

SOVEREIGN DEBT OVERHANG, EXPENDITURE COMPOSITION AND DEBT RESTRUCTURINGS

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DISCLAIMER

- The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management

OVERVIEW OF THE PAPER

- Empirical, theoretical and quantitative analysis of sovereign debt
- Two main contributions to the literature on sovereign debt:
 - New dataset on public expenditure composition and new stylized facts on sovereign debt overhang and restructurings
 - New theoretical explanations on sovereign debt overhang and delays in debt restructurings

MOTIVATION

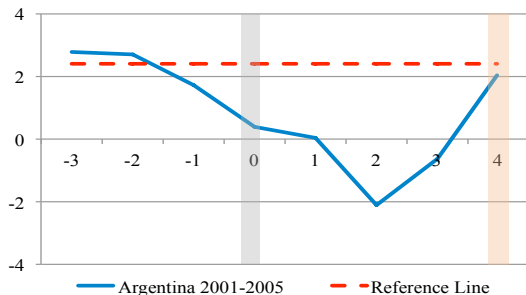


FIGURE: Public capital growth rate around Argentine debt restructuring 2001–05

- Deceleration of public capital accumulation
- Lengthy debt renegotiations (3.5 years)

NEW DATA ON PUBLIC EXPENDITURE COMPOSITION

- 179 privately-held external debt restructurings in 1978–2010 (Asonuma and Trebesch 2016)
- New dataset on public expenditure composition
 - Public consumption (public sector wage bills and consumption on final goods and service)
 - Public transfers
 - Public investment
 - Public capital (assets)
- Sources of our dataset
 - IMF Staff Reports from the IMF archives (more than 500 reports)
 - IMF FAD and WEO
 - WB WDI

STYLIZED FACTS ON RESTRUCTURINGS

- **Stylized Fact 1:** Public investment experiences a severe decline and slow recovery around restructurings

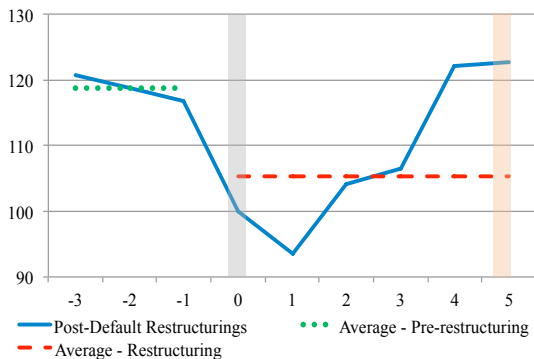


FIGURE: Around Start of Restructurings

STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

- **Stylized Fact 2:** Public consumption and transfers experience a short-lived decline and quick recovery

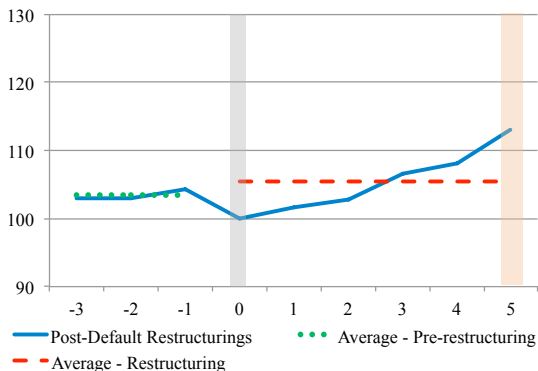
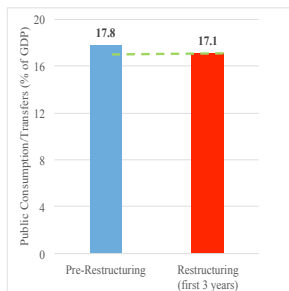


FIGURE: Around Start of Restructurings

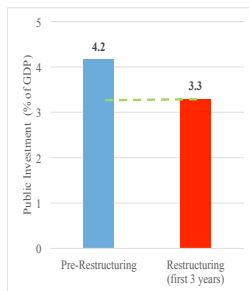
STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

- **Stylized Fact 3:** Public expenditure tilts remarkably towards consumption and transfers during debt crisis

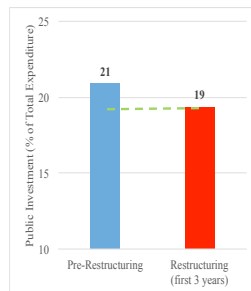
(a) Public Consumption/transfers
(percent of GDP)



(b) Public Investment
(percent of GDP)

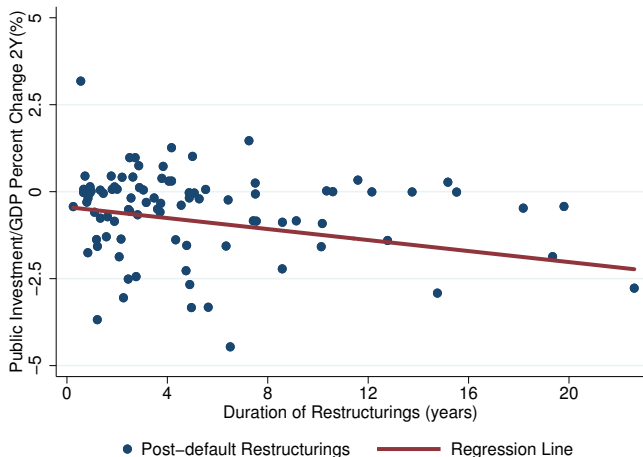


(c) Public Investment
(percent of public expenditure)



STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

- **Stylized Fact 4:** The sharp decline in public investment is associated with long restructuring duration



- **Main questions**

- Why public investment experiences a severe decline and slow recovery during debt crisis, but public consumption and transfers do not?
- What is the role of public capital accumulation (investment) on restructuring delays and debt settlements?

IMPLICATIONS OF THE PAPER

- New dataset on public expenditure composition and new stylized facts:
 - A severe decline in public investment
 - Different dynamics of public consumption and transfers
 - Public expenditure tilting towards consumption and transfers
 - Link between public investment decline and delays
- New theoretical explanations on debt overhang and restructurings:
 - Two phases of sovereign debt overhang
 - Role of public capital in debt renegotiations
- Quantitative analysis of model rationalizes the four stylized facts

LITERATURE REVIEW

- Sovereign debt overhang
 - Krugman (1989), Sach (1989), Aguiar et al. (2009), Ostry et al. (2014), Reinhart et al. (2012)
- Sovereign debt and fiscal policy
 - Kaminsky, Reinhart and Vegh (2005), Arellano and Bai (2017), Cuadra et al. (2010), Hatchondo et al. (2017), Bianchi et al. (2017), Azzimonti (2015), Mendoza et al. (2014), Pouzo and Presno (2016), Karantounias (2018)
- Sovereign defaults and renegotiations (multi-round)
 - Benjamin and Wright (2013), Kovrijnykh and Szentes (2007), Bai and Zhang (2012), Bi (2008), Asonuma and Joo (2017)
- Sovereign debt and “total” capital
 - Gordon and Guerron-Quantina (2018), Park (2017)

MODEL: GENERAL FEATURES

- Sovereign debt in a dynamic small open economy model:
 - Endogenous choice of default and repayment
 - Endogenous choice of settlement and delays conditional on default
 - Endogenous choice of public expenditure—public consumption, investment, transfers and debt repayments
 - Endogenous production with labor and public capital

INTUITION: KEY MECHANISM

- **Prior to default:** Low productivity, high debt payments, and consumption-smoothing motive (Aguiar et al. 2009)
 - On the one hand, sovereigns opt to smooth both private and public consumption (through transfers and public consumption)
 - On the other hand, sovereigns reduce public investment
- **During restructurings:** Deceleration of public capital accumulation and lengthy negotiations
 - Public capital accumulation is slow due to low productivity and no external borrowing
 - Sovereigns opt to delay the settlement since they prioritize investment to public capital (high MPK)
 - Sovereigns choose to settle with debt repayments after public capital accumulation (low MPK)

QUANTITATIVE ANALYSIS - PARAMETERS

- TFP process -AR(1) process:

$$\log(a_t) = \rho \log(a_{t-1}) + \epsilon_t, \quad (41)$$

- Household utility function - GHH, CRRA:

$$u(c_t, l_t) = \frac{(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{1-\sigma}}{1-\sigma}, \quad v(g_t) = \frac{g_t^{1-\sigma_g}}{1-\sigma_g} \quad (40)$$

Parameter	Value	Source
Risk aversion for private consumption	$\sigma = 3$	Previous studies
Risk aversion for public consumption	$\sigma_g = 3$	Hatchondo et al. (2017)
Risk-free interest rate	$r^* = 0.01$	US Treasury Bill
Labor elasticity	$\psi = 0.48$	Mendoza (1991)
Labor income share	$\alpha^L = 0.64$	Gordon and Guerron-Quintana (2018)
Public capital income share	$\alpha^K = 0.058$	Argentine public and private capital (1993–2005)
Public capital depreciation rate	$\delta = 0.04$	US BEA (1999)
Effective consumption tax rate	$\tau = 0.33$	Argentine tax revenues (1993–2005)
Auto-correlation of productivity shock	$\rho = 0.85$	Computed Argentine GDP- MECON
Standard deviation of productivity shock	$\sigma^a = 0.017$	Computed Argentine GDP- MECON
Direct productivity loss	$\lambda_d = 0.02$	Computed
Weight on public consumption	$\lambda = 0.8$	Computed
Public capital adjustment costs	$\Omega = 10$	Computed
Discount rate	$\beta = 0.90$	Computed
Bargaining power	$\phi = 0.9$	Computed

- Sovereign's choice among repayment, delay and settlement

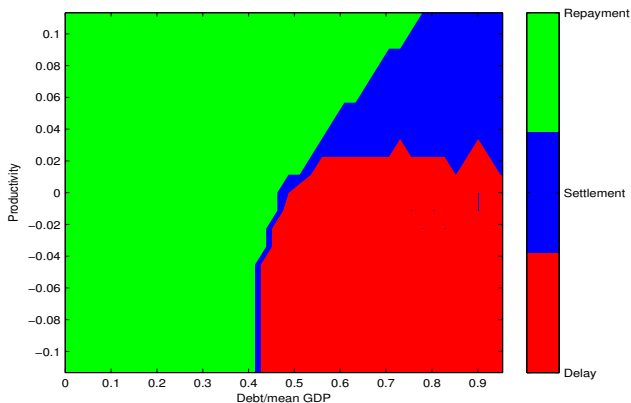


FIGURE: Mean public capital

- Sovereign's choice among repayment, delay and settlement

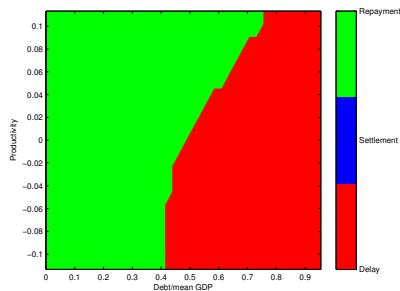


FIGURE: Low public capital

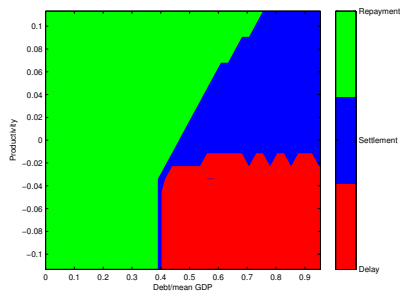


FIGURE: High public capital

- Public investment – Mean public capital

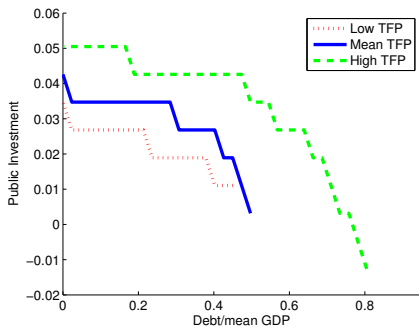


FIGURE: “Repayment” Region

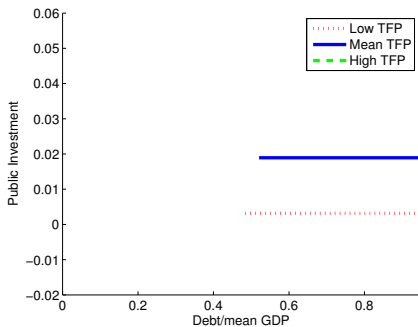


FIGURE: “Delay” Region

TABLE: Non-business Cycle Statistics

	Data	Model	Model with Fixed Public Capital	Model without Separation of Public/Private Sectors
Target statistics				
Default probability (%)	3.26	3.10	2.7	3.29
Average recovery rate (%)	25.0	27.1	22.5	35.2
Average debtor output deviation during debt renegotiation (%)	-4.45	-3.73	-4.5	-4.23
Pre-default periods				
Average debt/GDP ratio (%)	45.4	43.9	45.6	50.8
Bond spreads: average (%)	9.4	0.2	0.17	0.5
Bond spreads: std dev. (%)	7.6	0.3	0.34	0.9
Corr.(spreads, output)	-0.88	-0.1	-0.30	-0.13
Corr.(debt/GDP, spreads)	0.92	0.21	0.36	0.26
Corr.(debt/GDP, output)	-0.97	-0.69	-0.69	-0.65
Renegotiation periods				
Average debt/GDP ratio (%)	130.5	50.6	53.6	59.8
Corr.(debt/GDP, output)	-0.95	-0.99	-0.99	-0.99
Duration of renegotiation/ exclusion (quarters)	14.0	11.1	8.7	8.8
Corr.(cumulative change in public investment to GDP, duration)	-0.12	-0.08	-	-0.15 ^{1/}
Corr.(cumulative percent change in public investment, duration)	-0.16	-0.1	-	-0.14 ^{1/}
Public capital (percent change from the trough to the end)	2.31	1.7	-	0.5 ^{2/}

Notes: ^{1/} “Total” investment in the model without separation of private and public sectors

^{2/} “Total” capital in the model without separation of private and public sectors

QUANTITATIVE ANALYSIS - SIMULATION (CONT.)

- Public investment around debt restructurings

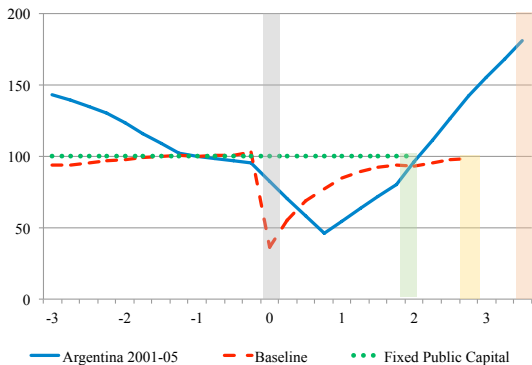


FIGURE: Around Start of Restructurings

QUANTITATIVE ANALYSIS - SIMULATION (CONT.)

- Public consumption and transfers around debt restructurings

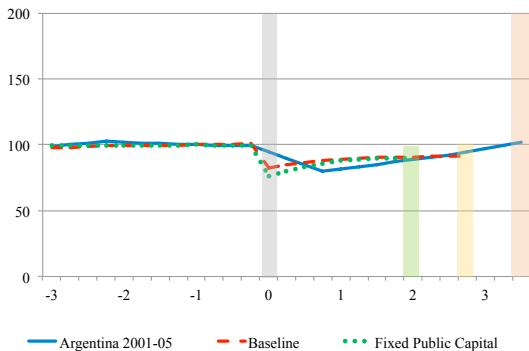


FIGURE: Around Start of Restructurings

FIGURE: Public Expenditure Composition

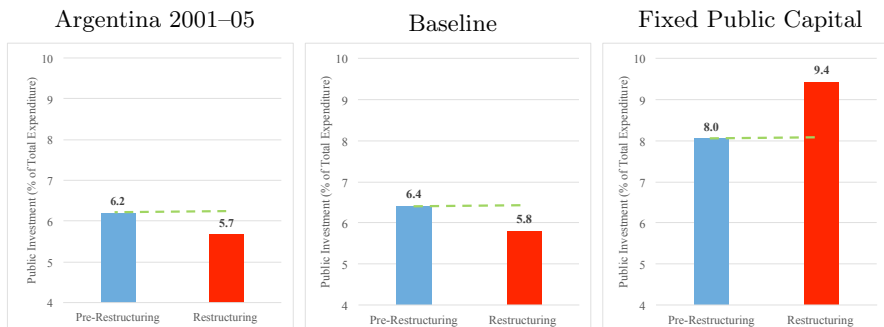


FIGURE: Public Investment Decline and Duration of Restructurings

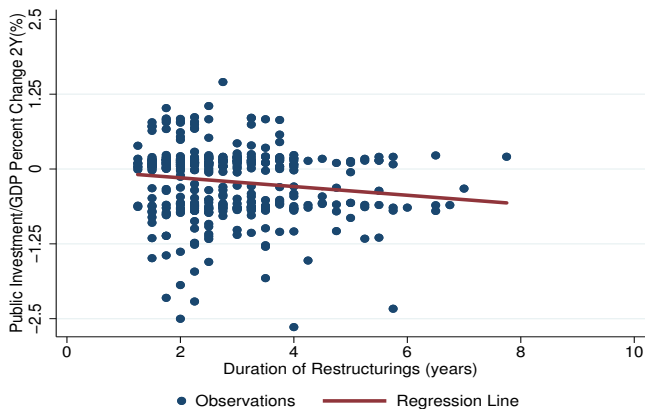


TABLE: Public Investment and Capital, and Debt Settlements

	Debt settlement (binary, current)			
	(1)	(1')	(2)	(2')
	coef/ se	dy/dx / Delta-method se	coef/ se	dy/dx / Delta-method se
Public investment (lagged, percent of mean TFP)	3.833*** (0.452)	0.471*** (0.055)	-	-
Public capital growth, annualized (lagged, percent)	-	-	0.055*** (0.019)	0.008*** (0.003)
External debt (lagged, percent of GDP)	-0.085*** (0.007)	-0.010*** (0.001)	-0.026*** (0.001)	-0.004*** (0.0001)
Constant	-	-	-	-
<hr/>				
Episode-specific Fixed effect	No		No	
Number of episodes	76		76	
Number of observations	831		831	
Wald χ^2	327.9		470.8	
Prob.> χ^2	0.000		0.000	

TESTING THE MODEL PREDICTION

TABLE: Public Investment and Capital, and Debt Settlements

	Debt Settlement (binary, current)			
	(1)	(1')	(2)	(2')
	coef/ se	dy/dx / Delta-method se	coef/ se	dy/dx / Delta-method se
Public investment, deviation from the trend (lagged, percent) ^{2/}	0.347* (0.195)	0.086* (0.048)	-	-
Public capital, cumulative growth rate (lagged, percent) ^{1/}	-	-	0.005* (0.003)	0.001* (0.0007)
PPG external debt (lagged, percent of GDP) ^{3/}	-0.003** (0.002)	-0.001** (0.0004)	-0.003* (0.002)	-0.0007*** (0.0004)
GDP, deviation from the trend (current, percent) ^{2/}	-0.017 (0.951)	-0.004 (0.236)	0.561 (0.864)	0.139 (0.215)
GDP, trend growth rate (current, percent) ^{2/}	-0.045 (0.042)	-0.011 (0.010)	-0.073 (0.045)	-0.018 (0.011)
LIBOR, 12-month average (current, percent)	-0.061*** (0.023)	-0.051*** (0.006)	-0.058** (0.024)	-0.014** (0.006)
World GDP, growth rate (current, percent)	0.207*** (0.069)	0.051*** (0.017)	0.206*** (0.069)	0.051*** (0.017)
Constant	-0.705* (0.369)	-	-0.722* (0.372)	-
Episode-specific Fixed effect		No		No
Number of episodes		90		89
Number of observations		496		492
Wald χ^2		21.01		20.74
Prob.> χ^2		0.002		0.002

CONCLUSION

- New dataset on public expenditure composition and new stylized facts:
 - A severe decline in public investment
 - Different dynamics of public consumption and transfers
 - Public expenditure tilting towards consumption and transfers
 - Link between public investment decline and delays
- New theoretical explanations on debt overhang and restructurings:
 - Two phases of sovereign debt overhang
 - Role of public capital in debt renegotiations
- Quantitative analysis of model rationalizes the four stylized facts

TABLE: Public Consumption, Investment, Transfers and Capital for Restructurings in 1978–2010

	Observation	Mean	Median	Std Dev.	Observation	Mean	Median	Std. Dev.
Restructuring Episodes	179				<i>Percent of GDP</i>			
		Pre-restructuring periods			Restructuring periods			
Public Consumption, average	124	13.1	11.2	9.4	124	12.0	10.6	7.4
Public Investment, average	151	4.7	3.4	4.3	151	3.7	3.0	3.3
Public Transfers, average	124	5.3	3.1	6.3	124	3.9	2.4	4.7
Public Capital, average	151	75.0	58.6	49.3	151	74.2	56.5	50.9
		Post-restructuring periods						
Public Consumption, average	124	11.7	10.1	7.5				
Public Investment, average	151	4.0	3.2	3.8				
Public Transfers, average	124	4.6	3.0	4.8				
Public Capital, average	151	74.9	61.7	48.4				

STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

TABLE: Public Investment, Capital, Consumption, and Transfers around Restructurings

	Public Investment	Public Capital	Public Consumption	Public Transfers
	deviation from trend, current ^{3/}	percentage change, current ^{4/}	deviation from trend, current ^{3/}	deviation from trend, current ^{3/}
	(1)	(2)	(3)	(4)
	coef/se	coef/se	coef/se	coef/se
Restructuring period (current, dummy) ^{1/}	-0.14*** (0.03)	-1.26*** (0.23)	0.007 (0.011)	-0.02 (0.03)
Post-restructuring period (current, dummy) ^{2/}	-0.07** (0.03)	-0.87*** (0.24)	0.003 (0.012)	-0.04 (0.03)
PPG external debt (lagged, percent of GDP)	-0.0007*** (0.003)	-0.01*** (0.002)	0.00003 (0.0001)	0.000005 (0.0002)
GDP, deviation from trend (current, percent) ^{3/}	0.03*** (0.003)	0.07*** (0.03)	0.011*** (0.001)	0.012*** (0.003)
Constant	0.10*** (0.03)	4.18*** (0.21)	-0.008* (0.01)	0.01 (0.03)
Episode-specific fixed effects	Yes	Yes	Yes	Yes
Number of restructuring episodes	97	96	92	81
Number of observations	1,043	996	949	747
F-statistics	37.94	21.39	19.59	4.02
R ²	0.139	0.087	0.084	0.024

STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

TABLE: Public Expenditure Composition around Restructurings

	Public Investment	Public Consumption	Public Transfers	Public Investment
	percent of GDP, current	percent of GDP, current	percent of GDP, current	percent of expenditure, current
	(1)	(2)	(3)	(4)
	coef/se	coef/se	coef/se	coef/se
Restructuring period (current, dummy) ^{1/}	-0.85*** (0.18)	-0.79* (0.44)	0.38** (0.18)	-2.23*** (0.80)
Restructuring period*PPG external debt (lagged, percent of GDP) ^{2/}	-0.003** (0.001)	-0.005 (0.003)	-0.009 (0.001)	-0.004 (0.005)
Post-restructuring period (current, dummy) ^{3/}	-0.47* (0.27)	-1.04 (0.65)	0.38 (0.26)	-0.58 (1.19)
Post-restructuring period*PPG external debt (lagged, percent of GDP) ^{4/}	-0.006* (0.003)	-0.015** (0.007)	-0.0006 (0.003)	-0.008 (0.013)
GDP, deviation from trend (current, percent) ^{5/}	0.03** (0.02)	0.02 (0.04)	0.03* (0.02)	0.08 (0.08)
Constant	4.23*** (0.12)	13.08*** (0.31)	2.99*** (0.12)	22.58*** (0.56)
Episode-specific fixed effects	Yes	Yes	Yes	Yes
Number of restructuring episodes	95	93	93	91
Number of observations	1,028	882	882	863
F-statistics	13.47	7.46	1.89	3.79
R ²	0.068	0.045	0.012	0.024

STYLIZED FACTS ON DEBT RESTRUCTURINGS (CONT.)

TABLE: Correlation between Public Investment Declines and Duration of Restructurings

	Duration of restructurings (years)			
	(1)	(2)	(3)	(4)
	coef/se	coef/se	coef/se	coef/se
Public investment/GDP, cumulative difference over first 2 years (percentage point)	-0.47* (0.25)	-0.73** (0.35)	-	-
Public investment/GDP, difference over first 1 year (percentage point)	-	-	-0.53* (0.29)	-
Public investment, cumulative change over first 2 years (%)	-	-	-	-0.03* (0.01)
Public capital/GDP, pre-restructuring (%)	-	-0.006 (0.008)	-	0.005 (0.006)
GDP, deviation from trend, end (%) ^{2/}	-	0.11* (0.06)	0.11* (0.06)	0.12** (0.06)
External debt, end (percent of GDP)	-	0.016 (0.01)	0.018* (0.01)	0.02* (0.01)
Export-to-debt service ratio, end (%)	-	0.13 (0.09)	0.12 (0.09)	0.13 (0.09)
LIBOR 12-month, end (%)	-	-0.69*** (0.16)	-0.67*** (0.16)	-0.64*** (0.16)
IMF-supported program, end (dummy) ^{3/}	-	-2.64*** (0.97)	-2.75*** (0.98)	-2.61*** (0.96)
Bond restructurings (dummy) ^{4/}	-	-4.33** (1.64)	-4.53*** (1.64)	-4.47*** (1.63)
Constant	4.83*** (0.535)	10.66*** (1.91)	10.42*** (1.88)	9.55*** (1.91)
Number of observations	95	86	86	87
Adjusted-R ²	0.027	0.36	0.35	0.35
Root MSE	4.79	4.05	4.08	4.06

STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

- **Stylized Fact 1:** Public investment experiences a severe decline and slow recovery around restructurings
- **Stylized Fact 2:** The dynamics of public investment differs from that of public consumption and transfers
- **Stylized Fact 3:** Public expenditure tilts remarkably towards consumption and transfers during debt crisis
- **Stylized Fact 4:** Sharp decline in public investment is associated with long restructuring duration

STYLIZED FACTS ON RESTRUCTURINGS (CONT.)

- Different dynamic of private investment (capital accumulation) with that of public investment

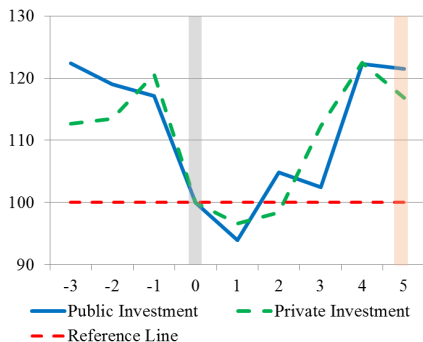


FIGURE: Public and private investment (level, start=100)

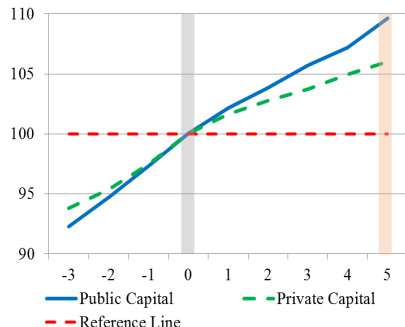
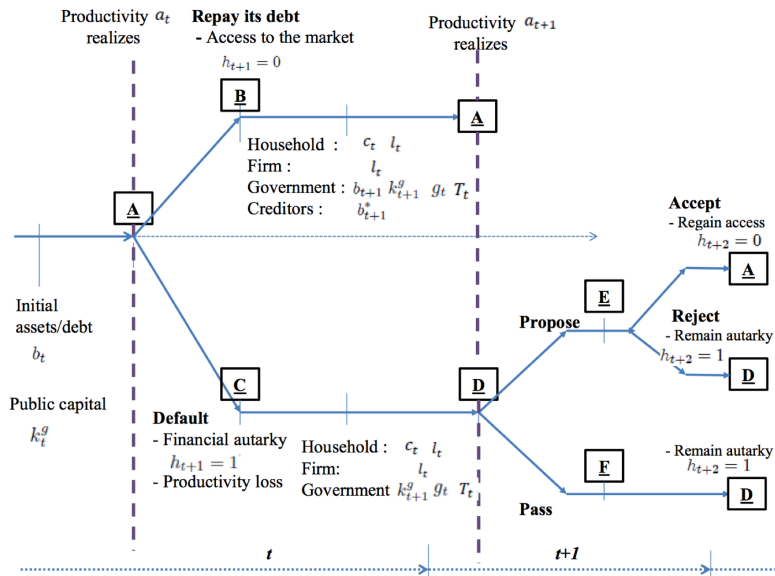


FIGURE: Public and private capital (level, start=100)

MODEL: GENERAL FEATURES (CONT.)

- A risk averse sovereign debtor, a household, a private firm and risk-neutral foreign creditors
- A stochastic TFP shock a_t
- Public expenditure choice: consumption, investment, transfers and debt repayments
- Distortionary consumption tax and no lump-sum tax
- Public capital accumulation and elastic labor supply
- Credit record h_t : indicating status of market access
- Incomplete capital market: one-period zero-coupon bonds
- One-side commitment
- Multi-round renegotiation upon the default choice

MODEL: TIMING



MODEL: HOUSEHOLD'S PROBLEM

- Household maximization problem

$$\max_{c_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t, g_t) \quad (1)$$

$$s.t. \quad (1 + \tau)c_t = w_t l_t + \pi_t^F + T_t \quad (2)$$

where $U(c_t, l_t, g_t) = (1 - \lambda)u(c_t, l_t) + \lambda v(g_t)$

- Optimality condition of household

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{w_t}{1 + \tau} \quad (3)$$

MODEL: FIRM'S PROBLEM

- Production function

$$y_t = a_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k} \quad (4)$$

where $\bar{k}^p = 1$

(Mendoza and Yue 2012, Azzimonti 2015)

- Private firm's profit maximization problem:

$$\max_{l_t} \pi_t^F = a_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k} - w_t l_t \quad (5)$$

- Optimality condition of the private firm

$$w_t = \alpha_l a_t(l_t)^{\alpha_l-1}(k_t^g)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k} \quad (6)$$

MODEL: SOVEREIGN'S PROBLEM

- If the sovereign has saving ($b_t \geq 0$)

$$V(b_t, k_t^g, 0, a_t) = \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) \\ + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1} | a_t) \quad (7)$$

$$s.t. \quad g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_{t+1}^g, 0, a_t)b_{t+1} = \tau c_t + (1 - \delta^g)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g + b_t \quad (8)$$

$$T_t \geq 0 \quad (9)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t (l_t)^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (10)$$

$$(1 + \tau)c_t = y_t + T_t \quad (11)$$

MODEL: SOVEREIGN'S PROBLEM (CONT.)

- Case of good credit record (market access - $h_t = 0$)
- If the sovereign has debt ($b_t < 0$)

$$V(b_t, k_t^g, 0, a_t) = \max \left[V^R(b_t, k_t^g, 0, a_t), V^D(b_t, k_t^g, 0, a_t) \right] \quad (12)$$

- Sovereign's value of repayment

$$\begin{aligned} V^R(b_t, k_t^g, 0, a_t) = & \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) \\ & + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1} | a_t) \end{aligned} \quad (7')$$

$$s.t. \quad g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_{t+1}^g, 0, a_t) b_{t+1} = \tau c_t + (1 - \delta^g) k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g + b_t \quad (8)$$

$$T_t \geq 0 \quad (9)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t (l_t)^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^P)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (10)$$

$$(1 + \tau) c_t = y_t + T_t \quad (11)$$

MODEL: SOVEREIGN'S PROBLEM (CONT.)

- Sovereign's value of defaulting (restructuring)

$$V^D(b_t, k_t^g, 0, a_t) = \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) \\ + \beta \int_A V((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1})d\mu(a_{t+1}|a_t) \quad (13)$$

$$s.t. \quad g_t + k_{t+1}^g + T_t = \tau c_t + (1 - \delta^g)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g \quad (8')$$

$$T_t \geq 0 \quad (9)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l \tilde{a}_t(l_t)^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (10')$$

$$(1 + \tau)c_t = \tilde{y}_t + T_t \quad (11')$$

- Case of bad credit record (loss in access - $h_t = 1$)

$$V(b_t, k_t^g, 1, a_t) = \Gamma(b_t, k_t^g, a_t) \quad (14)$$

MODEL: RENEGOTIATION PROBLEM

- Strategies of the proposer i and the other party j (for $i, j = B, L$) depending on state (b_t, k_t^g, h_t, a_t) and current offer:

$$\theta_i = \{1 \text{ (propose)}\} \quad \& \quad \theta_j = \{1 \text{ (accept)}\}$$

$$\theta_i = \{0 \text{ (pass)}\} \quad \& \quad \theta_j = \{0 \text{ (reject)}\}$$

- Case when the borrower B is the proposer
- If B proposes and the proposal is

$$V^{PRO}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V(0, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (18)$$

$$s.t. \quad g_t + k_{t+1}^g + T_t = \tau c_t + (1 - \delta^g)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g + \delta_t^B b_t \quad (8'')$$

$$T_t \geq 0 \quad (9)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l \tilde{a}_t(l_t)^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (10')$$

$$(1 + \tau)c_t = \tilde{y}_t + T_t \quad (11')$$

$$V^{*ACT}(b_t, k_t^g, a_t) = -\delta_t^B b_t \quad (19)$$

MODEL: RENEGOTIATION PROBLEM (CONT.)

- If B passes,

$$V^{PASS}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) \\ + \beta \int_A V((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1})d\mu(a_{t+1}|a_t) \quad (20)$$

$$s.t. \quad g_t + k_{t+1}^g + T_t = \tau c_t + (1 - \delta^g)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g \quad (8')$$

$$T_t \geq 0 \quad (9)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l \tilde{a}_t^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (10')$$

$$(1 + \tau)c_t = \tilde{y}_t + T_t \quad (11')$$

$$V^{*REJ}(b_t, k_t^g, a_t) = \frac{1}{1 + r^*} \int_A \Gamma^*((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1})d\mu(a_{t+1}|a_t) \quad (21)$$

MODEL: RENEGOTIATION PROBLEM (CONT.)

- Equilibrium

$$\begin{aligned} \delta_t^{B*} &= \operatorname{argmax} V^{PRO}(b_t, k_t^g, a_t) \\ \text{s.t. } V^{PRO}(b_t, k_t^g, a_t) &\geq V^{PASS}(b_t, k_t^g, a_t) \\ V^{*ACT}(b_t, k_t^g, a_t) &\geq V^{*REJ}(b_t, k_t^g, a_t) \end{aligned} \quad (22)$$

- If both parties reach an agreement,

$$\Gamma^B(b_t, k_t^g, a_t) = V^{PRO}(b_t, k_t^g, a_t) \quad (23)$$

$$\Gamma^{B*}(b_t, k_t^g, a_t) = V^{*ACT}(b_t, k_t^g, a_t) \quad (24)$$

- Otherwise,

$$\Gamma^B(b_t, k_t^g, a_t) = V^{PASS}(b_t, k_t^g, a_t) \quad (23')$$

$$\Gamma^{B*}(b_t, k_t^g, a_t) = V^{*REJ}(b_t, k_t^g, a_t) \quad (24')$$

MODEL: CREDITOR'S PROBLEM

- Expected profit

$$\pi^c(b_{t+1}, k_{t+1}^g, 0, a_t) = \begin{cases} q(b_{t+1}, k_{t+1}^g, 0, a_t)b_{t+1} - \frac{1}{1+r^*}b_{t+1}, & \text{if } b_{t+1} \geq 0 \\ \left[\frac{1-p^D(b_{t+1}, k_{t+1}^g, 0, a_t)}{1+r^*} + \frac{p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_t) d\mu(a_{t+1}|a_t)}{1+r^*} \right] (-b_{t+1}) & \text{otherwise} \end{cases} \quad (34)$$

- Equilibrium bond price

$$q(b_{t+1}, k_{t+1}^g, 0, a_t) = \begin{cases} \frac{1}{1+r^*} & \text{if } b_{t+1} \geq 0 \\ \left[\frac{1-p^D(b_{t+1}, k_{t+1}^g, 0, a_t)}{1+r^*} + \frac{p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_t) d\mu(a_{t+1}|a_t)}{1+r^*} \right] & \text{otherwise} \end{cases} \quad (35)$$

- Default probability

$$p^D(b_{t+1}, k_{t+1}^g, 0, a_t) = \int_{D(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t), \quad (36)$$

- Expected recovery rates

$$\begin{aligned} \gamma(b_{t+1}, k_{t+1}^g, 1, a_t) &= \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1}|a_t) \\ &= \int_A \left[\begin{array}{l} \phi 1_{a_{t+1} \in R^B(b_{t+1}, k_{t+1}^g)} \delta_t^{B*}(b_{t+1}, k_{t+1}^g, a_{t+1}) \\ + (1 - \phi) 1_{a_{t+1} \in R^L(b_{t+1}, k_{t+1}^g)} \delta_t^{L*}(b_{t+1}, k_{t+1}^g, a_{t+1}) \\ + \left(\begin{array}{l} \phi 1_{a_{t+1} \notin R^B(b_{t+1}, k_{t+1}^g)} \\ + (1 - \phi) 1_{a_{t+1} \notin R^L(b_{t+1}, k_{t+1}^g)} \end{array} \right) \gamma(b_{t+2}, k_{t+2}^g, 1, a_{t+1}) \end{array} \right] d\mu(a_{t+1}|a_t) \end{aligned} \quad (38)$$

EQUILIBRIUM

DEFINITION

A recursive equilibrium is defined as a set of functions for (a) the sovereign's value function, public consumption, transfers, capital, assets/debt, default set, (b) the household's consumption and labor supply, (c) the firm's labor demand, (d) the sovereign's and the creditors' decision functions, payoffs, two sets of recovery rates, and (e) sovereign bond prices such that

- [1]. the sovereign government's value function, public consumption, capital, transfers, assets/debt, and default set satisfy its optimization problem (7)–(15);
- [2]. the household's consumption and labor supply satisfy his optimization problem (1)–(3);
- [3]. the firm's labor demand satisfies its optimization problem (4)–(6);
- [4]. both parties' decisions, payoffs and recovery rates solve the multi-round debt renegotiation problem (16)–(33);
- [5]. sovereign prices satisfy the foreign creditors' optimization problem (34)–(35).

QUANTITATIVE ANALYSIS - ERGODIC DIST. (CONT.)

- Public consumption/transfers – Mean public capital

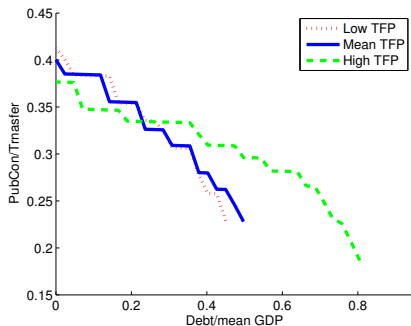


FIGURE: “Repayment” Region

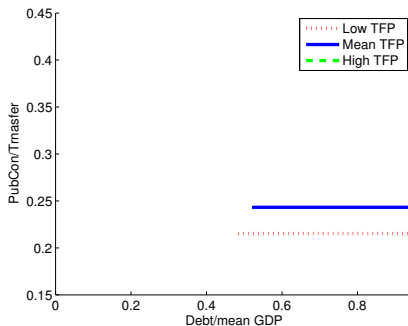


FIGURE: “Delay” Region

QUANTITATIVE ANALYSIS - SIMULATION (CONT.)

TABLE: Business Cycle Statistics

	Data	Model	Model with Fixed Public Capital	Model without Separation of Public/Private Sectors
Target statistics				
Pre-default periods				
Average public consumption & transfers/GDP ratio (%)	20.0	23.1	22.5	-
Public investment (std dev.)/output (std dev.) (%)	5.1	5.2	-	5.1
Non-target statistics				
Pre-default periods				
Private sector				
Private consumption (std dev.)/output (std dev.)	1.11	1.01	1.01	1.03
Trade balance/output: std dev. (%)	1.28	0.86	0.48	1.01
Corr.(trade balance, output)	-0.87	-0.18	-0.07	-0.23
Public sector				
Public consumption & transfers (std dev.)/output (std dev.)	1.26	1.23	1.22	-
Corr.(public consumption & transfers, output)	0.52	0.86	0.94	-
Average public investment/GDP ratio	1.31	1.60	2.01	1.58
Average public investment/public expenditure ratio	6.20	6.40	8.04	-
Corr.(public investment, output)	0.51	0.63	-	0.66
Renegotiation periods				
Private sector				
Private consumption (std dev.)/output (std dev.)	1.17	1.01	1.00	0.99
Trade balance/output: std dev. (%)	0.45	0.00	0.00	0.00
Corr.(trade balance, output)	-0.97	0.00	0.00	0.00
Debtor output deviation (diff. btw start and end, %) ^{6/}	12.6	21.2	22.58	21.47
Public sector				
Public consumption & transfers (std dev.)/output (std dev.)	0.99	2.23	1.07	-
Corr.(public consumption & transfers, Output)	0.99	0.77	0.67	-
Average public consumption & transfers/GDP ratio (%)	20.23	23.3	22.4	-
Average public investment/GDP ratio	1.19	1.47	2.36	1.24
Average public investment/GDP ratio (downward trend) ^{3/}	0.73	0.76	-	1.41
Average public investment/GDP ratio (upward trend) ^{3/}	1.64	1.65	-	0.91
Average public investment/public expenditure ratio	5.7	5.8	9.53	-
Corr.(public investment, output)	0.99	0.84	-	0.91

QUANTITATIVE ANALYSIS - COMPARISON WITH MODELS OF MULTI-ROUND NEGOTIATIONS

	Data	Baseline Model	Model with fixed public capital (case i) ^{1/}		Model with endogenous public capital (case ii) ^{2/}		Model with fixed public capital and no distortionary tax(case iii) ^{3/}
			With distortionary tax (case i-a)	With distortionary tax and lump-sum tax on income (case i-b)	No separation of public/private sectors (case ii-a)	With distortionary tax and lump-sum tax on income (case ii-b)	
Target statistics							
Default probability (%)	3.26	3.10	2.71	3.2	3.26	3.5	2.01
Average recovery rate (%)	25.0	27.1	22.4	33.2	39.0	54.9	32.0
Average debtor output deviation during debt renegotiation (%)	-4.45	-3.73	-4.50	-4.6	-4.62	-4.42	-5.80
Pre-default periods							
Average debt/GDP ratio (%)	45.4	43.9	45.6	62.5	40.0	24.5	52.5
Bond spreads: average (%)	9.4	0.2	0.2	0.1	0.2	0.4	0.15
Bond spreads: std dev. (%)	7.6	0.3	0.34	0.15	0.2	0.6	0.2
Corr.(spreads, output)	-0.88	-0.1	-0.31	-0.18	-0.17	-0.05	-0.32
Corr.(debt/GDP, spreads)	0.92	0.21	0.37	0.29	0.24	0.78	0.39
Corr.(debt/GDP, output)	-0.97	-0.69	-0.70	-0.68	-0.64	-0.1	-0.72
Renegotiation periods							
Average debt/GDP ratio (%)	130.5	50.6	53.7	73.5	47.1	30.0	63.7
Corr.(debt/GDP, output)	-0.95	-0.98	-0.99	-0.99	-0.98	-0.99	-0.99
Duration of renegotiation/ exclusion (quarters)	14.0	11.1	8.3	7.8	8.9	8.89	6.4
Debtor output deviation (diff. btw start and end, %) ^{4/}	12.6	21.2	22.7	22.1	22.2	17.1	21.6
Public capital (percent change from the trough to the end)	2.31	1.7	-	-	0.5	0	-
Corr.(cumulative change in public investment to GDP, duration)	-0.12	-0.08	-	-	-0.11	-0.17	-
Corr.(cumulative percent change in public investment, duration)	-0.16	-0.1	-	-	-0.15	-0.16	-

QUANTITATIVE ANALYSIS - ROBUSTNESS CHECK

	Adjustment costs			Depreciation rate			Weight on public cons			Risk aversion			Discount rate		
	5	10	15	0.025	0.04	0.075	0.7	0.8	0.9	2	3	4	0.85	0.90	0.93
Default probability (%)	4.22	3.1	1.9	5.2	3.1	4.9	3.7	3.1	5.2	5.7	3.1	4.9	3.3	3.1	3.9
Average recovery rate (%)	31.9	27.1	25.9	29.7	27.1	25.8	26.9	27.1	45.4	49.1	27.1	33.5	28.5	27.1	31.4
Public investment (std dev.)/output (std dev.) (%)	7.9	6.0	3.8	8.47	6.0	2.7	11.7	6.0	5.2	4.5	6.0	7.6	12.6	6.0	5.8
Non-target statistics															
Pre-default periods															
Public consumption & transfers (std dev.)/output (std dev.)	1.30	1.23	1.18	1.42	1.23	1.50	1.35	1.23	1.5	1.80	1.23	1.10	1.46	1.23	1.23
Corr.(public consumption & transfers, output)	0.83	0.89	0.92	0.78	0.89	0.84	0.87	0.89	0.81	0.78	0.89	0.87	0.85	0.89	0.84
Average public investment/GDP ratio	1.6	1.6	1.6	1.18	1.60	2.58	1.39	1.6	1.91	1.87	1.60	1.40	1.35	1.60	1.92
Corr.(public investment, output)	0.62	0.63	0.69	0.61	0.63	0.42	0.44	0.63	0.76	0.62	0.63	0.52	0.35	0.63	0.61
Average debt/GDP ratio (%)	40.2	43.9	43.6	40.2	43.9	44.9	42.4	43.9	25.0	25.9	43.9	23.4	42.7	43.9	36.4
Renegotiation periods															
Public consumption & transfers (std dev.)/output (std dev.)	2.28	2.23	2.30	2.30	2.23	2.20	2.0	2.23	3.54	3.85	2.23	1.30	2.10	2.23	2.14
Corr.(public consumption & transfers, output)	0.77	0.76	0.53	0.87	0.76	0.86	0.80	0.76	0.88	0.88	0.76	0.86	0.72	0.76	0.80
Average public investment/GDP ratio	1.58	1.47	1.46	0.85	1.47	2.95	1.58	1.47	1.39	1.60	1.47	1.51	1.61	1.47	1.73
Corr.(public investment, output)	0.80	0.84	0.78	0.8	0.84	0.81	0.83	0.84	0.9	0.85	0.84	0.67	0.73	0.84	0.80
Duration of restructurings & exclusion (quarters)	11.5	11.1	10.6	11.7	11.1	13.2	12.7	11.1	9.5	9.40	11.1	17.6	13.2	11.1	10.5
Public capital (percent change from the trough to the end, %)	5.0	1.7	0.4	0.56	1.7	7.49	6.4	1.7	0.11	4.21	1.7	7.9	7.1	1.7	0.8

- Probability of settling the deal

$$p^R(b_{t+1}, k_{t+1}^g, a_t) = \phi \int_{R^B(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t) + (1-\phi) \int_{R^L(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t) \quad (37)$$

- Sovereign bond spreads

$$s(b_{t+1}, k_{t+1}^g, 0, a_t) = \frac{1}{q(b_{t+1}, k_{t+1}^g, 0, a_t)} - (1 + r^*) \quad (39)$$

QUANTITATIVE ANALYSIS - ERGODIC DIST. (CONT.)

- Agreed recovery rates

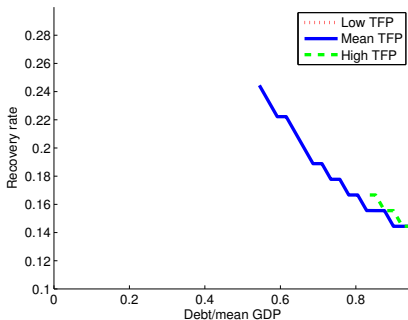


FIGURE: Mean Public Capital

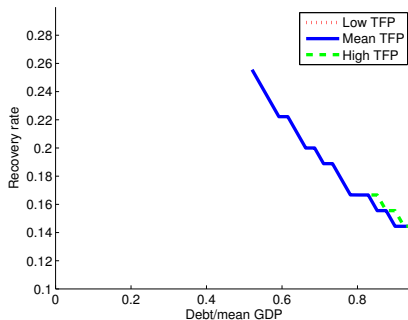


FIGURE: High Public Capital