

# Consumer Search with Imperfect Vertical-quality Information

Yijuan Chen<sup>1</sup>   Xiangting Hu<sup>2</sup>   Sanxi Li<sup>3</sup>

Harbin Institute of Technology, Shenzhen

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<sup>1</sup>Australian National University

<sup>2</sup>Harbin Institute of Technology, Shenzhen

<sup>3</sup>Renmin University of China

- Many previous studies explore the feature that online consumers incur **lower search costs** than in offline markets.
- On the other hand, online consumers can be **uncertain about product quality** because they cannot physically inspect a product before purchase.
- The uncertainty over product quality can be horizontal and / or **vertical**.
  - Horizontal quality is a product's match-value to a consumer, and can be idiosyncratic across consumers.
  - Vertical quality is a product's quality that is largely identical across consumers.
    - Examples: the effectiveness of noise-canceling headphones, materials used in furniture

- In this paper we develop a consumer-search model that incorporates consumer's (ex post) uncertainty over vertical product quality. We examine
  - Firms' pricing strategy and consumers' search behavior in equilibrium.
  - How the change of search cost affects price and consumer welfare.
  - How the change of informativeness affects price and consumer welfare.

- Baseline model:
  - Vertical uncertainty with binary quality signals
- Extended model:
  - Vertical uncertainty with continuous quality signals

- A reduced search cost will result in a lower consumer surplus and higher market prices.
- An increase in informativeness can lead to a higher consumer surplus.
- When the initial quality signal becomes less informative, consumers can still be better off if they increase the search intensity.

# Baseline Model

## Set-up: Firms, Consumers, and Information Structure

- There are a continuum of firms with measure 1 and a continuum of consumers with measure 1.
- Firms produce differentiated products at two quality levels:  
 $v_h > v_l > 0$ .
  - Each firm is high-quality with probability  $\mu$ .
  - The marginal cost of production is zero.
- Each consumer has a unitary demand and conducts sequential random search on the market with perfect recall and without replacement, at a search cost  $s$  per firm.

# Baseline Model

## Set-up: Firms, Consumers, and Information Structure

- At the beginning of the game, firms observe their qualities and set their prices.
- Upon sampling a store, a consumer observes the price but cannot directly observe the vertical quality. She observes a quality signal  $\eta \in \{\eta_l, \eta_h\}$  which follows the conditional distribution

$$\begin{aligned}\Pr(\eta = \eta_h | v = v_h) &= \beta_h & \Pr(\eta = \eta_l | v = v_h) &= 1 - \beta_h \\ \Pr(\eta = \eta_l | v = v_l) &= \beta_l & \Pr(\eta = \eta_h | v = v_l) &= 1 - \beta_l\end{aligned}$$

with  $\beta_l, \beta_h \in (\frac{1}{2}, 1)$ .

- We will say that  $\beta_l$  and  $\beta_h$  represent the *informativeness of quality signals*, or, the search quality.

# Baseline Model

## Set-up: Strategies and Payoffs

- A firm's pure strategy is a function  $P : \{v_l, v_h\} \rightarrow \mathbb{R}_+$ .
- The signal space for a consumer is

$$S = \{(\eta, p) \mid \eta \in \{\eta_l, \eta_h\}, p \in \mathbb{R}_+\}.$$

- We will be focused on pure strategies for consumers.
- Following a strategy profile, a consumer expects a reservation utility  $U$  by participating in searching on the market.
- The consumer will buy if  $E[v | (\eta, p)] - p \geq U$ .



- Equilibrium concept: Perfect Bayesian Equilibrium
- Three types of equilibrium:
  - An equilibrium is separating if different types of firms charge different prices, i.e.,  $p_l \neq p_h$ .
  - A pooling equilibrium entails  $p_l = p_h$ .
  - In a partial-pooling / partial-separating equilibrium, each type of firm can randomize over a distribution of prices, with the two distributions partially overlapped.

### Proposition

*There does not exist a separating equilibrium.*

- The existence of a separating equilibrium requires the distributions of quality signals, conditional on different quality levels, to have different supports.
- A key factor that leads to the Proposition is the assumption that  $\beta_h < 1$  and  $\beta_l < 1$ .
  - $\beta_h = 1$  and  $\beta_l = 1$ : A Diamond Paradox
  - $\beta_h = 1$  and  $\beta_l < 1$ : Wolinsky (1983)

# Baseline Model

## The partial-pooling equilibrium

- The partial-pooling equilibrium
  - The high-quality firm sets its price at  $p_h$ ,
  - The low-quality firm's price will be  $p_h$  with probability  $\sigma$  and  $p_l$  with probability  $1 - \sigma$ .
  - On the equilibrium path, the consumer will buy upon seeing  $(\eta_h, p_h)$ ,  $(\eta_l, p_l)$  or  $(\eta_h, p_l)$ , and will continue searching upon seeing  $(p_h, \eta_l)$ .
  - Off the equilibrium path, consumers holds the belief that  $\Pr(v = v_l | p \neq p_h, p \neq p_l) = 1$ , and will buy if  $p \leq v_l - U$ , and continue searching if  $p > v_l - U$ .
- As  $\sigma \rightarrow 1$ , the partial-pooling equilibrium becomes a pooling equilibrium.

# Baseline Model

## Partial-pooling equilibrium

- From the indifference condition of the low-quality firm we can derive a relationship between  $p_h$  and  $U$ , which we call the *Price Incentive Line*.

$$\begin{aligned}(1 - \beta_l)p_h &= p_l \\ p_l &= v_l - U \\ \implies (1 - \beta_l)p_h &= v_l - U\end{aligned}$$

- The Price Incentive Line characterizes the low-quality firm's pricing strategy in equilibrium, and implies a negative relationship between the price and consumer's reservation utility:
  - As consumer's reservation utility decreases by one unit, the sure-sell price  $v_l - U$  will increase by one unit;
  - hence the price of mimicing should increase by  $\frac{1}{1-\beta_l}$  unit to keep the indifference condition.

# Baseline Model

## Partial-pooling equilibrium

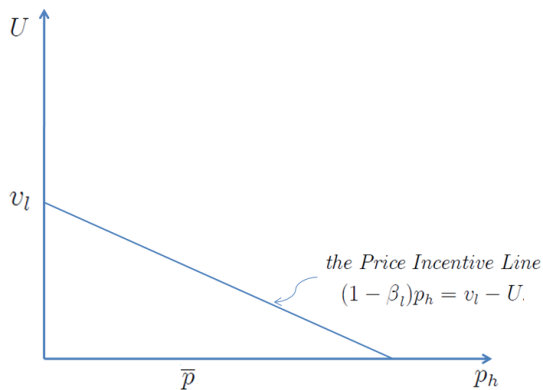


Figure 1a: The Price Incentive Line

# Baseline Model

## Partial-pooling equilibrium

- We can derive another relationship between  $p_h$  and  $U$ , from the consumer's searching problem, which we call *the Search Value Line*:

$$p_h + U = v_l + \phi(\beta_h, \beta_l, s; \sigma),$$

- The *Search Value Line* characterizes consumer's searching behavior.
- Given the value of search, the Search Value Line implies a negative relationship between  $p_h$  and consumer's reservation utility:
  - When  $p_h$  increases by one unit, consumer's reservation utility will decrease by one unit.

# Baseline Model

## Partial-pooling equilibrium

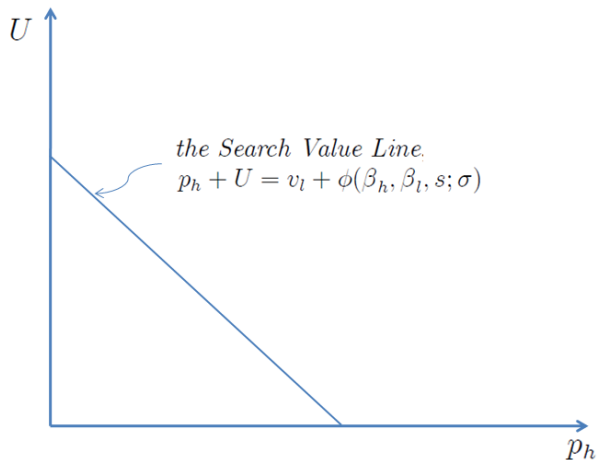


Figure 1b: The Search Value Line

# Baseline Model

## Partial-pooling equilibrium

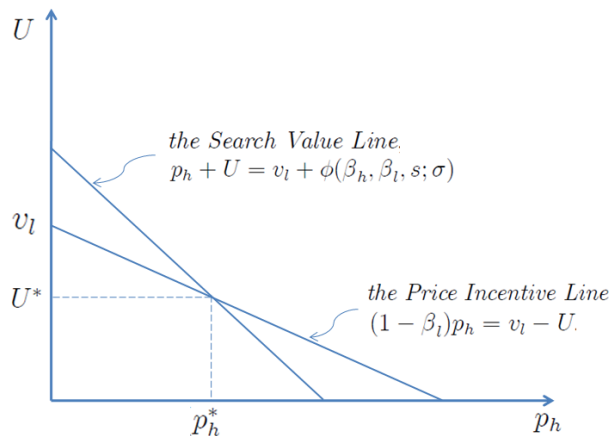


Figure 2: The partial-pooling equilibrium



# Baseline Model

## Equilibrium types

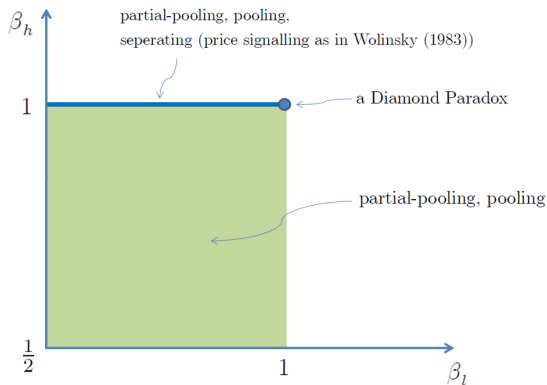


Figure 3: Summary of equilibrium types

# Baseline Model

## Impacts of search cost

- $s$  decrease  $\implies U$  decreases,  $p_h$  increases.

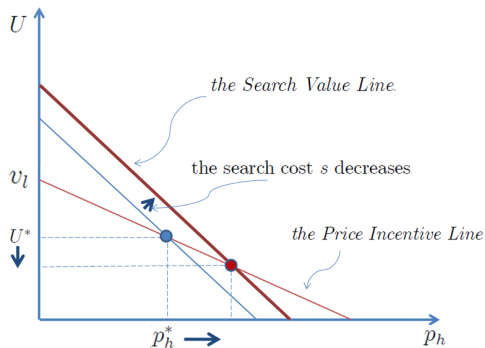


Figure 4: The impacts of a lower search cost on equilibrium

# Baseline Model

## Impacts of search quality

- $\beta_l$  increases  $\implies U$  increases,  $p_h$ : ambiguous

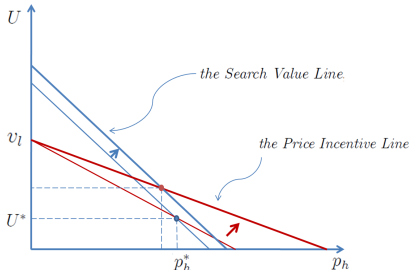


Figure 5a:  $\beta_l < \frac{1}{2} \left( \frac{\mu}{\sigma(1-\mu)} \beta_h + 1 \right)$

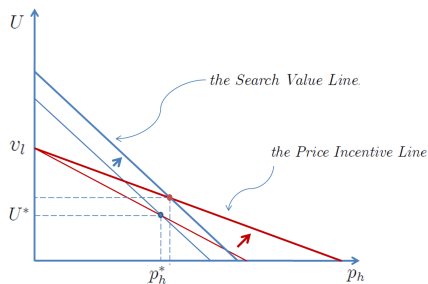


Figure 5b:  $\beta_l > \frac{1}{2} \left( \frac{\mu}{\sigma(1-\mu)} \beta_h + 1 \right)$

# Baseline Model

Simultaneous change of search cost and search quality

- Let  $\beta_h = \beta_l = \beta$ .
- Suppose there is a function  $s = g(\beta)$ , with  $g'(\beta) > 0$ , that characterizes the path of the change in  $(\beta, s)$ .
- Results:
  - $U^*$  decreases, and thus  $p_l^*$  increases, given any path.
  - The impacts on  $p_h^*$  depends on the primitives and the path.

# The extended model: continuous quality signals

## Set-up

- In the baseline model, the probability of buying from the high / low product is independent of search cost.
- However, as search costs are lower, consumers may search more intensively and will endogenously choose their purchase probability.
- We incorporate this feature into the extended model.

# The extended model: continuous quality signals

## Set-up

- Conditional on quality  $v_h$ , the distribution of  $\eta$  : CDF  $F_h(\eta)$  and PDF  $f_h(\eta)$  on  $(-\infty, \infty)$ .
- Conditional on quality  $v_l$ , the distribution of  $\eta$ : CDF  $F_l(\eta)$  and PDF  $f_l(\eta)$  on  $(-\infty, \infty)$ .
- MLRP: i.e.,  $\frac{f_h(\eta)}{f_l(\eta)}$  is increasing in  $\eta$ .
- Other settings replicate those in the benchmark model.

# The extended model: continuous quality signals

## The partial-pooling equilibrium

- On the equilibrium path:
  - The high-quality firm will charge  $p_h$  with probability 1, while the low-quality firm will charge  $p_h$  with probability  $\sigma$  and  $p_l$  with probability  $1 - \sigma$ .
  - Consumer:
    - The consumer will buy upon seeing  $p_l$ .
    - Upon seeing  $p_h$ , there exists a cutoff value of  $\eta$ ,  $\hat{\eta}$ , such that on the equilibrium path, the consumer will buy if  $\eta > \hat{\eta}$ .

# The extended model: continuous quality signals

## The partial-pooling equilibrium

- The cutoff  $\hat{\eta}$  represents the purchase probability in equilibrium.
- $\hat{\eta}$  also represents the informativeness upon purchase.
- We show that  $\hat{\eta}$  increases as  $s$  decreases.
- The consumer will endogenously adjust the search intensity and thus the ability to identify the high-quality product upon purchase.
- As the distributions of  $F_h$  and  $F_l$  denote the *exogenous* informativeness,  $\hat{\eta}$  represents the *endogenous* informativeness of search.

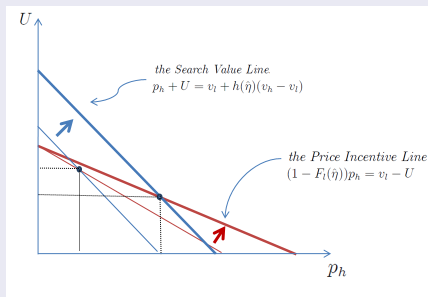


# The extended model: continuous quality signals

Comparative statics: search costs

## Corollary

*When search costs are lower,  $p_h^*$  will increase and  $U^*$  will decrease if the quality signals are sufficiently informative.*



# The extended model: continuous quality signals

Comparative statics: informativeness

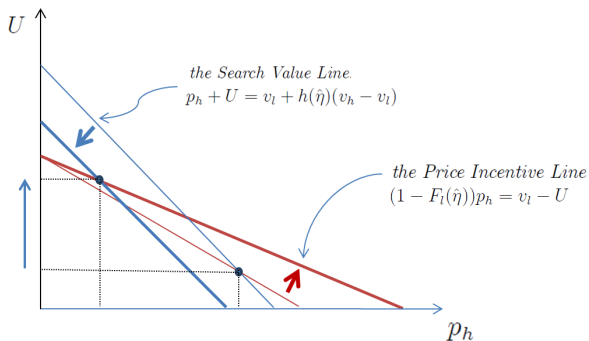
Suppose there are a family of distributions of  $f_h^\alpha$  associated with the index  $\alpha$ , which satisfy MLRP, i.e., for  $\alpha > \alpha'$ ,  $\frac{f_h^\alpha(\eta)}{f_h^{\alpha'}(\eta)}$  is increasing in  $\eta$ .

# The extended model: continuous quality signals

Comparative statics: informativeness

## Corollary

When the exogenous informativeness decreases,  $p_h^*$  will decrease while  $U^*$  will increase if  $\frac{d\hat{\eta}(\alpha)}{d\alpha} < 0$ .



- We incorporate consumer's uncertainty over vertical product quality into a consumer search model, and show that:
  - A separating perfect Bayesian equilibrium does not exist if the conditional distribution of the quality signal has full support;
  - A partial-pooling equilibrium can be characterized by the intersection of the search value line and the price incentive line;
  - Due to the search-value effect, the decrease in search costs can hurt consumers while benefit firms.
  - Due to the price-incentive effect, the increase in informativeness can benefit consumers while may lead to either higher or lower prices.
  - When the exogenous informativeness decreases, consumers can be better off if they respond by increasing the search intensity.
- A more comprehensive model with both horizontal and vertical uncertainty is working in progress.