

Gambling Traps

Anil Ari

International Monetary Fund

January 4, 2018

This Paper

Question: Why do banks become excessively exposed to aggregate risk?

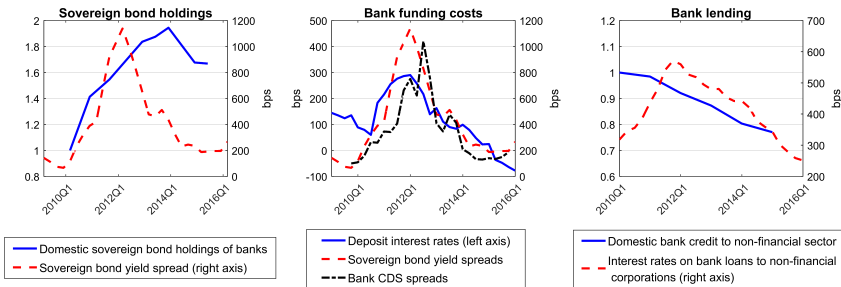
- Recent example: European sovereign debt crisis

This Paper

Question: Why do banks become excessively exposed to aggregate risk?

- Recent example: European sovereign debt crisis
- In European countries hit by the sovereign debt crisis, banks
 1. increased their holdings of domestic government debt
 2. faced rising financing costs
 3. reduced lending to the private sector

Example: Portugal



This Paper

Framework: Dynamic general equilibrium model with optimizing banks and depositors (and firms)

- Banks may optimally pursue risky portfolio, default in equilibrium
- Deposits priced according to expectations on bank risk-taking

This Paper

Framework: Dynamic general equilibrium model with optimizing banks and depositors (and firms)

- Banks may optimally pursue risky portfolio, default in equilibrium
- Deposits priced according to expectations on bank risk-taking

Contribution: Effects of funding costs on banks' risk-taking incentives

- Depositor sentiments about bank risk-taking may become self-fulfilling
- Bad equilibrium has implications for macroeconomic dynamics

This Paper

Framework: Dynamic general equilibrium model with optimizing banks and depositors (and firms)

- Banks may optimally pursue risky portfolio, default in equilibrium
- Deposits priced according to expectations on bank risk-taking

Contribution: Effects of funding costs on banks' risk-taking incentives

- Depositor sentiments about bank risk-taking may become self-fulfilling
- Bad equilibrium has implications for macroeconomic dynamics

Preview of Main Results:

- Financial fragility (risky banks & high funding costs)
- Endogenous persistence in crises (decline in bank lending & output)
- Accounts for macroeconomic dynamics in Portugal over 2010-2016
- Liquidity interventions may backfire and exacerbate multiplicity

Related Literature

- **Macro dynamics with bank lending:** Gertler & Kiyotaki (2010), Gertler & Karadi (2011), Krishnamurthy & He (2012), Brunnermeier & Sannikov (2014)
 - **Bank risk-taking:** Jensen & Meckling (1976), Kareken & Wallace (1978), Keeley (1990), Hellmann et al. (2000), Matutes & Vives (2000), Repullo (2004), Farhi & Tirole (2012)
 - **Sovereign default risk and banks:**
 - **Balance sheet effects:** Bolton & Jeanne (2011), Gennaioli et al. (2014), Perez (2015), Bocola (2016)
 - **Risk-shifting:** Acharya et al. (2014), Broner et al. (2014), Brunnermeier et al. (2016), Farhi & Tirole (2017), Crosignani (2017)
 - **Moral suasion:** Becker & Ivashina (2014), Uhlig (2014), Chari et al. (2016), De Marco & Macchiavelli (2016), Ongena et al. (2016)
- ⇒ **This paper:** Gambling on aggregate risk, role for funding costs

Overview: Model Environment

- Small open economy facing a sovereign debt crisis
 - ▶ Risky government debt
 - ▶ Incomplete (or non-credible) deposit insurance
 - ▶ Sovereign default costs

Overview: Model Environment

- Small open economy facing a sovereign debt crisis
 - ▶ Risky government debt
 - ▶ Incomplete (or non-credible) deposit insurance
 - ▶ Sovereign default costs
 - Banks optimally decide between two strategies
 - ▶ Safe strategy: precautionary, solvent after sovereign default
 - ▶ Gambling strategy: high sovereign exposure, insolvent after sov. default
- ⇒ Incentive to gamble because of limited liability

Overview: Model Environment

- Small open economy facing a sovereign debt crisis
 - ▶ Risky government debt
 - ▶ Incomplete (or non-credible) deposit insurance
 - ▶ Sovereign default costs
- Banks optimally decide between two strategies
 - ▶ Safe strategy: precautionary, solvent after sovereign default
 - ▶ Gambling strategy: high sovereign exposure, insolvent after sov. default

⇒ Incentive to gamble because of limited liability
- Depositors demand higher rates to compensate for insolvency risk
 - ▶ Banks cannot credibly commit to a safe strategy
non-contractibility of risk-taking decisions (sovereign exposure)
e.g. due to opacity of bank portfolios and/or timing

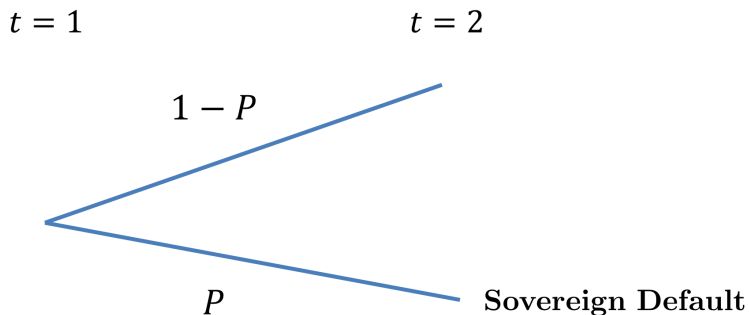
⇒ Depositor sentiments about gambling may become self-fulfilling

Roadmap

1. Motivation
2. **Two-period model**
3. Dynamic model
4. Policy

Two-period model

- Agents: households, banks, firms
- Sovereign default occurs with probability P in period 2



Assets & Portfolio Allocation (Period 1)

- Households: start with endowment, allocate savings between
 - ▶ Safe asset d^* at price q^*
 - ▶ Deposits d from domestic banks at price q
- Banks: use deposits and own net worth n to invest in
 - ▶ Domestic sovereign bonds b at price q^b
 - ▶ Loans l to firms at price q^l

Budget constraint

$$n + qd = q^l l + q^b b$$

- Firms: use loans to purchase capital, produce output (Cobb-Douglas)

Asset Payoffs & Bank Solvency (Period 2)

- Asset payoffs and bank profits contingent on sovereign default

$$\pi = b + l - d$$

$$\underline{\pi} = \max \left\{ \theta^b b + \theta^l l - d, 0 \right\}$$

⇒ Sovereign default reduces bank revenues

- Limited liability. Bank may become insolvent with recovery rate on deposits

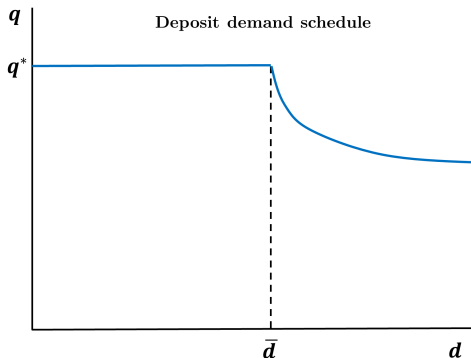
$$\theta = \min \left\{ \frac{\theta^b b + \theta^l l}{d}, 1 \right\}$$

⇒ Recovery rate depends on deposits, bank's portfolio choice $\{b, l\}$

Household's Portfolio Problem

- With prob. $(1 - P)$, there is no default and deposits are repaid fully
- With prob. P , households receive recovery rate θ
- Risk neutral: price deposits at expected return

$$q = q^*(1 - P + P\theta)$$

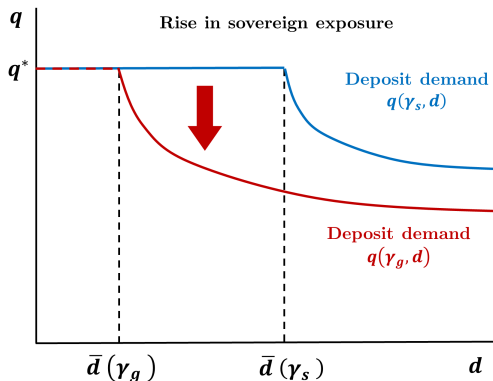


Household's Portfolio Problem

- Re-write portfolio choice $\{b, l\}$ in terms of sovereign exposure γ

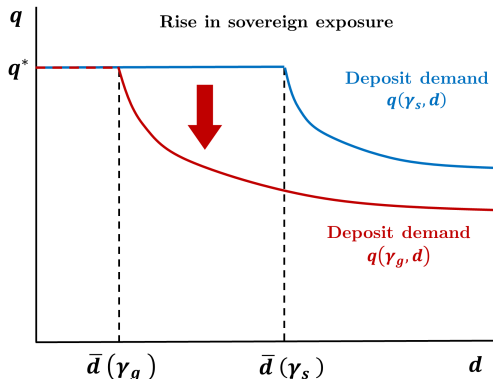
$$\gamma = \frac{q^b b}{n + qd}$$

- Higher sovereign exposure leads to inward shift of deposit threshold



Non-contractibility

- Sovereign exposure γ is non-contractible
 - Households form (rational) expectation $\tilde{\gamma}$
 - Banks cannot commit to exposure γ , take $\tilde{\gamma}$ as given
 - Bank's problem conditional on $\tilde{\gamma}$, determination of $\tilde{\gamma}$ explained later



Bank strategies

- Imperfect competition (Cournot). Discontinuity due to limited liability.

Safe Strategy

Limited liability never kicks in

$$E[\pi_s] = \max_{b,l,d} (1-P)(l+b-d) + P(\theta^l l + \theta^b b - d)$$

subject to

$$n + q(\tilde{y}, d)d = q^l l + q^b b \quad (\text{Budget constraint})$$

$$d \leq \theta^l l + \theta^b b \quad (\text{Solvency constraint for safe strategy})$$

Gambling Strategy

Limited liability after sovereign default

$$E[\pi_g] = \max_{b,l,d} (1-P)(l+b-d)$$

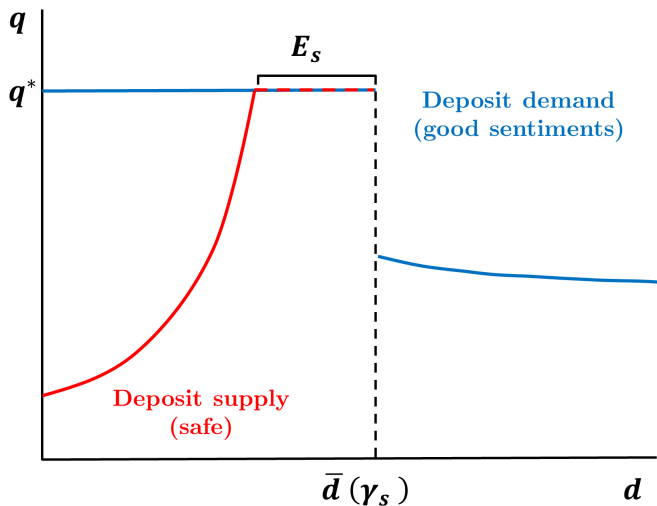
- Adopt gambling strategy if it has higher payoff $E[\pi_g] > E[\pi_s]$

Market expectations on sovereign exposure γ

- Deposit demand schedule depends on household expectations on γ
- One-to-one mapping between bank strategy and sovereign exposure
 - ▶ Good sentiments: safe strategy expected (γ_s)
 - ▶ Bad sentiments: gambling strategy expected (γ_g)
- Rational expectations equilibrium:
Sentiments can be realised only if they are self-confirming

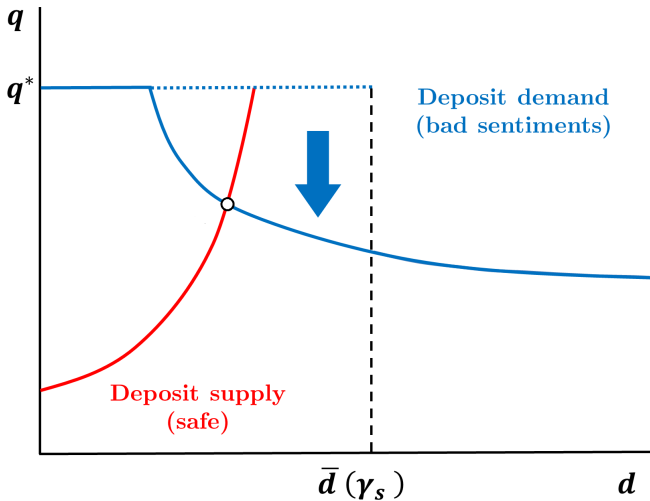
Multiplicity

Good sentiments: banks find safe strategy optimal



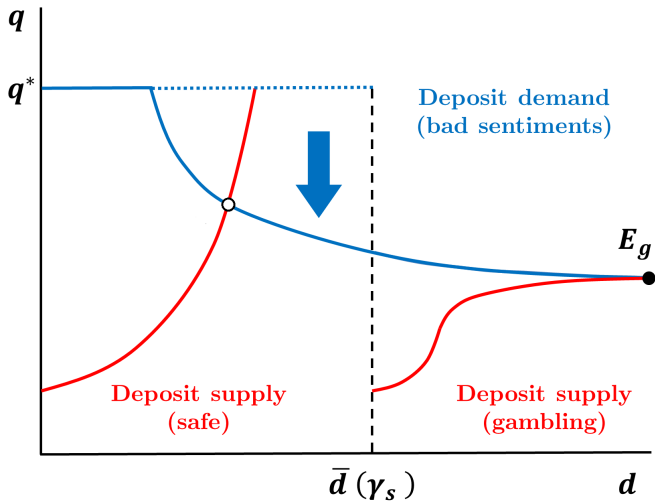
Multiplicity

Bad sentiments: deposit demand shifts inwards

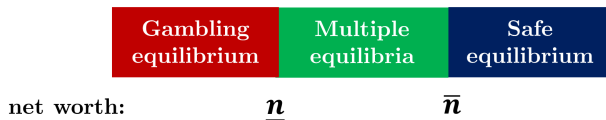


Multiplicity

Bad sentiments: gambling strategy optimal, sentiments confirmed



Equilibrium



- Safe equilibrium

- Banks have low leverage and sovereign bond exposure
- Banks are safe, borrow at risk-free rate

- Gambling equilibrium

- Banks pursue high leverage and sovereign exposure
- Sovereign default endogenously leads to a banking crisis
- Crowding out of bank lending, rise in bank funding costs

Dynamic Model

- Banks accumulate portion of payoff as future net worth
- Risk averse households
- Sovereign risk shocks

- Global solution, endogenous multiplicity region
 - ▶ 4 state variables: net worth \times 2, sovereign risk shock, sunspot
 - ▶ Steady (absorbing) state after sovereign default
 - ▶ Calibration targets Portugal over 2010-2016

computation

bank

household

shock

sunspots

steady state

calibration

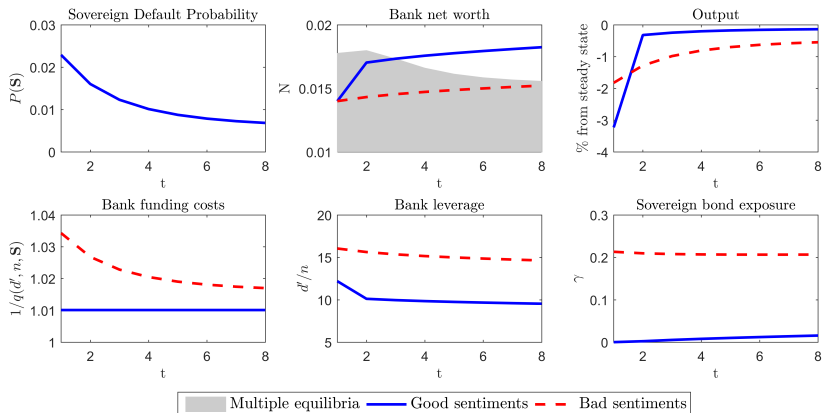
Sovereign risk shock

Good Sentiments

Rapid deleveraging, safe banks
Sharp but brief fall in lending

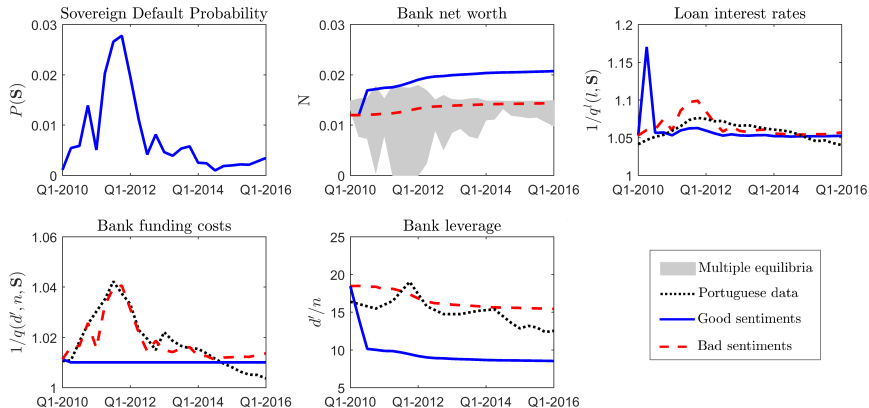
Bad Sentiments

Risky banks, high funding costs
Persistent fall in output



Portugal

- Series of sovereign risk shocks that match Portuguese sov. spreads
- Initial net worth set to Tier 1 capital of Portuguese banks

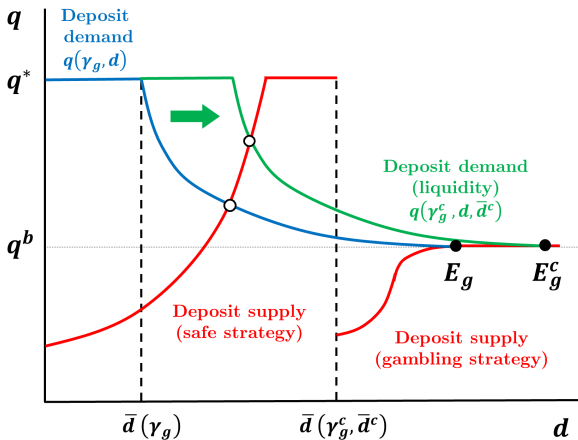


Policy

- Liquidity provision: Central bank allows banks to borrow up to a fixed amount at risk-free rate
 - ▶ Trade-off: alleviating funding conditions vs. incentivizing gambling
 - ▶ Backfires and expands multiplicity region

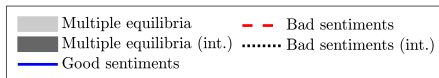
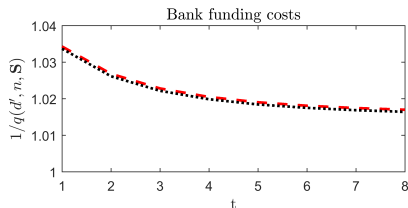
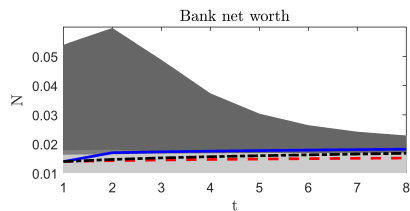
Policy

- Liquidity provision: Central bank allows banks to borrow up to a fixed amount at risk-free rate
 - ▶ Trade-off: alleviating funding conditions vs. incentivizing gambling
 - ▶ Backfires and expands multiplicity region



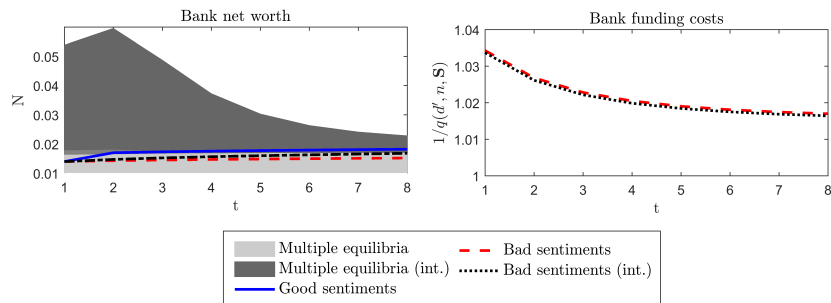
Policy

- Liquidity provision: Central bank allows banks to borrow up to a fixed amount at risk-free rate
 - ▶ Trade-off: alleviating funding conditions vs. incentivizing gambling
 - ▶ Backfires and expands multiplicity region



Policy

- Liquidity provision: Central bank allows banks to borrow up to a fixed amount at risk-free rate
 - ▶ Trade-off: alleviating funding conditions vs. incentivizing gambling
 - ▶ Backfires and expands multiplicity region



- Liquidity schedule conditional on bank leverage
 - ▶ Overcome trade-off: offer more liquidity for low leverage
 - ▶ Eliminates gambling equilibrium throughout multiplicity region
 - ▶ Central bank doesn't need to condition on sovereign exposure

Conclusion

Dynamic general equilibrium model with optimizing banks and depositors

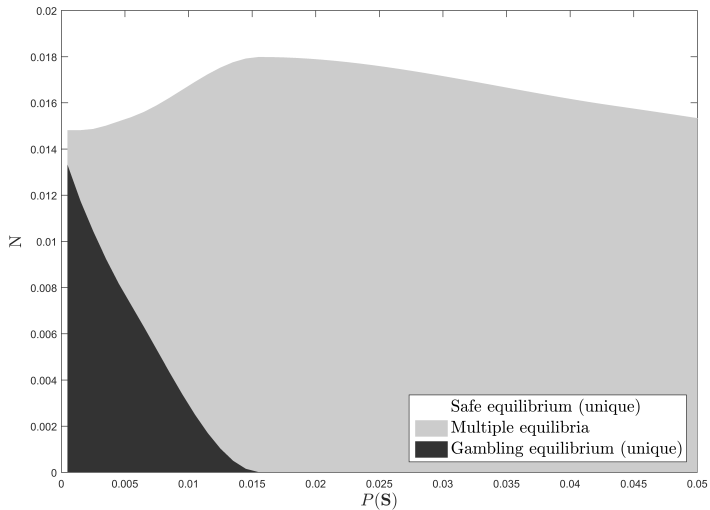
- Funding costs affect banks' risk-taking incentives
- Sentiments about bank risk-taking may become self-fulfilling
- Endogenous financial fragility and persistence in bad equilibrium
- Accounts for macroeconomic dynamics in Portugal over 2010-2016

Provides a framework for policy analysis and design

- Equilibrium-switching effects
- Success of intervention hinges on conditionalities

Thank you

Sovereign risk and equilibrium regions



Empirics

- Gambling: Battistini et al. (2014), Acharya & Steffen (2015), Altavilla et al. (2016)
- Decline in bank lending: Acharya et al. (2014), Becker & Ivashina (2014), De Marco (2014), Popov & Van Horen (2015)
- Rise in bank funding costs: Acharya & Steffen (2012), Acharya, Drechsler & Schnabl (2014)

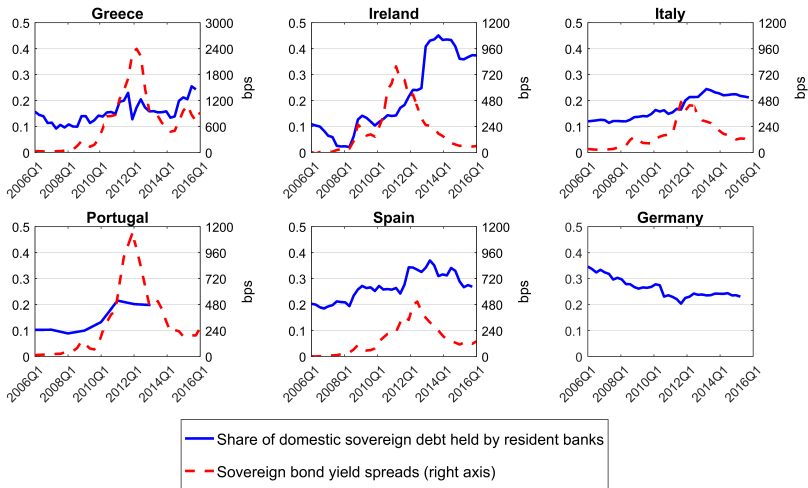
◀ Lending

◀ Cross section

Motivation

In European countries hit by the sovereign debt crisis, banks

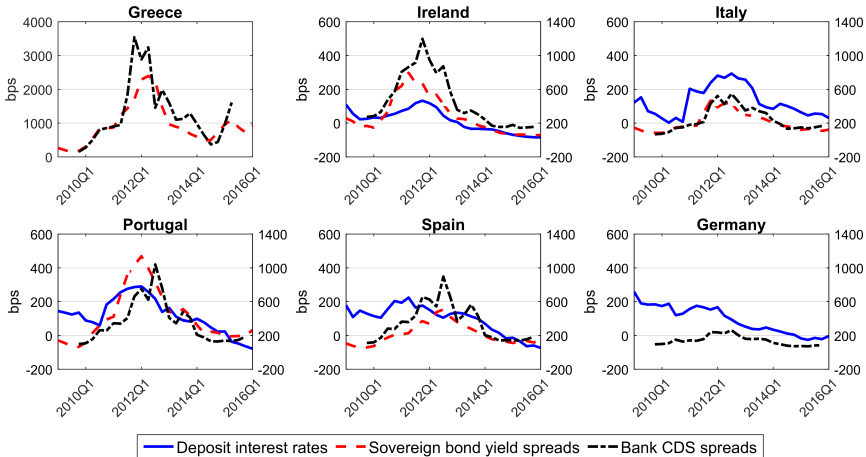
1. increased their holdings of domestic government debt



Motivation

In European countries hit by the sovereign debt crisis, banks

2. faced rising financing costs



Households

- Conditional on a sovereign exposure

$$q^b b = \gamma(n + qd) \longrightarrow \gamma = \frac{q^b b}{n + qd}$$

- Deposit threshold

$$\bar{d}(\gamma) = \frac{\gamma \frac{\theta^b}{q^b} + (1 - \gamma) \frac{\theta^l}{q^l}}{1 - q^* \left(\gamma \frac{\theta^b}{q^b} + (1 - \gamma) \frac{\theta^l}{q^l} \right)} n$$

- Optimal deposit demand schedule has a kink

$$q(\gamma, d) = \left\{ \begin{array}{ll} q^* & \text{if } d \leq \bar{d}(\gamma) \\ q^* \frac{1 - P + P \left(\gamma \frac{\theta^b}{q^b} + (1 - \gamma) \frac{\theta^l}{q^l} \right) \frac{n}{d}}{1 - q^* P \left(\gamma \frac{\theta^b}{q^b} + (1 - \gamma) \frac{\theta^l}{q^l} \right)} & \text{if } d > \bar{d}(\gamma) \end{array} \right\}$$

Gambling strategy

- First order condition for d

$$q^b = \frac{q^*(1-P)}{1 - q^*P \left(\gamma_g \frac{\theta^b}{q^b} + (1 - \gamma_g) \frac{\theta^l}{q_g^l} \right)}$$

- Risk neutrality: optimal to borrow until deposits have same recovery rate as sovereign bonds

$$\gamma_g = 1, q_g = q^b$$

- Corner solution due to risk neutrality. Interior solution under risk aversion.

Solvency constraint

- Occasionally binding solvency constraint

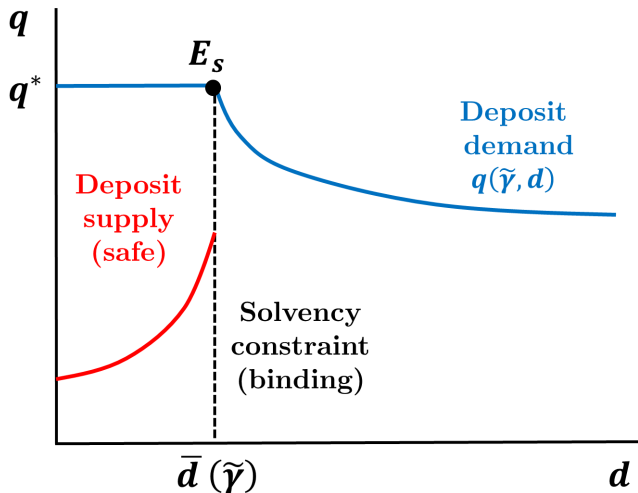
$$d \leq \theta^l l + \theta^b b, \lambda_s \geq 0$$

- First order conditions when the solvency constraint binds

$$q^l = \frac{1 - P + (P + \lambda) \theta^l}{1 + \lambda} (1 - \mu_l) q^*$$

$$q^b > \frac{1 - P + (P + \lambda) \theta^b}{1 + \lambda} q^*$$

Solvency constraint



Moral Suasion

- Moral suasion
 - ▶ Governments in need of funding incentivize or coerce domestic banks to purchase their debt
 - ▶ Theory: Chari, DAVIS & Kehoe (2016)
 - ▶ Empirics: Becker and Ivashina (2014), De Marco & Macchiavelli (2016), Ongena et al. (2016)

- Gambling & moral suasion are not mutually exclusive
 - ▶ Moral suasion can be conducive to gambling
 - ▶ Lax supervision of risky domestic sovereign bond holdings as a form of moral suasion
see e.g. Uhlig (2014), Crosignani (2015), Farhi & Tirole (2016)

Assets

1 Sovereign bonds (b)

- ▶ Sov. default in bad state, calibrated recovery rate $\theta^b < 1$
- ▶ Priced at expected return (deep pocketed marginal buyer)

$$q^b = (1 - P + P\theta^b) q^*$$

2 Loans to non-financial firms (l)

- ▶ Cobb-Douglas with working capital borrowed in advance

$$Y = AK^\alpha, \quad K = q^l L$$

- ▶ FOC: price of loans q^l increases in aggregate loans L

$$R^K \equiv \frac{1}{q^l} = \alpha AK^{\alpha-1} \rightarrow q^l = (\alpha A)^{-\frac{1}{\alpha}} L^{\frac{1-\alpha}{\alpha}}$$

- ▶ Productivity falls to $\underline{A} < A$ under sovereign default.
- ▶ Non-performing loan. Banks claim revenues after wage payments.

$$\theta^l = \frac{\underline{A}}{A}$$

Parameter restrictions

- Restriction on productivity decline

$$\frac{\alpha(1-P)}{\alpha(1-P) + \nu(1-\alpha)} > \frac{A}{A} > \frac{a\theta^b}{a + \nu(1-\alpha)}$$

- Rate of return to loans under sovereign default
 - 1 More than the return from sovereign bonds under default
 - 2 Less than the promised rate of return to deposits
- Justification:
 - ▶ Positive risk-weight in regulation for non-sovereign assets for (1)
 - ▶ Spillover effects of sovereign default on bank balance sheets for (2)

Sovereign risk shocks

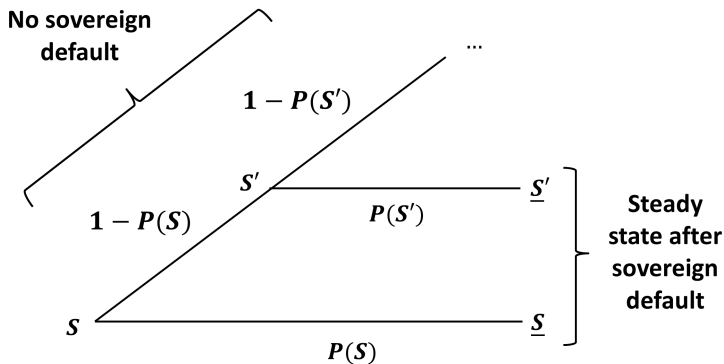
- Fiscal limit with standard logistic distribution

$$P(\mathbf{S}) = \Pr[\varepsilon \leq \Upsilon(\mathbf{S})] = \frac{\exp(\Upsilon(\mathbf{S}))}{1 + \exp(\Upsilon(\mathbf{S}))}$$

- Fiscal stress $\Upsilon(\mathbf{S})$ follows AR(1) shock process around a mean
- Can be linked to public debt or state of the economy as well but
 - ▶ Focus on propagation of sovereign risk rather than potential feedback loops, which are well understood (see e.g. Corsetti et al., 2013, 2014)
 - ▶ Factors orthogonal to domestic fundamentals played a major role in the European sovereign debt crisis (see e.g. Bahaj, 2014)

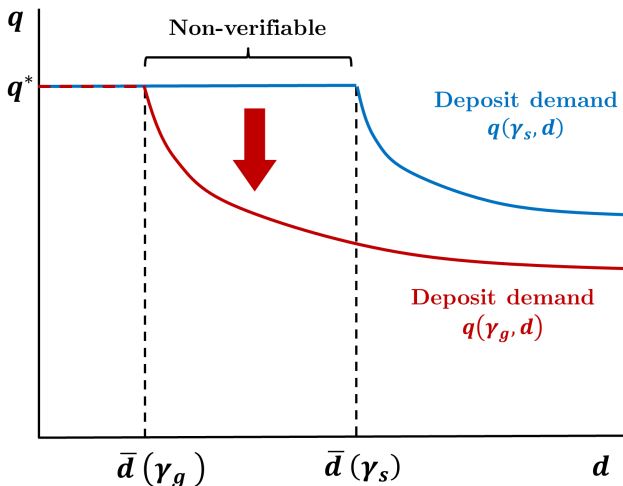
Steady state

- No more default risk once the government defaults
- Economy immediately moves to absorbing state



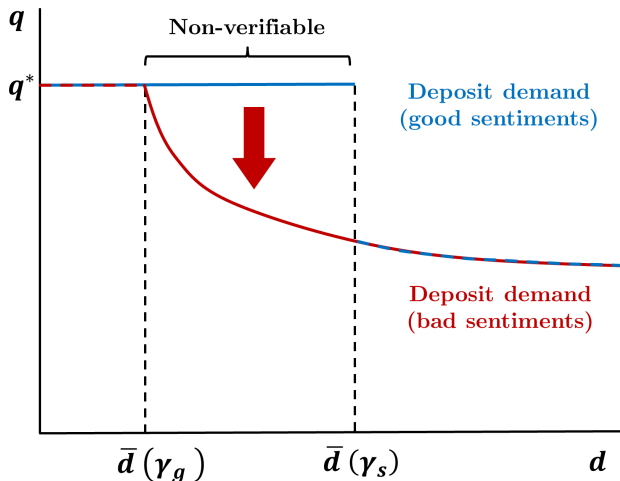
Sentiments

- Gambling strategy leads to higher sovereign exposure ($\gamma_g > \gamma_s$)
- Bad sentiments reduce deposit demand in non-verifiable region



Sentiments

- Gambling strategy leads to higher sovereign exposure ($\gamma_g > \gamma_s$)
- Bad sentiments reduce deposit demand in non-verifiable region



Household's Problem

$$v^h(D, D^*, \mathbf{S}) = \max_{D', D^{*'}, c} \left\{ \begin{array}{l} u(c) + \beta(1 - P(\mathbf{S})) \mathbb{E}_{\mathbf{S}} [v^h(D', D^{*'}, \mathbf{S}')] \\ + \beta P(\mathbf{S}) \underline{v}^h(D', D^{*'}, \mathbf{S}') \end{array} \right\},$$

s.t.

$$\begin{aligned} c + qD' + q^*D^{*'} &= D + D^* + w(\mathbf{S}) - T(\mathbf{S}) \\ \mathbf{S}' &= \Gamma(\mathbf{S}) \end{aligned}$$

First order condition

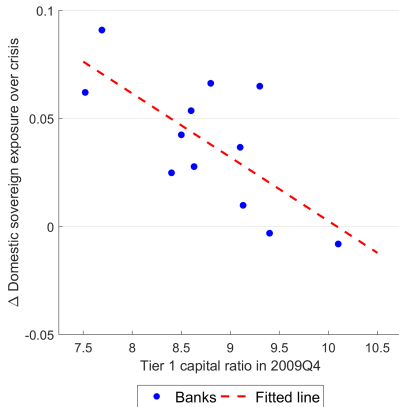
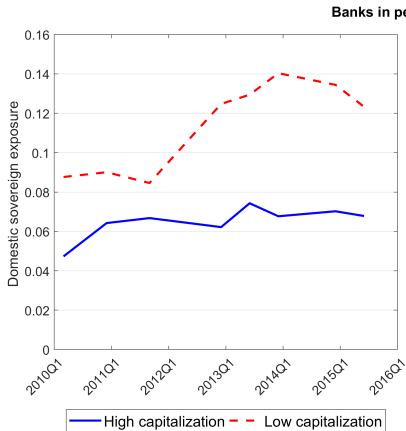
$$q = q^* \frac{(1 - P(\mathbf{S})) \mathbb{E}_{\mathbf{S}} [u_c(c')] + P(\mathbf{S}) \theta u_c(\underline{c})}{(1 - P(\mathbf{S})) \mathbb{E}_{\mathbf{S}} [u_c(c')] + P(\mathbf{S}) u_c(\underline{c})}$$

Abstract from wealth accumulation

$$D + D^* + w(\mathbf{S}) - T(\mathbf{S}) = \bar{E}$$

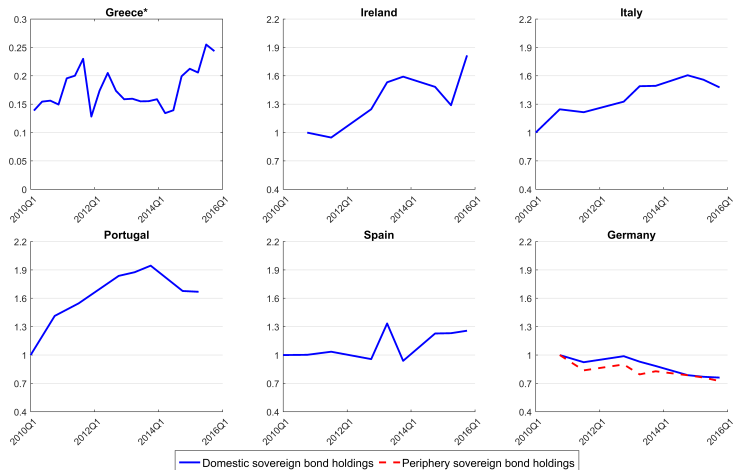
Cross-section of banks

- Under-capitalized banks increased their exposure to domestic sovereign bonds more than well capitalized banks



Same story in Ireland, Italy, Spain, Greece

1. Banks increased their holdings of domestic government debt



Source: ECB, EBA, Merler & Pisani-Ferry (2012)

*Greek data is on the share of sovereign debt held by resident banks

Shares

◀ Back

Same story in Ireland, Italy, Spain, Greece

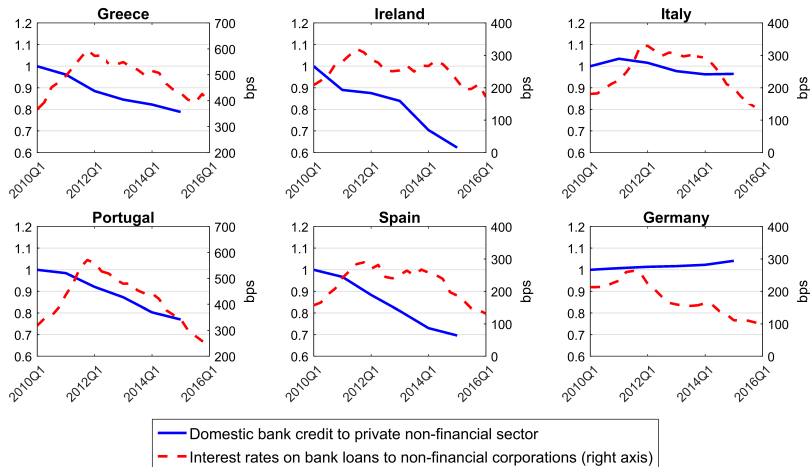
2. Bank financing costs co-move with sovereign bond yield spreads

Correlation with sovereign bond yield spreads over 2010-2015

	Greece	Ireland	Italy	Portugal	Spain
Bank CDS spreads	0.85	0.93	0.93	0.85	0.93
Deposit interest rates	-	0.84	0.84	0.74	0.37

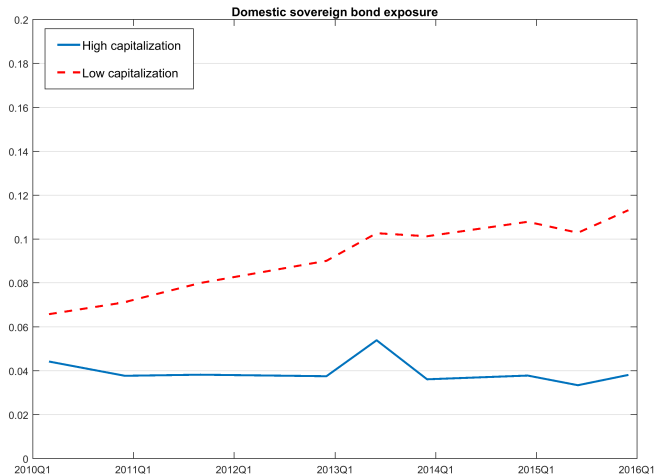
Same story in Ireland, Italy, Spain, Greece

3. Banks reduced lending to the private sector



Cross-section of banks

- Under-capitalized banks increased their exposure to domestic sovereign bonds more than well capitalized banks



First order conditions

Safe Strategy

Sovereign bond purchases b

$$q^b = (1 - P + P\theta^b)(1 - \mu_d)q$$

Loans to firms l

$$q^l = \frac{1 - P + P\theta^l}{1 - P + P\theta^b}(1 - \mu_l)q^b$$

Gambling Strategy

$$q^b = (1 - \mu_d)q$$

$$q^l = (1 - \mu_l)q^b$$

- Safe strategy prices assets according to expected return
- Gambling strategy prices assets according to return in good state
⇒ Bank lending crowded out by sov. bond purchases

Gambling strategy

Under the gambling strategy

- 1 High exposure to domestic sovereign bonds

$$\gamma_g > \gamma_s$$

- 2 Domestic sovereign bond purchases crowd out bank lending to firms

$$l_g < l_s$$

- 3 Bank funding costs mirror domestic sovereign bond yields

$$q_g < q_s = q^*$$

⇒ captures key characteristics of the sovereign debt crisis

Calibration

Target Portugal over 2010-2016. Each period is a quarter

	Description	Value	Source
θ^b	Recovery rate of sov. bonds	0.60	Cruces & Trebesch (2013)
-	Sovereign risk shock AR(1)	—	Match sov. spreads
ν	Market share of banks	0.005	Match loan interest rates
α	Share of capital income	0.33	Cobb-Douglas
β	Discount factor	$0.99^{1/4}$	-
σ	Coeff. of risk aversion	3	Thimme (2016)
A	Productivity (no default)	1	Normalized
\underline{A}	Productivity (default)	0.90	Schreger & Hébert (2015)
\bar{E}	Household wealth	$0.07 * 10^{-9}$	OECD
$\bar{\zeta}$	Prob. of bad sentiments	0.50	-

Policy analysis: Liquidity Provision

- Consider policy where central bank allows banks to borrow \bar{d}^c at risk-free rate q^*

$$q^l l + q^b b = n + qd + q^* \bar{d}^c$$

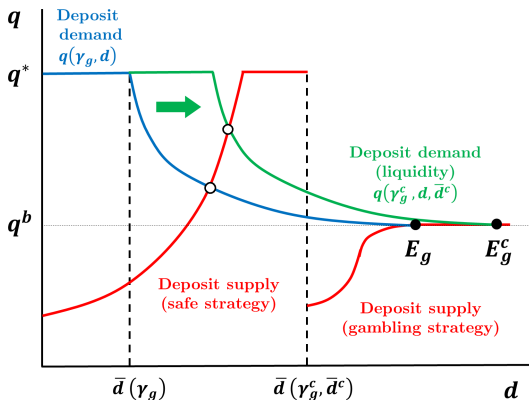
- Outcome depends on
 - 1 Risk sharing: If bank is insolvent, who gets paid first? (i.e. seniority)
Central bank vs. depositors
 - 2 Conditionalities attached to liquidity provision

Liquidity Provision (without risk sharing)

- In case of bank insolvency, central bank is repaid first
- Dilution effects
 - ▶ Depositors' claim to bank revenues diluted in case of insolvency
 - ▶ Recovery rate θ decreases. Fall in deposit demand in anticipation
 - ▶ Completely offsets central bank liquidity. Policy ineffective
- Conditionalities do not matter

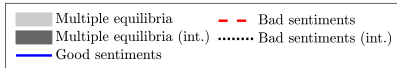
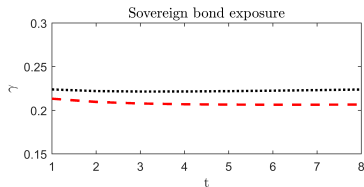
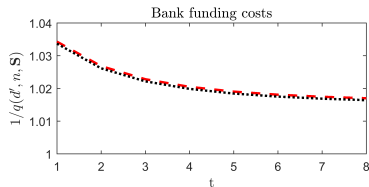
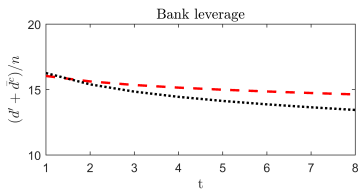
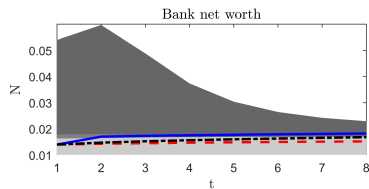
Liquidity Provision (risk sharing, unconditional)

- Risk sharing: Implicit risk transfer from depositors to central bank
 - ▶ Deposit demand shifts out
- Unconditional: borrow up to fixed amount without conditionalities
 - ▶ Trade-off: alleviate funding conditions vs. incentivize gambling



Liquidity Provision (risk sharing, unconditional)

- Multiplicity region shifts up
- Good sentiments: no longer escape from multiplicity region
- Sovereign exposure increases until funding costs return to pre-intervention level



Liquidity Provision (risk sharing, conditional)

- Provide liquidity conditional on deposits and bank net worth
- Design to insulate banks from shifts in depositor sentiment

$$\bar{d}^c(n, d) = \frac{\left(\frac{\theta^l}{q_s^l} - \frac{\theta^b}{q^b}\right) q_s^l l_s + \frac{\theta^b}{q^b} n}{1 - \frac{\theta^b}{q^b} q^*} - d$$

- Overcomes trade-off: no participation under gambling strategy
- Eliminates gambling equilibrium throughout multiplicity region
- Central bank resolves coordination problem of atomistic households
- No risk of realized losses to central bank

Deposit Insurance & Macroprudential Policy

- Insights can be generalized to wider set of policy interventions
- ① Deposit insurance on its own is equivalent to unconditional liquidity provision (with risk sharing)
 - ▶ Shifts out deposit demand schedule. Same trade-off.
- ② Deposit insurance & macroprudential policy can achieve same outcome as conditional liquidity provision (with risk sharing)
 - ▶ Regulatory constraint on bank liabilities or leverage ratio
 - ▶ Targeting bank assets and capital regulation also works, but only with positive risk-weight on domestic sovereign bond holdings

Bank's Problem

- When bank is solvent, portion $(1 - \psi)$ of bankers consume share of profits and exit. Remainder of profits accumulated as net worth after deducting operating costs

$$n' = \psi(\pi - \omega)$$

- Bank's problem

$$v^b(n; \mathbf{S}) = \max \left\{ v_s^b(n; \mathbf{S}), v_g^b(n; \mathbf{S}) \right\},$$

$$v_s^b(n; \mathbf{S}) = \max_{d', \gamma \in [0,1]} \left\{ \begin{array}{l} (1 - P(\mathbf{S})) \left((1 - \psi) \pi + \psi \mathbb{E}_{\mathbf{S}} [v^b(n'; \mathbf{S}')] \right) \\ + P(\mathbf{S}) \left((1 - \psi) \underline{\pi} + \psi v^b(\underline{n}; \mathbf{S}) \right) \end{array} \right\},$$

$$v_g^b(n; \mathbf{S}) = \max_{d', \gamma \in [0,1]} \left\{ (1 - P(\mathbf{S})) \left((1 - \psi) \pi + \psi \mathbb{E}_{\mathbf{S}} [v^b(n'; \mathbf{S}')] \right) \right\}$$

subject to the law of motion for n , aggregate state variables $\mathbf{S}' = \Gamma(\mathbf{S})$ and all the constraints in the simple model

Multiple equilibria and sunspots

- State space *endogenously* segmented to equilibrium regions:
 - ▶ Unique safe equilibrium
 - ▶ Unique gambling equilibrium
 - ▶ Multiple equilibria
- In multiple equilibria region, sunspot determines equilibrium type
 - ▶ Bad sentiments (i.e. gambling eq.) with probability $\bar{\zeta}$
 - ▶ Can determine probability by AR(1) process or dependent on government debt or any other state variable

Equilibrium

Recursive rational expectations equilibrium

- ① Value and policy functions of households and banks solve their respective optimization problems
- ② Market clearing for domestic deposits and loans
- ③ Segmentation of states into equilibrium regions is consistent with agents' optimal strategies and expectations

Computation

Problem: Bank's policy function is discontinuous

Difficult to have precise grid because

- Curse of dimensionality: 4 state variables
- Decentralized banks and households \rightarrow two iteration loops

Computation

Problem: Bank's policy function is discontinuous

Difficult to have precise grid because

- Curse of dimensionality: 4 state variables
- Decentralized banks and households \rightarrow two iteration loops

Solution: Take advantage of limited liability

- Bank's FOC trades off profits in good state against bad state

$$\begin{aligned} & P(\mathbf{S}) \left[(1 - \psi) + \psi \frac{\partial v^b(n'; \mathbf{S})}{\partial \pi} \right] \frac{\partial \pi}{\partial \gamma} \\ &= - (1 - P(\mathbf{S})) \left[(1 - \psi) + \psi \frac{\partial \mathbb{E}_{\mathbf{S}} [v^b(n'; \mathbf{S}')] }{\partial \pi} \right] \frac{\partial \pi}{\partial \gamma} \end{aligned}$$

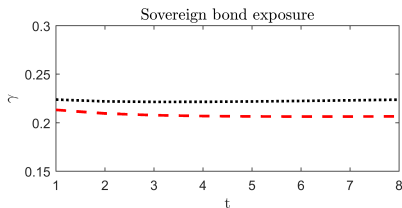
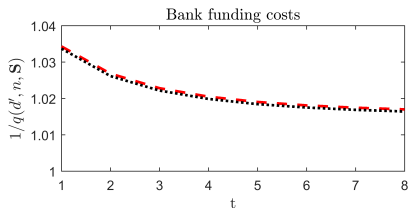
- Key unknown is $\mathbb{E}_{\mathbf{S}} [v^b(n'; \mathbf{S}')]$
 - Gambling strategy: $\frac{\partial \pi}{\partial \gamma} = 0$ due to limited liability
- \Rightarrow Pin down one end of discontinuity algebraically

Higher household wealth

- Bank's FOC pins down funding costs

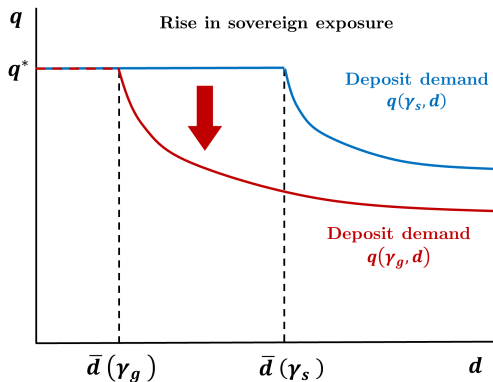
$$q^b = (1 - \mu_d) q$$

- Higher household wealth (less risk aversion) increases sovereign exposure and leverage



Household's Portfolio Problem: Transparency

- Banks internalize relation between γ and deposit demand schedule
 - ⇒ Market discipline offsets temptation to gamble
 - ⇒ Safe equilibrium unique outcome at all levels of net worth



Bank's Problem

- When bank is solvent, portion $(1 - \psi)$ of bankers consume share of profits and exit. Remainder of profits accumulated as net worth after deducting operating costs

$$n' = \psi(\pi - \omega)$$

- Bank's problem

$$v^b(n; \mathbf{S}) = \max \left\{ v_s^b(n; \mathbf{S}), v_g^b(n; \mathbf{S}) \right\}$$

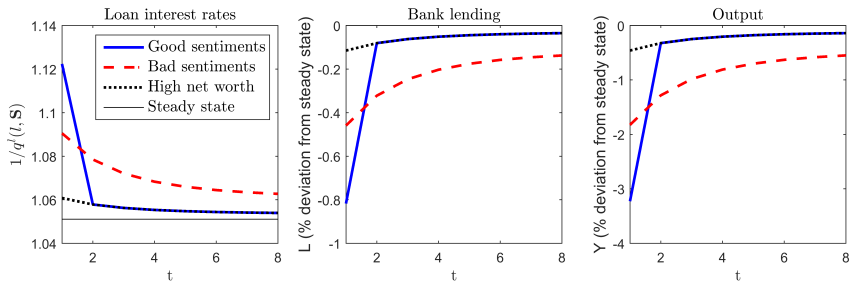
$$v_s^b(n; \mathbf{S}) = \max_{d', \gamma \in [0,1]} \left\{ \begin{array}{l} (1 - P(\mathbf{S})) \left((1 - \psi) \pi + \psi \mathbb{E}_{\mathbf{S}'} [v^b(n'; \mathbf{S}')] \right) \\ + P(\mathbf{S}) \left((1 - \psi) \underline{\pi} + \psi v^b(\underline{n}'; \mathbf{S}) \right) \end{array} \right\}$$

$$v_g^b(n; \mathbf{S}) = \max_{d', \gamma \in [0,1]} \left\{ (1 - P(\mathbf{S})) \left((1 - \psi) \pi + \psi \mathbb{E}_{\mathbf{S}'} [v^b(n'; \mathbf{S}')] \right) \right\}$$

subject to the law of motion for n , aggregate state variables $\mathbf{S}' = \Gamma(\mathbf{S})$ and all the constraints in the simple model

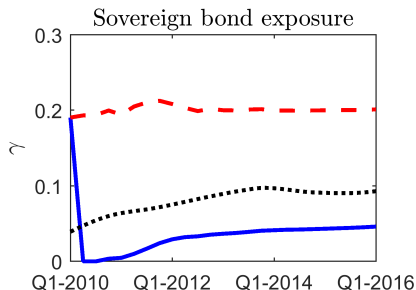
Bank lending and output

Persistent fall in lending and output under bad sentiments



Portugal

- Model over-estimates sovereign exposure
- Data only on government bond holdings
- There may also be indirect exposure



Multiple equilibria Good sentiments
Portuguese data Bad sentiments