Information Frictions and Real Exchange Rate Dynamics[†]

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Abstract

Real exchange rates are highly volatile and persistent. I provide a novel structural explanation for these facts using a model with dispersed information among firms. When producers face strategic complementarities in price-setting, uncertainty about competitors' beliefs results in sluggish price adjustment that can generate large and long-lived real exchange rate movements. I estimate the model using data from the US and Euro Area, and show that it successfully explains the *unconditional* volatility and persistence of the real exchange rate. The model also accounts for the persistent and hump-shaped real exchange rate behavior *conditional* on nominal disturbances documented by a structural VAR. About 50% of this persistence is due to the inertial dynamics of higher-order beliefs.

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1 Introduction

Real exchange rates have been extremely volatile and persistent since the end of the Bretton Woods system (Mussa, 1986). For many developed economies, real exchange rates are roughly four times as volatile as output, and their fluctuations exhibit a half-life in the range of three to five years.[1] Moreover, real and nominal exchange rates are highly correlated.[2] In principle, sticky-price models can explain this correlation and volatility: if price levels fail to adjust, changes in nominal exchange rates following nominal shocks will readily translate in real exchange rate movements. However, such models cannot produce the observed persistence under plausible nominal rigidities, as demonstrated by Bergin and Feenstra (2001) and Chari, Kehoe, and McGrattan (2002).[3]

In this paper, I demonstrate that a two-country, general-equilibrium model with noisy, dispersed information à la Woodford (2002) can reconcile these empirical features of real exchange rates. The framework is estimated via likelihood methods using data from the US and the Euro Area. I then show that the model explains the *unconditional* moments of the real exchange rate, as well as its moments *conditional* on nominal disturbances. In the model, monetary shocks have real effects because noisy information results in inertial dynamics of firms' beliefs. Furthermore, with strategic complementarities in price-setting, dispersed information makes a producer's optimal price depend not only on their beliefs about exogenous disturbances but also on their higher-order beliefs—i.e., beliefs about other producers' beliefs. The sluggish adjustment of higher-order beliefs rationalizes why nominal shocks have persistent real effects, and is quantitatively important, accounting for almost 50% of the conditional persistence of the real exchange rate.

The paper makes two main contributions to the literature. The first is to show that the estimated open-economy dispersed-information model explains the unconditional persistence and volatility of the real exchange rate, as well as its correlation with the nominal exchange rate. Closed-economy models in which agents are imperfectly informed are known to be quantitatively successful in accounting for the persistent effects of monetary disturbances on output and inflation (Melosi, 2014) documented by VAR studies (e.g. Christiano, Eichenbaum, and Evans, 2005).

[1] Following a shock, the half-life is the time that the exchange rate takes to fall below half the size of its initial response. A formal definition of half-life is in Section 6.3.

[2] For evidence, see Chari, Kehoe, and McGrattan (2002) and Steinsson (2008).

[3] Subsequent research partly addresses the persistence anomaly by introducing strategic complementarities (Bouakez, 2005), inertial Taylor rules (Benigno, 2004) and real shocks (Steinsson, 2008; Iversen and Söderström, 2014). While these features increase the persistence of the exchange rate, they are not sufficient to jointly explain the observed half life of the real exchange rate as well as its volatility.

However, little is known about these models' ability to explain the behavior of international relative prices. This paper fills this gap. First, I show analytically that an open-economy framework with dispersed information can deliver highly volatile and persistent real exchange rates. Second, by estimating it, I demonstrate that the model is quantitatively successful in accounting for these empirical features. The paper thus provides a novel explanation for the observed real exchange rate fluctuations.

The second main contribution is to demonstrate that the model accounts for the conditional behavior of the real exchange rate in response to nominal disturbances. Using a structural VAR, I document that real exchange rates display persistent and hump-shaped dynamics in response to nominal shocks, in line with previous evidence in Clarida and Gali (1994). I then show that the model successfully explains these features, generating endogenous persistence from the inertial dynamics of higher-order beliefs. A decomposition of this persistence shows that dispersed information is quantitatively important, almost doubling the half-life of the real exchange rate in comparison to a counterfactual economy with noisy information but no role for higher-order beliefs. These results contrast with those coming from an estimated benchmark model of sticky prices, which generates relatively low persistence in response to nominal shocks even in the presence of strategic complementarities.

The information friction that I model is motivated by the mounting evidence about heterogeneity in beliefs among decision makers. Coibion and Gorodnichenko (2012) document that there is considerable dispersion in inflation forecasts within the Federal Reserve's Survey of Professional Forecasters or the Michigan Survey of Consumers. Recent survey evidence shows that there is also widespread dispersion in beliefs of firms about macroeconomic conditions (Coibion, Gorodnichenko, and Kumar, 2015), suggesting that producers have their own "window on the world" (Amato and Shin, 2006). In such an environment, defending a firm's market share amounts to second-guessing a competitor's pricing strategy. This second-guessing game might prove particularly challenging in an open economy, where firms face competition not only from domestic firms but also from foreign exporters.

I follow Woodford (2002) and Melosi (2014) in modeling this heterogeneity in beliefs. In the baseline model, firms observe private, idiosyncratic signals about nominal aggregate demand and aggregate productivity in the two countries. They also face strategic complementarity in price-setting, which implies that a firm's optimal price depends positively on the prices set by competitors. Strategic interactions and heterogeneous information often result in tractability issues, as the state space becomes infinite dimensional when agents are required to forecast the forecast of others. This problem is more severe in open-economy models, which additionally feature heterogeneity of agents across countries and fluctuations in international prices. I exploit the symmetry of the model and the equilibrium characterization of the terms of trade to provide an analytical solution that only requires keeping track of two weighted-averages of higher-order beliefs. The solution, in turn, allows me to gain deep insights into the driving forces of the real exchange rate and to conduct a likelihood evaluation of the model.

My analytical results show that the volatility and persistence of the real exchange rate are higher (i) the lower is the precision of firms' signals about aggregate demand and (ii) the higher is the degree of strategic complementarity. Intuitively, when signals are not very precise, firms learn gradually about changes in nominal aggregate demand and sluggishly update their prices. When strategic complementarities are strong, firms place more weight on higher-order beliefs, which update slowly, as the private signals a firm receives provide relatively little information about the signals of other firms. Both of these channels slow down the price adjustment, delivering volatile and persistent real exchange rates following nominal shocks. Notably, strategic complementarity depends on the degree of the economies' openness and on the substitutability between domestic- and foreign-produced goods. Thus, foreign competition provides a channel through which the adjustment of prices might be delayed, one that is naturally absent in closed-economy models.

I then assess whether the framework can quantitatively explain the dynamics of the Euro-Dollar real exchange rate in the period between 1973 and 2008. The theoretical results suggest that the model would be able to generate a highly volatile and persistent real exchange rate with a sufficiently low signal precision. However, it is unclear which values of signal precision should be considered empirically relevant, given the scarce existing evidence on these parameters. To address this shortcoming, I estimate the model parameters via likelihood methods using real GDP and GDP deflator data for the US and the Euro Area. These data do not directly contain information on the real exchange rate, which is instead defined as the nominal exchange rate adjusted by consumption price indices.

The exclusion of the real exchange rate from the estimation allows me to conduct an out-of-sample test for the model. Specifically, I simulate the model at the estimated parameter values and ask whether it reproduces the unconditional dynamics of the Euro/Dollar real exchange rate, which were not targeted in the estimation. I show that the model successfully explains these dynamics, as measured by the volatility and persistence of the real exchange rate. In particular, the model delivers a half-life of roughly 4 years for the real exchange rate, which is very close to the empirical counterpart of 3.5 years. The framework also generates highly correlated nominal and real exchange rates. Additionally, the estimated signal-to-noise ratios suggest that signals about nominal aggregate conditions are less precise than those about productivity, which generates persistent and hump-shaped dynamics of the real exchange rate conditional on nominal shocks. Using a structural VAR approach, I show that nominal shocks do generate long-lived and hump-shaped responses for the Euro/Dollar real exchange rate, which are consistent with the model's propagation mechanism.

I also compare the model's predictions with those of a benchmark sticky-price framework, which is estimated using the same data on real GDP and GDP deflators. The sticky-price model deviates from the dispersed-information model in only two respects: (i) all agents are perfectly informed, and (ii) firms can optimally adjust their prices only in random periods, as in Calvo (1983). Two sets of results emerge. First, the dispersed-information model fits the data better than the sticky-price model, as suggested by a Bayesian model comparison. Second, the estimated Calvo model generates counterfactually low real exchange rate persistence following nominal shocks, confirming the intuition behind the results of Chari, Kehoe, and McGrattan (2002).

The nominal disturbances that I consider might arise from monetary policy shocks and from other exogenous disturbances that affect aggregate demand, including those that originate in the financial sector. A recent literature has emphasized the importance of financial shocks to account for exchange rate movements, as they can potentially address several exchange rate puzzles by driving a wedge into the risk-sharing and UIP conditions (Itskhoki and Mukhin, 2017). In the last part of the paper, I thus investigate the role of such shocks in my framework by explicitly introducing shocks to international asset demand and re-estimating the models to fit also the real exchange rate data. The results suggest that financial shocks account for the bulk of real and nominal exchange rate unconditional volatility, in line with the findings from the calibration exercise of Itskhoki and Mukhin (2017). Financial shocks also partially address other puzzles such as the exchange-rate disconnect and the UIP puzzle. At the same time, my previous results hold in this extended framework: the dispersed-information model still well explains the persistence of the real exchange rate, and outperforms the Calvo model in fitting the data as well as in reproducing the empirical propagation of nominal disturbances.

This paper contributes to the growing literature that focuses on the aggregate implications of dispersed information among price setters, such as Woodford (2002), Maćkowiak and Wiederholt (2009), Nimark (2008), and Melosi (2014), which builds on the seminal contributions of Phelps (1970) and Lucas (1972). In contrast to these closed-economy contributions, this paper studies the implications for international prices, where uncertainty about foreign demand as well as the actions of foreign competitors plays an important role. My analysis lends further empirical support to the dispersed-information theory and shows that it is central to the understanding of real exchange rate dynamics.

The paper is also naturally related to the literature that studies real exchange rate dynamics within monetary models, such as Johri and Lahiri (2008) and Carvalho and Nechio (2011), in addition to the contributions already mentioned. Relative to this literature, this paper highlights the importance of a source of endogenous persistence in real exchange rates—dispersed information in environments with strategic complementarities—that has so far not been studied in this context. Indeed, the estimated dispersed-information framework generates substantial persistence in the real exchange rate resorting to a modest degree of exogenous persistence in nominal aggregate demand. The model is thus able to explain the real exchange rate persistence without compromising its ability to account for other aspects of the data, such as the autocorrelation of nominal exchange rates.

Finally, the present study adds to the small body of literature that focuses on information frictions in open economies. Bacchetta and van Wincoop (2006, 2010) introduce information frictions in the foreign exchange market to study the exchange-rate disconnect and the forward-discount puzzle. While in these studies incomplete information is on the agents trading assets (i.e., investors/households), this paper studies the role of information frictions on the agents pricing goods (i.e., firms), thus switching the focus from assets markets to goods markets. Crucini, Shintani, and Tsuruga (2010) introduce sticky information à la Mankiw and Reis (2002) in a sticky-price model to explain the volatility and persistence of deviations from the law of one price. In contrast, this paper explains the substantially longer half-life of aggregate real exchange rates by relying only on dispersed information and Bayesian updating, which is consistent with recent evidence on the behavior of firms (Coibion, Gorodnichenko, and Kumar, 2015).

The paper proceeds as follows. Section 2 develops the dispersed-information model. Section 3 provides some analytical results. Section 4 discusses the solution method. Section 5 analyzes the model's impulse responses. Section 6 contains the empirical analysis. Section 7 traces a comparison with the sticky-price model. Section 8 investigates the role of financial shocks, and Section 9 concludes.

2 The Model

The framework is a two-country open-economy monetary model that follows the international macroeconomic tradition initiated by Obstfeld and Rogoff (1995). The setup is similar to Corsetti, Dedola, and Leduc (2010). The world economy consists of two countries of unit mass, denominated H (Home) and F (Foreign), each populated by households, a continuum of monopolistically competitive producers, and a monetary authority. Each country specializes in the production of one type of tradable goods, produced in a number of varieties or brands, with measure equal to the population size. All goods produced are traded and consumed in both countries.[4] Prices are set in the currency of the producer; therefore, the law of one price holds. Nevertheless, deviations of the real exchange rate from purchasing-power parity arise because of home bias in consumption preferences.

All information is, in principle, available to every agent; however, firms can only pay limited attention to the information available, owing to finite information-processing capacity (Sims, 2003). Following Woodford (2002) and Melosi (2014), this idea is modeled by assuming that firms do not perfectly observe current and past realizations of the variables in the model, but rather only observe private noisy signals about the state of nominal aggregate demand and technology.[5] Firms use these signals to draw inferences about other model variables. Households and the monetary authorities are assumed, for tractability, to observe the current and past realization of all the model variables. Below I present the structure of the Home economy in more detail. The Foreign economy is symmetric, and foreign variables will be denoted with an asterisk.

2.1 Preferences and Households

The utility function of the representative household in country H is

$$\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \int_0^1 \frac{L_{ht}^{1+1/\psi}}{1+1/\psi} dh \right] \right\}. \tag{1}$$

The representative household has full information; $\mathbb{E}(.)$ denotes the statistical expectations operator, and $\beta < 1$ is the discount factor. Households receive utility from consumption C_t and disutility from working, where L_{ht} indicates hours of labor input in the production of domestic variety $h \in [0, 1]$. The parameter $\psi > 0$ governs

[4]This modeling choice is consistent with the findings in Engel (1999), who documents that a large fraction of the US real exchange rate fluctuations can be accounted for by movements in the relative price of tradable goods across countries.

[5]The findings of the paper are robust to the inclusion of noisy endogenous signals about prices. These results are available upon request.

the Frisch elasticity of labor supply. Following Woodford (2003, Ch. 3), each variety, indexed by h (f) in the home (foreign) economy over the unit interval, uses a specialized labor input in its production. As noted by Woodford, specialized labor markets generate more strategic complementarities in price-setting.[6]

Households consume both types of traded goods. The consumption of these goods is denoted by C_{Ht} and C_{Ft}. For each type of goods, one brand or variety is an imperfect substitute for all the other brands, and γ is the elasticity of substitution between brands. Mathematically, consumption baskets of Home (Foreign) goods by Home agents are a CES aggregate of Home (Foreign) brands:

$$C_{Ht} \equiv \left(\int_0^1 C_t(h)^{\frac{\gamma-1}{\gamma}} dh\right)^{\frac{\gamma}{\gamma-1}} \qquad C_{Ft} \equiv \left(\int_0^1 C_t(f)^{\frac{\gamma-1}{\gamma}} df\right)^{\frac{\gamma}{\gamma-1}} \qquad \gamma > 1.$$

The overall consumption basket, C_t, is defined as

$$C_t \equiv \left[\alpha^{\frac{1}{\omega}}(C_{Ht})^{\frac{\omega-1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}}(C_{Ft})^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \qquad \omega > 0,$$

where α is the weight of the home consumption good and ω is the elasticity of substitution between home and foreign goods, which I alternatively refer to as the trade elasticity. The utility-based consumption price index (CPI) is

$$P_t = \left[\alpha P_{Ht}^{1-\omega} + (1-\alpha)P_{Ft}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

where P_{Ht} and P_{Ft} are the price sub-indices for the home- and foreign-produced goods, expressed in domestic currency

$$P_{Ht} = \left(\int_0^1 p_t(h)^{1-\gamma} dh\right)^{\frac{1}{1-\gamma}} \qquad P_{Ft} = \left(\int_0^1 p_t(f)^{1-\gamma} df\right)^{\frac{1}{1-\gamma}}.$$

Foreign prices are similarly defined. The Foreign CPI is

$$P_t^* = \left[(1-\alpha)(P_{Ht}^*)^{1-\omega} + \alpha(P_{Ft}^*)^{1-\omega}\right]^{\frac{1}{1-\omega}}.$$

Let $\mathcal{Q}_t$ denote the real exchange rate, that is, the relative price of consumption: $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$, where $\mathcal{E}_t$ is the nominal exchange rate expressed in domestic currency per foreign currency. Even if the law of one price holds at the individual good level (i.e., $P_t(h) = \mathcal{E}_t P_t(h)^*$, which implies $P_{Ht} = \mathcal{E}_t P_{Ht}^*$), the presence of home bias in consumption—that is $\alpha > 1/2$—implies that the price of consumption may not be

[6] Pricing decisions are strategic complements if, when other firms raise their prices, a given firm i wishes to raise its own price. It is closely related to the concept of real rigidities defined as the lack of sensitivity of desired relative prices to macroeconomic conditions. Strategic complementarities arise also in the presence of decreasing returns or input-output structures in production (Basu, 1995). For a theoretical discussion, see Ball and Romer (1990) and Woodford (2003, Ch. 3).

equalized across countries. Put differently, purchasing-power parity ($\mathcal{Q}_t = 1$) will generally not hold. The terms of trade are defined as the price of imports in terms of exports: $\mathcal{T}_t = \frac{P_{Ft}}{\mathcal{E}_t P^*_{Ht}}$. If the law of one price holds, the real exchange rate will be proportional to the terms of trade up to a first-order approximation:

$$q_t = (2\alpha - 1)\,t_t. \tag{2}$$

Throughout the paper, lower-case letters denote percentage deviations from steady state, assuming symmetric initial conditions. Equation (2) implies that an improvement in the terms of trade always appreciates the real exchange rate. This is consistent with the empirical evidence (Obstfeld and Rogoff, 2000). Minimizing expenditure over brands and over goods, one can derive the domestic household demand for a generic good h, produced in country H, and the demand for a good f, produced in country F:

$$C_t(h) = \left(\frac{P_t(h)}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\alpha C_t \qquad C_t(f) = \left(\frac{P_t(f)}{P_{Ft}}\right)^{-\gamma}\left(\frac{P_{Ft}}{P_t}\right)^{-\omega}(1-\alpha)C_t,$$

Assuming that the law of one price holds, total demand for a generic home variety h or foreign variety f may be written as

$$Y_t^d(h) = \left(\frac{P_t(h)}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\left[\alpha C_t + (1-\alpha)\mathcal{Q}_t^{\omega} C_t^*\right], \tag{3}$$

$$Y_t^d(f) = \left(\frac{P_t(f)^*}{P^*_{Ft}}\right)^{-\gamma}\left(\frac{P^*_{Ft}}{P^*_t}\right)^{-\omega}\left[(1-\alpha)\mathcal{Q}_t^{-\omega} C_t + \alpha C_t^*\right]. \tag{4}$$

2.2 Budget Constraint and First-Order Conditions

The representative household uses its revenues in every period to purchase consumption goods or invest in a full set of state-contingent securities. The domestic household's budget constraint can be written as

$$P_t C_t + \int q_{Ht}(s_{t+1})\mathcal{B}_{Ht}(s_{t+1})ds_{t+1} \leq \int_0^1 W_{ht}L_{ht}dh + \mathcal{B}_{Ht} + P_t\int_0^1 \Pi_{ht}dh + T_t. \tag{5}$$

The quantity $\mathcal{B}_{Ht}(s_{t+1})$ denotes the holding of state-contingent claims that pay off one unit of domestic currency if the realized state of the world at time $t+1$ is s_{t+1}, and $q_{Ht}(s_{t+1})$ is the time-t corresponding price. W_{ht} is the wage for the h-th type of labor input, Π_{ht} denotes the real profits of domestic firm h, and T_t is a lump-sum nominal transfer from the government. Maximizing (1) subject to (5) gives the static first-order condition:

$$C_t^{\sigma} L_{ht}^{1/\psi} = W_{ht}/P_t, \tag{6}$$

and the following Euler equation

$$1 = R_t \mathbb{E}_t \Theta_{t+1}, \qquad \Theta_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{7}$$

where $R_t \equiv \frac{1}{\int q_{Ht}(s_{t+1})}$ is the risk-free rate of return between t and $t+1$. Combining the Home and Foreign intertemporal conditions, one obtains the familiar complete-markets risk-sharing condition

$$\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{P_t}{P_{t+1}} = \left(\frac{C_t^*}{C_{t+1}^*} \right)^{\sigma} \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+1} P_{t+1}^*}.$$

This equation relates the cross-country differential in the growth rate of consumption to the depreciation of the exchange rate. Assuming symmetric initial conditions, this relationship implies

$$\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*} \right)^{\sigma}. \tag{8}$$

Equation (8) is an efficiency condition that equates the marginal rate of substitution between home and foreign consumption to their marginal rate of transformation, expressed as equilibrium prices, i.e., the real exchange rate. A key consequence is that home consumption can rise relative to foreign consumption only if the real exchange rate depreciates.[7] The nominal exchange rate $\mathcal{E}_t$ is determined by combining equation (8) with the processes for nominal aggregate demand

$$\mathcal{E}_t = \frac{M_t}{M_t^*} \mathcal{Q}_t^{1-\sigma^{-1}}. \tag{9}$$

2.3 The Government

The fiscal authority makes lump-sum transfers to households in every period, which are financed by printing money. The home-country government budget constraint is given by

$$T_t = M_t - M_{t-1}. \tag{10}$$

The variable M_t can be interpreted as a measure of money supply, such as M1 or M2, or more broadly as a measure of aggregate demand, such as nominal spending. Following Woodford (2002) and Carvalho and Nechio (2011), I leave the specification of monetary policy implicit, and assume that the growth rates of nominal aggregate demands, or money supplies, $M_t \equiv P_t C_t$ and $M_t^* \equiv P_t^* C_t^*$, follow exogenous

[7] This implication is known to be at odds with the the data, where real exchange rates and consumption differentials exhibit low or negative correlation (Backus and Smith, 1993). I return to this issue in Section 8.

autoregressive processes:

$$\Delta m_t = \rho_m \Delta m_{t-1} + u_t^m, \tag{11}$$

$$\Delta m_t^* = \rho_{m^*} \Delta m_{t-1}^* + u_t^{m^*}, \tag{12}$$

where $\Delta m_t \equiv \ln M_t - \ln M_{t-1}$ and the monetary shocks u_t^m and $u_t^{m^*}$ are $i.i.d.$, distributed as $\mathcal{N}(0, \sigma_m^2)$ and $\mathcal{N}(0, \sigma_{m^*}^2)$ and uncorrelated across countries.[8] I refer to these shocks as nominal demand or monetary shocks, with the understanding that they capture structural disturbances that move nominal expenditure. Thus, shocks to m_t and m_t^* should be viewed as incorporating monetary policy shocks as well as the many other disturbances that shift aggregate demand (Mankiw and Reis, 2002). This specification is widely used in the monetary literature and has been shown to be a good approximation of the process that implements estimated Taylor rules of the types studied in Christiano, Eichenbaum, and Evans (1998).

2.4 Price-Setting Decisions

Firms do not perfectly observe the state of aggregate demand and their marginal cost, but at each date they receive private signals about economic conditions. Prices are set in the producer's currency and there are no barriers to trade, so the law of one price always holds. Firm h's expected real profits in period t, conditional on the history of signals observed by that firm at time t, are given by

$$\Pi_{ht} = \mathbb{E}_{ht} \left[\Lambda_t \left(\frac{P_t(h)}{P_t} Y_t^d(h) - \frac{W_{ht}}{P_t} L_{ht} \right) \right], \tag{13}$$

where $\mathbb{E}_{ht} \equiv \mathbb{E}[\cdot | \mathcal{I}_h^t]$ is the expectation operator conditional on firm h's information set, $\mathcal{I}_h^t$, and $\Lambda_t = \beta^t C_t^{-\sigma}$ is the appropriate pricing kernel. The production function is given by

$$Y_t(h) = A_t L_{ht}. \tag{14}$$

Total factor productivity, A_t, in the two countries follows the processes

$$\ln A_t = \rho_a \ln A_{t-1} + u_t^a, \tag{15}$$

$$\ln A_t^* = \rho_{a^*} \ln A_{t-1}^* + u_t^{a^*}. \tag{16}$$

[8]This formulation of aggregate demand can also be justified by the presence of a cash-in-advance constraint.

The shocks u have mean zero and variances σ_a^2 and $\sigma_{a^*}^2$, respectively. Each firm in the home country receives the following signals:

$$Z_{h,t} = \begin{bmatrix} z_{h,t}^m \\ z_{h,t}^{m^*} \\ z_{h,t}^a \\ z_{h,t}^{a^*} \end{bmatrix} = \begin{bmatrix} m_t \\ m_t^* \\ a_t \\ a_t^* \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_m & 0 & 0 & 0 \\ 0 & \tilde{\sigma}_{m^*} & 0 & 0 \\ 0 & 0 & \tilde{\sigma}_a & 0 \\ 0 & 0 & 0 & \tilde{\sigma}_{a^*} \end{bmatrix} \begin{bmatrix} v_{h,t}^m \\ v_{h,t}^{m^*} \\ v_{h,t}^a \\ v_{h,t}^{a^*} \end{bmatrix}, \tag{17}$$

where $v_{h,t}^m, v_{h,t}^{m^*}, v_{h,t}^a, v_{h,t}^{a^*} \sim \mathcal{N}(0,1)$, $a_t = \ln A_t$ and $a_t^* = \ln A_t^*$. $m_t = \ln M_t$ and $m_t^* = \ln M_t^*$ represent the nominal aggregate demands, and the signal noises are assumed to be independently and identically distributed across firms and over time. Foreign firms receive similar signals drawn from the same distributions, so that domestic and foreign firms are equally well informed about a shock in a given country.[9] In every period t, firms observe the history of their signals Z_h^t (that is, their information set is $\mathcal{I}_{ht} = \{Z_{h,\tau}\}_{\tau=-\infty}^t$). At time t, firm h chooses $P_t(h)$ to maximize (13) subject to (3) and (14). The first-order condition yields

$$P_t(h) = \frac{\gamma}{\gamma-1} \frac{\mathbb{E}_{ht}\left[\Lambda_t \left(\frac{1}{P_{Ht}}\right)^{-\gamma-1} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} \left(\frac{C_t^W}{P_t}\right) \left(\frac{W_{ht}}{P_{Ht}A_t}\right)\right]}{\mathbb{E}_{ht}\left[\Lambda_t \left(\frac{1}{P_{Ht}}\right)^{-\gamma} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} \left(\frac{C_t^W}{P_t}\right)\right]}, \tag{18}$$

where $C_t^W \equiv \alpha C_t + (1-\alpha)\mathcal{Q}_t^\omega C_t^*$. Equation (18) states that a firm optimally sets its price to a markup, $\frac{\gamma}{\gamma-1}$, over its *perceived* marginal cost. Note that the above information structure implies that the realizations of signals, information sets, and beliefs will be firm-specific at each point in time. Therefore, producers will set heterogeneous prices even in response to purely aggregate disturbances.

Following the tradition in this literature, I log-linearize the price-setting equation around the deterministic steady state so that the transition equations of average prices are linear. I assume that firms use the log-linearized model, rather than the original nonlinear model, when addressing their signal-extraction problem. This assumption greatly simplifies the analysis, because it allows for the use of the Kalman filter to characterize the dynamics of firms' beliefs. Finally, I assume that at the beginning of time, firms are endowed with an infinite history of signals. This implies that the Kalman gain matrix is time-invariant and identical across firms.[10]

[9] As I argue below, imposing the signal-to-noise ratio to be the same for domestic and foreign shocks does not change the quantitative results of the paper.

[10] Kalman gains depend solely on the stochastic processes for structural disturbances and on the signal-to-noise ratios, which are common across firms. With a long enough history of signals the Kalman gain matrix converges to its time-invariant analogue.

2.5 Real Exchange Rate Dynamics

In this section I characterize the solution for the real exchange rate. To simplify the algebra and convey intuition, I henceforth assume logarithmic utility for consumption ($\sigma = 1$). Appendix A shows how the model can be solved also for a generic value of σ. Under the producer currency pricing (PCP) assumption, Appendix A also shows that the log-linearized first-order conditions for a generic h and f firm, combined with equation (2), are

$$p_t(h) = \mathbb{E}_{ht}\left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t - a_t)\right], \tag{19}$$

$$p_t^*(f) = \mathbb{E}_{ft}\left[(1-\xi)p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t^* - a_t^*)\right], \tag{20}$$

where $\xi = \frac{1+\psi}{\gamma+\psi}$. These equations show the interdependence of the optimal price with their foreign counterpart through the terms of trade, $t_t = p_{Ft}^* + e_t - p_{Ht}$, where $e_t = \log \mathcal{E}_t$. In particular, if home and foreign goods are substitutes ($\omega > 1$), other things equal, a rise in the relative price of foreign goods (that is, a rise in t_t) causes expenditure switching away from foreign goods toward home goods. The increased demand for home goods increases firm h's marginal cost and makes it optimal to raise $p_t(h)$. If goods are instead complements ($\omega < 1$), a rise in t_t decreases demand both for foreign and home output, hence the optimal price for a home good $p_t(h)$ falls.

The parameter $1-\xi$ is related to the degree of strategic complementarities in price-setting, i.e., it determines by how much the optimal price of an individual firm changes when all the other domestic competitors are changing their prices. Because $\gamma > 1$, then $0 \leq \xi < 1$.[11] Integrating (19) over domestic agents and (20) over foreign agents and noting that the log-linear price indices read as $p_{Ht} = \int_0^1 p_t(h)dh$ and $p_{Ft}^* = \int_0^1 p_t^*(f)df$, I obtain

$$p_{Ht} = \bar{\mathbb{E}}_t^{(1)}\left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t - a_t)\right], \tag{21}$$

$$p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)}\left[(1-\xi)p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t^* - a_t^*)\right], \tag{22}$$

where $\bar{\mathbb{E}}_t^{(1)}(\cdot) \equiv \int_0^1 \bar{\mathbb{E}}_{it}(\cdot)di$ for $i = h, f$ denotes a first-order average belief. Note that $\int_0^1 \bar{\mathbb{E}}_{ht}(\cdot)dh = \int_0^1 \bar{\mathbb{E}}_{ft}(\cdot)df$ follows from the symmetry of the information structure.

[11] I focus on cases in which $0 < \xi < 1$ holds strictly.

Equations (21) and (22) still depend on the terms of trade, t_t, which is an endogenous variable that would not appear in a closed-economy version of this framework. The presence of the terms of trade poses a challenge for solving the model, as one would need to keep track of higher-order-beliefs of this endogenous variable. Instead, I now show that one can eliminate the terms of trade from the two equations above, making the model solution analytically tractable. Taking the sum of (21) and (22) I obtain

$$p_{Ht} + p^*_{Ft} = \bar{\mathbb{E}}^{(1)}_t \left[(1-\xi)(p_{Ht} + p^*_{Ft}) + \xi m^W_t - \xi a^W_t\right], \tag{23}$$

where for any variable x_t, I define $x^W_t \equiv x_t + x^*_t$ and $x^D_t \equiv x_t - x^*_t$.

A key feature of the dispersed-information model is the absence of common knowledge. The presence of idiosyncratic, private information implies that no one knows whats others in the economy know. There is a role for higher-order beliefs, as each firm must form a belief of what other firms believe, as well as of what other firms believe that the firm believes, and so on. A consequence of this is that $\int_0^1 \mathbb{E}_{it}[\int_0^1 \mathbb{E}_{it}(\cdot)di]di \neq \int_0^1 \mathbb{E}_{it}(\cdot)di$ or that the second-order average belief is not equal to the first-order one. By recursive substitution in equation (23) we can make explicit the role of higher-order beliefs

$$p_{Ht} + p^*_{Ft} = \xi \sum_{k=1}^{\infty} (1-\xi)^{k-1} \bar{\mathbb{E}}^{(k)}_t (m^W_t - a^W_t). \tag{24}$$

Here $\bar{\mathbb{E}}^{(k)}_t(\cdot) \equiv \int_0^1 \bar{\mathbb{E}}^{(k-1)}_{it}(\cdot)di$ denotes the k-th-order average belief, i.e., the beliefs about other firms' beliefs about other firms' beliefs . . . about fundamentals. By taking the difference between (21) and (22) we obtain

$$p_{Ht} - p^*_{Ft} = \bar{\mathbb{E}}^{(1)}_t \left[(1-\xi)(p_{Ht} - p^*_{Ft}) + \frac{4\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi m^D_t - \xi a^D_t\right],$$

To eliminate the endogenous terms of trade $t_t = p^*_{Ft} + e_t - p_{Ht}$, note that with log-utility the risk-sharing condition (8), combined with the definition of nominal aggregate demand, implies that $e_t = m_t - m^*_t$.[12] Therefore, we can rewrite the above expression as

$$p_{Ht} - p^*_{Ft} = \bar{\mathbb{E}}^{(1)}_t \left[(1-\varphi)(p_{Ht} - p^*_{Ft}) + \varphi(m_t - m^*_t) - \xi(a_t - a^*_t)\right], \tag{25}$$

[12] Under the more general CRRA case used below for estimation, the nominal exchange rate also depends on the terms of trade as can be seen in equation (51) in Appendix A.

where $\varphi \equiv \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$. Recursively substituting $p_{Ht} - p^*_{Ft}$ yields

$$p_{Ht} - p^*_{Ft} = \varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D - \xi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} a_t^D. \tag{26}$$

The solution for p_{Ht} and p^*_{Ft} can be found by taking sums and differences of equations (24) and (26). Proposition 1 then follows from the proportionality of the real exchange rate to the terms of trade (see Appendix A for the formal proof).

Proposition 1 *Under the assumption of log-utility and complete asset markets, the real exchange rate is given by*

$$q_t = (2\alpha - 1)\left(m_t^D - \varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D + \xi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} a_t^D \right), \tag{27}$$

where $\varphi \equiv \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$ *governs the degree of strategic complementarity.*

The intuition behind this equation is straightforward. Focus on the first two terms on the right-hand side of (27) and consider a shock to relative nominal demand, m_t^D. Under full-information rational expectations, the shock has no effect on the real exchange rate, because nominal prices adjust one for one with nominal demand. Indeed, with full information $\bar{\mathbb{E}}_t^{(k)} m_t = m_t$ and $\bar{\mathbb{E}}_t^{(k)} m^*_t = m^*_t$ for every k so that $m_t^D = \varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D$, and the real exchange rate responds only to productivity shocks.

Instead, under imperfect information, the real exchange rate also responds to nominal shocks to the extent that higher-order expectations deviate from full-information rational expectations. Equation (27) shows also that the persistence of the real exchange rate to relative monetary shocks depends on how quickly the weighted average of higher-order expectations $\varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D$ adjusts. As shown in section 3, the speed of adjustment depends on the degree of strategic complementarities (φ for relative variables) and on the signal-to-noise ratios $\sigma_m/\tilde{\sigma}_m$ and $\sigma_{m^*}/\tilde{\sigma}_{m^*}$. Specifically, the signal-to-noise ratios determine how quickly the different order of expectations in the summation will adjust to shocks. The strategic-complementarity parameter determines the *weights* attached to the different orders. For instance, the average first-order expectation about m_t^D receives a weight φ, the second order receives a weight $\varphi(1-\varphi)$, the third $\varphi(1-\varphi)^2$, and so on.

The last term on the right-hand side of (27) indicates that the real exchange rate always responds to relative technology shocks or, in the presence of dispersed information, to the weighted-average of higher-order beliefs regarding the shock.

3 Analytical Results

3.1 Random Walk in Nominal Aggregate Demand

To gain intuition about the cyclical properties of the real exchange rate in response to monetary shocks, let us abstract from technological shocks and study the simple case in which money supplies follow a random walk. Precisely, for this section I assume that $A_t = A_t^* = A$, and

$$m_t = m_{t-1} + u_t^m,$$
$$m_t^* = m_{t-1} + u_t^{m^*},$$

which is obtained as a special case from equation (11) by setting $\rho_m = \rho_{m^*} = 0$. With random walks in nominal spending and linear updating implied by the signal-extraction problem, I can prove the following Proposition (see Appendix A).

Proposition 2 *Assuming random-walk processes for nominal spending and complete asset markets, the real exchange rate follows an AR(1) process*

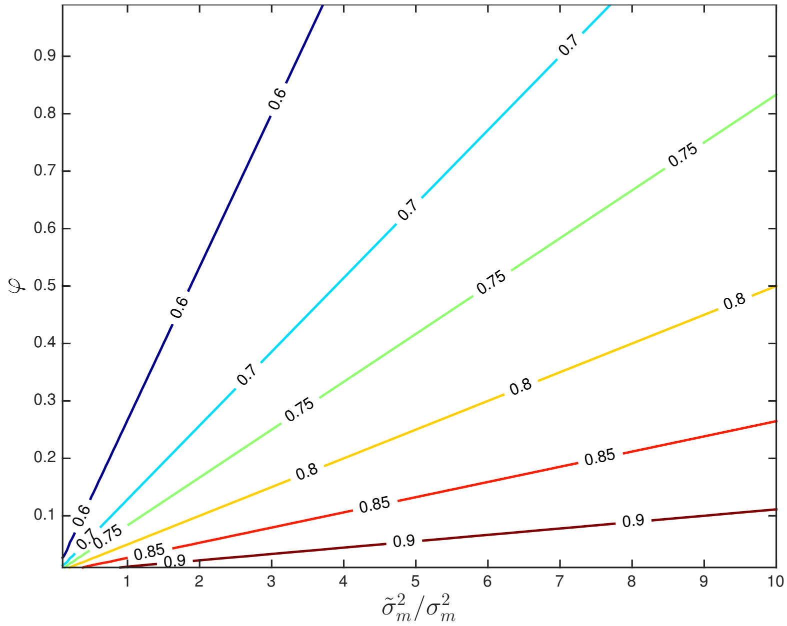
$$q_t = \nu q_{t-1} + (2\alpha - 1)\nu(u_t^m - u_t^{m^*}),$$

where $1 - \nu = \varphi \times \kappa_1 + (1 - \varphi) \times \kappa_2 \in (0, 1)$*, and* κ_1, κ_2 *are the non-zero elements of the Kalman gains matrix. The autocorrelation and variance of the real exchange rate are*

$$\rho_{\hat{Q}} = \nu \quad \sigma_{\hat{Q}}^2 = (2\alpha - 1)^2 \left(\frac{\nu}{1 - \nu}\right)^2 (\sigma_m^2 + \sigma_{m^*}^2).$$

Recall that with log-utility $e_t = m_t - m_t^*$, random walks in money supplies imply a random walk process for the nominal exchange rate. This special case is useful to explain the endogenous persistence of the real exchange rate. A one-time shock to the relative money supplies immediately and permanently depreciates the nominal exchange rate. If there is perfect information, domestic and foreign prices, p_{Ht} and p_{Ft}^* fully adjust at the time of shock, so that the real exchange rate does not react to the nominal shock. Proposition 2 shows instead that under imperfect information, the same shock implies a persistent depreciation of the real exchange rate, which now follows an AR(1) process. The Proposition highlights how this endogenous persistence, ν, depends on the relevant degree of strategic complementarity, φ, and the precision of the signals that determine the weights κ_1 and κ_2 in the Kalman gain matrix. Larger noise and more strategic complementarity increase the persistence of the exchange rate. This is illustrated in Figure 1, which depicts the iso-persistence of the real exchange rate as a function of φ and the inverse signal-to-noise ratios $\tilde{\sigma}_m^2/\sigma_m^2$, assumed to be identical for m_t and m_t^*.

Figure 1: Iso-persistence Curves of the Real Exchange Rate



Notes: Iso-persistence curves of the real exchange rate for different values of the inverse signal-to-noise ratio $\tilde{\sigma}_m^2/\sigma_m^2$ and of the strategic-complementarity parameter φ with random walk in money. Lower φ indicates more strategic complementarity.

A lower φ indicates a higher degree of strategic complementarities, which means that agents attribute a larger weight to what they believe to be others' actions (and beliefs about others' beliefs about others' actions) relative to what they believe to be the current state of nominal demand. This implies that higher-order beliefs receive a higher weight than lower-order beliefs. With high-order beliefs moving more sluggishly than low-order beliefs[13], prices adjust more slowly, which in turn implies slower movements in the real exchange rate following a money shock. Additionally, when the relative precision of the signal falls ($\tilde{\sigma}_m^2/\sigma_m^2 \downarrow$), agents will weight their prior more than their signals, failing to change prices and gradually updating their beliefs when monetary shocks hit the economy. While the shock immediately affects the nominal exchange rate, the slow movement in prices triggers a slow reversion of the real exchange rate to purchasing-power parity.

Finally, notice from Proposition 2 that a higher ν not only affects the persistence of the exchange rate, but also its volatility. To understand this, consider the response of prices when a monetary shock hits the Home economy. For the same reasons discussed above, the higher the value of ν, the smaller the adjustment of domestic prices at the impact of the shock. The small impact response of prices drives the amplification of monetary shocks onto the real exchange rate.

[13] See Woodford (2002) or Melosi (2014) for further explanation and graphical examples.

3.2 Strategic Complementarities in the Open Economy

The strategic-complementarity parameter $(1-\varphi)$ is an important determinant of the dynamics of the real exchange rate, as it affects the weights attached to different orders of expectations. In this section, I explain how this parameter crucially depends on the elasticity of substitution between home and foreign goods. In the case of logarithmic utility, we have

$$(1-\varphi) = 1 - \xi - \frac{4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}. \tag{28}$$

To build intuition let us focus on the case of a closed economy first, obtainable by setting the home-bias parameter α to one (which implies $\xi = \varphi$). In this case, the optimal pricing equations (19) and (20) would read

$$\begin{aligned} p_t(h) &= E_{ht}\left[(1-\xi)p_{Ht} + \xi(m_t - a_t)\right], \\ p_t^*(f) &= E_{ft}\left[(1-\xi)p_{Ft}^* + \xi(m_t^* - a_t^*)\right]. \end{aligned}$$

Here the degree of strategic complementarity is governed by $1-\xi = 1-\frac{1+\psi}{\gamma+\psi} \in (0,1)$. Consider the experiment of increasing p_{Ht}, while keeping all else constant. A domestic firm h responds to an increase in the average price p_{Ht} by increasing its own price. This happens because the increase in p_{Ht} shifts demand away from competitors towards firm h's output. Owing to specialized labor markets, the increase in firm h's demand results in an increase in its marginal cost. The strength of the increase in $p_t(h)$, measured by $(1-\xi)$, depends on the size of the change in firm's h demand, which is determined by the elasticity of substitution between domestic goods γ, and by the slope of the labor supply curve, governed by the Frisch elasticity ψ.

Now consider the same experiment as above if the economies are open. Using the solution for the terms of trade to rewrite the pricing equations (19) and (20) yields

$$\begin{aligned} p_t(h) &= E_{ht}\left\{\left(1-\xi-\frac{\varphi-\xi}{2}\right)p_{Ht} + \left(\frac{\varphi-\xi}{2}\right)(p_{Ft}^* + m_t^* - m_t) + \xi(m_t - a_t)\right\}, \\ p_t^*(f) &= E_{ft}\left\{\left(1-\xi-\frac{\varphi-\xi}{2}\right)p_{Ft}^* - \left(\frac{\varphi-\xi}{2}\right)(m_t^* - m_t - p_{Ht}) + \xi(m_t^* - a_t^*)\right\}. \end{aligned}$$

Now the response of $p_t(h)$ to an increase in the average domestic price, p_{Ht}, is determined by the strategic-complementarity parameter $\left(1-\xi-\frac{\varphi-\xi}{2}\right)$, which has the same sign as $(1-\varphi)$ in (28). Note from the same equation that this response might be smaller or larger than in the closed-economy case, depending on whether the value of ω is above or below unity. The intuition goes as follows. Under our maintained assumption of log utility in this section, when $\omega > 1$, home and foreign goods are net substitutes. This diminishes strategic complementarity relative to the closed economy, because an increase in p_{Ht} now shifts demand $away$ from all the other domestic goods, partly $toward$ firm h's good and partly toward foreign goods. Thus firm h experiences a milder increase in marginal cost and changes its price by a smaller amount than if it were to operate in a closed economy. Conversely, when $\omega < 1$, home and foreign goods are net complements. An increase in p_{Ht} induces a larger increase in firm h's marginal cost relative to the closed-economy case, and firm h raises its price by a larger amount. These additional effects are captured in the strategic-complementarity parameter $1-\varphi$ via ω. Thus, the substitutability between home and foreign goods has important implications for the degree of strategic complementarity, which in turn affects the dynamics of the real exchange rate by changing the importance of higher-order beliefs.

4 Model Solution

Models with dispersed information and strategic interactions are challenging to solve because they feature the "infinite regress" problem in which agents are required to forecast the forecast of others, which results in an infinite dimensional state space (Townsend, 1983). A number of approaches have been developed to solve this class of models. A numerical approach consists of guessing and verifying the laws of motion for the vector of higher-order beliefs. Since this vector is infinite-dimensional, in practice it is truncated at a sufficiently high order (e.g. Nimark, 2011). Another approach guesses and verifies the joint distribution of endogenous and exogenous variables and then computes conditional expectations to verify the guess (e.g. Lorenzoni, 2009; Maćkowiak and Wiederholt, 2009). Here the approximation is on the history dependence of the state variables, which is modeled using finite-order ARMA processes.

In some cases, one can exploit the fact that only a particular *weighted average* of higher-order expectations matters for the solution of the model (Woodford, 2002). Sufficient conditions for applicability of this method, which are met in the environment considered here, are that the nominal expenditure follows an exogenous process and no past and forward-looking endogenous variables enter the price-setting problem (Melosi, 2014). The advantage of this approach is that there is no need to truncate the state vector and its dimensionality becomes very small, allowing for a fast and accurate model solution, which is essential for estimation.

By looking at equations (24) and (26), it is clear that determining the dynamics of $\varphi \sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)} x_t^D$ and $\xi \sum_{k=1}^{\infty}(1-\xi)^{k-1}\bar{\mathbb{E}}_t^{(k)} x_t^D$ for $x = a, m$ is sufficient to determine the endogenous prices p_{Ht} and p^*_{Ft}. In turn, one can use these two variables together with the nominal exchange rate to solve for the rest of the model.

Hence, to solve the model, I guess that the state of the system includes the exogenous state variables plus the two specific weighted averages of high-order expectations implied by equations (24) and (26). In particular, I define $F_{\xi,t} \equiv \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$ and $F_{\varphi,t} \equiv \varphi \sum_{k=1}^{\infty}(1-\varphi)X_t^{(k)}$where $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*]'$ is the vector of exogenous state variables. $X_t^{(k)}$ is shorthand notations for $\bar{\mathbb{E}}_t^{(k)} X_t$. The transition equation for the model can be shown to be

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t, \tag{29}$$

where

$$\bar{X}_t = \left[\begin{array}{c} X_t \\ \hline F_{\xi,t} \\ \hline F_{\varphi,t} \end{array}\right] \quad \bar{B} = \left[\begin{array}{ccc} B_{6\times 6} & 0 & 0 \\ \Gamma^{\xi,x}_{6\times 6} & \Gamma^{\xi,\xi}_{6\times 6} & 0 \\ \Gamma^{\varphi,x}_{6\times 6} & 0 & \Gamma^{\varphi,\varphi}_{6\times 6} \end{array}\right] \quad \bar{b} = \left[\begin{array}{c} b_{6\times 4} \\ \hline \Gamma^{\xi,u}_{6\times 4} \\ \hline \Gamma^{\varphi,u}_{6\times 4} \end{array}\right] \quad u_t = \left[\begin{array}{c} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{array}\right].$$

Equation (29) is the state transition equation of the system. Firms in the model use the observation equation (17) and its foreign counterpart to form expectations about the state vector. The matrices B and b are given by the exogenous processes of the model, whereas matrices Γ are to be determined by solving the signal-extraction problem of the firms using the Kalman filter. These matrices are functions of the model parameters and the Kalman gain matrix associated with the signal-extraction problem of the firms. The algorithm used in Woodford (2002) can be easily extended to solve this model.[14]

5 Impulse Responses

In this section, I study the properties of the model in the more general case in which monetary processes can be autocorrelated ($\rho_m \neq 0$) and the economies are also subject to technology shocks. The model is parameterized using the calibration and prior means described in Section 6.2, unless otherwise noted.

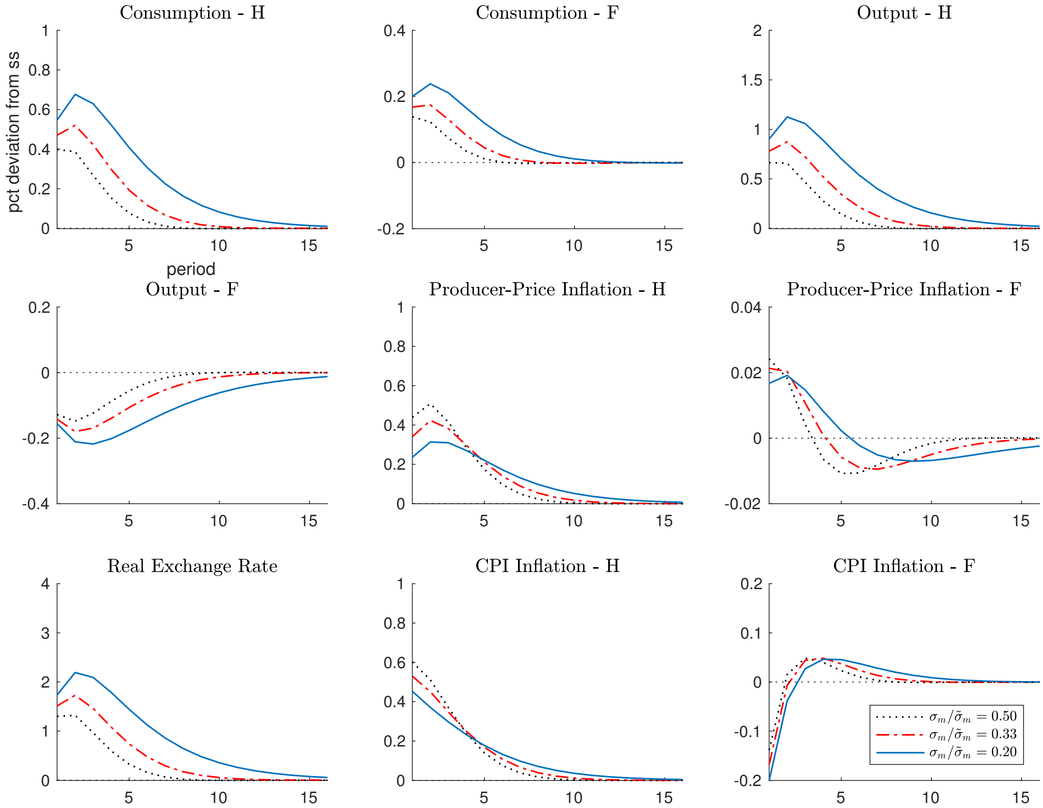
5.1 Monetary Shocks

Figure 2 shows the impulse responses of key variables to a positive monetary shock in the home country for a value of $\rho_m = 0.5$ and various signal-to-noise ratios. Prices for goods produced in the home country increase, but—because price adjustment is incomplete with imperfect information—domestic output also rises. Foreign output falls because, according to the parameterization used, home and foreign goods are net substitute. Consumption rises in both countries, more so in Home given the presence of home bias. The nominal exchange rate (not shown) depreciates as a

[14]Details are in Appendix B.

result of the monetary expansion. The difference between home and foreign goods' prices rises by less than the nominal exchange rate, resulting in a worsening of the terms of trade and in a real exchange rate depreciation (upward movement). Finally, domestic inflation rises as the prices of both home goods and foreign goods rise in domestic currency. Conversely, foreign inflation falls, as foreign goods' prices are almost unchanged, and home goods' prices in domestic currency fall.

Figure 2: Impulse Responses to a Home Monetary Shock



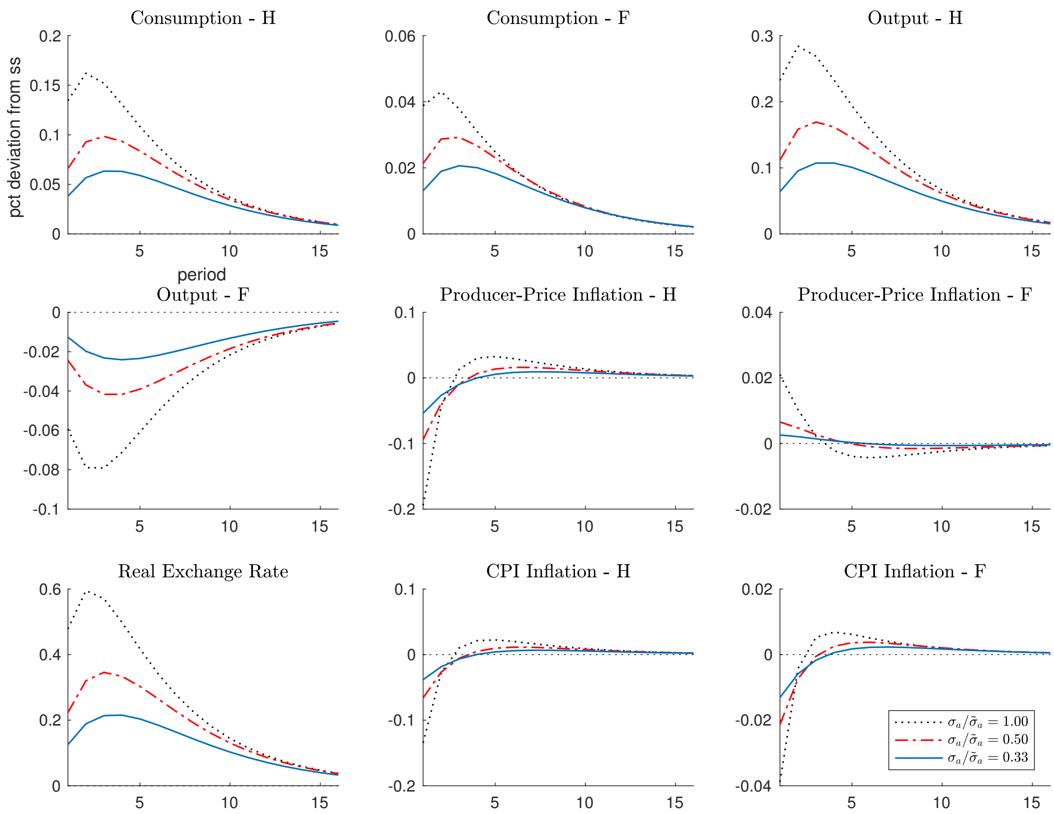
Notes: The figure depicts the impulse responses of key variables following a one-standard deviation (σ_m) Home monetary shock for different values of noise in the signals ($\tilde{\sigma}_m$).

The introduction of persistent monetary shocks results in hump-shaped responses for most key macro variables, including the real exchange rate. The hump in the response of the real exchange rate is consistent with the empirical literature (Steinsson, 2008). Interestingly, domestic producer-price inflation displays a hump for persistent monetary shocks. Hence, this model is consistent with the inertial behavior of inflation observed in the data (Christiano, Eichenbaum, and Evans, 2005). The Figure also shows that an increase in the noise of private signals delivers more volatility and persistence in the exchange rate. The intuition for this result is the same as that highlighted in the previous section, whereby with noisy signals, firms put little weight on new information and adjust prices slowly.

5.2 Technology Shocks

Figure 3 shows the impulse responses to a home technology shock with persistence $\rho_a = 0.80$ for different signal-to-noise ratios. A home technology shock raises domestic output and lowers the prices of home-produced goods. The shock is transmitted internationally via a depreciation of the real exchange rate. Consumption rises in both countries but more markedly in the home country. Varying the signal-to-noise ratio, we observe that more noise tends to dampen the effect of technology shocks, although it contributes to somewhat higher persistence. Intuitively, in this model, output can rise in response to technology shocks only if prices fall because nominal expenditure is fixed by the levels of money supplies.[15] When signals are more precise, firms change prices quickly and output can rise substantially. When signals are noisier, firms fail to lower prices enough, and therefore output increases by a smaller amount.

Figure 3: Impulse Responses to a Home Technology Shock



Notes: The figure depicts the impulse responses of key variables following a one-standard deviation (σ_a) Home technology shock for different values of noise in the signals ($\tilde{\sigma}_a$).

[15]This corresponds to the case in which monetary authorities do not accommodate technology shocks.

6 Empirical Analysis

This section contains the econometric analysis that evaluates whether the dispersed-information model can account for the empirical properties of the Euro/Dollar real exchange rate. The analysis proceeds as follows. First, I estimate the model parameters using Bayesian techniques. The estimation will help me pin down values for the parameters of the model, in particular the signal-to-noise ratios, for which empirical evidence is scarce. I then use the estimated model to test how well it captures the dynamics of the real exchange rate.

6.1 Data and Empirical Strategy

I estimate the parameters of the dispersed-information model using data on the US and Euro Area. The US data comes from the FRED database, while the European data are taken from the Area Wide Model database. I use the time series of GDP and GDP deflators that I map to the variables $[\Pi_t^H, \Pi_t^F, Y_t^H, Y_t^F]$ in the model, where $\Pi_t^H = \frac{P_{Ht}}{P_{H,t-1}}$ and $\Pi_t^F = \frac{P_{Ft}^*}{P_{F,t-1}^*}$. The sample period ranges from 1973:I to 2008:II. Details on the construction of the dataset and on the estimation are in Appendix C.

Using these four observables allows me to pin down the key parameters of the model, including the signal-to-noise ratios for monetary and technology shocks, for which there is scarce microeconomic evidence. Given the estimated parameters, I subsequently test whether the dispersed information model is quantitatively able to generate the volatility and persistence observed in the Euro/Dollar real exchange rate. This empirical strategy is analogous in spirit to the common practice of calibrating a model to fit certain moments (in this case, the moments of real GDP and GDP-deflator inflation rates included in the likelihood function) and testing how well the model reproduces other moments in the data (here, the moments of the real exchange rate). This setup effectively allows me to conduct an out-of-sample test on the real exchange rate, as none of its moments were directly used in the estimation.

6.2 Prior and Posterior Distributions

I fix the values of the parameters that are not well identified in the estimation process. Specifically, I set the home bias $\alpha = 0.9$ to match the average import-to-GDP ratio for the US over the sample. The parameter γ is set to 7, which implies a steady-state markup of 16.7%, in line with macro estimates (Basu and Fernald, 1997). Finally, I set σ to 5, as in Chari, Kehoe, and McGrattan (2002). I estimate the rest of the parameters.

Table 1 reports the prior and posterior distribution of the parameter estimates. The more novel and important parameters in the present analysis are the signal-to-noise ratios. The priors for the standard deviation of shocks and noise terms follow

Table 1: Priors and Posterior Estimates

	Prior				Baseline - Posterior			Financial - Posterior		
Name	Shape	Mean	0.05	0.95	Mean	0.05	0.95	Mean	0.05	0.95
ψ	$\mathcal{G}$	1.00	0.41	2.10	0.26	0.14	0.42	0.30	0.16	0.48
ω	$\mathcal{N}$	1.00	0.30	1.70	0.68	0.53	0.84	0.30	0.23	0.36
ρ_a	$\mathcal{B}$	0.80	0.44	0.95	0.96	0.94	0.98	0.96	0.94	0.98
ρ_a^*	$\mathcal{B}$	0.80	0.44	0.95	0.96	0.93	0.98	0.95	0.92	0.98
ρ_m	$\mathcal{B}$	0.50	0.25	0.75	0.45	0.33	0.57	0.39	0.27	0.51
ρ_m^*	$\mathcal{B}$	0.50	0.25	0.75	0.45	0.32	0.58	0.41	0.28	0.54
ρ_ζ	$\mathcal{B}$	0.50	0.17	0.83	—	—	—	0.94	0.90	0.97
$100\sigma_a$	$\mathcal{IG}$	0.70	0.52	1.02	0.92	0.72	1.17	0.99	0.76	1.27
$100\sigma_a^*$	$\mathcal{IG}$	0.70	0.52	1.02	0.79	0.64	0.97	0.80	0.65	1.00
$100\sigma_m$	$\mathcal{IG}$	1.60	0.83	6.64	0.56	0.49	0.63	0.63	0.56	0.70
$100\sigma_m^*$	$\mathcal{IG}$	1.60	0.83	6.64	0.54	0.48	0.61	0.61	0.54	0.68
$100\sigma_\zeta$	$\mathcal{U}$				—	—	—	9.68	8.44	11.05
$\sigma_a/\tilde{\sigma}_a$	$\mathcal{NA}$	0.66	0.18	0.94	0.86	0.65	1.10	0.79	0.60	1.01
$\sigma_a^*/\tilde{\sigma}_a^*$	$\mathcal{NA}$	0.66	0.18	0.94	1.36	1.03	1.74	1.29	0.98	1.64
$\sigma_m/\tilde{\sigma}_m$	$\mathcal{NA}$	0.32	0.25	0.71	0.12	0.08	0.16	0.12	0.09	0.16
$\sigma_m^*/\tilde{\sigma}_m^*$	$\mathcal{NA}$	0.32	0.25	0.71	0.14	0.11	0.19	0.15	0.11	0.20

$Notes$: The letters $\mathcal{B}$, $\mathcal{N}$, $\mathcal{G}$, $\mathcal{IG}$ denote the beta, normal, gamma, and inverse gamma distributions. $\mathcal{U}$ denotes a degenerate uniform distribution on $\mathbb{R}^+$. $\mathcal{NA}$ is used for implied priors, which do not belong to any family of theoretical distributions. Baseline refers to the model of Section 2 driven by monetary and productivity shocks. Financial refers to the models augmented with financial shocks introduced in Section 8.

Melosi (2014). These priors allow for a wide range of signal-to-noise ratios, letting the data select plausible empirical values. For monetary shocks, the mean estimates of the signal-to-noise ratios are $\sigma_m/\tilde{\sigma}_m = 0.12$ and $\sigma_m^*/\tilde{\sigma}_m^* = 0.14$. For the technology shocks, $\sigma_a/\tilde{\sigma}_a = 0.86$ and $\sigma_{a^*}/\tilde{\sigma}_a^* = 1.36$. These results indicate that firms are more informed about technology shocks than they are about nominal demand shocks by a factor of about seven for the US and nine for the Euro Area. Melosi (2014), who estimates a closed-economy model similar to the one used here, finds a comparable factor for the US economy and argues that this is consistent with the predictions of a rational inattention model (Sims, 2003) as well as with the median absolute sizes of price change found in micro evidence by Nakamura and Steinsson (2008).

While the signal-to-noise ratio for monetary shocks is very similar for the US and Euro Area, the estimates suggest that the Euro Area technology shocks are more precisely observed than the US ones. Empirically, this difference allows the model to better match the relative volatility of US to Euro Area output. I found that restricting the signal-to-noise to be the same in the US and in the Euro Area does not quantitatively affect the results presented below. I discuss the estimates of the more standard parameters of the model in Appendix D.

A posterior check. One additional way to check the plausibility of the estimated signal-to-noise ratio is to consider the answer to the following questions: when all firms are subject to information frictions, how much profit is a given firm losing from not being perfectly informed? The presence of information frictions implies that firms do not generally set their price equal to the profit-maximizing price, which is defined as the price a particular firm would set if it had complete information. Arguably, it would be implausible for a given firm to be imperfectly informed if that came at the cost of foregoing a sizable share of profits. Performing this profit-loss calculation at the parameters' posterior mean reveals that the estimated signal-to-noise ratios imply profit losses well below 1% of steady state revenues—0.097% (0.68%) of steady state revenues (profits) for a US firm and 0.075% (0.52%) for a European firm.[16] These profit losses are small, and can be compared to empirical estimates of the information cost of price adjustment, which is 1.22% of a firm's revenues according to the findings of Zbaracki et al. (2004).

6.3 How Well Does the Model Explain the Real Exchange Rate?

Next, I test how well the estimated model captures the dynamics of the real exchange rate observed in the data. The real exchange rate consists of the nominal exchange rate in U.S. dollars per Euros, converted to the real exchange rate index by multiplying it by the Euro area CPI (*HICP*) and dividing it by the U.S. CPI (*CPIAUCSL*). The "synthetic" US/Euro nominal exchange rate prior to the launch of the Euro also comes from the Area Wide Model Database. As for the model estimation, the sample period runs from 1973:I to 2008:II.

Following the empirical approach of Steinsson (2008) and Carvalho and Nechio (2011), I calculate measures of persistence for the real exchange rate based on the estimates of an AR(p) process of the form:

$$q_t = \mu + \alpha q_{t-1} + \sum_{j=1}^{p} \psi_j \Delta q_{t-j} + \epsilon_t, \tag{30}$$

where I calculate median unbiased estimates of μ, α, and ψ's using the grid-bootstrap method of Hansen (1999). The number of lags, p, is set to 5 as in Steinsson (2008).

The first two columns of Table 2 report several measures of persistence and volatility for the real exchange rate. In the top part of the table, I compute the half-life (HL), up-life (UP), and quarter-life (QL) following a unitary impulse response. The half-life is defined as the largest T such that the impulse response $IR(T-1)$ >0.5 and $IR(T)$ <0.5. The up-life and quarter-life are defined similarly, but with thresholds 1 and 0.25, respectively. All these measures are useful in captur-

[16] Details about the calculation of profit losses are in Appendix E.

ing the non-monotonically-decaying shape of the exchange rate impulse response. These statistics are reported in years. I also consider more traditional measures of persistence, such as the sum of autoregressive coefficients (captured by α) and the autocorrelation of the HP-filtered exchange rate. The second part of Table 2 reports the measures of volatility and cross-correlation of real and nominal exchange rates most extensively used in the literature (see, e.g., Bergin and Feenstra, 2001; Chari, Kehoe, and McGrattan, 2002): the relative standard deviation of the HP-filtered real exchange rate to output or consumption, and the contemporaneous correlation of HP-filtered real and nominal exchange rates.

Table 2: Properties of Real Exchange Rate

	Data		Baseline		Financial	
	Median	[0.05,0.95]	Median	[0.05, 0.95]	Median	[0.05, 0.95]
α	0.94	[0.89, 0.98]	0.96	[0.93, 1.00]	0.96	[0.93, 1.00]
Half-life	3.55	[2.22, ∞]	4.15	[2.47,∞]	4.46	[2.45, ∞]
Up-life	2.28	[1.19, ∞]	2.01	[1.04,∞]	1.86	[0.77, ∞]
Quarter-life	4.48	[2.72, ∞]	6.10	[3.45,∞]	7.12	[3.68, ∞]
UL/HL	0.64	[0.36, 0.76]	0.50	[0.24, 0.70]	0.42	[0.16, 0.62]
QL-HL	0.93	[0.40, 4.50]	1.95	[0.65, 12.67]	2.37	[0.92, 17.51]
$\rho(q_{hp})$	0.85	[0.79, 0.90]	0.85	[0.79, 0.89]	0.82	[0.75, 0.87]
$\frac{\sigma(q_{hp})}{\sigma(c_{hp})}$	6.29	-	4.20	[3.19, 5.59]	4.63	[3.48, 6.20]
$\frac{\sigma(q_{hp})}{\sigma(y_{hp})}$	5.12	-	3.14	[2.55 , 3.91]	4.25	[3.28, 5.58]
$\rho(q_{hp}, \varepsilon_{hp})$	0.99	-	0.97	[0.95 , 0.99]	0.99	[0.98,0.99]

$Notes$: the table reports median, 5th and 95th percentile for key real exchange rate statistics. Half-life: the largest T such that $IR(T-1) \geq 0.5$ and $IR(T) < 0.5$. Quarter-life: the largest T such that $IR(T-1) \geq 0.25$ and $IR(T) < 0.25$. Up-life: the largest time T such that $IR(T-1) \geq 1$ and $IR(T) < 1$. ρ and σ correspond to first-order autocorrelation/cross-correlation and standard deviation, respectively. Baseline refers to the model of Section 2 driven by monetary and productivity shocks. Financial refers to the models augmented with financial shocks introduced in Section 8.

The real exchange rate displays a typical hump-shaped behavior, initially rising over time and not falling below the initial impulse—the up-life—for 9 quarters. The half-life of the exchange rate is about 3.5 years, which is well in line with previous evidence. Finally, the quarter-life of the exchange rate is about 4.5 years, which implies that the time the exchange rate spends below one half of the initial response but above one quarter of it is 1 year, suggesting a moderate acceleration in the rate of decay when short-run dynamics start to die out. These findings are well in line with empirical evidence from other countries (Steinsson, 2008), and point to the presence of a hump shape in the impulse response of the Euro/Dollar real exchange rate as well. Moreover, Table 2 highlights that the real exchange rate is extremely volatile: 6.29 times as volatile as consumption and 5.12 times as volatile as output. Finally, the correlation between the real and nominal exchange rate is 0.99.

To assess the empirical success of the dispersed-information model, I simulate it fixing parameters at the posterior mean and then estimate an AR(5) process on the real exchange rate using the same method described above. Columns 3 and 4 of Table 2 show that the model is notably successful in matching the moments from the data, even though its parameters were not estimated by targeting these moments. The model predicts a median half-life of 4.1 years, which is very close to the 3.7 years observed in the data. The model captures the early dynamics of the impulse response well, predicting an up-life of 2 years, close to the 2.2 years found in the data. The model predicts a quarter-life of about 6 years, somewhat longer than the empirical counterpart. Finally, the autocorrelation of the HP-filtered exchange rate in the model matches the data exactly. The simulated real exchange rate exhibits high volatility and strong correlation with the nominal exchange rate as in the data.

Additional moments. While the focus of the paper is on the volatility and persistence of real exchange rates, I here consider how the model performs along other business cycle dimensions commonly considered in the literature. Columns 1 and 2 of Table 3 report some business cycle moments from the data and from the model developed above. The model produces reasonable results along several of the business-cycle dimensions considered, particularly in terms of volatilities and autocorrelations. As in the data, in the model consumption is less volatile than output. The model accurately predicts that nominal exchange rates are more volatile than real exchange rates, which are in turn much more volatile than the foreign to domestic price ratio. It also generates considerable persistence for prices and quantities. While still delivering the correct signs, the model somewhat overstates the positive cross-country consumption correlation and the negative correlation between real exchange rate and the price ratio.

The Table also highlights some limitations of the model, which relate to some of the assumptions made in order to keep it tractable enough to be estimated. In the data, the correlation between real exchange rate and macro variables such as relative output or relative consumption is close to zero, or negative — a feature that is often referred to as exchange-rate disconnect (Meese and Rogoff, 1983; Engel and West, 2005). In contrast, in the model real exchange rates display high correlation with such macro variables. This should not come as a surprise, because with complete markets and only monetary and productivity shocks, the risk sharing condition (8) implies a tight link between exchange rates and fundamentals, as first pointed out by Backus and Smith (1993). Relatedly, taking the first difference of the logarithm of this equation and combining it with the log-linearized Home and Foreign Euler equation results in Uncovered Interest Parity (UIP): $i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1}$. UIP explains the near-one coefficient in the Fama (1984) regression of nominal exchange rate changes on interest rate differentials. In the data, this coefficient is negative, consistently with a large empirical literature on UIP-violations that finds that countries with higher interest rates see their currency appreciate over short to medium horizons.

An appealing way of breaking the Backus-Smith condition while keeping the model tractable enough to be estimated is by allowing for an additional source of economic fluctuations that directly enters the risk-sharing condition. I return to this issue in Section 8, where I explore the robustness of the results to the introduction of a shock to international asset demand.

Table 3: Additional Moments

Moment	Data	Baseline	Financial
$\sigma(c_t)/\sigma(y_t)$	0.81	0.74	1.13
$\sigma(q_t)/\sigma(e_t)$	0.96	0.84	0.82
$\sigma(p_t^* - p_t)/\sigma(q_t)$	0.16	0.31	0.26
$\sigma(c_t - c_t^*)/\sigma(q_t)$	0.16	0.20	0.23
$\rho(c_t)$	0.86	0.84	0.77
$\rho(p_t^* - p_t)$	0.92	0.87	0.75
$\rho(e_t)$	0.86	0.84	0.72
$\rho(\Delta e_t)$	0.30	0.39	0.04
$corr(c_t, c_t^*)$	0.33	0.62	-0.04
$corr(q_t, c_t - c_t^*)$	-0.20	1.00	-0.44
$corr(q_t, y_{Ht} - y_{Ft})$	0.03	1.00	0.29
$corr(q_t, p_t^* - p_t)$	-0.18	-0.49	-0.80
Fama β	-0.75	0.99	0.19
Fama R^2	0.01	0.17	0.01
$\sigma(i_t - i_t^*)/\sigma(\Delta e_t)$	0.13	0.42	0.21

Notes: with the exception of nominal exchange rate depreciation, standard deviations and correlations in the table are based on logged and HP-filtered US and Euro Area data for the period 1973:I-2008:II. Nominal exchange rate depreciations are measured as log-changes in nominal exchange rates. Fama β and Fama R^2 are the OLS coefficient and R-squared of the projection of the exchange rate change Δe_{t+1} on the interest rate differential $i_t - i_t^*$. The model statistics in columns report the average across 2000 simulations of 142 periods. Baseline refers to the model of Section 2 driven by monetary and productivity shocks. Financial refers to the models augmented with financial shocks introduced in Section 8.

6.4 Monetary Shocks and Persistent Real Exchange Rates

The previous section showed how the dispersed-information model, driven by monetary and technology shocks, is able to capture the large and persistent fluctuations of real exchange rates. Those were statements about unconditional moments. This section instead focuses on the dynamics of the real exchange rate conditional on nominal shocks. It is well known that in models with sticky prices, these shocks cannot produce highly persistent real exchange rate dynamics under plausible nominal rigidities. Here we use impulse-response analysis to ask, on the one hand, whether from an empirical perspective real exchange rates are persistent following nominal disturbances, and, on the other hand, whether the empirical propagation of these shocks is properly accounted for by the model. The analysis thus allows us to test whether the dispersed-information model adequately captures the transmission of nominal shocks on the real exchange rate.

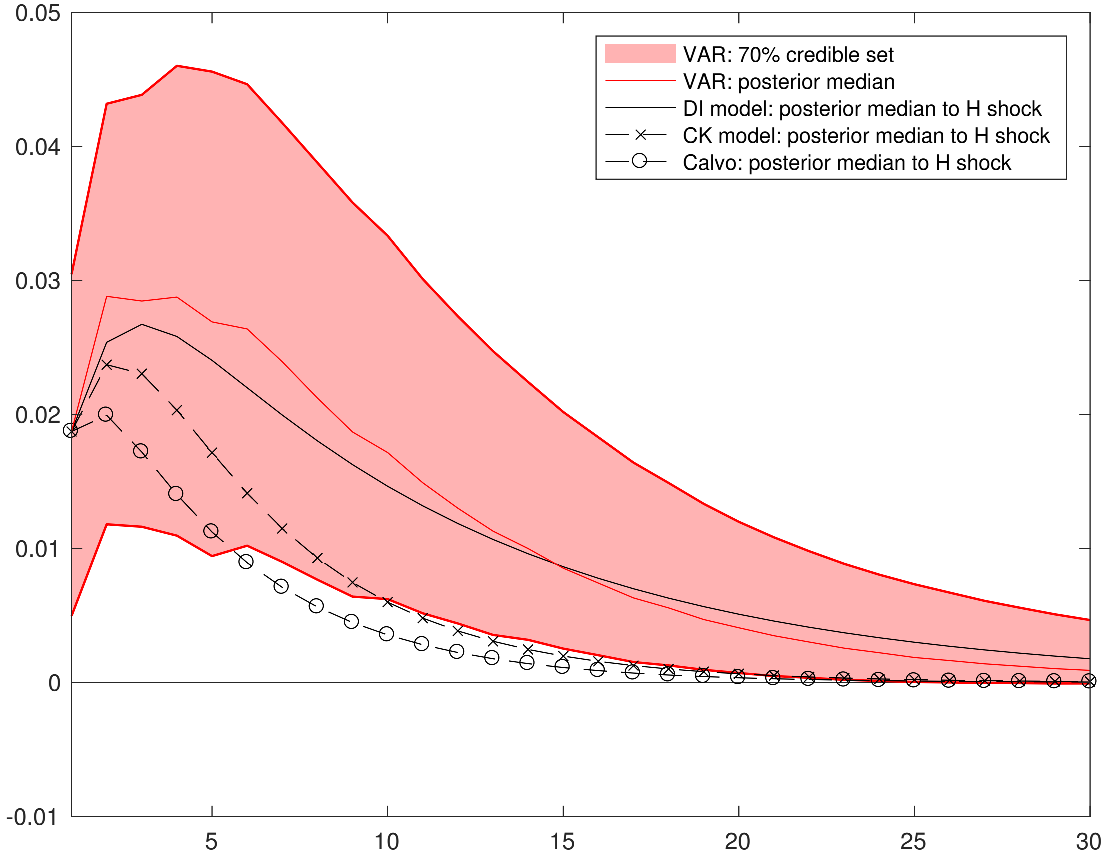
To study the empirical propagation of monetary shocks onto the real exchange rate I use a structural VAR approach. My findings complement those of a large literature, pioneered by Clarida and Gali (1994), that attempts to identify the effects of nominal shocks on real exchange rates by means of autoregressive models. I estimate a VAR with 4 lags that includes the real exchange rate and the CPI differential between the US and Euro Area. The variables are collected in the vector $X_t = [\Delta \ln RER_t, \Delta(\ln CPI_t^{US} - \ln CPI_t^{EU})]$. Identification is based on the restriction that nominal shocks have no long-run effects on the real exchange rate, in the spirit of Blanchard and Quah (1989). Long-run restrictions have been used before in this literature (Lastrapes, 1992; Clarida and Gali, 1994; Rogers, 1999; Chen, 2004) and are consistent with the predictions of the models examined in this study. In keeping with the Bayesian spirit of the paper, I follow Sims and Zha (1998) in specifying the prior distribution for the VAR parameters. I obtain 100,000 posterior draws using the Gibbs sampler.

Figure 4 reports the impulse response of the level of the real exchange rate to a monetary shock from the estimated VAR in red, along with the median impulse response to a home monetary shock in the estimated dispersed-information model in solid black. The impulse response from the VAR highlights the fact that monetary shocks have significant effects on the real exchange rate. The real exchange rate depreciates on impact and peaks two quarters after the impulse, displaying pronounced hump-shaped dynamics, like in the unconditional moments. Consistent with the findings in Clarida and Gali (1994), the dynamic response is very persistent, as it is indistinguishable from zero up to 6 years after the shock.

The dispersed-information model captures these dynamics remarkably well. The response from the model peaks three quarters after the impulse and then decays at a pace similar to the median response in the data, and well within the 70% posterior credible set throughout the horizon. The model naturally generates the hump-shaped dynamics observed in the data. It should again be highlighted that the parameters in the model were estimated without any explicit reference to real exchange rates.

The first column of Table 4 shows the properties of the real exchange rate in an economy driven only by nominal shocks. The table shows that the model generates

Figure 4: Real Exchange Rate Response to a Monetary Shock



Note: the red lines represent the median response and the 70% credible set of the real exchange rate to a one standard deviation monetary shock in the VAR. The black solid (dotted) line depicts the median response of the real exchange rate to a Home monetary shocks in the dispersed-information (Calvo) model.

highly persistent real exchange rates conditional on these shocks, with a half-life of roughly 3 years. The intuition for the persistent hump-shaped dynamics can be grasped by writing the real exchange rate depreciation as:

$$\Delta q_t = \frac{2\alpha - 1}{1 - \Psi} \Delta m_t^D - \frac{\Psi + (2\alpha - 1)(1 - \Psi)}{1 - \Psi} (\Delta p_{Ht} - \Delta p_{Ft}^*) \tag{31}$$

with $\Psi = (2\alpha - 1)(1 - \sigma^{-1}) \in (0, 1)$. This expression shows that after a initial relative money supply shock, the real exchange rate will keep depreciating as long as the "adjusted" growth rate of money exceeds that of relative prices. The latter in turn depends on the firms' expectations about the money supplies. The key here is that the dispersed-information model results in a delayed response of prices. Firms who suspect that nominal spending has increased will increase their prices to some extent right after the shock, but can plan to increase them later on. Only once firms have become fairly confident that (i) demand has actually increased and that (ii) others are convinced that demand has increased, they increase their prices at a faster rate than the growth rate of money, causing the real exchange rate to start reverting back to its mean. In Figure 4, this switching point occurs three quarters after the shock.[17]

Table 4: The Propagation of Monetary Shocks

	Baseline			Financial		
	DI	CK	Calvo	DI	CK	Calvo
Half-life	3.14	1.63	1.25	3.18	1.55	1.57
Up-life	1.51	0.80	0.42	1.38	0.70	0.48
Quarter-life	4.65	2.43	1.96	4.96	2.38	2.61
$\rho(q_{hp})$	0.84	0.81	0.75	0.83	0.79	0.76
$\frac{\sigma(q_{hp})}{\sigma(c_{hp})}$	4.05	3.92	4.13	4.18	3.95	4.44
$\frac{\sigma(q_{hp})}{\sigma(y_{hp})}$	3.04	2.96	2.96	3.77	3.61	4.00

Note: the table compares the properties of the real exchange rate conditioning on monetary shocks. Common Knowledge is a counterfactual economy where all firms have the same infomation as discussed in Section 6.5. Calvo refers to the sticky-price model of Section 7. Baseline refers to the model of Section 2 driven by monetary and productivity shocks. Financial refers to the models augmented with financial shocks introduced in Section 8.

6.5 The Role of Higher-Order Beliefs

The persistent effects of monetary shocks on the real exchange rate in the dispersed-information model are in part driven by the dynamics of higher-order beliefs. Unlike a model in which firms have common information, strategic complementarity makes a producer's price responsive to the higher-order beliefs (i.e., what they think other firms are thinking). A natural question is how much persistence is due to these higher-order beliefs. This section considers a counterfactual economy where producers have noisy but common information about the state of nominal aggregate demand. In this case, all firms share the same beliefs and as a result higher-order beliefs do not play any separate role. Comparing the behavior of the real exchange rate in this economy vis-à-vis the benchmark model will thus allow to disentangle the role of higher-order beliefs.[18]

In the counterfactual economy, every firm in both countries receives the following

[17] There is no simple analytical expression for the dynamics of relative prices when money growth is autocorrelated but from simulations I found that for values of $\rho_m > 0.05$ and of the signal-to-noise $\sigma_m/\tilde{\sigma}_m < 1.18$ the peak always occurs after the impact. The simulations were done varying one parameter at a time, and keeping the other ones at the posterior mean.

[18] There is an alternative counterfactual that one can consider: one in which firms have heterogeneous beliefs but do not face strategic complementarities. Also in this counterfactual economy, higher-order beliefs play no role as firms do not need to coordinate their actions and forecast the forecast of others. It turns out that the response of the real exchange rate is the same in both counterfactuals because in the model considered strategic complementarities matter if and only if there are heterogenous higher-order beliefs. I thank an anonymous referee for making this point.

common signals about the two nominal disturbances:

$$Z_t = \begin{bmatrix} z_t^m \\ z_t^{m^*} \end{bmatrix} = \begin{bmatrix} m_t \\ m_t^* \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_m & 0 \\ 0 & \tilde{\sigma}_{m^*} \end{bmatrix} \begin{bmatrix} \eta_t^m \\ \eta_t^{m^*} \end{bmatrix}. \tag{32}$$

It is easy to see that equation (25) that I report here for convenience

$$p_{Ht} - p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)} \left[(1-\varphi)(p_{Ht} - p_{Ft}^*) + \varphi(m_t - m_t^*) \right],$$

still holds up to a modification. When there is common knowledge about the structural shocks, firms in this economy become identical along every dimension, and therefore set the same price. Owing to common knowledge of rationality, those prices are also known to all producers. Therefore we obtain

$$p_{Ht} - p_{Ft}^* = (1-\varphi)(p_{Ht} - p_{Ft}^*) + \varphi \mathbb{E}_t^{(1)} m_t^D = \bar{\mathbb{E}}_t^{(1)} m_t^D, \tag{33}$$

where the expectation operator in this economy is the same as the average first-order expectation operator in the benchmark model. The real exchange rate in the counterfactual economy is given by

$$q_t = (2\alpha - 1)(m_t^D - \bar{\mathbb{E}}_t^{(1)} m_t^D). \tag{34}$$

The real exchange rate responds to shocks to the extent that firms' first-order beliefs deviate from the true monetary disturbance. So long as common signals are noisy, it will respond to monetary shocks but not via the dynamics of higher-order beliefs.

The second column of Table 4 reports the properties of the real exchange rate in this counterfactual economy. The table shows that the persistence of the real exchange rate drops considerably when there is common knowledge; the half-life falls to 1.65 years. Eliminating dispersed information also somewhat reduces the hump-shape behavior, as can be seen by the lower up-life of the real exchange rate in the Table as well as in Figure 4. The counterfactual exercise thus shows that dispersed information and higher-order beliefs play an important role in generating long-lived dynamics of the real exchange rate in the model, contributing to about 47% of its conditional half-life. Recall that the speed of price adjustment depends on (i) whether firms believe that demand has actually increased and (ii) whether they believe that others are convinced that demand has increased. By shutting down higher order beliefs we are eliminating the second channel. Because first-order beliefs adjust faster than higher-order beliefs, price adjustment will be faster, resulting in less-hump shaped and shorter lived real exchange rate responses.

7 Comparison with Sticky-Price Model

A natural question that arises in evaluating the success of the imperfect-information model is how well it performs relative to a more traditional sticky-price model à la Calvo (1983). I address this question in two ways. First, I estimate a model with sticky prices à la Calvo and compare its fit to the data on output and deflators used in the estimation relative to the dispersed-information model. Second, I compare the sticky-price model's ability to reproduce the VAR evidence on real exchange rate dynamics relative to the model with information frictions.

7.1 The Calvo Model

Households and monetary authorities are modeled in the same way as in the benchmark economy. Firms can perfectly observe the current and past realization of shocks, but can only reset their prices with a random probability $1-\theta$. The derivations of the model are standard and can be found, for instance, in Corsetti, Dedola, and Leduc (2010). The dynamics of inflation can be described by the New Keynesian Phillips Curves:

$$\pi_t^H = \kappa \left[\frac{\sigma\psi + 1}{\gamma + \psi} y_{H,t} - \frac{2(1-\alpha)\alpha\psi(\sigma\omega - 1)}{\gamma + \psi} t_t - \frac{1+\psi}{\gamma+\psi} a_t \right] + \beta \mathbb{E}_t \pi_{t+1}^H, \tag{35}$$

$$\pi_t^F = \kappa \left[\frac{\sigma\psi + 1}{\gamma + \psi} y_{F,t} + \frac{2(1-\alpha)\alpha\psi(\sigma\omega - 1)}{\gamma + \psi} t_t - \frac{1+\psi}{\gamma+\psi} a_t^* \right] + \beta \mathbb{E}_t \pi_{t+1}^F, \tag{36}$$

where $\pi_t^H = p_{H,t} - p_{H,t-1}$, $\pi_t^F = p_{F,t}^* - p_{F,t-1}^*$ and $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. These two equations replace equations (24) and (26) of the dispersed-information model.

Two additional remarks are in order. First, similarly to the dispersed-information model, also the Calvo model exhibits strategic complementarities as long as $\frac{\sigma\psi+1}{\gamma+\psi} < 1$, a restriction that will be supported by the data. Second, while one could certainly write down a more sophisticated model of nominal rigidities, the spirit of the current exercise is to compare two models that differ only along one dimension: the assumptions about price setting. By keeping both models simple, the comparison highlights how these assumptions—as opposed to other "bells and whistles"—directly impact the transmission of shocks in the open economy.

7.2 Estimation and Results

I estimate the Calvo model with the same observables used for the dispersed-information model. A first result that emerges from the estimation is that the model with information frictions tends to fit the data better than the Calvo model. A detailed discussion about the estimation and the Bayesian model comparison is in Appendix D. Here I focus instead on the propagation of nominal disturbances onto the real exchange rate.

The third column of Table 4 shows that the estimated Calvo model delivers counterfactually low persistence conditional on monetary shocks, predicting a half-life of the real exchange rate of 1.25 years. This point is further illustrated in Figure 4, which shows that the response of the real exchange rate in the Calvo model to a monetary shock falls short of explaining the persistence and the hump-shaped dynamics documented in the VAR. Intuitively, this happens because a Calvo firm that anticipates future rises in marginal cost increases prices today, out of fear of not being able to adjust them tomorrow. This results in a more front-loaded response of inflation and real exchange rates to nominal disturbances. In contrast, the dispersed-information model produces more realistic and longer-lived real exchange rate dynamics thanks to the gradual adjustment of higher-order beliefs.

For the purpose of highlighting the different mechanisms at play, this paper has considered separately how informational and nominal rigidities can explain the behavior of real exchange rates. It is important to note, however, that both mechanisms are present in the data and they likely contribute to the persistence of real exchange rates in a complementary way.

8 The Role of Financial Shocks

The analysis conducted so far has highlighted how information frictions can help explain the volatility and persistence of the real exchange rate as well as rationalize its response to nominal disturbances. Yet, we noted in Section 6.3 that the focus on nominal and productivity shocks, together with the assumption of complete markets, results in a tight relationship between exchange rates and macro variables, at odds with the exchange-rate disconnect puzzle. Moreover, these assumptions also result in UIP, thus preventing the model by construction to explain features of the data involving nominal exchange rates and interest rates, such as the UIP puzzle.

A recent literature has stressed the importance of shocks that originate in the financial sector to address the puzzling behavior associated with nominal and real exchange rates observed in the data. For example, Itskhoki and Mukhin (2017) show that a model featuring an international asset demand shock, coupled with a transmission mechanism that weakens the effect of exchange movements on local prices and quantities, can simultaneously account for the exchange rate disconnect puzzle as well as other puzzles associated with exchange rates, including the Purchasing Power Parity, the Backus-Smith, and the UIP puzzle. I here explore the role of financial shocks in my framework and investigate the robustness of my findings to the inclusion of this additional source of fluctuations.

8.1 Modeling Financial Shocks

Several microfoundations of financial shocks exist: noise traders and limits to arbitrage (Jeanne and Rose, 2002) or financial frictions (e.g. Hau and Rey, 2006; Gabaix and Maggiori, 2015) in currency markets, time-varying risk premia model, or departures from full-information rational expectations in currency markets as in Evans and Lyons (2002), Gourinchas and Tornell (2004), and Bacchetta and van Wincoop (2006). The latter class of models differs from the environment considered here, which instead assumes complete information for the agents trading in asset markets (i.e., the households) and incomplete information for the firms. Therefore here dispersed information affect the dynamics of price adjustment rather than portfolio decisions as in Bacchetta and van Wincoop (2006).

I follow one of the formalizations of financial shocks adopted by Itskhoki and Mukhin, which allows me to break the Backus-Smith condition, while at the same time preserving tractability and allowing for a likelihood analysis.[19] I now assume that only foreign-currency assets are traded internationally and that they are the only types of assets held by foreign households. Note that this is without loss of generality as long as a full set of state-contingent foreign-currency securities can be traded. I assume that the Home household faces an exogenous shock, ζ_t, for holding foreign currency assets at time t. Under these assumptions the Foreign nominal stochastic discount factor for foreign-currency assets is:

$$\Theta^*_{t+1} = \beta \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{P^*_t}{P^*_{t+1}}, \tag{37}$$

and the Home nominal stochastic discount factor for foreign-currency assets is:

$$\Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \exp \Delta\zeta_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \exp \Delta\zeta_{t+1}. \tag{38}$$

With a full set of state-contingent foreign-currency assets, the risk-sharing condition becomes:

$$\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \exp \Delta\zeta_{t+1} = \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{P^*_t}{P^*_{t+1}}, \tag{39}$$

which is equivalent to the following static condition (written in log-linear form):

$$q_t = \sigma(c_t - c^*_t) - \zeta_t. \tag{40}$$

From this expression it is clear that ζ_t acts as a wedge to the risk-sharing condition (8). Taking the first difference of this equation and combining it with the Home and

[19] This specification can be found in their Appendix A.6.

Foreign Euler equation gives:

$$i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \mathbb{E}_t \Delta \zeta_{t+1}. \tag{41}$$

Therefore $\Delta\zeta_{t+1}$ also introduces deviations from UIP. I assume that the financial shock follows the exogenous process:

$$\zeta_t = \rho_\zeta \zeta_{t-1} + u_t^\zeta, \qquad u_t^\zeta \sim \mathcal{N}(0, \sigma_\zeta^2). \tag{42}$$

In the sticky-price model of Section 7, there is full information so I only need to specify the stochastic process in (42). For the dispersed-information model, I also need to specify how firms form beliefs about the shock. I treat this symmetrically with respect to the other shocks and assume that Home and Foreign firms receive the following idiosyncratic signal $z_i^\zeta = \zeta_t + \nu_{i,t}^\zeta$ for $i \in \{h, f\}$ where $\nu_{i,t}^\zeta \sim \mathcal{N}(0, \tilde{\sigma}_\zeta^2)$.

8.2 Estimation and Results

I re-estimate the dispersed-information and Calvo models augmented with the financial shock ζ_t. Given that the shock has been introduced in the literature to explain the nominal and real exchange rate behavior, I add to the set of observables the log real exchange rate between the US and the Euro Area.[20] The last three columns of Table 1 report the estimates for the dispersed-information model with financial shocks.[21]

The additional parameters relative to the benchmark model are the persistence, ρ_ζ, the standard deviation, σ_ζ, of the financial shock, as well as the standard deviation of the noise in the signal, $\tilde{\sigma}_\zeta$. The latter turns out to be weakly identified in the estimation, although the data tend to prefer a low value of the signal-to-noise ratio, suggesting a relatively noisy signal. I have experimented with several values of this ratio and fixed it at the point where the posterior becomes flat relative to this parameter. I settled on a value of $\sigma_\zeta/\tilde{\sigma}_\zeta = 0.125$, similarly to the signal-to-noise for monetary shocks.[22] The other two parameters are estimated, using a loose Beta prior for the persistence parameter, and a uniform prior on $\mathbb{R}^+$ for the standard deviation of the innovations to ζ. The estimated standard deviation of the financial shock may seem large but equation (41) indicates that it is the expected first difference of this shock that matters for interest rate determination: $\psi_t \equiv \mathbb{E}_t \Delta\zeta_{t+1} = -(1-\rho_\zeta)\zeta_t$. The standard deviation of ψ_t is thus much smaller than that of ζ_t and the last column

[20] I have also estimated these models using as additional observable the nominal exchange rate depreciation instead of the real exchange rate. Results are very similar.

[21] The estimates for the sticky price model are in the Appendix D.

[22] The other parameter estimates as well as the propagation of the shock are not significantly affected by this choice.

of Table 3 shows that the model predicts a reasonable volatility of interest rates differentials.[23]

The last two columns of Table 2 report the exchange rates moments of interest for this version of the model. The dispersed information model augmented with the financial shock still accounts well for the unconditional behavior of the real exchange rate. In particular, this estimated version of the model improves on the volatility of the real exchange rate relative to consumption and output, and introduces even more persistence, with a median unconditional half-life of the real exchange rate of 4.68 years, roughly 1 year more than in the data. Table 3 shows that this version of the model brings the correlation of exchange rates and macro variables closer to the data, partly addressing the exchange rate disconnect puzzle. The β coefficient and R^2 on the Fama regression improve to some extent, although the estimation suggests that simply adding the financial shock is not sufficient to quantitatively address these issues. The introduction of the financial shock also helps ameliorate the volatility of interest rates relative to exchange exchange rates. All these findings go in the direction suggested by Itskhoki and Mukhin (2017). Adding more realistic features to the model that mute the response of local variables to exchange rate movement such as those mentioned by those authors could be an interesting avenue for future research.

Table 5: Theoretical Variance Decomposition

Shocks		DI		Calvo	
		q_t	Δe_t	q_t	Δe_t
Monetary	$u_t^m, u_t^{m^*}$	15%	16%	4%	13%
Productivity	$u_t^a, u_t^{a^*}$	20%	3%	34%	3%
Financial	u_t^ζ	65%	81%	62%	84%

Note: the table reports the theoretical variance decomposition of real exchange rates and nominal exchange rate changes for the dispersed-information (DI) and Calvo model calculated at the estimated posterior mean estimated reported in columns 4 and 5 of Table 6.

Given that financial shocks substantially help the model improve on the comovement between exchange rates and macro variables, we may expect these shocks to be an important source of exchange rate fluctuations. The variance decomposition of real and nominal exchange rates for the dispersed-information and Calvo models reported in Table 5 confirms this hypothesis. In both models, financial shocks play a dominant role, accounting for more than 60% and 80% or real and nominal exchange rate fluctuations, respectively. These results are consistent with the findings resulting from the calibration exercise in Itskhoki and Mukhin (2017), who argue

[23]The standard deviation of ψ_t is 0.58 percent. When compared to that of technology shocks, one obtains $\sigma_a/\sigma_\psi = 1.70$, which is close to the the ratio of 2.1 set in Itskhoki and Mukhin (2017).

that to explain the exchange-rate disconnect, the data requires financial shocks to be primarily driving the *unconditional* exchange rate dynamics. Monetary shocks in this version of the model explain about 15% of real and nominal exchange rate variance, which is consistent with the findings in the empirical literature about these shocks.

Lastly, I investigate the robustness of the transmission of nominal shock onto exchange rates in this version of the models. Section 6.5-6.6 stressed the importance of information frictions to account for the persistence effect of monetary shocks on the real exchange rates, highlighting how higher-order beliefs are essential to the slugglish propagation of these shocks and for the resulting hump-shaped behavior of real exchange rates. The last three columns of Table 4 confirm the robustness of this result to the inclusion of financial shocks. In particular, in column 4 we notice that the dispersed information model still deliver persistent and hump-shaped responses of real exchange rates following monetary disturbances that line up well with the VAR evidence.[24] Comparing the up-life in columns 4 and 5 shows that the presence of higher-order belief remains essential to deliver the delayed peak in the real exchange rate documented by the VAR.

9 Conclusions

Existing New-Keynesian models with sticky prices struggle to deliver the persistence in the real exchange rate observed in the data under plausible nominal rigidities. In this paper, I argue that the persistence of the real exchange rate, together with its other empirical features, can be explained by a model with strategic complementarity and dispersed information among price-setting firms. In this environment, firms' beliefs about economic conditions and about other firms' expectations become endogenous state variables that result in increased inertia in real exchange rates. Once taken to the data, the model is shown to successfully explain the volatility and persistence of the real exchange rate. The model also generates inertial real exchange rate dynamics following monetary shocks, which is consistent with the empirical evidence documented by a structural VAR. Taken together, my findings suggest that dispersed information is a quantitatively important channel for real exchange rate dynamics.

The framework developed in this paper relies on some assumptions that enhance the tractability of the model as well as the speed and accuracy of the solution. For instance, the model assumes that households are fully informed and that international asset markets are complete. These assumptions render the likelihood evaluation at

[24]The size of the response is also robust: in the baseline (financial) model, a 1% increase in output triggered by a monetary shock depreciates the real exchange rate by 2.24% (2.73%), increases producer-price inflation by 10 (12) basis points and consumer-price inflation by 40 (46) basis points.

different points in the parameter space fairly rapid, thus allowing a researcher to apply full-information estimation methods. Introducing incomplete markets and information frictions for households in the spirit of Bacchetta and van Wincoop (2006) would result in endogenous deviations from UIP, which may help rationalize the dynamic comovement between interest rates and exchange rates recently emphasized in Engel (2016) and Valchev (2017). More broadly, whether dispersed information can reconcile the many puzzles in the international macroeconomics and finance literature within a single framework remains an open question for future research.

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Appendix (For Online Publication)

A. Algebraic Results and Proofs

These derivations are based on the model of Section 8 that includes the shock to international asset demand, ζ_t, which replaces the risk-sharing condition (8) with (39). To recover the baseline model, it is sufficient to set $\zeta_t = 0$ for all t.

Solution for p_{Ht} and p^*_{Ft}

Log-linearizing the FOC, one obtains

$$p_t(h) = \mathbb{E}_{ht}(w_{ht} - a_t). \tag{43}$$

Add and subtract p_t inside the expectation

$$p_t(h) = \mathbb{E}_{ht}(w_{ht} - p_t + p_t - a_t). \tag{44}$$

Now substitute $w_{ht} - p_t$ from the log-linear version of (6) to obtain

$$p_t(h) = \mathbb{E}_{ht}(\sigma c_t + \frac{1}{\psi} l_{it} + p_t - a_t). \tag{45}$$

Substitute the production function for l_{ht}

$$p_t(h) = \mathbb{E}_{ht}(\sigma c_t + \frac{1}{\psi}(y_{ht} - a_t) + p_t - a_t). \tag{46}$$

Now substitute the log-linearized demand for y_{ht}

$$p_t(h) = \mathbb{E}_{ht}[\sigma c_t + p_t - (1 + \frac{1}{\psi})a_t + \frac{1}{\psi}(-\gamma(p_t(h) - p_{Ht}) - \omega(p_{Ht} - p_t) + \alpha c_t + (1-\alpha)(\omega q_t + c^*_t)].$$

Add and subtract p_{Ht} and rearrange to obtain

$$\begin{aligned} p_t(h) =& \mathbb{E}_{ht}\left[-\left(\frac{1+\psi}{\gamma+\psi}\right)a_t + p_{Ht} - \left(\frac{\psi+\omega}{\gamma+\psi}\right)(p_{Ht} - p_t) + \alpha\left(\frac{\alpha+\psi\sigma}{\gamma+\psi}\right)c_t\right] + \\ & \mathbb{E}_{ht}\left[(1-\alpha)\left(\frac{1}{\gamma+\psi}\right)(\omega q_t + c^*_t)\right]. \end{aligned}$$

Now recall that $p_{Ht} - p_t = -(1-\alpha)t_t$, so

$$\begin{aligned} p_t(h) =& \mathbb{E}_{it}\left[-\left(\frac{1+\psi}{\gamma+\psi}\right)a_t + p_{Ht} + (1-\alpha)\left(\frac{\psi+\omega}{\gamma+\psi}\right)t_t + \alpha\left(\frac{\alpha+\psi\sigma}{\gamma+\psi}\right)c_t\right] + \\ & \mathbb{E}_{ht}\left[(1-\alpha)\left(\frac{1}{\gamma+\psi}\right)(\omega q_t + c^*_t)\right]. \end{aligned}$$

Recall that $\boldsymbol{q}_t = (2\alpha - 1)\,\boldsymbol{t}_t$, hence

$$
\begin{aligned}
&p_t(h) = \mathbb{E}_{ht}\left\{p_{Ht}\right\} + \\
&(\gamma+\psi)^{-1}\mathbb{E}_{ht}\left\{(1-\alpha)\left(\psi+\omega+\omega(2\alpha-1)\right)\boldsymbol{t}_t + (\alpha+\psi\sigma)\,c_t + (1-\alpha)c_t^* - (1+\psi)\,a_t]\right\},
\end{aligned}
$$

$$
\begin{aligned}
&p_t(h) = \mathbb{E}_{ht}\left\{p_{Ht}\right\} + \\
&(\gamma+\psi)^{-1}\mathbb{E}_{ht}\left\{[(1-\alpha)\left(\psi+2\alpha\omega\right)\boldsymbol{t}_t + (\alpha+\psi\sigma)\,c_t + (1-\alpha)c_t^* - (1+\psi)\,a_t]\right\}.
\end{aligned}
$$

A similar equation can be derived for $p_t(f)$. Rewrite this as

$$
\begin{aligned}
&p_t(h) = \mathbb{E}_{ht}\left\{p_{Ht}\right\} + \\
&(\gamma+\psi)^{-1}\mathbb{E}_{ht}\left\{[(1-\alpha)\left(\psi+2\alpha\omega\right)\boldsymbol{t}_t + (1+\psi\sigma)\,c_t - (1-\alpha)(c_t - c_t^*) - (1+\psi)\,a_t]\right\}.
\end{aligned}
$$

Using the fact that $c_t - c_t^* = \sigma^{-1}(2\alpha-1)\,\boldsymbol{t}_t + \sigma^{-1}\zeta_t$, I can write

$$
\begin{aligned}
&p_t(h) = \mathbb{E}_{ht}\left\{p_{Ht}\right\} + \\
&(\gamma+\psi)^{-1}\mathbb{E}_{ht}\left\{[(1-\alpha)\left(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}\right)\boldsymbol{t}_t + (1+\psi\sigma)\,c_t - (1-\alpha)\sigma^{-1}\zeta_t - (1+\psi)\,a_t]\right\}.
\end{aligned}
$$

Use the money process and the link between relative prices to write

$$
\begin{aligned}
p_t(h) =& \mathbb{E}_{ht}\left\{p_{Ht} + (\gamma+\psi)^{-1}[(1-\alpha)\left(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}\right)\boldsymbol{t}_t\right\} + \\
&(\gamma+\psi)^{-1}\mathbb{E}_{ht}\left\{(1+\psi\sigma)\left(m_t - (1-\alpha)\,\boldsymbol{t}_t - p_{Ht}\right) - (1-\alpha)\sigma^{-1}\zeta_t - (1+\psi)\,a_t]\right\}.
\end{aligned}
$$

and finally

$$
\begin{aligned}
p_t(h) =& \mathbb{E}_{ht}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ht} + \left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\boldsymbol{t}_t\right\} + \\
&\mathbb{E}_{ht}\left\{\left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t - \left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right)\zeta_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a_t\right\}.
\end{aligned}
$$

Similarly, for the foreign country I have

$$
\begin{aligned}
p_t^*(f) =& \mathbb{E}_{ft}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ft}^* - \left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\boldsymbol{t}_t\right\} + \\
&\mathbb{E}_{ft}\left\{\left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t^* + \left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right)\zeta_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a_t^*\right\}.
\end{aligned}
$$

Notice that the last two equations collapse to (19) and (20) when $\sigma = 1$ and $\zeta_t = 0$. By averaging these two equation over firms one obtains

$$p_{Ht}(h) = \bar{\mathbb{E}}_t^{(1)} \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right) p_{Ht} + \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma+\psi}\right] t_t \right\} + \bar{\mathbb{E}}_t^{(1)} \left\{ \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m_t - \left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right) \zeta_t - \left(\frac{1+\psi}{\gamma+\psi}\right) a_t \right\}.$$

and

$$p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)} \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right) p_{Ft}^* - \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma+\psi}\right] t_t \right\} + \bar{\mathbb{E}}_t^{(1)} \left\{ \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m_t^* + \left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right) \zeta_t - \left(\frac{1+\psi}{\gamma+\psi}\right) a_t^* \right\}.$$

When taking the sum of these two equations the terms of trade cancel out

$$p_{Ht} + p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)} \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right) (p_{Ht} + p_{Ft}^*) + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m_t^W - \left(\frac{1+\psi}{\gamma+\psi}\right) a_t^W \right\}.$$

Recursively substituting $p_{Ht} + p_{Ft}^*$ on the right-hand side yields

$$p_{Ht} + p_{Ft}^* = \tilde{\xi} \sum_{k=1}^{\infty} (1-\tilde{\xi})^{k-1} E_t^{(k)} \left(m_t^W - \frac{1+\psi}{1+\psi\sigma} a_t^W \right), \tag{47}$$

where

$$\tilde{\xi} = \frac{1+\psi\sigma}{\gamma+\psi}. \tag{48}$$

Taking instead the difference of the average prices equations yields

$$p_{Ht} - p_{Ft}^* = \mathbb{E}_t^{(1)} \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right) (p_{Ht} - p_{Ft}^*) - 2\left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right) \zeta_t \right\} + \mathbb{E}_t^{(1)} \left\{ 2\left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma+\psi}\right] t_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m_t^D - \left(\frac{1+\psi}{\gamma+\psi}\right) a_{it}^D \right\}. \tag{49}$$

Now I need to solve for t_t in terms of $p_{Ht} - p_{Ft}^*$ and m_t^D. The nominal exchange rate is given by

$$e_t = q_t + p_t - p_t^* = (2\alpha - 1) t_t + m_t^D - c_t^D, \tag{50}$$

$$= (2\alpha-1) t_t - (2\alpha-1)\sigma^{-1} t_t - \sigma^{-1}\zeta_t + m_t^D = (2\alpha-1)(1-\sigma^{-1}) t_t - \sigma^{-1}\zeta_t + m_t^D. \tag{51}$$

So

$$t_t = p_{Ft}^* - p_{Ht} + e_t = p_{Ft}^* - p_{Ht} + (2\alpha-1)(1-\sigma^{-1}) t_t - \sigma^{-1}\zeta_t + m_t^D, \tag{52}$$

or

$$t_t = \frac{1}{(1-(2\alpha-1)(1-\sigma^{-1}))}(-(p_{Ht}-p^*_{Ft}) - \sigma^{-1}\zeta_t + m^D_t). \tag{53}$$

Substituting (53) into (51) we can write the nominal exchange rate as

$$e_t = \frac{(2\alpha-1)(1-\sigma^{-1})}{1-(2\alpha-1)(1-\sigma^{-1})}(-(p_{Ht}-p^*_{Ft}) - \sigma^{-1}\zeta_t + m^D_t) + m^D_t \tag{54}$$

$$= \frac{1}{1-\Psi}m^D_t - \frac{\Psi}{1-\Psi}(p_{Ht}-p^*_{Ft}) - \frac{\Psi}{\sigma(1-\Psi)}\zeta_t \tag{55}$$

where $\Psi = (2\alpha-1)(1-\sigma^{-1})$. Substituting (53) into (49)

$$p_t(h) - p_t(f)^* = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi} - 2\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\right)(p_{Ht}-p^*_{Ft})\right\}$$
$$+\mathbb{E}_{it}\left\{\left(\frac{1+\psi\sigma}{\gamma+\psi} + 2\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\right)m^D_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a^D_t\right\}$$
$$-\mathbb{E}_{it}\left\{2\left[\left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right) + \frac{\sigma^{-1}(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\zeta_t\right\}.$$

Hence the solution for the price difference can be expressed as

$$p_{Ht} - p^*_{Ft} = \tilde{\varphi}\sum_{k=1}^{\infty}(1-\tilde{\varphi})^{k-1}E^{(k)}_t\left(m^D_t - \frac{\tilde{\kappa}}{\tilde{\varphi}}\zeta_t - \frac{1+\psi}{(\gamma+\psi)\tilde{\varphi}}a^D_t\right), \tag{56}$$

where

$$\tilde{\varphi} = \left(\frac{1+\psi\sigma}{\gamma+\psi} + 2\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\right), \tag{57}$$

and

$$\tilde{\kappa} = 2\left[\left(\frac{(1-\alpha)\sigma^{-1}}{\gamma+\psi}\right) + \frac{\sigma^{-1}(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]. \tag{58}$$

Proof of Proposition 1

Substituting $\sigma = 1$ into equations (48) and (57), one obtains $\tilde{\xi} = \xi$ and $\tilde{\varphi} = \varphi$. With this substitution, equations (47) and (56) become (24) and (26) in the main text. Furthermore, using $\sigma = 1$ and $\zeta_t = 0$ in equation (53) one obtains $t_t = p^*_{Ft} + m^D_t - p_{Ht}$. Combining this expression with equation (26) and with $q_t = (2\alpha-1)t_t$ yields equation (27) in Proposition 1. ■

Proof of Proposition 2

The random-walk hypothesis implies that $m^D_t = m^D_{t-1} + u_t$, where $u_t \equiv u^m_t - u^{m^*}_t$. The proof follows the guess-and-verify approach used by Woodford (2002). The guess is that:

$$p_{Ht} - p^*_{Ft} = \nu(p_{H,t-1} - p^*_{F,t-1}) + (1-\nu)m^D_t, \tag{59}$$

Denote with the i subscript a generic firm in either Home or Foreign. Equation (17) shows that firms in each country receive two signals about the money supplies: one for the Home ($z^m_{i,t}$) and one for the Foreign ($z^{m^*}_{i,t}$) money supplies. Given the properties of the signals, it is as if firm i received one signal about the difference in money supplies: $s_{i,t} = m^D_t + \eta_{i,t}$ with $\eta_{i,t} = \upsilon^m_{i,t} - \upsilon^{m^*}_{i,t}$. The process for the money supplies, the guess for the price difference, and this signal can be written compactly in the following state-space representation:

$$\begin{aligned}
\begin{bmatrix} m^D_t \\ p_{Ht} - p^*_{Ft} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 1-\nu & \nu \end{bmatrix} \begin{bmatrix} m^D_{t-1} \\ p_{H,t-1} - p^*_{F,t-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 1-\nu \end{bmatrix} u_t & \implies x_t = Mx_{t-1} + du_t, \\
s_{i,t} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} m^D_t \\ p_{Ht} - p^*_{Ft} \end{bmatrix} + \eta_{i,t} & \implies s_{i,t} = ex_t + \eta_{i,t}.
\end{aligned}$$

Here I have defined the new vector x_t and matrices M, d and e to write the problem as a state-space system. The Kalman filter implies:

$$\mathbb{E}_{it}(x_t) = \mathbb{E}_{i,t-1}(x_{t-1}) + \kappa[s_{i,t} - eM\mathbb{E}_{i,t-1}(x_{t-1})], \tag{60}$$

with $\kappa = [\kappa_1, \kappa_2]'$ being a 2×1 vector of Kalman gains. Given the symmetry of signals across countries, integrating the last expression over the continuum of Home or Foreign firms yields:

$$\bar{\mathbb{E}}^{(1)}_t(x_t) = \kappa eMx_{t-1} + (M - \kappa eM)\bar{\mathbb{E}}^{(1)}_{t-1}(x_{t-1}) + \kappa edu_t. \tag{61}$$

Now note that equation (49), absent technology shocks, may be written as:

$$p_{Ht} - p^*_{Ft} = (1-\varphi)\bar{\mathbb{E}}^{(1)}_t(p_{Ht} - p^*_{Ft}) + \varphi\bar{\mathbb{E}}^{(1)}_t(m^D_t). \tag{62}$$

On the right-hand side of this expression, the average expectations of m^D_t and $p_{Ht} - p^*_{Ft}$ can be replaced using equation (61) after performing the matrix algebra. This yields:

$$\begin{aligned}
p_{Ht} - p^*_{Ft} = \nu(p_{H,t-1} - p^*_{F,t-1}) &+ [\varphi\kappa_1 + (1-\varphi)\kappa_2]m^D_t \\
&+ [(1-\nu) - \varphi\kappa_1 - (1-\varphi)\kappa_2]\bar{\mathbb{E}}^{(1)}_{t-1}(m^D_{t-1}).
\end{aligned}$$

This verifies the original guess in equation (59) and shows that $(1-\nu) = \varphi\kappa_1 + (1-\varphi)\kappa_2$. Now recall that with log utility the real exchange rate is given by $q_t = (2\alpha - 1)(p^*_{Ft} + e_t - p_{Ht}) = (2\alpha - 1)(m^D_t - p_{Ht} + p^*_{Ft})$. Using the solution for the price difference yields:

$$\begin{aligned} q_t &= (2\alpha - 1)[m_{t-1} + u_t - (1-\nu)m_{t-1} - \nu(p_{H,t-1} - p^*_{F,t-1}) - (1-\nu)u_t], \\ &= \nu q_{t-1} + (2\alpha - 1)\nu u_t. \end{aligned}$$

The expressions for the autocorrelation and the standard deviation of the real exchange rate immediately follow.

To find the Kalman gains, let us define the variance-covariance matrix of forecast errors:

$$\Sigma \equiv var\{x_t - \mathbb{E}_{i,t-1}x_t\}$$

Note that the matrix Σ are the same for all firms because the observation errors have the same stochastic processes for all firms. From standard Kalman filtering, the matrix Σ solves the Riccati equation

$$\Sigma = M\Sigma M' - (e\Sigma e' + \sigma^2_\eta)^{-1} M\Sigma e' e\Sigma M' + \sigma^2_u dd' \tag{63}$$

and the Kalman gain is given by

$$\kappa = (e\Sigma e' + \sigma^2_\eta)^{-1}\Sigma e' = (\Sigma_{11} + \sigma^2_\eta)^{-1} \begin{bmatrix} \Sigma_{11} \\ \Sigma_{21} \end{bmatrix} \tag{64}$$

We can use equation (63) to solve for Σ_{11} and Σ_{21}. The upper left equation of (63) yields

$$(\Sigma_{11} + \sigma^2_\eta)^{-1}\Sigma^2_{11} - \sigma^2_u = 0$$

Since Σ is a variance-covariance matrix, the positive solution is the only relevant one. This is given by

$$\Sigma_{11} = \frac{\sigma^2_u}{2}\left[1 + \sqrt{1 + 4\sigma^2_\eta/\sigma^2_u}\right] \tag{65}$$

Substituting this into the first element of κ in (64) we obtain

$$\kappa_1 = \frac{\frac{\sigma^2_u}{2}\left[1 + \sqrt{1 + 4\sigma^2_\eta/\sigma^2_u}\right]}{\frac{\sigma^2_u}{2}\left[1 + \sqrt{1 + 4\sigma^2_\eta/\sigma^2_u}\right] + \sigma^2_\eta} \tag{66}$$

Now we can use the lower left equation of (64) together with the solution for Σ_{11} to obtain

$$\Sigma_{21} = \sigma_u^2 \frac{1 + 2\sigma_\eta^2/\sigma_u^2 + \sqrt{1 + 4\sigma_\eta^2/\sigma_u^2}}{2/(1-\nu) - 1 + \sqrt{1 + 4\sigma_\eta^2/\sigma_u^2}} \tag{67}$$

Substituting (65) and (67) into (64) we obtain:

$$\sigma_\eta^2/\sigma_u^2(1-\nu)^2 + \varphi(1-\nu) - \varphi = 0 \tag{68}$$

The only solution of this expression in which the variances of the forecast error are finite and constant over time requires $0 < |1-\nu| < 1$. The corresponding root is:

$$(1-\nu) = \frac{1}{2}\left\{-\varphi\sigma_u^2/\sigma_\eta^2 + \sqrt{\left(\varphi\sigma_u^2/\sigma_\eta^2\right) + 4\varphi\sigma_u^2/\sigma_\eta^2}\right\} \tag{69}$$

So

$$\kappa_2 = (1-\varphi)^{-1}\left\{\frac{1}{2}\left\{-\varphi\sigma_u^2/\sigma_\eta^2 + \sqrt{\left(\varphi\sigma_u^2/\sigma_\eta^2\right) + 4\varphi\sigma_u^2/\sigma_\eta^2}\right\} - \varphi\kappa_1\right\} \tag{70}$$

■

B. Solving and Stationarizing the Model

The exogenous state variables for the model of section 2 are $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*]'$.[25] The state-transition equation and signal equations can be written compactly as

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{71}$$

$$\bar{Z}_{i,t} = D\bar{X}_t + v_t \text{ for } i = h, f, \tag{72}$$

where

$$D = \begin{bmatrix} D^\ddagger & \mathbf{0}_{4x12} \end{bmatrix} \quad D^\ddagger \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{X}_t = \begin{bmatrix} X_t \\ \hline F_{\xi,t} \\ \hline F_{\varphi,t} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_{6\times6} & 0 & 0 \\ \Gamma_{6\times6}^{\xi,x} & \Gamma_{6\times6}^{\xi,\xi} & 0 \\ \Gamma_{6\times6}^{\varphi,x} & 0 & \Gamma_{6\times6}^{\varphi,\varphi} \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b_{6\times4} \\ \hline \Gamma_{6\times4}^{\xi,u} \\ \hline \Gamma_{6\times4}^{\varphi,u} \end{bmatrix} \quad u_t = \begin{bmatrix} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{bmatrix},$$

[25] It is straightforward to extend these derivation to the model of Section 8 where $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*, \zeta_t]'$.

where $F_{\xi,t} \equiv \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$ and $F_{\varphi,t} \equiv \varphi \sum_{k=1}^{\infty}(1-\varphi)X_t^{(k)}$ are the weighted averages of higher-order beliefs that matter for solving the model, $u_t \sim \mathcal{N}(0, \Sigma_u)$ and $v_t \sim \mathcal{N}(0, \Sigma_v)$. The matrices B and b are given by

$$B = \begin{bmatrix} 1+\rho_m & -\rho_m & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\rho_{m^*} & -\rho_{m^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{a^*} \end{bmatrix} \quad b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the matrices Γ can be found as the solution to the fixed-point problem following the approach in Woodford (2002). Specifically

$$\begin{aligned} \Gamma^{\xi,x} &= \boldsymbol{\xi} k B^{\ddagger} \qquad \Gamma^{\xi,u} = \boldsymbol{\xi} k \qquad \Gamma^{\xi,\xi} = B - \boldsymbol{\xi} k B^{\ddagger} \\ \Gamma^{\varphi,x} &= \boldsymbol{\varphi} k B^{\ddagger} \qquad \Gamma^{\varphi,u} = \boldsymbol{\varphi} k \qquad \Gamma^{\varphi,\varphi} = B - \boldsymbol{\varphi} k B^{\ddagger} \\ \boldsymbol{\xi} &\equiv \begin{bmatrix} \tilde{\xi}\boldsymbol{I}_6 & (1-\tilde{\xi})\boldsymbol{I}_6 & \mathbf{0}_6 \end{bmatrix} \\ \boldsymbol{\varphi} &\equiv \begin{bmatrix} \tilde{\varphi} & \mathbf{0}_6 & (1-\tilde{\varphi})\boldsymbol{I}_6 \end{bmatrix}, \end{aligned}$$

where $B^{\ddagger} \equiv \begin{bmatrix} B_1' & B_3' & B_5' & B_6' \end{bmatrix}'$ and B_j is the j-th row of the matrix B. The steady-state Kalman gain matrix, k, is known to be

$$k = PD'[DPD' + \Sigma_v]^{-1}.$$

The variance-covariance matrix P solves the Riccati equation

$$P = \bar{B}\left[P - PD'[DPD' + \Sigma_v]^{-1}DP\right]\bar{B}' + \bar{b}\Sigma_v\bar{b}. \tag{73}$$

For a given set of parameters, the equilibrium can be computed by setting an initial guess for P, and calculating k. With the Kalman gain matrix, the matrices Γ, $\bar{B}$, and $\bar{b}$ can be computed. At this point, one can solve for a new matrix P^* solving the Riccati equation (73). If P and P^* are sufficiently close the fixed-point for the solution of the model has been found, otherwise one has to reiterate through the algorithm using the new P^* until convergence is achieved.

After finding the solution for equation (71), the model can be stationarized exploiting the following facts:

- The level of the money supply is nonstationary, but money growth is stationary.
- Price levels and, more generally, higher-order beliefs about money supplies are

non-stationary but deviations of these beliefs from the true levels of the money supplies are stationary.

For any exogenous variable, x_t, define the deviation of the variable itself from its weighted average of higher-order beliefs: $x_t^{-\xi} \equiv x_t - \xi \sum_{k=1}^{\infty}(1-\xi)x_t^{(k)}$ and the corresponding object for the weight φ, so that in vectors this is $X_t^{-\xi} = X_t - \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$. Because the weighted average of higher-order beliefs converges in the long run to the respective variables, the dynamics of $X_t^{-\xi}$ and $X_t^{-\varphi}$ will be stationary. Furthermore, notice from the Kalman filter iteration that the Γ matrices defined above imply that $\Gamma^{\xi,x} + \Gamma^{\xi,\xi} = \Gamma^{\varphi,x} + \Gamma^{\varphi,\varphi} = B$. Using this fact and equation (71), one can show that

$$X_t^{-\xi} = \Gamma^{\xi,\xi} X_{t-1}^{-\xi} + [b - \Gamma^{\xi,u}]u_t, \tag{74}$$

$$X_t^{-\varphi} = \Gamma^{\varphi,\varphi} X_{t-1}^{-\varphi} + [b - \Gamma^{\varphi,u}]u_t. \tag{75}$$

I stationarize the exogenous part of the system by rewriting it in terms of money growth rates

$$\underbrace{\begin{bmatrix} \Delta m_t \\ \Delta m_t^* \\ a_t \\ a_t^* \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} \rho_m & 0 & 0 & 0 \\ 0 & \rho_{m^*} & 0 & 0 \\ 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & \rho_{a^*} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta m_{t-1} \\ \Delta m_{t-1}^* \\ a_{t-1} \\ a_{t-1}^* \end{bmatrix}}_{Y_{t-1}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{a} \begin{bmatrix} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{bmatrix}. \tag{76}$$

So finally, the stationary state can be written as

$$\underbrace{\begin{bmatrix} Y_t \\ X_t^{-\xi} \\ X_t^{-\varphi} \end{bmatrix}}_{\bar{Y}_t} = \underbrace{\begin{bmatrix} A & 0 & 0 \\ 0 & \Gamma^{\xi,\xi} & 0 \\ 0 & 0 & \Gamma^{\varphi,\varphi} \end{bmatrix}}_{\Phi} \begin{bmatrix} Y_{t-1} \\ X_{t-1}^{-\xi} \\ X_{t-1}^{-\varphi} \end{bmatrix} + \underbrace{\begin{bmatrix} a \\ b - \Gamma^{\xi,u} \\ b - \Gamma^{\varphi,u} \end{bmatrix}}_{C} u_t. \tag{77}$$

Define the stationary variables: $mp_t \equiv m_t - p_{Ht}$ and $mp_t^* = m_t^* - p_{Ft}^*$. The solution for these variables can be written as

$$mp_t = e_h \bar{Y}_t, \tag{78}$$

$$mp_t^* = e_f \bar{Y}_t, \tag{79}$$

for appropriate selector vectors,e_h and e_f. The other model equations can be written as

$$
\begin{aligned}
t_t &= [1-(2\alpha-1)(1-\sigma^{-1})]^{-1}(mp_t - mp_t^*), && (80)\\
c_t &= mp_t - (1-\alpha)t_t, && (81)\\
c_t^* &= mp_t^* + (1-\alpha)t_t, && (82)\\
y_{Ht} &= \omega(1-\alpha)t_t + \alpha c_t + (1-\alpha)(\omega q_t + c_t^*), && (83)\\
y_{Ft} &= -\omega(1-\alpha)t_t + (1-\alpha)(c_t - \omega q_t) + \alpha c_t^*, && (84)\\
q_t &= \sigma(c_t - c_t^*), && (85)\\
\pi_t^H &= \Delta m_t - mp_t + mp_{t-1}, && (86)\\
\pi_t^F &= \Delta m_t^* - mp_t^* + mp_{t-1}^*, && (87)\\
\pi_t &= \alpha\pi_t^H + (1-\alpha)(\Delta\varepsilon_t + \pi_t^F), && (88)\\
\pi_t^* &= (1-\alpha)(\pi_t^H - \Delta\varepsilon_t) + \alpha\pi_t^F, && (89)\\
\Delta\varepsilon_t &= \Delta m_t - \Delta m_t^* + (2\alpha-1)(1-\sigma^{-1})(t_t - t_{t-1}), && (90)\\
\Delta m_t &= \rho_m \Delta m_{t-1} + u_t^m, && (91)\\
\Delta m_t &= \rho_m^* \Delta m_{t-1} + u_t^{m^*}, && (92)\\
a_t &= \rho_a a_{t-1} + u_t^a, && (93)\\
a_t^* &= \rho_a^* a_{t-1}^* + u_t^{a^*}. && (94)
\end{aligned}
$$

These equations can be written compactly as

$$
Z_t = \Xi \bar{Y}_t. \qquad (95)
$$

The matrix equations (77) and (95) form the stationary state-space representation of the model. Equation (80) follows directly from (53). Equations (81) and (82) are obtained by scaling the definitions of aggregate demand $M_t \equiv P_t C_t$ and $M_t^* \equiv P_t^* C_t^*$ by P_{Ht} and P_{Ft}^*, and by combining their log-linearized version with the log-linear definition of the price indices, $p_{Ht} - p_t = -(1-\alpha)t_t$ and $p_{Ft}^* - p_t^* = (1-\alpha)t_t$. Equations (83) and (84) for aggregate outputs are the log-linearized versions of (3) and (4) integrated across Home and Foreign producers, respectively. The real exchange rate in (85) corresponds to equation (8). Producer inflation rates in (86) and (87) follows from the first difference of the definition of the new variables mp_t and mp_t^* above. CPI inflation rates in (88) and (89) come from the log-linearization of the price indices, applying the law of one price. Equation (90) is the first difference of equation (51). Finally, the last four equations represent the exogenous stochastic processes.

C. Data and Estimation

The baseline model is estimated using four observables: GDP and GDP deflator inflation rate for the US and Euro Area. The extension of section 8 also uses the Euro/Dollar real exchange rate. The US data comes from the FRED database, while the European data are taken from the Area Wide Model database. For the US, I construct GDP by taking the logarithm of real GDP (*GDPC96*) divided by the civilian non institutional population over 16 (*CNP16OV*). The growth rate of the GDP deflator is the log-difference of *GDPDEF*. For the Euro Area, I use the logarithm of the real GDP (*YER*) divided by the population, and the log-difference of the GDP deflator (*YED*). Population data for the 17 countries in the Euro Area, consistent with the GDP series, is taken from the OECD database.[26] Finally, the real exchange rate is the log of the nominal exchange rate in U.S. dollars per Euros, converted to the real exchange rate index by multiplying it by the Euro area CPI (HICP) and dividing it by the U.S. CPI (CPIAUCSL). The nominal exchange rate is the "synthetic" US/Euro exchange rate in the Area Wide Model Database (EXR). To construct the additional business cycle moments of Table 3 I also use data on real personal consumption expenditure (PCECC96 for the US and PCR for the Euro Area) per capita and interest rates (FEDFUNDS for the US and STN for the Euro Area).

The sample period ranges from 1973:I to 2008:II, as the model is not intended to capture the macroeconomic developments of the Great Recession nor the ensuing period in which monetary policy was constrained by the zero lower bound. In the model, nominal exchange rates are flexible, therefore the starting date for the sample coincides with the end of the Bretton Woods system of fixed exchange rates. As the model is written in terms of deviations from a balanced growth path, the data series are linearly detrended for consistency purposes. The US is considered to be the home country. The model is stationarized before estimation as described in Appendix B.

The model is estimated using Bayesian techniques, as explained in Herbst and Schorfheide (2016). Specifically, I draw from the posterior distribution $p(\Theta|Y)$, where Θ is the parameter vector and Y the data, using a standard random walk Metropolis-Hasting algorithm. The variance-covariance matrix of the proposal distribution, Σ, is set to the variance-covariance matrix of the estimated parameters at the posterior mode. The estimation is implemented in Dynare. The results are obtained by drawing 1,000,000 parameter vectors from the posterior distribution. With this procedure, I obtain an acceptance rate for the Markov chain of about 25%. The first half of the chain is considered burn-in and it is therefore dropped.

[26] Population data are available only at annual frequency. I use linear interpolation to obtain the quarterly frequency.

D. Priors, Posterior Estimates, and Model Comparison

Dispersed-information models The priors for the dispersed-information model are presented in Table 1 in the main text. The priors for the standard deviations of the shocks and the noise in the signals were already discussed there. There is no clear evidence on the value of the trade elasticity, ω, although macro studies usually point toward low values.[27] For this parameter I set the prior mean to 1. The prior mean for the Frisch elasticity of labor supply, ψ, is also fixed at 1. The persistence parameters for technology shocks are centered at 0.80, whereas the persistence for monetary shocks is set at 0.5—although for these parameters, I let the data guide the estimation by leaving priors fairly loose.

The posterior mean for the parameters of all the estimated models is presented in Table 6. For the baseline dispersed-information model, The posterior mean of ψ is 0.26, lower than the prior mean and in between micro and macro estimates.[28] The posterior mean for the persistence of technology shocks in the two countries is 0.96, in line with many other studies. The persistence of money growth processes is 0.45 for both the US and the Euro Area.

When including the financial shocks in the dispersed information model (column 3) almost all estimates tend to be very similar, with somewhat lower persistence of monetary shocks. One exception is the trade elasticity, ω, which is estimated to be 0.3 in both the dispersed-information model and the Calvo extended models with financial shocks. This low estimate is likely to be driven by the inclusion of the real exchange rate among the observables, which is much more volatile than the output differential in the data. This value is consistent with the findings of Lubik and Schorfheide (2006) and Rabanal and Tuesta (2010) who use data from the same sources and include the real exchange rate in the estimation.[29]

Calvo models For the common parameters with the dispersed-information model, the priors are kept the same. The new parameters relative to the dispersed-information model are the discount factor β and the Calvo parameter θ. [30] These two cannot be separately identified in the estimation, as they both enter the slope of the Phillips Curve in a non-linear fashion. I calibrate the discount factor β to 0.99. I estimate

[27] See Corsetti, Dedola, and Leduc (2008) for a discussion.

[28] A discussion on micro and macro estimates of the Frisch elasticity of labor supply is in Rogerson and Wallenius (2009) or Chetty (2012).

[29] Lubik and Schorfheide (2006) using demeaned data find a trade elasticity of 0.3. Rabanal and Tuesta (2010) compare the fit of several two-country DSGE models under different assumptions on the asset market structure and the currency of pricing. When they consider a model with PCP and complete markets they estimate a trade elasticity of 0.16.

[30] The discount factor β does not appear in the linearized equations of the dispersed-information model because profit maximization is done period by period.

Table 6: Posterior Estimates

Name	Description	Baseline		Financial	
		DI	Calvo	DI	Calvo
κ	$(1-\beta\theta)(1-\theta)/\theta$	—	0.28	—	0.22
ψ	Frisch elasticity of labor supply	0.26	0.14	0.30	0.12
ω	Trade elasticity	0.68	0.78	0.29	0.30
ρ_a	Persistence of technology shock (H)	0.96	0.93	0.96	0.94
ρ_a^*	Persistence of technology shock (F)	0.96	0.91	0.95	0.91
ρ_m	Persistence of monetary shock (H)	0.45	0.27	0.39	0.22
ρ_m^*	Persistence of monetary shock (F)	0.45	0.34	0.41	0.29
ρ_ζ	Persistence of financial shock	—	—	0.94	0.94
$100\sigma_a$	Std of technology shock (H)	0.92	1.62	0.99	1.77
$100\sigma_a^*$	Std of technology shock (F)	0.79	2.00	0.80	2.29
$100\sigma_m$	Std of monetary shock (H)	0.56	0.55	0.63	0.64
$100\sigma_m^*$	Std of monetary shock (F)	0.54	0.51	0.61	0.60
$100\sigma_\zeta^*$	Std of financial shock	—	—	9.68	9.73
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	0.82	—	0.76	—
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	1.30	—	1.23	—
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	0.11	—	0.12	—
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	0.14	—	0.15	—
Observables		$y_{Ht}, \pi_t^H, y_{Ft}, \pi_t^F$		$y_{Ht}, \pi_t^H, y_{Ft}, \pi_t^F, q_t$	
Log Marginal Data Density		1776.34	1739.99	1938.45	1896.60

Notes : The table reports the mean estimates for the parameters of the dispersed-information (DI) model and the Calvo model. Baseline refers to the models driven by monetary and productivity shocks. Financial refers to the models augmented with financial shocks introduced in Section 8.

the parameter $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. I set the prior of κ using a Gamma distribution such that the median implies a value of the Calvo parameter $\theta = 0.69$, and the 5th and 95th percentile imply values for θ of approximately 0.5 and 0.90. This range broadly covers the micro and macro estimates for the frequency of price adjustment.

For the parameters that are common across models, the posterior estimates of the sticky-price model are quite similar to the estimates for the dispersed-information model. One exception is the standard deviation of technology shocks, discussed in more detail below in the Bayesian model comparison. The mean estimate for κ implies a value for the Calvo parameter θ of 0.61. This value is comparable to those obtained by Lubik and Schorfheide (2006), and imply that prices change approximately every three quarters, consistent with the observed frequency of price change (Nakamura and Steinsson, 2008). Also for the Calvo model, the trade elasticity drops to 0.3 when including the real exchange rate in the estimation. The rest of the estimates do not substantially change when estimating the Calvo model with financial shocks. Finally, the size and the persistence of financial shocks is also very similar when comparing the extended dispersed-information and Calvo model.

Bayesian Model Comparison I take a Bayesian approach to compare the dispersed-information model and the sticky-price model. The marginal data density (MDD) is a key statistics used in a Bayesian model comparison exercise, as it tells the econometrician how he would update his prior on which model is more likely to be the true one after having observed the data. Table 6 reveals that the dispersed-information model has a larger MDD than the sticky-price model by 35 log points. When considering the extended models with financial shocks this difference rises to 42 log points. From an econometric perspective, this difference constitutes decisive evidence in favor of the dispersed-information model. A 35 log-point difference implies that the prior probability ratio in favor of the Calvo model would need to be larger than $2.4e^{15}$ in order for the Calvo model to attain a higher posterior probability than the information-friction model. The fact that the Calvo model has fewer parameters than the dispersed-information model is not worrisome, because the MDD penalizes models for the number of parameters. These findings suggest that, when restricting attention to simple frameworks such as the ones considered here, the model with information frictions is better suited for explaining the joint dynamics of US and Euro Area key macro variables than the sticky-price model.

Where is this sizable difference in overall fit coming from? Two key observations emerge from the estimated parameters reported in Table 6. First, the mean estimate for κ implies a value for the Calvo parameter θ of 0.61, which implies that prices change approximately every three quarters. It is well known that with this frequency of price change the basic Calvo model falls short of generating enough persistence in macro variables. Second, the estimated standard deviations of productivity shocks in the Calvo model are two to three times as large as those of the benchmark model. Note that the latter estimates are consistent with a standard real business cycle calibration of these shocks (Kydland and Prescott, 1982) as well as with estimated medium-scale DSGE models (Smets and Wouters, 2005; Adolfson et al., 2007), while the Calvo estimates are considerably larger. These observations suggest that the Calvo model has to ascribe a very large role to productivity shocks to generate the persistence required to explain the data, in contrast with the dispersed-information model, which is able to generate persistent dynamics following both productivity and monetary shocks. The MDD comparison suggests that the data prefers a model with smaller productivity shocks, but that propagates nominal shocks more persistently.

E. Profit Losses

Modeling imperfect information with noisy signals is a simple way of formalizing the idea that a cost is associated with gathering and processing the information that is relevant for firms' optimal pricing decisions. In the context of the present model, that information relates to aggregate economic conditions and to the prices set by domestic and foreign competitors. One way to evaluate the plausibility of the estimated signal-to-noise ratios is to consider the individual profit loss that a firm incurs when they observe signals only with finite precision. Indeed, one may argue that if paying limited attention to macroeconomic conditions leads to high profit losses, a firm should pay more attention to those conditions. On the other hand, if profit losses are small, then a firm's cost of acquiring more information would outweigh the gain in profits that derive from obtaining more information.

I here explore this reasoning in the context of my model estimates. The calculation of the profit losses follows the approach of Maćkowiak and Wiederholt (2009). Recall that the price set by firm h in the home country in the full model with dispersed information is given by

$$p_t(h) = \mathbb{E}_{ht}\left\{(1-\tilde{\xi})p_{Ht} + r\,\boldsymbol{t}_t + \tilde{\xi}m_t - \xi a_t\right\}, \tag{96}$$

with

$$r = \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma+\psi}\right].$$

Similarly for a firm f in the foreign country

$$p^*_t(f) = \mathbb{E}_{ft}\left\{(1-\tilde{\xi})p^*_{Ft} - r\,\boldsymbol{t}_t + \tilde{\xi}m^*_t - \xi a^*_t\right\},$$

The prices that firms would set under full information, expressed in log-deviations from the steady state, are

$$p^{\diamond}_t(h) = (1-\tilde{\xi})p_{Ht} + r\,\boldsymbol{t}_t + \tilde{\xi}m_t - \xi a_t, \tag{97}$$

$$p^{*\diamond}_t(f) = (1-\tilde{\xi})p^*_{Ft} - r\,\boldsymbol{t}_t + \tilde{\xi}m^*_t - \xi a^*_t. \tag{98}$$

Firm's h expected per-period profit loss due to imperfect information is then given by

$$\mathbb{E}\left[\Pi_{h,t}(P_t(h),\cdot) - \Pi_{h,t}(P^{\diamond}_t(h),\cdot)\right].$$

After taking a log-quadratic approximation to the profit function, this expression simplifies to

$$-\frac{\Pi_{11}}{2}\mathbb{E}\left[(p_t(h) - p^{\diamond}_t(h))^2\right],$$

where Π_{11} is the curvature of the profit function with respect to the firm's own price, and

$$\mathbb{E}\left[(p_t^{\diamond}(h) - p_t(h))^2\right] = \mathbb{E}\left[(1-\tilde{\xi})\bar{p}_{Ht} + r\bar{\mathfrak{t}}_t + \tilde{\xi}\bar{m}_t - \xi\bar{a}_t\right]^2, \tag{99}$$

where $\bar{p}_{Ht} \equiv \mathbb{E}_{ht}p_{Ht} - p_{Ht}$, $\bar{\mathfrak{t}}_t \equiv \mathbb{E}_{ht}\mathfrak{t}_t - \mathfrak{t}_t$, $\bar{m}_t \equiv \mathbb{E}_{ht}m_t - m_t$, and $\bar{a}_t \equiv \mathbb{E}_{ht}a_t - a_t$. The expectation can be calculated as follows. Define the sector vectors v_h and v_f and v_τ such that $p_{Ht} = v_h\bar{X}_t$, $\tau_t = v_f\bar{X}_t$, $p^*_{Ft} = v_f\bar{X}_t$

$$v_h = 1/2[0,0,0,0,0,0,1,0,1,0-\chi_w,-\chi_w,1,0,-1,0,-\chi_d,+\chi_d],$$

$$v_\tau = \frac{1}{(1-(2\alpha-1)(1-\sigma^{-1}))}[1,0,-1,0,0,0,0,0,0,0,0,0,-1,0,1,0,\chi_d,-\chi_d],$$

$$v_f = 1/2[0,0,0,0,0,0,1,0,1,0-\chi_w,-\chi_w,-1,0,1,0,\chi_d,-\chi_d].$$

Then

$$\begin{aligned}
\mathbb{E}\left[(p_t^{\diamond}(h) - p_t(h))^2\right] = {} & (1-\tilde{\xi})^2 VAR_{h,t}(v_h\bar{X}_t) + r^2 VAR_{h,t}(v_\tau\bar{X}_t) + \tilde{\xi}^2 VAR_{h,t}(m_t) \\
& + \xi^2 VAR_{h,t}(a_t) + 2(1-\tilde{\xi})rCOV_{h,t}(v_h\bar{X}_t, v_\tau\bar{X}_t) \\
& + 2(1-\tilde{\xi})\tilde{\xi}COV_{h,t}(v_h\bar{X}_t, m_t) - 2(1-\tilde{\xi})\xi COV_{h,t}(v_h\bar{X}_t, a_t) \\
& + 2r\tilde{\xi}COV_{h,t}(v_\tau\bar{X}_t, m_t) - 2r\xi COV_{h,t}(v_\tau\bar{X}_t, a_t).
\end{aligned}$$

Defining the vectors e_m and e_a such that they select m_t and a_t from $\bar{X}_t$, I can solve for the variances and covariances above using

$$\begin{aligned}
VAR_{h,t}(v_h\bar{X}_t) &= v_h\tilde{P}v_h', \\
VAR_{h,t}(v_\tau\bar{X}_t) &= v_\tau\tilde{P}v_\tau', \\
VAR_{h,t}(m_t) &= e_m\tilde{P}e_m', \\
VAR_{h,t}(a_t) &= e_m\tilde{P}e_a', \\
COV_{h,t}(v_h\bar{X}_t, v_\tau\bar{X}_t) &= v_h\tilde{P}v_\tau', \\
COV_{h,t}(v_h\bar{X}_t, m_t) &= v_h\tilde{P}e_m', \\
COV_{h,t}(v_h\bar{X}_t, a_t) &= v_h\tilde{P}e_a', \\
COV_{h,t}(v_\tau\bar{X}_t, m_t) &= v_\tau\tilde{P}e_m', \\
COV_{h,t}(v_\tau\bar{X}_t, a_t) &= v_\tau\tilde{P}e_a',
\end{aligned}$$

where $\tilde{P}$ solves

$$\tilde{P} = P - PD'[DPD' + \Sigma_v]^{-1}DP. \tag{100}$$