The role of trade credit and bankruptcy in business fluctuations*

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Abstract

Although trade credit is the most important source of short-term firms' credit, the determinants of delinquency on trade credit, its interactions with bankruptcy, and its consequences for macroeconomic variables are not well understood. We build a quantitative general equilibrium heterogeneous firms model to assess the contribution of trade credit delinquency to observed fluctuations in bankruptcies and employment. In the model, an intermediate input is purchased by final goods producers partly on trade credit before observing the realisation of their productivity. A bad productivity shock may ex-post induce final good producers to skip payment to suppliers or, alternatively, liquidate via bankruptcy. The aggregate delinquency is taken into account by input suppliers; the individual liquidation risk is priced in by lenders supplying bank credit. We characterise firm-level patterns of trade credit delinquency, focusing on its association with bankruptcy risk, indebtedness and size. Within a firms dynamics, delinquency most often precedes, though it is not necessary for, bankruptcy. Delinquency may mitigate bankruptcies as it provides an alternative option for dealing with financial constraints. The response of delinquency provides an amplification mechanism to the effect of aggregate shocks.

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1 Introduction

Trade credit is a significant source of firm short-term finance, probably the most important source of short-term credit in the U.S. (e.g., Petersen and Rajan (1997)). A large proportion of contracts between suppliers and retailers specify the payment schedule, i.e., the time by which the firms purchasing inputs must pay their suppliers (35, 45 or more days). In the U.S., accounts payable to creditors represent about 15% of assets while debt is about 7% (Rajan and Zingales (1995)). In Europe, the picture is similar with cases where total amount of trade credit can be three times as large as the alternative sources of debt, like in Portugal (Giannetti (2003)).[1] Delinquency on trade credit is sizable. Jacobson and von Schedvin (2015) report that in Sweden it is between 10% and 15%.

There is recent compelling evidence (e.g. Jacobson and von Schedvin (2015)) that trade credit delinquency is a channel of transmission of aggregate fluctuations as it imposes losses on other firms, and interacts with bankruptcy and the liquidation of firms. There is an empirical association between bankruptcy and delinquency for Sweden between 1992 and 2011. Although this evidence shows that firm level trade credit delinquency might play an important role in the transmission of aggregate fluctuations, not much is known about its quantitative role in the propagation of aggregate shocks.

In this paper we set out to investigate quantitatively the determinants of trade credit delinquency, its interactions with bankruptcy, and its consequences for macroeconomic variables and the business cycle. We seek to assess quantitatively the contribution of trade-credit delinquency to observed fluctuations in GDP and employment, and also its association with firm bankruptcy risk and external financial conditions for firms, including spreads and credit constraints. Our primary focus will be on the U.S. economy as a benchmark. How important have been trade credit and repayment problems in recent recessions and recoveries?

Alongside these aggregate aspects, we also pursues characterising firm-level patterns of trade credit delinquency, focusing on its association with bankruptcy risk, indebtedness and size within the distribution of firms. The aim is to uncover testable empirical implications at the individual firm level. How does the incidence of trade credit delinquency and bankruptcy vary by firm size, productivity and levels of bank debt? How does the scope for defaulting on trade credit affect the number and type of firms that file for bankruptcy and liquidate?

In order to pursue these objectives, we build a quantitative general-equilibrium heterogeneous-

[1] For a useful survey on trade credit, see Cuñat and Garcia-Appendini (2012).

firms model. This model contains the elements that seem essential to capture the mechanisms at work based on previous empirical work. The model contains four types of agents: a representative intermediate-good producer that uses labor to produce and intermediate input; heterogeneous final-good producers use the input from intermediate suppliers to produce a final good; households act as consumers, shareholders, investors and workers; lenders/banks take deposits from households and lend to final-goods firms. Some proportion of inputs are purchased on within-period credit (inputs are delivered at the beginning and paid at the end after shocks are realised). Final-good firms' may also hold non-contingent bank debt. Existing final-good firms cannot issue new shares so dividends must be non-negative; we also assume non-positive dividends if there is bankruptcy or trade-credit delinquency. Trade credit debt is junior to bank credit debt.

Bankruptcy by a final-good firm means liquidation of the firm and exit. Trade-credit delinquency implies a loss of future output to the firm over a random number of periods. Final-good firms are subject to aggregate and idiosyncratic productivity shocks. These shocks are observed only after inputs are purchased. Because of this timing, final-goods producers might be unable to honor their financial obligations (i.e., they become liquidity constrained). The aggregate delinquency rate on trade-credit payments is factored in intermediate-producer's pricing of inputs. On the other hand, individual bankruptcy risk is reflected in the lending rates offered by lenders.

We provide some analytical results regarding final-good firm's outcomes within each of the possible discrete repayment options. But the main equilibrium implications will have to be studied numerically. To this end, parameter values are chosen so the model matches a number of data observations regarding macroeconomic aggregates, bankruptcy, firm indebtedness, the level of trade credit, and standard macroeconomic targets for the U.S. A central target is the proportion of sales lost to delinquency, of which direct measures may be hard to come by.

We have obtained preliminary results for the stationary equilibrium under reasonable parameterizations. The model is able to deliver levels of trade-credit delinquency and bankruptcy comparable with the data. In the individual firm's dynamics, delinquency most often precedes bankruptcy, although bankruptcy often happens without delinquency and, on the other hand, delinquency need not lead to bankruptcy. Overall, firms that are delinquent today are less likely to become bankrupt in the future than firms who currently meet all payments, but they are much more likely to remain delinquent in the future.

The model contains core macroeconomic interactions between final-good producers, input producers, and lenders. Delinquency by final-good firms imposes losses, directly on intermediate-input firms and indirectly to the other final-good firms via the subsequent increase in input prices. Thus trade-credit delinquency may amplify recessions and fuel further bankruptcies. Bankruptcies do also affect delinquency as credit constraints become tighter. On the other hand, however, delinquency may mitigate bankruptcies as it provides an alternative option to deal with tight financial constraints. These mechanisms do also interact meaningfully with the other variables in general equilibrium, including employment, wages and GDP. Our preliminary results show a role of delinquency in the amplification of aggregate shocks.

In the present preliminary draft, in order to focus on the basic core mechanisms, the quantitative analysis abstracts from the interaction with households in general equilibrium. These interactions are being studied at the time of writing.

The next Section 2 places the contribution of this paper within the literature. Section 3 sets out the model. Section 4 expresses the decision problems and equilibrium conditions as a recursive equilibrium, and presents some useful characterisation results. (Rewrite:) Then we specify and study versions of the model in increasing level of generality. Some preliminary results clearly indicate this model provides a reasonable setting to study the question at hand. Specific numerical outcomes are not included in this draft yet.

2 Related literature

Most closely related to the present paper is the recent analysis of firm bankruptcy in the quantitative macroeconomics literature and, in particular, the three papers Arellano, Bai, and Kehoe (2016), Khan, Senga, and Thomas (2014), and Corbae and D'Erasmo (2017). The questions these papers address concern firm bankruptcy and also cyclical implications of different types of shocks, including volatility shocks. Our analysis deals with similar variables but, in addition, introduces trade credit and the decision regarding default on this trade credit. Our modelling strategy, while distinctive, shares various aspects with the papers cited since it belongs in the same class of general equilibrium models with idiosyncratic shocks and heterogeneous firms. At a more specific level, like in Arellano, Bai, and Kehoe (2016), we introduce an information friction since key investment and input choices are made before the realization of productivity shocks. Hence hiring inputs to produce output is a risky endeavor if idiosyncratic shocks occur between the time of production and the receipt of payments. This risk has real consequences when firms cannot meet their financial obligations and must experience a costly bankruptcy. Our paper also shares features with the paper of Khan, Senga, and Thomas (2014). Firms' borrowing limits will depend on firm-level and aggregate state variables and arise from forward looking schedules for debt prices. Corbae and D'Erasmo (2017) model will also represent a benchmark to compare our analysis to, where the focus in on the fine details of the bankruptcy code. As already indicated earlier, our modelling innovation is in incorporating trade credit into this type of models.

A different strand of literature consists of works on credit chains pioneered by Kiyotaki, Moore et al. (1997). In their work they show how shocks propagate through chains of firms borrowing and lending to each other and argue that temporary small shocks to the liquidity of one of the firms in the chain can generate large aggregate fluctuations. Boissay (2006) studies financial contagion through trade credit in a one-period model and shows that sound firms might become insolvent when customers fail to honor their trade debts. Boissay and Gropp (2013) estimate the extent to which credit constrained firms pass on adverse liquidity shocks they face by defaulting on their suppliers. Altinoglu (2018) presents a one-period input-output model and argues linkages between firms in propagating liquidity shocks. He estimates such shocks to US industrial production industries between 1997 and 2013. All these are either theoretical partial equilibrium analysis or empirical works. On the other hand, the theory of granularity in economics (e.g., Gabaix (2011)) offers a different explanation of the role of idiosyncratic shocks in explaining business cycles and hence is somehow related to the present paper.

We take the existence of trade credit as given. There is however a literature explaining why trade credit arises. Petersen and Rajan (1997) are the first to test the different explanations that have been put forward on why firms resort to trade credit. They find evidence suggesting that the main reason for recurring to trade credit is unavailability of other forms of credit. They also find that a lower asymmetric information between a supplier (lender) and a client (borrower) constitutes an advantage in extending credit. Similarly, asset liquidation is more efficient in that case, and extending trade credit is equivalent for suppliers to having an equity stake in their clients. There is also a literature on the importance of the capital structure and evidence of distress through trade credit. Rajan and Zingales (1995), Cunat (2006), Cuñat and Garcia-Appendini (2012), and García-Appendini and Montoriol-Garriga (2015) belong in this strand of work.

3 Model

The model consists of four types of agents: producers of intermediate inputs, producers of final goods, financial intermediaries, and a representative households. Firms are all competitive and inputs and goods are homogeneous. Final-good firms face financial constraints on issuance of shares, and experience idiosyncratic shocks. They pay for inputs with some delay, only after shocks are realised. Since there is no commitment to this within-period trade credit, these firms may fail to pay suppliers of inputs. Final-goods firms can also borrow from banks, and may declare bankruptcy and liquidate.

3.1 Input producers

There is a continuum of intermediate producers with mass 1. The intermediate input x is produced from labour n on a one-for-one basis:

$$x = n.$$

The price of the input is p. Payments from retail customers are received with a delay. A proportion of sales τ is on trade credit and a proportion of those, given by the trade-credit default rate θ, will not receive payment. The remaining fraction of sales $1 - \tau$ are on cash and receives payment, except for a proportion η of the value of those cash sales to final good firms who fail in the sense that they are liquidated *and*, additionally, have no residual output. A retail firm might liquidate but carry out production in the current period, in which case cash sales always receive payment.[2] We are assuming that the representative intermediate producer cannot see the individual types and takes as given the market default rate θ and failure rate η by pooling all the individual retailers.

The cash-flow to the representative firm includes the costs of labour at the wage rate w, and it becomes

$$px - \theta\tau px - \eta(1 - \tau)px - wx.$$

In this description, trade credit does not command an explicit interest rate or discount price (as it would be if the firm could issue financial contracts).

[2] Cash sales in this paper are therefore not strictly cash. The idea is that they have a high degree of compliance compared to other liabilities. They are honoured except in very catastrophic cases. Specifically, cash sales will be senior to bank debt. An alternative model would explicitly have cash payments at the beginning of the period funded through debt or savings, but this would call for an additional individual state variable for the retail firm.

There is free entry in the input sector. The rates of default on trade credit sales θ and of failure on cash sales η will be stochastic, and their realization unknown at entry. Free entry thus implies zero profits in ex-ante, or expected, terms given the information available at the beginning of the period:

$$E[px - \theta\tau px - \eta(1-\tau)px - wx] = 0.$$

It follows that ex-post aggregate net profits from this sector can be positive or negative. Households, who own the firms, will absorb these rents.

(We could specify the timing of trade credit within the period, including the firm using an overdraft to pay workers in the meantime.)

3.2 Final-goods producers

Output from a firm i depends on aggregate productivity z, idiosyncratic productivity ϵ, and a purchased intermediate input x:

$$y = ze^{\epsilon}F(x),$$

where $F(x) = x^{\gamma}$ with $\gamma \leq 1$. (Note, no capital or direct labour.) We consider two models for shocks. In one case, the two productivity components follow independent Markov chains: $\epsilon \sim \psi_{\epsilon}(\epsilon'|\epsilon)$ and $z \sim \psi_z(z'|z)$. In the other case, the idiosyncratic components's dispersion σ is stochastic, and aggregate productivity is constant: $\epsilon \sim \psi_{\epsilon}(\epsilon'|\epsilon, \sigma)$, $\sigma \sim \psi_{\sigma}(\sigma'|\sigma)$, and $z = 1$.

There is an entry cost ξ paid initially by household/shareholder. Then the firm draws a realisation of the initial idiosyncratic shock ϵ_{-1} from an initial distribution $\overline{\psi}_{\epsilon}(\epsilon_{-1})$.

There is a fixed cost of operating the firm c_F in every period.

In any given period, the firm chooses the amount of intermediate input x before the realisation of the shocks. After the shocks are observed, the firm can issue one-period debt b at a discount price q, decide whether to repay input suppliers by choosing $d^x \in \{0, 1\}$, and whether to declare bankruptcy on debt $d^b \in \{0, 1\}$. Delinquency on input payments $d^x = 1$ implies a loss of a proportion of output $\tilde{\nu} > 0$ in future periods. The delinquency indicator is $\nu \in \{0, \tilde{\nu}\}$. If $\nu > 0$ the probability of forgiveness, conditional on not incurring further delinquency, is λ. Bankruptcy $d^b = 1$ leads to liquidation and exit. Under bankruptcy or delinquency, claimants (i.e., creditors or trade-credit suppliers) receive the residual value of the firm, r^b goes to banks and r^x goes to trade-credit suppliers. Because of the fixed cost

c_F and the cash input payments $(1-\tau)px$, the residual value may be negative in which case the firm does not pay the fixed cost nor cash supplies, and fails to operate so no residual is available.

The firm maximises the expected discounted value of dividends. The discount rate is ρ, which will be determined in equilibrium. A firm faces the financial constraint that it cannot issue new shares so dividends cannot be negative. A firm also faces a non-positive dividends constraint when deciding to default or to become delinquent. Finally, debt has seniority over trade credit sales, and cash sales have seniority over debt Hence we have the following institutional constraints:

Assumption 1: Firms cannot pay negative dividends.

Assumption 2: Firms cannot pay positive dividends when defaulting or becoming delinquent. (Therefore claimants receive the residual.)

Assumption 3: Bank debt is senior to trade credit sales; cash sales are senior to bank debt.

3.3 Lenders

Lenders issue one-period loans to final goods firms. They have the same information that is available to firms, so there is a contract for each type of loan in terms of size and characteristics of the firm. Competition drives the surplus for lenders on all loan types to zero. In this way, the discount prices of debt q reflects the default risk—implied by firm's decision d^b—over the market stochastic discount rate ρ:

$$q = \rho(1 - \text{default probability}).$$

These lenders fund their lending by selling state-contingent Arrow securities to households. There is a full set of securities against each aggregate productivity state z'.

3.4 Households

Households own the firms. There is a representative household who can borrow and lend freely contingent claims. The stochastic subjective discount rate will determine the firms' and banks' discount rate, ρ. Households supply labour optimally before the realisation of shocks.

4 Recursive equilibrium

The aggregate state at the beginning of a period, S, includes the past aggregate productivity shock, z_{-1} or σ_{-1}, the distribution μ of firms over (b, ν, ϵ_{-1}) and the vector of contingent claims held by households, A:

$$S = (z_{-1}, \mu, A).$$

The input price and wage rate can be written as $p(S)$ and $w(S)$.

The final-good firm's individual state before shocks are realised consists of (ϵ_{-1}, b, ν). After shocks are realised the individual state becomes (ϵ, b, ν, x), which includes the level of input x chosen previously.

The price of contingent claims against future states z' can be written $Q(z'|S, z)$.

After the shock z is observed, the discount rate of the firm can be written $\rho(S, z)$.

The discount price of debt is a function $q^{ND}(b', \epsilon, \nu|S, z)$ if there is no delinquency, and $q^x(b', \epsilon|S, z)$ if there is delinquency.

There is an equilibrium law of motion for the distribution measure and portfolio of claims such that

$$\mu' = H^{\mu}(S, z), \quad A' = H^A(S, z).$$

An equilibrium satisfies: decision rules maximise final-good firm's objective given debt prices and input prices; prices of inputs are such that intermediate producers make ex-ante zero profits given aggregate delinquency and wages; the wage and mass of firms is such that labour market clears and there is free entry; prices of loans satisfy zero profit for lenders given the decision rules of final-good firms; entry and exit flows reflect optimal decisions.

4.1 Decision problem of final-good firms

There are two stages. Denote by $V(\epsilon, b, \nu, x|S, z)$ the value function at the second stage, after the realisation of shocks, and by $W(\epsilon_{-1}, b, \nu|S)$ the value at the first stage before the shocks are observed.

In the second stage, the decision needs to evaluate the value from 3 different courses of action: $V^{ND}(\epsilon, b, \nu, x|S, z)$ if inputs and debts receive payment; $V^x(\epsilon, b, \nu, x|S, z)$ if debt receives payment but trade-credit inputs do not; $V^b(\epsilon, b, \nu, x|S, z)$ if neither trade-credit inputs nor debts receive payment. The values in each case are as follows:

Repayment. When honoring all payments

$$V^{ND}(\epsilon, b, \nu, x|S, z) = \max_{b'} \Big\{(1-\nu)ze^{\epsilon}F(x) - c_F - p(S)x + q^{ND}(b', \epsilon, \nu|S, z)b' - b + \\ \rho(S, z)[\mathcal{I}_{\nu>0}(\lambda W(\epsilon, b', \nu' = 0|S') + (1-\lambda)W(\epsilon, b', \nu' = \tilde{\nu}|S')) \\ + \mathcal{I}_{\nu=0}W(\epsilon, b', \nu' = 0|S')]\Big\}, \quad (1)$$

with $S' = H(S, z)$, which gives $b' = g^{ND}(\epsilon, b, \nu, x|S, z)$, and the value of dividends

$$\pi^{ND}(\epsilon, b, \nu, x|S, z) = (1-\nu)ze^{\epsilon}F(x) - c_F - p(S)x + \\ q^{ND}(g^{ND}(\epsilon, b, \nu, x|S, z), \epsilon, \nu|S, z)g^{ND}(\epsilon, b, \nu, x|S, z) - b. \quad (2)$$

Delinquency. When repudiating payments for trade-credit input supplies

$$V^{x}(\epsilon, b, \nu, x|S, z) = \max_{b', r^x \geq 0} \Big\{(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px - b + q^{x}(b', \epsilon|S, z)b' - r^x \\ + \rho(S, z)W(\epsilon, b', \nu' = \tilde{\nu}|S')\Big\}, \quad (3)$$

subject to dividends being both non-negative (Assumption 1) and non-positive (Assumption 2) so

$$(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px - b + q^{x}(b', \epsilon|S, z)b' - r^x = 0.$$

The solution gives $b' = g^{x}(\epsilon, b, \nu, x|S, z)$, and $r^x(\epsilon, b, \nu, x|S, z)$, and the value of dividends

$$\pi^{x}(\epsilon, b, \nu, x|S, z) = (1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px - b \\ + q^{x}(g^{x}(\epsilon, b, \nu, x|S, z), \epsilon|S, z)g^{x}(\epsilon, b, \nu, x|S, z) - r^{x}(\epsilon, b, \nu, x|S, z), \quad (4)$$

which, by assumption, must be zero so $\pi^{x}(\epsilon, b, \nu, x|S, z) = 0$.[3]

Bankruptcy. Finally, defaulting on debts means liquidation so

$$V^{b}(\epsilon, b, \nu, x|S, z) = \max\{0, (1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px - r^b\}, \quad (5)$$

[3]But might be positive in some off-equilibrium situation due to the discontinuity in the value of funds raised via borrowing. We can disregard this for now.

with the constraint, again from A1 and A2, that

$$(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px - r^b = 0.$$

This results in dividends $\pi^b(\epsilon, b, \nu, x|S, z) = 0$, and the residual value

$$r^b(\epsilon, b, \nu, x|S, z) = \max\{(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px, 0\}$$

being recovered by creditors, and $r^x = 0$ so trade-credit supplies recover nothing. (Here we are using the property proven below that default necessarily implies delinquency on suppliers.) In the event that $(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px < 0$, cash in hand is negative and the firm's output cannot cover the fixed cost and cash input and therefore cannot operate in the current period. In this case, in addition to trade-credit input and debt, also cash inputs fail to receive the payments $(1-\tau)px$. We represent this firm failure outcome by the indicator $d^f(\epsilon, b, \nu, x|S, z) = 1$. When otherwise cash in hand is positive $d^f(\epsilon, b, \nu, x|S, z) = 0$.

The optimal choice among the three options solves

$$V(\epsilon, b, \nu, x|S, z) = \max\left\{V^{ND}(\epsilon, b, \nu, x|S, z), V^x(\epsilon, b, \nu, x|S, z), V^b(\epsilon, b, \nu, x|S, z)\right\}, \tag{6}$$

and gives decision rules $d^x(\epsilon, b, \nu, x|S, z)$ and $d^b(\epsilon, b, \nu, x|S, z)$.

Turning now to the first stage, the optimal choice of input solves

$$W(\epsilon_{-1}, b, \nu|S) = \max_x \sum_{\epsilon, z} \psi_z(z|z_{-1})\psi_{\epsilon}(\epsilon|\epsilon_{-1})V(\epsilon, b, \nu, x|S, z), \tag{7}$$

yielding $x = x(\epsilon_{-1}, b, \nu|S)$.

4.2 Discount

After the shocks, firms discount future values expected before the realisation of future shocks. The appropriate rate for discounting is thus based on a risk-free portfolio so

$$\rho(S, z) = \sum_{z'} Q(z'|S, z)\psi_z(z'|z). \tag{8}$$

4.3 Lenders and debt prices

Lenders use firm's decision and shock transition probabilities to make projections about the probability of default. They also take into account that they recover the residual value of the firm in case of default.

The price of debt when there is no delinquency today can be written

$$q^{ND}(b', \epsilon, \nu | S, z) = \sum_{z'} \psi_z(z'|z) Q(z'|S, z)(1 - Ed^{b,ND}(b', \epsilon, \nu | S', z')), \tag{9}$$

where the components in S' are given by

$$\mu' = H^{\mu}(S, z), \quad A' = H^{A}(S, z),$$

and $Ed^{b,ND}(.)$ denotes the forecast of default losses or expected default for a given future aggregate state, which depends on the default decision $d^b(\cdot)$ and the recovery $r^b(\cdot)$ expressed as a rate. We define this expected recovery rate as

$$rec^b(\epsilon', b', \nu' \mid S', z') \equiv \frac{r^b(\epsilon', b', \nu', x(\epsilon, b', \nu'|S') \mid S', z')}{b'}.$$

Therefore the expected default $Ed^{b,ND}(.)$ in Eq. (9) can be written

$$\begin{aligned}
&Ed^{b,ND}(b', \epsilon, \nu | S', z') \equiv \\
&\quad \mathcal{I}_{\nu>0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ (1-\lambda) d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S', z')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S', z')) \\
&\qquad\qquad + \lambda d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S', z')(1 - rec^b(\epsilon', b', 0 \mid S', z')) \Big\} \\
&\qquad\quad + \mathcal{I}_{\nu=0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S', z')(1 - rec^b(\epsilon', b', 0 \mid S', z')) \Big\}
\end{aligned}$$

The price of debt when there is delinquency today can be written

$$q^x(b', \epsilon | S, z) = \sum_{z'} \psi_z(z'|z) Q(z'|S, z)(1 - Ed^{b,Dx}(b', \epsilon | S', z')), \tag{10}$$

where $S' = H(S, z)$ and $Ed^{b,Dx}$ is the conditional expected default loss

$$Ed^{b,Dx}(b', \epsilon | S', z') = \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S', z')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S', z')) \Big\}$$

where the components in S' are given by

$$\mu' = H^{\mu}(S,z), \quad A' = H^{A}(S,z).$$

4.4 Consumers

At the beginning of a period, the state for the representative consumer is (a, S), where $S = (z_{-1}, \mu, A)$ and a is the individual portfolio of state-contingent securities. Given the timing, the decision problem can be broken down into two stages: choice of labour supply l given (a, S); choice of portfolio $a'(z')$ given $(l, a(z) \mid S, z)$. Proceeding backwards, the choice of portfolio $a'(z')(l, a \mid S, z)$ and consumption $c(l, a \mid S, z)$ solve

$$U(l, a(z) \mid S, z) = \max_{\{a'(z')\}} \left\{ u(c,l) + \beta J(a', S') \right\} \tag{11}$$

subject to

$$c + \sum_{z'} Q(z' \mid S, z) a'(z') = w(S) l + a(z) + \Pi(S,z),$$

where $\Pi(S, z)$ is dividends paid, and the components of S' obey

$$\mu' = H^{\mu}(S,z), \quad A' = H^{A}(S,z).$$

The labour supply policy $l(a, S)$ solves

$$J(a,S) = \max_{l} \sum_{z} \psi_z(z \mid z_{-1}) U(l, a \mid S, z).$$

Note that $S' = (z, \mu', A')$ evolves deterministically given the information at the end of the current period when the consumer makes the portfolio decision. Therefore, a set of securities contingent on only the realisation of z', not including S', is appropriate.

The first-order condition for the portfolio decision, for a particular z', implies

$$u_c(c,l) Q(z' \mid S, z) = \beta \psi_z(z' \mid z) u_c(c'(a'(z'), S, z), l'(a'(z'), S, z))$$

with, using some liberal notation, $c'(a'(z'), S, z) \equiv c(l(a'(z'), S'), a'(z') \mid S', z')$ and $l'(a'(z'), S, z) \equiv l(a'(z'), S')$, and, in S', $\mu' = H^{\mu}(S, z)$ and $A' = H^{A}(S, z)$. This means that, for any two contingent states z_i' and z_j', it must be true that

$$\psi_z(z_i' \mid z)\frac{u_c(c'(a'(z_i'), S, z), l'(a'(z_i'), S, z))}{Q(z_i' \mid S, z)} = \psi_z(z_j' \mid z)\frac{u_c(c'(a'(z_j'), S, z), l'(a'(z_j'), S, z))}{Q(z_j' \mid S, z)} \quad (12)$$

Expression (12) is a no-arbitrage condition.

4.5 Aggregate variables

Here we specify endogenous variables that obtain from aggregating individual decision rules.

4.5.1 Delinquency rate

Producers of intermediate inputs take as given the expected aggregate delinquency rate on trade credit, or fraction of sales on trade credit lost, θ given the initial state S. The delinquency rate results from aggregating up the individual firm's delinquency decisions $d^x(\epsilon, b, \nu, x|S, z)$ and bankruptcy $d^b(\epsilon, b, \nu, x|S, z)$, given that their choice of inputs is determined by $x = x(\epsilon_{-1}, b, \nu|S)$. The delinquency rate also depends on the recovery from the delinquent firms' cash on hand left after repaying debts and cash inputs, which we have defined as $r^x(\epsilon, b, x|S, z)$. Specifically,

$$\begin{aligned}\theta(S) = \Big[\sum_z \psi_z(z \mid z_{-1}) \int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1})\Big(& d^x(\cdot)(\tau p(S)x(\cdot) - r^x(\cdot)) \\ &+ (1 - d^x(\cdot))d^b(\cdot)\tau p(S)x(\cdot)\Big)\mu(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)\Big] \\ &/\Big[\sum_z \psi_z(z \mid z_{-1}) \int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1})\tau p(S)x(\cdot)\mu(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)\Big] \quad (13)\end{aligned}$$

where, for convenience, we are using the shorthand notation $r^x(\cdot) \equiv r^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S, z)$, $d^x(\cdot) \equiv d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S, z)$, $d^b(\cdot) \equiv d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S, z)$, and $x(\cdot) \equiv x(\epsilon_{-1}, b, \nu|S)$.

4.5.2 Failure rate

The input producer also takes as given the rate at which payments on cash sales may fail to materialise because of firm failure, $\eta(S)$. This type of failure occurs when final-good firms declare bankruptcy but current revenues would not cover the fixed cost and payments to cash inputs thus rendering the residual negative. In this case the firm stops producing and even cash sales receive no payment. Specifically,

$$\begin{aligned}\eta(S) = \Big[\sum_z \psi_z(z \mid z_{-1}) \int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \\ (1 - d^x(\cdot))d^b(\cdot)d^f(\cdot)(1-\tau)p(S)x(\cdot)\mu(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)\Big] \\ / \Big[\sum_z \psi_z(z \mid z_{-1}) \int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \\ (1-\tau)p(S)x(\cdot)\mu(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)\Big] \quad (14)\end{aligned}$$

where $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S, z)$.

4.5.3 Dividends

Given the firms' dividend policies from (2) and (4), the aggregate dividend received by the household is given by

$$\begin{aligned}\Pi(S,z) = \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \Big[\int \mathcal{I}_{d^b(\ldots)=0, d^x(\ldots)=0} \pi^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z)\mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\ + \int \mathcal{I}_{d^b(\ldots)=0, d^x(\ldots)=1} \pi^x(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z)\mathrm{d}\mu(\epsilon_{-1}, b, \nu)\Big] - \xi M(S) \quad (15)\end{aligned}$$

4.5.4 Motion for the distribution

The distribution μ is defined over the ex-ante firm types (ϵ_{-1}, b, ν). It evolves according to $\mu' = H^\mu(S,z)$ where, as defined earlier, $S = (z_{-1}, \mu, A)$. We define the transition probabilities for existing firms $\mathrm{Prob}(\epsilon, \mathcal{B}', \nu'; b, \nu, \epsilon_{-1} \mid S, z)$, where $\mathcal{B}$ is a set containing elements b, and for entrants $\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1} \mid S, z)$. The transition function is

$$H^\mu(\epsilon, \mathcal{B}', \nu' \mid S, z) = \int \text{Prob}(\epsilon, \mathcal{B}', \nu'; b, \nu, \epsilon_{-1} \mid S, z) \mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\ + M(S) \int \text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1} \mid S, z) \mathrm{d}\mu^E(\epsilon_{-1}), \quad (16)$$

where $\mu^E(\epsilon_{-1})$ is the distribution of productivity for new entrants $\overline{\psi}_\epsilon(\epsilon_{-1})$.

The transition probabilities are given by the firms optimal decisions and the process for the delinquency flag. For existing firms

$$\text{Prob}(\epsilon, \mathcal{B}', \nu'; b, \nu, \epsilon_{-1} \mid S, z) = \begin{cases} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; \nu' = 0, \ \nu = 0 \\ \lambda\psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; \nu' = 0, \ \nu = \tilde{\nu} \\ (1-\lambda)\psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; \nu' = \tilde{\nu}, \ \nu = \tilde{\nu} \\ \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^x(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) \in \mathcal{B}' \\ & \quad\; d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 0 \\ & \quad\; d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z) = 1 \\ & \quad\; \nu' = \tilde{\nu} \\ 0 & \quad\quad\;\; \text{otherwise} \end{cases} \quad (17)$$

For new entrants, for whom $b = 0$ and $\nu = 0$,

$$\mathrm{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1} \mid S, z) = \begin{cases} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^{ND}(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S, z) \in \mathcal{B}' \\ & \quad\;\; d^x(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S, z) = 0 \\ & \quad\;\; \nu' = 0 \\ \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^x(\epsilon, b, x(\epsilon_{-1}, b, 0 \mid S) \mid S, z) \in \mathcal{B}' \\ & \quad\;\; d^x(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S, z) = 1 \\ & \quad\;\; \nu' = \tilde{\nu} \\ 0 & \quad\;\; \text{otherwise} \end{cases} \tag{18}$$

4.5.5 Motion for the portfolio

Given the individual policy functions $a'(z')(l(A, S), A \mid S, z)$ for all z' from (11), the transition function for the aggregate portfolio A' is

$$H^A(z' \mid S, z) = a'(z')(l(A, S), A \mid S, z) \tag{19}$$

4.6 Input producers

Given $\theta(S)$, $\eta(S)$, and $w(S)$, zero profits means that the price of inputs satisfies

$$p(S) = \frac{w(S)}{1 - \theta(S)\tau - \eta(S)(1 - \tau)}. \tag{20}$$

4.7 Entry and exit

The value of a new entrant $W^E(S)$ is the expectation of $W(.)$ over the unconditional distribution of the starting ϵ_{-1}:

$$W^E(S) = \sum_{\epsilon_{-1}} \overline{\psi}_\epsilon(\epsilon_{-1}) W(\epsilon_{-1}, b = 0, \nu = 0 | S). \tag{21}$$

The free-entry condition is

$$W^E(S) \leq \xi, \tag{22}$$

with strict inequality only when there is zero entry $M = 0$.

4.8 Market clearing

Labour market:

$$l(S) = \int x(\epsilon_{-1}, b, \nu, \mid S)\mathrm{d}\mu(\epsilon_{-1}, b, \nu) + M(S)\int x(\epsilon_{-1}, b = 0, \nu = 0 \mid S)\mathrm{d}\mu^E(\epsilon_{-1}) \quad (23)$$

Asset markets: Total household savings derived from $a'(\cdot)$ and Q, and total loans to firms derived from g^{ND}, g^x and x from $\mathcal{G}$, M, q^{ND} and q^x, are equal:

$$\begin{aligned}
&\sum_{z'} Q(z' \mid S, z)a'(z')(l(a, S), a \mid S, z) = \sum_{\epsilon}\psi_\epsilon(\epsilon \mid \epsilon_{-1}) \\
&\Big[\int \mathcal{I}_{d^b(\ldots)=0, d^x(\ldots)=0}\, g^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z)q^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z)\mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\
&+ \int \mathcal{I}_{d^b(\ldots)=0, d^x(\ldots)=1}\, g^x(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z)q^x(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S, z)\mathrm{d}\mu(\epsilon_{-1}, b, \nu)\Big] \\
&\qquad + M(S)\sum_{\epsilon}\psi_\epsilon(\epsilon \mid \epsilon_{-1}) \\
&\Big[\int \mathcal{I}_{d^x(\ldots)=0}\, g^{ND}(\epsilon, 0, 0, x(\epsilon_{-1}, 0, 0 \mid S) \mid S, z)q^{ND}(\epsilon, 0, 0, x(\epsilon_{-1}, 0, 0 \mid S) \mid S, z)\mathrm{d}\mu^E(\epsilon_{-1}) \\
&+ \int \mathcal{I}_{d^x(\ldots)=1}\, g^x(\epsilon, 0, x(\epsilon_{-1}, 0, 0 \mid S) \mid S, z)q^x(\epsilon, 0, x(\epsilon_{-1}, 0, 0 \mid S) \mid S, z)\mathrm{d}\mu^E(\epsilon_{-1})\Big] \quad (24)
\end{aligned}$$

Final goods:

$$\begin{aligned}
&c(l(a, S), a \mid S, z) + \xi M(S) = \\
&\sum_{\epsilon}\Big[\int \psi_\epsilon(\epsilon \mid \epsilon_{-1})((1-\nu)z\epsilon F(x(\epsilon_{-1}, b, \nu|S)) - c_F)(1 - d^f(\cdot))\mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\
&\qquad + M(S)\int (z\epsilon F(x(\epsilon_{-1}, 0, 0|S)) - c_F)\mathrm{d}\mu^E(\epsilon_{-1})\Big] \quad (25)
\end{aligned}$$

The two terms on the right correspond to existing and new entrants, respectively. The possibility of firm failure is captured by the failure indicator $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ which is 1 when the firm declares bankruptcy and cannot cover the fixed cost and payments for cash inputs.

4.9 Equilibrium

An equilibrium consists of the following functions:

- Policy functions for final-good firms: $\mathcal{G} = \{g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, x, \pi^{ND}, \pi^x, \pi^b\}$, and value functions V^{ND}, V^b, V^x and W.
- Discount rate function for firms and lenders ρ
- Price kernel function Q
- Input price function p
- Wage function w
- Loan price functions: q^{ND}, q^x
- Aggregate default and failure functions θ and η
- Aggregate dividends Π
- Policy functions for households: $a'(.), l, c$
- Transition functions for endogenous aggregate states: H^μ, H^A
- Mass of entrants M
- Value of entry W^E

In an equilibrium, these objects have to satisfy the following conditions:

1. Optimisation by final-good firms. Given q^{ND}, q^x, ρ, P, H^μ and H^A, final-good firms's decision $\mathcal{G}$ solve the problems in equations (1), (2), (3), (4), (5), (6), (7).
2. Discount. Given Q, ρ is determined by (8)
3. Arbitrage. Given $\mathcal{G}$ and H^μ and H^A, the price kernel Q across any two contingent states (e.g., given $Q(z_1|S, z)$, the prices $Q(z_2|S, z), ..., Q(z_{n_z}|S, z)$) obeys (12)
4. Input-producers. The functions p, w and θ and η are such that input production satisfies the zero-profit condition (20)

5. Labour market clearing. The functions M, l, and x (in $\mathcal{G}$) are such that the labour market clears according to (23)

6. Lenders zero profit. Given Q, and d^b, r^b, and x from $\mathcal{G}$, debt prices q^{ND} and q^x satisfy equations (9) and (10).

7. Aggregate default and failure. Given d^x, r^x and x, and d^b and d^f, from $\mathcal{G}$, aggregate delinquency θ and failure η is determined by equations (13) and (14).

8. Aggregate dividends. Given π^{ND} and π^x from $\mathcal{G}$, and M, aggregate dividends are determined by (15)

9. Transition functions for endogenous aggregate state μ. Given g^{ND} and g^x, d^b, x and d^x from $\mathcal{G}$, the transition for μ is determined as in (16), (17), (18).

10. Transition functions for endogenous aggregate state A. Given $a'(z')(.)$ the transition for A is determined in (19).

11. Consumer maximisation. Given Q, w, Π, H^μ and H^A, the household policy functions $a'(\cdot), l, c$ solve the problem in (11).

12. Clearing in final goods. Given c, l, M, x from $\mathcal{G}$, demand and supply of final goods coincide according to (25).

13. Asset markets clearing. The asset price kernel Q (i.e., $Q(z_1|S, z)$ given its other components) is such that household savings and total loans to firms are equal according to (24).

14. Free entry. Given W, the function W^E in (21) satisfies (22).

Note the n_z components of the asset pricing kernel Q must satisfy two types of equilibrium conditions, $n_z - 1$ arbitrage, and asset market clearing. Also one of the market clearing conditions is redundant, either in assets or in final goods. The wage w adjusts to satisfy the zero-profit entry condition (22), since w via p determines the profitability of final-good firms W. The mass of entrants M helps meet labour market clearing (23) as the mass determines the aggregate level of demand for intermediate inputs x.

4.10 Characterisation of final-good firms' problem

We consider the three choice options at the end of the period in turns. We will rely on the fact that a firm will borrow at most as much as needed to meet the financial constraint in assumption A1. Borrowing to pay dividends is suboptimal.

In this model, the price of debt and therefore the resources raised via borrowing may be discontinuous in the level of debt chosen. The reason is that at the level of debt where the firm may be delinquent with positive probability, the marginal cost of hiring inputs drops and the firm's chosen amount of x may jump. The probability of bankruptcy may drop since operating profits will increase, and the price of debt may drop at that level of debt as a consequence. The price of debt might jump however if the cost of delinquency were large enough. So the price of debt is in general discontinuous with an indeterminate sign. One consequence is that the firm's choice of debt b' may raise more resources that necessary to meet the needs for liquidity, and this residual has to be apportioned to creditors accordingly. The characterisation that follows takes this possible discontinuity into account.

(A more precise characterisation of discontinuities to be added.)

Consider the case that the firm chooses to repay creditors and suppliers. In this case, dividends are positive if and only if cash in hand is positive except for the possible discontinuity in the price. (In the other two cases dividends will be zero by A1 and A2.) Borrowing is positive only if cash in hand is negative.

Proposition 1 *(No-default no-delinquency case) Suppose the firm chooses option ND in state $(\epsilon, b, \nu, x|S, z)$.*

1. *If cash in hand is positive or zero, that is $(1-\nu)ze^{\epsilon}F(x) - c_F - px - b \geq 0$, then borrowing is zero $g^{ND}(\epsilon, b, \nu, x|S, z) = 0$ and dividends $\pi^{ND}(\epsilon, b, \nu, x|S, z)$ are positive and equal to the value of the cash in hand.*

2. *If cash in hand is negative, that is $(1-\nu)z\epsilon F(x) - c_F - px - b < 0$, and borrowing cannot meet this gap*

$$\max_{b'} b'q^{ND}(b', \epsilon, \nu|S, z) < px + b - (1-\nu)ze^{\epsilon}F(x) + c_F,$$

   the choice set is empty (in practice, $V^{ND} = -\infty$).

3. *Otherwise, if cash in hand is negative, that is $(1-\nu)z\epsilon F(x) - c_F - px - b < 0$, then*

borrowing $g^{ND}(\epsilon, b, \nu, x|S, z)$ *is positive and given by the smallest value of* b' *such that*

$$b' q^{ND}(b', \epsilon, \nu|S, z) \geq px + b - (1-\nu) z e^{\epsilon} F(x) + c_F,$$

and dividends $\pi^{ND}(\epsilon, b, \nu, x|S, z) = b' q^{ND}(b', \epsilon, \nu|S, z) - (px + b - (1-\nu) z e^{\epsilon} F(x) + c_F)$ *if this amount is positive, or zero zero otherwise.*

Consider now the option of becoming delinquent without defaulting. The dividend paid is zero. If cash in hand is negative, then borrowing is positive and the residual repaid is zero; otherwise, borrowing is zero and there is a positive residual repayment so delinquency is partial.

Proposition 2 *(Delinquency no-default case) Suppose the firm chooses option* x *in state* $(\epsilon, b, \nu, x|S, z)$. *Dividend* $\pi^x(\epsilon, b, \nu, x|S, z)$ *is zero.*

1. *If cash in hand is positive or zero, that is* $(1-\nu) z e^{\epsilon} F(x) - b - c_F - (1-\tau) px \geq 0$, *then borrowing is zero* $g^x(\epsilon, b, \nu, x|S, z) = 0$ *and the residual* $r^x(\epsilon, b, \nu, x|S, z)$ *is positive and equal to the value of the cash in hand.*

2. *If cash in hand is negative, that is* $(1-\nu) z e^{\epsilon} F(x) - b - c_F - (1-\tau) px < 0$, *and borrowing cannot meet this gap*

$$\max_{b'} b' q^x(b', \epsilon, \nu|S, z) < b - (1-\nu) z \epsilon F(x) + c_F + (1-\tau) px,$$

the choice set is empty (in practice, $V^x = -\infty$*).*

3. *Otherwise, if cash in hand is negative, that is* $(1-\nu) z e^{\epsilon} F(x) - b - c_F - (1-\tau) px < 0$, *then borrowing* $g^x(\epsilon, b, \nu, x|S, z)$ *is positive and given by the smallest value* b' *such that*

$$b' q^x(b', \epsilon, \nu|S, z) \geq b - (1-\nu) z e^{\epsilon} F(x) + c_F + (1-\tau) px,$$

and the residual is , $r^x(\epsilon, b, \nu, x|S, z) = b' q^x(b', \epsilon, \nu|S, z) - (b - (1-\nu) z e^{\epsilon} F(x) + c_F + (1-\tau) px)$ *if this value is positive, and the residual is zero otherwise.*

Consider finally the option to default. Dividends are zero, there is delinquency, and the residual cash in hand goes to banks. If the residual is negative because of $c_F + (1-\tau)px$, then banks recover nothing, the firm fails to produce, and even cash supplies fail to receive payment.

Proposition 3 *(Default case) Suppose the firm chooses option to default b in state* $(\epsilon, b, \nu, x|S, z)$*. The value of dividends* $\pi^b(\epsilon, b, \nu, x|S, z)$ *is zero, and the residual paid to lenders is positive and equal to the value of the cash in hand (output)*

$$r^b(\epsilon, b, \nu, x|S, z) = (1-\nu)z\epsilon F(x) - c_F - (1-\tau)px,$$

if positive, and then $d^f(...) = 0$*. Otherwise,* $r^b(\epsilon, b, \nu, x|S, z) = 0$*, the firm fails to undergo production, and cash supplies fail to receive payment* $(1-\tau)px$ *and failure ensues* $d^f(...) = 1$*. (And* $r^x = 0$*.)*

5 The core model

To begin with, let us focus on outcomes from equilibrium conditions 1, 4, 6, 7 and 9, and 5 and 14. Therefore the endogenous functions of interest will be:

- Policy functions for final-good firms $\mathcal{G} = \{g^{ND}, g^x, d^b, d^f, d^x, r^b, r^x, x, \pi^{ND}, \pi^x, \pi^b\}$ and W, given p, q^{ND}, and q^x. Equations (1), (3), (5), (6) and (7).
- Input price function p, given θ and η. Eq.(20).
- Loan price functions: q^{ND}, q^x, given $\mathcal{G}$. Equations (9) and (10).
- Aggregate delinquency function θ and failure η, given $\mathcal{G}$. Equation (13) and (14).
- Transition functions for endogenous aggregate states μ. Given g^{ND} and g^x, d^b, x and d^x from $\mathcal{G}$, the transition for μ is determined as in (16), (17), (18).
- Labour market clearing. The functions M, l, and x (in $\mathcal{G}$) are such that the labour market clears according to (23)
- Free entry. Given W, the function W^E in (21) satisfies (22).

To do this we take as exogenous ρ, Q, Π, (c, a', l) and H^A. Initially, we will be focusing on stationary outcomes for the calibrated baseline. Accordingly, we set $\psi_z(1 \mid z_{-1}) = 1$ all z_{-1} so aggregate exogenous productivity is a constant $z = 1$, and set any (inconsequential) constant portfolio A. The only endogenous aggregate state is now μ, which will also be constant. Since the aggregate state is constant in a stationary equilibrium, the endogenous equilibrium functions will therefore be constant functions. A stationary equilibrium also requires the mass size of firms be constant, thus determining a mass of entrants M that matches the mass of firms exiting the economy. We will also study the dynamics of the model outside of the steady state.

There exists a stationary equilibrium with zero default. In this equilibrium, any debt is priced at the risk-free rate and there are no borrowing constraints beyond the natural debt limit. However this no-default equilibrium is not the limit of a finite economy. The limit of a finite economy will generally deliver borrowing constraints and positive default. We focus on this type of equilibria. We characterise it by initialising the algorithm with functions corresponding to the equilibrium in the terminal period of a finite economy.

Throughout numerical explorations, we learn that the proportion of input sales on credit τ, is important for the final-good firm's problem definition. For example, if $\tau = 1$ and all sales are on credit, the first stage problem of choosing x might not be bounded in situations where delinquency in the second stage becomes highly likely. Anticipating delinquency, the firm has an unbound incentive to hire inputs, and thus repay bank debt, at no effective marginal cost. With the empirically plausible share of trade credit τ we use, the problem of the firm always remains bounded.

We will next analyse for the stationary equilibrium under a reasonable parameterisation. (The next draft will contain a more rigorous moment matching calibration exercise.) The model seems able to deliver levels of trade-credit delinquency and bankruptcy comparable with the data. The model contains the core macroeconomic interactions between final-good producers, input producers, and lenders. Delinquency by final-good firms imposes losses, directly on intermediate-input firms and indirectly to the other final-good firms via the subsequent increase in input prices. Thus trade-credit delinquency could possibly amplify recessions and fuel further bankruptcies. Bankruptcies do also affect delinquency as credit constraints become tighter. On the other hand, however, delinquency may mitigate bankruptcies as it provides an alternative option to deal with tight financial constraints. In the extended analysis of the transition, these mechanisms will interact with the other variables in general equilibrium, including employment and GDP.

5.1 Parameterization

Consider first te functional forms in the model. We have assumed a one-for-one linear technology for intermediate $x = n$. Final goods firm's production is $y = ze^{\epsilon}F(x)$, where a concave function $F(x) = x^{\gamma}$ with $\gamma < 1$. For the idiosyncratic shock, we have assumed a discrete Markov process for productivity ϵ, and will pin down its support and transition probabilities so that it approximates a continuous process specified as a first-order autoregressive process for $\log \epsilon$ where the innovations η' follow an iid Normal distribution,

$$\log \epsilon' = \rho \log \epsilon + \eta', \text{ with } \eta' \sim N(0, \sigma_\eta).$$

We choose a 301-state Markov chain to do this approximation following the discretization methods in Tauchen (1986).

A model's period corresponds to one year. In this version of the model, aggregate productivity is a fixed parameter that can be normalised to $z = 1$. The discount rate ρ and wage rate w are also included among the parameters. Several parameters are pinned down from direct observation.The remaining parameters will be chosen so the model matches a number of empirical targets.

The parameters set directly are summarised in Table 1. The discount rate ρ is equivalent to an annual rate of return of 4%. The curvature of the production of final goods γ corresponds to the labour share if we think of capital given and uniform across firms, and we pick a value common in the literature, for instance Corbae and D'Erasmo (2017), Khan and Thomas (2013) or Arellano, Bai, and Kehoe (2016). The parameters of the idiosyncratic productivity process, ρ_ϵ and σ_η, are chosen according to estimates in Corbae and D'Erasmo (2017), also consistent with Khan and Thomas (2013) or Arellano, Bai, and Kehoe (2016).

The one direct parameter which is less standard is the share of trade credit in the intermediate producer's revenues, τ. This trade credit parameter would correspond to the value of accounts receivable to sales for US firms. Our choice here is at the top end of the ratio of receivable accounts to sales for the US in Petersen and Rajan (1997) or Shenoy and Williams (2017), although for European economies this ratio could be much larger (see Ferrando and Mulier (2013)).[4]

Table 1: **Direct Parameters.**

parameter	value	observation
discount	$\rho = 0.9615$	4% annual return
curvature final goods	$\gamma = 0.60$	60% labour share
persistence	$\rho_\epsilon = 0.653$	est Corbae et al 2017
volatility innovation	$\sigma_\eta = 0.40$	higher than estimated 0.20
trade credit	$\tau = 0.35$	top of receivables to sales

[4]This is consistent with the measures of payable accounts in GDP from the Flow of Funds (see FRED data base).

The five remaining parameters are the fixed cost c_F, the size of the penalty for delinquency $\tilde{\nu}$, the probability of forgiveness λ, and the cost of entry ξ^E. The cost of entry ξ^E can be calibrated so that the free entry condition in eq (22) holds for the wage rate w that gives the p consistent with the targets. Operationally, given the intervention of delinquency rate and failure rate in Equation (20), we find first such a p and then back out the required w by Eq 20 and, given the resulting value function W, infer ξ^E by eq (22).

These parameters are chosen so that in equilibrium the model approximates targeted values for four variables. Three of these variables are based on financial aggregates for US firms from Compustat reported in Corbae and D'Erasmo (2017). Our model speaks more directly to unsecured debt so the corresponding target values are a ratio of unsecured debt to operating profits 1.35, unsecured debt to dividends 2.54, and an estimated bankruptcy probability of 2.2%. The remaining target variable is the measured losses on trade credit of firms resulting from missed payments, the delinquency rate θ in the model which includes losses from delinquency inly and also from bankruptcy. For now, we use the estimate of 8 per cent for Sweden based on Jacobson and von Schedvin (2015).

The calibration process shows that c_F helps target dividends and debt-to-dividends, p, and therefore, ξ^E, has an important effect on debt and bankruptcy, and the two penalty parameters can be used to target delinquency. For the moment, we have just picked an example that falls within a reasonable range of the targets. This calibration is summarised in Table 2.

Table 2: **Calibrated Parameters**

parameter	value	targets
fixed cost	$c_F = 0.75$	1.35 debt to operating income
entry cost	$\xi^E = 25.26$	2.54 debt to dividends
penalty size	$\tilde{\nu} = 0.30$	0.02 bankruptcy probability
penalty forgiveness	$\lambda = 0.30$	0.08 bankruptcy rate

Table 3 presents the results from the above choice of parameters. The top section contains the implications of the model for the target moments. This numerical example over estimates somehow debt measures, and the the probability of bankruptcy. Note the gap between w and p expresses the effect of delinquency on trade credits payments plus the small failure rate affecting also cash payments.

Table 3: **Moments in model and data**

Moments	Data	Model
debt to operating income	1.35	2.52
debt to dividends	2.54	3.24
bankruptcy probability	0.022	0.0473
trade credit default rate	0.080	0.076
intermediate price p		0.22
wage w		0.213
delinquency probability		0.0474
failure rate		0.0068
trade credit to GDP		0.167
mass of firms		0.065

5.2 Policy functions

We describe the equilibrium properties of this economy to uncover the behaviour and mechanisms at work. We begin with objects pertaining to the first part of the period, before the technology shocks occur.

Figure 1 displays first the distribution of final goods firms without a flag $\nu = 0$ over the initial individual state space (ϵ_{-1}, b). There is substantial mass of firms on a wide domain of previous productivity values, including well above the mean of 1, and this mass declines in the debt position. The bottom panel displays the corresponding optimal choice of intermediate input x. In terms of sign, the amount of x is an increasing function of initial productivity, thereby inheriting the properties of the first best. The demand for x is also increasing in the value of debt due, and more markedly so at low levels of initial productivity. Larger debt obligations make it optimal to increase production and operating profits to meet repayments. The downside risk of poor productivity is limited by the delinquency and bankruptcy options. This figure displays a discontinuity, a distinct possibility that as discussed earlier arises from the two local optimal in the objective as a function of x. The reason is that at a level of x where delinquency may happen at the end of the period with some probability, the marginal cost for the firm declines and creates a kink in the objective. As displayed in the figure, the discontinuity describes a mapping between ϵ_{-1} and b with a decreasing slope. One implication is that delinquency will occur in the second part of the period only for firms with x chosen past the discontinuous jump.

Consider now decisions at the end of the period, after the realisation of the shock ϵ. Figure 2 shows firm's outcomes in the case of no defaulting ND in the space of input and debt (x, b),

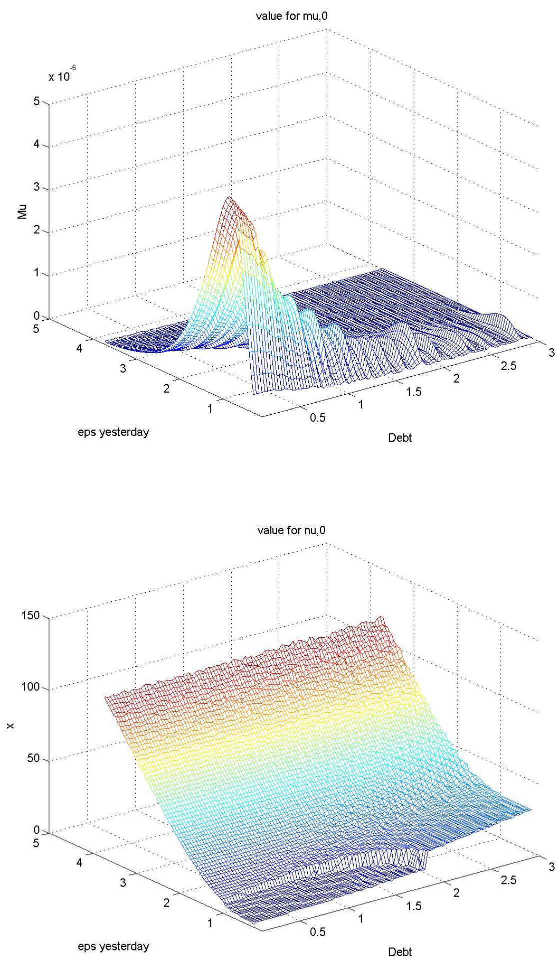

Figure 1: Pre-shocks - distrib μ (top) and demand for input x (bottom)

for a given level of the new productivity ϵ. The top graphs are for the average productivity 1; the bottom graphs for a lower productivity 0.76. The left graphs display the borrowing functions, with positive values where debt is large enough or small enough, u-shaped, with zero debt in a middle region of x, one which becomes wider with larger productivity. The outside areas where debt collapses corresponds to states where the option of no defaulting ND is unfeasible. The dividends policy is also non-monotonic in x, and positive in the inner region where borrowing is zero.

We consider now outcomes associated with the decision to be delinquent on trade credit payments, option Dx. The pattern is qualitatively similar to the one described for the

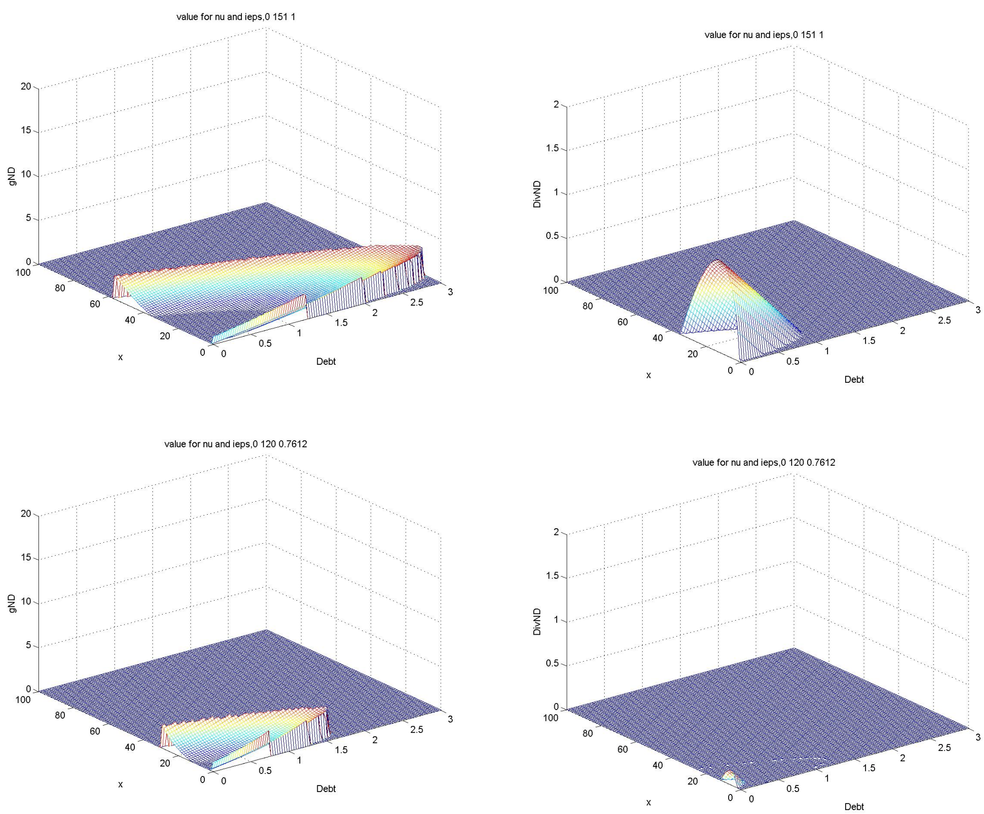

Figure 2: Post-shocks - debt g^{ND} and dividends π^{ND}

no default option. Quantitatively, however, there is less borrowing and more dividend payments, reflecting the fact that delinquency releases current resources for the firm. Note however that credit terms become less favourable compared to firms who do not default. Figure 3 shows the borrowing and recovery r^x function for the low level of productivity in the previous figure. There is recovery at intermediate values of x while borrowing is zero; borrowing is positive in a region where recovery becomes zero, and is again u-shaped. Dividends are zero.

Figure 4 displays the bankruptcy and delinquency regions in the space (x, b), for the low productivity realisation. Bankruptcy occurs when debt b is sufficiently large, and does not appear to depend strongly on the level of the input x that has to be repaid. Delinquency happens when x is sufficiently large and the range of debt positions is broader for larger x. Delinquency happens before bankruptcy in terms of debt. There is hint here that delinquency relieves pressures to liquidating.

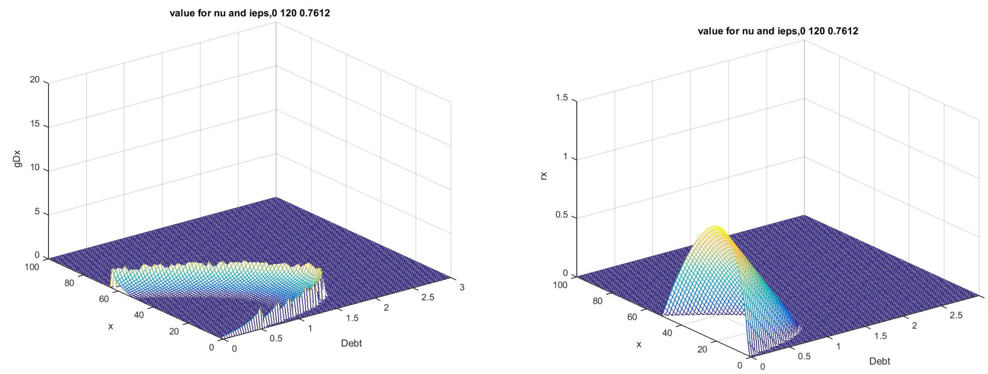

Figure 3: Post-shocks - debt g^{Dx} (top) and recovery r^x (bottom)

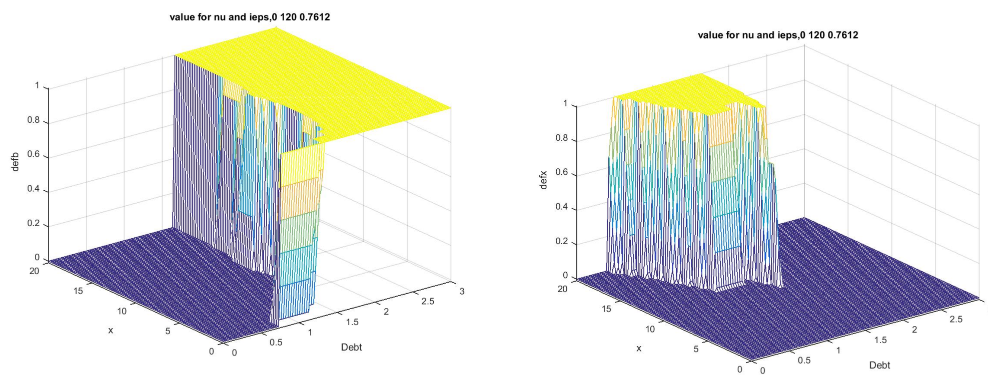

Figure 4: Post-shocks - bankruptcy d^b and delinquency d^x

The above decision rules on repayment determine the pricing of debt. Figure 5 displays the price of debt in the space (ϵ, b') for the case of no default q^{ND} and the case case of delinquency q^{Dx}. As expected, debt prices decline in debt borrowed and increase in productivity, and are lower in the event of the firm being currently delinquent. Delinquent firms face tougher conditions.

There is a mild discontinuity in q^{ND} at the points where the demand for input x is discontinuous. As discussed earlier, the reason is that delinquency releases resources to meet debt repayments and the switch to positions of possible delinquency has a discontinuous effect on the pricing. Figure 6 depicts the thresholds of discontinuity.

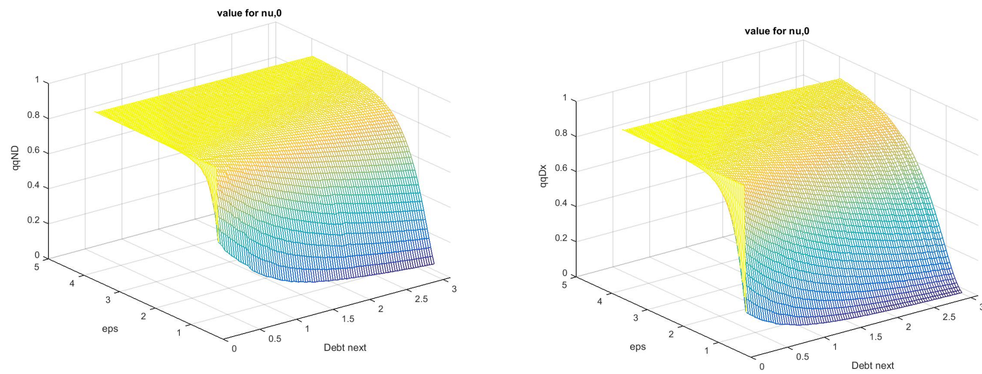

Figure 5: Post-shocks - debt prices q^{ND} and q^{Dx}

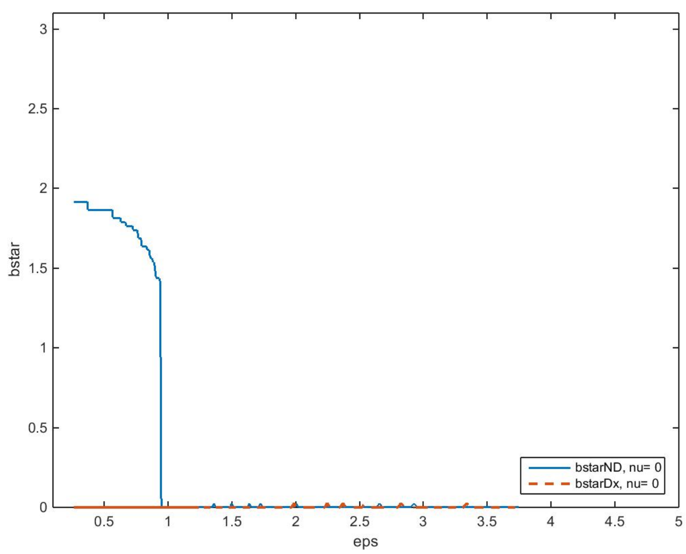

Figure 6: Discontinuity thresholds $b^*(\epsilon)$

5.3 The profile of delinquency and bankruptcy

Delinquent firms face tougher conditions in the market for credit and may find it harder to roll over debt. That might push them into liquidation. On the other hand, for the same reason delinquent firms may reduce their debts and be less likely to liquidate. We also saw that the demand for inputs x was non-decreasing in debt. So delinquency may precede bankruptcy or might avert it. The default decision also show that many bankruptcies at low x will happen without delinquency.

To investigate this matter, consider the stationary distribution. Compared to firms who do not default in any way, measures of debt are higher for delinquent firms and even larger for firms that declare bankruptcy. The figures are in Table 4. We should also compare the level of new borrowed debt for delinquent and non-delinquent firms.

Table 4: **Levels of debt** b

If debt positive	0.607
Non delinquent, non bankrupt	0.474
Delinquent, non bankrupt	0.729
Bankrupt	0.898

We also compute the average probabilities of transiting into bankruptcy and into delinquency, for firms who are and who are not currently delinquent. On average, delinquent firms are less likely to be bankrupt in the next period. This must be because delinquent firms tend to deleverage. On the other hand,, firms currently delinquent are to repeat delinquency next period much more likely than non-delinquent firms do. This must reflect that the deleveraging leading to debts debts that do not justify liquidation, but also a deteriorated productivity inducing the need to skip payments. Table 5 displays the figures.

Table 5: **Transition probabilities**

Probability to bankruptcy:		
	non-delinquent firms	0.0585
	delinquent firms	0.0322
Probability to delinquency:		
	non-delinquent firms	0.0293
	delinquent firms	0.1593

5.4 Transitional dynamics

In order to begin assessing the macroeconomic role of trade-credit default, we study the response to a temporary shock to final good firms' productivity. The economy is initially at the baseline stationary state, and at time zero $t = 0$ there is a reduction in the productivity state ϵ_{-1} for all firms (with the constraint that productivity can not fall below the lower bound). The individual productivity of each firm follows its specified Markov chain thereafter. The economy returns to the initial stationary state after the transitional adjustment.

To produce the transition for a candidate transition length period, we iterate on the path of input prices $\{p_t\}$ until convergence. For a given path for prices, we solve the firms value and policy functions backwards from the last period of the transition under the supposition that the economy is back at the steady state by then. Then we proceed starting from the first period of the transition to solve for the evolution of the distribution and the trade-credit default and failure rate in each period. We can then update the path for input prices. Finally, we increase the length of the period until the transition is completed within that time span.

To focus on the main forces at work, we impose restrictions in the equilibrium conditions during the transition. We do not require clearing of the labour market nor the free entry condition, and adjust entry to hold the number of firms constant. Since labour supply is for now assumed given, clearing in the labour market would kill much of the amplification action we want to illustrate. This is thus best interpreted as describing the short term response in the presence of wage rigidities and industry capacity constraints.

Figure 7 shows the resulting paths for the price of inputs p, final output y, and intermediate input x. The adjustment in the price p is necessarily driven by the changes in trade credit default. To asses this amplification mechanism, we also compute the transition assuming that p is constant, which removes the feedback effect of delinquency on the final good decisions, including the level of inputs and resulting output. We can see that rising delinquency raises the price of the intermediate inputs and contributes to the fall of output following the shock, accounting for about 1/10 of the initial impact.

The response of p reflects the levels and distribution of both trade-credit delinquency and bankruptcy. Figure 8 displays the proportions of firms undertaking these decisions along the transition. The shock causes a surge in the proportion of firms into liquidation via bankruptcy. This is the factor accounting for the rise in the price of inputs because the proportion nto delinquency actually declines on impact. The drop in productivity appears to lead firms into bankruptcy and away from simple delinquency.

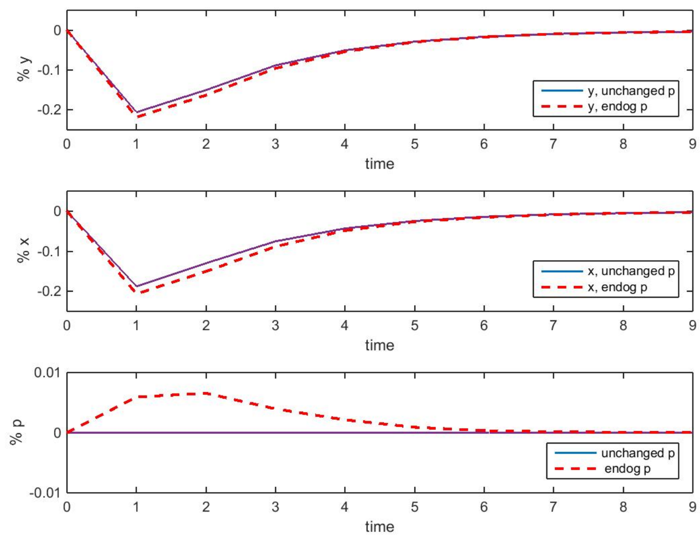

Figure 7: Transition to a fall in productivity

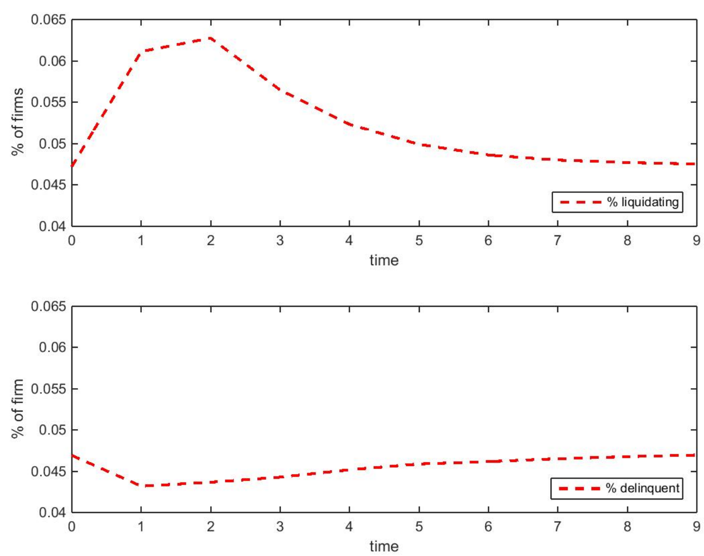

Figure 8: Transition to a fall in productivity

6 Aggregate uncertainty

Here we hold labour supply as given and constant. We focus on the firms' response to aggregate productivity shocks. Wages may be endogenous of exogenous depending on whether labour market clearing is required. Households' decisions are taken as given so, in this sense, we are studying partial equilibrium. Specifically, we can set the discount rate rho and the labour supply l. (ie, we ignore equilibrium conditions 2, 3, 8, 10, 11, and 12).

The equilibrium conditions we study here are (**note eq number when we fix them**):

- final-good firm optimality to get $\mathcal{G}$
- intermediate-input firm to get p
- lender to get debt prices q^{ND} and q^x
- delinquency aggregator to get θ
- labour-market clearing to get w (if applicable)
- transition functions to get H^μ and H^A
- entry value to get W^E
- free-entry to get M

In general equilibrium, decision rules and value functions are functions of the aggregate state $S = (z_{-1}, \mu, A)$ where μ is defined over (ϵ_{-1}, b, ν). In this partial equilibrium, we can drop A from S. The only transition function we need is $\mu' = H^\mu(S, z)$.

The distribution μ is a high-dimensional object. In order to make it manageable, we follow the macro literature and describe it in terms of a few moments in order to specify the forecasting function (e.g., Krusell and Smith 1998, Gomes and Michaelides 2008). Specifically, we start with the first moment of the distribution, the average level of firm debt $\overline{b}$.

The idea is to replace the 'true' transition H^μ as a function of state $(S, z) = (z_{-1}, \mu, z)$ by a forecasting rule Φ as a function of the relevant state which, given our first-moment approximation, becomes $(z_{-1}, \overline{b}, z)$. Following the literature, we choose a log linear specification on $\overline{b}$ for each pair of exogenous aggregate productivity realisations (z_{-1}, z):

$$\log \overline{b}' = \Phi(z_{-1}, \overline{b}, z) \equiv \phi_0(z_{-1}, z) + \phi_1(z_{-1}, z) \log \overline{b}.$$

In solving the model, we look for coefficients ϕ that most accurately describe the time-series produced by the model by aggregation based on the actual distribution. In this sense, we look for an approximate RE equilibrium. We will assess the quality of this approximation by evaluating the fit of the forecasting rule and running some tests (e.g., den Haan).

We will next describe the steps for solving the model in the two cases of exogenous wage and market-clearing endogenous wage.

6.1 Exogenous wage

In this case we take w as a given constant, for now.

The solution of this type of model relies on the simulation of the decision rules of firms and the equilibrium debt prices, and on solving, in each period of the simulation, the aggregate equilibrium conditions via aggregation based on the current distribution.

In the present case of exogenous wage, the aggregate equilibrium condition we need to solve is the pricing of the intermediate input $p(S)$. It requires calculating the state-dependent value of the aggregate delinquency rate θ from the actual distribution. One issue is that, in this way, p is dependent on the full actual distribution behind θ, which becomes unmanageable. (This is not unlike the difficulty motivating the reduced-form rule Φ.) In order to overcome this issue, we propose to approximate the input price by a log-linear forecasting rule of the approximate beginning-of-period aggregate state $\Psi(z_{-1}, \bar{b})$:

$$\log p = \Psi(z_{-1}, \bar{b}) \equiv \psi_0(z_{-1}) + \psi_1(z_{-1}) \log \bar{b}.$$

(A finite state such as $(\bar{b}, z)$ may not in general be sufficient for pinning down p. We will carry out accuracy tests accordingly.)

Given a specification of Φ and Ψ, the steps to solving the model are as follows:

1. Guess coefficients in Φ and Ψ
2. Solve final-good firms' decision rules and debt prices
3. Simulate over a T periods, starting from some initial actual distribution μ_0, and obtain time series $\bar{b}_t$, θ_t and p_t
4. Update coefficients in Φ and Ψ via OLS regressions, and back to 2

In point 2, we are solving the core model equilibrium given the aggregate state $(z_{t-1}, \bar{b}_t)$.

Since $\overline{b}_t$ need not be on the grid, we will interpolate. In point 3, the time series are obtained via aggregation using the distribution at each point in time μ_t.

6.2 Endogenous wage

The wage rate adjusts to clear the labour market at the given supply of labour. Computationally, this adds the task of clearing the market in each period of the simulation. With the original formulation of the model, however, the demand for intermediate inputs and hence of labour is written as a function of the state but not of prices like the wage. In order to clear the labour market, we will have to add the wage w as part of the state. Specifically, our forecasting rules are expressed:

$$\log \overline{b}' = \Phi(z_{-1}, \overline{b}, z, w)$$

$$\log p = \Psi(z_{-1}, \overline{b}, w)$$

We thus need a forecasting rule Ω for w as a state:

$$w' = \Omega(z_{-1}, \overline{b}, w).$$

In the algorithm, in each period of the iterations w_t will be determined by market clearing. With the time series for wages, and debt, the coefficients of Ω will be updated.

(TO BE ADDED)

7 Concluding remarks

(TO BE ADDED)

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