Bridge Burning and Escape Routes

Paul Pecorino
Department of Economics, Finance and Legal Studies
University of Alabama

If you are faced with an enemy who thinks you would turn and run if he kept advancing, and if the bridge is there to run across, he may keep advancing. He may advance to the point where, if you do not run, a clash is automatic. Calculating what is in your long-run interest, you may turn and cross the bridge. At least he may expect you to. But if you burn the bridge so that you cannot retreat, and in sheer desperation there is nothing you can do but defend yourself, he has a new calculation to make. Thomas Schelling (1966: 43).

Man, the way I been talking, if I didn’t back up my talk I’d have to leave town. I’d have to leave the country. … So I talk big and that just makes me fight harder. - Cassius Clay de Córdoba (2018) argues that potential prosecution by the ICC causes figures such as Nicolás Maduro in Venezuela and Daniel Ortega in Nicaragua to cling to power rather than flee to exile.

The issue here is the lack of an escape path.

Key modeling feature: In addition to having a payoff in the event of victory in the Tullock contest, there is a negative payoff in the event the contest is lost. Bridge burning is increasing one's own the negative payoff in the event of a loss. Leaving an escape route is raising your opponent's payoff (lowering his loss) in the event he loses the contest.

The Model

1.a. Player 2 chooses the loss she incurs in the event she is defeated in the contest, R_2^L. This loss has a minimum value of $\underline{R_2^L}$, but it may be freely increased above this level up to a finite maximum value of $\overline{R_2^L}$.

1.b. Player 2 chooses the loss player 1 incurs in the event he losses the contest R_1^L. This loss has an initial value of $\overline{R_1^L}$, but player 2 may reduce this down to a minimum of 0.

2. The contestants decide whether or not to incur an entry fee $\varepsilon > 0$ to potentially participate in the contest at stage 3, where ε is small. If neither player incurs the entry fee, they both receive a payoff of 0. If player 1 incurs the entry fee and player 2 does not, player 1 receives the positive payoff R_1^W, player 2 incurs the loss R_2^L, and no contest takes place. If player 2 incurs the entry fee and player 1 does not, player 1 incurs the loss R_1^L,player 2 receives the positive payoff R_2^W, and no contest takes place. If both incur the entry fee, the game proceeds to step 3.

3. Both players engage in a Tullock contest.

At most, one player's participation constraint will be violated. The work of Wang (2010), Ewerhart (2017) and Lu (2017) demonstrate that the following equilibrium would apply: the stronger player employs a pure strategy while the weaker player mixes between zero effort and a positive level of effort. Since the weaker player incurs the loss R_i^L when he employs zero effort in this mixed strategy equilibrium, his expected loss is R_i^L. Thus, if he needed to incur a small fixed cost to participate in the contest, he would strictly prefer not to do so, but would instead concede the contest to his stronger opponent. It is better to lose R_i^L than $R_i^L + \varepsilon$.

The ability to concede the contest is realistic. Absent an entry fee it would be necessary to address the mixed strategy equilibrium described previously. The opening quote by Schelling implies that bridge burning, if successful, may prevent an attack from occurring.

Contest Description

Let there be two contestants denoted by $i = 1, 2$ and let their respective efforts in the contest be denoted by X_i. The probability p that contestant 1 wins the contest is given by a standard Tullock (1980) function:

$$p = \frac{aX_1^r}{aX_1^r + X_2^r}, \tag{1}$$

where $a > 0$ is a bias parameter and $0 < r \leq 2$ is a scale parameter. The cost of a unit of effort X is normalized to 1.

Players obtain R_i^W when they win and incur the loss R_i^L when they lose.

The following substitutions will be utilized:

$$R_i^W + R_i^L = R_i,\ i = 1, 2. \tag{2a}$$

$$R_2 / R_1 = R. \tag{2b}$$

Note that R_i constitute the stakes of the contest for each player. This is the sum of the payoff in the event of victory and the loss in the event of defeat. The relative stakes are reflected by R, where higher values of R imply the relative stakes favor contestant 2.

With probability *p*, player 1 wins the contest and receives R_1^W and with probability 1-*p*, he loses and pays R_1^L. Making use of (1) and (2a), his payoff may be expressed as

$$\Pi_1 = \frac{aX_1^r}{aX_1^r + X_2^r} R_1 - R_1^L - X_1. \tag{3a}$$

Since player 2 wins with probability *1-p* and loses with probability *p* her expected payoff may be expressed as

$$\Pi_2 = R_2^W - \frac{aX_1^r}{aX_1^r + X_2^r} R_2 - X_2. \quad (3b)$$

Holding effort level constant, an increase in R_2^L will lower player 2's expected payoff through its increase in R_2. However, player 2's effort level is increasing in R_2^L and this is the source of the possible benefit to player 2 of increasing her own negative payoff in the event she loses the contest.

The first order conditions from (3a) and (3b) imply the following solutions:

$$X_1 = \frac{raR^r}{\left(a+R^r\right)^2} R_1, \tag{4a}$$

$$X_2 = \frac{raR^r}{\left(a+R^r\right)^2} R_2, \tag{4b}$$

$$\Pi_1 = \left(\frac{a}{a+R^r}\right)\left(1-\frac{rR^r}{a+R^r}\right)R_1 - R_1^L, \tag{5a}$$

$$\Pi_2 = R_2^W - \left(\frac{a}{a+R^r}\right)\left(1+\frac{rR^r}{a+R^r}\right)R_2, \tag{5b}$$

$$p = \frac{a}{a+R^r}. \tag{5c}$$

Participation Constraints

We can use the model solutions to derive the following participation constraints:

Player 1 Participation Constraint: $a \geq (r-1)R^r$ (6a)

Player 2 Participation Constraint: $R^r \geq (r-1)a$ (6b)

If $r \leq 1$, the constraints of both players are always satisfied. If $r > 1$, at most one participation constraint is violated.

Analysis

First, consider the impact on player 2's payoff of a marginal increase in R_2^L. This corresponds to the case of bridge burning.

$$\frac{d\Pi_2}{dR_2^L} = -\left(\frac{a}{\left(a+R^r\right)^3}\right)\left(a^2 + a(2+r^2)R^r + (1-r^2)R^{2r}\right). \qquad (8)$$

If $r < 1$, the expression in (8) is clearly negative. Making use of the participation constraints implies the following:

<u>Result 1:</u> At an interior equilibrium, bridge burning (raising R_2^L) will always reduce player 2's expected payoff.

Next suppose that player 2 can lower R_1^L.

$$\frac{d\Pi_2}{dR_1^L} = -\left(\frac{arR^{1+r}}{\left(a+R^r\right)^3}\right)\left(a(1-r)+R^r(1+r)\right). \tag{10}$$

Combined with the participation constraints, equation (7) implies the following:.

Result 2: At an interior equilibrium, leaving an escape path open (i.e., lowering R_1^L) always increases player 2's expected payoff.

What is the effect on player 1, when player 2 leaves him an open escape path?

$$\frac{d\Pi_1}{dR_1^L} = \left(\frac{a}{(a+R^r)^3}\right)\left[a^2 + a(2+r^2)R^r + (1-r^2)R^{2r}\right] - 1. \tag{12}$$

Combined with the participation constraints, equation (12) implies the following:

> Result 3: At an interior equilibrium, player 1 always benefits when player 2 leaves him an open escape path.

Bridge burning can succeed if it induces player 1 to concede the contest.

Consider the following conditions on the maximal loss player 2 can impose on herself if she loses the contest:

(i) $\overline{R_2^L} > \left(a/[r-1]\right)^{(1/r)}\left(R_1^W + \overline{R_1^L}\right) - R_2^W$ and

(ii) $\overline{R_2^L} \leq \left(a/[r-1]\right)^{(1/r)}\left(R_1^W + \overline{R_1^L}\right) - R_2^W$.

Combined with Result 1 and the participation constraints, the conditions in (i) and (ii) imply the following:

Result 4: (a) When $r < 1$, bridge burning is never a desirable strategy.

(b) Assume $r > 1$. When $\overline{R_2^L}$ is sufficiently large as defined by condition (i), it is always possible to identify a successful bridge burning strategy under which player 2's participation constraint is satisfied, while player 1's constraint is violated. When $\overline{R_2^L}$ is sufficiently small as defined by condition (ii), player 2 cannot induce player 1 to concede the contest and the bridge burning strategy is not employed.

Leaving an open escape path can also possibly induce player 1 to concede the contest. The following conditions are key:

(iii) $(r-1)\left(\frac{R_2^L + R_2^W}{R_1^W}\right)^r > a$

(iv) $(r-1)\left(\frac{R_2^L + R_2^W}{R_1^W}\right)^r \leq a$

Combining (iii) and (iv) with the participation constraints yields the following:

Result 5: (a) When condition (iii) holds, player 2 can induce player 1 to concede the contest by leaving an open escape path, i.e., by reducing R_1^L, possibly down to its minimum value of 0. A necessary condition for (iii) to hold is $r > 1$.

(b) When the condition in (iv) holds, player 2 cannot induce player 1 to concede the contest by leaving an open escape path. A sufficient condition for (iv) to hold is $r \leq 1$.

Conclusion

Leaving an escape route is always successful at an interior equilibrium and raises the expected payoff of both players. It is a positive sum strategy.

The comparative static effects of bridge burning are never favorable at an interior equilibrium. Bridge burning may possibly succeed when $r > 1$, and this works by inducing the other player to concede the contest.

Caveats: (i) Bridge burning may raise your opponent's payoff if there are potential future conflicts, because defeat becomes more total. Trash talking can make your opponent fight harder (Yip et al. 2018).

(ii) Leaving an escape path open will reduce your opponent's loss in the event he loses. However, it may reduce your own payoff in the event you win, if you need to battle your opponent again at some future point.

Dixit and Nalebuff (2008: 224) frame Sun Tzu's strategy as fooling the enemy into thinking there is an escape path, but then ambushing them during the retreat. This suggests a concern for future engagements.