Optimal Income Taxation with Endogenous Prices*

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Abstract

We study an optimal income taxation problem in a Mirrleesian setting with endogenous product prices and positive firm profits. In the presence of a progressive distribution of firm profit shares or foreign ownership, we show that the public authority favors lower equilibrium prices in competitive markets, which leads to more progressive taxation in the optimum. Using a calibrated model of the U.S. housing market, we quantify the price effect and show that it adds more than 4 percentage points on average to marginal income tax rates. In oligopolistic markets, market power creates an additional non-competitive effect that exerts downward pressure on optimal tax rates. Using the U.S. housing model, we show that, as the market structure varies, changes in the price effect and the non-competitive effect cancel each other out leading to a stable prediction of optimal income tax policy across markets. We also show that the price and non-competitive effects persist in the presence of commodity taxation.

Keywords: Optimal income taxation, endogenous price, pecuniary externality, competitive market, oligopolistic market, housing market.

JEL Classification: H21, H23, D43.

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1 Introduction

The design of income taxes is among the most important economic problems. At the same time, income taxation literature commonly assumes that product markets are perfectly competitive. While this assumption is convenient for analysis it is far from an accurate description of most markets. According to recent studies (Azar et al., 2017; Grullon et al., 2017), one third of U.S. industries are highly concentrated and over 75% of them are operating in markets that are more concentrated than 20 years ago.[1] Similar market structures are observed in Europe, where the top five food retailers typically have a 43% to 69% share of the market (European Commission, 2015).

What are the implications for optimal income taxation if markets are not perfectly competitive? What is the size of optimal income tax change due to the presence of endogenous prices? Our goal in this paper is to answer these questions and, thus, to bridge a gap in the public economics literature by incorporating endogenous prices into the analysis of income taxation.[2]

We consider the standard Mirrlees (1971) framework with a continuum of agents who differ in their productivity types. Agents earn labor income and care about the consumption of two types of goods: the numeraire good and the "main" good. The numeraire good is produced with a constant returns to scale technology and has a perfectly elastic supply function, whereas the main good is produced with a decreasing returns to scale technology and has a strictly increasing supply function. As a consequence, the price of the numeraire good is constant and the price of the main good is endogenously determined by the market equilibrium condition. Also, firms producing the numeraire good realize zero profits and firms producing the main good realize positive profits. Motivated by empirical evidence, we assume that firm profits are progressively distributed among agents in the economy with more productive agents receiving larger profit shares. We also allow for some firm profits to belong to foreigners or capitalists who have no labor income.[3]

The public authority problem is to design the optimal income tax schedule that maximizes a concave objective of agent utilities subject to three constraints: *the resource constraint*, which demands that the public authority raises a fixed level of public funds; *the incentive compatibility*

[1]See also articles in the popular press by Theo Francis and Ryan Knutson, "Wave of Megadeals Tests Antitrust Limits in U.S." *The Wall Street Journal*, October 18, 2015, and the Editorial Board, "How Mergers Damage the Economy" *The New York Times*, October 31, 2015.

[2]Atkinson (2012, p. 775) offers an extensive discussion of how the existing taxation literature mostly fails to take the underlying market structure and endogenous prices into account. One exception is Kaplow (2018) who analyzes income taxation for various market structure. We provide an extensive discussion of his and other relevant papers in literature review section.

[3]See Saez and Zucman (2016) for the data on the progressive distribution of dividends and capital income in the US. The foreign ownership assumption is motivated by empirical evidence that a substantial share of equities is held by foreigners in many countries. The share of foreign equity holdings amounts to 13.6% in the US (U.S. Treasury, 2017) and, on average, 38% in European countries (Davydoff et al., 2013). We show that the price effect associated with foreign ownership in U.S. is relatively small. We keep the foreign ownership assumption, however, as its influence could be much bigger in other countries. This assumption is also convenient for illustrating the effect of endogenous prices on optimal income taxation in complete information settings.

constraint, which requires agents to reveal their productivity types; and *the market equilibrium condition*, which determines the price level.

When agent productivity is perfectly observable, the public authority can impose a lump-sum tax tailored to each individual. If all firm profits are distributed among agents inside the economy, lump-sum taxation can achieve the first-best level of social welfare, because any Pareto-efficient allocation can be supported with competitive prices after some income redistribution (the second welfare theorem). However, if not all firm profits are distributed among agents inside the economy, then Pareto-efficient allocations can no longer be supported with competitive prices. The competitive equilibrium is associated with over-production (when the main good is normal). Motivated by efficiency concerns, the public authority then imposes a positive marginal income tax to correct for over-production.

When agent productivity is private information, the distribution of firm profits becomes an additional important factor in determining the optimal income tax policy. In this case, not any Pareto efficient allocation can be supported with competitive prices even in the absence of foreign ownership. To illustrate, let us consider an extreme example in which all agents have the same productivity type but different profit shares. If the agents have linear utility in income, the difference in profit shares does not create any difference in the agents' optimal labor supply. Then, all agents have the same labor income in equilibrium and are subject to the same income tax. Hence, their income cannot be freely redistributed. Overall, a tax policy based on labor income alone does not allow for full discretion in redistributing total income, which leads to the failure of the second welfare theorem.[4]

The failure of the second welfare theorem leads to a binding market equilibrium condition in the optimum. The public authority uses the price level as an additional redistributing tool: a decrease in price level benefits low-productivity agents as they can afford to consume more products, but hurts high-productivity agents as their utility is mainly influenced by the decrease in firm profits.[5] Motivated by equity concerns, the public authority then favors lower price levels and a more progressive income taxation in the optimum.

To understand whether the price effect can significantly alter the optimal income taxation policy, we estimate it using the U.S. housing market. This market is particularly suitable for illustrating our theoretical results because housing costs comprise the largest share of overall household expenditures.[6] The housing sector is also associated with large firm profits, which

[4]This result holds also if a tax policy is based on total income including profit shares (see Appendix A.4). The second welfare theorem can be restored only if the public authority can expropriate all firm profits.

[5]This argument is in line with recent empirical evidence that connects individual earnings growth with real stock returns. Guvenen et al. (2017) show that individual earnings growth is especially sensitive to a change in real stock returns for high- and low-income levels. This relationship for high-income levels supports our reasoning that a decrease in firm profits hurts more high-income agents than moderate- and low-income agents. For low-income levels, the sensitivity can be explained by a high correlation between real stock returns and GDP growth. That is, labor earnings are higher in booms and lower in recessions.

[6]See Consumer Expenditure Survey, 2017, Table 1203. *Income before taxes: Annual expenditure means, shares, standard errors, and coefficients of variation* and Bureau of Economic Analysis, 2016, Table 2.3.5U.

distribution is one of the drivers of the price effect on optimal income taxation. Using a calibrated model of the housing market (based on Miles and Sefton (2018) and Saiz (2010)), we found that the price effect increases optimal marginal income taxes by more than *4 percentage points* on average. We also identify that *the progressive distribution of firm profits* is responsible for most of the change with *foreign ownership* being responsible only for 0.5 percentage point increase.

We also consider markets with various forms of oligopolistic competition. In oligopolistic markets, the presence of market power leads to under-production in equilibrium. As a countermeasure, the public authority seeks to stimulate labor income and, thus, aggregate demand by decreasing marginal income taxes. This non-competitive effect works in the direction opposite to that of the price effect. Using the U.S. housing model, we show that the change in the price effect due to a change in the market structure is compensated by the change in the non-competitive effect. As a result, the optimal income taxation schedule barely varies across market structures.

Further, we investigate whether commodity and profit taxation can influence the effect of endogenous prices on income taxation. For competitive markets, we show that if firm profits are unequally distributed the price effect and commodity taxation coexist in the optimum. This finding is in contrast to Atkinson and Stiglitz (1976) who show that there is no need for commodity taxation in the presence of optimal income taxation. Their result, however, holds only when firm profits are taxed at 100%, which differs from our main assumption. If we allow for full profit extraction, the price effect vanishes in competitive markets. In this case, the competitive market equilibrium is constrained Pareto-efficient, which is also in line with the production efficiency theorem of Diamond and Mirrlees (1971). For oligopolistic markets, the price and anti-competitive effects vanish only when 100% profit taxation is coupled with optimal commodity taxation (see also Myles, 1996).

Next, we relate our findings to the three tests on policy relevance proposed by Diamond and Saez (2011). First, our results are based on an economic mechanism whereby the second welfare theorem does not hold in incomplete-information markets when only a limited set of tax instruments is available. Second, we show that the price effect is empirically relevant and of the first order by estimating it as equal to more than 4 percentage points based on the U.S. housing market (assuming that only a quarter of the economy has endogenous prices). As we show in appendix, our results are also robust to various market structures, model specifications, and simulation assumptions. Third, our tax policy recommendation favors more progressive income tax rates, which is socially acceptable on equity grounds. Lastly, we note that the price and non-competitive effects are relevant to any policy with implications for income redistribution

Personal Consumption Expenditures by Major Type of Product and by Major Function that report approximately 25% as the share of housing costs (including utilities) in U.S. household expenditure. Housing costs account for a similar average share of household expenditure in Europe (Eurostat, 2016).

such as the design of the minimum wage, basic income, welfare benefits, and pensions.

We postpone a detailed literature review until Section 6 and, here, only briefly outline our main contributions in relation to previous research. Compared to the growing literature on optimal income taxation with endogenous wages in labor markets (e.g., Rothschild and Scheuer, 2013; Sachs, Tsynvisky, and Werquin, 2016), we consider endogenous prices in product markets. The price effect is driven by the distribution of firm profits among agents inside and outside the economy rather than general equilibrium effects analyzed in this literature.

In relation to studies focused on optimal income taxation with externalities (e.g., Rothschild and Scheuer, 2016; Lockwood et al., 2017; Rothschild and Scheuer, 2014) agents in our paper do not impose any direct externality. Instead, in our model, agents impose only pecuniary externalities.[7] The welfare theorems imply that pecuniary externalities should not be corrected in complete information markets. If agent productivity is not perfectly observable and taxes are based on labor income, we show that a constrained Pareto-efficient allocation cannot be generally supported by a competitive equilibrium. This leads to a binding market equilibrium constraint and the price effect on optimal income taxation.

Finally, the literature analyzing taxation in the presence of imperfect competition is rather thin. To the best of our knowledge, Kaplow (2018) is the only paper that considers income taxation for various market structure. He studies income taxation policies in economies with *exogenously given firm markups*. Hence, he does not consider the influence of income taxation policy on equilibrium product prices, which is the main subject of our analysis. There are also a few important papers on *commodity taxation* in oligopolistic markets (Auerbach and Hines, 2001; Myles 1987; Reinhorn, 2005). In contrast to these papers, we consider an incomplete information setting and highlight the influence of profit distribution among agents on the optimal tax schedule.

The remainder of the paper is organized as follows. In Section 2, we introduce the model. In Section 3, we consider competitive markets with complete and incomplete information, analyze properties of the optimal marginal income tax schedule, and provide simulation results estimating the size of the price effect. In Section 4, we present our analysis for oligopolistic markets. We investigate the robustness of our results to the presence of commodity and profit taxation in Section 5. In Section 6, we provide a detailed literature review, and in Section 7, we present our conclusion. The omitted proofs are postponed to Appendix A.1. Appendices A.2-A.6 comprise extensions of our main model and some additional simulation results.

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[7] The effect of pecuniary externalities through wages on labor market on tax policy also arises in the analysis of optimal income and profit taxation in Scheuer (2014).

2 Model

There is a continuum of agents indexed by productivity type n and distributed according to the probability density function $f(n) > 0$ with support $[\underline{n}, \overline{n}]$. Agent n's labor income is given by $z = n\ell$, where ℓ is the number of hours worked. The labor cost is represented by an increasing and convex function $c(\ell)$. The labor income is taxed according to schedule $T(z)$. After tax, the agent's disposable income is equal to $y = z - T(z)$.

In the economy, there are two goods: a numeraire good and good X (referred to as the main good in the introduction). The numeraire good is produced with a constant returns to scale technology that results in a fixed price normalized to 1 and zero firm profits. Good X is produced with a decreasing returns to scale technology that yields positive profits. We denote p and $\Pi(p)$ as the price and the profit, respectively, of firms producing good X.[8]

We assume that firm profits are distributed to agents in the form of dividends with agent n receiving share $\xi(n) \geq 0$. Motivated by empirical evidence, we also assume that agents with higher labor income typically possess a larger share of firm profits $\xi'(n) \geq 0$ (see Saez and Zucman, 2016). Some firm profits can also be owned by foreigners or capitalists who have no labor income, i.e., $\int \xi(n)f(n)dn = \Xi \leq 1$ (U.S. Treasury, 2017; Davydoff et al., 2013). For the purpose of exposition, we do not consider profit taxation in our theoretical analysis.[9] In our simulations, however, we assume that firm profits are taxed at a fixed rate of 15% that corresponds to the U.S. tax rate on qualified dividends at most income levels.

Overall, an agent's income consists of labor income and dividends (profit shares) $\tilde{y}(n) = y(n) + \xi(n)\Pi(p)$. Agents' preferences for consumption are represented by an indirect utility function $v(p, \tilde{y})$, which is concave and increasing in $\tilde{y}$. The agent's net utility is defined by

$$U(p, \tilde{y}, \ell) = v(p, \tilde{y}) - c(\ell). \tag{1}$$

The social welfare function is given by[10]

$$\mathcal{W} = \int W(U(p, \tilde{y}(n), \ell(n)))f(n)dn, \tag{2}$$

where W is the social value of utility. Assuming that the public authority has equity concerns, we let W be a differentiable, increasing, and concave function.

The public authority wants to maximize the social welfare function $\mathcal{W}$ subject to three

[8] We do not model explicitly why firms that produce the numeraire good do not switch to producing the more profitable good X. A lack of technology and/or patents may function as a barrier to entry to an industry. The high degree of profit and performance differences even among similar firms is a well-documented phenomenon (see, for example, Syverson (2011) for a recent survey).

[9] See Scheuer (2014) for the analysis of optimal profit taxation under endogenous firm formation.

[10] In Appendix A.3, we consider a variant of the model where foreigners or capitalists who possess the remaining share of firm profits $(1 - \Xi)\Pi(p)$ receive a separate social weight ω. For the sake of clarity, we assume $\omega = 0$ in the main text.

constraints. The first is the resource constraint:

$$\int T(z(n))f(n)dn = \int (n\ell(n) - \tilde{y}(n) + \xi(n)\Pi(p))\, f(n)dn \geq R \tag{3}$$

that ensures the public authority covers its own expenses $R \geq 0$, which are spent solely on the numeraire good. The second is the incentive compatibility constraint:

$$U(p, z(n) - T(z(n)) + \xi(n)\Pi(p), \ell(n)) \geq U(p, z(m) - T(z(m)) + \xi(n)\Pi(p), z(m)/n)\,, \tag{4}$$

for all $n, m \in [\underline{n}, \overline{n}]$, which ensures that an agent with productivity n does no want to seek the labor income of an agent with productivity m.

The third is a *market equilibrium condition* that determines price p. This condition varies across market structures. In *competitive markets* (Section 3), where we assume that firms are price-takers, the market equilibrium condition requires the market supply to be equal to the market demand for good X. In *oligopolistic markets* (Section 4), where each firm takes into account its influence on the level of product price, the market equilibrium condition is determined by the firm profit maximization condition.

Overall, the main difference between our framework and the one of Mirrlees (1971) is that the price of one of the goods is endogenously determined. In addition, in our model, firms obtain positive profits that are distributed among agents inside and outside the economy. The effect of endogenous prices and profit distribution on optimal income taxation is the main subject of our analysis.

3 Competitive Market

In this section, we analyze the problem of optimal income taxation in competitive markets, in which the price of good X is determined by the market equilibrium condition

$$S(p) = \int x(p, \tilde{y}(n))f(n)dn.^{11} \tag{5}$$

On the left-hand side we have market supply $S(p)$, and on the right-hand side the market demand for good X, where $x(p, \tilde{y})$ is the Walrasian demand of agent with disposable income $\tilde{y}$. The Walrasian demand function can be determined using Roy's identity as $x(p, \tilde{y}) = -v_p(p, \tilde{y})/v_y(p, \tilde{y})$. We consider an increasing supply function $S'(p) > 0$ and zero fixed costs so that the total firm profits coincide with producer surplus: $\Pi(p) = \int_0^p S(\tilde{p})d\tilde{p}$. We also assume that the demand for good X satisfies the law of demand $x_p < 0$. In Appendix A.2, we show how our model can be supported with a labor market. We also explain there why condition (5)

[11] Note that the market equilibrium condition follows from our assumption that the foreigners and the government consume only the numeraire good (see Appendix A.2 for more details).

clears both product and labor markets in the economy where the foreign share of firm profits and government expenditures are spent solely on the numeraire good.

To highlight our main ideas, we start with the case of complete information wherein the agents' productivity is observable. We then proceed to the case of incomplete information.

3.1 Complete Information

Assume that productivity types and profit shares are observable and that the public authority can design type-specific taxes, i.e., $T(z,n)$. The public authority's problem is then to find a combination of price p, income schedule $\tilde{y}(n)$, and labor supply schedule $\ell(n)$ that maximizes[12]

$$\max_{p,\,\tilde{y}(n),\,\ell(n)} \int W(v(p,\tilde{y}(n)) - c(\ell(n)))f(n)dn \quad \text{subject to (3) and (5).}$$

The Lagrangian of the public authority's problem is given by

$$\int \{W(v(p,\tilde{y}(n)) - c(\ell(n))) + \lambda(n\ell(n) - \tilde{y}(n) + \xi(n)\Pi(p) - R) + \gamma(S(p) - x(p,\tilde{y}(n)))\}f(n)dn,$$

where λ and γ are multipliers corresponding to constraints (3) and (5), respectively. The first-order conditions are given by

$$\tilde{y}(n) : W_u v_y - \lambda - \gamma x_y = 0, \tag{6}$$

$$\ell(n) : -W_u c_\ell + \lambda n = 0, \tag{7}$$

$$p : \int (W_u v_p + \lambda \xi(n)\Pi'(p) + \gamma(S'(p) - x_p))f(n)dn = 0. \tag{8}$$

The optimal marginal income tax $t(z,n) = T_z(z,n)$ is determined by the individual utility maximization problem $\max_z U(p, z - T(z,n) + \xi(n)\Pi(p), z/n)$, which implies that $t = 1 - c_\ell/(nv_y)$. Taking into account that $\int \xi(n)f(n)dn = \Xi$ and $v_p = -xv_y$ (Roy's identity), we obtain the following result from the first-order conditions (6)–(8) and constraints (3) and (5):

Theorem 1. *In competitive markets with complete information, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{\gamma}{\lambda} x_y. \tag{9}$$

where

$$\gamma = \frac{\lambda S(p)(1-\Xi)}{S'(p) - \int (x_p + x_y x) f dn}.$$

The theorem implies that if all profits are distributed among agents in the economy ($\Xi = 1$), the optimal marginal income tax must be zero. In other words, the maximum value of the social

[12] We maximize over p because the price is implicitly determined by equation (5). If we had an explicit price function, we could introduce it into indirect utility and maximize only over $(\tilde{y}(n), \ell(n))$.

value function is achieved using lump-sum taxation targeted to each agent. This is a well-known result in the public economics literature (see Atkinson and Stiglitz, 2015) that also follows from the second welfare theorem. More specifically, let us consider a Pareto efficient allocation that maximizes objective (2) subject to budget constraint (3) for a given level of firm profits. Then, according to the second welfare theorem, it is always possible to find a redistribution of agents' income and price p that support the aforementioned allocation in a competitive equilibrium. Hence, the market equilibrium condition (5) is not binding that results in $\gamma = 0$.

When $\Xi < 1$, the second welfare theorem does not hold; i.e., Pareto efficient allocation can no longer be supported with a competitive equilibrium. Hence, the market equilibrium condition (5) is binding. Intuitively, labor supply contributes to firm profits that belong in part to foreigners. Hence, a competitive equilibrium results in an inefficient production level. To correct for this inefficiency, the public authority imposes an additional income tax with the aim of correcting the labor supply and thereby bringing the competitive equilibrium outcome closer to the Pareto efficiency.

We also note that in the presence of foreign ownership $\Xi < 1$ the market equilibrium multiplier is positive $\gamma > 0$. This follows from the strictly positive slope of the market supply $S'(p) > 0$ and the law of compensated demand $h_p = x_p + x_y x \leq 0$, where h is the Hicksian (compensated) demand function. Finally, as the multiplier of the resource constraint is positive $\lambda > 0$, the sign of the corrective tax term is determined by x_y.[13] Therefore, the optimal marginal income tax (9) is positive for normal goods ($x_y > 0$) and negative for inferior goods ($x_y < 0$). If we also assume that demand is either convex or concave in income ($x_{yy} \geq 0$ or $x_{yy} \leq 0$), we also obtain that x_y is increasing for luxury goods and decreasing for necessity goods.

Corollary 1. *In competitive markets with complete information and foreign ownership, the optimal marginal income tax is positive for normal goods and negative for inferior goods. If demand is either convex or concave at all income levels, the optimal marginal income tax is increasing for luxury goods and decreasing for necessity goods.*

3.2 Incomplete Information

Now we consider the case in which agent productivity is private information. The public authority cannot then condition tax payments on agent types and, therefore, has to take into account incentive compatibility constraints (4). When agent preferences are non-homothetic and firm profits unequally distributed, agent indirect utility (1) can violate the single-crossing property (see, e.g., Mirrlees (1976)). For better tractability, we consider only the case of *homothetic* preferences in the main text. This case also corresponds to our numerical simulations

[13] More precisely, $\lambda \geq 0$, but $\lambda > 0$ when the resource constraint is active, which we assume.

in Section 3.3, where we consider the constant elasticity of substitution (CES) preferences.[14] Homothetic preferences imply linear indirect utility that we express as

$$v(p, y) = a(p)y,$$

where $a(p)$ is some decreasing function. For the linear case, we also assume that welfare function W is strictly concave. We postpone the analysis of the non-homothetic case in Appendix A.3.

Let us denote an agent's utility from revealing his/her productivity type truthfully as

$$u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n).$$

If the truthful revelation is optimal, then

$$u(n) = \max_m \left(v(p, y(m) + \xi(n)\Pi(p)) - c(z(m)/n)\right). \tag{10}$$

Using the envelope theorem, we obtain the following first-order condition:

$$u'(n) = c_\ell z(n)/n^2 + \xi'(n)a(p)\Pi(p). \tag{11}$$

Note that the presence of profits does not change the incentive compatibility condition in the linear case, because the term with profit shares can be taken out of the maximization problem (10). Hence, the second-order condition that ensures truthtelling to be optimal is income schedule $y(n)$ being non-decreasing as in the standard case (see Mirrlees, 1976). At the same time, profit distribution $\xi(n)$ does influence the level of agent utility $u(n)$. This will be a quite important factor since the public authority has equity concerns (W is strictly concave).

The public authority's problem is now to maximize the social welfare function subject to the resource, market equilibrium, and incentive compatibility constraints:

$$\max_{p,\, \tilde{y}(n),\, \ell(n)} \int W(v(p, \tilde{y}(n)) - c(\ell(n)))f(n)dn \quad \text{subject to (3), (5), and (11).}$$

It is convenient to change optimization variables $\{p, \tilde{y}(n), \ell(n)\}$ to $\{p, u(n), \ell(n)\}$, where utility level is defined by $u(n) = a(p)\tilde{y}(n) - c(\ell(n))$. From the latter expression, we can invert the disposable income $\tilde{y}$ and express it as $\tilde{y} = r(p, u, \ell) \equiv \frac{u+c(\ell)}{a(p)}$. The maximization problem can then be written as

[14] Recent empirical studies found small income effects on individual labor supply (see a discussion in Sorensen, 2009), which also justifies the use of homothetic preferences in our analysis.

$$\max_{p,u(n),\ell(n)} \int W(u(n))f(n)dn$$

$$s.t.$$

$$\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)]f(n)dn \geq R \tag{12}$$

$$\int [S(p) - x(p, r(p, u(n), \ell(n)))]f(n)dn = 0 \tag{13}$$

$$u'(n) - \xi'(n)a(p)\Pi(p) - c_\ell \ell(n)/n = 0 \tag{14}$$

Let λ, γ, and $\mu(n)$ be multipliers corresponding to constraints (12), (13), and (14), respectively. After integration by parts and taking into account the transversality condition $\mu(\underline{n}) = \mu(\overline{n}) = 0$, we express the Lagrangian of the maximization problem as

$$\int \{[W(u(n)) + \lambda(n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p) - R)+$$

$$\gamma(S(p) - x(p, r(p, u(n), \ell(n))))]f(n) - \mu'(n)u(n) - \mu(n)(\xi'(n)a(p)\Pi(p) + c_\ell \ell(n)/n)\}dn.$$

Given $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_p = -v_p/v_y = x$, the first-order conditions are

$$u(n): \left[W_u - \frac{\lambda + \gamma x_y}{v_y}\right] f(n) - \mu'(n) = 0 \tag{15}$$

$$\ell(n): \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y}\right] f(n) - \mu(n)(c_\ell + c_{\ell\ell}\ell(n))/n = 0 \tag{16}$$

$$p: \int \{[\lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_y x)]f(n) - \tag{17}$$

$$\mu(n)\xi'(n)(a_p\Pi(p) + a\Pi'(p))\}dn = 0.$$

From the individual maximization problem, we have that the optimal marginal income tax satisfies $t = 1 - c_\ell/(nv_y)$ (see p. 8). Taking into account that $\Pi'(p) = S(p) = \int x(p, \tilde{y})f(n)dn$, $\int \xi(n)f(n)dn = \Xi$, that $1 + \ell c_{\ell\ell}/c_\ell = (1 + E^u)/E^c$, where E^c is the elasticity of the compensated labor supply, and that E^u is the elasticity of the uncompensated labor supply (see the proof of Theorem 2), we obtain the following result.

Theorem 2. *In competitive markets, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{v_y \mu}{\lambda n f} \frac{1 + E^u}{E^c} + \frac{\gamma x_y}{\lambda}, \text{ and} \tag{18}$$

$$\gamma = \frac{\lambda S(p)(1 - \Xi) + (a_p\Pi(p) + a\Pi'(p)) \int \mu(n)\xi'(n)dn}{S'(p) - \int (x_p + x_y x)f(n)dn}.[15] \tag{19}$$

[15]Multiplier $\mu(n) = \int_{\underline{n}}^{\overline{n}} [(\lambda + \gamma x_y)/v_y - W_u] f(n)dn$ is obtained from (15). Transversality conditions $\mu(\underline{n}) = \mu(\overline{n}) = 0$ also imply $\int W_u f(n)dn = \int (\lambda + \gamma x_y)/v_y f(n)dn$ that together with (19) determines γ and λ.

The optimal income tax formula in Theorem 2 has two terms. The first one is the standard Mirrleesian term that balances work incentives with the public authority's redistributive and budgetary objectives (see Mirrlees 1971, 1976). The second one captures the price effect that also appears in Theorem 1 for the complete information case.

The first part of the price effect $\lambda S(p)(1-\Xi)/(S'(p)-\int(x_p+x_y x)f dn)$ corrects inefficiency associated with the presence of foreign ownership. In contrast to the complete information case, however, the price effect does not vanish in the absence of foreign ownership, because the public authority has equity concerns (the second part). To see this, note that $\int \mu(n)\xi'(n)dn = \int W_u(\Xi-\xi(n))f(n)dn$.[16] The latter expression is positive when the public authority has equity concerns $W_{uu} < 0$ and profits are progressively distributed among agents $\xi'(n) > 0$. In addition, agent utility equals $a(p)(y+\xi(n)\Pi(p))$. Hence, $a_p\Pi + a\Pi_p$ corresponds to a change in utility associated with the presence of dividends, which is also positive given that

$$\Pi'(p) = S(p) = \int x(p,\tilde{y})f(n)dn = -\frac{a_p(p)}{a(p)}\int \tilde{y}(n)f(n)dn \geq -\frac{a_p(p)}{a(p)}pS(p) \geq -\frac{a_p(p)}{a(p)}\Pi(p),$$

where the first equation follows from the definition of firm profits $\Pi(p) = \int_0^p S(\tilde{p})d\tilde{p}$, the second from the market equilibrium condition, and the third from Roy's identity. The first inequality follows from an agent's individual budget constraint, and the second one follows from $pS(p) \geq \int_0^p S(\tilde{p})d\tilde{p}$. Intuitively, a decrease in equilibrium prices benefits low-productivity agents because they can afford to consume more products. At the same time, a decrease in equilibrium price hurts high-productivity agents, whose utility is mostly influenced by the level of firm profit shares. Overall, the public authority uses the equilibrium price level on par with income taxation to achieve its efficiency and equity objectives in the optimum.

Corollary 2. *In competitive markets with incomplete information, the price effect arises because of both efficiency and equity concerns. The price effect term does not vanish in the absence of foreign ownership.*

The price effect could also be explained in the absence of foreign ownership by the failure of the second welfare theorem in the presence of incomplete information. Note that agents' income consists of two parts: labor income and profit shares. However, if taxation can be based only on labor income, the second welfare theorem does not hold. To illustrate, consider a simple example in which all agents have the same level of productivity but different profit shares. Linear utility implies that they all choose the same level of labor and earn the same level of labor income. Then, each agent pays the same amount of income tax irrespective of her total income. Hence, the dependence of tax schedule on labor income only restricts the set of distributional objectives that the public authority can achieve.

[16]As agent indirect utility is linear, terms x_y and v_y are constants. Hence, (15) implies that $\int W_u f(n)dn = (\lambda+\gamma x_y)/v_y$ and $\int \mu(n)\xi'(n)dn = -\int \mu'(n)\xi(n)dn = \int W_u(\Xi-\xi(n))f(n)dn$.

Overall, Theorem 2 suggests that pecuniary externalities need to be corrected in markets where there is incomplete information, unequal distribution of firm profits, and equity concerns. In Appendix A.4, we also establish that this insight is robust if we assume that income tax is based on total agent income (labor income plus dividends) instead of labor income alone.[17]

The failure of the second welfare theorem renders the competitive equilibrium condition binding in the optimum. We now establish that $\gamma > 0$. As discussed, when $\xi'(n) > 0$ and $\Xi < 1$, we have $\lambda S(p)(1 - \Xi) > 0$ and $\int \mu(n)\xi'(n)dn = \int W_u(\Xi - \xi(n))f(n)dn > 0$. As in the case of complete information, we also have $S'(p) > 0$ and $h_p = x_p + x_y x \leq 0$. Hence, the properties of the price term are determined by x_y, and we obtain the following result.[18]

Corollary 3. *In competitive markets with incomplete information, the price effect term is positive for normal goods and negative for inferior goods.*

We do not have an analog of the second part of Corollary 1 because term x_y is constant in income when agent indirect utility is linear. Finally, we note that the profit redistribution term in (19) disappears only in the absence of foreign ownership $\Xi = 1$ and when firm profits are equally distributed $\xi'(n) = 0$. In this case, the income tax instrument can correct for the firm profit distribution by imposing an equal lump-sum tax on all agents.

Another corollary of Theorem 2 is that the seminal end-point results of Sadka (1976) and Seade (1977) no longer hold with endogenous prices. When prices are fixed and the support of productivity distribution is bounded, the optimal tax formula has only the incentive term and the transversality condition implies that the optimal marginal tax for the most productive agents is zero. Intuitively, if the marginal tax were positive, the public authority could instead impose zero tax on any income in excess of the current income of the most productive agents. Then, these agents would respond by exerting additional effort to achieve the previously not feasible level of utility. Since the amount of tax collected would not change while the most productive agents would obtain higher utility, the overall welfare would increase. With the binding market equilibrium constraint, the aforementioned tax relief would have implications for product prices. Therefore, the public authority cannot increase the utility of the most productive agents without influencing the other agents.

[17] This result assumes that profit shares cannot be taxed away. If firm profits can be taxed away, it is optimal to apply 100% profit tax, which would restore the constrained-Pareto efficiency of the competitive equilibrium.

[18] A similar analysis also applies to the environment when agents have non-labor endowments $e(n)$ instead of profit shares $\xi(n)\Pi(p)$. In the absence of foreign ownership $\Xi = 1$, we have $\gamma < 0$ because $(a(p)e(n))'_p < 0$, and the price effect on optimal income tax is negative. Intuitively, an additional endowment increases aggregate demand. Hence, it raises the equilibrium price and decreases the willingness to work on the part of agents. In the effort to increase agent labor supply, the public authority should then reduce marginal income taxes.

3.3 Numerical Simulations

In this subsection, we provide numerical simulations to estimate the size of the price effect on optimal income taxes. To accomplish this goal, we consider the U.S. housing market that suits particularly well to quantify our results. First of all, housing is the largest consumption item that accounts for about 25% of total household expenditure.[19] In addition, housing sector is non-competitive and associated with large firm profits, which distribution is the main driver of the price effect on optimal income tax schedule.

To incorporate housing demand, we consider a model estimated by Miles and Sefton (2018) with a constant elasticity of substitution (CES) utility function:

$$u(x,g) = \left(ag^{1-1/\rho} + (1-a)x^{1-1/\rho}\right)^{\frac{\rho}{\rho-1}}, \tag{20}$$

where x denotes the consumption of housing, g denotes the consumption of the other goods, a is the weight on the other goods (relative to housing consumption) in utility, and ρ is the elasticity of substitution between housing and the other goods. Following Miles and Sefton (2018), we take $a = 0.85$ and $\rho = 0.6$.[20] This utility specification results in the absolute value of the price elasticity of demand of 0.7 and the unit income elasticity of demand in our simulations, which are consistent with elasticity values estimated in the literature.[21]

We model the supply of housing using the standard constant price elasticity function $S(p) = sp^{\varepsilon}$, where s is a scale parameter and ε is the price elasticity of supply.[22] The estimates of the price elasticity of supply ε vary significantly across countries and even across cities. In particular, Saiz (2010) shows that ε highly depends on geographical and regulatory constraints within U.S. metropolitan areas. Drawing on his estimates for the average U.S. metropolitan area, we take $\varepsilon = 1.75$.[23] We calibrate scale parameter s to match the average share of housing expenditure of 25%, which renders $s = 0.65$. In Appendix A.6, we present the results for inelastic supply $\varepsilon = 0$ that better describe housing supply in large U.S. coastal cities (e.g., Boston and San-Francisco) and in countries with a rigid housing planning system (e.g., the UK).[24] We also present results for the price elasticity of supply $\varepsilon = 3$ that are closer to the estimates obtained in Epple and Romer (1991) and Green et al. (2005).

[19] See Bureau of Economic Analysis (2017, Table 2.3.5U) and Consumer Expenditure Survey (2017, Table 1203). Similar numbers are also observed in the European Union, for which Eurostat (2016) reports that on average housing accounts for 24.4% of household expenditure.

[20] For other estimates of the CES model for housing, see Määttänen and Terviö (2014) and Li et al. (2016).

[21] For the U.S. housing market, the estimates of price elasticity of demand are in the range 0.3 to 1 in absolute numbers and the estimates of income elasticity of demand are in the range of 0.4 to 1 (see Albouy et al., 2016). For the European housing market, the corresponding estimates are in the range of 0 to 1.43 and 0.74 to 1.23, respectively (see Arrazola et al., 2014).

[22] This functional form corresponds to the Cobb-Douglas production function (see, e.g., Epple et al. (2010)).

[23] For similar estimates, see also DiPasquale and Wheaton (1994) and Harter-Dreiman (2004).

[24] See Hilbert and Schöni (2016). For a discussion of the U.K. housing crisis in the popular press, see "How to solve Britain's Housing Crisis, " *The Economist*, 2017, August 5.

Distribution of dividends (in %) held by income percentiles:						
Bottom 90%	Top 10%	Top 5%	Top 1%	Top 0.5%	Top 0.1%	Top 0.01%
0.4%	99.6%	95.9%	76.0%	67.1%	50.0%	31.2%

Table 1: The Distribution of Dividends in the US, 2012

Note: The table shows the percentage shares of dividends held by the U.S. population. Source: Saez and Zucman (2016, Table B23).

We use lognormal function $\ln(m, \sigma)$ for the distribution of agent productivity $F(n)$. In particular, following Kanbur and Tuomala (2013), we consider the parameter values of mean $m = e^{-1}$ and standard deviation $\sigma = 0.7$, which have been found to offer a good match with the empirical distribution of personal income in the US. In Appendix A.6, we also show that our estimation of price effect remains almost unchanged if we consider a composite lognormal-Pareto distribution that is a better fit for the upper tail of income distribution (Saez, 2001; Diamond and Saez, 2011). We assume that 5% of agents are disabled, i.e., their productivity type is $n = 0$, which is approximately the percentage of total employees on public disability insurance in the US (see Mankiw et al., 2009). Following Saez et al. (2012) and Kanbur and Tuomala (2013), we assume a labor supply elasticity of 1/3 and cost function $c(\ell) = \ell^4/4$. We also assume logarithmic social welfare function $W(u) = \ln u$ to obtain results comparable to the previous literature, set public expenditures at $R = 0$.

Next, we set the share of profits held by agents in the economy at $\Xi = 0.85$, which approximately equals the 86.4% domestic share of equity holdings in the US (see US Treasury, 2017). This estimate should be considered an upper bound for developed countries. In Europe, on average, only 62% of equity shares are held by domestic investors, whereas the corresponding figure is 50% in the UK, 60% in France, and 70% in Germany (Davydoff et al., 2013).

Lastly, we approximate the distribution of firm profits among agents by the empirical distribution of dividends across U.S. households in 2012, as presented in Table 1.[25] To make our estimates more realistic, we slightly depart from our theoretical analysis and assume a flat profit tax at 15% (as in U.S. for most income levels). The proceeds from the profit tax go to financing government expenditures and they are included in the resource contraint.[26]

[25]The cumulative distribution of profits is approximated well by functional form $\Xi(n) = \exp(\widehat{b}(-logF(n))^{\frac{1}{2}})$, where $F(n)$ is the cumulative distribution of agent productivities and $\widehat{b} = -18.1$; the coefficient of determination equals $R^2 = 0.95$. We obtain function $\xi(n)$ as the derivative of $\Xi(n)$ and by normalizing $\int \xi(n) f(n) dn = 0.85$. Our estimation results of the price effect do not significantly change if we consider instead the distribution of capital income (Saez and Zucman, 2016, Table B21) or the distribution of wealth and financial resources (Wolff, 2017) to approximate the distribution of profit shares.

[26]Supplementary materials "OIT_simulation_documentation.pdf" present how the first order condition change in the presence of the profit tax.

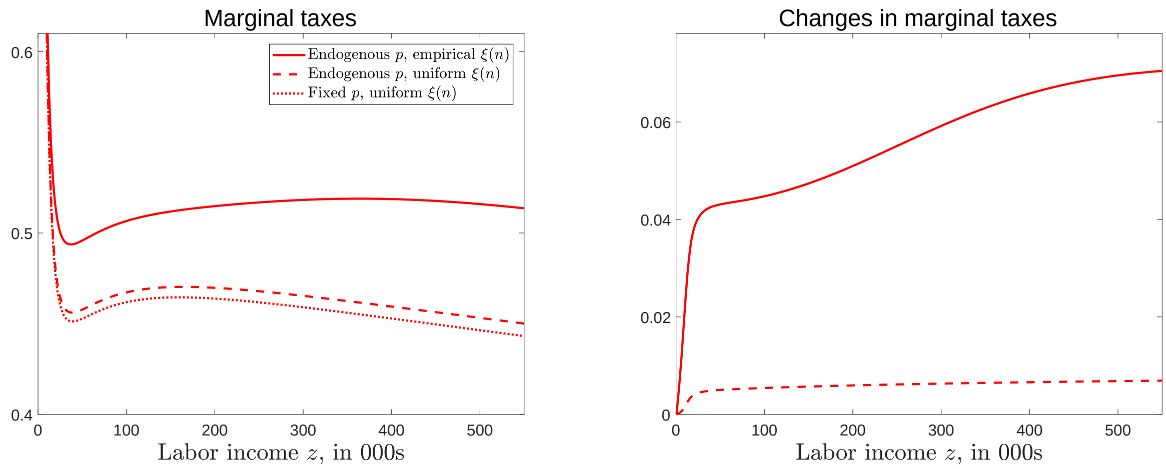

Figure 1. Optimal income taxation in the presence of endogenous prices.

Note: The left-hand diagram presents the optimal marginal income tax for an economy with a competitive housing market and (i) fixed price and equal profit distribution (dotted line; the benchmark case corresponding to Mirrlees, 1971); (ii) endogenous price and uniform profit distribution (dashed line); and (iii) endogenous price and empirical profit distribution (solid line). The right-hand diagram depicts the changes in tax rates for cases (ii) and (iii) in comparison with benchmark case (i).

Figure 1 presents our main simulation results. The left-hand diagram shows the optimal marginal tax schedules for cases with (i) fixed price and equal profit distribution (dotted line), (ii) endogenous price and uniform profit distribution (dashed line), and (iii) endogenous price and empirical profit distribution (solid line). In case (i), we take the price of housing equal to the equilibrium price of case (iii). Case (i) corresponds to the standard setting of Mirrlees (1971) and serves as a benchmark against which we estimate the size of the price effect. The right-hand diagram shows the change in optimal marginal income tax (ii)–(i) and (iii)–(i) compared to the benchmark case.

We note that the estimated optimal income tax schedules are of a shape similar to those obtained by previous studies. The high tax rates at the bottom correspond to the phasing-out of the guaranteed income level (Saez, 2001; Mankiw et al. 2009).[27] The decline of marginal income tax at high income levels reflects the lognormal distribution of agent productivity types at the upper tail. If we took Pareto distribution at the upper tail, however, the optimal marginal income tax would be constant and equal approximately 63% for the benchmark case (see Appendix A.6). The difference with the previous studies is that the minimum of the optimal marginal income tax is achieved around labor income of $\$40,000$, which is smaller than in Saez (2001) (around $\$75,000$). This is due to different assumptions on the distribution of agent productivity. Mankiw et al. (2009) have no minimum in optimal marginal income tax for the lognormal case due to a different agent utility specification.[28]

[27] The high optimal tax rate at the bottom disappears if one considers a labor participation decision of agents (see Saez, 2002).

[28] Our utility specification coupled with the logarithmic welfare function corresponds to Type I utility function of Saez (2001). However, we were not able to replicate the distribution of agent productivity considered in his

Our numerical simulations show that the endogenous price has an upward effect on optimal marginal income tax rates, adding *4.2 percentage points* on average (see the solid line in the right diagram of Figure 1). The overall increase in marginal income tax can be attributed to two factors: the presence of foreign ownership $\Xi < 1$, which leads to a tax correction based on efficiency concerns, and the presence of progressive distribution of firm profits $\xi'(n) > 0$, which leads to a tax correction due to equity concerns. The right diagram of Figure 1 shows that the factor of foreign ownership adds *0.5 percentage point* to the benchmark case compared to *3.7 percentage points* added by progressive firm profit distribution.[29]

The estimate of the price effect due to foreign ownership is rather small. This can be explained by the fact that the share of foreign ownership in US is less than 15%. With a larger share of foreign ownership, like 50% in the case of the UK, the effect of foreign ownership on income tax rates would become larger. On contrary, the degree of inequality of dividends in US is rather large, and we should expect the price effect due to progressive profit distribution to be smaller for UK and other European countries.

4 Oligopolistic Competition

In this section, we consider markets with various degrees of oligopolistic competition. In particular, we consider $M \geq 1$ identical firms with each firm i having a convex cost function $K(X_i)$ of producing X_i units of good X. Let us denote the inverse aggregate demand function by $p(X)$, where $X = \int x(p, \tilde{y}(n))f(n)dn$.[30] To get sharp analytical predictions, we also assume the second derivative of demand function is small (see p. 19 for more details).

We can write firm i's profit as $X_i p(X) - K(X_i)$, where the market clearing condition ensures $\sum_{i=1}^{M} X_i = X$. To model various forms of oligopolistic competition, we assume that when firm i maximizes profits it forms a belief, or *a conjectural variation*, about the other firms' responses to a unit change in its output level,

$$\frac{d(\sum_{j\neq i} X_j)}{dX_i} = \theta, \text{ where } -1 \leq \theta \leq M-1. \tag{21}$$

The first-order condition for the firm profit maximization problem can then be expressed by[31]

$$p(X) - K'(X_i) + (1+\theta)X_i\, p'(X) = 0. \tag{22}$$

paper. We cannot consider the utility format of Mankiw et al. (2009) because there is no calibrated housing model that corresponds to their utility specification.

[29] The estimated size of the price effect is robust to other modeling assumptions about labor supply elasticity and the distribution of productivity types as considered, for example, by Saez (2001), Kanbur and Tuomala (2013), and Mankiw et al. (2009). The simulation results are available upon request.

[30] The assumption $x_p < 0$ ensures that the inverse aggregate demand function $p(X)$ is well defined.

[31] We assume that the firm maximization problem is well-behaved and has a unique maximum. For an insightful discussion of problems associated with firm objective function being non-concave see Guesnerie and Laffont (1978).

The conjectural variation model was introduced by Bowley (1924) to capture a wide variety of oligopolistic competition models. For instance, the competitive equilibrium corresponds to $\theta = -1$ when firms expect the rest of the industry to absorb exactly its output expansion, the conjectural variation $\theta = 0$ represents the Cournot-Nash model where each firm expects the output of the other firms in its industry to remain unchanged, and the collusive behavior of firms maximizing their joint profits leads to $\theta = M - 1$.[32]

For the rest of our analysis, it is convenient to express the market equilibrium condition in terms of product price rather than quantity. We also limit our attention to symmetric equilibria $X_i = X/M$. The market equilibrium condition then is reduced to

$$J(X, X_p, p) \equiv -X_p \left(p - K'\left(\tfrac{X}{M}\right)\right) - (1+\theta)\tfrac{X}{M} = 0, \tag{23}$$

where $X_p = \int x_p(p, \tilde{y}(n)) f(n) dn$. We denote total firm profits as $\Pi(p, X) = pX - MK(\frac{X}{M})$.

Finding the optimal marginal income tax in oligopolistic markets with endogenous prices is generally a complicated problem with several effects interacting with each other. For the purpose of clarity and to illustrate a novel effect due to imperfect competition, we assume that all firm profits remain in the economy $\Xi = 1$ and that they are equally distributed among agents, i.e., $\xi(n) = 1$ for all n. A general analysis involving both the price effect and the non-competitive effect is considered in Appendix A.5. The public authority's problem for oligopolistic market can then be written as follows:

$$\max_{p, u(n), \ell(n)} \int W(u(n)) f(n) dn$$

$$s.t. \begin{cases} u'(n) - c_\ell \ell(n)/n = 0 & (\mu(n),\ \text{incentive compatibility}) \\ \int [n\ell(n) - r(p, u(n), \ell(n))] f(n) dn + \Pi(p, X) \geq R & (\lambda,\ \text{resource constraint}) \\ J(X, X_p, p) = 0 & (\gamma,\ \text{market equilibrium}) \\ X - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0 & (\alpha_1,\ \text{market demand}) \\ X_p - \int x_p(p, r(p, u(n), \ell(n))) f(n) dn = 0 & (\alpha_2,\ \text{market demand slope}) \end{cases}$$

where function r determines agent income $\tilde{y} = r(p, u, \ell) \equiv \frac{u + c(\ell)}{a(p)}$, and Lagrange multipliers are introduced next to the corresponding constraints. The Lagrangian can then be written as

$$\begin{aligned} \mathcal{L} = \int [(W(u(n)) + \lambda(n\ell(n) - r(p, u(n), \ell(n)) + \Pi(p, X) - R) + \gamma J(X, X_p, p) \\ + \alpha_1(X - x(p, r(p, u(n), \ell(n)))) + \alpha_2(X_p - x_p(p, r(p, u(n), \ell(n)))))f(n) \\ + \mu(n)(u'(n) - c_\ell(\ell(n))\ell(n)/n] dn. \end{aligned}$$

[32]For an excellent overview of various conjectural variational parameters see Perry (1982).

After integration by parts and with the transversality condition taken into account, $\mu(\underline{n}) = \mu(\overline{n}) = 0$, $\Pi_p(p, X) = X$, and $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_p = -v_p/v_y = x$, the first-order conditions are

$$u(n) : \left(W_u - \frac{\lambda + \alpha_1 x_y + \alpha_2 x_{py}}{v_y} \right) f(n) - \mu'(n) = 0, \tag{24}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \alpha_1 x_y + \alpha_2 x_{py}) \right) f(n) - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{25}$$

$$p : \gamma J_3 - \int (\alpha_1(x_p + x_y x) + \alpha_2(x_{pp} + x_{py} x)) f(n) dn = 0 \tag{26}$$

$$X : \alpha_1 + \gamma J_1 + \lambda \Pi_X(p, X) = 0, \tag{27}$$

$$X_p : \alpha_2 + \gamma J_2 = 0. \tag{28}$$

where J_1, J_2, and J_3 are partial derivatives of J. We note that $J_1 = X_p K''(\frac{X}{M})\frac{1}{M} - \frac{(1+\theta)}{M} < 0$, J_2 equals the negative of *firm's markup* $\Pi_X(p, X) = p - K'(\frac{X}{M}) \geq 0$, and $J_3 = -X_p > 0$. By substituting (27) and (28) in (25) and (26), we obtain the following result.

Theorem 3. *In oligopolistic markets, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{1 + E^u}{E^c} \frac{v_y \mu}{\lambda n f} - \frac{\gamma J_1 + \lambda \Pi_X}{\lambda} x_y - \frac{\gamma}{\lambda} J_2 x_{py},$$

where

$$\gamma = \frac{-\lambda \Pi_X(p, X) \int (x_p + x_y x) f(n) dn}{J_3 + J_1 \int (x_p + x_y x) f(n) dn + J_2 \int (x_{pp} + x_{py} x) f(n) dn}$$

We observe that in addition to the standard Mirrleesian term there are two corrective terms that together form *the non-competitive effect.* The first term corresponds to the influence of a change in income distribution on optimal marginal income tax through the market demand's level and the second term corresponds to the influence of this change through the market demand's slope. The second corrective term is absent in competitive markets where only the demand level determines the equilibrium price and quantity.

We now note that, if the second derivative of demand $\int x_{pp} f(n) dn$ is small, then the Lagrange multiplier corresponding to the market equilibrium is positive $\gamma \geq 0$. This follows from $\lambda > 0$, the firm's markup $\Pi_X \geq 0$, $\text{sign}(x_{py}) = \text{sign}(x_p) = -1$ for a linear indirect utility, and a non-positive derivative of aggregate compensated demand $\int (x_p + x_y x) f(n) dn \leq 0$. The fact that $\gamma \geq 0$ implies that both corrective terms are non-positive. In particular,

$$-\frac{\gamma J_1 + \lambda \Pi_X}{\lambda} x_y = -\frac{\Pi_X \left(J_3 + J_2 \int (x_{pp} + x_{py} x) f(n) dn \right) x_y}{J_3 + J_1 \int (x_p + x_y x) f(n) dn + J_2 \int (x_{pp} + x_{py} x) f(n) dn} \leq 0$$

because $x_y > 0$ and $\Pi_X \geq 0$, and the second term is negative because $\gamma J_2 \leq 0$. Finally, we note that the absolute value of the non-competitive effect is determined by the firm's markup

Π_X. In particular, the first term is proportional to the firm's markup given that $\gamma \sim \Pi_X$. The second term is, however, proportional to the firm's markup squared given that both $\gamma \sim \Pi_X$ and $|J_2| \sim \Pi_X$. Both terms disappear in competitive markets where the markup is zero.

Overall, in oligopolistic markets, the optimal income taxes feature the non-competitive effect which works in the opposite direction to that of the price effect. Intuitively, the equilibrium of an imperfectly competitive market is associated with under-production in relation to a constrained Pareto-efficient allocation. Hence, the public authority wants to stimulate agents to work more in order to increase market demand and thereby obtain an outcome closer to the constrained Pareto optimum. To relate our result to previous papers, the non-competitive term is similar to a subsidy in optimal commodity taxation literature (see Myles, 1989).

In Appendix A.5, we also analyze optimal income taxation in oligopolistic markets when both non-competitive and price effects are present for any value of $\Xi \in [0, 1]$ and progressive distribution of firm profits $\xi(n)$. These effects work in the opposite directions with the price effect advocating for higher marginal income taxes while the non-competitive effect for lower marginal income taxes. We illustrate both effects with numerical simulations in the following subsection.

Numerical Simulations: Oligopolistic Competition

We consider the numerical estimation of optimal income tax rates in oligopolistic markets using the U.S. housing market framework introduced in Section 3. We infer production cost function $K(\cdot)$ from the specification of the competitive market in the previous section. The profit maximization problem in competitive market yields $p - K'(\frac{X}{M_0}) = 0$, where X is equal to the market supply $S(p) = sp^{\varepsilon}$ in market equilibrium and M_0 is some fixed number of firms. Thus, for the marginal cost function, we obtain

$$K'(X_i) = \left(\frac{X_i M_0}{s}\right)^{\frac{1}{\varepsilon}},$$

where $s = 0.65$ and $\varepsilon = 1.75$ as used in our analysis of the competitive market. Further, we set $M_0 = 2$. All other parameters of the model remain the same.

To illustrate both the non-competitive effect and the price effect, we consider two cases: (i) equal profit distribution (only non-competitive effect) and (ii) empirical profit distribution (both price and non-competitive effects). Figure 2 presents our simulation results. In both cases, we vary the conjectural variation $\theta = 0$ (Cournot-Nash), $\theta = -0.5$, and $\theta = -1$ (perfect competition) in order to see how the non-competitive effect and the overall effect change with market structure. In the case of unequal distribution of firm profits, the public authority lowers income tax rates in order to stimulate labor supply and, thus, to increase aggregate market demand to offset the non-competitive effect of oligopolistic markets. The reduction in tax

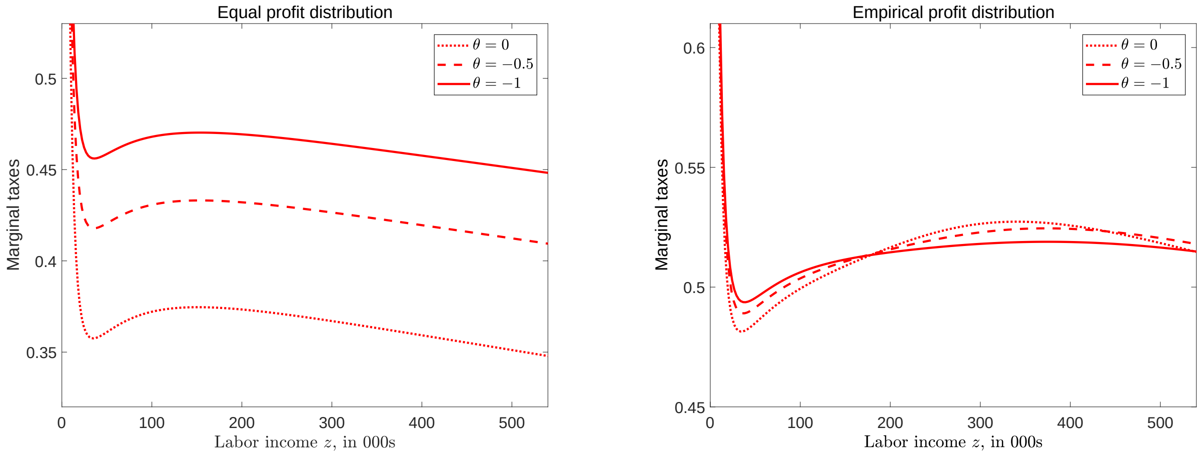

Figure 2. Optimal income taxation for various market structures.

Note: The figure illustrates the optimal marginal income tax schedules for various market structures $\theta \in \{0, -0.5, -1\}$, where $\theta = 0$ stands for the Cournot-Nash competition model and $\theta = -1$ for perfect competition. The left-hand diagram depicts the schedules for the case of the equal profit distribution (only non-competitive effect), and the right-hand diagram depicts the schedules for the case of the empirical profit distribution (both price effect and non-competitive effect).

rates is larger for less competitive markets, i.e., larger θ (left-hand diagram). In the case of empirical distribution of profits, we observe an opposing interplay between the price effect and the non-competitive effect that cancel each other at most income levels. Hence, the optimal marginal tax remains almost unchanged for various forms of market competition.

5 Commodity and Profit Taxation

In the previous sections, we demonstrated that the optimal income tax policy has to take into account market structure and firm profit distribution among agents inside and outside the economy. The question arises as to whether the price and non-competitive effects survive in the presence of other types of taxation. We study this question in this section.

Let us first consider commodity taxation in competitive markets. We assume that the public authority can impose a commodity tax b. In this case, consumer price q differs from producer price p with $b = q - p$. Negative commodity tax $b < 0$ is interpreted as a subsidy. As in the previous section we assume that the second derivative of demand function is small. We also assume that the derivative of the aggregated compensated demand is strictly negative $H_q = \int (x_q + x_y x) f(n) dn < 0$.

The public authority maximization problem can then be written as

$$\max_{p,q,u(n),\ell(n)} \int W(u(n))f(n)dn$$

$$s.t. \begin{cases} \int (n\ell(n) - r(q,u(n),\ell(n)))\, f(n)dn + \Xi\Pi(p) + (q-p)S(p) \geq R \\ S(p) = \int x(q,\tilde{y}(n))f(n)dn. \\ u'(n) - \xi'(n)a(q)\Pi(p) - c_\ell \ell(n)/n = 0 \end{cases}$$

After integration by parts and with the transversality condition $\mu(\underline{n}) = \mu(\overline{n}) = 0$ taken into account, the Lagrangian of the maximization equals

$$\int \{[W(u(n)) + \lambda(n\ell(n) - r(q,u(n),\ell(n)) + \Xi\Pi(p) + (q-p)S(p) - R)+$$

$$\gamma(S(p) - x(q, r(q,u(n),\ell(n))))]f(n) - \mu'(n)u(n) - \mu(n)(\xi'(n)a(q)\Pi(p) + c_\ell \ell(n)/n)\}dn.$$

When $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_q = -v_q/v_y = x$ are taken into account, the first-order conditions are

$$u(n): \left[W_u - \frac{\lambda + \gamma x_y}{v_y}\right] f(n) - \mu'(n) = 0 \tag{29}$$

$$\ell(n): \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y}\right] f(n) - \mu(n)(c_\ell + c_{\ell\ell}\ell(n))/n = 0 \tag{30}$$

$$p: \lambda(-S(p)(1-\Xi) + (q-p)S'(p)) + \gamma S'(p) - \int \mu(n)\xi'(n)dn\, a(q)\Pi'(p) = 0 \tag{31}$$

$$q: \int \{\gamma(-x_q - x_y x)f(n) - \mu(n)\xi'(n)(a_q\Pi(p))\}dn = 0. \tag{32}$$

Equation (30) implies that the optimal income tax formula is the same as in Theorem 2

$$\frac{t}{1-t} = \frac{\mu(n)v_y}{\lambda n f}\frac{1+E^u}{E^c} + \frac{\gamma x_y}{\lambda},$$

where μ is determined by (29) as previously. To derive the expression for γ, we use the following notation $H_q = \int (x_q + x_y x)f(n)dn$ and $M\xi = \int \mu(n)\xi'(n)dn$. Since $a_q, H_q < 0$ and $M\xi > 0$, equation (32) then implies

$$\gamma = \frac{a_q \Pi M\xi}{-H_q} < 0. \tag{33}$$

By substituting the latter expression into (31), we obtain an equation that determines commodity tax $b = q - p$:

$$\lambda(-S(1-\Xi) + (q-p)S_p) = \frac{M\xi}{H_q}(a_q\Pi S_p + a\Pi_p H_q) \geq 0 \tag{34}$$

We obtain that commodity and income taxation are both used by the public authority to achieve a socially desirable outcome. The commodity tax in the optimum decreases producer price leading to smaller firm profit. This decrease in profit mainly influences high-productivity agents who have large profit shares (equity concern). Commodity taxation also increases the consumer price, which leads to a decrease in utility for all agents. However, revenue from commodity taxation relaxes the resource constraint and leads to a decrease in income tax for all agents (as equation (33) shows) (efficiency concern). A proper balance of equity and efficiency concerns, therefore, determines the optimal tax policy. Overall, we obtain the following result:

Theorem 4. (Commodity taxation in competitive markets) *Commodity and income taxation are both used by the public authority in the optimum.*

Finally, we note that the price effect on optimal income tax vanishes if the public authority can impose 100% profit tax. Intuitively, full profit taxation implies that there is neither a loss of efficiency due to foreign ownership nor equity concerns due to unequal distribution of profits.

Let us now analyze commodity taxation in oligopolistic markets. For the purpose of exposition, we consider only the environment of Section 4 without the price effect, i.e., $\Xi = 1$ and $\xi'(n) = 0$. A complete analysis of the non-competitive effect and the price effect in oligopolistic markets is presented in the proof of Theorem 5 in Appendix A.1.

We recall that $\Pi(p, X) = pX - MK(\frac{X}{M})$ and now write the equilibrium condition as $J(X, X_q, p) \equiv -X_q \left(p - K'\left(\frac{X}{M}\right)\right) - (1+\theta)\frac{X}{M}$, where X_q is a slope of market demand for consumer price q. The public authority's problem for this environment is as follows:

$$\max_{p,q,u(n),\ell(n)} \int W(u(n))f(n)dn$$

s.t.

$$u'(n) - c_\ell \ell(n)/n = 0 \qquad (\mu(n),\ \text{incentice compatibility})$$

$$\int [n\ell(n) - r(q, u(n), \ell(n))]f(n)dn + \Pi(p, X) + (q-p)X \geq R \qquad (\lambda,\ \text{resource constraint})$$

$$J(X, X_q, p) = 0 \qquad (\gamma,\ \text{market equilibrium})$$

$$X - \int x(q, r(q, u(n), \ell(n)))f(n)dn = 0 \qquad (\alpha_1,\ \text{market demand})$$

$$X_q - \int x_q(q, r(q, u(n), \ell(n)))f(n)dn = 0 \qquad (\alpha_2,\ \text{market demand slope})$$

where function r determines agent income $\tilde{y} = r(q, u, \ell) = \frac{u+c(\ell)}{a(q)}$, and Largrange multipliers are introduced next to the corresponding constraints. The Lagrangian then equals

$$\mathcal{L} = \int [(W(u(n)) + \lambda(n\ell(n) - r(q, u(n), \ell(n)) + \Pi(p, X) + (q - p)X - R) + \gamma J(X, X_q, p)$$

$$+ \alpha_1(X - x(q, r(q, u(n), \ell(n)))) + \alpha_2(X_q - x_q(q, r(q, u(n), \ell(n)))))f(n)$$

$$+ \mu(n)(u'(n) - c_\ell \ell(n)/n)]dn.$$

After integration by parts and with the transversality condition taken into account, $\mu(\underline{n}) = \mu(\overline{n}) = 0$, and $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_q = -v_q/v_y = x$, the first-order conditions are

$$u(n) : \left(W_u - \frac{\lambda + \alpha_1 x_y + \alpha_2 x_{qy}}{v_y}\right) f(n) - \mu'(n) = 0, \tag{35}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \alpha_1 x_y + \alpha_2 x_{qy})\right) f(n) - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{36}$$

$$p : \gamma J_3 = 0 \tag{37}$$

$$q : \alpha_1 \int (x_q + x_y x) f(n) dn + \alpha_2 \int (x_{qq} + x_{qy} x) f(n) dn = 0 \tag{38}$$

$$X : \alpha_1 + \gamma J_1 + \lambda(\Pi_X + q - p) = 0, \tag{39}$$

$$X_p : \alpha_2 + \gamma J_2 = 0. \tag{40}$$

where J_1, J_2, and J_3 are the partial derivatives of J. From equation (36), we can determine the optimal income tax as

$$\frac{t}{1-t} = \frac{\mu(n) v_y}{\lambda n f} \frac{1 + E^u}{E^c} + \frac{\alpha_1 x_y}{\lambda} + \frac{\alpha_2 x_{qy}}{\lambda}.$$

Given that $J_3 = -X_q > 0$, we have $\gamma = 0$ and, as a consequence, $\alpha_1 = \alpha_2 = 0$. Equation (39) then implies that the public authority can eliminate welfare loss due to oligopolistic competition by imposing a subsidy that perfectly corrects for the presence of firm markup:

$$b = q - p = -\Pi_X = -(p - K'(X/M)) < 0. \tag{41}$$

This leads to the standard undistorted formula of optimal income taxation as in Mirrlees (1971). Note that when firm profits are equally distributed among agents, the public authority can finance the subsidy by imposing a constant uniform tax on all agents. This result is, however, specific to the case of $\Xi = 1$ and $\xi'(n) = 0$. In general, when foreign ownership is present or firm profits are unequally distributed, the public authority cannot achieve production efficiency.

Theorem 5. (Commodity taxation in oligopolistic markets) *In the presence of foreign ownership or unequal distribution of firm profits, both the price and non-competitive effects are present in the optimum. Only when there is no foreign ownership and firm profits are equally distributed (no price effect) can commodity taxation fully correct the non-competitive effect.*

Finally, if the public authority can impose 100% firm profit tax, we arrive again at equation (41). In this case, the public authority maximization problem coincides with the problem solved above independently of the distribution of firm profits. This result is in line with Myles (1996) who showed that a combination of ad valorem tax and commodity tax can eliminate the welfare loss that arises from oligopolistic competition.

Overall, commodity taxation alone cannot eliminate the effect of endogenous prices when firm profits are not fully taxed away. In this case, the optimal income taxation policy needs to account for the price and non-competitive effects.

6 Literature Review

The results of the present paper are connected to several strands in the literature. First, our analysis is closely related to the study of optimal income taxation in the presence of endogenous wages in labor markets. Stiglitz (1982) was one of the first to consider a setting in which workers are not perfect substitutes in production.[33] In this case, the general equilibrium effects imply that the optimal tax policy should subsidize high-talent workers and tax low-talent workers. Extending this analysis to a setting where workers have two-dimensional skill characteristics and an occupational choice, Rothschild and Scheuer (2013) found that the ability of workers to select their occupation involves a more progressive tax schedule than does a model without occupational choice. Ales, Kurnaz, and Sleet (2015) use a related multi-task assignment model with finite one-dimensional agent types to study a change in optimal income policy in response to a technical change. In a study of the continuum of one-dimensional agent types and general constant returns to scale production functions, Sachs, Tsyvinskiy, and Werquin (2016) show that the optimal labor supply in an equilibrium is determined by a complicated integral equation. Using this equation, they analyze the incidence of a tax reform with the actual U.S. tax code as a starting point. In contrast to these papers, the price effect in our model arises because of the distribution of firm profits among agents inside and outside the economy.

The literature in which endogenous wages in labor markets also studies the impact of externalities on the optimal income tax schedule.[34] Rothschild and Scheuer (2016) study the corrective role of income taxation in a setting where agents can engage in rent-seeking activity

[33] See also Feldstein (1973) and Stern (1982).

[34] Stantcheva (2014) also studies how adverse selection in labor markets influences optimal income taxation. Stantcheva (2017) provides a joint analysis of optimal income tax and optimal human capital policies.

whereby private returns are different from social returns. Lockwood et al. (2017) integrate tax considerations into an assignment model of agents to professions in which high-paying professions have negative externalities and low-paying professions have positive externalities. They estimate that the welfare gains from an optimal income taxation policy targeted to compensate for these externalities are small. See also Rothschild and Scheuer (2014) for a unifying framework to analyze the effects of externalities on optimal income taxation.

Compared to this literature, in the present study, agents do not impose direct externalities. Instead, agents impose only pecuniary externalities. One should not correct pecuniary externalities when agent types are perfectly observable and all firm profits remain in the economy. However, when agent types are not perfectly observable and tax policy can be based only on labor income, we show that pecuniary externalities should, in fact, be corrected.[35] In this case, a constrained Pareto-efficient allocation cannot be generally supported by a competitive equilibrium. Similarly, pecuniary externalities arise in the analysis of the optimal taxation of profits and labor income in Scheuer (2014). When profits and labor income are subject to the same tax schedule equilibrium, wages serve as an additional policy instrument to redistribute income across occupations.

Our analysis is also closely related to the seminal production efficiency theorem of Diamond and Mirrlees (1971), who assert that optimal income taxation entails efficient production in equilibrium.[36] This seminal result holds, however, only in the absence of firm profits or when firm profits are fully taxed.[37] The contribution of the present paper is to analyze optimal income taxation in the Mirrleesian setting in the presence of firm profits for both competitive and oligopolistic markets. Further, full taxation of firm profits is an important assumption in the commodity taxation literature. Atkinson and Stiglitz (1976) establish that commodity taxation is unnecessary in the presence of optimal income tax given weak separability of utility between labor and all consumption goods (see also Mirrlees, 1976; Kaplow, 2006). In contrast, we show that commodity taxation plays an important role on par with income taxation when firm profits cannot be fully taxed.

Only a few papers consider models in which firm profits are not fully taxed. Stiglitz and Dasgupta (1971) were among the first to show that production efficiency may not be desirable when the maximum profit tax rate is limited.[38] Specifically, they show that factor and commodity taxes may serve as a partial substitute for the tax on profits (see also Munk, 1978, 1980). Dasgupta and Stiglitz (1972) show that the government might not wish to tax all profits away in the economy with non-identical consumers when lump-sum taxes are not allowed.

[35] If tax policy is based on total income, pecuniary externalities should also be corrected (see Appendix A.4).

[36] See also Scheuer and Werning (2016) for an insightful connection between the seminal model of Diamond and Mirrlees (1971) on commodity taxation and Mirrlees (1971) on income taxation.

[37] Dasgupta and Stiglitz (1972) also establish that production efficiency is desirable if the government can set different percentage profit taxes for different producers; see also Mirrlees (1972).

[38] See also Atkinson and Stiglitz (1976, 2015) and Diamond and Mirrlees (1971).

Guesnerie and Laffont (1978) and Iwamoto and Konishi (1991) analyze the optimal commodity tax rule in a setting with firm profits and many consumers. In contrast to these papers, we consider an incomplete information environment of optimal income taxation. We both derive analytically and estimate numerically the influence of firm profit distribution on the optimal income tax schedule.

We also want to mention an important paper by Scheuer and Werning (2017) who consider an assignment model to study the taxation of top income earners.[39] They establish that the conditions for the efficiency of a marginal tax rate schedule do not depend on the production technology even when firm profits are not fully taxed. However, for a given social objective function the optimal marginal tax policy does depend on the production technology and the distribution of firm profits. Scheuer and Werning (2017) also do not consider endogenous product prices, which is the main subject of our analysis.

As mentioned in the introduction, there is almost no literature relating optimal income taxation to various market structures. The only exception is a recent work by Kaplow (2018) who studies the influence of market power on income taxation. One major difference between his approach and ours is that the former analyzes income taxation in an economy with *exogenously given* firm markups. Hence, Kaplow (2018) does not consider how income taxation policy influences equilibrium prices and firm markups, which is the main subject of our analysis.

Our paper is also related to a small body of literature focused on analyzing commodity taxation in oligopolistic markets. Auerbach and Hines (2001) and Myles (1987, 1989) show the benefit of a corrective subsidy to offset producer markups in the presence of market power.[40] Parallel to their findings, we show the benefit of a less progressive marginal income tax schedule. Similar to a corrective subsidy, a decrease in marginal income tax stimulates market demand that brings the equilibrium consumption closer to a constrained Pareto-efficiency frontier. Though our results can be seen as an extension of the previous analysis to optimal income taxation settings, we also study the distributional effect of firm profits, a consideration that is absent from previous papers. We also establish that the non-competitive effect persists even in the presence of commodity taxation.

Our paper is also closely related to an important strand of literature focused on the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Ireland (2001), Jinkins (2016), Kanbur and Tuomala (2013), Oswald (1983).[41] These papers are typically motivated by empirical observations showing that people care not only about their absolute level of consumption but also how it compares with that of others. In our paper, relativity concerns arise endogenously through equilibrium product prices.

[39] See also Ales and Sleet (2016), Kleven et al. (2013), Piketty, Saez, and Stantcheva (2014), and Shourideh (2014) for analyses of optimal taxation of top labor income and capital income.

[40] See also Reinhorn (2005, 2012).

[41] A similar idea according to which individuals may seek a more equal income distribution in order to improve the terms of trade is explored by Zubrickas (2012).

7 Conclusion

In this paper, we considered how endogenous prices affect optimal income taxation policy in both competitive and oligopolistic markets. We focused on a Mirrleesian framework with imperfectly observable agent productivity types and firm profits distributed among agents inside and outside the economy. We established that unequal distribution of firm profits is associated with the price effect on optimal income taxation. Using the U.S. housing market, we numerically estimated that the price effect increases optimal marginal income tax by more than 4 percentage points.

The presence of market power in oligopolistic markets is associated with the non-competitive effect. Using the model of the U.S. housing market, we showed that, as the market structure varies, changes in the price effect and the non-competitive effect almost cancel each out. Hence, the optimal income taxation policy remains robust across various market structures. We also studied how the price and non-competitive effects change in the presence of commodity and profit taxation.

Our study of the price effect is only the first step in incorporating an underlying market structure into the analysis of optimal income taxation. Spending on housing accounts for only one fourth of U.S. household expenditure. Hence, the size of the price effect may be significantly larger than 4 percentage points when other industries are taken into account. The exact size of the effect would depend, however, on the level and the distribution of firm profits in various industries and how the price effects interact across industries. Hence, to estimate the overall price effect on optimal income taxation one would need to develop and carefully calibrate a general equilibrium model. We leave this intriguing question for future research.

Our results highlight an important interaction between income and profit taxation. Scheuer (2014) presents an insightful analysis of optimal income and profit taxation in the presence of endogenous firm formation. Extending his analysis to the economies with various market structures and endogenous product prices is an important question for future investigation. In addition, incorporating endogenous prices into a dynamic setting (see Albanesi and Seet, 2006) would allow analyzing health care market (see Grossman, 1972) as well as many other major markets that cannot be modeled with a static framework.

Finally, we want to highlight that endogenous prices in product markets should be an important consideration beyond income taxation policies. The welfare assessment of subsidies, welfare benefits, pensions, and the minimum wage, etc., would be biased unless the public authority takes into account the price and non-competitive effects analyzed in this paper.

A Appendix

A.1 Proofs

Proof of Theorem 1. The statement follows from the argument in the main text. □

Proof of Corollary 1. The statement about normal and inferior goods follows from the argument presented in the main text. We now show that if demand for good X is either convex or concave at all income levels, x_y is increasing for luxury goods and x_y is decreasing for necessity goods.

Let us assume that good X is luxury. By the definition of luxury goods, its demand then increases more than proportionally as income rises, i.e., $px(p,y)/y$ is strictly increasing in y. From contrary, let us assume that function x is concave in income. Therefore,

$$\frac{d}{dy}\left(\frac{x(p,y)-x(p,0)}{y-0}\right) \leq 0.$$

Taken into account that $x(p,0)=0$ we arrive at the contradiction that $px(p,y)/y$ is strictly increasing. Therefore, demand x is convex in income and, hence, x_y is increasing in y. Therefore, the optimal marginal income tax is increasing for luxury goods. A similar argument applies if good X is necessity.[42] □

Proof of Theorem 2. The individual utility maximization problem $\max_z U(p, z - T(z,n) + \xi(n)\Pi(p), z/n)$ implies that $t = 1 - c_\ell/(nv_y)$. Taken this into account, condition (16) leads to

$$\frac{t}{1-t} = \left(1 + \frac{\ell c_{\ell\ell}}{c_\ell}\right)\frac{v_y \mu(n)}{\lambda n f} + \frac{\gamma}{\lambda} x_y,$$

with multiplier $\mu(n) = \int_n^{\overline{n}} \left(\frac{\lambda+\gamma x_y}{v_y} - W_u\right) f(n)dn$ found from (15).

Next, we demonstrate that $1 + \ell c_{\ell\ell}/c_\ell = (1+E^u)/E^c$, where E^c is the elasticity of the compensated labor supply and E^u is the elasticity of the uncompensated labor supply E^u. The individual utility maximization condition again implies $v_y(p,\tilde{y})\ n(1-t) - c_\ell(\ell) = 0$. Denoting $\tilde{y} = w\ell + \overline{y}$, where $w = n(1-t)$ is a net wage rate and $\overline{y}$ is non-labor income, we obtain

$$v_y(p, w\ell + \overline{y})\ w - c_\ell(\ell) = 0.$$

[42]The assumption that the demand for good X is either concave or convex in income at all income levels is generally indispensable for our result to hold. For example, consider function $f(y) = 0$ if $y \in [0,1]$ and $f(y) = y - 1/y$ if $y \geq 1$. Function f has increasing ratio $f(y)/y$ and decreasing derivative f_y at the same time.

Implicitly differentiating the above expression with respect to net wage we obtain

$$\frac{\partial \ell}{\partial w} = \frac{v_{yy} w \ell + v_y}{c_{\ell\ell} - v_{yy} w^2}.$$

Then, the elasticity of the uncompensated labor supply $E^u = \partial \ell / \partial w \ (w/\ell)$ is equal to

$$E^u = \frac{v_{yy} w \ell + v_y}{c_{\ell\ell} - v_{yy} w^2} \frac{w}{\ell} = \frac{v_{yy}(c_\ell/v_y)^2 + c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2},$$

where we use $w = c_\ell/v_y$. To obtain the elasticity of the compensated labor supply, E^c, we employ the Slutsky equation $E^c = E^u - E^m$, where $E^m = w(\partial \ell/\partial \overline{y})$ is the income effect parameter:

$$E^m = w \frac{\partial \ell}{\partial \overline{y}} = \frac{v_{yy}(c_\ell/v_y)^2}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2}.$$

Thus, the elasticity of the compensated labor supply, E^c, is given by

$$E^c = E^u - E^m = \frac{v_{yy}(c_\ell/v_y)^2 + c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2} - \frac{v_{yy}(c_\ell/v_y)^2}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2} = \frac{c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2}.$$

Thus, we obtain that $1 + \ell c_{\ell\ell}/c_\ell = (1 + E^u)/E^c$, which completes the derivation of the expression for optimal marginal income tax. Finally, the expression for the Lagrange multiplier γ follows directly from condition (17). $\square$

Proof of Corollaries 2 and 3. The statements follow from the argument in the main text. $\square$

Proof of Theorems 3 and 4. The statements follow from the argument in the main text. $\square$

Proof of Theorem 5. Let us consider commodity taxation in oligopolistic markets with general distribution of firm profits. The public authority problem can be written as follows:

$$\max_{p,q,u(n),\ell(n)} \int W(u(n)) f(n) dn$$

$s.t.$

$u'(n) - \xi'(n) a(q) \Pi(p, X) - c_\ell \ell(n)/n = 0$	($\mu(n)$, incentice compatibility)
$\int [n\ell(n) - r(q, u(n), \ell(n))] f(n) dn + \Xi \Pi(p, X) + (q - p) X \geq R$	(λ, resource constraint)
$J(X, X_q, p) = 0$	(γ, market equilibrium)
$X - \int x(q, r(q, u(n), \ell(n))) f(n) dn = 0$	(α_1, market demand)
$X_q - \int x_q(q, r(q, u(n), \ell(n))) f(n) dn = 0$	(α_2, market demand slope)

where function r determines agent income as a function of price, utility, and effort $\tilde{y} = r(q,u,\ell) = \frac{u+c(\ell)}{a(q)}$, and the Lagrange multipliers introduced next to the corresponding constraints. The Lagrangian can then be written as

$$\mathcal{L} = \int [(W(u(n)) + \lambda(n\ell(n) - r(q,u(n),\ell(n)) + \Xi\Pi(p,X) + (q-p)X - R) + \gamma J(X, X_q, p)$$

$$+ \alpha_1(X - x(q, r(q,u(n),\ell(n)))) + \alpha_2(X_q - x_q(q, r(q,u(n),\ell(n)))))f(n)$$

$$+ \mu(n)(u'(n) - \xi'(n)a(q)\Pi(p,X) - c_\ell \ell(n)/n)]dn$$

After the integration by parts and with $\mu(\underline{n}) = \mu(\overline{n}) = 0$, $\Pi_p(p,X) = X$, and $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_q = -v_q/v_y = x$ taken into account, the first-order conditions are

$$u(n) : \left(W_u - \frac{\lambda + \alpha_1 x_y + \alpha_2 x_{qy}}{v_y}\right) f(n) - \mu'(n) = 0, \tag{A.1}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \alpha_1 x_y + \alpha_2 x_{qy})\right) f(n) - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.2}$$

$$p : -a\Pi_p \int \mu(n)\xi'(n)dn + \lambda(-X(1-\Xi)) + \gamma J_3 = 0 \tag{A.3}$$

$$q : -a_q\Pi \int \mu(n)\xi'(n)dn - \alpha_1 \int (x_q + x_y x)f(n)dn - \alpha_2 \int (x_{qq} + x_{qy}x)f(n)dn = 0 \tag{A.4}$$

$$X : \alpha_1 - a\Pi_X \int \mu(n)\xi'(n)dn + \gamma J_1 + \lambda(\Xi\Pi_X + q - p) = 0, \tag{A.5}$$

$$X_p : \alpha_2 + \gamma J_2 = 0. \tag{A.6}$$

where partial derivatives of J are determined by

$$J_1 = X_q K''(\tfrac{X}{M})\tfrac{1}{M} - \tfrac{(1+\theta)}{M} < 0,$$

$$J_2 = -\Pi_X(p,X) = -(p - K'(\tfrac{X}{M})) \leq 0,$$

$$J_3 = -X_q > 0.$$

We also denote $H_{qq} = \int (x_{qq} + x_{qy}x)f(n)dn$. As a reminder, $H_q = \int (x_q + x_y x)f(n)dn < 0$ and $M\xi = \int \mu(n)\xi'(n)dn > 0$. We can then rewrite (A.4) and (A.6) as

$$\alpha_1 = \frac{a_q\Pi M\xi}{-H_q} - \gamma J_2 \frac{H_{qq}}{-H_q} = \frac{a_q\Pi M\xi}{-H_q} + (a\Pi_q M\xi + \lambda X(1-\Xi))\frac{J_2}{J_3}\frac{H_{qq}}{H_q} \tag{A.7}$$

$$\alpha_2 = -\gamma J_2 = (a\Pi_p M\xi + \lambda X(1-\Xi))\frac{J_2}{J_3}. \tag{A.8}$$

We now substitute the above expression into (A.5). Taking into account (A.3), we obtain

$$\frac{a_q \Pi M\xi}{-H_q} + (aM\xi + \lambda(1-\Xi))\left(\frac{XJ_2H_{qq} + XJ_1H_q}{J_3H_q} - \Pi_X\right) = -\lambda(\Pi_X + q - p) \tag{A.9}$$

This formula determines the optimal subsidy in oligopolistic markets. Note that the left-hand side of the above formula is non-positive if $H_{qq} \leq 0$ (which holds if the second derivative of demand x_{qq} is small). In particular, the first term is non-positive because $a_q, H_q < 0$ and $M\xi \geq 0$. The second term is also non-positive because $J_2 \leq 0$ and $J_1 < 0$. Hence, the optimal subsidy is generally higher or equal to the firm's markup Π_X in the optimum.

Overall, equation (A.9) shows that when foreign ownership is absent, $\Xi = 1$, and firm profits are equally distributed, $M\xi = 0$, the optimal commodity tax equals the size of the firm's markup leading to production efficiency. In the presence of foreign ownership or unequal distribution of firm profits, equations (A.7) and (A.8) imply that $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$. Moreover, if $H_{qq} < 0$, both $\alpha_1 < 0$ and $\alpha_2 \leq 0$, which is parallel to the result of Theorem 4 for competitive markets. $\square$

A.2 Supporting Equilibrium Model

In this section, we show that our model of competitive markets can be supported with a labor market and a consumer's utility maximization problem. In particular, we consider two competitive industries: one producing the numeraire good G and the other producing good X. We label these industries as G and X respectively. We assume that agents can earn wage w for effective labor hours supplied (i.e., $n\ell(n)$) in both industries, the price for the numeraire good is normalized to $p_g = 1$, and the price for good X is equal to p.

Industry G has a homogeneous of degree one production technology $F_g(L_g) \equiv L_g$, where L_g is the amount of labor used to produce good G. As the price of the numeraire good is normalized to 1, the profit maximization condition implies that $w = p_g = 1$, *zero profits*, and *any level of the equilibrium labor demand* L_g^d in industry G.

Industry X has a production technology with decreasing returns to scale $F_x(L_x)$, where L_x is the amount of labor supplied. To ensure that the firm profit maximization problem has a well-defined interior solution, we assume that F_x is differentiable and strictly concave, and that it satisfies the Inada conditions, e.g., $F_x(L_x) = AL_x^a$, where $0 < A$, $0 < a < 1$ are constants. Hence, the firm profit maximization problem

$$\max_{L_x} p \cdot F_x(L_x) - wL_x.$$

The solution to this maximization problem leads to the equilibrium labor demand $L_x^d(p)$ in industry X. If the production function is $F_x(L_x) = AL_x^a$, we have $L_x^d(p) = (aAp)^{\frac{1}{1-a}}$ (taking into account $w = 1$). The equilibrium market supply of good X then equals $S_x(p) = F(L_x^d(p))$ and firm profits equal $\Pi(p) = \int S_x(\tilde{p})d\tilde{p}$.

We assume that the share of firm profits Ξ is distributed among the agents in the economy according to distribution function $\xi(n)$ with $\int \xi(n)f(n)dn = \Xi$. The remaining share $1 - \Xi$ belongs to capitalists (or "foreigners") who spend it on the consumption of numeraire good G. The government also spends all its resources R on numeraire good G.

On the demand side of the economy, we assume that an agent's preferences can be summarized by utility function $u(x, g) - c(\ell)$, where (x, g) is the amount of good X and the numeraire good G consumed by the agent. Utility u is a continuous function representing locally non-satiated preferences.

Consider an agent with productivity n who works ℓ hours. With equilibrium wage $w = p_g = 1$ and tax schedule $T(n\ell)$ taken into account, her income equals $n\ell - T(n\ell)$. Hence, the agent's maximization problem is

$$\begin{aligned}
&\max_{x,\,g,\,\ell}\; u(x,g)-c(\ell) \\
&s.t.\;\; p\cdot x+g\le n\ell-T(n\ell)+\xi(n)\Pi(p)
\end{aligned} \tag{A.10}$$

The solution to the above problem is labor supply $\ell^*(n,p)$ and consumption bundle $(x^*(n,p),g^*(n,p))$. Overall, aggregate labor supply and consumer demand equal

$$L^s(p)=\int n\ell^*(n,p)f(n)dn,\quad X(p)=\int x^*(n,p)f(n)dn,\quad G(p)=\int g^*(n,p)f(n)dn.$$

The economy must satisfy three market clearing conditions:

$$S_x(p)=X(p) \tag{A.11}$$

$$S_g(p)=G(p)+(1-\Xi)\Pi(p)+R \tag{A.12}$$

$$L^s(p)=L^d_x(p)+L^d_g(p), \tag{A.13}$$

where the market clearing condition (A.12) requires the market supply for good G to equal the sum of the market demand for G, the share of capitalists' profits, and the extent of government spending.

Let us show that condition (A.11) is the only one that we should consider in the optimal income taxation problem. As the constant return to scale technology for the numeraire good ensures that any level $L^d_g(p)$ satisfies the firm maximization problem (when $w=p_g=1$), we are free to choose $L^d_g(p)=L^s(p)-L^d_x(p)$ to clear the labor market. Taking into account that $S_g(p)=L^d_g(p)$ and $\Pi(p)=pS_x(p)-L^d_x(p)$, we can rewrite condition (A.12) equivalently as

$$L^d_g(p)+L^d_x(p)=G(p)+pS_x(p)+R-\Xi\Pi(p)$$

Given conditions (A.11) and (A.13) this is equivalent to

$$\int n\ell^*(n,p)f(n)dn=G(p)+pX(p)+R-\Xi\Pi(p).$$

This condition follows from the budget constraint of the agent's maximization problem (A.10) when the government spending constraint is binding $\int T(n\ell^*(n,p))f(n)dn=R$ (as we assume). Overall, the only independent market clearing condition that we should take into account in the optimization problem is (A.11) – the market clearing condition for good X.

A.3 Competitive Markets: Non-Linear Indirect Utility

In this section, we consider the general case with non-homothetic preferences that lead to agent indirect utility being non-linear in income. We also analyze the public authority maximization problem where the remaining firm profits enter with a separate social weight.

Let us again consider an agent's utility from revealing his productivity type truthfully as

$$u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n).$$

If revealing the agent's type truthfully is optimal then

$$u(n) = \max_m U(p, y(m) + \xi(n)\Pi(p), z(m), n). \tag{A.14}$$

The envelope theorem implies the following first-order condition

$$u'(n) = U_n + U_y \xi'(n)\Pi(p). \tag{A.15}$$

We note that the single-crossing condition does not generally hold for a non-linear indirect utility. Hence, we need to derive the second-order condition when truth-telling is optimal.

Proposition A1. *Condition (A.15) ensures that truth-telling is an optimal solution of (A.14) if and only if for each productivity n schedule $\{y(n), z(n)\}$ satisfies*

$$[(c_{ll} z(n)/n + c_l)/n \frac{v_y}{c_l} + v_{yy}\xi'(n)\Pi(p)]y'(n) \geq 0. \tag{A.16}$$

Proof. Let us assume that $y(n)$ and $z(n)$ are differentiable. The second-order condition for maximization (A.14) is

$$u''(n) - U_{nn} - (U_{yy}\xi'(n)\Pi(p) + 2U_{yn})\xi'(n)\Pi(p) - U_y\xi''(n)\Pi(p) \geq 0. \tag{A.17}$$

Taking the derivative of (A.15) with respect to n we obtain

$$u''(n) = U_{nn} + U_{ny}(y'(n) + \xi'(n)\Pi(p)) + U_{nz}z'(n) +$$
$$(U_{yn} + U_{yy}(y'(n) + \xi'(n)\Pi(p)) + U_{yz}z'(n))\xi'(n)\Pi(p) + U_y\xi''(n)\Pi(p).$$

Hence, condition (A.17) is equivalent to

$$U_{ny}y'(n) + U_{nz}z'(n) + (U_{yy}y'(n) + U_{yz}z'(n))\xi'(n)\Pi(p) \geq 0.$$

Given our separable utility specification, this reduces to

$$U_{ny}y'(n) + U_{nz}z'(n) + (U_{yy}y'(n) + U_{yz}z'(n))\xi'(n)\Pi(p) =$$

$$U_{nz}z'(n) + U_{yy}y'(n)\xi'(n)\Pi(p) =$$

$$(c_{\ell\ell}z(n)/n + c_\ell)/n^2 z'(n) + v_{yy}y'(n)\xi'(n)\Pi(p) \geq 0.$$

Maximization (A.14) now implies $v_y y'(n) - c_\ell z'(n)/n = 0$, which allows rewriting the previous inequality in the form of (A.16). $\square$

Note that the first term in (A.16) is always positive because the cost function $c(\ell)$ is increasing and convex. Hence, the second-order condition reduces to income schedule $y(n)$ being non-decreasing if either profits are zero $\Pi(p) = 0$ or agent profits are equally distributed $\xi'(n) = 0$. When both $\Pi(p) > 0$ and $\xi'(n) > 0$ the second term in (A.16) is negative because the indirect utility function is concave.

We consider a version of the maximization problem that includes the remaining firm profits with a separate weight $\omega \geq 0$ in the objective function.

$$\max_{p,\,\tilde{y}(n),\,\ell(n)} \int W(v(p, \tilde{y}(n)) - c(\ell(n)))f(n)dn + \omega(1 - \Xi)\Pi(p) \quad \text{subject to (3), (5), and (A.15).}$$

It is again convenient to change the optimization variables $\{p, \tilde{y}(n), \ell(n)\}$ to $\{p, u(n), \ell(n)\}$, where the utility level is $u(n) = U(p, \tilde{y}(n), \ell(n))$. From the latter expression we can invert disposable income $\tilde{y}(n)$ and express it as a function of (p, u, ℓ), i.e. $\tilde{y} = r(p, u, \ell)$. Assuming that the second-order condition (A.16) is satisfied, the maximization problem can be written as

$$\begin{aligned}
&\max_{p,u(n),\ell(n)} \int W(u(n))f(n)dn + \omega(1 - \Xi)\Pi(p) \\
&s.t. \\
&\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)]f(n)dn \geq R \\
&\int [S(p) - x(p, r(p, u(n), \ell(n)))]f(n)dn = 0 \\
&u'(n) - v_y(p, r(p, u(n), \ell(n)))\xi'(n)\Pi(p) - c_\ell \ell(n)/n = 0
\end{aligned}$$

After integration by parts and taking into account the transversality condition $\mu(\underline{n}) = \mu(\overline{n}) = 0$, we can write the Lagrangian of the maximization problem as

$$\mathcal{L} = \int \{[W(u(n)) + \omega(1-\Xi)\Pi(p) + \lambda(n\ell(n) - r(p,u(n),\ell(n)) + \xi(n)\Pi(p) - R) +$$
$$\gamma(S(p) - x(p, r(p,u(n),\ell(n))))]f(n) -$$
$$\mu'(n)u(n) - \mu(n)(v_y(p, r(p,u(n),\ell(n)))\xi'(n)\Pi(p) + c_\ell \ell(n)/n\}dn.$$

With $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, $r_p = -v_p/v_y = x$ taken into account, the first-order conditions are

$$u(n): \left[W_u - \frac{\lambda + \gamma x_y}{v_y}\right] f(n) - \mu'(n) - \mu(n)\frac{v_{yy}}{v_y}\xi'(n)\Pi(p) = 0 \tag{A.18}$$

$$\ell(n): \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y}\right] f(n) - \mu(n)\frac{v_{yy}c_\ell}{v_y}\xi'(n)\Pi(p) - \mu(n)(c_\ell + c_{\ell\ell}\ell(n))/n = 0 \tag{A.19}$$

$$p: \int \{[\omega(1-\Xi)\Pi'(p) + \lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_y x)]f(n) - \tag{A.20}$$
$$\mu(n)((v_{yp} + v_{yy}x)\xi'(n)\Pi(p) + v_y\xi'(n)\Pi'(p))\}dn = 0.$$

To calculate the marginal income tax, we consider now the individual utility maximization problem $\max_\ell \ v(p, n\ell - T(n\ell) + \xi(n)\Pi(p)) - c(\ell)$, where $T(n\ell)$ is the gross income tax payment. The first-order condition with respect to ℓ yields $v_y n(1-t) = c_\ell$. Using the first-order condition (A.19) we can then write

$$\frac{t}{1-t} = \frac{nv_y}{c_\ell} - 1 = \frac{\mu(n)v_y}{\lambda n f}\frac{1+E^u}{E^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu(n)v_{yy}\xi'(n)\Pi(p)}{\lambda f}, \tag{A.21}$$

where μ is determined by (A.18) and multipliers λ and γ are determined by transversality conditions, equations (A.18) and (A.20). Using $\Pi'(p) = S(p) = \int x(n)f(n)dn$ and $\int \xi(n)f(n)dn = \Xi$, we obtain the following result.

Theorem A1. *In competitive markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{\mu(n)v_y}{\lambda n f}\frac{1+E^u}{E^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu(n)v_{yy}\xi'(n)\Pi(p)}{\lambda f},$$

where

$$\gamma = \frac{(\lambda - \omega)S(p)(1-\Xi) + \int (v_{yp}\Pi(p) + v_y\Pi'(p) + v_{yy}x\Pi(p))\mu(n)\xi'(n)dn]}{S'(p) - \int (x_p + x_y x)f(n)dn}.$$

Note that the optimal marginal income tax formula has three terms. The first is the standard Mirrleesian term. The second term corresponds to the price effect of the binding market equilibrium constraint. Both terms are present in the model with linear indirect utility. The third term, which is new, is responsible for the change of agent incentives due to profit distribution.

Hence, we refer to this term as an *incentive term.* When agent indirect utility is strictly concave, agents with a larger profit share realize a smaller marginal benefit from additional income. Hence, they are less willing to exert effort and the public authority can no longer impose a high marginal tax rate on them without disturbing their effort level. The third term is negative (as $v_{yy} < 0$), which makes the optimal marginal income tax less progressive in contrast to the price effect (when good X is normal). A similar incentive effect would occur if agents with higher productivity levels had larger endowments that are not related to firm profit shares.

Finally, we want to note that social weight ω influences the optimal marginal income tax through multiplier γ: the larger the social weight ω, the smaller the price effect. The price effect decreases with social weight ω and completely disappears when social weight ω is equal to the shadow costs of raising public funds λ.

A.4 Competitive Markets: Taxing Total Income

In this subsection, we consider the case when public authority can tax total agent income. We denote total income as

$$z(n) = n\ell(n) + \xi(n)\Pi(p).$$

The agent's disposable income then equals $y(n) = z(n) - T(z(n))$. Given that agents have different profit shares, it is now harder for agents to pretend to have different productivity levels compared to the case when taxes are based on labor income only. Now, if an agent of type n wants to pretend to be of type m, he must work more (or less) hours $\ell = (m\ell(m) + (\xi(m) - \xi(n))\Pi(p))/n$. Hence, the agent's utility from reporting type m is equal to

$$U(p, y(m), z(m), n) = v(p, y(m)) - c((z(m) - \xi(n)\Pi(p))/n).$$

We denote the agent's utility from revealing his productivity type truthfully as

$$u(n) \equiv U(p, y(n), z(n), n) = v(p, y(n)) - c((z(n) - \xi(n)\Pi(p))/n).$$

If revealing the agent's type truthfully is optimal, then

$$u(n) - U(p, y(n), z(n), n) = 0 \leq u(m) - U(p, y(n), z(n), m), \tag{A.22}$$

which leads to the following first-order condition

$$u'(n) = U_n = c_\ell \frac{\ell(n) + \xi'(n)\Pi(p)}{n}. \tag{A.23}$$

The latter condition coincides with the standard one when profits are zero $\Pi(p) = 0$ or $\xi'(n) = 0$. We also notice that the standard single-crossing condition is satisfied when agent total income is taxed. Hence, the second-order condition for truth-telling coincides with the one in the standard case, i.e., $z(n)$ must be increasing. In this case,

$$\frac{\partial}{\partial n \partial m} c((z(m) - \xi(n)\Pi(p))/n) < 0$$

Finally, the first-order condition of maximizing (A.22) implies $v_y y'(n) - c_\ell z'(n)/n = 0$. Hence, agent income $z(n)$ is increasing if and only if disposable agent income $y(n)$ is increasing. On the assumption that agent disposable income is increasing, the maximization problem of the public authority can be written as follows

$$\max_{p,u(n),\ell(n)} \int W(u(n))f(n)dn$$

$$s.t. \begin{cases} \int[n\ell(n) - r(p,u(n),\ell(n)) + \xi(n)\Pi(p)]f(n)dn \geq R \\ \int[S(p) - x(p, r(p,u(n),\ell(n)))]f(n)dn = 0 \\ u'(n) - c_\ell(\ell(n) + \xi'(n)\Pi(p))/n = 0, \end{cases}$$

where disposable income is determined by $y(n) = r(p,u,\ell)$ from equation $u(n) = U(p, y(n), \ell(n))$. After integration by parts with the transversality condition $\mu(\underline{n}) = \mu(\overline{n}) = 0$ taken into account, the Lagrangian of the maximization problem can be written as

$$\mathcal{L} = \int \{[W(u(n)) + \lambda(n\ell(n) - r(p,u(n),\ell(n)) + \xi(n)\Pi(p) - R) +$$

$$+ \ \gamma(S(p) - x(p, r(p,u(n),\ell(n))))]f(n) - \mu'(n)u(n) - \mu(n)(c_\ell(\ell(n) + \xi'(n)\Pi(p))/n)\}dn.$$

Given $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_p = -v_p/v_y = x$, the first-order conditions are

$$u(n): \left[W_u - \frac{\lambda + \gamma x_y}{v_y}\right] f(n) - \mu'(n) = 0 \tag{A.24}$$

$$\ell(n): \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y}\right] f(n) - \mu(n)(c_\ell + c_{\ell\ell}(\ell(n) + \xi'(n)\Pi(p)))/n = 0 \tag{A.25}$$

$$p: \int \{[\lambda(-x + \xi(n)\Pi'(p)) + \gamma(S'(p) - x_p - x_y x)]f(n) - \mu(n)\xi'(n)\Pi'(p)\}dn = 0. \tag{A.26}$$

From the individual utility maximization problem, optimal marginal income is determined by $v_y n(1-t) = c_\ell$. Using conditions (A.24)-(A.26), $\Pi'(p) = S(p) = \int x(n)f(n)dn$, $\int \xi(n)f(n)dn = \Xi$, and $1 + c_{\ell\ell}/c_\ell = \frac{1+E^u}{E^c}$, we obtain the following result:

Theorem A2. *In competitive markets with endogenous prices when agent total income is taxed, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{\mu v_y}{\lambda n f}\frac{1+E^u}{E^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu\xi'\Pi(p)c_{\ell\ell}}{\lambda n f c_\ell}, \tag{A.27}$$

where

$$\gamma = \frac{\lambda S(p)(1-\Xi) + S(p)\int \mu(n)\xi'(n)dn}{S'(p) - \int(x_p + x_y x)f(n)dn}. \tag{A.28}$$

Given that $\mu \geq 0$ and $\xi' \geq 0$ the profit redistribution leads to more progressive taxation for top income earners. Intuitively, when wealthy agents realize additional profit income, it becomes more difficult for them to deviate. Hence, it alleviates the incentive compatibility problem and allows for more progressive taxation. We also observe that the price effect is present even if all firm profits are distributed among agents $\Xi = 1$ (and $\xi'(n) \neq 0$).

A.5 Oligopolistic Markets: Firm Profits Distribution

In this section, we analyze optimal income taxation in oligopolistic markets where a part of firm profits can leave the economy $\Xi \leq 1$ and profits are distributed according to some progressive schedule, i.e., $\xi'(n) \geq 0$ for all n.

As a reminder, the market equilibrium condition in oligopolistic markets can be written as

$$J(X, X_p, p) \equiv -X_p \left(p - K'\left(\tfrac{X}{M}\right)\right) + (1+\theta)\tfrac{X}{M} = 0,$$

where θ is the firm's conjectural variation (see p. 17) and $X_p = \int x_p(p, \tilde{y}(n)) f(n) dn$. If we denote the total firm profits as $\Pi(p, X) = pX - MK(\frac{X}{M})$, the public authority's problem can be written as follows.

$$\max_{p, u(n), \ell(n)} \int W(u(n)) f(n) dn$$

$$s.t. \begin{cases} u'(n) - \xi'(n) a(p) \Pi(p, X) - c_\ell \ell(n)/n = 0 & (\mu(n),\ \text{incentive compatibility}) \\ \int [n\ell(n) - r(p, u(n), \ell(n))] f(n) dn + \Xi \Pi(p, X) \geq R & (\lambda,\ \text{resource constraint}) \\ J(X, X_p, p) = 0 & (\gamma,\ \text{oligopolistic market equilibrium}) \\ X - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0 & (\alpha_1,\ \text{market demand}) \\ X_p - \int x_p(p, r(p, u(n), \ell(n))) f(n) dn = 0 & (\alpha_2,\ \text{market demand slope}) \end{cases}$$

where function r determines total agent income $\tilde{y} = r(p, u, \ell) = \frac{u + c(\ell)}{a(p)}$ and the Lagrange multipliers are introduced next to their corresponding constraints. Note that the distribution of firm profits now enters both the incentive compatibility constraints as the agent's utility now depends on $\xi'(n)$ and the resource constraint as only share Ξ of firm profits remains in the economy. The Lagrangian can be written as

$$\begin{aligned} \mathcal{L} = \int & [(W(u(n)) + \lambda(n\ell(n) - r(p, u(n), \ell(n)) + \Xi \Pi(p, X) - R) + \gamma J(X, X_p, p) \\ & + \alpha_1 (X - x(p, r(p, u(n), \ell(n)))) + \alpha_2 (X_p - x_p(p, r(p, u(n), \ell(n)))))f(n) \\ & + \mu(n)(u'(n) - \xi'(n) a(p) \Pi(p, X) - c_\ell(\ell(n))\ell(n)/n)] dn \end{aligned}$$

After integration by parts and with the transversality condition $\mu(\underline{n}) = \mu(\overline{n}) = 0$, $r_u = 1/v_y$,

$r_\ell = c_\ell / v_y$, and $r_p = -v_p/v_y = x$ taken into account, the first-order conditions are

$$u(n) : \left(W_u - \frac{\lambda + \alpha_1 x_y + \alpha_2 x_{py}}{v_y} \right) f(n) - \mu'(n) = 0, \tag{A.29}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y} (\lambda + \alpha_1 x_y + \alpha_2 x_{py}) \right) f(n) - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.30}$$

$$p : -\lambda X(1 - \Xi) + \gamma J_3 - \int (\alpha_1 (x_p + x_y x) + \alpha_2 (x_{pp} + x_{py} x)) f(n) dn \tag{A.31}$$

$$- (a_p \Pi(p, X) + a(p) \Pi_p(p, X)) \int \mu(n) \xi'(n) dn = 0, \tag{A.32}$$

$$X : \alpha_1 + \gamma J_1 + \Pi_X(p, X) \left(\lambda \Xi - a(p) \int \mu(n) \xi'(n) dn \right) = 0, \tag{A.33}$$

$$X_p : \alpha_2 + \gamma J_2 = 0. \tag{A.34}$$

where $J_1 = X_p K''(\frac{X}{M})\frac{1}{M} - \frac{(1+\theta)}{M} < 0$, J_2 equals to the negative of the firm's markup $\Pi_X(p, X) = p - K'(\frac{X}{M})) > 0$, and $J_3 = -X_p > 0$. Condition (A.30) then implies the following result.

Theorem A3. *In oligopolistic markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{1 + E^u}{E^c} \frac{v_y \mu}{\lambda n f} + \frac{\alpha_1}{\lambda} x_y + \frac{\alpha_2}{\lambda} x_{py}.$$

Let us now discuss possible signs of the terms that enter the optimal income taxation formula. As $J_2 < 0$ the sign of α_2 coincides with the sign of γ (see (A.34)). However, the sign of α_1 is not fully determined by the sign of γ because of the third term in (A.33) depends on the distribution of profits in the economy. Term $\Pi_X \left(\lambda \Xi - a(p) \int \mu(n) \xi'(n) dn \right)$ is large and positive when a share of firm profits remaining in the economy is large ($\Xi \approx 1$) and when the distribution of profits is rather flat ($\xi'(n) \approx 0$). In particular, the third term is positive with equal distribution and can be negative with progressive distribution of profits.

To obtain the expression for γ, we incorporate (A.33) and (A.34) into (A.32)

$$\gamma = \left(\lambda X(1 - \Xi) - \lambda \Xi \Pi_X H_p + (a_p \Pi + a(p) \Pi_p + a(p) \Pi_X H_p) \int \mu(n) \xi'(n) dn \right) / J.$$

where $H_p = \int (x_p + x_y x) f(n) dn < 0$ is the derivative of the aggregate compensated demand and $J = J_3 + \int J_1(x_p + x_y x) + J_2(x_{pp} + x_{py} x) f(n) dn$. We note that the denominator is positive $J > 0$ when the second derivative of demand $\int x_{pp} f(n) dn$ is small. The first two terms in the numerator are also positive. However, the sign of the third term cannot be clearly determined because $a_p \Pi + a \Pi_p > 0$ and $a \Pi_X H_p < 0$. Overall, the sign and the magnitude of the tax corrective terms depend on specific profit distribution and on a share of firm profits remaining in the economy.

A.6 Numerical Simulations: Further Results

In this appendix, we provide additional numerical estimations of the size of the price effect on optimal income tax rates. Within the same framework of U.S. housing market, we study the robustness of the price effect to different forms of housing supply and to income distribution.

In Section 3.3 we consider the competitive market with supply function $S = sp^{\varepsilon}$ and price elasticity $\varepsilon = 1.75$ which corresponds to the price elasticity of the average U.S. metropolitan area (Saiz, 2010). However, as also noted earlier, the price elasticity of housing supply widely differs across various countries and regions and, therefore, we reestimate the size of the price effect for the cases of (i) inelastic supply $\varepsilon = 0$ and (ii) elastic supply with $\varepsilon = 3$. The first case better describes housing supply in large U.S. coastal cities (e.g., Boston, San-Francisco) and in countries with a rigid housing planning system, e.g., the UK (see Hilbert and Schoni (2016) and Saiz and Salazar (2018)). In the second case we draw on the estimates of the price elasticity of U.S. housing supply obtained by Green et al. (2005) and Epple and Romer (1991).

	$\varepsilon = 0$	$\varepsilon = 1.75$	$\varepsilon = 3$
Δt	16.3%	4.2%	2.8%

Table 2: The average change in optimal marginal income tax (in percentage points) between endogenous and fixed price regimes for various elasticities of housing supply.

Table 2 reports the average changes in the optimal income tax rate against the benchmark case of fixed prices and equal profit distribution.[43] For the case of inelastic supply $\varepsilon = 0$, we immediately note a massive increase in the size of the price effect compared to the case of the elastic supply, $\varepsilon = 1.75$, considered in Section 3.3. Intuitively, with fixed supply any change in aggregate demand is solely translated into price change, which calls for stronger price corrective measures on the part of the public authority. In contrast, in the case of an elastic supply of housing, $\varepsilon = 3$, we see a reduction in the size of the price effect compared to the case of price elasticity $\varepsilon = 1.75$ as changes in demand lead to smaller changes in price.

Lastly, we also reestimate the price effect for an alternative distribution of agent abilities. The lognormal distribution does not match well the upper tail of the income distribution in the United States. Drawing on Diamond and Saez (2011), we take the top 5% levels of types to follow the Pareto distribution with the shape parameter of 1.5. Specifically, using a standard kernel smoother we construct a composite distribution where types $n < 4.57$ follow the same lognormal distribution as before (with mean 0.368 and standard deviation 0.7) with a 5% atom at $n = 0$ and types $n \geq 4.57$ or the top 5% follow the Pareto distribution (with shape 1.5). We also adjust parameter $s = 0.75$ of supply function $S(p) = sp^{\varepsilon}$, $\varepsilon = 1.75$, in order to match the

[43] In our simulations, we calibrate parameter s of supply function $S(p) = sp^{\varepsilon}$ in order to match the average expenditure share of housing of 25%. In particular, we have $s = 0.56$ for $\varepsilon = 0$, $s = 0.65$ for $\varepsilon = 1.75$, and $s = 0.75$ for $\varepsilon = 3$.

average expenditure share of housing of 25%.

Figure 3 presents our findings. Similarly to Saez (2001), the effect of the Pareto distribution is an increase in income tax rates for top income levels. The change in marginal tax rates between endogenous and fixed price economies (the price effect) becomes smaller for top income levels. This change barely influences, however, the average change in marginal income tax rate that now becomes equal 4.1 percentage points.

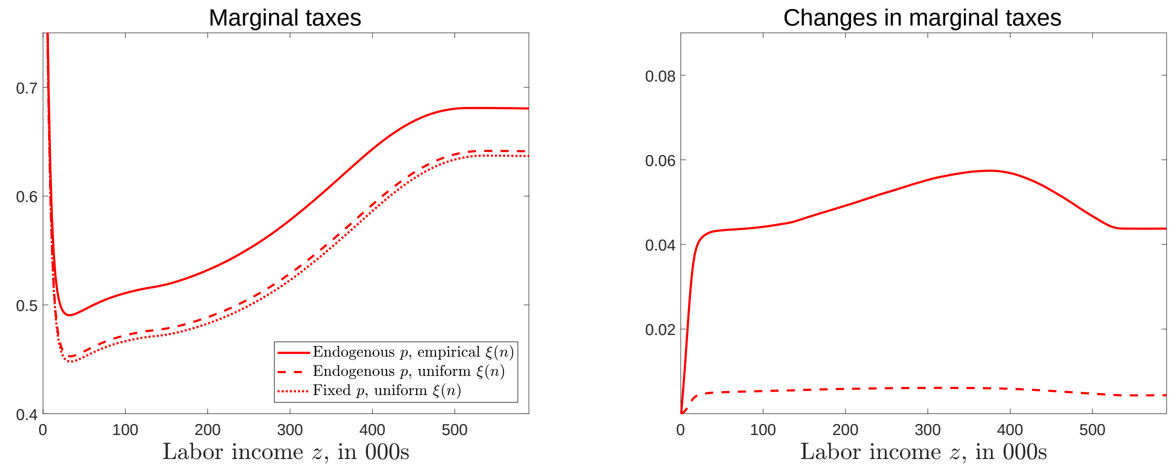

Figure 3. Optimal income taxation with lognormal-Pareto distribution of productivities.

Note: The left-hand diagram depicts the optimal marginal income tax rates for the economy with competitive housing market and (i) fixed price, equal profit distribution (dotted line); (ii) endogenous price, uniform profit distribution (dashed line); and (iii) endogenous price, empirical profit distribution (solid line). The right-hand diagram depicts the changes in tax rates for cases (ii) and (iii) in comparison with benchmark case (i). In case (i), we take the price of housing equal to the equilibrium price of case (iii).

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