Nonparametric Demand Estimation in Differentiated Products Markets

Giovanni Compiani

UC Berkeley – Haas School of Business

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Motivation

- Many questions in economics hinge on the shape of the market demand functions
- What are the effects of a merger?
- What is the pass-through of a tax?
- What are the sources of firm market power?

Example: tax pass-through

1. Estimate demand from data on quantities, prices and covariates
2. Get firm marginal costs from data or supply & demand
3. Given costs, use supply & demand to get equilibrium after tax

State of the art

- Focus on
  - differentiated products
  - endogenous prices
- The frontier is discrete choice with random coefficients (BLP)
  ▸ Literature
- Convenient, but arbitrary, parametric assumptions might drive results ▸ Ilustration
- Can we avoid these restrictions?

This paper

- Idea:
  - Minimize use of arbitrary restrictions
  - Impose constraints motivated by economics
- To do so, I combine:
  - large datasets
  - economic theory
  - frontier econometric tools

This paper

- The approach is nonparametric
  - No distributional assumptions on unobservables
  - Relax most functional form restrictions
- In addition, the model applies beyond discrete choice:
  - Complementarities
  - Consumer inattention
  - Continuous choices
  - Others

Applications

- Using grocery store data, estimate demand in two ways
  - standard mixed logit model
  - my approach
- What is the pass-through of a tax?
  - For one product, I find steeper own-price elasticity function $\Rightarrow$ pass-through is much lower ($\sim$30% vs $\sim$90% of tax)
- How much competition is internalized by multi-product firm? "Portfolio effect" (Nevo, 2001)
  - Two approaches give similar results
  - Not with more restrictive mixed logit model $\Rightarrow$ model selection

Roadmap

1. General Demand Model
   - Model and identification
   - Nonparametric estimation
   - Monte Carlo simulations
2. Applications
   - BLP demand estimates
   - Nonparametric demand estimates
   - Counterfactual 1: Tax pass-through
   - Counterfactual 2: Effect of two-product retailer

Model

- J goods plus the outside option
- $s = (s_1, ..., s_J)$: shares
- $p = (p_1, ..., p_J)$: endogenous prices
- $\xi = (\xi_1, ..., \xi_J)$: product- or market-level unobservables
- $z = (z_1, ..., z_J)$: excluded instruments for price
- $x = \left(x^{(1)}, x^{(2)}\right)$: exogenous demand shifters, with $x^{(1)} = \left(x_1^{(1)}, ..., x_J^{(1)}\right)$

Index Restriction

- Consider the general demand system

$$s = \sigma\left(x, \xi, p\right)$$

- I let $\delta_j = \beta_j x_j^{(1)} + \xi_j$ and require

$$s = \sigma\left(\delta, p, x^{(2)}\right)$$

Model subsumes discrete choice

- This discrete choice model satisfies the index restriction

$$\begin{aligned} u_{ij} &= \alpha_{p,i} p_j + \alpha_{\delta,i}\delta_j + \alpha_{x,i} x_j^{(2)} + \epsilon_{ij} \\ \delta_j &= \beta_j x_j^{(1)} + \xi_j \end{aligned}$$

- No need to assume distributions for ϵ_{ij} nor $(\alpha_{p,i}, \alpha_{\delta,i}, \alpha_{x,i})$
- u_{ij} need not be linear in $p_j, \delta_j, x_j^{(2)}$

Identification (Berry and Haile, 2014)

Assuming

- Index restriction
- Strict substitution *under some transformation of demand*
- The instruments (x, z) shift (s, p) 'enough',

Berry and Haile (2014) show that

$$s = \sigma\left(\delta, p, x^{(2)}\right)$$

is nonparametrically identified

Flexibility on consumer behavior

- The model is more general than discrete choice
- Focus on demand vs utility

  $\Rightarrow$ can be more agnostic about what consumers do
- Can accommodate
  - Complements
  - Consumer inattention
  - Consumer loss aversion
  - Continuous choice and multiple discrete choices

From Identification to Estimation

- Berry and Haile (2014) focus on nonparametric identification
  - What could we learn about demand if we observed the entire population of markets?
- Leveraging the identification results, I address estimation and inference
  - Can we estimate demand nonparametrically on datasets available to economists?
  - Can we obtain informative confidence sets for quantities of interest?
  - How can we test hypotheses on consumer behavior?
  - How do we choose among several parametric models?

Nonparametric Estimation

$$s_j = \sigma_j \left(\delta, p, x^{(2)} \right) \qquad j = 1, ..., J$$

Under identification assumptions,

$$x_j^{(1)} + \xi_j = \sigma_j^{-1} \left(s, p, x^{(2)} \right)$$

Also, we assume

$$\mathbb{E} \left(\xi_j | x, z \right) = 0 \qquad a.s.$$

$\Rightarrow$ Approximate σ_j^{-1} and project predicted residuals onto IVs

Nonparametric Estimation

- I approximate σ_j^{-1} using the method of sieves
- Basis functions: Bernstein polynomials

  Bernstein polynomials
- Easy to impose a number of economic constraints
- I obtain standard errors based on recent results (Chen and Pouzo, 2015; Chen and Christensen, 2018)

Inference

Theorem 1

Let the demand system σ be identified. Let f be a scalar functional of σ^{-1} and $\hat{v}_T(f)$ be a consistent estimator of the standard deviation of $f\left(\hat{\sigma}^{-1}\right)$. In addition, let Assumptions 1, 2, 3 and 4 hold. Then,

$$\sqrt{T}\frac{\left(f\left(\hat{\sigma}^{-1}\right)-f\left(\sigma^{-1}\right)\right)}{\hat{v}_T(f)} \xrightarrow{d} N(0,1).$$

Assumptions

Uses:

- Confidence intervals
- Hypothesis testing
- Model selection

Inference

- Proofs follow Chen and Christensen (2018), but in my setting there are multiple (J) equations and error terms
  - need to deal with correlation in the error terms
- I provide low-level conditions for functionals of interest:
  - Price elasticities ▸ Assumptions
  - (Counterfactual) equilibrium prices ▸ Assumptions

Curse of dimensionality

- The functions σ_j^{-1} have $2J$ arguments, plus extra covariates
- Number of parameters grows with number of goods and number of covariates

But

- Assumptions based on economics help alleviate that
- Large datasets (e.g. scanner data) are increasingly available
- Several interesting markets are low-dimensional ▸ Examples

Constraints

- Exchangeability (Pakes, 1994; BLP) ▸ Exchangeability
- Index restriction ▸ Index
- No income effects ▸ Symmetry
- Monotonicity ▸ M-Matrix

- I do not impose all of them in simulations/application

Computation

- Estimation is based on minimizing a quadratic form in the Bernstein coefficients
- If constraints are convex, standard algorithms converge to *global* minimizer
- BLP objective is typically non-convex

Monte Carlo Simulations

- Given the same data generating process, I estimate demand using
  - my approach
  - random coefficients logit
- Then compare own- and cross-price elasticities
- Show that
  - nonparametric approach works for reasonable sample sizes (3,000)
  - approach is applicable beyond discrete choice

Correctly-specified BLP

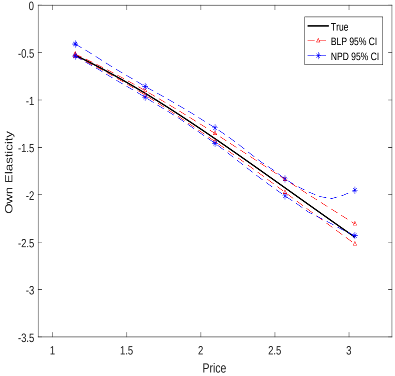


Figure: Own-price

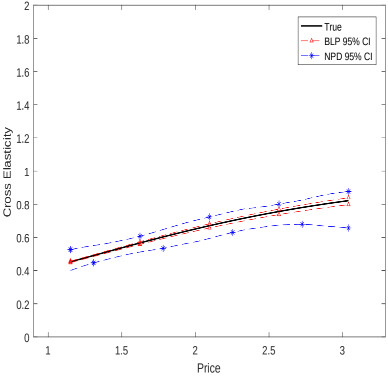


Figure: Cross-price

DGP

Inattention

- A fraction of consumers ignores good 1
- The fraction of inattentive consumer increases with p_1
- Otherwise, same model as before

Inattention

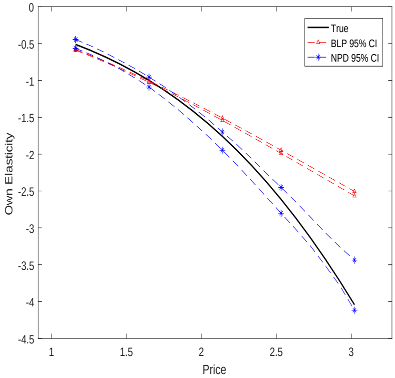


Figure: Own-price

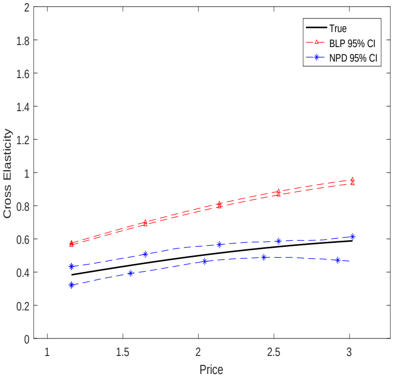


Figure: Cross-price

Loss Aversion

$$u_{ij} = -\alpha_i p_j - \alpha_{loss}(p_j - p_k) + x_j + \xi_j + \epsilon_{ij}$$

Loss Aversion

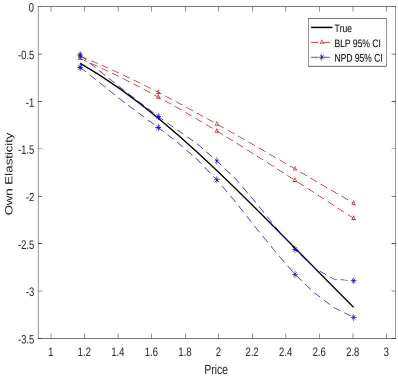


Figure: Own-price

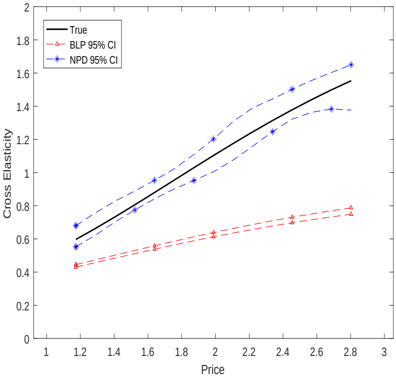


Figure: Cross-price

Complements

- Exogenous variables and prices are generated as above
- Let

$$q_j = 10 \frac{\delta_j}{p_j^2 p_k} \quad j = 1, 2; \quad k \neq j$$

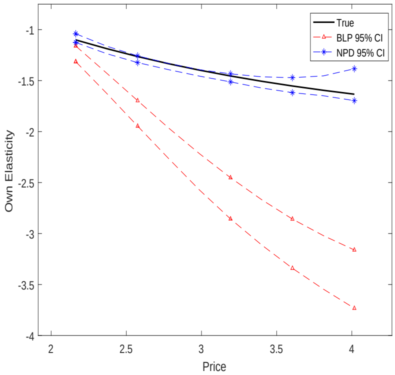
$\Rightarrow$ Good 1 and 2 are complements

- Define

$$s_j = \frac{q_j}{1 + q_1 + q_2} \quad j = 1, 2$$

$\Rightarrow$ Strict substitution assumption ✓

Complements



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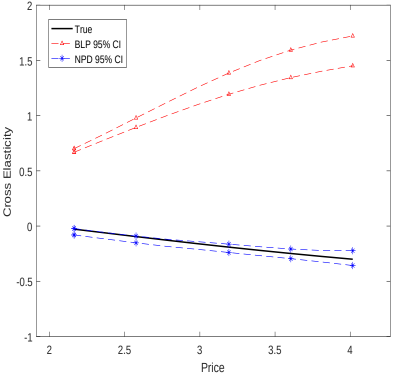

Figure: Cross-price

Additional Simulations

- Chi-square random coefficients [Chi-Square]
- Smaller sample size [T=500]
- Violation of index restriction [Index Violation]
- Sensitivity to tuning parameter [Sensitivity]
- $J > 2$ [$J > 2$]

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Empirical Setting

- I use scanner data from CA supermarkets
- A market is a store/week
- Look at sales of
  - non-organic strawberries (=1)
  - organic strawberries (=2)
  - other fruit (=0)
- Assume retailer is a monopolist *wrt strawberries*

Perishability simplifies framework

Empirical Model

$$s_1 = \sigma_1\left(\delta_{str}, \delta_{org}, p_0, p_1, p_2, x^{(2)}\right)$$

$$s_2 = \sigma_2\left(\delta_{str}, \delta_{org}, p_0, p_1, p_2, x^{(2)}\right)$$

where

- $x^{(2)} =$ Income
- $p_0, p_1, p_2 =$ Prices
- $\delta_{str}, \delta_{org} =$ Quality indices

Estimation Details | Fit | Elasticities

Exogenous Demand Shifters

$$\delta_{str} = \beta_{0,str} - \beta_{1,str} x_{str}^{(1)} + \xi_{str}$$

$$\delta_{org} = \beta_{0,org} + \beta_{1,org} x_{org}^{(1)} + \xi_{org}$$

where

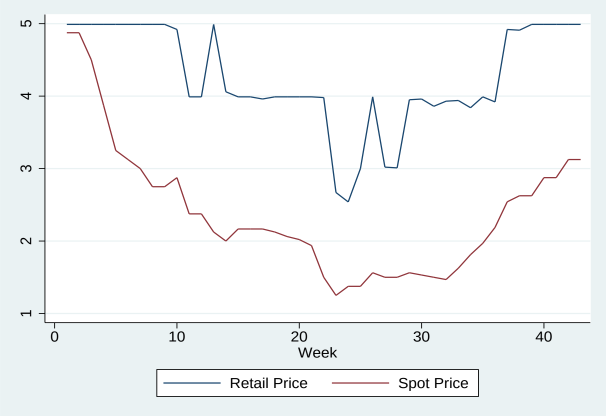
- $x_{str}^{(1)}$ is a proxy for richness of outside option
  - Captures substitution between inside and outside goods
- $x_{org}^{(1)}$ is a proxy for taste for organic products
  - Captures substitution between the two inside goods
- (ξ_{str}, ξ_{org}) = Unobservables varying across markets

Microfoundation

Endogenous Prices

- I instrument for retail prices using wholesale spot prices
- I also use retail prices of same products in other marketing areas (Hausman IVs)
    - Valid if unobservable demand shocks are independent across marketing areas, but retailer costs are not

Price Patterns



▸ Descriptive Stats ▸ First Stage

BLP Model

- For comparison, I also fit a logit model with a random coefficient on price
- I take a two-point distribution for the random coefficient

$$u_{i,1} = \beta_1 + \left(\beta_{p,i} + \beta_2 x^{(2)}\right) p_1 + \beta_{p,0} p_0 + \beta_{str}^{par} x_{str}^{(1)} + \xi_1 + \epsilon_{i,1}$$

$$u_{i,2} = \beta_2 + \left(\beta_{p,i} + \beta_2 x^{(2)}\right) p_2 + \beta_{p,0} p_0 + \beta_{str}^{par} x_{str}^{(1)} + \beta_{org}^{par} x_{org}^{(1)} + \xi_2 + \epsilon_{i,2}$$

BLP Estimates

Variable	Type I	Type II
Price	-7.58	-89.85
	(0.07)	(6.53)
Price$\times$Income	0.89	
	(0.06)	
Price other fruit	8.70	
	(0.23)	
Other fruit	-0.37	
	(0.01)	
Taste for organic	0.08	
	(0.06)	
Fraction of consumers	0.82	0.18
	(0.00)	(0.00)

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Counterfactual 1: Per-unit Tax

- Per-unit tax equal to 25% of the price
- Results depend on curvature of the demand function
- The faster the elasticity increases with price, the lower the pass-through

Significant difference in pass-through for organic

	NPD	Mixed Logit
Non-organic	0.84 (0.17)	0.53 ($5 \cdot 10^{-3}$)
Organic	0.33 (0.23)	0.91 ($5 \cdot 10^{-4}$)

Table: Median changes in prices as a percentage of the tax

Steeper own-price elasticity is consistent with lower pass-through

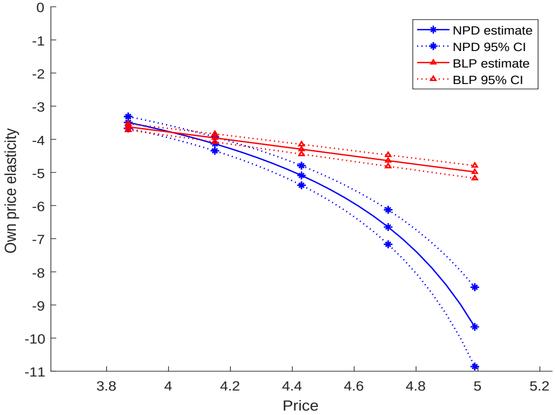

Figure: Estimated own-price elasticity function

Counterfactual 2: Portfolio Effect

- In each market, one retailer sells both types of strawberries
- Suppose there were two competing single-product retailers
- How much lower would markups be?

Portfolio Effect: Choice of parametric model matters

	NPD	MLog (I)	MLog (II)	MLog (III)
Non-organic	0.10	0.08	0.20	0.21
	$(3 \cdot 10^{-3})$	$(1 \cdot 10^{-3})$	$(8 \cdot 10^{-4})$	$(2 \cdot 10^{-3})$
Organic	0.43	0.42	0.54	0.55
	$(6 \cdot 10^{-3})$	$(2 \cdot 10^{-3})$	$(9 \cdot 10^{-4})$	$(1 \cdot 10^{-3})$

Table: Median decrease in prices as a percentage of markups

Portfolio Effect: Choice of parametric model matters

	NPD	MLog (I)	MLog (II)	MLog (III)
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Table: Median decrease in prices as a percentage of markups

Conclusion

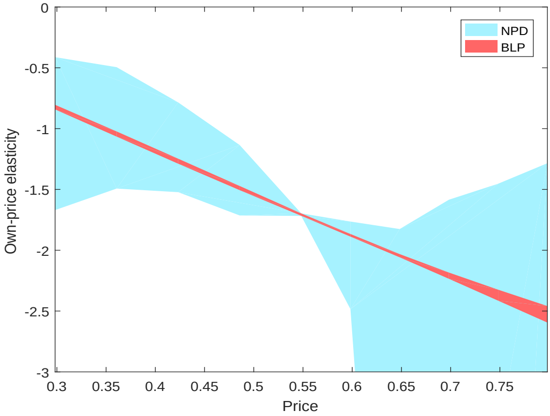
- First nonparametric approach to estimate demand for differentiated products
- Approach is applicable to data available to economists
- Flexibility matters for questions of interest

Related Literature

- Random coefficients discrete choice [Berry, Levinsohn and Pakes (1995); Nevo (2001); Petrin (2002); Berry, Levinsohn and Pakes (2004)]
- Discrete choice with inattention [Goeree (2008); Abaluck and Adams (2017)]
- Continuous and multiple discrete choice [Dubin and McFadden (1984); Hendel (1999); Dubé (2004)]
- Neoclassical demand [Deaton and Muellbauer (1980); Banks, Blundell and Lewbel (1997); Blundell, Chen and Kristensen (2007)]
- Complements [Gentzkow (2007)]
- Incomplete pass-through [Nakamura and Zerom (2010); Goldberg and Hellerstein (2013)]
- Multi-product firms and market power [Nevo (2001)]

Return

Illustration



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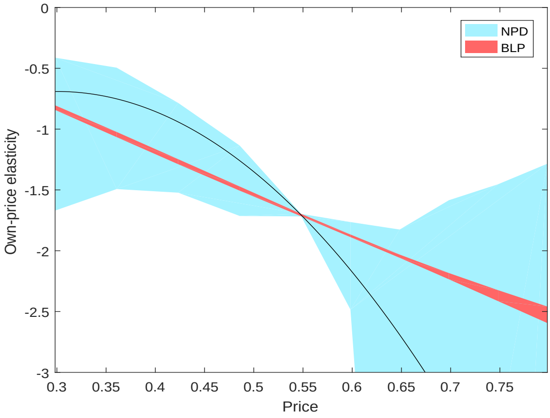

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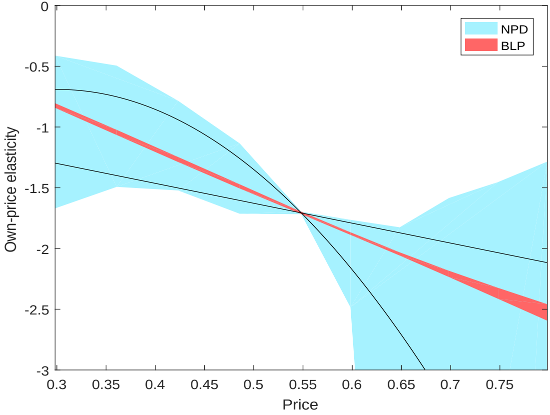

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Bernstein Polynomials

- The $k-$th Bernstein polynomial of degree m is defined as

$$b_{k,m}(u) \equiv \binom{m}{k} u^k (1-u)^{m-k}$$

  for $u \in [0,1]$ and $k = 0, \ldots, m$
- A continuous univariate function may be approximated by

$$\sum_{k=0}^{m} \theta_k b_{k,m}(u)$$

- For continuous multivariate functions, we may take the tensor product of the univariate Bernstein polynomials

Return

Bernstein Polynomials

Theorem (Uniform Approximation)

Let g *be a bounded real-valued function on* $[0,1]^N$ *and define*

$$B_m[g] = \sum_{v_1=0}^{m} \cdots \sum_{v_N=0}^{m} g\left(\frac{v_1}{m}, \cdots, \frac{v_N}{m}\right) b_{v_1,m}(u_1) \cdots b_{v_N,m}(u_N)$$

Then,

$$\sup_{\mathbf{u}\in[0,1]^N} \left| B_m[g](\mathbf{u}) - g(\mathbf{u}) \right| \to 0$$

as $m \to \infty$.

Return

Assumptions for Theorem 1

Assumption 1

For all $j, k \in \mathcal{J}, j \neq k$:

1. $\sup_{w \in \mathcal{W}} \mathbb{E}\left(\xi_j^2 | w\right) \leq \overline{\sigma}^2 < \infty$;
2. $\inf_{w \in \mathcal{W}} \mathbb{E}\left(\xi_j^2 | w\right) \geq \underline{\sigma}^2 > 0$;
3. $\sup_{w \in \mathcal{W}} \mathbb{E}\left(|\xi_j \xi_k| | w\right) \leq \overline{\sigma}_{cov} < \infty$;
4. $\sup_{w \in \mathcal{W}} \mathbb{E}\left[\xi_j^2 \mathbb{I}\left\{\sum_{i=1}^{J} |\xi_i| > \ell\left(T\right)\right\} | w\right] = o\left(1\right)$ *for any positive sequence* $\ell\left(T\right) \nearrow \infty$;
5. $\mathbb{E}\left(|\xi_j|^{2+\gamma^{(1)}}\right) < \infty$ *for some* $\gamma^{(1)} > 0$;
6. $\mathbb{E}\left(|\xi_j \xi_k|^{1+\gamma^{(2)}}\right) < \infty$ *for some* $\gamma^{(2)} > 0$.

Return

Assumptions for Theorem 1

Assumption 2

1. $\tau_M \zeta \sqrt{M(\log M)/T} = o(1)$;
2. $\zeta^{\left(2+\gamma^{(1)}\right)/\gamma^{(1)}} \sqrt{(\log K)/T} = o(1)$ *and* $\zeta^{\left(1+\gamma^{(2)}\right)/\gamma^{(2)}} \sqrt{(\log K)/T} = o(1)$, *where* $\gamma^{(1)}, \gamma^{(2)} > 0$ *are defined in Assumption 1;*
3. $K \asymp M$.

Assumption 3

The basis used for the instrument spaces is the same across all goods, i.e. $K_j = K_k$ *and* $a^{(j)}_{K_j}(\cdot) = a^{(k)}_{K_k}(\cdot)$ *for all* $j, k \in \mathcal{J}$.

Return

Assumptions for Theorem 1

Assumption 4

Let $\mathcal{H}_T \subset \mathcal{H}$ be a sequence of neighborhoods of h_0 with $\hat{h}, \tilde{h} \in \mathcal{H}_T$ wpa1 and assume $v_T(f) > 0$ for every T. Further, assume that:

1. $v \mapsto Df(h_0)[v]$ *is a linear functional and there exists* α *with* $|\alpha| \geq 0$ *s.t.* $|Df(h_0)[h - h_0]| \lesssim ||\partial^\alpha h - \partial^\alpha h_0||_\infty$ *for all* $h \in \mathcal{H}_T$;
2. *There are* α_1, α_2 *with* $|\alpha_1|, |\alpha_2| \geq 0$ *s.t.*
   1. $\left| f\left(\hat{h}\right) - f(h_0) - Df(h_0)[\hat{h} - h_0] \right| \lesssim ||\partial^{\alpha_1}\hat{h} - \partial^{\alpha_1} h_0||_\infty ||\partial^{\alpha_2}\hat{h} - \partial^{\alpha_2} h_0||_\infty$;
   2. $\frac{\sqrt{T}}{\sigma_T(f)}\left(||\partial^{\alpha_1}\hat{h} - \partial^{\alpha_1} h_0||_\infty ||\partial^{\alpha_2}\hat{h} - \partial^{\alpha_2} h_0||_\infty + ||\partial^{\alpha}\tilde{h} - \partial^{\alpha} h_0||_\infty\right) = O_p(\eta_T)$ *for a nonnegative sequence* η_T *such that* $\eta_T = o(1)$;
3. $\frac{1}{v_T(f)}\left|\left|\left(Df\left(\hat{h}\right)[\psi_M]' - Df(h_0)[\psi_M]'\right)\left(G_A^{-1/2} S\right)_l^-\right|\right| = o_p(1)$.

◂ Return

Elasticity Functional

Assumption 5

1. *P has bounded support and (P, S) have densities bounded away from 0 and ∞;*
2. *The basis used for both the sieve space and the instrument space is tensor-product Bernstein polynomials. Further, for the sieve space, the univariate Bernstein polynomials all have the same degree $M^{1/4}$;*
3. *$h_0 = [h_{0,1}, h_{0,2}]$ where $h_{0,1}$ and $h_{0,2}$ belong to the Hölder ball of smoothness $r \geq 4$ and finite radius L, and the order of the tensor-product Bernstein polynomials used for the sieve space is greater than r;*
4. $M^{\frac{2+\gamma^{(1)}}{2\gamma^{(1)}}} \sqrt{\frac{\log T}{T}} = o(1)$ *and* $M^{\frac{1+\gamma^{(2)}}{2\gamma^{(2)}}} \sqrt{\frac{\log T}{T}} = o(1)$;
5. $\frac{\sqrt{T}}{v_T(f_\epsilon)} \times \left(M^{\frac{3-r}{4}} + \tau_M^2 M^{9/4} \frac{\log M}{T} \right) = o(1)$.

◂ Return

Equilibrium Price Functional

Assumption 6

1. *P has bounded support and (P, S) have densities bounded away from 0 and ∞;*
2. *The basis used for both the sieve space and the instrument space is tensor-product Bernstein polynomials. Further, for the sieve space, the univariate Bernstein polynomials all have the same degree $M^{1/4}$;*
3. *$h_0 = [h_{0,1}, h_{0,2}]$ where $h_{0,1}$ and $h_{0,2}$ belong to the Hölder ball of smoothness $r \geq 5$ and finite radius L, and the order of the tensor-product Bernstein polynomials used for the sieve space is greater than r;*
4. $M^{\frac{2+\gamma^{(1)}}{2\gamma^{(1)}}} \sqrt{\frac{\log T}{T}} = o(1)$ *and* $M^{\frac{1+\gamma^{(2)}}{2\gamma^{(2)}}} \sqrt{\frac{\log T}{T}} = o(1)$;
5. $\frac{\sqrt{T}}{v_T\left(f_{p_1}\right)} \times \left(M^{\frac{4-r}{4}} + \tau_M^2 M^{9/4} \frac{\log M}{T}\right) = o(1).$

Return

Low-dimensional Settings

- International economics (Adao, Costinot and Donaldson, 2017)
- Market for news (Gentzkow, 2007)
- US presidential elections

Return

Exchangeability

Given a permutation $\pi : \{1, ..., J\} \to \{1, ..., J\}$, assume that $\forall j$

$$\sigma_j\left(\delta, p, x^{(2)}\right) = \sigma_{\pi(j)}\left(\delta_{\pi(1)}, ..., \delta_{\pi(J)}, p_{\pi(1)}, ..., p_{\pi(J)}, x^{(2)}_{\pi(1)}, ..., x^{(2)}_{\pi(J)}\right)$$

- In words, only the products' attributes—not their names—matter
- E.g. for $J = 3$ and no $x^{(2)}$,

$$\sigma_1\left(\delta_1, \underline{\delta}, \overline{\delta}, p_1, \underline{p}, \overline{p}\right) = \sigma_1\left(\delta_1, \overline{\delta}, \underline{\delta}, p_1, \overline{p}, \underline{p}\right)$$

- Implicit in most IO demand models
- Systematic differences between goods can be captured by product fixed effects

◂ Return

Exchangeability

- This assumption greatly reduces the number of parameters to be estimated. E.g. for univariate polynomials of degree 2 (and no $x^{(2)}$):

J	exchangeability	no exchangeability
3	324	729
4	900	6,561
5	2,025	59,049
10	27,225	3.4bn

- It can be shown that exchangeability of σ implies exchangeability of σ^{-1}
- Exchangeability of σ^{-1} can be imposed through linear restrictions on the Bernstein coefficients

Return

Index Restriction

- $x^{(2)}$ enters demand flexibly

$$\begin{aligned} s &= \sigma\left(\delta, p, x^{(2)}\right) \\ \delta_j &= \beta_j x_j^{(1)} + \xi_j \end{aligned}$$

- $x^{(2)}$ enters demand through $\delta \Rightarrow$ number of parameters drops

$$\begin{aligned} s &= \sigma(\delta, p) \\ \delta_j &= \beta_j^{(1)} x_j^{(1)} + \beta_j^{(2)} x_j^{(2)} + \xi_j \end{aligned}$$

- Same logic applies to p

Return

$M-$Matrix Properties

- Berry, Gandhi and Haile (2013) show that the Jacobian of the demand function, $\mathbb{J}_{\sigma}^{\delta}$ is an $M-$matrix
- This encapsulates restrictions from economic theory
  - e.g. shares increase (decrease) in own (competitors') δ
- By the implicit function theorem,

$$\mathbb{J}_{\sigma^{-1}}^{s} = \left[\mathbb{J}_{\sigma}^{\delta}\right]^{-1}$$

  i.e. $\mathbb{J}_{\sigma^{-1}}^{s}$ is an inverse $M-$matrix
- There is a large literature in linear algebra on properties of inverse $M-$matrices
- A number of them can be imposed through linear restrictions

Return

No Income Effects

- With no income effects, then Hicksian and Walrasian demands coincide
  $\Rightarrow \mathbb{J}^{p}_{\sigma}$ is symmetric

- By the implicit function theorem,

$$\mathbb{J}^{p}_{\sigma} = -\left[\mathbb{J}^{s}_{\sigma^{-1}}\right]^{-1} \mathbb{J}^{p}_{\sigma^{-1}}$$

- These are nonlinear constraints $\Rightarrow$ Use KNITRO

Return

Correctly-specified BLP

- Two goods with utility

$$u_{ij} = -\alpha_i p_j + x_j + \xi_j + \epsilon_{ij}$$

- Plus an outside option with utility $u_{i0} = \epsilon_{i0}$
- $\alpha_i \sim N\left(1, 0.15^2\right)$
- ϵ_{ij} is extreme value
- $x_j \sim \mathrm{U}\left[0, 2\right]$ independently across j
- $\xi_j \sim N\left(1, 0.15^2\right)$
- $z_j \sim \mathrm{U}\left[0, 1\right]$
- $p_j = 2\left(z_j + \eta_j\right) + \xi_j$, where $\eta_j \sim \mathrm{U}\left[0, 0.1\right]$
- Constraints: symmetry, $M-$matrix properties and exchangeability

Return

Chi-Square Random Coefficients

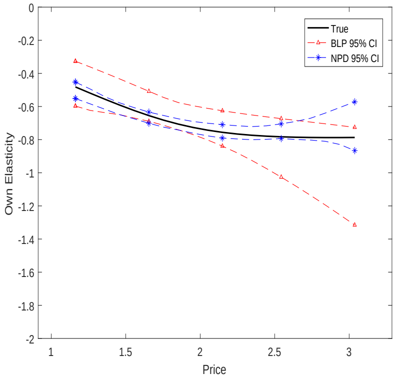

Figure: Own-price

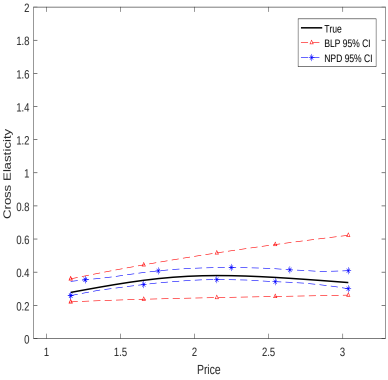

Figure: Cross-price

Return

Complements: T=500

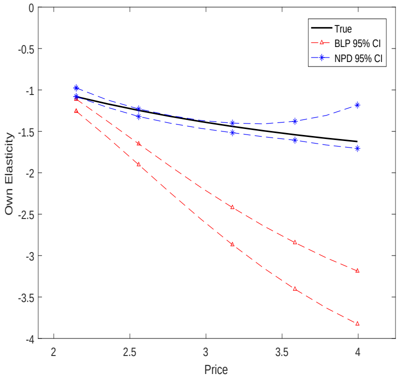

Figure: Own-price

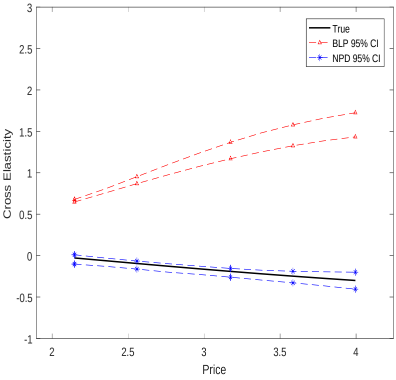

Figure: Cross-price

Return

Violation of index restriction: $st.dev. = 0.10$

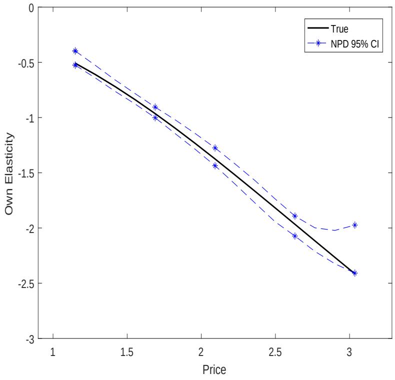

Figure: Own-price

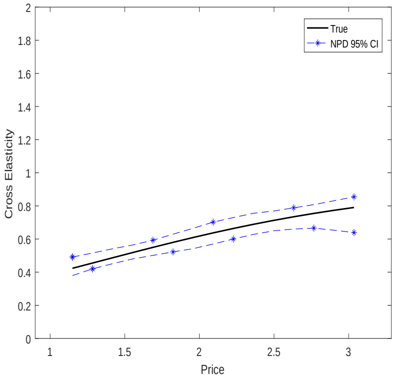

Figure: Cross-price

Return

Violation of index restriction: $st.dev. = 0.50$

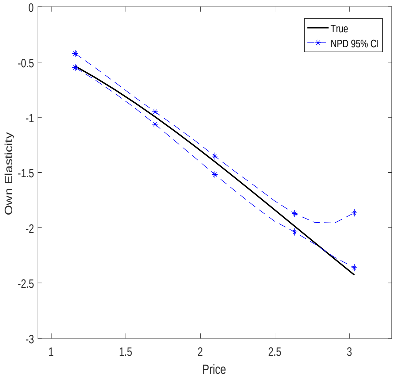

Figure: Own-price

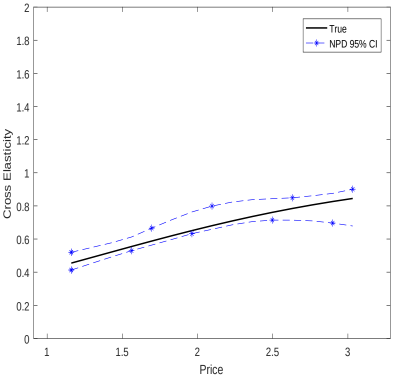

Figure: Cross-price

Return

Violation of index restriction: $st.dev. = 1.50$

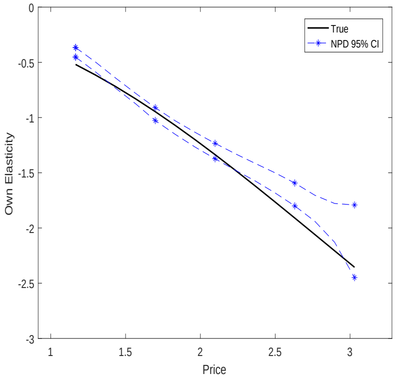

Figure: Own-price

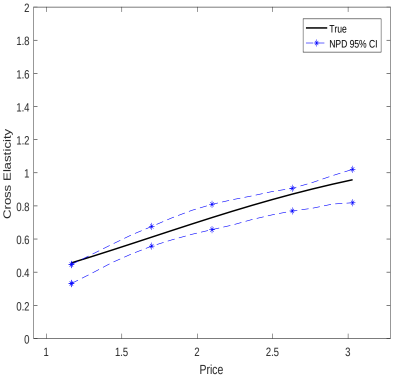

Figure: Cross-price

Return

Sensitivity: complements, degree=20

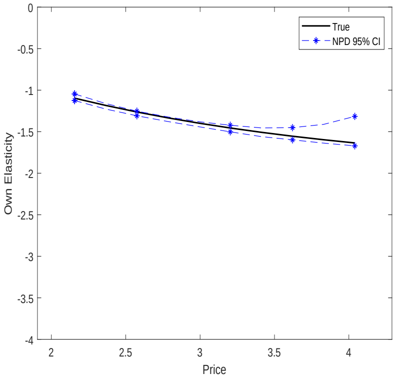

Figure: Own-price

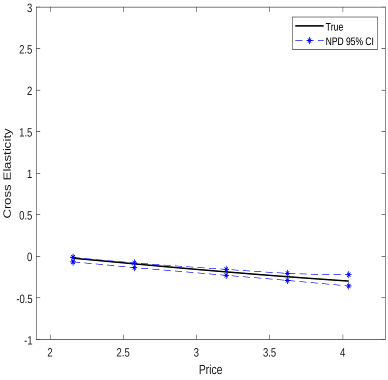

Figure: Cross-price

Return

Sensitivity: complements, degree=16

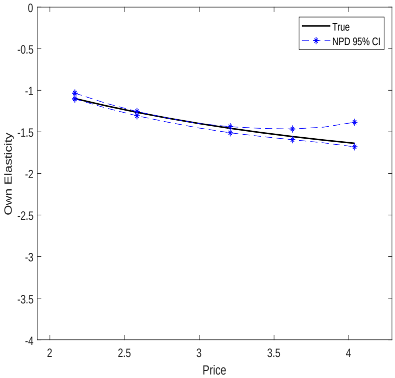

Figure: Own-price

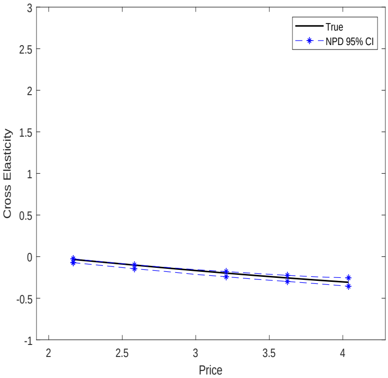

Figure: Cross-price

Return

Sensitivity: complements, degree=12

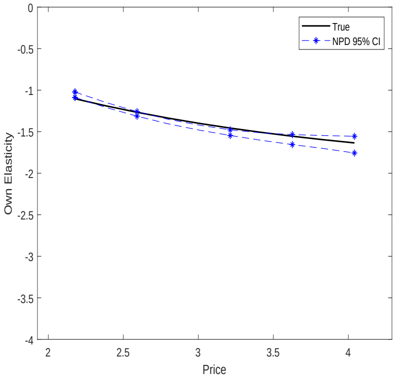

Figure: Own-price

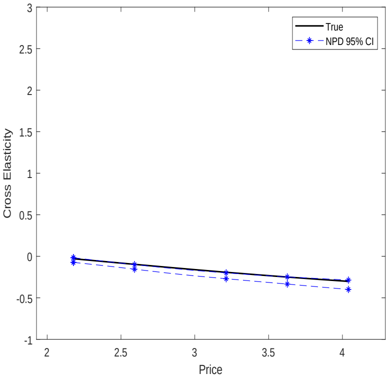

Figure: Cross-price

Return

Sensitivity: complements, degree=8

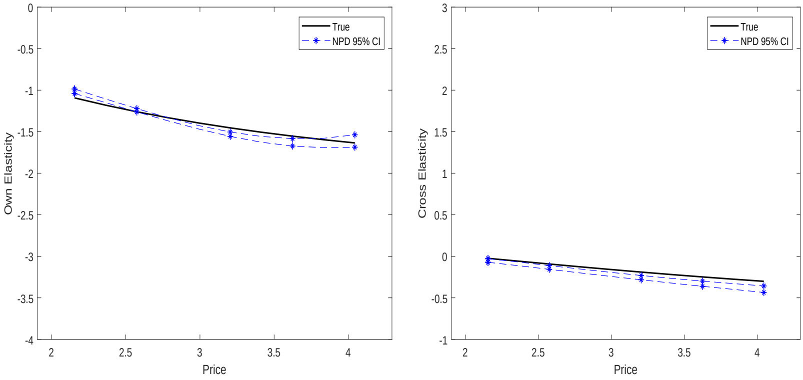

Figure: Own-price

Figure: Cross-price

Return

Sensitivity: complements, degree=6

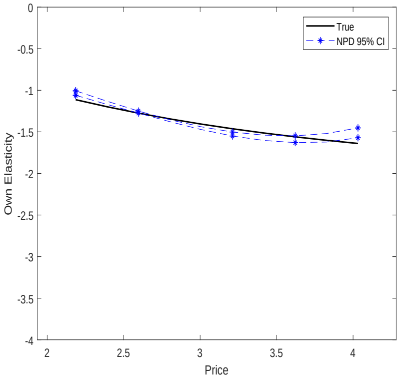

Figure: Own-price

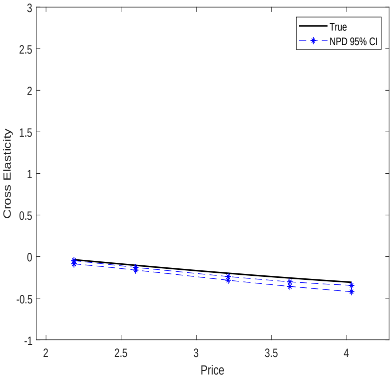

Figure: Cross-price

Return

Sensitivity: complements, degree=4

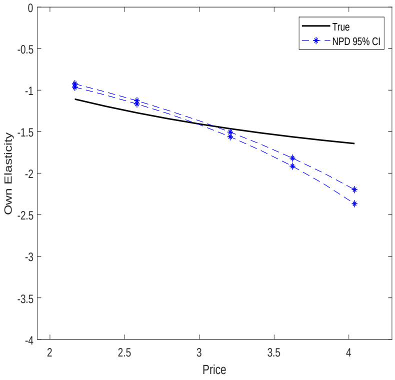

Figure: Own-price

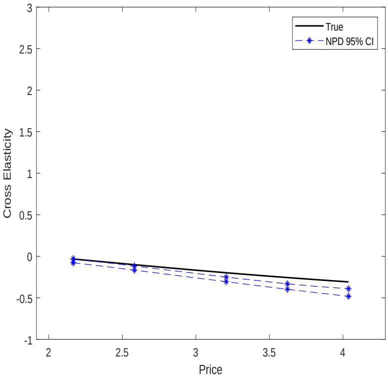

Figure: Cross-price

Return

Mixed Logit: $J = 3$

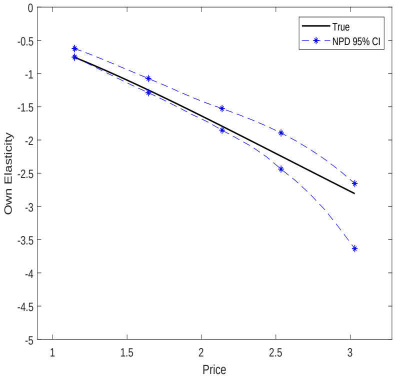

Figure: Own-price

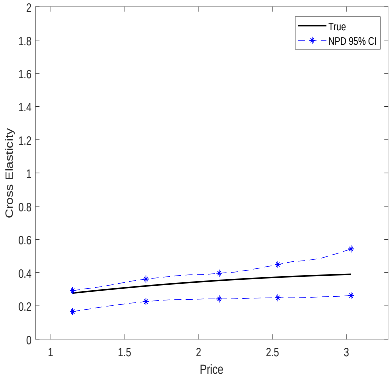

Figure: Cross-price

Return

Mixed Logit: $J = 5$

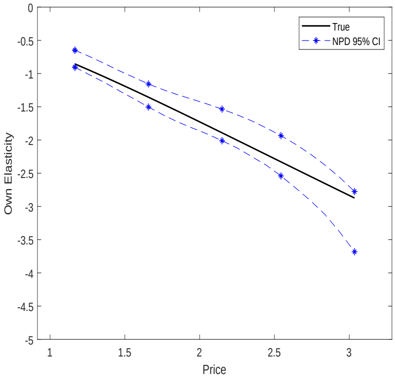

Figure: Own-price

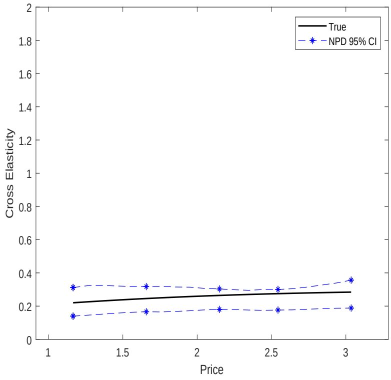

Figure: Cross-price

Return

Mixed Logit: $J = 7$

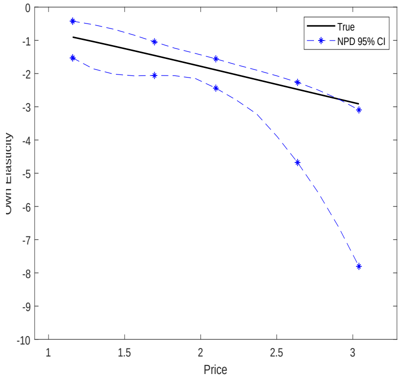

Figure: Own-price

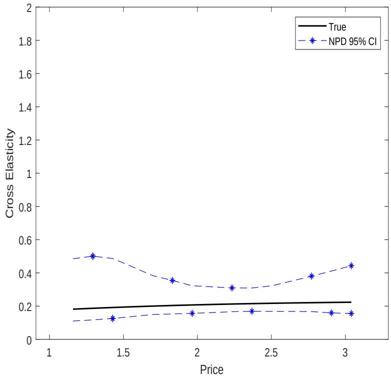

Figure: Cross-price

Return

Nonparametric Estimation

- Each of the functions to be estimated has six arguments $\Rightarrow$ a lot of parameters
- I impose $M-$matrix restrictions on the Jacobian
- Overall, I have 648 parameters
- The number of parameters could be reduced by including income and/or prices in the linear indices δ

Fit

Return

Fit

	NPD	Mixed Logit
MSE	0.93	2.38

Table: Two-Fold Cross-Validation Results

◂ Return

Median Nonparametric Elasticities

	Non-organic	Organic
Own-price elasticity	-1.402	-5.503
	(0.032)	(0.672)
Cross-price elasticity	0.699	1.097
	(0.044)	(0.177)

Return

Micro-foundation 1: discrete choice

$$u_{i1} = \theta_{str}\delta^*_{str} + \alpha_i p_1 + \epsilon_{i1}$$

$$u_{i2} = \theta_{str}\delta^*_{str} + \theta_{org}\delta^*_{org} + \alpha_i p_2 + \epsilon_{i2}$$

$$u_{i0} = \theta_{0,str}x^{(1)}_{str} + \theta_{0,org}\delta^*_{org} + \alpha_i p_0 + \epsilon_{i0}$$

where

$$\begin{aligned} \delta^*_{str} &= \xi_{str} \\ \delta^*_{org} &= \theta_{1,org}x^{(1)}_{org} + \xi_{org} \end{aligned}$$

Micro-foundation 2: continuous choice

$$\max_{q_0,q_1,q_2} q_0^{d_0\epsilon_{i,0}} q_1^{d_1\epsilon_{i,1}} q_2^{d_2\epsilon_{i,2}}$$

$$\text{s.t.} \quad p_0 q_0 + p_1 q_1 + p_2 q_2 \leq y_i^{inc}$$

where

$$\begin{aligned} d_0 &= \exp\left\{\theta_{0,org}\delta^*_{org} + \theta_{0,str} x^{(1)}_{str}\right\} \\ d_1 &= \exp\left\{\theta_{str}\delta^*_{str}\right\} \\ d_2 &= \exp\left\{\theta_{str}\delta^*_{str} + \theta_{org}\delta^*_{org}\right\} \end{aligned}$$

Return

Descriptive Statistics

	Mean	Median	Min	Max
Quantity non-organic	735.33	581.00	6.00	5,729.00
Quantity organic	128.91	78.00	1.00	2,647.00
Price non-organic	2.97	2.89	0.93	4.99
Price organic	4.26	3.99	1.24	6.99
Price other fruit	3.95	3.80	1.30	13.88
Hausman non-organic	3.00	2.98	2.09	4.05
Hausman organic	4.28	4.07	2.95	5.50
Hausman other fruit	4.50	3.79	1.19	13.33
Spot non-organic	1.46	1.35	0.99	2.32
Spot organic	2.38	2.17	1.25	4.88
Quantity other fruit (per capita)	0.83	0.82	0.60	1.08
Share organic lettuce	0.08	0.06	0.00	0.41
Income	82.54	72.61	33.44	405.09

First-Stage Regressions

	Non-organic		Organic	
	Price	Share	Price	Share
Spot price (own)	0.12**	−0.68**	0.35**	−0.26**
Spot price (other)	0.04**	0.10**	−0.21**	0.22**
Hausman (own)	0.70**	−1.30**	0.46**	−0.19**
Hausman (other)	−0.01	0.25**	0.13**	0.22**
Hausman (out)	−0.01**	0.11**	−0.10**	0.04**
Availability other fruit	−0.01**	−0.07**	−0.02**	−0.01**
Share organic lettuce	0.08**	−0.20**	−0.01**	0.10**
Income	−0.02**	0.00**	0.01**	0.04**
R^2	0.46	0.27	0.52	0.16

Return

Inattention

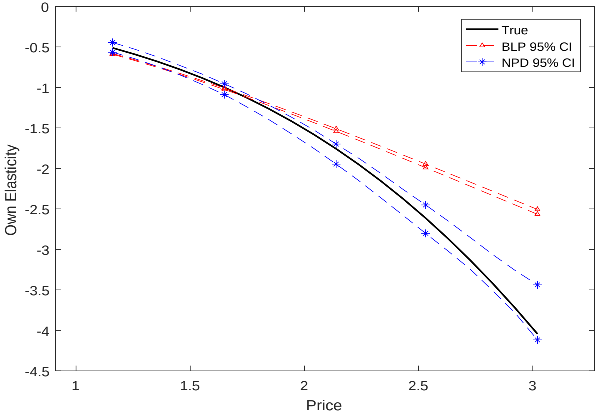

Figure: Own-price

◂ Return