Up-Cascaded Wisdom of the Crowd*

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November 17, 2017
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Abstract

Financing activities such as crowdfunding often involve both fund-raising and information production, and feature all-or-nothing (AoN) rules that contingent the financing upon achieving certain fundraising targets. Motivated by this observation, we introduce endogenous AoN target into a classical model of sequential sales and information cascade, and find that AoN leads to uni-directional cascades in which agents rationally ignore private signals and imitate preceding agents only if the preceding agents decide to *invest*. Consequently, an planner prices issuance more aggressively, and fundraising may succeed rapidly but never fails rapidly. Information production also becomes more efficient, especially with a large crowd of agents, yielding more probable financing of good projects, and the weeding-outs of bad projects that are absent in earlier models. More generally, endogenous pricing with AoN targets leads to greater financing feasibility and better harnessing of the wisdom of the crowd under informational frictions.

JEL Classification: D81, D83, G12, G14, L26

Keywords: Informational Cascade, Crowd-funding, All-or-nothing, plannerial Finance, Learning, Capital Markets, Information Efficiency.

*The authors thank Eliot Abrams, Will Gornall, Mark Westerfield, Ting Xu, and seminar and conference participants at UW Foster, UIC, CUNY Baruch, and Minnesota Carlson Junior Finance Conference for helpful comments. They also thank Ammon Lam for excellent research assistance.

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1 Introduction

Financial markets supposedly not only provide capital to planners, but also produce and aggregate information (Hayek (1945)). Yet with sequential sales and observational learning, information cascades emerge, underpricing of security issuance and reducing information production (Welch (1992); Bikhchandani, Hirshleifer, and Welch (1992)). Despite the large literature devoted to the study of information cascades, extant models leave out an important feature observed in real-life: in activities such as crowdfunding and IPO underwriting, the planner typically sets a funding target and gets the capital if and only if the target is reached. How does this "all-or-nothing" (AoN) feature affect information aggregation and financing? How should planners set the AoN target? How does it change our understanding of information cascades from the classical theory?

To answer these questions, we incorporate endogenous pricing and AoN target-setting into a standard model of sequential sales and dynamic learning. We characterize equilibrium pricing, optimal AoN targets, and information production, and find that the simple addition of AoN leads to uni-directional cascades in which agents rationally ignore private signals and imitate preceding agents *only if* the preceding agents decide to *invest*. Consequently, an planner prices issuance more aggressively, and fundraising may succeed rapidly but never fails rapidly. Relative to the standard setting of sequential sales with information cascades, information production now becomes more efficient, especially with a large crowd of agents, because an episode in which agents rely on their private information always proceeds information cascades (if there is one), leading to more successes of good projects and failures of bad projects, and more generally a better harnessing of the wisdom of the crowd under informational frictions.

Before delving into the details of the model and discussion on economic intuition, it is useful to discuss the main motivation and application of our model — crowdfunding. Since its inception in the arts and creativity-based industries (e.g., recorded music, film, video games), crowdfunding has quickly become a mainstream source of capital for early planners.[1] Importantly, crowdfunding exhibit the two salient features that motivates our model. First, potential backers often randomly chance upon crowdfunding websites or products within the window of offering. agents making decisions later can thus infer from earlier agents, or at least observe how well an offering has sold to date, or sold relative to offerings undertaken in the past.[2] Second, the most common type of crowdfunding scheme involves AoN implementation.[3] Moreover, recent empirical studies provide convincing evidence that planners use crowdfunding as an information aggregation mechanism (Xu (2017), Viotto da Cruz (2016), and Mollick and Kuppuswamy (2014)). Reduction of search and matching costs through the Internet allows divisibility of funding and low communication costs and facilitates greater outreach, decentralized participation, timely disclosure and monitoring. As such, the key advantage of crowdfunding platforms lies in aggregating information and

[1] In the span of a few years, its total annual volume has reached a whopping 34.4 billion USD globally at the dawn of 2017. It has surpassed the market size for angel funds in 2015, and the World Bank Report estimates that global investment through crowdfunding will reach $93 billion in 2025 ($http://www.infodev.org/infodev-files/wb_crowdfundingreport-v12.pdf$) The US deregulation also passed the law to allow non-accredited agents to join equity-based crowdfunding, further fueling the development. Specifically, on April 5, 2012, President Obama signed into law the Jumpstart Our Business Startups (JOBS) Act. Adding to then extant donation and reward based crowdfunding platforms, the JOBS Act Title III legalized crowdfunding for equity by relaxing various requirements concerning the sale of securities in May 2016. What is more, with the rise of initial coin offerings, alternative corporate crowdfunding emerges, with over two billion dollars raised in the US in the first half of 2017. In Appendix A, we provide two examples from well-known crowdfunding platforms.

[2] Take Kickstarter, for example. The planner is typically asked to provide the following pieces of information: (1) a description of the reward to the consumer, typically the planner's final product; (2) a pledge level ; (3) a target level. The crowdfunding campaign lasts typically for a fixed period of time – usually 30 days. During the campaign, Kickstarter provides information on the aggregate level of pledges, therefore a supporter can condition his decision based on previous consumers actions.

[3] The Crowdfund Act also indicates that AoN feature will likely be mandated, because intermediaries need to ensure that all offering proceeds are only provided to the issuer when the aggregate capital raised from all agents is equal to or greater than a target offering amount, and allow all agents to cancel their commitments to invest, as the Commission shall, by rule, determine appropriate (Sec. 4A.a.7). See http: // beta.congress .gov / bill / 112th- congress / senate- bill / 2190 / text.

harnessing the wisdom from the crowd, in addition to financing.[4]

Beside the recent rise of Internet-based crowdfunding, other examples of sequential selling and aggregating dispersed information under frictions abound. One important example is venture financing of startups: when raising series A and B rounds, planners often seek financing from multiple agents whom they approach sequentially. agents approached later learn which other agents have supported the project before them, and many agents condition their contributions on the fundraising reaching the target the planners specify.[5] Another oft-discussed example involves initial public offerings (IPOs): when agents are more informed than the issuer, for example, about the general market demand for shares and the after-market value, then the issuer faces an unknown demand for its stock and aggregates information from sequential agents about the demand curve (e.g., Ritter and Welch (2002)), and exhibits AoN feature (e.g., Welch (1992)).[6] In many elections a candidate is only voted into the office if the number of votes passes a threshold. Disclosure, accounting, and reporting practices may exhibit similar features.[7] Finally, as a solution to the coordination and free-riding issues in the provision of public goods, provision-point mechanism, alternatively known as assurance contract or crowdaction, is also defined by sequential decision-making

[4]In fact, SEC also recognizes in its final rule of regulating crowdfunding that "individuals interested in the crowdfunding campaign members of the 'crowd'$\cdots$ fund the campaign based on the collective 'wisdom of the crowd' " (17 CFR Parts 200, 227, 232, 239, 240, 249).

[5]For example, the blockchain startup String Labs approached multiple agents such as IDG capital and Zhenfund sequentially, many of whom decided to invest after observing Amino Capital's investment decision, and conditioned the pledge on the founders' "successfully fundraising" in the round (meeting the AoN target). Syndicates involving both incumbent agents from earlier rounds and new agents are also common.

[6] With limited distribution channels by investment banks, it takes the underwriter times to approach interested agents, who are typically institutions that do not communicate amongst one another. Strong initial sales encourage subsequent support while slow initial sales discourage subsequent investing. During an IPO, the issuer may decide to not continue with its proposed offering of securities if he observes a poor agent interest. IPO is therefore also characterized by sequential arrival and AoN. In both Internet-based crowdfunding and IPO, there is no market for agents to trade, and prices are fixed by planners or the underwriter.

[7]Scharfstein and Stein (1990) argue that managers imitate the investment decisions of other managers to appear to be informed. If new attempts have no cost upon failure, but can benefit the firms if there is a critical mass that triggers regulatory changes, then it is essentially an AoN implementation.

and the AoN feature (e.g., Bagnoli and Lipman (1989)).

Our model builds on the framework of Bikhchandani, Hirshleifer, and Welch (1992) and Welch (1992): an planner approaches sequentially N agents who choose to support or reject the planner's startup. Supporters pay a fixed price pre-determined by the planner and gets a payoff normalized to one if the project is good. All agents are risk-neutral and have a common prior on the project's quality. agents receive private, informative signals, and observe the decisions of preceding agents. Deviating from the standard setup, the planner also decides on AoN target—supporters only pay the price and enjoy the project payoff if the fundraising reaches a target number of supporters.

We show that in equilibrium the aggregation of private information only stops upon an UP cascade, in which case the public Bayesian posterior belief is so positive that agents always support the project regardless of their private signals. The intuition is that an AoN target encourages agents to invest even when the eventual aggregated information may be negative. In particular, agents with positive private signals always find it optimal to support because they only pay the price when the total support reaches the AoN target, which suggests a high posterior on the project's quality. On the other hand, agents with negative private signals are reluctant to support even before an AoN target is reached, because in equilibrium their actions may be misinterpreted as positive signals and causes either a too-early UP cascade or reaching the AoN target without enough number of positive signals, both implying a not-high-enough posterior on the project's quality. Therefore, DOWN cascades (where agents ignore positive private signals to reject) do not occur because they are all interrupted by agents with positive signals who do not care about DOWN cascades before AoN is reached.

After AoN is reached, the situation returns to the standard cascade setting. Higher AoN target excludes more DOWN cascades while it is less likely to be reached. To maximize the proceeds, the planner endogenously sets the AoN target to the smallest number that in equilibrium completely excludes DOWN cascades a la Welch (1992), with the caveat that the planner does not need to rely on price alone to avoid DOWN cascade. Consequently, there is no DOWN cascade which stops private information aggregation, and good projects are financed almost surely when the crowd base N is very large.

The exclusion of DOWN cascades has important implications on the availability of financing. In standard financial market models with information cascades, the feasible price range is limited because the price must be lower than the posterior of the first agent with a positive signal to prevent an early DOWN cascade. This limited price range makes it impossible to finance costly projects with potentially high qualities. With AoN target, planner can charge a sufficiently high price to cover the project implementation cost without worrying about DOWN cascades. Uni-directional cascades thus enlarge the feasible pricing range for fundraising. As a result, crowdfunding and the like can lead to financing of projects that would not have been funded by centralized experts, consistent with empirical findings in Mollick and Nanda (2015).[8] In particular, as we move from smaller agent base such as venture financing, to intermediate agent base such as IPO, to large agent base such as Internet-based crowdfunding, the issuance becomes increasingly less under-priced.

The exclusion of DOWN cascades also affects the optimal pricing. In the standard information cascade setting, Welch (1992) shows that the planner endogenously charges a low price to induce an UP cascade from the very beginning, preventing the potential arrival of DOWN cascades. This underpricing thus destroys information aggregation in financial market because information cascades start very early. Our model demonstrates that AoN

[8]Mollick and Nanda (2015) find that of the projects where there is no agreement, the crowd is much more likely to have funded a project that the judge did not like than the reverse. Around 75% of the projects where there is a disagreement are ones where the crowd funded a project but the expert would not have funded it. This is consistent with uni-directional cascades.

provides the planner an additional tool to avoid DOWN cascades. On the one hand, a higher price increases the profit the planner collects from each supporting agent. On the other hand, high price sets a higher bar for implementation and associated UP cascades, resulting a smaller chances of project implementation and the delay of UP cascades. Since the delay of UP cascade is less costly given a large agent base, the planner facing a large base of potential agents will charge a higher price for issuance, and the information aggregation continues until an UP cascade arrives. Uni-directional cascades thus reduces underpricing, and partially restores information aggregation by avoiding information cascades from the very beginning.

By aggregating information before investment is sunk, crowdfunding platforms adds an option value to experimentation, which can facilitate plannerial entry and innovation (Manso (2016)). In a sense, pre-selling through crowdfunding platforms can be viewed as credible surveys on consumer demand. Chemla and Tinn (2016) find that even for a failed crowdfunding, because the target is higher than the optimal investment threshold, the firm may still invest. Moreover, more successful at crowdfunding stage typically leads to greater success later for product implementation and future performance (Xu (2017)).

Literature

Our paper foremost relates to the large literature on information cascades, social learning, and rational herding. Bikhchandani, Hirshleifer, and Welch (1998) and Chamley (2004) provide comprehensive surveys. Our model is largely built on Bikhchandani, Hirshleifer, and Welch (1992) which discusses informational cascade as a general phenomenon. Welch (1992) relates information cascade to IPO underpricing, and serves as a natural benchmark for our model implications on pricing. Studies such as Anderson and Holt (1997), Çelen and Kariv (2004), and Hung and Plott (2001) provide experimental evidence for information cascades. We add to the literature by introducing AoN into sequential sales and learning, and show that the resulting directional cascades reduces underpricing, reduces the detriments of information cascades, and facilitate financing and harnessing the wisdom of the crowd.

Related are Guarino, Harmgart, and Huck (2011) and Herrera and Hörner (2013) that consider information cascades when only one of the binary actions is observable, and either the agents do not know their position or they have Poisson arrivals. While Herrera and Hörner (2013) find under certain signal distributions welfare could improve over that in Bikhchandani, Hirshleifer, and Welch (1992) and Guarino, Harmgart, and Huck (2011) show cascades only occur in one direction, they do not consider endogenous pricing. Moreover, they compare equilibrium outcomes across two exogenous environments, whereas we study the consequence of endogenous AoN under the standard cascade setting.

The paper also adds to an emerging literature on crowdfunding. Agrawal, Catalini, and Goldfarb (2014) comment on the basic patterns and economic tradeoffs of crowdfunding. Belleflamme, Lambert, and Schwienbacher (2014) provides an early theoretical comparison of reward-based and equity-based crowdfunding. Morse (2015) surveys informational issues in peer-to-peer crowdfunding. Liu (2017) and Chen (2017) discuss agent heterogeneity and endogenous timing of investment, but do not emphasize endogenous AoN. Strausz (2017) and Chemla and Tinn (2016) analyze demand uncertainty and moral hazard, and find that AoN is crucial in mitigating moral hazard, and Pareto-dominates the alternative "keep-it-all" (KiA) mechanism. Chang (2016) shows under common-value assumptions AoN generates more profit by making the expected payments positively correlated with values. Moreover, Cumming, Leboeuf, and Schwienbacher (2014) and Lau (2013, 2015) find that AoN performs better than KiA based on comparison between the two largest crowdfunding platforms, Kickcstarter and Indiegogo, and by comparing projects within Indiegogo. Like Strausz (2017), Ellman and Hurkens (2015) discuss optimal crowdfunding design, in the absence of moral hazard, but with a focus on price discrimination and demand uncertainty. Finally, Li (2017) similarly examines contract designs that harness the wisdom of the crowd and find profit-sharing to be optimal. Instead of introducing moral hazard or financial constraint, or derives optimal designs in static settings, we focus on pricing and information production, especially under endogenous AoN arrangements and with dynamic learning.

Empirically evidence on harnessing the wisdom of the crowd and on information cascades abound. Bond, Edmans, and Goldstein (2012) survey recent contributions related to the informational role of market prices for real decisions. Mollick and Nanda (2015) find significant agreement between the funding decisions of crowds and experts, and find no qualitative or quantitative differences in the long-term outcomes of projects selected by the two groups. Agrawal, Catalini, and Goldfarb (2011) finds suggestive empirical evidence of funding propensity increasing with accumulated capital on Sellaband, an Amsterdam based music-only platform started in 2006. Zhang and Liu (2012) documents rational herding on P2P lending on Prosper.com. Burtch, Ghose, and Wattal (2013) examine social influence in a crowd-funded marketplace for online journalism projects, and demonstrate that the decisions of others provide an informative signal of quality. Xu (2017) and Viotto da Cruz (2016) demonstrate the wisdom of the crowd benefits planners' ex post decisions and real option exercises. Our paper complements these studies by providing a formal framework to rationalize these phenomena.

Given our focus on financing efficiency, pricing efficiency, and informational efficiency, closely related is Brown and Davies (2017) which shows that when agents make decisions simultaneously, an exogenous AoN leads to loser's blessing, and scarce profits create a winner's curse, both adversely affecting financing efficiency for crowdfunding. We endogenize AoN target and demonstrate gains in informational efficiency as well as financing efficiency relative to the standard dynamic information-cascade benchmark. Also closely related is Hakenes and Schlegel (2014) which, along the same line, argues that endogenous loan rates and AoN targets encourage information acquisition by individual households in lending-based crowdfunding, and enable more good projects to receive financing. We focus on information aggregation and observational learning instead of agents' costly information acquisition. Moreover, we differ from these studies in our focus on dynamic learning and sequential investment instead of simultaneous investment games. Whereas those studies discuss the loss and gain in efficiency relative to the standard static auction benchmark, our setup allows us to uncover the benefits of setting AoN in a dynamic environment, in a spirit akin to how commitment helps improve informational efficiency in Bagnoli and Lipman (1989) and Bond and Goldstein (2015).

Our paper is also broadly related to innovation and plannerial finance. Startup firms receive venture funding often to experiment and uncover more information about the project's viability and future profitability (Gompers and Lerner (2004) and Kerr, Nanda, and Rhodes-Kropf (2014)). To the extent that such information can be gleaned from consumer surveys or aggregated from crowds, the planner can potentially reduce experimentation or learning costs. Moreover, crowdfunding arguably reduces the barrier to entry for planners. Yet it may not select or monitor projects as well as VC does (Gompers, Gornall, Kaplan, and Strebulaev (2016) show that VCs mainly add value through selection). It thus serves as a complement to the traditional venture capital (e.g., Chemla and Tinn (2016)). Abrams (2017) document initial empirical evidence on how the US securities crwodfunding market provides a new way to finance quality startups. We add to the literature by showing how

AoN rules commonly observed in crowdfunding help mitigate inefficiencies typically associated with information cascades, therefore further demonstrating the benefits and costs of these innovations in plannerial financing and information aggregation from dispersed agents and consumers.

The rest of the paper is organized as follows: Section 2 sets up the modeling framework and analyzes the main mechanism of uni-directional cascades under endogenous pricing and AoN targeting; Section 3 discusses pricing implications; Section 4 demonstrates how AoN better utilizes the wisdom of the crowd to improve financing and information production efficiency; Section 5 concludes.

2 A Model of Directional Cascades

2.1 Setup

Consider a planner deciding whether to press forward with a project. He visits a sequence of agents $i = 1, 2, \ldots, N$, each can potentially support or reject the proposal. The action of agent i is $A_i \in \{S, R\}$, where S denotes a support and R a rejection. If the proposal is implemented eventually, then every supporting agent incurs a predetermined adoption cost m, and receives the benefit V, which can be either 0 or 1. In non financial scenarios such as voting or fashion, m is the adoption cost. In fund-raising scenarios such as crowdfunding or series A and B rounds of venture financing, the planner is the entrepreneur, and cost m can be interpreted as the amount of money that each supporting agent pays. To focus on the impact of AoN rule, m is assumed to be exogenous in the baseline model. We allow the planner to determine m when we discuss pricing implications of AoN rule latter.

All agents including the planner are rational, risk-neutral, and share the same prior that the project type can be either $V = 0$ and $V = 1$ with equal probability. Each agent i observes one conditionally independent private signal $X_i \in \{H, L\}$. Signals are informative in the following sense:

$$Pr(X_i = H|V = 1) = Pr(X_i = L|V = 0) = p \in (\frac{1}{2}, 1); \tag{1}$$

$$Pr(X_i = L|V = 1) = Pr(X_i = H|V = 0) = q \equiv (1 - p) \in (0, \frac{1}{2}). \tag{2}$$

We depart from the literature by incorporating the observed "all-or-nothing" (AoN) scheme into this setup: the planner receives "all" if the campaign succeeds in reaching a pre-specified target number of supporters, and "nothing" if it fails to do so. In other words, before agents make decisions, the planner decides the amount of each contribution m and an AoN target T_N; the proposal is implemented if and only if more than T_N agents support. In this section, we take the AoN target T_N as exogenous. We later endogenize it.

The order of agents is exogenous and is known to all.[9] This is equivalent to observing both supporting and rejecting actions of previous agents, a standard assumption in the literature on information cascades. In other words, when agent i makes her decision, she observes her own private signal X_i and decisions made by all those ahead of her, that is, $\{A_1, A_2, \ldots, A_{i-1}\}$.[10] Agents Bayesian update their beliefs using their private information and inferences from the observed actions of their predecessors in the sequence. Let $H_i \equiv \{A_1, A_2 \ldots, A_i\}$ be the action history till agent i, and N_S be the total number of supporting

[9]While real world examples such as crowdfunding may involve endogenous orders of agents, our abstract and simplified setup allows us to relate and compare to the large literature on information cascades which typically has exogenous orders of agents. We show in extension section that our fundamental result is robust when agents have options to wait.

[10]In the application in crowdfunding, this information set is equivalent to observing fund raised to-date (and time) and knowing the starting time of fundraising and the agent arrival rate. Evidence that funders rely heavily on accumulated capital as a signal of quality is abundant (Agrawal, Catalini, and Goldfarb (2011); Zhang and Liu (2012), and Burtch, Ghose, and Wattal (2013)).

agents. Agent i's problem is:

$$\max_{A_i} \left[E\left(V|X_i, H_{i-1}, N_S \geq T_N\right) - m\right]\mathbb{1}_{\{A_i=S\}}, \tag{3}$$

where $\mathbb{1}_{\{A_i=S\}}$ is the indicator function for supporting. If $E\left(V|X_i, H_{i-1}, N_S \geq T_N\right) > m$, an agent chooses $A_i = S$. When $E\left(V|X_i, H_{i-1}, N_S \geq T_N\right) = m$, we assume that:

Assumption 1 (Tie-breaking)**.** *When indifferent between supporting and rejecting, an agent supports if the AoN target is possible to reach with all subsequent agents' supporting.*

This assumption states that agents, whenever indifferent in terms of payoff consideration, supports the project if it is still possible to reach the target threshold $T_N(m)$. It is natural because the planner can always lower m by an arbitrarily small amount to induce the contribution.

2.2 Solution

We start our analysis with the posterior dynamics. The following lemma characterizes the posterior belief given a series of signals.

Lemma 1. *Given a series of signals $X \equiv \{X_1, X_2, \ldots, X_n\}$, the ratio of the posterior probability of $V = 1$ to that of $V = 0$ is*

$$\frac{Pr(V=1|X)}{Pr(V=0|X)} = \frac{p^k}{q^k},$$

where $k = \#of\ H\ signals - \#of\ L\ signals$.

Lemma 1 states that the posterior belief of project type only depends on the difference between numbers of H and L signals so far, but not on the total number of observations.

This result suggests that observing one H and one L signals does not change the posterior belief. In other words, opposing H and L signals cancel each other and have no effect in forming posterior, a convenient feature also in Bikhchandani, Hirshleifer, and Welch (1992). Given Lemma 1, an agent's expected project value conditional on observing k more H signals is then,

$$E(V|k \text{ more } H \text{ signals}) = \frac{p^k}{p^k + q^k}. \tag{4}$$

It is apparent that the expected project payoff is strictly monotonically increasing in k.

When agents act regardless of their private signals, the market fails to aggregate dispersed information. Our notion of informational cascade follows the literature standard (e.g. Bikhchandani, Hirshleifer, and Welch (1992)).

Definition 1. *An information cascade occurs if a subsequent agent's action does not depend on her private information signal. An* UP *cascade occurs if a subsequent agent supports the project regardless of her private signal. A* DOWN *cascade occurs if she rejects the project regardless of her private signal.*

Notice that we have taken the convention of calling it a cascade as long as the NEXT agent ignores the private information, even though the current agent may still use private signal. This is immaterial for our theory but simplifies exposition in the proof. In standard models of informational cascades, both UP and DOWN cascades are possible. If a few early agents observe H signals, their contributions may push the posterior so high that the project remains attractive even with a private L signal. Similarly, a series of L signals may doom the offering. An early preponderance towards support or rejection causes all subsequent individuals to ignore their private signals, which thus are never reflected in the public pool of knowledge. The first main result in our paper is to show that with the AoN feature, there exists an equilibrium such that before the AoN target is reached, only UP cascades may exist.

Proposition 1. *There exists an equilibrium such that:*

1. *When there are more than $T_N - 1$ supporting predecessors:*

    - *Each agent i chooses to support if and only if*

$$E(V|X_i, H_{i-1}) \geq m. \tag{5}$$

2. *When there are strictly less than $T_N - 1$ supporting predecessors:*

    - *Agents with signal H always support the project;*
    - *Agent i with signal L contributes if and only if:*

$$E(V|k-1 \text{ more } H \text{ signals}) \geq m^*, \tag{6}$$

    where k is difference between the numbers of supporting and rejecting predecessors before agent i.

Proposition 1 describes adoption strategies for agents. Let $m_k \equiv E(V|k \text{ more } H \text{ signals})$. The proof for Proposition 1 suggests both the possibility and arrival time of cascades, as summarized in the following corollary.

Corollary 1. *When $m \in (m_{k-1}, m_k]$, an UP cascade starts whenever there are $k+1$ more agents supporting rather than rejecting, a DOWN cascade starts whenever there are $k-2$ more agents supporting rather than rejecting and there are more than $T_N - 1$ supporting predecessors.*

One can interpret the equilibrium in two steps. First, when the AoN target would be reached when one more agent decides to support (that is to say, there are more than $T_N - 1$ supporting predecessors), the agent knows that the project would be implemented if she supports, and she faces exactly the same optimization problem as in standard cascade model. Second, before the AoN target is reached (that is to say, there are strictly less than $T_N - 1$ supporting predecessors), in the equilibrium agents with good signals support regardless of history they observe while agents with bad signals only choose to support when there is an UP-cascade. When there are strictly less than $T_N^* - 1$ supporting predecessors, agents with good signals find it optimal to choose support because it essentially delegates their decisions to the follow agent facing $T_N^* - 1$ supporting predecessors. Since that follow agent shares the same interest with other agents and observes a longer history, she makes a better decision. However, for agents with bad signals, they have no incentive to deviate to support because all follow agents would misinterpret her action and form wrong posteriors. Given her true bad private signal, all follow agents are over optimistic and they either start an UP cascade too early or reach the AoN target when the true posterior is not high enough. Taking that into account, agents with bad signals find deviation unattractive. Second, when there are more than $T_N - 1$, agents know that the project would be implemented for sure if they invest, and their optimal adoption decision problem is exactly the same as in standard herding models, and both UP and DOWN cascades are possible.

One can interpret UP cascades as the source of type I error in information aggregation since it may falsely accept the project when it is bad. On the other hand, DOWN cascades introduce type II error, rejecting the proposal when it is actually good. Intuitively, if the agent base and AoN target is large, DOWN cascades do not occur and the type II error completely disappears, that is to say, all good project are implemented.

Proposition 2. *As $T_N \to \infty$, a good project with $V = 1$ is implemented almost surely with an UP-cascade.*

3 Endogenous Pricing and AoN Target

In baseline model we investigate the effect of AoN rule on agents' strategies. In many scenarios, especially fundraising activities such as Crowdfunding, VC series A and B round of financing etc., the planner can commit the price of each contribution and the AoN target to maximize his revenue. In this section, we extend our analysis to the planner's decision. We first define the planner's revenue maximization problem and the equilibrium. We then start our analysis by characterizing the optimal price in the standard information cascade model (without AoN) as a benchmark (most analysis from Welch (1992) but in our framework). Pricing implications of informational cascade is important because underpricing or overpricing may affect the success or failure of the issuance, resulting in an important and direct impact on the real economy. This is especially salient in the case of IPO with limited distribution channels of investment banks (Welch (1992)).

3.1 Planner's Optimization Problem

Let $0 \leq \nu < 1$ be the per contribution cost for the planner. In the context of reward-based crowd-funding, this could be the production cost of each reward product. In the IPO process, ν can be interpreted as the issuer's share reservation value. The planner chooses price m and AoN target T_N to solve the following problem:

$$\max_{m,T_N} \pi(m, T_N, N) = E[(m - \nu)N_S \mathbb{1}_{\{N_S \geq T_N\}}], \tag{7}$$

where $\mathbb{1}_{\{N_S \geq T_N\}}$ is the indicator function of project implementation. In fund-raising scenarios, the planner tries to maximize his expected profit. In non-financial scenarios, with exogenous transition cost m the planner is promoting the proposal and maximizing the expected number of supporting agents.

3.2 Equilibrium

We use the concept of perfect Bayesian Nash equilibrium (PBNE), which is defined as:

Definition 2. *An equilibrium consists of planner's choice of* $\{m^*, T_N^*\}$*, adoption strategies for agents* $\{A_i^*(X_i, H_{i-1}, m^*, T_N^*)\}_{i=1,2\ldots,N}$ *such that:*

1. *For each agent* i*, given the required contribution* m^* *and* T_N^**, associated* T_N^* *and other agents' investment strategies* $\{A_j^*(X_j, H_{j-1}, m^*, T_N^*)\}_{j=1,2,\ldots,i-1,i+1,\ldots,N}$*, investment strategy* $A_i^*(X_i, H_{i-1}, m^*, T_N^*)$ *solves her optimal problem:*

$$A_i^* \in \text{argmax } [E\left(V - m|X_i, H_{i-1}, N_S \geq T_N\right)]\mathbb{1}_{A_i=S}; \tag{8}$$

2. *Given investment strategies* $\{A_i^*(X_i, H_{i-1}, m^*, T_N^*)\}_{i=1,2\ldots,N}$*,* m^* *and* T_N^* *solve planner's problem:*

$$\{m^*, T_N^*\} \in \text{argmax} \quad \pi(m, T_N, N). \tag{9}$$

3.3 Standard Cascades without AoN Target

If there is no AoN (or set $T_N = 1$), then for each agent, her payoffs do not depend on what later agents do. Thus, the equilibrium is essentially the same as the one characterized in Bikhchandani, Hirshleifer, and Welch (1992) and Welch (1992). That is, each agent i

chooses to support if and only if

$$E(V|X_i, H_{i-1}) \geq m. \tag{10}$$

In this equilibrium, both UP and Down cascades may occur. The aggregation of public information stops once one cascade arrives. As discussed in Bikhchandani, Hirshleifer, and Welch (1992), the impact of cascades largely depends on the private information precision. If the information is precise, then cascades would not be a big concern since a cascade only occurs when the aggregated public information is sufficiently informative to dominate one's private signal, suggesting a high probability of correct cascades. When the private signal is noisy, cascades become a serious concern since a slightly more informative public pool of knowledge is enough to cause individuals to disregard their private signals. The following proposition shows that without AoN target, the contribution is under-priced when the precision of private signals is not too high.

Lemma 2. *The planner always charges $m \leq p$. When $\nu = 0$ and $p \leq \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})$, the optimal contribution is $m^* = 1 - p < \frac{1}{2} = E(V)$.*

The lemma is basically a restatement of the underpricing result in Welch (1992), especially Theorem 5.[11] We assume $\nu = 0$ to match the setup in Welch (1992). The first pricing upper bound comes from the concern for potential DOWN cascades. If planner charges $m > p$, then even with a H signal, the first agent choose rejection and so does every subsequent agent, leading to a DOWN cascade starts at the very beginning, which yields 0 benefit for

[11]Several articles such as Benveniste and Spindt (1989) argue that the common practice of "bookbuilding" allows underwriters to obtain information from informed agents. This information-gathering perspective of bookbuilding is certainly useful, but the information provided by one incremental agent is not very valuable when the investment banker can canvas hundreds of potential agents in an IPO. Thus, it is not obvious that this book-building framework is capable of fully explaining the average underpricing of about 50 percent, conditional on the offer price having been revised upward.

sure.

The second result concerns optimal pricing when the individual signal is not very precise and cascades are a relevant concern. UP and DOWN cascades, even though they both reduce the information aggregation among agents, affect the planner's profit asymmetrically. While the planner benefited from UP cascades by attracting contributions from late agents with L signals, he is concerned with DOWN cascades since a few early rejections may doom the offering. When the private information precision is low, the concern of DOWN cascades pushes down the price to the level such that given the low price the UP cascade starts at the very beginning with probability 1. Because $m^* < \mathbb{E}[V]$, the optimal pricing entails underpricing ex ante so that the first agent finds it attractive even with a L signal. To be clear, depending on the true project quality, we still have overpricing (if $V = 0$) ex post.

3.4 Pricing with AoN Target

Now we move to the optimal pricing problem with the AoN target $T_N(m)$. As we shown in this section, the AoN target changes both pricing upper bound and the underpricing results ith AoN there would be no DOWN cascade in the equilibrium. Similar to Proposition 1, with the AoN feature, there exists an equilibrium such that only UP cascades may exist.

Proposition 3. *There exists an equilibrium such that:*

1. *Given the investment contribution (price) $m^* \in (0, 1)$, the corresponding AoN target $T_N^* \leq N$ satisfies:*

$$E(V|T_N^*, N) \leq m^* < E(V|T_N^* + 1, N), \tag{11}$$

where $E(V|x, N)$ is the posterior mean of V given there are x number of H signals out of N observations;

2. *agents with signal H always support the project;*

3. *agent i with signal L contributes if and only if:*

$$E(V|k-1 \text{ more } H \text{ signals}) \geq m^*, \tag{12}$$

where k is difference between the numbers of supporting and rejecting predecessors before agent i.

Proposition 3 describes agent strategies and the planner's endogenous AoN target choice in the equilibrium. Let $m_k \equiv E(V|k \text{ more } H \text{ signals})$. The proof for Proposition 3 suggests both the possibility and arrival time of cascades, as summarized in the following corollary.

Corollary 2. *In the equilibrium characterized in Proposition 3, there would be no DOWN cascades. If $m \in (m_{k-1}, m_k]$, an UP cascade starts whenever there are $k+1$ more agents supporting rather than rejecting.*

In the equilibrium the planner chooses the optimal level of AoN target to exclude DOWN cascades. A higher AoN target excludes DOWN cascades before the target is reached but a higher target itself is more difficult to reach. In the equilibrium, the concern for DOWN cascades dominates the implementation likelihood concern and the planner chooses the lowest level of AoN target to completely exclude DOWN cascades. Intuitively, with the AoN target, DOWN cascades do not occur and the type II error completely disappears if the aggregated information is precise enough, because the endogenous price and AoN target always ensure good projects are financed when N is large.

Proposition 4. *As $N \to \infty$, a good project with $V = 1$ is financed almost surely with an UP-cascade.*

Next, we examine the informational environment in such an up-cascaded equilibrium, and its pricing implications. First, if $\nu > m_N$, then the marginal production cost is higher than the highest possible posterior, and the planner charges $m = \nu$ and get 0 profit.

Now we moves to the case $0 \leq \nu \leq m_N$. Lemma 1 and equation (4) show that the posterior only depends on the difference between numbers of H and L signals. If the price is m_{k-1}, then an UP cascade starts once there are k more H signals. Since each agent will observe either H or L signal and in the equilibrium her decision perfectly reveals her private signal before an UP cascade starts, the arrival of an UP cascade is equivalent to the first passage time of a one-dimension biased random walk. The following lemma lays the foundation for our analysis on the distribution of UP-cascades' arrival time.

Lemma 3 (Hitting Time Theorem)**.** *For a random walk starting at $k \geq 1$ with i.i.d. steps $\{Y_i\}_{i=1}^{\infty}$ satisfying $Y_i \geq -1$ almost surely, the distribution of the stopping time $\tau_0 = \inf\{n : S_n = k + \sum_{i=1}^{n} Y_i\}$ is given by*

$$Pr(\tau_0 = n) = \frac{k}{n} Pr(S_n = 0). \tag{13}$$

Proof. See Van der Hofstad and Keane (2008). □

To characterize the distribution of UP cascades arrival time, let $\varphi_{k,i}$ be the probability that an UP cascade starts at agent i, then

Lemma 4. *If the price $m \in (m_{k-2}, m_{k-1}]$, then the probability that an UP cascade starts at agent i is*

$$\varphi_{k,i} = \frac{k}{i} \begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} (pq)^{\frac{i-k}{2}} \frac{p^k + q^k}{2} \tag{14}$$

where

$$\left(\begin{array}{c} i \\ \frac{i+k}{2} \end{array} \right) = \begin{cases} \frac{i!}{\frac{i+k}{2}!\frac{i-k}{2}!} & \textit{if } i \geq k \textit{ and } k+i \textit{ even;} \\ 0 & \textit{otherwise.} \end{cases} \tag{15}$$

Since for any $m \in (m_{k-1}, m_k]$, all agents make the same investment decisions, the planner can always charge $m = m_k$ and receives a higher profit. Without loss of generality, we focus our pricing analysis on $m \in \{m_{-1}, m_0, \ldots, m_N\}$. We exclude cases for $k < -1$ because $m_{-1} = 1 - p$ is low enough to induce an UP cascade from the very beginning for sure.

Now we consider the optimal pricing. An UP cascade only occurs when the posterior given another L signal is higher than m, and all subsequent agents support the project. The project is eventually implemented once an UP cascade starts. On the other hand, for any agent $i \leq N - 1$, if the UP cascade has not started yet, then there is a strictly positive possibility that the project will not be implemented. So a project is eventually funded if and only if either 1) There is an UP cascade; or 2) agent N supports the project and the total number of supporting agents is exactly T_N. In either cases, we can compute the profit associated with m, as formalized in Proposition 5. But before going there, we illustrate the two scenarios in Figure 1, which plots the difference between supporting agents and rejecting agents when n agents have arrived. The figure also includes a sample path that leads to funding failure because AoN target is not reached.

Proposition 5. *When the price is $m = m_{-1} = 1 - p$, the planner's expected profit is $(1 - p - \nu)N$. More generally, given a price $m = m_{k-1}$, $k \in \{1, 2, \ldots, N\}$, the planner's*

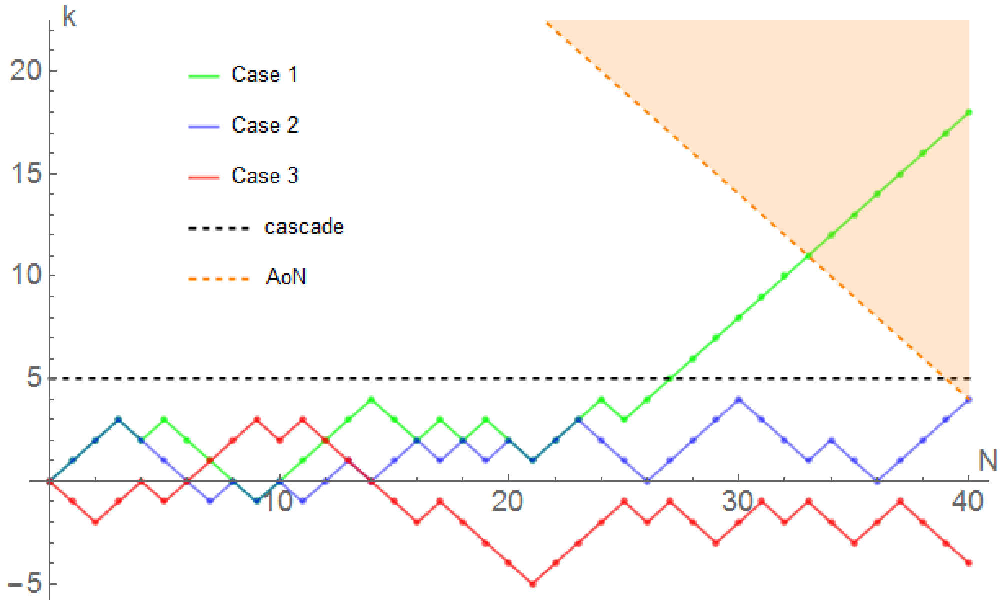

Figure 1: Evolution of Support-Reject Differential

Simulated paths for $N = 40$, $p = 0.7$, $m^* = m_5 = 0.9673$, and AoN target $T^*(N) = 22$. Case 1 indicates a path that crosses the cascade trigger $k = 5$ at the 26th agent and all subsequent agents support regardless of their private signal; case 2 indicates a path with no cascade, but the project is still funded by the end of the fundraising; case 3 indicates a path where AoN target is not reached and the project is not funded. The orange shaded region above the AoN line indicates that the project is funded.

expected profit is

$$\pi(m_{k-1}, N) = \begin{cases} (m_{k-1} - \nu) \left[\sum_{i}^{N} \varphi_{k,i} (N - \frac{i-k}{2}) + \frac{p^{k-1}q + pq^{k-1}}{p^k + q^k} \varphi_{k,N} \frac{N+k-2}{2} \right] & \text{if } k + N \text{ even;} \\ (m_{k-1} - \nu) \left[\sum_{i}^{N-1} \varphi_{k,i} (N - \frac{i-k}{2}) + \frac{p^k + q^k}{p^{k+1} + q^{k+1}} \varphi_{k,N+1} \frac{N+k-1}{2} \right] & \text{if } k + N \text{ odd.} \end{cases} \tag{16}$$

Let $k_\nu \in \{0, 1, 2, \dots\}$ be the smallest integer satisfying $m_{k_\nu} \geq \nu$. For each $k \in \{k_\nu, k_\nu+1, k_\nu+2, \dots\}$, there exists a finite positive integer $\underline{N}(k)$ such that for $\forall\ N \geq \underline{N}(k)$, $\pi(m_k, N) > \pi(m_{k-1}, N)$.

Proposition 5 gives an explicit characterization of planner's expected profit as a function of price m_k and number of potential agents N. Figure 2 provides an illustration on how the profit depends on m.

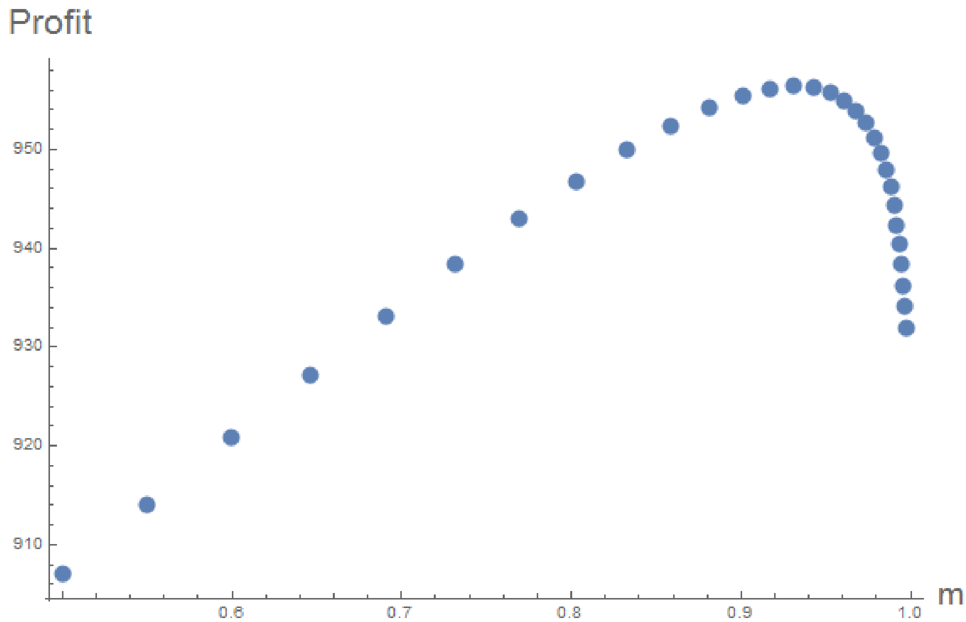

Figure 2: Optimal Pricing: An Illustration with $N = 2000$, $\nu = 0$ and $p = 0.55$.

More importantly, the result on $\underline{N}(k)$ suggests that, different from Lemma 2, the optimal price depends on the number of potential agents N. A financial technology (Internet-based platforms) that can allow us to reach a greater N thus has a fundamental impact. In the standard cascades models, a DOWN cascade hurts the planner significantly because subsequent agents all reject. The concern for DOWN cascades pushes down the optimal price, and can cause immediate start of an UP cascade, *independent of the number of agents* because the decisions of later agents have no impact on the first agent's payoffs (Welch (1992)). With the AoN target, in the equilibrium there would be no DOWN cascades and one early rejection is not a big concern since all agents with H signals would still support the project. Those supporting agents may trigger an UP cascade later, especially when there are many potential agents in the market. The following corollary shows the increasing trend of optimal price m^* as the number of potential agents N grows.

Corollary 3. *For* $\forall\ m_k$*, there exists a a finite positive integer* $N_\pi(m_k)$ *such that for* $\forall\ N \geq$

$N_\pi(m_k)$, $m^* > m_k$.

Proof. Let $N_\pi(m_k) = \max\{\underline{N}(0), \underline{N}(1), \ldots, \underline{N}(k), \underline{N}(k+1)\}$. Then for $\forall\ N \geq N_\pi(m_k)$, $\pi(m_{k+1}, N) > \pi(m_k, N) > \cdots > \pi(m_{-1}, N)$. So $m^* \geq m_{k+1} > m_k$. $\square$

This corollary has two implications novel to the literature: first, as we reach out to more and more agents through technological innovations such as the Internet, the planner can charge a higher price; second, unconditionally there would be less underpricing but more overpricing as N becomes big. The left panel in Figure 3 shows the optimal starting point of UP cascades (kth agent) when N differs, and right panel plots the optimal pricing as a function of N. We note that $m > \mathbb{E}[V]$ in these cases.

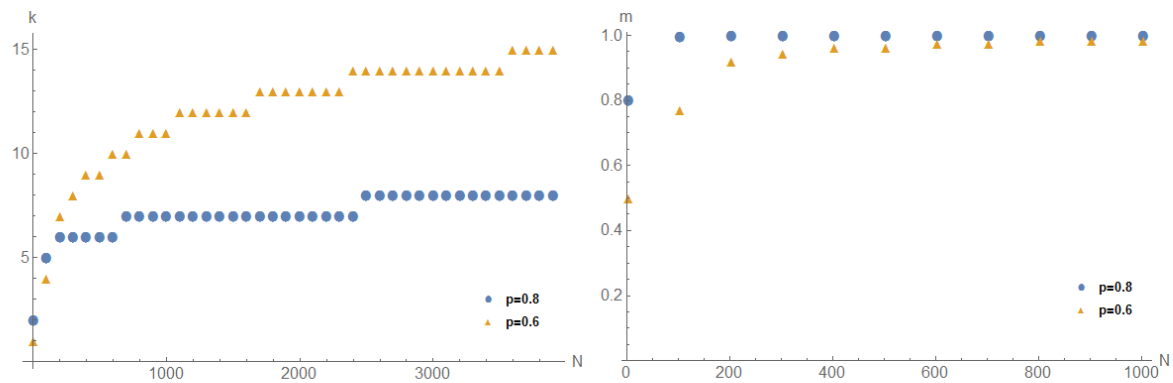

Figure 3: Cascades and optimal prices as N increases

Since for any finite integer $N \geq 2$, $m^*(N) \in \{-1, 0, 1, \ldots, N\}$. Corollary 3 implies that m^* shows an increasing trend. Since m_k is a monotonic increasing function in k and $\lim_{k\uparrow\infty} m_k = 1$, it is straightforward to see that

Corollary 4. $\lim_{N\to\infty} m^*(N) = 1$

That is to say, when there is a large base of potential agents, the optimal price approaches the highest possible value, leading to unconditional overpricing rather than the underpricing result in Welch (1992).

4 Wisdom of the Crowd (and AoN)

This section discusses the effect of AoN scheme on information aggregation. With AoN scheme, the uni-directional cascade result is robust to the option to wait. Aon scheme fundamentally changes the feasibility of harnessing the wisdom of the crowd, and the resulting informational environment. We also allow the planner to carry out the project even if the target is missed, or to give up the project even if the target is met.

4.1 Options to Wait and Information Aggregation

One common concern for standard information cascade models is the assumption of exogenous order of decision-making. In reality, agents may choose to wait in the hope that they may observe more information. Most results in standard information cascade models fail to hold if one introduces the option to wait. One particular feature of AoN is that the information aggregation pattern in our model is robust to the option to wait.

To be more specific, we enlarge each agent's action set to $\{S, R, W\}$, where W is the decision to wait and make decision after observing agent $i + 1$'s decision. The option to wait results in multiple equilibria due to the coordination problem on waiting decisions and off equilibrium path beliefs. That said, the following proposition shows that, in terms of information aggregation, there exists an equilibrium that is essentially the same as the one characterized in Proposition 3.

Proposition 6. *There exists an equilibrium such that:*

1. *Given the investment contribution (price)* $m^* \in (0, 1)$*, the corresponding AoN target* $T_N^* \leq N$ *satisfies:*

$$E(V|T_N^*, N) \leq m^* < E(V|T_N^* + 1, N), \tag{17}$$

where $E(V|x, N)$ is the posterior mean of V given there are x number of H signals out of N observations;

2. *agents with signal H always support the project;*

3. *agent i with signal L contributes if there is already an UP cascade, that is:*

$$E(V|k-1 \text{ more } H \text{ signals}) \geq m^*, \tag{18}$$

where k is difference between the numbers of agents whose first time decision is support and agents whose first time decision is to wait. Otherwise, agent i with signal L chooses to wait until all agents has made a decision at least once. Let N_S be the number of agents that chooses to support as her first decision. Then agent i chooses to support if:

$$E(V|N_S, N) \geq m^*, \tag{19}$$

and rejects otherwise.

In terms of information aggregation, this equilibrium is equivalent to the one in Proposition 3. In the equilibrium, those agents who wait upon their first decision-making are exactly those who reject the project in the baseline model, and those who support upon their first decision-making are exactly those supporting agents in Proposition 3.

To see this, consider first if there is already an UP cascade then no one wants to deviate (if everyone chooses to invest once there is an UP cascade). Now for agents with H signals, supporting always weakly dominates rejection and thus there is no need to wait. For agents with L signals, waiting till the end weakly dominates rejection and they will wait till the end. Observational learning still works since agents with different signals choose different actions. In the equilibrium, before the arrival of an UP cascade, all agents infer support action as a good signal and the decision to wait as a bad signal, resulting exactly the same information aggregation process as we described in the baseline model.[12]

4.2 Feasibility of Fundraising and Information Aggregation

From Lemma 2, we see that there is a pricing upper bound in order for the fundraising or offering to be feasible. This bound becomes a serious concern when the cost ν is non-zero. In particular, when ν is too high, traditional cascade models predict a failure (rejection cascade for sure) while in our model the planner can still charge a high price and is able to implement the project when aggregated information is good. The following proposition is immediate.

Proposition 7. *Without AoN, no project with $\nu > p$ is financed and information aggregation is infeasible; committing to an AoN target enables fundraising and information aggregation even when $\nu > p$.*

Because of DOWN cascades, agents certainly do not finance any project with $\nu > p$. In such cases, not only do we fail to raise financing, there is also no way the planner can harness the wisdom of the crowd because no information is aggregated. This result roots from the fact that the concern for DOWN cascades imposes an upper bound on possible prices, and any project with a high cost will charge a high price and thus triggers a DOWN cascade and financing failure for sure.

The exclusion of DOWN cascades therefore has an important impact on the pricing upper bound, and hence the availability of finance. With AoN target, any price $m < 1$ is possible and there would be a strictly positive possibility that the project would be financed given there is a large enough potential agent base. Moreover, from Proposition 2 we know that the good type of project ($V = 1$) will be financed almost surely as the number of agents goes

[12] The option to wait may affect the optimal price m^*, because with the option to wait agents with L signal still contribute if the posterior after the information aggregation is good.

to infinity. In this sense, AoN target drives the discrete jump in financing and information aggregation feasibility.

4.3 Harnessing the Wisdom and Social Welfare

Even when the fundraising is feasible, it serves little for information aggregation in most extant models of information cascade. For example, in Welch (1992), cascade always starts from the very beginning, and no private signals are aggregated because once a cascade starts, public information stops accumulating. Nor does the public pool of knowledge have to be very informative to cause individuals to disregard their private signals. As soon as the public pool becomes slightly more informative than the signal of a single individual, individuals defer to the actions of predecessors and a cascade begins.

With AoN target, however, the downside risk is removed, and optimal pricing does not necessarily result in information cascades from the very beginning (Lemma 4). Therefore, as long as $m^* > 1 - p$, the fundraising also aggregates some private information from the agents, allowing us to harness the wisdom of the crowd to some extent.

What is more, from Lemma 4, the probability that a cascade is correct (UP cascade when $V = 1$) is given by

$$Pr(V = 1|cascade\ at\ i^{th}\ agent) = \frac{p^k}{p^k + q^k}\mathbb{I}_{\{i \geq k \& k+i\ is\ even\}}$$

where k satisfies $m_{k-1} < m \leq m_{k-1}$. Because k is weakly increasing in the pricing m and the optimal pricing is weakly increasing in N (Proposition 5), the following proposition ensues.

Proposition 8. *A cascade starts weakly later with higher pricing m, and thus with a larger crowd (larger N) when pricing is endogenous. The probability of a cascade being correct is* *increasing in p, weakly increasing in the pricing m, and weakly increasing in N when pricing is endogenous.*

AoN reduces underpricing, which in turn delays cascade and increases the probability of correct cascades. More importantly, whereas N does not matter in standard cascade models, AoN links the timing and correctness of cascades to the size of the crowd. With a large N as is the case for Internet-based crowdfunding, information cascades has a less detrimental effect, allowing better harnessing of the wisdom of the crowd.

Information efficiency is closely related to social welfare. In our model, for any strictly positive production cost $\nu \in (0, 1)$, it is socially costly to finance a type 0 project and socially beneficial to finance a type 1 project. As we discussed above, harnessing the wisdom from the crowd increases the information efficiency, resulting more efficient investment decision and thus improve the social welfare. Uni-directional cascade also means that offerings in the cascade model can fail whereas in the baseline in Welch (1992), offerings never fail. This would help us explain why some offerings fail occasionally and/or are withdrawn, without invoking insider information as Welch (1992) did in his model extension. By allowing some projects to fail when N is large (Proposition 2), we put the wisdom of the crowd to use to increase social welfare. To be specific, when N goes to ∞, the probability that a good project being financed goes to 1 while the probability that a bad project being implemented goes to 0.

It should be noted that our findings complement rather than contradict those in Brown and Davies (2017). In their setup, agents bid more aggressively because the project is only implemented when the total investment reaches an exogenously given AoN target, leading to "loser's blessing" and failures of aggregating information from the crowd, relative to standard auction benchmarks. We focus on sequential investments in the presence of dynamic observational learning, and the gains in informational and financing efficiency are all benchmarked to standard settings outlined in Section 3.3.

4.4 Planner's Real Option

So far in our analysis we have required the planner to implement the project according to the AoN target. In some cases in reality, especially when the planner also learns about the project's promise from crowdfunding (not knowing the true V in our model), he commits to AoN in fundraising, but still holds the real option on how to use the capital and information aggregated. For example, an entrepreneur successful on Kickstarter or Indigogo can still decide on the scale of the project and how much effort to put into developing the product. On some crowdfunding platforms, the entrepreneur can decide whether to use the capital raised explicitly or implicitly (by postponing product development indefinitely, which results in refunding the agents). Xu (2017) and Viotto da Cruz (2016) provide strong empirical evidence that the planner indeed use the information aggregated from crowdfunding platforms for real decisions.

The real option embedded in the eventual investment often comes from the fact that crowdfunding is one way to learn about aggregate demand, which is obvious in reward-based platforms. Even for equity-based crowdfunding, agents reveals information on future product demand and profit. Similarly, in IPOs, firms unsuccessful at issuance may still find alternative sources of public financing. An IPO's initial pricing and trading also generates valuable information and feedback for managers. For example, van Bommel (2002) and Corwin and Schultz (2005) discuss information production at IPO through choices on pricing and underwriting syndicates.

In our baseline model, the planner's investment marginal cost ν is largely muted. One could imagine that ν is significant or there is also a fixed cost of investment for the planner. There could also be additional benefit to carrying out the project, such as the planner's private benefit of control or empire building. These forces distort the planner's ex post incentive on whether and how to implement the project. Other factors such as marketing, network effect, etc. also play a role.

Specifically, V can be interpreted as a transformation of the aggregate demand, which could be high ($V = 1$) or low ($V = 0$). Suppose that after the crowdfunding, the planner considers commercialization or abandoning the project (upon crowdfunding failure), and for simplicity the commercialization or continuation decision pays V (after normalization), but incurs an effort or reputation or monetary cost represented in reduced-form by I. Then the planner's expected payoff for the real option is

$$\max\{\mathbb{E}[V - I|H_N], 0\} \tag{20}$$

recall H_N is the entire crowdfunding history, including information on the total number of supports out of N agents, and when an UP-cascade starts if there is one, etc. For a given pricing and AoN target, the final amount raised is directly informative on the quality of the project V:

Proposition 9. *The posterior belief on V is increasing in the equilibrium support observed. Conditional on failing to reach AoN target, the planner more positively updates the belief with more supporting agents.*

Even with a successful crowdfunding, the planner may still choose to forgo commercialization if his belief on V after crowdfunding is not sufficiently optimistic; likewise, despite crowdfunding failure, the planner may continue pursuing the project. Our model further predicts that the sensitivity of the update on V based on incremental supports is smaller conditional on fundraising success (reaching AoN target), because it likely involves an UP-cascade and information aggregation is more limited.

Indeed, Xu (2017) documents in a survey of 262 unfunded Kickstarter planners that after failing, 33% continued as planned. He also finds that a 50% increase in pledged amount leads to a 9% increase in the probability of commercialization outside the crowdfunding platform, which indicates a why smaller sensitivity. It would be interesting to understand how the planner designs AoN and pricing to not only maximize profit from the crowdfunding, but also increase the real option value, which constitutes interesting future work.

5 Conclusion

Financial processes such as Internet-based crowdfunding and IPO underwriting involve aggregating information from diverse agents, sequential sales, observational learning, and most interestingly, all-or-nothing (AoN) rules that contingent the financing upon achieving certain fundraising targets. We incorporate these features into a classical model of information cascade, and find that AoN leads to uni-directional cascades in which agents rationally ignore private signals and imitate preceding agents only if the preceding agents decide to *invest*. Consequently, an planner prices issuance more aggressively, and information production also becomes more efficient, especially with a large crowd of agents. In general, financial technologies such as Internet-based funding platforms can help planners reach out to a greater agent base. But whether they can improve financing feasibility and better harness the wisdom of the crowd, as envisioned by the regulatory authorities, may depend on specific features and designs such as endogenous AoN targets, especially with sequential sales and informational frictions.

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Appendix

A Crowdfunding Platforms

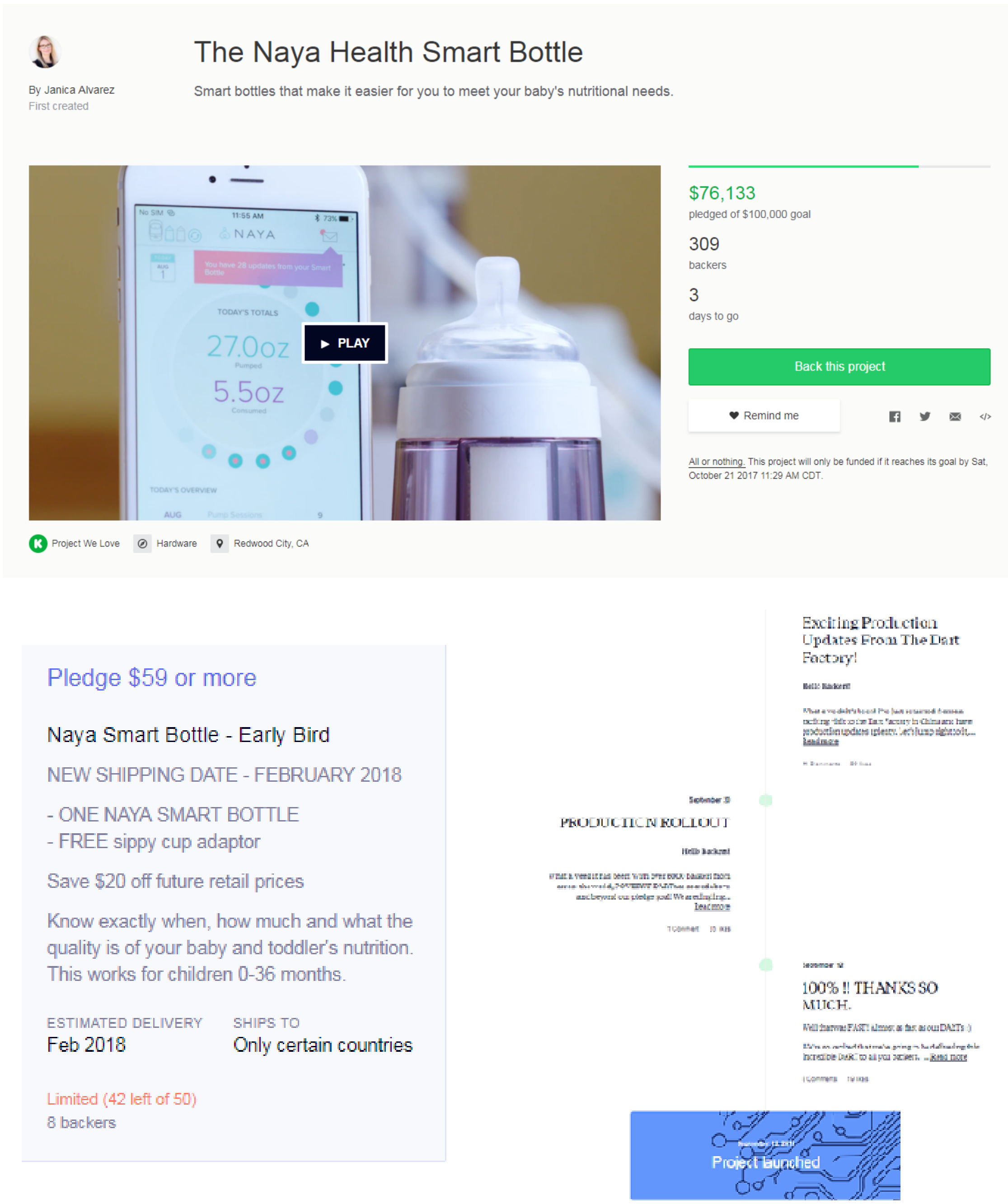

Figure 4: Example One: Kickstarter

Aside from all the details about the product, agent observes the target amount, fundraising start and end dates, pledged amount to date, and number of backers. They can also see a timeline of updates to the project (when it starts, factory production progress, etc.)

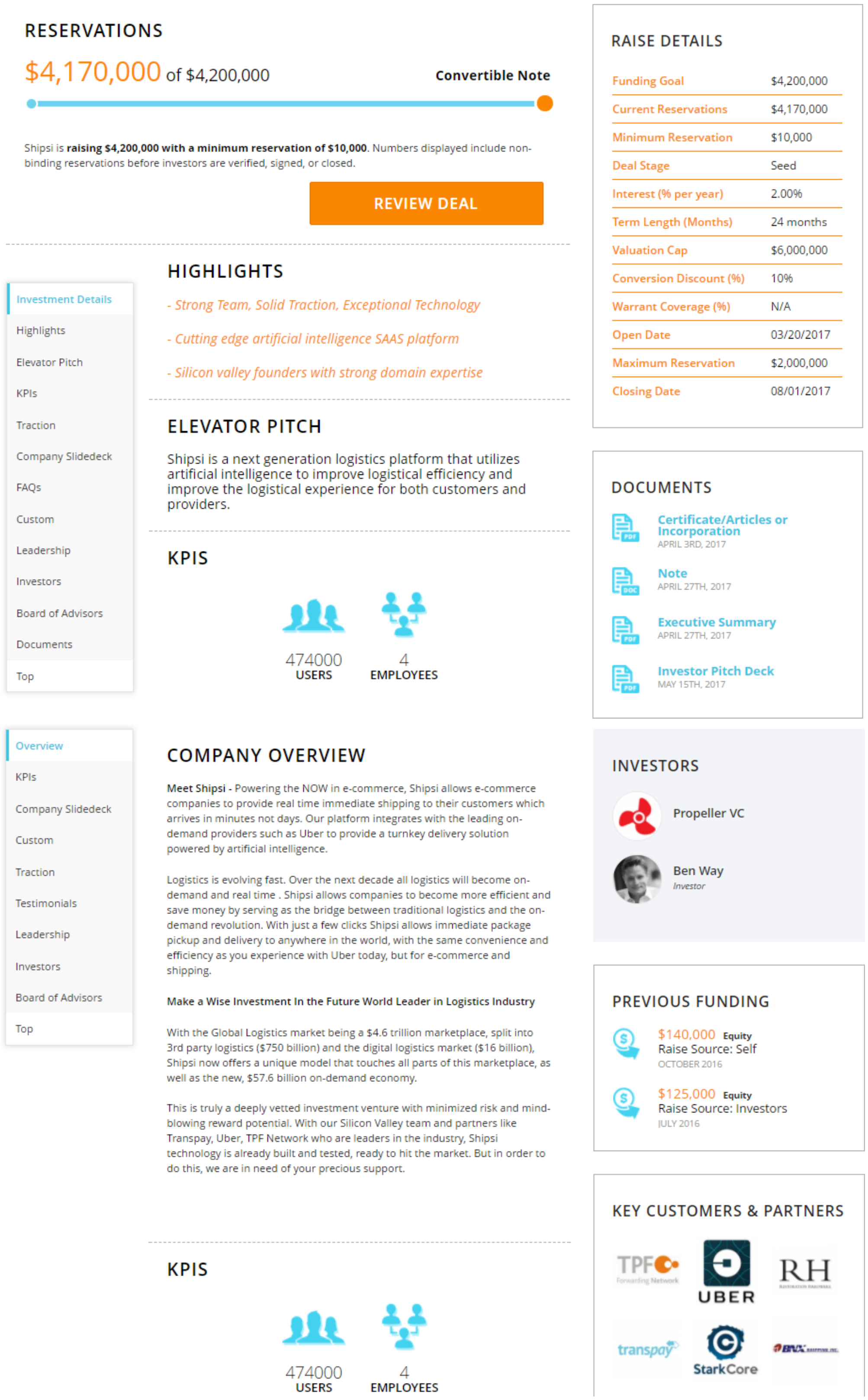

Figure 5: Example Two: Crowdfunder

Aside from all the details about the company including the equity investment contract, the companys previous funding, key customers and partners, and existing agents (only the VCs and the big players), agents also observe the target amount, fundraising start and end dates, reservation amount to date, etc.

B Derivations and Proofs

Proof of Lemma 1

Proof. Let k_n be the difference of numbers of H and L signals till the nth observation. For the prior, $k_0 = 0$, and $\frac{Pr(V=1)}{Pr(V=0)} = \frac{0.5}{0.5} = \frac{p^0}{q^0}$.

Suppose $\frac{Pr(V=1)|X}{Pr(V=0|X)} = \frac{p^{k_n}}{q^{k_n}}$ holds for $n \geq 0$, then for $n+1$:

1. If $X_{n+1} = H$, then $k_{n+1} = k_n + 1$, and

$$\begin{aligned}\frac{Pr(V=1|X)}{Pr(V=0|X)} &= \frac{\frac{Pr(X_{n+1}=H|V=1)Pr(V=1|X_1,X_2,\ldots,X_n)}{Pr(X_{n+1}=H)}}{\frac{Pr(X_{n+1}=H|V=0)Pr(V=0|X_1,X_2,\ldots,X_n)}{Pr(X_{n+1}=H)}} \\ &= \frac{Pr(X_{n+1}=H|V=1)p^{k_n}}{Pr(X_{n+1}=H|V=0)p^{k_n}} \\ &= \frac{p^{k_{n+1}}}{q^{k_{n+1}}};\end{aligned}$$

2. Similarly, if $X_{n+1} = L$, then $k_{n+1} = k_n - 1$, and

$$\begin{aligned}\frac{Pr(V=1|X)}{Pr(V=0|X)} &= \frac{\frac{Pr(X_{n+1}=L|V=1)Pr(V=1|X_1,X_2,\ldots,X_n)}{Pr(X_{n+1}=L)}}{\frac{Pr(X_{n+1}=L|V=0)Pr(V=0|X_1,X_2,\ldots,X_n)}{Pr(X_{n+1}=L)}} \\ &= \frac{Pr(X_{n+1}=L|V=1)p^{k_n}}{Pr(X_{n+1}=L|V=0)p^{k_n}} \\ &= \frac{p^{k_{n+1}}}{q^{k_{n+1}}};\end{aligned}$$

So $\frac{Pr(V=1|X)}{Pr(V=0|X)} = \frac{p^{k_{n+1}}}{q^{k_{n+1}}}$ holds for $n+1$. The lemma follows by induction. □

Proof of Proposition 1

Proof. When there are more than $T_N - 1$ supporting predecessors, if agent i supports, the project would be implemented for sure, so each agent is facing exactly the same problem as in Bikhchandani, Hirshleifer, and Welch (1992), and all agents behave as characterized by equation 3.3. So we only focus on the case when there are strictly less than $T_N - 1$ supporting predecessors. Let k_m be the minimal difference of numbers of H and L signals so that $E(V|k_m$ more H signals$) \geq m$. Without loss of generality we only consider the cases when $k_m \leq N$.

When agent i observes signal H, she receives 0 if she rejects the proposal. If she chooses support, then she only pays m when there is a follow agent j who makes a contribution to be the T_Nth supporting agent. The follow agent j only makes contribution when

$$E(V|H_{j-1}, X_j) \geq m.$$

Otherwise, the AoN target would not be reached and agent i does not pay m. So support weakly dominates rejection.

When agent i observes signal L, she receives 0 if she rejects the proposal. An UP cascade starts once there are $k_m + 1$ more H signals, because even if the agent currently making decision has private signal L, overall there are still k_m more H signals, and the weakly dominating strategy is to support. If there is no UP cascade yet and agent i deviates by choosing support, then all follow agents misinterpret her action and form the belief that agent i observes signal H. Then if follow agent j decides to be the T_Nth supporting agent, from agent i's perspective (she does not know agent j's private signal but her action) the posterior conditional on agent j's support decision is at best $E(V|k_m + 1 \text{ more supporting agents}) = E(V|k_m - 1 \text{ more } H \text{ signals}) < m$. The first term comes from the fact that there would an UP cascade once there are $k_m + 1$ more supporting agents, and $E(V|k_m - 1 \text{ more } H \text{ signals})$ is the true value because the fact that she has L means there are two less H than what others perceive in equilibrium. This is not a profitable action. Therefore it is optimal to reject instead. □

Proof of Proposition 3

Proof. The proof proceeds in two steps. We first show that the investment strategies in Proposition 3 is a sub-game equilibrium for a chosen price m^* and the corresponding AoN target T_N^* characterized by Equation (17). We then show that for any possible m^* in the equilibrium, the corresponding AoN target T_N^* characterized by Equation (17) is optimal.

Step 1: agent strategy with given AoN target

Given the price m^* and the corresponding AoN target T_N^* characterized by Equation (17). Let k_m be the minimal difference of numbers of H and L signals so that $E(V|k_m \text{ more } H \text{ signals}) \geq m^*$. Without loss of generality we only consider the cases when $k_m \leq N$. It is straightforward to see that $T_N^* - (N - T_N^*) = k_m$ when $k_m + N$ is even and $T_N^* - (N - T_N^*) = k_m - 1$ when $k_m + N$ is odd. An UP cascade starts once there are $k_m + 1$ more H signals, because even if the agent currently making decision has private signal L, overall there are still k_m more H signals, and the weakly undominated strategy is to support. When there is an UP cascade, because all subsequent agents would support, there must be more than T_N^* agents supporting the project and the AoN target is reached. If there is no UP cascade, the project will also be implemented when there are exactly T_N^* more supporting agents (if there are more than T_N^* supporting agents, then an UP cascade (recall our definition) starts at the latest at agent N).

Now consider agent $i \in \{1, 2, \ldots, N\}$, given investment strategies of other agents, if there is already an UP cascade before her turn, then the project will be implemented for sure and the conditional expected payoff given her private signal is weakly higher than $E(V|k_m \text{ more } H \text{ signals}) \geq m^*$. Her optimal decision is to support regardless of her private signal.

If there is no UP cascade yet and she chooses to support with a private H observation, then the project is implemented either when there is an UP cascade later or when there is no cascade but exactly T_N^* supporting agents.

1. When k_m+N is even, then with an UP cascade, the expected payoff is $E(V|k_m+1 \text{ more } H \text{ signals})-m^* > 0$. If there is no cascade later but exactly T_N^* supporting agents, the conditional expected payoff given her private signal is $E(V|T_N^*, N) - m^* \geq 0$.

2. When k_m+N is odd, similar to the even case, the expected payoff is $E(V|k_m+1 \text{ more } H \text{ signals}) - m^* > 0$ if an UP cascade arrives before agent N. Now consider the scenario when no UP cascade has yet arrived at agent $N-1$ and the project is implemented in the end. Since $T_N^* - (N-1-T_N^*) = k_m - 1 + 1 = k_m$, if there are less than T_N^* supporting agents until agent $N-1$, then there are at most k_m-2 more supporting agents and the project would not be implemented for sure. If there are more than T_N^* supporting agents, then there must be an UP cascade, a contradiction. Moreover, if agent $N-1$ chooses rejection, then there must be k_m+1 more supporting agents at agent $N-2$, suggesting an UP cascade. If no UP cascade has yet arrived at agent $N-1$ and the project is implemented in the end, then it must be the case that agent $N-1$ supports and there are exactly T_N^* supporting decisions from the first $N-1$ agents. Conditional expected payoff given her private signal is $E(V|k_m \text{ more } H \text{ signals}) - m^* \geq 0$. For agent N, of course she supports if and only if her private signal is good.

Therefore it is optimal to support.

If there is no UP cascade yet and $E(V|k_m-1 \text{ more } H \text{ signals}) < m^*$, and consider a deviation that she chooses to support with a private L observation, then the project is implemented either when 1) There is an Up cascade later or 2) There are exactly T_N^* supporting agents. In the first case, others interpret her action as having H signal on equilibrium, the conditional expected project payoff given her private signal is $E(V|k_m+1 \text{ more supporting agents}) = E(V|k_m-1 \text{ more } H \text{ signals}) < m^*$, where the first expression is the point of cascade start, and $E(V|k_m-1 \text{ more } H \text{ signals})$ is the true value because the fact that she has L means there are two less H than what others perceive in equilibrium. This is not a profitable action. Similarly, in the second case, the conditional expected payoff given her private signal is $E(V|T_N^*-1, N) - m^* < 0$. Therefore it is optimal to reject instead.

Step 2: Optimal AoN target

Notice that $m^* \geq E(V|1 \text{ less } H \text{ signal})$, since $m^* = E(V|1 \text{ less } H \text{ signal})$ guarantees an UP cascade from the very beginning and thus strictly dominates any $m < E(V|1 \text{ less } H \text{ signal})$. Let $T_N(m^*)$ be the target that satisfies $E(V|T_N(m^*), N) \leq m^* < E(V|T_N(m^*)+1, N)$.

When $m^* = E(V|1 \text{ less } H \text{ signal})$, the UP cascade starts from the first agent for sure, so any AoN target $T_N^* \leq N$ is optimal.

When $m^* > E(V|1 \text{ less } H \text{ signal})$, and $T_N^* = T_N(m^*)$: Following the proof in step 1, the project will be implemented whenever there is an UP cascade (that is to say, at some agents there are $k_m + 1$ more H signals), or when no UP cascades occur and there are exactly T_N^* supporting agents in total.

When $m^* > E(V|1 \text{ less } H \text{ signal})$, and $T_N^* > T_N(m^*)$: Since $T_N^* \geq T_N(m^*) + 1$, $T_N^* - (N - T_N^*) \geq T_N(m^*) + 1 - (N - T_N(m^*) - 1) \geq k_m + 1$. Suppose all agents choose the same investment strategies discussed in step 1. Because there would be an UP cascade once there are $k_m + 1$ more supporting agents, if there are no less than T_N^* supporting agents in total, then there would always be an UP cascade. That is to say, the project will be financed only when there is an UP cascade and the total number of supporting agents is at least T_N^*. Similar to the discussion in step 1, it is optimal for agents to support once an UP cascade starts. If there is no cascade yet and agent i chooses support, then the conditional expected project payoff given her private H signal is at least $E(V|k_m + 1 \text{ more supporting agents}) = E(V|k_m + 1 \text{ more } H \text{ signals}) > m^*$, and the conditional expected project payoff given her private L signal is $E(V|k_m + 1 \text{ more supporting agents}) = E(V|k_m - 1 \text{ more } H \text{ signals}) < m^*$. So agent i finds it optimal to choose support with a H observation and rejection with a L signal. Now consider the deviation to the same m^* but $T_N^* = T_N(m^*)$, each of the financing success scenario with UP cascades is still there, but the planner's strategy now strictly dominates $T_N^* > T_N(m^*)$ because it yields positive profit absent UP cascade but with exactly $T_N(m^*)$ supporting agents.

Finally, we have the case of $m^* > E(V|1 \text{ less } H \text{ signal})$, and $T_N^* < T_N(m^*)$, which is further divided into two sub-cases:

1. $m^* > E(V|0 \text{ more } H \text{ signals})$, and $T_N^* < T_N(m^*)$:

   Similar to previous discussions, the project is implemented whenever there is an UP cascade before the T_N^*th supporting decision. And agents choose the same investment decisions before the total number of supporting agents reaches T_N^*. For the pivotal agent who makes the T_N^*th support decision and all subsequent agents, because the project is implemented for sure if they invest, we are back to the standard cascade models without AoN. Now we show that any $T_N^* < T_N(m^*)$ is dominated by $T_N^* + 1$, using the following arguments.

   (a) For any $T_N^* < T_N(m^*)$, let agent i be the T_N^*th agent observing a good signal. Then she invests only if there are at least $k_m - 1$ more supporting actions than rejection actions before her. If there are at least k_m more supporting actions than rejection actions before agent i, then there must already be an UP cascade before the next agent, and thus the project can also be implemented with $T_N^* + 1$ as the AoN target (note $T_N^* < N$) because the planner profit is the same. Without loss of generality, we focus on the paths when there are exactly k_m more H signals at agent i with an UP cascade.

   (b) Given equation (17) and $T_N^* < T_N(m^*)$, it must be $i \leq N - 2$ by the definition of $T_N(m^*)$. Consider agent $i+1$, if she observes a good signal, then in both T_N^* and $T_N^* + 1$ cases she chooses support and the project would be implemented, and planner receives the same profit. If agent $i+1$ observes signal L, she rejects in both cases because including her own signal, there are only $k_m - 1$ more H signals. Conditional on the rejection at agent $i+1$, agent $i+2$ chooses support if and only if her signal is good in both cases. For the support case the project would also be implemented with $T_N^* + 1$ target and planner receives the same profit. However, when agent $i+2$ chooses rejection, it becomes a DOWN cascade and the project would be abandoned for sure. When there are $k_m - 1$ more good signals before agent i, and agents i, $i+1$ and $i+2$ observe H, L and L, respectively, we call this path HLL, and it is the only path along which T_N^* target dominates $T_N^* + 1$ target.

(c) To show $T_N^* + 1$ target dominates T_N^* in expectation, it suffices to consider the path LHH, which refers to the scenario that there are $k_m - 1$ more good signals before agent i, agents i, $i+1$ and $i+2$ observe L, H and H, respectively. With $T_N^* + 1$, this path meets AoN target, because agent $i+1$ with H signal invests, knowing that she only needs to pay if agent $i+2$, or a subsequent agent also has H signal and supports; with T_N^*, agent $i+1$ rejects even with H because she is the T_N^*th agent and she has to pay if she supports, yet there are only $k_m - 1$ more H signals (including hers). Conditional on there are $k_m - 1$ more good signals before agent i, when $k_m \geq 1$, the probability of LHH case is larger than the probability of HLL. Also notice that HLL suggests a DOWN cascade at agent $i+2$, so the expected profit of LHH is higher than the expected profit of HLL.

2. $m^* = E(V|0 \text{ more } H \text{ signals})$ (which equals $\frac{1}{2}$ in our setup) and $T_N^* < T_N(m^*)$:

Similar to the $m^* > E(V|0 \text{ more } H \text{ signals})$ case, T_N^* target only dominates $T_N^* + 1$ target along the HLL path. Let $Q_{T_N^*}$ be the event that there is no UP cascade yet and at the T_N^*th supporting agent there are exactly equal numbers of supporting and rejecting agents (that is to say, the T_N^*th supporting agent is the $2T_N^*$th agent). Let $U_{2T_N^*+1}$ be the event that the UP cascade arrives at the $2T_N^* + 1$th agent. Event $U_{2T_N^*+1}$ happens if and only if $Q_{T_N^*}$ happens and the $2T_N^* + 1$th agent observes a good signal, because by this point AoN target is already met and we are back to the standard cascade setting. Based on Lemma 4 for the case of $k = 1$, we have:

$$
\begin{aligned}
P(Q_{T_N^*}) =& \frac{1}{2p} P(U_{2T_N^*+1}|V=1) + \frac{1}{2q} P(U_{2T_N^*+1}|V=0) \\
=& \frac{1}{2T_N^*+1} \begin{pmatrix} 2T_N^*+1 \\ T_N^*+1 \end{pmatrix} (pq)^{T_N^*},
\end{aligned}
$$

and the expected profit from HLL path (implementable with T_N^* but not with $T_N^* + 1$) is:

$$
E_{HLL} \equiv m^* T_N^* P(Q_{T_N^*}) P(LL \textit{ for } 2T_N^*+1 \textit{ and } 2T_N^*+2) = \frac{1}{2} T_N^* P(Q_{T_N^*}) \frac{p^2+q^2}{2}.
$$

Similarly, let $Q_{T_N^*+1}$ be the event that there is no UP cascade yet and at the T_N^*+1th supporting agent there are exactly equal numbers of supporting and rejecting agents (that is to say, the (T_N^*+1)th supporting agent is the $(2T_N^*+2)$th agent). When the target is T_N^*+1, the probability that the project would be implemented with the T_N^*+1 target but fails in the T_N^* target (since the T_N^*th H signal agent behave differently given different AoN target) is:

$$P_1 \equiv P(Q_{T_N^*+1}) - P(Q_{T_N^*})pq,$$

where the second term is the case that the event $Q_{T_N^*}$ realizes and agent $i+1$ and $i+2$ observe L and H, respectively. Note that $Q_{T_N^*+1}$ indicates the T_N^*+1th supporter sees equal number of supporting and rejection actions (including her own), thus HLH meetings both funding target T_N^*+1 and T_N^* with the same payoff to the planner.

The ratio of the expected profit from HLL path that meets T_N^* but not T_N^*+1, to that from paths implemented with T_N^*+1 target but not T_N^* is:

$$\frac{E_{HLL}}{m^*(T_N^*+1)P_1} = \frac{(p^2+q^2)(T_N^*+2)}{6pq(T_N^*+1)} \leq \frac{p^2+q^2}{4pq},$$

where the last inequality comes from the fact that $T_N^* \geq 1$. Since $p^2+q^2+2pq = (p+q)^2 = 1$, $\frac{p^2+q^2}{4pq} < 1$ is equivalent to $pq > \frac{1}{6}$. So when $pq > \frac{1}{6}$, any $T_N^* < T_N(m^* = \frac{1}{2})$ is strictly dominated by T_N^*+1.

When $pq \leq \frac{1}{6}$, we have $p \geq \frac{1}{2} + \frac{\sqrt{3}}{6} > \frac{3}{4}$. We now show that any target $T_N^* < T_N(\frac{1}{2})$ is strictly dominated by alternative strategy $m^* = p$ (so $k_m = 1$) and AoN target T_N^*+1. For $m^* = \frac{1}{2}$ and AoN target $T_N^* < T_N(\frac{1}{2})$, we have shown earlier that the project would be implemented either there is an UP cascade before/at agent $2T_N^*-1$ or there is no UP cascade before $2T_N^*$ but the $2T_N^*$th agent is the T_N^*th supporting agent. It suffices to show that in either scenario, the alternative strategy fares better for the planner.

(a) When there is an UP cascade before $2T_N^*$, consider the case that right after the cascade the next agent observes H signal and support. This would also result in an UP cascade for $(m^* = p, T_N^* + 1)$ and the same number of supporting agents. The conditional probability that the next agent observes H is $E(V=1|1 \text{ more } H \text{ signals})p + E(V=0|1 \text{ more } H \text{ signals})q = p^2+q^2 = 1-2pq \geq \frac{2}{3}$. For the case $(m^* = p, T_N^*+1)$, for each contribution the planner charges p instead of $\frac{1}{2}$. The planner receives higher expected payoffs from UP cascades because $p(p^2+q^2) > \frac{1}{4}(p^2+q^2) \geq \frac{1}{2}$.

(b) When there is no UP cascade before $2T_N^*$ but the $2T_N^*$th agent is the T_N^*th supporting agent (event $Q_{T_N^*}$), consider two corresponding scenarios in $(m^* = p, T_N^*+1)$: (i) Event $Q_{T_N^*}$ happens and the next agent observes H and support; (ii) There is no UP cascade (corresponding to

$m^* = p$, that is to say, $k_m + 1 = 2$) yet, but there is one more supporting agent by (and including) the $2T_N^* - 1$th agent, and the $2T_N^*$th and $2T_N^* + 1$th agents observe L and H, respectively.

In both cases, funding target $T_N^* + 1$ is met and there are at least the same number of supporting agents as in $Q_{T_N^*}$. For (i), conditional on there are equal number of supporting and rejecting agents at $2T_N^*$, the conditional probability that the next agent observes H is $E(V = 1|0 \text{ more } H \text{ signals})p + E(V = 0|0 \text{ more } H \text{ signals})q = \frac{1}{2}$. For (ii), similar to the discussion on $P(Q_{T_N^*})$, the probability of scenario (ii) is:

$$\frac{1}{2T_N^*}\begin{pmatrix} 2T_N^* \\ \frac{2T_N^*+2}{2} \end{pmatrix}(pq)^{T_N^*} = \frac{1}{2}P(Q_{T_N^*}).$$

The probability that either (i) or (ii) happens equals $P(Q_{T_N^*})$, and in either case there are at least the same number of supporting agents paying $p > \frac{1}{2}$. So for $(m^* = p, T_N^* + 1)$ the planner receives more payoffs when there is no UP cascade before $2T_N^*$. Thus the planner is strictly better off with strategy $(m^* = p, T_N^* + 1)$.

In conclusion, $T_N^* = T_N(m^*)$ is the planner's weakly dominant strategy. □

Proof of Proposition 4

Proof. When $m^* = E(V|1 \text{ less } H \text{ signal})$, an UP cascade starts from the beginning and the project will be implemented for sure. When $m^* > E(V|1 \text{ less } H \text{ signal})$, an UP cascade starts once there are $k_m + 1 \geq 1$ more H signals. Then for arbitrary positive integer $k_m + 1$, the starting time of an UP cascade is equivalent to the standard gambler's ruin problem with asymmetric simple random walk. Because when $V = 1$, $Pr(X = H|V = 1) = p > q$, it is known that (Feller (1968), page 348 equation 2.8):

$$Pr(\text{an UP cascade starts at some finite time}) = 1.$$

□

Proof of Lemma 2

Proof. For agent 1, her posterior after observing H is $E(V|X_1 = H) = p$. If $m > p$, then agent 1 chooses rejection regardless of her private signal and a DOWN cascade starts from the beginning for sure.

Since $m = 1 - p = E(V|1 \text{ less } H \text{ signal})$ will induce an UP cascade starting form the beginning for sure, the planner has no incentive to charge $m < 1 - p$. Combine with $m \leq p$ we have $m \in [1 - p, p]$. Also, for each $m \in (m_{k-1}, m_k]$, $m = m_k$ induces exactly the same number of supporting agents, so in the equilibrium planner always charges $m^* = m_k$ for some $k \in \{-1, 0, 1, \ldots, N\}$. Without loss of generality, only three prices are possible: $m_{-1} = 1-p$, $m_0 = \frac{1}{2}$ and $m_1 = p$. Let $S(m, N)$ be the expected profit when the price is m and there are $N \geq 2$ potential agents. It is clear that $S(m, N)$ is increasing in N.

$m = 1-p$: The total profit is $(1-p)N$;

$m = \frac{1}{2}$: After the first two observations, LL induces a DOWN cascade, HL and HH both induce an UP cascade at agent 1, and LH does not change the belief. The expected profit is $S(m,N) = \frac{p+q}{2}\frac{1}{2}N + \frac{qp+pq}{2}(\frac{1}{2} + S(m, N-2)) \leq \frac{1}{4}N + pq(\frac{1}{2} + S(m,N))$. Thus $m = \frac{1}{2}$ is dominated by $m = 1-p$ if:

$$S(m,N) \leq \frac{\frac{N}{4} + \frac{pq}{2}}{1-pq} \leq (1-p)N \ for \ N = 2, 3, \ldots \tag{21}$$

One can verify that the inequality holds for $p \in (\frac{1}{2}, \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})]$;

$m = p$: After the first two observations, HH induces an UP cascade, LL and LH both induce a DOWN cascade at agent 1, and LH does not change the belief. The expected profit is $S(m,N) = \frac{p^2+q^2}{2}pN + \frac{qp+pq}{2}(p + S(m, N-2)) \leq \frac{p^2+q^2}{2}pN + pq(p + S(m,N))$. Thus $m = p$ is dominated by $m = 1-p$ if:

$$S(m,N) \leq \frac{\frac{p^2+q^2}{2}pN + p^2q}{1-pq} \leq (1-p)N \ for \ N = 2, 3, \ldots \tag{22}$$

One can verify that the inequality holds for $p \in (\frac{1}{2}, \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})]$. □

Proof of Lemma 4

Proof. Since an UP cascade starts once there are k more H signals. Exactly k more H signals at agent i implies $\frac{i-k}{2}$ L signals and $\frac{i+k}{2}$ H signals. The number of L signals suggests that $i \geq K$, and the number of H signals implies that $i+k$ must be even. There are $C_i^{\frac{i+k}{2}}$ different potential paths to reach exactly k more H signals, and for each path, the possibility is $p^{\frac{i+k}{2}}q^{\frac{i-k}{2}}$ conditional on $V = 1$ and $q^{\frac{i+k}{2}}p^{\frac{i-k}{2}}$ conditional on $V = 0$. Then:

$$Pr(\text{exactly } k \text{ more } H \text{ signals at agent } i) = \begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} (pq)^{\frac{i-k}{2}} \frac{p^k + q^k}{2}$$

By the reflection principle and Lemma 3 one can infer that $\varphi_{k,i} = \frac{k}{i} Pr(\text{exactly } k \text{ more } H \text{ signals at agent } i)$. That is:

$$\varphi_{k,i} = \frac{k}{i} \begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} (pq)^{\frac{i-k}{2}} \frac{p^k + q^k}{2}.$$

□

Proof of Proposition 5

Proof. For $m = m_{-1} = 1-p$, the project will be financed for sure. For $m = m_{k-1}$ $k \in \{1, 2, \ldots, N\}$, an UP cascade starts once there are k more supporting agents. When an UP cascade occurs at agent i, all subsequent agents support the project and the financing is successful, there would be in total $N - \frac{i-k}{2}$ supporting agents, and each contributes $m = m_{k-1}$. An UP cascade occurs only when $i+k$ is even. If $N+k$ is odd and there is no UP cascade yet, then the project may still reach the AoN target if there are exactly $k-1$ more supporting agents at agent N. Suppose there is one more round $N+1$, then an UP cascade starts at agent $N+1$ if and only if there are exactly $k-1$ more supporting agents at agent N and agent $N+1$ observes H. That is to say, when $k+N$ is odd, the probability that there is no UP cascade and the project reaches the AoN target is $\frac{p^k+q^k}{p^{k+1}+q^{k+1}}\varphi_{k,N+1}$, and there would be $\frac{N+k-1}{2}$ supporting agents in total. Similarly, if $N+k$ is even and there is no UP cascade until agent $N-1$ yet, then the project may still reach the AoN target if there are exactly $k-1$ more supporting agents at agent $N-1$. The event can be further decomposed into two parts. The first event is that the UP cascade starts at agent N, and the corresponding probability is $\frac{p^{k-1}+q^{k-1}}{p^k+q^k}\varphi_{k,N}$, and there would be $\frac{N+k}{2}$ supporting agents in total. The second event is that there is no UP cascade and there are exactly T^*_N supporting agents at agent $N-1$ (so the last agent observes L and rejects), and the corresponding probability is $\frac{qp^{k-1}+q^{k-1}p}{p^k+q^k}\varphi_{k,N}$, and there would be $\frac{N+k-2}{2}$ supporting agents in total.

To show the existence of $\underline{N}(k)$, we first prove the existence of $\underline{N}(0)$, then proceed to the $k \geq 1$ case. $\pi(m_{-1}, N) = (1-p-\nu)N$. When $m = m_0 = \frac{1}{2}$, an UP cascade starts once there are more than 1 H signals. From standard Gambler's ruin problem we know that the conditional probability that an UP cascade occurs at sometime is 1 if $V=1$, and $\frac{q}{p}$ if $V=0$ (Feller (1968), page 348 equation 2.8). Because $pq = p(1-p) < \frac{1}{4}$, we have:

$$\begin{aligned}(m_0-\nu)(Pr(V=1)+Pr(V=0)\frac{q}{p}) &= (\frac{1}{2}-\nu)(\frac{1}{2}+\frac{1+\frac{1-p}{p}}{2})\\ &= (\frac{1}{2}-\nu)\frac{1}{2p}\\ &> 1-p-\nu\\ &= m_{-1}.\end{aligned}$$

Since $\varphi_{0,i}$ is strictly positive, there exists a strictly positive integer $N_1(0)$ such that:

$$(m_0-\nu)\sum_{i=1}^{N_1(0)}\varphi_{0,i} > 1-p-\nu.$$

Let $D = (m_0-\nu)\sum_{i=1}^{N_1(0)}\varphi_{0,i} - (1-p-\nu) > 0$, $Q = (m_0-\nu)\sum_{i=1}^{N_1(0)}\varphi_{0,i}\frac{i}{2}$, and $\underline{N}(0)$ be the smallest integer that is larger than $\max\{N_1(0), \frac{Q}{D}\}$. Then for any $N \geq \underline{N}(0)$:

$$\begin{aligned}
\pi(m_0, N) &\geq (m_0 - \nu) \sum_{i=1}^{\underline{N}(0)} \varphi_{0,i}(N - \frac{i}{2}) \\
&= N(m_0 - \nu) \sum_{i=1}^{\underline{N}(0)} \varphi_{0,i} - Q \\
&\geq \frac{Q}{D} D + (1 - p - \nu)N - Q \\
&= (1 - p - \nu)N.
\end{aligned}$$

Now consider the case $k \geq 1$ (when $\nu \leq m_k$). When the price is m_{k-1}, an UP cascade starts once there are more than k H signals. It occurs once there are more than $k+1$ H signals when the price is m_k. For both cases, the conditional probability that an UP cascade occurs at sometime is 1 if $V = 1$. When $V = 0$, the conditional probability that an UP cascade occurs at sometime is $\frac{q^k}{p^k}$ for m_{k-1} and $\frac{q^{k+1}}{p^{k+1}}$ for m_k, respectively.

For each $k \geq 1$, and the time i arrival rate $\varphi_{k,i}$, there exists a corresponding $\varphi_{k+1,i+1}$ for price m_k. For each i, we have:

$$\begin{aligned}
\frac{(m_k - \nu)\varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}} \geq \frac{(m_k - \nu)\varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}} &= \frac{m_k \frac{k+1}{i+1} \frac{(i+1)!}{\frac{i+k+2}{2}!\frac{i-k}{2}!}(pq)^{\frac{i-k}{2}} \frac{p^{k+1}+q^{k+1}}{2}}{m_{k-1} \frac{k}{i} \frac{i!}{\frac{i+k}{2}!\frac{i-k}{2}!}(pq)^{\frac{i-k}{2}} \frac{p^k+q^k}{2}} \\
&= p\frac{k+1}{k} \frac{i}{\frac{i+k}{2}+1}(1 + \frac{(pq)^{k-1}(p-q)^2}{(p^k+q^k)^2}).
\end{aligned}$$

Since $\lim_{i\uparrow\infty} p\frac{i}{\frac{i+k}{2}+1} = 2p > 1$, for each k, the ratio $\frac{m_k\varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}}$ is monotonically increasing in i and there exists an integer N_1 that $\frac{m_k\varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}} \geq 1$ whenever $i \geq N_1$.

Because

$$\begin{aligned}
(p^{k+1} + q^{k+1})(p^{k-1} + q^{k-1}) &= p^{2k} + q^{2k} + p^{k+1}q^{k-1} + p^{k-1}q^{k+1} \\
&= p^{2k} + q^{2k} + p^{k-1}q^{k-1}(p^2 + q^2) \\
&> p^{2k} + q^{2k} + p^{k-1}q^{k-1}(2pq) \\
&= (p^k + q^k)^2.
\end{aligned}$$

We have

$$\begin{aligned}
\lim_{N\uparrow\infty}(m_k-\nu)\sum_{i=1}^{N-1}\varphi_{k+1,i+1} &= (m_k-\nu)(\frac{1}{2}+\frac{\frac{q^{k+1}}{p^{k+1}}}{2}) \\
&= \frac{m_k-\nu}{m_k}\frac{1}{2}\frac{p^k}{p^k+q^k}\frac{p^{k+1}+q^{k+1}}{p^{k+1}} \\
&= \frac{m_k-\nu}{m_k}\frac{1}{2p}\frac{p^{k+1}+q^{k+1}}{p^k+q^k} \\
&> \frac{m_k-\nu}{m_k}\frac{1}{2p}\frac{p^k+q^k}{p^{k-1}+q^{k-1}} \\
&= \frac{m_k-\nu}{m_k}m_{k-1}(\frac{1}{2}+\frac{\frac{q^k}{p^k}}{2}) \\
&\geq (m_{k-1}-\nu)(\frac{1}{2}+\frac{\frac{q^k}{p^k}}{2}) \\
&= \lim_{N\uparrow\infty} m_{k-1}\sum_{i=1}^{N}\varphi_{k,i}.
\end{aligned}$$

Given $\lim_{N\uparrow\infty}(m_k-\nu)\varphi_{k+1,i+1}\downarrow 0$, there exists an integer $N_2\geq N_1$ such that:

$$D\equiv(m_k-\nu)\sum_{i=1}^{N_2-1}\varphi_{k+1,i+1}-(m_{k-1}-\nu)\sum_{i=1}^{N_2}\varphi_{k,i}-(m_{k-1}-\nu)\tfrac{pq^{k-1}}{p^k+q^k}\varphi_{k,N_2}-(m_{k-1}-\nu)\tfrac{p^k+q^k}{p^{k+1}+q^{k+1}}\varphi_{k,N_2+1}>0$$

Let $Q\equiv(m_{k-1}-\nu)\sum_{i=1}^{N_2}\varphi_{k,i}\frac{i-k}{2}-(m_k-\nu)\sum_{i=1}^{N_2-1}\varphi_{k+1,i+1}\frac{i-k}{2}$. Then for each k, let $\underline{N}(k)$ be the smallest integer that is larger than $\max\{N_2,\frac{Q}{D}\}$. Then for any $N\geq\underline{N}(0)$:

$$\begin{aligned}
\pi(m_k,N)-\pi(m_{k-1},N) >&\pi(m_k,\underline{N}(k))-\pi(m_{k-1},\underline{N}(k)) \\
>&\underline{N}(k)(m_k-\nu)\sum_{i=1}^{N_2-1}\varphi_{k+1,i+1}-(m_{k-1}-\nu)\tfrac{p^k+q^k}{p^{k+1}+q^{k+1}}\varphi_{k,\underline{N}(k)+1}\tfrac{\underline{N}(k)+k-1}{2} \\
&-(m_{k-1}-\nu)\sum_{i=1}^{N_2}\varphi_{k,i}-Q-(m_{k-1}-\nu)\tfrac{p^{k-1}q+pq^{k-1}}{p^k+q^k}\varphi_{k,\underline{N}(k)}\tfrac{\underline{N}(k)+k-2}{2} \\
>&\underline{N}(k)D-Q \\
\geq&\frac{Q}{D}D-Q \\
=&0.
\end{aligned}$$

□

Proof of Proposition 6

Proof. First, suppose agent i observes H information, she has no incentive to deviate. If she chooses rejection or waiting, then all follow agents misinterpret her action and update their beliefs as if i observes L. This results in failures for some project that should be financed if i correctly reveals her information.

If agent i observes L, as we discussed in the baseline model, if there is an UP cascade she chooses to invest. When there is no UP cascade yet, she has no incentive to invest, and waiting is a weakly dominating strategy since she can always reject latter. Thus her first action still reveals her information. □

Proof of Proposition 9

Proof. Given N, m^*, and T_N^*, we show that an planner's posterior belief on V is indeed increasing in the total amount raised.

We first note that before entering a cascade, the number of supporting agents equals the number of H signals. We use n to denote the the first n agents, and h the number of H signals up to that point.Then $k = 2h - n$.

Case1: $N + k_m$ is even (which implies $k_m = 2T_N^* - N$).

Before or right at reaching the AoN target, $h \leq T_N^*$. We get k is at most $2T_N^* - N = k_m$, there is no cascade yet. k is increasing in h and $k = k_m$ when $h = T_N^*$. The posterior of V according to equation (7) is thus increasing in the number of supporters. This means if a project is not financed or barely reaches the AoN target, the planner learns most substantially from the fundraising outcome about the true type of V.

After reaching the AoN target, $k > 2(T_N^* + 1) - N = k_m + 2$, a cascade must have started at the last agent or earlier. Since no information is accumulated during cascade, $k = k_m + 1$. This implies that $\mathbb{E}[V] = \mathbb{E}[V|k = k_m + 1]$ is flat for all $h > T_N^*$. Therefore, for projects that exceed the AoN target by a large margin, the planner would not significantly positively update the belief on V beyond $\mathbb{E}[V|k = k_m + 1]$.

Case2: $N + k_m$ is odd (which implies $k_m + 1 = 2T_N^* - N$).

Before or right at reaching the AoN target, $h < T_N^*$. We get k is at most $2(T_N^* - 1) - N = k_m - 1$, there is no cascade yet. k is increasing in h and $k = k_m - 1$ when $h = T_N^* - 1$. The posterior of V according to equation (7) is thus increasing in the number of supporters. This means if a project is not financed, the planner learns most substantially from the fundraising outcome about the true type of V.

After reaching the AoN target, $k \geq 2(T_N^* + 1) - N = k_m + 1$, a cascade must have started at the last agent or earlier. Since no information is accumulated during cascade, $k = k_m + 1$. This implies that $\mathbb{E}[V] = \mathbb{E}[V|k = k_m + 1]$ is weakly increasing for $h \geq T_N^*$.

Taking all these into consideration, we conclude that the posterior of V is weakly increasing in total amount of supports observed (not necessarily received by the planner). The sensitivity of the posterior belief on the total support is greater when the fundraising actually fails. $\square$