An Active-Learning Exercise for Syllabus Day in Intermediate Microeconomics

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ABSTRACT: Syllabus Day provides the wrong signals for the first day of class. Gannon (2016) argues that: "Ideally, the first day gives students a taste of everything they'll be expected to do during the semester." In microeconomics classes, this could be translated into the admonition: *DO SOME MARGINAL ANALYSIS*. This active-learning exercise highlights marginal analysis and reminds students of the tradeoffs they learned about in introductory microeconomics. The exercise has both geometric and algebraic components.

BACKGROUND and CONTENT MAP

- Marginal analysis is the centerpiece of microeconomics
  - Partial derivatives (partials for short) are the underlying mathematical concept behind marginal value
- Students having a single course in calculus often do not encounter partials
  - It is relatively straightforward to teach such students the algebra behind partials, but the geometric interpretation often confounds students
- This 40-minute exercise, based on Erfle (2019), highlights the geometric foundation of marginal analysis using a balloon model
  - Slides from a PowerPoint file guide students through four interrelated tasks (see ***Exercise Goals*** to the right)
    - Three slides below describe ***Building the Model***
    - Additional slides guide students through each task
- The color-coded ***Algebraic Counterparts*** handout provides a further roadmap to the exercise
- Instructor notes are included with the PowerPoint slides

Exercise Goals (color-coded to Four Tasks)

- The overarching goal is to provide an intuitive introduction to the tradeoffs inherent in marginal analysis
  - The objective, z, represents utility, profit, or output produced while x and y are outputs or inputs, depending on economic situation under analysis

1
- To visualize **marginal value of x** as slope in the x direction
  - This slope holds y fixed
    - This is the basis of the economists' notion of *ceteris paribus*
  - Marginal value is marginal utility, marginal profit, or marginal product, depending on economic situation under analysis

2
- To visualize **marginal value of y** as slope in the y direction
  - This slope holds x fixed using the economists' notion of *ceteris paribus*

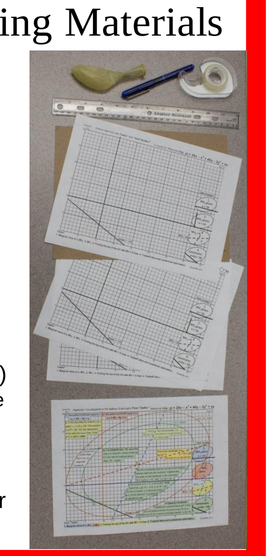
3
- To find the top of a hill using $MV_x = 0$ and $MV_y = 0$ lines
  - To see that points where $MV_x = 0$ are *horizontals* of the level sets and points where $MV_y = 0$ are *verticals* of the level sets

4
- To find a point where y is twice as valuable as x using **tradeoff ratios** ($TR = MV_x/MV_y$ is the slope of the level set)
- To visualize constrained optimization on the balloon and on the graph as tangency between constraint and level set
  - Relate tangency to marginal values and *Equal Bang-for-the-Buck* rule

Building the Balloon Model

Each Group needs the following Materials

- One clear 11-inch or 12-inch balloon (don't blow it up yet)
- Ruler (clear is best)
- At least one gel pen (although different colors help)
- Tape for attaching graph paper to backing board and balloon to the base
  - 12 pieces of tape (8 one-inch pieces to attach paper to base, 4 two-inch pieces to attach balloon to base)
- One piece of backing board (8.5" × 11" or a bit larger) to act as the base
- A sheet of graph paper for each student and an additional one for the group as a whole (Slide 4)
  - One provides a base for the balloon, the others are for 2-D graphing
  - The sheets have lines and "Four Task Guides" to use during the exercise
- An *Algebraic Counterparts* sheet of graph paper (Slide 5, printed in color) for each student

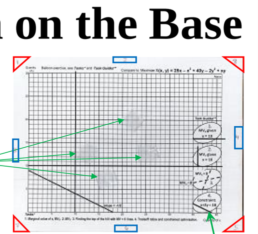


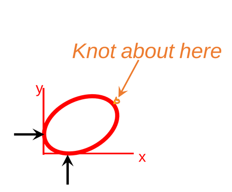
How to Create an Oval Balloon from a Round Balloon

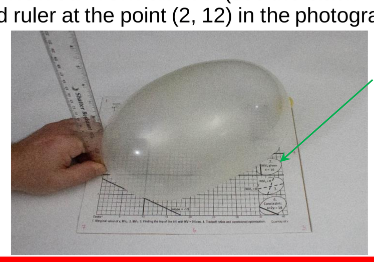
- Round balloons are readily available but not as interesting as ovals for representing economic models
  - Luckily, you can turn a round balloon into an oval one pretty easily
  - The height of the balloon above the base, z, can be used to represent utility, profit, or production (if x and y are inputs)
- Blow up the balloon while restricting the balloon in the middle with your fingers [Watch Here]
  - Think of two balloon ends and restricted middle like a barbell
  - *Note:* when doing this, it is a bit harder to blow up the balloon
- You should end up with a balloon that is 9 to 10 inches long, and 6 to 7 inches wide

Positioning your Balloon on the Base

- Tape the **corners**, and **edges**, of a piece of graph paper to the backing board to create a base for the balloon
- Attach 4 small ***tape loops*** to the paper and use them to fix the balloon to the base like shown in the ***Task Guides*** *with knot in NE corner as shown below*
- The balloon should be approximately over the x (bottom) and y (left) axes when viewed from above (see the black arrows in the mock-up and ruler at the point (2, 12) in the photograph)

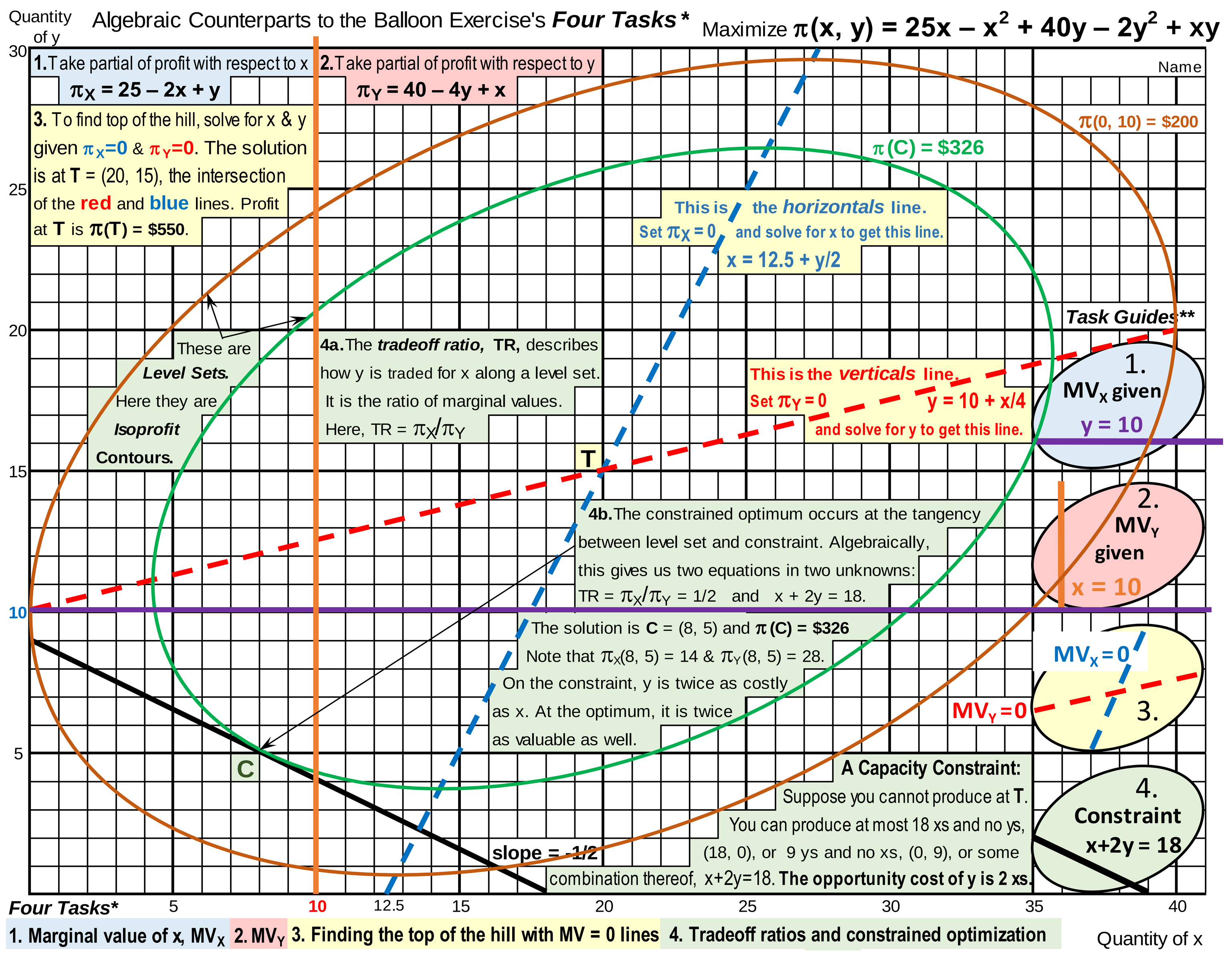






The four ***Task Guides*** highlight lines used in the main analytical tasks in the exercise

The profit function: $\pi(x, y) = 25x - x^2 + 40y - 2y^2 + xy$ provides **Algebraic Counterparts** to the 4 Balloon Tasks



- This exercise works in intermediate microeconomics classes without calculus
  - The geometric analysis on the balloon does not use calculus and the algebraic analysis can be done without calculus by asserting the marginal values of x and y
- The balloon is an ellipsoid, but paraboloids provide easier algebraic modeling
  - This exercise works best if the geometric and algebraic analyses are interwoven

Erfle, S. 2019. An Active-Learning Approach to Visualizing Multivariate Functions using Balloons. *Spreadsheets in Education*, Vol. 12, Issue 1, pp. 1-16.

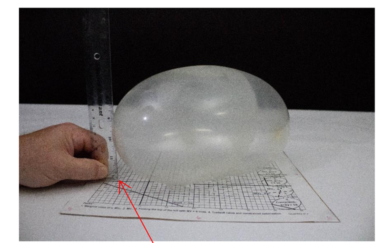
Gannon, K. 2016. The Absolute Worst Way to Start the Semester. *Chronicle of Higher Education, Views Section*.

1. Visualizing the Marginal Value of x

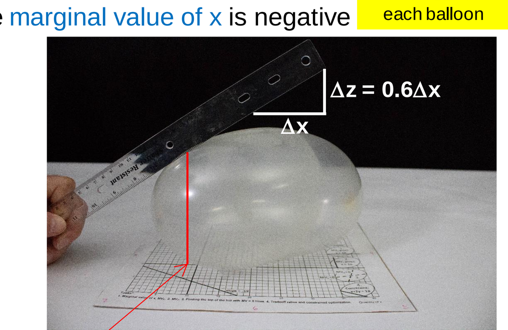
Task 1. Visualizing z Slope in the x Direction

- Marginal value of x, MV_X, is $\Delta z/\Delta x$, the z slope in the x direction, *holding y fixed* (Imagine walking W to E)*
  - Here, hold y fixed at y = 10 (the black horizontal line on graph):
    - The marginal value of x is infinite (i.e., the ruler is vertical) at x = 3
    - From there, marginal value of x declines as x increases
    - At x = 18, given y = 10, the marginal value of x is zero
    - For x > 18, given y = 10, the marginal value of x is negative

Note: This will occur at different x values for each balloon



This panel depicts the vertical edge of the balloon at (3, 10) where $MV_X = \infty$



At (8, 10), MV_X = 0.6 (slope of the ruler above)

*In terms of calculus, MV_X is the partial derivative with respect to x, $\partial z/\partial x$

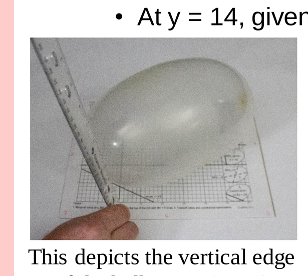
- The surface of the balloon z(x, y) can represent a number of economic concepts including profits attained by a multi-product firm, $\pi(x, y)$
- A partial derivative calculates slope a specific direction, in this instance, the x direction
  - This treats other variables (here y) as constants
  - This is denoted using a del operator, ∂, or a subscript rather than d or ʹ
- If $\pi(x, y) = 25x - x^2 + 40y - 2y^2 + xy$, the partial with respect to x is: $\partial\pi/\partial x = \pi_x = 25 - 2x + y$
  - Note that π_x varies as x varies for fixed y

- The π-maximizing x for a given y occurs when $\pi_x = 0$
  - $\pi_x = 25 - 2x + y = 0$ occurs when $x = 25/2 + y/2$
  - This is the dashed-blue *horizontals* line from **B** to **T**

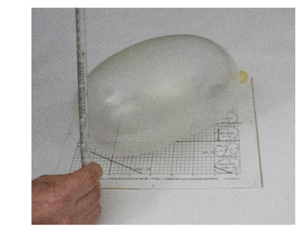
2. Visualizing the Marginal Value of y

Task 2. Visualizing z Slope in the y Direction

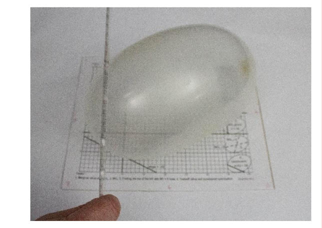
- Marginal value of y, MV_Y, is $\Delta z/\Delta y$, the z slope in the y direction, holding x fixed (Imagine walking S to N)*
  - This is done at x = 10 in the figures below
    - The marginal value of y is infinite (i.e., the ruler is vertical) at y = 0
    - From there, marginal value of y declines as y increases
    - At y = 14, given x = 10, the marginal value of y is zero



This depicts the vertical edge of the balloon at (10, 0) where $MV_Y = \infty$



MV_Y is positive for 0 < y < 14, given x = 10



MV_Y declines as y increases

Note: The above observations will occur at different y values for each group's balloon.

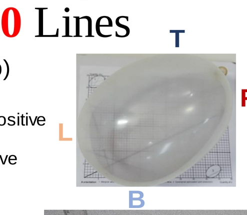
*Similarly, MV_Y is the partial derivative with respect to y, $\partial z/\partial y$, is written in shorthand as z_y

- The slope in the y direction at any value of y, MV_y, depends on the specific value of x that is being held fixed (as was the case for y when visualizing MV_x)
  - It is important to rotate the model clockwise 90° to look at the slope in the y direction from E to W
    - This ensures that y increases to the right, just as x increased to the right when we viewed the balloon from S to N in visualizing MV_x in Task 1
- If $\pi(x, y) = 25x - x^2 + 40y - 2y^2 + xy$, the partial with respect to y is: $\partial\pi/\partial y = \pi_y = 40 - 4y + x$
  - Note that π_y varies as y varies for fixed x

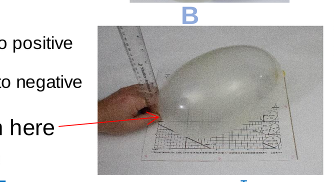
- The π-maximizing y for a given x occurs when $\pi_y = 0$
  - $\pi_y = 40 - 4y + x = 0$ occurs when $y = 10 + x/4$
  - This is the dashed-red *verticals* line from **L** to **R**

3. Using MV to find the Top of a Hill

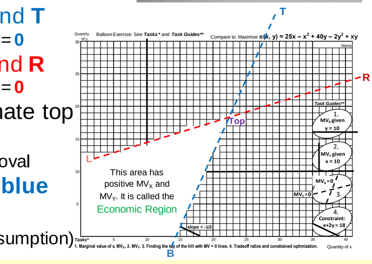
Task 3. *Finding and Graphing* the Top of a Hill using $MV_X = 0$ & $MV_Y = 0$ Lines

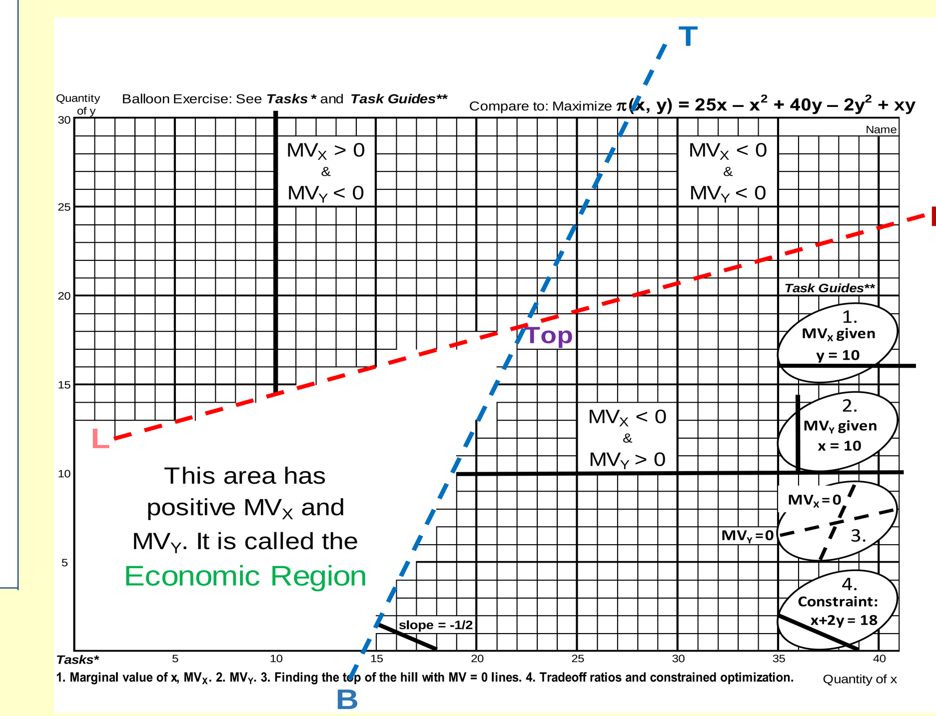


- To find MV = 0 lines, view the balloon from above (stand up)
- Find the point where $MV_X = 0$ at the bottom and top
  - The Bottom is where the outline of the balloon goes from negative to positive slope as x increases (See B)
  - The Top is where the outline of the balloon goes from positive to negative slope as x increases (See T)
- Find the point where $MV_Y = 0$ at the left and right
  - The Left is where the outline of the balloon goes from negative to positive slope as y increases (See L)
  - The Right is where the outline of the balloon goes from positive to negative slope as y increases (See R)
- Use ruler to locate each point on the base: L is shown here



- Transfer this information to graph paper (bottom right)
- Use your ruler to draw a dashed line between **B** and **T**
  - This line approximates the set of (x, y) bundles where $MV_X = 0$
- Use your ruler to draw a dashed line between **L** and **R**
  - This line approximates the set of (x, y) bundles where $MV_Y = 0$
- The intersection of these two lines is the approximate top of the hill
  - This will be more accurate, the closer your balloon is to an oval
- The area below red $MV_Y = 0$ and to the left of the blue $MV_X = 0$ lines has both $MV_X > 0$ and $MV_Y > 0$
  - This is called the *Economic Region* (of production or consumption)





- The MV = 0 lines are the ridge lines denoting four quadrants on the balloon
  - The third quadrant is called the "economic region"
- The top of the hill occurs when $MV_x = 0$ **and** $MV_y = 0$
  - The (x, y) value where this occurs will differ from balloon to balloon
- Algebraically, we must solve two equations in two unknowns to find the top of the hill
  - There are many ways to solve for x and y, one involves using the *horizontals* and *verticals* lines
    - $x = 25/2 + y/2$ and $y = 10 + x/4$ implies
    - $x = 12.5 + (10 + x/4)/2 = 12.5 + 5 + x/8 = 17.5 + x/8$
    - Subtract x/8 from both sides: $(7/8)\cdot x = 17.5$
    - So, $x = 17.5\cdot 8/7 = 20$ and $y = 10 + 20/4 = 15$

Task 4. Level Sets, Tradeoff Ratios, and Constrained Optimization

4A.1 Estimating Δz in any Direction

- Tasks 1 and 2 focused on understanding how one describes slope in two specific directions: x, MV_x; and y, MV_y
- We can use these slopes to describe the change in height on the balloon that would occur if we move in **any** direction
- One way to describe the move uses points of a compass; another uses change coordinates $(\Delta x, \Delta y)$
  - For example, a move to the North East (along a 45° line) involves moving $(\Delta x, \Delta y)$ where $\Delta x = \Delta y > 0$
- What would happen to z if we move Δx in the x direction and Δy in the y direction on the surface of the balloon?
  - For small changes in x and y, we (approximately) would have:

$$\Delta z = \Delta x\cdot MV_x + \Delta y\cdot MV_y$$

  - This change would be exact if the surface was a plane
- The first component is the change in z due to the change in x and the second is the change in z due to the change in y

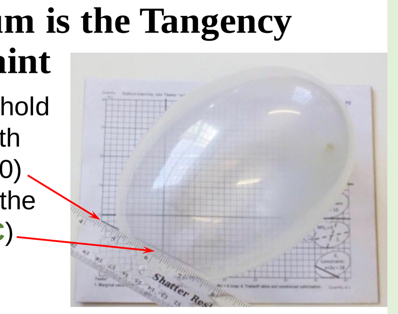
4A.2 Level Sets and Tradeoff Ratios

- If the change in z due to changes of Δx and Δy cancel one another, the net change in z is zero
  - In this case, (x_0, y_0) and $(x_0+\Delta x, y_0+\Delta y)$ are on the same ***level set***
- Economists have a variety of terms for these level sets
  - *Indifference curves*, *isoquants* and *isoprofit contours* are the terms given for equal utility, equal production, and equal profit curves
- Consider an initial point in the Economic Region (where both marginal values are positive): If x increases from there then y must decrease if Δz = 0
  - Put another way, there must be a **tradeoff** between y and x
- Minus the slope of the level set describes the ***tradeoff ratio***
  - Set Δz = 0 and solve for $-\Delta y/\Delta x$:

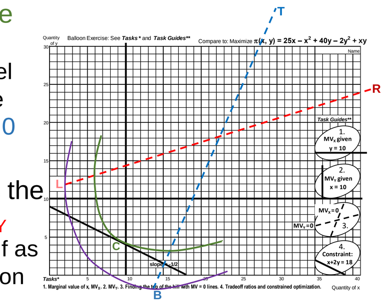
$$0 = \Delta x\cdot MV_x + \Delta y\cdot MV_y \text{ implies } TR = -\Delta y/\Delta x = MV_x / MV_y$$

  - The minus is added so we can talk about this tradeoff in positive terms (2 units of x for 1 unit of y, for example, means TR = 1/2)
- The ***tradeoff ratio*** tells us the rate at which y must be traded for x in order to maintain utility, production, or profit at a given level
  - Economists often call this the *Marginal Rate of Substitution, MRS* (although other letters are often added to distinguish scenarios)

4B. The Constrained Optimum is the Tangency between Balloon and Constraint



- Looking at the balloon from overhead, hold the ruler, parallel to the base, in line with the line on the base from (0, 9) to (18, 0) and lower the ruler until it just touches the balloon at the **6"** mark (call this point **C**)
- At point **C**, the tradeoff ratio = ½
- For this balloon, this is at **C** = (9, 4.5)
- Sketch the tangency at **C** (done with the green curve in this graph)
  - The green curve is constrained optimal level (and the purple one is the outer edge of the balloon. Both are horizontal at blue $MV_X = 0$ and vertical at red $MV_Y = 0$ lines)
- In 4A.2 we saw that the tradeoff ratio is the ratio of Marginal Values, $TR = MV_X/MV_Y$
  - Check that the slope in the x direction is half as steep as the slope in the y at **C** on the balloon



- The algebraic counterpart requires solving two equations in two unknowns:

a) constraint function & b) Tradeoff ratio = price ratio

a) $x + 2y = 18$ b) $TR = \pi_X/\pi_Y = (25 - 2x + y)/(40 - 4y + x) = ½$

-Solve for x in a) we obtain: $x = 18 - 2y$

-Cross-multiplying b) we obtain: $50 - 4x + 2y = 40 - 4y + x$

This simplifies to: $10 - 5x + 6y = 0$

-Substitute for x and solve for y: $10 - 5(18 - 2y) + 6y = 0$

$10 - 90 + 10y + 6y = 0$, $16y = 80$, or $y = 5$ & $x = 18 - 2\cdot 5 = 8$

- At C = (8, 5), the MV_y, $\pi_y = 28$, is twice as much as the MV_x, $\pi_x = 14$, just as required because y is twice as costly as x (the opportunity cost implicit in the constraint)
- And on the balloon, MV_y is twice as steep as MV_x at **C**
- Both results are simply restatements of the *Equal Bang-for-the-Buck* rule learned in introductory economics

This question tests the algebraic counterparts to Tasks 1 – 4A

36 points total) FOC Enterprises estimates its profit, π, as a function of the two goods it produces, x and y, is:

$$\pi(x, y) = 400x - 20x^2 + 180y - 10y^2 + 10xy - 1{,}240.$$

A;8) How many units of x and y should FOC produce in order to maximize profits? What profits result?

B;4) FOC wishes to verify that it is, indeed, producing at the profit maximizing output level. To do this, FOC wishes to consider what profits will be if it produces Δx = 6 units more or less x than the amount determined in part A. FOC chooses the level of y that maximizes profits given these alternative x values, y(x). What profits result in each instance? Fill in the table and **provide the equation for y(x).**

6 Δx	Part B	x	y(x)	π(x,y)
x is Δx more than x_{max}				
π maximizing x = x_{max}				
x is Δx less than x_{max}				

y(x) =

C;3) FOC decides instead to consider what profits will be if it produces Δy = 4 units more or less y than the amount determined in part A. What level of x maximizes profits given these alternative y values, x(y)? What profits result in each instance? Fill in the table and **provide the equation for x(y).**

4 Δy	Part C	x(y)	y	π(x,y)
y is Δy more than y_{max}				
π maximizing y = y_{max}				
y is Δy less than y_{max}				

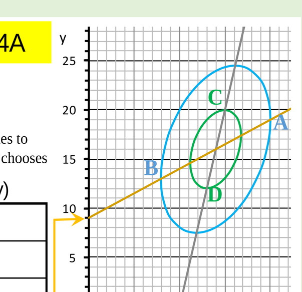
x(y) =

D;6) What is the tradeoff ratio at (9, 9) and at (10, 8)? Provide both answers to the nearest 0.01 or as a fraction. Imagine x and y are measured in "thousands" so that a "one" unit change would be represented by 9.001 or 9.999 from 9 or 10 respectively. Use the tradeoff ratio to show what y would be (your answer **MUST BE** to the nearest 0.0001) that would produce the same profits as (9, 9) or (10, 8). Briefly explain why each answer is correct (one sentence, or a simple equation, will do, but do not say: "I used Goal Seek").

Point	Tradeoff Ratio	New x	New y (to nearest **0.0001**)	Why is this the new y value?
(9, 9)		9.001		
(10, 8)		9.999		

E;13) Graph the profit hill using the graph paper provided. Make sure to include the iso-profit contours associated with your answers to B and C. Provide two lines on this graph: $\pi_x = 0$; $\pi_y = 0$ and label these lines.

(9, 9)	13/9 or 1.44	9.001	8.9986	=9-13/9*0.001
(10, 8)	2/3 or 0.67	9.999	8.0007	=8+2/3*0.001



x	y	π(x,y)	
20	19	$2,370	A
14	16	$ 3,000	
8	13	$2,370	B

0.5 * x + 9 = y(x)

x	y	π(x,y)	
15	20	$2,860	C
14	16	$ 3,000	
13	12	$2,860	D

0.25 * y + 10 = x(y)

Blended Learning Video Hyperlinks: 1 and 2

Answers above and at the left

Presented at the January 2020 ASSA, AEA Committee on Economic Education Annual Conference, San Diego, CA