Household Sorting in an Ancient Setting*

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December 31, 2019

Evolving

Abstract

We use archaeological data from three different ancient settlements of different historical eras on a Greek island to construct novel measures of consumption. Using these, we show that households that lived closer to the center of the settlements consumed more luxurious goods. We build a monocentric city model with heterogeneous households, luxury goods and endogenous labor choices that is consistent with the rich living closer to the center and consuming more luxuries. This result holds when transportation costs within the model are predominately time costs, as they mostly were in ancient history.

1 Introduction

What causes different households to choose to live in different parts of a city is one of the key questions in urban economics. We use archaeological data from within several ancient agglomerations to estimate how households sorted in early "urban" settlements. The data include precise locations for finds of several different qualities of a consumption good. We extend the canonical Muth-Mills model with heterogeneous households to include luxury goods in a tractable way and provide some conditions under which household location sorting in the model can be inferred from our data.

*Gupta's research is supported by UK Economic and Social Research Council (ESRC) grant number ES/R006032/1. Halket's research is supported by the ESRC through the Centre for Microdata Methods and Practice (CeMMAP grant number ES/I034021/1).

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The nature of household sorting can have dramatic implications for a wide variety of urban public policies on, e.g., transportation, public housing, public amenity provisions and the environment. Yet, inferring the way various amenities and technologies shape household location decisions even within variations of the static monocentric city model is challenging; “small” changes to the model environment can impart large qualitative changes on the model’s outcomes.

If inferences are based on modern cities, these challenges are exacerbated by the pre-existing, “sticky” built and settled environment. Today’s households make their choices conditional on the existing transportation network and character of the housing stock in various locations, which themselves are partially or wholly a product of the technologies and preferences of the past. Indeed, the possibility for differing equilibria based on legacy conditions are a feature of studies like Brueckner et al. [1999], Brueckner and Rosenthal [2009], Lee and Lin [2017].

Our work complements several strands of literature. It adds to the growing use of archaeological data to test economic theory. For instance Maurer et al. [2017] uses data from a similar period to ours to document trade and development patterns across settlements. Our paper is the first to our knowledge to look within settlements. We are able to do this by using detailed archaeological data collected at a fine spatial resolution and dated using cutting edge archaeological techniques. Both parametric and non-parametric estimates offer fairly clear pictures that the concentrations of consumption were highest closest to the settlements’ centers.

We also add to the evidence on sorting in early cities cited in LeRoy and Sonstelie [1983] (who focus on census data from 19th century, North American cities). In their paper, the authors build a model with transportation choices. When the rich choose different transportation modes than the poor (e.g. the rich use an automobile and the poor walk), then the rich may choose to live in the suburbs. In addition to the alternative setting, our model adds to their work by providing various differing sorting conditions even when the transportation modes for all households are the same and also by explicitly including luxury goods in the model.

2 Data and Empirics

2.1 The Island and Data

The data are publicly available data[1] collected from the Greek island of Antikythera. Bevan and Conolly [2012] provide the following description of the island: *“Antikythera is a small island (ca. 20.8 sq.km) in the Mediterranean Sea. Despite being comparatively remote*

[1] see Bevan and Conolly [2014]

from larger land masses in Mediterranean terms, it lies along important routes of maritime interaction between the Peloponnese and Crete, and between the eastern and central Mediterranean. This geographical position has contributed to its very episodic history of human exploitation stretching back some 7,000 years, but with periods of substantial settlement followed by others of near complete abandonment. Highlights of this long-term history include evidence visits by Neolithic hunters from the Cyclades, Bronze Age farms with cultural links to Crete during the period of the Minoan palaces, a fortified settlement of Hellenistic pirates, a clutch of Late Roman communities, some glimpses of Middle Byzantine settlement and a recolonisation by west Cretan families in the late 18th century AD."

Between 2005-07, the Antikythera Survey Project (ASP), co-directed by Andrew Bevan, James Conolly and Aris Tsaravopoulos (Greek Archaeological Service) conducted an intensive pedestrian survey of the island. The uniqueness of this exercise lay in the coverage of an entire island in a uniform manner with intensive survey methods.[2]The data offer a unique level of detail in both the individual finds and their precise spatial locations.

We focus on pottery in our study. In the data, each piece of pottery is given a classification by Bevan and Conolly [2014] according to its fabrication: "Fine", "Medium" or "Coarse". In addition, as mentioned above, for each piece of pottery Bevan and Conolly [2014] assign a probability to it belonging to a particular chronological phase, using state-of the-art methods in Bevan et al. [2013]. Finer pottery tends to reflect the quality of its fabrication but also correlates with usage. For instance coarse pottery in the Hellenistic era is more likely cooking ware, whereas finer pottery from that era on Antikythera might be tableware. Exact usage would obviously change with era. [3]

Our study focuses on three major historical periods in the history of Antikythera: the

[2] Quoting from the description in Bevan and Conolly [2012] "...the entire island was fieldwalked in parallel lines 15-m apart. For certain interesting or problematic surface artefact scatters (particularly those of prehistoric date) this stage-one survey was followed by more detailed stage-two collections on a 10×10-m grid. In terms of digital recording, this project was unusual for the detail of its treatment of the location, dating and other attributes of its artefacts. First, all artefacts and standing structures were entered individually in a database (with information on shape, size, decoration, fabric, date, location, etc.), rather than in aggregate, and these records were all the result of sustained laboratory study rather than decisions in the field. Second, the project sought to standardise the recording of the spatial location of all material culture, regardless of the survey method by which it was observed, such that all finds and observations had an effective spatial precision of ±10 m. Third and finally, it was the first substantial fieldwork project, to our knowledge, to adopt a probabilistic approach to assigning dates to individual collected artefacts."

[3]The phases are: Middle to Late Neolithic (pre-4500 BC), Final Neolithic to Early Bronze 1 (ca. 4500-2700 BC), Early Bronze 2 (ca. 2700-2200 BC), Cretan late Prepalatial (ca. 2200-1950 BC), First Palace or Cretan Protopalatial (ca. 1950-1750 BC), Second Palace or Cretan Neopalatial (ca. 1750-1450 BC), Third Palace or Mycenaean (ca. 1450-1200 BC), Post Palatial to Protogeometric phases (1200-900 BC), Geometric phase (900-600 BC), Archaic phase (600-500 BC), Classical phase (500-325 BC), Hellenistic phase (325-0 AD), Early Roman phase (0-200 AD), Middle Roman phase (200-350 AD), Late Roman phase (350-650 AD), Early Byzantine phase (650-900 AD), Middle Byzantine phase (900-1200 AD), Early Venetian phase (1200-1400 AD), Middle Venetian phase (1400-1600 AD), Late Venetian phase (1600-1800 AD), Recent phase (1800-present), any other chronological phase.

Minoan period, the Hellenistic period and the Late Roman. The Minoan period covers the time period between 2700-1200 BC when Antikythera was influenced by the Cretan civilization, thus ranging from the Early Bronze 2 phase to the Third Palace or Mycenaean phase in the Bevan et al. [2013] classification. In the Bevan et al. [2013] classification the Hellenistic period covers 325 BC-0 AD, while the Late Roman period covers 350 AD-650 AD. We choose these three distinct time periods for our study because of the vastly different characteristics of settlement observed on Antikythera during them and because the island seemed to be relatively abandonded for large spells between these periods. Antikythera is well-known in the archaeological literature for exhibiting a high degree of historical variance in its settlement. Bevan et al. [2006] describe this phenomenon as one of "rollercoaster demographics", and present a rather complete history of the island.

For the purposes of our study we highlight several elements of the island's history. The Minoan period is dominated by mostly by "cultivators" living in the fertile central part of the island who may have colonized the island from its larger neighbor, Crete. After the Minoan period, archaeologists have yet to find "good evidence... for much activity;" (Bevan et al. [2006]) in other words, it may have been abandoned (a situation comparable to its current lightly inhabited state). During the Hellenistic period Antikythera was resettled but in a different part of the island. The island was, as Bevan et al. [2006] notes, "dominated by a fortified town at a strategic position on its northern coast, overlooking a natural protected harbor. Documentary evidence suggests its role in piracy. Our survey indicates the presence of one or two other Hellenistic scatters on the island" which may have been "in some manner, part of the logistical and economic agenda of the fortified town itself." Subsequent to the sack of this fortified town by the Romans in 69-67 BC the island once again suffered a near abandonment before settlements appeared in and around the town of Potamos and in the fertile area of the island culminating in a peak in the Late Roman era. Thus, Antikthera appears to have been primarily an agricultural economy in Minoan times, a maritime economy in Hellenistic times and a combination of maritime and agrarian in Late Roman times.

Thus our choice of the three time periods is motivated precisely by archaeological and historical observations: these three periods correspond to distinct and prosperous phases in Antikythera's history. The discontinuity in settlement also makes the task of distinguishing between historical phases much simpler; in the words of Bevan et al. [2006] the discontinuity makes the landscape "a less complicated palimpsest than in most other Mediterranean locations."

Our interest lies in estimating consumption gradients relative to a "central" location, in the sense of being the center of economic activity. This center changed across the three time periods we focus on. Figure 1 shows the island in its entirety, together with the location of the fertile center of the island, where most economic activity took place during the Minoan era, the port of Kastro in the northern part of Antikythera, which was the economic hub during the Hellenistic heyday of the island, and the port of Potamos, which saw considerable economic activity in the Late Roman epoch. Potamos is now the largest modern-day settlement on Antikythera.

Table 1: Pottery summary statistics

	Pottery piece counts					
	Minoan		Hellenistic		Late Roman	
	Raw	Weighted	Raw	Weighted	Raw	Weighted
Coarse	5497	5342.85	32	16.10	49	19.50
Medium	906	833	818	433.66	1173	882
Fine	226	192.60	856	506.55	1369	1029.30
Total	6629	6368.45	1706	956.31	2591	1930.80

Number of cells with at least one pottery piece by grade				
	Minoan	Hellenistic		Late Roman
		Grid Cell Size		
		$633m^2$	$70m^2$	
Coarse	541	29	22	42
Medium	364	172	377	704
Fine	108	243	399	835
Number of cells with at least one pottery piece by grade				
	684	309	566	1310

2.2 Measuring consumption

In this section we detail how we measure consumption gradients using the ASP data set. Our method covers the island of Antikythera with a fine grid of cells, and then measures pottery counts and consumption ratios for each of these cells. The cells are approximately 633 sq. metres each, and Table 1 provides details about how many of these cells contain finds. The exercise is conducted separately for all three eras of settlement, although based on our initial analyses we refine our gridding strategy for the Hellenistic era, as we detail below.

Given our data on both quantity of pottery as well as quality (fine, medium, coarse), we already have a natural separation of total unweighted consumption and relative expenditure. Raw total pottery counts may be taken to be proxies for total consumption, unweighted by goods' relative prices. Meanwhile, relative gradients of pottery counts by quality can measure the relative consumption of higher quality goods by location.

More precisely, suppose that in a given cell $\mathcal{C}$ in era $\mathscr{E}$we observe $p^{\mathscr{E}}_{\mathcal{C},f}, p^{\mathscr{E}}_{\mathcal{C},m}$ and $p^{\mathscr{E}}_{\mathcal{C},c}$ pieces of fine, medium and coarse pottery respectively with each individual piece denoted with i subscript. Denoting by $\pi_{\mathscr{E},i,f}$ the probability of the ith piece of fine pottery belonging to era $\mathscr{E}$ as computed by Bevan et al. [2013], with similar probability notations for other pottery qualities, the probability-weighted consumption measure in cell $\mathcal{C}$ is

$$P^{\mathscr{E}}_{\mathcal{C}} = \sum_{i=1}^{p^{\mathscr{E}}_f} \pi^{\mathscr{E}}_{i,f} p^{\mathscr{E}}_{\mathcal{C},i,f} + \sum_{i=1}^{p^{\mathscr{E}}_m} \pi^{\mathscr{E}}_{i,m} p^{\mathscr{E}}_{\mathcal{C},i,m} + \sum_{i=1}^{p^{\mathscr{E}}_c} \pi^{\mathscr{E}}_{i,c} p^{\mathscr{E}}_{\mathcal{C},i,c}, \mathscr{E} = \text{Minoan, Hellenistic, Late Roman} \tag{1}$$

while raw consumption measures can be constructed without the use of probability weighting.

In Figures 2-4, we illustrate the distribution of pottery over the island, separately for each era. In each figure, the left panel shows pottery locations from the Minoan era while the right panel does so for the Hellenistic era. Each pottery location corresponds to a piece that belongs to the relevant era with nonzero probability. Figure 2 plots the distribution of coarse pottery over the island, with the panels corresponding to Minoan, Hellenstic and Late Roman era respectively from left to right. Table 1 presents some summary statistics about the data, and these are visualized in the presented maps. There are 5,497 pottery pieces of coarse quality in the Minoan era, and seem mostly concentrated around the fertile center of the island, while the 32 coarse pottery pieces that correspond to the Hellenistic era are almost entirely concentrated around the port of Kastro. The 49 pieces of coarse for the Late Roman era are more broadly scattered but noticeably absent from the Potamos area. Thus, we already see some evidence of the vastly changed economic structure of the island across eras.

The comparison between medium pottery quantities is somewhat closer: 906 Minoan pieces, 818 Hellenistic pieces and 1173 Late Roman pieces. We plot these finds in Figure 3, noticing a similar pattern to the one observed for coarse pottery, with one exception. Examination of the rightmost panel reveals an abundance of medium grade pottery in the Potamos area in the Late Roman era, while coarse pottery was noticeably absent. Nevertheless, there are further scatters that suggest the presence of some prosperous farmsteads in the "hinterland" of the island even while the bulk of economic activity takes place around the port. Indeed, in Figure 4, we plot the finds of fine pottery and find much the same patterns. The overall distributions between eras still reflects the stark contrasts observed earlier, and the fact that there are 856 (1369) pieces of fine Hellenestic (Late Roman) pottery as opposed to just 226 Minoan pieces provide some evidence of the technological advances that accompanied the structural economic changes on the island.

The eyeballing exercise for the previous paragraph can be improved by using the cell-wise consumption measures defined in equation (1) to obtain a smooth estimate of consumption over the island by plotting kernel density estimates. The results are displayed in 3D in Figures 5-7. The color scheme runs low-medium-high as green-yellow-red and is buttressed further with vertical heights measuring consumption densities. The figures are plotted in a northeasterly perspective from an elevated southwestern viewpoint. We observe the concentrations of consumption in the areas we saw previously in both eras. As the earlier figures suggested, the consumption distributions in the Minoan and Late Roman eras are substantially less skewed than the Hellenistic distribution. The latter is quite distinctive in the exclusivity of economic activity around Kastro in a fairly small radius, and this factor will influence our choice of gridding strategy for this era.

2.3 Consumption gradients relative to center of economic activity

Our analysis in the previous section indicates the presence of consumption gradients. In this section we estimate these gradients and discuss our finding in relation to the figures we have already discussed. We show nonparametric gradient estimates: by a statement 'y is regressed on x', or similar, we mean that we fit a regression model $y = m(x) + \epsilon$, where $m(\cdot)$ is an unknown nonparametric function. We use the series or sieve estimation method which approximates the regression function $m(x)$ by a linear combination of, say, ℓ basis functions, which we choose to be splines. Thus the regressions estimated are of the form $y = \sum_{j=1}^{\ell} s_j(x)\beta_j + e$, where $e = \epsilon + m(x) - \sum_{j=1}^{\ell} s_j(x)\beta_j \equiv \epsilon + r(x)$, say. The remainder $r(x)$ is the approximation error which is negligible under various technical conditions involving the smoothness of $m(\cdot)$, see e.g. Chen [2007]. The estimation is implemented using the GAM package in R.

As we will see below, nonparametric gradients allows us to capture nonlinearities in the gradients that reflect economic features of the island's consumption distribution. Solid lines correspond to gradients while asymptotic 95% confidence intervals (i.e. based on a standard normal critical value of 1.96) are traced out with dashed lines in each figure. Distance from the economic centers, defined as the fertile centre, Kastro and Potamos in the Minoan, Hellenistic and Late Roman eras respectively, is in metres on the horizontal axes. For the Late Roman era we will also examine the situation where two separate economic centers, Potamos (maritime) and the fertile center (agrarian) are considered.

2.3.1 Absolute consumption gradients

Estimated gradients of total consumption, obtained from the probability-weighted formula of equation (1) and its unweighted version are displayed in Figure 8. Plotted in each are spline based nonparametric fits; red lines correspond to the probability-weighted measure as in equation (1) while the green lines correspond to the unweighted versions. The origin is a centre of economic activity for each era: the fertile heart of the island for the Minoan era, Kastro for the Hellenistic era and Potamos for the Late Roman era.

Nonlinearity in the gradients is captured by the nonparametric fits, which show humps in the gradients in the Minoan and Late Roman eras. The former is a smaller peak than at the origin and corresponds to other fertile areas of the island that saw some amount of economic activity but less than that observed in the fertile center that constitutes the origin. The hump is more pronounced, and the gradients generally less steep, in the Late Roman era. This reflects the more equitable distribution of economic activity on the island during this phase, as both maritime and agrarian activity co-existed. Thus in our analysis of relative consumption gradients below, we analyze the two centers as separate economic hubs. On the other hand, the nonparametric fits for the Hellenistic era essentially plummet to zero at just about one kilometre from Kastro.

The slight upwards bend observed in both fits for the Hellenistic era at large distances could be ascribed to the presence of isolated communities in the coastal areas of the island, as seen be the presence of small quantities of pottery in some coastal areas in the leftmost panels of Figures 2-4. Note though that confidence bands become rather wide at the extremities of distance (as in the other two era considered), so this upwards bend could as much reflect the imprecision of these estimates due to sparse data.

2.3.2 Relative consumption gradients

We now plot gradients of pottery quality counts as a function of distance from economic center. From our examination of absolute consumption gradients above we see no qualitative difference between considering probability weighted and unweighted counts, so we focus on the latter. Plots with the former lead to no difference in interpretations. Our prior analysis has revealed the highly local nature of pottery concentration in the Hellenistic era. Thus, in order to better utilize the data and obtain clearer insights we adopt a finer spatial resolution for this phase. We do this by gridding the data with cells of approximately 70 sq. metres, as compared to the 633 sq. metres used earlier. Such 'zoomed-in' smaller cells are not very useful in the other two eras with pottery scatters ranging over a much wider area, but are feasible and indeed useful in the Hellenistic era. Table 1 includes summary statistics for the Hellenistic era with this finer resolution.

Using these grids, fit a non-parametric spline to the logarithm of pottery counts in each cell by quality. As our goal is to measure the relative consumption of each type of pottery across space, we wish to avoid unsettled regions contaminating any inference, so we exclude cells which contain no pottery of any type. As there remains some cells which contain some types of pottery but not all types of pottery, for our logarithms, we take the logarithm of $1 + p^{\mathscr{E}}_{\mathcal{C},q}, q =$ Fine, Medium or Coarse. Figures 9- 11 plot the fitted curves, which are normalized to be unity at the origin. As discussed above we present separate gradients relative to the two distinc centers observed in the Late Roman era.

For settlements such as Potamos and the fertile center in the Late Roman era and Kastro in the Hellenistic, consumption of fine and medium pottery decreases noticeably from the settled center (approximately the cell with the highest total pottery count). Meanwhile coarse pottery consumption remains relatively flat with distance. Note that the lower panel of Figure 11 shows a hump for both fine and medium pottery corresponding to the fertile center, with gradients increasing (decreasing) as one gets closer to (farther from) the hump, while coarse pottery shows no such pattern. This is consistent with greater 'luxury' consumption in the economic center.

The Minoan era, which comprised agrarian settlements with little commerical (non-farming) activity constitues a more primitive economy and acts as a kind of informal placebo. We expect that it should show no discernible differences in the spatial orientation of luxury and and non-luxury consumption. Examination of Figure 9 confirms this, with all three types of pottery gradients moving in tandem. The upward bend in fine pottery for the Hellenistic era at the farthest distance is due to the presence of a coastal temple of Apollo, which does not correspond to an economic settlement.

We also compute some correlations that buttress our visual analysis. Table 2 shows the correlation between $\log(1 + p^{\mathscr{E}}_{\mathcal{C},q})$ and $\log(P^{\mathscr{E}}_{\mathcal{C}}), q =$ Fine, Medium or Coarse, i.e. between pottery coarseness and total consumption within cells in different eras. We observe that there is no difference in the pattern of these correlations for the Minoan era that suggest a decline in luxury consumption witha decline in total consumption: all correlations are strong and because coarse pottery consitutes the majority of all consumption its correlation is strongest. On the other hand, for the Hellenistic and Late Roman eras the correlations are clearly stronger for the higher grades of pottery, implying that a higher share of luxury consumption is higher associated with higher consumption. This is consistent with the sorting predicted by our model below. The equality of the correlations for fine and medium pottery in the Late Roman era is simply the result of chance.

Table 2: Correlation between type of consumption and total consumption

	Minoan	Hellenistic	Late Roman
$corr(\log(1+p^{\mathscr{E}}_{\mathcal{C},c}), \log(P^{\mathscr{E}}_{\mathcal{C}}))$	0.9661	0.1410	0.0612
$corr(\log(1+p^{\mathscr{E}}_{\mathcal{C},m}), \log(P^{\mathscr{E}}_{\mathcal{C}}))$	0.6956	0.7464	0.6343
$corr(\log(1+p^{\mathscr{E}}_{\mathcal{C},f}), \log(P^{\mathscr{E}}_{\mathcal{C}}))$	0.5666	0.6939	0.6343

3 A monocentric city model with household heterogeneity and luxury goods

3.1 Introduction

We build a monocentric city model with household heterogeneity, a set of consumption goods, land and leisure. Even though the model uses preferences with meaningful consumption and substitution effects from leisure, we are able to obtain some sorting results for the competitive equilibria. We show that when commuting costs are dominated by time costs, households sort such that high ability (high income) households live close to the city center whereas when commuting costs are mostly in goods, households sort in opposite fashion. In the former case, luxury goods consumption is higher in the city center. In the latter case, luxury goods consumption will be higher in the periphery as long as substitution effects between land and non-durable goods are not too strong.

3.2 Setup

Households have preferences over a vector of I non-durable consumption goods, land and leisure (c, a, l, respectively) denoted by $u(c, a, l)$. Households are endowed with one unit of time which they may use for work, commuting or leisure. We normalize the population to 1. Households are heterogeneous in the productivity of their work time, denoted by $z \sim F_z$, where F_z is the distribution of population abilities with support Z and density f_z. A unit of work time is converted into z units of any in a set of non-durable goods $i \in I$. In equilibrium this will mean that the relative price of each good i is the same. We normalize this price to 1.

All households live in a monocentric city and "commute" into the center of the city.[4] Commuting from home a distance r costs $t_1(r)$ in time and $t_0(r)$ in the numeraire good. The supply of land at a distance r in the economy is given by the density $f_r : \Re_+ \to \Re_+$.

We assume the initial endowment of land is equal for all households. In competitive

[4]We leave unspecified whether this is to consume location based amenities, buy goods or work.

equilibrium it must be the case, for each household, that

$$\sum_i c_i + ap(r) + lz \leq \Lambda + z(1 - t_1(r)) - t_0(r) \tag{2}$$

where $p(r)$ is the price of land,

$$\Lambda = \int p(r) f_r(r)\, dr$$

is the total value of land in the city and ((2)) is the household's budget constraint.

3.3 Preferences

We assume that households have a constant elasticity of substitution preferences over land, leisure and a composite $g : I \to \Re$ of the non-durable goods:

$$u(c, a, l) = \frac{\left(\omega_1 g(c)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_2 a^{\frac{\varepsilon-1}{\varepsilon}} + \omega_3 l^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\gamma\varepsilon}{\varepsilon-1}}}{\gamma}.$$

with $\varepsilon, \gamma < 1$.We assume the composite g is:

$$g(c) = \prod_{i \in I} (c_i - \beta_i)^{\alpha_i} + \sum_{i \in I} \beta_i \tag{3}$$

where $\beta_i \geq 0$ are preference parameters and $\sum_{i \in I} \alpha_i = 1$ with $\alpha_i > 0$.

3.4 First order conditions

3.4.1 Non-durable consumption choices

The first order conditions for the household imply that

$$\frac{(c_i - \beta_i)}{\alpha_i} = \frac{(c_j - \beta_j)}{\alpha_j} = v(x)$$

where v is the indirect sub-utility function for preferences g given total spending on non-durable goods x. Using the budget constraint for this sub-problem ($x = \sum_{i \in I} c_i$) we get

$$v(x) = x$$

and

$$c_i = \alpha_i (x - \sum_{j \in I} \beta_j) + \beta_i \tag{4}$$

If the goods can be ordered such that $(\beta_i - \alpha_i \sum_{j \in I} \beta_j)$ is decreasing in i, than higher

i goods are more "luxuriousness." (I.e. the elasticity of good i's consumption share of total good spending x with respect to x is increasing in i.)

3.4.2 Consumption expenditures, land, leisure and location

The first order conditions for the rest of the household's problem, using the fact that $g(c) = x$ where x is the amount the household will spend on non-durables, are:

$$\begin{aligned}
\lambda_c &= u_c(x,a,l) && (5)\\
\lambda_c p_r(r) &= u_a(x,a,l) && (6)\\
\lambda_L(r,z) + \lambda_c z &= u_l(x,a,l) && (7)\\
a(r,z)\frac{dp}{dr} &= -z\frac{\partial t_1(r)}{\partial r} - \frac{\partial t_0(r)}{\partial r} && (8)
\end{aligned}$$

Equation 8 becomes the Alonso-Muth condition by examining the slope of the bid-rent curves $\Psi(r,z;\bar{u})$. In equilibrium, $\frac{dp}{dr}(r) = \Psi_r(r^*(z),z;u^*)$.

Strict sorting occurs if everywhere:

$$\frac{\partial^2\Psi}{\partial r\partial z}(r,z) \gtrless 0$$

Subbing in we get that land demand is:

$$a(r,z) = \frac{\Lambda + z(1-t_1(r)) - t_0(r)}{\underbrace{\omega_1\left(\frac{1}{\omega_1}\right)^{1-\varepsilon} + \omega_2\left(\frac{p(r)}{\omega_2}\right)^{1-\varepsilon} + \omega_3\left(\frac{z+\lambda_L(r,z)}{\omega_3}\right)^{1-\varepsilon}}_{\equiv P(r,z)}}\left(\frac{p(r)}{\omega_2}\right)^{-\varepsilon} \tag{9}$$

so

$$\frac{\partial^2\Psi(r,z)}{\partial r\partial z} = \frac{\frac{dt_1(r)}{dr}}{a(r,z)}\left[\frac{z}{a(r,z)}\frac{\partial a(r,z)}{\partial z} - 1\right] + \frac{\frac{dt_0(r)}{dr}}{(a(r,z))^2}\frac{\partial a(r,z)}{\partial z}. \tag{10}$$

Note that

$$\frac{\partial P(r,z)}{\partial z} \geq 0,$$

strictly so if $1 - t_1(r) > l(r,z)$. Differentiating 9:

$$\begin{aligned}
\frac{\partial a(r,z)}{\partial z} &= \frac{1-t_1(r)}{P(r,z)}\left(\frac{p(r)}{\omega_2}\right)^{-\varepsilon} - \frac{\Lambda + z(1-t_1(r)) - t_0(r)}{(P(r,z))^2}\left(\frac{p(r)}{\omega_2}\right)^{-\varepsilon}\frac{\partial P(r,z)}{\partial z}\\
&= a(r,z)\left(\frac{1-t_1(r)}{\Lambda + z(1-t_1(r)) - t_0(r)} - \frac{\partial P(r,z)}{\partial z}\frac{1}{P(r,z)}\right).
\end{aligned}$$

Case 1. Commuting costs are only in time: $t_0 \approx 0$.

The cross-derivative of the bid-rent curve then becomes:

$$\frac{\partial^2 \Psi(r,z)}{\partial r \partial z} = \frac{\frac{dt_1(r)}{dr}}{a(r,z)} \left[\frac{z(1-t_1(r))}{\Lambda + z(1-t_1(r))} - \frac{\partial P(r,z)}{\partial z} \frac{z}{P(r,z)} - 1 \right] < 0$$

where the inequality follows because $\frac{z(1-t_1(r))}{\Lambda+z(1-t_1(r))} < 1$. So the highest types live strictly closer to the city center.

Non-durable expenditures $x(r,z)$ are similarly

$$x(r,z) = \frac{\Lambda + z(1-t_1(r))}{P(r,z)} \left(\frac{1}{\omega_1} \right)^{-\varepsilon}. \tag{11}$$

The first-order condition for expenditures can be rewritten as:

$$\omega_1 (x(r,z))^{-\frac{1}{\varepsilon}} P^{\frac{\gamma\varepsilon - \varepsilon + 1}{(\varepsilon-1)(1-\gamma)\varepsilon}} = \lambda_c.$$

If $\gamma\varepsilon - \varepsilon + 1 < 0$ (i.e. if $\varepsilon > \frac{1}{1-\gamma}$) then $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial z} > 0$,which in turn implies that $\frac{\partial x(r,z)}{\partial z} > 0$. Note that a conventional estimate of risk aversion and elasticity of substitution have $\gamma \approx -1$ and $\varepsilon \approx 0.9$, which would satisfy the inequality. Under the same parameter conditions, $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial r} < 0$ and then the first-order condition similarly implies that $\frac{\partial x(r,z)}{\partial r} < 0$. Given negative assortative matching on location, the total derivative $\frac{dx(r,z)}{dz} = \frac{\partial x(r,z)}{\partial z} + \frac{\partial x(r,z)}{\partial r} \frac{\partial r(z)}{\partial z} > 0$ follows. Therefore, higher types live closer to the center and spend more on non-durables.

Case 2. Commuting costs are only in goods: $t_1 \approx 0$.

The cross-derivative of the bid-rent curve then becomes:

$$\frac{\partial^2 \Psi(r,z)}{\partial r \partial z} = \frac{\frac{dt_0(r)}{dr}}{a(r,z)} \left[\frac{1}{\Lambda + z - t_0(r)} - \frac{\partial P(r,z)}{\partial z} \frac{z}{P(r,z)} \right]$$

Non-durable expenditures $x(r,z)$ are similarly

$$x(r,z) = \frac{\Lambda + z - t_0(r)}{P(r,z)} \left(\frac{1}{\omega_1} \right)^{-\varepsilon}. \tag{12}$$

The first-order condition for expenditures can still be rewritten as:

$$\omega_1 (x(r,z))^{-\frac{1}{\varepsilon}} P^{\frac{\gamma\varepsilon - \varepsilon + 1}{(\varepsilon-1)(1-\gamma)\varepsilon}} = \lambda_c.$$

As in the case above, if $\gamma\varepsilon - \varepsilon + 1 < 0$ (i.e. if $\varepsilon > \frac{1}{1-\gamma}$) then $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial z} > 0$ and $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial r} < 0$ which in turn imply that $\frac{\partial x(r,z)}{\partial z} > 0$ and $\frac{\partial x(r,z)}{\partial r} < 0$. It also implies $\frac{\partial a(r,z)}{\partial z} > 0$ and thus $\frac{\partial^2 \Psi(r,z)}{\partial r \partial z} > 0$. Which means households positively sort. However such sorting is not sufficient to determine which types of households spend more on luxuries. Higher land prices close to the center encourage households living there to spend more of their income on the non-durables. On the other hand the households living further away are more productive and have higher incomes. Which effect is stronger will depend on parameterizations.

4 Conclusion

Modern cities are shaped by amalgam of forces, some present and some from historical sources. Modern transportation networks often are partially molded by historic networks (in part to reduce frictions to rights of way). Modern public goods often have explicit links to the location preferences of past generations (the Louvre and the Frick Museums were formally residences of their patrons).

Ancient settlements, especially those that were built without meaningful antecedents, offer a different laboratory to test urban economics models. Here we integrate archaeological data from two past settlements into a simple monocentric city model. We show how to infer the spatial distribution of consumption from the data and then how the simple model, calibrated with modern preferences but ancient transportation costs, matches the data.

5 Figures

Figure 1: Antikythera

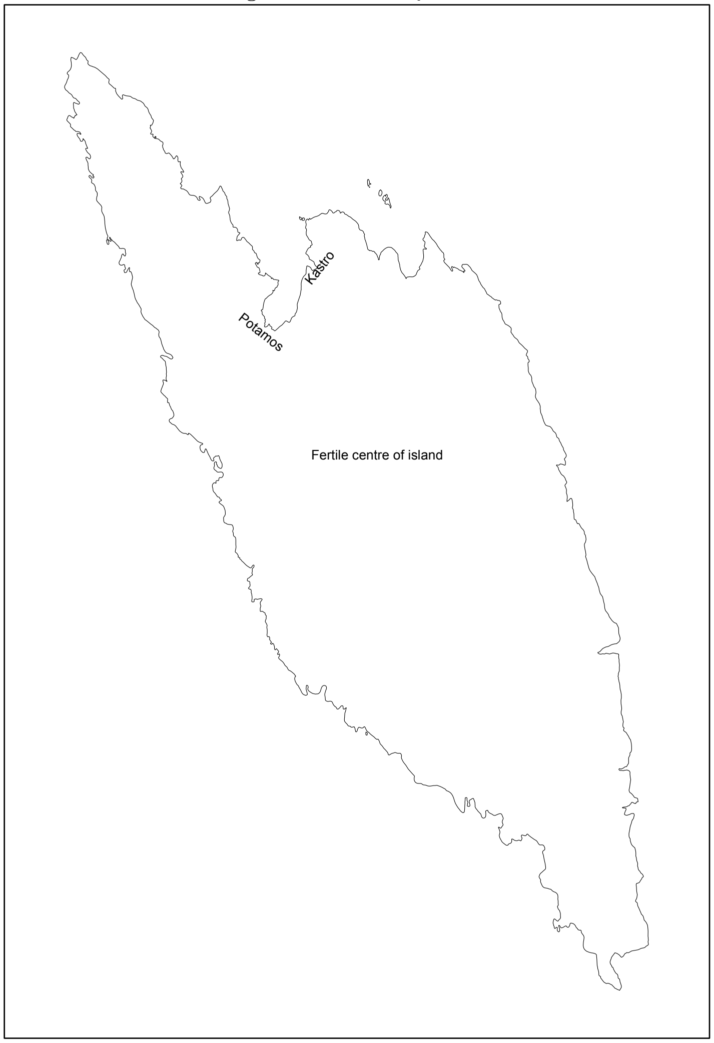


Figure 2: Coarse pottery locations: Minoan, Hellenistic, Late Roman eras (left to right)

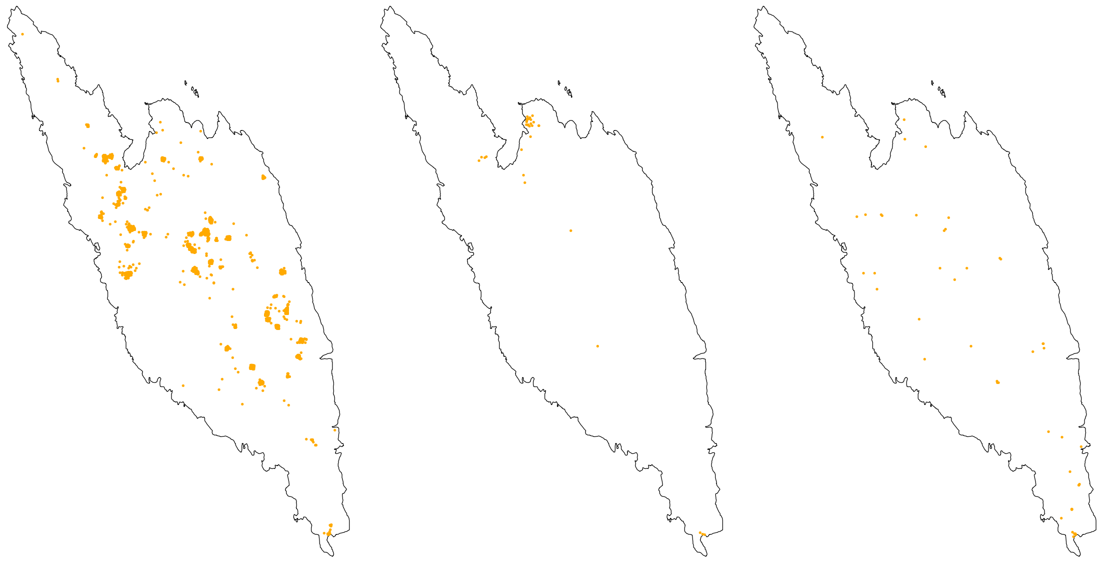


Figure 3: Medium pottery locations: Minoan, Hellenistic, Late Roman eras (left to right)

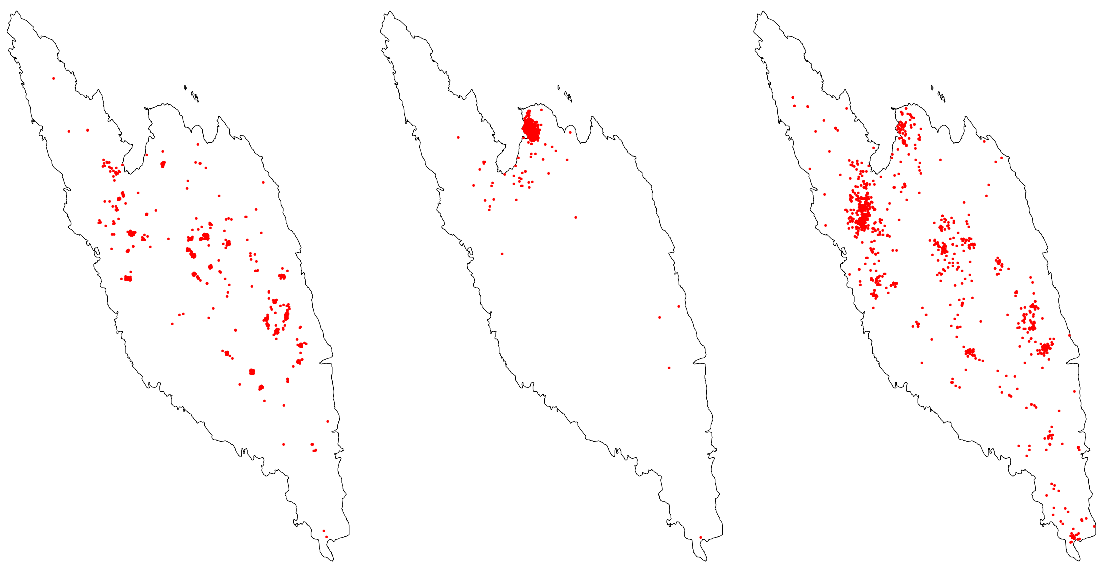


Figure 4: Fine pottery locations: Minoan, Hellenistic, Late Roman eras (left to right)

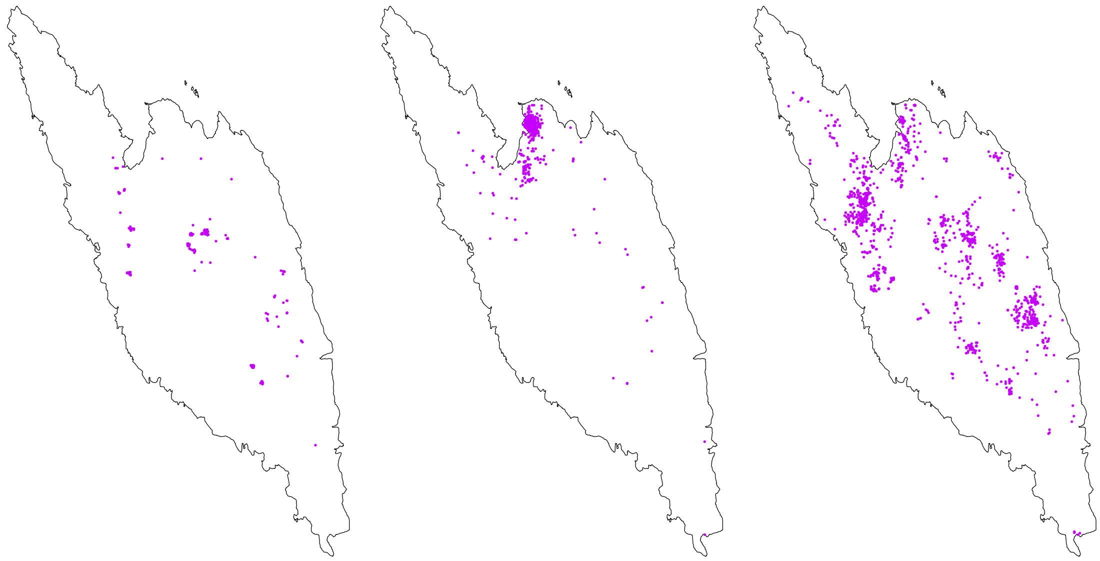


Figure 5: Density of raw counts: Minoan era

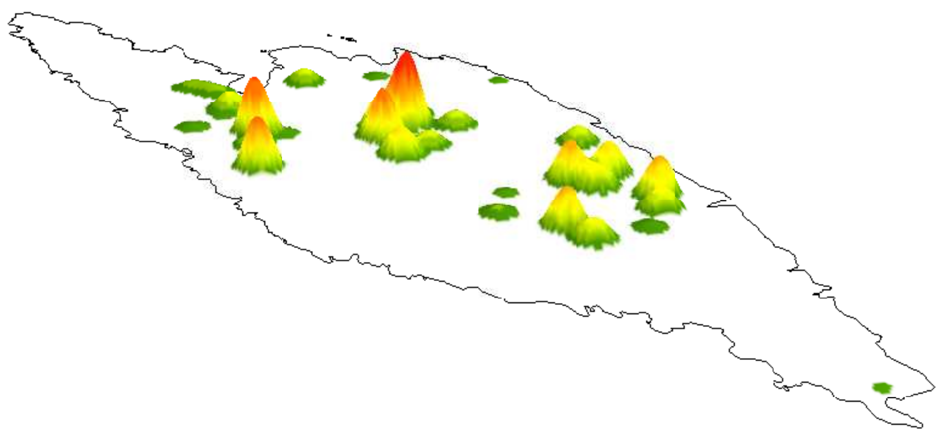


Figure 6: Density of raw counts: Hellenistic era

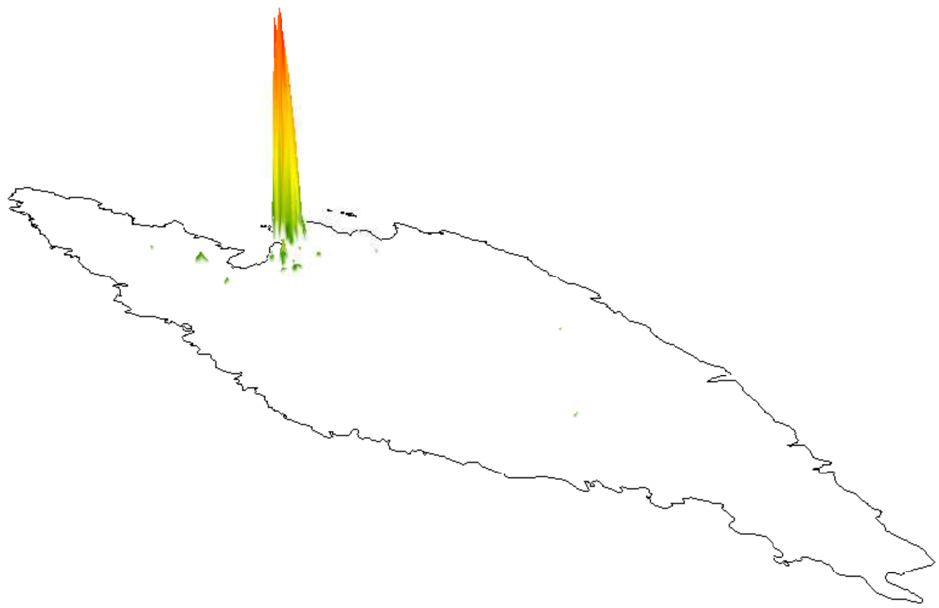


Figure 7: Density of raw counts: Late Roman era

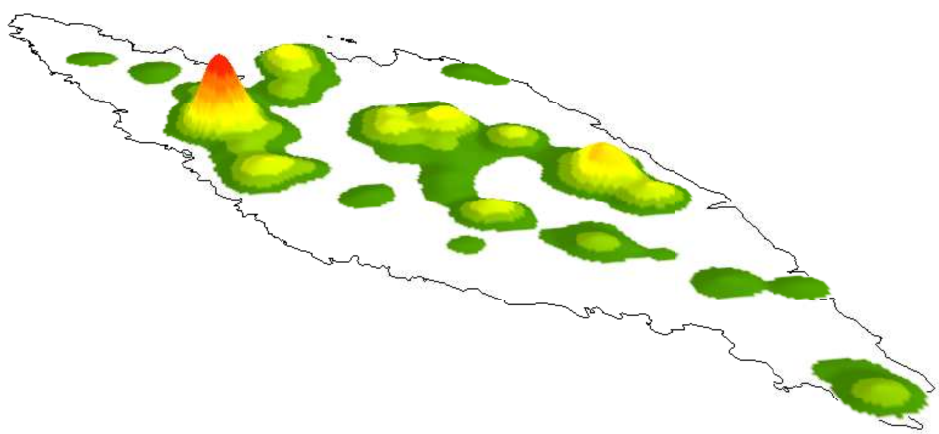


Figure 8: Nonparametric absolute consumption gradients by era

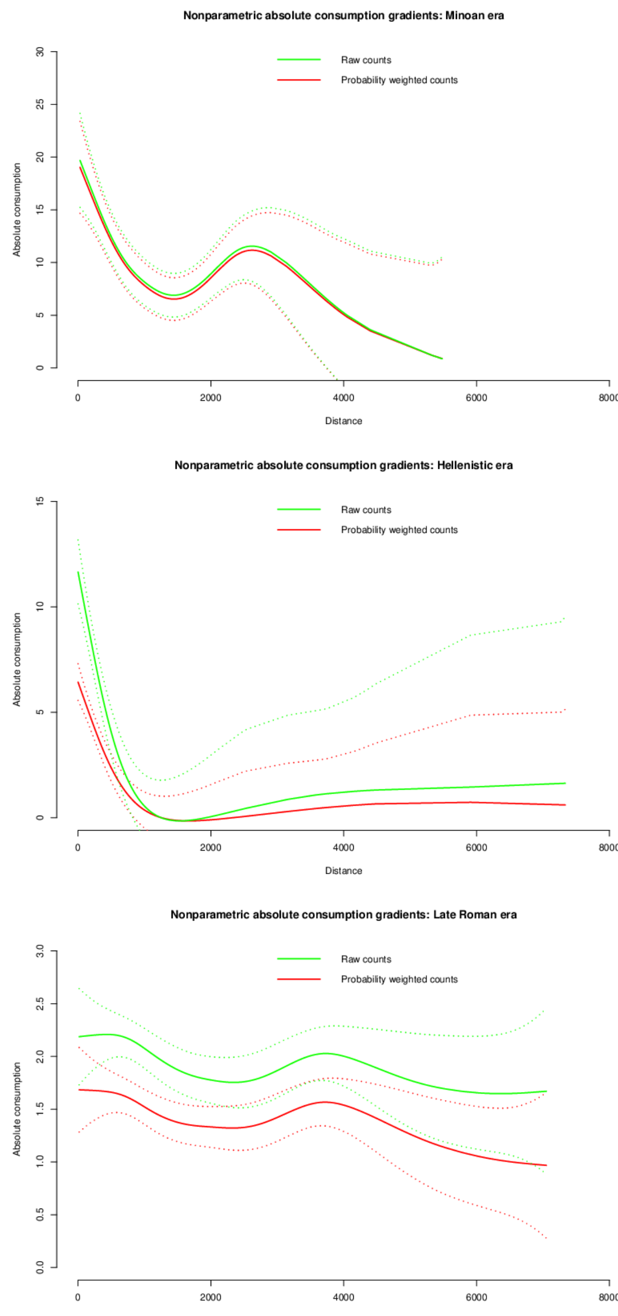


Figure 9: Relative consumption gradients: Minoan era

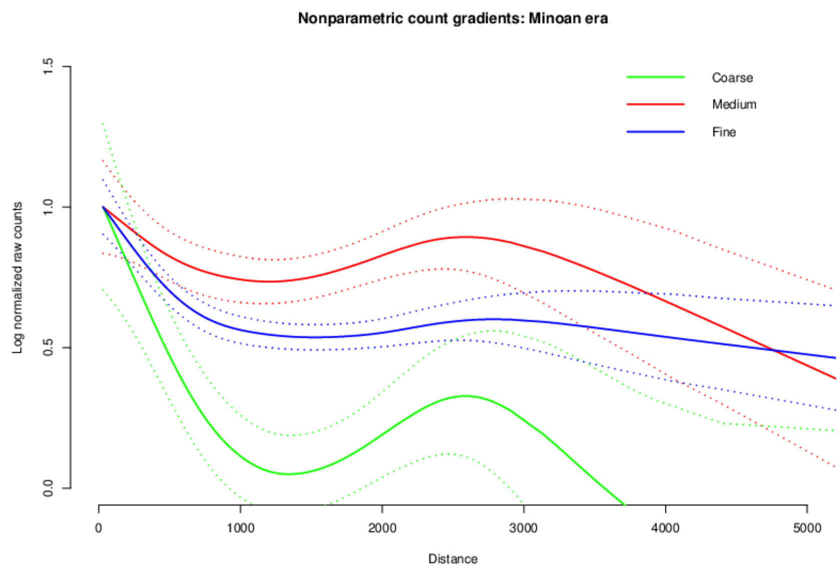


Figure 10: Relative consumption gradients: Hellenistic era

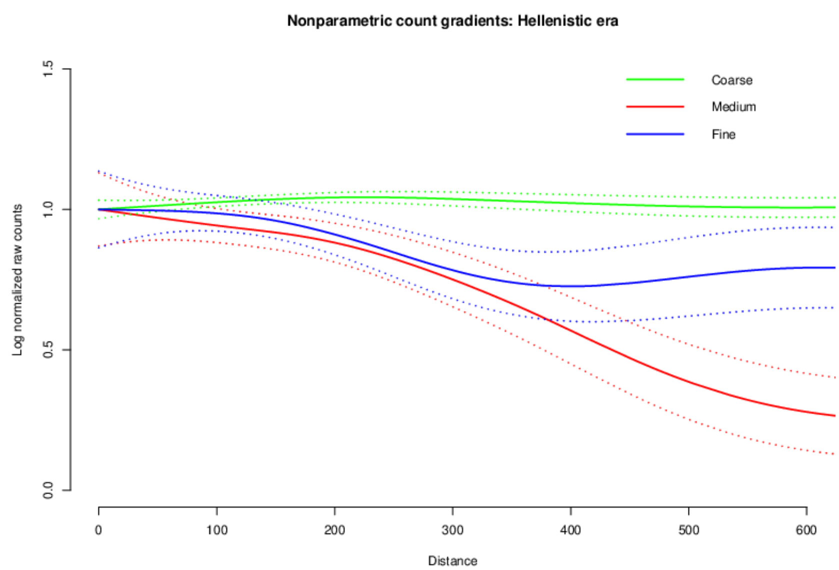
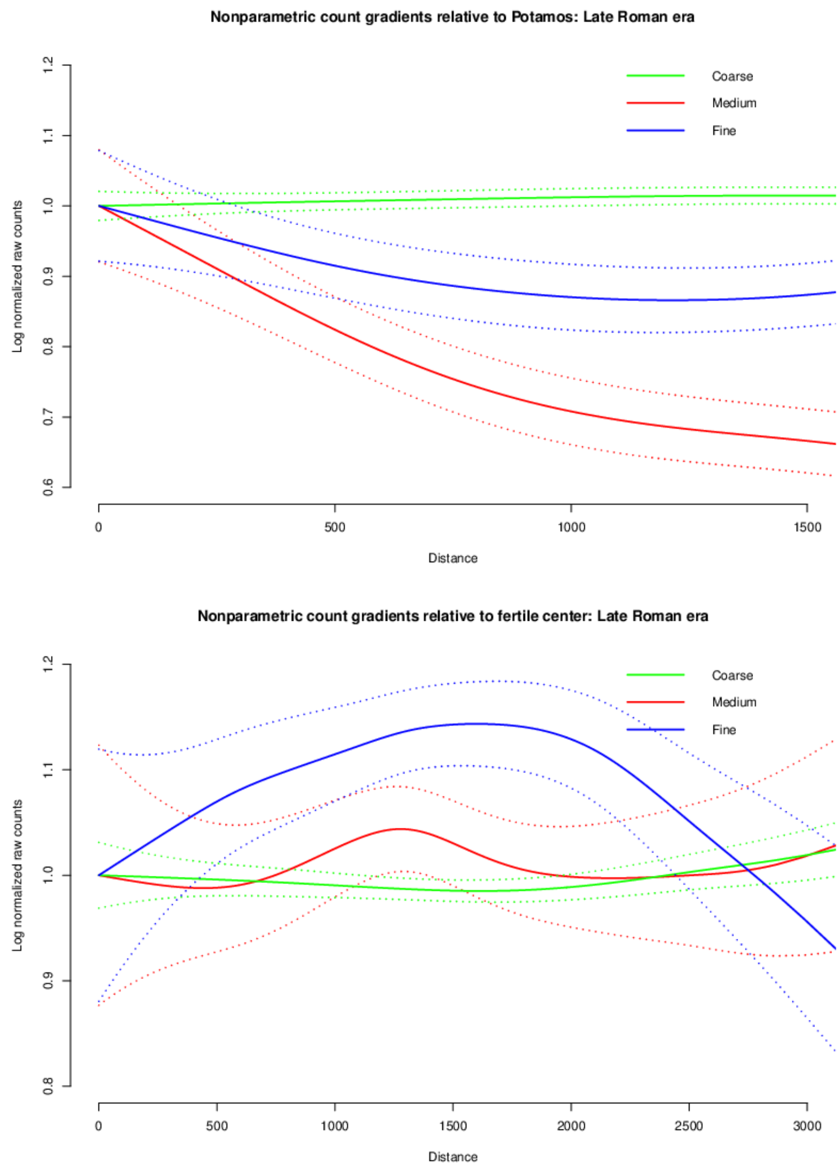


Figure 11: Relative consumption gradients: Late Roman era



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