

# Asset Pricing Anomalies and the Low-risk Puzzle<sup>†</sup>

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## Abstract

The original observation in [Black, Jensen and Scholes \(1972\)](#) that the security market line is too flat – the beta anomaly – is a driving force behind a number of well-documented cross-sectional asset pricing puzzles. I document that returns to a broad set of anomaly portfolios are negatively correlated with the contemporaneous market excess return. I show that this negative covariance implicitly embeds the beta anomaly in these cross-sectional return puzzles. Taking into account the exposure to the beta anomaly attenuates the economic and statistical significance of the risk-adjusted returns to a large set of asset pricing anomalies.

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Defying both theory and intuition, low beta assets have consistently outperformed high beta assets, both over time and across various asset markets (Baker, Bradley and Wurgler, 2011; Frazzini and Pedersen, 2014). This observation has come to be known as the beta anomaly. A trading strategy of buying low beta stocks and shorting high beta stocks constructed in Frazzini and Pedersen (2014) produces a monthly CAPM alpha of 73 basis points and a  $t$ -statistic above 7. In this paper, I present evidence that the beta anomaly is embedded in a broad set of cross-sectional asset pricing puzzles. I document that anomaly portfolio returns share a striking and peculiar pattern: returns are positive and peak in market downfalls, but are negative when the market rises. I verify that this negative covariance is empirically equivalent to the long portfolios holding stocks with lower betas relative to the short portfolios. Mitigating the exposure to the beta anomaly either attenuates or eliminates the economic and statistical significance of the risk-adjusted returns to numerous cross-sectional anomalies.

This paper analyzes a set of twelve asset pricing puzzles representative of different types of cross-sectional return predictors documented in the literature. The set is taken from Fama and French (2016), and is complemented by those examined in Stambaugh, Yu and Yuan (2012)<sup>1</sup>. The sample includes anomalies that are firm operation-based (total accruals, return on assets, profitability, investment-to-asset, asset growth), stock return-based (momentum), risk-based (O-score, default probability, total return volatility, idiosyncratic return volatility), as well as stock issuance-based (net stock issues, composite equity issues). It is remarkable that portfolios formed on such a wide range of characteristics all have returns that are negatively correlated with the market. The observed negative covariance has two immediate implications. First, to the extent that these anomaly portfolios hold “quality” stocks (profitable, high past return, mature, low probability of failure, etc.), the fact that they pay off in bad states of the world is consistent with flight

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<sup>1</sup>Absent from the Stambaugh et al. (2012) list is *net operating assets* anomaly. Section 5 discusses *net operating assets* along with the size and value effects.

to quality in market downturns. The negative covariance between “quality” stocks and the market points to the beta anomaly as an explanation for why “quality” stocks have high average returns (Asness, Frazzini and Pedersen, 2014). Second, the shared negative covariance with the market is suggestive of the data mining concern in the empirical asset pricing literature (see for instance Harvey, Liu and Zhu, 2016; Chordia, Goyal and Saretto, 2017, among others.). The search for cross-sectional return predictability has led to different dimensions to slice the data, many of which somehow seem to implicitly take advantage of the beta anomaly.

To show that the beta anomaly adds to the risk-adjusted returns of the cross-sectional anomalies, I mitigate the long-short portfolios’ exposure to the beta anomaly in two complementary ways. First, I consider an alternative weighting scheme when aggregating individual stock returns to the portfolio level: I shift weights from low (high) beta stocks to high (low) beta stocks in long (short) leg portfolios. This way of constructing portfolios keeps the original anomaly portfolio constituents, but deviates from the value-weighting portfolio construction. The second approach complements the first by keeping the value-weighting scheme from the original portfolio construction, but removes stocks with low betas in the long leg, and stocks with high betas in the short leg. Together these two approaches allow me to separate the effect of beta exposure from the effect of anomaly characteristics on long-short portfolios’ risk-adjusted returns. Both modified portfolio construction methods by design reduce or remove portfolio exposure to the beta anomaly, and lead to reduced CAPM alphas for the anomaly trading strategies. The reduction in trading profitability varies: the risk-based anomalies (O-score, default probability, return volatilities) see reductions of 40% to 70%. In contrast, investment and stock issuance-related anomaly portfolios remain robust despite reductions in trading profitability, with  $t$ -statistics of their CAPM alpha estimates above 3 in most cases, the significance threshold suggested by Harvey, Liu and Zhu (2016) to account for data mining concerns. The results hold after ensuring that the resulting anomaly portfolios are ex post market neutral, in

different time periods, and are robust to alternative beta measures. Falsification tests show that reductions of such magnitude are difficult to replicate through random adjustments to the anomaly portfolios.

There are three alternative ways to control the extent to which the long-short portfolios are susceptible to the beta anomaly. However, none is effective for the purpose of this paper. The first alternative approach is a regression specification where the return to the beta-sorted portfolio is added to the CAPM as an explanatory variable. I show that this regression specification suffers from multicollinearity: by construction, the beta-sorted portfolio is highly negatively correlated with the market excess return. I verify this negative correlation and find a long-run correlation coefficient of -0.76. Therefore it should not be expected that adding the beta-sorted portfolio return to the CAPM significantly improves the explanatory power of the CAPM. Moreover, the regression specification implies that returns to the beta-sorted portfolio proxy for a systematic risk factor, while this paper analyzes individual stock betas as characteristics. The second approach is to form anomaly portfolios from independent double-sorts on beta and an anomaly characteristic. However, this method does not effectively adjust the ex-post beta estimates of the long-short portfolios: even within each beta quintile, there is still significant variation in portfolio betas between extreme quintiles. In some cases this variation is comparable in magnitude to that from the univariate sort on the anomaly characteristic alone. Lastly, leverages can be applied to long and short leg portfolios separately to make the overall long-short portfolio close to being market-neutral. However, as discussed in [Frazzini and Pedersen \(2014\)](#), re-scaling investments in long and short legs does not change the fact that long portfolios on average hold lower beta stocks relative to securities held in short portfolios.

The CAPM beta is one of many common measures of risk. The literature (see for instance [Ang, Hodrick, Xing and Zhang, 2006](#); [Baker, Bradley and Wurgler, 2011](#)) has identified a number of alternative risk measures that are also negatively related to expected returns in the cross-section. I analyze return volatility as a model-free alternative measure

of risk. I find a positive average cross-sectional correlation between beta and return volatility of about 0.32. I show that over the sample period, the anomaly long portfolios have lower realized return volatility relative to the short portfolios. Moreover, removing the return volatility anomaly imbalance in the long-short strategies has similar effects as mitigating the long-short portfolios' beta anomaly exposure. This is suggestive evidence that more general than the beta anomaly, the low-risk puzzle adds to the cross-sectional return anomalies.

The paper proceeds as follows. The next section discusses the relevant literature. Section 2 motivates the hypothesis that the beta anomaly is embedded in many cross-sectional asset pricing puzzles. Section 3 discusses the data and the empirical measures used in the paper. Section 4 presents the main empirical findings. Section 5 discusses a set of anomalies not covered in the paper. Section 6 discusses alternative ways to mitigate the long-short portfolios' exposure to the beta anomaly. Section 7 concludes.

## 1. Literature

The beta anomaly has been documented as early as [Black, Jensen and Scholes \(1972\)](#): high (low) beta stocks tend to have low (high) risk-adjusted returns under the CAPM, resulting in a security market line flatter than predicted by the CAPM. The beta anomaly since then has been extended in a number of ways to the more general low-risk puzzle. [Ang, Hodrick, Xing and Zhang \(2006\)](#) consider alternative measures of risk, and find that return volatility and idiosyncratic volatility are negatively correlated with expected returns in the cross-section. [Bali, Cakici and Whitelaw \(2011\)](#) find that investors have preferences for the risky lottery-like assets by documenting a negative relation between a stock's recent maximum daily return and expected returns. [Asness, Frazzini and Pedersen \(2014\)](#) show that "quality" stocks offer high average returns relative to "junks". [Kapadia, Ostdiek, Weston and Zekhnini \(2015\)](#) extend the literature by showing that stocks that are predicted to hedge market downturns out-of-sample significantly outperform those that do

not.

The literature proposes several explanations for the beta anomaly, most of which rely on some type of investor preference for risk. Such preferences could arise due to behavioral reasons (Karceski, 2002; Baker, Bradley and Wurgler, 2011; Bali, Brown, Murray and Tang, 2016; Hong and Sraer, 2016), or due to institutional constraints (Frazzini and Pedersen, 2014). Independent from the investor preference argument, Cederburg and O'Doherty (2016) find no consistently significant alpha from the beta-sorted portfolio after accounting for the time-variation in beta under a conditional CAPM framework. In contrast, I study the asset pricing anomalies in an unconditional model. This paper does not take a stance on the source of the beta anomaly. Rather, I verify its empirical validity in an unconditional setting, and show that the beta anomaly is embedded in the other cross-sectional anomalies. Therefore, to the extent the beta anomaly is explained in the literature, my results suggest that we have explanations for a wide range of other anomalies as well.

In a related paper, Novy-Marx (2014) attributes the abnormal performance of the defensive minus aggressive (DMA) strategy to small, growth, and unprofitable stocks, and argues that the converse does not hold. The converse is studied by analyzing alphas from time-series regressions of anomaly portfolio returns on a model where the DMA return is added to the market excess return. While this paper is silent on the source of the beta anomaly, I find a correlation coefficient of -0.77 between the return to the beta-sorted portfolio and the market. Therefore in the context of my paper, it might not be surprising that adding the beta anomaly return to the market model does not significantly improve its performance explaining anomaly returns. Moreover, the regression specification by design implies that the DMA portfolio return proxies for a systematic risk factor, while I study stock betas as characteristics.

This paper relates to the literature connecting the cross-sectional anomalies with mispricing and limits to arbitrage. Market-wide sentiment causes mispricing (Baker and Wurgler, 2006; Stambaugh, Yu and Yuan, 2012; Stambaugh and Yuan, 2016), which in combination

with some form of limits to arbitrage (for example high short-selling fees in [Drechsler and Drechsler, 2014](#)), lead to the observed cross-sectional anomalies. Two papers in this literature relate most closely to my work. [Stambaugh, Yu and Yuan \(2015\)](#) explains the idiosyncratic volatility (IVOL) puzzle with mispricing by arguing that arbitrage asymmetry makes overpricing more difficult to correct compared to underpricing, rendering the negative IVOL-return relation among overpriced stocks more prevalent. [Liu, Stambaugh and Yuan \(2016\)](#) explain the beta anomaly by showing that it exists through positive cross-sectional correlation with IVOL. Together the above two papers suggest mispricing with arbitrage asymmetry should be the cause of the low-risk puzzle. The result in this paper that the low-risk puzzle is embedded in other anomalies is consistent with the mispricing explanation of the anomalies. My work adds to the literature by presenting direct evidence that the low-risk puzzle is a channel through which mispricing contributes to the anomalies.

Given the plethora of the cross-sectional asset pricing puzzles ([Harvey, Liu and Zhu, 2016](#); [McLean and Pontiff, 2016](#)), [Cochrane \(2011\)](#) calls for consolidation. A burgeoning literature in the intersection of asset pricing and econometrics aims at reducing the set of cross-sectional anomalies, or “risk factors.” This literature employs machine-learning techniques to evaluate the explanatory power of new factors in addition to existing ones ([Feng, Giglio and Xiu, 2017](#); [Freyberger, Neuhierl and Weber, 2017](#); [Kozak, Nagel and Santosh, 2017](#)). My paper adds to this literature by taking an empirically-motivated approach, and shedding light on a viable dimension along which the space of cross-sectional anomalies could be reduced.

## 2. Motivation

### 2.1. The Beta Anomaly

[Black, Jensen and Scholes \(1972\)](#) make the observation that the CAPM alphas “are consistently negative for the high-risk portfolios ( $\beta > 1$ ) and consistently positive for the low-risk portfolios ( $\beta < 1$ ). Thus the high-risk securities earned less on average ... than the

amount predicted by the traditional form of the asset pricing model. At the same time, the low-risk securities earned more than the amount predicted by the model.”<sup>2</sup>

Figure 1 presents a visual illustration of the beta anomaly. CRSP stocks in each month are sorted into quintiles based on their trailing 12-month beta estimated using daily returns. The plots show the cumulative excess returns from a \$1 investment in 1927 in each of the two extreme quintiles, where “excess” means in excess of one-month Treasury bill rates.

[Insert figure 1 here]

Returns in the plots are not adjusted for inflation, and do not take into account transaction costs. The inflation adjustment would be the same for both portfolios. Transaction costs, if anything, should only be higher for the top quintile, resulting in an even lower cost-adjusted cumulative return. What matters here is the contrast in cumulative returns. A \$1 invested in the low beta portfolio in 1927 increased to \$103.99 in 2017, whereas \$1 invested in the high beta portfolio only increased in nominal terms to \$15.51 in the same time period. Such a sharp contrast is inconsistent with either theory or intuition: investors holding high beta assets do not get compensated in expected returns commensurate with the risk they bear.

## 2.2. Hypothesis Development

Table 1 reports the long-short anomaly portfolio returns conditional on the market excess return.

[Insert table 1 here]

The monotonically increasing pattern of the market excess return from the worst to the best months is perfectly reversed for all the anomaly portfolios. For all twelve of the tabulated anomaly portfolios, the best performing months are in fact the months in which

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<sup>2</sup>See Black, Jensen and Scholes (1972) table 2.



the market performs the worst, whereas their best performing months are the ones in which the market rises the most. Take the composite equity issues portfolio for example. The average return in months with extreme market downfalls is almost 3% higher than that in months with the highest market increases. Furthermore, the decreasing trend in each row going from the left to the right indicates that on average, the higher the overall market return is in a given month, the worse are returns to anomaly portfolios, suggesting a negative covariance between returns to the anomaly portfolios and the contemporaneous market excess return. The same pattern holds under daily returns: the anomaly portfolios perform the best in the 20% of trading days when the market falls the most, and perform the worst when the market rises the most. Results from table 1 are robust to the exclusion of the great depression, the dot-com bubble, and the housing crisis periods.

Conventionally, the anomaly portfolio return is taken as the return to the long leg minus the return to the short leg, where the long and short portfolios have the same weight. A long-short portfolio having a negative beta is equivalent to the condition that the long portfolio has a lower beta relative to the short portfolio

$$\beta_{LS} \equiv \beta_L - \beta_S < 0 \iff \beta_L < \beta_S.$$

$\beta_L < \beta_S$  then is equivalent to the condition that on average, the long portfolios hold stocks of lower betas compared to the stocks in the short portfolio

$$\beta_L < \beta_S \iff \sum_{i \in L} \omega_i \cdot \beta_i - \sum_{j \in S} \omega_j \cdot \beta_j < 0, \quad (1)$$

where  $i$  denotes a stock in the long portfolio,  $j$  denotes a stock in the short portfolio, and  $\omega_k$  denotes the weight a stock carries when returns are aggregated to the portfolio level. To the extent the beta anomaly holds in the data, it is most likely that the same stocks that are heavily-weighted in the long-leg should have lower alpha compared to the heavily-weighted stocks in the short-leg. Taking the difference between the long and short portfolio returns

then results in a positive alpha for the anomaly portfolio<sup>3</sup>. Hence the long-short portfolio has a source of positive alpha that is independent of the intended anomaly characteristic.

### 3. Data and Empirical Measures

#### 3.1. Return and Accounting Data

The sample of stocks comes from the Center or Research in Security Prices (CRSP) and Compustat. The stock return data cover the period from 1927 to 2017. The accounting data cover the period from 1964 to 2017. The sample of stocks consists of common stocks (shrcd 10 and 11) that are listed on NYSE, AMEX, or NASDAQ (exchcd 1, 2, 3). I require that all firms in the sample must have existed at least 24 months. I carry a firm’s accounting data forward up to the earliest occurrence of any of the following three conditions. First, the next annual financial statement is available. Second, the firm is delisted. Third, 24 months have passed in between the firm’s two consecutive financial statement releases. Returns are adjusted for delisting bias wherever applicable.

Because market betas are of primary interest in this study, the 2% of stocks with extreme beta estimates each month are excluded from the sample (1% on each end) in an attempt to reduce the impact of outliers. The sample includes all stocks surviving the restrictions outlined above. Robustness tests are done with microcap stocks excluded from the sample. To the extent the original anomalies survive the exclusion of microcap stocks, all main results remain. The proxy for the market return is the CRSP value-weighted

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<sup>3</sup>This argument relies on the function that maps a stock’s beta to its alpha being “regular.” A class of functions that are sufficient for this argument are those that are monotonically decreasing and affine. For example, suppose  $f : \beta \rightarrow \alpha$  satisfies

$$f' < 0, \quad f\left(\sum_i k_i \cdot x_i\right) = \sum_i k_i \cdot f(x_i),$$

for  $\sum_i k_i = 1$  and  $x_i \in \mathbb{R}$ . Then we have that

$$\alpha_i \equiv \sum_{i \in L} \omega_i \cdot f(\beta_i) = f\left(\sum_{i \in L} \omega_i \cdot \beta_i\right) > f\left(\sum_{j \in S} \omega_j \cdot \beta_j\right) = \sum_{j \in S} \omega_j \cdot f(\beta_j) \equiv \alpha_j,$$

where the inequality in the middle follows from condition (1).

index. The proxy for the risk-free rate is the one month T-bill rate, obtained from Ken French's data library.

### 3.2. Beta Estimates

At the beginning of every month I estimate a stock's CAPM  $\beta$  using its daily excess returns (gross return minus one-month T-bill rate) in the past 12 months, with a minimum of 150 observations of non-missing returns required. To limit the impact of non-synchronous trading, I estimate a stock's  $\beta$  using the sum of coefficients method following [Dimson \(1979\)](#). The rolling window regression specification is

$$r_{i,t} = \alpha_i + \sum_{l=0}^5 \beta_{i,t-l} R_{m,t-l} + \epsilon_{i,t},$$

where  $r_{i,t}$  denotes the excess return on stock  $i$  on day  $t$ , and  $R_{m,t}$  denotes the market excess return on day  $t$ .

The stock's beta estimate for month  $t$  is then calculated as

$$\hat{\beta}_{i,t} = \sum_{l=0}^5 \beta_{i,t-l}.$$

I measure individual stock betas in two alternative ways. First, I use rolling windows of monthly returns in the past five years requiring at least 24 non-missing return observations, and estimate the specification

$$r_{i,t} = \alpha_i + \beta_{i,t} R_{m,t} + \beta_{i,t-1} R_{m,t-1} + \epsilon_{i,t},$$

where  $r_{i,t}$  denotes the excess return on stock  $i$  in month  $t$ , and  $R_{m,t}$  denotes the market excess return in month  $t$ .

The stock's beta estimate for month  $t$  is then calculated as

$$\hat{\beta}_{i,t} = \beta_{i,t} + \beta_{i,t-1}.$$

The second alternative is to estimate individual stock betas as in [Frazzini and Pedersen \(2014\)](#). Specifically,

$$\hat{\beta}_{i,t} = \hat{\rho}_{i,t} \cdot \frac{\hat{\sigma}_{i,t}}{\hat{\sigma}_{m,t}},$$

where  $\hat{\sigma}_{i,t}$  and  $\hat{\sigma}_{m,t}$  denote the estimated volatility for stock  $i$  and the market;  $\hat{\rho}_{i,t}$  is their correlation. Details about the parameter estimations are given in section 3.1 in [Frazzini and Pedersen \(2014\)](#).

All analyses in this paper use betas estimates from daily returns. However, results remain qualitatively similar across all three beta estimation methods.

### *3.3. Anomalies and Long-short Strategies*

I focus on twelve asset pricing anomalies that are based on both accounting data and past stock return information. The list is taken from the union of the sets of anomalies studied by [Fama and French \(2016\)](#) and [Stambaugh, Yu and Yuan \(2012\)](#), and is representative of different types of cross-sectional return puzzles documented in the literature. Specifically, I consider anomalies on profitability ([Novy-Marx, 2013](#); [Fama and French, 2016](#)), momentum ([Jegadeesh and Titman, 1993](#)), composite equity issues ([Daniel and Titman, 2006](#)), net stock issues ([Loughran and Ritter, 1995](#)), financial distress ([Ohlson, 1980](#)), default probability [Campbell, Hilscher and Szilagyi \(2008\)](#), total accruals ([Sloan, 1996](#)), asset growth [Cooper, Gulen and Schill \(2008\)](#), investment-to-assets ([Titman, Wei and Xie, 2004](#); [Xing, 2008](#)), return on assets ([Chen, Novy-Marx and Zhang, 2011](#); [Fama and French, 2006](#)), total return volatility and idiosyncratic return volatility ([Ang, Hodrick, Xing and Zhang, 2006](#)). Note asset growth is interpreted as a measure of investment in the five-factor model in [Hou et al. \(2015\)](#). For anomaly characteristic calculations, I follow constructions outlined in [Fama and French \(2016\)](#) wherever possible, and then that in [Stambaugh et al. \(2012\)](#). A detailed description and the calculations of the anomalies are in [Appendix A](#). Absent from the [Stambaugh et al. \(2012\)](#) and [Fama and French \(2016\)](#) lists is the net operating assets anomaly. I address this omission as well as the size and value effects in section 5.

I consider monthly-rebalanced long-short trading strategies. For all accounting-based anomalies, stocks in each month are sorted into quintiles by the most recently available accounting variable. All accounting data are assumed to be available four months after the end of the fiscal period. For NSI, stocks with net repurchases make up group 1, and all stocks with zero net issuance constitute group 2. Stocks with positive net issuances are then sorted into three quintiles. For momentum, I require a one-month gap between the six-month window in which momentum is measured and the month in which the momentum measure becomes available. I measure return volatility using the standard deviation of stocks' daily excess returns in the past 60 days. For idiosyncratic volatility, I use the standard deviation of the CAPM residuals estimated using daily returns in the past 60 days. All anomalies are traded using zero-cost long-short portfolios, defined as the difference in value-weighted returns between extreme quintiles. I require each extreme quintile portfolio to have at least 50 stocks in any given month to be included in the sample. The summary statistics are presented in table 2.

[Insert table 2 here]

Note that the starting years for the time-series of the anomaly portfolios vary. The starting year for each portfolio is determined by two exogenous constraints. The first is data availability: variables that require only CRSP data to compute are available as early as 1927, while variables that require Compustat data only go back to 1964. The second constraint is that I require both the long and short leg portfolios to hold at least 50 stocks each month. The row minimum holdings in panel A of table 2 shows that this constraint appears to be binding for the total accruals and default probability portfolios, leading to different starting years (1968 and 1973) relative to starting years to the other accounting-based anomalies.

The parameter  $\gamma$  presented in panel B of table 2 is estimated in

$$R_{i,t} = \alpha_i + \gamma_i \cdot R_{b,t} + \epsilon_{i,t},$$

where  $R_{i,t}$  denotes the return to a zero-cost portfolio  $i$  in month  $t$ , and  $R_{b,t}$  denotes the month  $t$  return to the beta-sorted portfolio. Thus  $\gamma$  captures the sensitivity of the anomaly portfolio returns to the return of trading on the beta anomaly. All portfolios in the table have returns that are positively related to  $R_{b,t}$ , although the magnitude of the covariance varies. The return volatility, idiosyncratic volatility and default probability portfolios exhibit the highest sensitivity, while the investment growth and total accruals portfolios show the lowest.

The bottom panel reports the realized (post-formation) beta estimates for each characteristic-sorted quintile portfolio. The row “L-S” presents the differences in beta estimates between the long and short portfolio betas, along with the corresponding  $t$ -statistics below. Because non-synchronous trading is less of a concern for portfolios of stocks, the realized portfolio betas are estimated in the CAPM without lagged market excess returns on the right-hand-side. For all anomalies, the long leg portfolios exhibit lower beta estimates relative to their short legs. The differences in long and short portfolio beta estimates are all statistically significant at 1% level. Long portfolios generally show beta estimates at or below 1. The asset growth, investment-to-asset, and return-on-asset portfolios show beta estimates just above 1. In contrast, in all cases short leg portfolios exhibit beta estimates above 1.1. Due to the high beta estimates from long legs, the investment-related portfolios (asset growth AG and investment-to-assets ITA) exhibit the lowest variation in realized betas between the extreme quintiles. In comparison, the volatility-related and default probability portfolios show the greatest variation in realized betas between the extreme quintiles. The strong beta imbalance in the volatility-related anomaly portfolio is consistent with the observation in [Liu et al. \(2016\)](#) that beta is positively related to IVOL in the cross-section.

Note also in panel C that despite the difference in betas between extreme portfolios, the cross-sectional relations between beta and characteristics are not always strictly monotone: beta estimates exhibit a U-shape pattern across characteristic-sorted portfolios formed on profitability, asset growth, net stock issues, total accruals, return on assets, Ohlson score,

and momentum. This muddles the average cross-sectional relation between beta and these characteristics that one would obtain from Fama-MacBeth regressions.

#### 4. Empirical Results: Examining Cross-sectional Anomalies After Mitigating the Beta Imbalance

I mitigate the long-short portfolio’s exposure to the beta anomaly in two complementary ways. This section discusses and implements these two approaches, and examines the profitability of the resulting anomaly long-short trading strategies.

##### 4.1. Mitigating Exposure to the Beta Anomaly: Shifting Weights

First, I shift portfolio weights from low (high) beta to high (low) beta stocks in long (short) portfolios, relative to the benchmark weights assigned by market capitalization. Specifically, pick an arbitray stock  $i \in \{1, \dots, M\}$  from an anomaly long portfolio with a beginning-of-month beta estimate  $\beta_i < 1$ , and let  $vw_i$  be stock  $i$ ’s portfolio weight as determined by its market capitalization at the beginning of the month. Instead of using  $vw_i$  as its weight, stock  $i$  is assigned a new weight  $wt_i$  as in

$$wt_i = vw_i - vw_i \cdot w,$$

where the parameter  $0 < w \leq 0.7$  is unique to each anomaly portfolio, and is determined so that the realized time-series return to the anomaly long-short portfolio is close to being market neutral. The upper bound is set at 0.7 so that each stock still carries a substantial part of their original weight as determined by market capitalization. The parameter values for all anomalies are presented in panel A in table 3. The total weight reduction then is equally assigned to all stocks  $j$  with beginning-of-month beta estimates  $\beta_j > 1$ , so that stock  $j$  is assigned the new weight  $wt_j$  as in

$$wt_j = vw_j + \frac{1}{N} \sum_i vw_i \cdot w,$$

where  $j \in \{M + 1, \dots, N\}$  denotes the set of stocks with beta estimates  $\beta_j > 1$ . Recall that  $i \in \{1, \dots, M\}$  denotes an arbitrary stock with  $\beta_i < 1$ . A symmetric procedure is applied to the short portfolios, with weights being shifted from stocks with beta estimates greater than 1 to those with betas less than 1.

This approach of balancing long and short portfolio betas has two advantages. The first is that by design it preserves all original portfolio constituents, so that the overall portfolio still has a long position on the entire set of stocks with the desirable anomaly characteristics, and a short position on the entire set of stocks with undesirable characteristic measures. This is done through imposing the constraint  $0 < w < 0.7$ , which only modifies the weight that each stock carries when returns are aggregated to the portfolio level. The second advantage is that it only considers the binary outcome of whether a stock's beta estimate is above or below the cross-sectional mean of 1, rather than relying on the specific values of the beta estimates, which can be rather noisy and have extreme values.

I obtain the CAPM estimates for anomaly long-short portfolios both before and after shifting weights. The results are presented in table 3. The top row in each of the panel B, C, and D presents the whole-sample CAPM estimates for the value-weighted long-short anomaly portfolios, hence the subscript  $vw$ . The second row in each panel presents the whole-sample CAPM estimates after shifting weights, and is denoted by the subscript  $wt$ .

[Insert table 3 here.]

In panel B, the first row indicates that the CAPM beta estimates are negative for all value-weighted anomaly portfolios, with those for the default probability and volatility-related anomalies being of the highest magnitude. The negative betas are consistent with the negative return covariance documented in table 1. By construction, shifting weights towards high (low) beta stocks in long (short) portfolios, however, effectively increases the beta estimates for all anomaly portfolios, as shown in the second row in panel B. Note that despite the increase, the post-formation beta estimates for a number



of anomaly portfolios, in particular the return volatility, idiosyncratic volatility, and the default probability portfolios remain negative.

The first row in panel C shows that, not surprisingly, all value-weighted portfolios produce both economically and statistically significant alpha estimates. Shifting weights, however, reduces both the economic and statistical significance of the alpha estimates across the board. In terms of economic magnitude, the reduction in trading profitability ranges from 74% for the idiosyncratic volatility portfolio, to a mere 1.25% for the net stock issues portfolio. The CAPM-adjusted returns for investment (AG<sup>4</sup> and ITA) and stock issuance-related portfolios (NSI and CEI) are least affected by the weight-shifting exercise in terms of their alpha point estimates. The reductions in statistical significance of risk-adjusted returns also show variation. The  $t$ -statistic for the portfolio alpha of the return-on-asset portfolio is no longer significant at 10% level. However, the risk-adjusted returns for the investment-related and NSI portfolios exhibit an increase in statistical significance. The investment and issuance-related portfolios also continue to show risk-adjusted returns with  $t$ -statistics above 3, the threshold suggested by [Harvey, Liu and Zhu \(2016\)](#). The alpha estimates for all other anomaly portfolios are no longer significant at the 1% level.

Panel D presents results on annualized portfolio information ratios, which are defined as

$$IR = \sqrt{12} \cdot \frac{\alpha}{RMSE},$$

where  $\alpha$  denotes the monthly whole-sample portfolio alpha estimate, and  $RMSE$  denotes the regression root mean-square error. The factor  $\sqrt{12}$  serves to annualize the information ratio.  $IR$  can be interpreted as the portfolio Sharpe ratio after hedging out the market risk, and is a commonly-used metric to measure portfolio performance<sup>5</sup>. By construction, it rewards value added ( $\alpha$ ) on top of the benchmark returns, and punishes high tracking

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<sup>4</sup>Return to portfolios formed on Asset growth is interpreted as the investment factor in [Hou et al. \(2015\)](#).

<sup>5</sup>For a more detailed discussion on *information ratio*, see [Goodwin \(1998\)](#).

error, or equivalently, residual risk.

For all but the investment and issuance-related anomaly portfolios, the information ratio estimates tell a similar story: increasing the beta loadings of the long-leg portfolio and decreasing that of the short-leg portfolio reduces the trading profitability of the anomaly portfolios. The magnitude of the reduction in  $IR$  is comparable to that in alphas for each of the anomaly. This means shifting weights from low (high) to high (low) beta stocks in long (short) portfolios results in similar portfolio residual volatility relative to value-weighting<sup>6</sup>. In the period from 1927 to 2017, the US equity market exhibits an annualized Sharpe ratio of roughly 0.421. After weight-shifting, all of the above portfolios show information ratios below this level. However, the same cannot be said about the investment-related and net stock issues portfolios. The investment-related portfolios in fact exhibit an increase in  $IR$  after more weight is put on high (low) beta stocks in long (short) portfolios. This suggests that shifting weights reduces the residual risk of these portfolios.

Taken together, table 3 presents evidence indicating that 8 out of 12 anomaly portfolios, when traded in the direction suggested direction but also in a way that mitigates the negative beta exposure, exhibit risk-adjusted returns of both lower economic and statistical significance. The 4 portfolios that are based on sorts of investment and stock issuance-related characteristics show greater risk-adjusted returns beyond the long-short portfolios' beta imbalance.

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<sup>6</sup>To see this, note  $IR = \alpha/RMSE$ . So the change  $(IR_{wt} - IR_{vw})/IR_{vw}$  simplifies to

$$\frac{RMSE_{vw} \cdot \alpha_{wt}}{RMSE_{wt} \cdot \alpha_{vw}} - 1,$$

which differs from the change in alpha

$$\frac{\alpha_{wt}}{\alpha_{vw}} - 1$$

only by the multiplicative fraction  $RMSE_{vw}/RMSE_{wt}$ . Therefore similar changes in alphas and  $IR$ 's necessarily means that  $RMSE_{vw}/RMSE_{wt}$  is close to 1.

#### 4.2. *Balancing Portfolio Betas: Removing Low (High) Beta Stocks in the Short (Long) Leg Portfolio*

The second approach also starts with an independent double-sort each month on the pre-ranking betas and the anomaly characteristic into quintiles. Long-short portfolios are still taken as the extreme quintiles based on the anomaly characteristic sort. In the long (short) leg, stocks whose betas are ranked in the bottom (top)  $P \leq 50\%$  in the cross-section are removed, so that the long (short) leg essentially holds stocks that are both predicted to have high (low) returns by the anomaly characteristic and have high (low) betas. The choice of the parameter  $P\%$  is specific to each anomaly, and is set so that anomaly portfolios are roughly market neutral. The upper bound is set at  $50\%$  in an effort to keep at least half of the original portfolio composition. The parameter values  $P\%$  for all anomalies are reported in panel A in table 4. To complement the weight-shifting method, in this exercise stocks remain weighted by their one-month lagged market capitalization, the same as in the original anomaly portfolio construction. The whole-sample CAPM estimates are presented in table 4. The top row in each of the panel B, C, and D presents estimates for the original *value-weighted* portfolios, hence the subscript *vw*. The second row in each panel presents the whole-sample CAPM estimates for anomaly portfolios after *eliminating* the low (high) beta stocks in the long (short) leg, hence the subscript *el*.

[Insert table 4 here.]

Panel B presents the beta estimate before and after eliminating stocks in each leg. As intended, removing stocks with low (high) beta in the long (short) portfolios results in increases in beta estimates, so that the long-short anomaly portfolios are roughly market neutral.

Panel C presents the alpha estimates. Across the twelve anomalies analyzed in the paper, there is consistent reduction in the economic significance of the alpha estimates. The magnitude ranges from 63% for the default probability and return volatility portfolios

to about 15% for the net stock issues portfolio. The  $t$ -statistics for alpha estimates for all but the net stock issues portfolio exhibit a decrease after elimination. However, the risk-adjusted returns for investment and issuance-related anomalies remain above 3. The statistical significance of the risk-adjusted return to the net stock issues portfolio in fact shows an increase after eliminating stocks in the aforementioned manner.

The information ratio estimates show reductions of very similar magnitudes compared to the reductions in alphas, suggesting significant reductions in the anomaly portfolios' benchmark-adjusted performance. This again means that the anomaly portfolios formed after elimination has similar residual risk relative to the original portfolios. The net stock issues anomaly, however, does not exhibit material change in information ratio after elimination.

#### *4.3. Falsification Tests*

To test whether the presented reductions in the anomaly portfolio performance are due to chance, I perform falsification tests of both methods of balancing the long-short portfolio betas.

For the weight-shifting method, I simulate betas for all stocks in each month following a normal distribution with mean 1 and a standard deviation of 0.73. The standard deviation is obtained from the time-series mean of the monthly standard deviations of individual stock betas estimated in this paper, from 1927 to 2017. The weight that each stock carries then is calculated as in section 4.1, but with simulated betas. This process is repeated independently 1,000 times<sup>7</sup>. The distributions of the alpha estimates after random weighting are summarized in figure 2.

[Insert figure 2 here.]

In each subplot, the 1,000 alpha estimates are put into 100 bins, denoted by the green

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<sup>7</sup>The number of runs is limited only by the computing time this procedure requires.

bars. The red vertical lines denote alpha estimates from portfolios using actual beta estimates as the basis to shift weight from low (high) to high (low) beta stocks in long (short) portfolios. For each anomaly, the test result indicates that none of the simulated portfolios produces an alpha estimate as small as the one from weight-shifted portfolios.

For the elimination method, I randomly eliminate  $P\%$  of the long and short portfolio holdings each month, and then compute the unconditional alpha for the new long-short portfolios. The parameter value  $P\%$  is taken from panel A in table 4. This process is also repeated 1,000 times. The distributions of alpha estimates after random elimination are summarized in figure 3.

[Insert figure 3 here.]

The results indicate that almost no simulation run produces alpha estimate as small as the one from eliminating low (high) beta stocks from the long (short) portfolios. Only the total accruals anomaly has a few out of 1,000 estimates falling to the left of the alpha estimate after inflating long-short portfolio betas as intended.

Taken together, the falsification tests suggest that it is difficult to replicate reductions in portfolio performance of comparable magnitude to the ones presented in the previous section. Similar results would be difficult, if not impossible, to reproduce.

#### *4.4. The Low-risk Puzzle as Explanation*

The average cross-sectional correlation between beta and return volatility is about 32%<sup>8</sup> in the sample period from 1927 to 2017. In light of this positive cross-sectional correlation, I test the hypothesis that the results from section 4 are more general than the beta anomaly: the more general low-risk puzzle is behind the high risk-adjusted returns to the cross-sectional anomalies examined in this study. In table 5, I tabulate the realized return volatility for anomaly long and short portfolios.

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<sup>8</sup>This average cross-sectional correlation is obtained after trimming stocks with extreme beta and return volatility estimates in each cross-section (month) on both ends at 1%.

[Insert table 5 here.]

In the sample period from 1927 to 2017, long portfolios for all anomalies exhibit lower realized return volatility relative to short legs. The difference is statistically significant. The Bartlett tests reject the null hypothesis that the long and short portfolios for each anomaly have equal variances with low  $p$ -values.

I then repeat the tests from the previous section, but replace beta with return volatility. The choice of using return volatility instead of idiosyncratic volatility to study the low risk anomaly is due to the fact that return volatility estimation is model-free. Results remain qualitatively similar under idiosyncratic volatility. Test results are presented in table 6, where volatility-related anomaly portfolios are excluded. This is because under the elimination method, given the strong cross-sectional correlation between idiosyncratic volatility and return volatility, removing stocks with low (high) return volatility would almost empty the long (short) portfolio formed on idiosyncratic volatility.

[Insert table 6 here.]

Under elimination, low (high) return volatility stocks are removed from long (short) leg portfolios. Under the alternative method, weight is shifted from low (high) to high (low) return volatility stocks in the long (short) leg portfolios. The adjustment parameters  $P\%$  and  $wt$  are under the same constraints as those imposed in section 4:  $P\% \leq 50\%$  and  $wt \leq 0.7$ . Panel A shows that both methods effectively balances the realized return volatilities for anomaly long and short portfolios.

Results in the top half of panel B show that eliminating low (high) return volatility stocks in the long (short) portfolios generally leads to reductions in the magnitude of the alpha estimates. However, similar to results in section 4, investment and issuance-related portfolios (AG, ITA, NSI, and CEI) show low attenuation in both the economic and statistical significance. The top half of panel C shows that elimination leads to an

comparable reduction in information ratio across all anomaly portfolios as well, indicating similar portfolio residual risk levels before and after elimination.

The method of shifting weights has more varied effects on alpha estimates. The profitability, total accruals, return-on-asset, Ohlson score, and momentum portfolios show consistent reductions in both the economic and statistical significance of their risk-adjusted returns. The investment and insurance-related anomalies still show robust alphas after more weights are put on high (low) return volatility stocks in long (short) portfolios. The default probability portfolio, strangely, exhibits an increase in the magnitude of the alpha point estimate. However, the bottom half of panel C indicates that shifting weights consistently leads to reductions in portfolio information ratios. This suggests that shifting weights to high (low) stocks in long (short) legs increases the portfolio residual risk in cases where the portfolio alpha estimates show minimal (AG) or no reduction (ITA, NSI, CEI, DP).

Overall adjusting return volatilities in the anomaly long-short portfolios leads to reductions in the anomaly portfolios' trading profitability in most cases. Under both methods, the investment and stock insurance-related portfolios appear to have persistent and strong benchmark-adjusted performance, measured in terms of both alpha and information ratio.

## 5. On Other Cross-sectional Anomalies

The list of all anomalies examined also includes size ([Banz, 1981](#)), value ([Fama and French, 1992](#)), and net operating assets ([Hirshleifer, Hou, Teoh and Zhang, 2004](#)). This section addresses these anomalies.

The long-short portfolios formed on size, value (book-to-market equity), and net operating assets are not negatively correlated with the contemporaneous market excess return. The size anomaly, when value-weighted, has returns that are positively related to the market. This suggests that the long portfolio has on average a higher beta relative to the short portfolio, and is consistent with the observation in [Fama and French \(1992\)](#) that on average beta and size are negatively correlated in the cross-section. Moreover, the value-weighted

size portfolio does not produce significant CAPM-adjusted returns in the sample period of 1927 to 2017.

On the value effect, I find that when book-to-market equity ratio is computed using the most recently available market value of equity, the monthly BM-sorted portfolio is roughly market-neutral. This could be due to the observation in [Fama and French \(1995\)](#) that “high BM is typical of firms that are relatively distressed.” If investors exhibit tendency to move to quality holdings in times of market downturns, then value stocks as measured by high BM ratios might not be ideal candidates to hold. In addition, the CAPM-adjusted return to the monthly BM-sorted portfolio has a statistical significance below 5% level.

In the case of the portfolio sorted on net operating assets, there is no clear relation between its time-series of returns and the market excess return, and therefore not surprisingly, no significant difference between its long and short portfolio betas. [Hirshleifer, Hou, Teoh and Zhang \(2004\)](#) argue that net operating assets captures investors’ biased tendency to focus on accounting profitability as opposed to cash profitability as a result of limited attention. This tendency is biased because high net operating assets is a sign that recent earnings performance cannot be sustained, but investors do not fully account for it when valuing firms. Thus if it is the accounting profitability that catches investors’ attention, then it is not immediately clear whether investors are more likely to hold high or low net operating assets stocks in market downturns. This nebulates the relation between NOA portfolio returns and the market excess return.

Overall, the size and value effects are not as pronounced on value-weighted and monthly re-balanced portfolios. The trading strategy formed on sorts of net operating assets appears to be free of the long-short portfolio imbalance on beta, and have substantial returns that are not attributable to the beta anomaly.



## 6. On the Methodology to Mitigate Exposure to Beta Anomaly

In this section I discuss three alternative ways to mitigate the long-short portfolios' exposure to the beta anomaly.

### 6.1. Regression / Factor Model

The first alternative is the regression specification where return to the long-short portfolio formed on sorts of stocks' pre-formation betas, in similar spirit to the 'betting against beta' factor in [Frazzini and Pedersen \(2014\)](#), is added to the CAPM, as in

$$R_{i,t} = \alpha_i + \beta_i \cdot R_{m,t} + \gamma_i \cdot R_{b,t} + \epsilon_{i,t}. \quad (2)$$

In specification (2)  $R_{b,t}$  denotes the return to the portfolio formed on sorts of pre-formation betas. There are two reasons why this specification might not be appropriate for the purpose of this study. The first reason is that specification 2 necessitates the interpretation that  $R_{b,t}$  proxies for a systematic risk factor. However, this paper considers betas as characteristics, and looks to adjust long and short anomaly portfolios in order to mitigate their imbalance in this characteristic. Specification 2 is not well-suited for adjustment in characteristics.

The second reason is that this regression specification suffers from multicollinearity. Intuitively, the market excess return  $R_{m,t}$  and the beta portfolio return  $R_{b,t}$  should be negatively correlated. This is because the portfolio that buys low beta stocks and sells high beta stocks has returns that, by design, negatively covary with the market. In the sample period from 1927 to 2017, the time-series of  $R_{m,t}$  and  $R_{b,t}$  have a correlation coefficient of  $-0.76$ . Projecting  $R_{b,t}$  on  $R_{m,t}$  produces a regression coefficient of  $-1.03$  with a  $t$ -statistic of  $-22.14$ . Together the correlation coefficient and the regression coefficient estimate suggest that  $R_{m,t}$  and  $R_{b,t}$  are highly negatively correlated, making interpreting the coefficient estimates  $\gamma_i$  in 2 difficult. Moreover, the high correlation makes the  $R_{b,t}$  have limited marginal explanatory power beyond that of  $R_{m,t}$  in the CAPM.

## 6.2. Double Sorts

The second alternative way to mitigate the beta anomaly exposure is an independent double-sort, which is a common approach to control for one characteristic while studying the effect of another (Fama and French, 1993, 2006, 2016). However, independent double-sorts on pre-formation betas and an anomaly characteristic do not effectively mitigate the exposure to beta anomaly. I perform this exercise of double sort and tabulate the resulting portfolio betas in table 7.

[Insert table 7 here]

To construct table 7, stocks in each month are sorted into quintiles independently by an anomaly characteristic and by pre-formation betas. The intersections form 25 portfolios for each anomaly. The table reports the difference in post-formation betas between extreme anomaly quintile portfolios within each beta quintile. The row ‘all’ presents the difference in post-formation betas between extreme anomaly quintiles unconditional on the pre-formation betas.

It is evident in table 7 that even within each beta quintile, extreme anomaly portfolios still exhibit significant variation in beta estimates. Take return volatility (VOL) for example. Except in quintile 3 (row Beta3), in each of the other four beta quintiles, the magnitude of the variation in betas between extreme return volatility quintiles is more than half of that from the univariate sort. Similar lack of sufficient reductions in beta variation across extreme quintiles is observed among the other anomalies. The only few exceptions seem to be the two investment-related and net stock issues long-short portfolios formed within the extreme beta quintiles (row Beta1 and Beta5).

### 6.3. Applying Leverage to Long and Short Portfolios

In [Frazzini and Pedersen \(2014\)](#), leverages are applied to the construction of the long and short beta-sorted portfolios, as in

$$R_t = \frac{1}{\beta_{L,t}} \cdot (R_{L,t} - r_{f,t}) - \frac{1}{\beta_{S,t}} \cdot (R_{S,t} - r_{f,t}), \quad (3)$$

where  $R_{L,t}$  ( $R_{S,t}$ ) is the return to the low (high) beta portfolio, and  $\beta_{L,t}$  ( $\beta_{S,t}$ ) is the weighted beta estimate for the long (short) leg portfolio, all in month  $t$ . By construction,  $R_t$  is the return to an ex ante beta-neutral portfolio that is long low beta stocks, and short high beta stocks. Note the overall long-short portfolio still is self-financing, because the net proceeds from buying  $R_{L,t}$  and shorting  $R_{S,t}$  is invested at the risk-free rate  $r_{f,t}$ .

Applying leverage ensures that the  $R_t$  is the return to a zero-beta portfolio. However, it does not alter the fact that the long portfolio consists of low beta securities, and that the short portfolio consists of high beta securities. This permits the interpretation in [Frazzini and Pedersen \(2014\)](#) that  $R_t$  represents returns to a portfolio that “bets against beta.” For this reason, applying leverages as in (6.3) to long-short anomaly portfolios does not effectively change the fact that the long leg of anomaly portfolios on average hold lower beta stocks relative to the short legs.

## 7. Conclusion

Returns to long-short portfolios formed on a broad set of cross-sectional puzzles are negatively correlated with the contemporaneous market excess return. This negative covariance implies that the anomaly portfolios hold low beta assets and sell high beta assets, thus taking advantage of the well-documented beta anomaly. Mitigating the long-short portfolios’ imbalance in beta either attenuates or eliminates the risk-adjusted returns to the asset pricing puzzles, and leads to worse anomaly portfolio performance as measured by information ratios.

This paper suggests a new direction towards understanding the cross-section of expected returns. Results shed light on an empirically-motivated way of consolidating a large set of documented anomalies, thereby reducing the number of cross-sectional puzzles in the literature. To the extent the beta anomaly can be explained by investor preferences or trading constraints, this paper suggests possible extensions of the same explanations to the other cross-sectional puzzles. At the same time, the negative covariance between the long-short portfolios and the market excess return presents a challenge to the risk-based interpretation of these cross-sectional anomalies.

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## Tables and Figures

Table 1: Anomaly portfolio returns conditional on market excess returns

Each column reports the average gross returns of the corresponding portfolios (row titles) in months in which the market excess return falls into the corresponding quintile (column titles). The sample period is 1927 to 2017 for all portfolios whose calculations only require CRSP data (MOM, CEI, VOL, IVOL), 1964 to 2017 for most accounting-based portfolios (PROF, AG, NSI, ITA, ROA, O-SCORE), 1968 to 2017 for TAC, and 1973 to 2017 for DP. In each month stocks are sorted into quintiles based on their anomaly characteristics, where extreme quintiles make up the long-short portfolios. Monthly returns are reported in percents. Reported in square brackets are the  $t$ -statistics.

	Bottom 20%	20% to 40%	40% to 60%	60% to 80%	Top 20%	Hi-Lo
Market Excess Return (MKTRF)	-6.527 [-24.67]	-1.371 [-25.37]	0.988 [27.42]	3.09 [68.83]	7.169 [23.48]	13.761 [33.22]
Profitability (PROF)	3.156 [6.09]	1.559 [5.11]	0.198 [0.75]	-1.156 [-2.62]	-1.735 [-3.27]	-4.879 [-6.64]
Asset Growth (AG)	1.768 [5.28]	0.532 [2.51]	0.435 [1.77]	-0.485 [-2.0]	-0.422 [-1.22]	-2.27 [-4.74]
Net Stock Issues (NSI)	1.76 [6.56]	0.995 [5.95]	0.318 [2.02]	-0.213 [-1.06]	-0.747 [-2.91]	-2.541 [-6.87]
Total Accruals (TAC)	2.016 [5.13]	0.931 [3.39]	0.444 [1.56]	-0.806 [-2.81]	-1.427 [-3.16]	-3.497 [-5.94]
Investment-to-assets (ITA)	1.14 [3.73]	0.621 [3.13]	0.624 [3.12]	0.081 [0.38]	-0.405 [-1.48]	-1.55 [-3.77]
Return on Assets (ROA)	2.901 [6.32]	1.161 [3.69]	-0.14 [-0.57]	-1.081 [-3.32]	-1.501 [-2.96]	-4.42 [-6.56]
O Score (O-SCORE)	3.05 [6.72]	1.21 [3.62]	-0.074 [-0.24]	-0.978 [-2.73]	-1.726 [-3.05]	-4.756 [-6.72]
Default Probability (DP)	5.22 [7.13]	1.599 [3.65]	0.419 [1.06]	-1.12 [-2.05]	-3.829 [-4.8]	-8.979 [-8.41]
Momentum (MOM)	1.712 [3.98]	1.044 [3.56]	1.312 [5.27]	0.718 [2.16]	-2.212 [-3.32]	-3.871 [-4.92]
Composite Equity Issues (CEI)	2.618 [10.22]	1.029 [6.11]	0.077 [0.49]	-0.687 [-4.24]	-1.611 [-5.34]	-4.217 [-10.75]
Return Volatility (VOL)	6.095 [12.36]	1.843 [5.07]	0.221 [0.68]	-1.875 [-4.34]	-4.355 [-7.17]	-10.414 [-13.4]
Idiosyncratic Volatility (IVOL)	4.796 [10.43]	1.648 [4.52]	0.274 [0.86]	-1.519 [-3.41]	-2.994 [-4.91]	-7.78 [-10.24]

$$t \geq 2.58 \iff p \leq 1\%, \quad t \geq 1.96 \iff p \leq 5\%, \quad t \geq 1.64 \iff p \leq 10\%$$

Table 2: Summary Statistics

Reported in this table are the summary statistics of the long-short anomaly portfolios. Monthly returns are reported as percents. Return volatility is the standard deviation of the time-series of portfolio returns. Mean (min, max) holdings is the average (minimum, maximum) number of stocks in a quintile in a month.  $\gamma$  is estimated in the specification  $R_{i,t} = \alpha_i + \gamma_i \cdot R_{b,t} + \epsilon_{i,t}$ , where  $R_{i,t}$  denotes the return to a zero-cost portfolio  $i$  in month  $t$ , and  $R_{b,t}$  denotes the return in month  $t$  to the beta-sorted portfolio. The  $t$ -statistics are computed using standard errors adjusted for heteroskedasticity.

	PROF	AG	NSI	TAC	ITA	ROA	O-SCORE	DP	MOM	CEI	VOL	IVOL
Panel A: Summary												
Starting year	1964	1964	1964	1968	1964	1964	1964	1973	1927	1927	1927	1927
Monthly return	0.457	0.378	0.453	0.269	0.438	0.302	0.339	0.521	0.518	0.285	0.412	0.452
Return volatility	4.999	3.214	2.522	3.885	2.736	4.475	4.856	6.539	6.323	3.494	7.516	7.092
Mean holdings	735.7	755.7	817.5	651.0	670.3	755.0	642.7	719.7	547.9	544.0	523.5	522.5
Min holdings	143	149	142	50	129	149	136	56	90	88	94	93
Panel B: Covariance with returns to beta anomaly												
$\gamma$	0.406	0.184	0.218	0.218	0.14	0.341	0.358	0.492	0.344	0.313	0.718	0.574
$t$	[16.7]	[10.67]	[18.38]	[10.2]	[9.39]	[15.26]	[14.64]	[13.63]	[14.22]	[28.37]	[31.9]	[23.9]
Panel C: Characteristic-sorted portfolio betas												
1	1.379	1.057	0.919	1.002	1.068	1.382	0.976	0.897	1.395	0.93	0.766	0.858
2	1.079	0.891	0.819	0.881	0.995	1.142	0.95	0.989	1.201	0.925	1.124	1.176
3	0.953	0.899	0.959	0.933	0.897	0.928	0.969	1.105	1.029	1.092	1.372	1.315
4	1.001	1.018	1.087	1.043	1.004	0.94	1.116	1.316	0.959	1.286	1.524	1.45
5	0.972	1.233	1.125	1.278	1.187	1.002	1.387	1.611	0.99	1.264	1.453	1.361
L-S	-0.408	-0.176	-0.205	-0.276	-0.119	-0.379	-0.412	-0.714	-0.405	-0.334	-0.687	-0.503
$t$	[-9.82]	[-6.3]	[-9.8]	[-8.24]	[-4.96]	[-10.27]	[-10.27]	[-12.51]	[-12.05]	[-19.65]	[-18.53]	[-13.52]

Table 3: CAPM estimates for long-short anomaly portfolios after shifting weights

Reported in this table are the whole-sample CAPM estimates of long-short portfolios on cross-sectional anomalies and the corresponding  $t$ -statistics. The sample period is 1927 to 2017 for all portfolios whose calculations only require CRSP data (MOM, CEI, VOL, IVOL), 1964 to 2017 for most accounting-based portfolios (PROF, AG, NSI, ITA, ROA, O-SCORE), 1968 to 2017 for TAC, and 1973 to 2017 for DP. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted average returns of extreme quintiles.  $\alpha_{vw}$  ( $\beta_{vw}$ ) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios.  $\alpha_{wt}$  ( $\beta_{wt}$ ) is the CAPM alpha estimate of the long-short portfolios after weights are shifted from low (high) to high (low) vol stocks in the long (short) leg portfolios.  $\Delta_\alpha$  ( $\Delta_\beta$ ) is the difference between  $\alpha_{vw}$  ( $\beta_{vw}$ ) and  $\alpha_{wt}$  ( $\beta_{wt}$ ). The  $t$ -statistics are computed using standard errors adjusted for heteroskedasticity.

	PROF	AG	NSI	TAC	ITA	ROA	O-SCORE	DP	MOM	CEI	VOL	IVOL
Panel A: Percentage of weight shifted												
Wt %	50.0%	30.0%	40.0%	40.0%	20.0%	50.0%	60.0%	70.0%	70.0%	70.0%	70.0%	70.0%
Panel B: $\beta$ estimates												
$\beta_{vw}$	-0.409	-0.176	-0.206	-0.275	-0.12	-0.381	-0.414	-0.713	-0.407	-0.334	-0.688	-0.504
$t$	[-8.13]	[-5.08]	[-7.48]	[-6.82]	[-4.11]	[-8.2]	[-7.95]	[-9.11]	[-4.95]	[-8.64]	[-11.39]	[-8.02]
$\beta_{wt}$	-0.036	0.004	0.032	-0.001	-0.009	-0.031	0.025	-0.144	-0.039	-0.022	-0.373	-0.215
$t$	[-0.88]	[0.13]	[1.6]	[-0.03]	[-0.35]	[-0.77]	[0.52]	[-1.99]	[-0.48]	[-0.31]	[-4.85]	[-2.62]
Panel C: $\alpha$ estimates												
$\alpha_{vw}$	0.67	0.47	0.56	0.413	0.501	0.5	0.554	0.944	0.78	0.5	0.839	0.77
$t$	[3.6]	[3.77]	[5.88]	[2.72]	[4.61]	[3.01]	[3.14]	[3.68]	[4.55]	[5.5]	[4.15]	[3.86]
$\alpha_{wt}$	0.374	0.411	0.553	0.294	0.491	0.217	0.263	0.427	0.35	0.274	0.333	0.198
$t$	[2.45]	[3.89]	[7.83]	[2.34]	[5.06]	[1.49]	[1.72]	[1.96]	[2.2]	[3.13]	[1.8]	[1.08]
$\Delta_\alpha$	-44.15%	-12.59%	-1.25%	-28.92%	-2.0%	-56.59%	-52.55%	-54.78%	-55.15%	-45.14%	-60.26%	-74.24%
Panel D: Information ratios												
$IR_{vw}$	0.498	0.522	0.825	0.388	0.646	0.417	0.426	0.573	0.455	0.577	0.445	0.407
$t$	[3.649]	[3.824]	[6.046]	[2.744]	[4.736]	[3.058]	[3.127]	[3.687]	[4.318]	[5.48]	[4.178]	[3.819]
$IR_{wt}$	0.339	0.529	1.097	0.333	0.707	0.204	0.236	0.291	0.212	0.275	0.18	0.107
$t$	[2.488]	[3.879]	[8.041]	[2.352]	[5.182]	[1.496]	[1.73]	[1.874]	[2.01]	[2.615]	[1.692]	[1.002]
$\Delta_{IR}$	-31.82%	1.44%	33.0%	-14.3%	9.42%	-51.08%	-44.67%	-49.16%	-53.45%	-52.28%	-59.5%	-73.75%

Table 4: CAPM estimates for long-short anomaly portfolios after elimination

Reported in this table are the whole-sample CAPM estimates of long-short portfolios on cross-sectional anomalies and the corresponding  $t$ -statistics. The sample period is 1927 to 2017 for all portfolios whose calculations only require CRSP data (MOM, CEI, VOL, IVOL), 1964 to 2017 for most accounting-based portfolios (PROF, AG, NSI, ITA, ROA, O-SCORE), 1968 to 2017 for TAC, and 1973 to 2017 for DP. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted average returns of extreme quintiles.  $\alpha_{vw}$  ( $\beta_{vw}$ ) is the CAPM alpha (beta) estimate of the value-weighted long-short portfolios.  $\alpha_{el}$  ( $\beta_{el}$ ) is the CAPM alpha estimate of the long-short portfolios after eliminating low beta stocks in the long legs, and high beta stocks in short legs.  $\Delta_\alpha$  ( $\Delta_\beta$ ) is the difference between  $\alpha_{vw}$  ( $\beta_{vw}$ ) and  $\alpha_{el}$  ( $\beta_{el}$ ). The  $t$ -statistics are computed using standard errors adjusted for heteroskedasticity.

	PROF	AG	NSI	TAC	ITA	ROA	O-SCORE	DP	MOM	CEI	VOL	IVOL
Panel A: Percentage eliminated												
El %	30%	20%	20%	30%	20%	30%	30%	50%	30%	30%	50%	40%
Panel B: $\beta$ estimates												
$\beta_{vw}$	-0.409	-0.176	-0.206	-0.275	-0.12	-0.381	-0.414	-0.713	-0.407	-0.334	-0.688	-0.504
$t$	[-8.13]	[-5.08]	[-7.48]	[-6.82]	[-4.11]	[-8.2]	[-7.95]	[-9.11]	[-4.95]	[-8.64]	[-11.39]	[-8.02]
$\beta_{el}$	0.013	0.003	-0.015	0.084	0.055	0.033	0.025	-0.012	-0.042	0.046	-0.146	-0.006
$t$	[0.28]	[0.1]	[-0.73]	[1.89]	[1.92]	[0.69]	[0.45]	[-0.14]	[-0.68]	[1.25]	[-1.71]	[-0.08]
Panel C: $\alpha$ estimates												
$\alpha_{vw}$	0.67	0.47	0.56	0.413	0.501	0.5	0.554	0.944	0.78	0.5	0.839	0.77
$t$	[3.6]	[3.77]	[5.88]	[2.72]	[4.61]	[3.01]	[3.14]	[3.68]	[4.55]	[5.5]	[4.15]	[3.86]
$\alpha_{el}$	0.374	0.353	0.473	0.228	0.402	0.239	0.323	0.343	0.52	0.316	0.31	0.387
$t$	[2.32]	[3.03]	[6.06]	[1.52]	[3.83]	[1.45]	[1.88]	[1.35]	[3.17]	[3.67]	[1.64]	[2.01]
$\Delta_\alpha$	-44.15%	-24.83%	-15.59%	-44.87%	-19.82%	-52.23%	-41.73%	-63.62%	-33.24%	-36.89%	-63.05%	-49.71%
Panel D: Information ratios												
$IR_{vw}$	0.498	0.522	0.825	0.388	0.646	0.417	0.426	0.573	0.455	0.577	0.445	0.407
$t$	[3.649]	[3.824]	[6.046]	[2.744]	[4.736]	[3.058]	[3.127]	[3.687]	[4.318]	[5.48]	[4.178]	[3.819]
$IR_{el}$	0.324	0.41	0.828	0.218	0.533	0.201	0.264	0.209	0.324	0.385	0.182	0.213
$t$	[2.379]	[3.003]	[6.073]	[1.54]	[3.909]	[1.471]	[1.932]	[1.344]	[3.066]	[3.645]	[1.62]	[1.994]
$\Delta_{IR}$	-34.81%	-21.46%	0.45%	-43.88%	-17.45%	-51.89%	-38.21%	-63.56%	-28.87%	-33.36%	-59.07%	-47.69%

Table 5: Realized return volatility for anomaly long and short portfolios

Reported in this table are the realized return volatility estimates of anomaly long (Long Vol) and short (Short Vol) portfolios. Return volatility is measured as the standard deviation of monthly returns. The sample period is 1927 to 2017 for all portfolios whose calculations only require CRSP data (MOM, CEI, VOL, IVOL), 1964 to 2017 for most accounting-based portfolios (PROF, AG, NSI, ITA, ROA, O-SCORE), 1968 to 2017 for TAC, and 1973 to 2017 for DP. The row  $p$  reports the  $p$ -value from the Bartlett's test of the null hypothesis that the long and short portfolio variances are the same.

	PROF	AG	NSI	TAC	ITA	ROA	O-SCORE	DP	MOM	CEI	VOL	IVOL
Long Vol	0.044	0.051	0.042	0.051	0.05	0.046	0.044	0.042	0.059	0.053	0.043	0.048
Short Vol	0.073	0.057	0.052	0.062	0.056	0.072	0.074	0.087	0.085	0.071	0.097	0.094
Diff	-0.029	-0.006	-0.01	-0.011	-0.006	-0.026	-0.03	-0.045	-0.026	-0.018	-0.054	-0.046
$p$ -value	0.0	0.00022	0.0	0.0	0.00052	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 6: CAPM estimates for long-short anomaly portfolios after mitigating exposure to return volatility

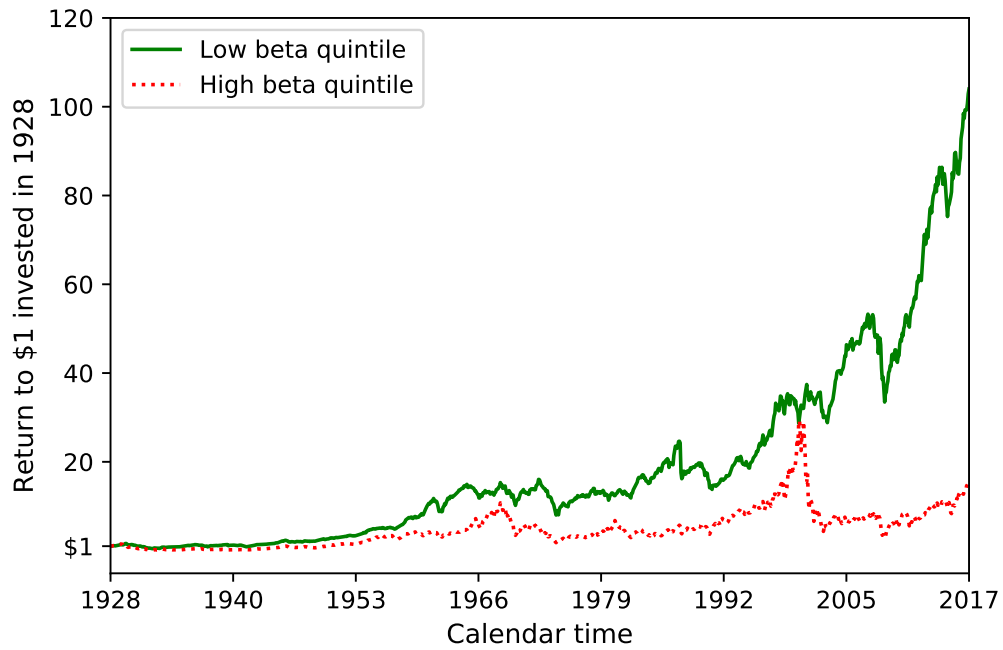
Reported in this table are the whole-sample CAPM estimates of long-short portfolios on cross-sectional anomalies and the corresponding  $t$ -statistics. The sample period is 1927 to 2017 for all portfolios whose calculations only require CRSP data (MOM, CEI, VOL, IVOL), 1964 to 2017 for most accounting-based portfolios (PROF, AG, NSI, ITA, ROA, O-SCORE), 1968 to 2017 for TAC, and 1973 to 2017 for DP. In each month, value-weighted anomaly portfolios are formed from univariate sorts into quintiles of all NYSE, AMEX and NASDAQ stocks. The monthly anomaly portfolio returns are defined as the difference between value-weighted average returns of extreme quintiles. The subscript  $vw$  denotes the CAPM estimates of the value-weighted long-short portfolios. The subscript  $el$  denotes the estimates of the long-short portfolios after eliminating low return volatility stocks in the long-legs, and high return volatility stocks in the short-leg. The subscript  $wt$  denotes the estimates of the long-short portfolios after weights are shifted from low (high) to high (low) vol stocks in the long (short) leg portfolios.  $\Delta_{el}$  ( $\Delta_{wt}$ ) is the difference between the estimate of the original  $vw$  portfolio and the  $el$  ( $wt$ ) portfolio. The row  $p$  reports the  $p$ -value from the Bartlett's test of the null hypothesis that the long and short portfolio variances are the same. The  $t$ -statistics are computed using standard errors adjusted for heteroskedasticity.

	PROF	AG	NSI	TAC	ITA	ROA	O-SCORE	DP	MOM	CEI
Panel A: Difference in realized volatilities between long and short portfolios										
Vol diff $_{vw}$	-0.029	-0.006	-0.009	-0.011	-0.006	-0.026	-0.03	-0.044	-0.025	-0.018
$p$	0.0	0.0043	0.0	0.0	0.0074	0.0	0.0	0.0	0.0	0.0
Vol diff $_{el}$	0.003	-0.002	0.001	0.0	-0.001	0.005	-0.003	-0.004	0.002	0.003
$p$	0.2562	0.3715	0.6831	0.9742	0.7314	0.078	0.2017	0.1668	0.3167	0.1327
Vol diff $_{wt}$	0.007	0.0	-0.001	-0.001	0.0	0.003	-0.004	-0.001	0.005	0.001
$p$	0.0181	0.9034	0.6377	0.8245	0.863	0.3492	0.1972	0.6953	0.0429	0.662
Panel B: $\alpha$ estimates										
$\alpha_{vw}$	0.67	0.47	0.56	0.413	0.501	0.5	0.554	0.944	0.78	0.5
$t$	[3.6]	[3.77]	[5.88]	[2.72]	[4.61]	[3.01]	[3.14]	[3.68]	[4.55]	[5.5]
$\alpha_{el}$	0.079	0.447	0.467	0.224	0.455	0.056	-0.018	0.15	0.188	0.422
$t$	[0.44]	[3.36]	[4.89]	[1.44]	[3.94]	[0.32]	[-0.11]	[0.56]	[0.96]	[4.51]
$\Delta_{\alpha}$	-88.24%	-4.9%	-16.72%	-45.7%	-9.19%	-88.85%	-103.18%	-84.12%	-75.91%	-15.69%
$\alpha_{wt}$	0.542	0.445	0.573	0.312	0.501	0.105	0.215	1.101	0.694	0.501
$t$	[2.32]	[3.26]	[5.44]	[1.88]	[4.08]	[0.52]	[1.14]	[2.83]	[2.75]	[3.45]
$\Delta_{\alpha}$	-19.15%	-5.29%	2.26%	-24.41%	0.07%	-78.97%	-61.19%	16.7%	-11.01%	0.26%
Panel C: Information ratios										
IR $_{vw}$	0.498	0.522	0.825	0.388	0.646	0.417	0.426	0.573	0.455	0.577
$t$	[3.649]	[3.824]	[6.046]	[2.744]	[4.736]	[3.058]	[3.127]	[3.687]	[4.318]	[5.48]
IR $_{el}$	0.063	0.477	0.693	0.207	0.558	0.046	-0.015	0.089	0.097	0.481
$t$	[0.456]	[3.431]	[4.988]	[1.433]	[4.02]	[0.328]	[-0.109]	[0.568]	[0.902]	[4.515]
$\Delta_{IR}$	-87.28%	-8.64%	-15.99%	-46.75%	-13.57%	-89.07%	-103.54%	-84.49%	-78.72%	-16.71%
IR $_{wt}$	0.325	0.452	0.762	0.268	0.57	0.073	0.158	0.469	0.296	0.358
$t$	[2.343]	[3.255]	[5.483]	[1.86]	[4.105]	[0.522]	[1.138]	[3.018]	[2.776]	[3.359]
$\Delta_{IR}$	-34.61%	-13.32%	-7.66%	-30.88%	-11.73%	-82.61%	-62.94%	-18.13%	-35.01%	-38.04%

Table 7: Post-formation beta estimates for the long-short anomaly portfolios within beta quintiles

Each month stocks are sorted into quintiles independently by an anomaly characteristic and pre-formation betas. The intersections form 25 portfolios. The table reports the difference in post-formation betas between extreme anomaly quintile portfolios within each beta quintile. The row ‘all’ presents the difference in post-formation betas between extreme anomaly quintiles unconditional on the pre-formation betas.

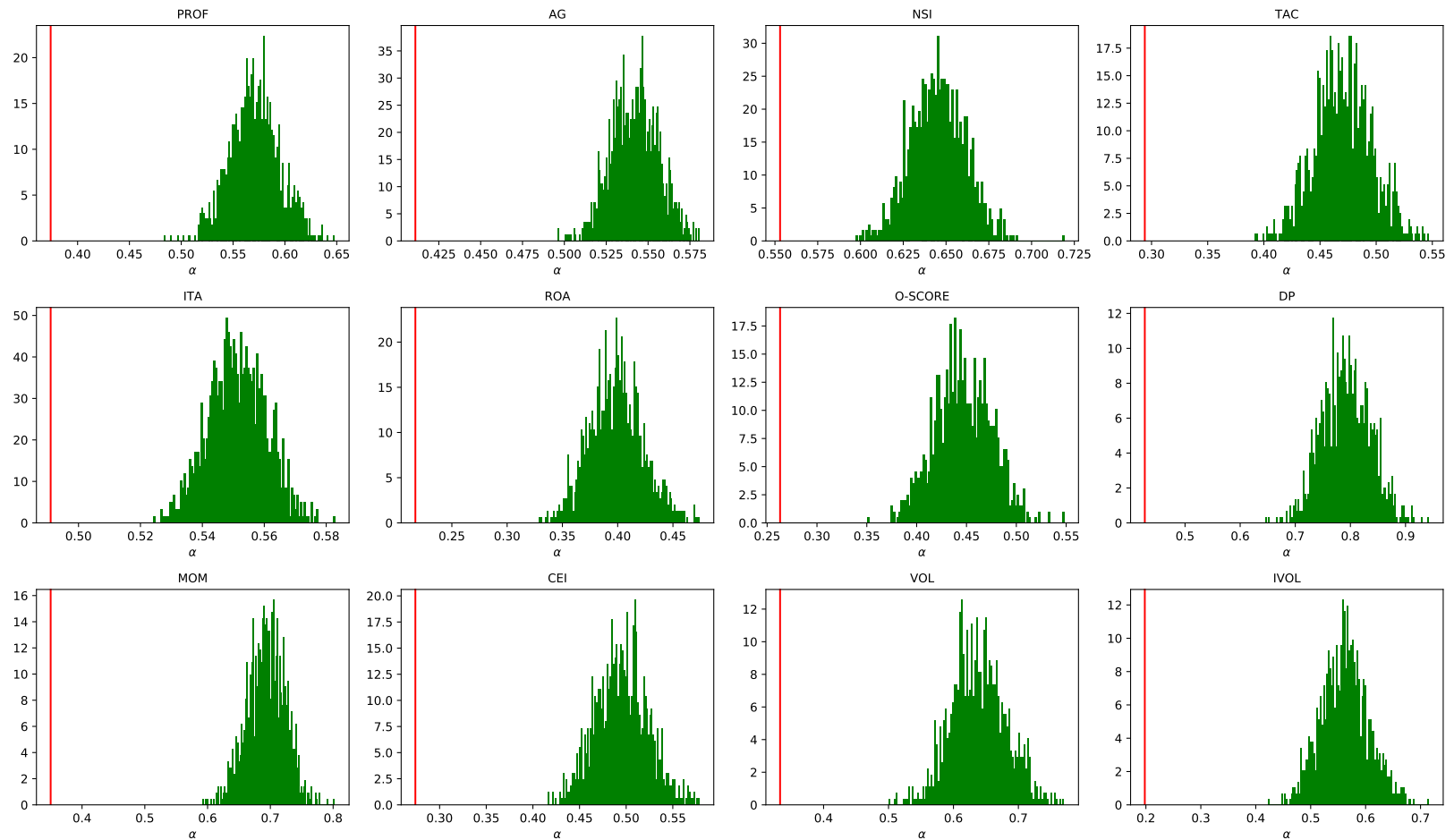
	PROF	AG	NSI	TAC	ITA	ROA	O-SCORE	DP	MOM	CEI	VOL	IVOL
Beta1	-0.193	-0.025	0.034	-0.06	-0.079	-0.135	-0.247	-0.383	-0.239	-0.114	-0.479	-0.441
Beta2	-0.217	-0.064	-0.05	-0.171	-0.082	-0.191	-0.181	-0.461	-0.208	-0.094	-0.59	-0.496
Beta3	-0.182	-0.087	-0.081	-0.132	-0.039	-0.184	-0.29	-0.39	-0.222	-0.216	-0.276	-0.191
Beta4	-0.124	-0.147	-0.156	-0.183	-0.077	-0.167	-0.211	-0.315	-0.244	-0.149	-0.425	-0.287
Beta5	-0.197	-0.02	-0.116	-0.117	-0.057	-0.184	-0.317	-0.497	-0.361	-0.273	-0.386	-0.277
All	-0.408	-0.176	-0.206	-0.277	-0.119	-0.38	-0.412	-0.713	-0.405	-0.334	-0.688	-0.504



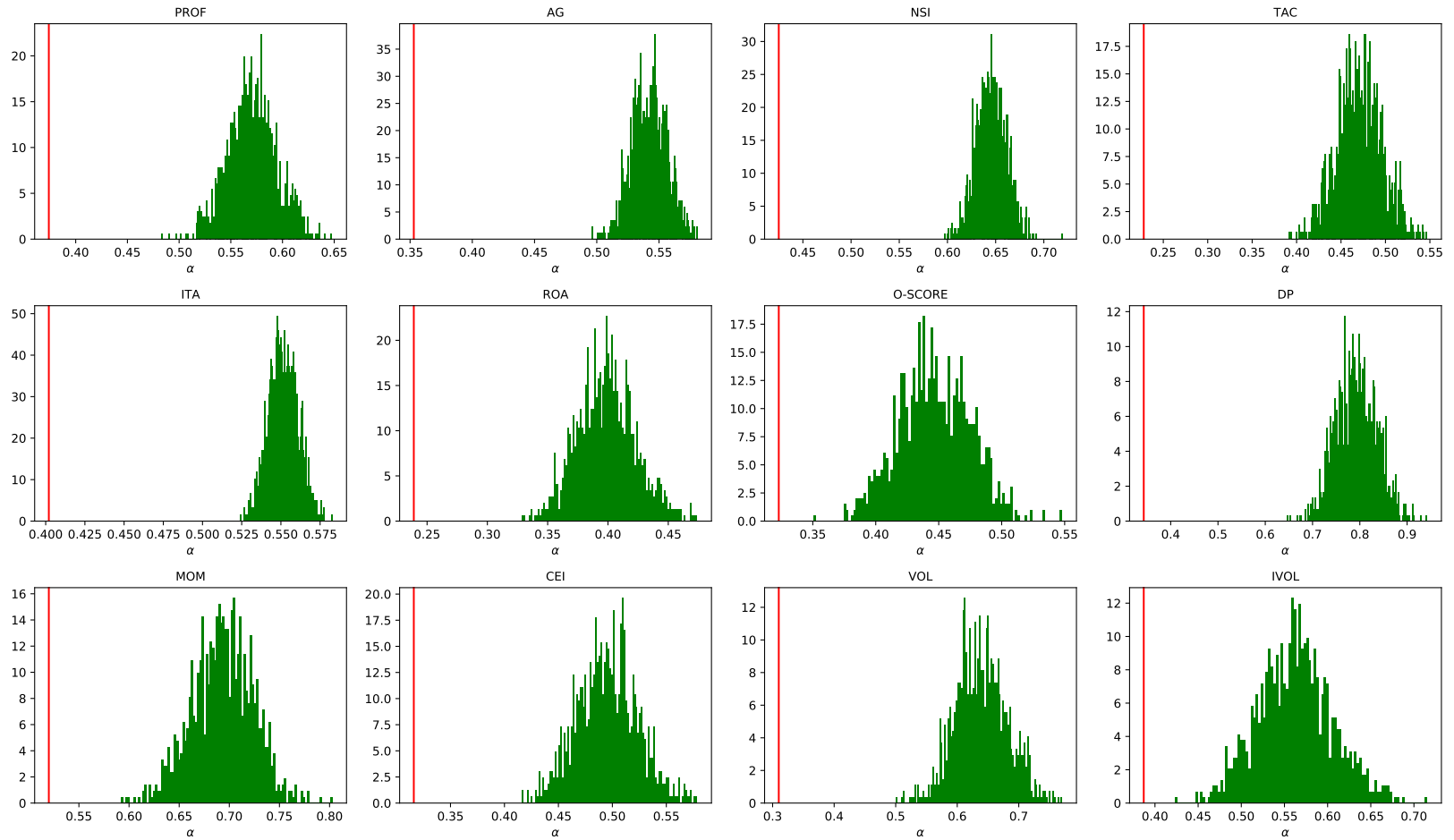
**Figure 1: Cumulative Excess Return from Trading on Beta** This figure plots cumulative excess returns (relative to Treasury Bill monthly short rate) to monthly-rebalanced value-weighted portfolios sorted on individual stock betas, from 1927 to 2017. Each month stocks are sorted into quintiles based on their betas estimated using the trailing 12 months of daily returns. The green plot denotes the cumulative excess returns to the portfolio that holds stocks of betas in the bottom quintile each month. The red plot denotes the cumulative excess returns to the portfolio that holds stocks with betas in the top quintile each month. Excess returns are defined relative to one-month T-bill rates.







**Figure 2: Falsification test on the weight-shifting method** This set of falsification tests simulate individual stock betas from a normal distribution in each cross-section (month) with a mean of 1 and standard deviation of 0.73, obtained from the empirical moments of average cross-sectional distributions of beta estimates. The histograms plot the distributions of 500 anomaly portfolios' CAPM alphas after weights are shifted from stocks with low (high) to high (low) *simulated* betas in long (short) leg portfolios, relative to market capitalization-based weights. The red vertical lines indicate CAPM alpha estimates from anomaly portfolios after weights are shifted based on empirical beta estimates, as described in section 4.1.



**Figure 3: Falsification test on the elimination method** The falsification test assigns stocks in the long and short portfolios into 10 groups independently. The histograms plot the distributions of 500 anomaly portfolios' CAPM alpha estimates after eliminating  $El$  groups of the long and short portfolio constituents. The percentage of elimination  $El$  is taken from panel A in table 4. The red vertical lines indicate CAPM alpha estimates from the anomaly portfolios obtained after eliminating low (high) beta stocks in long (short) leg portfolios, as described in section 4.2.

## Appendix A. List of Anomalies

This appendix details the list of anomalies studied in this paper. The list consists of those studied in [Fama and French \(2016\)](#) in union with those in [Stambaugh et al. \(2012\)](#).

Anomaly	Construction	Reference
Composite Equity Issues (CEI)	CEI is calculated as the change in the market capitalization of the firm in the past 12 months minus the cumulative stock return in the past 12 months. Buy bottom quintile. Sell Top quintile.	<a href="#">Daniel and Titman (2006)</a>
Net Stock Issues (NSI)	NSI is calculated as the annual change in split-adjusted shares outstanding in the previous fiscal year. Firms with net shares repurchases are put into bottom quintile; firms with no change in shares outstanding are put into quintile 2; all other firms are sorted into quintile 3, 4, and 5. Buy bottom quintile. Sell Top quintile.	<a href="#">Daniel and Titman (2006)</a> ; <a href="#">Fama and French (2016)</a>
Return on Assets (ROA)	ROA is calculated as the income before extraordinary items divided by lagged total assets. Buy top quintile. Sell bottom quintile.	<a href="#">Fama and French (2006)</a>
Profitability (PROF)	PROF is calculated as sales minus cost of goods sold minus selling, general and administrative expenses minus total interest and related expense, all divided by book value of equity. Buy top quintile. Sell bottom quintile.	<a href="#">Fama and French (2016)</a> ; <a href="#">Chen, Novy-Marx and Zhang (2011)</a>

Asset Growth (AG)	AG is calculated as the annual growth rate of total assets. Buy bottom quintile. Sell top quintile.	<a href="#">Cooper, Gulen and Schill (2008)</a>
Investment-to-Assets (ITA)	ITA is calculated as the change in property, plant, and equipment plus changes in inventory, all divided by total assets from the previous fiscal year end. Buy bottom quintile. Sell top quintile.	<a href="#">Titman, Wei and Xie (2004)</a> ; <a href="#">Xing (2008)</a>
Momentum (MOM)	MOM is calculated as the cumulative stock return in the past 6 months, with a one-month gap between the end of measurement period and the portfolio formation date. Buy top quintile. Sell bottom quintile.	<a href="#">Jegadeesh and Titman (1993)</a>
Return Volatility (VOL)	VOL is calculated as the standard deviation of the daily gross stock return in the past 60 days, with a one-month gap between the end of measurement period and the portfolio formation date. Buy top quintile. Sell bottom quintile.	<a href="#">Ang, Hodrick, Xing and Zhang (2006)</a> ; <a href="#">Fama and French (2016)</a>
Idiosyncratic Volatility (IVOL)	IVOL is calculated as the root mean square error from the regression of the stock's daily return in the past 60 days onto the market excess return in the same time period, with a one-month gap between the end of measurement period and the portfolio formation date. Buy bottom quintile. Sell top quintile.	<a href="#">Ang, Hodrick, Xing and Zhang (2006)</a> ; <a href="#">Fama and French (2016)</a>

Total Accruals (TAC)	See below for details. Buy bottom quintile. Sell top quintile.	(Sloan, 1996)
O-Score (OSCORE)	Calculation is obtained from <a href="#">Stambaugh and Yuan (2016)</a> . See below for details. Buy bottom quintile. Sell top quintile.	<a href="#">Ohlson (1980)</a>
Default Probability (DP)	Calculation is obtained from <a href="#">Stambaugh and Yuan (2016)</a> . See below for details. Buy bottom quintile. Sell top quintile.	<a href="#">Campbell, Hilscher and Szilagyi (2008)</a>

$$TAC = [DIF(ACT) - DIF(CHE) - (DIF(LCT) - DIF(DLC) - DIF(TXP)) - DP] / ((AT + AT_{t-1}) / 2),$$

where ACT = total current assets; AT = total assets; DLC = total debt in current liabilities; LCT = total current liabilities; CHE = cash and short-term investments; TXP = income taxes payable; DP = depreciation and amortization.

$$OSCORE = -1.32 - 0.407 \cdot \log AT + 6.03 \cdot (DLC + DLTT) / AT - 1.43 \cdot (ACT - LCT) / AT + 0.076 \cdot LCT / ACT - 1.72 \cdot OENEG - 2.37 \cdot NI / AT - 1.83 \cdot PI / LT + 0.285 \cdot INTWO - 0.521 \cdot (NI_t - NI_{t-1}) / (|NI| + |NI_{t-1}|),$$

where AT = total assets; DLC = total debt in current liabilities; DLTT = total long-term debt; ACT = total current assets; LCT = total current liabilities; NI = net income; PI = pretax income; LT = total liabilities; ONEEG is an indicator function that equals 1 if total liabilities (LT) > total

assets (AT); INTWO is an indicator function that equals 1 if the net income (NI) in year  $t - 1$  and year  $t - 2$  are both negative.

$$\begin{aligned} DP = & -9.16 - 20.26 \cdot NIMTAAVG + 1.42 \cdot TLMTA - 7.13 \cdot EXRETAVG + 1.41 \cdot SIGMA \\ & - 0.045 \cdot RELSIZE - 2.13 \cdot CASHMTA + 0.075 \cdot MB - 0.058 \cdot PRICE, \end{aligned}$$

where

$$NIMTAAVG_t = \frac{1 - \phi^3}{1 - \phi^{12}} \cdot \sum_{i=0}^9 \phi^i \cdot NIMTA_{t-i, t-i-2};$$

$$EXRETAVG_t = \frac{1 - \phi}{1 - \phi^{12}} \cdot \sum_{i=0}^{11} \phi^i \cdot EXRET_{t-i};$$

$\phi = 2^{-1/3}$ ;  $NIMTA = NIQ/(ME + LTQ)$ ;  $NIQ$  is the quarterly net income;  $ME$  is the firm's market capitalization;  $LTQ$  is the quarterly total liabilities;  $EXRET = \log R_i - \log R_{S\&P500}$ ;  $R_i$  is the firm's stock return in a month, and  $R_{S\&P500}$  is the return to the S&P500 index in the same month;  $TLMTA = LTQ/(ME + LTQ)$ ;  $SIGMA$  is the annualized standard deviation of the stock's daily return in the most recent 3 months;  $RELSIZE = \log(ME/USDVAL_{t-1})$ ;  $USDVAL_{t-1}$  is the market cap of the S&P500 index in the previous month;  $CASHMTA = CHEQ/(ME + LTQ)$ ;  $CHEQ$  is the quarterly cash and cash equivalents;  $MB = ME/ADJBEQ$ ;  $ADJBEQ$  is the adjusted book equity, obtained by increasing the Compustat book equity value ( $BEQ$ ) by 10% of the difference between market equity and book equity;  $PRICE$  is the lagged stock price.