The macroprudential toolkit: Effectiveness and interactions

Stephen Millard* Margarita Rubio† Alexandra Varadi‡

Abstract

We use a DSGE model with financial frictions, leverage limits on banks, loan-to-value limits and debt-service ratio (DSR) limits on mortgage borrowing, to examine: i) the effects of different macroprudential policies on key macro aggregates; ii) the interaction of DSR limits with the rest of the macroprudential toolkit and with monetary policy; and iii) the effects of various macroprudential tools on welfare. We find that both capital requirements and DSR limits reduce the need for monetary policy to react to a housing demand shock, and that DSR limits also contribute to monetary stability when the economy is hit by technology shocks. Additionally, we find that introducing capital requirements on banks reduces the volatility of lending, house prices, output and inflation only marginally relative to an LTV ratio on mortgage borrowing. Finally, we show that DSR limits lead to an increase in the volatility of real house prices and to a significant decrease in the volatility of lending, consumption and inflation, since they remove the link between house price shocks and mortgage borrowing. Overall, DSR limits are welfare improving relative to any other macroprudential tool.

Keywords: Macroprudential Policy, Monetary Policy, Leverage Ratio, Affordability Constraint, Collateral Constraint.

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*Bank of England, Durham University Business School and Centre for Macroeconomics; Email: stephen.millard@bankofengland.co.uk.

†Nottingham University; Email: Margarita.Rubio@nottingham.ac.uk.

‡Wadham College, Oxford; Email: alexandra.varadi@wadham.ox.ac.uk.

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1 Introduction and motivation

Since the global financial crisis, policymakers have designed macroprudential policies that help stabilise debt and prevent or lessen the impact of future financial shocks. However, with many of these policies still untested, policymakers are facing the challenge of understanding their interactions with monetary policy or with the rest of the macroprudential toolkit. The task is even harder when, unlike for monetary policy, the objectives of macroprudential policy are much broader in nature and cannot be defined numerically. For example, in the UK the objectives of the macroprudential policymaker, the Financial Policy Committee (FPC), is to identify and address all risks to financial stability while remaining mindful to the effects it has on the wider economy.

The ample range of potential risks to be monitored and addressed as well as the availability of multiple macroprudential tools adds complexity to the task of choosing optimal policy by central bankers. Additionally, setting policy is even more complicated if, as in the UK, the remit of the macroprudential policymaker includes risks from both the financial sector and household balance sheets. As household behaviour can affect both the resilience of financial intermediaries and the wider economy via aggregate demand effects, they will also be of interest to the monetary policymaker. Thus, these risks could be addressed by both monetary and macroprudential tools. This raises the importance of optimal policy interaction to achieve both financial and price stability.

This paper contributes to the existing literature on the optimal use of monetary and macroprudential policy by considering a comprehensive macroprudential toolkit that includes collateral constraints, capital requirements for banks and affordability constraints on mortgage borrowers. Our setup allows us to explore a rich set of interactions between policies acting on bank balance sheets, household balance sheet and firms' production decisions. To the standard DSGE model of Smets and Wouters (2007), we follow Iacoviello (2015) and add household borrowing subject to a collateral constraint in the form of a loan-to-value (LTV) limit. We also add an endogenous leverage constraint on banks, resulting from the possibility of bank runs a la Gertler and Karadi (2105). The financial and real frictions in the model give rise to meaningful roles for macroprudential policy and monetary policy. However, unlike the existing academic literature we model the actual policy toolkit used by central banks at the moment. We do this by augmenting the model in two important ways.

First, we add capital requirements on banks. We do this via a maximum leverage ratio set by the policy maker. Further, we assume that banks see leverage limits as an absolute maximum and they will expend effort (ie, incur costs) to avoid reaching it. This approach ties in with the data, as in practice banks keep excess capital buffers over and above their capital requirements.

Second, in addition to the LTV limit, we examine the role of affordability constraints on mortgage lending and their interaction with monetary policy. Most of the existing literature on household and bank leverage has considered the policy design of either LTV limits or capital requirements. But affordability constraints can be used to stress test households' debt levels. We follow the current macroprudential framework in the UK and model affordability constraints as stressed debt-service ratios (DSR)[2] on households' balance sheets. We augment the standard DSR measure which captures debt repayments as a proportion of labour income, by adding a fixed buffer on top of the mortgage interest rate. This tests whether borrowers can still afford their mortgage payments should credit conditions tighten. Additionally, a change in the monetary policy rate will have a direct effect on DSR ratios by increasing interest repayments. As such, adding this tool in the model introduces an additional channel of monetary and macroprudential policy interaction, which is missing in the literature with just collateral constraints.

Affordability constraints were introduced in the UK in June 2014 (Financial Stability Report, 2017). The FPC argued that this tool allows them to guard against an increase in the number of highly-indebted households. A high proportion of highly leveraged households can lead to demand externalities if they are forced to deleverage following a negative aggregate shock, cutting back on spending and amplifying the economic bust. The FPC did not expect their recommendation to restrain housing market activity unless lending standards declined. We interpret this as implying that the LTV limit will be the usual binding constraint on lending but that the affordability constraint would 'kick in' if lending rose too strongly relative to income.

There are two key issues we examine in this paper. First, we investigate the interaction of macroprudential tools with each other and with monetary policy. Second, we examine the gains from adding each policy to the macroprudential toolkit in terms of reducing the volatility of key macroeconomic variables. In order to assess the impact of the different macroprudential policy tools and their interaction with each other, we adopt the following approach. We first develop a baseline model in which we have frictions in the banking and the housing sectors. We then consider the impact of adding a maximum leverage ratio on banks imposed by the macroprudential policymaker. Next, we examine the impact of introducing DSR limits on household borrowing either as a sole macroprudential policy, or together with capital requirements. In each case we examine the volatilities of household borrowing, house prices, output and inflation as well as welfare. To understand the interaction between different tools, we examine the responses of macroeconomic variables to productivity, housing demand and monetary policy shocks.

The remainder of the paper is structured as follows. In the next section, we briefly review the literature that is most relevant to our paper before going on to describe the model in Section 3. Section 4 derives a welfare-based loss function against which we can assess our macroprudential policy tools. Section 5 describes our quantitative experiments, starting with a description of the calibration and the simulation methodology before describing our results. Section 6 concludes.

[2]We use affordability constraints and debt-service ratios limits interchangeably throughout the paper

2 Literature review

In this section, we review some of the existing literature on macroprudential policy tools that is most relevant to this paper.

A substantial corpus of evidence establishes the existence of quantitatively relevant channels through which macroprudential tools might influence aggregate demand and through which monetary policy might influence bank profitability and risk-taking (eg, Woodford (2011), Curdia and Woodford (2009), Korinek and Simsek (2014) and Farhi and Werning (2016)). In particular some authors (Angelini et al (2014), Rubio and Carrasco-Gallego (2015), Rubio and Yao (2019), De Paoli and Paustian (2017) and Carrillo, Mendoza, Nuguer and Roldan-Pena (2017)) have explicitly turned to the question of how monetary and macroprudential policies should be coordinated in a world featuring both nominal rigidities and financial frictions. These papers evaluate the optimal policy response of monetary policy and macroprudential actions either on LTV limits or on capital requirements when the economy is faced with aggregate shocks, such as to productivity or monetary policy. In most of these papers, the objective of the macroprudential policy is to avoid excessive lending, that is, to minimize the variances of total lending or the ratio of loans to output. The extent to which policies are complementary or substitutes for each other, depends on the nature of the shock. For example, shocks to net worth or productivity create no tension between policies targeting output and inflation on the one side and bank lending on the other. However, there are welfare losses when the committees are non-cooperative in the case of cost-push shocks. In this case monetary and macroprudential policies become strategic complements with both policies tightened more than in the case of coordination.

Our model contributes to this literature in two important ways. First, we introduce DSR limits on household balance sheets to limit mortgage borrowing. This tool acts to reduce the overall indebtedness of the household sector relative to nominal income. It is different from collateral constraints because it is not related to house prices. By modeling this tool as a regulatory stress rate buffer on existing mortgage rates rather than a standard loan-to-income limit, we introduce additional interactions between macroprudential and monetary policy. Second, we consider the interaction of monetary policy with a rich macroprudential toolkit. This allows us to examine not only the coordination between macroprudential and monetary policy tools, but also the optimal interaction of policies within the macroprudential toolkit.

To our knowledge, affordability constraints have not been addressed in the literature so far, although some authors have examined tools acting on limiting household debt relative to income. Ingholt (2017) compares LTV limits on mortgage lending with LTI limits in terms of smoothing responses to shocks. Greenwald (2018) also examines a mortgage-payments-to-income limit in a DSGE model, and finds that it amplifies the transmission mechanism from policy rates to debt, house prices and economic activity. The paper also finds that a relaxation of payments-to-income standards is essential to match the recent boom. Fazio et al. (2019) study the impact of debt limits on housing markets and find that they might have distributional effects.

However, unlike our model, neither of these papers has a banking sector.

In terms of model setup, there are two papers that use a similar model to ours in the literature on policy coordination. First, Ferrero, Harrison and Nelson (2017) introduce a DSGE model with housing, heterogeneous households, macroprudential loan-to-value and capital tools on financial intermediaries, to study how monetary and macroprudential policies should optimally respond to shocks. The authors derive a welfare-based loss function containing five (quadratic) terms. Two of them stem from the standard NK model where the policymaker seeks to stabilize the output gap and inflation. The remaining terms stem from the desire of the policymaker to stabilize the distribution of non-durable consumption and housing consumption between borrowers and savers. Monetary policy is constrained by the zero bound. In a similar fashion, Rubio and Yao (2019) also study optimal macroprudential and monetary policy in a low interest-rate environment.

Second, Gelain and Ilbas (2017) study the implications of macroprudential policy in the context of an estimated Smets and Wouters (2007) type DSGE model for the United States, featuring a financial intermediation sector, subject to Gertler and Karadi (2011) financial frictions. Macroprudential policy aims at stabilizing nominal credit growth and the output gap by setting a lump-sum levy on bank capital. Monetary policy pursues a standard inflation targeting mandate using the short term interest rate. The paper focuses on testing how the variations in the macroprudential objectives affect the coordination between macro and monetary policies. In addition, the paper derives optimal policy rules and optimal weights under the assumption that the two policy makers have no possibility to coordinate. In both papers macroprudential policy is always binding and the interaction between various macroprudential policy tools is not considered.

3 Model

We start by describing our baseline model. The household and housing sectors follows Iacoviello (2015). We have two types of households: patient ones, who save via bank deposits, and impatient ones, who borrow from banks against a housing collateral. Patient households have a higher discount factor than impatient households. Hence, they value future consumption relative to current consumption by more than the impatient households. Both types of households obtain utility from consumption, housing and leisure. In line with typical DSGE models (eg, Smets and Wouters (2007)), we have a perfectly competitive final-goods sector whose firms combine intermediate goods to produce the final good. Intermediate-goods-producing firms combine labour and capital to produce intermediate goods. They face investment adjustment costs and price adjustment costs and have to borrow from banks to finance their investment and working capital (ie, wage payment) needs. Finally, we have a banking sector that accepts deposits from the patient households and lends money to impatient households and firms. Following Gertler and Karadi (2011), banks face a costly enforcement problem. Specifically, we assume that banks are able to divert a fraction of their assets to their owners, albeit at the expense of not being able to continue as a bank. To stop this from happening, it must always be more profitable for the banks to continue operating than to divert funds. This incentive constraint acts as a friction in the banking sector that limits leverage and creates a spread between loan and deposit rates.

3.1 Patient Households

We start by describing the problem faced by patient households. We assume that there is a unit continuum of these households and that they maximise the present discounted value of their current and future streams of utility subject to a budget constraint. They obtain utility from consumption, housing and leisure - ie. obtain disutility from working. We can write the problem facing patient household i mathematically a

$$Maximise\ E_0 \sum_{t=0}^{\infty} \beta_P^t \left[ln(c_{P,i,t}) + jA_{H,t} ln(H_{i,t}) - \frac{1}{1+\xi} h_{i,t}^{1+\xi} \right]$$

$$Subject\, to:\ D_{i,t} + Q_t H_{P,i,t} = Q_t H_{P,i,t-1} + R_{t-1} D_{i,t-1} + W_{P,t} h_{P,i,t} + \Pi_t - P_t c_{P,i,t} - P_t T_P - \tau_H Q_t H_{P,i,t}$$

Where c_i denotes consumption of household i, H_i denotes housing held by household i, h_i denotes hours worked by household i, D_i denotes bank deposits held by household i, Q denotes the price of a unit of housing, R denotes the interest rate paid on bank deposits (which will be equal to the central bank's policy rate), W_P denotes the wage paid to patient households, P denotes the aggregate price level, Π denotes profits of the firms and banks returned to the patient households, who we assume own them, net of money used by patient households to provide initial capital to new banks, and T_P denotes lump-sum taxes. In order to deliver an efficient steady state in the housing market, we introduce a constant tax/subsidy on saver's housing denoted by τ_H. In order to generate volatility in house prices, we introduce a 'housing demand' shock common to all (ie, both patient and impatient) households, denoted by A_H.

Assuming all patient households are identical, the first-order conditions for this problem imply:

$$\frac{1}{c_{P,t}} = \beta_P R_t E_t \frac{1}{(1+\pi_{t+1}) c_{P,t+1}} \tag{1}$$

$$\frac{(1+\tau_H) q_t}{c_{P,t}} - \frac{jA_{j,t}}{H_{P,t}} = \beta_P E_t \frac{q_{t+1}}{c_{P,t+1}} \tag{2}$$

$$w_{P,t} = h_{P,t}^{\xi} c_{P,t} \tag{3}$$

Where c_P denotes aggregate consumption by patient households, H_P denotes the aggregate housing stock owned by patient households, π denotes the rate of inflation, q denotes real house prices and w_P denotes the real wage paid to patient households. Equation (1) is the familiar patient household's intertemporal Euler equation, relating consumption today to the real interest rates and expected consumption tomorrow. Equation (2) is the housing demand equation for patient households, which shows that the higher is the real cost of housing, the less housing will be demanded. Finally, equation (3) is the labour supply equation for patient households, which shows that the higher is the real wage paid to patient households, the more hours of labour they will supply.

3.2 Impatient households

We assume that there is a unit continuum of impatient households who also maximise the present discounted value of their current and future streams of utility. Again, they obtain utility from consumption, housing and leisure (ie, obtain disutility from working). In addition to a budget constraint, however, they also face both a collateral (loan-to-value) constraint on their borrowing. Following Iacoviello (2015), we assume that impatient households discount the future at a greater rate than the patient households, ie, $\beta_I < \beta_P$. We can write the problem facing impatient household i mathematically as:

$$Maximise\ E_0 \sum_{t=0}^{\infty} \beta_I^t \left[ln(c_{I,i,t}) + jA_{H,t} ln(H_{i,t}) - \frac{1}{1+\xi} h_{i,t}^{1+\xi} \right]$$

$$Subject\,to:\ L_{M,i,t} = Q_t(H_{I,i,t} - H_{I,i,t-1}) + R_{L,t-1} L_{M,i,t-1} - W_{I,t} h_{I,i,t} + P_t c_{I,i,t} + P_t T_I \tag{4}$$

$$L_{M,j,t} = \rho_L L_{M,i,t-1} + (1-\rho_L) LTV\ H_{I,i,t} E_t Q_{t+1} \tag{5}$$

Where c_i denotes consumption of impatient household i, H_i denotes housing held by household i, h_i denotes hours worked by household i, $L_{M,i}$ denotes bank lending to household i, $R_{L,i}$ denotes the interest rate charged on bank loans, w_I denotes the wage paid to impatient households, LTV is the loan-to-value limit targeted by the banks on their lending, and T_I denotes lump-sum taxes, including those used to achieve an efficient allocation of consumption in steady state.[3]Note that, following Iacoviello (2015), we assume that impatient households only adjust slowly to their borrowing limits. There are, at least, two intuitive justifications for allowing impatient consumers to adjust slowly to the mortgage borrowing limits. The first is that these limits are typically imposed when mortgages are taken out; thus they will not effectively apply

[3]In the United Kingdom, the Financial Policy Committee has the power to direct banks to set LTV limits at levels of their choosing for owner-occupier and/or buy-to-let mortgages. But, as the Committee has not used these powers yet, we set the LTV ratio at the average across all UK owner-occupied mortgage lending between 2005-2018.

to all mortgage lending. Since, in our model, there are only one-period loans, imposing the LTV limit at all times would mean that the limit was applying counterfactually to all mortgage lending. Given this intuition, we can interpret ρ_L as the proportion of existing mortgages and $1-\rho_L$ as the proportion of new mortgages. A second justification is that we can think of the banks as setting a range of LTVs across their lending to households based on other (unobservable) household and bank characteristics. So, we can think of the LTV constraint applying on average across the banks' mortgage books. And it would seem reasonable to suggest that banks adjust slowly towards these limits on their 'average' lending, as forcing them to adjust immediately every period would lead to too much volatility in lending. Given this intuition, we can interpret ρ_L as capturing the speed of adjustment of banks towards their LTV target.

The first-order conditions for this problem imply:

$$\frac{1}{c_{I,t}}(1-\mu_t) = \beta_I E_t \frac{R_{L,t}-\rho_t\mu_{t+1}}{(1+\pi_{t+1})c_{I,t+1}} \tag{6}$$

$$\frac{jA_{j,t}}{H_{I,t}} = \frac{q_t}{c_{I,t}} - \frac{\mu_t(1-\rho_t)LTV\ E_t[q_{t+1}(1+\pi_{t+1})]}{c_{I,t}} - \beta_I E_t \frac{q_{t+1}}{c_{I,t+1}} \tag{7}$$

$$w_{I,t} = h^{\xi}_{I,t} c_{I,t} \tag{8}$$

Where c_I denotes aggregate consumption by impatient households, H_I denotes the aggregate housing stock owned by impatient households and w_I denotes the real wage paid to impatient households. Equation (6) is the intertemporal Euler equation for impatient households. Note that in addition to the real interest rate they pay on their borrowing and their expected future consumption, the consumption of impatient households will also depend on the tightness of the loan-to-value constraint on their borrowing, as picked up by the Lagrange multiplier, μ. Equation (7) is the housing demand equation for impatient households. This equation shows that in addition to its utility value, a marginal unit of housing yields extra value to impatient households by loosening their collateral constraint, enabling them to borrow and consume more. This effect is picked up by the term: $\frac{\mu_t(1-\rho_t)LTV\ E_t[q_{t+1}(1+\pi_{t+1})]}{c_{I,t}}$. Equation (8) is the labour supply equation for impatient households showing that the higher is the real wage, the more hours of labour they will supply.

3.3 Firms

As is standard in the New Keynesian literature, we assume that there is a unit continuum of monopolistically-competitive intermediate-goods-producing firms and a representative perfectly-competitive firm that combines intermediate goods to produce a final good. We assume that the intermediate-goods-producing firm faces costs of adjusting prices a la Rotemberg (1982). They also have to borrow to finance their working capital needs. In what follows we present the optimisation problem for the two types of firms.

3.3.1 Final-goods-producing firms

The representative final goods firm operates in a perfectly-competitive market and produces a final good by combining inputs of intermediate goods. These final goods are then consumed or invested. We can write the problem for this firm mathematically as follows:

$$Maximise\, P_t y_t - \int_{l=0}^{1} P_{l,t} y_{l,t} dl$$

$$Subject\, to: \; y_t \quad = \quad (\int_{l=0}^{1} y_{l,t}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$$

Where y denotes final goods output, y_l denotes output of intermediate firm l and P_l denotes the price of output for intermediate firm l.

The first-order condition for this firm gives the demand function for the output of individual firms:

$$y_{l,t} \quad = \quad (\frac{P_t}{P_{j,t}})^{\epsilon} y_t \tag{9}$$

3.3.2 Intermediate-goods-producing firms

We assume a unit continuum of firms producing differentiated intermediate goods in a monopolistically-competitive market. These firms face costs of adjusting prices. In addition, they have to borrow to finance their wage bill (what we think of as working capital). Since the firms are owned by the patient households, they discount their profits using the patient households' stochastic discount rate. We can write the problem facing intermediate firm l mathematically as:

$$\begin{aligned} Maximise \sum_{t=0}^{\infty} \frac{\beta_P^t}{P_t c_{P,t}} [(1+\tau_P) P_{l,t} y_{l,t} - W_{P,t} h_{P,l,t} - W_{I,t} h_{I,l,t} \\ + L_{E,l,t} - R_{L,t-1} L_{l,t-1} - \frac{\chi}{2} \left(\frac{P_{l,t}}{P_{l,t-1}} - 1 \right)^2 P_t y_t)] \end{aligned}$$

Subject to:

$$L_{E,l,t} = W_{P,t} h_{P,l,t} + W_{I,t} h_{I,l,t} \tag{10}$$

$$y_{l,t} = A_{z,t} h_{P,l,t}^{(1-\sigma)} h_{I,l,t}^{\sigma} \tag{11}$$

$$y_{l,t} = \left(\frac{P_t}{P_{l,t}}\right)^{\epsilon} y_t$$

Where τ_P is a subsidy to make steady-state production efficient, $h_{P,l}$ is the labour input of patient households within firm l, $h_{I,l}$ is the labour input of impatient households within firm l and $L_{E,l}$ is borrowing by firm l. All intermediate firms are subject to an aggregate technology shock, A_Z.

If we assume a symmetric equilibrium, the first-order conditions for this problem imply:

$$\frac{(1-\sigma)y_t}{h_{P,t}} rmc_t = \frac{R_{L,t}}{R_t} w_{P,t} \tag{12}$$

$$\frac{\sigma y_t}{h_{I,t}} rmc_t = \frac{R_{L,t}}{R_t} w_{I,t} \tag{13}$$

$$\pi_t(1+\pi_t) = \frac{(1-\epsilon)(1+\tau_p)}{\chi} + \frac{\epsilon}{\chi} rmc_t + \frac{1}{R_t} E_t \pi_{t+1}(1+\pi_{t+1})^2 \frac{y_{t+1}}{y_t} \tag{14}$$

Equations (12) and (13) represent the demand for each type of labour; in each case, the lower is the wage, the more labour is demanded. Note that the wage is multiplied by the interest rate spread, reflecting the fact that firms have to borrow to pay their wage bill. Equation (14) is the New Keynesian Phillips curve, which relates inflation today to expected future inflation, expected future output growth and real marginal cost.

3.4 Banks

Our modeling of the banking sector follows Gertler and Karadi (2011) with an endogenously-generated interest rate spread and leverage ratio. We assume that banks issue loans to firms and finance these out of patient household deposits and their own net worth, n. To ensure that banks cannot accumulate retained earnings to achieve full equity finance, we follow Gertler and Karadi (2011) and assume that each period banks have an *iid* probability $1-\zeta$ of exiting. Hence, the expected lifetime of a bank is $1/(1-\zeta)$. When banks exit, their accumulated net worth is distributed as dividends to the patient households. Each period, exiting banks are replaced with an equal number of new banks which initially start with a net worth of $L\nu$, where L is the steady state value of the banking sector's assets, provided by the patient households. A bank that survived from the previous period – bank b, say – will have net worth, n_b, given by:

$$n_{b,t} = R_{L,t-1} L_{b,t-1}(1+\tau_b) - R_{t-1} D_{b,t-1} \tag{15}$$

where τ_b is a subsidy which ensures a steady-state spread of zero (the efficient level), L_b is the total lending of bank b to impatient households and firms and D_b are deposits from patient households held at bank b.

Total net worth, n of the banking sector will be given by:

$$n_t = \zeta(R_{L,t-1}L_{t-1}(1+\tau_b) - R_{t-1}D_{t-1}) + (1-\zeta)L\nu \tag{16}$$

Each period banks (whether new or existing) finance their loan book with newly issued deposits and net worth:

$$L_{b,t} = D_{b,t} + n_{b,t} \tag{17}$$

Following Gertler and Karadi (2011) we introduce the following friction into the banks' ability to issue deposits. After accepting deposits and issuing loans, banks have the ability to divert some of their assets for the personal use of their owners. Although the patient households are both the owners of the banks and the depositors in the model, we assume that each household is 'large' enough that we could imagine the banks owners and depositors being separate individuals, with the owners prepared to divert assets towards their own personal use. Specifically, they can sell up to a fraction θ of their loans in period t and spend the proceeds during period$t$. But, if they do, their depositors will force them into bankruptcy at the beginning of period $t+1$. When deciding whether or not to divert funds, bank b, will compare the franchise value of the bank, V_b, against the gain from diverting funds, θL_b. Hence, depositors will ensure that banks satisfy the following incentive constraint:

$$\theta L_{b,t} < V_{b,t} \tag{18}$$

The problem for bank b is to choose L_b and D_b each period to maximise its franchise value subject to its incentive constraint, equation (18), its balance sheet constraint (17) and the evolution of its net worth (15).

$$Maximise\, V_{b,t} = P_t E_t \sum_{j=1}^{\infty} \left(\beta_P^j \zeta^{j-1}(1-\zeta)\frac{1}{c_{P,t+j}P_{t+j}}(R_{L,t+j-1}L_{b,t+j-1}(1+\tau_b) - R_{t+j-1}D_{t+j-1} \right)$$

We can note that both the objective and constraints of the bank are constant returns to scale. As a result, we can rewrite the optimisation problem for bank$b$ in terms of choosing its leverage ratio, $\varphi_b = \frac{L_b}{n_b}$, to maximise the ratio of its franchise value to net worth, $\psi_b = \frac{V_b}{n_b}$. Given constant returns to scale, we can aggregate up across all bank. Doing so, we obtain the aggregate Bellman equation for the franchise value of the banking sector as a whole:

$$\psi_t = \beta_H E_t(\frac{P_t}{P_{t+1}})\frac{c_{P,t}}{c_{P,t+1}}(1-\zeta+\zeta\psi_{t+1})((R_{L,t}(1+\tau_b)-R_t)\varphi_t+R_t)) \quad (19)$$

$$Subject\,to:\ \theta\varphi_t \leq \psi_t \quad (20)$$

where we note that constant returns to scale implies that all banks will choose the same leverage ratio, φ.

3.5 Monetary policy

The central bank operates a Taylor Rule of the form:

$$lnR_t = (1-\rho_R)ln(R) + \rho_R lnR_{t-1} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y ln(\frac{y_t}{y})] + \epsilon_{R,t} \quad (21)$$

where y denotes the steady-state level of output and ϵ_R is a white-noise shock.

3.6 Market clearing

Aggregating the budget constraints for each sector implies the goods market clearing condition:

$$y_t = \frac{c_t}{1-\frac{\chi}{2}\pi_t^2} \quad (22)$$

We assume a fixed stock of housing equal to unity:

$$H_{P,t} + H_{I,t} = 1 \quad (23)$$

And:

$$L_{M,t} + L_{E,t} = L_{b,t} \quad (24)$$

3.7 Augmenting the baseline model with additional macroprudential tools

Relative to the baseline model described above, we add two more macroprudential policies.

First we consider the effects of adding a maximum leverage ratio constraint on banks as a way of capturing capital requirements. Specifically, we suppose that the macroprudential policy maker sets a maximum leverage ratio *Lev*. Banks regard *Lev* as an absolute maximum expending efforts which incur costs in order to avoid reaching it. These costs get larger the closer the bank gets to the maximum leverage limit. Specifically, we suppose that banks face the following cost function:

$$\left(\frac{\phi_b}{(Lev-\varphi_t)}-\frac{\phi_b}{(Lev-\varphi)}\right)n_t \tag{25}$$

where φ_t is their leverage in period t and φ is steady-state leverage.

Banking sector net worth will evolve according to:

$$n_t \;=\; \zeta\left(R_{L,t-1}L_{t-1}(1+\tau_b)-R_{t-1}D_{t-1}-\left(\frac{\phi_b}{(Lev-\varphi_t)}-\frac{\phi_b}{(Lev-\varphi)}\right)n_{t-1}\right)+(1-\zeta)\nu \tag{26}$$

And the Bellman equation for the banking sector will now be given by:

$$\begin{aligned}\psi_t \;=\; & \beta_P E_t \\ & \left(\frac{P_t}{P_{t+1}}\frac{c_{P,t}}{c_{P,t+1}}(1-\zeta+\zeta\psi_{t+1})\left((R_{L,t}(1+\tau_b)-R_t)\varphi_t+R_t-\frac{\phi_b}{(Lev-\varphi_t)}+\frac{\phi_b}{(Lev-\varphi)}\right)\right)\end{aligned} \tag{27}$$

Subject to equation (20).

The first-order conditions for this problem imply:

$$\varphi_t \;=\; Lev-\sqrt{\frac{\phi_b}{R_{L,t}(1+\tau_b)-R_t}}\; and\; \theta\varphi_t<\psi_t \tag{28}$$

We assume that the imposition of a maximum leverage ratio (with associated penalty cost function) results in the diversion risk constraint always being slack.

Second, we add an affordability constraint on household lending, which in essence is a debt-service-ratio (DSR) limit on impatient households' balance sheets. Specifically, we assume that impatient households face the following constraint:

$$L_{M,j,t}=\rho_L L_{M,j,t-1}+(1-\rho_L)\frac{DSR\,h_{I,t}w_{I,t}}{R_{L,t}-1+stress} \tag{29}$$

where DSR is a measure of a debt service ratio - i.e. the proportion of impatient households' wage income being used to pay back the principal and interest on a loan. *stress* denotes the assumed stress interest rates set by the macroprudential policymaker at which the debt-service ratio is being stressed. Intuitively, the constraint checks whether a borrower would still be able to afford the interest payments on their loan if the interest rate they had to pay were to rise by the amount implied by the *stress* parameter.

In this paper we assume that either the LTV constraint from equation 5 binds or the affordability constraint binds. There are no occasionally binding constraints. As such, we assume that the imposition of an affordability constraint on household lending renders the LTV limit slack in all periods.

The addition of an affordability constraint and the assumed slackness of the LTV constraint results in the following first-order conditions for the impatient households:

$$\frac{1}{c_{I,t}}(1-\mu_t) \quad = \quad \beta_I E_t \frac{R_{L,t} - \rho_L \mu_{t+1}}{(1+\pi_{t+1})c_{I,t+1}} \tag{30}$$

$$\frac{jA_{j,t}}{H_{I,t}} \quad = \quad \frac{q_t}{c_{I,t}} - \beta_I E_t \frac{q_{t+1}}{c_{I,t+1}} \tag{31}$$

$$w_{I,t}(1+\frac{\mu_t(1-\rho_L)DSR}{R_{L,t}-1+stress}) \quad = \quad h^{\xi}_{I,t} \tag{32}$$

Where μ is now the lagrange multiplier on the affordability constraint. The housing demand equation is now simplified as impatient borrowers no longer benefit from having more housing to relax their collateral constraint. Against that, impatient households are now prepared to supply more labour for a given wage since doing so will relax their affordability constraint.

4 Loss function

In this section, we derive the welfare-based loss function for our model, which we use in Section 5, to evaluate different macroprudential policy tools . Our discussion of the loss function follows Ferrero et al. (2018) and Rubio and Yao (2019). We derive the loss function by taking a weighted-average of the per-period utility functions of patient and impatient households where the savers are given an arbitrary weight of β_P. We assume that the planner discounts the future at the discount rate of the savers, w_P. A second-order approximation of the resulting objective function around a zero-inflation steady state in which the loan-to-value constraint is assumed to bind gives:

$$L \approx \frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta_P^t (\hat{y}_t^2 + \lambda_\pi \pi_t^2 + \lambda_c \tilde{c}_t^2 + \lambda_H \tilde{H}_t^2) \tag{33}$$

where$\hat{y}$ denotes the log deviation of output from its efficient steady-state level, $\tilde{c}$ denotes the consumption gap, defined as the log difference in consumption between patient and impatient households relative to the log difference between their consumption levels in the efficient steady state, and $\tilde{H}$ denotes the housing gap, defined as the log difference in housing held by patient and impatient households relative to the log difference between their housing levels in the efficient steady state. The efficient steady state is defined and derived in Annex 1 of this paper.

The weights on inflation, the consumption gap and the housing gap are derived in Annex 2 of this paper and are given by:

$$\lambda_\pi = \frac{\chi}{1+\xi},\ \lambda_c = \frac{1+\xi-4\sigma(1-\sigma)}{4(1+\xi)^2}\ and\ \lambda_H = \frac{j}{4(1+\xi)}$$

As in Ferrero et al. (2019), the loss function adds terms in the consumption and housing gaps to the standard output gap and inflation terms found in standard New Keynesian macroeconomic models. These terms are generated by incomplete financial markets where households are unable to completely share consumption and housing risk between them. Risk-sharing is further limited by the collateral constraint faced by impatient households. The goal of the macroprudential policy is to limit the welfare losses that arise out of the incomplete risk-sharing.

5 Quantitative experiments

Before discussing our quantitative experiments, we first discuss our calibration and what this means for the implied steady-state relationships in our model.

5.1 Calibration

We calibrate the parameters of the model either to match the previous literature or to hit steady-state targets. Our parameter choices for the baseline model are shown in Table A.

Table 1: Parameter values

Parameter	**Description**	**Value**
β_P	Discount rate for patient households	0.9925
β_I	Discount rate for impatient households	0.985
j	Weight on housing in utility function	0.1062
ξ	Inverse Frisch elasticity of labour supply	1.83
σ	Proportion of total wage bill going to impatient households	0.33
ε	Elasticity of demand for differentiated intermediate goods	6
χ	Size of price adjustment costs	70.4225
ρ	Inertia in loan-to-value constraint	0.7
θ	Proportion of assets that can be diverted	0.1262
ζ	Bank survival rate	0.975
ν	Capital of newly-formed banks as a fraction of bank assets	0.05
φ	Steady-state leverage ratio	10
φ_{max}	Maximum leverage ratio	20
$\frac{qH}{y}$	Housing wealth to GDP	10
$\frac{L}{y}$	Mortgage debt to GDP	3
DSR	Debt-service ratio	0.14
$stress$	Stress rate	3

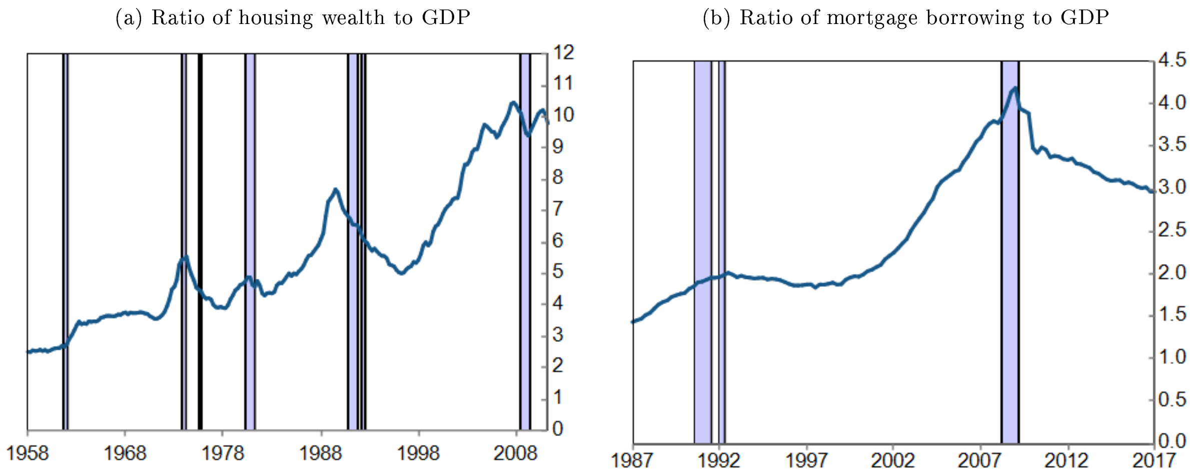
The discount rates for patient households is 0.9925 implying a risk-free rate of 3% per annum. The discount rate for impatient households is set to 0.985, following Ferrero et al. (2018). The steady-state version of equation (6), implies the following steady-state value for the Lagrange multiplier on the impatient households' borrowing constraint:

$$\mu = \frac{1 - \beta_I R_L}{1 - \beta_I \rho_L} \tag{34}$$

Given the calibration of the two discount factors, the impatient households will be constrained in their ability to borrow. However, we set the banking subsidy, τ_b, to ensure a zero spread in steady state.

Based on the estimation results reported in Smets and Wouters (2007), we set the inverse Frisch elasticity to 1.83. Following Iacoviello (2015), we set the inertia in the LTV constraint equal to 0.7 and the share of the total wage bill going to impatient households equal to 0.33. We set the elasticity of substitution, ϵ, equal to 6. Absent the production subsidy, this would imply a mark-up of 1.2 in the intermediate goods sector, in line with the results in Macallan et al. (2008). We then set the size of the price adjustment costs, χ, such that the coefficient on real marginal cost in the New Keynesian Phillips curve, $\frac{\epsilon}{\chi}$, was equal to 0.0852. This is the value that would be obtained in a Calvo (1982) model of price-setting with prices assumed to be adjusted once a year on average. We set the survival rate for banks equal to 0.975, implying an average expected life for a retail bank of 10 years, and the amount of capital that new banks start off with equal to 1/20 of the steady-state assets of the banking sector. Finally, we used standard values for the Taylor rule. The remaining parameters ensure that the steady-state of our model implies targeted values for the steady-state leverage ratio and housing wealth to GDP ratio. In particular, we target a steady-state leverage ratio of 10, roughly in line with the average leverage in the UK banking sector. We turn to the data to choose a target for the steady-state housing wealth to output ratio. Panel (a) of Figure 1 shows that this ratio has risen over time from around 3 $\frac{1}{2}$ to 4 in the 1960s to around 10 over the past few years. Hence, we target a steady-state value for the housing wealth to output ratio of 10. These choices imply values for the parameters θ and j in the model.

Figure 1: UK Data



Panel (b) of Figure 1 shows that in the UK, the ratio of mortgage borrowing to GDP is currently around 3. Given that, we set the LTV ratio to 0.6 ensuring that the steady-state ratio of mortgage borrowing to GDP in our model is also equal to 3. This is roughly in line with the average LTV ratio on the outstanding stock of mortgages in the UK. Over 2005 to 2018, the average LTV ratio on new owner-occupied loans was approximately 0.65.

To calibrate the macroprudential tools, we ensure that the steady-state is identical across four versions of the model we consider later on in our results – ie. baseline, baseline plus capital requirements, baseline plus capital requirements and the DSR ratio and baseline with the DSR ratio only. We set the maximum leverage ratio to 20, implying a minimum capital requirement of 5%.

Next, the steady-state version of equation (28) implies:

$$\phi_b = \frac{\tau_b(Lev - \varphi)^2}{\beta_P} \tag{35}$$

where the subsidy, τ_b, has been set to ensure a zero spread in steady state. Hence, setting Lev to 20 implies a value for ϕ_b of 0.0526. As we mentioned earlier, once capital requirements are imposed, the ‘diversion risk’ constraint does not bind. To ensure that this is the case, we assume that θ is set low enough for this to hold.

For the affordability constraint, we set the stress buffer to 0.0075. This applies a 3% buffer per annum on top of the current interest rate when assessing principal and interest repayments for mortgage borrowing relative to labour income. Given that we set the subsidy to firms so as to ensure that real marginal cost is unity in steady state and the subsidy to banks to ensure that the interest rate spread is zero in steady state, the steady-state versions of equations (13) and (29) imply:

$$\frac{L_M}{y} = \frac{\sigma DSR}{\frac{1}{\beta_P} - 1 + stress} \tag{36}$$

Given our other parameters, we set DSR limit to ensure that the steady-state ratio of mortgage borrowing to GDP, $\frac{L_M}{y}$, is equal to 3 as in the baseline model. This implies a value for DSR of 0.1369. This value for DSR is low relative to the value of 0.4 that is applied in the United Kingdom in practice. However, this is a result of having only one-period loans in our model. For a long-term mortgage, the DSRs fall over the lifetime of the mortgage as income rises.

5.2 Estimation of shock processes

In order to simulate tour model, we need to calibrate the two shock processes: productivity, A_z and housing demand, A_H. In each case, we assume that the shock follows an AR(1) process. We estimate the standard deviations and first-order autocorrelation coefficients of the shocks using Bayesian techniques and quarterly UK data for GDP growth, real house prices and the spread of effective mortgage interest rates over 1999 – 2018. Table 2 shows the priors and the full results from the estimation. We set parameter values in line with mean estimated values. As such, the standard deviation of the productivity shock is set to 1.41% and its autocorrelation to 0.97, which is in line with existing literature (e.g. Smets and Wouters, 2007). We set the standard deviation of the housing demand shock to 8.16% and its autocorrelation to 0.98.

Table 2: Estimation of shock processes

	Prior			Estimated Max Posterior		Posterior
Parameter	**Type**	**Mean**	**Std. error**	**Mode**	**Std. error**	**Mean**
σ productivity shock	Inv gamma	0.01	∞	0.0136	0.0014	0.0141
σ housing demand shock	Inv gamma	0.035	∞	0.0586	0.0291	0.0816
ρ productivity shock	Beta	0.5	0.2	0.9718	0.0151	0.9662
ρ housing demand shock	Beta	0.5	0.2	0.9855	0.0119	0.9761

5.3 Results

We simulate four versions of the model with 4 macroprudential policies in place: i) an LTV ratio of 60% (the baseline model); ii) an LTV ratio of 60% and capital requirements (or a maximum leverage constraint); iii) capital requirements and affordability constraints; and iv) a version with affordability constraints only. In each case, we use Dynare to calculate the volatilities of key macroeconomic variables and their impulse responses to aggregate shocks.

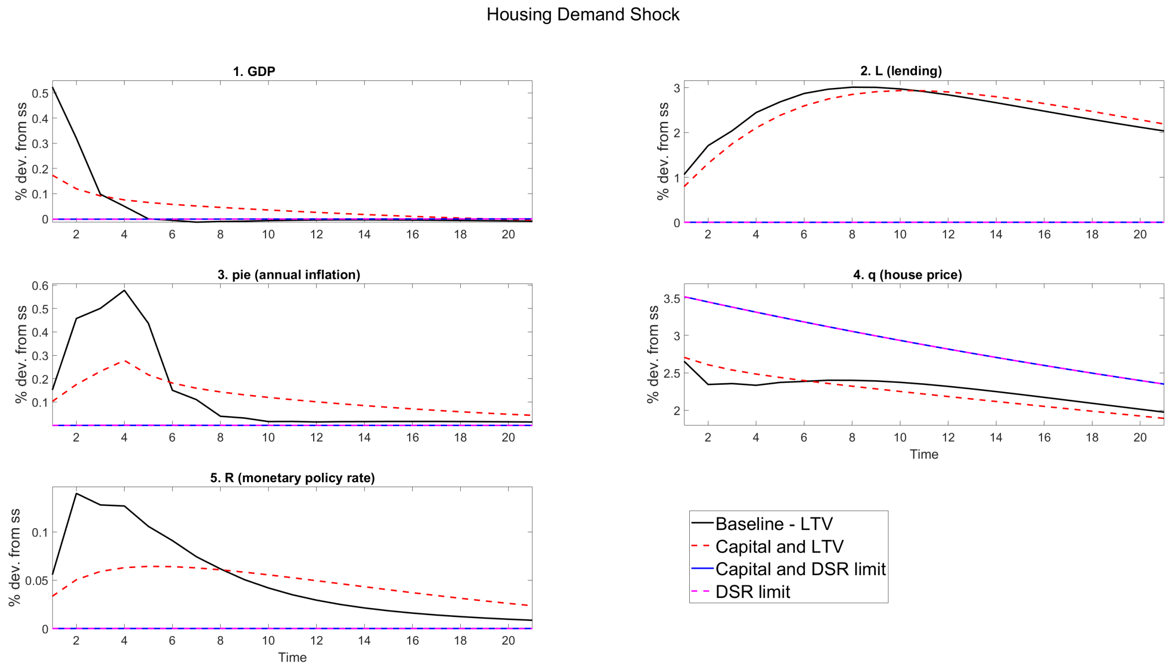
5.3.1 The interaction of housing tools with capital requirements and monetary policy

The key question in this paper is how different macroprudential tools interact with each other and with monetary policy. To investigate this question we gradually switch on different policies and examine their impact on output, lending, inflation, house prices and the interest rate following a housing demand shock, a technology shock and a monetary policy shock.

Figure 2 plots the impulse response functions to a housing demand shock that leads to an approximately 3% rise in house prices. There are two important results coming out of this experiment. First, when mortgage lending is constrained by DSR limits (the blue and magenta dotted lines), the economy does not respond to the housing demand shock, except for an increase in house prices. Affordability constraints disconnect the housing market from mortgage borrowing, thus ensuring that housing demand shocks are not transmitted to the real economy. This result is intuitive. When borrowing is not backed by housing wealth, a shock to house prices does not influence credit constraints or how much households can borrow.

Second, capital requirements interact differently with monetary policy compared to LTV limits, as shown in plot 5 of Figure 2. Monetary policy responds less to the housing demand shock when capital requirements are switched on, compared the the baseline case with a 60% LTV ratio. That occurs because, capital requirements dampen the effect of the house price shock on lending, which decreases the effect of the shock on GDP and inflation. Hence, macroprudential policy acting through capital requirements contributes to price stability in the face of a housing demand shock, helping monetary policy achieve its primary objective.

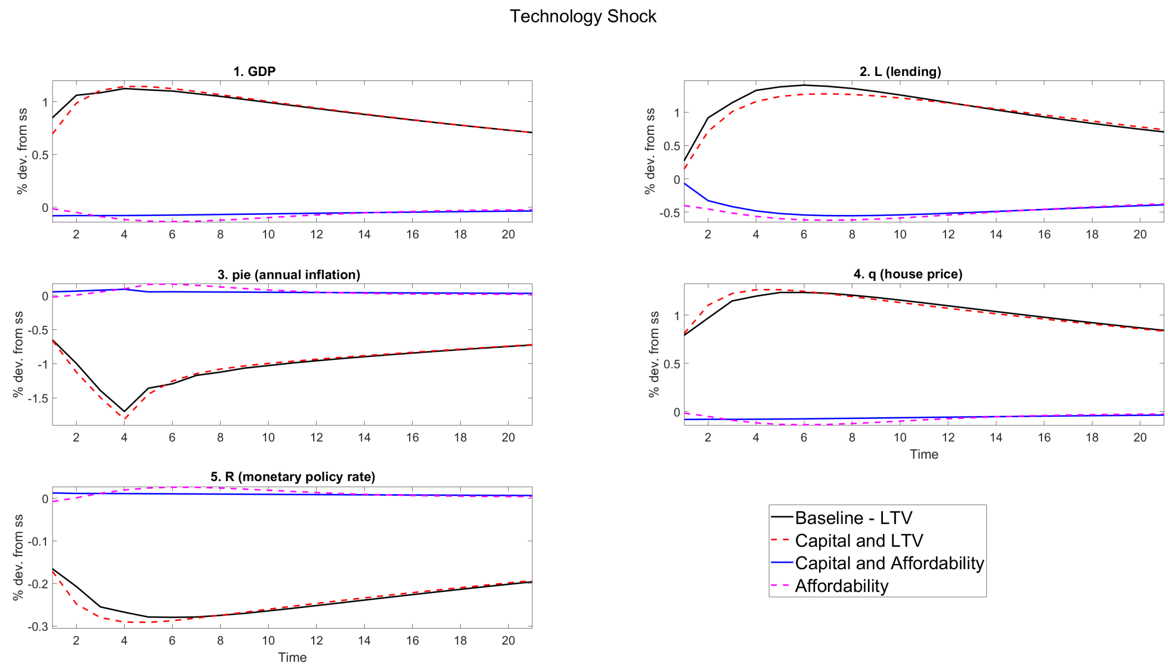
Figure 2: Responses to a housing demand shock (≈3% rise in prices)



Next we investigate the impulse response functions of variables to a positive technology shock, shown in Figure 3. The plots show that LTV ratios and DSR limits on mortgage borrowing deliver different responses of variables to the shock, but that adding capital requirements leads only to additional marginal changes. In the models with LTV ratios in place (black and red dotted lines), the productivity shock leads to positive responses of output and consumption. This incentivises borrowers to purchase more housing and leads to a rise in house prices and lending. However, when affordability constraints are switched on (the blue and magenta lines), the link between house price movements and borrowing is muted, leading to very modest effects of the productivity shock on the economy.

Plot 5 of Figure 3 shows the interaction of macroprudential tools with monetary policy. When affordability constraints are switched on, interest rates respond very little to a productivity shock. This suggests that, when faced with a technology shock, macroprudential DSR policy implemented via DSR ratios may also support the objectives of the monetary policymaker. However, if financial policy is instead introduced via LTV ratios, monetary policy has to be more active and responds by decreasing interest rates in order to bring inflation back to normal.

Figure 3: Technology shock (1.41%)

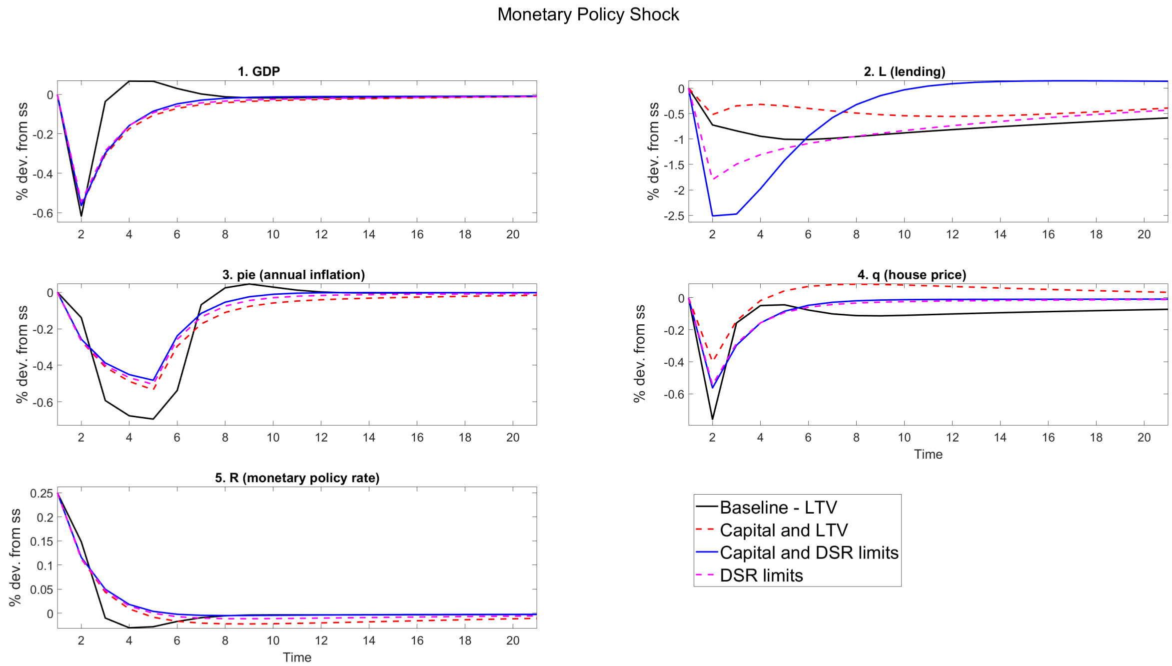


To further examine the interaction of macroprudential and monetary policies, we plot the impulse responses of macro variables to a monetary policy shock which leads to a 1% annual rise in rates. Figure 4 shows that, the introduction of capital requirements increases the persistence of interest rate movements on output and inflation and reduces the maximum effect of the monetary policy shock on inflation. Additionally, the monetary policy shock leads to a large contraction in lending when affordability constraints are switched on (blue and magenta dotted lines) . This effect occurs for two reasons. First, the monetary policy contraction leads to a drop in GDP which results in lower household income. As borrowing is backed by household earnings, a loss of income leads to an immediate tightening of credit constraints and of overall lending. Second, the rise in risk-free rates leads to a subsequent rise in the mortgage lending rate. This further tightens households' credit constraints by increasing the proportion of interest payments that households have to pay back for any given loan size - i.e. increases the denominator in equation (29).

However, despite the more significant contraction in lending, output and inflation do not fall by more in the presence of affordability constraints relative to other policies. That occurs because DSR limits raise the shadow value of work, since working an additional hour will relax the constraint. This leads to hours worked falling by less in response to the monetary policy tightening when affordability constraints are switched on.

These results suggests that capital requirements, DSR limits and monetary policy can have important spill-overs on each other, highlighting the importance of coordination between policymakers.

Figure 4: Monetary policy shock (1% rise in rates)



5.3.2 The interaction of housing tools with each other

The previous section described the interaction of our two housing tools - i.e. LTV ratios and affordability constraints - with capital tools and with monetary policy. This section provides more details on how the two housing tools may interact with each other.

To understand how LTV and DSR ratios evolve following economic shocks and how imposing macroprudential limits on one affects the other, we conduct the following experiment. For the versions of the model where the LTV limit is switched on - i.e the baseline with and without capital requirements, we calculate the unconstrained DSR ratio. Similarly, for the versions of the model where the DSR is switched on, we calculate the unconstrained LTV ratio. For each shock, we then examine how the unconstrained ratios compare to the macroprudential limits we calibrate in Section 5.1 - i.e. a 60% limit for the LTV ratio and a 0.14 limit for the DSR ratio. This exercise allows us to investigate whether different housing tools are complements to each other - i.e. they are both binding or tightener at the same time, or substitutes to each other - i.e. when one is looser the other one is tightener. This is an important excercise for policymaking. For instance, if we find that the two housing tools are complements, then a collateral constraint (DSR limit) will interact with and have spill-overs for borrowers' debt-service ratios (LTV ratios) in which case the macroprudential policymaker can address risks coming from the housing market using only one housing tool. However, if collateral constraints and affordability tools are substitutes, then they will respond to boom-bust cycles differently and hence the policymaker may need to assess the effectiveness of each tool separately. This case is more likely to occur in boom periods, when a relaxation in house prices relaxes LTV but not DSR constraints, since the latter is linked to the borrowers' incomes rather than collateral values. For instance, Greenwald (2018) finds that a cap on debt-to-income ratios, not LTV ratios, is the more effective policy for limiting boom-bust cycles and that debt-to-income limits would have reduced the size of the 2007 boom by nearly 60%. And Ingholt (2019) finds that a lower LTV limit could not have prevented the 2007 boom since soaring house prices slackened the constraint.

The implied unconstrained DSR ratio for models where the LTV tool is switched on, is calculated using equation 29. Ignoring the slow adjustment of loans in the economy gives:

$$DSR = \frac{L_{M,j,t}(R_{L,t} - 1 + stress)}{h_{I,t}w_{I,t}} \tag{37}$$

The implied unconstrained LTV ratio for the models where the DSR is switched on, is determined using equation 5 which, ignoring the slow adjustment of loans in the economy gives:

$$LTV = \frac{L_{M,j,t}}{H_{I,i,t}E_tQ_{t+1}} \tag{38}$$

Figure 5 shows the results for the housing demand shock. The blue lines use equation 37 to compute the implied DSR ratios in the models with the LTV limit switched on, given the responses of borrowing, interest rates and labour income to the shock. The red lines use equation 38 to compute the implied LTV ratio when the affordability constraint is switched on, given the effects of the shock on housing wealth. The figure shows that DSR ratios increase to 0.3 in the top-left panel, when macroprudential policy is implemented solely through a collateral constraint. The response of the DSR ratio is nearly twice as big as the 0.14 macroprudential DSR limit we impose in the version of the model where the affordability constraint is switched on. The impact on DSRs is explained by a larger increase in borrowing and mortgage rates relative to income in the baseline model, following the shock. As shown in Figure 2, the house prices appreciate by nearly 3%, which relaxes LTV constraints and allows households to access more debt. The LTV limit remains constant over time, due to the adjustment in borrowing, but the additional debt in the economy raises debt service ratios. As a result, a macroprudential LTV tool is not sufficient on its own to constrain debt levels when the economy is hit by a housing demand shocks. The feedback mechanism between house prices and borrowing imply that the LTV tool acts procyclically. Instead, a macroprudential constraint on DSR ratios would lean against the wind in a countercyclical manner.

The role for a macroprudential DSR limit on top of a collateral constraints is less important however when capital requirements on banks are also in place. The top-right panel shows that implied DSR ratios increase by very little when capital requirements are added as an additional macroprudential policy in the baseline model. As shown in Figure 2, lending and interest rates respond less to the shock when capital is added, which lead DSR ratios to respond only modestly to the shock. This suggests that capital requirements, if calibrated correctly, could make a DSR tool obsolete. However, increasing capital requirements in the face of a housing demand shock could be costly since it is a blunt tool that affects all types of lending, not just mortgage lending. Affordability tools are a more natural substitute to LTV limits when dealing with housing booms as they are specifically targeted at mortgage credit and have fewer spillovers on other sectors of the economy.

The bottom two panels of Figure 5 show the unconstrained LTV ratio when in the versions of the model where macroprudential policy operates through an affordability constraints and capital requirements. The red lines show that LTV ratios decrease following the house price shock. This result occurs because macroprudential DSR tools break the link between collateral values and mortgage borrowing. As a result, the LTV ratios decrease since its numerator - i.e. loan amount - remains unchanged, while its denominator - i.e. house prices - increases by approximately 3%.

Figure 5: Behaviour of housing tools following a housing demand shock

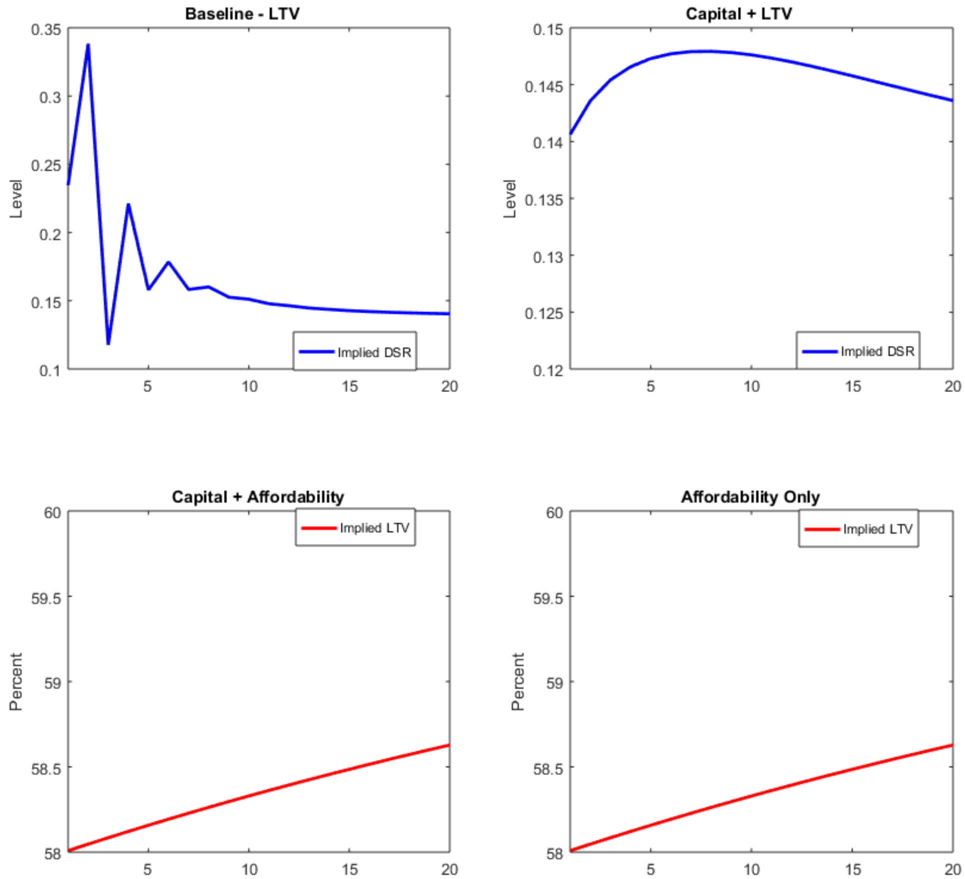
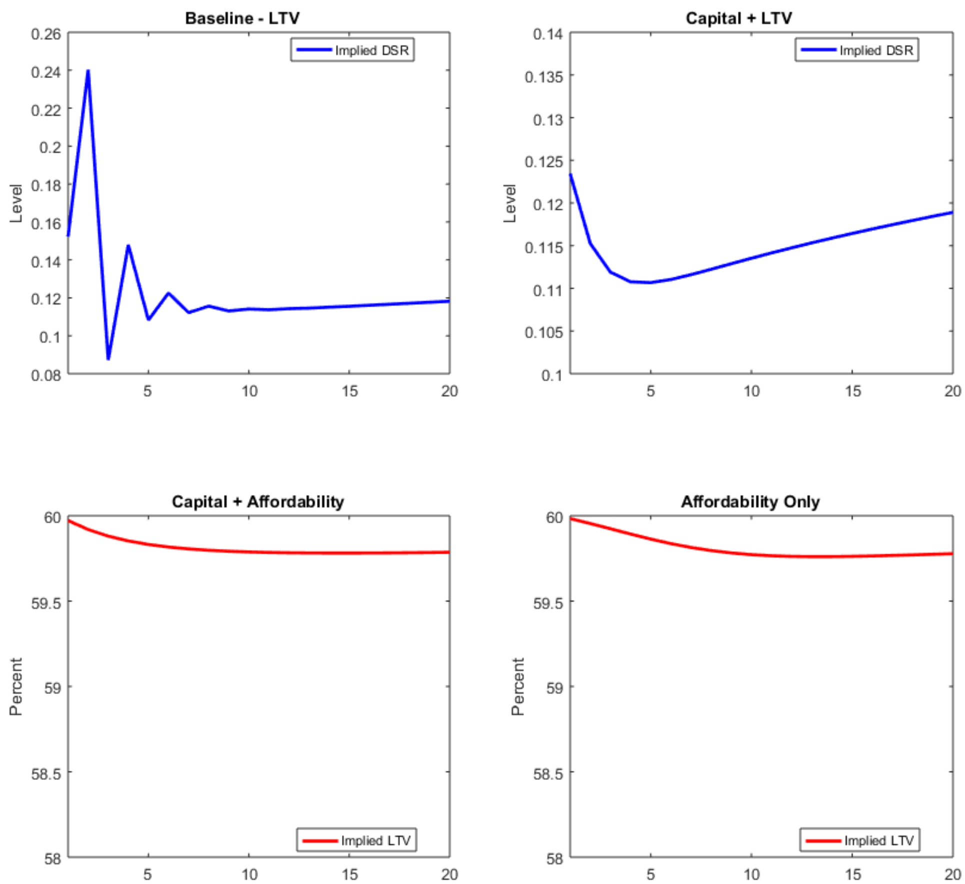


Figure 6 follows the same rationale as Figure 5 for a technology shock. Similar to before, the DSR nearly doubles and becomes very volatile in the baseline case. This occurs because, the technology shock increases household borrowing and decreases hours worked by the impatient household. Thus, higher debt serviced by lower labour incomes leads to a rise in debt-service ratios. However, when adding capital requirements to the baseline model in the top-right panel, the DSR ratio actually decreases. Compared to the baseline case, adding capital requirements leads to a larger loosening in the monetary policy rate and to a more muted decrease in labour supply, both of which outweigh the increase in borrowing. As shown in equation 37, these effects weigh down on debt-service ratios. This suggests that a capital requirement may constrain household leverage in the face of a technology shock. Similar to Figure 5, the LTV ratios remain mostly stable over time when affordability constrains are imposed in the bottom panels.

Figure 6: Behaviour of housing tools following a technology shock



5.3.3 The impact of macroprudential tools on the volatility of key macroeconomic variables

Another question we are interested in, is the extent to which the adoption of macroprudential policy tools can improve welfare by stabilising output, inflation, lending and house prices. In particular, we are interested in examining which tool is better for smoothing lending and house prices, as broad measures of financial stability and how these tools affect the ability of monetary policy makers to smooth output and inflation, which are the monetary policy targets.

Table 3 shows the results of our stochastic simulations. For each policy tool we show the standard deviations of total bank lending, L, output, y, inflation, π and real house prices, q in response to technology and housing demand shocks. In addition, we show the implications on the welfare loss. Relative to the baseline model, imposing capital requirements leads to marginal reductions in the volatilities of macro variables, including on welfare loss. This suggests that, when LTV ratios in the economy remain at relatively low levels - i.e. 60% in our calibration - capital requirements have very little additional benefit. Nonetheless, switching on affordability constraints on mortgage lending leads to an increase in the volatility of real house prices, and to a large decrease in the volatilities of lending, inflation and output. This results in a substantial improvement in welfare.

Table 3: Volatility of macro variables

	$\sigma_{House\,Prices}$ (%)	$\sigma_{Lending}$ (%)	σ_{π} (%)	σ_{y} (%)	Welfare loss
Baseline: 60% LTV ratio	15.48	14.88	5.67	5.19	0.070
Baseline and CR	15.29	14.66	5.66	5.16	0.069
CR and DSR	17.69	2.61	0.24	0.31	0.00

To investigate these results further we decompose the variance in lending, real house prices, output and inflation into the proportions driven by each our shocks. The results are shown in Table 4. The introduction of capital requirements reduces the effect of the housing demand shocks on house prices, output and inflation and the effects of productivity shocks on the lending. The introduction of affordability constraints wipes out any effect of the housing demand shocks on all variables other than house prices. This is because affordability constraints ensure that borrowing is no longer linked to house prices via the LTV constraint. Housing demand shocks result in volatile house prices with no impact on borrowing and, hence, on the rest of the economy.

Table 4: Variance decomposition

	LTV		**LTV and CR**		**CR and DSR**	
	technology	housing demand	technology	housing demand	technology	housing demand
Lending	13.24	86.76	12.67	87.33	100.00	0.00
Output	98.50	1.50	99.64	0.36	99.96	0.04
Inflation	96.73	3.27	98.75	1.25	99.99	0.01
House prices	14.67	85.33	15.14	84.86	0.03	99.97

6 Conclusion

In this paper, we examine three macroprudential policies: LTV ratios, capital requirements on banks and affordability constraints on mortgage borrowing. We consider the interaction of macroprudential policies with each other as well as with monetary policy. Additionally, we assess the effects of each policy on macroeconomic stability, as measured by the standard deviations of output and inflation, on financial stability, as measured by the standard deviations of bank lending and house prices, and on welfare.

We find that both capital requirements and DSR limits reduce the need for monetary policy to react to a housing demand shock, and that DSR limits also contribute to monetary stability when the economy is hit by technology shocks. Additionally, we find that introducing capital requirements on banks reduces the volatility of lending, house prices, output and inflation only marginally relative to an LTV ratio. Finally, we show that DSR limits lead to an increase in the volatility of real house prices and to a significant decrease in the volatility of lending, consumption and inflation, since they disconnect the housing market from the real economy. Overall, DSR limits are welfare improving relative to any other macroprudential tool.

In future versions of this paper, we intend to allow our policy tools to vary over the cycle and work out the welfare implications of optimal simple macroprudential policy rules. For instance, we plan to examine the optimal degree of countercyclicality in capital requirements or the DSR stress buffer holding the Taylor rule coefficients fixed. This would allow us to assess the impact of different calibrations of macroprudential tools and to better inform macroprudential policymakers on the effectiveness of different tools in smoothing aggregate shocks over the business cycle.

References

[1] Adrian, T and Duarte, F (2018), 'Financial vulnerability and monetary policy', Centre for Economic Policy Research Discussion Paper No. 12,680.

[2] Angelini, P, Neri, S and Panetta, F (2014), 'The interaction between capital requirements and monetary policy', Journal of Money, Credit and Banking, Vol. 46, No.6

[3] Benigno, Otrok, Chen, Rebucci and Young (2013), Financial crises and macro-prudential policies, Journal of International Economics, Vol. 89, page 453-70.

[4] Bianchi (2010), Credit externalities: Macroeconomic effects and policy implications, American Economic Review: Papers and Proceedings, Vol. 100, pages 398-402.

[5] Carrillo, J A, Mendoza, E G, Nuguer, V and Roldan-Pena, J (2017), 'Tight money-tight credit: Coordination failure in the conduct of monetary and financial policies', National Bureau of Economic Research Working Paper No. 23,151.

[6] Collard, F, Dellas, H, Diba, B and Loisel, O (2017), 'Optimal monetary and prudential policies', American Economic Journal: Macroeconomics, Vol. 9, pages 40-87.

[7] Curdia, V and Woodford, M (2009), 'Credit frictions and optimal monetary policy', Bank for International Settlements Working Paper No. 27,811.

[8] De Paoli, B and Paustian, M (2017), 'Coordinating monetary and macroprudential policies', Journal of Money, Credit, and Banking, Vol. 49, pages 319–49.

[9] Farhi, E and Werning, I (2016), 'A Theory of macroprudential policies in the presence of nominal rigidities', Econometrica, Vol. 84, pages 1,645-704.

[10] Ferrero, A, Harrison, R and Nelson, B (2018), 'House price dynamics, optimal LTV limits and the liquidity trap', Centre for Economic Policy Research Discussion Paper No. 13,400.

[11] Gelain and Ilbas (2017), 'Monetary and macroprudential policies in an estimated model with financial intermediation', Journal of Economic Dynamics and Control, Vol. 78, pages 164-89.

[12] Gertler, M and Karadi, P (2011), 'A model of unconventional monetary policy', Journal of Monetary Economics, Vol. 58, pages 17-34.

[13] Gertler, M and Karadi, P (2015), 'Monetary policy surprises, credit costs and economic activity', American Economic Journal: Macroeconomics, Vol. 7, pages 44-76.

[14] Greenwald, D L (2018), 'The mortgage credit channel of macroeconomic transmission', Sloan School of Management.

[15] Ingholt, M M (2017), 'LTV vs. LTI constraints: When did they bind and how do they interact?, University of Copenhagen.

[16] Macallan, C, Millard, S P and Parker, M (2008), 'The cyclicality of mark-ups and profit margins for the United Kingdom: Some new evidence', Bank of England Working Paper No. 351.

[17] Rubio, M and Carresco-Gallego, J A (2015), 'Macroprudential and monetary policy rules: A welfare analysis', Manchester School, Vol. 83, pages 127-52.

[18] Rubio, M and Yao, F (2019), 'Macroprudential policies in a low interest rate environment', Journal of Money, Credit and Banking, forthcoming.

[19] Smets, F and Wouters, R (2007), 'Shocks and frictions in US business cycles: A Bayesian DSGE approach', American Economic Review, Vol. 97, pages 586-606.

[20] Van der Ghote, A (2017), 'Coordinating monetary and financial regulatory policies', European Central Bank mimeo.

[21] Woodford M (2011), 'Optimal monetary stabilization policy', in Friedman, B and Woodford, M (eds.) Handbook of Monetary Economics, Vol. 3B, pages 724-828, Elsevier: Amsterdam.

Annex 1: The efficient steady state

In this annex, we define the conditions under which a zero-inflation steady state is efficient and show that we can obtain an efficient steady state in our decentralised economy by setting taxes and subsidies.

Consider a social planner who maximises a weighted average of patient and impatient households' period utility function, subject to the aggregate resource constraint and market clearing in the housing and labour markets. Price adjustment costs are zero in a zero inflation steady state.

Maximise:

$$U = \omega U(c_P, H_P, h_P) + (1-\omega)U(c_I, H_I, h_I)$$

Subject to

$$h_P^{(1-\sigma)} h_I^{\sigma} = c_P + c_I$$

And

$$H_P + H_I = 1$$

Let μ_1 and μ_2 be the Lagrange multipliers on the resource and housing constraints, respectively. Then the first-order conditions will imply:

$$\omega U_{c,P} = \mu_1 \tag{39}$$

$$(1-\omega)U_{c,I} = \mu_1 \tag{40}$$

$$\omega U_{H,P} = \mu_2 \tag{41}$$

$$(1-\omega)U_{H,I} = \mu_2 \tag{42}$$

$$\omega U_{h,P} = -\mu_1(1-\sigma)\frac{y}{h_P} \tag{43}$$

$$(1-\omega)U_{h,I} = -\mu_1 \frac{\sigma y}{h_I} \tag{44}$$

Where Uc, U_H and U_h are the marginal utilities of consumption, housing and hours worked, respectively, for household type j. Combining equations 37,38, 39 and 40 gives:

$$\frac{U_{c,P}}{U_{H,P}} = \frac{U_{c,I}}{U_{H,I}} = \frac{\mu_1}{\mu_2} \tag{45}$$

In addition, equations 41 and 42 imply that the marginal rate of substitution between consumption and each type of labour is equal to the marginal rate of transformation between each type of labour and output.

$$\frac{U_{h,P}}{U_{c,P}} = (1-\sigma)\frac{y}{h_P} \tag{46}$$

$$\frac{U_{h,I}}{U_{c,I}} = \sigma\frac{y}{h_I} \tag{47}$$

Furthermore, if Pareto weights are set to match the population weights, i.e. $\omega = 1/2$ then in the efficient steady state:

$$c_P = c_I = \frac{y}{2} \tag{48}$$

$$H_P = H_I = \frac{1}{2} \tag{49}$$

Next, we show that by choosing taxes and subsidies we can achieve the efficient steady state in the decentralised economy. We set the subsidy to firms, τ_P, equal to $\frac{1}{(\varepsilon-1)}$. The zero-inflation steady-state version of the New Keynesian Phillips curve now implies:

$$rmc = \frac{(\varepsilon-1)(1+\tau_p)}{\varepsilon} = 1 \tag{50}$$

This implies:

$$R_L = \frac{1}{\beta_P}\frac{(\beta_P + \zeta(\varphi-1) - (1-\zeta)\varphi\nu\beta_P)}{\zeta\varphi(1+\tau_b)} \tag{51}$$

If we set the subsidy to banks, τ_b, equal to $\frac{\beta_P}{\zeta\varphi^*}(1 - \frac{\zeta}{\beta_P} - (1-\zeta)\varphi^*\nu)$ where φ^* is the degree of leverage in the efficient steady state, then:

$$R = R_L = \frac{1}{\beta_P} \tag{52}$$

And:

$$\theta\varphi^* = (1-\zeta+\zeta\theta\varphi^*)\left(\frac{\beta_P}{\zeta}(1-\frac{\zeta}{\beta_P}-(1-\zeta)\varphi^*\nu)+1\right) \tag{53}$$

Which can be used to solve for φ^*.

The steady-state versions of equations 3, 8, 12 and 13 imply:

$$\frac{U_{h,P}}{U_{c,P}} = w_P = (1-\sigma)\frac{y}{h_P} \tag{54}$$

$$\frac{U_{h,I}}{U_{c,I}} = w_I = \sigma\frac{y}{h_I} \tag{55}$$

Evaluating the Euler equation for impatient households at the efficient steady state gives:

$$\mu = \frac{1-\frac{\beta_I}{\beta_P}}{1-\beta_I\rho_L} \tag{56}$$

The Lagrange multiplier will be positive in the efficient steady state so long as $\beta_P > \beta_I$. Hence, the housing demand equation for impatient households in steady state implies:

$$\frac{c_I}{H_I} = \frac{(1-\beta_I-\mu(1-\rho_L)LTV)\,q}{j} = \frac{\left(1-\beta_I-\frac{1-\frac{\beta_I}{\beta_P}}{1-\beta_I\rho_L}(1-\rho_L)LTV\right)q}{j} \tag{57}$$

Similarly, for patient households we obtain:

$$\frac{c_P}{H_P} = \frac{(1+\tau_H-\beta_P)q}{j} \tag{58}$$

Equation 43 then implies that to obtain an efficient steady state, we need to set the housing tax equal to:

$$\tau_H = \beta_P - \beta_I - \frac{(1-\frac{\beta_I}{\beta_P})}{(1-\beta_I\rho_L)}(1-\rho_L)LTV \tag{59}$$

The LTV constraint then implies the efficient household debt to GDP ratio:

$$\frac{L_M}{y} = LTV\frac{q}{2y} \tag{60}$$

From the steady-state budget constraint for the impatient households we have:

$$\sigma = \frac{1-\beta_P}{\beta_P}\frac{L_M}{y} + \frac{c_I}{y} + \frac{T_I}{y} \Rightarrow \frac{T_I}{y} = -(\frac{1-\beta_P}{\beta_P}LTV\frac{q}{2y} + \frac{1}{2} - \sigma) \tag{61}$$

The impatient households need to receive a subsidy (net of taxes) proportional to GDP given by the term in brackets on the right-hand side of equation 59. Given such a subsidy, they will enjoy the same consumption and housing as the patient households, in line with our efficiency conditions 46 and 47.

Annex 2: Derivation of the loss function

This annex describes the derivation of the loss function shown in Section 4 of the paper. Following Ferrero et al. (2018), the welfare objective of the policymaker is defined as the present discounted value of the of the utility of the two types of household, weighted by arbitrary weights, ω and $1-\omega$, and discounted at the patient households' discount rate, β_P:

$$W_0 = E_0\sum_{t=0}^{\infty}\beta_P^t(\omega U_{P,t} + (1-\omega)U_{I,t})$$

Given the functional forms:

$$U_{P,t} = lnc_{P,t} + jlnH_{P,t} - \frac{1}{(1+\xi)}h_{P,t}^{1+\xi}$$

$$U_{I,t} = lnc_{I,t} + jlnH_{I,t} - \frac{1}{(1+\xi)}h_{I,t}^{1+\xi}$$

A second order approximation of U around the efficient steady state gives:

$$\begin{aligned}
U_t - U \approx \omega U_c\left(c_{P,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{P,t} - \frac{y}{2})^2\right) + (1-\omega)U_c\left(c_{I,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{I,t} - \frac{y}{2})^2\right) + \\
\omega U_H\left(H_{P,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{P,t} - \frac{1}{2})^2\right) + (1-\omega)U_H\left(H_{I,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{I,t} - \frac{1}{2})^2\right) + \\
\omega U_h\left(h_{P,t} - h_P + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{P,t} - h_P)^2\right) + (1-\omega)U_h\left(h_{I,t} - h_I + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{I,t} - h_I)^2\right)
\end{aligned}$$

Using the first-order conditions for the efficient steady state derived in Annex 1 we obtain:

$$U_t - U \approx \mu_1 \left(c_{P,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{P,t} - \frac{y}{2})^2 \right) + \mu_1 \left(c_{I,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{I,t} - \frac{y}{2})^2 \right) +$$
$$\mu_2 \left(H_{P,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{P,t} - \frac{1}{2})^2 \right) + \mu_2 \left(H_{I,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{I,t} - \frac{1}{2})^2 \right) -$$
$$\mu_1(1-\sigma)\frac{y}{h_P} \left(h_{P,t} - h_P + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{P,t} - h_P)^2 \right) - \mu_1 \sigma \frac{y}{h_I} \left(h_{I,t} - h_I + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{I,t} - h_I)^2 \right)$$

Given the functional form for preferences, we note that:

$$\frac{U_{cc}}{U_c} = -\frac{2}{y}$$

$$\frac{U_{HH}}{U_H} = -2$$

$$\frac{U_{hh}}{U_h} = \frac{\xi}{h}$$

Substituting in gives:

$$U_t - U \approx \mu_1 \left(c_{P,t} - \frac{y}{2} - \frac{1}{y}(c_{P,t} - \frac{y}{2})^2 \right) + \mu_1 \left(c_{I,t} - \frac{y}{2} - \frac{1}{y}(c_{I,t} - \frac{y}{2})^2 \right) +$$
$$\mu_2 \left(H_{P,t} - \frac{1}{2} - (H_{P,t} - \frac{1}{2})^2 \right) + \mu_2 \left(H_{I,t} - \frac{1}{2} - (H_{I,t} - \frac{1}{2})^2 \right) -$$
$$\mu_1(1-\sigma)\frac{y}{h_P} \left(h_{P,t} - h_P + \frac{1}{2}\frac{\xi}{h}(h_{P,t} - h_P)^2 \right) - \mu_1 \sigma \frac{y}{h_I} \left(h_{I,t} - h_I + \frac{1}{2}\frac{\xi}{h}(h_{I,t} - h_I)^2 \right) \quad (62)$$

Now the aggregate resource constraint is given by:

$$c_{P,t} + c_{I,t} = y_t(1 - \frac{\chi}{2}\pi_t^2) \quad (63)$$

We approximate any variable x using $x_t = x(1 + \hat{x}_t + \frac{1}{2}\hat{x}_t{}^2)$. Taking a second-order approximation of equation 61 and ignoring terms independent of policy gives:

$$c_{P,t} + c_{I,t} - y = y(\hat{y}_t + \frac{1}{2}\hat{y}_t{}^2 - \frac{1}{2}\chi\pi_t^2) \quad (64)$$

We can also note that:

$$H_{P,t} - \frac{1}{2} + H_{I,t} - \frac{1}{2} = 0 \tag{65}$$

Substituting equations 62 and 63 into equation 60 gives:

$$\begin{aligned} U_t - U \approx & \mu_1 y \left(\hat{y}_t + \frac{1}{2}\hat{y_t}^2 - \frac{1}{2}\chi\pi_t^2 \right) - \frac{\mu_1}{y} \left((c_{P,t} - \frac{y}{2})^2 + (c_{I,t} - \frac{y}{2})^2 \right) - \\ \mu_2 \left((H_{P,t} - \frac{1}{2})^2 + (H_{I,t} - \frac{1}{2})^2 \right) & - \mu_1(1-\sigma) y \left(\frac{h_{P,t} - h_P}{h_P} + \frac{\xi}{2} (\frac{h_{P,t} - h_P}{h})^2 \right) - \\ & \mu_1 \sigma y \left(\frac{h_{I,t} - h_I}{h_I} + \frac{\xi}{2} (\frac{h_{I,t} - h_I}{h_I})^2 \right) \end{aligned} \tag{66}$$

To eliminate the remaining first order terms from equation 64, we express variables in terms of log-deviations from the efficient steady-state values and drop terms of order 3 and higher:

$$\begin{aligned} U_t - U \approx & \mu_1 y \left(\hat{y}_t + \frac{1}{2}\hat{y_t}^2 - \frac{1}{2}\chi\pi_t^2 \right) - \frac{\mu_1 y}{4} (\hat{c}_{P,t}^2 - \hat{c}_{I,t}^2) - \\ \mu_1 y \left((1-\sigma)\hat{h}_{P,t} + \sigma \hat{h}_{I,t} \right) & - \mu_1 y \left(\frac{(1-\sigma)}{2} \hat{h}_{P,t}^2 + \frac{\sigma}{2} \hat{h}_{I,t}^2 \right) - \\ \frac{\mu_1 \xi y}{2} & \left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2 \right) - \frac{\mu_2}{4} (\hat{H}_{P,t}^2 + \hat{H}_{I,t}^2) \end{aligned} \tag{67}$$

Log-linearising the production function around the efficient steady state implies:

$$\hat{y}_t = \hat{A}_{z,t} + (1-\sigma)\hat{h}_{P,t} + \sigma \hat{h}_{I,t}$$

Substituting into equation 65 and dropping the term in $\hat{A}_{z,t}$, as it is independent of policy, implies:

$$\begin{aligned} U_t - U \approx & \frac{\mu_1 y}{2} (\hat{y}_t^2 - \chi\pi_t^2) - \frac{\mu_1 y}{4} (\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2) - \\ \frac{\mu_1 (1+\xi) y}{2} & \left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2 \right) - \frac{\mu_2}{4} (\hat{H}_{P,t}^2 + \hat{H}_{I,t}^2) \end{aligned} \tag{68}$$

The log-linearised version of the housing market equilibrium condition around the efficient steady state implies:

$$\hat{H}_{P,t} = -\hat{H}_{I,t} \Rightarrow \hat{H}_{P,t}^2 + \hat{H}_{I,t}^2 = \frac{1}{2} (\hat{H}_{P,t} - \hat{H}_{I,t})^2$$

Substituting back into equation 66 and collecting the output, consumption and labour terms implies:

$$
\begin{aligned}
U_t - U \approx & \frac{-\mu_1 y}{2}\left(\frac{1}{2}(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2) - \hat{y}_t^2 + (1+\xi)\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2\right)\right) \\
& - \frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{(\mu_1 y \chi)}{2}\pi_t^2
\end{aligned} \tag{69}
$$

Next, use:

$$
\begin{aligned}
\frac{1}{2}(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2) - \hat{y}_t^2 &= \frac{1}{2}(\hat{c}_{P,t}^2 - \hat{y}_t^2) + \frac{1}{2}(\hat{c}_{I,t}^2 - \hat{y}_t^2) \\
&= \frac{1}{2}\left((\hat{c}_{P,t} + \hat{y}_t)(\hat{c}_{P,t} - \hat{y}_t) + (\hat{c}_{I,t} + \hat{y}_t)(\hat{c}_{I,t} - \hat{y}_t)\right) \\
&= \frac{1}{2}\left((\frac{3}{2}\hat{c}_{P,t} + \frac{1}{2}\hat{c}_{I,t})(\frac{1}{2}\hat{c}_{P,t} - \frac{1}{2}\hat{c}_{I,t}) - (\frac{3}{2}\hat{c}_{I,t} + \frac{1}{2}\hat{c}_{P,t})(\frac{1}{2}\hat{c}_{P,t} - \frac{1}{2}\hat{c}_{I,t})\right) \\
&= \frac{1}{4}(\hat{c}_{P,t} - \hat{c}_{I,t})^2
\end{aligned}
$$

Substituting back into equation 67 implies:

$$
\begin{aligned}
U_t - U \approx & \frac{-\mu_1 y}{2}\left(\frac{1}{4}(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + (1+\xi)\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2\right)\right) \\
& - \frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{\mu_1 y \chi}{2}\pi_t^2
\end{aligned} \tag{70}
$$

Next, the labour supply equations imply:

$$
\frac{w_{P,t} h_{P,t}}{w_{I,t} h_{I,t}} = \frac{1-\sigma}{\sigma}
$$

Combining implies:

$$
h_{I,t} = (\frac{\sigma}{1-\sigma})^{\frac{1}{1+\xi}} h_{P,t} (\frac{c_{P,t}}{c_{I,t}})^{\frac{1}{1+\xi}}
$$

Combining with the production function implies:

$$
\begin{aligned}
y_t &= A_{z,t} h_{P,t} (\frac{\sigma}{1-\sigma}\frac{c_{P,t}}{c_{I,t}})^{\frac{\sigma}{1+\xi}} \\
\Rightarrow \hat{h}_{P,t} &= \hat{y}_t - \hat{A}_{z,t} - \frac{\sigma}{1+\xi}(\hat{c}_{P,t} - \hat{c}_{I,t}) \\
\Rightarrow \hat{h}_{I,t} &= \hat{y}_t - \hat{A}_{z,t} - \frac{1-\sigma}{1+\xi}(\hat{c}_{P,t} - \hat{c}_{I,t})
\end{aligned}
$$

Hence:

$$(1-\sigma)\hat{h}_{P,t}^2+\sigma\hat{h}_{I,t}^2$$
$$= (1-\sigma)\left(\hat{y}_t-\hat{A}_{z,t}-\frac{\sigma}{(1+\xi)}(\hat{c}_{P,t}-\hat{c}_{I,t})\right)^2+\sigma\left(\hat{y}_t-\hat{A}_{z,t}-\frac{1-\sigma}{(1+\xi)}(\hat{c}_{P,t}-\hat{c}_{I,t})\right)^2$$
$$= (\hat{y}_t-\hat{A}_{z,t})^2+\frac{\sigma(1-\sigma)}{(1+\xi)^2}(\hat{c}_{P,t}-\hat{c}_{I,t})^2$$

Substituting back into equation 68 and ignoring terms independent of policy gives:

$$U_t-U\approx\frac{-\mu_1 y}{2}\left(\frac{1+\xi+4\sigma(1-\sigma)}{4(1+\xi)}(\hat{c}_{P,t}-\hat{c}_{I,t})^2+(1+\xi)\hat{y}_t^2\right)$$
$$-\frac{\mu_2}{8}(\hat{H}_{P,t}-\hat{H}_{I,t})^2-\frac{\mu_1 y\chi}{2}\pi_t^2 \tag{71}$$

Using the first-order conditions for the efficient steady state to express μ_2 in terms of $\mu_1 y$:

$$\mu_2=\frac{\mu_1 U_{H,I}}{U_{C,I}}=\mu_1 yj$$

Substituting into equation 69 gives:

$$U_t-U\approx\frac{-\mu_1 y}{2}\left(\frac{1+\xi+4\sigma(1-\sigma)}{4(1+\xi)}(\hat{c}_{P,t}-\hat{c}_{I,t})^2+(1+\xi)\hat{y}_t^2\right)-\frac{j}{4}(\hat{H}_{P,t}-\hat{H}_{I,t})^2-\chi\pi_t^2$$

The welfare-based loss function can be expressed in terms of quadratic and gap variables as:

$$W_0=\frac{-\mu_1 y}{2}(1+\xi)E_0\sum_{t=0}^{\infty}\beta_P^t\left(\hat{y}_t^2+\lambda_1\pi_t^2+\lambda_2(\hat{c}_{P,t}-\hat{c}_{I,t})^2+\lambda_3(\hat{H}_{P,t}-\hat{H}_{I,t})^2\right)$$

Where $\lambda_1=\frac{\chi}{(1+\xi)}$, $\lambda_2=\frac{(1+\xi+4\sigma(1-\sigma))}{4(1+\xi)^2}$ and $\lambda_3=\frac{j}{4(1+\xi)}$.

Annex 3: Log-linear equations of the model

This annex presents the log-linearised version of the model based on a Taylor series expansion of the equations of the model around the efficient non-stochastic steady state derived in Annex 2.

$$\hat{c}_{P,t}=E_t\hat{c}_{P,t+1}-(\hat{R}_t-E_t\pi_{t+1})$$

$$\hat{H}_{P,t}=\frac{\beta_P}{(1+\tau_H-\beta_P)}E_t(\hat{q}_{t+1}-\hat{c}_{P,t+1})-\frac{(1+\tau_H)}{(1+\tau_H-\beta_P)}(\hat{q}_t-\hat{c}_{P,t})+\hat{A}_{j,t}$$

$$\hat{w}_{P,t} = \xi \hat{h}_{P,t} + \hat{c}_{P,t}$$

$$\sigma(\hat{w}_{I,t} + \hat{h}_{I,t}) + \frac{L_M}{y}\left(\hat{L}_{M,t} - \frac{1}{\beta_P}(\hat{L}_{M,t-1} + \hat{R}_{L,t-1} - \pi_t)\right) - \frac{q}{2y}(\hat{H}_{I,t} - \hat{H}_{I,t-1}) = \frac{1}{2}\hat{c}_{I,t}$$

$$\hat{L}_{M,t} = \rho_L(\hat{L}_{M,t-1} - \pi_t) + (1-\rho_L)E_t(\hat{q}_{t+1} + \pi_{t+1} + \hat{H}_{I,t})$$

$$\hat{c}_{I,t} = E_t\hat{c}_{I,t+1} - \left(\frac{1}{1-\beta_P\rho_L\mu}\hat{R}_{L,t} - \frac{\beta_P\rho_L\mu}{(1-\beta_P\rho_L\mu)}\hat{\mu}_{t+1} - E_t\pi_{t+1} + \frac{\mu}{(1-\mu)\hat{\mu}_t}\right)$$

$$\begin{aligned}
\hat{H}_{I,t} = & \frac{\beta_I}{(1-\mu(1-\rho_L)LTV-\beta_I)}E_t(\hat{q}_{t+1} - \hat{c}_{I,t+1}) \\
& + \frac{\mu(1-\rho_L)LTV}{(1-\mu(1-\rho_L)LTV-\beta_I)}E_t(\hat{\mu}_t + \hat{q}_{t+1} + \pi_{t+1} - \hat{c}_{I,t}) \\
& - \frac{1}{(1-\mu(1-\rho_L)LTV-\beta_I)}(\hat{q}_t - \hat{c}_{I,t}) + \hat{A}_{j,t}
\end{aligned}$$

$$\hat{w}_{I,t} = \xi \hat{h}_{I,t} + \hat{c}_{I,t}$$

$$\hat{L}_{E,t} = (1-\sigma)(\hat{w}_{P,t} + \hat{h}_{P,t}) + \sigma(\hat{w}_{I,t} + \hat{h}_{I,t})$$

$$\hat{y}_t = \hat{A}_t + (1-\sigma)\hat{h}_{P,t} + \sigma\hat{h}_{I,t}$$

$$\hat{w}_{P,t} = r\hat{m}c_t + \hat{y}_t - \hat{h}_{P,t} + \hat{R}_t - \hat{R}_{L,t}$$

$$\hat{w}_{I,t} = r\hat{m}c_t + \hat{y}_t - \hat{h}_{I,t} + \hat{R}_t - \hat{R}_{L,t}$$

$$\pi_t = \beta_P E_t\pi_{t+1} + \frac{\varepsilon}{\chi}r\hat{m}c_t$$

$$\hat{n}_t = \frac{\zeta\varphi(1+\tau_b)}{\beta_P}(\hat{R}_{L,t-1} + \hat{L}_{t-1}) - \zeta\frac{(\varphi-1)}{\beta_P}(\hat{R}_{t-1} + \hat{D}_{t-1}) - \frac{\zeta}{\beta_P}(1+\varphi\tau_b)\pi_t$$

$$\hat{n_t} = \varphi\hat{L}_t - (\varphi-1)\hat{D}_t$$

$$\hat{\varphi}_t = \hat{L}_t - \hat{n_t}$$

$$\hat{\psi}_t = \hat{\varphi}_t$$

$$\hat{\psi}_t = \frac{\varphi\tau_b}{(\varphi\tau_b+1)}\hat{\varphi}_t + \frac{\varphi(1+\tau_b)}{(\varphi\tau_b+1)}(\hat{R}_{L,t} - \hat{R}_t) + \frac{\zeta\psi}{(1-\zeta+\zeta\psi)}\hat{\psi_{t+1}}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(\nu_\pi\pi_t + \nu_y\hat{y}_t) + \varepsilon_{R,t}$$

$$\hat{y}_t = \frac{1}{2}(\hat{c}_{P,t} + \hat{c}_{I,t})$$

$$\hat{H}_{P,t} + \hat{H}_{I,t} = 0$$

$$\frac{L}{y}\hat{L}_t = \frac{L_M}{y}\hat{L}_{M,t} + \hat{L}_{E,t}$$