Surender Contagion in Life Insurance: Modeling and Valuation[1]

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Abstract

This paper incorporates contagious surrender behavior into the valuation and risk management of participating life insurance contracts, allowing for structural default of the insurance company. The insurance pool features a financially sophisticated (professional) policyholder and many retail (non-professional) policyholders. A surrender-history-dependent intensity process is introduced to capture the non-professionals' contagious surrender behavior. While contagion aligns the non-professionals' surrender behavior closer to the optimal surrender of the professional, it jeopardizes the non-professionals' financial position in favor of equity holders as a strict regulatory intervention or a riskier investment strategy is imposed.

Keywords: Contagion, Bounded Rationality, Insurance Regulation, Early Default.

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1 Introduction

A mass surrender, which features a temporary and drastic raise of surrender rates, and can be regarded as an equivalent "bank run" event in the insurance sector, namely an insurance run, has been empirically widely documented. Brewer, Mondschean, and Strahan (1993) recorded the bankruptcy of two US life insurers, First Executive Corporation and Travelers Corporation, caused by liquidity shortages due to $900 millions insurance policy terminations after the two insurers announced their investment losses in 1990. And one year later, the Mutual Benefit Life Insurance Company was seized by New Jersey regulators after $500 millions insurance policy runs over three months. Moreover, Brown and Balasingham (2013) also studied life insurers' failures and their causes, for example, Equitable Life (UK, 2000), Mannheimer Insurance (Germany, 2004), and Ethias (Belgium, 2008), and discovered that these insurers all experienced surrender spikes after they suffered from some financial difficulties. Additionally, in the aftermath of the Asian crisis, twelve Korean life insurers failed over liquidity problems caused by massive surrenders during the period 1998-2002 (Brown and Balasingham, 2013).

A mass surrender imposes a significant risk to insurance companies, and the discussion on triggers of a mass surrender has attracted much attention in insurance literature. Undoubtedly, changes on financial markets (e.g., raises in interest rates, etc.) and changes on insurers' financial performance affect policyholders' surrender decision making. By assuming that policyholders are capable of analyzing those changes and react to them by terminating their contract, a mass surrender can occur. However, in practice policyholders are not always timely reacting to such changes, see the inclusion of limited monetary rationality into the insurance policy surrender literature by De Giovanni (2010) and Bernard and Lemieux (2008). The CEIOPS[1] emphasized this issue by pointing out a difference in surrender risk for heterogeneous policyholders. For example, well-informed institutional policyholders react quicker to financial changes, which results in better surrender timing compared to non-professional policyholders. Given that institutional policyholders take up a small fraction of a policyholder pool, apart from panics which are the cause of bank runs and can be borrowed to explain insurance runs, the occurrence of a mass surrender can also imply a surrender-herding phenomenon.[2] Here, we summarize panic surrender behavior and surrender-herding behavior as contagious surrender behavior,[3] which has

[1]The CEIOPS refers to the Committee of European Insurance and Occupational Pensions Supervisors. It was replaced by the European Insurance and Occupational Pensions Authority (EIOPA) since 2011.

[2]Liu (2015) provides empirical evidence for insurance purchase herding behavior of policyholders in insurance markets. Duflo and Saez (2002, 2003) document herding behavior in retirement planning decisions both experimentally and with field data.

[3]The concept of contagion is used with different definitions in economics literature. For example, Morris

not been well studied in insurance literature, in particular in insurance pricing literature.

In the present paper, we introduce a life insurance pricing model with contagious surrender behavior among a heterogeneous pool of policyholder. To be specific, we consider two groups of policyholders in the heterogeneous pool: The first group of policyholders, whose contracts are handled by a financially professional agent, for example a broker managing his or her clients' policies,[4] are labeled "professionals". The second group of policyholders, whom we label "non-professionals", deal with their contract individually and when they are left alone to make a surrender decision, they only surrender their contract for exogenous reasons, for example, financial liquidity problems, that is, they use their policy as an emergency fund (Outreville, 1990). While professional policyholders fully use their informational advantage to maximize their contract's monetary value by surrendering optimally, non-professional policyholders, apart from terminating their contract early for dealing with their own financial problems, observe past surrenders by other policyholders on the market and may hurry to do the same if their impression on past surrenders is sufficiently salient. In this paper, we model non-professional policyholders' contagious surrender behavior from the perspective of insurers by specifying a counting process with a surrender-history-dependent intensity process. Specifically, we consider that past surrenders on the market first need to accumulate to a certain level to draw non-professional policyholders' attention to other policyholders' actions. Second, as in practice it is not revealed to insurers that past surrenders accumulating to a certain level guarantees a mimicking surrender action of non-professionals, a binary random variable with the function of indicating the burst of their mimicking surrender actions is included into our surrender-history-dependent intensity process. Past surrenders alter non-professional policyholders' surrender behavior in favor of terminating their contract earlier, resulting in an increase in surrender intensity. The more salient past surrenders are, the higher the surrender intensity becomes. Moreover, as the memory of non-professional policyholders for past surrenders fades over time, our surrender-history-dependent intensity process also allows non-professionals' surrender behavior to calm down along time, converging to a constant surrender reversion level again.

Accordingly, we price life insurance contracts for the two types of policyholders, namely professionals and professionals, from the perspective of insurers. Since professional policyholders can optimally terminate their contract, the surrender option carries the

(2000) uses contagion to describe synchronization of strategies in games. By summarizing panic surrender behavior and surrender-herding behavior as contagious surrender behavior, we have contagion defined as actions which trigger further actions.

[4]Another example is a professional insurance dealer who purchases policies of policyholders on life settlement markets, see Gatzert (2010) and Hilpert, Li, and Szimayer (2014).

interpretation of an American option, and their contract value is then given by solving an optimal stopping problem. However, for non-professional policyholders, after they observe that surrenders on the market accumulate to a certain level, once they start mimicking those decisions, they obtain surrender benefits, and their contract value is then given as the expectation of discounted contingent payoffs. Moreover, we allow for structural default of the insurance company caused by regulatory authorities' intervention, which is imposed to reserve a certain amount of capital in the insurance company for protecting policyholders. We numerically study the impacts of early default regulation and the insurer's investment strategy on contract valuation in the setting where contagious surrenders are triggered by professionals' actions on the market, while in general, our model also allows for non-professionals to be self-excited, which then forces professionals to adapt their surrender behavior. We find that following up professional policyholders' optimal surrender decisions does not necessarily improve non-professional policyholders' financial position, and it does not for sure lead to a higher default risk for the insurer neither. Regulatory authorities' frequent-enough intervention and the insurer's risky-enough investment can stimulate professional policyholders' surrenders at fairly high asset values of the insurance company. By paying out relatively low surrender benefits to the professionals, the company's asset value per policyholder gets actually pushed upward, which consequently lowers the company's default risk and boosts the contract value. Therefore in this case, it is actually beneficial for non-professional policyholders to stay in the insurance pool.

The only work, as far as we are aware, that models contagious surrender behavior of policyholders is Barsotti, Milhaud, and Salhi (2016). Following the idea of extending the Hawkes (1971)-process in Dassios and Zhao (2011), the authors introduce a mathematical framework which embeds the dependence of policyholders' surrender behavior on both macroeconomic conditions and other policyholders' surrenders in a stochastic surrender intensity contagion process with external market-driven and internal self-excited jumps. In their paper, policyholders are assumed to be homogeneous, and by using internal self-excited jumps in the surrender intensity process, policyholders' surrender probability changes after each single other policyholder surrender, which simplifies policyholders' copycat behaviors in reality. By relaxing the assumption of homogeneity of policyholders and recognizing that in reality policyholders' copycat behaviors can appear only after past surrenders have accumulated to a certain level, our present paper for the first time presents close-to-reality structural changes of a heterogeneous pool of policyholders with copycat behaviors over time. In addition, the paper of Barsotti, Milhaud, and Salhi (2016) takes on insurers' risk management perspective and focuses on estimating default risk with different measures, whereby it does not give insights into pricing life insurance policies with contagious surrender behavior taken into account.

Moreover, recently, contagion modeling based on extensions of the Hawkes process, see Hawkes (1971), as an alternative to Levy processes has gained much attention in finance. For example, Aït-Sahalia, Laeven, and Pelizzon (2014) develop a credit risk swap pricing model with credit default events described by both self-excited and cross-excited Hawkes processes, also called multivariate Hawkes jump processes. In the papers of Aït-Sahalia, Cacho-Diaz, and Laeven (2015) and Aït-Sahalia and Hurd (2016), multivariate Hawkes jump processes are modeled together with diffusion. This so-called multivariate Hawkes jump diffusion is then used to capture the dynamics of asset returns with financial contagion effects on markets. Boswijk, Laeven, and Yang (2018) explore the empirical implications of a self-excited Hawkes jump process in an option pricing model. All those papers employ self-excited and cross-excited Hawkes processes to model contemporaneous financial contagion on markets, whereas our paper studies policyholders' contagious surrender behavior, being subject to past surrenders accumulated to a certain level and "impulse"-behavior-type uncertainty, which restricts us from directly using a self-exciting or cross-exciting component in a contagion process, but requires designing a new surrender-history-dependent contagion process.

2 Model Framework

This section presents the model framework of surrender contagion by reviewing the economy's financial and insurance markets followed by policyholders and equity holders. Policyholders differ in the way of handling their contract, by which we mean that some of them make their surrender decision based on professional agents' advice, and the rest of them make their surrender decision on their own. In our model, we assume that there is a representative professional agent who handles the contracts of some policyholders, and under this assumption we consider that these policyholders are financially rational in making their surrender decision and call them professionals.[5] The rest of policyholders, on the contrary, are named as non-professionals, and in our model we assume that when the non-professionals are left alone to make their surrender decision, they surrender their contract only for exogenous reasons, for example, for dealing with their own liquidity

[5] In order to focus on mimicking surrender behaviors of non-professional policyholders after enough-many rational policyholders' surrender move, we abstract from the complex reality where there will be more than one agent handling policyholders' contracts and also some professionals who are capable of making rational surrender decisions on their own, and probably some strategic surrender decisions among financially rational policyholders on the market.

Table 1: Insurance company's balance sheet at $t = 0$.

Assets	Liabilities & Equity
A_0	$L_0 \equiv P_0(Z^{pro} + Z^{non})$ E_0

shortage problems.

2.1 Financial and Insurance Markets

In the present paper, we fix a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$, where $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ reflects the information on both the financial market and the insurance market. We consider an insurance company which is initially founded by professional policyholders, non-professional policyholders, and equity holders, at time $t = 0$.

Each policyholder pays a same premium P_0 to purchase an identical participating life insurance policy embedded with a surrender option which can be exercised anytime before the contract's maturity date T. We use Z^{pro} and Z^{non} to denote the initial number of policies held by the professional and non-professional policyholders, respectively. Hence, $Z = Z^{pro} + Z^{non}$ is the initial total number of outstanding policies and the total contribution from the policyholders then sums up to $L_0 = P_0 Z$. Additionally, the equity holders inject a total initial payment of E_0. Therefore, the company's initial asset value is $A_0 = L_0 + E_0$. We summarize the insurance company's financial status at $t = 0$ in a balance sheet in Table 1.

We consider that at time $t = 0$ the company invests all its initial assets A_0 in traded risky and risk-free securities on the financial market, which is assumed to be arbitrage-free, so that a risk-neutral probability measure $\mathbb{Q}$ exists. For simplicity, we model the financial and insurance markets directly under this risk-neutral measure. We assume that during the time when no liquidation of the company's asset portfolio is required, that is, while no policyholder terminates his or her contract prematurely, the insurance company's asset value follows a geometric Brownian motion. As policyholders decide to surrender the contract before maturity, they receive surrender benefits, which will reduce the asset value of the company correspondingly. For simplicity, we assume that the company fixes its investment strategy at the initial time $t = 0$ and follows it during the insuring period.

Now, let N_t denote the total number of surrenders up to time t, and $S(t, A_{t-}, \mathrm{d}N_t)$ denote the surrender benefits paid to policyholders who surrender their contract at time t, which is a function of t, the company's asset value shortly before time t, that is, A_{t-}, and the number of surrenders at time t, that is $\mathrm{d}N_t$.[6] Under the risk-neutral probability measure $\mathbb{Q}$, the company's asset price process $A = (A_t)_{t\geq 0}$ satisfies the following stochastic differential equation

$$\mathrm{d}A_t = rA_t\,\mathrm{d}t + \sigma A_t\,\mathrm{d}W_t - S(t, A_{t-}, \mathrm{d}N_t)\,\mathrm{d}N_t\,, \tag{1}$$

where W is a standard Brownian motion, and $r \in \mathbb{R}$ and $\sigma > 0$ refer to the risk-free rate of return and the volatility of the asset price process, respectively.[7]

2.2 Payment Structure of Participating Life Insurance

In this paper, we consider point-to-point participating life insurance contracts embedded with a surrender option, which have a similar payoff structure as in Briys and de Varenne (1994), Grosen and Jorgensen (2002), and Cheng and Li (2018). For simplicity, we abstract from the death event of the policyholders in order to allow us to focus on their contagious surrender behaviors. If policyholders hold their contract till maturity T, namely, they stay in the insurance pool till maturity, first, they are entitled to minimum guaranteed benefits, $G_T^{r_g} = P_0 e^{r_g T}$, where r_g is the minimum guaranteed interest rate. Second, after serving all minimum guaranteed claims to the policyholders, a share of the company's profits is also promised by the insurer to them, which takes the form of $\delta[\alpha_T A_T - G_T^{r_g}]^+$, $\alpha_T = \frac{P_0}{A_0 - P_0 N_T}$. δ is the profit participation rate and α_T is the share of the company's asset value attributed to the policyholders left in the pool according their initial contribution for calculating the asset surplus. Third, if the company's assets fall short of the policyholder's minimum guaranteed claims, they are split equally between the remaining policyholders, yielding individual payoffs $\frac{A_T}{Z - N_T}$. To sum up, policyholders who hold their contract till maturity T receive maturity benefits, which are also called survival benefits and have the form of

$$\Phi(T, A_T, N_T) = G_T^{r_g} + \delta\left[\alpha_T A_T - G_T^{r_g}\right]^+ - \left[G_T^{r_g} - \frac{1}{Z - N_T} A_T\right]^+ . \tag{2}$$

If policyholders exercise the surrender option written in their contract before maturity T, surrender benefits are paid out, which do not contain a bonus payment. At surrender, policyholders receive a payment based on a minimum guarantee, which is $G_t^{r_s} = P_0 e^{r_s t}$, where r_s is the guaranteed interest rate in the surrender case. For penalizing policyholders

[6] We will discuss about the payment form of the surrender benefits in the next subsection.

[7] This asset price process is assumed as long as the company is on-going. As we later in Section 2.4 introduce an external regulator's intervention, we will see that once the company is shut down by a regulator, all company's assets are liquidated and distributed to the company's stakeholders. Hence, the asset price process given in Equation (1) is subject to a regulator's early default intervention, that is, at early default time, the company's asset value turns to be zero.

for early terminating their contract, a penalty parameter κ_t, which is assumed to be a decreasing function of time t, is applied as a discount on the minimum surrender guarantee. Moreover, similar to the survival case, we consider that when the insurer does not have enough assets to cover surrender benefits, the policyholders who surrender the contract at time t equally share what is left in the company and the company declares bankruptcy. [8] To sum up, at time $t < T$ when the surrender option is exercised, policyholders receive surrender benefits which take the form of

$$S(t, A_{t-}, \mathrm{d}N_t) = (1-\kappa_t)G_t^{r_s} - \left[(1-\kappa_t)G_t^{r_s} - \frac{1}{\mathrm{d}N_t}A_{t-}\right]^+ , \tag{3}$$

where $\mathrm{d}N_t = N_t - N_{t-}$ refers to the number of surrenders at time t.[9]

2.3 Contagious Surrender Behavior

As we have mentioned in the beginning of Section 2, in the insurance pool, there are two types of policyholders, namely policyholders whose contracts are managed by a professional agent and therefore whom we consider to be professionals, being capable of terminating their contract at the optimal time, and non-professional policyholders who handle their contract on their own. Let τ_P denote the optimal surrender time, at which the professionals surrender their contract. If the professionals do not surrender before maturity T, we set τ_P to T, therefore, we have $\tau_P \in (0, T]$. Before time τ_P, all surrenders are caused by non-professional policyholders. The process N_t^{non} counts non-professional policyholders' surrenders till time t. Hence, the number of total surrenders N_t is equal to N_t^{non} for $t \in [0, \tau_P)$, and at time τ_P, the total surrenders consist of two parts, that is, $\mathrm{d}N_{\tau_P} = \mathrm{d}N_{\tau_P}^{non} + Z^{pro}$. To sum up, the number of total surrenders satisfies

$$\mathrm{d}N_t = \mathrm{d}N_t^{non} + Z^{pro}\mathbb{1}_{\{t=\tau_P\}} ,\ N_0 = n. \tag{4}$$

We now turn to model the surrender behavior of non-professional policyholders, who we consider to be not capable of valuing their contract for carrying out a financially optimal

[8] There must be other surrender benefit forms which can be considered in the case when the company's asset value is insufficient at any surrender time t. For example, in order to protect policyholders who are still holding the contract, the insurer uses only the asset value after deducting the minimum survival guarantees promised to those contract-holding policyholders to pay surrender benefits. Since in our paper, we focus on contagious surrender behaviors of policyholders, in particular, non-professional policyholders, having the surrender benefit form presented in our model with minimum protection from the insurer to the policyholders left in the pool makes our study and explanation of results comparable to contagious deposit withdraws.

[9] For $\mathrm{d}N_t = 0$, we set $S(t, A_{t-}, \mathrm{d}N_t) = 0$ to ensure that the latter is well-defined.

surrender decision. Instead, they surrender their contract only for personal exogenous reasons. From a financial point of view, surrenders of non-professional policyholders are purely random. We model them through single jump Poisson processes $N^i = (N_t^i)_{t\geq 0}$ with associated intensity processes of the form $\lambda_t \mathbb{1}_{\{N_t^i=0\}}$, for $i = 1, \ldots, Z^{non}$. Accordingly, the counting process N_t^{non} is given by the accumulated single jump Poisson processes, that is, $N_t^{non} = \sum_{i=1}^{Z^{non}} N_t^i$, and, thus, its associated intensity process is $\Lambda_t = (Z^{non} - N_t^{non})\lambda_t$.[10]

Moreover, in our paper, we consider that the non-professionals' surrender decision can be influenced by the surrender activity of their peers holding the same policy. For example, if the number of recent surrenders on the market, denoted by M_t, exceeds a certain level B, this active surrender event can become public due to media coverage or statements of supervisory authorities. As a result, non-professionals might follow up and surrender more frequently since they believe that the resulting financial distress for the insurance could harm their policies. This, again, maintains the public attention on high surrender rates, so that non-professionals stick to their excessive surrender behavior. We refer this scenario as surrender contagion. Of course, it can also be that non-professionals believe in the financial stability of the insurance, so that their surrender intensity remains unaffected although they take note of the public attention on the past active surrender activity. In this case, the recent active surrender event fails to hit non-professionals' nerve to run, and for simplicity, in this scenario we assume that the past surrender activity M_t and the public then wait for the next active surrender event.

Mathematically, as the past surrender activity M_t hits the so called contagion threshold B at time t, in our model, the Bernoulli random variable B_t announces with probability $Pr(B_t = 1) = p$ the occurrence of surrender contagion. If surrender contagion remains absent, we reset the surrender history M_t to zero. Let $\tau^{(n)}$ denote the n-th time at which M_t is reset. Before the first reset, the surrender history is simply given by the discounted past surrenders, that is, $M_t = \int_0^t e^{-\beta(t-s)}\,\mathrm{d}N_s$ for $t < \tau^{(1)}$. Now, suppose the process M_t was reset n times, which means that the surrender history takes into account only surrenders that occurred after the last reset, that is, for $t > \tau^{(n)}$. Accordingly, we compose M_t of processes $M_t^{(n)}$ that capture the surrender history between the $n-1$-th and n-th reset, that is, $M_t = \sum_{n=1}^{\infty} \mathbb{1}_{t\in[\tau^{(n-1)},\tau^{(n)})} M_t^{(n)}$. Formally, those processes are defined by

$$M_t^{(n)} = \int_{[\tau^{(n-1)}\wedge t,\ \tau^{(n)}\wedge t]} e^{-\beta(t-s)}\,\mathrm{d}N_s\,, \qquad \text{for } n \in \mathbb{N},$$

where, $\tau^{(n)} = \inf\{t \geq 0 \mid M_t^{(n-1)} \geq B,\ M_{t-}^{(n-1)} < B,\ B_t = 0\}$ denotes the n-th time

[10] For technical justifications, see Lemma 7.1.1 in Bielecki and Rutkowski, 2004.

at which the non-professionals' surrenders are not affected by the surrender history M_t hitting the critical contagion threshold B, while $\tau^{(0)} = 0$.

Note that, with this definition non-professional policyholders' contagious surrender behavior is successfully triggered if the surrender history process M_t crosses the contagion threshold B. The excess surrender intensity caused by contagion is denoted λ_c, so that we define the surrender intensity for non-professionals by

$$\lambda_t = \lambda(M_t) = \lambda_0 + \lambda_c \mathbb{1}_{\{M_t \geq B\}}, \text{ for } t \geq 0, \tag{5}$$

where $\lambda_0 \geq 0$ is the constant exogenous surrender intensity level.

2.4 Early Default of the Insurance Company

Motivated by real-world solvency rules, for example, the Swiss Solvency Test and Solvency II, in our paper, we consider that there exists an external regulator, who continuously monitors insurance company's financial status and closes it when it is necessary. We assume that an early default barrier based on the minimum interest rate guarantees promised to policyholders is imposed by the regulator, which has the form of

$$D_t = \theta \, G_t^{r_g} \, (Z - N_t), \tag{6}$$

where $G_t^{r_g}$ was the minimum interest rate guarantee promised to each policyholder up to time t, $Z - N_t$ refers to the number of remaining policyholders in the insurance pool at time t, and $\theta \geq 0$ is a default multiplier. As the company's asset value drops to or below the early default barrier D_t, the regulator intervenes by early closing the company. In order to rule out the case that the company is closed immediately after it is founded, θ has to be smaller than $\frac{A_0}{P_0 Z}$. Therefore, we have $\theta \in [0, \frac{A_0}{P_0 Z})$. The higher the default multiplier, the more likely the insurance company is going to be closed, and the more intensively the regulator intends to protect policyholders. We use τ_D to denote the early default time, which is given by $\tau_D = \inf\{t \geq 0 \mid A_t \leq D_t\}$. At default, policyholders who are still in the pool receive default benefits, which take the form of

$$\Upsilon(t, A_t, N_t) = \min\left(G_t^{r_g}, \; \frac{1}{Z - N_t} A_t \right). \tag{7}$$

If the company has enough asset value to cover all minimum interest guarantees promised to the remaining policyholders, each of them receives the minimum guaranteed amount, otherwise, all these policyholders equally share what is left in the company.

3 Two-Step Contract Valuation

Given that the professional policyholders and the non-professional policyholders follow different contract surrendering rules, specifically the professionals will surrender their contract at a financially optimal time, however, the non-professionals do not necessarily do so, their contract values will differ. The lack of optimal surrender decision making of the non-professionals reduces the expected value of discounted cash flows emerging from their contract, which leads to a lower contract value than for the professionals. In this section, we present a two-step contract valuation scheme to price the professional and non-professional policyholders' contracts.

3.1 Professional Policyholders' Contract Valuation

This section presents the first valuation step: Pricing the professional policyholders' contract on a market with surrenders solely from non-professional policyholders. In our risk-neutral setting, the professional policyholders aim at maximizing their contract's expected discounted payoff by timing their surrender optimally, and therefore, we describe the respective contract valuation as an optimal stopping problem. Since in our model we assume that all professional policyholders' contracts are managed by one professional agent, the professionals follow the same surrender rule, namely the same optimal stopping rule, which allows us to conduct the pricing from the perspective of a representative professional policyholder. So, as long as the representative professional policyholder holds the contract, surrenders appearing on the market are exclusively from the non-professionals. Then, we modify our insurance market in the way of having it collect only surrenders from the non-professional policyholders to determine the professional policyholders' optimal surrender rule τ_P and price their contract.

For the moment, we set the surrender time of the representative professional to infinity, that is $\tau_P = \infty$, to ensure that surrenders on the market up to time t are exclusively from non-professionals. In this setting, we then have the non-professional policyholders' surrender counting process N_t^{non} as the total surrender process, and here we denote it by N_t^∞. By replacing the total surrender process N_t by N_t^∞, we update the insurance company's asset price process A_t and the surrender history process M_t accordingly, and denote them by A_t^∞ and M_t^∞, respectively. We summarize all processes above and the time parameter in $X^\infty = (X_t^\infty)_{t\geq 0} = (t, A_t^\infty, M_t^\infty, N_t^\infty)_{t\geq 0}$ with $X_t^\infty = x = (t, a, m, n) \in E$, where $E = [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+ \times \mathbb{N}_0$.

Next, we define the contract value for the representative professional policyholder by solving an optimal stopping problem. Let $\mathcal{T}(t)$ be the set of stopping times $t < \tau < T$

that represent all admissible surrender strategies of the representative professional, and, let $\mathbb{E}_x$ denote the expected value conditioned on $X_t^\infty = x$.[11] Then, the representative professional's contract value is defined as the supremum over all expected discounted payoffs which are accessible by the admissible surrender strategies τ, that is,

$$\begin{aligned} v^{pro}(x) = \sup_{\tau \in \mathcal{T}(t)} \mathbb{E}_x \Big[e^{-r(\tau \wedge \tau_D - t)} \Big(& S(\tau, A_\tau^\infty, Z^{pro}) \mathbb{1}_{\{\tau < \tau_D \wedge T\}} \\ & + \Upsilon(\tau_D, A_{\tau_D}^\infty, N_{\tau_D}^\infty) \mathbb{1}_{\{\tau_D \leq \tau, \tau_D < T\}} + \Phi(T, A_T^\infty, N_T^\infty) \mathbb{1}_{\{T = \tau \wedge \tau_D\}} \Big) \Big] . \end{aligned} \tag{8}$$

We rewrite the above optimal stopping problem in a more familiar form by using a reward function which we define as

$$\begin{aligned} g(X_t^\infty) = & S(t, A_t^\infty, Z^{pro}) \mathbb{1}_{\{t < T,\, A_t > D(t, N_t)\}} + \Upsilon(t, A_t^\infty, N_t^\infty) \mathbb{1}_{\{t < T,\, A_t \leq D(t, N_t)\}} \\ & + \Phi(t, A_t^\infty, N_t^\infty) \mathbb{1}_{\{t \geq T\}} , \end{aligned} \tag{9}$$

By considering $(X_{s \wedge \tau_D}^\infty)_{s \geq t}$ as the underlying process for the optimal stopping problem, we have

$$v^{pro}(x) = \sup_{\tau \in \mathcal{T}(t)} \mathbb{E}_x \left[e^{-r(\tau \wedge \tau_D - t)} g(X_{\tau \wedge \tau_D}^\infty) \right] . \tag{10}$$

For technical tractability, we assume v^{pro} to be lower semi-continuous. Then, there exists a smallest admissible optimal surrender rule $\tau_P = \inf\{t \geq 0 \mid X_t^\infty \in P\}$ with $P = \{x \mid v^{pro}(x) = g(x)\}$, see Peskir and Shiryaev (2006, Section 1). The optimal stopping region P includes the early default region $D = \{x \in E \mid t < T, a < D(t, n)\}$ and each terminal state with $t = T$, implying $\tau_P \leq \tau_D \wedge T$. At early default or maturity, that is, $\tau_P = \tau_D$ or $\tau_P = T$, the representative professional policyholder receives the default benefits or the survival benefits, respectively.[12] The value function of the above optimal stopping problem is given as the solution of the free boundary value problem, which we summarize in Result 3.1, and its derivation is provided in Appendix A.1.

Result 3.1. *Let the value function v^{pro} be sufficiently smooth. Then, it satisfies*

$$\mathcal{L}_X^\infty v^{pro}(x) = r\, v^{pro}(x) , \qquad \text{for } v^{pro}(x) > g(x) , \tag{11}$$

$$v^{pro}(x) = g(x) , \qquad \text{else} , \tag{12}$$

[11] Note, that $\mathcal{T}(t)$ are stopping times of the process $(X_s^\infty)_{s \geq t}$.

[12] Notice that for time $t < \tau_P$, we have $X_t^\infty = X_t$. At time τ_P, the asset price drops by the amount of surrender benefits paid to the professionals and the remaining processes adjust accordingly, that is, $A_{\tau_P} = A_{\tau_P}^\infty - S(\tau_P, A_{\tau_P -}, Z^{pro}) Z^{pro}$, $M_{\tau_P} = M_{\tau_P}^\infty + Z^{pro}$ and $N_{\tau_P} = N_{\tau_P}^\infty + Z^{pro}$.

where $\mathcal{L}_X^\infty$ is the infinitesimal generator of $(X_t^\infty)_{t\geq 0}$. For all $(t,a,m,n) = x \in E$ with $v^{pro}(x) > g(x)$ and $n \leq Z^{non}$ the generator applied to the contract value takes the form of

$$\mathcal{L}_X^\infty v^{pro} = \frac{\partial v^{pro}}{\partial t} + r\,a\frac{\partial v^{pro}}{\partial a} + \frac{1}{2}a^2\sigma^2\frac{\partial^2 v^{pro}}{\partial a^2} - \beta\,m\frac{\partial v^{pro}}{\partial m} + (Z^{non} - n)\,\lambda(m)\Delta v^{pro}\,, \tag{13}$$

where

$$\begin{aligned}\Delta v^{pro}(x) = &\mathbb{1}_{m\in[B-1,B)}\big(p v^{pro}(t, a - S(t,a,1), m+1, n+1) \\ &+ (1-p)v^{pro}(t, a - S(t,a,1), 0, n+1)\big) \\ &+ \mathbb{1}_{m\notin[B-1,B)}\big(v^{pro}(t, a - S(t,a,1), m+1, n+1) - v^{pro}(t,a,m,n)\big)\,.\end{aligned} \tag{14}$$

Moreover, the value function v^{pro}, see its definition in (10)*, is subject to the following two fixed boundary conditions:*

$$v^{pro}(x) = g(x) = \Upsilon(t,a,n)\,, \qquad \textit{for all } (t,a,n,m) = x \in D \textit{ and} \tag{15}$$

$$v^{pro}(x) = g(x) = \Phi(T,a,n)\,, \qquad \textit{for all } (t,a,n,m) = x \in E \textit{ with } t = T\,. \tag{16}$$

3.2 Non-Professional Policyholders' Contract Valuation

In this section, as our second valuation step, we take the perspective of a representative non-professional policyholder and price his contract by taking account of the optimal mutual surrender strategy of the professionals. Without loss of generality, let the jump time τ_1 of N^1 denote the representative non-professional policyholder's surrender time.[13] We use N_t^* to denote the underlying dynamics with respect to the total number of surrenders on the market facing the representative non-professional policyholder, namely the surrender counting process excluding the surrender of the representative non-professional policyholder. By taking account of the surrender action of all professionals at τ_P, we have $\mathrm{d}N_t^* = \sum_{i=2}^{Z^{non}} \mathrm{d}N_t^i + Z^{pro}\mathbb{1}_{\{t=\tau_P\}}$. The same as in Section 3.1, we can update the asset price process as well as the surrender history process facing the representative non-professional policyholder by substituting the total number of surrenders N_t with N_t^*, and we use $X_t^* = (t, A_t^*, N_t^*, M_t^*)$ to denote these updated underlying dynamics, which satisfies $X_t^* = X_t$ for $t < \tau_1$.[14]

[13] Same as for the professional policyholders' surrender time τ_P, we set the non-professional policyholders' surrender time τ_i to T if they do not surrender until time T. Hence, we have $\tau_1 \in (0,T]$.

[14] Note that at the representative non-professional's surrender time τ_1, the drop of the asset price caused by his own surrender is not taken into account for determining his surrender payout, but A_{τ_1-}, and we have $A_{\tau_1} = A_{\tau_1}^* - S(\tau_1, A_{\tau_1-}, 1)$, $M_{\tau_1} = M_{\tau_1}^* + 1$ and $N_{\tau_1} = N_{\tau_1}^* + 1$.

The respective policy value for some initial market state $x = (t, a, m, n) \in E$ with $n < Z^{non}$ is given by

$$v^{non}(x) = \mathbb{E}_x\left[e^{-r(\tau_1 \wedge \tau_D - t)} g(X^*_{\tau_1 \wedge \tau_D}) \mid N^1_t = 0\right], \tag{17}$$

where $\mathbb{E}_x$ denotes the expectation conditioned on $X^*_{t-} = x$. The above value function satisfies a nested characterization in terms of two standard boundary value problems given in the following result, where $\tilde{v}^{non}$ describes the contract value of our representative non-professional policyholder given that all professional policyholders have surrendered already. Appendix A.2 provides a derivation of Result 3.2.

Result 3.2. *Let the policy value function v^{non} be sufficiently smooth. Then, it satisfies*

$$\begin{aligned} &\frac{\partial v^{non}}{\partial t} + r\,a \frac{\partial v^{non}}{\partial a} + \frac{1}{2} a^2\,\sigma^2 \frac{\partial^2 v^{non}}{\partial a^2} - \beta\,m \frac{\partial v^{non}}{\partial m} \\ &\qquad + (Z^{non} - n - 1)\,\lambda(m) \Delta v^{non}(x) + \lambda(m)(S(t, a, 1) - v^{non}) = r v^{non}, \end{aligned} \tag{18}$$

with boundary conditions (15), (16)*, where we define $\Delta v^{non}(x)$ as in* (14)*. Further,*

$$v^{non}(x) = \tilde{v}^{non}\,(t, a - S(t, a, Z^{pro}) Z^{pro}, m + Z^{pro}, n + Z^{pro}) \tag{19}$$

for $(t, a, m, n) = x \in P \setminus D$, where $\tilde{v}^{non}$ satisfies

$$\begin{aligned} &\frac{\partial \tilde{v}^{non}}{\partial t} + r\,a \frac{\partial \tilde{v}^{non}}{\partial a} + \frac{1}{2} a^2\,\sigma^2 \frac{\partial^2 \tilde{v}^{non}}{\partial a^2} - \beta\,m \frac{\partial \tilde{v}^{non}}{\partial m} \\ &\qquad + (Z - n - 1)\,\lambda(m) \Delta \tilde{v}^{non}(x) + \lambda(m)(S(t, a, 1) - \tilde{v}^{non}) = r \tilde{v}^{non}, \end{aligned} \tag{20}$$

with boundary conditions (15) *and* (16) *but without* (19)*, where, again, we define $\tilde{v}^{non}$ as in* (14).

4 Numerical Analysis

This section presents the numerical computation of the contract values from the perspective of both policyholder types corresponding to the two-step contract valuation procedure from the previous section. For computing professional policyholders' contract, we employ the Least Squares Monte Carlo method by Longstaff and Schwartz (2001) with monomials as basis functions to compute contracts' continuation value for making surrender decisions, following Glasserman (2004). As result, we obtain pathwise realizations of the optimal surrender rule applied by professionals. The simulated paths of the first valuation step together with the optimal surrender rule are then used to determine the price of non-

Table 2: Model parameters for the base case simulation.

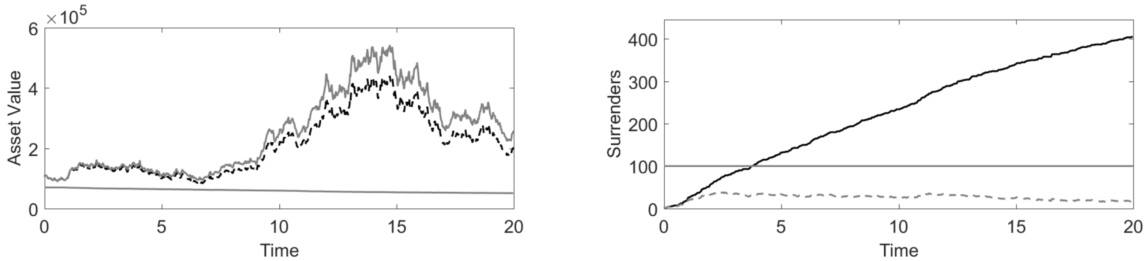
Parameter	Value	Parameter	Value
Risk-free rate	$r = 2\%$	Guaranteed rate	$r_g = r_s = 2\%$
Risky asset volatility	$\sigma = 20\%$	Maturity	$T = 20$
Default multiplier	$\theta = 70\%$	Premium	$P_0 = 100$
Initial equity	$E_0 = 10,000$	Participation coefficient	$\delta = 90\%$
Surrender fees	κ: Lin. decay: 10% to 0%	Non-professional policyholders	$Z^{non} = 900$
Professional policyholders	$Z^{pro} = 100$	Contagion probability	$p = 50\%$
Critical surrender boundary	$B = 100$	Initial surrender intensity	$\lambda_0 = 3\%$
Contagion impact	$\lambda_c = 1.2$	Surrender discount rate	$\beta = 75\%$
Number of time steps	240	Number of paths	1,000,000

professionals' contracts with a classical Monte Carlo approach. Using the fair values of the professional and non-professional contracts, we calculate the fair equity value for equity holders. Both the Least Squares Monte Carlo and classical Monte Carlo algorithms work on the same equidistant Euler discretization.

This section proceeds by outlining the base case parametrization which serves as a benchmark. Then, we present the impacts of surrender contagion on the insurance company's asset value and the surrender decision making by non-professional policyholders in three sample scenarios. The following subsection provides comparative statics for the contagion process. It studies the impacts of non-professionals' tendency to surrender contagiously on the pricing of both professional and non-professional policyholders' contracts. The final sections analyze the impact of regulatory intervention and volatility on both contract values and the equity value.

4.1 Model Parametrization and Base Case

The base case concerns a financial market featuring a risk-free rate of $r = 2\%$ and a volatility of the risky asset of $\sigma = 20\%$. The insurance company has initial equity of 10,000 and faces 900 non-professional policyholders. Additionally, there are 100 professional policyholders whose policies are managed by one professional agent. All policies sold to the policyholders are identical, which cost $P_0 = 100$ and mature in $T = 20$ years. The policyholders are entitled to a constant guaranteed interest rate for both the survival and surrender cases, that is, $r_g = r_s = 2\%$. They also participate in the company's profits with a participation coefficient of $\delta = 90\%$. There exists an external regulator who steps in once the company's asset value falls below $\theta = 70\%$ the minimum interest rate guarantees. Table 2 summarizes the parameter values in the base case.



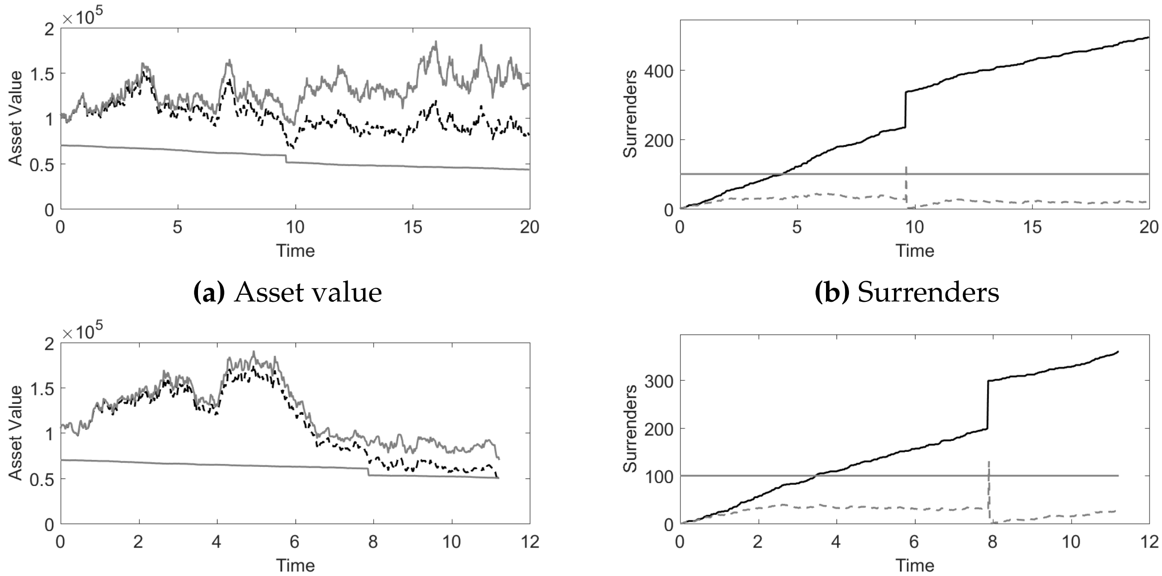
(a) Asset value **(b)** Surrenders

Figure 1: Scenario 1 – Neither Contagion, Professional Surrender nor Default. This figure displays the asset and surrender development for a sample path with base case parameters that features neither contagion, professional surrender, nor default. Panel (a) shows the original asset price (solid line) and surrender-adjusted asset price process (dashed line) of the insurance company. The gray line indicates the point of regulatory intervention. Panel (b) presents the number of total surrenders (solid line) and the discounted surrender history (dashed line). The gray line shows the contagion threshold for the surrender history, indicating contagion.

4.2 An Example of Surrender Contagion

This section turns to the mechanics of the surrender-history-dependent contagion process. Non-professional policyholders' contagious surrenders can severely influence the insurer squeeze it into early bankruptcy in an extreme case. As an illustration, we present three possible scenarios. They differ in the occurrence of professionals surrender and non-professional policyholders' contagious surrender behavior and its impacts on the operation of the insurance company. First, there could be neither professional policyholders' surrenders nor a contagion, leaving the overall development of the insurance company's assets unimpaired besides non-professional policyholders' unsystematic exogenous surrenders. Second, professionals observe an unfavorable asset performance of the insurance company and all decide to surrender their contract at that time. It does not trigger contagion Later, it may or may not cause regulatory intervention. Third, as in the second case, professionals deem the surrender appropriate at a point before maturity. In this case, it also triggers contagion. The regulator may or may not intervene at a later point.

The first scenario is illustrated in Figure 1 which shows the asset and surrender developments in two panels. Panel 1(a) draws the unimpaired asset development (solid line) and the surrender-fees-deducted assets (dashed line) of the insurance company as well as the default triggering barrier (gray line). Next, Panel 1(b) summarizes policyholders' surrender results: The solid line shows the absolute number of surrenders by policyholders, and the dashed line shows the perceived surrenders M_t, which are the accumulated discounted past surrenders, accounting for the surrender history facing non-professional policyholders. In gray we have the contagion threshold B. This scenario features a positive development of the insurance company's assets, due to which professional policyholders



(a) Asset value

(b) Surrenders

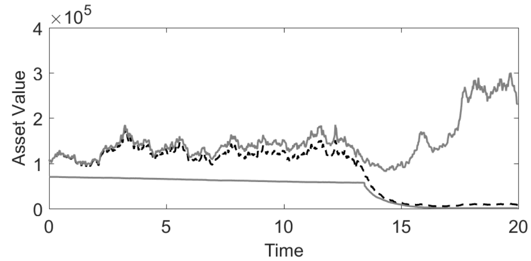
(c) Asset value

(d) Surrenders

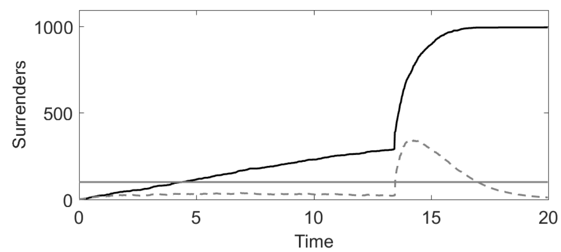
Figure 2: Scenario 2 – Professional Surrender without Contagion. This figure displays the asset and surrender development for sample paths with base case parameters that features no contagion after professionals surrender. Panels (a) and (c) show the original asset price (solid line) and surrender-adjusted asset price process (dashed line) of the insurance company. The gray line indicates the point of regulatory intervention. Panels (b) and (d) present the number of total surrenders (solid line) and the discounted surrender history (dashed line). The gray line shows the contagion threshold for the surrender history, indicating contagion.

do not surrender their contract during the whole insurance period. Surrenders on the market are the exogenous surrenders from non-professional policyholders, which accumulate with a rather slow speed. Therefore, the accumulated discounted past surrenders line never crosses the contagion threshold, and there is no contagion triggered in this scenario. Accordingly, we observe only limited draw-down on the company's asset value in Panel 1(a), which in this scenario accounts only for the exogenous surrenders on the market, and the draw-down is too small to trigger an early default intervention.

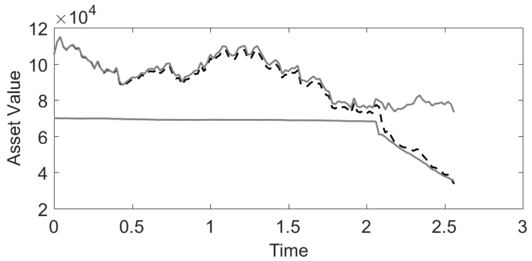
Now we turn to the second scenario illustrated by Figure 2. This scenario features an unattractive asset development, which causes professional policyholders to initiate surrender before maturity, see the spike in the total number of surrenders in Panel 2(a) and Panel 2(c), respectively. Professional policyholders' surrender action drives up the perceived surrenders line to cross the contagion threshold B (see the gray line in Panels 2(b) and 2(d)). In this scenario, non-professional policyholders' surrender do not spiral. Consequently, the surrender history up to this point is reset. Non-professionals deem this information irrelevant. For the lower figure, the regulator prevents further asset losses and intervenes. The insurance company defaults early after professionals' surrender move. In the upper case, the company remains solvent despite professionals surrender.



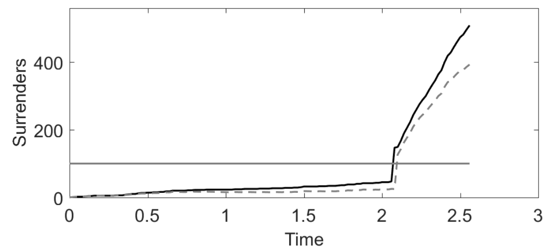
(a) Asset value



(b) Surrenders



(c) Asset value



(d) Surrenders

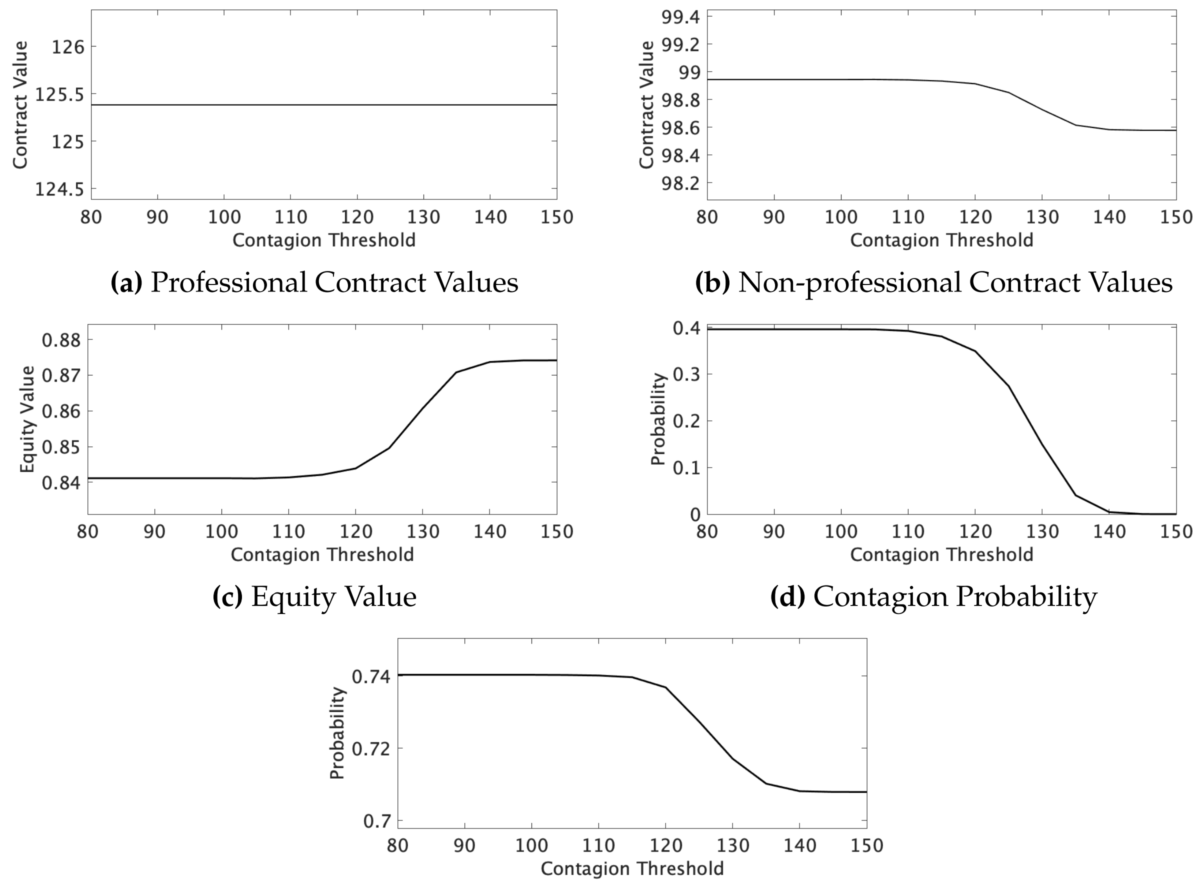
Figure 3: Scenario 3 – Professional Surrender with Contagion. This figure displays the asset and surrender development for sample paths with base case parameters that features contagion after professionals surrender. Panels (a) and (c) show the original asset price (solid line) and surrender-adjusted asset price process (dashed line) of the insurance company. The gray line indicates the point of regulatory intervention. Panels (b) and (d) present the number of total surrenders (solid line) and the discounted surrender history (dashed line). The gray line shows the contagion threshold for the surrender history, indicating contagion.

Figure 3 presents the third scenario, which is similar to Figure 2. An unfavorable asset development causes professional policyholders to cancel their contract at a point before maturity. In this case, it triggers non-professional policyholders' contagious surrender behavior, see the perceived surrenders line crossing the contagion threshold in Panel 3(b) and Panel 3(b), respectively. Again, the spiraling surrenders and the implied asset drain can cause regulatory intervention (upper case) or not (lower case).

4.3 Surrender contagion analysis

In this section, we study the impacts of the surrender contagion, represented by the contagion threshold B, on the fair value of professional and non-professional policyholders' contacts, the equity value and the insurance company's default probability. Since we are not interested in scenarios in which non-professional policyholders' random surrenders alone trigger market contagious surrenders, we let the contagion threshold B start varying from 80 and plot the corresponding contract value of professionals and non-professionals, contagion probability, and insurer's default probability till $B = 150$ at which the surrender contagion can not be triggered any more.

We see in Panel 4(d) that as the contagion threshold B is smaller or equal to 100, since



(a) Professional Contract Values

(b) Non-professional Contract Values

(c) Equity Value

(d) Contagion Probability

(e) Default Probability

Figure 4: Comparative Statics for the Contagion Threshold. Panel (a) presents the impact of the contagion threshold B on the contract value of professional policyholders. Panel (b) shows the non-professional policyholders' value if contagion is impossible (solid line) or caused by professional surrender (dashed line). Panel (c) features the equity value without (solid line) and with contagion (dashed line). Panel (d) shows the probability for contagion if caused by professionals. Panel (e) displays the default probability if contagion is impossible (solid line) or caused by professional surrender (dashed line). All other parameters follow the base case.

professionals' surrenders alone are sufficient for potentially causing market contagious surrenders, the contagion probability stays constant, being above 30%. As B increases above 100, non-professionals need to join professionals' actions to trigger the market surrender contagion. The higher the contagion threshold, the more non-professional policyholders' random surrenders need to contribute to the past surrenders for triggering the contagion, which results in a decreasing contagion probability in B in Panel 4(d). When the contagion threshold gets higher, namely above 140, contagion can no longer be triggered on the market and its probability reaches 0.

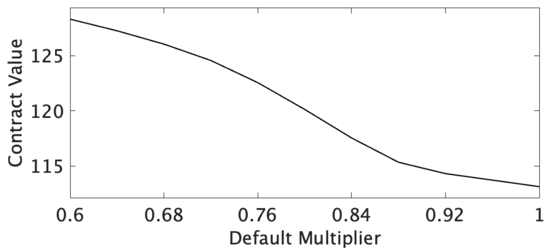
To professional policyholders, since the market surrender contagion cannot be triggered without their surrender actions, the potential contagion event will have no influence on their surrender decision making, which implies that it has no impact on their contract value, see the constant contract value of professional policyholders as the contagion threshold B varies between 80 and 150 in Panel 4(a). But to non-professional policyholders who originally surrender their contract purely random on the market, having the chance of following professional policyholders' surrender actions, which are optimal in the sense of securing the contract's highest payment, can improve their financial position, see the increase in the contract value of non-professional policyholders as the contagion threshold B decreases in Panel 4(b). But that comes at the expense of equity holders, see the corresponding decrease in the equity value in Panel 4(c), and at the same time, heavy early payments to not only professionals but also following-up non-professionals before maturity drive up the insurance company's default probability, see Panel 4(e).

4.4 The influence of default regulation on the contagion effects

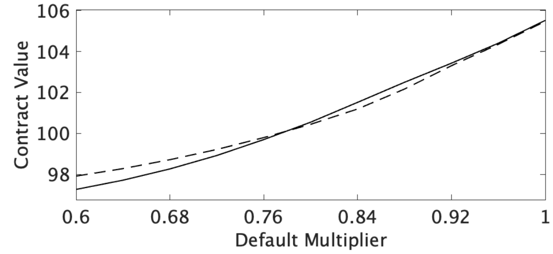
This section studies the interaction effects of early default regulation and surrender contagion. Here, the contagion threshold B has the base value of 100. Figure 5 presents the impacts of the default multiplier θ on the contract value of both professional and non-professional policyholders, the equity value, the default probability, and the contagion probability. We consider two scenarios, namely, without surrender contagion, indicated by the solid line, and with surrender contagion, indicated by the dashed line. Since contagion has no influence on professional policyholders' surrender decision making (see Section 4.3), the contract values of professionals with and without contagion coincide in Panel 5(a).

First, independent of surrender contagion, the higher the default multiplier, the more likely the insurance company is prematurely closed by the regulator, see Panel 5(e), which harms both professionals and equity holders but benefits non-professional policyholders, see Panels 5(a), 5(b), and 5(c). While stricter early default regulation constrains professional policyholders' surrender actions, which can be confirmed by the decrease in the contagion probability as the default multiplier increases in Panel 5(d), it supports non-professional policyholders, who surrender randomly, by closing the insurance company early and securing them at a fraction of their minimum guaranteed benefits. Moreover, the regulator's frequent intervention reduces equity holders' chance of receiving any residual capital of the insurance, that is, their probability of receiving a zero payoff increases. Therefore, regulatory intervention induces a wealth transfer from both professionals and equity holders to non-professionals, which increases in the strictness of the regulatory rule.

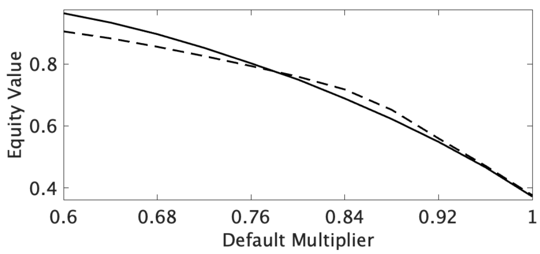
Next, the effects of contagion on the contract value of non-professionals, the equity value, and the insurance company's early default probability highly depend on the regu-



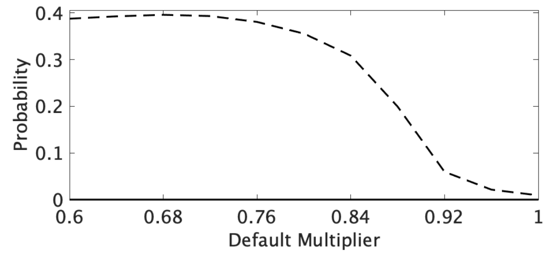
(a) Professional Contract Values



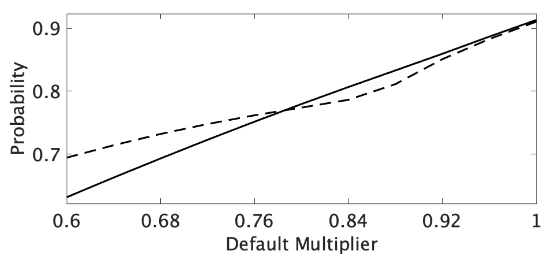
(b) Non-professional Contract Values



(c) Equity Value



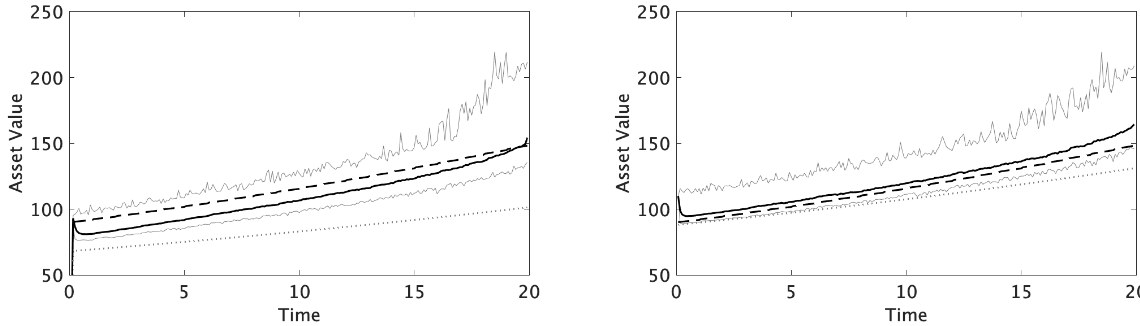
(d) Contagion Probability



(e) Early Default Probability

Figure 5: Default multiplier impact on contagion. Panel (a) presents the impact of the default multiplier θ on the contract value of professional policyholders. Panel (b) shows the non-professional policyholders' value if contagion is impossible (solid line) or caused by professional surrender (dashed line). Panel (c) features the equity value without (solid line) and with contagion (dashed line). Panel (d) shows the probability for contagion if caused by professionals. Panel (e) displays the early default probability if contagion is impossible (solid line) or caused by professional surrender (dashed line). All other parameters follow the base case.

latory environment, see Panels 5(b), 5(c), and 5(e). Within a lax regulatory environment, that is, the default multiplier is lower than about 77% in Figure 5, contagion increases the contract value of non-professionals and the insurer's early default probability, but it decreases the equity value, which are consistent to the results that we obtained in Section 4.3. However, as the default multiplier increases above about 77%, we see that, on the contrary, contagion reduces the contract value of non-professionals and the insurer's early default probability, but it enhances the equity value. Strict regulatory intervention stops the positive signaling role of contagion to non-professionals which improves their financial position by aligning their surrender behavior to professionals' optimal surrender strategy. To understand how that happens, we need to discover the impact of the regula-



(a) Assets at Surrender, $\theta = 68\%$ **(b)** Assets at Surrender, $\theta = 88\%$

Figure 6: Professional surrender decision. Panels (a) and (b) display the surrender decision of professional policyholders as a function of time and asset values for a low default threshold $\theta = 68\%$ and for a high default multiplier $\theta = 88\%$, respectively. The solid black line and gray lines indicate the median asset value per policyholder, the minimum and maximum asset values per policyholder, respectively, at professionals' surrender over the term of the insurance contract. The surrender benefits and the early default threshold are represented by the dashed black line and the dotted gray line, respectively.

tor's intervention on professional policyholders' surrender conditions, which eventually form non-professional policyholders' contagious surrender conditions.

Figure 6 presents the distribution of the insurance company's assets, including the median asset value per policyholder (solid black line) and the minimum and maximum asset values per policyholder (solid gray lines), the surrender benefits (dashed black line), and the early default threshold (dotted gray line) over the contract term at professional policyholders' surrender in both lax ($\theta = 68\%$) and strict ($\theta = 88\%$) regulatory environments. The asset value boundaries at which professionals surrender share the same pattern in both the lax and strict regulatory environments, which move upward and spread out along time. Intuitively, as the insurance contract approaches maturity, compared to holding the contract to participate in the insurer's profits at maturity, professionals are more likely to surrender their contract early when they notice the insurance company's asset value is dropping in order to secure the minimum guaranteed benefits.

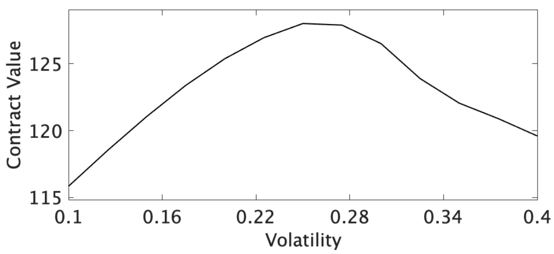
Professional policyholders' surrender, independent of the regulatory environment, is optimal from the financial point of view as they take out surrender benefits that are higher than their contract continuation value, following professionals' surrenders does not necessarily guarantee a better financial position for non-professionals. In the lax regulatory environment, see Panel 6(a), most of the distribution of the company's assets at professionals' surrender lies below the surrender benefits (compare the location of the dashed black line to the location of the solid black line). In this case, professional policyholders' surrenders push the company's assets per policyholder downward, which further lowers the contract's continuation value and calls for an early termination of the contract. However, in the strict regulatory environment, see Panel 6(b), the distribution of the company's assets at professionals' surrender lies above the surrender benefits. As professionals cash out surrender benefits, the company's assets value per policyholder gets pushed upward, which then increases the contract's continuation value and (at least) reduces the necessity of early terminating the contract. Following professionals' surrenders on the market by non-professionals can further enhance the contract's continuation value, which eventually renders early termination of the contract not optimal. Moreover, as the company's asset value per policyholder is pushed away from the early default threshold by contagion, its early default probability is automatically lowered.

4.5 The influence of investment risk on the contagion effect

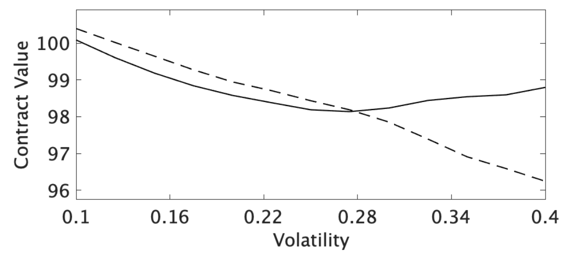
This section turns to the interaction effects of investment risk in the guise of volatility σ and surrender contagion. Again, the contagion threshold equals $B = 100$ and the default multiplier takes value $\theta = 70\%$. Figure 7 presents the impacts of the volatility σ, ranging from 10% to 40%, on the contract value of both professional and non-professional policyholders, the equity value, the default probability, and the contagion probability. Similar to the previous section, the two scenarios under consideration consist of no surrender contagion, indicated by the solid line, and surrender contagion, indicated by the dashed line. As previously, contagion does not influence professional's surrenders.

Professional policyholders benefit from the increase in the volatility as long as it is not too high, namely, lower than 28%, see Panel 7(a). Facing higher investment risk, professional policyholders will surrender their contract more likely, which can be confirmed by the increase in the contagion probability in Panel 7(d), and also earlier, see the forward moving surrender timing in Figure 8 as σ increases from 12.5% to 30%. Hence, professionals more often receive surrender benefits as σ increases, which their share of the potentially higher expected rate of return of the insurance company more than compensates. However, when the investment risk of the insurance company increases above 28%, the dominance of the higher expected rate of return disappears since the insurance company is more likely running into default with professionals staying in the insurance pool; compare the residual probabilities in the last bar in Panels 8(b) and 8(d), and in that scenario the professionals will obtain default benefits instead.

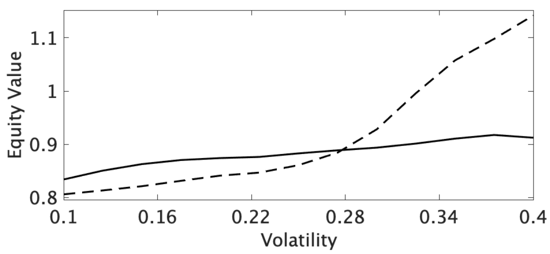
As non-professional policyholders surrender their contract independently for pure exogenous reasons, they receive roughly the opposite payoff from professional policyholders, see the reversed shape of their contract value indicated by the black line in Panel 7(b). With a rather mild increase in the equity value as σ increases, we see a clear wealth transfer



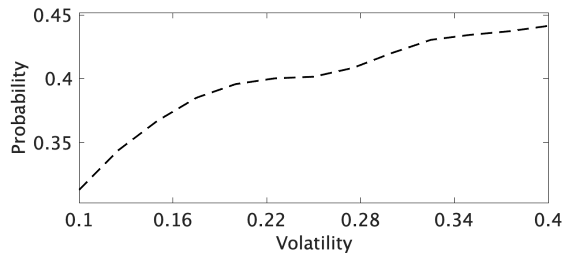
(a) Professional Contract Values



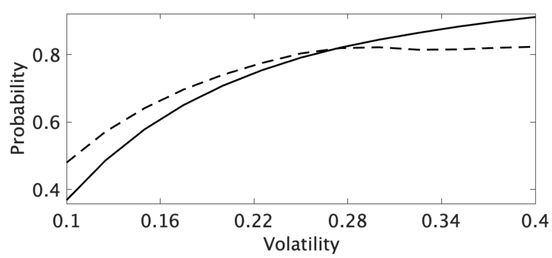
(b) Non-professional Contract Values



(c) Equity Value



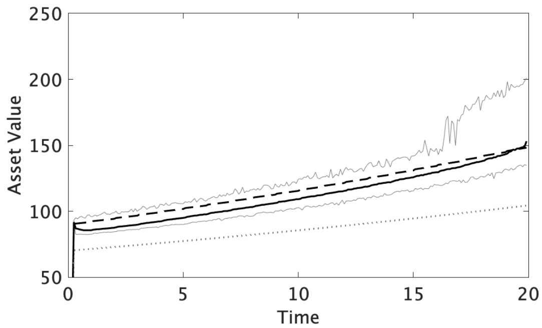
(d) Contagion Probability



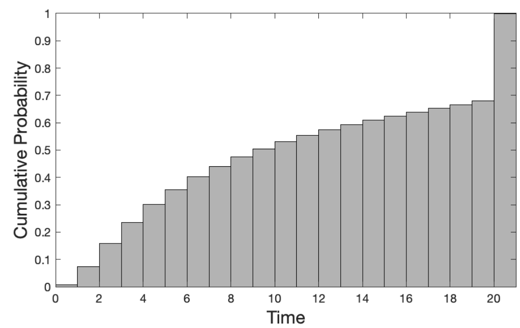
(e) Early Default Probability

Figure 7: Volatility impact on contagion. Panel (a) presents the impact of the volatility σ on the contract value of professional policyholders. Panel (b) shows the non-professional policyholders' value if contagion is impossible (solid line) or caused by professional surrender (dashed line). Panel (c) features the equity value without (solid line) and with contagion (dashed line). Panel (d) shows the probability for contagion if caused by professionals. Panel (e) displays the early default probability if contagion is impossible (solid line) or caused by professional surrender (dashed line). All other parameters follow the base case.

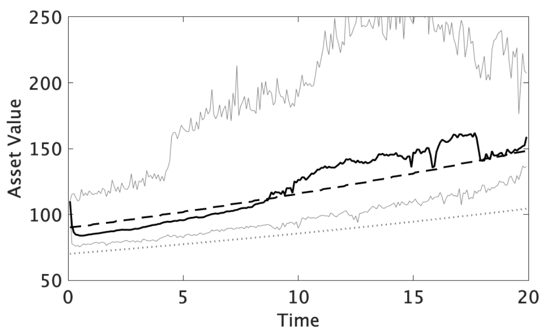
between professional policyholders and non-professionals. And again, following up professional policyholders' optimal surrenders on the market does not necessarily guarantee an improvement of non-professional policyholders' financial position, see the decrease in their contract value for volatility values above 28% as contagion is included for pricing in Panel 7(b). Similar to the previous section, we see that in Figure 8 at the higher volatility value $\sigma = 30\%$, most of the company's asset value distribution at professionals' surrender lies above the surrender benefits. Professional policyholders' action pushes up asset value per policyholder, which then lowers the company's early default probability, see Panel 7(e). The contract becomes more valuable to the remaining non-professional policyholders, and following up professional policyholders in this case will reveal a wealth transfer from non-professional policyholders to equity holders, see the increase in the equity value for



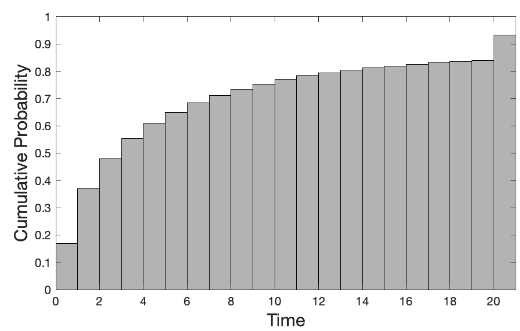
(a) Assets at Surrender, $\sigma = 12.5\%$



(b) Surrender Timing, $\sigma = 12.5\%$



(c) Assets at Surrender, $\sigma = 30\%$



(d) Surrender Timing, $\sigma = 30\%$

Figure 8: Professional surrender decision. Panel (a) and Panel (c) show the professionals' surrender decision as a function of time and the insurance company's asset value for a low volatility $\sigma = 12.5\%$ and a high volatility $\sigma = 30\%$, respectively. The solid black line and gray lines indicate the median asset value per policyholder, the minimum and maximum asset values per policyholder, respectively, at professionals' surrender over the term of the insurance contract. The surrender benefits and the early default threshold are represented by the dashed black line and the dotted gray line, respectively. Panel (b) and Panel 8(d)display the professionals' surrender timing for a low volatility $\sigma = 12.5\%$ and a high volatility $\sigma = 30\%$, respectively. The last bar represents the probability of the professionals reaching maturity.

higher volatility values in Panel 7(c). Hence, in the presence of contagion, the insurer is highly motivated to conduct riskier investment, which not only reduces its early default probability, but also boosts its equity value.

5 Conclusion

In this paper, we introduce a surrender-history-dependent intensity process to describe policyholders' contagious surrender behavior observed on insurance markets and study its impacts on the pricing and risk management of participating life insurance contracts within a heterogeneous insurance pool. The insurance pool consists of so called professional policyholders whose contracts are handled by a professional agent and non-professional policyholders who manage their contract on their own. All professional policyholders surrender by following the optimal stopping rule which maximizes their contract value while non-professionals surrender due to exogenous reasons and surrender contagion. Within a two-stage valuation scheme, first, an optimal stopping problem describes the value of professionally managed contacts, which we solve numerically by a Least Squares Monte-Carlo algorithm. Second, we obtain the contract value of non-professionals by a simple Monte-Carlo simulation which incorporates the optimal surrender strategy of the professionals obtained in the first stage.

Our numerical results show that the impacts of contagion on policy and equity values crucially depend on the strictness of the early default regulatory framework and the insurance company's investment strategy. In a moderate regulatory framework, professionals' surrender poses an informative signal to non-professional policyholders. Contagion leads non-professionals to surrender in states where the surrender benefits exceed their contract's expected future payoffs, which, reduces the asset value per policyholder, increases the probability of early default, and, eventually harms the value of equity. However, in a strict regulatory framework, contagion slightly reduces the value of non-professionals' contract. Since strict early default regulation induces professionals to surrender (most of time) in states where the asset value per policyholder lies above the surrender value, their leave helps the insurer accumulate more assets to be shared between the remaining non-professional policyholders and equity holders, which lowers not only the probability of early default but also the value of the surrender option. Following up professionals triggers a wealth transfer from non-professionals to equity holders. As for investment risk in the form of volatility, we find that in most cases, namely with relatively low volatility values, contagion benefits non-professionals by enabling them to profit from following the professionals' surrender fast enough, which renders professionals' surrender as a valuable signal. However, as the insurer conducts risky investment, the same as in a strict regulatory framework, we find that contagious surrenders not only shift value from non-professional policyholders' contracts to equity, but also lower the company's early default probability.

Concerning future research, one could calibrate the model to assess the actual impacts of solvency regulation and capital requirements on the policy as well as equity values. This could quantify surrender risk more accurately by allowing for contagion dynamics among heterogeneous policyholders. Moreover, for technical tractability, we assumed one optimal surrender strategy suggested by a financially professional agent and followed by all professional policyholders in the present paper. Relaxing the assumption by assuming multiple financially professional agents, analyzing strategic interactions among them from a game-theory perspective, and incorporating the strategic interactions into pricing life insurance policies also deserve much analysis in this line of research.

A Proofs

A.1 Proof for Result 3.1

With the smoothness assumption on the value function v^{pro}, its characterization in terms of a free boundary value problem stated in Result 3.1 follows directly from the Markov approach of optimal stopping, see Peskir and Shiryaev (2006, Section 8). This proof gives the reader an intuition about the link between the free boundary value problem and the value function of the corresponding optimal stopping problem.

Let us consider some time $t \in [0, T]$. Conditioned on $\{t < \tau_D \wedge \tau_P\}$ and by no-arbitrage, the value process satisfies

$$rv^{pro}(X_t^\infty)\,\mathrm{d}t = \mathbb{E}[\mathrm{d}v^{pro}(X_t^\infty)|\mathcal{F}_t]\,, \tag{21}$$

where $\mathrm{d}v^{pro}(X_t^\infty)$ denotes the forward looking marginal dynamics of the value process. The Itô formula for semi-martingales provides

$$\begin{aligned} v^{pro}(X_u^\infty) - v^{pro}(X_t^\infty) = &\int_t^u \Big(\partial_t + r\,a\partial_a + \frac{a^2\sigma^2}{2}\partial_a^2 - \beta\, m\partial_m\Big) v^{pro}(X_s^\infty)\,\mathrm{d}s \\ &+ \int_t^u \sigma a \partial_a v^{pro}(X_s^\infty)\,\mathrm{d}W_s \\ &+ \int_t^u \Big(\mathbb{1}_{\{M_{s-}^\infty \in [B-1,\,B),\ B_s=1\}} v^{pro}(t, A_{s-}^\infty - S(t, A_{s-}^\infty, 1), M_{s-}^\infty + 1, N_{s-}^\infty + 1) \\ &+ \mathbb{1}_{\{M_{s-}^\infty \in [B-1,\,B),\ B_s=1\}^c}\ v^{pro}(t, A_{s-}^\infty - S(t, A_{s-}^\infty, 1), 0, N_{s-}^\infty + 1) - v^{pro}(X_{s-}^\infty)\Big)\,\mathrm{d}N_s^\infty\,, \end{aligned} \tag{22}$$

where, in expectation, the martingale parts vanish, the jumps of N_s^∞ are averaged out, and realizations of B_t are weighted by their respective probabilities so that

$$\begin{aligned} \mathbb{E}[v^{pro}(X_u^\infty) - v^{pro}(X_t^\infty)|\mathcal{F}_t] = \mathbb{E}\Big[\int_t^u \Big(&\big(\partial_t + r\,a\partial_a + \frac{a^2\sigma^2}{2}\partial_a^2 - \beta\, m\partial_m\big) v^{pro}(X_s^\infty) \\ &+ (Z^{non} - n)\lambda(M_t^\infty)\Delta v^{pro}(X_{s-}^\infty)\Big)\,\mathrm{d}s\Big|\mathcal{F}_t\Big]\,. \end{aligned} \tag{23}$$

As u tends to t, we obtain the differential characterization

$$\begin{aligned} &\mathbb{E}[\mathrm{d}v^{pro}(X_t^\infty)|\mathcal{F}_t] = \\ &\quad\Big[\Big(\partial_t + ra\partial_a + \frac{a^2\sigma^2}{2}\partial_a^2 - \beta m\partial_m\Big) v^{pro}(X_t^\infty) + (Z^{non} - n)\lambda(M_t^\infty)\Delta v^{pro}(X_t^\infty)\Big]\,\mathrm{d}t\,. \end{aligned} \tag{24}$$

Each state $x = X_t^\infty$ with $v^{pro}(x) > g(x)$ implicitly satisfies $t < \tau_D \wedge \tau_P$. Hence, (21) and (24) together yield the partial differential equation in (11). States x with $v^{pro}(x) \leq g(x)$ are in the optimal stopping region P, and, since τ_P is an optimal stopping time in (10), (12) follows with

$$v^{pro}(x) = \mathbb{E}_x\left[e^{-r(\tau_P\wedge\tau_D - t)} g(X^\infty_{\tau_P\wedge\tau_D})\right] = \mathbb{E}_x\left[g(X_t^\infty)\right] = g(x)\,. \tag{25}$$

□

A.2 Proof for Result 3.2

First, we rewrite the value function for non-professionals given in (17). By grouping the contract's payoffs according to the time that they occur before or after the professional policyholders' surrender time τ_P, we can rewrite the contract value of the representative non-professional policyholder as follows:

$$v^{non}(x) = \mathbb{E}_x\left[e^{-r(\tau_1\wedge\tau_D - t)}\left(g(\tau_1, X^*_{\tau_1\wedge\tau_D})\mathbb{1}_{\{\tau_1\wedge\tau_D\leq\tau_P\}} + \tilde{v}^{non}(X^*_{\tau_P})\mathbb{1}_{\{\tau_1\wedge\tau_D>\tau_P\}}\right) \mid N_t^1 = 0\right],$$

where $\tilde{v}^{non}$ presents the contract value for the non-professionals after the professionals have surrendered. Now, we introduce the process $N_t^{**} = \sum_{i=2}^{Z^{non}} N^i$ and denote the adjusted processes $X_t^{**} = (t, A_t^{**}, N_t^{**}, M_t^{**})$ where N_t is replaced by N_t^{**}. Hence, X_t^{**} describes the underlying dynamics after the departure of the professionals. The contract value $\tilde{v}^{non}$ for non-professionals given that the processionals have departed is

$$\tilde{v}^{non}(x) = \mathbb{E}_x\left[e^{-r(\tau_1\wedge\tau_D - t)} g(\tau_1, X^{**}_{\tau_1\wedge\tau_D}) \mid N_t^1 = 0\right], \tag{26}$$

for initial states $x = (t, a, n, m)$ with $n \geq Z^{pro}$.

We argue that $\tilde{v}^{non}$ satisfies (20) and, then, show that v^{non} satisfies (18). Consider the following contract value process, $\tilde{V}_t = \tilde{v}^{non}(X_t^{**}) + \mathbb{1}_{\tau_1\leq t}(S(t, A_t^{**}, 1) - \tilde{v}^{non}(X_t^{**}))$ for $t < T$, at which the contract value is given by the value function $\tilde{v}^{non}(X_t^{**})$ if the representative non-professional policyholder does not surrender, and equals the surrender value otherwise. Du to no-arbitrage, the value process satisfies

$$r\tilde{v}^{non}(X_t^{**})\,\mathrm{d}t = \mathbb{E}[\mathrm{d}v^{non}(X_t^{**}) + \left(S(t, A_t^{**}, 1) - \tilde{v}^{non}(X_t^{**})\right)\mathrm{d}\mathbb{1}_{\tau_1\leq t}|\mathcal{F}_t]\,, \tag{27}$$

conditioned on $\{t < \tau_D\}$. Itô's Lemma provides

$$\begin{aligned}\mathbb{E}[\mathrm{d}\tilde{v}^{non}(X_t^{**})|\mathcal{F}_t] =\Big[&\frac{\partial \tilde{v}^{non}}{\partial t} + r\, A_t^{**}\frac{\partial \tilde{v}^{non}}{\partial a} + \frac{1}{2}A_t^{**2}\,\sigma^2\frac{\partial^2 \tilde{v}^{non}}{\partial a^2} - \beta\, M_t^{**}\frac{\partial \tilde{v}^{non}}{\partial m} \\ &+ (Z - N_t^{**} - 1)\,\lambda(M_t^{**})\Delta\tilde{v}^{non}(X_t^{**})\Big]\,\mathrm{d}t\,,\end{aligned} \tag{28}$$

where the jump component is averaged out and, thus, replaced by its corresponding jump intensity. Additionally, by Bielecki and Rutkowski (2004, Proposition 5.1.3), we have

$$\mathbb{E}[\mathrm{d}(S(t, A_t^{**}, 1) - \tilde{v}^{non}(X_t^{**})\,\mathrm{d}\mathbb{1}_{\tau_1 \le t}|\mathcal{F}_t] = \big(S(t, A_t^{**}, 1) - \tilde{v}^{non}(X_t^{**})\big)\lambda(M_t^{**})\,\mathrm{d}t\,, \tag{29}$$

so that (29) and (28) inserted to (27) provide the differential characterization (20) of $\tilde{v}^{non}$. By its definition, $\tilde{v}^{non}$ must satisfy the boundary conditions (15) and (16). Finally, consider the value process for non-professionals given that professionals have not surrendered their contracts, so that the contract value process takes the form $V_t = v^{non}(X_t^{**}) + \mathbb{1}_{\tau_1 \le t}\big(S(t, A_t^{**}, 1) - v^{non}(X_t^{**})\big)$ for $t < T \wedge \tau_P$. Note, that we do not include the jump caused by professional surrenders at time τ_P to the underlying dynamics. However, by definition, v^{non} satisfies (19) on $P \setminus D$, which incorporates the surrender event of professionals. With the arguments used in the first part of the proof we obtain

$$\begin{aligned}rv^{non}(X_t^{**})\,\mathrm{d}t = \Big[&\frac{\partial v^{non}}{\partial t} + r\, A_t^{**}\frac{\partial v^{non}}{\partial a} + \frac{1}{2}A_t^{**2}\,\sigma^2\frac{\partial^2 v^{non}}{\partial a^2} - \beta\, M_t^{**}\frac{\partial v^{non}}{\partial m} \\ &+ (Z - N_t^{**} - 1)\,\lambda(M_t^{**})\Delta v^{non}(X_t^{**})\big(S(t, A_t^{**}, 1) - v^{non}(X_t^{**})\big)\lambda(M_t^{**})\Big]\,\mathrm{d}t,\end{aligned} \tag{30}$$

which justifies that v^{non} satisfies (18) for $x \in E \setminus (D \cup P)$. Note that, if the underlying dynamics enter the optimal surrender region of the professionals P, the boundary condition (19) induces the professional surrenders and shifts the valuation problem to $\tilde{v}^{non}$. In total, the boundary conditions (15) and (16) account for default and maturity payoffs.

□

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