analyst 2			
Drifts:	$\theta_1^{(1)}$	$\theta_2^{(1)}$	$\kappa_1^{(1)}$	$\kappa_2^{(2)}$	$\theta_1^{(2)}$	$\theta_2^{(2)}$	$\kappa_1^{(2)}$	$\kappa_2^{(2)}$
	-0.2440	0.0828	0.0280	0.0280	-0.2305	0.0795	0.0280	0.0280
	(0.0194)	(0.0289)	(0.0103)	(0.0103)	(0.0192)	(0.0258)	(0.0103)	(0.0103)
Generator Elements:	$\lambda_{12}^{(1)}$	$\lambda_{21}^{(1)}$	$P_{12}{}^{(1)}$	$P_{21}{}^{(1)}$	$\lambda_{12}^{(2)}$	$\lambda_{21}^{(2)}$	$P_{12}{}^{(2)}$	$P_{21}{}^{(2)}$
	0.5061	0.3427	0.1611	0.0772	1.3194	0.3727	0.2749	0.0776
			(0.0612)	(0.0444)			(0.0656)	(0.0462)
Volatilities:	$\sigma_{q,1}$	$\sigma_{x,1}$	$\sigma_{x,2}$					
	0.0833	0.0109	0.0200					
	(0.0003)	(0.0001)	(0.0001)					

Model Fits: $\Delta \log(q)\,(t) = \alpha + \beta \cdot (\theta_1^{(m)}\pi_1^{(m)}(t|t) + \theta_2^{(m)}\pi_1^{(m)}(t|t)) + \epsilon(t)$, *m* = 1,2

Analyst 1			Analyst 2		
$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2
0.0932	1.4885	0.6476	-0.2116	1.8240	0.6737
(0.2224)	(8.8857)		(-0.5248)	(10.0718)	

Dispersion: $d(t,4) = \alpha + \beta \cdot d(\pi^{(1)}(t,4), \pi^{(2)}(t,4)) + \epsilon(t)$

$\hat{\alpha}$	$\hat{\beta}$	R^2
4.1301	0.7190	0.1982
(12.1873)	(4.0921)	

Figure 1: The figure plots the monthly aggregate dispersion measured as the value-weighted average of the stock-level dispersion. Stock-level dispersion is measured as the dispersion in analyst forecasts of the earnings-per-share (EPS) long-term growth rate. We define low dispersion months as the months where aggregate dispersion is lower then the average aggregate dispersion minus one standard deviation, shown in the figure as the low horizontal line. We define high dispersion months as the months where aggregate dispersion is higher then the average aggregate dispersion plus one standard deviation, shown in the figure as the high horizontal line.

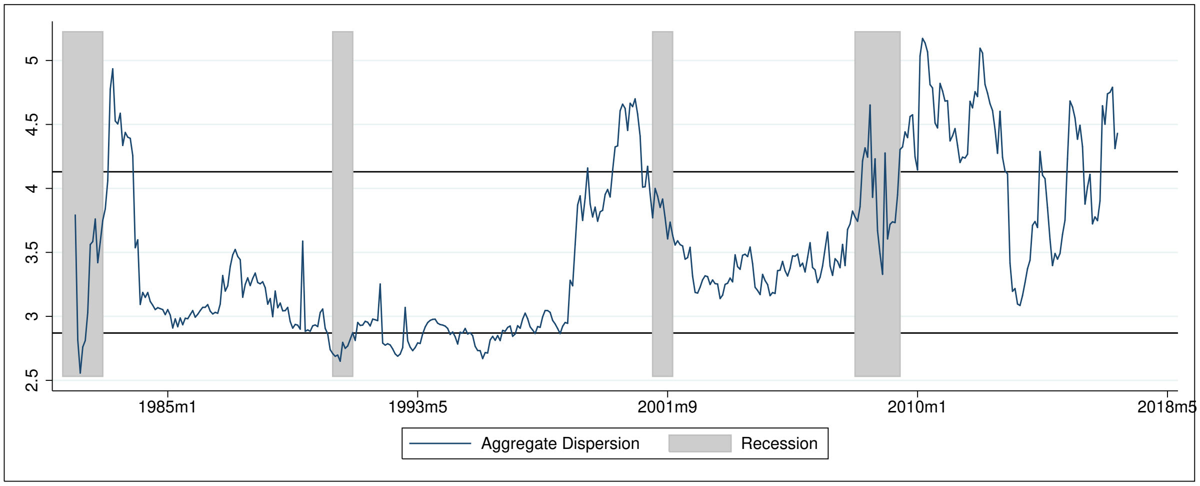


Figure 2: This figure plots the average monthly value weighted portfolio returns formed on aggregate dispersion loading. The dispersion loadings ,pre-ranking δs, are obtained using one year rolling regression of individual stock return on the market excess return and one-month lagged aggregate dispersion:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \delta_{i,t} \times Agg\_disp_{t-1} + \epsilon_{i,t}.$$

Panel A reports the sorting results using the full sample, Panel B shows the results using low dispersion months, Panel C presents the results using medium dispersion months and Panel D reports the results using high dispersion months.

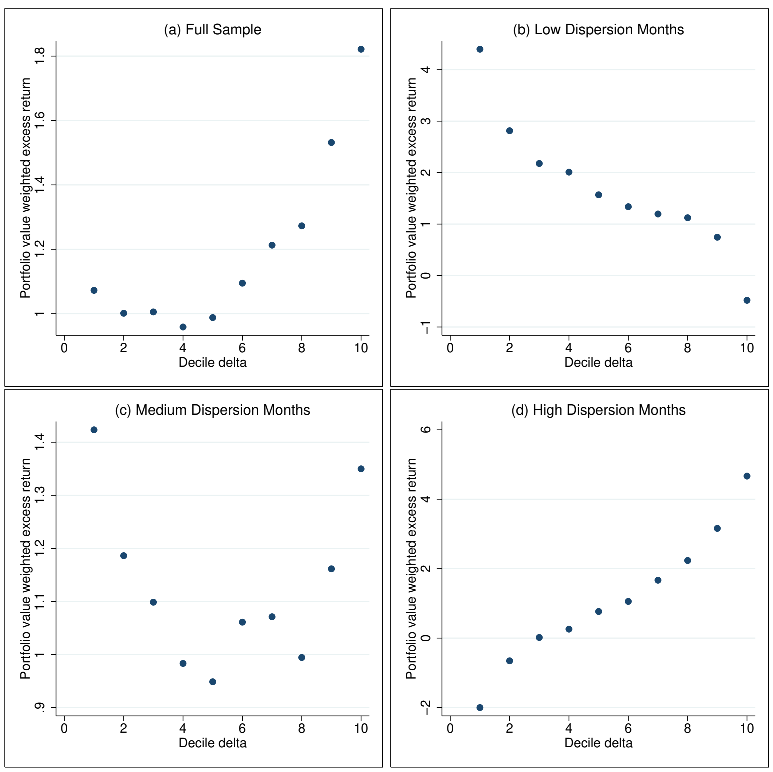


Figure 3: The figure plots the time series of the price of dispersion, ω_t, obtained form the following regression:
$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \epsilon_{t,P}$
where $P = 1, ..., 100$. R_t^P is the value weighted returns of stocks in the P^{th} $\beta - \delta$-sorted portfolio at t and β_P^{MKT} , δ_P, are the time series averages of the post-ranking portfolio loadings. Light grey shaded area refer to low dispersion periods and dark grey to high dispersion periods. Month t is considered low (high) dispersion month if the aggregate dispersion at $t-1$ is lower (higher) then the average aggregate dispersion minus (plus) one standard deviation.

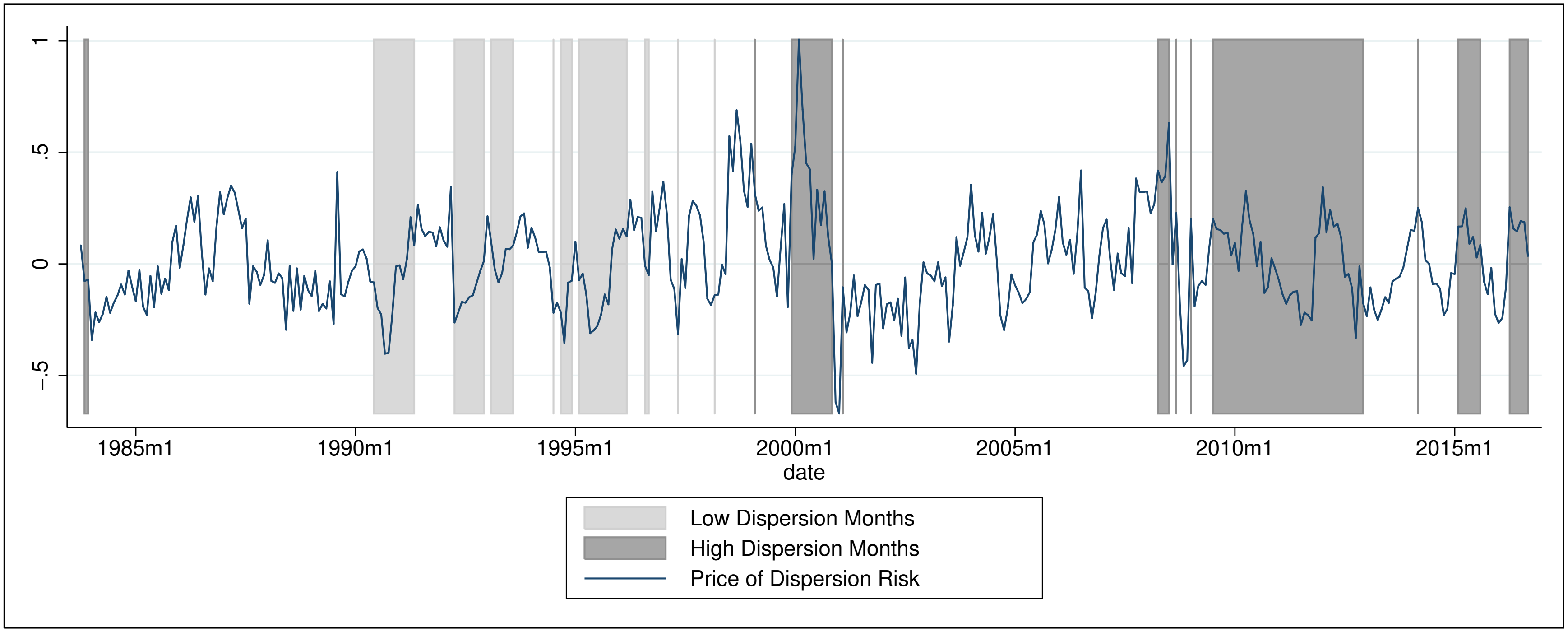
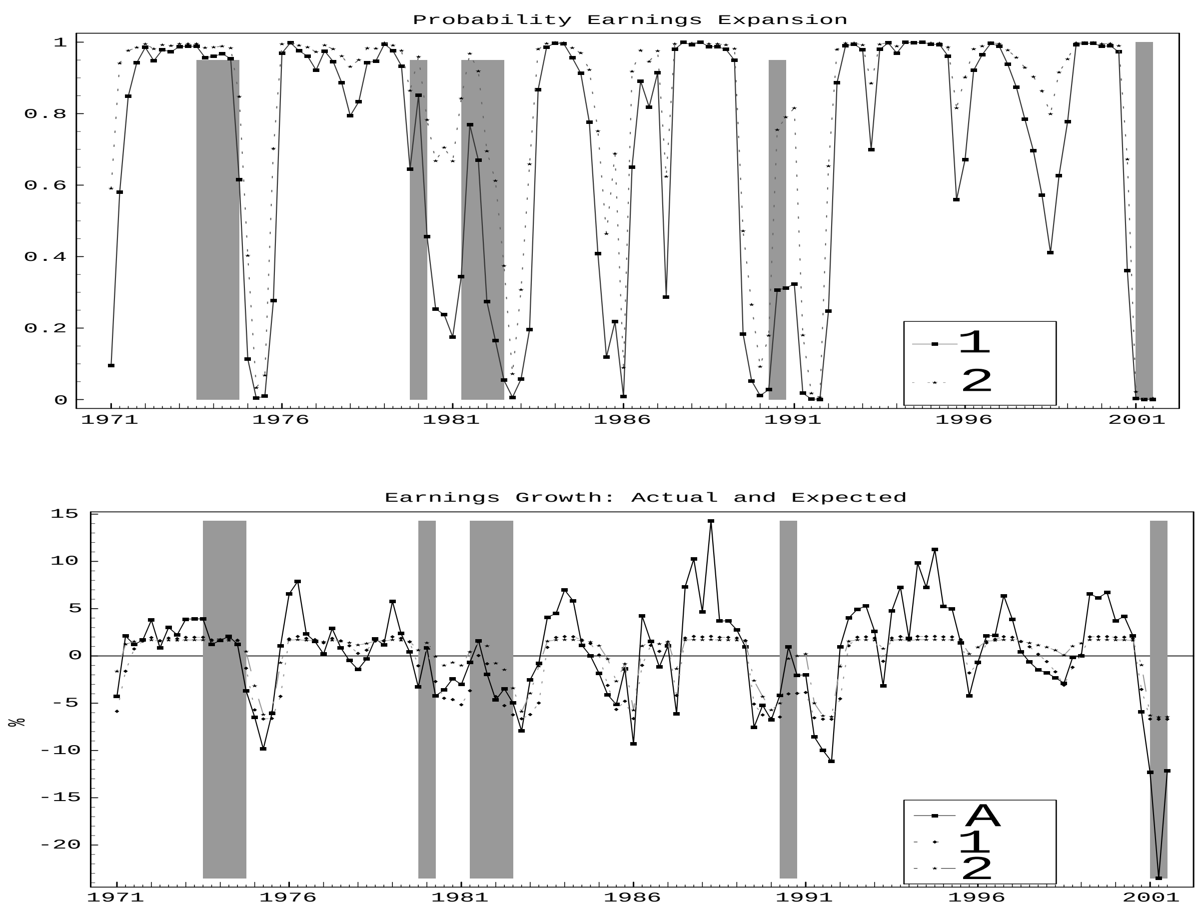
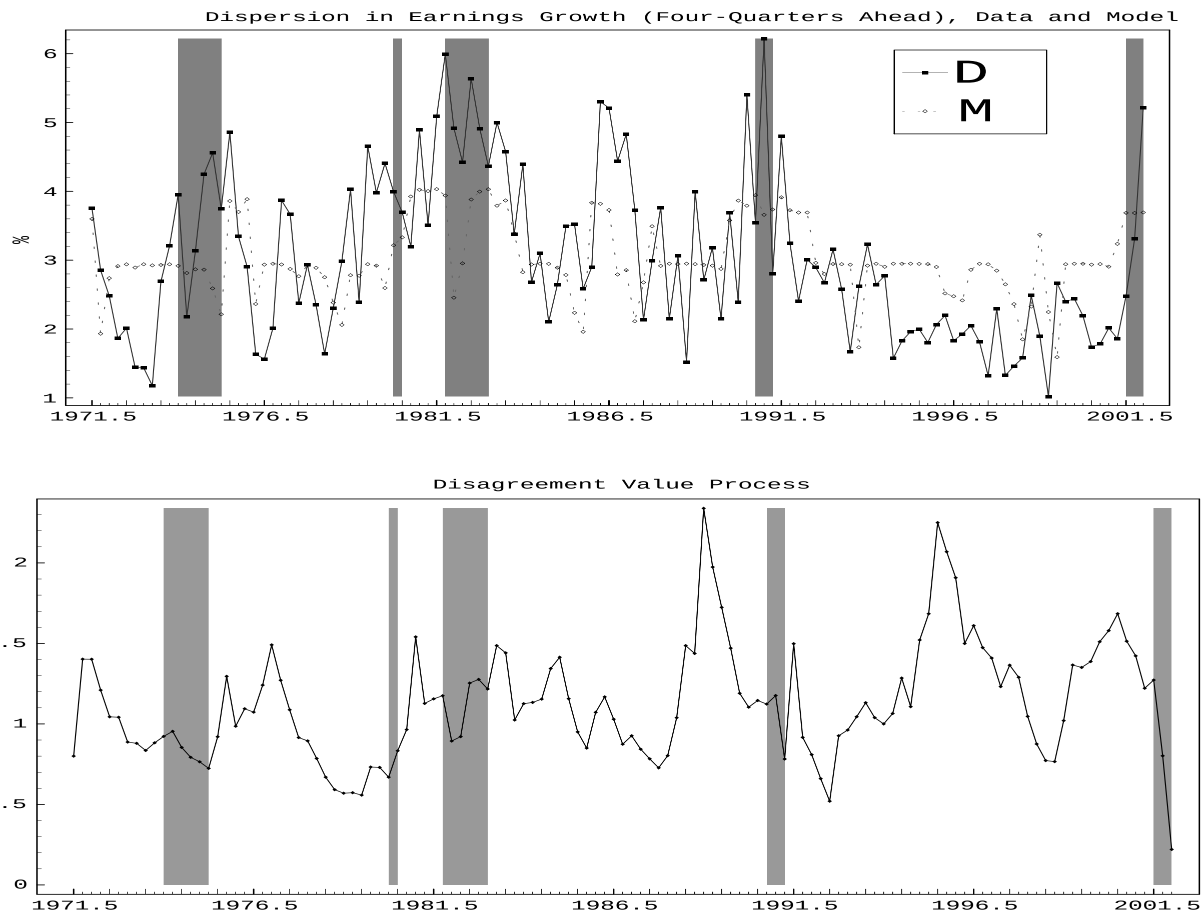


Figure 4: Investor Beliefs, Expected Growth Rates of Earnings From Calibrated Model (1971 – 2001)



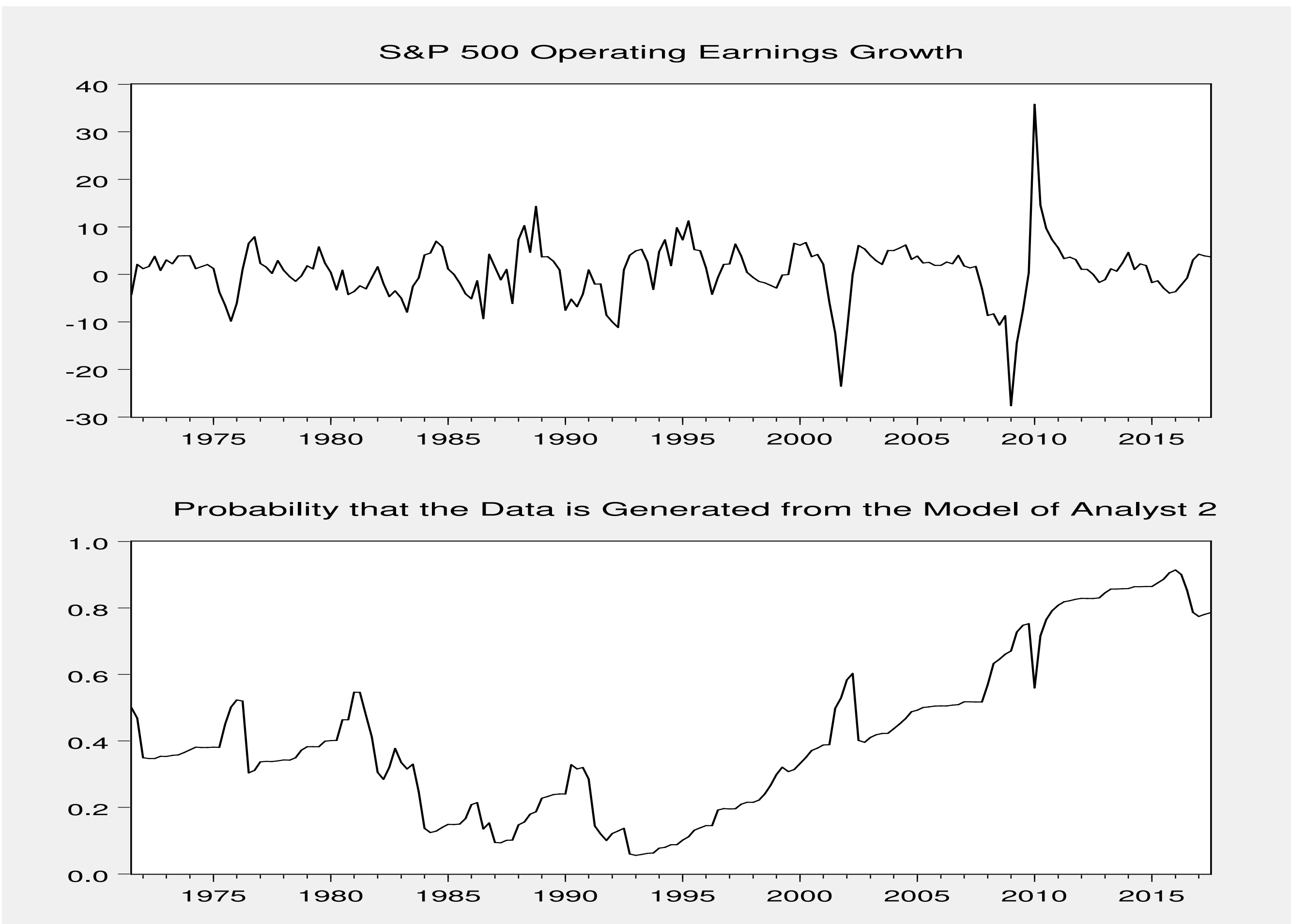
The top panel has the time series of filtered beliefs about real earnings growth of the two types of agents. Filtered beliefs of the two agents are obtained from the discretized version of the belief processes as shown in equation (5). The calibrated parameters for each type of agent shown in Table 7. The second panel displays the actual and expected earnings growth of the two types of agents using these filtered beliefs.

Figure 5: Dispersion in Earning Growth and the Disagreement Value Process (1971 - 2001)



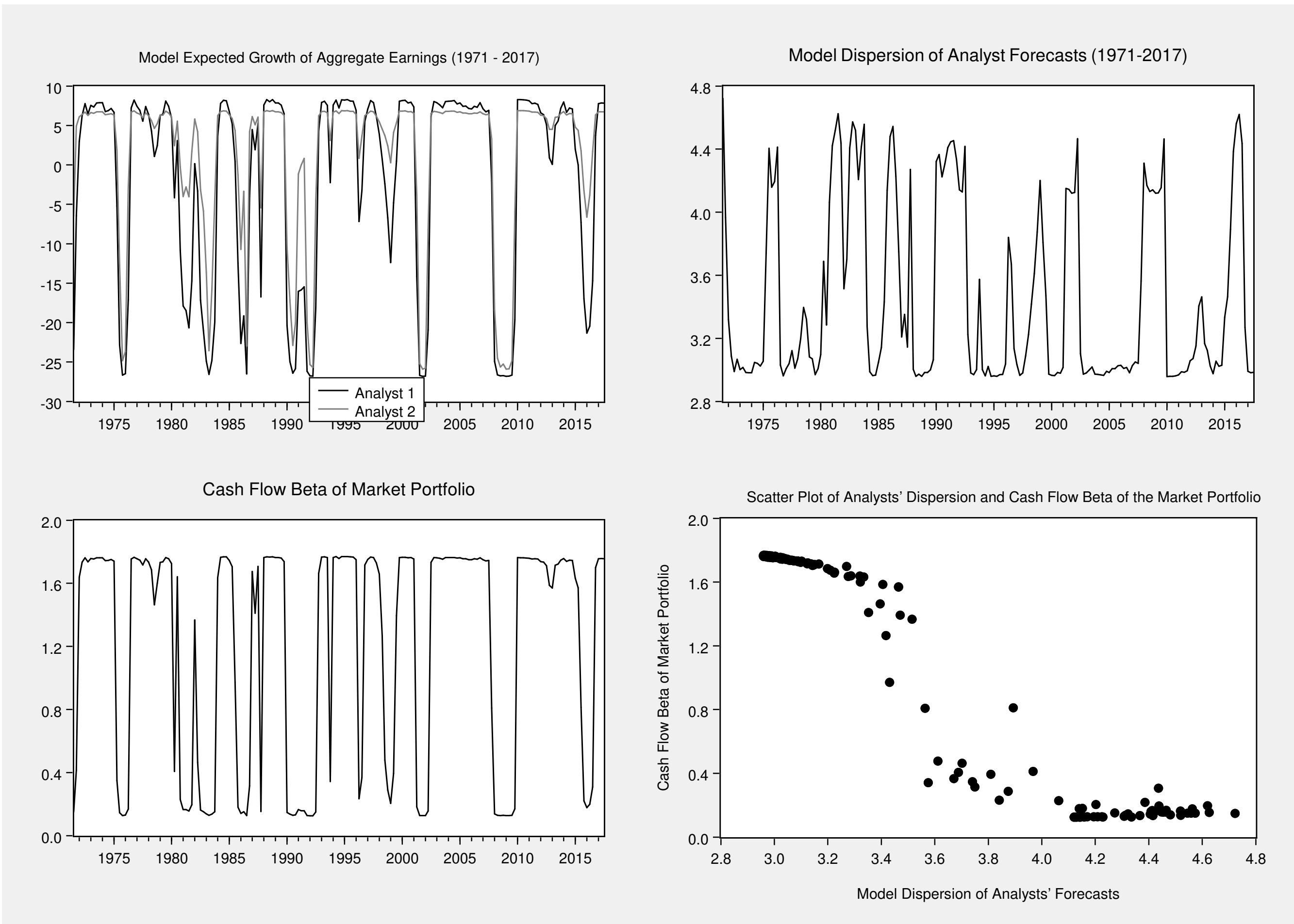
The top panel shows the dispersion of the analysts expectations in the calibrated model with parameters in Table 7, as well as the dispersion of forecasted growth from the Philadelphia Fed Survey of Professional Forecasters. The bottom panel shows the disagreement value η_t, which is formulated using the two analyst's beliefs using equation (11).

Figure 6: S&P 500 Operating Earnings Growth and the Weight Given to the Forecast of Analyst 2 (1971 - 2017)



Historical S&P 500 operating earnings growth is obtained from Standard and Poor's. The weight given to Analyst 2 is is formulated in equation. (14).

Figure 7: Model Forecasted Growth, Dispersion, and the Cash Flow Beta of the Market Portfolio, An (1971 - 2017)



Using the parameters in Table 7 and realized earnings growth from (1971-2017), we formulate the expected the 1-year ahead forecasted earnings growth of each analyst type, and then report the dispersion as the standard deviation of their forecasts. The cash flow beta of the market portfolio is formulated using the market portfolio weights in Proposition 1.

Figure 8: Market Prices of Risk From Simulated 2nd Stage Regression (1971 - 2017)

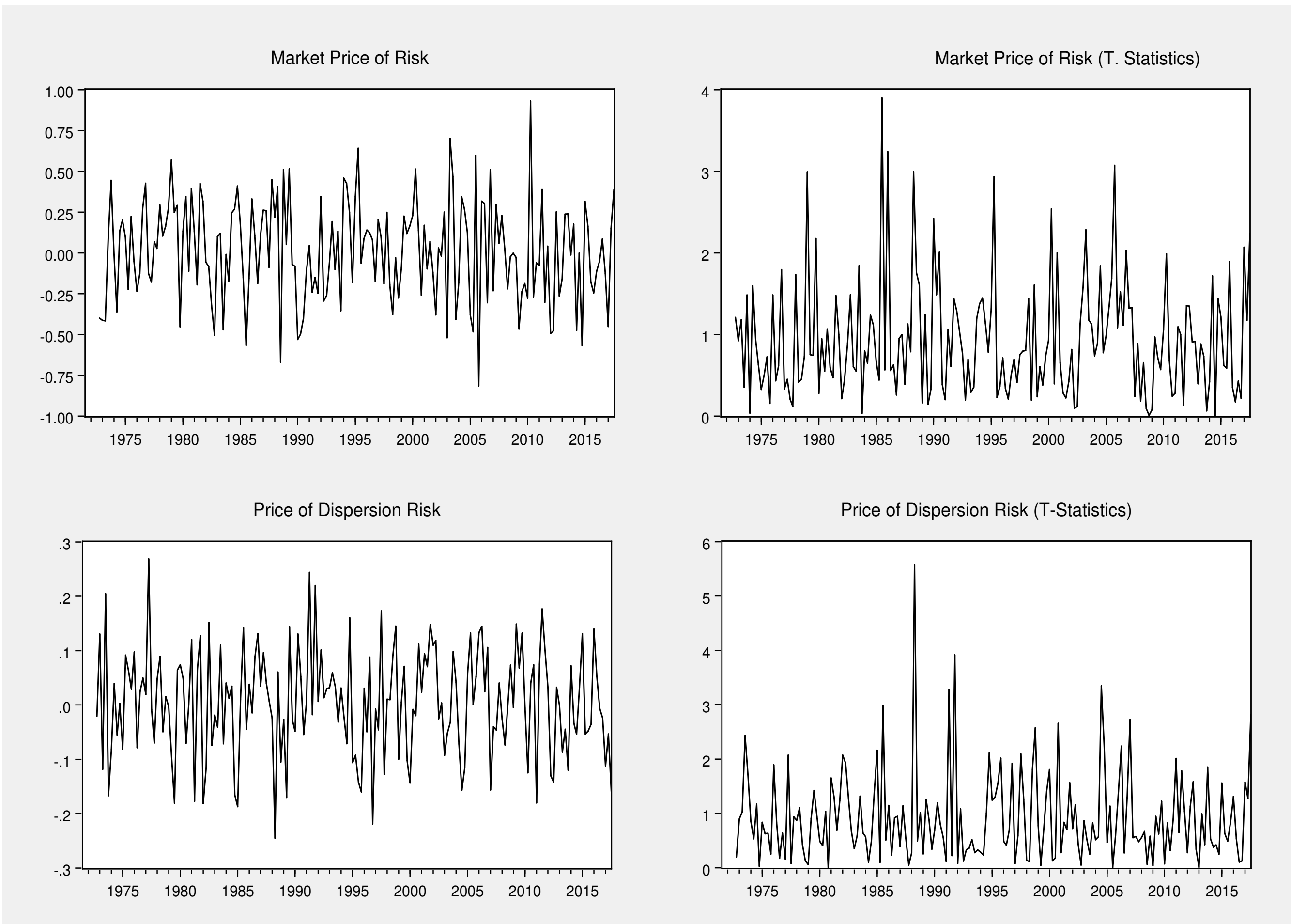
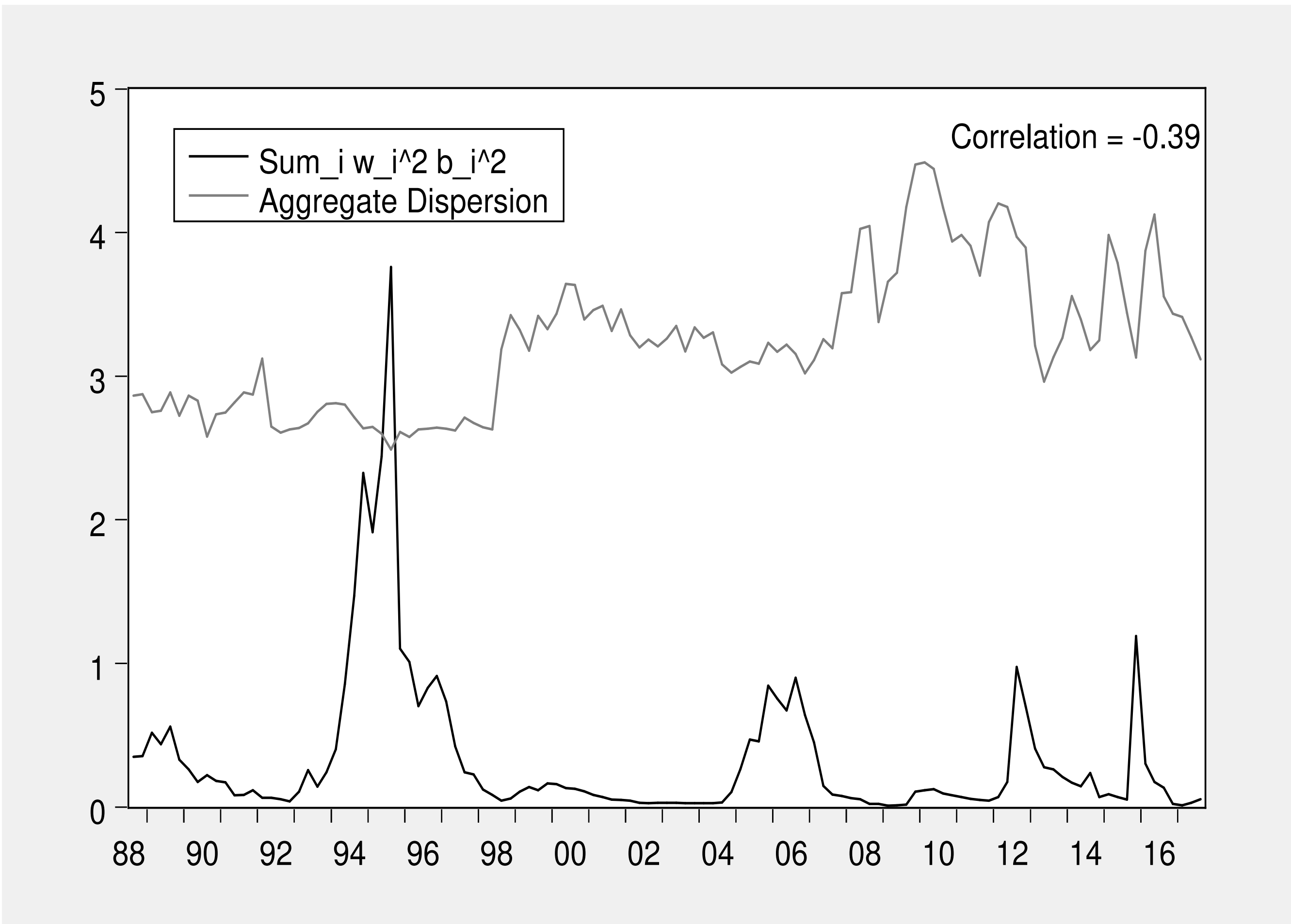


Figure 9: The Flight-to-Safety Effect in the Data



The dispersion series is the value-weighted average of the stock-level dispersion. For each stock, i, the cash flow beta, b_i is estimated using 5-year rolling windows of firm level earnings growth on aggregate earnings growth. The weight, w_i is the market capitalization weights of each stock in the CRSP database.

A Appendix

Table A.1: The table reports results from second-stage regression using observations from December 1981 to September 2016 for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from from Ken French's data web site. The table reports the results from:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \epsilon_{t,P}$$

, R_t^P is the value weighted Portfolio return at t, $\beta_P^{MKT}, isthetimeseriesaverageofthepost-rankingportfoliomarketriskloading.t$-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-0.794	-0.772	0.202	0.282	-0.691	-0.948	-0.236	-0.372	-0.828	-0.455
	(-3.24)	(-2.30)	(0.91)	(0.61)	(-3.37)	(-4.00)	(-1.71)	(-1.05)	(-1.40)	(-1.82)
Constant	1.588	1.571	0.484	0.450	1.493	1.740	0.943	1.084	1.504	1.190
	(6.00)	(4.07)	(1.97)	(1.00)	(6.72)	(6.71)	(6.60)	(3.19)	(2.39)	(4.51)
R^2	0.314	0.187	0.095	0.045	0.331	0.141	0.095	0.122	0.196	0.126
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	396	396	396	396	396	396	396	396	396	396

Table A.2: The table reports results from second-stage regression using observations from December 1981 to September 2016 for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from from Ken French's data web site. The table reports the results from:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \epsilon_{t,P}$$

. R_t^P is the value weighted Portfolio return at t, β_P^{MKT} and γ_P are the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-0.918	-1.168	-0.149	0.294	-0.671	-0.810	-0.183	-0.141	-2.035	0.0445
	(-2.52)	(-2.52)	(-0.67)	(0.63)	(-2.66)	(-3.09)	(-1.22)	(-0.19)	(-1.55)	(0.16)
$\bar{\omega}$	-0.954	-5.102	-6.196	4.808	1.365	3.931	2.240	2.775	-15.85	7.289
	(-0.25)	(-1.06)	(-2.69)	(1.02)	(0.43)	(1.43)	(1.26)	(0.39)	(-0.99)	(3.14)
Constant	1.714	1.982	0.836	0.414	1.470	1.607	0.879	0.852	2.747	0.657
	(4.42)	(3.89)	(3.53)	(0.91)	(5.51)	(5.75)	(5.62)	(1.14)	(2.02)	(2.26)
R^2	0.316	0.238	0.560	0.181	0.337	0.141	0.146	0.132	0.308	0.385
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	396	396	396	396	396	396	396	396	396	396

Table A.3: The table reports results from second-stage regression using observations from December 1981 to September 2016 for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from from Ken French's data web site. The table reports the results from

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P}$$

. R_t^P is the value weighted Portfolio return at t, β_P^{MKT}, β_P^{SMB}, β_P^{HML} and β_P^{umd} are the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-0.423	-0.423	0.752	2.059	0.0266	-0.903	-0.0985	-0.804	-0.572	1.559
	(-0.75)	(-0.57)	(1.72)	(3.65)	(0.06)	(-2.06)	(-0.36)	(-1.28)	(-0.68)	(3.14)
$\bar{\phi}^{SMB}$	-0.0388	0.000397	0.00969	0.0575	-0.123	0.181	-0.245	0.554	-1.075	-0.620
	(-0.52)	(0.00)	(0.14)	(0.19)	(-1.98)	(2.01)	(-1.61)	(1.70)	(-2.19)	(-3.23)
$\bar{\phi}^{HML}$	0.283	0.452	-0.200	-0.0196	0.553	-0.176	-0.180	0.400	-1.120	0.233
	(3.14)	(1.40)	(-0.49)	(-0.26)	(3.53)	(-0.61)	(-1.84)	(4.00)	(-2.72)	(3.35)
$\bar{\phi}^{UMD}$	1.676	4.045	-1.101	0.741	1.578	1.136	-0.0452	0.990	0.878	0.787
	(1.61)	(2.53)	(-1.04)	(1.99)	(1.92)	(1.96)	(-0.16)	(1.55)	(0.65)	(1.29)
Constant	1.159	1.181	-0.0560	-1.347	0.731	1.568	0.841	1.520	1.281	-0.855
	(2.00)	(1.53)	(-0.13)	(-2.40)	(1.66)	(3.43)	(3.22)	(2.43)	(1.54)	(-1.72)
R^2	0.434	0.383	0.746	0.838	0.565	0.127	0.327	0.901	0.862	0.641
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	396	396	396	396	396	396	396	396	396	396

Table A.4: The table reports results from second-stage regression using observations from December 1981 to September 2016 for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from from Ken French's data web site. The table reports the results from

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P}.$$

R_t^P is the value weighted Portfolio return at t, β_P^{MKT}, δ_P, β_P^{SMB}, β_P^{HML} and β_P^{umd} are the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-0.593	-0.972	0.856	2.097	0.0754	-0.421	-0.0923	-1.012	-1.907	1.583
	(-1.05)	(-1.36)	(1.61)	(2.72)	(0.17)	(-1.03)	(-0.35)	(-1.85)	(-1.62)	(3.01)
$\bar{\omega}$	-0.469	-5.122	0.525	-1.871	0.668	2.087	2.443	-2.120	-9.904	-1.281
	(-0.09)	(-0.90)	(0.10)	(-0.35)	(0.18)	(0.82)	(1.80)	(-0.54)	(-1.10)	(-0.45)
$\bar{\phi}^{SMB}$	-0.0218	0.00395	0.0506	0.117	-0.102	0.110	-0.217	0.543	-0.642	-0.585
	(-0.31)	(0.04)	(0.49)	(0.17)	(-1.66)	(1.23)	(-1.44)	(1.53)	(-1.26)	(-2.72)
$\bar{\phi}^{HML}$	0.301	0.482	-0.523	-0.0345	0.549	0.296	-0.192	0.422	-0.674	0.242
	(3.56)	(1.56)	(-0.85)	(-0.37)	(3.23)	(1.18)	(-2.00)	(4.36)	(-1.00)	(3.30)
$\bar{\phi}^{UMD}$	1.987	3.061	-1.398	0.667	1.745	1.695	-0.0886	1.081	0.0496	1.071
	(2.13)	(3.06)	(-1.12)	(1.55)	(2.64)	(3.49)	(-0.29)	(1.72)	(0.03)	(1.72)
Constant	1.322	1.722	-0.151	-1.385	0.672	1.073	0.821	1.724	2.595	-0.881
	(2.29)	(2.33)	(-0.28)	(-1.80)	(1.53)	(2.55)	(3.18)	(3.17)	(2.23)	(-1.67)
R^2	0.536	0.504	0.746	0.881	0.615	0.226	0.401	0.945	0.896	0.663
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	396	396	396	396	396	396	396	396	396	396

Table A.5: The table reports results from second-stage regression using using three sub-samples: low (Panel A), medium (Panel B) and high (Panel C) dispersion months for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from from Ken French's data web site. The table reports the results from:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \epsilon_{t,P}.$$

R_t^P is the value weighted Portfolio return at t, $\beta_P^{MKT}$is the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

Panel A: Low Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	0.164	0.756	1.181	-0.0554	0.418	0.183	0.452	0.108	0.348	0.193
	(0.23)	(1.03)	(1.32)	(-0.05)	(0.57)	(0.26)	(0.47)	(0.14)	(0.40)	(0.25)
Constant	1.122	0.433	0.0498	1.374	0.856	1.148	0.628	1.244	0.894	1.117
	(1.97)	(0.77)	(0.07)	(1.47)	(1.74)	(2.73)	(0.83)	(1.89)	(1.16)	(1.86)
R^2	0.173	0.121	0.232	0.187	0.193	0.081	0.133	0.174	0.208	0.194
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	51	51	51	51	51	51	51	51	51	267

Table A.6: *

—Continued

Panel B: Medium Dispersion Months										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-0.904	-0.781	-0.211	0.643	-0.802	-0.855	-0.0830	-0.393	-0.427	-0.692
	(-1.83)	(-1.54)	(-0.30)	(0.84)	(-1.84)	(-2.02)	(-0.20)	(-0.71)	(-0.86)	(-1.53)
Constant	1.599	1.489	0.851	0.0387	1.526	1.560	0.754	1.042	1.037	1.344
	(3.50)	(3.32)	(1.31)	(0.05)	(4.48)	(4.41)	(2.29)	(2.00)	(2.28)	(3.23)
R^2	0.231	0.190	0.332	0.203	0.207	0.109	0.116	0.222	0.210	0.180
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	267	267	267	267	267	267	267	267	267	267
Panel C: High Dispersion Months										
$\bar{\pi}$	0.00601	-0.641	1.341	-0.202	-0.170	-1.517	-0.405	-0.251	-2.352	-0.203
	(0.00)	(-0.37)	(0.81)	(-0.19)	(-0.11)	(-1.24)	(-0.51)	(-0.21)	(-1.73)	(-0.21)
Constant	0.727	1.371	-0.903	0.754	0.880	2.233	0.981	0.771	2.886	0.820
	(0.37)	(0.82)	(-0.58)	(0.87)	(0.66)	(2.04)	(1.70)	(0.72)	(2.39)	(0.95)
R^2	0.218	0.209	0.376	0.313	0.248	0.099	0.220	0.221	0.220	0.166
Num. time period	78	78	78	78	78	78	78	78	78	78

Table A.7: The table reports results from second-stage regression using using three sub-samples: low (Panel A), medium (Panel B) and high (Panel C) dispersion months for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. We obtain value weighted monthly returns on these portfolios from from Ken French's data web site. The table reports the results from:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \epsilon_{t,P}$$

. R_t^P is the value weighted Portfolio return at t, β_P^{MKT}, δ_P are the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

Panel A: Low Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-0.00770	0.0723	0.648	-0.0549	0.184	-0.114	0.828	0.103	0.0861	0.188
	(-0.01)	(0.08)	(0.50)	(-0.05)	(0.20)	(-0.15)	(0.80)	(0.13)	(0.10)	(0.24)
$\bar{\omega}$	-1.648	-6.121	-6.572	-0.759	-3.345	-4.687	-7.701	0.980	-4.640	-0.242
	(-0.64)	(-2.33)	(-0.86)	(-0.22)	(-0.79)	(-1.98)	(-2.28)	(0.24)	(-1.21)	(-0.08)
Constant	1.329	1.233	0.665	1.385	1.139	1.496	0.397	1.251	1.174	1.124*
	(2.71)	(1.85)	(0.62)	(1.41)	(1.75)	(3.21)	(0.49)	(1.78)	(1.58)	(2.07)
R^2	0.272	0.197	0.549	0.445	0.310	0.120	0.203	0.373	0.330	0.317
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	51	51	51	51	51	51	51	51	51	267

Table A.8: *

—Continued

Panel B: Medium Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-1.151*	-1.220	-0.500	0.772	-0.843	-0.871*	0.0880	-1.415	-0.697	-0.185
	(-2.00)	(-1.89)	(-0.49)	(1.02)	(-1.83)	(-2.01)	(0.22)	(-1.36)	(-0.77)	(-0.39)
$\bar{\omega}$	-3.377	-6.337	-4.072	-6.494	-0.328	0.423	3.596	-8.698	-2.803	6.647**
	(-0.98)	(-1.60)	(-0.60)	(-1.21)	(-0.12)	(0.23)	(1.50)	(-1.33)	(-0.32)	(2.61)
Constant	1.834***	1.912**	1.128	-0.0709	1.560***	1.571***	0.573	2.067*	1.308	0.833
	(3.41)	(3.28)	(1.17)	(-0.09)	(4.37)	(4.37)	(1.87)	(2.04)	(1.51)	(1.95)
R^2	0.283	0.265	0.551	0.303	0.276	0.135	0.170	0.317	0.313	0.241
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	267	267	267	267	267	267	267	267	267	267

Panel C: High Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	0.717	-0.232	3.022	-0.475	0.183	-0.981	-0.328	0.384	-2.262	-0.129
	(0.51)	(-0.17)	(1.47)	(-0.47)	(0.13)	(-0.81)	(-0.43)	(0.35)	(-1.69)	(-0.14)
$\bar{\omega}$	15.87	15.09	41.97	21.14	20.79	-13.69	8.186	33.74	33.53**	10.92
	(0.73)	(0.90)	(1.58)	(1.17)	(1.74)	(-1.41)	(0.75)	(1.08)	(2.77)	(0.65)
Constant	-0.0941	0.910	-2.691	0.898	0.493	1.657	0.907	0.0215	2.677*	0.667
	(-0.08)	(0.74)	(-1.37)	(1.04)	(0.40)	(1.58)	(1.68)	(0.02)	(2.28)	(0.86)
R^2	0.377	0.248	0.440	0.500	0.336	0.111	0.268	0.407	0.303	0.283
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	78	78	78	78	78	78	78	78	78	78

Table A.9: The table reports results from second-stage regression using using three sub-samples: low (Panel A), medium (Panel B) and high (Panel C) dispersion months for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. the table reports the results from:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P}.$$

R_t^P is the value weighted Portfolio return at t, β_P^{MKT}, β_P^{SMB}, β_P^{HML} and β_P^{umd} are the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

Panel A: Low Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	0.496	0.768	1.348	1.311	0.855	0.662	1.678	-0.350	1.226	0.739
	(0.95)	(1.19)	(1.36)	(0.95)	(1.62)	(1.15)	(1.52)	(-0.29)	(0.79)	(1.03)
$\bar{\phi}^{SMB}$	-0.174	-0.126	-0.186	-0.467	-0.165	-0.269	-0.823	0.630	-0.0725	-0.192
	(-0.52)	(-0.36)	(-0.57)	(-0.77)	(-0.46)	(-0.81)	(-1.80)	(0.72)	(-0.11)	(-0.45)
$\bar{\phi}^{HML}$	0.00618	-0.425	0.316	-0.0404	-0.178	-0.253	-0.340	0.468	0.482	-0.00167
	(0.02)	(-1.23)	(0.41)	(-0.13)	(-0.43)	(-0.78)	(-0.91)	(1.21)	(0.40)	(-0.01)
$\bar{\phi}^{UMD}$	0.867	1.482	0.837	0.639	1.480	0.339	0.435	0.863	2.183	0.330
	(1.72)	(2.62)	(1.78)	(1.01)	(2.85)	(1.06)	(0.97)	(0.96)	(1.16)	(0.65)
Constant	0.882	0.590	0.0254	0.00779	0.520	0.826	-0.506	1.706	0.0716	0.574
	(1.90)	(1.01)	(0.03)	(0.01)	(1.04)	(1.94)	(-0.55)	(1.30)	(0.05)	(0.94)
R^2	0.582	0.471	0.757	0.621	0.550	0.255	0.319	0.688	0.569	0.428
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	51	51	51	51	51	51	51	51	51	267

Table A.10: *

—Continued

Panel B: Medium Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-1.076	-0.511	0.427	2.658	-0.956	-0.820	-0.0619	-1.349	-1.273	1.327
	(-2.12)	(-0.88)	(0.51)	(3.11)	(-2.06)	(-1.92)	(-0.12)	(-1.18)	(-1.21)	(2.02)
$\bar{\phi}^{SMB}$	-0.107	-0.160	-0.0814	0.440	-0.140	-0.0717	-0.197	0.866	-0.600	-0.681
	(-0.64)	(-0.85)	(-0.47)	(0.73)	(-0.80)	(-0.44)	(-0.68)	(1.92)	(-1.26)	(-2.38)
$\bar{\phi}^{HML}$	0.262	0.395	-0.00966	-0.139	0.290	0.142	-0.113	0.409	-0.625	0.240
	(1.65)	(1.32)	(-0.02)	(-0.70)	(1.24)	(0.64)	(-0.56)	(1.70)	(-1.37)	(1.37)
$\bar{\phi}^{UMD}$	0.476	0.745	-0.0770	0.344	-0.548	0.647	-0.0766	1.865	-1.191	0.530
	(0.77)	(1.01)	(-0.10)	(0.71)	(-0.95)	(2.10)	(-0.16)	(1.65)	(-0.96)	(1.14)
Constant	1.736	1.210	0.228	-2.007	1.649	1.497	0.763	2.014	1.903	-0.682
	(4.24)	(2.28)	(0.29)	(-2.36)	(4.32)	(4.38)	(1.61)	(1.85)	(1.99)	(-1.10)
R^2	0.531	0.469	0.766	0.605	0.541	0.246	0.325	0.592	0.518	0.401
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	267	267	267	267	267	267	267	267	267	267

Panel C: High Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	0.477	-2.008	1.039	0.473	0.566	0.0812	-0.771	0.522	-0.250	0.273
	(0.54)	(-2.01)	(0.59)	(0.22)	(0.45)	(0.10)	(-0.80)	(0.26)	(-0.15)	(0.23)
$\bar{\phi}^{SMB}$	0.189	0.483	0.292	1.678	0.0201	1.674	-0.0162	-0.597	-0.696	-0.0413
	(0.32)	(0.85)	(0.65)	(1.36)	(0.04)	(3.44)	(-0.03)	(-0.32)	(-0.62)	(-0.07)
$\bar{\phi}^{HML}$	0.464	0.119	0.234	0.0356	1.326	-1.745	-0.201	0.308	-1.472	0.362
	(0.99)	(0.23)	(0.27)	(0.08)	(2.69)	(-3.20)	(-0.37)	(0.52)	(-2.04)	(0.75)
$\bar{\phi}^{UMD}$	1.871	4.708	2.104	2.160	2.691	-3.686	-0.792	0.665	3.442	1.444
	(2.25)	(3.13)	(0.74)	(2.26)	(2.53)	(-2.78)	(-0.69)	(0.71)	(2.35)	(1.52)
Constant	0.0542	2.489	-0.624	0.0209	-0.0826	-0.269	1.279	-0.00517	0.640	0.261
	(0.07)	(3.11)	(-0.37)	(0.01)	(-0.08)	(-0.36)	(1.68)	(-0.00)	(0.42)	(0.24)
R^2	0.557	0.437	0.745	0.629	0.560	0.236	0.377	0.626	0.541	0.460
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	78	78	78	78	78	78	78	78	78	78

Table A.11: The table reports results from second-stage regression using using three sub-samples: low (Panel A), medium (Panel B) and high (Panel C) dispersion months for ten test assets: (1) 25 Value Size Portfolios, (2) 25 Net Share Issues Size Portfolios , (3) 10 Size Portfolios, (4) 10 Value Portfolios, (5) 25 Size Investment Portfolios, (6) 100 Size Operating Prof Portfolios, (7) 30 Industry Portfolios, (8) 10 E/P Portfolios, (9) 10 Net Share Issues Portfolios and (10) 25 Book-to-Market and Operating Profitability Portfolios. The table reports the results from:

$$R_t^P = \kappa_t + \pi_t \times \beta_P^{MKT} + \omega_t \times \delta_P + \phi_t^{SMB} \times \beta_P^{SMB} + \phi_t^{HML} \times \beta_P^{HML} + \phi_t^{UMD} \times \beta_P^{UMD} + \epsilon_{t,P}.$$

R_t^P is the value weighted Portfolio return at t, β_P^{MKT}, δ_P, β_P^{SMB}, β_P^{HML} and β_P^{umd} are the time series average of the post-ranking portfolio risk loadings. t-statistics are reported in parenthesis and are corrected for estimation error as formulated by Shanken (1992)

Panel A: Low Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	0.184	0.264	0.941	1.220	0.531	0.386	2.496*	-0.601	0.641	0.928
	(0.34)	(0.44)	(0.93)	(0.85)	(1.16)	(0.67)	(2.09)	(-0.53)	(0.65)	(1.07)
$\bar{\omega}$	-2.042	-2.821	-5.151	-1.900	-0.00406	-2.356	-10.56***	1.233	-9.434*	-2.914
	(-1.24)	(-1.58)	(-1.42)	(-0.61)	(-0.00)	(-1.57)	(-3.97)	(0.44)	(-2.17)	(-1.50)
$\bar{\phi}^{SMB}$	-0.159	-0.127	0.104	-0.446	-0.0916	-0.219	-0.628	1.034	-0.347	-0.412
	(-0.46)	(-0.35)	(0.28)	(-0.81)	(-0.24)	(-0.66)	(-1.44)	(1.38)	(-0.63)	(-1.11)
$\bar{\phi}^{HML}$	0.0664	-0.350	-1.765	-0.0319	-0.142	-0.393	-0.522	0.478	1.191	0.00487
	(0.23)	(-1.01)	(-1.37)	(-0.10)	(-0.36)	(-1.33)	(-1.45)	(1.26)	(1.31)	(0.02)
$\bar{\phi}^{UMD}$	1.243*	1.477**	0.378	0.639	1.231*	0.490	1.166*	1.222	1.181	0.249
	(2.34)	(2.99)	(0.80)	(1.08)	(2.51)	(1.70)	(2.48)	(1.37)	(0.71)	(0.56)
Constant	1.179*	1.096	0.408	0.1000	0.817	1.075*	-1.180	1.980	0.613	0.378
	(2.48)	(1.95)	(0.47)	(0.07)	(1.75)	(2.61)	(-1.18)	(1.63)	(0.62)	(0.56)
R^2	0.624	0.516	0.830	0.714	0.585	0.279	0.361	0.734	0.673	0.475
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	51	51	51	51	51	51	51	51	51	51

Table A.12: *

—Continued

Panel B: Medium Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	-1.407**	-0.938	0.732	2.629**	-0.858	-0.746	-0.00291	-2.110	-3.795**	1.297
	(-2.78)	(-1.70)	(0.93)	(3.05)	(-1.90)	(-1.80)	(-0.01)	(-1.62)	(-2.75)	(1.93)
$\bar{\omega}$	-5.989	-6.080*	2.438	-6.809	-0.0798	-0.631	2.986	-0.486	-13.67	1.293
	(-1.57)	(-2.22)	(0.37)	(-1.24)	(-0.03)	(-0.48)	(1.38)	(-0.09)	(-1.60)	(0.43)
$\bar{\phi}^{SMB}$	-0.0763	-0.0843	-0.0877	-0.0159	-0.138	-0.0577	-0.133	1.174*	0.303	-0.607*
	(-0.46)	(-0.46)	(-0.50)	(-0.02)	(-0.80)	(-0.37)	(-0.46)	(2.44)	(0.53)	(-2.03)
$\bar{\phi}^{HML}$	0.284	0.521	-0.0486	-0.0837	0.298	0.132	-0.151	0.435	-0.159	0.215
	(1.76)	(1.65)	(-0.11)	(-0.40)	(1.24)	(0.63)	(-0.76)	(1.66)	(-0.32)	(1.20)
$\bar{\phi}^{UMD}$	0.705	1.762*	0.0852	-0.102	-0.104	0.891*	-0.162	2.495*	0.121	0.585
	(1.19)	(2.33)	(0.11)	(-0.15)	(-0.17)	(2.49)	(-0.32)	(2.09)	(0.10)	(1.27)
Constant	2.051***	1.617**	-0.0692	-1.979*	1.558***	1.412***	0.697	2.768*	4.368**	-0.653
	(4.79)	(3.21)	(-0.10)	(-2.35)	(4.12)	(4.10)	(1.53)	(2.18)	(3.29)	(-1.04)
R^2	0.582	0.505	0.830	0.683	0.599	0.272	0.363	0.683	0.623	0.443
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	267	267	267	267	267	267	267	267	267	267

Panel C: High Dispersion Months	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\bar{\pi}$	0.338	-1.530	1.266	0.976	-0.349	0.831	-0.474	0.922	-0.991	0.157
	(0.38)	(-1.72)	(0.70)	(0.45)	(-0.34)	(1.07)	(-0.51)	(0.36)	(-0.61)	(0.15)
$\bar{\omega}$	13.56	19.96	26.81	18.62	23.83*	-42.23*	11.39	12.42	34.38*	11.92
	(1.10)	(1.65)	(1.84)	(0.68)	(2.37)	(-2.57)	(1.14)	(0.35)	(2.18)	(1.05)
$\bar{\phi}^{SMB}$	0.197	0.465	0.360	1.564	0.142	0.446	-0.128	-1.209	-1.277	-0.0184
	(0.34)	(0.83)	(0.82)	(0.84)	(0.26)	(1.06)	(-0.27)	(-0.47)	(-1.04)	(-0.03)
$\bar{\phi}^{HML}$	0.477	0.195	0.359	0.00531	0.891	1.195*	-0.212	0.178	-1.644*	0.406
	(1.08)	(0.40)	(0.46)	(0.01)	(1.64)	(2.49)	(-0.38)	(0.27)	(-2.38)	(0.91)
$\bar{\phi}^{UMD}$	1.870*	4.138**	3.147	2.184*	3.027**	-4.722**	-0.398	0.797	2.990*	1.239
	(2.48)	(3.31)	(1.14)	(2.00)	(2.88)	(-3.07)	(-0.43)	(0.74)	(2.14)	(1.28)
Constant	0.185	1.931*	-0.941	-0.468	0.813	-0.710	0.989	-0.393	1.389	0.352
	(0.25)	(2.64)	(-0.53)	(-0.23)	(0.98)	(-0.93)	(1.34)	(-0.16)	(0.91)	(0.37)
R^2	0.583	0.467	0.824	0.729	0.602	0.299	0.443	0.704	0.601	0.512
Num. portfolios	25	25	10	10	25	100	30	10	10	25
Num. time period	78	78	78	78	78	78	78	78	78	78

Appendix 2

Proof of Proposition 1

The following moments are used to formulate the value function of consumers below:

$$E[\frac{dX_{it}}{X_{it}}\, d\pi_{1t}^{(m)}] = b_i\, \sigma_{1t}^{(m)}\, \sigma_Y\, dt, \qquad \text{for } i=1,\cdots,N, m=1,2 \tag{28}$$

$$E[\left(\frac{dX_{it}}{X_{it}}\right)^2] = b_i^2\, \sigma_Y^2 + \sigma_i^2, \qquad \text{for } \ i=1,\cdots,N, \tag{29}$$

$$E[d\varrho_t\, d\pi_{1t}^{(m)}] = -\varrho_t(1-\varrho_t)\sigma_{\eta t}\sigma_{1t}^{(m)}, \text{for } \ m=1,2. \tag{30}$$

The value function $J(W_t, \pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t, t)$ under the measure of analyst 1 satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}
0 = \max_{C, w_i, s.t. w_i \geq 0, \sum_{i=1}^N w_i = 1} \Bigg[& U(C) + J_t - J_W C \\
& + J_W W \left(\sum_{i=1}^N w_i \alpha_i\right) + J_{\pi^{(1)}}\mu_1^{(1)} + J_{\pi^{(2)}}(\mu_1^{(2)} + \sigma_1^{(2)}\sigma_\eta) + J_\varrho \varrho(1-\varrho)^2\sigma_\eta \\
& + J_{W\pi^{(1)}} W \left(\sum_{i=1}^N w_i b_i\right) \sigma_1^{(1)}\sigma_Y + J_{W\pi^{(2)}} W \left(\sum_{i=1}^N w_i b_i\right) \sigma_1^{(2)}\sigma_Y - J_{W\varrho} W \left(\sum_{i=1}^N w_i b_i\right) \varrho(1-\varrho)\sigma_\eta \\
& + \frac{1}{2} J_{\pi_1^{(1)}\pi_1^{(1)}}(\sigma_1^{(1)})^2 + \frac{1}{2} J_{\pi_1^{(2)}\pi_1^{(2)}}(\sigma_1^{(2)})^2 + \frac{1}{2} J_{\varrho\varrho}\varrho^2(1-\varrho)^2\sigma_\eta^2 \\
& - J_{\pi^{(1)}\varrho}\sigma_1^{(1)}\varrho(1-\varrho)\sigma_\eta - J_{\pi^{(2)}\varrho}\sigma_1^{(2)}\varrho(1-\varrho)\sigma_\eta + J_{\pi^{(1)}\pi^{(2)}}\sigma_1^{(1)}\sigma_1^{(2)} \\
& + \frac{1}{2} J_{WW} W^2 \left(\sum_{i=1}^N w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2)\right) \Bigg].
\end{aligned} \tag{31}$$

The envelope optimality condition for consumption is:

$$U_C = J_W \tag{32}$$

Given the CRRA preference of the representative consumer, her value function takes the form:

$$J(W_t, \pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t, t) = \exp(-\rho t)\frac{W_t^\gamma}{\gamma} I(\pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t). \tag{33}$$

The partial derivatives of J therefore satisfy: $J_t = -\rho J$; $J_W = (\gamma J)/W$; $J_{WW} = (\gamma(\gamma - 1)J)/W^2$; $J_{\pi_1^{(m)}} = (I_{\pi_1^{(m)}}/I)J$, for $m = 1,2$; $J_{\pi_1^{(m)}\pi_1^{(m)}} = (I_{\pi_1^{(m)}\pi_1^{(m)}}/I)J$, for $m = 1,2$; $J_\varrho = (I_\varrho/I)J$, $J_{\varrho\varrho} = (I_{\varrho\varrho}/I)J$; $J_{W\pi_1^{(m)}} = (\gamma J)/W(I_{\pi_1^{(m)}}/I)$, for $m = 1,2$; $J_{W\varrho} = (\gamma J)/W(I_\varrho/I)$.

Substituting these and the optimality condition for consumption (32) into the HJB equation (31) implies the PDE in (20). The Kuhn-Tucker first-order conditions for the portfolio choices of the consumer follow. ■

Proof of Proposition 2

First consider the derived utility of wealth function in the decentralized economy. By following the same steps as for the central planner's problem, the function $I(\pi_{1t}^{(1)}, \pi_{1t}^{(2)}, \varrho_t)$ satisfies the PDE:

$$\begin{aligned}
0 = \max_{w_i, s.t. w_i \geq 0, \text{for } i=1,\cdots,N,\ \sum_{i=0}^{N} w_i = 1} \Bigg[& \left(\frac{1}{\gamma} - 1\right) I^{\frac{\gamma}{\gamma-1}} - \frac{\rho}{\gamma} I \\
& + I \left(w_0\, r + \sum_{i=1}^{N} w_i b_i ((1-\varrho)\bar{\nu}^{(1)} + \varrho \bar{\nu}^{(2)}) + \frac{1}{2}(\gamma - 1) \sum_{i=1}^{N} w_i^2 (b_i^2 \sigma_Y^2 + \sigma_i^2) \right) \\
& + I_{\pi_1^{(1)}} \left(\frac{\mu_1^{(1)}}{\gamma} + (\sum_{i=1}^{N} w_i b_i) \sigma_1^{(1)} \sigma_Y \right) + I_{\pi_1^{(2)}} \left(\frac{\mu_1^{(2)} + \sigma_2^{(2)} \sigma_\eta}{\gamma} + (\sum_{i=1}^{N} w_i b_i) \sigma_1^{(2)} \sigma_Y \right) \\
& + I_\varrho \left(\frac{\varrho (1-\varrho)^2 \sigma_\eta}{\gamma} - (\sum_{i=1}^{N} w_i b_i) \varrho (1-\varrho) \sigma_\eta \right) \\
& \frac{1}{2} I_{\pi_1^{(1)} \pi_1^{(1)}} (\sigma_1^{(1)})^2 + \frac{1}{2} I_{\pi_1^{(2)} \pi_1^{(2)}} (\sigma_1^{(2)})^2 + \frac{1}{2} I_{\varrho\varrho} \varrho^2 (1-\varrho)^2 \sigma_\eta^2 \\
& - I_{\pi^{(1)}\varrho} \sigma_1^{(1)} \varrho (1-\varrho) \sigma_\eta - I_{\pi^{(2)}\varrho} \sigma_1^{(2)} \varrho (1-\varrho) \sigma_\eta + I_{\pi^{(1)}\pi^{(2)}} \sigma_1^{(1)} \sigma_1^{(2)} \Bigg]. \quad (34)
\end{aligned}$$

where now w_0 is the portfolio choice in the riskless asset. We allow for w_0 to be positive (riskless lending) or negative (riskless borrowing). The first order condition for w_0 is

$$r = \frac{\lambda^{(1)}}{I}. \quad (35)$$

Now set the optimal choices for w_i for $i = 1, \cdots, N$, as for the central planner, and in addition, let r satisfy (35) Then, $w_0 = 0$ is optimal in the decentralized choice. Now, summing (22) over $i = 1, \cdots, N$, and using (23) and (25) implies that

$$\begin{aligned}
\left(\sum_{i=1}^{N} w_i b_i \right) \left[((1-\varrho)\bar{\nu}^{(1)} + \varrho \bar{\nu}^{(2)}) + \frac{I_{\pi_1^{(1)}}}{I} \sigma_1^{(1)} \sigma_Y + \frac{I_{\pi_1^{(2)}}}{I} \sigma_1^{(2)} \sigma_Y - \frac{I_\varrho}{I} \varrho (1-\varrho) \sigma_\eta \sigma_Y \right] \\
+ (\gamma - 1) \sum_{i=1}^{N} w_i^2 (b_i^2 \sigma_Y + \sigma_i^2) = \frac{\lambda^{(1)}}{I} = r \quad (36)
\end{aligned}$$

Now using (21), which holds with equality for any asset with $w_i > 0$, and (36), implies (26) once we recognize that $\alpha_i = b_i((1-\varrho)\bar{\nu}^{(1)} + \varrho \bar{\nu}^{(2)})$. ■

References

Abel, Andrew B., 1989, *Asset Prices under Heterogeneous Beliefs: Implications for the Equity Premium* (Rodney L. White Center for Financial Research).

Anderson, Evan W., Eric Ghysels, and Jennifer L. Juergens, 2005, Do heterogeneous beliefs matter for asset pricing, *Review of Financial Studies* 18, 875–924.

———, 2009, The impact of risk and uncertainty on expected return, *Journal of Financial Economics* 94, 233–263.

Baker, Steven D., Burton Hollifield, and Emilio Osambela, 2016, Disagreement, speculation, and aggregate investment, *Journal of Financial Economics* 119, 210–225.

Basak, Suleyman, 2000, A model of dynamic equilibrium asset pricing with heterogeneous beliefs and extraneous risk, *Journal of Economic Dynamics and Control* 24, 63–95.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, *The journal of business* 45, 444–455.

Burashi, Andrea, Fabio Trojani, and Andrea Vedolin, 2014, When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia, *Journal of Finance* 69, 101–137.

Carlin, Bruce I, Francis A Longstaff, and Kyle Matoba, 2014, Disagreement and asset prices, *Journal of Financial Economics* 114, 226–238.

Chen, Joseph, Harrison Hong, and Jeremy C Stein, 2002, Breadth of ownership and stock returns, *Journal of financial Economics* 66, 171–205.

Cochrane, John H, 2009, *Asset Pricing:(Revised Edition)* (Princeton university press).

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross, 1985, An intertemporal general equilibrium model of asset prices, *Econometrica* 53, 363–384.

David, Alexander, 1997, Fluctuating confidence in stock markets: Implications for returns and volatility, *Journal of Financial and Quantitative Analysis* 32, 427–462.

———, 2008, Heterogeneous beliefs, speculation, and the equity premium, *Journal of Finance* 63, 41–84.

Detemple, Jerome, and Shashidhar Murthy, 1994, Intertemporal asset pricing with heterogeneous beliefs, *Journal of Economic Theory* 62, 292–320.

Diether, Karl B, Christopher J Malloy, and Anna Scherbina, 2002, Differences of opinion and the cross section of stock returns, *The Journal of Finance* 57, 2113–2141.

Doukas, John A, Chansog Kim, and Christos Pantzalis, 2004, Divergent opinions and the performance of value stocks, *Financial Analysts Journal* 60, 55–64.

Dumas, Bernard, Alexander Kurshev, and Raman Uppal, 2009, Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility, *Journal of Finance* 64, 579–629.

Fama, Eugene F, and Kenneth R French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of financial economics* 33, 3–56.

Fama, Eugene F, and James D MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of political economy* 81, 607–636.

Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

Gallmeyer, Michael, and Burton Hollifield, 2008, An examination of heterogeneous beliefs with a short-sale constraint in a dynamic economy, *Review of Finance* 12, 323–364.

Hong, Harrison, and David A Sraer, 2016, Speculative betas, *The Journal of Finance* 71, 2095–2144.

Israel, Robert B., Jeffrey S. Rosenthal, and Jason Z. Wei, 2001, Finding generators from markov chains via empirical transition matrices with applications to credit ratings, *Mathematical Finance* 11, 245–265.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *The Journal of finance* 48, 65–91.

Jensen, Michael C, Fischer Black, and Myron S Scholes, 1972, The capital asset pricing model: Some empirical tests, .

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2010, A skeptical appraisal of asset pricing tests, *Journal of Financial economics* 96, 175–194.

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *The review of economics and statistics* pp. 13–37.

Miller, Edward M., 1977, Risk, uncertainty, and divergence of opinion, *The Journal of Finance* 32, 1151–1168.

Newey, Whitney R., and Kenneth D. West, 1987, A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Park, Cheolbeom, 2005, Stock return predictability and the dispersion in earnings forecasts, *The Journal of Business* 78, 2351–2376.

Qu, Shisheng, Laura Starks, and Hong Yan, 2003, Risk, dispersion of analyst forecasts and stock returns, *University of Texas at Austin Working Paper* pp. 1–33.

Sadka, Ronnie, and Anna Scherbina, 2007, Analyst disagreement, mispricing, and liquidity, *The Journal of Finance* 62, 2367–2403.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *The Review of Financial Studies* 5, 1–33.

Sharpe, William F, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *The journal of finance* 19, 425–442.

Varian, Hal R, 1985, Divergence of opinion in complete markets: A note, *The Journal of Finance* 40, 309–317.

——— , et al., 1989, Differences of opinion in financial markets, in *Financial Risk: Theory, Evidence, and Implications: Proceedings of the 11th Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis* pp. 3–40. Kluwer Academic Publishers Boston, Massachusetts.

Yu, Jialin, 2011, Disagreement and return predictability of stock portfolios, *Journal of Financial Economics* 99, 162–183.