Optimal Taxation of Inheritance and Retirement Savings

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[Preliminary and Incomplete]

Abstract

We study optimal inheritance tax in a model where the bequest and saving motives are driven by the differences in medical expenses, mortality risk, and patience, as well as heterogeneous productivity. We show that the correlations between these factors and the earning productivities are the key for the marginal inheritance taxation — the sign and the magnitude of the tax. Positive inheritance taxes are optimal when rich people face higher medical expenses and are more patient. In the presence of heterogeneous mortality risk, optimal taxation of inheritance and retirement savings should be designed jointly. Longer life expectancy of productive workers will increase the tax on retirement savings and decrease the tax on inherited wealth. Higher patience for more productive people could be another reason for taxing inherited wealth.

Keywords: Optimal Taxation, Inheritance, Retirement Savings
JEL Classification: E21, D64, H21

1 Introduction

Optimal inheritance tax has been a highly controversial issue for both policy makers and economists. There has been investigation on the desirability and the properties of optimal wealth transfer taxation, but there is little consensus on the theoretical justification of taxing bequest. Application of the celebrated uniform-taxation results (Atkinson and Stiglitz (1976)) to the bequest taxation implies zero bequest taxes if the social welfare function sorely takes the utility from the parent's perspective.[1] On the other hand, if the society attaches direct welfare weight to future generations, there are positive externalities of bequest which are not fully internalized by the parents' bequest decision in the most widely used model of bequest — altruistic bequest model or the warm-glow bequest model (Farhi and Werning (2010), Kopczuk (2010)). For example, with the altruistic bequest motive, the society directly values the children's utility from bequest in addition to the indirect utility through the parent's altruism. Although these analyses provide important implications, a *negative* bequest tax result implied by the externalities seems to go in an opposite direction of policy debate in developed countries.

The main policy debate on this issue centers around the equity versus efficiency trade-off. Recently, Piketty and Saez (2013) made an important progress along this line. They analyze optimal linear inheritance tax formulas in terms of sufficient statistics which reflect the equity-efficiency trade-off. They show that the optimal tax rate is positive if the society has a meritocratic preferences — society cares mostly about people receiving little inheritance. However, it is not satisfactory to find a rationales of the bequest taxation sorely based on the meritocratic preferences. Without meritocratic preferences, does the society still want to redistribute beyond the income redistribution? What could be a rationale for taxing inheritance in the presence of the nonlinear earning taxation? Does the society has additional redistribution objective across different inherited wealth? Farhi and Werning (2013) consider one scenario where the society has preferences for redistribution across heterogeneous altruism. However, it is not obvious why the the society wants to redistribute across different altruism.

In this article, we analyze optimal inheritance tax in the world where there is a theoretical justification of taxing bequest without assumiing any meritocratic preferences of the society. In the real world, the important factors determining the saving motives and the bequest motive in the later stage of life includes medical expenses, uncertain lifespan, and discounting factors. It is

[1] Farhi and Werning (2010) shows this formally with a two-generation model with altruistic bequest motive.

not so surprising that these factors are not completely independent of earnings ability. We show that a positive (negative) correlation between these factors and earning abilities can generate a rationale for positive (negative) redistribution across the inherited wealth. These factors generate preferences for bequest that vary with ability, and thus if the bequest is more (less) preferred by the high ability worker, taxing (subsidizing) bequest can improve the equity-efficiency trade-off. This is because leaving bequest becomes a source of indiret evidnece about who has higher abilities and thus more efficient redistributive taxation can be achieved. Thus, even if we do abstract from the merticratic preferences of the sociiety, we can provide a theoretical justification for taxing (subsidizing) the inherited wealth. To show this, we extend the canonical Mirrleesian tax model — the model with heterogeneous earning productivity and the nonlinear labor income tax — by adding bequest decision. We investigate the constrained efficient allocation and its implications for the marginal inheritance tax.

First of all, we examine the role of the heterogeneous need for medical expenses on optimal inheritance taxation. It is now well known that the medical expenses are important for the elderly savings and the bequest. Especially near death, serious health shock and illness can easily deplete the wealth of most retirees and reduce the amount of bequest significantly. Another important feature of the medical expenses is that the average out-of-pocket medical expenditures rise very rapidly with age and income (De Nardi, French, and Jones (2010), De Nardi, French, and Jones (2016)). High ability individuals tend to live longer, and the medical expense risk gets highr with age, which generates a positive correlation between the ability and the medical expenses. We show that this positive correlation generates a higher valuation for the bequest for more productive parents and thus taxing the inheritance can relax the cost of redistribution — the cost of redistributing across income generating abilities.

We also show that the heterogeneous mortality risk generates more subtle implications for the optimal tax system between the inheritance tax and the taxes on the retirement savings. In reality, parent's retirement savings can simultaneously serve both a precautionary savings for longer lifespan and a bequest to children, and thus the two motives of savings cannot be clearly distinguished (Dynan, Skinner, and Zeldes (2002), Lockwood (2014)). When the savings can serve for both the precautionary retirement saving and the bequest, we need to design the optimal taxation/subsidy on the inheritance and the retirement savings jointly. Empirical evidences strongly show that people with higher earning ability tend to live longer (e.g., Cristia (2009) and Waldron (2013)). This positive correlation between the earning abilities and longevity implies that more productive parents have relatively higher valuations for the consumption after retirement than those for the bequest. Then taxing the returns of the retirement savings and subsidizing the inherited wealth can contribute to more efficient redistributive taxation — taxation of redistributing across heterogenous skills.

Finally, the heterogeneous discount factor can be another rationale of the positive bequest taxation. The intuition for this result is consistent with that in Saez (2002). Saez (2002) shows that when those with higher earning abilities save more, taxation of saving can help with the equity-efficiency trade-off because savings can be an indirect evidence for higher earning abilities. The same argument can be applied to the bequest. Parents with altruistic bequest motive values the bequest more when they have higher discount factor. If there is a positive correlation between the patience and productivities, then positive inheritance taxation can be optimal.

Thus, in order to determine the sign and shape of the inheritance taxation, we need to carefully set the relationship among the earning ability, medical expenses, mortality, discount factor. We also need to study the optimal taxes on the inherited wealth and the retirement savings together. We plan to carry out careful quantitative investigation by carefully calibrating the model to the data.

Literature Review

[List of papers that need to be discussed:]

- Piketty and Saez (2013)
- Farhi and Werning (2010) and Farhi and Werning (2013)
- Pavoni and Yazici (2017)
- Laitner and Juster (1996)
- Lockwood (2014)
- Hosseini and Shourideh (2016)

2 The Model: Two Period Economy

In this section, we use a simple two period economy to provide a theoretical analysis of optimal taxation on bequest and retirement savings.

There are two generations, parents born at $t = 1$ and children born at $t = 2$. A continuum of parents live for at most two periods. Upon birth, each parent draws a type $\theta \in \Theta$ from a continuous distribution $F(\theta)$ that has density $f(\theta)$. This type determines various characteristics of the individual: 1. θ determines labor productivity when parents are young ($t = 1$), 2. θ also determines the average medical expenses near death $m(\theta)$, 3. θ also determines the mortality risk which is captured by the survival probability at $t = 2$, $P(\theta)$, 4. It also determines the average discount factor of future $\beta(\theta)$.

For the theoretical analysis, we do not have to make strong assumptions on the shape of these functions $m(\theta)$, $P(\theta)$, and $\beta(\theta)$. However, to get the sharp policy implications, we do make assumptions on the slope of these functions based on the empirical evidences. We assume that the average medical expenses $m(\theta)$ are increasing in productivity θ, reflecting the universal Medicare and Medicaid program of the US government. Below, we will discuss the relationship between the inheritance taxation and the government medical program in more detail. Rich people are healthier and tend to live longer, and thus we assume that the average probability of living long $P(\theta)$ is higher for parents with higher productivity ($\dot{P}(\theta) > 0$). Finally, the empirical evidences on the relationship between the discount factor and the productivity is more subtle, because there are many other factors than can make rich people to save more and bequest more. For most of the theoretical analysis, we assume that $\dot{\beta}(\theta) > 0$, but we also discuss other possibilities below.

Later, in the quantitative analysis, the shape of $m(\theta)$, $P(\theta)$, and $\beta(\theta)$ will be determined to match the empirical correlation between the earnings productivity and medical expenses, the correlation between the income and the mortality, and the correlation between the income and the savings/bequest.

We can consider period one as young and period two as old. At $t = 1$, parents first learn their type θ and then provide $l(\theta)$ units of work effort to produce efficiency units of labor $y(\theta) = \theta \cdot l(\theta)$. With probability $P(\theta)$, the parents with productivity θ live long — survive in period 2. Near death, parents face negative health shock which causes the average medical expenses $m(\theta)$. After the parents die, remaining wealth will be inherited to the child, but the medical expenses near death will reduce the inherited wealth to the child. The utility of the parent with productivity θ is given by

$$U(c_1, c_2, c_c, y; \theta) \quad = \quad u(c_1) - \psi\left(\frac{y}{\theta}\right) + P(\theta)\beta(\theta)\left[u(c_2) + \beta(\theta)V_c(c_{c3}^*)\right] + (1 - P(\theta))\beta(\theta)V_c(c_{c2}^*),$$

where c_1 is consumption when parents are young and c_2 are the consumption after the retirement if they live long, and c_{c2}^* and c_{c3}^* are the consumption of children. We assume that the flow utility from parent's consumption $u(\cdot)$ and the child's utility $V_c(\cdot)$ are strictly concave and the disutility of parent's working $\psi(\cdot)$ is strictly convex.

In the presence of uncertainty in mortality, parents' savings in period $t = 1$ serve both purposes of savings — a precautionary savings fore retirement and a bequest. Parents save against the future risk of living a long time after the retirement, which happens with probability $P(\theta)$. In the event they do not survive, however, bequests are given to their children, and the parents value the bequest. Thus, in this model, the parents have an altruistic bequest motive, but the motive is operative only in the state of the death. This implies that the precautionary savings motive for the life-cycle after retirement and the bequest motives are overlapping and cannot be clearly distinguished, and thus optimal tax system on the bequest and retirement savings should be designed together. This operative bequest motive was first pointed out by Dynan, Skinner, and Zeldes (2002) and has been adopted in the papers studying the elderly savings and bequest (De Nardi, French, and Jones (2010),Lockwood (2014)). Lockwood (2014) has shown that this overlapping bequest motives can explain retiree's puzzling decisions on saving and insurance. In this paper, we study the optimal inheritance taxation in the economy where the two motives of the bequest are overlapping due to the uncertainty in life-cycle.

In addition to the production, there is an endowment of goods I_1 and I_2 to each parent in period $t = 1$ and in period $t = 2$ (if they survive). Goods can be transferred between periods $t = 1$ and $t = 2$ with a savings technology with rate of return $R > 0$. The total cost for the child's consumption and the medical expenses are denoted by $c_{c2}(\theta) = c_{c2}^*(\theta) + m(\theta)$ and $c_{c3}(\theta) = c_{c3}^*(\theta) + m(\theta)$ respectively. Then, an allocation is resource feasible if

$$\int \left[c_1(\theta) + P(\theta)\left\{\frac{1}{R}c_2(\theta) + \frac{1}{R^2}c_{c3}(\theta)\right\} + (1 - P(\theta))\frac{1}{R}c_{c2}(\theta)\right] dF(\theta) \leq \int y(\theta)dF(\theta) + I, \qquad (1)$$

where $I = I_1 + \frac{I_2}{R}$.

We assume that each parent's θ type is private information. We can restrict attention to direct mechanisms applying the revelation principle — parents report their type and receive allocation as a function of this report. An allocation is incentive compatible if

$$
\begin{aligned}
&u(c_1(\theta)) - \psi\left(\frac{y(\theta)}{\theta}\right) + P(\theta)\beta(\theta)\left[u(c_2(\theta)) + \beta(\theta)V_c(c_{c3}^*(\theta))\right] + (1-P(\theta))\beta(\theta)V_c(c_{c2}^*(\theta)) \geq \\
&\quad u(c_1(\theta')) - \psi\left(\frac{y(\theta')}{\theta}\right) + P(\theta)\beta(\theta)\left[u(c_2(\theta')) + \beta(\theta)V_c(c_{c3}^*(\theta'))\right] + (1-P(\theta))\beta(\theta)V_c(c_{c2}^*(\theta')), \quad \forall\theta,\theta'. \quad (2)
\end{aligned}
$$

An allocation is incentive feasible if it satisfies the resource constraint (1) and the incentive compatibility constraint (2).

In the benchmark analysis, we assume that the social planner only cares for the utility of the parents and the utility of the children are valued by the planner only indirectly through the altruism of the parents. Later we will extend our analysis to the case where the social planner values the children's utility directly. We consider a weighted Utilitarian criterion:

$$\int \lambda(\theta)U(c_1, c_2, c_c, y; \theta)f(\theta)d\theta, \tag{3}$$

where $\lambda(\theta)$ is the weight on the parents of type θ. We can interpret $\lambda(\theta)$ as the Pareto weight, and by varying the weight $\lambda(\theta)$, we can trace out the Pareto frontier. Later, in the quantitative analysis, we will consider a class of Pareto weight function that can invert the U.S. policymakers' tastes for redistribution from the current US tax system.

We analyze optimal taxation following a Mirrleesian approach. That is, we do not impose any restriction on the policy instruments of the government except that the type θ is private information. Our first goal is to study the constrained efficient allocation to wedges implied by the optimal allocation. The social planner solves the following mechanism design problem of maximizing the social welfare function (3) subject to the resource constraint (1) and the incentive constraints (2).

We now simplify the incentive compatibility constraint (2) to rewrite the planner's problem. A necessary condition for an allocation (c_1, c_2, c_c, y) to be incentive compatible is given by the envelope condition:

$$
\begin{aligned}
\dot{v}(\theta) \quad = \quad & \psi'\left(\frac{y(\theta)}{\theta}\right)\frac{y(\theta)}{\theta^2} + \dot{P}(\theta)\left[\beta(\theta)u(c_2(\theta)) + \beta(\theta)^2 V_c(c_{c3}^*(\theta))\right] - \dot{P}(\theta)\beta(\theta)V_c(c_{c2}^*) \\
& -P(\theta)\beta(\theta)V_c'(c_{c3}^*(\theta))\dot{m}(\theta) - (1-P(\theta))\beta(\theta)V_c'(c_{c2}^*(\theta))\dot{m}(\theta) \\
& +P(\theta)\dot{\beta}(\theta)\left[(\theta)u(c_2(\theta)) + 2\beta(\theta)V_c(c_{c3}^*(\theta))\right] + (1-P(\theta))\dot{\beta}(\theta)V_c(c_{c2}^*(\theta)), \quad \forall\theta, \qquad (4)
\end{aligned}
$$

where $v(\theta)$ is the associated indirect utility function. As in the standard Mirrleesian taxation, we can show that the envelope condition (4) is also sufficient for an allocation to be incentive compatible, if the allocation satisfies additional monotonicity conditions.[2]

Condition 1. ***(MON)***

$c_2(\theta)$, $c^*_{c3}(\theta)$, and $y(\theta)$ are nondecreasing in θ, and $c^*_{c2}(\theta)$ satisfies the following:

$$\begin{cases} \dot{c}^*_{c2}(\theta) \geq 0 & \textit{if } \left\{ \frac{\dot{P}(\theta)}{1-P(\theta)} - \frac{\dot{\beta}(\theta)}{\beta(\theta)} \right\} + \frac{V_c''(c^*_{c2}(\theta))}{V_c'(c^*_{c2}(\theta))}\dot{m}(\theta) \geq 0 \\ \dot{c}^*_{c2}(\theta) \leq 0 & \textit{otherwise.} \end{cases}$$

Lemma 2. *If an allocation $(c_1, c_2, c^*_{c2}, c^*_{c3}, y)$ satisfies (4) and the monotonicity condition, then the allocation is incentive compatible.*[3]

Proof We denote the utility of a θ-type agent who reports $\hat{\theta}$ by

$$U(\hat{\theta}, \theta) = u(c_1(\hat{\theta})) - \psi(\frac{y(\hat{\theta})}{\theta}) + P(\theta)\beta(\theta)\left[u(c_2(\hat{\theta})) + \beta(\theta)V_c(c^*_{c3}(\hat{\theta}))\right] + (1-P(\theta))\beta(\theta)V_c(c^*_{c2}(\hat{\theta})).$$

Then the first order condition $\frac{\partial U(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$ implies the envelope condition (4). Using the first order condition, we obtain $\frac{\partial^2 U(\hat{\theta},\theta)}{\partial \theta \partial \hat{\theta}} + \frac{\partial^2 U(\hat{\theta},\theta)}{\partial \hat{\theta}^2} = 0$, and thus the second order condition of the agent's problem ($\frac{\partial^2 U(\hat{\theta},\theta)}{\partial \hat{\theta}^2} \leq 0$) is equivalent to $\frac{\partial^2 U(\hat{\theta},\theta)}{\partial \theta \partial \hat{\theta}} \geq 0$, where

$$\begin{aligned} \frac{\partial^2 U(\hat{\theta}, \theta)}{\partial \theta \partial \hat{\theta}} = & \left\{ \frac{1}{\theta^2}\psi'(\frac{y(\theta)}{\theta}) + \frac{y(\theta)}{\theta^3}\psi''(\frac{y(\theta)}{\theta}) \right\} \dot{y}(\theta) \\ & + \dot{P}(\theta)\beta(\theta)\left[u'(c_2(\theta))\dot{c}_2(\theta) + \beta(\theta)V_c'(c^*_{c3}(\theta))\dot{c}^*_{c3}(\theta)\right] \\ & - P(\theta)\beta(\theta)V_c''(c^*_{c3}(\theta))\dot{m}(\theta)\dot{c}^*_{c3}(\theta) - (1-P(\theta))\beta(\theta)V_c''(c^*_{c2}(\theta))\dot{m}(\theta)\dot{c}^*_{c2}(\theta) \\ & + P(\theta)\dot{\beta}(\theta)\left[u'(c_2(\theta))\dot{c}_2(\theta) + 2\beta(\theta)V_c'(c^*_{c3}(\theta))\dot{c}_{c3^*}(\theta)\right] + (1-P(\theta))\dot{\beta}(\theta)V_c'(c^*_{c2}(\theta))\dot{c}_{c2^*}(\theta). \end{aligned}$$

Then, the monotonicity condition proves the second order condition, and thus (4) is sufficient. ■

Note that the envelope condition (4) and the monotonicity conditions are sufficient, but the monotonicity conditions are not necessary, which is different from the standard Mirrlees optimal taxation analysis.[4] For example, even if $c^*_{c3}(\theta)$ violates the monotonicity condition **(MON)**, if the absolute value of the $\left\{ \frac{\dot{P}(\theta)}{1-P(\theta)} - \frac{\dot{\beta}(\theta)}{\beta(\theta)} \right\} + \frac{V_c''(c^*_{c2}(\theta))}{V_c'(c^*_{c2}(\theta))}\dot{m}(\theta)$ is relatively small, then the second order condition is still satisfied. Thus, for most of the theoretical analysis, we focus on the solution of the relaxed problem — the planner's problem without the monotonicity conditions, and then we

[2]In the standard Mirrleesian taxation literature, this is the case under the preferences that satisfy the single-crossing property. Note that in our simple model, the single crossing is always satisfied because we only consider the additively separable utility functions. Our analysis can be extended to general utility functions with the additional single crossing conditions.

[3]If assumption **??** does not hold for all θ, then for those θ-interval, the monotonicity condition changes the sign : $\dot{c}_c(\theta) \geq 0$

[4]In Mirrlees optimal taxation (Mirrlees (1971)), the envelope condition and the monotonicity condition are both necessary and sufficient.

provide some more investigation when the bunching occurs — when the monotonicity conditions are binding. In the quantitative analysis, we plan to solve the relaxed problem and to check the monotonicity conditions ex post.

We now recast the planning problem as the problem whose incentive constraints (4) are replaced by the envelope condition (4) and the monotonicity conditions (**MON**).

$$
\begin{aligned}
\textbf{(P)} \quad \max_{c_1,c_2,c_c,y,v} \quad & \int \lambda(\theta)v(\theta)f(\theta)d\theta && (5)\\
s.t. \quad & v(\theta) = u(c_1(\theta)) - \psi\left(\frac{y(\theta)}{\theta}\right) + P(\theta)u(c_2(\theta)) + (1-P(\theta))\gamma(\theta)V_c(c_c(\theta)) && (6)\\
& (1), (4) \text{ and } (\textbf{MON}).
\end{aligned}
$$

We call the planning problem **(P)** *without* the monotonicity constraint (**MON**) the relaxed problem **(RP)**. Note that the solution of the problem **(P)** and the relaxed problem are the same except for the productivity types whose monotonicity conditions are binding.

We first characterize the optimal allocation — the solution to the relaxed planning problem **(RP)** — by analyzing the wedges implied by the solution. Then, we also discuss how to implement the optimal allocation using a nonlinear tax system.

3 Main Result: Role of Medical Expenses, Mortality, and Discount Factor

3.1 Optimal Wedges

To analyze the implications for the optimal inheritance taxation, we now analyze the wedges implied by the optimal allocation. To analyze the role of the heterogeneous medical expenses, mortality, and the discount factor on the optimal wedges, we solve the planning problem by incorporating these factors one by one. By following these steps, we will be able to clearly understand the role of the each factor on the implicit inheritance taxation.

Before getting into the inheritance tax analysis, we start with the intratemporal wedge, which shows the standard results as in canonical Mirrleesian tax theory.

3.1.1 Intratemporal wedge

Given an allocation $(c_1, c_2, c^*_{c2}, c^*_{c3}, y)$ and a type θ, we define the intratemporal wedges

$$\tau_{intra}(\theta) = 1 - \frac{\psi'(\frac{y(\theta)}{\theta})}{\theta u'(c_1(\theta))}.$$

The intratemporal wedge is the labor wedge, and the optimal labor wedge is characterized as usual.

Proposition 3. *Suppose that (c_1, c_2, c_c, y) solves the relaxed planning problem. Then the intratemporal wedge is given by*

$$\tau_{intra}(\theta) = -\frac{\mu(\theta)}{\theta^2 \eta f(\theta)} \left[\psi''(\frac{y(\theta)}{\theta})\frac{y(\theta)}{\theta} + \psi'(\frac{y(\theta)}{\theta})\right],$$

where $\mu(\theta)$ and η are the multipliers on the envelope condition (4) and the resource constraint (1), respectively.
If we assume that Pareto weight $\lambda(\theta)$ is nonincreasing in θ, then $\mu(\theta) < 0$, which implies that when there is no bunching,[5]

$$\tau_{intra}(\theta) > 0.$$

Proof See the appendix. ■

Note that the sign of $\mu(\theta)$ determines the sign of the labor wedge, and the sign of $\mu(\theta)$ does depend on the Pareto weight function $\lambda(\theta)$. Proposition 3 shows that $\mu(\theta)$ is negative if $\lambda(\theta)$ is nonincreasing, including the utilitarian case, which is commonly used in the literature. We also want to remark that $\mu(\theta)$ can be negative even if $\lambda(\theta)$ is increasing as long as the slope of $\lambda(\theta)$ is not too high — as long as the social preferences for redistribution across earning ability is not too weak, and we will discuss this in more detail when the sign of $\mu(\theta)$ is crucial for the savings wedges and the bequest wedges.

Proposition 3 hold even if $\lambda(\theta)$ is increasing, as long as the multiplier on the envelope condition $\mu(\theta)$ is negative. To formalize this argument, we consider the special parametric functional form assumption on the Pareto weight $\lambda(\theta)$ as in Heathcote and Tsujiyama (2017):

$$\lambda(\theta) = \frac{exp(-\alpha\theta)}{\int exp(-\alpha\theta) f(\theta) d\theta} \tag{7}$$

for some constant α which reflects the planner's preferences for redistribution. A negative α implies that the planner puts higher weight on more productive parents ($\dot{\lambda}(\theta) > 0$), while a positive α implies the planner prefers less productive parents ($\dot{\lambda}(\theta) < 0$). Under this special Pareto weight function, we have the following proposition.

Proposition 4. *Suppose that the Pareto weight $\lambda(\theta)$ takes the form of (7). Then,*

[5] See the appendix for the investigation on the sign of the intratemporal wedge when there is bunching.

1. *There exists a cutoff* $\hat{\alpha} < 0$ *such that* $\mu(\theta) \leq 0$ *for all* $\alpha \geq \hat{\alpha}$ *and* $\mu(\theta) > 0$ *for all* $\alpha < \hat{\alpha}$.

2. *If* $\alpha \geq \hat{\alpha}$, *then* $\tau_{intra}(\theta) > 0$.

Proof See the appendix. ■

Proposition 4 shows that even if the Pareto weight $\lambda(\theta)$ is increasing with negative α, the characterization of the intertemporal wedge above still applies, as long as the planner's preferences for redistribution is not too weak ($\alpha > \hat{\alpha}$).

3.1.2 Atkinson and Stiglitz

As a benchmark, we show that if there is no heterogeneity in medical expenses, mortality, and discount factor, then it is optimal not to have any intertemporal taxes. Thus, the inheritance taxes and any taxes on saving are not useful.

Proposition 5. *Suppose that* $\dot{m}(\theta) = \dot{P}(\theta) = \dot{\beta}(\theta) = 0$ *for all* θ *and* (c_1, c^*_{c2}, y) *solves the planning problem (**P**). Then the following intertemporal conditions hold: for all* θ,

$$u'(c_1(\theta)) = R\beta u'(c_2(\theta)), \quad u'(c_1(\theta)) = R\beta V'_c(c^*_{c2}(\theta)), \quad u'(c_2(\theta)) = R\beta V'_c(c^*_{c3}(\theta)).$$

Proof See the appendix. ■

3.1.3 Role of Medical Expenses

We now start analyzing the role of each factor on the optimal inheritance tax. We begin with the effects of heterogeneous medical expenses. As we briefly discussed in the introduction, empirical evidences show that the average medical expenses near death can be very large. Mariacristina, Eric, Bailey, and Jeremy (2016) find that medical expenses more than double between ages 70 and 90. A long stay in nursing home near death can easily exhaust the wealth even for the upper half of the wealth distribution (See Lockwood (2014) and the references there in).

Another important feature of the medical expenses in the data is that the average medical expenses are increasing as income increases because of the government health insurance program. Empirical evidences show that the permanent income has a large effect on average medical expenses, especially at older ages. People with low income and wealth pay a smaller share of the total medical costs due to the means-tested programs such as Medicaid (See De Nardi, French, and Jones (2016) andMariacristina, Eric, Bailey, and Jeremy (2016)). We now show that this positive correlation between the medical expenses and the income can generate a rationale for the positive inheritance taxation.

To focus on the role of medical expenses, we assume that $P(\theta) = 0$ for all θ. That is, every parent lives for 1 period dies after the first period, and thus there is no heterogeneity in mortality risk. We also assume that there is no heterogeneity in discount factor : $\beta(\theta) = 1$ for all θ. Then the indirect utility for parents is simply

$$v(\theta) = u(c_1(\theta)) - \psi(\frac{y(\theta)}{\theta}) + V_c(c^*_{c2}(\theta)),$$

and the only relevant intertemporal distortion we can consider is the inheritance wedge:

$$t_b(\theta) = 1 - \frac{u'(c_1(\theta))}{RV'_c(c^*_{c2}(\theta))}.$$

Next proposition clearly shows the role of medical expenses on the inheritance wedge.

Proposition 6. *Suppose that $P(\theta) = 0$ and $\beta(\theta) = 1$ for all θ. Suppose that $\lambda(\theta)$ is nonincreasing and (c_1, c^*_{c2}, y) solves the relaxed problem (**RP**). Then the optimal inheritance wedge is*

$$t_b(\theta) = \frac{\mu(\theta)}{\eta f(\theta)} \frac{V''_c(c^*_{c2}(\theta))}{V'_c(c^*_{c2}(\theta))} u'(c_1(\theta))\dot{m}(\theta),$$

which implies that $t_b(\theta) > 0$ if $\dot{m}(\theta) > 0$ ($t_b(\theta) < 0$ if $\dot{m}(\theta) < 0$).

Proof See the appendix. ■

If the parent with type θ saves $A(\theta)$ for the medical expenses and bequest, the inherited wealth to children is $R \cdot A(\theta) - m(\theta)$. Thus, higher medical expenses reduces the inheritance, which increases the marginal value of bequest. Since the average medical expenses are increasing as the income (or θ) increases, the parents with higher productivity have a relatively higher valuation for child's consumption. Thus, reducing the children's consumption by taxing inherited wealth can relax the cost of redistribution, and thus positive inheritance is optimal.

We want to remark that the important feature of the higher medical expenses for the higher income is endogenous object driven by the government medical program. Without the government program, we can conjecture that the medical expenses would decrease in θ. This is because in our model, θ captures the comprehensive income-generating abilities including the skills and the health status. Without redistributive medical program of the government, the parents with good health and high income will face lower risk of getting serious medical shock, which decreases the average medical expenses for more productive parents. This observation implies that the government medical program and the inheritance taxes should be considered together. More redistributive medical program provides a rationale for the positive inheritance taxation.

3.1.4 Role of Mortality

We now introduce heterogeneous mortality to analyze the effects of correlation between the mortality and the income on the inheritance taxes. Empirical evidences show that the uncertainty of the lifespan is huge. When there is uncertainty after the retirement, parents save for both the precautionary motive and the bequest motive. Despite many empirical evidences of showing the importance of the bequest motive, most of the survey on the elderly saving reveals that the most important reasons of savings are life-cycle or precautionary considerations. Dynan, Skinner, and Zeldes (2002) pointed out that this seemingly contradictory survey evidences could be explained by the overlapping motive of the savings. That is, one unit of saving can serve for two purposes, but the first purpose is to insure against the uncertainties (such as living a long time or high health expenditure risk), and the second purpose — bequest motive — becomes operative in the event the parents die early with positive wealth.[6]

Another important feature of the mortality risk is that the rich tend to live long. Many empirical studies show that rich people live much longer than the poor (For example, see Cristia (2009), Waldron (2013), De Nardi, French, and Jones (2016)). The income-generating ability θ in our model is a comprehensive productivity including the health status, and thus the positive correlation between the probability of living long and the productivities are partially reflects the fact that health people live longer. However, the empirical evidences show that even when we control for the health status, the rich tend to live longer.

We show that the mortality risk generate the positive intertemporal wedge, but the operative bequest motive and the negative correlation between the mortality has implication toward the positive taxes on the return of the retirement savings and the negative taxes on the inheritance taxes.

[6]Recently, Lockwood (2014) also showed that this operative bequest motive is crucial to explain the retiree's saving behavior — why many retirees self-insure late-life risk.

To show the role of mortality on the optimal wedge, we now assume that the probability of living long is higher for more productive parents: $\dot{P}(\theta) > 0$ for all θ. To focus on the role of mortality, we assume $m(\theta) = 0$ — no medical expenses at the end of life — and $\beta(\theta) = 1$ for all θ.

Ex ante Intertemporal Wedge

In the presence of life-span uncertainty, we can defined the intertemporal distortions for both ex ante and ex post. The ex ante intertemporal distortion is instructive for the optimal level of saving. However, the intertemporal wedge is the combination of the bequest distortion and the asset distortion, and thus the sign of the intertemporal wedge does not tell us the sign of each distortion. We first investigate the ex ante intertemporal wedge and then study each distortion. We can define the the ex ante intertemporal wedge as usual:

$$\tau_{inter}(\theta) = 1 - \frac{u'(c_1(\theta))}{R[P(\theta)u'(c_2(\theta)) + (1 - P(\theta))V_c'(c_{c2}(\theta))]}.$$

Next proposition shows that the well known inverse Euler equation holds when there is no bunching.

Proposition 7. *Suppose that $\dot{P}(\theta) > 0$, $m(\theta) = 0$, and $\beta(\theta) = 1$ for all θ. Suppose that (c_1, c_2, c_{c2}, y) solves the relaxed planning problem. Then the inverse Euler equation holds:*

$$\frac{P(\theta)}{Ru'(c_2(\theta))} + \frac{1 - P(\theta)}{RV_c'(c_{c2}(\theta))} = \frac{1}{u'(c_1(\theta))}, \quad \forall \theta \in \Theta. \tag{8}$$

Proof See the appendix. ■

By applying the Jensen's inequality to (8) those parents, we get the following inequality :

$$u'(c_1(\theta)) \leq P(\theta)Ru'(c_2(\theta)) + (1 - P(\theta))RV_c'(c_c(\theta)), \quad \forall \theta,$$

which implies the positive intertemporal wedges :

$$\tau_{inter}(\theta) \geq 0, \quad \forall \theta.$$

Note that the inequality holds for any Pareto weight function $\lambda(\theta)$. We also note that the strict inequality holds as long as $u'(c_2(\theta)) \neq V_c'(c_{c2}(\theta))$. That is, the intertemporal wedge is positive, as long as the the constrained efficient allocation does not achieve the perfect insurance and there is no bunching. As long as the Pareto weight $\lambda(\theta)$ is set to guarantee that $\mu(\theta) \neq 0$, perfect insurance cannot be achieved .

As in the Dynamic Mirrleesian tax literature, the positive intertemporal wedge result is to provide better insurance which is the first order benefit at the second-order cost of reducing the consumption smoothing. The insurance benefit in this economy, however is not the insurance against the future productivity uncertainty, but against the life-cycle uncertainty — the mortality risk.

In general, a positive intertemporal distortion implies that there is need to tax the return of savings. As we discussed above, however, it does not mean that both the bequest distortion and the retirement savings distortion should be positive. To see the implication of each distortion, we now investigate the ex post wedges.

Ex post Wedges

We now define ex post intertemporal distortions, the retirement savings wedge and the bequest wedges. Given an allocation $(c_1, c_2, c_{c2}, c_{c3}, y)$ and a type θ, we define the ex post retirement savings wedge

$$t_a(\theta) = 1 - \frac{u'(c_1)}{Ru'(c_2)}$$

and the ex post bequest wedges

$$t_{b2}(\theta) = 1 - \frac{u'(c_1)}{RV_c'(c_{c2})}, \qquad t_{b3}(\theta) = 1 - \frac{u'(c_2)}{RV_c'(c_{c3})}.$$

These ex post wedges can be understood as the implicit tax on the return to the retirement savings when surviving and the the implicit tax on the inheritance when parents are dying.

Next proposition shows that in the absence of heterogeneity in altruism, the retirement saving wedge and inheritance wedge have the opposite sings.

Proposition 8. *Suppose that $\dot{P}(\theta) > 0$, $m(\theta) = 0$, and $\beta(\theta) = 1$ for all θ. Suppose that $\lambda(\theta)$ is nonincreasing and $(c_1, c_2, c_{c2}, c_{c3}, y)$ solves the relaxed planning problem. Then,*

$$t_a(\theta) > 0, \qquad t_{b2}(\theta) < 0, \quad \forall\theta.$$

Proof See the appendix. ■

This proposition shows that in the presence of mortality risk which is negatively correlated with productivity, the planner who has preferences for redistribution (with nonincreasing $\lambda(\theta)$) would tax the return of the retirement savings and subsidize the inherited wealth. This is because the parents who are likely to live longer values the consumption after the retirement relatively more than the children's consumption. Thus, the planner who wants to redistribute from the high productive parents to low productive parents can achieve this redistribution at the lower efficiency cost by taxing the asset income after the retirement and subsidizing the inherited wealth. In other words, in the presence of a negative correlation between the mortality and productivity, the planner can accomplish distinction among different productivities beyond what can be done by labor income taxes.

Thus, the planner wants to subsidize bequest and tax retirement savings so that the planner can redistribute from the more productive parents who are likely to live longer to the less productive parents whose bequest motive becomes operative with high probability. Note that this redistribution is possible, because the government can observe the realization of the survival and death, and can tax the returns of the saving with contingency. The taxes on the retirement savings are the taxes contingent on the survival, while the inheritance subsidy is the subsidy contingent on the death.

Once again, we want to note that proposition 8 includes the utilitarian case and it is also applied to the case with increasing $\lambda(\theta)$, as long as the preferences for redistribution are not too weak. With the special parametric Pareto weight function (7) we discussed above, proposition 8 holds as long as $\alpha > \hat{\alpha}$.

Note that in this two period model, the ex post bequest wedge is zero for the parents who live long: $t_{b3} = 0$. However, this is because of the simplification assumptions that parents die for sure after living two periods. If we extend the analysis to the economy with multi-periods, then this zero bequest wedge result will only apply to the last periods with no probability of survive.

Another important take-away message is that in the world where the savings can serve for both the precautionary life-cycle function and the bequest function, the asset taxes and the inheritance taxes should be considered together. Proposition 8 shows that the implication for these two distortions are the opposite in this economy with contingent bequest motive.

Retirement Saving Wedge and Bequest Wedge and their Decomposition

The ex post wedges defined above can be interpreted as the tax imposed on the return of state contingent assets. These ex post wedges are helpful for understanding the mechanisms of the intertemporal distortion, but these wedges are hypothetical, since the incomplete market does not allow state contingent asset holdings against the uncertainty of the life cycle. To interpret the retirement saving wedges and the bequest wedges as the taxes imposed on the existing asset in this economy — state non-contingent bond, we now define new wedges.

Given an allocation $c_1, c_2, c_{c2}, c_{c3}, y)$ and a type θ, we define the noncontingnet retirement savings wedge $\tau_a(\theta)$ and the noncontingent bequest wedge $\tau_b(\theta)$ as follows:

$$\tau_a(\theta) \;=\; \left(1-\frac{1}{P(\theta)}\right)+\frac{1}{P(\theta)}\underbrace{\left\{1-\frac{u'(c_1(\theta))}{Ru'(c_2(\theta))}\right\}}_{=t_a(\theta)}+(1-\tau_b(\theta))\frac{1-P(\theta)}{P(\theta)}\frac{V_c'(c_{c2}(\theta))}{u'(c_2(\theta))}, \tag{9}$$

$$\tau_b(\theta) \;=\; \left(1-\frac{1}{1-P(\theta)}\right)+\frac{1}{1-P(\theta)}\underbrace{\left\{1-\frac{u'(c_1(\theta))}{RV_c'(c_{c2}(\theta))}\right\}}_{=t_b(\theta)}+(1-\tau_a(\theta))\frac{P(\theta)}{1-P(\theta)}\frac{u'(c_2(\theta))}{V_c'(c_{c2}(\theta))}. \tag{10}$$

We can understand the retirement savings wedge $\tau_a(\theta)$ as the tax imposed on the return of the non-contingent saving if the parents live long, while we can understand the bequest wedge $\tau_b(\theta)$ as the tax imposed on the return of the same non-contingent saving if the parents die early.

Note that the retirement saving wedge (9) is composed of the three components. We want to remark that this decomposition of the wedge is based on the definition of the wedge given any allocation, not necessarily the optimal allocation. The first term $1-\frac{1}{P(\theta)}$, which is negative, is a component arising due to non-existence of annuity market. The second term $\frac{1}{P(\theta)}t_a(\theta)$ is the normalized ex post retirement saving wedge. The third term $(1-\tau_b(\theta))\frac{1-P(\theta)}{P(\theta)}\frac{V_c'(c_{c2}(\theta))}{u'(c_2(\theta))}$ captures the interaction of the asset and the inheritance due to overlapping functions of saving.

We can better understand the third term by rewriting the third term:

$$(1-\tau_b(\theta))\tfrac{1-P(\theta)}{P(\theta)}\tfrac{V_c'(c_{c2}(\theta))}{u'(c_2(\theta))}=-\left(1-\tfrac{1}{P(\theta)}\right)+\tfrac{1-P(\theta)}{P(\theta)}\left(\tfrac{V_c'(c_{c2}(\theta))}{u'(c_2(\theta))}-1\right)-\tau_B(\theta)\left(\tfrac{1-P(\theta)}{P(\theta)}\right)\tfrac{V_c'(c_{c2}(\theta))}{u'(c_2(\theta))}.$$

The third term is again decomposed into the three component: (i) undoing the subsidy, (ii) adjusting the risk — which arises due to no state-contingent asset, (iii) undoing the indirect distortion of the inheritance tax $\tau_b(\theta)$ on the retirement saving.

Notice that the net cost/benefit of the no contingent market is $\frac{1-P(\theta)}{P(\theta)}\left(\frac{V_c'(c_c(\theta))}{u'(c_2(\theta))}-1\right)$, which is the component (ii) in the third term. For example, if the marginal utility the children in the event of the parent's death is smaller larger than the marginal utility of the parent in the event of their surviving, then we can adjust this risk by increasing tax on the parent's asset when they survive.

It is instructive to compare the retirement savings wedge in this economy with that of an economy without the bequest motive (with $V_c(\cdot)=0$), which is typically considered in the literature on the retirement financing and the annuity. In an economy without the bequest motive, the saving wedge is only composed of the first two terms in (9) because there is no interaction between the retirement saving and the inheritance. Hosseini and Shourideh (2016) argue that there should be a huge asset subsidy using the model without bequest motive, because the first component — subsidy to complete the annuity market — dominates the second component in their quantitative analysis. Our results show that when there is an overlapping motive of the retirement saving, the cost of the no annuity market is lower, and thus there is less need for the subsidy.

The three components of the bequest wedge (10) can be also interpreted in the similar way. The first term $1-\frac{1}{1-P(\theta)}$ is to complete the incomplete market, and the second term $\frac{1}{1-P(\theta)}t_b(\theta)$ is the normalized ex post bequest wedge. The third term $(1-\tau_a(\theta))\frac{P(\theta)}{1-P(\theta)}\frac{u'(c_2(\theta))}{V_c'(c_c(\theta))}$ captures the interaction of the asset and the inheritance — adjustment of the subsidy due to less costly incomplete market and the compensation for the indirect distortion caused by the asset tax $\tau_a(\theta)$.

As we mentioned above, the definition of $\tau_a(\theta)$ and $\tau_b(\theta)$ is given for any allocation, and there are many combinations of $\tau_a(\theta)$ and $\tau_b(\theta)$ that satisfy (9) and (10), since (9) and (10) are essentially the same Euler equations. To guarantee the optimal ex post wedges $t_a(\theta)$ and $t_b(\theta)$, however, the retirement savings wedge $\tau_a(\theta)$ and the bequest wedge $\tau_b(\theta)$ should guarantee that the after distortion marginal utility should be equalized:

$$(1-\tau_a(\theta))Ru'(c_2(\theta)) = (1-\tau_b(\theta))RV_c'(c_c(\theta)). \tag{11}$$

Then the wedges that can satisfy both the Euler equation (9) (or equivalently (10)) and (11) are :

$$\tau_a(\theta) = t_a(\theta), \qquad \tau_b(\theta) = t_b(\theta), \quad \forall\theta.$$

3.1.5 Role of Discount Factor

Finally, we now briefly analyze the role of the heterogeneous discount factor. Empirical evidences show that rich people tend to save more with higher saving propensity (e.g., Dynan, Skinner, and Zeldes (2004)), and one of the reason for this higher saving could be higher discount factor for the rich. Saez (2002) pointed out that the heterogeneous discount factor which is increasing in productivity can be a rationale for the positive taxes on saving. Here, we show that the same argument can be applied to the rationale for the positive taxes on inheritance. We can easily see that by assuming $\dot{\beta}(\theta) > 0$ for all θ. To focus on the role of discount factor, we assume that every parent lives for two period for sure and there is no medical expenses.

Proposition 9. *Suppose that $P(\theta) = 1$, $m(\theta) = 0$, and $\dot{\beta}(\theta) > 0$ for all θ. Suppose that $\lambda(\theta)$ is nonincreasing and (c_1, c_2, c_{c3}, y) solves the relaxed planning problem. Then,*

$$t_a(\theta) > 0, \qquad t_{b3}(\theta) > 0, \quad \forall \theta.$$

Proposition 9 is the straightforward extension of the result in Saez (2002). The positive ex post retirement saving wedge ($t_a(\theta) > 0$) is exactly because of the same reason in Saez (2002). Parents with higher productivity has higher propensity to save because of higher discount factor, then taxation of saving can relax the efficiency cost of redistribution across θ, because higher saving can be an indirect evidence about who has higher productivity. The same argument can be applied to the taxation of inherited wealth if the bequest motive is based on pure altruism with the exactly same discount factor $\beta(\theta)$ for both parent's own future utility and child's utility.

There can be other reasons of why the rich save more and bequest more such as uncertainties about future earnings and medical expenses, bequest motive when the bequest is luxury good, and the asset tested program of the government (See Dynan, Skinner, and Zeldes (2004)). Thus, higher propensity to save for the rich cannot be the direct evidence for the increasing discount factor, and we need to investigate this more seriously in the quantitative analysis.

3.1.6 Summary and Discussion

We now sum up the analyses so far by putting all factors together. Suppose that $\dot{m}(\theta) > 0$, $\dot{P(\theta)} > 0$, and $\dot{\beta}(\theta) > 0$. We also assume that the preferences for redistribution are not too weak so that $\mu(\theta) < 0$. Under this assumption, the ex post wedges can be expressed by

$$\begin{aligned}
t_a(\theta) &= -\frac{\mu(\theta)}{\eta f(\theta)} u'(c_1(\theta)) \left[\frac{\dot{P}(\theta)}{P(\theta)} + \frac{\dot{\beta}(\theta)}{\beta(\theta)} \right] > 0 \\
t_{b2}(\theta) &= \underbrace{-\frac{\mu(\theta)}{\eta f(\theta)} u'(c_1(\theta))}_{+} \Bigg[\underbrace{-\frac{V_c''(c_{c2}^*(\theta))}{V_c'(c_{c2}^*(\theta))} \dot{m}(\theta)}_{\equiv t_{b2,m} > 0} \underbrace{-\frac{\dot{P}(\theta)}{1 - P(\theta)}}_{\equiv t_{b2,P} < 0} \underbrace{+\frac{\dot{\beta}(\theta)}{\beta(\theta)}}_{\equiv t_{b2,\beta} > 0} \Bigg] \\
t_{b3}(\theta) &= -\frac{\mu(\theta)}{\eta f(\theta)} R\beta(\theta) u'(c_2(\theta)) \left[-\frac{V_c''(c_{c3}^*(\theta))}{V_c'(c_{c3}^*(\theta))} \dot{m}(\theta) + \frac{\dot{\beta}(\theta)}{\beta(\theta)} \right] > 0.
\end{aligned}$$

This expression of the wedge shows that the ex post intertemporal wedges are decomposed into the components, where each of them reflects the effects of each channel highlighted above — the medical expense channel, the mortality channel, and the (discounting) preference channel.

The implicit tax on the return of retirement savings $t_a(\theta)$ is positive because more productive parents tend to save more because they tend to live longer and be more patient. Thus taxing the retirement saving makes the redistributive tax system more efficient. Taxation of savings at lower income are also positive because this prevents productive parents from under save.

On the other hand, the implicit tax on the inherited wealth $t_b(\theta)$ can be either positive or negative because the mortality channel has the opposite implication from those of the medical expense channel and the β-preference channel. Both increasing medical expenses and increasing discount factor make the more productive parents value the bequest more, implying that the inheritance taxation is useful for redistribution. However, uncertainty in life span has more subtle implication. The mortality risk makes the ex ante intertemporal wedge positive, which implies that the return of saving should be taxed in general. However, the negative correlation between the mortality and productivity has the opposite implications on the taxation of saving and the taxation of inheritance. If this mortality channel is strong enough, then it can be optimal to subsidize the inheritance for the efficient redistribution.

Previous studies found the economic reasons for either taxing or subsidizing the bequest from the positive externality of the bequest or the planner's direct preferences for altruism. When the government puts a direct Pareto weight for children in addition to indirect valuation through altruism there is positive externality of the bequest — which is rationale for subsidy (Farhi and Werning (2010)).[7] On the other hand, the heterogeneity in altruism and the planner's direct preferences across the altruism can generate rationales for both subsidy and tax (Farhi and Werning (2013)). We show that even if the planner does not have direct preferences over the bequest or altruism — the planner's preferences for redistribution is mainly based on the heterogeneity in earning, the correlation between the earning ability and the key factors which determines the bequest motive could be another reason for either taxing or subsidizing bequest.

There can be other factors that can affect the propensity to bequest for rich people. For example, the rich might have higher altruism than the poor, or the bequest is luxury good. What are the impact of these factors on the optimal inheritance taxation? Heterogeneity in altruism is another type of preference heterogeneity which is very similar to the discount factor heterogeneity. Thus, if the degree of altruism is increasing in income, then the same argument as in the β-preference

[7] Farhi and Werning (2010) showed that if the planner does not put a direct Pareto weight for children and value the children's utility only indirectly through the altruism of the parents, then there is no need to distort the bequest decision.

channel applies : the positive correlation between the altruism and the income can provide a rationale for the positive inheritance taxation. However, we do not have clear empirical evidences on this correlation, and thus the heterogeneity in altruism might not be the first order concern for the inheritance tax.

Many previous studies have pointed out that bequests are luxury good, and this is very important to explain why a large fraction of poor parents do not leave bequest at all (or leave only insignificant amount). Typical way of modeling this feature is assuming that there is some consumption floor in the utility from bequests : $V_c(c_c) = U(\underline{c} + c_c)$, where $\underline{c} > 0$ is the threshold consumption level under which people do not leave bequest. It is important to have this factor to match the bequest behavior in the data, but this is not important for the optimal inheritance tax unless the consumption floor $\underline{c}$ has some correlation with the income-generating abilities. This is rather straightforward implication of uniform commodity taxation results of Atkinson and Stiglitz (1976).

3.2 Nonlinear Taxation : Implementation

to be filled out

4 Extensions

4.1 Intergenerational Redistribution: Pareto-Efficient Allocation

So far, we have analyzed the constrained efficient allocation when the planner only cares about the utility of the parents. We now extend our analysis to the planner who also cares about children's directly.

[to be continued...]

4.2 Infinite Horizon Model with Productivity Shock

We now extend our anaysis to the infinite horizon economy with overlapping generations. The economy lasts for the infinite periods: $t = 1, \cdots, \infty$. In eacy period t, a continuum of new generation is born, and we assume that each individual in any generation can live for at most 90 years, and we also assume that each individual gives a birth to his child at age 30. Thus, the age gap between parent and child is 30 years.

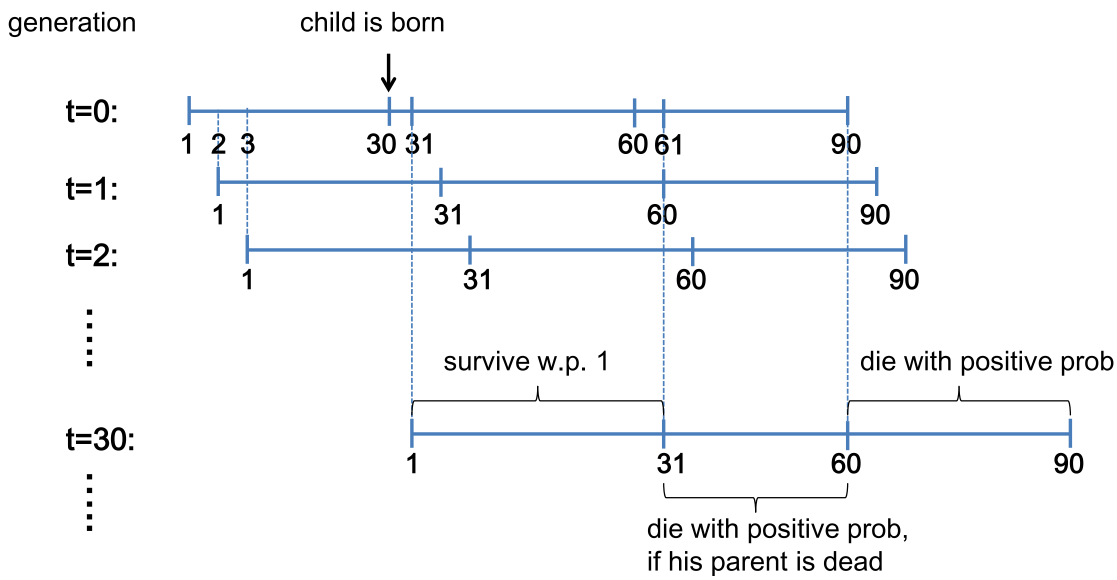

Figure 1: Timeline

In the two period model, we have maintained the assumption that only parents have working productivity and the parent's productivity is perfectly inherited to the consumption of the child. We now consider the case where there is a productivity shock for each generation. That is, an incividual in generation t is born with productivity type θ_t, where θ_t is drawn from the distribution $F(\theta)$, which is i.i.d. across people within generation and across generations. In this economy, the parent's income-generating ability is not perfectly inherited to the consumption of the children, and there will be some mean reversion of consumption across generation.

The working productivity of the individaul wih θ_t at age j is $\varphi_j(\theta)$. In addition, θ_t is correlated with the factors of the bequest motive. We assume perfect correlation, by assuming that the medical expenses, mortality, and the discount factors are function of θ_t. The survival probability of individaul with θ at age j conditional on the survival by the age $j-1$ is denoted by $p_j(\theta_t|S_{j+29}^{t-30})$, where S_j^t is the indicator function of survival — $S_j^t = 1$, if t-th generation with θ_t is alive at age j.

[to be continued...]

5 Numerical Analysis

From the theoretical analysis with the simple two period model, we could see that in theory, various range of results are possible and the sign and the shape of the optimal tax system does depend on the calibration of the mortality and the altruism and the welfare criterion. We thus need to investigate these important determinant seriously to get the quantitative results.

[to be continued...]

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Appendix

to be filled out