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Asset Prices and Unemployment Fluctuations*

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ABSTRACT

Recent critiques have demonstrated that existing attempts to account for the unemployment volatility puzzle of search models are inconsistent with the procylicality of the opportunity cost of employment, the cyclicality of wages, and the volatility of risk-free rates. We propose a model that is immune to these critiques and solves this puzzle by allowing for preferences that generate *time-varying risk* over the cycle, and so account for observed asset pricing fluctuations, and for *human capital accumulation on the job*, consistent with existing estimates of returns to labor market experience. Our model reproduces the observed fluctuations in unemployment because hiring a worker is a risky investment with long-duration surplus flows. Intuitively, since the price of risk in our model sharply increases in recessions as observed in the data, the benefit from creating new matches greatly drops, leading to a large decline in job vacancies and an increase in unemployment of the same magnitude as in the data.

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The most important theoretical contribution of search models of the labor market to the study of business cycles is that they interpret involuntary unemployment as an equilibrium phenomenon. The key insight of these models is that involuntary unemployment can arise even without any assumed inefficiencies in contracting, such as rigid wages. Despite its great promise, though, Shimer (2005) showed that the textbook search model cannot generate anywhere near the observed magnitude of the fluctuations in the job-finding rate and unemployment in response to shocks of plausible magnitude. A large body of work has attempted to address the *unemployment volatility puzzle* of Shimer (2005) but, as we later discuss, recent critiques of it have demonstrated that existing attempts are inconsistent with the procylicality of the opportunity cost of employment, the cyclicality of wages, and the volatility of risk-free rates. Hence, in this precise sense, the puzzle has not been solved.

In this paper, we propose a model that reproduces these features of the data, respects the original promise of search models by generating involuntary equilibrium unemployment without relying on inefficient contracting or wage rigidities, and solves this puzzle. We do so by allowing for preferences that give rise to *time-varying risk* over the cycle, as consistent with observed fluctuations in asset prices, and for *human capital accumulation on the job*, in line with the documented growth of wages with labor market experience. Throughout most of our analysis, we abstract from physical capital simply to help illustrate our mechanism in the most transparent way. Unlike much of the literature, however, we also incorporate physical capital and show that such an augmented model matches key observed patterns of job-finding rates, unemployment, output, investment, and asset prices. In contrast to the classic separation result by Tallarini (2000) that introducing asset pricing preferences into an otherwise standard real business cycle model has no effect on the fluctuations of real variables, here introducing such preferences, either in the presence or absence of physical capital, creates an important interaction between the financial and real sides of an economy and greatly amplifies fluctuations.

The main idea of our model is that hiring a worker is akin to investing in an asset with *risky* dividend flows that have *long* durations. In our model, as in the data, the price of risk rises sharply in downturns. Owing to human capital accumulation on the job, the surplus flows to matches between workers and firms have long durations and so are sensitive to variation in the price of risk. These two features then imply that the benefits of creating matches sharply drop in downturns, which induces firms to substantially reduce the number of job vacancies they create and, correspondingly, leads unemployment to increase as much as it does in the data.

Our model adds to the textbook search model two simple ingredients that make it consistent with two salient aspects of the data: asset prices fluctuate over the cycle and wages increase with experience in the labor market. To accommodate the first feature, we augment the textbook model with preferences that generate time-varying risk, whereas to accommodate the second feature, we introduce human capital accumulation on the job and depreciation off the job. We choose parameters for preferences and technology that are consistent with key observed properties of asset prices and wage-experience profiles, and show that the resulting allocations display fluctuations in unemployment that are as large as those in the data.

We generate involuntary unemployment without exploiting inefficiencies in wage contracting by focusing on labor market outcomes generated by a competitive search equilibrium. We find this concept appealing relative to common bargaining concepts such as Nash bargaining or alternating offer bargaining, since these bargaining schemes give rise to inefficient wage setting unless the parameters that characterize the bargaining process are chosen appropriately. For instance, a well-known result is that equilibrium wage setting under Nash bargaining is efficient and, hence, leads to the same outcomes that arise under competitive search when the Hosios's (1990) condition holds. Similarly, we show that under alternating offer bargaining, wage setting is efficient when two conditions hold: the exogenous rate of breakdown of bargaining between workers and firms converges to one and the probability that a worker makes the first offer equals the elasticity of the matching function with respect to the measure of unemployed workers. In light of these results, we can interpret our work as focused on economies with efficient wage setting, which can be achieved under any of the three most popular wage determination schemes: competitive search and, as long as suitably parametrized, Nash bargaining and alternating offer bargaining. In this sense, our results do not depend on the specific wage determination scheme chosen.

We argue that our two simple ingredients are necessary to account for the observed volatility of unemployment. In particular, we show that if we retain human capital accumulation but replace our asset pricing preferences with standard constant relative risk aversion preferences, then the model generates no fluctuations in unemployment regardless of the degree of human capital accumulation. Conversely, if we retain our asset pricing preferences but abstract from human capital accumulation, then the model generates almost no fluctuations in unemployment.

We turn to providing further details about our two additional ingredients. Consider first preferences. The asset pricing literature has developed several classes of preferences and stochastic processes for exogenous shocks that give rise to large increases in the price of risk in downturns and, hence, reproduce key features of the fluctuations of asset prices. As Cochrane (2011) emphasizes, all of these preferences and shocks generate variation in asset prices from variation in risk premia, as observed in the data. To emphasize that our results are robust to the specific details of the preferences and shocks that achieve this variation, we show that our results hold for a wide range of the most popular specifications.

Specifically, we begin with a variant of the original preferences in Campbell and Cochrane (1999) in which we eliminate the resulting consumption externality by making the habit in consumption a function of exogenous shocks. We find these preferences appealing because they incorporate the idea that the price of risk rises in recessions in a transparent and intuitive way. Moreover, as we show, their implications for asset prices and unemployment fluctuations are nearly identical to those of the original preferences in Campbell and Cochrane (1999). We use this model as a baseline for later comparisons.

We then examine two versions of the preferences in Epstein and Zin (1989). We first consider a version of the long-run risk setup of Bansal and Yaron (2004) with preferences as in Epstein and Zin

(1989), modified along the lines suggested by Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018). Albuquerque et al. (2016) show that a model that combines long-run risk with preference shocks replicates well observed features of asset prices. Following the setup of Wachter (2013), we next consider the preferences in Epstein and Zin (1989) augmented with a time-varying risk of disasters, defined as episodes of unusually large decreases in aggregate consumption triggered by marked declines in productivity. Finally, given the large class of reduced-form asset pricing models that simply specify a discount factor as a function of shocks, we explore a version of the affine discount factor model of Ang and Piazzesi (2003) as a representative model in this class. We show that all these preference and shock structures imply analogous results for the volatility of the job-finding rate and unemployment.

Consider now human capital accumulation. For simplicity, we first assume that a worker's human capital grows at a constant rate during employment and depreciates at a constant rate during unemployment, and that market production, home production, and the cost of posting job vacancies are proportional to human capital. This formulation is particularly convenient because it implies that only the aggregate levels of human capital of employed and unemployed workers, rather than their distributions, need to be recorded as state variables. We also consider a more general formulation of the human capital process in which the rates of human capital accumulation and depreciation are stochastic and vary with the level of acquired human capital. This richer version of the model better reproduces the shape of empirical wage-experience profiles and yields very similar results to our baseline for the volatility of the job-finding rate and unemployment.

After exploring the quantitative results implied by alternative preferences and parametrizations of the human capital process, we characterize the mechanism generating them. Namely, we show that the job-finding rate is proportional to the present value of the surplus flows from matches between workers and firms. This present value, in turn, can be expressed as a weighted average of the prices of the stream of dividends from each match that are proportional to aggregate productivity, in short *claims to aggregate productivity*, at each time horizon. In this weighted average, the *weights* are determined by the degree of human capital accumulation whereas the *prices* of these claims are determined by the chosen preference and shock structure. We refer to these claims as *strips*. Intuitively, since human capital accumulation increases the duration of surplus flows, the greater is the amount of human capital accumulation, the slower is the decay of the surplus flows from matches between workers and firms, and, hence, the slower is the decay of the weights attached to strips at different horizons. The slow rate of decay of these weights is key to the amplification of aggregate shocks, since it affects the sensitivity of the job-finding rate to changes in the exogenous stochastic state of an economy.

Formally, we prove that the volatility of the job-finding rate can be well approximated by a single sufficient statistic: a weighted average, $\sum_n \omega_n b_n \sigma(s_t)$, over different horizons $\{n\}$ of the elasticity b_n of the price of a strip with respect to the relevant exogenous stochastic state s_t of an economy, multiplied by the volatility $\sigma(s_t)$ of this state.[1] The weights $\{\omega_n\}$ decay more slowly the greater is human capital accumulation and the elasticity b_n increases with n so that strips become more sensitive to the exogenous state as the horizon of a strip increases. We show that although the five asset pricing models we consider may have very different implications for various asset pricing moments, their implications for the volatility of unemployment only depend on our single sufficient statistic, which captures the volatility of the exogenous state, the implied variation in the price of risk, and the persistence of the returns to hiring workers. This result thus explains why all of these structures generate similar results for unemployment volatility.

The sufficient statistic that we identify further allows us to characterize the roles of time-varying risk and human capital accumulation in our results. First, we show that when there is little time-varying risk, the elasticity b_n of the price of strips with respect to the state s_t is small regardless of the horizon n. Hence, the model cannot generate much volatility in the job-finding rate regardless of the weights $\{\omega_n\}$ on strips. Second, we show that when there is little or no human capital accumulation on the job, the weights $\{\omega_n\}$ are nearly all concentrated on short-horizon claims, which display little volatility under all of our asset pricing specifications. Therefore, the model cannot generate much volatility in the job-finding rate in this case either. Only when both features are present—time variation in the price of risk and human capital accumulation—can our model produce sizable volatility in the job-finding rate and unemployment.

We conclude by considering two extensions. First, we examine a more general model of human capital along the lines of Ljungqvist and Sargent (1998, 2008) and Kehoe, Midrigan, and Pastorino (2019), which captures the feature that observed wages grow faster at low levels than at high levels of labor market experience. We show that our results are robust to accounting for this aspect of the data. Second, we augment our model with physical capital, subject to adjustment costs along the lines of Jermann (1998), and construct a business cycle model in the spirit of the seminal work by Merz (1995) and Andolfatto (1996). As Shimer (2005) points out, though, both of these papers miss a key feature of the data, namely, the strong negative correlation between vacancies and unemployment. Our model, instead, not only reproduces this feature but also matches salient patterns of job-finding rates, unemployment, output, investment, and asset prices in the data.

Based on these results, we view our exercise as a promising first step toward developing an integrated theory of real and financial business cycles.

1. Relation to the Literature

Our model relies on a fundamentally different mechanism than that isolated by Ljungqvist and Sargent (2017) in their survey of early attempts to solve the unemployment volatility puzzle, which include Hagedorn and Manovskii (2008), Hall and Milgrom (2008), and Pissarides (2009). In particular, Ljungqvist

[1] Note that s_t is the surplus consumption ratio for Campbell-Cochrane preferences with exogenous habit, the long-run risk factor for Epstein-Zin preferences with long-run risk, the probability of a disaster for Epstein-Zin preferences with time-varying disaster risk, and an abstract one for the Ang-Piazzesi discount factor. For Campbell- Cochrane preferences with external habit, the same statistic applies but the relevant stochastic state, namely, the surplus consumption ratio, is not exogenous.

and Sargent (2017) show that all of these attempts feature an acyclical opportunity cost of employment. In a recent paper, though, Chodorow-Reich and Karabarbounis (2016) critique this literature and argue that none of these attempts are consistent with the data. Namely, these authors document that the opportunity cost of employment in the data is procyclical with an elasticity close to one rather than, as assumed in these models, zero. These authors further demonstrate that once these models are made consistent with this aspect of the data, they are incapable of producing volatile unemployment.

A second critique of the literature that has addressed this puzzle by introducing some form of wage rigidity is by Kudlyak (2014). This work builds on the insight of Becker's (1962) classic paper that only the present value of the wages paid by firms to workers over the course of an employment relationship is allocative for employment. Specifically, Kudlyak (2014) establishes that for a large class of search models, the appropriate measure of rigidity of the allocative wage is the cyclicality of the user cost of labor, defined as the difference in the present values of wages between two firm-worker matches that are formed in two consecutive periods. As Kudlyak (2014) estimates and Basu and House (2016) confirm, the user cost of labor is highly cyclical in that it sharply falls when unemployment rises. Both of these papers also argue that reproducing the observed cyclicality of the user cost of labor is the key litmus test for the cyclicality of wages implied by any business cycle model. Early attempts to solve the unemployment volatility puzzle fail this test, as these authors discuss. Here we show that our model, instead, passes it.

Finally, a third critique has been formulated by Borovicka and Borovickova (2019), who argue that the literature is grossly at odds with robust patterns of asset prices. In contrast to existing attempts, our model incorporates standard asset pricing preferences, which avoid counterfactual movements in risk-free rates and risk premia. In this sense, our model overcomes this final critique as well.

The important related contribution of Hall (2017) accounts for the observed volatility of unemployment within a model that features alternating wage offer bargaining, a reduced-form discount factor, and no human capital accumulation. This paper is immune to the critique by Chodorow-Reich and Karabarbounis (2016) but not to those by Kudlyak (2014) and Borovicka and Borovickova (2019). In particular, as we show, Hall (2017) relies on a parametrization of wage setting that yields highly inefficient allocations associated with a counterfactually low degree of cyclicality of the user cost of labor. Hence, in this precise sense, the wages in Hall (2017) are much more rigid than those in the data. Moreover, Borovicka and Borovickova (2019) show that Hall's model generates fluctuations in unemployment not from time-variation in the price of risk, as our model does, but rather from strongly countercyclical movements in the risk-free rate, which are counterfactual. Thus, although Hall (2017) provides critical insights, it fails the litmus test of Kudlyak (2014) and Basu and House (2016) and is subject to the critique by Borovicka and Borovickova (2019).

Also related to ours is the work of Kilic and Wachter (2018). These authors embed a reduced-form version of the mechanism in Hall (2017) within a model with preferences as in Epstein and Zin (1989) with variable disaster risk. Although this model's pricing kernel does not generate a risk-free rate puzzle, it generates volatile unemployment by heavily relying on a form of inefficient real wage stickiness. In contrast, we show that variable disaster risk can generate realistic fluctuations in the job-finding rate under efficient wage setting without rigid wages, provided human capital is incorporated.

To show that our mechanism is fundamentally different from those in the large literature discussed by Ljungqvist and Sargent (2017) that addresses the unemployment volatility puzzle, we revisit this literature but modify the relevant models to be consistent with the critique by Chodorow-Reich and Karabarbounis (2016) as well as with the insight in Shimer (2010). By this insight, if recruiting workers or bargaining takes time away from production, then the cost of doing so is proportional to the opportunity cost of a worker's time in production. Under these assumptions on the opportunity costs of employment, recruiting, and bargaining, we prove that the job-finding rate and unemployment are exactly constant in response to changes in productivity. In contrast, for the same specification of these opportunity costs, our model generates fluctuations in the job-finding rate and unemployment of the same magnitudes as in the data. In short, our model seems to offer a counterexample to the claim that in matching models, "the fundamental surplus is the single intermediate channel through which economic forces generating a high elasticity of market tightness with respect to productivity must operate" (Ljungqvist and Sargent 2017, p. 2663). As such, our model provides a qualitatively different mechanism for the amplification and transmission of productivity shocks.

2. Economy

We embed a Diamond-Mortenson-Pissarides (DMP) model of the labor market with competitive search within a general equilibrium model of an economy in which households are composed of employed and unemployed workers and own firms. The economy is subject to both aggregate shocks, including productivity shocks, and idiosyncratic shocks. We extend the DMP model to include two key features: asset-pricing preferences that generate time-varying risk and human capital accumulation during employment. We consider some of the most popular classes of asset pricing preferences. For concreteness only, we begin with a simple one leading to exogenous time-varying risk.

The economy consists of a continuum of firms and consumers. Each consumer survives from one period to the next with probability ϕ. In each period, a measure $1 - \phi$ of new consumers is born so that there is a constant measure one of consumers in the economy. Individual consumers accumulate human capital. Firms post vacancies to hire consumers with any level of human capital they desire. Each consumer belongs to one of a large number of families that own firms and insure their members against idiosyncratic risks.

We consider five specifications of preferences and processes for shocks that include most of the popular ones in the macro-finance literature. We do so to emphasize that our results apply to a wide array of specifications that generate quantitatively relevant asset-pricing implications primarily from time-varying risk. These include a simple specification of Campbell-Cochrane preferences with exogenous habit, Campbell-Cochrane preferences with external habit, Epstein-Zin preferences with long-run risk, Epstein-Zin preferences with variable disaster risk, and an affine discount factor. The first four of these preference specifications are special cases of recursive preferences of the Epstein-Zin form,

$$(1) \quad V_t = \left[(1-\beta)u(C_t, \bar{C}_t, S_{1t}) + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}},$$

where α is the coefficient of relative risk aversion and ρ is the inverse elasticity of intertemporal substitution. In (1), individual consumption C_t, aggregate consumption $\bar{C}_t$, and a shock S_{1t}, which follows an autoregressive process that captures either an exogenous habit in consumption, an external habit in consumption, or a demand shock depending on the specific model considered, enter the period utility function $u(\cdot)$. The growth rate of the log of aggregate productivity A_t follows

$$(2) \quad \Delta a_{t+1} = g_a + \log(S_{2t}) + \sigma_a \varepsilon_{at+1} - \theta j_{t+1} \text{ with } \quad j_{t+1} \sim Poisson(S_{3t}),$$

where S_{2t} and S_{3t} are governed by autoregressive processes determining long-run risk and disaster risk, respectively. The fifth specification is a reduced-form affine discount factor of the Ang-Piazzesi form. We discuss these specifications in detail later on. Omitted proofs and details are collected in the Appendix.

A. Technologies and Resource Constraints

Consumers are indexed by a state variable that summarizes their ability to produce output. The variable z_t, referred to as *human capital*, captures returns to experience in the labor market. A consumer with state variable z_t produces $A_t z_t$ units of output when employed and $bA_t z_t$ units of output when unemployed in period t. Hence, the opportunity cost of employment is $bA_t z_t$ with an elasticity to aggregate productivity of one, consistent with the findings in Chodorow-Reich and Karabarbounis (2016). Here we follow Hall (2017), who incorporates these findings by assuming that the opportunity cost of employment is proportional to aggregate productivity; see the discussion in Hall (2017, p. 324). We assume that aggregate productivity follows a random walk process with drift g_a given by

$$(3) \quad \log(A_{t+1}) = g_a + \log(A_t) + \sigma_a \varepsilon_{at+1},$$

where here and throughout $\varepsilon_{at+1} \sim N(0, 1)$. Relative to (2), here we drop S_{2t} and S_{3t} and thus abstract from long-run risk and disaster risk. Newly born consumers draw their initial human capital from a distribution $n(z)$ with mean 1 and enter the labor market as unemployed. After that, when a consumer is employed, human capital evolves according to

$$(4) \quad z_{t+1} = (1 + g_e)z_t,$$

and when a consumer is not employed, it evolves according to

$$(5) \quad z_{t+1} = (1 + g_u)z_t,$$

where $g_e \geq 0$ and $g_u \leq 0$ are constant rates of human capital accumulation on the job and depreciation off the job. Posting a vacancy directed at a consumer with human capital z costs a firm $\kappa A_t z$ in lost production in period t. This specification of the cost of posting vacancies is consistent with the argument in Shimer (2010) that to recruit workers, existing workers must reduce their time devoted to production, which costs a firm lost output. Under this view, the cost of recruiting workers moves one-for-one with the productivity of a worker engaged in market production.[2] Note that scaling home production and vacancy posting costs by z is convenient because, as we show later, it implies that all value functions are linear in z. This scaling assumption, though, is not necessary for our results and is purely motivated by analytical tractability and computational convenience.

The realization of the productivity innovation ε_t is the aggregate event. Let $\varepsilon^t = (\varepsilon_0, \ldots, \varepsilon_t)$ be the history of aggregate events at time t. An allocation is a set of stochastic processes for consumption $\{C(\varepsilon^t)\}$ and measures of employed consumers, unemployed consumers, and vacancies posted for each level of human capital z, $\left\{e(z,\varepsilon^t), u(z,\varepsilon^t), v(z,\varepsilon^t)\right\}$. For notational simplicity, from now on we suppress any explicit dependence on ε^t and express these allocations in shorthand notation as $\{C_t, e_t(z), u_t(z), v_t(z)\}$. The measures of employed and unemployed consumers satisfy

$$(6) \qquad \int_z \left[e_t(z) + u_t(z)\right] dz = 1.$$

The timing of events is as follows. At the beginning of period t, current productivity A_t is realized, firms make offers and post vacancies, and unemployed workers from the end of period $t-1$ search for jobs. Then, new matches are formed and employed consumers immediately begin to work. At the end of the period, a fraction σ of employed consumers separate and enter the unemployment pool of period t, and consumption takes place.

To understand the law of motion for the measure of employed and unemployed consumers, consider unemployed consumers searching for a job at the beginning of period t with human capital z, denoted by $u_{bt}(z)$. These consumers were unemployed at the end of period $t-1$, had human capital $z/(1+g_u)$ that grew at rate $1+g_u$ to z between $t-1$ and t, and survived. Therefore,

$$(7) \qquad u_{bt}(z) \equiv \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right).$$

The term $1/(1+g_u)$ that multiplies u_{t-1} in (7) arises from the change of variable in the density over $z/(1+g_u)$ to derive the density over z. At the beginning of period t, firms post a measure of vacancies $v_t(z)$ that targets consumers with human capital z thus creating a measure $m_t(u_{bt}(z), v_t(z))$ of matches, where $m_t(\cdot)$ is a constant returns-to-scale matching function increasing in both arguments. The transition laws for employed and unemployed workers' human capital are then given, respectively, by

$$(8) \qquad e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_{wt}\left(\theta_t(z)\right) u_{bt}(z)$$

[2] Since we maintain that productivity follows a random walk with positive drift, it would not make sense to assume that home production b and the vacancy cost κ are constant, because then the ratios b/A_t and κ/A_t would (in a precise stochastic sense) converge to zero and all agents would always work.

and

$$(9)\quad u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1 - \lambda_{wt}(\theta_t(z))]u_{bt}(z) + (1-\phi)\, n(z),$$

where $\lambda_{wt}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/u_{bt}(z)$ is the *job-finding rate* of an unemployed consumer with human capital z and *market tightness* for consumers with human capital z is $\theta_t(z) = v_t(z)/u_{bt}(z)$.

To understand these expressions, consider (9). Observe first that new entrants into the unemployment pool include the measure $\phi\sigma e_{t-1}(z/(1+g_e))/(1+g_e)$ of consumers with $z/(1+g_e)$ units of human capital in $t-1$ and z units of human capital in t who worked in period $t-1$, separated from their firms at the end of the period (an event with probability σ), and survived (an event with probability ϕ). New entrants into unemployment also include all newborn consumers with human capital z, $(1-\phi)n(z)$. Note that a proportion $1 - \lambda_{wt}(\theta_t(z))$ of unemployed consumers at the beginning of period t remain unemployed.

For later use, it is convenient to define the *job-filling rate* for a firm that posts a vacancy for consumers with human capital z as $\lambda_{ft}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/v_t(z)$. It follows that $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$. We also define the elasticity of the job-filling rate with respect to $\theta_t(z)$ as $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$ so that $1 - \eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(\theta_t(z))/\lambda_{wt}(\theta_t(z))$.[3] Note that when we later assume a Cobb-Douglas matching function, the elasticity $\eta_t(\theta_t(z))$ is a constant. The aggregate resource constraint in period t is

$$(10)\quad C_t \leq A_t \int_z z e_t(z)dz + bA_t \int_z z u_t(z)dz - \kappa A_t \int_z z v_t(z)dz,$$

where the terms on the right side of this constraint are the total output of the employed, the total output of the unemployed, and the total cost of posting vacancies.

B. A Family's Problem

We represent the insurance arrangements in the economy by assuming that each consumer belongs to one of a large number of identical families, each with a continuum of household members, who have access to complete one-period contingent claims against aggregate risk. Risk sharing within a family implies that at date t, each household member consumes the same amount C_t of goods regardless of the idiosyncratic shocks that such a member experiences. (This type of risk-sharing arrangement is familiar from the work of Merz 1995 and Andolfatto 1996.)

Given this setup, we can separate a family's problem into two parts. The first part is at the level of the family and determines the family's choice of assets and the common consumption level of each member. The second part is at the level of individual consumers and firms in the family. The individual consumer problem determines the employment and unemployment status of each consumer in the family whereas the individual firm problem determines the vacancies created and the matches formed by the firms that the family owns.

[3] This is easy to see by substituting $\lambda_{ft}(\theta_t) = \lambda_{wt}(\theta_t)/\theta_t$ and $\theta_t\lambda'_{ft}(\theta_t) = \lambda'_{wt}(\theta_t) - \lambda_{wt}(\theta_t)/\theta_t$ into the expression for $1-\eta_t$.

We begin with the simplest and most transparent of our preference specifications, in which we replace the external habit in Campbell and Cochrane (1999) with an exogenous habit. We do so in order to eliminate the consumption externality generated by their external habit but retain the desirable asset pricing properties of their specification. (See Ljungqvist and Uhlig 2015 for the implications of this externality.) We show below that our specification implies nearly identical results to those implied by their specification. Specifically, with *exogenous habit* a family's utility is given by

$$(11) \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\alpha}}{1-\alpha}.$$

In a symmetric equilibrium, each consumer's consumption C_t equals aggregate consumption $\bar{C}_t$ and we can define the *aggregate surplus consumption ratio* as $S_t = \left(\bar{C}_t - X_t\right)/\bar{C}_t$ so that aggregate marginal utility is $\beta^t(\bar{C}_t - X_t)^{-\alpha} = \beta^t \bar{C}_t^{-\alpha} S_t^{-\alpha}$. As in Campbell and Cochrane (1999), we specify the law of motion for the exogenous habit X_t indirectly by specifying a law of motion for the aggregate surplus consumption ratio S_t. Here we assume S_t is an autoregressive process with $s_t = \log(S_t)$ given by

$$(12) \quad s_{t+1} = (1-\rho_s)\, s + \rho_s s_t + \lambda_a(s_t)(\Delta a_{t+1} - \mathbb{E}_t \Delta a_{t+1}),$$

where $a_t = \log(A_t)$ and s denotes the mean of s_t.[4] The *sensitivity function* $\lambda_a(s_t)$ is defined as

$$(13) \quad \lambda_a(s_t) = \frac{1}{S}\left[1 - 2\,(s_t - s)\right]^{1/2} - 1,$$

if the right side of (13) is nonnegative and zero otherwise. Here, as in Campbell and Cochrane (1999), the function $\lambda_a(s_t)$ is chosen so that in any downturn induced by a technology shock, risk aversion rises sharply but the risk-free rate does not. The *pricing kernel* for the economy is

$$(14) \quad Q_{t,t+1} = \beta \left(\frac{S_{t+1}}{S_t} \frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\alpha}.$$

This kernel determines the intertemporal price of consumption goods and is the discount factor used by individual consumers and firms in the same family. Using similar notation, we let $Q_{t,r} = \beta^{r-t} \left(\frac{S_r}{S_t} \frac{\bar{C}_r}{\bar{C}_t} \right)^{-\alpha}$ denote the discount factor for period $r \geq t+1$ in units of the period-t consumption good.

Since each family is identical, has access to complete one-period contingent claims against aggregate risk, and the prices of contingent claims are related in the usual fashion to the marginal rate of substitution in (14), for notational simplicity we do not explicitly include them in the budget constraint of a family, which can then be written as

$$(15) \quad C_t + I_t = W_t + \Pi_t + H_t,$$

where I_t are the total resources invested in new vacancies, W_t are the total wages of employed consumers

[4] We have specialized (1) by setting $\alpha = \rho$ so that preferences are additively separable over time and by assuming that the utility function does not depend on aggregate consumption $\bar{C}_t$ in that $u(C_t, S_t) = S_t^{-\alpha} C_t^{1-\alpha}$ with $S_t = S_{1t}$. An equivalent specification is obtained when the period utility is $\delta_t C_t^{1-\alpha}$ and $\delta_t = \beta^t S_t^{-\alpha}$ is the time-varying discount factor.

of the family, Π_t is the flow of profits from the firms the family owns, and H_t is the total home production of unemployed consumers of the family. In equilibrium, $I_t = \kappa A_t \int z v_t(z)dz$, $W_t + \Pi_t = A_t \int z e_t(z)dz$, and $H_t = bA_t \int z u_t(z)dz$.

Note for later that the risk-free rate $R_{ft} = 1 + r_{ft}$ in this economy, namely, the return on a claim purchased at t to one unit of consumption in all states at $t+1$, is $R_{ft} = 1/\mathbb{E}_t Q_{t,t+1}$. More generally, the return R_{t+1} on any asset in $t+1$ must satisfy the first-order condition $1 = \mathbb{E}_t Q_{t,t+1} R_{t+1}$. By a standard argument in Hansen and Jagannathan (1991), this fact implies that the (log) *Sharpe ratio* of any asset, defined here as the ratio of the log of the conditional mean excess return on the asset, $\log(\mathbb{E}_t(R_{t+1}/R_{ft}))$, to the conditional standard deviation of the log excess return, $\sigma_t(\log(R_{t+1}/R_{ft}))$, must satisfy

$$(16) \quad \left| \frac{\log(\mathbb{E}_t(R_{t+1}/R_{ft}))}{\sigma_t(\log(R_{t+1}/R_{ft}))} \right| \leq \ \sigma_t(\log(Q_{t,t+1})) = \alpha[1 + \lambda_a(s_t)]\sigma_t(\Delta c_{t+1}),$$

if returns are lognormally distributed.[5] The right side of this Hansen-Jagannathan bound, namely, $\alpha[1 + \lambda_a(s_t)]\sigma_t(\Delta c_{t+1})$, is the highest possible Sharpe ratio in this economy, the *maximum Sharpe ratio*, which is a common measure of the *price of risk*. As Campbell and Cochrane (1999) showed, a critical feature of these type of preferences is that the price of risk varies with the exogenous state s_t so that when the state is low, the price of risk is high, and risky investments are not too attractive. This feature of the price of risk will prove critical to generating volatility in the job-finding rate and so in unemployment in our model.

C. Comparison with Original Campbell-Cochrane Preferences

The preferences with exogenous habit are very similar to those in Campbell and Cochrane (1999). The differences are that the exogenous habit X_t in the utility function (11) is replaced by the *external habit* $\bar{X}_t$ in Campbell and Cochrane (1999), with a law of motion indirectly determined by the process for the corresponding aggregate surplus consumption ratio $\bar{S}_t = \left(\bar{C}_t - \bar{X}_t\right)/\bar{C}_t$,

$$(17) \quad \bar{s}_{t+1} = (1 - \rho_s)\,\bar{s} + \rho_s \bar{s}_t + \lambda(\bar{s}_t)(\Delta \bar{c}_{t+1} - \mathbb{E}_t \Delta \bar{c}_{t+1}),$$

where $\bar{s}_t = \log(\bar{S}_t)$ and the corresponding sensitivity function is $\lambda(\bar{s}_t) = \frac{1}{\bar{S}}\left[1 - 2\left(\bar{s}_t - \bar{s}\right)\right]^{1/2} - 1$ as long as $\lambda(\bar{s}_t)$ is nonnegative and zero otherwise. Note that the law of motion for surplus consumption (12) in the exogenous habit specification is driven by innovations in the growth rate of productivity, Δa_t, whereas the corresponding law of motion in Campbell and Cochrane (1999) is driven by innovations in the growth rate of aggregate consumption, $\Delta \bar{c}_t$. In the economy in Campbell and Cochrane (1999), consumption is

[5] Alternatively, the same first-order condition implies that the *level* of the Sharpe ratio for any asset return satisfies

$$E_t(R^e_{t+1})/\sigma_t(R^e_{t+1}) = -Corr_t(Q_{t,t+1}, R^e_{t+1})\sigma_t(Q_{t,t+1})/E_t(Q_{t,t+1}) \leq \sigma_t(Q_{t,t+1})/E_t(Q_{t,t+1}),$$

where $R^e_{t+1} = R_{t+1} - R_{ft+1}$. Since $Q_{t,t+1}$ is conditionally lognormal, the maximal Sharpe ratio in levels is

$$\max_{\{all\ assets\}} [E_t(R^e_{t+1})/\sigma_t(R^e_{t+1})] = \{e^{\alpha^2[1+\lambda_a(s_t)]^2\sigma_t^2(\Delta c_{t+1})} - 1\}^{1/2} \cong \alpha[1 + \lambda_a(s_t)]\sigma_t(\Delta c_{t+1}).$$

Our definition of the (log) Sharpe ratio implies $\alpha[1+\lambda_a(s_t)]\sigma_t(\Delta c_{t+1})$ is an exact, rather than an approximate, upper bound.

exogenous so that $\Delta \bar{c}_t = \Delta a_t$ and these two specifications are identical. In our production economy, in contrast, consumption is not identical to productivity. As we show later, though, these two specifications lead to nearly identical quantitative results. In this precise sense, one can think of our baseline model as having *either* Campbell-Cochrane preferences with exogenous habit *or* Campbell-Cochrane preferences with external habit.

D. Competitive Search Equilibrium

We set up a competitive search equilibrium in the spirit of the market utility approach in Montgomery (1991). (See also Moen (1997) and, for an extensive review of the literature, Wright et al. (forthcoming).) Let $\mathbb{Z}_t$ be the set of human capital levels among the unemployed in period t. Since we assume free entry, we can think of there being a large number of firms that target workers with any given level of human capital $z \in \mathbb{Z}_t$.[6] In each period t, there are two stages. In stage 1, any firm that targets workers with human capital z commits to a *wage offer* of $W_t(z)$ for the present value of payments to any worker of type z it hires and posts vacancies for such workers. In stage 2, after having observed all offers, workers of type z choose which market to search in. A market is defined by $(z, W_t(z))$, namely, a skill level and a wage offer for that skill level.[7] These two stages should be thought of as occurring at the beginning of each period t right after aggregate productivity is realized.[8] Then, matches are formed, output is produced, and, at the end of the period, consumption takes place.

We now turn to set up and characterize a symmetric equilibrium starting from stage 2.

Stage 2: Consumers Choose Market to Search

We start by considering symmetric histories in which all firms have made the same offers in stage 1 of period t, so that there is only one wage offer $W_t(z)$ for each level of human capital z. We refer to the present value of all payments to a worker with human capital z from future home production or future employment after a match formed at t dissolves at any future date as the *post-match value* at t and denote it by $P_t(z)$, which is given recursively by

(18) $$P_t(z) = \sigma \mathbb{E}_t Q_{t,t+1} U_{t+1}(z') + (1-\sigma)\mathbb{E}_t Q_{t,t+1} P_{t+1}(z')$$

with $z' = (1 + g_e)z$. Of course, the total value of a new match to a worker is $W_t(z) + P_t(z)$, since the current match pays $W_t(z)$ over its course and the worker's post-match value is $P_t(z)$. (We decompose the total value of a match to a worker into these two pieces to keep clear what a firm chooses and what a firm

[6] If a firm targeted workers with human capital z in some period and with human capital z' in some other period, we would simply count that firm as two firms: one that targets z and one that targets z'.

[7] Rather than envisioning one large market with many firms that make the same wage offer, we find it useful to think of every firm as potentially creating its own market through its wage offer and of workers as freely flowing between these markets until the value of search $\mathcal{W}_t(z)$, defined below, is equated across them. Given a set of wage offers for all markets, the associated levels of market tightness are determined by the equality of the value of search across markets. As a convention, we interpret two or more markets with identical human capital and offers as belonging to the same market.

[8] In a monthly model like ours, one might think of these stages as all occurring early on the morning of the first day of the month. Then, on the same day consumers and firms match and produce that day and for the rest of the month.

takes as given.) The value of unemployment $U_t(z)$ is given by

(19) $$U_t(z) = bA_t z + \mathbb{E}_t Q_{t,t+1}\{\lambda_{wt+1}(\theta_{t+1}(z'))[W_{t+1}(z') + P_{t+1}(z')] + [1 - \lambda_{wt+1}(\theta_{t+1}(z'))]U_{t+1}(z')\}$$

with $z' = (1 + g_u)z$. The *value of search* for a worker with human capital z in market $(z, W_t(z))$ is

(20) $$\mathcal{W}_t(z) = \lambda_{wt}(\theta_t(z))[W_t(z) + P_t(z)] + [1 - \lambda_{wt}(\theta_t(z))]U_t(z).$$

Since a firm needs to anticipate workers' behavior in stage 2 when it contemplates an arbitrary wage offer in stage 1, we also need to determine outcomes in stage 2 for any such offer. Given that we focus on a symmetric equilibrium, we need only consider asymmetric histories at the beginning of stage 2 in which all firms but one have offered $W_t(z)$ and one has offered, say, $\tilde{W}_t(z)$.

Consider then markets $(z, W_t(z))$ and $(z, \tilde{W}_t(z))$. The tightness $\theta_t(z)$ of market $(z, W_t(z))$ satisfies the free-entry condition defined below in (24). The tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_t(z))$ is determined as follows. As long as the wage offer $\tilde{W}_t(z)$ is sufficiently attractive, workers flow between markets $(z, W_t(z))$ and $(z, \tilde{W}_t(z))$ until the value of search in the two markets is equated. Hence, $\tilde{\theta}_t(z)$ is determined by the *worker participation constraint* $\widetilde{\mathcal{W}}_t(z) = \mathcal{W}_t(z)$, which can be written as

(21) $$\lambda_{wt}(\tilde{\theta}_t(z))[\tilde{W}_t(z) + P_t(z)] + [1 - \lambda_{wt}(\tilde{\theta}_t(z))]U_t(z) = \lambda_{wt}(\theta_t(z))[W_t(z) + P_t(z)] + [1 - \lambda_{wt}(\theta_t(z))]U_t(z)$$

with $\widetilde{\mathcal{W}}_t(z)$ defined by the left side of this equality. By the one-shot deviation principle, we have maintained that after period t, regardless of whether a worker accepts the offer $W_t(z)$ in market $(z, W_t(z))$ or the offer $\tilde{W}_t(z)$ in market $(z, \tilde{W}_t(z))$, the worker takes as given the same set of present values $\{U_r(z)\}_{r=t}^{\infty}$ and so $\{P_r(z)\}_{r=t}^{\infty}$ to be received in any period r from a combination of future home production and future employment. Note for later that if a firm makes the symmetric wage offer $\tilde{W}_t(z) = W_t(z)$, then by the participation constraint (21), the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_t(z))$ is the symmetric one $\theta_t(z)$.

If the wage offer $\tilde{W}_t(z)$ is sufficiently unattractive, then the value of search in market $(z, \tilde{W}_t(z))$ is strictly lower than that of search in market $(z, W_t(z))$, even if a worker with human capital z finds a job in market $(z, \tilde{W}_t(z))$ with probability one. This situation occurs when the wage offer $\tilde{W}_t(z)$ is so low that $\mathcal{W}_t(z) > \tilde{W}_t(z) + P_t(z)$. As a result, no workers search in such a market and $\lambda_{ft}(\tilde{\theta}_t(z))$ is zero—clearly, it is pointless for a firm to make an offer that attracts no workers. Then, we can think of workers' optimal search strategies as specifying the behavior that firms in stage 1 anticipate will determine the tightness $\tilde{\theta}_t(z)$ of market $(z, \tilde{W}_t(z))$ from any offer $\tilde{W}_t(z)$ such that $\mathcal{W}_t(z) \leq \tilde{W}_t(z) + P_t(z)$. Finally, at the end of stage 2 of period t, each family consumes C_t.

Stage 1: Firms Choose Contingent Wage Offers and Post Vacancies

Consider the problem of any given firm targeting a worker with human capital z in stage 1 of period t with state ε^t and current productivity $A_t = A(\varepsilon^t)$. To set up this problem, given that we focus on a symmetric equilibrium, we allow a firm to choose any possible wage offer $\tilde{W}_t(z)$ when all other firms that target workers with human capital z make the *symmetric wage offer* $W_t(z)$.

Consider market $(z, W_t(z))$. Any firm targeting a worker of type $z \in \mathbb{Z}_t$ incurs the cost $\kappa A_t z$ to post a vacancy. Denote by $Y_t(z)$ the present value of output produced by a match between a firm and a worker of type z and let $z' = (1+g_e)z$. Since a match dissolves with exogenous probability σ, the present value $Y_t(z)$ can be expressed recursively as

$$(22)\quad Y_t(z) = A_t z + (1-\sigma)\mathbb{E}_t Q_{t,t+1} Y_{t+1}(z').$$

Given a wage offer $W_t(z)$ for workers of type z, the value of a vacancy aimed at such workers is

$$(23)\quad V_t(z) = -\kappa A_t z + \lambda_{ft}(\theta_t(z))[Y_t(z) - W_t(z)] + [1 - \lambda_{ft}(\theta_t(z))]\mathbb{E}_t Q_{t,t+1} V_{t+1}(z).$$

Note that the last term in (23) captures the idea that if a firm is unsuccessful in hiring a worker in market z in period t, then the firm can search again in period $t+1$. Free entry into market $(z, W_t(z))$ implies that $V_t(z) = 0$ for any t and z so that

$$(24)\quad \kappa A_t z = \lambda_{ft}(\theta_t(z))[Y_t(z) - W_t(z)].$$

Consider now the problem of a firm choosing an offer $\tilde{W}_t(z)$ possibly different from $W_t(z)$. We use the specification of workers' behavior in stage 2 to derive the tightness $\tilde{\theta}_t(z)$ associated with market $(z, \tilde{W}_t(z))$ and restrict attention to *serious offers*, namely, offers that satisfy

$$(25)\quad \mathcal{W}_t(z) \leq \tilde{W}_t(z) + P_t(z)$$

and hence lead to a positive job-filling rate, as discussed earlier. When a firm makes a (serious) offer of $\tilde{W}_t(z)$, the value of a vacancy is

$$(26)\quad \tilde{V}_t(z) = -\kappa A_t z + \lambda_{ft}(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_t(z)] + [1 - \lambda_{ft}(\tilde{\theta}_t(z))]\mathbb{E}_t Q_{t,t+1}\widetilde{V}_{t+1}((1+g_e)z),$$

where $\lambda_{ft}(\tilde{\theta}_t(z))$, determined by the worker participation constraint (21) from $\lambda_{wt}(\tilde{\theta}_t(z))$, is the job-filling rate in market $(z, \tilde{W}_t(z))$. The problem of a firm that posts a vacancy for a worker of type z is then

$$(27)\quad \max_{\{\tilde{W}_t(z), \tilde{\theta}_t(z)\}} \tilde{V}_t(z),$$

subject to the participation constraint (21) and the serious offer constraint (25). Taking the first-order conditions for this problem and using the free-entry condition for period $t+1$, namely, $V_{t+1}(z) = 0$, gives

$$(28)\quad \frac{\lambda'_{ft}(\tilde{\theta}_t(z))}{\lambda_{ft}(\tilde{\theta}_t(z))}[Y_t(z) - \tilde{W}_t(z)] = -\frac{\lambda'_{wt}(\tilde{\theta}_t(z))}{\lambda_{wt}(\tilde{\theta}_t(z))}[\tilde{W}_t(z) + P_t(z) - U_t(z)].$$

In a symmetric equilibrium, this condition becomes

$$(29)\quad \frac{\lambda'_{ft}(\theta_t(z))}{\lambda_{ft}(\theta_t(z))}[Y_t(z) - W_t(z)] = -\frac{\lambda'_{wt}(\theta_t(z))}{\lambda_{wt}(\theta_t(z))}[W_t(z) + P_t(z) - U_t(z)]$$

for all firms. Note that this first-order condition, which determines $\theta_t(z)$ given the values $Y_t(z)$, $W_t(z)$,

$P_t(z)$, and $U_t(z)$, is the key condition that guarantees efficiency of a competitive search equilibrium. A simple way to see this result is to note that if we multiply both sides of (29) by $\theta_t(z)$, and use $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$ and $1-\eta_t(\theta_t(z)) = \theta_t(z)\lambda'_{wt}(\theta_t(z))/\lambda_{wt}(\theta_t(z))$, then this condition is equivalent to the Hosios condition for Nash bargaining, which in turn implies efficiency.

E. Equilibrium: Definition and Characterization

A collection of state-contingent sequences $\{C_t, Q_{t,t+1}, S_t\}_{t=0}^{\infty}$ and state- and z-contingent sequences $\{W_t(z), P_t(z), U_t(z), \mathcal{W}_t(z), Y_t(z), V_t(z), \theta_t(z), e_t(z), u_t(z), v_t(z)\}_{t=0}^{\infty}$ is a *competitive search equilibrium* if: *i)* for each t, taking as given $P_t(z)$, $U_t(z)$, $\mathcal{W}_t(z)$, $Y_t(z)$, $V_t(z)$, and $Q_{t,t+1}$, the wage offer $W_t(z)$ and market tightness $\theta_t(z)$ solve the firm's problem (27), *ii)* the collection of state-contingent sequences $\{P_t(z), U_t(z), \mathcal{W}_t(z), Y_t(z), V_t(z)\}_{t=0}^{\infty}$ satisfy the valuation equations (18), (19), (20), (22), and (23), *iii)* the law of motions for employment and unemployment satisfy (8) and (9), *iv)* the free-entry condition (24) holds, *v)* the resource constraint (10) holds, and *vi)* the pricing kernel $\{Q_{t,t+1}\}$ satisfies (14).

Notice that our first four preference structures satisfy this definition and only differ in the form of the intertemporal marginal rate of substitution that defines the stochastic discount factor of the family in (14). In contrast, the reduced-form affine discount factor simply posits a discount factor that is not derived from marginal utility. For that specification, we define a *competitive search equilibrium given* $\{Q_{t,t+1}\}$ and the same definition applies, but we simply drop condition *vi*).

We turn now to characterizing equilibrium. We first show that since market production, home production, and the cost of posting vacancies all scale with z, all equilibrium value functions are linear in z and market tightness, job-finding rates, and job-filling rates are independent of z. In establishing this result, we let Y_t denote $Y_t(1)$ and use similar notation for the remaining values.

Lemma 1 (Linearity of Competitive Search Equilibrium)**.** *In a competitive search equilibrium, labor market tightness $\theta_t(z)$, the job-finding rate $\lambda_{wt}(\theta_t(z))$, and the job-filling rate $\lambda_{ft}(\theta_t(z))$ are independent of z, and values are linear in z in that $Y_t(z) = Y_t z$, $U_t(z) = U_t z$, $P_t(z) = P_t z$, and $W_t(z) = W_t z$.*

This result immediately implies that to solve for equilibrium values, we do not need to record the measures $e_t(z)$ and $u_t(z)$ but, rather, only the aggregate human capital of employed and unemployed workers given by $Z_{et} = \int z e_t(z) \mathrm{d}z$ and $Z_{ut} = \int z u_t(z) \mathrm{d}z$. Integrating (8) and (9) gives the transitions laws for the aggregate human capital of employed and unemployed workers,

(30) $Z_{et} = \phi(1-\sigma)(1+g_e) Z_{et-1} + \phi\lambda_{wt}(1+g_u) Z_{ut-1},$

(31) $Z_{ut} = \phi\sigma(1+g_e) Z_{et-1} + \phi(1-\lambda_{wt})(1+g_u) Z_{ut-1} + 1 - \phi,$

which can be used to express the aggregate resource constraint as

(32) $C_t \leq A_t Z_{et} + b A_t Z_{ut} - \kappa A_t \phi \theta_t (1+g_u) Z_{ut-1},$

where we have used that aggregate vacancy costs satisfy $Z_{vt} = \int z v_t(z) \mathrm{d}z = \phi\theta_t(1+g_u)Z_{ut-1}$. In light of Lemma 1, we denote the job-finding rate and the job-filling rate by λ_{wt} and λ_{ft}, respectively.

The next proposition establishes that for our five specifications of preferences, namely, Campbell-Cochrane preferences with exogenous habit, Campbell-Cochrane preferences with external habit, Epstein-Zin preferences with long-run risk, Epstein-Zin preferences with variable disaster risk, and reduced-form preferences summarized by an affine discount factor, the allocations are constrained efficient in that they solve the following *restricted planning problem*, namely, given the process for the date-zero discount factors $\{Q_{0,t}\}$ from the competitive search equilibrium and the initial conditions for aggregate human capital Z_{e-1} and Z_{u-1}, the allocations $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ maximize $E_0 \sum_{t=0}^{\infty} Q_{0,t} C_t$ subject to (30)-(32).

Proposition 1. *For all five specifications of preferences, competitive search equilibrium allocations solve the restricted planning problem.*

The idea behind this result is that since the competitive search wage setting mechanism leads to an efficient labor market equilibrium given the pricing kernel, the equilibrium is efficient conditional on the consumption process. There are several features to notice about this characterization. First, it holds even for Campbell-Cochrane preferences, which lead to consumption externalities. Thus, this result shows the precise sense in which the search-side of the model implies a type of efficient wage setting and so constrained efficiency. Second, for the affine discount factor specification of preferences, the precise statement of this result is that allocations in a competitive search equilibrium given $\{Q_{t,t+1}\}$ solve this problem, where $\{Q_{0,t}\}$ in the restricted planning problem are implied by the equilibrium $\{Q_{t,t+1}\}$ as $Q_{0,t} = Q_{0,1} \ldots Q_{t-1,t}$. Third, it is easy to show that for Campbell-Cochrane preferences with exogenous habit, Epstein-Zin preferences with long-run risk, and Epstein-Zin preferences with variable disaster risk, equilibrium allocations not only solve this restricted planning problem, but also solve a standard planning problem of maximizing utility—rather than the present value of consumption given a discount factor—subject to the same constraints and, hence, are efficient. Proposition 1 will prove helpful to shed light on the mechanism underlying our results. Specifically, it will allow us to isolate the contributions of movements in the discount factor and human capital in generating fluctuations in the job-finding rate and unemployment.

F. Characterizing the Job-Finding Rate

Consider now the first-order conditions for the restricted planning problem given by

$$(33) \quad \mu_{et} = A_t + \phi(1+g_e)\mathbb{E}_t Q_{t,t+1}\left[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}\right],$$

$$(34) \quad \mu_{ut} = bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\left[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + \left(1-\eta_{t+1}\lambda_{wt+1}\right)\mu_{ut+1}\right],$$

$$(35) \quad \kappa A_t = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}),$$

where μ_{et} and μ_{ut} are the multipliers associated with the transition laws for the aggregate human capital of employed and unemployed workers, (30) and (31), and so describe the shadow values of increasing the stocks of human capital of employed and unemployed workers by one unit. Conditions (33) to (35) are similar to those that arise in random search models. In particular, equation (33) is analogous to the sum of the value of an employed worker and the value of an employing firm, (34) is analogous to sum of the value of an unemployed worker and the value of an unmatched firm, and (35) is analogous the free-entry condition in those models. The key difference is that in our competitive search equilibrium, the planner takes into account the impact of vacancy creation on job-finding and job-filling rates and, hence, internalizes the search externality generated by firms posting vacancies. We can rewrite (35) as

$$(36)\quad \log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right)\log\left(\frac{\mu_{et}-\mu_{ut}}{A_t}\right),$$

using that the job-filling rate λ_{ft} and the job-finding rate λ_{wt} are determined by the Cobb-Douglas matching function $m(u,v) = Bu^{\eta}v^{1-\eta}$ we use in our quantitative analysis, which implies that $\lambda_{ft}^{1-\eta} = B\lambda_{wt}^{-\eta}$. Expression (36) makes it clear that the job-finding rate λ_{wt} is completely determined, up to constants, by the value $\mu_{et}-\mu_{ut}$ of hiring a worker relative to productivity, A_t. In turn, given $\{Q_{t,t+1}\}$, the values μ_{et} and μ_{ut} are solutions to the dynamical system determined by (33) and (34).

To develop intuition for the solution to this system, we consider an approximation to it in which we ignore the variation in future job-finding rates ($\lambda_{ws} = \lambda_w$ for $s > t$) and, after imposing the appropriate limiting condition, solve the dynamical system forward to obtain

$$(37)\quad \begin{bmatrix}\mu_{et}\\ \mu_{ut}\end{bmatrix} = \sum_{n=0}^{\infty}\phi^n \begin{bmatrix}(1+g_e)(1-\sigma) & (1+g_e)\sigma \\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w)\end{bmatrix}^n \begin{bmatrix}1\\ b\end{bmatrix}\mathbb{E}_t Q_{t,t+n}A_{t+n}.$$

It is apparent from (37) that the value $\mu_{et}-\mu_{ut}$ of hiring a worker on the right side of (36) depends on the present value of aggregate productivity. This value can be expressed as the present value of the surplus flows from a match between a worker and a firm, namely,

$$(38)\quad \mu_{et}-\mu_{ut} = \sum_{n=0}^{\infty}\mathbb{E}_t Q_{t,t+n}v_{t+n},$$

where $v_{t+n} = (c_\ell\delta_\ell^n + c_s\delta_s^n)A_{t+n}$ is the *surplus flow* in period $t+n$ from a match formed in period t and δ_ℓ and δ_s are the large and small roots of the vector difference equation given by (33) and (34) with corresponding weights c_ℓ and c_s derived below. Observe that the surplus flow v_{t+n} in period $t+n$ is proportional to productivity in that period. The present value of these flows in (38) decays with the length of time since a match is formed, because an employed worker can lose a job and an unemployed worker can find one. Critically, as we elaborate below, the present value of these flows decays more slowly the larger is the growth of human capital when a consumer is employed and the larger is the decay of human capital when a consumer is unemployed. The persistence that the presence of human capital imparts to surplus flows will imply that these flows have long durations. This feature will prove critical in amplifying the impact of any aggregate shock on the labor market.

3. Quantification

We next describe how we choose parameters for our quantitative analysis and discuss the model's steady-state implications. The model is monthly and its parameters are listed in Table 1: six parameters, $\{b, \sigma, \eta, \phi, g_e, \rho_s\}$, are assigned and the remaining seven, $\{g_a, \sigma_a, B, \kappa, \beta, S, \alpha\}$, are chosen to match seven moments from the data. Following Ljungqvist and Sargent (2017), we set the home production parameter b to 0.6 and the matching function elasticity η to 0.5. We choose the separation rate σ to match the Abowd-Zellner corrected estimate of a separation rate of 2.8% by Krusell et al. (2017) based on CPS data.[9] We set the survival probability ϕ to be consistent with an average working life of 30 years and the growth rate of human capital during employment, g_e, to 3.5% per year. Note that taking into account an aggregate productivity growth of 2.2% per year, this rate matches the average annual growth rate of real hourly wages documented by Rubinstein and Weiss (2006, Table 2b) based on the 1979-2000 waves of the National Longitudinal Survey of Youth (NLSY) for workers with up to 25 years of labor market experience. To make clear that our results do not rely on the depreciation of human capital during unemployment, we set g_u to zero in our baseline. We later explore the sensitivity of our findings to lower rates of human capital accumulation and higher rates of human capital depreciation. As we will discuss, our results hold for a wide range of values for g_e and g_u. In particular, there exists a locus of pairs (g_e, g_u) with identical predictions for the job-finding rate. We discuss this point further below.

To pin down the persistence ρ_s of the discount factor shock, we follow Mehra and Prescott (1985), Campbell and Cochrane (1999), and Wachter (2006) and interpret dividends as claims to aggregate consumption in the model and as claims to aggregate dividends from CRSP in the data. Based on this strategy, we choose ρ_s to match the observed autocorrelation of price-dividend ratios. We note for later that when we do so, the standard deviation of the price-consumption ratio is 82% of the standard deviation of the price-dividend ratio in the CRSP data.

We turn now to the endogenously chosen parameters. We choose the parameters g_a and σ_a of the exogenous productivity process to match the mean and standard deviation of labor productivity growth from the Bureau of Labor Statistics (BLS) for the period between January 1947 and December 2007.[10] To pin down the match efficiency parameter B and the vacancy posting cost κ, we normalize the mean value of market tightness θ to 1, as in Shimer (2005), and then choose B and κ to reproduce two moments of the data: a mean job-finding rate of 46% as in Shimer (2012) and a mean unemployment rate of 5.9% from

[9] This statistic is lower than the 3.4% monthly separation rate used by Shimer (2005) due to our correction for potential misclassification. We also experimented with a recalibration in which we used the higher separation rate in Shimer (2005) and found very similar results. As it will become evident, employment responses in our model are mainly determined by the duration of surplus flows from a match rather than by the length of time a worker spends in any given match.

[10] We use the variable "Nonfarm Business Sector: Real Output Per Hour of All Persons." Note that we use data starting from 1947 to guarantee that the time series for productivity *growth* conforms to our time series for unemployment, which extends from 1948 to 2007 as in Shimer (2012).

BLS data between January 1948 and December 2007.

Consider next the preference parameters, $\{\beta, S, \alpha\}$. We choose the rate of time preference β and the mean S of the state S_t to match the mean and the standard deviation of the real risk-free rate r_{ft} constructed as $i_t - \mathbb{E}_t\pi_{t+1}$, where i_t is the one-month Treasury bill rate.[11] To see how the mean S of the process governing the state S_t can be chosen to generate only a modest volatility in the risk-free rate, note that when consumption is conditionally lognormally distributed, the real risk-free rate satisfies

$$(39)\quad r_{ft} \cong -\log(\beta) + \alpha E_t \Delta c_{t+1} + \alpha E_t \Delta s_{t+1} - \frac{\alpha^2[1+\lambda_a(s_t)]^2}{2}\sigma_t^2\left(\varepsilon_{ct+1}\right)$$

since $\Delta c_{t+1} \approx \Delta a_{t+1}$, where $\sigma_t(\varepsilon_{ct+1})$ is the conditional standard deviation of the innovation to consumption growth. Thus, the impact of $\sigma_t(\varepsilon_{ct+1})$ on r_{ft} is affected by the level of S through $\lambda_a(s_t)$ by (13).

Finally, as explained, by interpreting dividends as claims to aggregate consumption in the model and to aggregate dividends from CRSP in the data, we choose the inverse elasticity of intertemporal substitution α in the model to match the (mean) maximum Sharpe ratio of the aggregate stock market return measured from the CRSP value-weighted stock index, which covers all firms continuously listed on NYSE, AMEX, and NASDAQ.[12] This strategy is similar to that used by Campbell and Cochrane (1999) and Wachter (2006) in a very related context.

4. Findings: Job-Finding Rate and Unemployment

Shimer (2012) has argued that fluctuations in the job-finding rate account for over two-thirds of the observed fluctuations in unemployment and that a key issue confronting existing search models is that they generate much too little variation in the job-finding rate. Our study is focused solely on a mechanism that increases the volatility of the job-finding rate. For this reason, we purposely abstract from fluctuations in the job-separation rate. Thus, the most obvious statistics to compare in the model and in the data are those on the job-finding rate. We turn to do so next. (We solve all versions of our model by a global numerical strategy described in the Appendix.)

As Table 1 shows, our model produces a volatility of the job-finding rate (6.60) very similar to that in the data (6.66). The autocorrelation of the job-finding rate in the model (0.98) is also close to that in the data (0.94). Note, though, that even if our model exactly matched the observed time series for the job-finding rate, it would not be able to match the observed time series for the unemployment rate, because the separation rate in the data varies whereas it is constant in our model. To address this issue, we follow Shimer (2012) and construct a *constant-separation unemployment rate* series $\{\bar{u}_t\}$ from data from the BLS between 1948 and 2007 with law of motion $\bar{u}_{t+1} = \sigma(1-\bar{u}_t) + (1-\lambda_{wt})\bar{u}_t$ where σ is set as in our baseline

[11] To compute $\mathbb{E}_t\pi_{t+1}$, we use an updated version of the Fama and French's (1993) data available from Kenneth French's website and proxy $\mathbb{E}_t\pi_{t+1}$ by the projection of monthly CPI inflation on twelve of its lags. Fitting a univariate AR(1) specification for $\mathbb{E}_t\pi_{t+1}$ with lags up to one year is standard. See, for instance, Hur, Kondo, and Perri (2019).

[12] Note that we would have obtained similar results by using data from the Flow of Funds, since, as shown by Larrain and Yogo (2008), the returns measured from CRSP are highly correlated with the returns on the aggregate stock market measured from the Flow of Funds. In our sample, this correlation is of 0.97.

(2.8%), which implies an average unemployment rate of 5.9%; see Shimer (2012) for details. For brevity, both in Table 1 and hereafter, we refer to this series as simply the *unemployment rate*. Table 1 shows that our model successfully matches the volatility of this constant-separation unemployment rate in the data (0.75) and implies a serial correlation for it (0.99) that is very similar to that in data (0.97).

Finally, as Table 1 shows, our model also reproduces well the highly negative correlation between job-finding and unemployment rates: −0.98 in the model and −0.96 in the data. This result is consistent with Shimer's (2005) emphasis that unemployment rises in recessions because the job-finding rate falls due to a decline in vacancy creation.

Based on all of these statistics, we conclude that our model solves the unemployment volatility puzzle.

5. Two Critical Ingredients: Time-Varying Risk and Human Capital

Here we demonstrate the critical roles played by preferences associated with time-varying risk and by human capital accumulation for our results. Specifically, without either time-varying risk or human capital accumulation, the model does not generate volatile job-finding or unemployment rates. As for preferences, we show that with standard constant relative risk aversion (CRRA) preferences, the model implies no volatility in the job-finding rate and unemployment. As for human capital accumulation, we show that even if we allow for both human capital accumulation on the job and depreciation off the job, the accumulation of new human capital on the job is the main quantitative force. See Tables 2 for a summary of the parametrization of all models considered and their implications.

A. Role of Time-Varying Risk

To highlight the importance of time-varying risk, we contrast our results to those from a model with CRRA preferences in which we keep all parameters the same except that we set $S_t = 1$, where utility is

$$(40) \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\alpha}}{1-\alpha} \right).$$

Table 2 shows that the resulting fluctuations in both the job-finding rate and unemployment are identically zero. We can prove this result analytically.

Proposition 2. *Starting from the steady-state values of the total human capital of employed and unemployed workers, Z_e and Z_u, with preferences of the form in (40), both the job-finding rate and unemployment are constant.*

In interpreting this result, it is important to note that, by construction, we have abstracted from the standard mechanism of *differential productivity across sectors* of search models, which implies that an increase in aggregate productivity A_t raises a consumer's productivity in market production but leaves a consumer's productivity in home production and the cost of posting vacancies unaffected. In our model, instead, an increase in A_t increases equally a worker's productivity in market and home production as well as the cost of posting vacancies. In particular, a consumer with human capital z produces $A_t z$ when employed and bA_tz when unemployed, and it costs a firm $\kappa A_t z$ to post a vacancy for such a consumer. Therefore, the only effect of a change in aggregate productivity in our model is that it changes the expected discounted value of the surplus from a match scaled by current productivity, as the right side of (36) shows. We elaborate on this point in Section 12.

B. Role of Human Capital

Consider next the role of human capital in generating the fluctuations in the job-finding rate and in unemployment that we have discussed. Here we present some quantitative examples. In the next section, we develop further intuition for our findings by characterizing the elasticity of the job-finding rate with respect to the exogenous state s_t, by showing analytically how this elasticity is affected by human capital acquisition, and by illustrating the amplification effect of human capital by way of simple examples.

In Table 2, we compare our baseline model to one in which we set $g_e = g_u = 0$. We refer to this latter model as the *DMP with baseline preference*. In this latter model, as well as in the other variations that we will consider in later sections, we maintain the same parametrization as in the baseline model with the exception of the hiring cost parameter κ, which is chosen to ensure that the model exactly reproduces the mean unemployment rate in panel B of Table 1. As Table 2 shows, the volatility of job-finding rate in this model drops to about 2% of that in the data (0.15/6.66). Thus, absent human capital, the unemployment rate barely moves. In the last column of Table 2, we consider the *baseline model with* $g_e = g_u = 3.5$ so that human capital grows at the same rate regardless of whether a consumer is employed or unemployed. We see that in this case too the volatility of the job-finding rate is quite low, about 2% of that in the data (0.15/6.66). This finding makes it clear that it is not the presence of human capital in and of itself that is important for our result, but, rather, the differential growth of human capital on the job and off the job, which makes hiring a worker an investment with long duration payoffs.

The two panels of Figure 1 plot the impulse responses of the job-finding rate and the unemployment rate to a one-percent decrease in productivity, starting from the ergodic mean of the state variables S_t, Z_{et}, and Z_{ut}, for two versions of our model: the baseline model and the DMP with baseline preferences.[13] Clearly, the responses of both the job-finding rate and the unemployment rate are much larger in the presence of human capital than in the absence of it.

So far we have considered an extreme scenario in which all of the duration in surplus flows is due to the growth of human capital on the job, captured by g_e, by setting the depreciation of human capital off the job, captured by g_u, to zero. A variety of studies, though, have documented that the wage losses following a spell of unemployment can be substantial. In light of this evidence, we now argue that we can lower the degree of human capital accumulation on the job and, as consistent with the evidence on wage

[13] Note that since the model is nonlinear, the response to a shock depends on the levels of the state variables and the size of the shock. As is standard, we compute the impulse response for, say, the job-finding rate at $t+n$ as $E_t(\lambda_{wt+n}|\varepsilon_t = \Delta, S_t, Z_{ut}, Z_{et}) - E_t(\lambda_{wt+n}|\varepsilon_t = 0, S_t, Z_{ut}, Z_{et})$ with S_t, Z_{ut}, and Z_{et} all set to their ergodic means.

losses after unemployment, correspondingly increase the degree of human capital depreciation off the job and obtain nearly identical results as under our baseline parametrization.

For example, a conservative estimate of the degree of human capital depreciation off the job is $g_u = -5.7\%$, which matches the average wage loss of workers with fewer than 35 years of labor market experience after up to one year of nonemployment in the PSID.[14] If we set $g_u = -5.7\%$ per year, then we need only a value of g_e of 2.11% per year to generate the same standard deviations for the job-finding rate and unemployment as under our baseline parametrization—in doing so, we adjust κ to keep mean unemployment at its value in the baseline. To make this point more generally, in Figure 2 we graph the locus of values for (g_e, g_u) that give rise to the same standard deviations for the job-finding rate and unemployment as under our baseline parametrization. We trace this locus by varying g_e and g_u while keeping all other parameters fixed at their baseline values except for κ, which we adjust to keep the mean unemployment rate unchanged. Clearly, our amplification results hold for very modest rates of human capital accumulation on the job and depreciation off the job relative to standard estimates in the literature.

6. Inspecting the Mechanism

Here we inspect the details of our mechanism by deriving a closed-form solution for the job-finding rate and its dependence on the relevant exogenous state based on the simple approximation for the multipliers μ_{et} and μ_{ut} in (37). We also identify a sufficient statistic for the volatility of the job-finding rate that will turn out to be common across all preference structures we will consider.

To this purpose, recall that the surplus flow in the n-th period after a match is formed is $v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$ and rewrite the expected present discounted value of this flow as $\mathbb{E}_t Q_{t,t+n} v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) P_{nt}$, where $P_{nt} \equiv \mathbb{E}_t Q_{t,t+n} A_{t+n}$ is the price of a claim to an asset that pays a one-time dividend of A_{t+n} in period $t+n$. We refer to this asset as a claim to productivity in n periods or simply a *productivity strip*. Next, consider the roots and the weights associated with the solution for μ_{et} and μ_{ut} in (37). To keep the algebra simple, we set the survival probability $\phi = 1$ so agents do not die. Then, the large root $\delta_\ell > 1$ and the small root $\delta_s < 1$ are given by

$$\delta_\ell = 1 + \frac{1}{2}\left[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}\right] \text{ and } \delta_s = \lambda - \frac{1}{2}\left[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}\right].$$

The corresponding weights on these roots are $c_\ell = \left[(1-b)(\lambda - \delta_s) + b g_e\right]/(\delta_\ell - \delta_s)$ and $c_s = 1 - b - c_\ell$ with $\lambda \equiv (1-\sigma)(1+g_e) - \eta\lambda_w < 1$.[15] Note that these roots and weights do not depend on the utility function or the process for technology. Combining these formulae with (36), we then have:

[14] We computed this value of g_u using the same sample used by Buchinsky, Fougère, Kramarz, and Tchernis (2010).

[15] In the general case with $d = \phi(1+g_u)$ and $e = \phi(1+g_u)\eta\lambda_w$, we obtain $\delta_{\ell,s} = \phi(1+g_u+\lambda)/2 \pm \phi[(1+g_u-\lambda)^2 + 4\eta\lambda_w(1+g_u)(g_e-g_u)]^{1/2}/2$, $c_\ell = [(\delta_\ell - d)/(\delta_\ell - \delta_s)]\left[1 - b - (\delta_s - d)/e\right]$, and $c_s = -\left[(\delta_s - d)/(\delta_\ell - \delta_s)\right]\left[1 - b - (\delta_\ell - d)/e\right]$.

Proposition 3. *The job-finding rate approximately satisfies*

$$(41) \quad \log(\lambda_{wt}) = \chi + \left(\frac{1-\eta}{\eta}\right) \log\left[\sum_{n=0}^{\infty}(c_\ell \delta_\ell^n + c_s \delta_s^n)\frac{P_{nt}}{A_t}\right],$$

where δ_ℓ, δ_s, c_ℓ, and c_s are given above and χ is a constant.

This proposition shows that the job-finding rate is a weighted average of the prices of claims to future productivity. Hence, all movements in the job-finding rate are *only* due to movements in the prices of these claims. As we will show, this result applies as stated to all preferences considered here. In particular, since the weights $(c_\ell \delta_\ell^n + c_s \delta_s^n)$ are determined solely by the search side of the model and remain fixed as we vary preferences, we will show that the formula for the job-finding rate for all five preferences we examine has this form and differs across models only in terms of the expression for P_{nt}/A_t, which we characterize next.

We simplify the calculation of the terms $\{P_{nt}\}$ in (41) by approximating the growth rate of consumption by the growth rate of productivity, $\Delta c_{t+1} \approx \Delta a_{t+1}$. Under this approximation, the pricing kernel becomes $Q_{t,t+1} = \beta\left(\frac{S_{t+1}}{S_t}\frac{A_{t+1}}{A_t}\right)^{-\alpha}$. In the next lemma, we derive a risk-adjusted log-linear approximation to P_{nt}/A_t based on a first-order perturbation of the price of strips around the risky steady state; see Lopez et al. (2017) for details. Note that since A_t follows a random walk process with drift and, hence, is nonstationary, the price $P_{nt} = \mathbb{E}_t Q_{t,t+n} A_{t+n}$ grows over time whereas the *scaled price* $P_{nt}/A_t \equiv \mathbb{E}_t Q_{t,t+n} A_{t+n}/A_t$, which is the price of a claim to the growth rate of productivity A_{t+n}/A_t in period $t+n$, is stationary. Therefore, we characterize here the dependence of P_{nt}/A_t on the exogenous state s_t.

Lemma 2. *The price of a claim to productivity in n periods approximately satisfies*

$$(42) \quad \log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n(s_t - s),$$

where $a_0 = b_0 = 0$, $a_n = \log(\beta) + (1-\alpha)g_a + a_{n-1} + \left[1 - b_{n-1} - (\alpha - b_{n-1})/S\right]^2 \sigma_a^2/2$, and b_n satisfies

$$(43) \quad b_n = \alpha(1-\rho_s) + \rho_s b_{n-1} + \left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\sigma_a^2.$$

Note that the constant a_n in (42) corresponds to the log of the discount factor β^n adjusted for productivity growth and risk, as captured by g_a and σ_a respectively. The elasticity b_n of the (scaled) price P_{nt}/A_t with respect to the exogenous state s_t, instead, captures how this price moves with s_t. The constants $\{a_n\}$ decrease with n as long as the drift rate g_a is not too large whereas the elasticities $\{b_n\}$ increase monotonically from 0 to α provided that $1 - \rho_s + (1 - \alpha/S)\sigma_a^2/S > 0$. This condition is satisfied for any reasonable parametrization of our preferences, as the variance of the innovation to productivity σ_a^2 is only about 0.003%.

Since the elasticity b_n increases with the maturity n of a claim, the longer is the maturity of a claim, the more sensitive is the price at horizon n to the exogenous state s_t, and so the lower is the price of a long-maturity claim relative to a short-maturity one when $s_t < s$. To understand why $\{b_n\}$ are upward sloping, consider first an economy without risk ($\sigma_a = 0$). In this case, b_n equals $\alpha(1 - \rho_s^n)$ and increases with n since ρ_s^n decreases with n. This result is due to intertemporal substitution motives. Intuitively, when the exogenous state s_t is below its mean s and expected to revert to it, consumers value current consumption more and so are willing to pay relatively *more* for a claim in the near future, when the state is expected to be close to s_t, and relatively *less* for a claim far into the future, when the state is expected to be much closer to its mean. The third term in (43) is an adjustment factor for risk. With risk, b_n still increases with the maturity n, albeit at a lower rate, because, all else equal, a precautionary saving motive makes consumers more willing to save, which attenuates the intertemporal substitution motive just discussed. For our purposes, the key implication of $\{b_n\}$ increasing with n is that the response of the job-finding rate to a given shock to s_t is larger, the larger are the weights on long-maturity claims. We formalize this intuition in the following proposition, where $\sigma(s_t)$ denotes the standard deviation of s_t.

Proposition 4. *Under the approximation in Lemma 2, the response of the job-finding rate with respect to a change in s_t evaluated at a risky steady-state is given by*

$$(44)\quad \frac{d\log(\lambda_{wt})}{ds_t} = \left(\frac{1-\eta}{\eta}\right)\sum_{n=0}^{\infty}\omega_n b_n \quad \textit{with} \quad \omega_n = \frac{e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)}{\sum_{n=0}^{\infty} e^{a_n}(c_\ell\delta_\ell^n + c_s\delta_s^n)},$$

where a_n and b_n are given in Lemma 2 and the standard deviation of the job-finding rate $\sigma(\lambda_{wt})$ satisfies

$$(45)\quad \sigma(\lambda_{wt}) = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(s_t).$$

Since the elasticities $\{b_n\}$ of claims to productivity increase with the horizon of a claim, a change in the exogenous state s_t leads to a large change in the job-finding rate only if the weights ω_n on long-term claims to productivity are large. In Figure 3, we graph the exact (scaled) prices of productivity strips against the state using neither the assumption that $\Delta c_{t+1} \approx \Delta a_{t+1}$ nor the risk-adjusted log-linear approximation of Lemma 2. Note that the prices of longer-maturity strips are much more sensitive to changes in the state than those of shorter-maturity ones. Moreover, as the figure makes clear, the log of these prices are indeed approximately linear in the state. In Figure 4a, we show the impulse responses of these strips to a 1% drop in productivity. Clearly, the price of short-horizon strips falls little whereas the price of long-horizon strips falls greatly after this shock. Thus, these figures together with Proposition 4 illustrate that our model generates large variations in the job-finding rate only when the weights $\{\omega_n\}$ are sufficiently large for large n. We now turn to showing that without human capital accumulation, these weights decay very quickly. Moreover, the larger the rate of human capital accumulation during employment relative to that during unemployment $(g_e - g_u)$, the slower these weights decay.

DMP Model with Baseline Preferences. Consider first the DMP model with our baseline preferences and $g_e = g_u = 0$. Here the constant c_ℓ on the large root is zero and the small root, referred to as the *DMP root*, is given $\delta_{DMP} = 1-\sigma-\eta\lambda_w$, where σ is the separation rate, η is the elasticity of the matching function with respect to the measure of unemployed workers, and λ_w is the worker's job-finding rate. Thus, in the DMP version of our model, surplus flows n periods after a match is formed follow a first-order difference equation with the surplus flow at n proportional to $\delta_{DMP}^n A_{t+n}$. The weight in the analogous expression for $d\log(\lambda_{wt})/ds_t$ is $\omega_n = e^{a_n}\delta_{DMP}^n / \sum_{n=0}^{\infty} e^{a_n}\delta_{DMP}^n$. For standard parametrizations, the DMP root is substantially smaller than one so that surplus flows decay quickly at a rate of about 25% per month. To see why, note that with $\delta_{DMP} = 1 - \sigma - \eta\lambda_w$, $\sigma = 2.8\%$, $\eta = 0.5$, and $\lambda_w = 46\%$, which is the mean job-finding rate in the data, it follows that $\delta_{DMP} = 74.2\%$, which amounts to a decay rate of over 25% per month. Hence, after only two years, $(\delta_{DMP})^{24}$ is 0.08%. Correspondingly, the weights assigned to long-maturity productivity strips are essentially zero.

Baseline Model. In our baseline model, surplus flows follow a second-order difference equation whose solution is such that n-th flow is proportional to $(c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$. By our above formula for the roots, the large root δ_ℓ is bigger than one and the weight c_ℓ on this root is positive so that the discounted value of surplus flows decays slowly over time. In turn, this fact implies that the job-finding rate in (41) assigns sizable weights to long-maturity productivity strips, which fluctuate a lot with the exogenous state s_t and, hence, greatly move with current productivity shocks. In Figure 4b, we plot the cumulative weights implied by the DMP model with baseline preferences and our baseline model without any approximation. Clearly, the weights in the DMP model with baseline preferences decay very quickly relative to those in our baseline model. For a sense of the magnitude of the decay of these weights with the horizon, define the (Macaulay) *duration* of these weights as $\sum_{n=0}^{\infty} \omega_n n$ and note that the duration of the weights is 3.6 months in the DMP model with baseline preferences and 11 years in the baseline model. The expression in (44) implies that a more relevant measure of duration is the elasticity of the job-finding rate with respect to the exogenous state s_t defined by $\sum_{n=0}^{\infty} \omega_n b_n$.[16] For the DMP model with baseline preferences, this modified duration is 0.03 and for the baseline model, it is 0.89.

7. Implications for Wages and Stock Market Returns

Here we discuss additional implications for our model for wages and stock market returns.

A. Implications for Wages

Our competitive search equilibrium determines the present value of wages paid to a worker over the course of a match with a firm, but not the flow wage in each period. More generally, in any model with complete markets and commitment by both workers and firms to a state-contingent employment contract, many alternative sequences of flow wages give rise to the same present value of wages. Hence, in this precise sense, our model does not have specific predictions for flow wages.

Given this indeterminacy, one possible strategy to evaluating our model is to simply refrain from any comparison between our model and the data in terms of statistics that rely on auxiliary assumptions that

[16] The term $\sum_{n=0}^{\infty} \omega_n b_n$ is a type of Macaulay duration alternative to the standard one, where instead of weighting the *horizon length* n by the fraction ω_n of the present value of surplus flows accruing at that horizon at the risky steady-state, we weight the *elasticity of the price of a claim to productivity at horizon* n to the state, namely, b_n.

pin down flow wages. Another strategy, which we adopt here, is to follow the approach popularized by Barlevy (2008) and Bagger et al. (2014), who assume that when a match is formed at t, a firm commits to pay a worker each period a share ϱ_t of the period output for the duration of the match so that $w_{t,\tau} = \varrho_t A_\tau z_\tau$ is the wage paid in period $\tau \geq t$. Accordingly, we determine flow wages in our model as follows. For any present value of wages $W_t(z_t) = W_t z_t$ implied by our model for a match that starts at t, we choose ϱ_t so that the present value of the wages $w_{t,t} = \varrho_t A_t z_t$, $w_{t,t+1} = \varrho_t A_{t+1} z_{t+1}$, and so on, calculated using our stochastic discount factor, exactly equals $W_t(z_t)$. Using this approach, we examine our baseline model's implications for wages.

We first discuss additional evidence on wage growth in support of our parametrization of the human capital process. We then argue that our model is robust to the critique by Kudlyak (2014) of the degree of rigidity of the wage process implied by prominent solutions to the unemployment volatility puzzle. Specifically, we find that our model is consistent with the estimated degree of cyclicality of wages by Kudlyak (2014) and, hence, does not rely on counterfactually rigid wages.

Consider first wage growth, which is the moment that pins down the rate of human capital accumulation in our model. As noted, we have set the growth rate of human capital, g_e, so as to match *longitudinal* wage growth with experience. We now argue that the wage process implied by our model, under the parametrization discussed earlier, also matches the evidence on *cross-sectional* wage growth with experience documented by Elsby and Shapiro (2012). These authors report that the difference in the log real wages of workers with 30 years of experience and those with 1 year of experience is 1.2 in the data.[17] Our baseline model is consistent with this untargeted statistic as it implies a difference of 1.0.

Consider next wage rigidity. As Becker (1962) emphasizes, the present discounted value of the wages paid to a worker over the course of an employment relationship is allocative for employment, not the flow wage. Kudlyak (2014) proves that for a large class of search models, the appropriate allocative wage is the difference in the present values of wages between two matches that start in two consecutive periods, as captured by the *user cost of labor*. Intuitively, in a search model, hiring a work is akin to acquiring a long-term asset subject to adjustment costs. Thus, by capturing the rental price of the services of a worker potentially employed for many years, the user cost of labor is a more suitable measure of the cost of hiring a worker than the current wage.

Kudlyak (2014) and Basu and House (2016) measure the cyclicality of the user cost of labor as the semi-elasticity of the user cost to unemployment and, based on NLSY data, estimate it to be highly procyclical. These authors estimate the user cost of labor as $UC_t \equiv PDV_t - \beta(1-\sigma)PDV_{t+1}$, where PDV_t is an empirical measure of the present value of wages associated with a match that starts at t defined as $PDV_t = w_{t,t} + \sum_{\tau=t+1}^{T}[\beta(1-\sigma)]^{\tau-t}w_{t,\tau}$ given the fixed discount factor $\beta(1-\sigma)$, which takes into account the real interest rate and the separation rate, where $w_{t,\tau}$ is the wage in period $\tau \geq t$. (See Kudlyak 2014

[17] We consider the census years of 1980, 1990, and 2000 for consistency with the panel horizon of the data of Rubinstein and Weiss (2006), who use the 1979-2000 waves of NLSY in their analysis of wage growth.

and Basu and House 2016 for details.) Intuitively, this empirical measure of the user cost is the shadow wage that would make a risk-neutral firm indifferent between hiring a worker today, who survives in a match with probability $1-\sigma$, or tomorrow. Importantly, the user cost of labor at t does not just count the flow wage of new hires at t but also the difference in the present value of wages from $t+1$ on between a worker hired at t and a worker hired at $t+1$. Hence, the user cost incorporates the potential extra cost or benefit of committing at t to a (possibly state-contingent) sequence of wage payments from $t+1$ on, relative to waiting and hiring an identical worker at $t+1$ at the present value of wages prevailing at $t+1$. If recessions are times of *scarring* in that workers hired in downturns not only obtain a lower wage in the period they are hired, but also a lower wage in any subsequent period, relative to workers hired in upturns, then it is clear that the user cost can be much more cyclical than the flow wage.

Kudlyak (2014) and Basu and House (2016) estimate a semi-elasticity of the user cost of labor to the unemployment rate of -5.2% and -5.8%, respectively, which implies that a one percentage point increase in the unemployment rate is associated with an approximately 6% decrease in the user cost of labor. Hence, the user cost is quite procyclical. When we compute the user cost in our model, we treat the empirical measure of the user cost in Kudlyak (2014) as simply a particular statistic on the allocative wage that takes as inputs a sequence of flow wages, $\{w_{t,\tau}\}$, and the fixed discount factor $\beta(1-\sigma)$ according to the above formulae for UC_t and PDV_t. Based on the flow wages constructed as described, our model implies a cyclicality of the user cost of labor of -6.4%. Hence, the user cost of labor in baseline model, although untargeted, is in line with the data—it only falls slightly more than in the data when unemployment rises. Thus, our mechanism for unemployment volatility does not rely on a counterfactual degree of wage rigidity.

B. Implications for Stock Market Returns

In the data, flows of payments to equity or debt holders are mostly payments for physical and intangible capital, and depend on firm leverage. Our simple model without either physical or intangible capital features none of these payments and abstracts from leverage. Indeed, as the free-entry condition makes clear, equity flows in our model are simply payments for the upfront costs of posting job vacancies. For these reasons, we follow the simple approach used in the asset pricing literature that dates back at least to Mehra and Prescott (1985), which interprets stocks as claims to streams of aggregate consumption—see, for instance, Campbell and Cochrane (1999) and Wachter (2006). Following this approach, we price claims to streams of aggregate consumption in the model and contrast them to stock prices in the data. In Table 3, we compare the mean and standard deviation of the excess return, their ratio, and the mean and standard deviation of the price-dividend ratio computed from the Flow of Funds to the corresponding statistics on consumption claims implied by our baseline model. As apparent from the table, the two sets of statistics are indeed close. The key point of this exercise is that our baseline model has similar implications for the prices of such claims as does the model by Campbell and Cochrane (1999), who follow the same strategy that we have used here.

8. A More General Human Capital Process

So far, we have considered a simple process of human capital accumulation such that human capital grows at a constant rate when a consumer is employed and decays at a constant rate when a consumer is unemployed. In the data, though, wage growth tends to decline as experience in the labor market accumulates. To accommodate this feature of the data, we consider a more general human capital process as in Kehoe, Midrigan, and Pastorino (2019) in the spirit of that in Ljungqvist and Sargent (1998, 2008), whereby human capital z_t evolves according to the autoregressive process

$$\log(z_{t+1}) = (1-\rho_z)\log(\bar{z}_e) + \rho_z \log(z_t) + \sigma_z \varepsilon_{zt+1} \tag{46}$$

when a consumer is employed, whereas it evolves according to

$$\log(z_{t+1}) = (1-\rho_z)\log(\bar{z}_u) + \rho_z \log(z_t) + \sigma_z \varepsilon_{zt+1} \tag{47}$$

when a consumer is unemployed, where ε_{zt+1} is a standard Normal random variable. Newborn consumers start as unemployed with general human capital z, where $\log(z)$ is drawn from the normal distribution $N(\log(\bar{z}_u), \sigma_z^2/(1-\rho_z^2))$. We assume that $\bar{z}_u < \bar{z}_e$ so that when a consumer is employed, on average, human capital z_t drifts up toward a high level of productivity $\bar{z}_e$ from the low average level of productivity $\bar{z}_u$ of newborn consumers. Analogously, when a consumer is unemployed, on average, human capital z_t depreciates and hence drifts down toward a low level of productivity, $\bar{z}_u$, which we normalize to 1 so that $\log(\bar{z}_u) = 0$. The parameter ρ_z governs the rate at which human capital converges toward $\bar{z}_e$ when a consumer is employed and toward $\bar{z}_u$ when a consumer is unemployed. Hence, the higher ρ_z is, the slower human capital accumulates during employment, the slower it depreciates during unemployment, and the slower wages grow with experience. Incorporating idiosyncratic shocks ε_{zt+1} allows the model to reproduce the dispersion in wage growth rates observed in the data. (See Rubinstein and Weiss 2006.)

A consumer with human capital z_t produces $A_t z_t$ when employed but, in contrast to our baseline model, bA_t when unemployed. Also in contrast to our baseline model, we assume that a firm incurs the cost κA_t to recruit a consumer with any level of human capital. (Recall that the earlier scaling of home production and the cost of posting vacancies by z_t was purely motivated by analytical convenience to allow the model to aggregate.) To ensure that the job-finding rate $\lambda_{wt}(z)$ lies between zero and one, we assume that the matching function is $m_t(u_{bt}(z), v_t(z)) = \min\{u_{bt}(z), Bu_{bt}(z)^{\eta} v_t(z)^{1-\eta}\}$. A competitive search equilibrium is defined as before with the free-entry condition for market z now given by

$$\kappa A_t \geq \lambda_{ft}(\theta_t(z))[Y_t(z) - W_t(z)], \tag{48}$$

with equality if vacancies are created in an active market z in that the measure of vacancies $v_t(z)$ is strictly positive. Here we focus on our baseline preferences with exogenous habit. It is easy to show that for any of the preferences we consider, the competitive search allocations solve the restricted planning problem described above.

We parametrize the model as before with few modifications. With $\bar{z}_u$ normalized to one, the parameters of the human capital process are $\bar{z}_e$, ρ_z, and σ_z. We target a net annual wage growth over the first 10 years in the labor market of 5.5%, based on the estimates by Rubinstein and Weiss (2006) discussed earlier, and a difference in the log real wages between workers with 30 years of experience and those with 1 year of experience of 1.2, based on the estimates by Elsby and Shapiro (2012) also discussed earlier. These two targets help pin down ρ_z and $\bar{z}_e$. We choose σ_z to match the standard deviation of annual wage growth for workers with up to 10 years of labor market experience, which is 1.2 percentage points according to the estimates by Rubinstein and Weiss (2006) from the NLSY.

Since, unlike our baseline model, this version of the model is not amenable to aggregation, we need to record the measures of human capital among employed and unemployed workers, $(e_t(z), u_t(z))$, as part of the endogenous state of the economy. This feature makes the model much more difficult to solve numerically than our baseline model. For this reason, we use a variant of the algorithm by Krusell and Smith (1996) that, unlike in typical applications such as those in Winberry (2018), needs to accurately capture time-varying risk in aggregate variables.

Notwithstanding this complexity, this version of the model too successfully solves the unemployment volatility puzzle. In particular, in Table 4 we see that the model produces only a slightly lower volatility for the job-finding rate and unemployment than in the data, respectively, 6.38 versus 6.66 and 0.65 versus 0.75. In this sense, our earlier results based on a simple model of human capital accumulation are robust to extensions that capture additional features of the micro data on returns to labor market experience.

9. Toward a Real and Financial Business Cycle Model

The well-known early business cycle work by Merz (1995) and Andolfatto (1996) integrated search theory into real business cycle models. While ambitious, those contributions did not attempt to make their models consistent with any asset pricing patterns. Since those early contributions, the subsequent literature has mostly shied away from doing so and, instead, focused on models without physical capital. Here we embed our mechanism into a real business cycle model with such capital retaining the preferences considered so far. We thus construct a simple real and financial business cycle model that solves the unemployment volatility puzzle and is in line with key patterns of job-finding rates, unemployment, output, investment, and asset prices in the data. Note that in contrast to the classic separation result between the real and financial sides of an economy by Tallarini (2000), here including time-varying risk greatly amplifies the fluctuations of real variables.

Consider then the following extension of our baseline model. The production functions for a consumer with human capital z when paired with physical capital depend on whether the consumer produces goods or vacancies in the market or goods at home. We assume that a consumer with human capital z paired with $K_{et}(z)$ units of physical capital produces $(A_t z)^{1-\gamma} K_{et}(z)^\gamma$ units of goods when employed, whereas when paired with $K_{ut}(z)$ units of physical capital produces $(bA_t z)^{1-\gamma} K_{ut}(z)^\gamma$ units of goods at home. We follow

Shimer (2011, p. 100) by assuming that the activity of hiring workers uses only labor in that a consumer with human capital z produces $\kappa A_t z$ vacancies. There are costs of adjustment for the aggregate capital stock, but for a given level of the aggregate capital stock, capital can be costlessly moved between the market production and the home production of goods. We assume that the aggregate investment decision is made at the end of period $t-1$ and that the aggregate capital stock K_t that enters period t is divided between its two uses after the time t aggregate shock is realized.

We consider our baseline preferences and examine the competitive search allocations that solve the natural planning problem. It is immediate that the economy aggregates in a similar fashion as does the economy of our baseline model. The aggregate resource constraint can then be written as

$$C_t + I_t \leq (A_t Z_{et})^{1-\gamma} K_{et}^{\gamma} + (bA_t Z_{ut})^{1-\gamma} K_{ut}^{\gamma} - \kappa A_t Z_{vt},$$

where $K_{et} = \int K_{et}(z) e_t(z) \mathrm{d}z$ is the measure of physical capital used by employed consumers; we use a similar notation for K_{ut}. Aggregate vacancy costs are given by $Z_{vt} = \int z v_t(z) \mathrm{d}z = \phi\theta_t(1+g_u)Z_{ut-1}$. The aggregate capital stock follows the accumulation law $K_{t+1} = (1-\delta)K_t + \Phi(I_t/K_t) K_t$ with $K_{et}+K_{ut} \leq K_t$. We choose $\Phi(I/K) = \delta[(I/\delta K)^{1-1/\xi} - 1]/(1-1/\xi)$ as in Jermann (1998). We set $\gamma = 1/4$, $\delta = 0.10/12$, and the curvature parameter ξ of the adjustment cost function so that the model produces a standard deviation of investment growth relative to consumption growth equal to that in the data.

We turn now to the results, reported in Table 5. Note that relative to our baseline model without physical capital, agents in this model have another way to smooth consumption, namely, by decreasing investment in physical capital in downturns and increasing it in upturns. Doing so decreases consumption risk and, therefore, slightly dampens the fluctuations in the price of risk, which, in turn, partly reduces the fluctuations in the present value of surplus flows and so the resulting fluctuations in the job-finding rate. Overall, though, this augmented model gives rise to a standard deviation of unemployment that is very similar to that in the baseline model and is 95% (0.71/0.75) of that in the data.

10. Results for Alternative Preferences

We first show that we obtain quantitative results for our other preferences similar to those obtained for our baseline preferences. We then inspect the mechanism generating them and emphasize that they formally all work in a nearly identical way.

A. Quantitative Results for Alternative Preferences

Here we present results for Campbell-Cochrane preferences with external habit, Epstein-Zin preferences with long-run risk, Epstein-Zin preferences with variable disaster risk, and an affine discount factor.

Campbell-Cochrane Preferences with External Habit

We adapt the setup of Campbell and Cochrane (1999) with external habit designed for a pure exchange economy, discussed earlier, to our production economy. The only difference from the original Campbell-

Cochrane specification is that we replace the sensitivity function with

$$(49)\quad \lambda_t(\bar{s}_t) = \frac{\sigma(\varepsilon_{ct+1})}{\sigma_t(\varepsilon_{ct+1})}\frac{1}{\bar{S}}\left[1-2\left(\bar{s}_t-\bar{s}\right)\right]^{1/2}-1.$$

Here $\sigma(\varepsilon_{ct+1})$ and $\sigma_t(\varepsilon_{ct+1})$ are, respectively, the unconditional and conditional standard deviations of the innovations to aggregate consumption growth $\varepsilon_{ct+1} = \Delta\bar{c}_{t+1} - \mathbb{E}_t\Delta\bar{c}_{t+1}$. This sensitivity function is slightly different from that in Campbell and Cochrane (1999) and Wachter (2006), who consider economies in which consumption is exogenous and exhibits a constant conditional variance so $\sigma(\varepsilon_{ct+1})/\sigma_t(\varepsilon_{ct+1}) = 1$. When this is the case, our sensitivity function reduces to theirs. Our production economy, instead, features endogenous consumption with time-varying conditional volatility. The term $\sigma(\varepsilon_{ct+1})/\sigma_t(\varepsilon_{ct+1})$ in (49) adjusts for this time-varying conditional volatility, and so helps the model generate stable interest rates over time by ensuring that intertemporal substitution motives and precautionary saving motives almost offset each other to replicate the observed volatility of interest rates. We choose the parameters of this model to equal those of the baseline model except that we slightly adjust the mean surplus consumption ratio from 0.2066 to 0.2087 to match the standard deviation of the risk-free rate. Tables 2 and 6 confirm that this model produces nearly identical results to those produced by the baseline model.

Epstein-Zin Preferences with Long-Run Risk

We consider next a model with Epstein-Zin preferences, a slow-moving predictable component in productivity as in Bansal and Yaron (2004), and discount rate shocks as in Albuquerque et al. (2016) and Schorfheide et al. (2018). In particular, preferences are now given by

$$V_t = \left[(1-\beta)S_tC_t^{1-\rho} + \beta\left(\mathbb{E}_tV_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}.$$

Productivity growth now has a long-run risk component x_t in that

$$(50)\quad \Delta a_{t+1} = g_a + x_t + \sigma_a\varepsilon_{at+1} \text{ and } x_{t+1} = \rho_x x_t + \phi_x\sigma_a\varepsilon_{xt+1},$$

where the shocks ε_{at} and ε_{xt} are standard normal i.i.d. and orthogonal to each other. The growth rate of discount factor shocks $\Delta\log(S_t) = \Delta s_t$ follows an autoregressive process given by

$$(51)\quad \Delta s_{t+1} = \rho_s\Delta s_t + \phi_s\sigma_a\varepsilon_{st+1},$$

where the shock ε_{st} is standard normal i.i.d. and orthogonal to the other shocks. The pricing kernel is

$$Q_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{S_{t+1}}{S_t}\right)\left[\frac{V_{t+1}}{\left(\mathbb{E}_tV_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}}\right]^{\rho-\alpha}.$$

We set the model's parameters as follows. We select the mean and standard deviation of the productivity growth process to match those in the data. We choose the parameter ϕ_s governing the volatility of the discount factor shock to match the standard deviation of the risk-free rate. We select a risk-aversion coefficient α of 4.3 to match a maximum Sharpe ratio of 0.45 for the consumption portfolio.

For ease of comparison with the baseline model, we set the persistence ρ_s of the process for s_t equal to that in baseline, and choose the persistence ρ_x of the long-run risk state x_t so that the model generates the same standard deviation of the price-consumption ratio as in the baseline model. We pick a large elasticity of intertemporal substitution of 10 ($\rho = 0.1$). To understand this choice, note first that with an elasticity of intertemporal substitution equal to one, the volatility of the job-finding rate is exactly zero—see the Appendix for a proof of this claim. As noted by Kilic and Wachter (2018) in a related context, a large elasticity parameter is not necessarily inconsistent with the available evidence of a low elasticity of intertemporal substitution of consumption, which reflects the weak correlation between consumption growth and interest rates. Indeed, when we estimate the contemporaneous elasticity of consumption growth with respect to interest rates based on data simulated from our model, using powers of the states s_t and x_t and lagged consumption growth as instruments, we find a coefficient of around 0.2, which is consistent with estimates in the literature (see, for instance, Hall 1988 and Beeler and Campbell 2012).

Lastly, note that the volatility of the productivity process $\sigma_a^2 + \sigma_x^2$ is the sum of the volatility of the i.i.d. component, ε_{at}, and the persistent component, x_t, with $\sigma_x^2 \equiv \phi_x^2\sigma_a^2/(1-\rho_x^2)$. We assume that the persistent component accounts for the same share $\sigma_x^2/(\sigma_a^2+\sigma_x^2) = 0.0445$ of volatility of productivity growth as that chosen by Bansal and Yaron (2004), which pins down the value of ϕ_x.

In Tables 2 and 7, we show that with these preferences, the model can produce around 92% of the observed volatility of unemployment (0.69 in the model versus 0.75 in the data respectively).

Epstein-Zin Preferences with Variable Disaster Risk

We adopt a discrete-time version of the model of Wachter (2013) with Epstein-Zin preferences and a slow-moving probability of rare disasters. In this case, the specification of preferences becomes

$$(52)\quad V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}.$$

The process for productivity growth, now driven by a discrete-valued jump component j_{t+1}, is given by

$$\Delta a_{t+1} = g_a + \sigma_a\varepsilon_{at+1} - \theta j_{t+1},$$

where the disaster component j_{t+1} is a Poisson random variable with intensity s_t, which evolves as

$$(53)\quad s_{t+1} = (1-\rho_s)s + \rho_s s_t + \sqrt{s_t}\sigma_s\varepsilon_{st+1}.$$

As before, we choose the mean and the standard deviation of the productivity growth process to match those in the data. We choose an ergodic mean disaster intensity s of 3.55% per year as in Wachter (2013) to match the mean disaster intensity in the data. We select values for the remaining parameters so as to generate a volatility σ_s for the disaster intensity of 0.0083 to reproduce the standard deviation of the risk-free rate, and a risk aversion coefficient of 2.65 to target a maximum Sharpe ratio of 0.45. For ease of comparison with the baseline model, we choose a persistence ρ_s of the disaster intensity of 0.9966 to generate the same standard deviation of the price-consumption ratio as in the baseline model. Lastly, as in Wachter (2013), we set the disaster impact θ to 0.26 and the elasticity of intertemporal substitution to 10 ($\rho = 0.1$).[18]

In Tables 2 and 8, we show that the version of the model with these preferences produces only a slightly higher volatility of unemployment in normal times (0.77), that is, in times without a disaster, than in the data (0.75). Importantly, these results are derived under the assumption of competitive search and thus do not rely on either inefficient real wage stickiness or exogenous movements in scaled hiring costs as in Kilic and Wachter (2018).

An Affine Discount Factor

So far we have studied consumption-based discount factors derived from underlying utility functions. Here we argue that our results also hold for reduced-form discount factors of the type considered by Ang and Piazzesi (2003) among others, which are specified as functions of an exogenous state whose innovations are also innovations to productivity. This approach is similar to that in Hall (2017), who also specifies a reduced-form discount factor. Specifically, we assume that

$$\log(Q_{t,t+1}) = -(\mu_0 - \mu_1 s_t) - \frac{1}{2}(\gamma_0 - \gamma_1 s_t)^2 \sigma_a^2 - (\gamma_0 - \gamma_1 s_t)\sigma_a \varepsilon_{at+1},$$

where the exogenous state s_t follows the autoregressive process

$$s_{t+1} = \rho_s s_t + \sigma_a \varepsilon_{at+1} \tag{54}$$

and is driven by fluctuations in productivity, ε_{at+1}. Productivity growth still follows a random walk given by $\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{at+1}$. This discount factor is termed *affine* because it implies that both the risk-free rate r_{ft} and the conditional standard deviation of the log of the pricing kernel $\sigma_t(\log(Q_{t,t+1}))$ are affine in the exogenous state s_t, since $r_{ft} = \mu_0 - \mu_1 s_t$ and $\sigma_t(\log(Q_{t,t+1})) = (\gamma_0 - \gamma_1 s_t)\sigma_a$. Recall from (16) that $\sigma_t(\log(Q_{t,t+1}))$ is also the maximum Sharpe ratio for continuously compounded lognormally distributed returns. Hence, the parameters μ_0 and μ_1 control the mean and the volatility of the risk-free rate, whereas the parameters γ_0 and γ_1 control the mean maximum Sharpe ratio and the volatility of the excess return.

We investigate the quantitative properties of this affine discount factor model for the volatility of the job-finding rate and unemployment by keeping the parameters for the mean and standard deviation of productivity growth, g_a and σ_a, as in the baseline model and by choosing the four parameters $(\mu_0, \mu_1, \gamma_0, \gamma_1)$ to reproduce the mean and standard deviation of the risk-free rate, the maximum Sharpe ratio, and the volatility of the excess return. For ease of comparison with the baseline model, we choose the persistence ρ_s of the exogenous state to generate the same standard deviation of the price-consumption ratio as in the baseline model.

[18] When we estimate the contemporaneous elasticity of consumption growth to interest rates on data simulated from our model using powers of s_t and lagged consumption growth as instruments, we find estimates between 0.01 and 0.5 despite the assumption that $\rho = 0.1$.

In Tables 2 and 9, we show that this model produces about 97% of the volatility of unemployment in the data (0.73 in the model and 0.75 in the data).

B. The Mechanism for Other Preferences

Note first that Proposition 3 holds as stated for our models with Campbell-Cochrane preferences with external habit, Epstein-Zin preferences with long-run risk, Epstein-Zin preferences with variable disaster risk, and the affine discount factor. The reason is simply that this result depends only on the search side of the model and not on the discount factor that a particular preference and shock structure implies. It turns out that an analogue of Lemma 2 holds for each of these preferences as well. For Campbell-Cochrane preferences with external habit, the log-linear approximation in (42) holds with the same constants given in Lemma 2 except that the constant S in b_n is replaced by $\bar{S}$. Proposition 4 then applies as stated. For Epstein-Zin preferences with long-run risk, the analogue of Lemma 2 holds with $\log(P_{nt}/A_t) = a_n + b_n \Delta s_t + c_n x_t$, where $b_n = \rho_s(1-\rho_s^n)/(1-\rho_s)$, $c_n = (1-\rho)(1-\rho_x^n)/(1-\rho_x)$, and the constants a_n are given in the Appendix. For the remaining preferences, the prices of claims to strips have the same form as (42) with constants provided in the Appendix. Then, Proposition 4 applies as stated.

In order to provide some intuition as to how these elasticities and the associated weights vary across models, in Figure 5, we graph these elasticities scaled by the volatility of the relevant state, and the corresponding weights. Notice that in all these models, these scaled elasticities increase with the horizon n. Hence, the intuition for the role of human capital is the same for all these models: the greater the degree of human capital accumulation, the larger the weights placed on long-horizon claims, which are relatively more sensitive to changes in the state, and so the larger the volatility of the job-finding rate. Therefore, as far as the volatility of the job-finding rate is concerned, all of these models work in the same way.

11. Distinguishing Mechanisms: A Comparison with Hall (2017)

Here we compare our model with competitive search to the model of Hall (2017) with alternating offer bargaining. Our model and Hall's model emphasize distinct mechanisms that generate volatility in unemployment. The key mechanism in our model relies on the interaction between time-varying risk and human capital accumulation. In contrast, the key mechanism in Hall (2017) relies on the interaction between time-varying discount factors (rather than time-varying risk) and a type of real wage stickiness resulting from inefficient wage setting, which implies a counterfactual degree of wage rigidity.

It has long been known that one way to reproduce the observed fluctuations in unemployment is to impose a form of wage stickiness. Intuitively, if the cost of employing a worker does not decrease much in downturns, then firms' incentives to hire workers are greatly reduced and, as a result, unemployment sharply declines. Recent evidence on the extent of actual wage rigidity, though, has challenged the relevance of this mechanism. For instance, Kudlyak (2014) and Basu and House (2016) document that the user cost of labor is highly procyclical. Here we show that the cyclicality of the user cost of labor in Hall (2017) is much lower than that estimated by Kudlyak (2014) and Basu and House (2016). In this sense, the allocative wage in Hall's model is much stickier than in the data.

We then show that the alternating offer bargaining game of Hall (2017) and Hall and Milgrom (2008) can support efficient allocations and, hence, the competitive search allocations, as long as the parameters of the bargaining game are chosen appropriately. The key condition to achieve efficiency is that the *duration of a job opportunity*, defined as the mean length of time available to form a match if bargaining continues until it exogenously breaks down, be short (one month). We emphasize that the values of the parameters of bargaining in Hall (2017) needed to reproduce the observed volatility of unemployment are very far from those that yield efficient outcomes and, according to Christiano, Eichenbaum, and Trabandt (2016), imply a somewhat implausible duration of a job opportunity of over 6 years. Namely, workers and firms must believe that bargaining can continue for over 6 years if no agreement is reached, for the model to produce sizable fluctuations in unemployment. In contrast, Christiano et al. (2016) argue that a reasonable length of time for the duration of a job opportunity with such alternating offer bargaining is at most one quarter.

A. Alternating Offer Bargaining

We briefly lay out the alternating offer bargaining game of Hall (2017). Using our notation, the formulae for the resource constraint, the post-match value P_t, the unemployment value U_t, the present value of output in a match Y_t, the value of a vacancy V_t, and the free-entry condition in the alternating offer bargaining equilibrium are identical to those in the competitive search equilibrium, namely, (10), (18), (19), (22), (23), and (24), but without human capital accumulation in that $g_e = g_u = 0$ and $z = 1$ for all consumers. The only two remaining differences between Hall's model and our model is that wages in Hall's model are set in an imperfectly competitive rather than a competitive way and the stochastic discount factor $Q_{t,t+1}$ is an exogenous rather than an endogenous one.

The game can be described as follows. The worker makes the first wage offer with probability p and the firm makes the first wage offer with probability $1 - p$. In each subsequent period, firms and workers deterministically alternate making offers each period, if bargaining has not broken down, until an offer is accepted. If period t is one in which the firm makes the offer, we denote the offer by W_{ft}, whereas if period t is one in which the worker makes the offer, we denote it by W_{wt}—these offers are contingent on the exogenous state ε^t, but we have suppressed their explicit dependence on ε^t for simplicity. In each period, with probability δ bargaining exogenously breaks down, in which case the firm returns to the market with an unfilled vacancy and the worker enters unemployment. When the firm offers W_{ft} in period t, then the worker can either accept it, reject it and make a counteroffer W_{wt+1} in period $t + 1$ if bargaining does not exogenously break down, or abandon negotiations and immediately return to unemployment. The firm has symmetric options if it is the worker's turn to make an offer. The cost of bargaining to the worker is that in each period of bargaining, the worker only receives the value of home production bA_t rather than a wage, so the implicit delay cost is the difference between foregone wages and home production. The cost of bargaining to the firm is the cost ψA_t of making a counteroffer to the worker at t; we refer to ψ as the *haggling cost*. Thus, the three parameters that characterize this bargaining scheme are (p, δ, ψ).

As explained in Hall and Milgrom (2008), standard recursive logic implies that the firm will make the best possible offer from its perspective so that the worker will prefer to accept it rather than to make a counteroffer, in the event of no exogenous breakdown, or to abandon negotiations. Thus, the firm's offer W_{ft} satisfies

$$W_{ft} + P_t = \max \left\{ bA_t + \phi(1-\delta)\mathbb{E}_t Q_{t,t+1}(W_{wt+1} + P_{t+1}) + \phi\delta\mathbb{E}_t Q_{t,t+1} U_{t+1}, U_t \right\}, \tag{55}$$

where the maximum ensures that the worker does not strictly prefer unemployment today to accepting such an offer. Of course, the firm's offer W_{ft} must be smaller than the discounted value of output from the match with the worker, Y_t, or else the firm would prefer to stay idle. Thus, $W_{ft} \leq Y_t$. In turn, the worker will make the best possible offer from the worker's perspective so that the firm will prefer to accept it rather than to make a counteroffer, in the event of no exogenous breakdown, or to abandon negotiations. Therefore, the worker's offer satisfies

$$Y_t - W_{wt} = \max\{-\psi A_t + \phi(1-\delta)\mathbb{E}_t Q_{t,t+1}(Y_{t+1} - W_{ft+1}), 0\}, \tag{56}$$

where the maximum ensures that the firm does not strictly prefer to abandon negotiations rather than to accept the offer. Clearly, the worker will only make offers such that employment is preferable to unemployment, that is, $W_{wt} + P_t \geq U_t$ must hold. Since a family consists of a large number of consumers who are independently drawn to make the first offer in bargaining, the value to a family of the wages of all its consumers who are bargaining at t is $W_t = pW_{wt} + (1-p)W_{ft}$. Likewise, W_t is the value to the firm of the present value of wages from bargaining.

B. The Cyclicality of the User Cost of Labor

Here we show that Hall (2017) generates sizable fluctuations in unemployment only under a parametrization of wage setting that yields very rigid and, as we will discuss in the next subsection, inefficient wages. It turns our that the critical parameter governing the stickiness of wages in Hall (2017) is the probability of exogenous breakdown of bargaining, δ. It is not easy to interpret this exogenous breakdown probability based on actual bargaining behavior because, in equilibrium, the first offer is accepted regardless of the value of δ. We find it therefore useful to translate δ into units of time by calculating the mean duration of the opportunity to bargain to form a match, if bargaining continues until it exogenously breaks down. Correspondingly, we refer to $1/\delta$ as the duration of a job opportunity during bargaining. It turns out that the longer is the duration of a job opportunity, the stickier are real wages. In Hall's baseline model this duration is 77 months.

In Table 10, the third column illustrates the parameters and results in Hall (2017) reproduced from the computer code on Hall's website. Note that when the duration of a job opportunity is 77 months, the cyclicality of the user cost of labor is 0.1%. That is, after a one percentage point increase in the unemployment rate, the user cost of labor actually slightly *increases*. Recall that Kudlyak (2014) estimates that after a one percentage point increase in the unemployment rate, the user cost of labor *falls* by 5.2%—Basu and House (2016) obtain a similar estimate of 5.8%. In this sense, Hall's model generates an extreme degree of wage rigidity that is at odds with the estimated cyclicality of the user cost of labor.

We now turn to determine the duration of a job opportunity that generates the observed degree of wage cyclicality. As the second column in Table 10 shows, at $1/\delta = 2.6$ months, the model generates the observed cyclicality of the user cost of labor. With this degree of stickiness, however, the model generates 1/25th of the volatility of unemployment in the data (0.03/0.75). (For this exercise, as we vary the duration of a job opportunity, we adjust the vacancy posting cost in Hall's code to keep the mean unemployment rate unchanged.)

The idea behind Hall's mechanism is simple: in downturns the user cost of labor does not fall, even though the present value of what a worker will produce over the course of a match greatly falls. Hence, firms greatly contract their vacancies in recessions. Such a mechanism, though, is inconsistent with the evidence on the cyclicality of the user cost of labor.

C. Efficiency under Alternating Offer Bargaining

We have shown that the results in Hall (2017) depend critically on the duration of a job opportunity, $1/\delta$. When this duration is short, the model generate very small fluctuations in unemployment, whereas when it is long, the model generates large fluctuations. Here we link this key parameter to the efficiency of the resulting allocations: when the duration is short, allocations are close to efficient and thus close to the competitive search ones, but when the duration is long, allocations are very inefficient.

Proposition 5. *When the probability p that the worker makes the first offer equals the elasticity of the matching function with respect to the measure of unemployed workers, then the allocations in a sequence of bargaining games indexed by the breakdown probabilities $\{\delta_n\}_{n=1}^{\infty}$ converge to the constrained efficient allocations as δ_n converges to one.*

Recall that an allocation is constrained efficient if it solves the restricted planning problem of Proposition 1. Proposition 5 directly applies to the model in Hall (2017) with an exogenous discount factor. It also applies to all five versions of our model if we modify the equilibrium concept from that of competitive search equilibrium to that of alternating offer bargaining equilibrium. For our model with baseline preferences, Epstein-Zin preferences with long-run risk, and Epstein-Zin preferences with disaster risk, these allocations also converge to the efficient allocations.

This result offers an additional interpretation of the results in Table 10, namely, that inefficiencies are central to the amplification mechanism in Hall (2017): the lower is $1/\delta$, the more efficient are the allocations in Hall (2017), the smaller is the impact of changes in the stochastic discount factor on the volatility of the job-finding rate, and so the lower is the volatility of unemployment—by Proposition 5, allocations are efficient when $\delta = 1$. Indeed, for Hall's model to generate the observed volatility of unemployment, the economy has to be very inefficient in that the duration of a job opportunity has to be 6.2 years rather than one month.

We can shed further light on the mechanism in Hall (2017) by solving for the time-varying Nash bargaining weights of workers and firms that produce the job-finding rates in the alternating offer bargaining equilibrium. Recall that the efficient allocations are achieved under Nash bargaining with a constant bargaining weight equal to η, which equals 1/2 in both Hall's and our parametrizations. In Figure 7, we plot this time-varying Nash bargaining weight for a worker in Hall's economy. We see that in deep downturns, the worker's bargaining weight *increases* sharply relative to its level in booms. Thus, a key intuition for Hall's mechanism is that firms understand that during downturns workers will demand much larger surplus shares in order to accept a job. Anticipating such behavior, firms drastically cut vacancies and so unemployment drops.

12. Comparison with the Differential Productivity Mechanism of Search Models

We now show that the mechanism of our model works quite differently from those of the models addressing the Shimer puzzle examined by Ljungqvist and Sargent (2017). That literature builds in a *differential productivity across sectors* mechanism. Specifically, it assumes that an increase in productivity leads to an increase in the productivity of working in the market relative to both the productivity of working at home and the cost of posting vacancies. Then, as Shimer (2005, p. 25) explains, "an increase in labor productivity relative to the value of nonmarket activity and to the cost of advertising a job vacancy makes unemployment relatively expensive and vacancies relatively cheap. The market substitutes toward vacancies." That is, in a boom, the differential increase in productivity in the market draws workers out of nonmarket activity and into the market.

In that literature, authors compute the steady-state response of the job-finding rate and unemployment to a steady-state change in aggregate productivity. We show that our model works differently by proving two results. First, if we perform the same steady-state experiment in our model, we obtain no change in the job-finding rate. Second, once we modify the models in Ljungqvist and Sargent (2017) so that productivity enters those models as it does ours, then in the basic matching model and the alternating offer bargaining model of Hall and Milgrom (2008), a change in steady-state productivity has similarly no effect on the job-finding rate. In the Appendix, we show an analogous result for the training cost model of Pissarides (2009), also reviewed by Ljungqvist and Sargent (2017). (Note that our results are reminiscent of the result on the neutrality of productivity shocks by Shimer 2010. See also a related intuition by Ljungqvist and Sargent 2017 in footnote 28 of their paper, page 2664.)

A Steady-State Change in Aggregate Productivity in our Baseline Model. We consider the experiment conducted by Ljungqvist and Sargent (2017) in our model, namely a steady-state increase in A, and obtain the following result. For simplicity, we abstract from growth.

Proposition 6. *In our baseline model, the steady-state levels of the job-finding rate and unemployment are independent of steady-state productivity, A.*

To see why, note that $Q_{t,t+1} = \beta$ at a steady state where $S_t = S$ and $C_t = C$ by (14). Evaluating the expression for the job-finding rate in (36) at a steady state gives $\log(\lambda_w) = \chi + (1-\eta)\log\left((\mu_e - \mu_u)/A\right)/\eta$, where μ_e and μ_e are the steady-state versions of (33) and (34), namely,

$$(57)\quad \frac{\mu_e}{A} = 1 + \phi(1+g_e)\beta\Big[(1-\sigma)\left(\frac{\mu_e}{A}\right) + \sigma\left(\frac{\mu_u}{A}\right)\Big] \text{ and } \left(\frac{\mu_u}{A}\right) = b + \phi(1+g_u)\beta\Big[\eta\lambda_w\left(\frac{\mu_e}{A}\right) + (1-\eta\lambda_w)\left(\frac{\mu_u}{A}\right)\Big].$$

Clearly, $(\mu_e - \mu_u)/A$ is independent of A and so is the job-finding rate. Notice that key to this result is that the steady-state value of the discount factor does not vary with the steady-state value of A. Since this same property holds for a broad class of consumption-based discount factors, including all of those considered here, all of these discount factors are consistent with Proposition 6.

Basic Matching Model. Consider the *basic matching model* in Ljungqvist and Sargent (2017). Using notation similar to ours, in this model consumers are risk neutral with discount factor β. A consumer produces A units of output when employed and b units of output when unemployed. The cost of posting a vacancy is κ, the exogenous separation rate is σ, the worker's bargaining share is γ, and the job-filling rate for a firm is $\lambda_f(\theta)$ given market tightness θ. Equation (12) in Ljungqvist and Sargent (2017, p. 2635) shows that the equilibrium value of market tightness is determined by the free-entry condition, which we rearrange and express as

$$(58)\quad \kappa = (1-\gamma)\lambda_f(\theta)\frac{\beta(A-b)}{1-\beta[1-\sigma-\gamma\theta\lambda_f(\theta)]}.$$

These authors then differentiate this equation to derive $d\log(\theta)/d\log(A)$ and explain how their measure of *fundamental surplus* given by $A-b$ is critical for understanding the magnitude of this derivative. In contrast, in our model the output produced in the market and the cost of posting a vacancy are proportional to productivity so that b and κ are replaced by bA and κA, respectively. Observe that scaling home production b by A is consistent with the findings in Chodorow-Reich and Karabarbounis (2016), as discussed earlier. Scaling κ by A is consistent with the view in Shimer (2010) that posting vacancies absorbs a fixed amount of workers' time in recruiting that could otherwise be devoted to producing goods. When this is the case, the free-entry condition becomes

$$(59)\quad \kappa A = (1-\gamma)\lambda_f(\theta)\frac{\beta(1-b)A}{1-\beta[1-\sigma-\gamma\theta\lambda_f(\theta)]}.$$

Since A cancels out from both sides of this equality, θ is constant and thus $d\log(\theta)/d\log(A) = 0$.

Proposition 7. *In the basic matching model, if home produced output and the cost of posting a vacancy are proportional to productivity, then the change in steady-state unemployment with respect to a change in steady-state productivity is zero regardless of all other parameters.*

Note that this result holds regardless of the size of the home production parameter b, which plays an important role in the debate that originated with Shimer (2005) and Hagedorn and Manovskii (2008). More generally, this property holds independently of the size of the fundamental surplus, which, instead, is central to the analysis in Ljungqvist and Sargent (2017).

Hall and Milgrom (2008): Alternating Offer Bargaining Model. A similar result also applies to alternating offer bargaining models. Consider the exposition in Ljungqvist and Sargent (2017) of Hall and Milgrom (2008). In this model, firms and workers make alternating offers and after each unsuccessful bargaining round, the firm incurs a haggling cost of ψ of making a new offer while the worker receives b. There is a probability δ that the job opportunity exogenously expires across bargaining rounds and the worker reenters unemployment. Ljungqvist and Sargent (2017) assume that $\delta = \sigma$ so the probability that a job opportunity expires equals the probability of exogenous separation between a firm and a worker. Under this assumption, the free-entry condition (equation (36), p. 2648 of Ljungqvist and Sargent 2017) can be rearranged to obtain

$$(60) \quad \kappa = \frac{\lambda_f(\theta)\beta}{1-\beta(1-\sigma)}\left[A - \frac{b+\beta(1-\sigma)(A+\psi)}{1+\beta(1-\sigma)}\right].$$

Now, suppose we extend the earlier idea in Shimer (2010) that recruiting workers takes a fixed amount of an existing worker's time to the idea that each round of bargaining also absorbs a fixed amount of a worker's time in haggling. Under this interpretation, it is natural to scale both κ and ψ by A, since both parameters reflect the foregone opportunity of producing goods for a worker engaged in either recruiting or bargaining. Hence, (60) becomes

$$(61) \quad \kappa A = \frac{\lambda_f(\theta)\beta}{1-\beta(1-\sigma)}\left[1 - \frac{b+\beta(1-\sigma)(1+\psi)}{1+\beta(1-\sigma)}\right]A.$$

Since A cancels out from both sides of this equality, θ is constant and so $d\log(\theta)/d\log(A) = 0$. Note that this same result holds even if δ does not equal σ because all value functions are proportional to A.

Proposition 8. *In the alternating offer bargaining model, if home produced output, the cost of posting a vacancy, and the haggling cost are proportional to productivity, then the change in steady-state unemployment with respect to a change in steady-state productivity is zero regardless of all other parameters.*

In sum, our model produces large movements in response to productivity changes but works differently from those analyzed by Ljungqvist and Sargent (2017) in their excellent synthesis of the work on the unemployment volatility puzzle. All of these models depend critically on the differential productivity mechanism, while ours does not.

13. Conclusion

We propose a new mechanism that allows search models to reproduce the observed fluctuations in the job-finding rate and unemployment at business cycle frequencies. Our model not only solves the unemployment volatility puzzle of Shimer (2005) but also is immune to the critiques of existing mechanisms that address it, namely, those by Chodorow-Reich and Karabarbounis (2016) on the cyclicality of the opportunity cost of employment, by Kudlyak (2014) and Basu and House (2016) on the cyclicality of wages, and by Borovicka and Borovickova (2019) on the asset pricing implications of existing mechanisms.

To this purpose, we augment the textbook search model with two features: preferences from the macro-finance literature that match the observed variation in asset prices, and human capital accumulation on the job that is consistent with longitudinal and cross-sectional evidence on wage growth with experience. In such a framework, investing in hiring workers becomes a risky endeavor with long duration flows of the surplus from a match between a worker and a firm. Hence, shocks to either productivity or directly to discount factors make the present value of these surplus flows fluctuate sharply over the cycle. In turn, fluctuations in the present value of these surplus flows imply that investments in hiring workers are highly cyclical and, hence, that job-finding rates and unemployment are as volatile as in the data. We show that both new features we introduce play a critical role: if we abstract from either preferences that generate time-varying risk or human capital accumulation, the model generates only negligible movements in unemployment. We show that the same intuition applies once we augment the model with physical capital. Overall, our results show that re-integrating search and business cycle theory is both a tractable and promising avenue of future research.

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Table 1: Parametrization and Results for Model with Campbell-Cochrane Preferences with Exogenous Habit

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
B, efficiency of matching technology	0.455	Mean job-finding rate	0.46	0.46
κ, hiring cost	0.975	Mean unemployment rate	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean of state S_t	0.2066	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	5.0	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.60
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.98
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.75
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, human capital growth when employed (%p.a.)	3.5	Elasticity user cost labor to u (Kudlyak)	-5.2	-3.6

Note: Labor productivity in the data is measured as nonfarm business sector real output per hour from the BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ.

Table 2: Role of Preferences and Human Capital Accumulation

			Alternative Preferences					Alternative Human Capital	
	Data	Baseline	CRRA	CC	EZ w/ LRR	EZ w/ Disasters	Affine SDF	$g_e = 0$ $g_u = 0$	$g_e = .035$ $g_u = .035$
S.d. job-finding rate	6.66	6.60	0.00	6.69	6.36	5.66	7.52	0.15	0.15
Autocorr. job-finding rate	0.94	0.98	—	0.99	0.99	0.99	0.99	0.99	0.99
S.d. unemployment rate	0.75	0.75	0.00	0.75	0.69	0.77	0.73	0.02	0.02
Autocorr. unemployment rate	0.97	0.99	—	0.99	0.99	0.99	0.99	0.99	0.99
Correlation u, job-finding rate	-0.96	-0.98	—	-0.98	-0.98	-0.98	-0.97	-0.98	-0.98

Note: For the results in the last two columns, we adjust the value of κ to keep mean unemployment constant.

Table 3: Implications of Baseline Model for Stock Prices

Statistics	Data	Model
Mean excess return (%p.a.)	6.96	6.30
S.d. excess return (%p.a.)	15.6	14.1
Mean excess return / s.d. excess return (p.a.)	0.45	0.45
Mean log price-dividend ratio	3.51	3.36
S.d. log price-dividend ratio	0.44	0.36

Note: The data cover January 1947 to December 2007 and refers to statistics for the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX and NASDAQ and statistics for the market value of outstanding equities and net equity payouts from the Flow of Funds. A consumption claim is a claim to the aggregate consumption process. The equity index claim is a claim to aggregate net equity payouts.

Table 4: Parameterization and Results for More General Human Capital Model and Preferences with Exogenous Habit

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
κ, hiring cost	1.60	Mean unemployment rate	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean of state S_t	0.1779	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	4.5	Maximum Sharpe ratio (p.a.)	0.45	0.45
$\bar{z}_e$, human capital growth when employed	1.90	Log wage difference 0-30 years of experience	1.21	1.21
ρ_z, persistence human capital	0.9969	Log wages difference 0-10 years of experience (%)	0.55	0.55
σ_z, std. dev. of human capital shocks	0.033	Std. dev. annual w growth 1-10 years exp. (p.p.p.a.)	1.19	1.19
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.38
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.65
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.99
B, efficiency of matching technology	0.455			

Note: The standard deviation of annual wage growth is computed in percentage points per annum (p.p.p.a.).

Table 5: Parametrization and Results for Model with Physical Capital and Preferences with Exogenous Habit

Panel A: Parameters		Panel B: Moments		
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	1.36	Mean productivity growth (%p.a.)	1.36	1.36
σ_a, s.d. productivity growth (%p.a.)	1.79	S.d. productivity growth (%p.a.)	1.79	1.79
B, efficiency of matching technology	0.455	Mean job-finding rate	0.46	0.46
κ, hiring cost	2.0	Mean unemployment rate	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (%p.a.)	0.92	0.92
S, mean surplus consumption	0.2887	S.d. risk-free rate (%p.a.)	2.31	2.31
α, inverse EIS	7.0	Maximum Sharpe ratio (p.a.)	0.45	0.45
γ, curvature of production function	0.25	Mean labor share of output	0.70	0.70
ξ, curvature of adjustment cost	0.25	Ratio s.d. invest. to consumption growth	4.5	4.5
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.66	6.45
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.71
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, state persistence	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.98
δ, physical capital depreciation rate	0.1/12			
g_e, human capital growth when employed (%p.a.)	3.5			

Note: Labor productivity in the data is measured as total factor productivity from John Fernald's website. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ.

Table 6: Parametrization and Results for Model with Campbell-Cochrane Preferences with External Habit

Panel A: Parameters

Endogenously Chosen	
g_a, mean productivity growth (%p.a.)	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84
B, efficiency of matching technology	0.455
κ, hiring cost	0.975
β, time preference factor	0.999
$\bar{S}$, mean surplus consumption	0.2087
α, inverse EIS	5.0
Assigned	
b, home production parameter	0.6
σ, probability of separation	0.028
η, matching function elasticity	0.5
ϕ, survival probability	0.9972
ρ_s, habit persistence	0.9944
g_e, human capital growth when employed (%p.a.)	3.5

Panel B: Moments

Targeted	Data	Model
Mean productivity growth (%p.a.)	2.22	2.22
S.d. productivity growth (%p.a.)	1.84	1.84
Mean job-finding rate	0.46	0.46
Mean unemployment rate	5.9	5.9
Mean risk-free rate (%p.a.)	0.92	0.92
S.d. risk-free rate (%p.a.)	2.31	2.31
Maximum Sharpe ratio (p.a.)	0.45	0.45
Results		
S.d. job-finding rate	6.66	6.69
Autocorrelation job-finding rate	0.94	0.99
S.d. unemployment rate	0.75	0.75
Autocorrelation unemployment rate	0.97	0.99
Correlation unemployment, job-finding rate	-0.96	-0.98

Note: Labor productivity in the data is measured as nonfarm business sector real output per hour from the BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ.

Table 7: Parametrization and Results for Model with Epstein-Zin Preferences with Long-Run Risk

Panel A: Parameters

Endogenously Chosen	
g_a, mean productivity growth (%p.a.)	2.22
σ_a, s.d. productivity growth (%p.a.)	1.80
B, efficiency of matching technology	0.455
κ, hiring cost	1.31
β, time preference factor	0.998
ϕ_s, relative s.d. s_t	0.0379
α, risk aversion coefficient	4.3
Assigned	
b, home production parameter	0.6
σ, probability of separation	0.028
η, matching function elasticity	0.5
ϕ, survival probability	0.9972
ρ_x, persistence of x_t	0.9977
ρ_s, persistence of s_t	0.9944
g_e, human capital growth when employed (%p.a.)	3.5
ρ, inverse EIS	0.1
rel. size x_t component of productivity	0.0445

Panel B: Moments

Targeted	Data	Model
Mean productivity growth (%p.a.)	2.22	2.22
S.d. productivity growth (%p.a.)	1.84	1.84
Mean job-finding rate	0.46	0.46
Mean unemployment rate	5.9	5.9
Mean risk-free rate (%p.a.)	0.92	0.92
S.d. risk-free rate (%p.a.)	2.31	2.31
Maximum Sharpe ratio (p.a.)	0.45	0.45
Results		
S.d. job-finding rate	6.66	6.36
Autocorrelation job-finding rate	0.94	0.99
S.d. unemployment rate	0.75	0.69
Autocorrelation unemployment rate	0.97	0.99
Correlation unemployment, job-finding rate	-0.96	-0.98

Note: Labor productivity in the data is measured as nonfarm business sector real output per hour from the BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ. The productivity process is governed by $\Delta a_{t+1} = g_a + x_t + \sigma_a \varepsilon_{at+1}$ where $x_{t+1} = \rho_x x_t + \phi_x \sigma_a \varepsilon_{xt+1}$ and the preference shock is governed by $\Delta s_{t+1} = \rho_s \Delta s_t + \phi_s \sigma_a \varepsilon_{st+1}$.

Table 8: Parametrization and Results for Model with Epstein-Zin Preferences with Variable Disaster Risk

Panel A: Parameters		Panel B: Moments	Data	Model
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
B, efficiency of matching technology	0.455	Mean job-finding rate	0.46	0.46
κ, hiring cost	1.22	Mean unemployment rate	5.9	5.9
β, time preference factor	0.998	Mean risk-free rate (%p.a.)	0.92	0.92
σ_s, disaster intensity volatility parameter	0.0083	S.d. risk-free rate (%p.a.)	2.31	2.31
α, risk aversion coefficient	2.65	Maximum Sharpe ratio (p.a.)	0.45	0.45
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.66	5.66
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.77
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, persistence disaster intensity	0.9966	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, human capital growth when employed (%p.a.)	3.5			
ρ, inverse EIS	0.1			
s, disaster intensity (%p.a.)	3.55			
θ, disaster impact	0.26			

Note: Labor productivity in the data is measured as nonfarm business sector real output per hour from the BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ. The productivity process is governed by $\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{at+1} - \theta j_{t+1}$ with variable disaster intensity and the state process is governed by $s_{t+1} = (1-\rho_s)s + \rho_s s_t + \sqrt{s_t}\sigma_s \varepsilon_{st+1}$.

Table 9: Parametrization and Results for Model with Affine Stochastic Discounts

Panel A: Parameters		Panel B: Moments	Data	Model
Endogenously Chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.84	S.d. productivity growth (%p.a.)	1.84	1.84
B, efficiency of matching technology	0.455	Mean job-finding rate	0.46	0.46
κ, hiring cost	0.90	Mean unemployment rate	5.9	5.9
μ_0	.00077	Mean risk-free rate (%p.a.)	0.92	0.92
μ_1	-.042	S.d. risk-free rate (%p.a.)	2.31	2.31
γ_0	25.6	Maximum Sharpe ratio (p.a.)	0.45	0.45
γ_1	0.83	S.d. excess return	15.6	15.6
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.66	7.52
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate	0.75	0.73
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.97	0.99
ρ_s, state persistence	0.9944	Correlation unemployment, job-finding rate	-0.96	-0.98
g_e, human capital growth when employed (%p.a.)	3.5			

Note: Labor productivity in the data is measured as nonfarm business sector real output per hour from the BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ.

Table 10: Hall (2017) with Alternative Durations of Job Opportunities

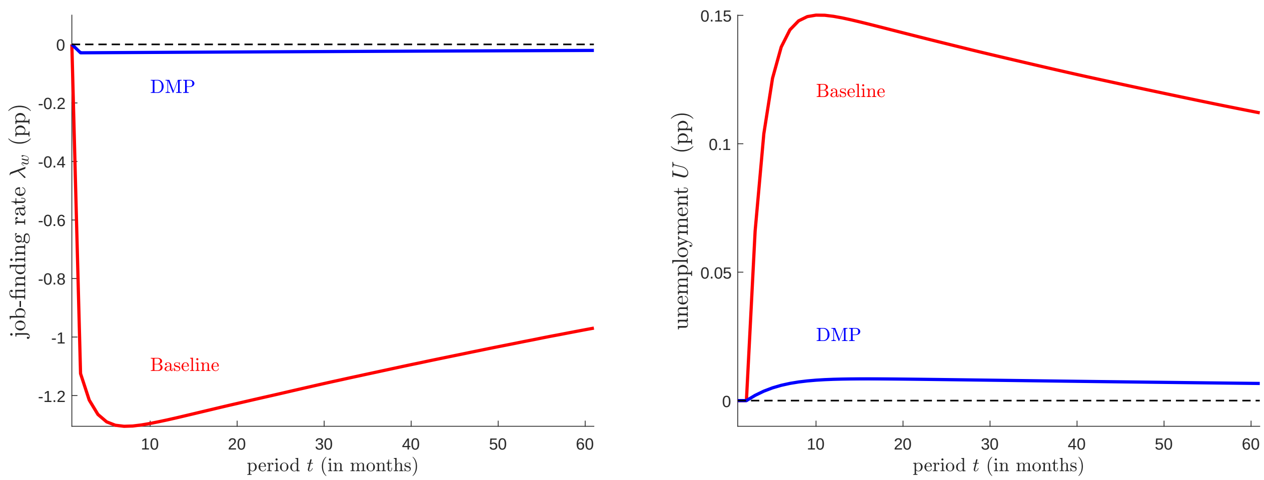
	Data	Model in Hall (2017)	
		$1/\delta = 2.6$	Original
Parameters			
Avg. duration of job opportunity during bargaining (in months)	–	2.6	77
Per-round probability bargaining ends, δ	–	1/2.6	1/77
Bargaining delay cost, ψ	–	1.01	0.57
Results			
S.d. quarterly unemployment rate (in pp)	0.75	0.03	0.97
Cyclicality of user cost of labor to unemployment (%)	-5.2	-5.2	0.10

Note: The probability that a job opportunity breaks down after n rounds of bargaining is $\delta(1-\delta)^n$ so the expected duration of a job opportunity during bargaining is $\delta + 2\delta(1-\delta) + \ldots + n\delta(1-\delta)^{n-1} + \ldots = 1/\delta$ rounds.

Figure 1: Responses to Productivity Shock for Preferences with Exogenous Habit

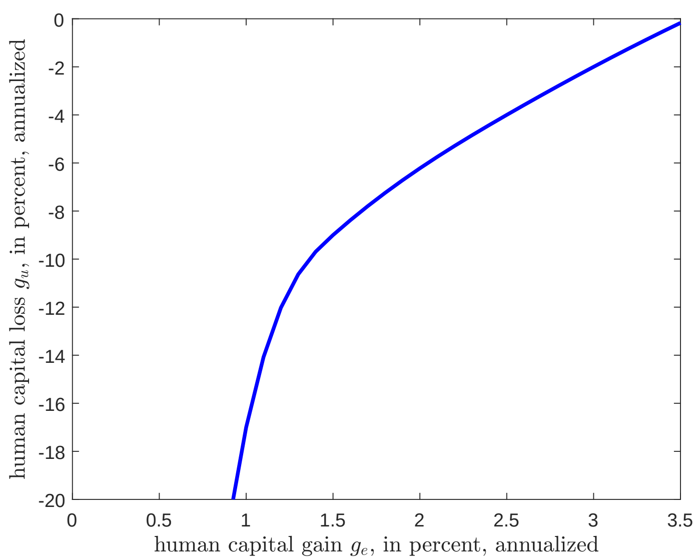
(a) Job-Finding Rate

(b) Unemployment



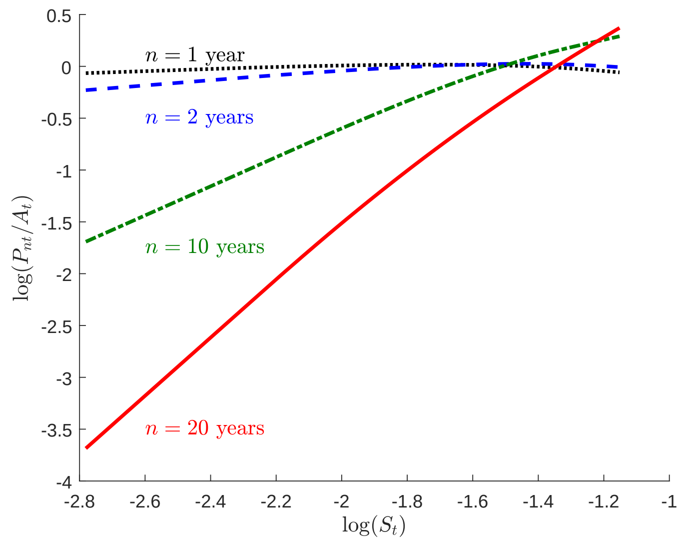
Note: Impulse responses of the job-finding rate and unemployment to a -1% permanent productivity shock. Generalized impulse response functions are based on 10,000 simulations.

Figure 2: Locus of (g_e, g_u) with Same Job-Finding Rate Volatility as in Baseline Model



Note: We vary g_e and g_u while keeping the remaining parameters at their baseline values except for κ, which is adjusted to reproduce the same mean unemployment rate across parametrizations.

Figure 3: Prices of Productivity Strips for Preferences with Exogenous Habit

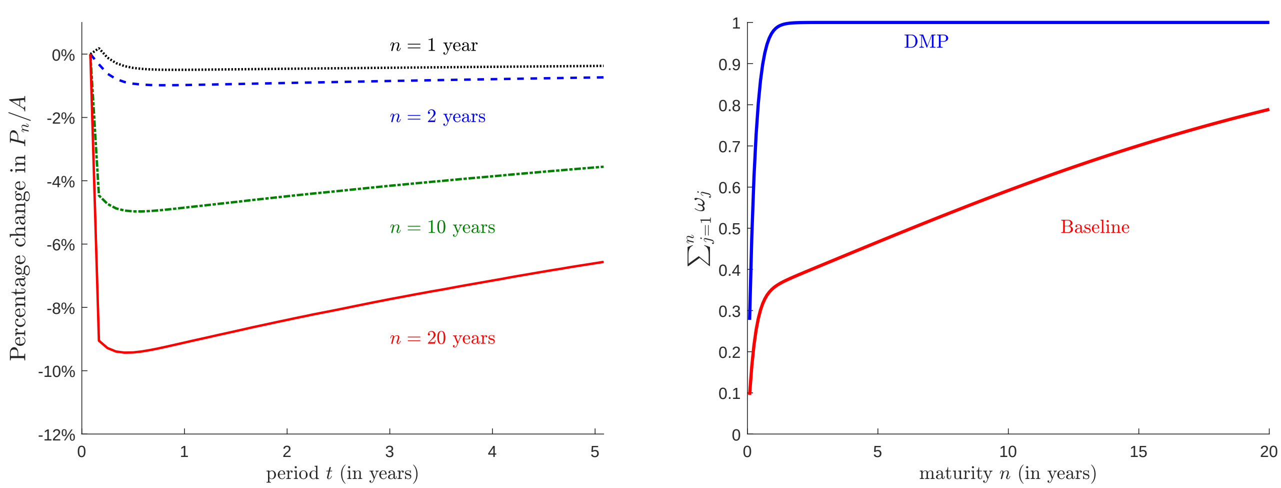


Note: Price of underlying claims to productivity at different horizons as function of the state.

Figure 4: Responses to Productivity Shock for Preferences with Exogenous Habit

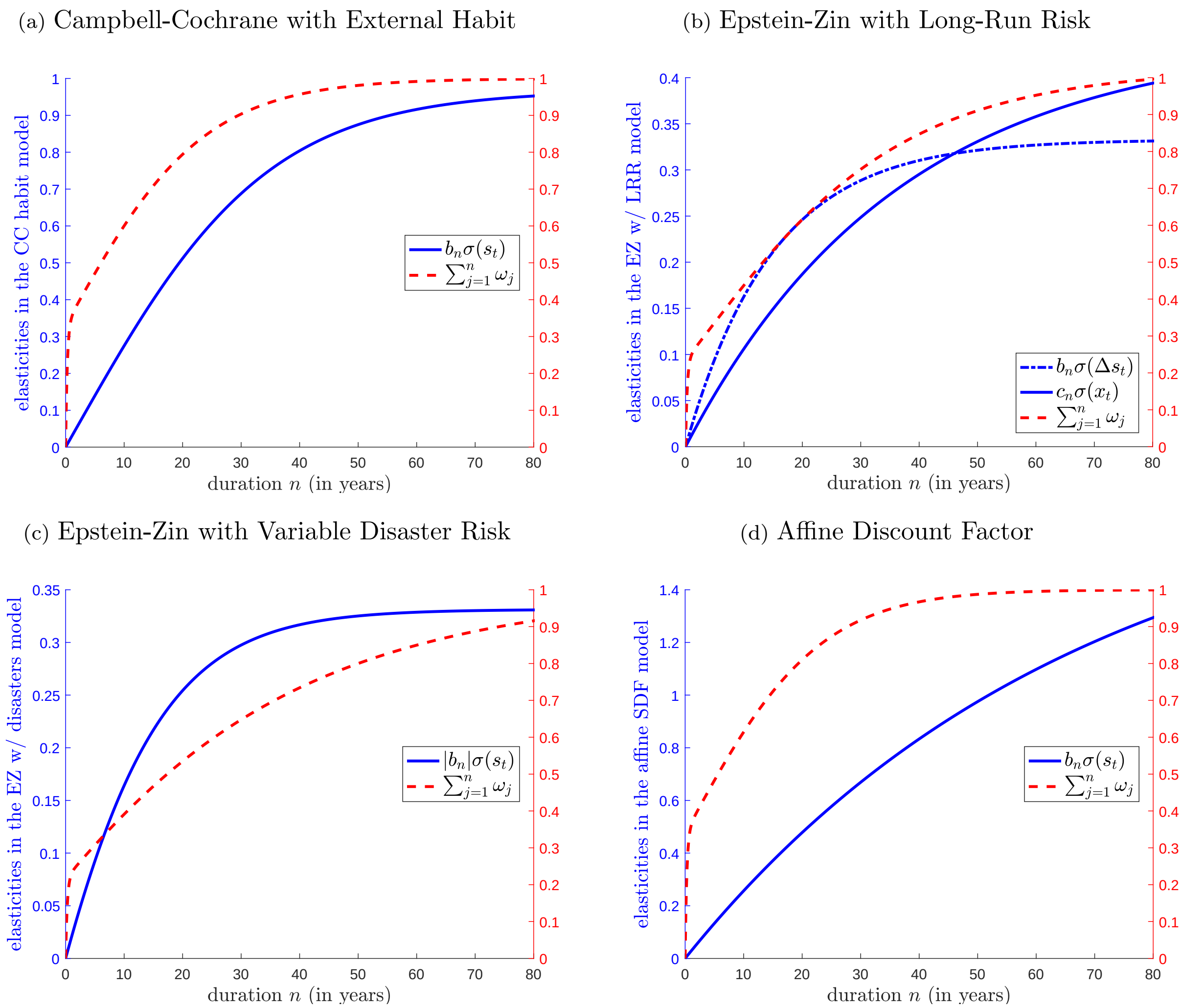
(a) Prices of Productivity Strips by Maturity

(b) Cumulative Weights by Maturity



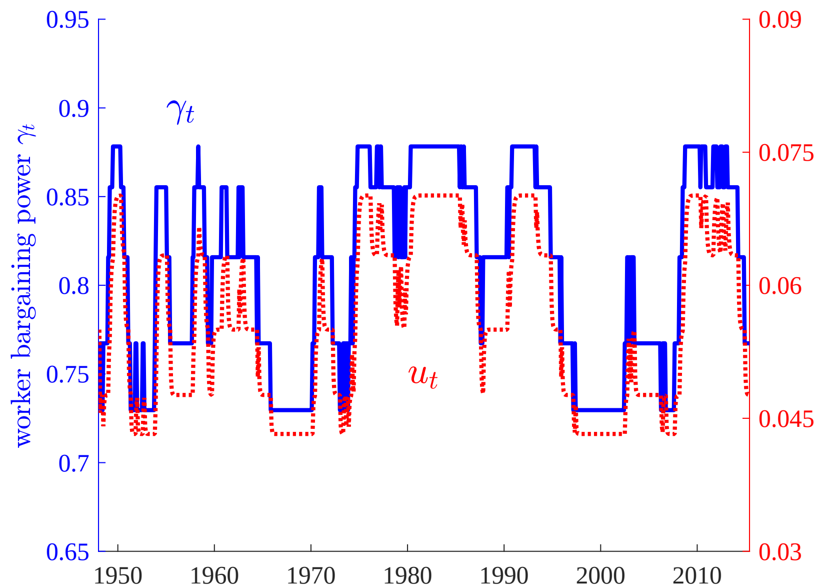
Note: Impulse responses of the prices of productivity strips to a -1% permanent productivity shock. Generalized impulse response functions are based on 10,000 simulations.

Figure 5: Determinants of Volatility of Job-Finding Rate



Note: $\sigma(\lambda_{wt}) = |\sum_{n=1}^{\infty}\omega_n b_n|\,\sigma(s_t)$ for Campbell-Cochrane preferences with external habit, affine stochastic discount factor, and Epstein-Zin preferences with variable disaster risk, and $\sigma(\lambda_{wt}) = |\sum_{n=1}^{\infty}\omega_n b_n|\,\sigma(\Delta s_t) + |\sum_{n=1}^{\infty}\omega_n c_n|\,\sigma(x_t)$ for Epstein-Zin preferences with long-run risk.

Figure 6: Time-Varying Worker Bargaining Power in Hall (2017)



Note: Constructed from data in Hall (2017).