Substitution Between Private and Government Consumption in African Economies*

Taufiq Carnegie Dawood[a] and John Nana Francois[†,b]

[a]Economics and Business Department, Syiah Kuala University, Darussalam, Banda Aceh, Indonesia (23111). Email: taufiq.dawood@unsyiah.ac.id
[b]College of Business, West Texas A&M University, 2501 4th Avenue, Canyon, TX 79015, United States. Email: jfrancois@wtamu.edu

Abstract

In a context of elevated public debt and slower global growth, a number of countries in Africa are facing the prospect of sustained declines in public consumption. The macroeconomic impact of such adjustments will depend importantly on whether a decline in government consumption increases or decreases the marginal utility of private consumption. Employing a cointegration-panel approach, we estimate the intratemporal elasticity of substitution between private and government consumption in 24 African countries. Our estimates suggest that for plausible values of the relevant intertemporal elasticity, private and public consumption are Edgeworth substitutes in private utility. Countries facing fiscal consolidation can therefore expect some degree of demand-side offset to reductions in public consumption, and some resulting moderation in the impact of austerity on real GDP. In the presence of fungibility, our results also imply a labor-supply offset to declines in foreign aid for public investment. Country-level analysis suggests that these impacts of declines in public consumption may be heterogeneous across countries.

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†Corresponding author.

1 Introduction

The active use of countercyclical fiscal policy over the past decade has stimulated renewed interest in the impact of changes in public spending on aggregate economic activity (Jha et al., 2014). This impact may soon acquire urgency in a number of African countries, where a combination of countercyclical policies and development ambitions has led to an erosion of the fiscal space these countries enjoyed at the outset of the global financial crisis (World Bank, 2017). In the context of elevated public debts, a period of higher global interest rates and continued weakness in commodity prices and global growth would likely require sustained declines in public consumption.

An extensive literature predicts that increases in government consumption raise output in the short run. Both the short- and long-run impacts, however, are mediated by the response of private consumption to changes in government consumption. The size and direction of this response remain a source of debate in both the empirical and theoretical literatures (Leeper et al., 2017). At the core of this issue is whether public and private consumption are Edgeworth substitutes, in which case reductions in public consumption increase the marginal utility of private consumption *ceteris paribus*, or Edgeworth complements. In the substitutes case, a decline in public consumption generates at least a partial offset in private consumption, which may be accompanied by an increase in labor supply. These responses moderate the aggregate impact on output, both in the short run and over time. In the complements case, private-sector responses exacerbate the impact on output of the decline in public consumption (Ganelli and Tervala, 2009).

This paper examines the empirical relationship between government and private consumption in African economies. Following the approach of Ogaki (1992), we estimate the intratemporal elasticity of substitution between government and private consumption in a panel of 24 countries. When combined with plausible values for the relevant intertemporal

elasticity, our pooled results imply that government and private consumption are Edgeworth substitutes in African countries. Countries facing fiscal consolidation can therefore expect some degree of demand-side offset to reductions in public consumption, and some resulting moderation in the impact of austerity on real GDP.

Foreign aid finances a substantial share of government expenditures in African countries, leaving a number of countries in our sample also vulnerable to declines in aid-financed public spending. In an extension of the fungibility literature, we show that the intratemporal elasticities estimated in this paper play an important role in determining the growth impact not only of undifferentiated budget support but also of aid that is earmarked for domestic investment. In the presence of what Morrissey (2006) and others call general fungibility, changes in investment-earmarked aid affect the level of government consumption, altering the marginal utility of private consumption and activating the channels emphasized in this paper. In the Edgeworth substitutes case, fungibility that is high enough to undermine the growth impact of investment-earmarked aid will for the same reason substantially soften the growth impact of a decline in aid.

In what follows, Section 2 reviews the relevant fiscal policy and aid literatures. Section 3 presents the econometric methodology, which is based on a cointegrating regression model interpreted as an equilibrium condition in an optimal fiscal policy setting. Section 4 presents and discusses the policy implication of the results, and Section 5 concludes.

2 Related Literature

Two goods are Edgeworth substitutes (complements) in utility if their cross-partial derivative is negative (positive), so that *ceteris paribus*, an increase in consumption of the first good reduces (increases) the marginal utility of the second (Amano and Wirjanto, 1998; Karras, 1994). A well-known implication of Edgeworth complementarity is that changes in government consumption can alter the demand for private spending in a direction that offsets the

conventional wealth effect of the fiscal policy (Ercolani and e Azevedo, 2014). Intuitively, when private and government consumption are complements (substitutes), an increase in government consumption increases (decreases) the marginal utility of private consumption, which then mitigates (reinforces) the negative wealth effect of higher public spending. If complementarity is sufficiently strong, the marginal utility effect can fully outweigh the negative wealth effect, leading to an increase in private consumption (Bouakez and Rebei, 2007). In contrast, substitutability is a sufficient condition for increases in public consumption to crowd out private consumption.

While there is an extensive empirical literature on the relationship between private and government consumption, that literature has focused predominantly on non-African countries (Amano and Wirjanto, 1997, 1998; Aschauer, 1985; Auteri and Costantini, 2010; Brown and Wells, 2008; Chiu, 2001; Ho, 2001; Karras, 1994; Kormendi, 1983; Kwan, 2009; Okubo, 2003). These studies suggest that patterns of substitutability vary across countries and regions and may be correlated with structural features that include development levels and the composition of public spending (Karras, 1994). The central aim of this paper is to extend the literature to African countries by using time-series methods to uncover the intratemporal elasticity of substitution between private and public consumption in a sample of 24 African countries during the period 1981-2013. To our knowledge, this paper is the first to present systematic evidence on this key substitution elasticity in African countries.

Our approach follows Amano and Wirjanto (1997, 1998) and more closely Kwan (2009) and Brown and Wells (2008), who employ the cointegration-based strategy of Ogaki (1992) and Ogaki and Park (1997) to estimate the elasticity of substitution in private utility between government and private consumption. This strategy uses the long-run restriction imposed by the intraperiod first-order condition that characterizes the optimal choice of private and government consumption from the perspective of a social planner. Taken together with plausible values of intertemporal elasticity of substitution, our pooled estimates show that

private and public consumption are Edgeworth substitutes, implying that increases in public consumption reduce the marginal utility of private consumption.

The pattern of Edgeworth substitutability or complementarity is an issue of potential urgency in the context of African countries. While aggregate growth in the continent is projected to rise in the near term, with projected growth rates of 3.2 and 3.5 percent in 2018 and 2019 (IMF, 2017), public debt as a share of GDP has been increasing. In close to half of the economies in the sub-Saharan African region, public debt as a ratio to GDP stands above 50 percent (IMF, 2017). As global interest rates rise and particularly if global outcomes falter, these countries may face a period of fiscal consolidation in order to ensure macroeconomic stability and sustained growth. In the past, fiscal consolidations in developing economies have generally been associated with negative effects on output.[1] A number of studies find, moreover, that adjustments accomplished through reductions in government investment have larger contractionary effects than adjustments based on cuts in government consumption (Arizala et al., 2017; Mallick, 2006). These observations suggest that fiscal consolidations have to be well designed to avoid highly adverse impacts on output. Given the substantial reliance on publicly-provided goods and services in African countries, it is important to investigate the relationship between private and government consumption and how a fiscal consolidation involving cuts in government consumption will impact aggregate output. As developed further below, our pooled estimates suggest that in Africa, substitutability patterns will typically moderate rather than exacerbate the contractionary impact of cuts in government consumption.

A distinctive feature of African economies is their dependence on foreign aid to finance public expenditures. The role of intratemporal substitutability in mediating the economy-wide impact of changes in aid provides a final and novel motivation for our analy-

[1]See, IMF (2017) for further discussion. In the context of developing Asia, it was observed that in the aftermath of the Asian financial crisis, IMF-supported fiscal austerity measures contributed to output collapse in the first year of the stabilization programmes (Mallick, 2006).

sis. McGillivray and Morrissey (2001b) note that most of the aid that is intended to expand productive capacity and improve long-term growth in Africa goes to the public sector, where it is managed by the recipient government. The growth impact of such aid depends among other things on the degree of general fungibility, defined as the share of such aid that ends up financing an increase in government consumption rather than the intended increase in government investment. The growth impact of investment-earmarked aid, in the McGillivray and Morrissey analysis, is a declining function of the degree of general fungibility. Morrissey (2015) argues, in contrast, that government consumption typically complements public investment and enhances its productivity, so that concerns about general fungibility are misplaced.[2] Neither study, however, examines how intratemporal substitutability interacts with general fungibility to determine the effectiveness of investment-earmarked aid in raising growth. In an analytical appendix, we develop and numerically simulate a simple theoretical model to illustrate the implications of our empirical findings for the aggregate impact of aid in African countries.

3 Empirical Evidence

In this section we investigate the intratemporal elasticity of substitution between private and public consumption by estimating a cointegrating regression of the form

$$\ln(C_t/G_t) = \beta_0 + \beta_1 \ln(P_t^g/P_t^c) + \nu_t, \tag{1}$$

where $\ln(C_t/G_t)$ is the logarithm of the ratio of private to government consumption and $\ln(P_t^g/P_t^c)$ is the logarithm of the relevant inverse price ratio. Both $\ln(C_t/G_t)$ and $\ln(P_t^g/P_t^c)$ are difference-stationary I(1) process, and ν_t is a stationary I(0) process, implying that the

[2]Other authors have argued that fungibility does not undermine aid effectiveness in an environment of sound policies, appropriate allocations and effective services (e.g., Hauck et al., 2005; McGillivray and Morrissey, 2004; Morrissey, 2006; Pettersson, 2007; Wagstaff, 2011).

two variables are cointegrated. We provide formal statistical evidence for the cointegration property below. The gradient parameter β_1 governs the elasticity of substitution between private and public consumption.

The parameter β_1 can be estimated consistently from Eq.(1) even though there may be measurement errors or stationary omitted variables. This is possible because with cointegration regression, gradient parameters can be estimated consistently without the assumption that the regressors are econometrically exogenous.

Eq.(1) is a reduced form specification and does not have any structural interpretations. To lend a structural interpretation to Eq.(1), we follow Ogaki and Park (1997), Ogaki and Reinhart (1998), Ogaki (1992) and more closely Kwan (2009) and assume a representative consumer who gains utility from two goods, private and public. The agent's expected lifetime utility function is governed by Eq.(2) and is subject to stationary preference shocks:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^e) \tag{2}$$

where $u(.)$ takes the constant relative risk aversion (CRRA) form $u(C^e) = \frac{(C^e)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}$, with $1/\gamma$ representing the intertemporal elasticity of substitution. Effective consumption C^e is a constant elasticity of substitution (CES) aggregate of private and public consumption:[3]

$$C_t^e = [\lambda \varepsilon_t C_t^{1-(1/\eta)} + (1-\lambda)\epsilon_t G_t^{1-(1/\eta)}]^{1/(1-(1/\eta))} \tag{3}$$

where the random preference shocks $(\varepsilon_t, \epsilon_t)$ are strictly stationary with unit means. These stationarity assumptions imply that preferences are stable in the long run. The preference parameters $\lambda \in [0,1]$ and $\eta \geq 0$ represent the relative weight assigned to private goods

[3]Earlier empirical studies such as Kormendi (1983), Aschauer (1985), and Evans and Karras (1996) specify effective consumption as a weighted average of private and government consumption: $C^e = C_t + \alpha_g G_t$, where each unit of government goods is equivalent to α_g units of private goods, irrespective of the current consumption level of the two goods. While this functional form is convenient for analytical tractability it is empirically restrictive (see, Kwan (2009) for a detailed discussion).

and the intratemporal elasticity of substitution, respectively. An intratemporal elasticity of substitution that is greater (less) than one implies gross substitutability (complementarity) between private and public consumption. When η is equal to zero, the two goods are perfect gross complements. Finally, estimated values of η less than zero are theoretically implausible as they violate standard properties of the consumer utility function (Ogaki et al., 1996).

With the assumption that the agent's utility function is time separable, the optimal consumption bundle satisfies an equality condition between the marginal rate of substitution (MRS) and the relevant relative price. Hence, we obtain the condition:

$$\frac{\partial U_t/\partial G_t}{\partial U_t/\partial C_t} \equiv \frac{\epsilon_t \lambda C_t^{1/\eta}}{\varepsilon_t (1-\lambda) G_t^{1/\eta}} = \frac{P_t^g}{P_t^c} \tag{4}$$

Taking logs in Eq.(4), we obtain:

$$\ln\left(\frac{C_t}{G_t}\right) = -\eta \ln\left(\frac{1-\lambda}{\lambda}\right) + \eta \ln\left(\frac{P_t^g}{P_t^c}\right) - \eta \ln\left(\frac{\epsilon_t}{\varepsilon_t}\right) \tag{5}$$

As mentioned earlier, stability of preferences implies that the residual term $-\eta \ln(\epsilon_t/\varepsilon_t)$ is stationary, and thus that Eq.(5) is a cointegrating regression provided that the log price ratio $\ln(P_t^g/P_t^c)$ and the log consumption ratio $\ln(C_t/G_t)$ are both I(1) processes. The combination of stable preferences and the optimality condition in Eq.(4) therefore imposes a cointegration restriction on the co-movements of the log consumption ratio and log price ratio series.[4]

Equation (5) gives us structural interpretation for the parameters in equation (1), with $\beta_0 = -\eta \ln\left(\frac{1-\lambda}{\lambda}\right)$, $\beta_1 = \eta$, and $\nu_t = -\eta \ln\left(\frac{\varepsilon}{\epsilon}\right)$. Gross complementarity between private and government consumption corresponds to estimates of β_1 between zero and one, while estimates of β_1 greater than or equal 1 imply gross substitutability. It is important to point

[4] Kwan (2009) offers detailed discussion of the stable-preferences assumption based on formal results in Ogaki (1992).

out that the estimation equation is void of the intertemporal elasticity of substitution. This allows us to focus on uncovering the intratemporal elasticity of substitution without having to make any stringent assumptions on the intertemporal elasticity of substitution.

In panel form, Eq.(1) can be written

$$\ln\left(\frac{C}{G}\right)_{i,t} = \beta_{i,0} + \beta_1 \ln\left(\frac{P^g}{P^c}\right)_{i,t} + \nu_{i,t} \tag{6}$$

where the inclusion of country fixed effects allows for cross-country heterogeneity in the weight λ on the private-consumption component in the CES aggregate.

3.1 Data and Pre-Testing

We employ annual data for 1981 to 2013 from the World Development Indicators (World Bank 2017) for 24 African countries. The initial dataset contains all 54 African countries and covers the period 1960 to 2015. However, 30 of of the countries had to be dropped due to data availability. Additionally, the start and end dates for the data are driven by missing observation for years leading up to 1980 and after 2013. The 24 countries that remain in the sample collectively account for approximately 55 and 80 percent of total African population and GDP, respectively.[5] To obtain the the consumption ratios C_t/G_t, we divide household final consumption expenditure by general government final consumption expenditure, both in 2010 constant dollars. The corresponding prices P_t^c and P_t^g are computed as the implicit price deflators, which are constructed by dividing the nominal private and government consumption series by their respective constant price series.

[5]Due to data availability, the GDP for Eriteria, Niger, Somalia, and Lybia were not included in the total African GDP. However, it is worth noting that even if the GDP data for these countries was included in the total share of GDP, the calculated share in GDP for the countries considered in this study will still exceed 50 percent.

3.1.1 Unit Root Tests

It has been widely shown that most of the unit root tests for time series have low power and therefore accept the null of a unit root too often. The extension of unit root tests to a panel framework improves the power of unit root testing by incorporating information contained in the cross-sectional dimension. We therefore pool data from the 24 countries to perform four panel unit root tests (Levin, Lin, and Chu (2002), Breitung (2000), Im, Pesaran, and Shin (2003) (IPS), and Maddala and Wu (1999)). The first two tests assume a common autoregressive coefficient across all cross-sections, while the final two (IPS and Maddala and Wu) allow more flexibility by permiting the autoregressive coefficient to vary across cross-sections. This flexibility makes the IPS and Maddala and Wu tests more desirable and hence bear more weight than the results of the LLC and Breitung tests.

Table 1: Panel Unit Root Tests

	Level		First Difference	
	$\ln(C_t/G_t)$	$\ln(P_t^g/P_t^c)$	$\Delta\ln(C_t/G_t)$	$\Delta\ln(P_t^g/P_t^c)$
Levin, Lin and Chu	-0.22482	1.0838	-22.2083	-15.0288
	(0.4111)	(0.8608)	(0.0000)	(0.0000)
Brietung t-stat	-0.73580	2.2504	-12.5302	-9.0609
	(0.2309)	(0.9878)	(0.0000)	(0.0000)
IPS W-statistic	0.0066	1.9094	-204829	-16.1227
	(0.5026)	(0.9719)	(0.0000)	(0.0000)
Maddala and Wu (ADF-Fisher)	44.7273	32.4306	520.609	341.699
	(0.6077)	(0.9585)	(0.0000)	(0.0000)

Notes:p-values are in parenthesis. Null: Unit root. The Levin, Lin, and Chu and Brietung tests assume common unit root process. The IPS and augmented Dickey-Fuller (ADF)-Fisher tests assume individual unit root process. Exogenous variables include individual country effects and country-specific linear trends. Automatic lag length selection is based on the modified Akaike information criteria (MAIC) with a maximum lag of 4.

Table 1 reports the formal panel unit root test results. It is evident from the table that all four tests fail dramatically to reject the unit root null hypothesis for the level series. However, the unit root null hypothesis is strongly rejected when we employ the first differenced series. The results therefore show that the log price and consumption ratio series

are non-stationary I(1) processes.

Furthermore, the p-values obtained for the case of the level series suggests that the log consumption ratio has a weaker random walk component than the log price ratio. That is, the log consumption ratio is less integrated than the log price ratio. As proposed by Ng and Perron (1997) and applied in Kwan (2009), it is more desirable to put the more integrated series as the regressor and the less integrated series as the regressand. Applying the Ng and Perron rule to our case means that the cointegrating regression specification in Eq.(6) is indeed the right choice for estimation.

4 Results

Table 2 reports the panel cointegrating regression results with country-specific fixed effects. Additionally, the bottom part of the table reports seven panel cointegration tests proposed by Pedroni (1999) to determine whether a cointegrating relationship exist in Eq.(6). The test employs panel statistics and group panel statistics to test the null hypothesis of no cointegration against the alternative hypothesis of cointegration. The third column under the cointegration test presents panel and group statistics for the baseline model in Eq.(6). As shown from the results, there is strong evidence of panel cointegration as all the tests decisively reject the null hypothesis of no cointegration. All tests are conducted at a 5 percent level of signficance.

We now turn our focus to the estimation results in the top panel of Table 2. The parameter of interest is the coefficient of the log price ratio, β_1, which is the intratemporal elasticity of substitution between government and private consumption. β_1 is estimated to be between 0.53 to 0.59, and it is statistically significant at all conventional levels across the different estimation methods employed. These estimated values suggest that government and private consumption are on average gross complements in Africa.

Table 2: Panel Regression Results and Cointegration Tests

	Regressors		
	$\ln(P_t^g/P_t^c)$	$\ln(P_t^g)$	$\ln(P_t^c)$
Estimation (DOLS)	0.5861^{***}	–	–
	(0.0571)		
	–	0.5491^{***}	-0.5601^{***}
		(0.0646)	(0.0689)
Estimation (OLS)	0.5293^{***}	–	–
	(0.0270)		
	–	0.5321^{***}	-0.5192^{***}
		(0.0293)	(0.0313)

Cointegration Test	Statistic	Baseline Est. Eq.	Robust Est. Eq.
Panel Test	v-stats:	$2.4427^{\dagger}$	$2.0182^{\dagger}$
	rho-stats:	$-4.0277^{\dagger}$	$-2.9218^{\dagger}$
	PP-stats:	$-4.3636^{\dagger}$	$-4.3570^{\dagger}$
	ADF-Stats:	$-4.4149^{\dagger}$	$-4.3667^{\dagger}$
Group Test	rho-stats:	$-2.6307^{\dagger}$	$-1.7630^{\dagger}$
	PP-stats:	$-4.5578^{\dagger}$	$-4.9581^{\dagger}$
	ADF-stats:	$-3.7612^{\dagger}$	$-4.2734^{\dagger}$

[a] Top panel notes: Regressand is $\ln(C_t/G_t)$. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. All regressions include country-specific fixed effects. DOLS is Kao and Chiang (2000) panel dynamic ordinary least squares. The DOLS regression is augmented with one lead and one lag of the first difference of the regressors. [b] Bottom panel notes: Null hypothesis: No cointegration. † indicates rejection at the 5% level or better. The test results in the third column are related to equation (6). The last column is related to the estimation equation $\ln(C_{i,t}/G_{i,t}) = \alpha_{i,0} + \alpha_1 \ln(P_{i,t}^g) + \alpha_2 \ln(P_t^c) + \nu_{i,t}$. Automatic lag length selection is based on AIC with a maximum lag of 4.

For purposes of robustness and specification check, we also estimate the following unrestricted version of the baseline equation:

$$\ln(C_{i,t}/G_{i,t}) = \alpha_{i,0} + \alpha_1 \ln(P_{i,t}^g) + \alpha_2 \ln(P_{i,t}^c) + \nu_{i,t} \tag{7}$$

Estimates of α_1 and α_2 that are opposite in sign but have identical magnitudes support the restriction employed in Eq.(6). The results in Table 2 favor the restriction: specifically, we can not reject the null hypothesis that α_1 and α_2 are similar in magnitude and have opposite signs. The data are therefore consistent with a CES aggregator function for effective consumption as given in Eq.(3).

Our pooled estimates are considerably smaller than elasticities of around 1.06 estimated

by Kwan (2009) for a set of East Asian countries. Kwan's estimates imply gross substitutability between private and government consumption in East Asia, while our estimates for Africa imply gross complementarity. In constrast, Brown and Wells (2008) find an intratemporal elasticity of substitution of 0.17 for a panel of six Austrialian states suggesting stronger gross complementarity in Australian states than what our pooled estimates suggest for African countries.

To further appreciate the implications of these estimates for policy, we consider whether the uncovered intratemporal elasticity of substitution implies Edgeworth substitutability or complementarity between private and government consumption. Given the utility function defined in Eq. (2), Amano and Wirjanto (1998) show that the sign of the cross-partial derivative $u_{C,G} = \partial(\partial u/\partial C)/\partial G$ depends on the difference between the intratemporal elasticity of substitution and the intertemporal elasticity of substitution, so that $sgn[u_{C,G}] = sgn[\frac{1}{\gamma} - \beta_1]$. Private and government consumption are therefore Edgeworth complements (substitutes) if the intertemporal elasticity of substitution is greater (less) than the intratemporal elasticity of substitution. If the two preference parameters are equal, then changes in government consumption have no impact on the marginal utility of private consumption.

Relative to advanced economies, plausible values of the intertemporal elasticity of substitution has consistently been found to be small and less than 0.5 for African countries (Ogaki et al., 1996). In this study, we take our baseline intertemporal elasticity to be 0.34, based on estimates for developing countries in Ogaki, Ostry, and Reinhart (1996). We combine this value of the intertemporal elasticity of substitution with our DOLS estimate of the pooled intratemporal elasticity to determine the estimated direction of Edgeworth substitutability.

The sign of the cross partial implied by the pooled estimate is negative (i.e. $sgn[u_{C,G}] = sgn[-0.246]$ using $\beta_1 = 0.5861$). This suggests that private and public consumption are best described as Edgeworth substitutes in most African countries. This results imply that a decrease (increase) in government consumption raises (reduces) the marginal utility of

private consumption. These conclusions are of course robust to higher assumed values of the intertemporoal elasticity up to a maximum value between 0.5 and 0.6.

If elasticities of substitution between private and public consumption differ across countries, a country-by-country analysis on the topic at hand may be more appropriate from a policy perspective. To address this concern, we estimate the intratemporal elasticity of substitution between private and government consumption for each individual country in the panel. We employ three different estimation methods for the country-specific regressions: dynamic ordinary least squares (DOLS) following Stock and Watson (1993), fully modified ordinary least squares (FM-OLS) following Phillips and Hansen (1990), and canonical cointegration regression (CCR) following Park (1992). All three estimation methods are asymptotically efficient procedures used in estimating cointegrating regressions. Employing the three methods ensures robust estimates.

Table 3 reports the country-specific estimates of the elasticity of substitution between private and government consumption for each of the 24 countries in our panel. Generally, the estimated elasticity of substitution between private and government are stable across the estimation methods employed. For ease of discussion, we focus on the results from the DOLS estimates. The results from the table can be grouped into three categories: (1) positive and significant estimates of β_1, comprising 13 countries;[6] (2) negative and significant estimates of β_1 consisting of 2 countries;[7] and (3) insignificant estimates of β_1.[8]

[6]The countries are: Algeria, Benin, Congo, Rep., Kenya, Madagascar, Mauritius, Morocco, Namibia, Nigeria, Sierra Leone, South Africa, Swaziland, and Tunisia.

[7]Gambia and Mauritania

[8]The countries include Botswana, Burkina Faso, Cameroon, Egypt, Gabon, Mozambique, Rwanda, Senegal, and Sudan.

Table 3: Country-Specific Regression Results

	Estimation		
	FM-OLS	CCR	DOLS
	$\ln(P_t^g/P_t^c)$	$\ln(P_t^g/P_t^c)$	$\ln(P_t^g/P_t^c)$
Algeria	0.257*	0.258*	0.612***
	(0.147)	(0.138)	(0.137)
Benin	1.082**	1.046***	1.089***
	(0.473)	(0.361)	(0.355)
Botswana	-0.570	-0.595	-0.641
	(0.560)	(0.576)	(0.529)
Burkina Faso	0.0832	0.0827	0.116
	(0.200)	(0.185)	(0.129)
Cameroon	0.330*	0.331*	0.126
	(0.175)	(0.184)	(0.107)
Congo, Rep.	0.698***	0.647***	0.638***
	(0.169)	(0.194)	(0.162)
Egypt	-0.350	-0.370*	-0.374
	(0.252)	(0.223)	(0.382)
Gabon	-0.187	-0.169	-0.0930
	(0.212)	(0.200)	(0.286)
Gambia, The	-0.459	-0.364*	-0.253*
	(0.318)	(0.217)	(0.153)
Kenya	0.340***	0.346***	0.382***
	(0.0735)	(0.0682)	(0.0685)
Madagascar	0.292**	0.300**	0.256***
	(0.137)	(0.151)	(0.0954)
Mauritania	-0.859**	-0.921*	-1.481**
	(0.429)	(0.477)	(0.626)
Mauritius	0.694***	0.680***	0.625***
	(0.145)	(0.121)	(0.174)
Morocco	0.892***	0.885***	0.919***
	(0.0523)	(0.0458)	(0.0302)
Mozambique	0.340	0.307	0.164
	(0.327)	(0.305)	(0.471)
Namibia	0.635	0.608	0.875***
	(0.447)	(0.504)	(0.272)
Nigeria	0.875***	0.876***	0.921***
	(0.101)	(0.101)	(0.0845)
Rwanda	0.0927	0.0946	0.0440
	(0.163)	(0.160)	(0.220)
Senegal	0.171	0.168	-0.0607
	(0.586)	(0.578)	(0.702)
Sierra Leone	0.664***	0.664***	0.639***
	(0.136)	(0.137)	(0.163)
South Africa	0.684***	0.680***	0.786***
	(0.127)	(0.125)	(0.0638)
Sudan	0.618	0.638	-0.115
	(0.379)	(0.399)	(0.359)
Swaziland	0.589***	0.586***	0.560***
	(0.0950)	(0.0886)	(0.0595)
Tunisia	0.565***	0.566***	0.512***
	(0.129)	(0.130)	(0.100)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. FM-OLS is Fully Modified OLS; CCR is Canonical Cointegrating Regression and DOLS is Dynamic OLS. CCR and FM-OLS use Andrews' automatic bandwith selection method in computing the long-run variance matrix. DOLS includes one lead and one lag of the first difference of the regressors in the augmented regression.

Recall in Section 3 that the intratemporal elasticity of substitution is theoretically re-

stricted to non-negative values (i.e. $\beta_1 \geq 0$). Thus, estimates of the intratemporal elasticity of substitution that fall in category two are theoretically inadmissible as they violate standard preference properties and are therefore ignored.[9] The third category, which represents statistically insignificant estimates of β_1, suggests that we can not reject the null hypothesis that β_1 is equal to zero. This implies that either private and government consumption are perfect complements or that the identifying assumptions in this study do not hold for these countries.

In contrast, the first category of estimates meets the theoretical criteria in that estimates of β_1 are positive and significant. The results show that the intratemporal elasticity of substitution is less than unity for 12 out of the 13 countries, with estimated values ranging between 0.26 in Madagascar to 0.92 in Morocco. This means that for those countries, private and government consumption are gross complements. For one country (Benin) the estimated elasticity of substitution is greater than one, implying that public and private consumption are gross substitutes.

Table 4: Implied Edgeworth Substitutability/Complementarity

	DOLS of Estimates, $\hat{\beta}_1$	$[\frac{1}{\gamma} - \hat{\beta}_1]$ ($\frac{1}{\gamma} = 0.34$)	Substitutability/Complementarity in Edgeworth Pareto Sense
Pooled Estimate	0.586	-0.246	substitutes
Algeria	0.612	-0.272	substitutes
Benin	1.089	-0.749	substitutes
Congo, Rep.	0.638	-0.298	substitutes
Kenya	0.382	-0.042	substitutes
Madagascar	0.256	0.084	complements
Mauritius	0.625	-0.285	substitutes
Morocco	0.919	-0.579	substitutes
Namibia	0.875	-0.535	substitutes
Nigeria	0.921	-0.581	substitutes
Sierra Leone	0.639	-0.299	substitutes
South Africa	0.786	-0.446	substitutes
Swaziland	0.560	-0.220	substitutes
Tunisia	0.512	-0.172	substitutes

Notes: Countries were selected based on estimates satisfying the properties of the utility function, i.e. statistically significant and $\hat{\beta} \geq 0$. The intertemporal elasticity of substitution, $1/\gamma$ is set to 0.34 as in Ogaki, Ostry, and Reinhart (1996).
Source: Authors' Computation.

[9] Ogaki, Ostry, and Reinhart (1996) for further discussion on why the negative estimated values of β_1 are dropped.

Similar to the panel analysis we determine whether the country-by-country estimates imply Edgeworth substitutability/complementarity in individual countries. Table 4 reports the sign of the cross partials implied by our pooled estimates and the country-level estimates of the intratemporal elasticity of substitution. As the table shows, the sign of the cross partial implied by the pooled estimate is negative. Moreover, this sign is negative for all the countries except for Madagascar where the sign is positive. This means that for the given value of the intertemporal elasticity of substitution and the estimated intratemporal elasticities, private and public consumption are best described as Edgeworth substitutes in most African countries. This results imply that a decrease (increase) in government consumption raises (reduces) the marginal utility of private consumption in these economies. In the case of Madagascar, the computed sign of the cross partial is positive suggesting that private and public consumption are Edgeworth complements.

What might explain the observed heterogeneity in country-level patterns of Edgeworth substitutability? Evans and Karras (1996) argues that the degree of complementarity is likely to be an increasing function of the share of public goods in government consumption. In particular, other things equal complementarity should be correlated with a high share of defense spending in government consumption. Based on this logic our finding that public consumption substitutes for private consumption in Africa is not all that surprising as the defense spending share in government expenditure remains small. In particular, compared to other advanced countries such as the U.S where recent studies have found that public and private consumption are complements[10], defense spending as a share of government expenditure in African countries remains low. More specifically, the average share of defense spending in government expenditure between 2005 and 2013 was as low as 0.74 in Mauritania and only as high as 10.3 percent in Burkina Faso. Meanwhile, this value is approximately 17.3 percent in the U.S.[11]

[10]See for instance, Fève, Matheron, and Sahuc (2013)

[11]The shares of defense spending in government expenditure between 2005 and 2013 are obtained from the World Development Indicators (World Bank, 2018)

While the defense spending explanation sheds some light on the relationship between public and private consumption in Africa, there are clearly additional factors at work. This is highlighted by Madagascar and Benin, with very similar defense shares (7.1 and 7.4 percent, respectively) but sharply different patterns of Edgeworth substitutability. Obvious potential explanations include (1) possible heterogeneity in the intertemporal elasticities of substitution $1/\gamma$, and (2) other compositional features beyond defense spending. An understanding of the institutional details of each country in regards to the composition of government consumption might help elucidate the observed differences in the estimates across countries. However, the lack of high-quality disaggregated data on government consumption in African countries limits the scope for a comprehensive analysis of the observed heterogeneity in the relationship between private and public consumption.

4.1 Policy Implications

In this section we discuss the policy implication of our results. Our pooled estimate characterizes private and government consumption as Edgeworth substitutes. Thus, on average, an increase (decrease) in government consumption will reduce (increase) the marginal utility of private consumption. The implication of this result for fiscal policy is that, countries facing fiscal consolidation can therefore expect some degree of private consumption offset to reductions in public consumption, and some resulting moderation in the impact of austerity on real GDP. This finding in the paper is in support of arguments that fiscal adjustments using cuts in public consumption have smaller negative impacts on output than adjustments led by public investment (Arizala et al., 2017; Mallick, 2001, 2006). By symmetry, the negative wealth effect of an increase in public consumption would be reinforced by a fall the marginal utility of private consumption. Expansionary fiscal policy will therefore have a smaller expansionary impact on aggregate demand in most African countries when the impact on the marginal utility of private consumption is accounted for, and might possibly even induce

a net contraction. The marginal utility channel studied in this paper therefore provides an additional mechanism for why fiscal multipliers are small and in some cases negative in African economies (see for instance Ilzetzki et al., 2013).

Beyond fiscal policy, our results have direct implications for the effectiveness of foreign aid in African countries in the presence of general fungibility. Specifically, given that the countries we study in this paper are aid-dependent, we discuss the immediate implication of our results for aid effectiveness when general fungibility exists. To see this connection, consider a simple economic environment where a representative household chooses private consumption and labor hours to maximize consumption, given the level of government spending which also enters the household's utility function. There is also a perfectly competitive firm that maximizes profits taking prices and wages as given. Finally, government investment spending, which is co-financed by foreign aid and lump-sum taxes, is productive. In this simple economic environment, the relationship between private and public consumption affects the marginal disutility of labor. More precisely, if government and private consumption are Edgeworth substitutes (complements), an increase in government consumption–through the marginal disutility channel– will generate some fall (increase) in labor supply. This is because private consumption has become less (more) attractive relative to leisure.

With this channel, a policy concern such as general fungibility can interact with the existing relationship between public and private consumption in an aid-recipient country to impact aid effectiveness, defined here as the long-run effect of aid on output. In particular, when private and government consumption are Edgeworth substitutes, the increase in government consumption that accompanies aid earmarked for public investment will produce a labor-supply offset that diminishes the growth impact of the aid. In contrast, complementarity between private and governnmet consumption would mitigate the negative effects of general fungibility on aid effectiveness. In the Appendix we show that with a sufficiently high degree of complementarity, general fungibility can actually enhance the effectiveness of aid

that is earmarked for public consumption.[12] Our pooled results therefore imply that since private and public consumption are Edgeworth substitutes, an increase in government consumption due to general fungibility will generate a labor-supply offset to falls in foreign aid for public investment. Consequently, if present, general fungibility will on average diminish some of the positive effect of aid on ouput and soften the blow to output if aid falls.

The heterogeneity in our country-level estimates suggests that the impacts of fiscal policy and aid fungibility may differ across countries. Consider for instance the cases of Benin and Madagascar where our estimates suggest that private and public consumption are Edgeworth substitutes and complements, respectively. All else equal, a fiscal consolidation involving cuts in government consumption will have smaller negative effects on real GDP in Benin compared to Madagascar. In a similar vein, a given degree of general fungibility will have a larger impact on diminishing aid effectiveness in Benin than in Madagascar. This strongly suggests that policymakers should account for this heterogeneity in the relationship between private and government consumption across countries when implementing or assessing the potential effects of a policy that changes public consumption.

5 Conclusion

In a context of elevated public debt and slower global growth, a number of countries in Africa are facing the prospect of sustained declines in public consumption. The macroeconomic impact of such adjustments will depend importantly on whether a decline in government consumption increases or decreases the marginal utility of private consumption. This paper employed the cointegration approach of Ogaki (1992) and Ogaki and Park (1997) to estimate the intratemporal elasticity of substitution between government and private consumption in African economies. Our estimates reveal that for plausible values of the relevant

[12] In the analytical/numerical appendix, Appendix A, we develop a simple theoretical model in support of these channels.

intertermporal elasticity, private and public consumption are Edgeworth substitutes in private utility. These results are robust across several estimation methods. They imply that a decrease (increase) in government consumption increases (reduces) the marginal utility of private consumption.

From a policy perspective, the results suggests that countries facing fiscal consolidation can expect some degree of demand-side offset to reductions in public consumption and some resulting moderation in the impact of austerity on real GDP. Furthermore, the results can be novelly applied to aid effectiveness in Africa. More precisely, in a flexible-price setting with general fungibility, our results also imply a labor-supply offset to declines in foreign aid for public investment. Such aid will therefore be less effective in raising GDP when it increases, and less damaing to GDP when it declines. However, country-level analysis further shows that due to the diversity in the estimated intratemporal elasticities of substitution, changes in government consumption will have heterogenous effects across countries.

Although this paper is the first to provide systematic evidence of the intratemporal elasticity of substitution between private and public consumption in African countries, there is scope for further research. Two natural extensions would be to allow for heterogeneity in the intertemporal elasticity, and to develop country-level data on expenditure composition with a view to explaining the cross-country heterogeneity in the estimated intratemporal elasticities. A final extension would address the sizable fraction of consumers who are subject to liquidity constraints in African countries. Private consumption by these agents depends not only on the relative price of public and private consumption, but also possibly on current or transitory disposable income. Future research can therefore investigate how liquidity constraints influence the relationship between private and government consumption across individual countries in Africa.

References

Amano, R. A. and T. S. Wirjanto (1997). Intratemporal substitution and government spending. *The Review of Economics and Statistics 79*(4), 605–609.

Amano, R. A. and T. S. Wirjanto (1998). Government expenditures and the permanent-income model. *Review of Economic Dynamics 1*(3), 719–730.

Arizala, F., M. J. R. Gonzalez-Garcia, M. C. G. Tsangarides, and M. Yenice (2017). *The Impact of Fiscal Consolidations on Growth in Sub-Saharan Africa*. International Monetary Fund.

Aschauer, D. A. (1985). Fiscal policy and aggregate demand. *The American Economic Review 75*(1), 117–127.

Auteri, M. and M. Costantini (2010). A panel cointegration approach to estimating substitution elasticities in consumption. *Economic Modelling 27*(3), 782–787.

Bouakez, H. and N. Rebei (2007). Why does private consumption rise after a government spending shock? *Canadian Journal of Economics/Revue canadienne d'économique 40*(3), 954–979.

Breitung, J. (2000). The local power of some unit root tests for panel data. In B. H. Baltagi (Ed.), *Advances in Econometrics, Vol. 15: Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, pp. 161–178. Amsterdam: JAI Press.

Brown, A. and G. Wells (2008). Substitution between public and private consumption in Australian states. In *Australasian Meeting of the Econometric Society (ESAM08)*, pp. 1–18.

Chiu, R.-L. (2001). The intratemporal substitution between government spending and private consumption: Empirical evidence from Taiwan. *Asian Economic Journal 15*(3), 313–323.

Christiano, L. J. and M. Eichenbaum (1992). Current real-business-cycle theories and aggregate labor-market fluctuations. *The American Economic Review*, 430–450.

Ercolani, V. and J. V. e Azevedo (2014). The effects of public spending externalities. *Journal of Economic Dynamics and Control 46*, 173–199.

Evans, P. and G. Karras (1996). Private and government consumption with liquidity constraints. *Journal of International Money and Finance 15*(2), 255–266.

Fève, P., J. Matheron, and J.-G. Sahuc (2013). A pitfall with estimated dsge-based government spending multipliers. *American Economic Journal: Macroeconomics 5*(4), 141–178.

Galiani, S., S. Knack, L. C. Xu, and B. Zou (2017). The effect of aid on growth: Evidence from a quasi-experiment. *Journal of Economic Growth 22*(1), 1–33.

Ganelli, G. and J. Tervala (2009). Can government spending increase private consumption? the role of complementarity. *Economics Letters 103*(1), 5–7.

Hauck, V., O. Hasse, and M. Koppensteiner (2005). Ec budget support: thumbs up or down? European Centre for Development Policy Management (ECDPM).

Ho, T.-w. (2001). The government spending and private consumption: a panel cointegration analysis. *International Review of Economics & Finance 10*(1), 95–108.

Ilzetzki, E., E. G. Mendoza, and C. A. Végh (2013). How big (small?) are fiscal multipliers? *Journal of Monetary Economics 60*(2), 239–254.

Im, K. S., M. H. Pesaran, and Y. Shin (2003). Testing for unit roots in heterogeneous panels. *Journal of econometrics 115*(1), 53–74.

IMF (2017). World Economic and Financial Surveys: Sub-saharan Africa Fiscal Adjustment and Economic Diversification. IMF, Washington.

Jha, S., S. K. Mallick, D. Park, and P. F. Quising (2014). Effectiveness of countercyclical fiscal policy: Evidence from developing asia. *Journal of Macroeconomics 40*, 82–98.

Juselius, K., N. F. Møller, and F. Tarp (2014). The long-run impact of foreign aid in 36 African countries: Insights from multivariate time series analysis. *Oxford Bulletin of Economics and Statistics 76*(2), 153–184.

Kao, C. and M.-H. Chiang (2000). On the estimation and inference of a cointegrated regression in panel data. *Advances in Econometrics 20*, 179–222.

Karras, G. (1994). Government spending and private consumption: some international evidence. *Journal of Money, Credit and Banking 26*(1), 9–22.

Kormendi, R. C. (1983). Government debt, government spending, and private sector behavior. *The American Economic Review 73*(5), 994–1010.

Kwan, Y. K. (2009). The direct substitution between government and private consumption in East Asia. *Fiscal Policy and Management in East Asia 16*, 45.

Leeper, E. M., N. Traum, and T. B. Walker (2017, August). Clearing up the fiscal multiplier morass. *American Economic Review 107*(8), 2409–54.

Levin, A., C.-F. Lin, and C.-S. J. Chu (2002). Unit root tests in panel data: asymptotic and finite-sample properties. *Journal of Econometrics 108*(1), 1–24.

Linnemann, L. and A. Schabert (2006). Productive government expenditure in monetary business cycle models. *Scottish Journal of Political Economy 53*(1), 28–46.

Maddala, G. S. and S. Wu (1999). A comparative study of unit root tests with panel data and a new simple test. *Oxford Bulletin of Economics and statistics 61*(S1), 631–652.

Mallick, S. and T. Moore (2008). Foreign capital in a growth model. *Review of Development Economics 12*(1), 143–159.

Mallick, S. K. (2001). Dynamics of macroeconomic adjustment with growth: some simulation results. *International Economic Journal 15*(1), 115–139.

Mallick, S. K. (2006). Policy instruments to avoid output collapse: an optimal control model for India. *Applied Financial Economics 16*(10), 761–776.

McGillivray, M. and O. Morrissey (2001a). Fiscal effects of aid. Discussion Paper Series 2001/061. Helsinki: UNU-WIDER.

McGillivray, M. and O. Morrissey (2001b). A review of evidence on the fiscal effects of aid. Technical report, CREDIT Research Paper.

McGillivray, M. and O. Morrissey (2004). *Fiscal effects of aid*. UNU-WIDER/Palgrave, Basingstoke.

Morrissey, O. (2006). Fungibility, prior actions, and eligibility for budget support. *Budget support as more effective aid*, 333–350.

Morrissey, O. (2015). Aid and government fiscal behavior: Assessing recent evidence. *World Development 69*, 98–105.

Ng, S. and P. Perron (1997). Estimation and inference in nearly unbalanced nearly cointegrated systems. *Journal of Econometrics 79*(1), 53–81.

Ogaki, M. (1992). Engel's law and cointegration. *Journal of Political Economy 100*(5), 1027–1046.

Ogaki, M., J. D. Ostry, and C. M. Reinhart (1996). Saving behavior in low-and middle-income developing countries a comparison. *IMF Staff Papers 43*(1), 38–71.

Ogaki, M. and J. Y. Park (1997). A cointegration approach to estimating preference parameters. *Journal of Econometrics 82*(1), 107–134.

Ogaki, M. and C. M. Reinhart (1998). Measuring intertemporal substitution: The role of durable goods. *Journal of Political Economy 106*(5), 1078–1098.

Okubo, M. (2003). Intratemporal substitution between private and government consumption: the case of Japan. *Economics Letters 79*(1), 75–81.

Park, J. Y. (1992). Canonical cointegrating regressions. *Econometrica: Journal of the Econometric Society*, 119–143.

Pedroni, P. (1999). Critical values for cointegration tests in heterogeneous panels with multiple regressors. *Oxford Bulletin of Economics and statistics 61*(S1), 653–670.

Pettersson, J. (2007). Foreign sectoral aid fungibility, growth and poverty reduction. *Journal of International Development 19*(8), 1074–1098.

Phillips, P. C. and B. E. Hansen (1990). Statistical inference in instrumental variables regression with i (1) processes. *The Review of Economic Studies 57*(1), 99–125.

Stock, J. H. and M. W. Watson (1993). A simple estimator of cointegrating vectors in higher order integrated systems. *Econometrica: Journal of the Econometric Society*, 783–820.

Tervala, J. (2009). Productive government spending and private consumption. *Economics Bulletin*.

Thompson, H. (2008). Economic growth with foreign capital. *Review of Development Economics 12*(4), 694–701.

Wagstaff, A. (2011). Fungibility and the impact of development assistance: Evidence from Vietnam's health sector. *Journal of Development Economics 94*(1), 62–73.

World Bank (2017, oct). Africa's Pulse: An Analysis of Issues Shaping Africa's Economic Future. Technical report.

Zanna, L.-F., M. A. Berg, T. Mirzoev, and R. Portillo (2010). *The short-run macroeconomics of aid inflows: understanding the interaction of fiscal and reserve policy*. Number 10-65. International Monetary Fund.

APPENDICES

A A Theoretical Backdrop: Fungibility and Edgeworth Substitutability

We work out a simple general equilibrium model tractable enough to obtain a closed-form formula that describes how Edgeworth substitutability interacts with general fungibility to impact aid effectiveness. Our model therefore abstracts from openness in growth models as described in Mallick and Moore (2008) and Thompson (2008). We define aid effectiveness here as the long-run effect of foreign aid on output.

A.1 The Stylized Model

Consider an economy populated by a unit measure of identical households. Each household maximizes lifetime utility function which depends on consumption c_t, hours worked n_t, and a given level of government provided consumption goods $g_{c,t}$:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s} + \theta g_{c,t+s}) - \frac{n_{t+s}^{1+\varphi}}{1+\varphi}$$

subject to the flow budget constraints with $t \geq 0$,

$$p_t c_t = w_t n_t - p_t \tau_t, \tag{A.1}$$

where $\mathrm{E}\{.\}$ represents the expectation operator. The parameter $\beta \in (0,1)$ is the subjective discount factor, p_t is the general price level in the economy, w_t denotes the nominal wage rate, and τ_t is lump-sum real taxes levied by government. The inverse Frisch elasticity of labor supply is $\varphi \geq 0$.

This functional form for effective consumption $(c_t + \theta g_{c,t})$ follows from Ganelli and Tervala (2009), and Fève, Matheron, and Sahuc (2013) among others. It allows for analytical tractability and offers a short cut to capturing the concept of Edgeworth substitutability between private and government consumption compared to the more general and empirically flexible functional form employed in Section 3. The parameter of interest θ governs the Edgeworth complementarity/substitutability between private (c_t) and government consumption $(g_{c,t})$ such that $\theta < 0$ and $\theta \geq 0$ imply Edgeworth complementarity and substitutability, respectively. For the case where $\theta = 1$, a permanent increase in government consumption has no effect on output and labor hours, but impacts private consumption negatively through full crowding out (Christiano and Eichenbaum, 1992). When $\theta = 0$, government consumption does not affect the marginal utility of private consumption.

Maximization of household utility yields the intratemporal condition

$$n_t^{\varphi} = \frac{w_t}{p_t(c_t + \theta g_{c,t})} \tag{A.2}$$

As can be seen in Eq.(A.2) labor hours depend on the effective consumption in the economy and thus on the sign and size of θ. In particular, if government and private consumption are substitutes in the utility function and there is an increase in government consumption, labor supply will fall because private consumption has become less attractive relative to leisure. In constrast, if the two goods are complements the opposite effect will hold.

The representative firm in the economy produces a final good y_t using labor supplied by households and the flow of services from a public investment good g_I such as infrastructure, through the production function:[13]

$$y_t = n_t g_{I,t}^{\alpha}, \tag{A.3}$$

where $\alpha > 0$ measures the productivity of goverment spending. The assumption that the flow of government investment spending affects the production function is in line with Linnemann and Schabert (2006) and Tervala (2009).

The firm's profit maximization problem is therefore given as

$$\max_{n_t} \pi_t = p_t n_t g_{I,t}^{\alpha} - w_t n_t$$

Maximization of the profit function of the firm relates the price level to the marginal cost in the economy as follows,

$$p_t = \frac{w_t}{g_{I,t}^{\alpha}} \tag{A.4}$$

We assume that the government maintains a balanced budget for each time period t, so that the government budget is given by

$$g_{I,t} + g_{c,t} = \tau_t + a_t \tag{A.5}$$

where a_t is foreign aid, disbursed through the recipient's government budget as a source of financing for public investment. To ensure that the equilibrium system is identified, we assume that the recipient country takes foreign aid as given and that aid follows a simple AR(1) process with an i.i.d shock.[14]

General Fungibility Donors intend public investment good g_I to be co-financed by a combination of foreign aid and domestic revenue. However, given the existence of general fungibility in our model, a fraction ($\phi^F > 0$) of aid earmarked for public investment is diverted to fund government consumption. The following equations describe allocation of domestic resource and foreign aid in the economy.

$$g_{I,t} = (1 - \phi^F) a_t + g_{I,t}^d \tag{A.6}$$

$$g_{c,t} = \phi^F a_t + g_{c,t}^d \tag{A.7}$$

where $g_{I,t}^d$ and $g_{c,t}^d$ are domestic resources used to finance public investment and consumption

[13] Since we focus on intratemporal effects in our model, we abstract from private capital (investment). Investment in the economy is therefore equal to public investment.

[14] Modelling the endogeneity of aid would require a more elaborate model. This assumption follows Zanna, Berg, Mirzoev, and Portillo (2010).

respectively, so that $g^d_{I,t} + g^d_{c,t} = \tau_t$. When $\phi^F = 0$, general fungibility is absent and aid is allocated solely to public investment, as intended by donors.

Finally, the disbursement of aid by donors means that aggregate resource constraint in the economy is given as,

$$y_t + a_t = c_t + g_{c,t} + g_{I,t} \tag{A.8}$$

Combining the household's first order condition (A.2) with the firm's profit maximization condition (A.4) we obtain,

$$y_t^{\varphi} = \frac{g_{I,t}^{\alpha+\alpha\varphi}}{c_t + \theta g_{c,t}} \tag{A.9}$$

The complete equilibrium system of the economy described above consists of Eqs. (A.6), (A.7), (A.8), and (A.9).

A.2 General Fungibility, Edgeworth Substitution and Aid Effectiveness

From the equilibrium system described above, the long-run impact of aid on output in this economy is defined in the following manner.

DEFINITION 1. *The long-run effect of aid on output (i.e. aid effectiveness), denoted by $\bar{y}_a$ is the increase in steady-state output following an increase in steady-state foreign aid given formally by*

$$\bar{y}_a = \frac{\mathrm{d}y}{\mathrm{d}a}$$

Given the definition of aid effectiveness and the simplifying assumption of $\varphi = \frac{dg^d_c}{da} = \frac{dg^d_I}{da} = 0$, the equation below (derived in Appendix B) describes the key properties of the long-run effect of aid on output:

$$\bar{y}_a = \frac{\alpha}{s_{g_I}}(1 + \theta s_{g_c}) - \frac{\phi^F}{s_{g_I}}(\alpha + \theta\nu) \tag{A.10}$$

where $\nu = s_{g_I} + \alpha s_{g_c} > 0$ and $s_{g_c} > 0$ and $s_{g_I} > 0$ are the ratios of steady state public investment and consumption to steady state output, respectively. The second term in Eq. (A.10) is the effect of general fungibility on aid effectiveness. In the absence of fungibility, $\phi^F = 0$ and the long-run effect of aid on output is given in the first term in Eq. (A.10). Without loss of generality, we assume that the effect of aid in the absence of fungibility is positive, by imposing the restriction, $-\theta s_{g_c} < 1$. This assumption is in line with recent empirical evidence that aid has a positive effect on growth (See for instance Galiani, Knack, Xu, and Zou, 2017; Juselius, Møller, and Tarp, 2014). More importantly, when $\phi^F > 0$ the effect of fungibility on aid effectiveness depends on the sign and size of θ, the parameter that governs the Edgeworth substitution between private and government consumption. In particular, if government and private consumption are Edgeworth substitutes then labor supply falls because private consumption has become less attractive relative to leisure. Hence, any

positive effect of fungibility under the complementarity of private and government consumption comes about via an increase in labor supply, and vice-versa in the case of substitutability between private and government consumption (See Eq.(A.2) and Eq.(A.9) for this channel).

The following propositions therefore emerge from Eq. (A.10):

PROPOSITION. *Given the preceeding assumptions and* $\forall\alpha > 0$ *and* $\forall\nu > 0$*:*

1. *The impact of general fungibility on aid effectiveness is,*

$$\frac{\partial\bar{y}_a}{\partial\phi^F} = -\frac{\alpha + \theta\nu}{s_{g_I}} \tag{A.11}$$

where $\frac{\partial\bar{y}_a}{\partial\phi^F} < 0$, $\frac{\partial\bar{y}_a}{\partial\phi^F} > 0$, *and* $\frac{\partial\bar{y}_a}{\partial\phi^F} = 0$ *mean general fungibility has a negative, positive, and no impact on aid effectiveness, respectively.*

2. *Whenever private and government consumption are substitutes (i.e.* $\theta \geq 0$*) or complements with* $-\frac{\alpha}{\nu} < \theta < 0$*, general fungibility has a negative effect on aid effectiveness and* $\frac{\partial\bar{y}_a}{\partial\phi^F}\Big|_{\theta\geq 0} < \frac{\partial\bar{y}_a}{\partial\phi^F}\Big|_{-\frac{\alpha}{\nu}<\theta<0}$

3. *If the relationship between private and government consumption is complementary and* $\theta \leq -\frac{\alpha}{\nu}$*, general fungibility has a non-negative effect on aid effectiveness (i.e.* $\frac{\partial\bar{y}_a}{\partial\phi^F}\Big|_{\theta\leq-\frac{\alpha}{\nu}} \geq 0$*)*

Proof. See Appendix C. ∎

From *Proposition 1* we can see that the effect of fungibility is decreasing in θ. That is, higher Edgeworth substitutability (complementarity) between government and private consumption exacerbates (mitigates) the negative effects of general fungibility on aid effectiveness. Moreover, if the degree of complementarity between the two goods is high enough, fungibility has a positive impact on aid effectiveness. These results emerge because when the two goods are substitutes an increase in government consumption crowds out private consumption which in turn negatively affects output. In contrast, when complementarity exists between the two goods an increase in government consumption crowds in private consumption, which offsets the negative effect generated by diverting aid from public investment. If the degree of complementarity is high enough, general fungibility will have a positive impact on aid effectiveness.

B Proof of Equation A.10

Given the model described in Appendix A, the equilibrium system can be characterized by the four equations

$$y^{\varphi} = \frac{g_I^{\alpha+\alpha\varphi}}{c + \theta g_c} \tag{B.1}$$

$$y + a = c + g_I + g_c \tag{B.2}$$

$$g_I = (1 - \phi^F)a + g_I^d \tag{B.3}$$

$$g_c = \phi^F a + g_c^d \tag{B.4}$$

We can rewrite Eq. (B.1) as:

$$F = y^{\varphi} - g_I^{\alpha+\alpha\varphi}(c + \theta g_c)^{-1} \tag{B.5}$$

Totally differentiating Eq.(B.2), Eq.(B.3), Eq.(B.4), and Eq.(B.5) we arrive at:

$$\mathrm{d}y = \frac{1}{(1 + \varphi(1 + \theta s_{g_c}))}\left[\left(\frac{\alpha + \alpha\varphi}{s_{g_I}}\right)\mathrm{d}g_I - \theta \mathrm{d}g_c\right] \tag{B.6}$$

$$\mathrm{d}g_I = (1 - \phi^F)\mathrm{d}a + \mathrm{d}g_I^d \tag{B.7}$$

$$\mathrm{d}g_I = \phi^F \mathrm{d}a + \mathrm{d}g_c^d \tag{B.8}$$

$$\mathrm{d}y + \mathrm{d}a = \mathrm{d}c + \mathrm{d}g_I + \mathrm{d}g_c \tag{B.9}$$

Based on the definition of general fungibility, we can conveniently set $\mathrm{d}g_c^d/\mathrm{d}a = \mathrm{d}g_I^d/\mathrm{d}a = 0$. We also assume $\varphi = 0$. Combining Eqs.(B.6), (B.7), (B.8) and (B.9) we arrive at:

$$\bar{y}_a = \frac{\mathrm{d}y}{\mathrm{d}a} = \frac{\alpha}{s_{g_I}}(1 + \theta s_{g_c}) - \frac{\phi^F}{s_{g_I}}(\alpha + \theta\nu) \tag{B.11}$$

where $\nu = s_{g_I} + \alpha s_{g_c}$.

C Proof of Proposition

Part 1: It is straigtforward from the definition of the long-run impact of aid on output that the effect of fungibility is:

$$\frac{\partial y_a}{\partial \phi^F} = -\frac{(\alpha + \theta\nu)}{s_{g_I}} \tag{C.1}$$

Part 2: First suppose that government and private consumption are Edgeworth substitutes so that $\theta \geq 0$. This implies that with $\nu > 0$, $s_{g_I} > 0$ and $\alpha > 0$ it easy to show that,

$$-\frac{(\alpha + \theta\nu)}{s_{g_I}} < -\frac{\alpha}{s_{g_I}} < 0 \tag{C.2}$$

Second, suppose that government and private consumption are complements with $-\frac{\alpha}{\nu} < \theta < 0$, and let $\nu > 0$ and $\alpha > 0$ so that

$$-\alpha < \nu\theta < 0 \implies 0 < \alpha + \nu\theta < \alpha \tag{C.3}$$

Multiplying the expression in (C.3) through by $-\frac{1}{s_{g_I}} < 0$ yields

$$-\frac{\alpha}{s_{g_I}} < -\frac{(\alpha+\nu\theta)}{s_{g_I}} < 0 \tag{C.4}$$

The inequalities in (C.2) and (C.4) together complete the proof for part 2.

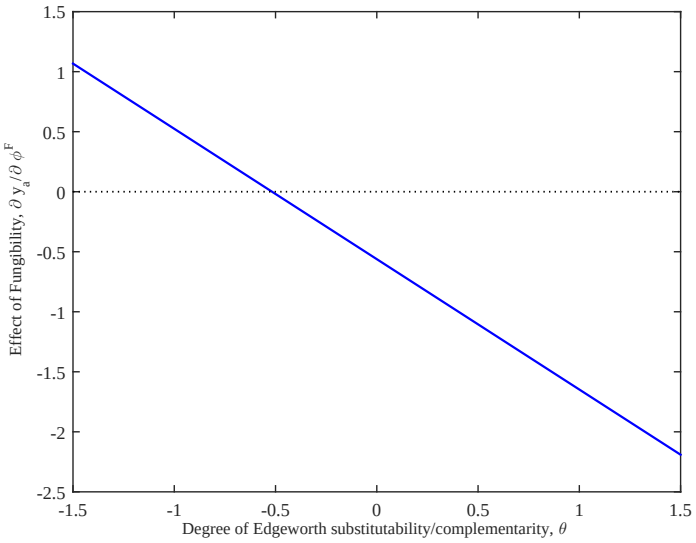
Part 3: Suppose that government and private consumption are complement and $\theta \leq -\frac{\alpha}{\nu}$. Setting $\nu > 0$, $s_{g_I} >$ and $\alpha > 0$ it is straightforward to show that,

$$\theta \leq -\frac{\alpha}{\nu} \implies -\frac{(\alpha+\theta\nu)}{s_{g_I}} \geq 0 \tag{C.5}$$

D Numerical Example from Model

Figure 1 provides a numerical example of the relationship between Edgeworth substitutability and the impact of general fungibility on the effectiveness of aid that is earmarked for public investment. We calibrate the parameters in Eq.(A.11) in a standard way. We rely on Linnemann and Schabert (2006) for α and compute steady-state values using the World Banks' World Development Indicator data for African countries. Accordingly, $\alpha = 0.1$ and the fungibility parameter ϕ^F is set to 0.5 which means for every 1 dollar of aid intended for public investment, 50 cents is diverted to government consumption. The ratio of government consumption to GDP is set to $s_{g_c} = 15.3\%$ and the ratio of government investment to GDP to $s_{g_I} = 17.9\%$. Figure 1 illustrates our analytical result (in *Proposition 1*) that the effect of fungibility on aid effectiveness is diminishing in θ.

Figure 1: Edgeworth Substitutability and the Effect of General Fungibility on Aid Effectiveness



Notes: The effect of fungibility on aid effectiveness is decreasing in the size of θ.