Self-Enforcing Peace Agreements that Preserve the Status Quo

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Abstract: On the basis of a single-period, guns-versus-butter, complete-information model in which two agents dispute control over an insecure portion of their combined output, we study the choice between a peace agreement that maintains the status quo without arming (or unarmed peace) and open conflict (or war) that is possibly destructive. With a focus on outcomes that are immune to both unilateral deviations and coalitional deviations, we find that, depending on war's destructive effects, the degree of output security and the initial distribution of resources, peace can, but need not, emerge in equilibrium. We also find that, while *ex ante* resource transfers without commitment can improve the prospects for peace, war remains the unique equilibrium in pure strategies when the initial distribution of resources is sufficiently uneven.

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1 Introduction

Why do nearly all countries build military forces even when doing so represents a significant diversion of productive resources?[1] An obvious answer is that the absence of a higher authority to bring disputes to a peaceful resolution compels countries to prepare for the possibility of war. Yet, the decision to go to war is itself endogenous, and the fact that countries choose war when alternative, less costly means of conflict resolution are available would seem even more puzzling. One prominent explanation of this puzzle centers on commitment problems (Fearon, 1995).[2] In particular, the non-enforceabilty of a current negotiated settlement regarding the future distribution of resources or output can, under some circumstances, undermine the possibility of peace today. As analyzed by Powell (2006) among others, when it is not possible to enforce deals for the future and there are expected future shifts in power, the party expecting to lose power could be driven to engage in war today, despite a short-run preference for peace due to war's destructive effects. Others have emphasized that negotiated agreements, made today to divide whatever is being contested on the basis of resources allocated to arms by each side, is welfare improving in a single-period setting. But, such negotiations can settle only a current dispute. Insofar as the dispute is ongoing, the maintenance of peace over time requires repeated negotiations in the future and, with that, the diversion of additional resources from production to arming. Given the implied costs, one or both sides might prefer to end the dispute today once and for all (or at least to severely weaken the opponent in future interactions) by declaring war, again despite war's destructive effects (Garfinkel and Skaperdas, 2000; McBride and Skaperdas, 2014).

In this paper, we further analyze commitment problems associated with peace. Our analysis departs from much of the existing literature in that we focus on short-run commitment problems and on the sort of peace that amounts to the preservation of the status quo. Specifically, we consider a single-period, complete information setting, with pre-play communication, in which imperfect security of output gives rise to a possible conflict between two risk-neutral agents (or countries). Agents simultaneously choose the allocation of their respective resource endowments to arming and to the production of consumables, and whether or not to declare war in an effort to seize all of the insecure output produced by both. If at least one agent chooses war, then war emerges, possibly resulting in the destruction of insecure and possibly secure output. Peace, which in contrast requires both agents to choose it, results in no destruction, allowing each agent to enjoy all of his/her own output. It turns out that, in this setting, war is always a Nash equilibrium.

[1] According to Barbey (2015), only 26 countries have no armies of their own; and, of those countries, 7 have "defense or friendship treaties" with other countries.

[2] See Jackson and Morelli (2011) for a relatively recent survey of the literature, including contributions by economists as well as political scientists, on this and alternative rationales for war.

Our primary objective in this paper is to explore the conditions under which peace is a *stable* equilibrium in the sense of being "coalition-proof" (Bernheim et al., 1987)—i.e., a Nash equilbrium that is immune not only to unilateral deviations but also to coalitional deviations. We show that there exist sufficiently uneven distributions of resource endowments for which the less affluent agent strictly prefers war, whereas the richer agent prefers peace. But, even when the distribution is sufficiently even to render peace Pareto optimal and the agents can communicate before they make their decisions, peace need not emerge as the stable equilibrium. In particular, because peace preserves the status quo and, as a consequence, the contending agents do not benefit from arming, no agent has an incentive to arm under peace. Herein lies an important problem: if an agent does not arm in anticipation of peace, his rival could find it appealing to expand his military capacity and declare war. The appeal of such a deviation, which is more likely to hold for the less affluent agent, naturally undermines the stability of unarmed peace.[3]

We show how the degree of output security, the pattern of war's destructive effects on secure vs. insecure output, and the configuration of initial resource endowments matter for equilibrium arming decisions and the choice between war and peace. In the extreme case where war involves no destruction, peace is simply ruled out for any degree of (imperfect) output security and any distribution of resource endowments across agents.[4] However, even when war is destructive, peace could be vulnerable to unilateral deviations for all resource distributions, thereby wiping out the possibility of self-enforcing peace agreements. Such an outcome is more likely when war's destructive effects are mild and the degree of output insecurity is high. Otherwise, unarmed peace can emerge as the stable equilibrium. What is required additionally in this case is that the distribution of initial resource endowments is sufficiently even. The greater is the extent of war's destruction, the wider is the range of resource distributions that make unilateral deviations unprofitable. Likewise, greater output security makes peace more likely to emerge in equilibrium.

In our setting where the contending agents can communicate with each other prior to making decisions, it seem natural to also consider the possible role of *ex ante* resource transfers that do not involve any sort of commitment by either agent to subsequently choose peace. We find that, by reducing the disparity in resource endowments between agents, such transfers from the more affluent agent to the poorer agent can expand the range of initial resource distributions under which peace is immune to unilateral devia-

[3]Without denying the relevance of deterrence motives for arming as considered by Powell (1993), Jackson and Morelli (2009), and De Luca and Sekeris (2013), we are particularly interested in studying the stability of unarmed peace and thus the conditions under which arming is not necessary for deterring one's rival.

[4]It might seem a bit far fetched to suppose that war could be nondestructive. However, we can think of conflict more broadly as it manifests itself in, for example, disputes between lobbyists who contest political influence or between litigants in court cases. The possibility of no destruction in such contexts as in the present paper means that the cost of "war" is limited to the resources used to "fight."

tions and thus under which peace agreements are self-enforcing. However, this pacifying effect is operational only when peace without transfers is possible for at least some resource distributions. Moreover, this effect could be smaller when output is more secure, which suggests that higher output security need not be conducive to unarmed peace.

Our analysis is most closely related to Beviá and Corchón (2010), who similarly explore the choice between war and peace that preserves the status quo in a one-period setting. Some of the assumptions and thus the central results are quite different, however. For example, they do not consider how destruction and partial security of output matter for peace. More importantly, these authors assume that the contending agents first make their peace/war choice; and, when at least one agent chooses war, both agents arm for war. As a result, Beviá and Corchón (2010) do not need to consider the possibility that one or both agents could find a unilateral deviation appealing. Put differently, they abstract from the commitment problem that can undermine peace and that is central to our analysis. In this light, it is perhaps not surprising that they find that, even when war is not destructive, peace with (as well as without) *ex ante* resource transfers can prevail for at least sufficiently symmetric distributions of resources. Similarly, Jackson and Morelli (2007) study the choice between war and peace identified with the status quo, though with an aim to explore the role of political biases within a country that arise when the decision maker of the country stands to gain relatively more from a victory. In any case, while accounting for war's destructive effects, they abstract from the agents' arming decisions and thus a key component of the commitment problem that tends to detract from the possibility of peace in our setting. Indeed, in the baseline version of their model that includes destruction but not the distorting effects of political biases on decision making, peace can always be supported by transfers, in contrast to what we find.

In what follows, the next section presents a basic one-period model of conflict over output, including the two modes of conflict resolution (war and peace). In Section 3, we study arming incentives and payoffs under each of these modes. The analysis in Section 4 identifies and characterizes the conditions, with and without transfers, under which peace arises in this one-period setting as the stable equilibrium outcome. Section 5 considers a variety of extensions (including diminishing returns, opportunity for mutually advantageous trade, and preexisting military capabilities) to check the robustness of our results. Section 6 concludes. All technical details appear in appendices.

2 Disputing the Distribution of Insecure Output

Consider a one-period, complete-information setting in which there are two risk-neutral agents, $i = 1, 2$. Each agent can be thought of as an individual or a collectivity (e.g., a group or nation).[5] At the beginning of the period, agent i is endowed with R^i units of a productive resource, where $\overline{R} \equiv R^1 + R^2$ denotes the aggregate amount of the resource across the two agents. This resource can be transformed, on a one-to-one basis, into X^i units of "butter" for consumption. But, not all output is secure. Specifically, while a fraction $\sigma \in [0,1)$ of each agent's butter X^i is secure, the remaining fraction $1-\sigma \in (0,1]$ is insecure and contestable. The contestability of output possibly motivates war with its associated production of "guns" (or arms), denoted by G^i and also produced on a one-to-one basis using R^i.

The game is structured as follows: Each agent i simultaneously chooses whether or not to declare war and the allocation of his resource R^i to the production of G^i units of guns, leaving $X^i = R^i - G^i \geq 0$ units of the resource to produce butter.[6] The outcome of peace, which requires that neither agent declares war, supports the status quo, letting each agent consume his entire output of butter, X^i. However, if at least one agent declares war, then each agent deploys his guns to contest the sum of insecure output $(1-\sigma)\overline{X}$, where $\overline{X} \equiv X^1 + X^2$ denotes total output. Importantly, war results in the destruction of a fraction $1-\gamma\beta \in [0,1)$ of insecure output and a fraction of $1-\beta \in [0,1)$ of secure output. Hence, $\beta \in (0,1]$ represents the overall rate at which output generally survives war and $\beta\gamma \in (0,1]$ indicates the survival rate of contested output. The parameter $\gamma \in (0,1]$, then, reflects the possible difference in survival rates of contested and uncontested output, with $\gamma = 1$ implying no difference at all and decreases in γ implying greater differential destruction of contested output.[7]

In this setting, we model war as a "winner-take-all" contest over $(1-\sigma)\overline{X}$. The probability that agent i wins depends on the arming choices by both agents, $\phi^i = \phi^i(G^i, G^j)$ for $i = 1,2$ and $j \neq i$. More precisely, letting $\overline{G} \equiv G^1 + G^2$ denote the aggregate quantity of guns chosen, agent i's probability of winning is specified as follows:

$$\phi^i = \phi^i(G^i, G^j) = \begin{cases} G^i/\overline{G} & \text{if} \quad \overline{G} > 0 \\ R^i/\overline{R} & \text{if} \quad \overline{G} = 0 \end{cases}, \; i \neq j = 1,2. \tag{1}$$

According to this specification of the conflict technology (also referred to as the "contest

[5]In the case that each agent represents group of individuals, we assume that the decision maker acts in the interest of the collectivity, thereby abstracting from the political biases studied in Jackson and Morelli (2007) as well as from collective action problems.

[6]With a focus on a single-stage game, our analysis abstracts from some issues that can arise if agents are allowed to communicate between the arming and war decisions. While these issues, which are taken up in Section 5, do not affect our characterization of the conditions under which unarmed peace is stable, they do matter in the case of war. Note further that there is no "first-mover" advantage in war.

[7]This parameterization is sufficiently flexible to capture several interesting cases, including one where no output is subject to destruction ($\beta = \gamma = 1$) and another where only contested output is subject to destruction ($\beta = 1$ and $\gamma < 1$).

success function," CSF), when $\overline{G} > 0$, the winning probability for agent i is increasing in his own guns ($\phi^i_{G^i} > 0$) and decreasing in the guns of his rival ($\phi^i_{G^j} < 0$). Equation (1) also implies that $\phi^i(G^i, G^j)$ is symmetric, so that $G^i = G^j = G > 0$ implies $\phi^i = \phi^j = \frac{1}{2}$, and is concave in G^i. For $\overline{G} > 0$, it implies that $\phi^i_{G^iG^j} \gtreqless 0$ as $G^i \gtreqless G^j$ for $i \neq j = 1, 2$.[8] In contrast, when $\overline{G} = 0$ so that no guns are deployed, there is no destruction and each agent's winning probability (given war is declared) is determined by his initial resource relative to the aggregate resource.

For any given guns chosen in the first stage (G^i, G^j), agent i's payoff under peace V^i is

$$V^i = X^i, \quad \text{for } i = 1, 2, \tag{2}$$

where $X^i = R^i - G^i$. As this expression shows, agent i derives no benefit from guns G^i in the case of peace, and the production of guns is costly as it diverts resources away from producing butter. Matters differ, however, in the case of war. In particular, agent i's expected payoff under war U^i is

$$U^i = \phi^i \beta\gamma(1-\sigma)\overline{X} + \beta\sigma X^i, \quad \text{for } i = 1, 2, \tag{3}$$

where $\overline{X} = \sum_i X^i = \sum_i (R^i - G^i) = \overline{R} - \overline{G}$. Guns production by agent i positively influences his payoff through his probability of winning ϕ^i as shown in (1), but also negatively through his residual resource X^i that also negatively impacts $\overline{X}$. Importantly, when $G^i = G^j = 0$ so that, by assumption, $\gamma = \beta = 1$, our specification in (1) implies $V^i = U^i$.

3 Arming and Payoffs Given War and Peace

In this section, we first characterize the agents' optimizing choices of arming under peace and war. We then turn to analyze the resulting payoffs, which provide the groundwork for our analysis of the stability of peace when agents are allowed to communicate with each other prior to making any decisions.

3.1 Arming Incentives Under Peace and War

Arming is always costly in that it draws resources away from the production of butter. However, its benefits depend on whether peace prevails or war breaks out. Let G^i_k denote equilibrium arming when peace ($k = p$) or war ($k = w$) is anticipated in the second stage. From (2), when peace is anticipated by both agents, arming yields no benefits; and, as such,

[8] See Tullock (1980), who first introduced this functional form. Skaperdas (1996) axiomatizes a general class of such functions, $\phi(G^i, G^j) = f(G^i) / \sum_{k=1}^{2} f(G^k)$, assuming only that $f(\cdot)$ is non-negative and increasing. One commonly used specification, studied by Hirshleifer (1989), is the "ratio success function," where $f(G) = G^m$ with $m > 0$. The results to follow remain qualitatively unchanged under this more general specification with $m \in (0, 1]$. But, to maintain clarity, we focus on the specification in (1) with $m = 1$.

neither agent arms: $G^i_p = 0$ for $i = 1, 2$.[9]

In the case of war, by contrast, arming does generate a benefit, as well as a cost. The extent to which agent i arms in this case depends on the solution to $\max_{G^i} U^i$ subject to $X^i = R^i - G^i \geq 0$. Differentiating (3) with respect to G^i shows:

$$\frac{\partial U^i}{\partial G^i} = \phi^i_{G^i} \beta\gamma(1-\sigma)\overline{X} - \left[\phi^i \beta\gamma(1-\sigma) + \beta\sigma\right] \quad \text{for } i = 1, 2. \tag{4}$$

The first term on the right-hand side (RHS) of the expression above represents the marginal benefit of arming for agent i (MB^i), due to the effect of G^i (given G^j) to increase his probability of winning the pool of insecure butter net of destruction. MB^i is increasing in the survival rate of contested output ($\gamma\beta \uparrow$) and in the insecurity of output ($\sigma \downarrow$), as well as in the aggregate resource ($\overline{R}$) through its influence on $\overline{X}$. The second term on the RHS of the expression represents the agent's marginal cost of arming (MC^i) in terms of foregone production of butter that would otherwise add to the pool of insecure output $(1-\sigma)\overline{X}$ and to the agent's own secure output σX^i. Like MB^i, MC^i is increasing in the survival rate of contested output ($\gamma\beta \uparrow$). But, an increase in the overall rate of destruction alone ($\beta \downarrow$) does not influence the net marginal benefit of arming in (4). What matters instead is the rate of differential destruction of insecure output, reflected in $\gamma \leq 1$. Specifically, as will become clear shortly, an increase in γ amplifies arming incentives. In addition, MC^i is increasing in the security of output ($\sigma \uparrow$). Combining this effect with the aforementioned effect on MB^i shows that an increase in output security dampens an agent's incentive to arm.

Based on (4) along with the resource constraint $X^i = R^i - G^i \geq 0$ and the conflict technology (1), agent i's best reply to agent j's arming choice can be written as follows:

$$B^i_w(G^j; \gamma, \sigma, R^i, \overline{R}) = \min\left\{R^i, \widetilde{B}^i_w(G^j)\right\}, \quad \text{for } i \neq j = 1, 2, \tag{5a}$$

where $\widetilde{B}^i_w(G^j)$ denotes agent i's *unconstrained* best-response function, implicitly defined by the condition $\partial U^i / \partial G^i = 0$ and given by

$$\widetilde{B}^i_w(G^j) = -G^j + \sqrt{G^j \theta \overline{R}}, \tag{5b}$$

where

$$\theta \equiv \frac{\gamma(1-\sigma)}{\gamma(1-\sigma)+\sigma} \in (0, 1] \tag{5c}$$

[9] As shown by De Luca and Sekeris (2013), when each agent arms under peace to deter the other agent from declaring war, no pure-strategy equilibrium exists. We focus on the possibility of unarmed peace to maintain comparability with Beviá and Corchón (2010), while studying the commitment problem discussed in the introduction.

reflects the importance of his contribution to the pool of contested output net of destruction relative to his total output, again net of destruction.[10] Consistent with our discussion above in relation to (4), an increase in the overall rate of output destruction ($\beta \downarrow$) has no consequences for equilibrium arming since it affects the marginal benefit and marginal cost of arming equi-proportionately. By contrast, differential destruction does matter along with the insecurity of output. In particular, an increase in θ, due to a smaller differential between the rates of destruction of contested and uncontested output ($\gamma \uparrow$) and/or greater output insecurity ($\sigma \downarrow$), fuels arming incentives. All else the same, such incentives are largest when $\sigma = 0$ implying $\theta = 1$ or, given some output security ($\sigma > 0$), when $\gamma = 1$ implying $\theta = 1 - \sigma$.

Of course, resource constraints matter here as well for arming choices. Using (5b) while explicitly taking into account agent i's resource constraint, we define the following:

$$R_L = \tfrac{1}{4}\theta\overline{R} \leq \tfrac{1}{4}\overline{R} \quad \text{and} \quad R_H = (1 - \tfrac{1}{4}\theta)\overline{R}, \tag{6}$$

where the subscripts "L" and "H" are used to designate the "low "and "high" threshold levels of resources. Together, these threshold levels define the parameter space for which one or neither agent is resource constrained in the production of guns. In particular, when $R^i, R^j \in [R_L, R_H]$, neither agent is resource constrained. If, however, $R^i \in (0, R_L)$ implying $R^j \in (R_H, \overline{R})$, then agent i is constrained, while his rival (j) is not.[11]

Equations (5) and (6) give us the following:

Proposition 1 (Arming under war.) *Assume output is not perfectly secure ($\sigma < 1$) and both agents anticipate war. Then, there exists a unique equilibrium in arming, with positive quantities of guns produced by both agents $G_w^i > 0$, $i = 1, 2$. For any given $\overline{R}$ such that $R^i + R^j = \overline{R}$ ($i \neq j = 1, 2$), these quantities have the following properties:*

(a) *If $R^i \in [R_L, R_H]$ for $i = 1, 2$, then $G_w^i = R_L$, with $dG_w^i/d\theta > 0$.*

(b) *If $R^i \in (0, R_L)$ for $i \neq j = 1$ or 2, then $G_w^i = R^i$ and $G_w^j = \widetilde{B}_w^j(R^i) > G_w^i$, with $dG_w^j/d\theta > 0$.*

Clearly, the distribution of $\overline{R}$ across the two agents can matter for their equilibrium arming choices. However, as established in part (a), if the distribution of initial resources is sufficiently even such that neither agent is resource constrained, then they choose an identical

[10] To avoid notational cluttering, we suppress the dependence of agent i's unconstrained best-response function on γ, σ, R^i and $\overline{R}$.

[11] In our setting, both agents cannot be resource constrained at the same time. However, that would be possible, if we were to follow Beviá and Corchón (2010) in supposing that, after resource allocations to guns have been made and the outcome of war is determined, some fraction guns $1 - k > 0$ could be "recovered" and consumed by the victor, provided k is sufficiently small. But, as indicated below, our central results that effectively assume $k = 1$ would remain intact if we assumed instead that $k < 1$.

amount of guns. What's more, transfers of the initial resource from one agent to the other (leaving $\overline{R}$ unchanged) have no effect on equilibrium arming choices, provided the transfer does not make one of them resource constrained. By contrast, as shown in part (b), when one agent is constrained (i), the equilibrium is asymmetric, with the unconstrained agent (j) naturally arming by more. In this case, an exogenous transfer of resources from the unconstrained agent (j) to the constrained agent (i) tends to dampen differences in their arming choices, whereas transfers in the other direction tend to amplify such differences. Whether the distribution is sufficiently even or uneven, equilibrium arming by an unconstrained agent depends positively on θ—or, more precisely, positively on the survival rate of contested output in war ($\gamma \uparrow$ given $\beta \leq 1$) and on the insecurity of output ($\sigma \downarrow$). Finally, observe from (6) with (5c) that such parameter changes also shrink the range of distributions $R^i \in [R_L, R_H]$ for which neither agent is resource constrained.

3.2 Payoffs under Peace and War

We now turn to explore the implications of the above for payoffs, again given war or peace. The finding that neither agent arms under peace implies, from (2), that

$$V_p^i = R^i, \ \text{ for } i \neq j = 1, 2, \tag{7}$$

which depends only on R^i, positively and linearly so.

Under war where the two agents arm according to Proposition 1, their expected payoffs U_w^i depend on the distribution of the resource $\overline{R}$ as well as on the survival rate of output under war (reflected in β and γ) and the degree of security of output (σ). In particular, as shown in Appendix A, an application of Proposition 1 to (3) using the technology of conflict (1) while keeping in mind that $R^j = \overline{R} - R^i$ yields

$$U_w^i(R^i) = \begin{cases} \beta\gamma(1-\sigma)R^i\left(\sqrt{\frac{\overline{R}}{\theta R^i}} - 1\right) & \text{if } R^i \in (0, R_L) \\ \frac{1}{4}\beta\gamma\,(1-\sigma)\,\overline{R} + \beta\sigma R^i & \text{if } R^i \in [R_L, R_H] \\ [\beta\gamma(1-\sigma) + \beta\sigma]\,\overline{R}\left(1 - \sqrt{\frac{R^j\theta}{\overline{R}}}\right)^2 & \text{if } R^i \in (R_H, \overline{R}), \end{cases} \tag{8}$$

for $i \neq j = 1, 2$, which in turn gives us:

Proposition 2 (Payoffs under war.) *Assuming both agents anticipate war and arm accordingly, their payoffs have the following properties.*

(a) *If $R^i \in [R_L, R_H]$ for $i = 1, 2$, then:* (i) $dU_w^i/dR^i \geq 0$ *as* $\sigma \geq 0$ *with* $d^2U_w^i/(dR^i)^2 = 0$*;* (ii) $dU_w^i/d\beta > 0$*;* (iii) $dU_w^i/d\gamma > 0$*; and* (iv) $dU_w^i/d\sigma \geq 0$ *when* $\gamma \leq \frac{1-2\sigma}{1-\sigma}$ *and otherwise* ($\gamma > \frac{1-2\sigma}{1-\sigma}$) $dU_w^i/d\sigma < 0$ *for R^i sufficiently close to R_L.*

(b) *If $R^i \in (0, R_L)$ for $i \neq j = 1$ or 2, then:* (i) $dU_w^i/dR^i > 0$ *with* $d^2U_w^i/(dR^i)^2 < 0$ *and* $\lim_{R^i \to 0} U_w^i = 0$, *whereas* $dU_w^j/dR^j > 0$ *with* $d^2U_w^j/(dR^j)^2 > 0$ *and* $\lim_{R^j \to \overline{R}} U_w^j = [\beta\gamma(1-\sigma) + \beta\sigma]\,\overline{R}$; (ii) $dU_w^i/d\beta > 0$ *and* $dU_w^j/d\beta > 0$; (iii) $dU_w^i/d\gamma > 0$ *and* $dU_w^j/d\gamma > 0$; *and* (iv) $dU_w^i/d\sigma > 0$ *when* $\gamma \leq \frac{1-2\sigma}{2(1-\sigma)}$ *and otherwise* ($\gamma > \frac{1-2\sigma}{2(1-\sigma)}$) $dU_w^i/d\sigma > 0$ *only when* $\gamma < \frac{1-2\sigma}{1-\sigma}$ *and* R^i *is sufficiently close to* R_L, *while* $dU_w^j/d\sigma > 0$ *for all* σ *and* γ.

This proposition shows that an agent's payoff under war depends positively on his resource endowment. Specifically, the first component of part (a) shows that, even when the distribution of resources is sufficiently even such that neither agent is constrained in his arming and they arm identically, their payoffs will differ provided that some fraction of output is secure ($\sigma > 0$). A transfer of resources from agent i to j has no effect on equilibrium arming, but makes i worse off and j better off, again provided $\sigma > 0$. Similarly, the first component of part (b) shows that when agent i is resource constrained in his arming, he is generally worse off than his unconstrained opponent j, and the difference in payoffs increases as the distribution of $\overline{R}$ shifts towards j. The second and third components of both parts establish that , whether or not an agent is resource constrained, his payoff falls as war becomes more destructive ($\beta \downarrow$ and/or $\gamma \downarrow$). Finally, the fourth components of parts (a) and (b) taken together show that an improvement in the security of output ($\sigma \uparrow$) always increases the payoffs of the more affluent agent. In addition, when differential destruction is sufficiently large $\gamma \leq \frac{1-2\sigma}{2(1-\sigma)}$, such improvements also benefit the less affluent agent. Otherwise, the effect on the less affluent agent depends on parameter values as well as the distribution of resource endowments.[12] In particular, there exists a critical value of R^i above which improvements in output security make agent i better off and below which the agent is worse off. In the case that $\gamma \in (\frac{1-2\sigma}{2(1-\sigma)}, \frac{1-2\sigma}{1-\sigma})$, this critical value falls within the range $(0, R_L)$; and, in the case that $\gamma \geq \frac{1-2\sigma}{1-\sigma}$, it falls within the range $[R_L, \frac{1}{4}\bar{R}]$.

The effects of σ and γ on U_w^i are illustrated in Fig. 1, which depicts the payoffs under war under various distributions of resources in pink.[13] Panel (a) focuses on the benchmark case where there is no destruction ($\gamma = \beta = 1$), showing that an improvement in output security ($\sigma \uparrow$) results in a counterclockwise rotation of the payoff function $U_w^i(R^i)$ at the initial value of R_L.[14] Panel (b) shows the effect of a decrease in the differential survival rate γ (given $\beta = \frac{9}{10}$) to rotate $U_w^i(R^i)$ in a clockwise direction at $R^i = 0$.[15]

[12]The condition $\gamma > (1-2\sigma)/2(1-\sigma)$ is necessarily satisfied if either output is moderately secure ($\sigma \geq \frac{1}{2}$) or the differential survival rate of contested output is moderately high ($\gamma > \frac{1}{2}$).

[13]Ignore the blue and green curves for now.

[14]Observe, from (5c) and (6), that an increase in σ also decreases θ and hence R_L, and accordingly increases R_H, as illustrated in Fig. 1(a).

[15]One can similarly visualize the effect of a decrease in β as a clockwise rotation of $U_w^i(R^i)$ at $R^i = 0$.

3.3 Comparing Payoffs under Peace and War

Drawing on our analysis above, we now compare payoffs for each agent $i = 1, 2$ across the peace and war outcomes as they depend on the distribution of resource endowments $\overline{R}$, the security of output σ, and the survival rate of output in war determined jointly by β and γ. Clearly, agent i prefers war when $U_w^i(R^i) > V_p^i(R^i)$ and otherwise prefers peace. Taking into account that the shape of $U_w^i(R^i)$ depends on where R^i falls within the distribution of $\overline{R}$ while $V_p^i(R^i) = R^i$ for all $R^i \in (0, \overline{R})$, we establish the following:

Proposition 3 (Comparison of payoffs.) *There exists a unique threshold level of R^i, denoted by $\hat{R}$ for $i = 1, 2$ and given by*

$$\hat{R} = \begin{cases} \hat{R}_L \equiv \left[\frac{\beta(\gamma(1-\sigma)+\sigma)}{\beta\gamma(1-\sigma)+1}\right]^2 \theta\overline{R} \in (0, R_L) & \text{if } \gamma < \frac{1-2\beta\sigma}{\beta(1-\sigma)}, \text{ or} \\ \hat{R}_H \equiv \frac{\beta\gamma(1-\sigma)}{4(1-\beta\sigma)}\overline{R} \in [R_L, \frac{1}{4}\overline{R}] & \text{otherwise,} \end{cases} \tag{9}$$

above which agent i prefers peace and below which he strictly prefers war. These threshold points are increasing in γ and β. In addition, $d\hat{R}_H/d\sigma < 0$, and $d\hat{R}_L/d\sigma \lesseqgtr 0$ as $\gamma \gtreqless \frac{1-2\sigma}{(2-\beta)(1-\sigma)}$.

For all feasible values of $\beta \in (0, 1]$, $\gamma \in (0, 1]$, and $\sigma \in [0, 1)$, the threshold value $\hat{R} \in \{\hat{R}_L, \hat{R}_H\}$ is less than half of the aggregate resource $\hat{R} < \frac{1}{2}\overline{R}$. Intuitively, when $R^i = \frac{1}{2}\overline{R}$ for $i = 1, 2$, each agent would enjoy one-half of whatever output is available for consumption regardless of whether war or peace prevails; however, under war that induces arming and possibly destruction, the total amount of output available is strictly less than what would be available under peace, to imply $V_p^i(\frac{1}{2}\overline{R}) > U_w^i(\frac{1}{2}\overline{R})$ for both $i = 1, 2$. Thus, Proposition 3 establishes that there exists a non-empty subset of resource distributions $R^i \in [\hat{R}, \overline{R} - \hat{R}] \in (0, \overline{R})$ under which peace Pareto dominates war (i.e., $V_p^i(R^i) \geq U_w^i(R^i)$ for $i = 1, 2$), and the size of that range expands as $\hat{R}$ falls.

Which threshold applies depends on the configuration of parameters. For example, if war is not destructive at all ($\beta = \gamma = 1$), the higher threshold $\hat{R}_H$ applies, with $\hat{R}_H = \frac{1}{4}\overline{R} \geq R_L = \frac{1}{4}(1-\sigma)\overline{R}$ as $\sigma \geq 0$. This case is illustrated in Fig. 1(a), where the green line represents V_p^i for all possible initial distributions $R^i \in [0, \overline{R}]$.[16] As war's overall destruction becomes sufficiently large ($\beta < \frac{1}{2}$), the lower threshold $\hat{R}_L$ would apply for any $\sigma \in [0, 1)$ and $\gamma \in (0, 1]$. In the case of perfect output insecurity ($\sigma = 0$), $\hat{R}_L$ applies for any degree of destruction, $\beta, \gamma \in (0, 1]$. Fig. 1(b) illustrates the case where $\hat{R} = \hat{R}_L$, though under less extreme circumstances.

[16] Note that the figure is not drawn to scale. If it were, the green line, depicting V_p^i as a function of R^i, would be a 45° line from the origin.

But, whether $\hat{R}_L$ or $\hat{R}_H$ applies, the size of the range $[\hat{R}, \overline{R} - \hat{R}]$ expands (and therefore the condition for peace to Pareto dominate war is more likely to be satisfied) when war is more destructive ($\beta \downarrow$ and/or $\gamma \downarrow$). Intuitively, from Proposition 2, an increase in destruction reduces the payoffs to both agents under war without affecting their payoffs under peace.[17] Likewise, an increase in output security σ tends to reduce the war payoff of the less affluent agent provided that the differential survival rate of output exceeds a critical value conditioned on β and σ and thus tends to decrease the threshold level $\hat{R}$. By contrast, if the differential survival rate is sufficiently small, an increase in σ raises the threshold level of resource.[18] This latter possibility suggests that, when the initial value of σ is sufficiently low, an improvement in output security can reduce the parameter space for which peace Pareto dominates war.

4 Equilibrium Choice Between War and Peace

To be sure, war is always a pure-strategy, Nash equilibrium for the following reason: if an agent's rival declares war and arms accordingly, then the agent's best reply is to do the same. Moreover, the Pareto dominance of unarmed peace does not guarantee its emergence as another Nash equilibrium even when agents communicate with each other prior to their decisions. What is required, in addition, is that neither agent have an incentive to deviate unilaterally from the choices which support that outcome. In this section, we explore the circumstances under which unarmed peace emerges as the stable equilibrium outcome that is immune to unilateral deviations as well as to coalitional deviations, first without transfers and then with transfers.[19]

4.1 Without Transfers

In this setting without transfers, the optimal unilateral deviation from peace for either agent i given $G^j = G_p = 0$ is to produce an infinitesimal amount of guns $G_d^i = \epsilon > 0$ and declare war. To be more precise, given that the opponent j anticipates peace and chooses $G^j = 0$, our specification for the conflict technology (1) implies such a deviation brings

[17]See Fig. 1(b) that illustrates the effect of an increase in differential destruction ($\gamma \downarrow$).

[18]The parameter values that imply $\hat{R} = \hat{R}_H$ also imply that the payoff under war for agent i with R^i less than or equal to that threshold always falls with an increase in σ. Put differently, the value of $R^i \in [R_L, R_H]$ for which $dU_w^i/d\sigma = 0$ (and below which $dU_w^i/d\sigma < 0$) is greater than $\hat{R}_H$. (In the special case that those two points coincide, as shown in Fig. 1(a), an increase in σ has no effect on $\hat{R}_H$.) Similarly, the restriction on the parameter values (stated in the proposition) for $d\hat{R}_L/d\sigma < 0$ to hold is precisely the necessary and sufficient condition for the critical value of R^i at which $dU_w^i/d\sigma = 0$ and below which $dU_w^i/d\sigma < 0$ to be greater than $\hat{R}_L$. Otherwise, the counterclockwise rotation in $U_w^i(R^i)$ induced by an increase in σ occurs at a value of R^i which is less than the initial $\hat{R}_L$, implying a new intersection of $U_w^i(R^i)$ with R^i at a larger value of $\hat{R}_L$.

[19]While the equilibrium concept we employ here follows Bernheim et al.'s (1987) notion of coalition-proof equilibrium in that it requires immunity to both sorts of deviations, the concept is weaker than that of "perfect coalition-proofness" that would be relevant in the context of a sequential game, allowing unlimited communication throughout. We return to this issue below in Section 5.

agent i a certain victory and an associated payoff $U_d^i(R^i)$ equal to

$$U_d^i(R^i) = \beta\gamma(1-\sigma)[\overline{R} - \epsilon] + \beta\sigma[R^i - \epsilon] \approx \beta\gamma(1-\sigma)\overline{R} + \beta\sigma R^i, \quad \text{for } i = 1,2, \tag{10}$$

where the second expression on the RHS follows for ϵ arbitrarily close to zero. Like the payoff under war $U_w^i(R^i)$ shown in (8), $U_d^i(R^i)$ increases linearly in R^i (provided $\sigma > 0$) and is also increasing in the survival rate of output ($\gamma \uparrow$ and/or $\beta \uparrow$).[20] In addition, U_d^i is increasing in output security ($\sigma \uparrow$) if the agent's resource is sufficiently large ($R^i \geq \gamma\overline{R}$) and is otherwise strictly decreasing in σ. Panels (a) and (b) of Fig. 1, where the blue line depicts $U_d^i(R^i)$, illustrates the dependence of $U_d^i(R^i)$ on R^i as well as on σ and γ.[21] Observe further that $U_d^i(R^i) > U_w^i(R^i)$ for any given distribution $R^i \in (0, \overline{R})$, while $\beta\gamma(1-\sigma)\overline{R} = \lim_{R^i\to 0} U_d^i \geq \lim_{R^i \to 0} U_w^i = 0$ (which holds with equality if $\beta\gamma = 0$ and/or $\sigma = 1$) whereas $\beta[\gamma(1-\sigma)+\sigma]\overline{R} = \lim_{R^i\to\overline{R}} U_d^i = \lim_{R^i\to\overline{R}} U_w^i \leq \overline{R}$ (which holds with equality only if $\beta = \gamma = 1$).

Turning to the comparison of payoffs under peace and under an optimal (unilateral) deviation from it, one can see from (7) and (10) that, provided there is some destruction (i.e., $\beta\gamma \in (0,1)$), we have: (i) $\lim_{R^i\to 0} U_d^i > \lim_{R^i\to 0} V_p^i$ while $\lim_{R^i\to\overline{R}} U_d^i < \lim_{R^i\to\overline{R}} V_p^i$; and (ii) $\partial U_d^i/\partial R^i < \partial V_p^i/\partial R^i$. We thus arrive at

Lemma 1 *For any $\sigma \in [0,1)$ and $\beta\gamma \in (0,1)$, there exists a unique allocation of resources*

$$R_* = \frac{\beta\gamma(1-\sigma)}{1-\beta\sigma}\overline{R} \in (0, \overline{R}), \tag{11}$$

such that $R^i \lesseqgtr R_*$ as $U_d^i(R^i) \gtreqless V_p^i(R^i)$, with the following properties:

(a) *$sign\{\partial R_*/\partial\beta\} = sign\{\partial R_*/\partial\gamma\} > 0$ whereas $\partial R_*/\partial\sigma \leq 0$ (with equality if $\beta = 1$);*

(b) *$R_* \lesseqgtr \frac{1}{2}\overline{R}$ as $\gamma \lesseqgtr \gamma_{NT}$, where $\gamma_{NT} \equiv \gamma_{NT}(\sigma;\beta) = \frac{1-\beta\sigma}{2\beta(1-\sigma)} > 0$.*

This lemma establishes that, provided war is destructive, agent i finds a unilateral deviation from peace to be unprofitable (profitable) when his initial resource allocation is suffi-

[20] One might object to our implicit assumption here that a unilateral deviation involving the deployment of only an infinitesimal quantity guns results in the destruction of some output as in the case where both agents arm and fight. Below in Section 5, we consider an extension of the analysis that, following Slantchev (2011), supposes each agent holds an initial stock of guns that can be deployed, possibly along with additional guns produced; in this extension, where our assumption of destruction in the case of a unilateral deviation seems quite reasonable, the various threshold values do change, but the central results remain qualitatively intact. Another possible approach would be to assume that the rate of destruction is increasing in each agent's arming, along the lines of Chang and Luo (2017). In such an extension, the rate of destruction of a unilateral deviation would be smaller though still strictly positive. We conjecture that, while this modification would once again influence the various thresholds, there would be no qualitative differences in our key insights.

[21] In the case that $\beta = \gamma = 1$, an increase in output security results an a counterclockwise rotation of $U_d^i(R^i)$ at $R^i = \overline{R}$ as shown in panel (a) of the figure, such that $U_d^i(R^i)$ falls with increases in σ for all $R^i \in (0, \overline{R})$.

ciently large (small).[22] Part (a) shows that the threshold R_* is decreasing in destruction ($\beta \downarrow$ and/or $\gamma \downarrow$), as would be expected since the deviation payoff $U_d^i(R^i)$ is decreasing in destruction, while the peace payoff $V_p^i(R^i)$ is independent of destruction. Similarly, because $U_d^i(R^i)$ falls (rises) with improvements in output security ($\sigma \uparrow$) when R^i is sufficiently small (large) whereas $V_p^i(R^i)$ is independent of σ, an increase in σ reduces the relative appeal of a unilateral deviation to the less affluent agent and thus reduces R_* (provided $\beta < 1$).[23]

Of course, for peace to arise as a stable equilibrium, both agents must view a unilateral deviation as being unprofitable—i.e., $R^i \leq R_*$ for $i = 1, 2$.[24] Such stability requires as a necessary (but not sufficient) condition that $R_* \leq \frac{1}{2}\overline{R}$.[25] Part (b) of the lemma helps us to identify the circumstances under which this condition is satisfied and when it is not. To proceed, observe the parameter γ_{NT} introduced in this part of Lemma 1 gives the critical value of the differential survival rate of contested output, conditioned on σ and β, above which we have $R_* > \frac{1}{2}\overline{R}$. Thus, when $\gamma > \gamma_{NT}$, at least one agent would optimally choose to deviate given his rival anticipates peace and so does not arm for any distribution $R^i \in (0, \overline{R})$. (The subscript "$NT$" indicates the case of no transfers.) This critical value is decreasing and convex in β but increasing and concave in σ. Furthermore, depending on the values of β and σ, γ_{NT} could exceed 1.

To flesh out the implications, consider first the special case where $\beta = 1$, which implies that only the insecure portion of an agent's output is subject to destruction under war. In this case, $\gamma_{NT} = \frac{1}{2}$, and thus $R_* \leq \frac{1}{2}\overline{R}$ holds for any $\gamma \leq \frac{1}{2}$ and all $\sigma \in [0, 1)$. Exactly the opposite is true for $\gamma > \frac{1}{2}$, implying peace cannot be a stable equilibrium for such parameter values. This case is illustrated by the pink horizontal line in Fig. 2(a) that cuts the (σ, γ) plane into the just described subsets. Now, let us consider values of $\beta < 1$ which, for any given σ, causes γ_{NT} to rise. Indeed, there exist (β, σ) pairs that imply $\gamma_{NT} \geq 1$, such that $\gamma \leq \gamma_{NT}$ and hence, by part (b), $R_* \leq \frac{1}{2}\overline{R}$ always holds. One can verify that a sufficient condition for this possibility is that $\beta \leq \frac{1}{2-\sigma}$ (or, equivalently, $\sigma \geq \sigma_{NT}(\beta) \equiv 2 - \frac{1}{\beta}$).

[22] The threshold R_* shown in (11) is strictly greater than the threshold $\hat{R}$ shown in (9) that defines the parameter space for which each agent i is better off under peace. Thus, as suggested by our earlier discussion, even when an agent prefers the outcome under peace to that under war, he could have a strictly positive incentive to deviate unilaterally from the peaceful outcome.

[23] Fig. 1(a) illustrates the effects of an increase in σ, but does so under the assumption that $\beta = 1$; in this extreme case, $R_* = \overline{R}$ and $U_d^i(R^i)$ rotates counterclockwise at $R^i = \overline{R}$ as σ rises, such that there is no effect on $R_* = \overline{R}$. However, Fig. 1(b) shows how a decrease in γ shifts $U_d^i(R^i)$ downward, resulting in a decrease in R_*, from the value of R^i associated with point A to that associated with point B.

[24] As suggested earlier (see footnote 11), allowing for the possibility that the agents could "recover" some portion, $(1-k)$ with $k < 1$, of their guns for consumption as in Beviá and Corchón (2010) would not matter for this condition. To be sure, the possibility of such recovery would reduce the effective marginal cost of arming and thus affect the payoffs under war U_w^i. However, it would have no consequences for the payoffs under peace V_p^i (since there is no arming at all) or under a unilateral deviation U_d^i (since arming by the deviating agent is infinitesimal). As such, allowing $k < 1$ would have no relevance for the determination of R_*.

[25] Without any loss of generality, we assume that an agent chooses peace at the point of indifference.

Clearly, this condition is always satisfied for $\beta \leq \frac{1}{2}$ because $\sigma \in [0,1)$. Thus, if the overall rate of destruction is sufficiently strong (i.e., $\beta \leq \frac{1}{2}$), then $\gamma_{NT} \geq 1$ and $R_* \leq \frac{1}{2}\overline{R}$ for any $\sigma \in [0,1)$ and $\gamma \in (0,1]$. When $\beta \in (\frac{1}{2},1)$, the values of both σ and γ matter because $\sigma < \sigma_{NT}$ implies $\gamma_{NT} < 1$, thereby raising the possibility that $\gamma > \gamma_{NT}$ and thus $R_* > \frac{1}{2}\overline{R}$. Nevertheless, $R_* \leq \frac{1}{2}\overline{R}$ will hold provided the differential rate of destruction γ satisfies $\gamma \leq \gamma_{NT}$. For additional insight, we illustrate this case with the blue curve in Fig. 2(a) which depicts γ_{NT} for $\beta = \frac{9}{10}$. The green curve above the blue one arises when a lower value of β (specifically, $\beta = \frac{2}{3}$) is considered. In summary, we have $R_* \leq \frac{1}{2}\overline{R}$ at all (σ,γ) pairs on or below γ_{NT} and $R_* > \frac{1}{2}\overline{R}$ at all pairs above γ_{NT}.[26]

Let us define $R^* \equiv \overline{R} - R_*$. Then, using Lemma 1 under the assumption that *ex ante* resource transfers are not possible, the next proposition establishes the conditions under which peace can and cannot arise as the stable equilibrium outcome:

Proposition 4 (Stability of peace without transfers.) *For all values of $\beta \in (0,1]$ and $\sigma \in [0,1)$, if $\gamma > \gamma_{NT}(\sigma;\beta)$, then war emerges as the unique pure-strategy, Nash equilibrium for all distributions. However, if $\gamma \leq \gamma_{NT}(\sigma;\beta)$, then*

(a) *there exists a non-empty subset $[R_*, R^*] \subset (0,\overline{R})$ of initial resource distributions that imply unarmed peace is the stable equilibrium for any R^i in this subset;*

(b) *war is the unique pure-strategy, Nash equilibrium for all other (sufficiently uneven) distribution of resources.*

Higher degrees of output security ($\sigma \uparrow$) and larger overall and/or differential rates of destruction ($\beta \downarrow$ and/or $\gamma \downarrow$) enlarge the subset $[R_*, R^*]$ of endowments that support peace.

This proposition shows that there exist certain combinations of parameter values (β,γ,σ) for which unarmed peace (without transfers) is not possible for any feasible resource distribution, implying war is the unique, pure-strategy Nash equilibrium.[27] Such an outcome is more likely to materialize when war is less destructive ($\beta \uparrow$ and/or $\gamma \uparrow$) and output is less secure ($\sigma \downarrow$).[28] Outside that parameter space, which implies $R_* \leq \frac{1}{2}\overline{R}$, unarmed peace can be another equilibrium outcome, but only provided the distribution of resources is sufficiently even. Because $R^i \geq R_* > \hat{R}$ for $i = 1,2$ for such distributions, peace Pareto

[26] Of course, the value of $\frac{1-\beta\sigma}{2\beta(1-\sigma)}$ can exceed 1 (specifically, when $\sigma > \sigma_{NT}$), as discussed above and illustrated by the thin dotted-line extensions of γ_{NT} in Fig. 2(a). Since γ cannot exceed 1, the critical value of γ, γ_{NT}, could be written more precisely as $\min\{\frac{1-\beta\sigma}{2\beta(1-\sigma)}, 1\}$ as shown in the figure. But, to avoid clutter in the text, we simply write γ_{NT} as specified in the lemma.

[27] Although mixed-strategy equilibria that dominate the war outcome could exist in this case, we focus on pure-strategy equilibria.

[28] Our earlier discussion in connection with Lemma 1 suggests that a necessary (but not sufficient) condition for this possibility is that $\gamma > \frac{1}{2}$ and $\beta > \frac{1}{2}$, which includes the case of no destruction at all.

dominates war (i.e., $V_p^i(R^i) \geq U_w^i(R^i)$); and, under the reasonable assumption that the two agents can communicate before they act, they would naturally coordinate on peace, making that outcome the stable or coalition-proof equilibrium in the absence of transfers.[29]

To illustrate, let W_{NT}^i denote agent i's (pure-strategy) equilibrium payoff in the absence of transfers. Now, consider combinations of the degree of output security σ and the rates of destruction of output β and γ, such as the ones depicted in Fig. 2(c) below the γ_{NT} curve. The two panels of Fig. 3 show $W_{NT}^i(R^i)$ (captured by the thick, black and discontinuous curve) associated with each of these points. Proposition 4 implies $W_{NT}^i(R^i) = V_P^i(R^i)$ for all $R^i \in [R_*, R^*]$, whereas $W_{NT}^i(R^i) = U_w^i(R^i) < V_p^i(R^i)$ for all $R^i \notin [R_*, R^*]$. Thus, peace emerges as the equilibrium outcome only if the initial distribution of resources is sufficiently even. The discontinuity at point A arises because agent i has an incentive to deviate unilaterally from peace as soon as R^i falls below R_*. Since the payoff $W_{NT}^j(R^i)$ to agent $j \neq i$ (not drawn to avoid cluttering) mirrors $W_{NT}^i(R^i)$, it should be clear that the discontinuity at point B arises because agent j ($\neq i$) undermines peace as R^i rises above R^* (or equivalently, as R^j falls below R_*). This logic suggests that, in the absence of transfers, war arises as the pure-strategy equilibrium for sufficiently uneven distributions of resources, where peace fails to be immune to unilateral deviations. A comparison of the two panels in Fig. 3 illustrates the result that an increase in the differential rate of destruction ($\gamma \downarrow$) expands the size of $[R_*, R^*]$, thereby making peace a more likely equilibrium outcome.

4.2 With Transfers

We now turn to explore how resource transfers affect the stability of unarmed peace when the initial distribution is such that peace without transfers is not possible to begin with—i.e., $R^i \notin [R_*, R^*]$. Following Beviá and Corchón (2010) and Jackson and Morelli (2007) among others, we assume such transfers are made in advance of the agents' arming and war/peace decisions and without any commitments.[30] That is to say, an agent's acceptance or delivery of a transfer in no way precludes him from arming and declaring war; peace in this setting, as in the case of no transfers, has to be self-enforcing.

To fix ideas, suppose that $R^i > R^j$, so that agent i more affluent. From our discussion above, it should be clear that such transfers can improve the stability of peace only if they

[29]More formally, following Bernheim et al. (1987), this equilibrium concept requires that (i) neither agent views a unilateral deviation from the outcome to be profitable and (ii) coalitional deviations are unprofitable as well. Note that, while mixed-strategy equilibria with arming by at least one agent could exist, such equilibria are Pareto dominated by unarmed peace. Thus, provided unarmed peace is a stable outcome satisfying conditions (i) and (ii), it is the unique stable equilibrium. [See the proof of Proposition 4 presented in Appendix A for some details.]

[30]This *ex ante* resource transfer differs sharply from an *ex post* transfer resulting from a division contested output conditioned on arming choices by each agent under peace. As shown by Garfinkel and Syropoulos (2019) among others, in single-period settings, there always exists a division that (given arming choices) can induce a peaceful outcome; however, that sort of peace comes at the cost of arming by both agents.

make the *ex post* distribution of resources more even. But, for the transfer to support peace, the resulting distribution of resources must not leave either agent with an incentive to deviate unilaterally from that outcome. More precisely, the transfer from the more affluent agent i must be sufficiently large to render a unilateral deviation from peace unprofitable and thus unappealing to the less affluent agent (j). Assuming that transferred resources are subject to destruction in the event of war, this condition can be written as

$$V_p^j(R^j+T) \geq U_d^j(R^j+T) \longrightarrow R^j+T \geq \beta\gamma(1-\sigma)\overline{R}+\beta\sigma(R^j+T).$$

Rearranging the second inequality using the definition of R_* in (11) shows that this constraint imposes an lower bound on the transfer, denoted by $T_{\min}$:

$$T \geq T_{\min} \equiv \frac{\beta\gamma(1-\sigma)}{1-\beta\sigma}\overline{R}-R^j = R_*-R^j. \tag{12}$$

Since $R^j+T_{\min}=R_*$, a transfer of $T_{\min}$ from i to j makes agent j just indifferent between peace and a unilateral deviation from it. By the same token, the transfer should not be too large so as to make a unilateral deviation by the more affluent agent (i) profitable:

$$V_p^i(R^i-T) \geq U_d^i(R^i-T) \longrightarrow R^i-T \geq \beta\gamma(1-\sigma)\overline{R}+\beta\sigma(R^i-T),$$

which places an upper bound on the transfer, denoted by $T_{\max}$:

$$T \leq T_{\max} \equiv R^i-\frac{\beta\gamma(1-\sigma)}{1-\beta\sigma}\overline{R} = R^i-R_*. \tag{13}$$

But to have $T \in [T_{\min}, T_{\max}]$ requires $T_{\min} \leq T_{\max}$. One can now verify from (12) and (13), that this condition brings us back to the necessary condition for peace without transfers: $R_* \leq \frac{1}{2}\overline{R}$. Thus, transfers can support peace only if peace can be supported in the absence of transfers for at least some resource distributions. But, even if there exists a transfer T that simultaneously satisfies (12) and (13), it need not support peace. In addition, the transfer should not be so large as to make the more affluent agent (i) worse off under peace than his fallback payoff under war with no transfers: $V_p^i(R^i-T)=R^i-T \geq U_w^i(R^i)$.

Building on this last condition that must hold for the more affluent agent i, we establish the next lemma that lays the groundwork for our characterization of parameter values under which unarmed peace with transfers is stable. To start, observe that, when the more affluent agent i donates the minimum transfer that supports peace ($T_{\min}=R_*-R^j$) to agent j, agent i is left with R^* $(=\overline{R}-R_*)$ resources and his payoff is $V_p^i(R^*)=[1-\frac{\beta\gamma(1-\sigma)}{1-\beta\sigma}]\overline{R}$. Next, recall that the payoffs under war and under an unilateral deviation from peace satisfy $\lim_{R^i\to\overline{R}} U_w^i(R^i) = \lim_{R^i\to\overline{R}} U_d^i(R^i) = \beta\left[\gamma\left(1-\sigma\right)+\sigma\right]\overline{R}$. Based on this equality, we now identify another critical value of γ, denoted by γ_T (where the "T" subscript indicates when transfers are possible) and conditioned on σ and β:

Lemma 2 *There exists a critical value of γ, given by*

$$\gamma_T \equiv \gamma_T(\sigma;\beta) = \frac{(1-\beta\sigma)^2}{\beta(1-\sigma)(2-\beta\sigma)}, \tag{14}$$

that implies $V_p^i(R^*) = \lim_{R^i\to\overline{R}} U_w^i(R^i)$ and $V_p^i(R^*) \gtreqless \lim_{R^i\to\overline{R}} U_w^i$ as $\gamma \lesseqgtr \gamma_T$. For any $\sigma \in [0,1)$ and $\beta \in (0,1]$, $\gamma_T(\sigma;\beta) \le \gamma_{NT}(\sigma;\beta)$ (with equality for $\sigma = 0$) and depends on β and σ as follows:

(a) *If $\beta = 1$, then $\gamma_T = \frac{1-\sigma}{2-\sigma}$ and $\partial\gamma_T/\partial\sigma < 0$ with $\gamma_T|_{\sigma=0} = \frac{1}{2}$ and $\lim_{\sigma\to1}\gamma_T = 0$.*

(b) *If $\beta \in (\frac{1}{2},1)$, then*

  (i) *there exists a unique $\sigma_T = 2\left(\beta-\frac{1}{2}\right)/\beta^2 \in (0,1)$ s.t. $\gamma_T(\sigma;\gamma) > 1$ for all $\sigma > \sigma_T$;*

  (ii) *γ_T is strictly quasi-convex in σ with $\arg\min_\sigma \gamma_T = \min\{0, 3\left(\beta-\frac{2}{3}\right)/\beta^2\}$ and $\gamma_T < 1$ for all $\sigma \in (0,\sigma_T)$.*

(c) *If $\beta \in (0,\frac{1}{2}]$, then $\gamma_T \ge 1$.*

The properties of γ_T highlighted in parts (a) and (b) are illustrated in Fig. 2(b). The pink, dashed-line curve in panel (b) of Fig. 2 shows the γ_T schedule when $\beta = 1$ as characterized in Lemma 2(a). The blue and green curves illustrate γ_T for $\beta = \frac{9}{10}$ and $\beta = \frac{2}{3}$ as characterized in part (b).[31] Lastly, panel (c) of Fig. 2 shows γ_{NT} in relation to γ_T for $\beta = \frac{9}{10}$.

Let us now give these figures more context. We have already described how the partition of the $[0,1]\times[0,1]$ space of (σ,γ) by γ_{NT} matters for our characterization of the equilibrium without transfers. Specifically, war is the unique pure-strategy, Nash equilibrium outcome for all parameter values in the subset above γ_{NT} (shown in Fig. 2(c) as the pink-shaded area). In contrast, peace is possible when the distribution of resources is sufficiently even for all parameter values in the subset below γ_{NT}. The γ_T $(\le \gamma_{NT})$ schedule partitions the latter (green-shaded) subset into two additional subsets: (σ,γ) pairs on and below γ_T that support peace for all resource distributions $R^i \in (0,\overline{R})$ (in the area shaded with a darker green), whereas those pairs above γ_T (but below or on γ_{NT}, in the area shaded with a lighter green) that support peace only for a subset of resource distributions.

Building on these ideas with Proposition 4 and Lemma 2, we can now establish the following:

Proposition 5 (Stability of peace with transfers.) *Suppose that ex ante resource transfers between agents are possible. Such transfers are relevant in supporting peace only for those*

[31] In the latter case, $\arg\min_\sigma \gamma_T = 0$, as noted in part (b.ii). It is worth noting that, even though $\gamma_T < 1$ for $\sigma \in (0,\sigma_T)$, γ_T is increasing in $\sigma \in [0,1)$ when $\beta \in (0,\frac{2}{3}]$.

(β, γ, σ) parameter values that also ensure peace can arise as a stable equilibrium in the absence of transfers (i.e., $\gamma \leq \gamma_{NT}$). Under such circumstances given $\beta \in (0,1]$ and $\sigma \in [0,1)$, transfers expand the distribution of resources under which unarmed peace is stable as follows:

(a) *If $\gamma \in (\gamma_T(\sigma;\beta), \gamma_{NT}(\sigma;\beta)]$, there exist a unique $R_{**} \in (0, R_*)$ and corresponding $R^{**} \equiv \overline{R} - R_{**}$ that satisfy $V_p^i(R^*) = U_w^i(R^{**})$ and imply the following:*

   (i) *unarmed peace arises as the stable equilibrium for all $R^i \in [R_{**}, R^{**}]$;*

   (ii) *war is the unique pure-strategy, Nash equilibrium for all $R^i \notin [R_{**}, R^{**}]$.*

(b) *If $\gamma \in (0, \gamma_T(\sigma;\beta)]$, unarmed peace is the stable equilibrium for all $R^i \in (0, \overline{R})$.*

If $\beta \in (\frac{2}{3}, 1]$, then improvements in output security ($\sigma \uparrow$) can reduce the range of resources $R^i \in [R_{**}, R^{**}]$ for which peace can be supported when transfers are possible.

This proposition identifies the conditions for which transfers between agents do not matter and those for which they do and how. Clearly, as argued earlier, if unarmed peace is not possible in the absence of transfers for any initial distribution of resources (i.e., $\gamma > \gamma_{NT}$), then transfers can do nothing to support peace. We illustrate parts (a) and (b) of the proposition, with the help of Figs. 3 and 4, assuming $\gamma \leq \gamma_{NT}$. As established earlier in Proposition 4 and illustrated in both panels of Fig. 3, when $\gamma \leq \gamma_{NT}$, peace can arise as the stable equilibrium in the absence of transfers, but only for a subset of resource distributions $R^i \in [R^*, R^*] \in (0, \bar{R})$, since $R^i > R^*$ implies $R^j < R_*$ and which, in turn, implies agent j has an incentive to deviate unilaterally. However, for an allocation R^i just above R^* (or point B in the figure), agent i could make an *ex ante* resource transfer to agent j, $T = T_{\min} = R_* - R^j$ shown in (12), to give the less affluent agent j the minimum payoff, $V_p^j(R_*) = R_* = U_d^j(R^j) > U_w^j(R^j)$, required to keep him from deviating from the peace outcome. At the same time, with a final endowment of $R^* = R^i - R_* + R^j$ after the transfer, the more affluent agent i continues to enjoy the higher payoff under peace, $V_p^i(R^*)$, that exceeds his fallback payoff of not making a transfer and tolerating war, $U_w^i(R^i)$, as illustrated in Fig. 3. Importantly, additional increases in R^i (above R^*) cause agent i's fallback payoff U_w^i to rise, such that the gains from making a transfer $V_p^i(R^*) - U_w(R^i)$ falls.

The lower bound on γ specified in part (a) implies that $V_p^i(R^*) < \lim_{R^i \to \overline{R}} U_w^i$. It then follows that there exists an allocation $R^i = R^{**} > 0$ that implies $V_p^i(R^*)$ represented by point B in Fig. 3(a) equals $U_w^i(R^{**})$ represented by point C in the same figure. Thus, for any $R^i \in (R^*, R^{**}]$, agent i would prefer to make a resource transfer and avoid war. However, for allocations $R^i > R^{**}$ (just beyond point C), agent i views the required transfer as too costly and so is willing to tolerate war. Considering all possible endowment distributions, we illustrate agent i's (pure-strategy) equilibrium payoff in the presence of *ex ante* transfers, $W_T^i(R^i)$, with the thick black curve in panel (a) of Fig. 4. Clearly, when transfers are possible, peace is sustainable for a larger set of endowment distributions (i.e., $R^{**} > R^*$), and both agents obtain payoffs that are at least as large as the ones associated without transfers. But, for the set of parameter values considered here, war does emerge as the unique, pure-strategy equilibrium outcome if the initial distribution of resources is sufficiently uneven: $R^i > R^{**}$ that impies $R^j < R_{**}$.

Part (b) establishes that it is possible for peace to arise as the stable equilibrium in the presence of *ex ante* resource transfers for all possible endowment distributions, as illustrated in panel (b) of Fig. 4. In particular, by the definition of γ_T, we have if $\gamma \leq \gamma_T$, then $V_p^i(R^*) \geq \lim_{R^i \to \overline{R}} U_w^i$. Thus, although the gains to agent i of making an *ex ante* resource transfer to agent j for $R^i > R^*$ diminish as his initial resource endowment increases, they remain non-negative for all $R^i \in (R^*, \bar{R})$. Accordingly, $R^{**} = \overline{R}$, and peace emerges as the stable equilibrium outcome for all $R^i \in (0, \overline{R})$ when transfers are possible.

Finally, Proposition 5 shows that improvements in output security ($\sigma \uparrow$) can make transfers less effective in promoting unarmed peace. To gain some intuition here, recall from Lemma 1 that an increase in σ reduces the critical value of the endowment R_*, above which unilateral deviations are viewed as being unprofitable, and thus reduces the minimum transfer required to keep the less affluent agent (j) from deviating. As a consequence, an increase in σ enhances the more affluent agent's (i) payoff under peace with transfers, $V_p^i(R^*)$. This effect alone tends to increase R^{**}. However, by Proposition 2, an increase in σ also raises the more affluent agent's fallback payoff under war $U_w^i(R^i)$ at any given R^*, and that effect alone tends to decrease R^{**}, thereby weakening the power of transfers to promote peace.[32] As shown in Appendix A, which effect dominates depends on the shape of $\gamma_T(\sigma)$ and the initial value of σ; but, for the latter effect to dominate, overall destruction must not be too severe as stated in the proposition.[33]

[32] Allowing for some fraction $(1-k)$ of guns to be recoverable for consumption that influences U_w^i (as mentioned in footnote 24) naturally affects the determination of R^{**}. In particular, one can confirm that, relative to the case where $k = 1$ as assumed throughout our analysis, allowing $k < 1$ implies a higher (lower) payoff under war for the more (less) affluent agent. Since $k < 1$ has no effect on V_p^i, this positive influence on U_w^i for the more affluent agent tends to reduce R^{**}. Nonetheless, since the value of γ_{NT} is independent of $k \leq 1$, the parameter values that preclude peace without transfers for any $R^i \in (0, \overline{R})$ (i.e., $\gamma > \gamma_{NT}$) also preclude peace with transfers.

[33] One can visualize the possibility that an increase in σ makes transfers less effective in supporting peace in Fig. 2(c) that assumes $\beta = \frac{9}{10}$. In particular, suppose that $\gamma = \frac{1}{2}$. As shown in the figure, for relatively small values of σ, we have $\gamma\ (= \frac{1}{2}) < \gamma_T$, implying that, when transfers are possible, peace can be supported for all resource distributions (i.e., $R_{**} = 0$). As we consider larger values of σ with $\gamma = \frac{1}{2}$, we eventually cross over the γ_T schedule, where $\gamma \in (\gamma_T, \gamma_{NT})$ and thus peace with transfers is possible only for a subset of resource distributions, implying $R_{**} > 0$. But, with further security improvements, ultimately the inequality $\gamma\ (= \frac{1}{2}) < \gamma_T$ is restored, such that $R_{**} = 0$ again.

5 Extensions: Generalizations and Limitations

That the stability of unarmed peace (identified with the status quo) for any feasible distribution of resource endowments requires war to be destructive is noteworthy, and stands in sharp contrast to what happens when peace is modeled as bargaining process to divide whatever is contested, with the agents arming to gain leverage in that process. Specifically, as argued by Garfinkel and Syropoulos (2018), peace in the latter case (or "armed peace") in a one-period setting is immune to unilateral deviations provided that, for any given guns, the payoffs to both contenders are greater under peace than under war. That is to say, peace (identified with a division of contested goods) generates a "dividend" that extends beyond the savings in resources allocated to arming. As long as guns are chosen before the war/peace decision, peace necessarily arises as a possible equilibrium outcome.[34]

Although war's destructive effects represent one reason for the presence of such a peace dividend as has been argued by Fearon (1995) and Powell (1993) among others, alternative factors similarly render peace better than war given arming choices—for example, risk aversion, diminishing returns in production, and mutually advantageous trade that is possible only when war is avoided.[35] Could the introduction of these other factors in our analysis restore the possible stability of unarmed peace (identified with the status quo) for at least some distributions of initial resource endowments, even when war (including unilateral deviations) is not destructive? In this section, we consider four extensions that, while interesting in their own right, allow us to check the robustness of our finding to such factors and others. In particular, we consider (i) diminishing returns in the production of butter and (ii) the possibility of trade when the final goods produced by the two agents are differentiated. In addition, we consider (iii) the possibility that the agents have an initial stock of guns carried over at no cost to the present. Such an extension allows us to address the possible objection to our analysis in which a unilateral deviation from unarmed peace by one agent would allow him capture, with certainty, all of the contested output upon producing only an infinitesimal quantity of guns. Finally, we show, under what conditions, our results extend to (iv) a sequential-move game.[36]

[34]However, there could exist resource distributions for which the *ex ante* equilibrium payoff of one agent under war exceeds his *ex ante* equilibrium payoff under peace, such that peace does not dominate war in a Pareto sense. As demonstrated by Garfinkel and Syropoulos (2019), this possibility can arise in the presence of bargaining for a division of contested output under peace due to the different levels of arming by the contenders that war and peace induce. In this case, it is the more affluent agent who could view war from an *ex ante* perspective to be more appealing than peace (also see Schaller and Skaperdas, 2019). Then, if guns are chosen after the war/peace decision as assumed in Beviá and Corchón (2010), armed peace, though immune to unilateral deviations, need not arise as a stable equilibrium even in a one-period setting.

[35]See Garfinkel and Skaperdas (2007) for a general discussion. Also, see Anbarci (2002) on rules of division in the presence of diminishing returns and Garfinkel and Syropoulos (2018) on such rules in the presence of trade.

[36]More technical details regarding these extensions can be found in Appendix B.

5.1 Diminishing Returns

Here, we focus on a simple form of diminishing returns, one that helps us show clearly how the analysis of our baseline model generalizes. Specifically, we modify the technology of butter as follows: $X^i = (R^i - G^i)^\alpha$, where $\alpha \in (0,1]$.[37] An agent's payoff functions under peace $V^i = X^i$ and war U^i remain precisely as shown respectively in (2) and in (3). Thus, the structure of the contest is identical to that in the baseline model. Nevertheless, the strict concavity of X^i in $R^i - G^i$ (i.e., for $\alpha < 1$) has some distinct analytical implications. First, because an agent's marginal product in butter is infinitely large when $R^i - G^i$ is infinitesimal (i.e., $\lim_{G^i \to R^i} \partial X^i/\partial R^i = \lim_{G^i \to R^i} \alpha(R^i - G^i)^{\alpha-1} = \infty$), the agent's opportunity cost to producing guns becomes infinitely large as $G^i \to R^i$. As a consequence, an agent never chooses to allocate his entire resource endowment to the production of guns. Second, closed-form solutions for the agents' best-response functions and the associated Nash equilibrium in arming no longer exist. Nonetheless, it is possible to characterize these functions (and equilibrium) and show that they have properties similar to the those in the baseline model. Third, while the dependence of an agent i's equilibrium payoff under war U^i_w on the distribution of resources is similar to that in the baseline model it differs in that U^i_w is smooth in R^i. Finally, although the payoffs under unarmed peace $V^i_p = (R^i)^\alpha$ and a unilateral deviation from it U^i_d are similar to the ones in the baseline model in that they continue to be smooth, they are, in addition, strictly concave in R^i.

To see the implications of diminishing returns for the stability of peace, observe that agent i's payoff under a unilateral deviation from peace evaluated at G^i arbitrarily close to zero is given by

$$U^i_d \approx \beta\gamma(1-\sigma)[(R^i)^\alpha + (R^j)^\alpha] + \beta\sigma(R^i)^\alpha.$$

Thus, the critical value of R^i, above which agent i finds a unilateral deviation unprofitable (R_*), is now

$$R_* = \frac{\overline{R}}{1 + \left[\frac{1-\beta\sigma}{\beta\gamma(1-\sigma)} - 1\right]^{1/\alpha}} \in (0, \overline{R}), \tag{11'}$$

which simplifies to the value of R_* in (11) when $\alpha = 1$. Indeed, in the absence of destruction under war or under unilateral deviations from peace (i.e., $\beta\gamma = 1$), we have $V^i_p = \left(R^i\right)^\alpha < U^i_d = \left(R^i\right)^\alpha + (1-\sigma)\left(R^j\right)^\alpha$ for all $R^i \in (0, \overline{R})$. As such, $R_* = \overline{R}$ holds, implying (as in the

[37]Results analogous to those that follow hold when we consider instead risk aversion. Alternatively, to capture the importance of complementarities between multiple inputs in production, one could consider the CES production function $X^i = [\alpha(R^i - G^i)^r + (1-\alpha)(K^i)^r]^{1/r}$ for $\alpha \in (0,1]$ and $r < 1$, where K^i is a specific factor (e.g., land) that remains fixed in the background.This function becomes very similar to the one considered in the text for $K^i = K$ ($i = 1, 2$) as $r \to 0$, and simplifies to the specification in our baseline model when $\alpha = 1$.

baseline model) that peace cannot emerge as a stable equilibrium for any distribution $R^i \in (0, \overline{R})$ when war is not destructive. Even when war is destructive ($\beta\gamma < 1$), a comparison of R_* in (11′) with $\frac{1}{2}\overline{R}$ reveals that γ_{NT} in the case of diminishing returns is identical to the one obtained in the baseline model (as shown in Lemma 1(b)), implying that the degree of output insecurity and the nature of destruction alone determine γ_{NT}. Thus, the parameter α plays no role in determining the possible existence of at least some resource distributions under which unarmed peace can emerge as a stable equilibrium.[38]

This is not to say that the presence of diminishing returns is inconsequential. Suppose, for any given $\sigma \in [0, 1)$ and $\beta \in (0, 1)$, that peace without transfers is sustainable for some allocations of R^i (i.e., $\gamma < \gamma_{NT}$). As can be confirmed from (11′), stronger diminishing returns (i.e., a smaller value of α) are associated with a lower value of R_* and, therefore, a larger range of (sufficiently even) resource distributions that support peace. Furthermore, one can verify that the presence of diminishing returns makes *ex ante* resource transfers more effective in expanding the range of allocations that support peace.[39]

5.2 Trade

In this section we consider the possible implications of trade for the stability of peace. We do so in a very simple setting along the lines of Armington (1969), in which each agent produces a differentiated good ($j = 1, 2$) that can be traded under peace. Although we assume trade is not possible in the case of war, the winner of the conflict takes possession of the contested portion of the rival's good and thus can enjoy consumption of both goods, whereas the defeated side can consume only the secure portion of what he produces. To derive the corresponding payoffs, let preferences defined over these two consumption goods take the following symmetric CES form: $F \equiv F(D_1, D_2) = [D_1^\rho + D_2^\rho]^{1/\rho}$, where $\rho \in (0, 1]$ and D_j denotes the quantity of good $j = 1, 2$ consumed. The elasticity of substitution in consumption is given by $\varepsilon \equiv \frac{1}{1-\rho}$, implying that the two goods are perfect substitutes (as in our baseline model) if $\rho \to 1$ or $\varepsilon \to \infty$. Based on these preferences, agent i's payoff function under war is given by

$$U^i = \phi^i \left[\left(\beta\gamma(1-\sigma)X^i + \beta\sigma X^i \right)^\rho + \left(\beta\gamma\,(1-\sigma)\,X^j \right)^\rho \right]^{1/\rho} + (1-\phi^i)\beta\sigma X^i.$$

The first term represents agent i's consumption contingent on victory, weighted by his probability of winning. The second term equals his consumption in the case of defeat

[38] To see this more clearly, note that we can rewrite R^* in (11′) as a function of $\gamma_{NT} = (1-\beta\sigma)/2\beta(1-\sigma)$:

$$R_* = \frac{\overline{R}}{1 + (1 + 2[\gamma_{NT}/\gamma - 1])^{1/\alpha}}.$$

For any $\alpha \in (0, 1]$, if $\gamma > \gamma_{NT}$, then $R_* > \frac{1}{2}\overline{R}$, such that peace is not possible for any distribution $R^i \in (0, \overline{R})$.

[39] See Appendix B.

weighted by that probability. Define $\eta \equiv \gamma(1-\sigma)+\sigma$, so that $\theta = \gamma(1-\sigma)/\eta$. Then, agent i's payoff under war can be written more compactly as

$$U^i = \beta\eta\left(\phi^i\left[(X^i)^\rho + (\theta X^j)^\rho\right]^{1/\rho} + (1-\phi^i)(1-\theta)X^i\right), \tag{15}$$

which simplifies to that in the baseline model (3) if $\rho = 1$.[40]

Agent i's payoff under peace and perfectly competitive trade (with no trade costs) can be written as:[41]

$$V^i = \psi^i(R^i,R^j)F(R^i,R^j), \quad \text{where} \quad \psi^i \equiv \psi^i(R^i,R^j) = \frac{(R^i)^\rho}{(R^i)^\rho + (R^j)^\rho}. \tag{16}$$

ψ^i represents agent i's share of total utility available to the two players when each agent devotes his entire resource endowment to produce his (differentiated) good. This payoff, too, simplifies to that of the baseline model when $\rho = 1$, as shown in (7). Keeping in mind that $R^i + R^j = \overline{R}$, one can demonstrate the following: First, for $\rho \in (0,1)$, $F(\cdot)$ is strictly concave in the allocation R^i, reaching its maximum at $R^i_F = \frac{1}{2}\overline{R}$. Second, $\psi^i(\cdot)$ is increasing in $R^i \in (0,\overline{R})$ for $\rho \in (0,1]$. As a result, $V^i_p = \psi^i F$ attains a maximum at some $R^i_p > R^i_F$. In addition, $\lim_{R^i\to\overline{R}} V^i_p = \overline{R}$ and $\lim_{R^i\to\overline{R}}(dV^i_p/dR^i) < 0$ that imply $R^i_p \in (\frac{1}{2}\overline{R},\overline{R})$.

The payoff to agent i under a unilateral deviation from unarmed peace can be derived from (15), recognizing that his optimal deviation is $G^i_d = \epsilon$, which is arbitarily close to zero and, in turn, implies (given $G^j = 0$) that $\phi^i = 1$, as well as $X^j = R^j$ and $X^i \approx R^i$:

$$U^i_d \approx \beta\eta\left[(R^i)^\rho + (\theta R^j)^\rho\right]^{1/\rho}. \tag{17}$$

U^i_d in (17) simplifies to the deviation payoff when $\rho = 1$ as shown in (10). For $\rho \in (0,1)$, this payoff is strictly concave in the allocation R^i, reaching its maximum at some $R^i_d \geq \frac{1}{2}\overline{R}$, with equality in the case of perfectly insecure property (i.e., $\sigma = 0$). Furthermore, we have $\lim_{R^i\to\overline{R}} U^i_d = \beta\eta\overline{R}$ and $\lim_{R^i\to\overline{R}}\left(dU^i_d/dR^i\right) < 0$, such that $R^i_d \in [\frac{1}{2}\overline{R},\overline{R})$.

As in the baseline model, $U^i_d > U^i_w$ holds. Thus, to examine how trade affects the stability of peace (absent transfers), we must compare V^i_p with U^i_d. We start with the following two observations:

(i) $\lim_{R^i\to 0} V^i_p = 0$ whereas $\lim_{R^i\to 0} U^i_d = \beta\gamma(1-\sigma)\overline{R}$. Therefore, $\lim_{R^i\to 0} U^i_d > \lim_{R^i\to 0} V^i_p$ for $\beta\gamma \in (0,1]$.

[40] As in the case of diminishing returns, it is not possible to find closed-form solutions for the best-response functions or the Nash equilibrium; however, we can characterize the payoff functions.

[41] For brevity, the derivation of this expression, based on a standard analysis of the Armington (1969) model of trade, is presented in Appendix B.

(ii) $\lim_{R^i \to \overline{R}} V_p^i = \overline{R}$ whereas $\lim_{R^i \to \overline{R}} U_d^i = \beta\eta\overline{R}$. Therefore, $\lim_{R^i \to \overline{R}} U_d^i \leq \lim_{R^i \to \overline{R}} V_p^i$ with equality if there is no destruction under war or under a unilateral deviation from peace (i.e., if $\beta\gamma = 1$).

Suppose war is destructive (i.e., $\beta\gamma < 1$). Since both V_p^i and U_d^i are continuous in $R^i \in (0, \overline{R})$, observations (i) and (ii) imply that U_d^i will cross V_p^i from above at some R^i. Exhaustive numerical analysis confirms that this crossing occurs at most once at some $R^i \in (0, \overline{R})$. As a consequence, agent i will prefer a unilateral deviation over peace, if his initial endowment is sufficiently small and conversely if his endowment is sufficiently large . One can show further that the larger is the overall and/or differential rates of destruction the more likely that the crossing will occur at some $R^i < \frac{1}{2}\overline{R}$. Taken together, these results establish the existence of sufficiently even distributions that support peace provided war and unilateral deviations are sufficiently destructive as in our baseline model.

But, the more important question for our purposes here, assuming agents capitalize on the opportunity that exists under peace to engage in mutually advantageous free trade (i.e., with $\rho \in (0, 1)$), is whether peace can arise as a stable equilibrium when there is no destruction under war at all (i.e., $\beta\gamma = 1$). To establish the possibility that it can, we must show the crossing of U_d^i and V_p^i can occur at some $R^i \leq \frac{1}{2}\overline{R}$ when $\beta = \gamma = 1$. To that end, consider the ratio $(V_p^i / U_d^i)|_{R^i = \frac{1}{2}\overline{R}}$. Using (16) and (17), one can show that this ratio evaluated at $\beta = \gamma = 1$ is given by

$$\Omega \equiv \Omega(\sigma, \rho) = \left.\frac{V_p^i}{U_d^i}\right|_{R^i = \frac{1}{2}\overline{R}} = \left[\frac{2^{1-\rho}}{1 + (1-\sigma)^\rho}\right]^{1/\rho}.$$

For any $\rho \in (0, 1)$ which inversely reflects the possible gains from trade, there exists a $\sigma_\Omega \equiv \sigma_\Omega(\rho) = 1 - (2^{1-\rho} - 1)^{1/\rho} \in (0, 1)$, where $\sigma'_\Omega > 0$ and $\sigma''_\Omega > 0$, such that $\Omega \geq 1$ for all $\sigma \geq \sigma_\Omega$.[42] The condition $\sigma \geq \sigma_\Omega$ ensures that $V_p^i \geq U_d^i$ for both i when resource endowments are identical across agents; and, when $\sigma > \sigma_\Omega$, the allocation where the less affluent strictly is indifferent between peace and a unilateral deviation from it is at some allocation $R^i < \frac{1}{2}\overline{R}$. These findings imply that war need not be destructive for the emergence of unarmed peace as the stable equilibrium when peace gives rise to the possibility for mutually beneficial trade. Even in the absence of war's destructive effects, provided that output is sufficiently secure, trade can ensure the existence of sufficiently even distributions of resources that support peace as a stable equilibrium. What's more, since $\sigma'_\Omega > 0$, greater gains from trade (i.e., lower values of ρ) weaken the requirement on the security of output.

While the above analysis is encouraging for the prospects of unarmed peace when war is not destructive and more generally, it is important not to lose sight of the converse im-

[42] One can also show that $\lim_{\rho \to 0} \sigma_\Omega = \frac{3}{4}$ while $\lim_{\rho \to 1} \sigma_\Omega = 1$.

plication, that higher degrees of insecurity ($\sigma \downarrow$) together with lower gains from trade ($\rho \uparrow$) weaken the effectiveness of trade to sustain unarmed peace. Put differently, one would be incorrect in asserting that trade can always help support unarmed peace. Institutions that shape the degree of security in property rights and the distribution of resource ownership play important roles here.

5.3 Preexisting Military Capabilities

Next we turn to the possibility of preexisting military capabilities. As has been argued by Slantchev (2011) among others, preexisting military capabilities are empirically relevant. In particular, modern wars are typically of short duration and, when of a limited nature, often are fought with the contenders' exiting military apparatus alone (i.e., without producing additional weaponry). Applied to our setting, preexisting military capabilities mean that, when contemplating a unilateral deviation from peace, each agent would have to take into account that his rival is already armed, so that the production of some infinitesimal quantity of guns would no longer suffice to assure victory.[43] More generally, they affect both the initial and the final distributions of power through their possible impact on current arming decisions. Although this possibility does not affect the payoffs under peace, it does matter for arming incentives and, thus, for the payoffs under unilateral deviations as well as under war; and, this influence naturally matters for the stability of peace. Our specific focus here is to examine whether peace can arise as the stable equilibrium outcome in the presence of preexisting arms, but no destruction.

Let $G_0^i > 0$ denote the initial quantity of guns each agent i holds at the beginning of their interactions, and define $\overline{G}_0 = G_0^1 + G_0^2$ as the total quantity of initial holdings. For simplicity, we assume G_0^i is a perfect substitute for current guns G^i and that agents need not incur any additional costs to maintain or put their preexisting arms into use. However, these arms affect power through the conflict technology, which we modify as follows:

$$\phi^i \equiv \phi^i(G^i, G^j) = \frac{G^i + G_0^i}{\overline{G} + \overline{G}_0}, \quad \text{for } i \neq j = 1, 2 \text{ and } \overline{G}_0 > 0. \tag{18}$$

Clearly, when neither agent produces any guns, the initial distribution of power is given by $\phi^i(0,0) = G_0^i/\overline{G}_0$. A key issue here is whether and if so how agent i chooses to adjust his military capacity through his current arming decisions, given $\overline{G}_0$ and G^j.

Applying the conflict technology in (18) to (4), while incorporating the relevant resource and non-negativity constraints, yields the following modified best-response func-

[43] In addition, as mentioned previously, the notion that a unilateral deviation could be destructive seems more reasonable when each agent holds some quantity of guns before any decisions are made.

tion for agent i:

$$B_w^i = B_w^i(G^j;\cdot) \equiv \min\left\{R^i, \max\left[\widetilde{B}_w^i(G^j;\cdot), 0\right]\right\},\ i = 1, 2. \tag{19a}$$

where $\widetilde{B}_w^i(G^j;\cdot)$ denotes agent i's unconstrained best-response function; this function satisfies $\partial U^i/\partial G^i = 0$ and is given by

$$\widetilde{B}_w^i = \widetilde{B}_w^i(G^j;\cdot) \equiv -(G^j + \overline{G}_0) + \sqrt{\theta(G^j + G_0^j)(\overline{G}_0 + \overline{R})}. \tag{19b}$$

Observe from (19) that, as in the baseline model, agent i's arming choice could be constrained by his available resources. Furthermore, it is possible for agent i to choose to arm even when his rival produces no additional arms (i.e., $G^j = 0$). Finally, agent i could choose to produce no additional arms at all. Henceforth, to streamline the analysis, we assume $G_0^i = \lambda R^i$ for $\lambda \in [0, 1]$, implying there exists only one source of asymmetry between agents—namely, in resource endowments. This formulation, which aims to capture the reasonable idea that arming decisions are limited by agents' endowments, nests our baseline model (i.e., with $\lambda = 0$). Note especially, it implies, with our modified conflict technology (18) and consistent with our specification in the baseline model (1), that $\phi^i(0, 0) = R^i/\overline{R}$.[44]

Given our primary interest in how the presence of preexisting arms influences the stability of peace when neither war nor unilateral deviations from peace cause destruction, we now impose the condition that $\beta\gamma = 1$, which implies $\theta = 1 - \sigma \in (0, 1]$, and evaluate agent i's net marginal benefit of arming (4) using (18) at $G^i = G^j = 0$ and $G_0^i = \lambda R^i$ for $i = 1, 2$:[45]

$$\left.\frac{\partial U_d^i}{\partial G^i}\right|_{G^i=G^j=0} = \frac{\delta\overline{R} - R^i}{\overline{R}(1-\delta)} \gtreqless 0 \quad \text{as} \quad R^i \lesseqgtr \delta\overline{R}, \tag{20}$$

where $\delta \equiv \delta(\lambda, \sigma) = 1 - \frac{\lambda}{(1+\lambda)(1-\sigma)} < 1$ for $\lambda > 0$. The function $\delta(\lambda, \theta)$ indicates the threshold value of an agent's resource endowment as a fraction of the total resource base, $R^i/\overline{R}$, above which he chooses not to add to his preexisting holdings of guns—i.e., if $R^i \geq \delta\overline{R}$, then $B_w^i(G^j = 0;\cdot) = G_d^i = 0$ holds. This threshold is decreasing in λ that positively indicates preexisting guns (given $\overline{R}$) and in the security of output σ that reduces the prize from conflict. As a result, the condition for $G_d^i = 0$ (i.e., $\partial U^i/\partial G^i|_{G^i=G^j=0} \leq 0$) is more

[44] We could dispense with this assumption, though at the cost of added complexity due to an expanded number of possible cases to consider. In any case, note that Jackson and Morelli (2007) also employ this assumption, but do not allow agents to make adjustments in their guns.

[45] In Appendix B, we provide a more detailed analysis of this benchmark case as well as when there is some destruction.

easily satisfied when either λ or σ is larger.

Based on our findings above, we now identify the conditions under which agent i views a unilateral deviation as unappealing (and conversely) when war is not destructive. To start, observe that $\delta(\lambda, \sigma) \leq 0$ holds whenever the quantity of preexisting guns is sufficiently large: $\lambda \geq \frac{1-\sigma}{\sigma}$ (which also requires sufficiently secure output, $\sigma > \frac{1}{2}$). In such cases, from (20), neither agent i has an incentive to add to his military capacity with a unilateral deviation (i.e., $G_d^i = 0$ under all feasible resource distributions $R^i \in (0, \overline{R})$). Because $G_d^i = G_p^j = 0$ ($j \neq i$), the modified conflict technology in (18) implies agent i's probability of victory when he deviates unilaterally by declaring war is $\phi^i = R^i/\overline{R}$. Furthermore, since (like his rival j) agent i uses his entire endowment to produce butter (i.e., $X^i = R^i$ for $i \neq j = 1, 2$), his deviation payoff is given by $U_d^i = R^i$, which equals his payoff under peace $V_p^i = R^i$. Thus, when $\lambda \geq \frac{1-\sigma}{\sigma}$, neither agent i has an incentive to deviate from peace unilaterally. Even when $\lambda < \frac{1-\sigma}{\sigma}$ such that $\delta > 0$ holds, moderate quantities of preexiting guns (more precisely, $\lambda \in [\frac{1-\sigma}{1+\sigma}, \frac{1-\sigma}{\sigma})$) imply $\delta \leq \frac{1}{2}$. For such parameter values, there exists a nonempty subset of distributions $R^i \in [\delta\overline{R}, (1-\delta)\overline{R}]$ for which, once again, $G_d^i = 0$ holds for both i, and neither agent has an incentive to deviate from peace. However, if $R^i \notin [\delta\overline{R}, (1-\delta)\overline{R}]$, the less affluent agent i has an incentive to add to his preexisting guns ($G_d^i > 0$ given $G^j = 0$) and declare war, whereby he can obtain a higher payoff $U_d^i > R^i$. By the same token, if the quantities of preexisting guns are sufficiently small ($\lambda < \frac{1-\sigma}{1+\sigma}$) to imply that $\delta > \frac{1}{2}$ holds, at least one agent (the less affluent one) has an incentive to deviate unilaterally for any feasible resource distribution.[46] In either of these two latter cases, the profitability of a unilateral deviation undermines the stability of peace.

Nonetheless, this discussion would seem to suggest that, even when war is not destructive, provided that either (i) preexisting guns are sufficiently large or (ii) preexisting guns are moderately large and the distribution of resources is sufficiently even, peace is immune to unilateral deviations. But, in such cases, neither agent would have an incentive to arm when under war either (i.e., $G_w^i = G_d^i = 0$ for $i = 1, 2$), implying $U_w^i = U_d^i = V_p^i$.[47] Hence, when war is not destructive, the conditions that ensure that unilateral deviations are unprofitable for both agents are precisely the conditions under which there is essentially no difference between the war and peace payoffs.

Preexisting guns do matter, however, when war is destructive. First, we can show that a greater quantity of preexisting guns ($\lambda \uparrow$) decreases the payoff to an agent who deviates unilaterally (through an adverse strategic payoff effect), to expand the parameter space

[46] Observe that, if $\lambda < 1$, a sufficient condition for a unilateral deviation to be profitable for at least one agent, absent destruction under war, is that output is perfectly insecure, $\sigma = 0$.

[47] Of course, the realized outcomes will differ. To confirm that $G_w^i = 0$ for $i = 1, 2$ in such cases, one can evaluate $\partial U^i / \partial G^i$, using (4) with (18), at $G^j > 0$ and $G^i = 0$ to find this expression is non-positive for any $R^i \in (0, \overline{R})$ when $\delta \leq 0$ and for any $R^i \in [\delta\overline{R}, (1-\delta)\overline{R}]$ when $\delta \leq \frac{1}{2}$.

(β, γ, σ) under which peace without transfers can emerge as the stable equilibrium outcome for some resource distributions. Second, given that peace without transfers is possible for some resource distributions, an increase in λ expands the parameter space (β, γ, σ) under which transfers can support peace for all feasible initial resource distributions.[48]

5.4 Sequential Moves

If communication were absent in our baseline model, whether agents make their arming and war/peace choices simultaneously or sequentially would not matter. But, when communication is possible, the timing of choices can have important implications. In this section, we briefly analyze the conditions under which our analysis above of the simultaneous-move game (without transfers) extends to sequential-move version of the game.[49] Suppose, in particular, that there are two stages. In stage one, each agent i arms and these choices are made simultaneously; in stage two, after having observed the rival's arming choice, each agent chooses whether to declare "peace" or "war." We allow for communication between the two agents throughout.

When the conditions of Proposition 4(a) are satisfied, then unarmed peace continues to be the stable equilibrium.[50] When those conditions are not satisfied for a given distribution of resources, then war could be the unique subgame perfect, Nash equilibrium in pure strategies, but not necessarily. The potential problem here is that once the two agents arm in anticipation of war, both could be better off if they agreed not to fight.

To dig a little deeper, let us suppose that $R^i < R_*$ for at least the less affluent agent, and compare his payoffs under war with those under peace for given guns chosen in the first stage (G^i, G^j). Using (2) and (3) while keeping in mind that $R^j = \overline{R} - R^i$ and $G^j = \overline{G} - G^i$, one can confirm $V^i(G^i, G^j) - U^i(G^i, G^j) \geq 0$, so that agent i (at least weakly) prefers peace in the second stage, if and only if

$$(R^i - G^i)\left[1 - \phi^i \beta\gamma(1-\sigma) - \beta\sigma\right] - (R^j - G^j)\left[\phi^i \beta\gamma(1-\sigma)\right] \geq 0.$$

From Proposition 1, when $R^i \leq R_L$, $G^i = G^i_w = R^i$ holds, whereas $G^j = \widetilde{B}^j_w(R^i) < R^j$. Thus, when one agent i is constrained in his arming choice, the inequality above cannot be satisfied. Agent i, who finds a unilateral deviation from unarmed peace in the first stage to be profitable, also strictly prefers war in the second stage given both agents have armed. In this case, war is the unique subgame perfect, Nash equilibrium in pure strategies. Now

[48] See Appendix B for details regarding both claims.

[49] Following Bernheim et al. (1987), the relevant equilibrium concept in this version of the game is "perfect coalition proofness" that imposes the condition of dynamic consistency on strategies.

[50] Although there could exist mixed-strategy equilibria with positive arming, the conditions that ensure unarmed peace is stable also ensure that unarmed peace dominates any such mixed-strategy equilibrium, by the same logic we spell out in the proof to Proposition 4.

suppose neither agent is constrained, in which case $G_w^i = R_L \equiv \frac{1}{4}\theta\overline{R} \leq R^i$ and $\phi^i = \frac{1}{2}$ for both agents i. In this case, there exists a threshold level of the distribution of resources, denoted by $\widehat{\widehat{R}}$, such that for $R^i > \widehat{\widehat{R}}$, $V^i(G_w, G_w) - U^i(G_w, G_w) > 0$ holds and agent i is strictly better off by declaring "peace" in the second stage:

$$\widehat{\widehat{R}} \equiv \left[\frac{\beta\gamma(1-\sigma)+\theta}{4(1-\beta\sigma)}\right]\overline{R} \in \left(R_L, \tfrac{1}{2}\overline{R}\,\right].$$

As one can verify, absent destruction (i.e., $\beta\gamma = 1$) that implies $\theta = 1 - \sigma$, $\widehat{\widehat{R}} = \frac{1}{2}\overline{R}$. In this special case where $R_* = \overline{R}$, war remains the unique, pure-strategy equilibrium for all feasible resource distributions.[51]

But, when war is destructive such that $\widehat{\widehat{R}} < \frac{1}{2}\overline{R}$ holds, it is possible that $R^i < R_*$ for at least one agent, whereas $R^i > \widehat{\widehat{R}}$ for both agents. For such moderately even distributions $R^i \in [\widehat{\widehat{R}}, R_*] \subset (R_L, \frac{1}{2}\overline{R})$, the less affluent agent, who views a unilateral deviation from unarmed peace in the first stage to be profitable, prefers peace in the second stage like his more affluent rival. Thus, both agents who are ready for war in the second stage would be willing to agree to (armed) peace. Of course, in anticipation of such coordination in the second stage, each agent would want to adjust his first-period arming choice. Accordingly, neither war nor unarmed peace would constitute a stable equilibrium in the sequential-move version of this model.[52]

Yet, it is important to emphasize that this possibility need not arise when peace is not a stable equilibrium outcome in the sequential-move game. In particular, we have already pointed out that, when one agent is constrained in his arms production, his preference for war remains intact in the second stage. Even when neither agent is constrained, it is possible that either $R^i < \widehat{\widehat{R}} < R_*$ or that $R^i < R_* < \widehat{\widehat{R}}$. The former case can arise when war is not sufficiently destructive (i.e., $\gamma > \gamma_{NT}$) such that $R_* > \frac{1}{2}\overline{R}$, meaning that unarmed peace can be ruled out for any feasible resource distribution. The latter case, which is a bit stronger and ensures that war necessarily arises as the unique subgame perfect, Nash equilibrium in pure strategies, requires that $\gamma < \frac{1-3\beta\sigma}{3\beta(1-\sigma)}$, which itself requires $\beta\sigma < \frac{1}{3}$ and implies $\gamma < \gamma_{NT}$. These two cases point to a more nuanced relation between the destructiveness of war and its emergence in equilibrium. In either case, war remains the unique subgame-perfect, Nash equilibrium in pure strategies provided that unilateral deviations from peace are profitable.

[51] At $R^i = \frac{1}{2}\overline{R}$, both agents would be indifferent between war and peace given their arming choices $G_w = R_L$; however, when war is not destructive, the two outcomes are indistinguishable.

[52] In such cases, there likely exist mixed-strategy equilibria, as studied in Jackson and Morelli (2009) and De Luca and Sekeris (2013).

6 Concluding Remarks

Disputes over such things as resources, output, technology, and spheres of influence are common. While some are resolved by fighting that generates large social losses, others are resolved peacefully through negotiations and a division of whatever is being disputed or more simply by letting the status quo stand. To be sure, as has been studied in the literature, peace through negotiation to divide whatever is being contested is not costless, since each contender arms to improve his bargaining position vis-à-vis his rival.[53] Nevertheless, at least in a one-period setting, such armed peace is always welfare-improving given the contenders' guns choices, insofar as it allows them to avoid at least some of the social losses or enjoy a surplus relative to war—due to, for example, the avoidance of violence and uncertainty that comes with war and/or the opportunity of mutually advantageous trade that would not exist in the case of war.

In this paper, we study peace identified with the status quo in a single-period setting. Since no bargaining is involved, agents have no incentive to arm under peace, thereby freeing up resources to produce more goods for consumption. However, since there is no division of contestable output, one agent (the less affluent one) could find war relatively more appealing than this form of peace. Moreover, while a sufficiently even distribution of resource endowments could make peace relatively appealing to both agents, the absence of arming under peace possibly leaves the agents unable to commit to sustain it. Thus, the Pareto dominance of peace is not sufficient for its emergence as a stable equilibrium outcome in this setting. We must also check that the outcome is immune to unilateral deviations, where one agent arms and declares war while his rival remains unarmed in anticipation of peace. Put differently, we compare not only the payoffs under war and peace, but also the payoffs under peace and unilateral deviations.

These comparisons in our baseline model reveal that the pattern of war's destructive effects and the security of output matter. Indeed, a necessary condition for unarmed peace to be stable in the baseline model for any distribution of resource endowments is that the alternative (i.e., war) be destructive. In this case, and more generally when war is only mildly destructive and output insecurity is high, war is the unique equilibrium outcome in pure strategies. Otherwise, unarmed peace is possible, but only for sufficiently even distributions of resource endowments.[54] While *ex ante* resource transfers made from the more affluent agent to his rival render peace stable for a wider range of resource endowments, their effectiveness requires that peace without transfers be stable for at least some

[53]See Garfinkel and Syropoulos (2018, 2019) and references cited therein.

[54]Although war's effect to preclude mutually beneficial trade can "substitute" for war's destructive effects to support unarmed peace, the requirement that such peace be immune to unilateral deviations remains relevant and is what gives rise to the requirement that the distribution of resource endowments be sufficiently even.

resource distributions. Given that condition is satisfied, greater destruction enhances the power of transfers in the sense of making peace possible for a wider range of resource distributions. Furthermore, although improvements in output security tend to make peace without transfers more likely, if the destructive effects of war are not too severe, there are cases where increased security could shrink that range.

One interesting and important area for additional study involves a more comprehensive comparison of unarmed peace identified with the status quo (and possibly including transfers) with armed peace that involves negotiations and a division of contested output (that, as noted earlier, could be viewed as *ex post* transfers). When studied within a common, single-period framework, one could identify the conditions under which one form or both forms of peace can possibly emerge in equilibrium. In the case where both forms are possible, one could then study their Pareto ranking.

Another potentially fruitful avenue for further study builds on the one-period setting with preexisting military capacities. Specifically, instead of assuming that the quantity arms brought into the period depends on the contenders' resource endowments, one could suppose they are provided by a third party. Depending on its objectives (e.g., to promote peace or to favor one contender over the other), a third party could decide to provide both sides, one side, or neither side with guns; alternatively, third-party intervention need not involve the provision of arms, but rather provision of productive resources. One central issue here is how such intervention (whatever form it takes) influences the stability of unarmed peace.

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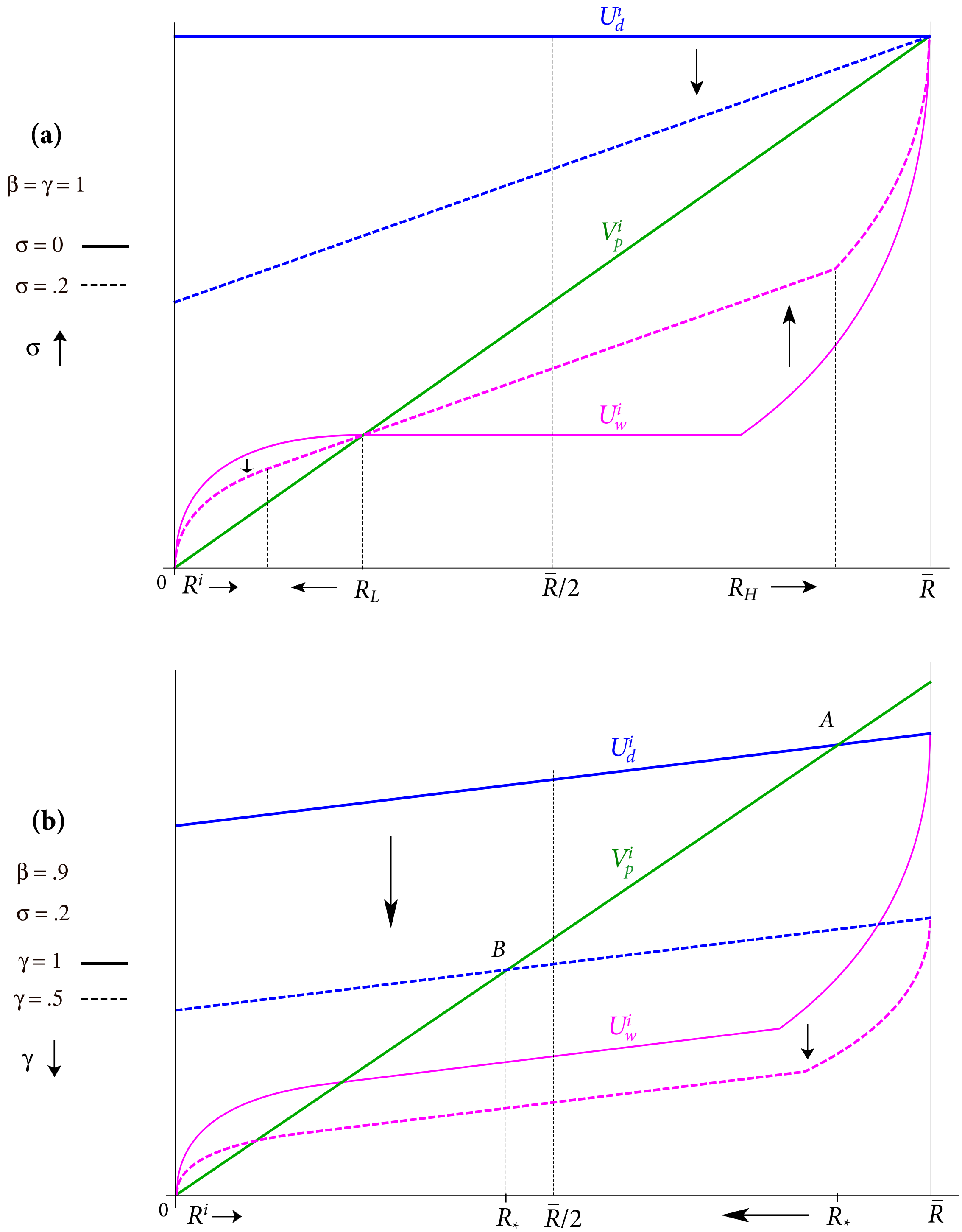

Figure 1: Payoffs under war for alternative distributions of resource endowments and values of security σ and differential destruction γ

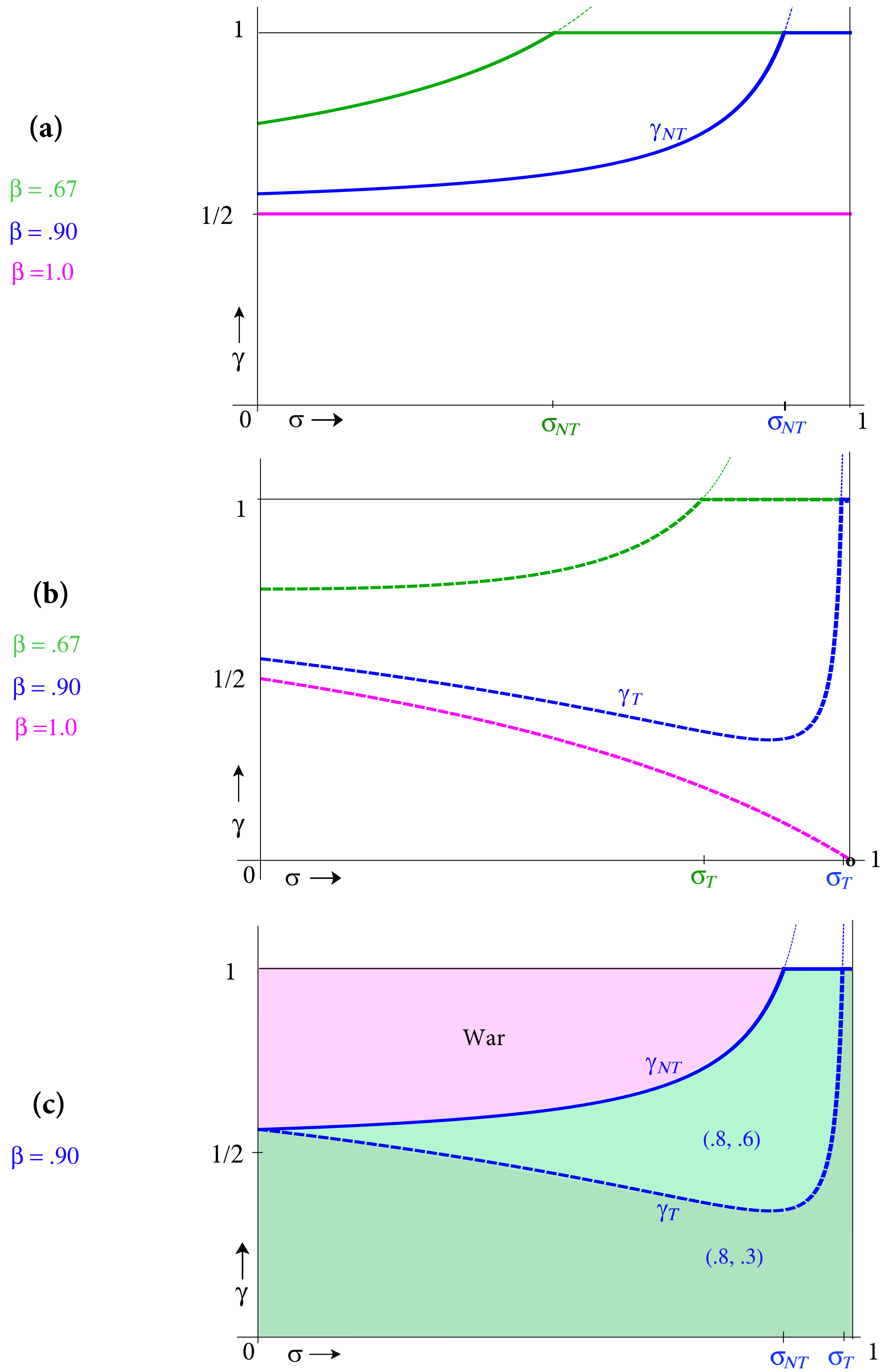

Figure 2: Combinations of γ and σ for which peace can arise under some resource distributions

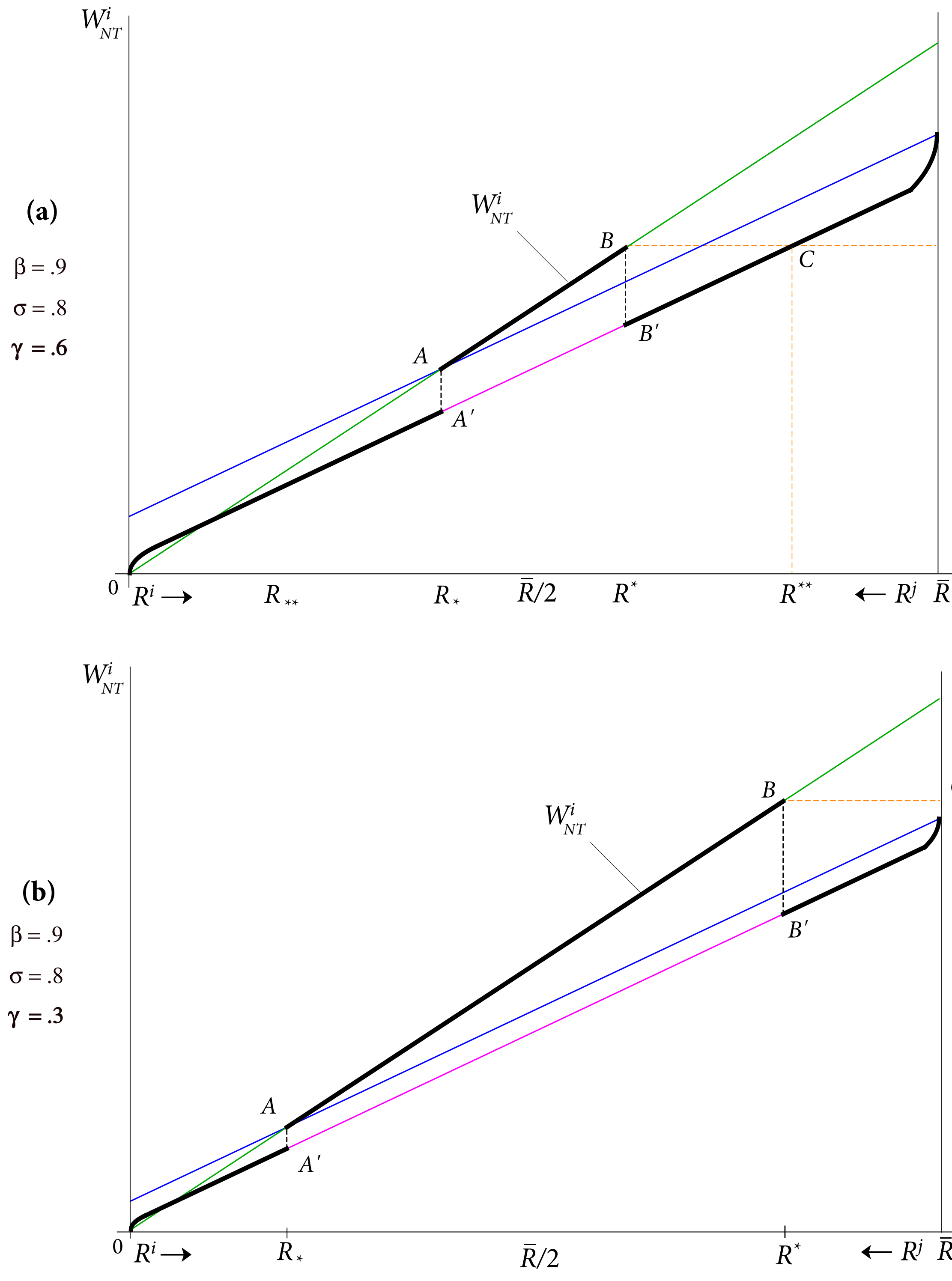

Figure 3: Equilibrium payoffs in the extended game without resource transfers

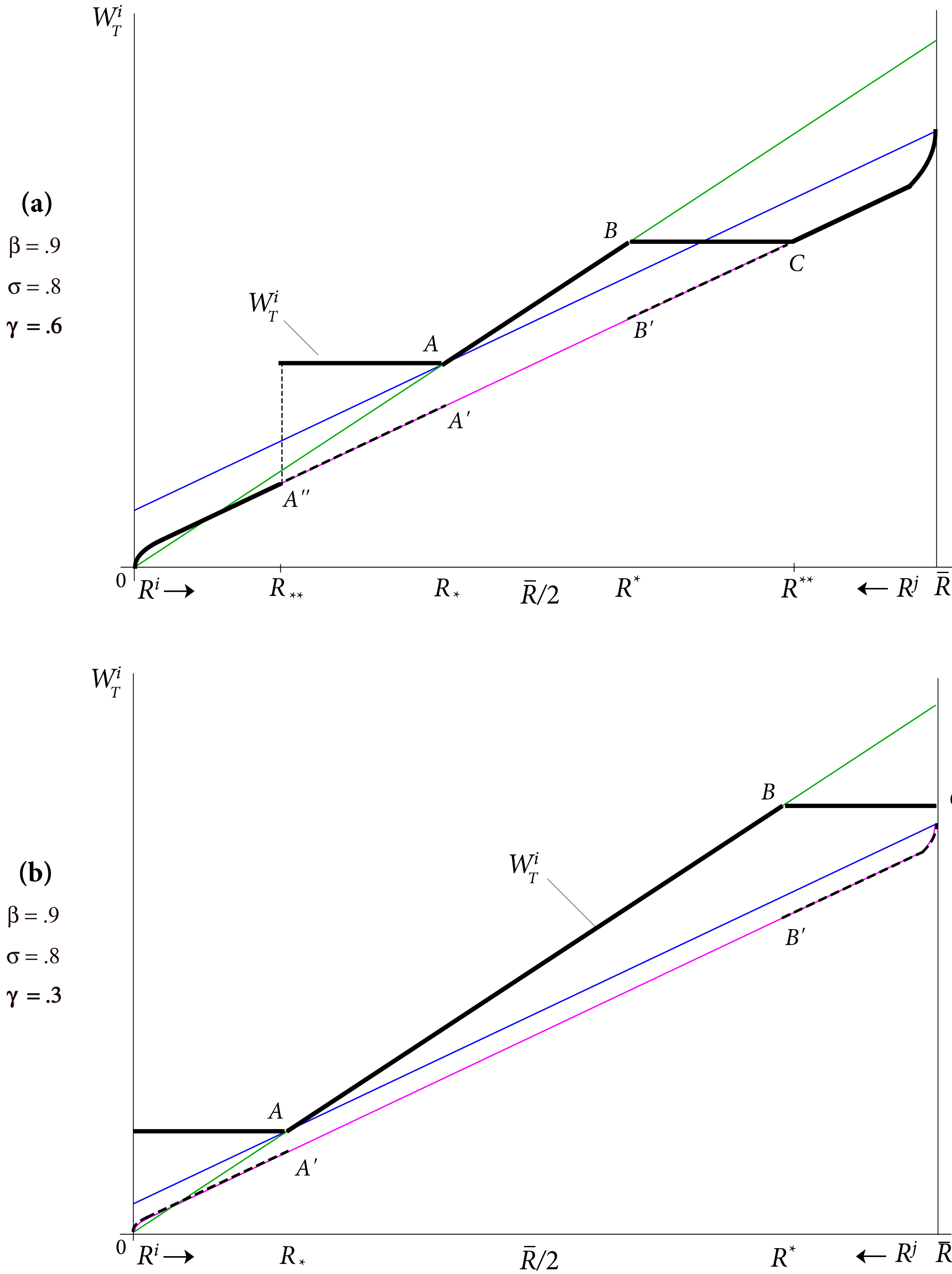

Figure 4: Equilibrium payoffs in the extended game with resource transfers

Appendix

A Proofs of Lemmas and Propositions

Proof of Proposition 1.

Part (a). The first-order conditions (FOCs) associated with $U^i_{G^i} = 0$ for $i \neq j = 1,2$ (from (4)), imply that $G^i_w = \frac{1}{4}\theta\overline{R}$. Since this outcome requires $G^i_w \leq R^i$ for $i = 1,2$, the threshold levels of the resource are given by $R_L \equiv \frac{1}{4}\theta\overline{R}$ and $R_H \equiv \overline{R} - R_L = \left[1 - \frac{1}{4}\theta\right]\overline{R}$, as shown in (6). From the expression for R_L, it follows immediately that $dR_L/d\theta > 0$.

Part (b). When agent i is constrained by his endowment, $G^i_w = R^i$ while $G^j_w = \widetilde{B}^j_w(R^i) > R^i$, where $\widetilde{B}^j_w(\cdot)$ is shown in (5b). In turn, differentiating $\widetilde{B}^j_w(\cdot)$ appropriately shows that increases in θ increase the unconstrained agent's optimal arming. ||

Proof of Proposition 2.

Part (a). Assuming $R^i \in [R_L, R_H]$ where $R_L \equiv \frac{1}{4}\theta\overline{R}$, Proposition 1(a) shows $G^i_w = R_L$ for $i = 1,2$, which implies (from (1)) $\phi^i = \frac{1}{2}$, $X^i = R^i - \frac{1}{4}\theta\overline{R}$, and $\overline{X} = [1 - \frac{1}{2}\theta]\overline{R}$ for $i = 1,2$. Substituting these values into (3) gives U^i_w as shown in the second line of (8). This payoff is clearly increasing in agent i's own resource R^i (given $\overline{R}$ and $\sigma > 0$), β, and γ. It is also increasing (decreasing) in σ for $R^i > \frac{1}{4}\gamma\overline{R}$ $(R^i < \frac{1}{4}\gamma\overline{R})$. When $\frac{1}{4}\gamma\overline{R} < R_L = \frac{1}{4}\theta\overline{R}$ or equivalently when $\gamma < \frac{1-\sigma}{2-\sigma}$, both agents $(i = 1,2)$ would benefit from an increase in σ for any distribution $R^i \in [R_L, R_H]$. Otherwise, an increase in σ would reduce the payoff of the less affluent agent i if his endowment R^i is sufficiently close to R_L.

Part (b). If $R^i \in (0, R_L)$, then from Proposition 1(b), $G^i_w = R^i$ and from (5b) $G^j_w = -R^i + \sqrt{\theta R^i \overline{R}}$, which from (1) imply $\phi^i = R^i/\sqrt{\theta R^i \overline{R}}$. Furthermore, we have $\overline{X} = X^j = \overline{R} - \sqrt{\theta R^i \overline{R}}$. Substitution of these values into (3) gives the payoff function for constrained agent $i \neq j = 1$ or 2 shown in the first line of (8). Clearly, $\lim_{R^i \to 0} U^i_w = 0$. Differentiating the expression for U^i_w with respect to R^i, γ, β, and σ, while using the definition of θ in (5c), shows respectively

$$\frac{dU^i_w}{dR^i} = \beta(1-\sigma)\left(\sqrt{\frac{\overline{R}}{4\theta R^i}} - 1\right) \geq 0, \quad \frac{d^2 U^i_w}{(dR^i)^2} < 0 \tag{A.1a}$$

$$\frac{dU^i_w}{d\gamma} = \tfrac{1}{2}\beta(1-\sigma)\sqrt{\frac{\overline{R}R^i}{\theta}}\left[1 + \theta\left(1 - \sqrt{\frac{4R^i}{\theta\overline{R}}}\right)\right] > 0, \quad \frac{d^2 U^i_w}{d\gamma^2} < 0 \tag{A.1b}$$

$$\frac{dU^i_w}{d\beta} = \frac{U^i_w}{\beta} > 0, \quad \frac{d^2 U^i_w}{d\beta^2} = 0. \tag{A.1c}$$

$$\frac{dU^i_w}{d\sigma} = \frac{\beta\sqrt{R^i\overline{R}\theta}}{2(1-\sigma)}\left[1 - 2\left[\gamma(1-\sigma) + \sigma\right]\left(1 - \sqrt{\frac{R^i\theta}{\overline{R}}}\right)\right], \quad \frac{d^2 U^i_w}{d\sigma^2} < 0. \tag{A.1d}$$

The first inequality in (A.1a) follows from the restriction that $R^i < R_L = \frac{1}{4}\theta\overline{R}$ and the fact that $\frac{1}{4}\theta\overline{R} < \frac{1}{4}\overline{R}/\theta$. The first inequality in (A.1b) follows directly from the requirement that $R^i < \frac{1}{4}\theta\overline{R}$, and the inequality in (A.1c) follows immediately. Turning to the payoff effects of an increase in σ, the RHS of the first expression in (A.1d) can be rearranged to show $dU^i_w/d\sigma < 0$ if and only if

$$\sqrt{\frac{R^i\theta}{\overline{R}}} < 1 - \frac{1}{2[\gamma(1-\sigma)+\sigma]}.$$

In the case that $\gamma(1-\sigma)+\sigma \leq \frac{1}{2}$ or equivalently $\gamma \leq \frac{1-2\sigma}{2(1-\sigma)}$, the inequality above cannot be satisfied, implying that $dU^i_w/d\sigma > 0$ for all $R^i \in (0, R_L)$. Alternatively, when $\gamma(1-\sigma)+\sigma > \frac{1}{2}$, the above inequality can be written as

$$R^i < \frac{\overline{R}}{\theta}\left(1 - \frac{1}{2[\gamma(1-\sigma)+\sigma]}\right)^2. \tag{A.2}$$

Consistent with our finding from part (a), this critical value of R^i is strictly less than $R_L = \frac{1}{4}\theta\overline{R}$ when $\gamma \in (\frac{1-2\sigma}{2(1-\sigma)}, \frac{1-\sigma}{2-\sigma})$, implying that, for R^i sufficiently close to R_L, an increase in σ is welfare-improving for the resource-constrained agent. By contrast, if $\gamma \geq \frac{1-\sigma}{2-\sigma}$, then an increase in σ necessarily makes the constrained agent i worse off.

Next consider the expected payoff for the unconstrained agent $j \neq i = 1$ or 2 under war. Substituting the solutions above into (3) gives U^j_w shown in the last line of (8). The envelope theorem implies that the payoff effects of an exogenous change in $\chi \in \{R^j, \sigma, \beta\}$ for the unconstrained agent j can be decomposed into a direct effect and an indirect effect as follows:

$$dU^j_w/d\chi = U^j_\chi + U^j_{G^i}\left(\partial B^i_w/\partial\chi\right), \quad j \neq i.$$

Starting with $\chi = R^j$, equation (3) implies that $U^j_{R^j} = U^j_{X^j} > 0$; with the CSF specification in (1), it also implies $U^j_{G^i} < 0$. Since $B^i_w = R^i$ for $R^i \in (0, R_L)$ and $dR^j = -dR^i$, it follows that $U^j_{G^i}(\partial B^i_w/\partial R^j) > 0$ and thus $dU^j_w/dR^j > 0$. Additionally, inspection of the expression for payoffs in the last line of (8) reveals that U^j_w is convex in R^j. The remaining parameters $\chi \in \{\sigma, \beta\}$ have no influence on the rival's arming ($B^i_w = R^i$), implying that only their respective direct effects matter for j's payoff. The sign of these effects can be easily identified upon inspection of U^j in (3). $||$

Proof of Proposition 3. In what follows, we consider the condition for $V^i_p > U^i_w$ depending on whether neither agent or one agent is resource constrained in his arms production.

Case 1: $R^i \in [R_L, R_H]$ *for* $i = 1, 2$. From (7) and the second line in (8), when neither agent is resource constrained, the condition for agent i to strictly prefer peace is that

$$R^i > \tfrac{1}{4}\beta\gamma\,(1-\sigma)\,\overline{R} + \beta\sigma R^i,$$

which requires

$$R^i > \hat{R}_H \equiv \frac{\frac{1}{4}\beta\gamma(1-\sigma)}{1-\beta\sigma}\overline{R}.$$

One can confirm that $\hat{R}_H \geq R_L = \frac{1}{4}\theta\overline{R}$ when $\gamma \geq \frac{1-2\beta\sigma}{\beta(1-\sigma)}$ as required by the proposition. (Otherwise, we have $V_p^i > U_w^i$ for both agents whenever neither one is resource constrained, which is to say $\hat{R} \notin [R_L, R_H]$.) Using the expression above for $\hat{R}_H$, one can easily verify that $\hat{R}_H \leq \frac{1}{4}\overline{R}$. Finally, straightforward differentiation of $\hat{R}_H$ shows that this threshold falls as $\beta \downarrow$ and/or $\gamma \downarrow$ and as $\sigma \uparrow$.

Case 2: $R^i \in (0, R_L)$ for $i = 1$ or 2. Since payoffs under war are increasing in the agents' respective resource endowments, we focus on the constrained agent, i. From (7) and the first line of (8), the condition for him to strictly prefer peace is that

$$R^i > \gamma\beta(1-\sigma)R^i\left(\sqrt{\frac{\overline{R}}{\theta R^i}} - 1\right),$$

which after some manipulation can be shown to require

$$R^i > \hat{R}_L \equiv \left[\frac{\beta\gamma(1-\sigma)}{\beta\gamma(1-\sigma)+1}\right]^2 \overline{R}/\theta = \left[\frac{\beta(\gamma(1-\sigma)+\sigma)}{\beta\gamma(1-\sigma)+1}\right]^2 \theta\overline{R}.$$

One can readily verify, using (6), that $\hat{R}_L < R_L$ provided $\gamma < \frac{1-2\beta\sigma}{\beta(1-\sigma)}$ holds, as required by the proposition. By appropriately differentiating the RHS of the expression immediately above, one can verify, in addition, that this threshold falls with increases in destruction ($\beta \downarrow$ and/or $\gamma \downarrow$). Furthermore, differentiating the expression above with respect to σ shows

$$\frac{d\hat{R}_L}{d\sigma} = \frac{\hat{R}_L}{1-\sigma}\left[\frac{1-\gamma(2-\beta)-\sigma(2-\gamma(2-\beta))}{(\beta\gamma(1-\sigma)+1)(\gamma(1-\sigma)+\sigma)}\right],$$

which is negative iff

$$\gamma > \frac{1-2\sigma}{(2-\beta)(1-\sigma)}.$$

This condition on γ is precisely the necessary and sufficient condition that ensures the initial value of $\hat{R}_L$ is less than the critical value of $R^i \in (0, R_L)$ (given $\gamma < \frac{1-\sigma}{2-\sigma}$), above (below) which $dU_w^i/d\sigma > 0$ ($dU_w^i/d\sigma < 0$). (This critical value is shown in (A.2).) Conversely, when the inequality above is reversed (which also implies that the initial value of $\hat{R}_L$ is greater than the critical value of R^i shown in (A.2)), $d\hat{R}_L/d\sigma > 0$.[55] ||

Proof of Lemma 1. Since $\beta\gamma \in (0,1)$ implies $\lim_{R^i \to 0} U_d^i(R^i) > \lim_{R^i \to 0} V_p^i(R^i) = 0$, while $\lim_{R^i \to \overline{R}} U_d^i(R^i) < \lim_{R^i \to \overline{R}} V_p^i(R^i) = \bar{R}$ and $\partial U_d^i/\partial R^i < \partial V_p^i/\partial R^i$, the value of R_* shown in (11) is the unique value of R^i that equates $V_p^i = R^i$ to $U_d^i(R^i)$ shown in (10), from which the relative rankings of payoffs follow.

Part a. This part can be confirmed by differentiating R_* in (11) with respect to β, γ and σ. Note that $R_*|_{\beta=1} = \gamma\overline{R}$, which explains why $\partial R_*/\partial\sigma|_{\beta=1} = 0$.

Part b. The value of $\gamma_{NT} \equiv \gamma_{NT}(\sigma;\beta)$ defined in this part of the lemma is obtained by equating R_* in (11) to $\frac{1}{2}\overline{R}$ and then solving for γ. Observe that this value can be greater than 1, whereas $\gamma \in (0,1]$. Our claim in part (b) follows from definition of γ_{NT} and (11) that together imply $R_* - \frac{1}{2}\overline{R} = \frac{R_*\overline{R}}{\gamma}(\gamma - \gamma_{NT})$. ||

Proof of Proposition 4. The first part of the proposition showing that war might be the only possible pure-strategy equilibrium for all $R^i \in (0,\overline{R})$ follows from Lemma 1(b). In particular, $\gamma > \gamma_{NT}$ implies that $R_* > \frac{1}{2}\overline{R}$. In this case, it is not possible to have $R^i \geq R_*$ for both $i = 1,2$. Conversely, when $\gamma \leq \gamma_{NT}$, $R_* \leq \frac{1}{2}\overline{R}$, thereby leaving open the possibility that $R^i \geq R_*$ for both $i = 1,2$. But, even in such cases since $R_* > 0$, peace (without transfers) arises only for a subset of distributions $R^i \in [R_*, R^*] \in (0,\overline{R})$.

Nevertheless, for any distribution satisfying $R^i \in [R_*, R^*] \in (0,\overline{R})$, the pure-strategy equilibrium involving unarmed peace Pareto dominates not only the pure strategy equilibrium of war, but also any mixed-strategy equilibrium with arming by at least one agent. To confirm this, suppose one agent j chooses $G^j > 0$. Suppose further there exists a threshold G_T^j that satisfies the following:[56]

(i) Agent i prefers war for $G^j < G_T^j$, and thus chooses war and $G^i = B_w^i(G^j)$ as shown in (5);

(ii) Agent i prefers peace for $G^j > G_T^j$ and thus chooses peace and $G^i = D_p^i(G^j)$, where $D_p^i(G^j)$ is the level of guns by agent i that makes agent j (having armed by G^j) indifferent between war and peace ("D" stands for "deterrence");

(iii) Agent i is indifferent between war and peace at $G^j = G_T^j$.

In case (iii), agent i's best reply is to randomize over the two pure strategies of war with $G^i = B_w^i(G_T^j)$ and peace with $G^i = D_p(G_T^j)$. In a mixed-strategy equilibrium, each agent

[55] This prediction also holds true when $\gamma < (1-2\sigma)/2(1-\sigma)$, such that $dU_w^i/d\sigma > 0$ for all $R^i \in (0,\overline{R})$.

[56] There could exist more such thresholds, in which case there would exist multiple equilibria in mixed strategies. However, our argument to follow would apply to those as well.

obtains an expected payoff equal to a weighted average of the payoff under peace supported by deterrence V_p^i and under war U_w^i, with weights equal to the probabilities that agent i chooses respectively those two pure-strategies. Now observe that $V_p^i(G^i)$ is decreasing in G^i and $U_w^i(G^i, G^j)$ is decreasing in G^j along $B_w^i(G^j)$, by the envelope condition and the negative strategic payoff effect of rival j's arming. In addition, observe that $D_p^i(G^j)$ is implicitly defined by $V_p^j(G^j) - U_w^j(G^j, G^i) = 0$, such that $dD_p^i/dG^j > 0$. This last finding implies that agent i's payoff under peace as supported by deterrence (V_p^i) is decreasing as G^j increases. With the above points, it implies that agent i's expected payoff in a mixed-strategy equilibrium is also decreasing in G^j. The same logic applies to agent j. Thus, provided that unarmed peace as a pure-strategy equilibrium is stable, it Pareto dominates any mixed-strategy equilibrium that involves arming by at least one agent, and thus represents the unique stable equilibrium.[57]

The last point of the proposition follows from Lemma 1, which establishes that decreases in β and/or γ and increases in σ reduce R_* and raise R^*, thereby expanding the range $[R_*, R^*]$ and increasing the subset of resource distributions centered on $\frac{1}{2}\overline{R}$ that can support unarmed peace. $||$

Proof of Lemma 2. Solving for the value of γ that equates $V_p^i(R^*)$ to $\lim_{R^i \to \overline{R}} U_w^i(R^i)$ gives the value of γ_T shown in (14). Using that expression one can then easily verify

$$V_p^i(R^*) - \lim_{R^i \to \overline{R}} U_w^i(R^i) = \frac{1 - \beta\sigma}{\gamma_T} (\gamma_T - \gamma) \overline{R}, \tag{A.3}$$

which readily confirms the payoff rankings stated in the lemma. That $\gamma_T(\sigma; \beta) \leq \gamma_{NT}(\sigma; \beta)$ (with equality when $\sigma = 0$) follows from (14) and the definition of γ_{NT} in Lemma 1(b).

Part (a). One can easily confirm this part after substituting in $\beta = 1$ into (14).

Parts (b) and (c). The value of σ_T shown in part (b) equals the value of σ that makes $\gamma_T = 1$, such that for $\sigma > \sigma_T$, $\gamma_T > 1$. Focusing on $\beta \in (\frac{1}{2}, 1)$ ensures $\sigma_T > 0$. The second component of part (b) can easily be confirmed with straightforward calculus. (Note that $\arg\min_\sigma \gamma_T < \sigma_T$.) To confirm part (c), one can evaluate the expression for σ_T in part (b) at any $\beta \in (0, \frac{1}{2}]$. The resulting expression is 0 for $\beta = \frac{1}{2}$ and strictly negative for $\beta < \frac{1}{2}$. Thus, for all $\beta \in (0, \frac{1}{2}]$ and $\sigma \in [0, 1)$, $\gamma_T \geq 1$. $||$

Proof of Proposition 5. By Lemma 1(b), if $\gamma > \gamma_{NT}$, then $R_* > \frac{1}{2}\overline{R}$; therefore, there is no transfer T that satisfies both (12) and (13), which is required to make unilateral deviations following the transfer unprofitable to both agents. Hence, transfers are inconsequential in this case.

[57] Of course, as noted in the text, when unilateral deviations from unarmed peace are profitable to at least one agent, a Nash equilibrium in mixed strategies could dominate the war equilibrium in pure strategies.

Part a. From Proposition 4(a), $\gamma \leq \gamma_{NT}$ implies there exists an allocation $R^j = R_* \leq \frac{1}{2}\overline{R}$, with $R^i = R^* \equiv \overline{R} - R_*$, such that $V_p^j(R_*) = U_d^j(R_*) > U_w^j(R_*)$, whereas $V_p^i(R^*) > U_d^i(R^*) > U_w^i(R^*)$. Part (b) of that proposition establishes further that, for $R^j < R_*$, we have $U_d^j(R^j) > V_p^j(R^j)$, giving agent j a strictly positive incentive to deviate unilaterally from peace in the absence of transfer. However, for such distributions, agent i could be willing to offer agent j a transfer $T_{\min} = R_* - R^j$, leaving agent j with an *ex post* endowment of R_* and agent i with $R^* = \overline{R} - R_*$. Such a transfer restores the equality that prevents agent j from deviating unilaterally from peace, $V_p^j(R_*) = U_d^i(R_*)$, and makes agent i better off than under war provided $V_p^i(R^*) \geq U_w(R^i)$ holds. Now observe that the difference $V_p^i(R^*) - U_w(R^i)$ is strictly positive at $R^i = R^*$. Since $U_w^i(R^i)$ is increasing in R^i whereas $V_p^i(R^*)$ is independent of R^i, this difference decreases as R^i rises (or equivalently $R^j = \overline{R} - R^i$ falls). Furthermore, by definition, $\gamma > \gamma_T$ implies $V_p^i(R^*) - \lim_{R^i \to \overline{R}} U_w^i < 0$. Then, by the continuity of U_w^i, there exists a unique value of R^j denoted by $R_{**} \in (0, R_*)$ and thus $R^i = R^{**} \equiv \overline{R} - R_{**}$ that implies $V_p^i(R^*) \gtreqless U_w^i(R^i)$ for $R^i \lesseqgtr R^{**}$.

Part b. The assumption that $\gamma \leq \gamma_T$ implies $V_p^i(R^*) \geq \lim_{R^i \to \overline{R}} U_w^i$, such that $V_p^i(R^*) > U_w(R^i)$ for $R^i \in [R^*, \overline{R})$. In this case, $R^{**} = \overline{R}$, and transfers can support peace for all distributions $R^i \in (0, \overline{R})$.

Finally, we turn to the claim that increases in σ can lower the effectiveness of transfers in supporting peace—i.e., reduce R^{**} $(= \overline{R} - R_{**})$. From Lemma 2(c), we know $\beta \in (0, \frac{1}{2}]$ implies $\gamma_T \geq 1$ or $R^{**} = \overline{R}$ for all $\sigma \in [0, 1)$. Thus, we consider only values of $\beta \in (\frac{1}{2}, 1]$. Let us define $\sigma_{\min} \equiv \arg\min_\sigma \gamma_T$. Based on parts (a) and (b) of Lemma 2, we distinguish between three cases:

(i) If $\beta \in (\frac{1}{2}, \frac{2}{3}]$, then $d\gamma_T/d\sigma \geq 0$ for all $\sigma \in [0, 1)$, implying $\sigma_{\min} = 0$.

(ii) If $\beta \in (\frac{2}{3}, 1)$, then γ_T is strictly quasi-concave in σ and $\sigma_{\min} = \frac{3}{\beta^2}\left(\beta - \frac{2}{3}\right)$.

(iii) If $\beta = 1$, then $d\gamma_T/d\sigma < 0$ for all $\sigma \in [0, 1)$.

Thus, given any $\beta > \frac{1}{2}$ and $\gamma > \gamma_T(\sigma_{\min})$, there exists a unique $\hat{\sigma} = \gamma_T^{-1}(\gamma)$ in cases (i) and (iii). By contrast, in case (ii), there exists two values $\hat{\sigma}_J = \gamma_T^{-1}(\gamma)$ for $J \in \{A, B\}$ with $\hat{\sigma}_A < \hat{\sigma}_B$, as indicated by points A and B in Fig. A.1(a) for $\gamma = \frac{1}{2}$, assuming $\beta = \frac{9}{10}$. In that figure, we partition the space further with the schedule γ_{TT}, defined by values of γ (given σ and β) that satisfy $V_p^i(R^*) = U_w^i(R_H)$, where $U_w^i(R^i)$ is given by the second line in (8). For given σ and β, values of $\gamma \in (\gamma_{TT}, \gamma_{NT})$ imply $R^{**} \in [\frac{1}{2}\overline{R}, R_H)$, with R^{**} implicitly defined by $V_p^i(R^*) = U_w^i(R^{**})$, where again the payoff under war is given by the second line in (8). For values of $\gamma \in (\gamma_T, \min\{\gamma_{NT}, \gamma_{TT}\})$—again, given σ and β— we have $R^{**} \in [R_H, \overline{R})$, which is implicitly defined by $V_p^i(R^*) = U_w^i(R^{**})$, where $U_w^i(R^i)$ is shown in the third line in (8).

For our purposes here, it suffices to consider values of σ that ensure $\gamma_T(\sigma) = \gamma$, such as points A and B along the pink horizontal line where $\gamma = \frac{1}{2}$ in Fig. A.1(a). Then, we apply the implicit function theorem to the condition $V_p^i(R^*) = U_w^i(R^{**})$ to study the local effect of a change in σ on R^{**}:

$$dR^{**}/d\sigma|_{\gamma=\gamma_T(\sigma)} = \frac{(\partial V_p^i/\partial\sigma) - (\partial U_w^i/\partial\sigma)|_{R^i=\overline{R}}}{(\partial U_w^i/\partial R^i)|_{R^i=\overline{R}}}.$$

Notice that we evaluate the effects on U_w^i at $\gamma = \gamma_T(\sigma)$, which implies $R^i = R^{**} = \overline{R}$. Since by Proposition 2 $\partial U_w^i/\partial R^i > 0$, we have that $\text{sign}\{dR^{**}/d\sigma|_{\gamma=\gamma_T(\sigma)}\}$ equals the sign of the expression in the numerator. As briefly discussed in the main text, an increase in σ, by Lemma 1(a), lowers the value of R_*; it, therefore, also reduces the minimum transfer required to keep agent j from deviating unilaterally from peace, thereby giving agent i (the donor) a higher payoff under peace (i.e., $R^* = \overline{R} - R_* \uparrow$). Thus, $\partial V_p^i/\partial\sigma > 0$. At the same time, however, an increase in σ also raises the payoff under war for sufficiently large R^i, implying $\partial U_w^i/\partial\sigma|_{R^i=\overline{R}} > 0$ (see Proposition 2).

Although it is not immediately clear which effect dominates, we can gain more insight by relating the expression we have for $\text{sign}\{dR^{**}/d\sigma|_{R^i=\overline{R}}\}$ to the shape of γ_T in (γ, σ) space. Recall that $\gamma_T(\sigma)$ as defined in Lemma 2 solves $V_p^i(R^*) = U_w^i(R^i)|_{R^i=\overline{R}}$. Then, we can apply the implicit function theorem to that condition to find

$$d\gamma_T/d\sigma = -\frac{\left(\partial V_p^i/\partial\sigma\right) - \left(\partial U_w^i/\partial\sigma\right)|_{R^i=\overline{R}}}{\left(\partial V_p^i/\partial\gamma\right) - \left(\partial U_w^i/\partial\gamma\right)|_{R^i=\overline{R}}}.$$

From Lemma 1, an increase in γ raises R_* and thus the minimum transfer required to induce agent j not to deviate from peace to imply $\partial V_p^i/\partial\gamma < 0$. Furthermore, by Proposition 2, an increase in γ indicates less destruction and thus greater payoffs under war, $\partial U_w^i/\partial\gamma > 0$. Thus, we have that the sign of the numerator of the above expression gives us the sign of $d\gamma_T/d\sigma$. Moreover, combining this result with that above implies

$$\text{sign}\left\{dR^{**}/d\sigma|_{\gamma=\gamma_T(\sigma)}\right\} = \text{sign}\left\{d\gamma_T/d\sigma\right\} = \text{sign}\left\{\left(\partial V_p^i/\partial\sigma\right) - \left(\partial U_w^i/\partial\sigma\right)|_{R^i=\overline{R}}\right\}.$$

Thus, the condition that indicates whether or not the beneficial effect of an increase in σ on V_p^i is swamped by the negative effect through U_w^i to induce a decrease in R^{**} is directly linked to the sign of $d\gamma_T/d\sigma$. We see, in particular, that $d\gamma_T/d\sigma < 0$ must hold (for some σ). But $d\gamma_T/d\sigma < 0$ can hold only in cases (ii) and (iii) discussed earlier and that requires $\beta \in (\frac{2}{3}, 1]$. Panel (b) of Fig. A.1 illustrates the dependence of R^{**} on σ for $\beta = \frac{9}{10}$ and $\gamma = \frac{1}{2}$.[58] $\quad ||$

[58]Note that the figure is not drawn to scale; in particular, the value of R^{**} at $\arg\min_\sigma R^{**} > \frac{1}{2}\overline{R}$.

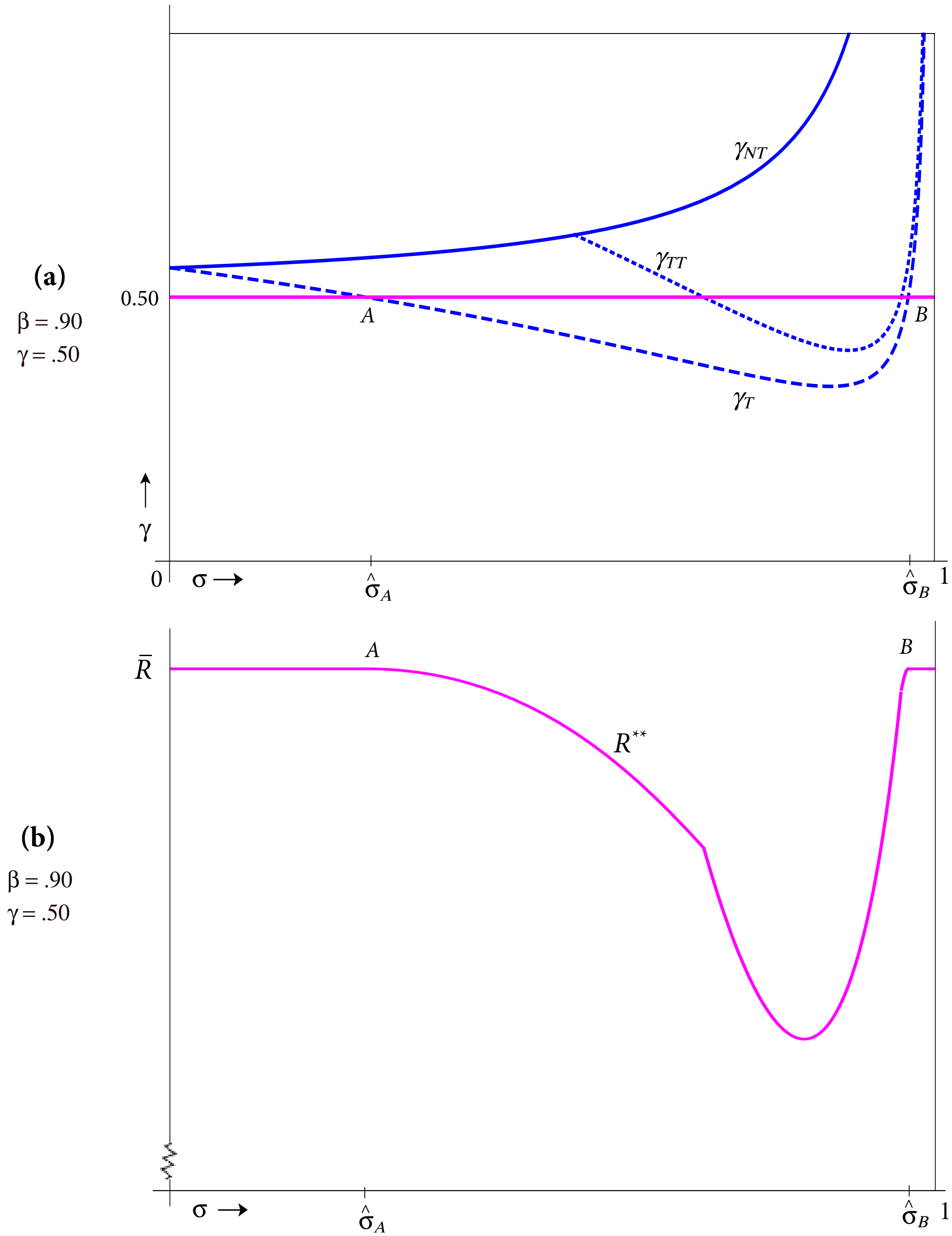

Figure A.1: Non-monotonicity of $\overline{R} - R^{**}$ in output security σ

B More Details on the Extensions

Case of diminishing returns. Here we provide a sketch of a proof to our claim in the main text that the presence of diminishing returns can enhance the effectiveness of transfers to support unarmed peace provided that peace without transfers is possible for at least some initial resource distributions. Let us focus on a set of parameter values that, for $\alpha = 1$, imply (i) $R_* < \frac{1}{2}\overline{R}$ and (ii) $R^{**} \in (\frac{1}{2}\overline{R}, \overline{R})$. Since R_* is decreasing in α as can be confirmed by (11′), the first assumption ensures that unarmed peace without *ex ante* resource transfers is possible, though only for sufficiently even distributions, in the presence of diminishing returns $\alpha < 1$. The second assumption implies that, absent diminishing returns, transfers can support peace for some but not all initial resource distributions. To fix ideas, let agent i be the more affluent agent and define $1 + K^{1/\alpha}$ as the denominator of R_* shown in (11′) Then, we evaluate $V_p^i(R^i) = (R^i)^\alpha$ at $R^* = \overline{R} - R_*$, to find agent i's payoff under peace:

$$H_p \equiv V_p^i(R^*) = \overline{R}^\alpha K/(1 + K^{1/\alpha})^\alpha.$$

Recall that for agent i to be willing to make a transfer to agent j at a given resource distribution $R^i > R^*$, it must be the case that his fallback payoff of U_w^i evaluated at that distribution be no greater than $V_p^i(R^*)$ shown above. Although we have not characterized the payoff function under war $U_w^i(R^i)$ for all R^i, we do know that this payoff and that under a unilateral deviation by agent i $U_d^i(R^i)$ approach each other as $R^i \to \overline{R}$. Thus, using (10), agent i's fallback payoff $U_w^i(R^i)$ in the limit as $R^i \to \overline{R}$ can be written as

$$H_w \equiv \lim_{R^i \to \overline{R}} U_w^i(R^i) = \lim_{R^i \to \overline{R}} U_d^i(R^i) = \overline{R}^\alpha \beta\eta,$$

where as previously defined in the main text $\eta \equiv \gamma(1 - \sigma) + \sigma$. A necessary and sufficient condition for *ex ante* transfers to support peace for *all* feasible resource distributions (i.e., $R^{**} = \overline{R}$) is that $H_p \geq H_w$. Hence, consider the ratio,

$$H_p/H_w = K/(1 + K^{1/\alpha})^\alpha \beta\eta.$$

As one can verify, this ratio is increasing in the degree of diminishing returns (or decreasing in α) and rises above 1 for sufficiently small $\alpha < 1$. Thus, for sufficiently small α, transfers can support peace for all resource distributions. Numerical analysis shows further that, under our assumptions made above, a decrease in α increases R^{**}, thereby expanding the range of initial resource distributions under which peace with transfers is possible.

Case of competitive trade: equilibrium prices and payoffs. Suppose that agent i produces good i, whereas agent j produces the other good (j). Now, let p^i_j denote the price agent i pays for good $j \neq i$. Absent trade costs and under the condition of perfectly competitive markets, p^i_j also equals the price received by agent j for supplying good j. Given the linear specification for transforming resource endowments into goods for consumption and with each agent allocating all of his resource endowment to produce his good $X^i = R^i$ under unarmed peace, agent i's income is $p^i_i R^i$. In turn, the specification for preferences implies that agent i's demand for good $j = 1, 2$ is given by $D^i_j = s^i_j p^i_i R^i / p^i_j$, where $s^i_j \equiv (p^i_j)^{1-\varepsilon} / [(p^i_j)^{1-\varepsilon} + (p^i_i)^{1-\varepsilon}]$, represents agent i's expenditure share on good $j = 1, 2$ and where, as defined in the text, $\varepsilon = 1/(1-\rho)$ represents the constant elasticity of substitution in consumption. Then, the market-clearing condition, which requires $p^i_j D^i_j = p^i_i D^j_i$, pins down the price of good j in terms of good i: $\pi^i \equiv p^i_j / p^i_i = (R^i/R^j)^{1/\varepsilon}$.

Using the expression for $F(D^i_i, D^i_j)$ in the text with the demand functions above, agent i's indirect utility can be written as a function of prices and his endowment as follows: $V^i = [1 + (\pi^i)^{1-\varepsilon}]^{1/(\varepsilon-1)} R^i$. Substituting in $\pi^i = (R^i/R^j)^{1/\varepsilon}$ gives, after some manipulation, the equilibrium payoff under unarmed peace with competitive trade:

$$V^i_p = \left[(R^i)^{(\varepsilon-1)/\varepsilon} + (R^j)^{(\varepsilon-1)/\varepsilon} \right]^{1/(\varepsilon-1)} (R^i)^{(\varepsilon-1)/\varepsilon}. \tag{B.1}$$

From this expression for V^i and an analogous one for agent j, one can find

$$V^i_p + V^j_p = \left[(R^i)^{(\varepsilon-1)/\varepsilon} + (R^j)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}, \tag{B.2}$$

which upon substituting in $\rho = (\varepsilon - 1)/\varepsilon$ confirms that $V^i_p + V^j_p = F(R^i, R^j)$. Finally, multiply and divide V^i_p in (B.1) by $V^i + V^j$ in (B.2). Then, after rearranging terms , one can verify the expression for V^i_p, shown in (16), as a share $\psi^i = (R^i)^\rho / [(R^i)^\rho + (R^j)^\rho]$ of total utility $V^i_p + V^j_p = F(R^i, R^j)$.

We now establish some claims made in the main text regarding the effects of an increase in R^i on agent i's payoffs under peace and a unilateral deviation as $R^i \to \overline{R}$ that helped to establish V^i_p and U^i_d reach their respective maximum values at resource distributions strictly less than $\overline{R}$ (i.e., $R_p < \overline{R}$ and $R_d < \overline{R}$). Differentiation of V^i_p in (16) shows

$$dV^i_p / dR^i = V^i_p \left[\frac{\rho \overline{R} - R^i + (R^i)^\rho (R^j)^{1-\rho}}{R^i R^j \left[(R^i)^\rho + (R^j)^\rho \right]} \right]. \tag{B.3}$$

Since $\lim_{R^i \to \overline{R}} V^i_p = \overline{R}$ is positive and finite, the expression above goes to $-\infty$ as $R^i \to \overline{R}$ as claimed in observation (ii) in the text. Similarly, by differentiating U^i_d in (17), one can confirm that

$$dU_d^i/dR^i = U_d^i \frac{\left[(R^j)^{1-\rho} - \theta^\rho \left(R^i\right)^{1-\rho}\right]}{\left(R^i R^j\right)^{1-\rho} \left[(R^i)^\rho + \left(\theta R^j\right)^\rho\right]}. \tag{B.4}$$

Now recall our definition of $\eta \equiv \gamma(1-\sigma) + \sigma$ in the text, so that $\theta = \gamma(1-\sigma)/\eta$. Since $\lim_{R^i \to \overline{R}} U_d^i = \beta\eta\overline{R}$ is positive and finite, the expression above goes to $-\infty$ as $R^i \to \overline{R}$.[59] Thus, when $\beta\gamma = 1$, both U_d^i and V_p^i are decreasing and approaching $\overline{R}$ as $R^i \to \overline{R}$.

We can now establish that, for sufficiently secure output, V_p^i is falling faster than U_d^i as $R^i \to \overline{R}$, which implies $V_p^i > U_d^i$ for some $R^i < \overline{R}$. Using (B.3) and (B.4), one can confirm $\lim_{R^i \to \overline{R}} \frac{dV_p^i/dR^i}{dU_d^i/dR^i} = \frac{1-\rho}{\beta\eta\theta^\rho}$ that, when evaluated at $\beta = \gamma = 1$, gives

$$\Upsilon \equiv \Upsilon(\sigma, \rho) = \lim_{R^i \to \overline{R}} \frac{dV_p^i/dR^i}{dU_d^i/dR^i}\bigg|_{\beta=\gamma=1} = \frac{1-\rho}{(1-\sigma)^\rho}.$$

Hence, for any given $\rho \in (0,1)$, there exists a unique $\sigma_\Upsilon(\rho) \equiv 1 - (1-\rho)^{1/\rho} \in (0,1)$, where $\sigma_\Upsilon' > 0$ and $\sigma_\Upsilon'' > 0$, that implies $\Upsilon > 1$ for all $\sigma > \sigma_\Upsilon$.[60] Of course, this condition is necessary, but not sufficient, for U_d^i to cross V_p^i from above at some $R^i < \frac{1}{2}\overline{R}$, as we characterized in the main text with the function $\Omega(\sigma, \rho) = V_p^i / U_d^i|_{R^i = \frac{1}{2}\overline{R}}$ and the implied condition that $\sigma > \sigma_\Omega(\rho)$. Thus, it should not be surprising that $\sigma_\Upsilon(\rho) < \sigma_\Omega(\rho)$ for any $\rho \in (0,1)$.

Case of preexisting military capabilities. Using (19), we provide more details regarding the profitability of unilateral deviations from peace, allowing for the possibility of destruction ($\beta\gamma \leq 1$) while maintaining our assumption that $G_0^i = \lambda R^i$ for $\lambda \in [0,1]$. We proceed in two parts. First, we provide a fuller characterization of an agent's incentive to add to his preexisting guns under a unilateral deviation, and identify some implications for the profitability of such deviations. Second, building on this characterization, we examine more generally the incentives for unilateral deviations and show how the presence of preexisting guns matters for the stability of peace.

Recall that, under peace, both agents produce no additional guns. Thus, agent i's optimal arming under a unilateral deviation is given by $G_d^i \equiv B_w^i(0; \cdot)$. Applying our assumption that $G_0^i = \lambda R^i = 0$ for $i = 1, 2$ and the fact that $R^i = \overline{R} - R^j$ to (19b), we find

$$\widetilde{B}_w^i(0; \cdot) = -\overline{G}_0 + \sqrt{\theta G_0^j \left(\overline{G}_0 + \overline{R}\right)} = -\lambda\overline{R} + \sqrt{\left(\overline{R} - R^i\right)\theta(1+\lambda)\lambda\overline{R}}. \tag{B.5}$$

[59] Observe that setting $dU_d^i/dR^i = 0$ implies $R_d^i = [1 + \theta^{\rho/(1-\rho)}]^{-1}\overline{R} \geq \frac{1}{2}\overline{R}$ with equality when $\sigma = 0$, as claimed in the text.

[60] One can also show that $\lim_{\rho \to 0} \sigma_\Upsilon = 1 - \frac{1}{e} \approx 0.632$ while $\lim_{\rho \to 1} \sigma_\Upsilon = 1$.

Observe that $\lim_{\lambda\to 0}\widetilde{B}^i_w(0;\cdot)=0$ (and thus $\lim_{\lambda\to 0}B^i_w(0;\cdot)=0$) for all feasible endowment distributions. Indeed, this is the case of the baseline model, which led us to conclude that $G^i_d\approx 0$ for $\lambda=0$.

Using (B.5) with (19a) shows that G^i_d takes the following form, contingent on R^i:

$$G^i_d=\begin{cases} R^i & \text{if} \quad R^i\in(0,\mu_L\overline{R}) \\ \widetilde{B}^i_w(0;\cdot) & \text{if} \quad R^i\in(\mu_L\overline{R},\mu_H\overline{R}) \\ 0 & \text{if} \quad R^i\in[\mu_H\overline{R},\overline{R}), \end{cases} \tag{B.6}$$

where

$$\begin{aligned} \mu_L &\equiv \mu_L(\lambda,\theta)=\frac{1}{1+\frac{1}{(1+\lambda)\theta}+\frac{1}{2}\left(\sqrt{1+\frac{4}{\lambda\theta}}-1\right)}\left[1-\frac{\lambda}{(1+\lambda)\theta}\right] \\ \mu_H &\equiv \mu_H(\lambda,\theta)=1-\frac{\lambda}{(1+\lambda)\theta}. \end{aligned}$$

The function μ_L (resp., μ_H) is derived by searching for the value of $R^i=\mu\overline{R}$ that solves $\widetilde{B}^i_w\left(0;R^i\right)=R^i$ (resp., $\widetilde{B}^i_w\left(0;R^i\right)=0$), naturally with $\mu_L<\mu_H$.[61] One can confirm $\mu_L=\mu_L(\lambda,\theta)$ depends positively on θ and is concave in λ, reaching a maximum value of $\theta/4$ so that $\mu_L\leq\frac{1}{4}$. Furthermore, $\mu_H=\mu_H(\lambda,\theta)\leq 1$ depends positively on θ and negatively on λ. Figs. B.1(a) and B.2(a) show G^i_d in the absence of destruction ($\beta=\gamma=1$) and in the presence of destruction ($\beta=1$ and $\gamma=0.8$), respectively. The colors blue, pink and orange depict the function in each of the three ranges indicated in (B.6).[62]

In general, decreases in differential destruction ($\gamma\uparrow$) and in output security ($\sigma\downarrow$), which tend to fuel arming incentives under war (i.e., cause $\theta\uparrow$), also fuel incentives to arm under a unilateral deviation in this extension (with $\lambda>0$). Here, there are two implications. First, since $d\mu_L/d\theta>0$, the deviating agent i is constrained in his arming over a larger range of distributions $R^i\in(0,\mu_L\overline{R}]$, with an increase in peak in G^i_d (at $R^i=\mu_L\overline{R}$). Second, because $d\mu_H/d\theta>0$, the range of resource distributions under which agent i arms in a unilateral deviation (i.e., $R^i\in(0,\mu_H\overline{R}]$) expands as well. Turning to the implications of an increase in preexisting arms ($\lambda\uparrow$), observe that, since $d\mu_H/d\lambda<0$, an increase in λ expands the range of resource endowments $R^i\in[\mu_H\overline{R},\overline{R})$ for which agent i does not adjust his arms in a unilateral deviation.[63] The implications of an increase in λ for μ_L are a

[61] One can obtain $\mu_L=\frac{1}{2}[(1+\lambda)\sqrt{\lambda\theta(4+\lambda\theta)}-\lambda(2+\theta+\lambda\theta)]$ directly from the condition that $\widetilde{B}^i_w(0;R^i)=R^i$, and then with some algebraic manipulation of terms find the expression shown in the text. Note, the function μ_H evaluated at $\gamma=1$, which implies $\theta=1-\sigma$, corresponds to $\delta(\lambda,\sigma)$ used in the main text.

[62] Note that (both panels of) these figures are not drawn to scale. But, the line designated as the "45^o line" should help give the proper perspective.

[63] It is easy to verify that $\lambda\to 0$ implies $\mu_L\overline{R}\to 0$, $\mu_H\overline{R}\to\overline{R}$ and $G^i_d\to 0$ for all $R^i\in(0,\overline{R})$, as in our baseline

little more nuanced. Specifically, the non-monotonicity of μ_L in λ described above means that an increase in λ starting at a small value initially shifts the peak of the G_d^i schedule upward (e.g., from point A to point A' in the figures); eventually, however, at sufficiently high values of λ, it begins to shift that peak downward (from point A' to point A'').

Following our strategy in the text, let us consider first the conditions under which neither agent would choose to add to his preexisting holdings of guns. From (B.6), $G_d^i = 0$ whenever $R^i > \mu_H \overline{R}$. One possibility is that $\lambda \geq \frac{\gamma(1-\sigma)}{\sigma}$, which implies $\mu_H \leq 0$, such that for any distribution $R^i \in (0, \overline{R})$, $G_d^i = 0$ for $i = 1, 2$. The other (weaker) condition is that $\lambda \geq \frac{\gamma(1-\sigma)}{\gamma(1-\sigma)+2\sigma}$, which implies $\mu_H \in (0, \frac{1}{2}]$, such that for distributions $R^i \in [\mu_H \overline{R}, (1 - \mu_H)\overline{R}]$, $G_d^i = 0$ holds again for $i = 1, 2$. Observe these conditions simplify to those stated in the text when there is no destruction ($\gamma = 1$). Furthermore, as in the case of no destruction, when these conditions are satisfied, the unique equilibrium in arming under war involves no additional production of guns: $G_w^i = 0$ for both i.[64] However, in the case where war is destructive (i.e., $\beta\gamma < 1$), the payoff under peace is strictly greater than that under a unilateral deviation for both agents: $V_p^i = R^i > U_d^i = \beta\eta R^i$ for $i = 1, 2$, where as defined in the main text $\eta \equiv \gamma(1-\sigma) + \sigma$.[65] Indeed, these conditions that ensure $G_d^i = 0$ for $i = 1, 2$ when war is destructive are sufficient, but not necessary, to render unilateral deviations from peace unprofitable for both agents.

As such, to get a more complete picture of when peace is stable, we now turn to the corresponding payoffs under a unilateral deviation, U_d^i. Upon substituting $G^j = 0$ and the values of G_d^i shown in (B.6) into the expression for U^i shown in (3), using the modified conflict technology (18) and simplifying, we obtain:

$$U_d^i = \begin{cases} U_{d1}^i = \beta\eta\theta\,(1+\lambda)\,R^i \left[\frac{\overline{R}-R^i}{R^i+\lambda\overline{R}}\right] & \text{if} \quad R^i \in (0, \mu_L \overline{R}) \\ U_{d2}^i = \beta\eta \Big[-(1-\theta)(\overline{R} - R^i) & \\ \qquad + \left([(1+\lambda)\overline{R}]^{\frac{1}{2}} - [\theta\lambda(\overline{R} - R^i)]^{\frac{1}{2}} \right)^2 \Big] & \text{if} \quad R^i \in (\mu_L \overline{R}, \mu_H \overline{R}) \\ U_{d3}^i = \beta\eta R^i & \text{if} \quad R^i \in [\mu_H \overline{R}, \overline{R}). \end{cases} \tag{B.7}$$

Obviously, U_d^i is a piecewise function of the resource allocation R^i, as illustrated in Figs. B.1(b) and B.2(b), respectively for the benchmark case of no destruction and the case of destruction. A noteworthy difference between the case of preexisting arms ($\lambda > 0$) and our baseline model without such arms ($\lambda = 0$) is that U_d^i is no longer linear in $R^i \in (0, \overline{R})$

model.

[64]This claim can be confirmed by evaluating the net marginal value of arming, $\partial U^i / \partial G^i$, using (4) with (18), at $G^j > 0$ and $G^i = 0$. The resulting expression is non-positive for any $R^i \in (0, \overline{R})$ when $\mu_H \leq 0$ and for any $R^i \in [\mu_H \overline{R}, (1 - \mu_H)\overline{R}]$ when $\mu_H \leq \frac{1}{2}$.

[65]The expression for U_d^i in this case can be confirmed using (3) and (18) with $G^i = G^j = 0$. Also see below.

in the former case. In particular, as one can verify, U^i_{d1} (the blue curve) is increasing and concave in $R^i \in (0, \mu_L \overline{R}]$; U^i_{d2} (the pink curve) is increasing and convex in $R^i \in (\mu_L \overline{R}, \mu_H \overline{R})$; and U^i_{d3} (the orange curve) is increasing and linear in $R^i \in [\mu_H \overline{R}, \overline{R})$.

Turning to our comparison of U^i_d with V^i_p, let us start with resource distributions $R^i \in [\mu_H \overline{R}, \overline{R})$ that imply $U^i_d = U^i_{d3}$ as shown in (B.7). Clearly, $U^i_{d3} \leq V^i_p$ $(= R^i)$ holds as a strict inequality, if $\beta\gamma < 1$. What's more, $dU^i_{d3}/dR^i < dV^i_p/dR^i = 1$. When $R^i \in (\mu_L \overline{R}, \mu_H \overline{R})$ such that $U^i_d = U^i_{d2}$ shown in (B.7), the fact that $\lim_{R^i \nearrow \mu_H \overline{R}} U^i_{d2} = \lim_{R^i \searrow \mu_H \overline{R}} U^i_{d3}$ together with the just outlined properties of monotonicity and convexity of U^i_{d2} in R^i imply that $dU^i_{d2}/dR^i < \beta\eta$, so that U^i_{d2} is flatter than U^i_{d3} and thus approaches V^i_p from above. Lastly, we note that $\lim_{R^i \to 0} dU^i_{d1}/dR^i = \beta\eta\theta\frac{1+\lambda}{\lambda}$, whereas $\lim_{R^i \to 0} dV^i_p/dR^i = 1$; thus, we have $\lim_{R^i \to 0} dU^i_d/dR^i \gtreqless \lim_{R^i \to 0} dV^i_p/dR^i$ as $\lambda \lesseqgtr \frac{\beta\gamma(1-\sigma)}{1-\beta\gamma(1-\sigma)}$.[66] Since $\lim_{R^i \to 0} U_{d1} = \lim_{R^i \to 0} V^i_p = 0$, it follows that, if λ is sufficiently small such that $\lim_{R^i \to 0} dU^i_d/dR^i > \lim_{R^i \to 0} dV^i_p/dR^i$, then $U^i_d > V^i_p$ at least for allocations close to $R^i = 0$.

We now stitch together the above description to obtain a more complete picture of how U^i_d compares with V^i_p for all R^i. If λ is sufficiently large (specifically, if $\lambda \geq \frac{\beta\gamma(1-\sigma)}{1-\beta\gamma(1-\sigma)}$), then $U^i_d \leq V^i_p$ holds for all feasible R^i $(i = 1, 2)$, so that peace is never threatened.[67] When $\lambda < \frac{\beta\gamma(1-\sigma)}{1-\beta\gamma(1-\sigma)}$, the distribution of resource endowments matters. In particular, as we have just shown, the sufficiently small value of λ implies that $U^i_d > V^i_p$ for values of R^i close to zero. As R^i increases, both V^i_p and U^i_d rise as well. But, the properties of U_{d2} described above and the fact that $U^i_d < V^i_p$ (provided $\beta\gamma < 1$ holds) when $R^i \in (\mu_H \overline{R}, \overline{R})$ imply that there exists a unique point $R_* \in (0, \mu_H \overline{R})$ such that $V^i_p \gtreqless U^i_d$ as $R^i \gtreqless R_*$. By the same logic in the main text where we assumed $\lambda = 0$, if $R_* \leq \frac{1}{2}\overline{R}$, then, there exists a non-empty subset of resource distributions $R^i \in [R_*, R^*] \in (0, \overline{R})$ under which peace is immune to unilateral deviations, whereas war is the equilibrium for $R^i \notin [R_*, R^*] \in (0, \overline{R})$. However, if $R_* > \frac{1}{2}\overline{R}$, war is the unique, pure-strategy equilibrium outcome for all resource distributions.

Based on the above analysis, we now turn to sketch out the proofs to the last two claims we make in the main text regarding the effects of the quantity of preexisting arms or more precisely λ $(< \frac{\beta\gamma(1-\sigma)}{1-\beta\gamma(1-\sigma)})$ relative to our baseline model where $\lambda = 0$. To show the first claim that the the threshold value R_* depends negatively on λ, we let agent i be the less affluent one, and apply the implicit function theorem to $U^i_d\left(R^i, \lambda\right) = V^i_p(R^i)$ evaluated at $R^i = R_* \in (0, \mu_H \overline{R})$, while recognizing that V^i_p is independent of λ, to find:

$$dR_*/d\lambda = -\frac{dU^i_d/d\lambda}{dU^i_d/dR^i - dV^i_p/dR^i}.$$

[66] In the special case of $\beta = \gamma = 1$, the last inequality becomes $\lambda \lesseqgtr \frac{1-\sigma}{\sigma}$ that implies $\mu_H \gtreqless 0$.

[67] Of course, as discussed in the text, peace and war are distinct outcomes only when war is destructive (i.e., $\beta\gamma < 1$).

Since U_d^i approaches V_p^i from above as R^i increases, the denominator of the above expression is negative. Thus, the sign of $dR_*/d\lambda$ equals the sign of the numerator. If $R_* \in (0, \mu_L \overline{R})$, then $G_d^i = R^i$ holds, in which case $dG_d^i/d\lambda = 0$. Alternatively, if $R_* \in [\mu_L \overline{R}, \mu_H \overline{R}]$ so that $G_d^i = \widetilde{B}_w^i(0;\cdot) > 0$, we can invoke the envelope theorem. In both cases, the numerator of the expression above, using (3) with (18), $G_0^i = \lambda R^i$ (for $i = 1, 2$), $G^i = G_d^i$ and $G^j = 0$, can be written as

$$dU_d^i/d\lambda = \beta\eta\theta\left(\overline{R} - G_d^i\right)\frac{d\phi^i}{d\lambda} = -\beta\eta\theta\left(\overline{R} - G_d^i\right)\frac{G_d^i R^j}{\left(G_d^i + \lambda\overline{R}\right)^2} < 0,$$

which implies $dR_*/d\lambda < 0$. Thus, an increase in λ expands the range of resource endowments $R^i \in [R_*, R^*]$ under which peace emerges as the stable equilibrium outcome. This result is illustrated in Fig. B.2(b) in the case of destruction. Points C, C' and C'' are associated with $R^i = R_*$ for increasing values of λ. Depending on parameter values, R_* will lie either in $\left(0, \mu_L\overline{R}\right)$ (if λ is sufficiently large) or in $\left[\mu_L\overline{R}, \mu_H\overline{R}\right)$ (if λ is sufficiently small).

Finally, we show how an increase in the quantity of preexisting guns λ can enhance the effectiveness of transfers to support peace. Now let agent i be the more affluent agent. Our finding above that $dR_*/d\lambda < 0$ implies that $dR^*/d\lambda > 0$ and thus agent i's payoff under peace is increasing in λ: $dV_p^i(R^*)/\lambda > 0$. The effect of an increase in λ on the payoff under war $U_w^i(R^i)$ includes both a direct effect through the conflict technology (18) and a strategic effect through the rival j's arming. Numerical analysis indicates that the combined effect is positive for the affluent agent (i). Hence, the effect of an increase in λ on R^{**}, implicitly defined by the condition $V_p^i(R^*) - U_w^i(R^{**}) = 0$, would appear to be ambiguous. However, since U_w^i and the payoff to agent i when he deviates unilaterally $U_d^i(R^i)$ approach each other as R^i approaches $\overline{R}$, we can focus on what happens as R^i approaches $\overline{R}$ as we did in the case of diminishing returns. Specifically, one can confirm

$$\lim_{R^i \to \overline{R}} U_w^i(R^i) = \lim_{R^i \to \overline{R}} U_d^i(R^i) = \overline{R}\beta\eta,$$

which is independent of λ. Since $dV_p^i(R^*)/\lambda > 0$, the necessary and sufficient condition for *ex ante* transfers to support peace for *all* feasible resource distributions (i.e., $R^{**} = \overline{R}$) is more likely to be satisfied as the quantity of preexisting guns increases ($\lambda \uparrow$).

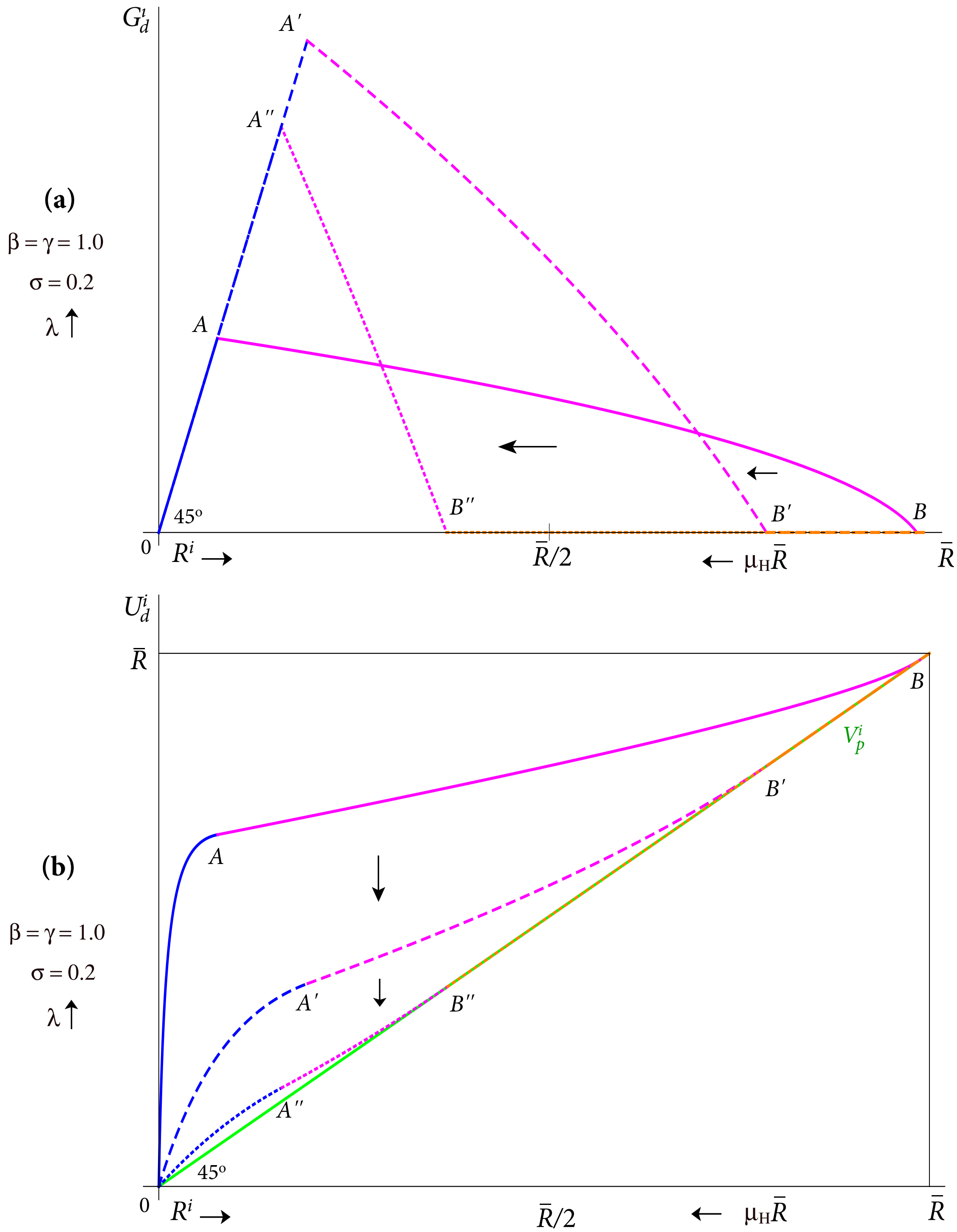

Figure B.1: Optimizing arming and payoffs under a unilateral deviation and various resource distributions: no destruction

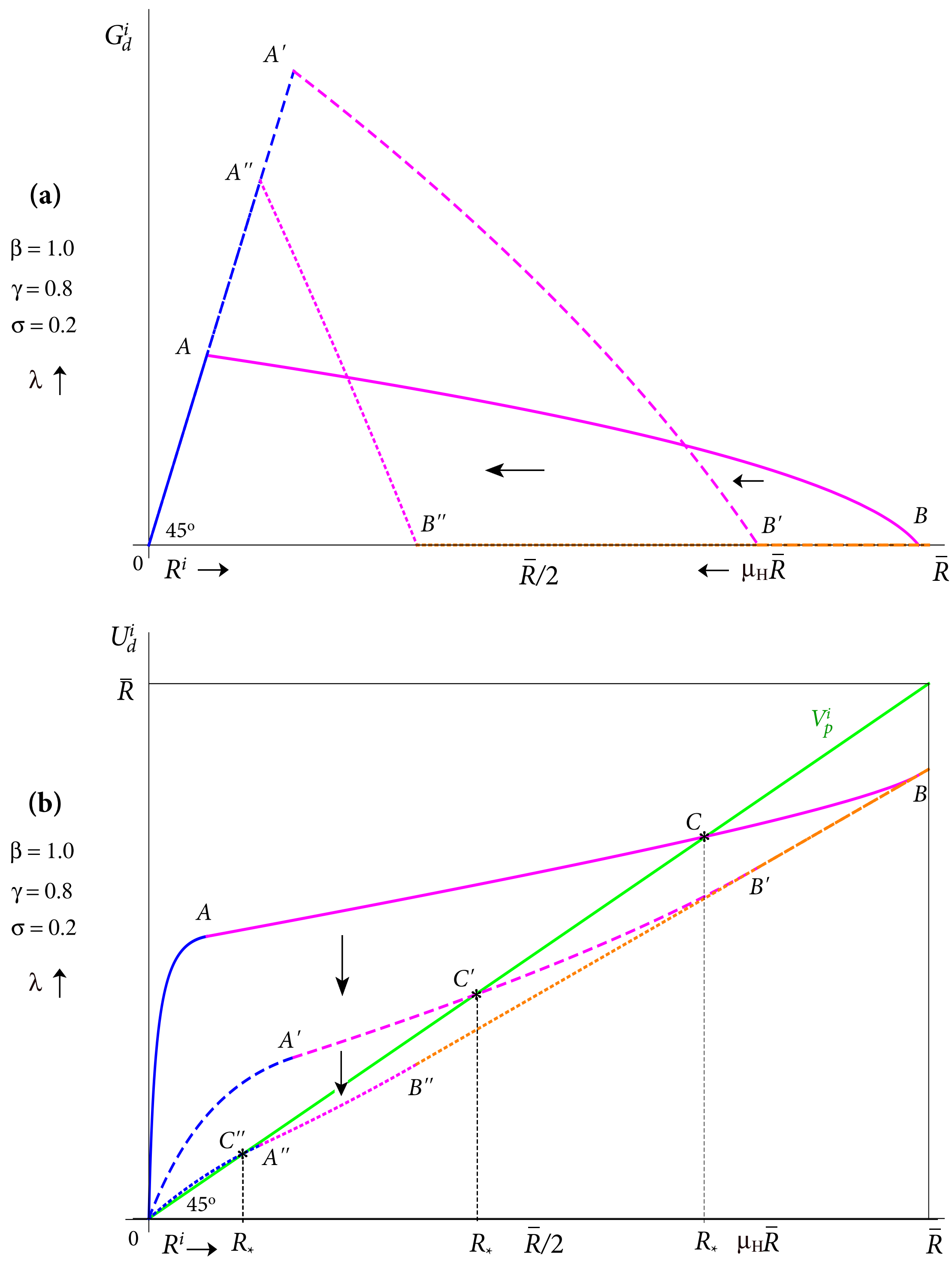

Figure B.2: Optimizing arming and payoffs under a unilateral deviation and various resource distributions: destruction