A Unified Model of Distress Risk Puzzles

Abstract

We build a dynamic model to link two empirical patterns: the negative failure probability-return relation (Campbell, Hilscher, and Szilagyi, 2008) and the positive distress risk premium-return relation (Friewald, Wagner, and Zechner, 2014). We show analytically and quantitatively that (i) equity-to-debt ratios and levered equity betas negatively covary with the market risk premium in highly distressed firms; (ii) the negative covariance generates low stock returns and negative alphas among those firms; and (iii) firms with a lower distress risk premium endogenously choose higher leverage, so they are more likely to become distressed and earn negative returns. We provide empirical evidence to support our model predictions.

1 Introduction

Distress risk plays an important role in corporate financing choices and asset prices. Even though distress risk deters debt taking, empirical evidence on the *equity* distress risk premium in asset prices is mixed.[1] Recently, while Campbell et al. (2008) document a negative relation between failure probabilities and stock returns, Friewald et al. (2014) find a positive relation between distress risk premium (from credit default swaps) and stock returns. Moreover, firms with a high failure probability or a low distress risk premium have high equity beta but low stock returns on average. In this study, we develop a unified framework to explicitly link seemingly contradicting observations, i.e., the negative failure probability-return relation and the positive distress risk premium-return relation.

Endogenous covariance between levered equity beta and the market risk premium helps us to understand the failure risk premium, in the framework of conditional capital asset pricing model (CAPM). The intuition for the negative covariance is as follows. First, debt issuance is procyclical in general. Firms issue more debt in expansions than in recessions (Bhamra, Kuehn, and Strebulaev, 2010a), which implies that the debt ratio is procyclical around debt issuance. Second, when firms do not issue debt because of transaction costs (Strebulaev, 2007), debt value decreases more than equity value in response to the increased market risk premium in distressed firms but not in healthy firms. Intuitively, firms are more likely to become distressed and incur addition costs, when the economy slides into the bad states where the market risk premium is high. When the market risk premium and distress costs increase simultaneously in the bad states, the sharing of the decreased asset value is asymmetry. While equity holders are able to walk away at bankruptcy due to limited liability, debt holders in distressed firms will take over the residuals of assets and bear most of the losses in assets. Thus, the debt value decreases much more than the equity value when the market risk premium and distress cost rise in the bad states. In other words, debt-to-equity ratios and therefore levered equity betas decrease with the increased market risk premium. The resulting negative covariance between them decreases stock returns among distressed firms. Combined with

[1] Dichev (1998), Griffin and Lemmon (2002), Campbell, Hilscher, and Szilagyi (2008) and Avramov, Chordia, Jostova, and Philipov (2013) find a negative relation between various proxies of default likelihood and stock returns. Vassalou and Xing (2004) and Chava and Purnanandam (2010) find a positive relation between the default probability and stock returns. Garlappi and Yan (2011) show there is a hump-shaped relation between default probability and stock returns.

the fact that distressed firms have high leverage and high equity betas, we produce simultaneously high unconditional CAPM beta and low stock returns among them, as documented by Campbell et al. (2008).

Endogenous distress status helps understand the positive relation between the distress risk premium and stock returns, i.e., heterogeneity in the exposure to market risk has a first order effect on the endogenous debt choice and therefore the firm's future financial status. That is, firms with a low exposure to aggregate distress risk (cash flow betas) choose higher leverage *ex ante*.[2] When hit by a large market-wide shock, those firms are more likely to become distressed relative to their counterparts. In other words, firms with a low distress risk premium are more likely to be distressed and hence earn low stock returns in Friewald et al. (2014).[3] To our knowledge, we are the first to model the endogenous distress status in the class of dynamic capital structure/credit risk models, and demonstrate its profound implications for the distress risk premium puzzle.

To facilitate our understanding of the negative relation between failure probability and stock returns, we simplify the model and derive closed-form solutions to show that the negative failure probability-return relation is due to the negative covariance of equity beta and market premium. Then, we take advantage of the dynamic model and demonstrate it can generate the sizable failure risk premium quantitatively. Following Campbell et al. (2008), we perform logit regressions and construct failure probabilities using our simulated data panels. When sorting firms on the failure probabilities, highly distressed firms exhibit high leverage and default probability, but have low returns and negative CAPM alphas. We provide empirical evidence to confirm the novel economic channel in our model. We first show that, debt-to-equity ratios of distressed firms are more negatively associated with the market risk premium than those in healthy firms. Then, we follow Lewellen and Nagel (2006), construct time-varying equity betas, and confirm that levered equity be-

[2]Following Andrade and Kaplan (1998), we define a firm as "distressed" when its cash flow level falls below its contractual interest payment. Higher coupon payments imply an earlier time of entering distress. Hence, the distress threshold is endogenously chosen in our model, because debt levels are endogenously chosen over the business cycle. Distress is exogenous in prior studies. For example, Elkamhi, Ericsson, and Parsons (2012) are the first who explicitly study financial distress in a Leland-type model (Leland, 1994). They take the distress threshold as exogenous and calibrate the threshold to match the firm characteristic before and after the downgrades of credit rating, and find a small flow distress cost before liquidation substantially helps to explain the low financial leverage puzzle.

[3]In our model, firms choose optimal financing policies over the business cycle. The endogeneity of debt financing becomes more severe when the economy fluctuates between good and bad states. Equity holders are concerned about bad states even when they finance in the good states, because the economy may suddenly switch into the bad states, in which they will face a higher distress cost. Thus, they choose even lower leverage *ex ante* (Hackbarth, Miao, and Morellec, 2006) even if financing in the good states.

tas are negatively (positively) correlated with expected market risk premium in distressed (healthy) firms. Finally, the negative covariance between levered equity beta and market risk premium helps explain about 50% of the distress risk premium in the conditional CAPM.

To understand the positive relation between the implied distress risk premium and returns, we propose a simple procedure to imply the distress risk premium in spirit of Almeida and Philippon (2007). Motivated by our analytical solution for the simplified model, we use as proxy of distress risk premium the log-difference between risk-neutral and actual default probability in our calibrated economies. In the model, we have two types of firms, i.e., low- and high-beta firms. The high-beta firms face a greater exposure to the aggregate distress risk premium and therefore have a high firm-specific distress risk premium. We mimic standard empirical procedures, imply the firm-specific distress risk premium from our simulated data, and form portfolios on the implied risk premium. Consistent with findings of Friewald et al. (2014), firms with a lower implied distress risk premium, on average, tend to have higher leverage ratios, higher expected default probabilities, and higher realized distressed frequencies, receiving negative stock returns.

Taken together, we connect the two seemingly contradicting observations by explicitly showing that the two ranking variables, failure probability and implied distress risk premium, are negatively correlated *ex post*.[4]

Our works belongs to the literature of dynamic models of debt refinancing (i.e., Goldstein, Ju, and Leland (2001) and Strebulaev (2007)). Recent literature has introduced macroeconomic risk on corporate financing and investment decisions as well as credit risk. Hackbarth et al. (2006) are the first to introduce macroeconomic dynamics to dynamic capital structure/credit risk models. Along this line, Chen (2010) seeks to explain the observed credit spreads and leverage ratios, Bhamra, Kuehn, and Strebulaev (2010b) focus on a levered equity premium and Bhamra et al. (2010a) focus on the dynamics of leverage in an economy with macroeconomic risk. Chen and Manso (2016) and Chen and Strebulaev (2018) study the debt overhang problem and the risk-shifting problem over the business cycle, respectively.

[4]Friewald et al. (2014) show that the distress risk premium from CDS data and equity risk premium are positively correlated in the Merton (1974) model. Thus, physical or risk-neutral default probability alone is insufficient to correctly assess the distress risk premium. However, they do not explain why firms with a low distress risk premium have a high default probability and low credit rating. We complement their point and explicitly establish an *ex post* negative relation between distress risk premium and default probability, because of the endogenous distress status.

Our work also adds to the literature on how financial or real frictions affect asset prices.[5] Gomes and Schmid (2010) and Kuehn and Schmid (2014) examine the interaction between investment and financing, and their implications for the levered equity risk and default risk. Ozdagli (2012) and Choi (2013) demonstrate that the value premium is mainly driven by financial leverage. Recently, Kuehn and Schmid (2014) study the investment-based corporate bond pricing, Koijen, Lustig, and Van Nieuwerburgh (2017) show that bond factors from different business cycle horizons are priced in the cross-section of stock returns, and Chaderina, Weiss, and Zechner (2018) show that firms with more long-term debt earn greater stock returns.[6]

Our paper relates to recent risk-based theories to explain the distress puzzles. A partial list includes George and Hwang (2010), Garlappi and Yan (2011), O'Doherty (2012), and Boualam, Gomes, and Ward (2017).[7] All the aforementioned theories appeal to the decline in the equity beta among the highly distressed firms. However, distressed stocks have high volatility and high unconditional equity betas in the data. Thus, our work differs in at least two perspectives. First, we illustrate the importance of endogenous debt financing, and the negative covariance between the equity beta and market premium in the closed-form solution and in the calibrated economies, and verify its quantitative implications in the data. Second, we explicitly show that the default probability and distress risk premium are negatively connected. That is, firms with a low distress risk premium choose high leverage *ex ante*, which likely cause them to become distressed *ex post*. The endogenous connection between them allows us to explain the *negative* failure probability-return relation and the *positive* distress risk premium and stock return relation jointly.

[5]Several papers study the implications of corporate investment on stock returns, such as Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), Hackbarth and Johnson (2015), and Ai and Kiku (2013).

[6]Slightly different, Chen, Collin-Dufresne, and Goldstein (2009), Arnold, Wagner, and Westermann (2013), Chen, Xu, and Yang (2019) and Chen, Cui, He, and Milbradt (2014) examine credit spreads in the framework of credit risk over the business cycles.

[7]George and Hwang (2010) also argue that firms with a low distress cost choose to issue more debt and consequently become distressed. Therefore, these distressed firms earn *low* stock returns because of their *low* asset/cash flow beta. Johnson, Chebonenko, Cunha, D'Almeida, and Spencer (2011) point out two problems in their argument. First, the expected equity returns in Proposition 1 of George and Hwang (2010) pertain to the *asset* beta and return instead of the *equity* beta and return. In other words, because distressed equity has a high equity beta, the low asset beta is not sufficient to explain the default/failure risk premium. Second, because of transaction costs, the firms do not adjust their debt every period, given their asset betas. Moreover, Garlappi and Yan (2011) argue that equity holders with more bargain power receive more at bankruptcy, which in turn alleviates their downside risk and results in low equity returns. O'Doherty (2012) shows that information risk attributes to the negative failure probability-return relation in the conditional CAPM framework, but without explicitly modeling the time-varying market risk premium. Lastly, Boualam et al. (2017) argue that measurement errors due to the mean-reverting earnings growth induces the failure risk premium.

The rest of the paper proceeds as follows. Section 2 develops the fully fledged model. Section 3 derives closed-form solution and generate three predictions from the simplified model. Then, we calibrate the model and study the quantitative implications of the model in Section 4. Section 5 provides empirical evidence in support of the calibrated model and its implications. Finally, Section 6 concludes.

2 The Model

We build a dynamic capital structure model that endogenizes a firm's financing, distress, and default decisions in an environment with time-varying macroeconomic risk. Our model is built on the recent development of credit risk models, including Chen (2010), Bhamra et al. (2010a,b).

Considering an economy with business-cycle fluctuations, and without loss of generality, we assume the economy has two aggregate states, i.e., $s_t = \{G, B\}$ for good (G) and bad (B) states, respectively. The state s_t follows a continuous-time Markov chain as follows:

$$\begin{bmatrix} 1-\hat{p}_B & \hat{p}_B \\ \hat{p}_G & 1-\hat{p}_G \end{bmatrix} \tag{1}$$

where $\hat{p}_{s_t} \in (0,1)$ is the rate of leaving the current state of s_t for another state. The probability of switching states, s_t, within a small interval Δt is approximately $\hat{p}_{s_t}\Delta t$. While the long-run duration of the economy in the bad state is $\hat{p}_G/(\hat{p}_G + \hat{p}_B)$, the duration of the economy in the good state is $\hat{p}_B/(\hat{p}_G + \hat{p}_B)$. Recall that we use ˆ to denote the physical measure throughout the paper.

The model is partial equilibrium with pricing kernel, m_t, as follows:

$$\frac{dm_t}{m_t} = -r_{s_t}dt - \theta_{s_t}d\hat{W}_t^m + (e^{\kappa_{s_t}} - 1)dM_t, \tag{2}$$

where r_{s_t} is the risk-free rate, θ_{s_t} is the market price of risk of small shocks, κ_{s_t} is the relative jump size of the stochastic discount factor, $\hat{W}_t^m$ is a standard Brownian motion, and M_t is a compensated Poisson process with intensity $\hat{p}_{s_t}$ that follows the Markov chain specified in equation (1). κ_{s_t} determines the market price of large shocks in the aggregate economy: $\kappa_B = -\kappa_G$ and $\kappa_G > 0$.

2.1 Firm

Firms operate in one of two aggregate states, s_t. In each state, their firm-specific financial status (w) can be healthy (H) or distressed (D), i.e., $w = H, D$. When solvent, the firms produce instantaneous cash flows X_t governed by the following stochastic process:

$$\frac{dX_t}{X_t} = \hat{\mu}_{s_t,w}dt + \beta\sigma^m_{s_t}d\hat{W}^m_t + \sigma^{i,X}_{s_t}d\hat{W}^i_t, \tag{3}$$

where $\hat{\mu}_{s_t,w} = \mu_{s_t,w} + \beta\lambda^m_{s_t}$ is the physical growth rate, $\mu_{s_t,w}$ is the risk-neutral growth rate, $\lambda^m_{s_t} = \theta_{s_t}\sigma^m_{s_t}$ is the countercyclical market risk premium (i.e., $\lambda^m_B > \lambda^m_G$), β is the firm's cash flow beta, $\sigma^{i,X}_{s_t}$ is the idiosyncratic cash flow volatility, and $\hat{W}^i_t$ is a standard Brownian motion. The total volatility of cash flow rates $\sigma^{T,X}_t = \sqrt{(\beta\sigma^m_{s_t})^2 + (\sigma^{i,X}_{s_t})^2}$.

The difference in the growth rate, $\hat{\mu}_{s_t,H} - \hat{\mu}_{s_t,D}$, is the distress cost, η_{s_t}, in the state s_t. The distress cost $\eta_{s_t} \geq 0$ is a deadweight loss due to the loss of reputation, customers, suppliers, and productive workers when the firms become distressed.[8] Additionally, because Almeida and Philippon (2007) document the distress cost is countercyclical (i.e., low in good aggregate states, but high in bad aggregate states), we assume $\eta_B > \eta_G$.

We consider two types of firms, i.e., low- and high-beta firms. Because the difference in the market risk premium across the bad and good states, $\lambda^m_B - \lambda^m_G$, is mainly due to the aggregate "distress", the high-beta firms have more exposure to the aggregate distress market risk premium, and therefore a high firm-specific distress risk premium, $\beta(\lambda^m_B - \lambda^m_G)$. This distress risk premium can be considered as systematic distress costs related to the market, because it lowers the risk-neutral (or risk-adjusted) growth rate $\mu_{s_t,w}$.

2.2 Financing and Default Decisions

The timeline is as follows. At time 0 in the initial state s_0, a firm finances its investments with a mix of equity and debt. As in Leland (1994), the debt issued at the initial state s_0 is perpetual with fixed coupon payments $c(s_0)$ and a par value of $P(s_0)$. The issuance cost is a constant fraction

[8] For example, according to Titman (1984), firms start to lose reputation, capable workers, and customers and suppliers when they started entering distress, but well before officially filing for bankruptcy. Specifically, capable workers have incentives to seek a more stable position when they observe that their firm is sinking. Customers (Suppliers) are reluctant to buy (sell) products from (to) a troubled firm because they are worried about replacement of parts or services (payments).

ϕ of the amount of issued debt. The coupon payment is fixed until equity holders choose to default or restructure. The firm is operating between the good and bad state s_t, but both its default and restructuring decisions will depend on the initial aggregate state s_0 it enters, in addition to the current state s_t.

[Insert Figure 1 Here]

Following Goldstein et al. (2001), we assume that when restructuring its debt, a firm can only adjust debt levels upward. When cash flow increases to a high threshold $X_u(s_t; s_0)$ at the aggregate state s_t, the firm first calls the outstanding debt at par $P(s_0)$ and then issues more debt to enjoy more tax benefits. When cash flow X_t declines to a low threshold $X_s(s_t; s_0)$ at either state s_t, the firm is entering distress and incurs a distress cost η_{s_t} in the form of a depressed growth rate. Following Andrade and Kaplan (1998), we assume that the firm enters distress when cash flow X_t falls below its required coupon payment $c(s_0)$ issued at the initial state s_0, i.e., $X_s(s_t; s_0) = c(s_0)$, regardless of the current state s_t the firm is in.

When cash flows cannot cover the coupon payments, the firm may be able to issue equity to cover the shortfalls. As the additional distress cost lowers the firms cash flow growth rate, its cash flow may continue to deteriorate to the threshold $X_d(s_t; s_0)$, at which equity holders are no longer willing to inject capital, and decide to go bankrupt. Bankruptcy leads to immediate liquidation. While debt holders take over the firm and pay the liquidation costs α_{s_t}, equity holders receive nothing.

2.3 Firm's Problems

The firm makes optimal financing and default decisions to maximize equity value. Specifically, it chooses optimal bankruptcy and restructuring timing, as well as the optimal coupon.

When the firm is distressed ($w = D$) in either aggregate state s_t, equity holders choose the optimal bankruptcy timing $X_d(s_t; s_0)$, by making a tradeoff between the costs of keeping the firm alive and the tax benefits (Leland, 1994). We have the following smooth-pasting conditions in both states for distressed firms:

$$\lim_{X_t \downarrow X_d(B;s_0)} E^{'}(X_t, B, D; s_0) = 0, \tag{4}$$

$$\lim_{X_t \downarrow X_d(G;s_0)} E^{'}(X_t, G, D; s_0) = 0. \tag{5}$$

where $E(X_t, s_t, w; s_0)$ and $E^{'}(X_t, s_t, w; s_0)$ are the equity value function and its first derivative in the aggregate state s_t and in the financial condition w, conditional on the initial state s_0, respectively.

At time 0 when the firm is healthy (i.e., $w = H$) in an initial aggregate economic state, s_0, equity holders choose the optimal coupon $\mathbf{c}(s_0)$, debt $\mathbf{P}(s_0)$ and the optimal threshold of restructuring $\mathbf{X_u}(s_0)$ to maximize the firm value (Goldstein et al., 2001), where the vectors $\mathbf{c}(s_0) = \{c(B), c(G)\}$, $\mathbf{P}(s_0) = \{P(B), P(G)\}$, and $\mathbf{X_u}(s_0) = \{X_u(B; s_0), X_u(G; s_0)\}$, respectively. In choosing its capital structure, the firm makes a tradeoff between tax benefits and the expected cost of default, as well as the expected cost of distress, as follows:

$$\max_{\mathbf{c}(s_0), \mathbf{P}(s_0), \mathbf{X_u}(s_0)} \quad E(X_0, s_0, H; s_0) + (1 - \phi)D(X_0, s_0, H; s_0). \tag{6}$$

$$\text{subject to equations (4), (5), and } P(s_0) = D(X_0, s_0, H; s_0).$$

where $D(X_0, s_t, w; s_0)$ denotes the debt value function in the aggregate state s_t and in the financial condition w, conditional on the initial state s_0. All the valuation functions of equity and debt in different regions are expressed in Appendix A.

It is worth noting that the distress threshold is endogenous in our model, because the coupon $c(s_0)$ is endogenously chosen. The greater the exposure to the aggregate distress risk premium, i.e., $\beta(\lambda_B^m - \lambda_G^m)$, the less debt issued, and the smaller the coupon c. The smaller coupon implies a lower distress threshold. In other words, a firm with a high distress cost is less likely to become distressed if it optimally chooses less debt *ex ante*. In this two-state model, the firm is more precautionary in its debt policies. That is, even if the firm enters at the good state, $s_0 = G$, it issues less debt than in the single-state model, because it anticipates to carry the debt and make the contractual coupon payment in the future bad state $s_t = B$ where the distress cost η_B is even higher than that in the good state.

Given an initial state s_0, we impose the following order of thresholds:

$$X_d(G; s_0) < X_d(B; s_0) < X_s(G; s_0) = X_s(B; s_0) < X_0 < X_u(G; s_0) < X_u(B; s_0). \tag{7}$$

[Insert Figure 2 Here]

Figure 2 illustrates the order of the optimal thresholds in both states. It is intuitive that the firm goes bankruptcy earlier in the bad state than they are in the good state. That is, $X_d(G; s_0) < X_d(B; s_0)$. With the reasonable parameter values, we assume that the firm refinances debt earlier in the good state than in the bad state, $X_u(G; s_0) < X_u(B; s_0)$. As we explain for the distress thresholds, we assume they are the same in both current states and are endogenously determined by the initial coupon, i.e., $X_s(G; s_0) = X_s(B; s_0) = c(s_0)$. It is worth noting that if firms finance in a good state, $s_0 = G$, they tend to borrow more and have a high endogenous distressed threshold, i.e., $X_s(s_t; G) > X_s(s_t; B)$.

2.4 Scaling Property

Goldstein et al. (2001) show that the geometric process in equation, debt retirement at par value, and proportional debt issuance costs ensure the scaling property. Thus, the dynamic problem reduces to a static problem. The scaling property states that, given the state of the economy, the coupon, default, distress and restructuring thresholds as well as the value of debt and equity at the restructuring points are all homogeneous of degree one in cash flow. Notably, the firm at two adjacent restructuring points faces an identical problem, except that the cash flow levels are scaled by a constant; e.g., if cash flow has doubled, it is optimal to double default, distress and restructuring boundaries.

Chen (2010) and Bhamra et al. (2010b) extend the scaling property of Goldstein et al. (2001) across different aggregate states. Our structural model preserves the scaling property across two aggregate states, because it is particularly useful when we calibrate the model. Across two initial states, due to the homogeneity, the optimal thresholds are proportional to the coupons issued at the initial states s_0 at time 0 as follow:

$$\frac{X_d(s_t; G)}{X_d(s_t; B)} = \frac{X_s(s_t; G)}{X_s(s_t; B)} = \frac{X_u(s_t; G)}{X_u(s_t; B)} = \frac{c(G)}{c(B)}. \tag{8}$$

If a firm restructures at the new refinancing threshold $\bar{X}_t$ in the same good state ($s_t = G$), the new set of optimal policies are $X_d(s_t; G)\frac{\bar{X}_t}{X_0}$, $X_s(s_t; G)\frac{\bar{X}_t}{X_0}$, and $X_u(s_t; G)\frac{\bar{X}_t}{X_0}$. If it does in the bad state ($s_t = B$), the new set of optimal policies are $X_d(s_t; B)\frac{\bar{X}_t}{X_0}$, $X_s(s_t; B)\frac{\bar{X}_t}{X_0}$, and $X_u(s_t; B)\frac{\bar{X}_t}{X_0}$. Therefore, even though the new thresholds are scalped up by $\frac{\bar{X}_t}{X_0}$, the cross-state ratios between each pair of thresholds in equation (8) remain the same whenever the firm restructure their debt.

With the convenient scaling property, we do not have to solve for the optimal policies whenever the firms refinance their debt and increase their equity size repeatedly in simulations. We assume all the firms start at the good state $s_0 = G$, and solve the optimal coupon $\mathbf{c}(G)$, debt $\mathbf{P}(G)$ and the optimal threshold of restructuring $\mathbf{X_u}(G)$, as well as their counterparts $\mathbf{c}(B)$, debt $\mathbf{P}(B)$ and the optimal threshold of restructuring $\mathbf{X_u}(B)$ according to equation (8), only once.

3 Asset Pricing Implications

We start with presenting the general formula for the expected excess stock return in the two-state economy. Then, we simplify the model and derive closed-form solutions to illustrate our intuition and develop three predictions.

The following proposition shows that the expected excess stock return differs across the two states $s_t \in (G, B)$ and the two financial status, $w \in (H, D)$.

Proposition 1 *In the two-state economy, the firm then operates in the two aggregate states s_t, and have two financial status w within each state. Its conditional expected excess return of equity is*

$$r^e_{s_t,w} = \mathbb{E}_t[r^E_{s_t,w}] - rdt = \beta^E_{s_t,w}\lambda^m_{s_t}dt + \psi_{s_t,w}\hat{p}_{s_t}(1 - \kappa_{s_t})dt, \tag{9}$$

where $\beta^E_{s_t,w} = \gamma_{s_t,w}\beta$ and $\gamma_{s_t,w} = \frac{\partial E_{s_t,w;s_0}/E_{s_t,w;s_0}}{\partial X_t/X_t}$, which measures the elasticity of equity to the cash flow X_t, $\psi_{s_t,w} = (\frac{E_{s_t^+,w;s_0}}{E_{s_t,w;s_0}} - 1)$, which measures the percentage change in equity value in response to the changes in the aggregate economy from the state s_t to the other state s_t^+, and $E_{s_t,w;s_0} \equiv E(X_t, s_t, w; s_0)$.

Proof: See Section A.3 in the online appendix.

Equation (9) shows that the expected excess return, $r^{ex}_{t,w}$, is the product of the market risk premium, $\lambda^m_{s_t}$, the elasticity of stocks to underlying cash flows, $\gamma_{t,w}$, and the cash flow beta, β. The market risk premium $\lambda^m_{s_t} = \theta_{s_t}\sigma^m_{s_t}$, is countercyclical because the market price of risk $\theta_B > \theta_G$ and the market volatility $\sigma^m_B > \sigma^m_G$ (see e.g., Bhamra et al. (2010b) and Chen et al. (2009)).

The second component, $\psi_{s_t,v_t}(1-\kappa_{s_t})\hat{p}_{s_t}dt$, captures macroeconomic uncertainty risk. The price of the uncertainty risk, $\hat{p}_{s_t}(1-\kappa_{s_t})$, is countercyclical, because the equity holders prefer an early resolution of macroeconomic state-switching uncertainty (Epstein and Zin, 1989). According to Bhamra et al. (2010b), among others, the preference for an early resolution implies $\kappa_G > 1$. That is, when the economy is in the good state, $s_t = G$, investors like this good state and are willing to charge (pay) a negative (positive) risk premium for staying in the good state. In contrast, when the economy is in the bad state, investors do not like this bad state, and demand a positive risk premium for staying this state. In other words, the state-switching premium is negative in good times ($1 - \kappa_G \leq 0$), but positive in bad times.

3.1 Simplified Model

To gain preliminary insights, we first simplify the baseline model and use closed-form solutions to illustrate the interaction between levered equity risk and the market risk premium in the conditional CAPM framework. Our discussion on the equity returns focuses on distressed firms (i.e,. for $w = D$).

In the simplified model, the economy has only one state. The firm has no option to refinance its debt, but has one option to go bankrupt after it becomes distressed. The following proposition provides semi-closed-form solutions for equity betas. We drop the subscript s_t as we have only one state.

Proposition 2 *Outside of distress, $w = H$ for $X_t > X_s$, the equity beta, $\beta^E_{t,H} = \gamma_{t,H}\beta$, and the elasticity*

$$\gamma_{t,H} \;=\; 1 + \underbrace{\frac{D_{t,H}}{E_{t,H}}}_{} \;=\; 1 + \underbrace{\frac{\frac{c}{r}(1-\tau)}{E_{t,H}}}_{\textit{Book Leverage}} \underbrace{-(1-\omega_{H,1})\frac{(\frac{c}{r} - (E(X_s,D) - A(X_s,H)))}{E_{t,H}}(\frac{X_t}{X_s})^{\omega_{H,1}}(1-\tau)}_{\textit{Put Option of Deleveraging (+)}} \tag{10}$$

In distress, $w = D$ for $X_s > X_t > X_d$, the equity beta, $\beta^E_{t,D} = \gamma_{t,D}\beta$, and the elasticity

$$\gamma_{t,D} = 1 + \underbrace{\frac{\frac{c}{r}(1-\tau)}{E_{t,D}}}_{\text{Book Leverage}} - \underbrace{\frac{(\frac{c}{r} - A(X_d, D))(1-\tau)\pi_t}{E_{t,D}}}_{\text{Put Option of Going Bankruptcy (+)}}, \tag{11}$$

where π_t is the risk-neutral default probability and defined in equation (B8) *in Section B of the online appendix.*

Proof: See Section B of the online appendix.

When the firm is healthy, the elasticity, $\gamma_{t,H}$, in equation (10) has three components. Compared with the financial leverage component of the distressed firm, the leverage component of the healthy firm is greater because it has more debt in place. Everything else equal, the leveraged beta of a healthy firm is greater than that of a distressed firm. However, if the cash flow X_t of this healthy firm is declining, it has an American put option of deleveraging. Therefore, this option helps to reduce the equity risk when the firm is approaching distress.

When the firm is distressed, the elasticity, $\gamma_{t,D}$, increases with the market debt-to-equity ratio, $\frac{D_{t,D}}{E_{t,D}}$, as in the textbook. The market value of debt $D_{t,D}$ is the sum of the book value of debt and a put option. Specifically, the book value of debt in the second component in equation (11) is the perpetual value of the coupon payment $\frac{c}{r}$. The put the option of delaying bankruptcy in the third component protects equity holders from downside risk and decreases the equity-cash flow elasticity. Given limited liability, equity holders choose to go bankrupt only when the asset value $A(X_d, D)$ falls below the risk-free equivalent debt c/r. Hence, $\frac{c}{r} - A(X_d, D) > 0$.

This put option is particularly valuable to equity holders in the bad states either when the market risk premium is high, or when distressed firm incur additional costs in the form of a depressed growth rate $\hat{\mu}_B$, or both. Indeed, both conditions are very likely to occur simultaneously because firms are more likely to become distressed in the bad aggregate states. Finally, the increased put option value lowers the market value of debt and debt-to-equity ratio, resulting a negative relation between levered betas and the market risk premium in distressed firms.

3.2 Debt-to-Equity Ratio and Levered Beta

Given debt plays an important role in equity beta, we further discuss the levered equity beta for the distressed firm. Alternatively, if we combine the numerators of the second and third items in equation (11), we can express the market value of debt $D_{t,D}$ as follows:

$$D_{t,D} = \left(\frac{c}{r}(1-\pi_t) + A(X_d, D)\pi_t\right)(1-\tau), \tag{12}$$

which is a weighted average of the present value of coupon c and asset value at bankruptcy $A(X_{d,D})$. Their relative weights are a probability of staying in business, $1-\pi_t$, and a probability of default, π_t, respectively. This expression is essentially the market value of Merton (1974) without bankruptcy cost α (i.e., equation (13) on page 454). For a highly distressed firm with a default probability $\pi_t \to 1$, its debt value is approximating the residual assets taken over by debt holders, i.e., $D_{t,D} \to A(X_d, D)(1-\tau)$.

[Insert Figure 3 Here]

To illustrate the debt-to-equity ratio among distressed firms, Figure 3 plots the equity value E_t (in left panel) and the debt value D_t (in right panel) against the cash flow X_t. According to Merton (1974), equity (debt) is a convex (concave) function of the underlying assets, which generate the cash flow X_t. Interestingly, when the firm is in distress, i.e., $X_t < c$, its equity value is flat, close to zero and has no room to further decrease if the asset value further declines. In contrast, its debt value can decreases substantially in response to the same shock.

The intuition is as follows. The increase in market risk premium decreases the asset value across the cash flow level for $X_t \leq c$, particularly the asset value $A(X_d)$ at bankruptcy X_d. At bankruptcy, the loss due to the increased market risk premium is not shared between two parties. Because debt holders bear almost all the losses in assets, debt value decreases. In contrast, equity holders are insensitive to any loss in the asset value because they receive nothing at bankruptcy, i.e., $E(X_d, D) = 0$. Taken together, given the level of cash flow X_t, debt-to-equity ratios decline with the increased market risk premium in distressed firms, but no in healthy firms. In fact, using the same logic, we can easily reason a positive relation between them among healthy firms.[9]

[9]Alternatively, we can understand the negative covariance $\mathbf{cov}(\beta^E_{t,D}, \lambda^m_{s_t})$ via the equity-cash flow elasticity, $\gamma_{t,D} =$

Moreover, the distress cost η, that depresses the growth rate, further amplifies the adverse effect from the increased market risk premium on the debt value. As shown in equation (12), the debt value further declines, because the depressed growth rate increases the default probability π_t and reduces the residual asset value $A(X_d)$. More important, because both the market risk premium and the distress costs are likely to increase simultaneously in the bad states, i.e., $\lambda_B^m > \lambda_G^m$ and $\eta_B > \eta_G$, the negative relation between debt ratios/levered betas and the market risk premium can be further strengthened.

3.3 Predictions

We use comparative static analysis to illustrate the impact of endogenous leverage on stock returns via comparative statics and generate three predictions.

3.3.1 First Prediction

To illustrate the covariance between the market risk premium and the levered beta, we first change the market risk premium exogenously. We assume that the market price of risk θ increases from 0.40 to 0.44 by 10%, when the economy state switches from the good to the bad state. For a firm that is operating in this economy, at time 0 when $X_0 = 1$, the firm chooses an optimal level of debt P and coupon c, given the market volatility and market price of risk. In this heuristic example, the financial distress is mostly driven by the sudden switch from the good to bad state. That is, the financial distress threshold X_s is right next to the threshold where the aggregate economy enters the bad state. Indeed, the heuristic example is consistent with our discussion on the positive correlation of the market risk premium $\lambda_{s_t}^m$ and distress costs η_{s_t}.

We start with the case in which there is no distress cost (i.e., $\eta = 0$). We plot equity beta against the cash flow X_t in Panel A of Figure 4. When the market risk premium is constant, i.e., $\theta_G = \theta_B = 0.400$, the equity betas in the dotted, blue line monotonically increase with the declining cash flows. In other words, the more distressed the firm, the greater the equity betas.

However, in the second case where the market risk premium is assumed to increase in the bad state, i.e., $\theta_B = \theta_G + 0.04$, the equity betas in the solid, red line are parallelly shifted down by

$\frac{\partial E_D/E_D}{\partial X_t/X_t}$. The increase in the market risk premium λ reduces E_D and $\partial E_D/E_D$, but its does not affects X_t or $\partial X_t/X_t$. Thus, in response to the increased market risk premium, debt-to-equity ratio and levered beta decline, i.e., $\mathbf{cov}(\beta^E_{t,D}, \lambda^m_{s_t}) = \mathbf{cov}(\beta\gamma_{t,D}, \lambda^m_{s_t}) \leq 0$, because its equity value is inactive and its debt value declines substantially.

0.858 for $X_t \leq X_s = 0.359$, in response to the increase in the market risk premium. Therefore, controlling for the cash flow level X_t or profitability, the equity betas negatively correlate with the market risk premium.

[Insert Figure 4 Here]

In Panel B where we assume the distressed firms incur additional costs, i.e., $\eta = 0.01$,[10] the levered betas substantially decrease by 4.324, about five times more than the decrease of 0.858 in Panel A. This confirms our observation on the depressed debt value in Figure 3 and equation (12), which in turn lowers the levered beta in equation (11). Therefore, the distress cost significantly amplifies the adverse effect on equity betas from the increased market risk premium.

The following prediction summarizes the effect from the market risk premium on the levered equity betas.

Prediction 1 *Highly distressed firms have high levered equity betas, which negatively covary with the market risk premium.*

3.3.2 Second Prediction

The following proposition is based on Jagannathan and Wang (1996). They argue that the covariance between the market beta and the market risk premium plays an important role in the conditional CAPM.

Proposition 3 *The unconditional expected excess return of a distressed firm is:*

$$\mathbb{E}r^{ex}_{s_t,D} = \mathbb{E}\beta^E_{s_t,D}\mathbb{E}\lambda^m_{s_t}dt + \underbrace{\mathbf{cov}(\beta^E_{s_t,D}, \lambda^m_{s_t})dt}_{\leq 0}. \tag{13}$$

We have demonstrated in our first prediction that in distressed firms, levered equity betas and the market risk premium covary negatively, i.e., $\mathbf{cov}(\beta^E_{s_t,D}, \lambda^m_{s_t}) < 0$. Therefore, the negative covariance results in a reduction in the unconditional expected excess returns for distressed firms.

Next, we discuss the negative alphas of distressed firms in the conditional CAPM. Lewellen and

[10]Elkamhi et al. (2012) find the flow distress cost is about 1-2% of the growth rate.

Nagel (2006) show that, if the conditional CAPM holds, the unconditional alpha α^u is

$$\alpha^u \approx \underbrace{\mathbf{cov}(\beta_t^E, \lambda_{s_t}^m)dt}_{\leq 0} - \frac{\mathbf{E}[\lambda_{s_t}^m]}{(\mathbf{E}[\sigma_t^m])^2 \mathbf{cov}(\beta_t^E, (\sigma_t^m)^2)} < 0, \tag{14}$$

where σ_t^m is the time-varying market volatility.[11] Recall that $\mathbf{cov}(\beta_{s_t,D}^E, \lambda_t^m) < 0$ in equation (13).

In our study, this negative covariance generates a negative unconditional alpha α_D^u among distressed firms. The following prediction summarizes our discussion on the effect of the covariance between levered equity beta and market risk premium on stock returns and unconditional CAPM alphas.

Prediction 2 *The negative covariance between equity beta and the market risk premium causes low unconditional stock returns as well as negative CAPM alphas in highly distressed firms.*

3.3.3 Third Prediction

To illustrate the effect from heterogeneous distress risk premium on the endogenous debt financing and distress status, we set the beta of the low- and high-beta firms to be 1 and 1.5, respectively.

As shown in Panel A of Figure 5, compared with Firm 2, Firm 1 with a low cash flow beta β chooses a high leverage and coupon. After the debt is in place, both firms become distressed when their cash flow X_t level falls below the coupon level c, respectively. It is evident that the greater the debt, the earlier the firm becomes distressed (X_s), and the earlier the bankruptcy and liquidation (X_d). Hence, the exposure to the market risk, cash flow beta β, determines the optimal level of debt, which in turn determine the distressed status and stock returns.

Moreover, the leverage increases when the cash flow level declines or the distress becomes more acute, if the distressed firms have difficulties to adjust their debt. This is consistent with what we observe in the data: distressed firms have high leverage.

Prediction 3 *Firms with a low distress risk premium choose more debt ex ante and are more likely to become distressed ex post, having high betas but low stock returns and negative CAPM alphas.*

In summary, we derive the closed-form solution and use comparative statics to demonstrate that the countercyclical market risk premium results in procyclical financial leverage among distressed

[11]Specifically, Lewellen and Nagel (2006) demonstrate that the third item $\frac{\mathbf{E}[r_t^m]}{(\mathbf{E}[\sigma_t^m]^2 \mathbf{cov}(\beta_t^E, (r_t^m - \mathbf{E}[r_t^m])^2)}$ is trivial.

firms. The negative covariance between them causes low stock returns, and negative CAPM alphas. Then, we show that firms with a low distress risk premium choose high debt, which results in a high likelihood of distress and low stock returns.

4 Calibration

We use calibration to examine the quantitative implications of our dynamic model, with in mind that comparative statistic analysis we present in the last section does not speak to the dynamics of cash flows as well as path-dependent finance leverage and default decisions.

4.1 Data

We collect data from different sources. We use quarterly earnings data from National Income and Product Accounts (NIPA) table provided by the Bureau of Economic Analysis (BEA), and obtain firm-level stock returns from the Center for Research in Security Prices (CRSP) and accounting information from quarterly Compustat industrial data.

We obtain accounting information from quarterly Compustat industrial data. Due to the availability of quarterly ComputStat data, our sample period is from January 1974 to December 2015. We restrict the sample to firm-quarter observations with non-missing values for operating income and total assets, with positive total assets. We include common stocks listed on the NYSE, AMEX, and NASDAQ with CRSP share code 10 or 11, while excluding firms from the financial and utility sectors.

Debt is the sum of current liabilities (Compustat item DLCQ) and long term debt (item DLTTQ). Market leverage is the ratio of book value of debt to the sum of debt and equity (PRCCQ*CSHOQ). We winsorize the outliers at the top and bottom two percentiles. Following Strebulaev and Whited (2012), we remove the heterogeneity of financial leverage by demeaning the time-series mean of the variables, and adding the sample mean of each variable, because we do not allow heterogeneity of parameter values in our model simulation, except for the cash flow betas.

4.2 Optimal Policies and Model Generated Moments

To begin, we set commonly used parameters to predetermined values similar to prior studies. The parameter values are listed in Table 1, and are largely based on the literature (Bhamra et al. (2010b), Bhamra et al. (2010a), Chen et al. (2009) and Chen et al. (2014)).

[Insert Table 1 Here]

Starting with the macroeconomic variables, we set the risk free rate $r_G = r_B = 4\%$ in both aggregate states to abstract away from any term structure effects. The market volatility $\sigma^m_{s_t}$ is 0.1 and 0.12 in the good and bad states, respectively. The countercyclical market price of risk θ_{s_t} is 0.22 and 0.38 in the good and bad states, respectively. The transition intensities of the Markov chain are chosen to match average duration of NBER-dated expansions and recessions, i.e., $\hat{p}_G = 0.5$ and $\hat{p}_B = 0.1$, which gives the average durations 10 years for expansions and 2 years for recessions over the business cycle. We set $\kappa_G = 1/\kappa_B = 2$, which implies the risk-neutral probability of switching from the good state to the bad state is tow times as high as the actual probability.

The only heterogeneity we allow in our setup is the firm's exposure to market risk. We set the cash flow beta β of the low- and high-beta firm to be 0.6 and 1.1, respectively. Those two cash flow betas imply an average beta of 0.85, which is close to the median of cash flow or asset betas in the data (Chen et al., 2019). Moreover, we set idiosyncratic cash flow volatility to $\sigma^{i,X}_{s_t}$ to 0.17, which we calibrate to match the total volatility of earning growth rates obtained from the NIPA table. We set debt issuance cost ϕ_{s_t} to 0.75%. The effective tax rate is 0.175.

Notably, our study differentiates distress costs from liquidation costs. Most prior models require large liquidation costs to match low observed leverage. That is, the liquidation cost ranges from 0.3 to 0.45 in the capital structure literature.[12] To focus on the distress cost, we assume a much smaller liquidation cost of 10%. We assume the distress cost $\eta_{s_t} = 1\%$ and 1.5%, respectively, in the good and bad states, based on their estimates by Elkamhi et al. (2012), who find a small distress cost of 1-2% helps to generate low leverage ratios.

Panel B presents the optimal policies for low- and high-beta firms that start in the initial state

[12] Glover (2014) uses the simulated method of moments (SMM) to estimate the expected cost of default across 2,505 firms without considering the expected cost of distress. He does not separate the distress cost from the liquidation because he needs to keep the model parsimonious for structural estimation. We explicitly model the endogenous financial distress.

$s_0 = G$, given the predetermined parameters.[13] First, consistent with our intuition, firms borrow more debt in the good state than in the bad state, because the coupon in the good state $c(G)$ is greater than in the bad state $c(B)$. Second, the difference in coupon $c(s_t)$ across the states among the low beta firms is 0.398 (0.581 – 0.183), greater than the difference of 0.241 (0.417 – 0.176) among the high beta firms. This implies that those low-beta firms that commit to more contractual coupon payments are likely to become distressed, if they are not able to reduce their debt or coupon payments from 0.581 to the optimal level 0.183 immediately when the economy slides into the bad state because of debt issuance costs.

[Insert Table 2 Here]

Table 2 reports the model-generated moments averaged across 100 simulated economies. We discipline our model mainly using the moments of financial leverage, because debt is our important choice variable. We include the median, mean, and interquarterile of quasi-market leverage (QML_{t-1}) in each quarter, as well as the first order autocorrelation (AC(1)) of the mean of QML_{t-1}. The mean and median of leverage are mainly used to quantify the distress costs, because financial leverage decrease with distress costs. Its interquartile helps us to ensure the spread of cash flow betas we choose is reasonable. The first order autocorrelation help us to pin down debt issuance cost. Overall, the model-generated moments match the data reasonably well, with a median of 0.294, a mean of 0.314, an interquartile of 0.145, and a first order auto-correlation of 0.929.

Second, we follow Chen et al. (2019) and match the moments of earning growth rates from the NIPA table from 1974 to 2015. The growth rate $\hat{\mu}_{s_t}$ in the good and bad aggregate states in our simulated data are –0.041 and 0.085, respectively, very close to those in the data. Additionally, the cash flow volatility is 0.198, slightly below 0.212 in the actual data. Lastly, the market excess stock return is 5.0%, close to the historical market risk premium of 5.1%.

Overall, all the targeted moments are well matched with actual data moments, because the differences between data moments and model-generated moments are small in the last column. Thus, our model delivers a reasonable job in matching the dynamics of the actual data.

[13]The optimal policies for the initial state $s_0 = B$ can be obtained via the cross-state scaling property in equation (8).

4.3 Portfolios Sorted on the Failure Probability

Following Campbell et al. (2008), we apply logit regressions to each simulated data panel and calculate the failure probability each period.[14] Then, we sort the firms into deciles based on the constructed failure probability and rebalance the decile portfolios each year.

We report cross-sectional averages of key portfolio characteristics for our simulated data panels, including leverage, one-year default probability of Merton, fraction of distressed firms in Table 3. The one-year default probability of Merton (1974) is calculated using equation (C3) in the appendix, and the fraction of distressed firms is the ratio of the number of distressed firms to the total number of firms each period. As in Andrade and Kaplan (1998), we classify a firm as "distressed" if its cash flow falls below the coupon payment or if it defaults on coupon payments.

[Insert Table 3 Here]

In panel A, we calculate the cross-sectional averages of the above characteristics. First, the simple average of financial leverage increases monotonically with the failure probability. Second, the one-year expected default probability of Merton increases from 0.02% to 20.88%. Consequently, the realized fraction of distressed firms increases with failure probability. Thus, the failure probability is consistent with the two alternative measures of default, confirming the predictive power of the failure probability in our simulated data panels.

Then, we use the calibrated economy to examine the cross-sectional stock returns quantitatively. Panel B reports the value-weighted returns of portfolios sorted on the failure probability. It is evident that average excess returns monotonically decrease from 7.26% to –2.94% by 10.20% per year. In addition, the alphas from the unconditional CAPM follow the same pattern but display a more significant decline. This confirms our second prediction on the negative relation between failure probability and low stock returns. Moreover, the unconditional betas rise from 1.08 to 1.26, confirming our first prediction that distressed firms have high leverage and high levered equity risk.

Taken together, our model generated moments and stock returns are largely consistent with the empirical evidence by Campbell et al. (2008). To the best of our knowledge, this is the first paper that can generate simultaneously the increasing unconditional equity betas and the decreasing

[14]The construction of the variables can be found in Section C of the online appendix. Our results are very similar if we apply the estimates of Campbell et al. (2008) to our simulated data directly.

average returns.

4.4 Portfolios Sorted on the Implied Distress Risk Premium

To understand the positive relation between the implied distress risk premium and stock returns, we use as proxy of distress risk premium the log-difference between risk-neutral and actual default probability in our calibrated economies.[15] This measure is consistent with the general framework proposed by Friewald et al. (2014), in which they use credit risk premium to proxy for the default risk preimum.[16]

In Panel A of Table 4, we sort firms based on our proposed proxy for the distressed risk premium. The key observation is that financial leverage monotonically declines from 0.55 to 0.23 with our distress risk premium proxy. This confirms our third prediction that firms with a higher distress risk premium choose lower leverage *ex ante*, and also confirms that our proxy captures the distress risk premium well. Second, the expected Merton's default probability decreases with the implied distress risk premium. This observation is in line with the results by Friewald et al. (2014). In their Tables II to VII, firms with a low distress risk premium actually have a low credit rating. Lastly, the realized fraction of distressed firms decreases, consistent with the declining leverage.

The intuition for the negative relation between failure probability and implied distress risk premium is as follows. When issuing debt, firms with a low cash flow beta or risk exposure choose to borrow more *ex ante*. When the bad states are realized, large negative shock hit those firms with more extant debt. Therefore, when we sort the firms based on the distress risk premium, those firms with a low distress risk premium will appear to have a high leverage and a high default probability in our standard portfolio formation procedure, manifesting the endogenous relation between the two ranking variables in our standard portfolio sorting procedure in our simulated economies.

[Insert Table 4 Here]

As shown in Panel B, excess stock returns monotonically increase from –1.54% to 6.44% by

[15] The detail on the construction and justification of this measure can be found in Appendix C.

[16] They extract credit risk premium from credit default swaps and find that the credit risk premium is U-shaped with the risk-neutral and objective probabilities. Additionally, Almeida and Philippon (2007) use the ratio between risk-neutral and physical probability to proxy for the distress premium and find that the expected cost of default is larger than previously thought.

8.04%, in line with the pattern in Friewald et al. (2014).[17] So do the unconditional alphas. The low returns in the firms with a low implied asset risk premium can be explained by the endogeneity in the firms' financial status, as discussed in our third prediction. That is, firms with a low distress risk premium issue more debt and have a high default probability. After becoming distressed, they have low stock returns on average because of the negative covariance between their equity beta and market risk premium.

In summary, although we are not able to extract default risk premium from the CDS in our simulated economies, we complement Friewald et al. (2014) by showing that there is an endogenous connection between distress risk premium and the objective/risk-neutral default probability. we provide an internally coherent framework that connects two seemingly contradicting puzzles in our calibrated dynamic model, and demonstrate quantitatively the large stock return spread when we sort the firms based on the failure probability or the implied distress risk premium.

4.5 Counterfactual Analysis

We have used comparative statics analysis in Section 3.3 to qualitatively show the distress costs strengthen the negative relation between levered betas and the market risk premium. In this section, we use counterfactual experiments to assess quantitatively the importance of distress costs and heterogeneous exposure to aggregate distress risk (or cash flow betas). As mentioned before, the cash flow beta is a proxy for corporate exposure to aggregate distress risk premium, and the firm-specific distress risk premium can be regarded as systematic distress costs related to the market.

In the first counterfactual experiment, we examine the importance of the distress costs, by setting the distress cost η_{s_t} to zero. We simulate the model and re-calculate equity and equity returns, using the optimal default, debt restructuring, and coupons from the benchmark model. By using the optimal policies from the original models, we essentially calculate the values of equity and debt in the form of European options instead of American options, because the default and restructuring thresholds are exogenously given. Thus, this counterfactual experiment only examines the pricing effect of distress costs on equity and equity returns. As shown in Panel A1 of Table

[17] Our results are slightly different from theirs in that we have positive returns for firms with a high distressed risk premium, because we calibrate the model to large sample of Compustat firms, while their data limited to a much smaller sample of firms that have CDS prices.

5, the spread in the stock portfolios sorted on the failure probability decreases to substantially decreases to 3.97% from 10.20% in the benchmark model in Table 3. Similarly, when we sort the firms by the implied distress risk premium, the return spread in Panel A2 declines to 3.59% from 8.04% in the benchmark model. Therefore, the distress costs account for more than 60% of the failure risk premium and 55% of the distress risk premium.

[Insert Table 5 Here]

In the second counterfactual experiment, we examine how heterogeneous exposures to aggregate distress risk (i.e., cash flow betas) affect endogenous distress status and eventually stock returns. Recall our explanation for the distress risk premium puzzle (Friewald et al., 2014) is via the endogenous distress status, i.e., firms with a low beta (or low distress risk premium) choose to issue more debt and are more likely to become distressed, receiving low stock returns. In this experiment, we set the cash flow betas of both low- and high-beta firms to be the same, i.e., $\beta = 0.85$, and solve for all the optimal coupon, default and restructuring policies.[18] Then, we simulate the model using the new optimal policies, which are the same across the low- and high-beta firms. Because we have removed the heterogeneity in target debt at the refinancing points, the optimal policies of debt are the same for both types of firms, and the only heterogeneity left in the simulation is realized idiosyncratic cash flow shocks. Although the target debt ratios are the same at all the refinancing points across the two types of firms, the amount of debts and debt-to-equity ratios are path-dependent because firms restructure their debt at different time, depending on whether their realized shocks are large enough to cross the restructuring threshold.

As shown in Panel B1, the average excess returns are hump-shaped when we sort the firms based on the failure probability, and the spread in returns decreases to 5.64% from 10.20% in the benchmark model by nearly 50%. When we sort the firms on the implied distress risk premium, the return spread in Panel B2 decreases to 5.15% from 8.04% by 36%. This confirms that heterogeneous debt levels, determined by the heterogeneous risk exposures, at the restructuring thresholds has a persistent effect on the off-the-target debt ratios, the endogenous distress status and consequently stock returns.

[18] Different from the first counterfactual experiment, we need to re-solve for the optimal policies, because our purpose is to remove the impact of heterogeneous debt financing policies in this counterfactual experiment.

In short, our counterfactual experiments show that both distress costs and heterogeneous cash flow betas have a large explanatory power for the negative failure probability-return relation and the positive distress risk premium-return relation, although the distress costs are relatively more important.

5 Empirical Results

In this section, we first provide empirical evidence for highly distressed firms that debt-to-equity ratio negatively covaries with market risk premium. Then, we confirm the key mechanism of our model: the negative covariance between levered equity beta and market risk premium in highly distressed firms. Lastly, we compute the unconditional excess returns and CAPM alpha from the conditional CAPM.

Our empirical tests in this section focus on testing the first two theoretical predictions, related to the failure probability, listed in Section 3. We choose not to provide additional empirical tests on our third prediction on the endogenous distress status and resulting stock returns, because Friewald et al. (2014) have shown that firms with low distress risk premiums have high debt, low credit rating/high default probability, and low stock returns and negative CAPM alphas on average.

5.1 Debt-to-Equity Ratio and Market Risk Premium

We start with investigating how debt-to-equity ratios covary with the market risk premium, because the debt ratio is the key determinant of the levered equity beta.

To stay close to our analytical results in equation (11) and (10), we use this equity-to-debt ratio, which is calculated as total debt (Compustat item DLCCQ plus DLTQ) divided by equity value. We require the price greater than one to mitigate the microstructure bias. We obtain the quarterly market risk premium from Haddad, Loualiche, and Plosser (2017) from 1952 to 2015.[19]

[19] We thank Erik Loualiche to share his R codes with us to update their risk premium. They use the predicted expected excess equity returns to proxy for the aggregate risk premium. That is, they regress excess equity returns on the dividend-price ratio, D/P, consumption-wealth ratio, cay (Lettau and Ludvigson, 2001), and the three-month T-bill yield, $T\text{-}Bill$, to predict excess returns. Their regression yields the expected market risk premium as follows:

$$E_t(R^e_{M,t}) = -0.76 + 2.89D/P_{t-1} + 2.54cay_{t-1} - 0.97T\text{-}Bill_{t-1},$$

where $R^e_{M,t}$ is the annualized return of the value-weighted market portfolio over the next three years in excess of the current three-month T-bill yield. The dividend-price ratio $(D/P)_{t-1}$ is constructed using CRSP data on monthly returns and the variable cay_{t-1} is an empirical proxy for the log consumption-wealth ratio. Interest rates are constant

We follow Begenau and Salomao (2018) and examine the impact of the market risk premium on debt ratios at the portfolio level. As Strebulaev (2007) points out, not all the firms adjust their debt immediately because idiosyncratic profitability shocks they receive are not necessarily large enough to cross the optimal thresholds. However, the effect of inactive debt adjustment can be mitigated at the portfolio level, because some firms within the portfolio might respond substantially. Using the same procedure as in the standard portfolio formation for stock returns, we form leverage portfolios based on the failure probability at the end of the previous quarter.

We start with a visual inspection into the relation between the debt ratio and market risk premium in Figure 6. The gray shade areas are for the NBER recession times. The black line with stars depicts the expected market risk premium, which is notably countercyclical.[20] Panel A shows that the debt-to-equity ratios of the most distressed firms (in the red line) are much higher than those in the least distressed firms (in the blue line). Second, the debt ratios of the most distressed firms negatively correlates the market risk premium significantly. For example, during the early 1980s recession, the debt-to-equity ratio declines substantially after the market risk premium increases. Third, the ratios of the least distressed firms are nearly flat and are not very unresponsive to the changing market risk premium. Therefore, this contrast in the debt ratio across the most and least distressed firms are consistent with our discussion for Figure 3: debt holders of distress firms bear almost all the loss in the asset values induced by the increased market risk premium and prepossessed growth rate.

Then, we proceed to provide statistical confirmation. The first two rows of Table 6 report the time series average during the NBER recession and expansion times for each failure probability portfolio. It is evident that the debt-to-equity ratio increases with the failure probability for both subsamples, and that the average ratios in recessions are greater than that in expansions, particularly in highly distressed firms.

[Insert Table 6 Here]

To examine the association between debt-to-equity ratio and market risk premium, we run time

maturity rates according to the Federal Reserve's H.15 release.

[20]Except for the early 1990s recession, the market risk premium is close to zero before the onset of the recessions, and then increases substantially during the recession times.

series regressions as follows:

$$y_{j,t} = a_j + b_{j,Y} MRP_t + cX_{j,t} + e_{j,t}, \tag{15}$$

where $y_{j,t}$ is the quarterly times series of the value-weighted averages of debt-to-equity ratios, MRP_t is the quarterly market risk premium, and $X_{j,t}$ is the vector of control variables that include the equal-weighted book assets as well as value-weighted profitability and Tobin's Q. The market risk premium (MRP) is proxied by predicted market excess returns (Haddad et al., 2017) in Panel A and actual market excess returns in Panel B.

Controlling for the profitability in the regressions is important for our study. Because the decrease in the profitability and the increase in the market risk premium occur simultaneously in bad times, they have the opposite effects on the debt ratios, i.e., the positive effect from the decreased profitability and the negative effect from the increased market risk premium. First, forming the failure probability portfolio naturally allows us to control for the cross-sectional effect from individual profitability shocks, because profitability is one of the main determinants of the failure probability. Second, when running the time series regression, we include the profitability as one of the control variables as well.

Panel A of Table 6 shows the association between debt-to-equity ratios with the market premium. They become increasingly negative from –0.14 (t-statistic = –1.62) to –5.31(t-statistic = –3.50). In Panel B where we use an alternative measure for the market risk premium, the actual market excess returns, the increasing negative relation between the equity-to-debt ratio and market risk premium remains the same.

Interestingly, most of the estimated coefficients of profitability are positive, particularly in the top three decile portfolios. This observation is largely different from the well known negative profitability-financial leverage relation at the firm level.[21]

[21] In unreported results, we confirm the negative relation at the firm-level using standard panel regressions in the same data sample, consistent with Strebulaev (2007) who advances the idea that transaction costs cause inactive debt financing and therefore the negative profitability-leverage relation at the firm level. In contrast, the positive relation between profitability and debt ratio at the portfolio-level can be understood as follows. First, this effect from inactive debt adjustment is weaken at the portfolio level because some of the firms within the portfolio might react to profitability shocks substantially. Second, when we form the failure probability portfolios, we have essentially controlled for the cross-sectional profitability effect. Thus, the negative relation becomes much weaker and become positive in the time series regression after we control for the cross-sectional effect. To our knowledge, we are first to document the positive relation between profitability and debt ratio at the portfolio level.

In sum, we provide novel, supportive evidence for our prediction that debt-to-equity ratios are negatively correlate with the market risk premium in highly distressed firms. Our results are robust when we use different measure of debt and use equal-weighted debt-ratio portfolios, as shown Tables D1 and D2 in the online appendix.

5.2 Conditional CAPM

We now proceed to assess our second prediction: a negative covariance between levered betas and market risk premium generates low excess returns and negative CAPM alphas in distressed firms.

We follow Lewellen and Nagel (2006), use the monthly excess stock market return r_t^m to proxy for the market risk premium $\lambda_{s_t}^m$, and use the monthly CAPM beta to proxy for the time-varying market beta β_t^E. The monthly CAPM beta is obtained by regressing daily returns on daily excess market returns within each month. We also use the procedure of Lewellen and Nagel (2006) to mitigate microstructure noises. Empirically, the unconditional expected stock excess return is:

$$\mathbf{E}[r_{i,t}^{ex}] \;=\; \mathbf{E}[r_{i,t}^{E}] - r \;=\; \mathbf{E}[\beta_t^E r_t^m] \;=\; \mathbf{E}[\beta_{i,t}^E]\mathbf{E}[r_t^m] + \mathbf{cov}(\beta_{i,t}^E, r_t^m). \tag{16}$$

and the unconditional CAPM alpha is

$$\alpha^u \approx \mathbf{cov}(\beta_{i,t}^E, r_t^m)dt - \frac{\mathbf{E}[r_t^m]}{(\mathbf{E}[\sigma_t^m])^2}\mathbf{cov}(\beta_{i,t}^E, (\sigma_t^m)^2). \tag{17}$$

Table 7 confirms the findings by Campbell et al. (2008) in our sample from 1974 to 2015. Panel A presents the average excess return in percent, $r_{i,t}^{ex}$, and unconditional CAPM alpha, α^u, for the value-weighted portfolios, sorted on the prior month's failure probability. The excess return in the most distressed portfolio is much lower than the healthy portfolio by 9.63% with a t-statistic of –3.46. This is puzzling because we expect the firms with a high failure risk earn higher excess returns. The unconditional CAPM alphas substantially decrease from 3.02% to 11.16%, with a difference of –14.20%. Moreover, the unconditional beta monotonically increases from 0.94 to 1.59, which makes the lowest returns in the most distress firms even more puzzling.

[Insert Table 7 Here]

To assess our model prediction, we calculate the excess return and the unconditional alpha implied from the conditional CAPM, using $\beta^E_{i,t}$ from the data and following the procedure of Lewellen and Nagel (2006). The first row of Panel B presents the value-weighted average of the conditional market equity betas, $\beta^E_{i,t}$. The conditional equity betas are increasing with the failure probability, with a small drop at the top decile portfolio, largely confirming the leverage effect in our first prediction. The difference in beta is small with a value of 0.30. The next two rows report the two components of the excess return, $\mathbf{E}[\beta^E_{i,t}]\mathbf{E}[r^m_t]$ and $\mathbf{cov}(\beta^E_{i,t}, r^m_t)$, respectively. Without the covariance, $\mathbf{E}[\beta^E_{i,t}]\mathbf{E}[r^m_t]$ is simply the unconditional CAPM alpha. The spread in $\mathbf{E}[\beta^E_{i,t}]\mathbf{E}[r^m_t]$ in the second row is 2.11% per year.

The covariance, $\mathbf{cov}(\beta^E_{i,t}, r^m_t)$, in the third row decreases monotonically from 0.62% to –5.96%, a total fall of 6.58%. This substantial fall in the covariance dominates the effect from the equity beta in the model-implied excess returns and unconditional CAPM alpha. Specifically, the model-implied CAPM alpha decreases from 1.19% to –7.96%, a drop of 9.15%, which is slightly more than 50% of the 14.2% in the data. It is worth noting that the monotonically decreasing α^u is mainly due to the $\mathbf{cov}(\beta^E_{i,t}, r^m_t)$ because the second item in the α^u formula is small, the spreads in the model-implied excess returns and unconditional alphas account for more than 50% of their empirical counterparts.

In short, our empirical evidence supports our analytical and quantitative model results that the negative covariance between market risk premium and equity beta generates low stock returns for highly distressed firms, which helps to produce the negative relation between the failure probability and stock returns, even though the highly distressed firms have a high equity beta.

6 Concluding Remarks

Empirical evidence on the equity distress risk premium is mixed. While Campbell, Hilscher, and Szilagyi (2008) find the negative failure probability-return relation, Friewald, Wagner, and Zechner (2014) document a positive distress risk premium-return relation. While a basic extension of Merton's (1974) framework can rationalize the empirical finding on the positive relation between distress risk premium and stock returns, what remains puzzling in Friewald et al.'s (2014) setting is why firms with a low distress risk premium have low credit ratings, and are highly likely to default. In a unified, dynamic model, we establish that optimal debt dynamics and, as a result, endogenous distress status over the business cycle can explain the two seemingly contradicting puzzles simultaneously. Specifically, we connect them by explicitly showing that the failure probability and distress risk premium are *negatively* correlated because of the endogenous debt financing.

In the simplified model, we derive closed-form solutions that reveal three main results. First, distressed firms with high failure probabilities are more sensitive to business cycle conditions, and reduce their debt more aggressively during economic downturns. Second, the countercyclical distress costs and endogenous debt financing induce distressed firms' debt-to-equity ratios and levered equity betas to covary negatively with countercyclical market risk premiums. Importantly, the negative covariance effect generates low equity returns among distressed firms. Third, firms that have low exposure to distress risk choose higher debt levels and hence are more likely to become distressed, receiving low returns and negative CAPM alphas.

When calibrating the model that features the countercyclical market risk premium, we demonstrate quantitatively that firms with a lower implied distressed premiums select higher leverage and display higher default probabilities, but earn lower returns. Finally, we provide empirical evidence in support of our setting, namely, we document empirically that debt-to-equity ratios are negatively related to the market risk premium, especially for more distressed firms in our setting. We also demonstrate a negative covariance between levered equity beta and market risk premium among highly distressed firms.

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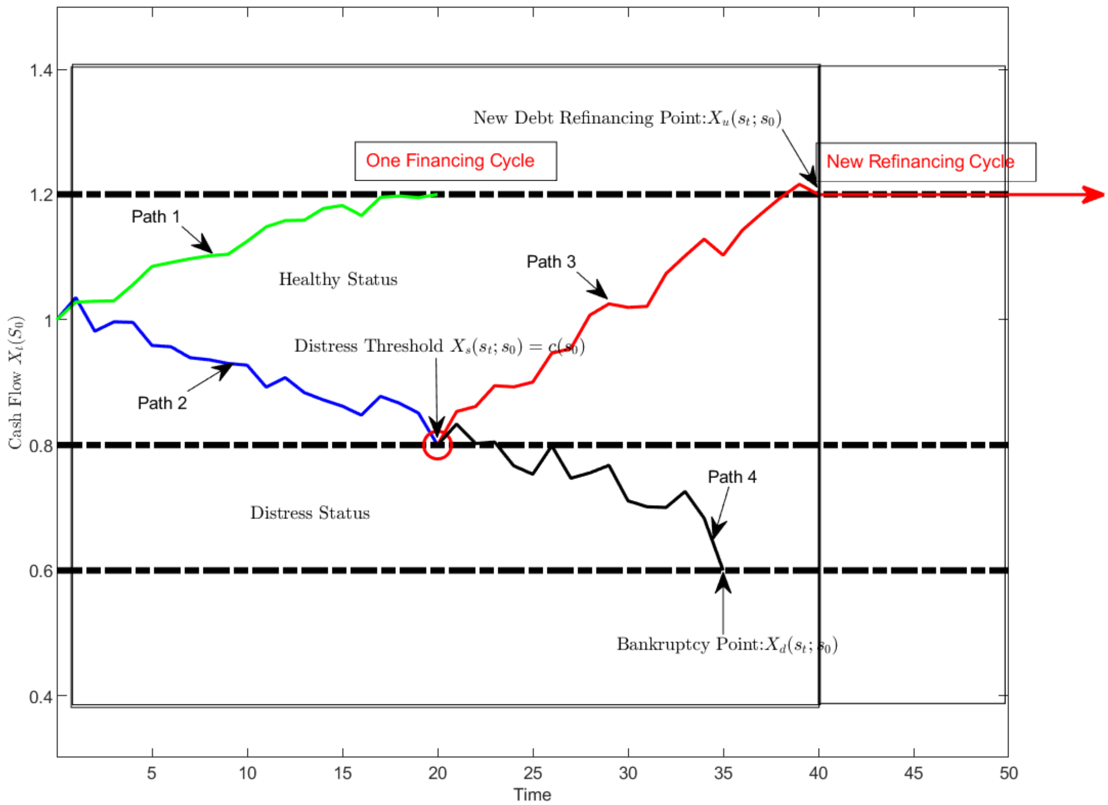

Figure 1. Dynamic Paths
This figure plots four possible paths that a firm could take within one refinancing cycle, which can be repeated infinitely. At time 0 in the initial state s_0, the firm enters the market and issue a mix of equity and debt. It is operating between the good and bad state s_t, but both its default and restructuring decisions partially depend on the initial aggregate state s_0 it enters and the coupon payment $c(s_0)$ it promises to pay, in addition to the current cash flow level X_t and state s_t. In observing its dynamic cash flow X_t and new state s_t, the firm makes financing and default decisions. Path 1 (green line) shows that, when its cash flows reach an upper threshold $X_u(s_t; s_0)$, the firm decides to issue more debt to take advantage of tax benefits. In contrast, if the cash flows X_t decline to a low threshold $X_s(s_t; s_0)$ along Path 2 (blue line), the firm becomes distressed. This distressed firm might survive and rebound, leading to a subsequent debt restructuring at the *same* upper threshold X_u, as shown in Path 3 (red line). In contrast, additional distress costs might induce a more severe cash flow shortfall through a depressed growth rate. If the firm continues to deteriorate, equity holders will no longer be willing to inject more capital, and decide to go bankrupt at $X_d(s_t; s_0)$, as shown in Path 4 (black line). Bankruptcy leads to immediate liquidation.

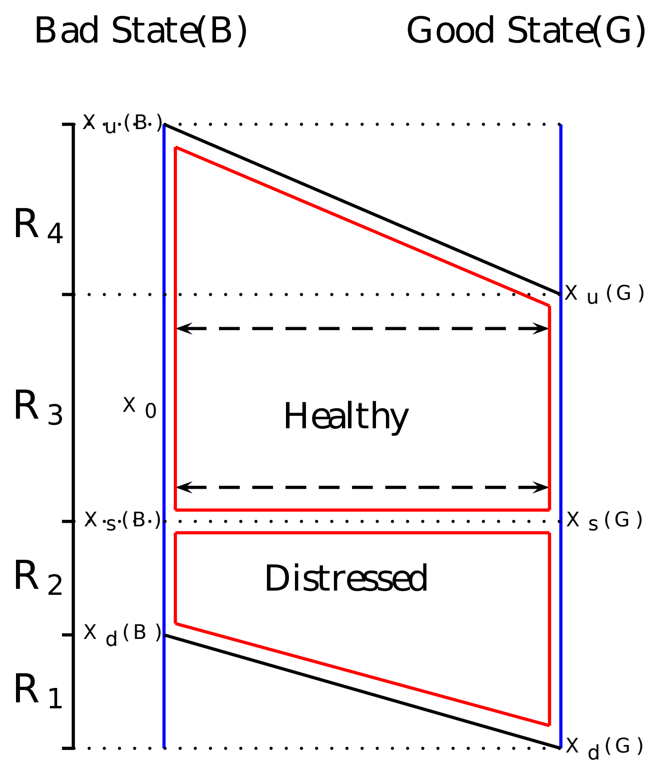

Figure 2. Optimal Thresholds in Two States
This figure plots the optimal default thresholds $X_d(s_t; s_0)$, distress threshold $X_s(s_t; s_0)$ and refinancing threshold $X_u(s_t; s_0)$ across the two states over four regions of cash flow. $X_s(s_t; s_0)$ is the same in both states and equals to the initial coupon $c(s_0)$.

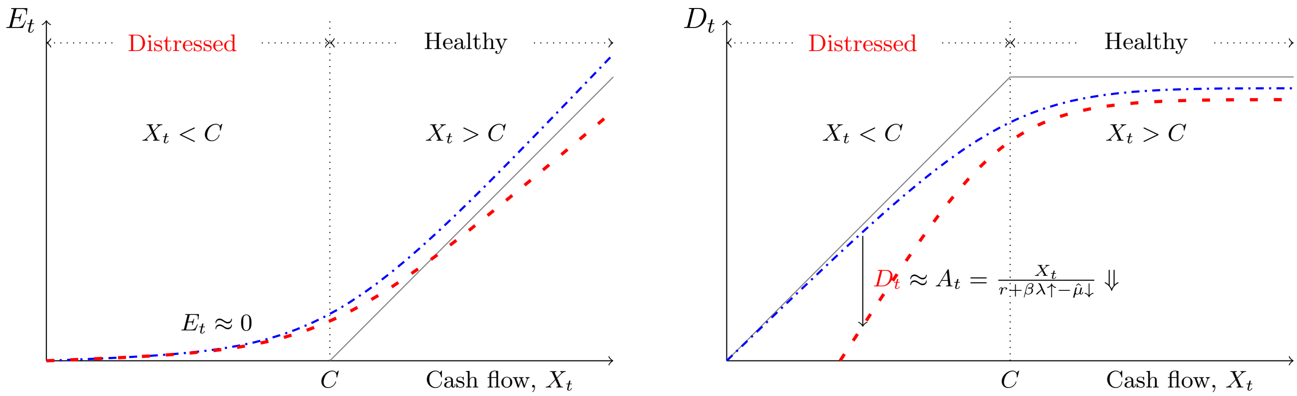

Figure 3. Equity and Debt Values

This figure plots the equity value E_t (in left panel) and the debt value D_t (in right panel) against the cash flow X_t. Equity (debt) is a convex (concave) function of the underlying assets, which generate the cash flow X_t. A firm is considered distress if $X_t < c$ or healthy otherwise. The gray lines plot the payoff for the case of European option as in (Merton, 1974), and the blue and red lines for the American option case as in our model. The blue, dotted dash lines are for the equity and debt values before the increase of the market risk premium λ^m or the decrease in the growth rate $\hat{\mu}$, and the red, dash lines are for those after the change in the market risk premium and growth rate.

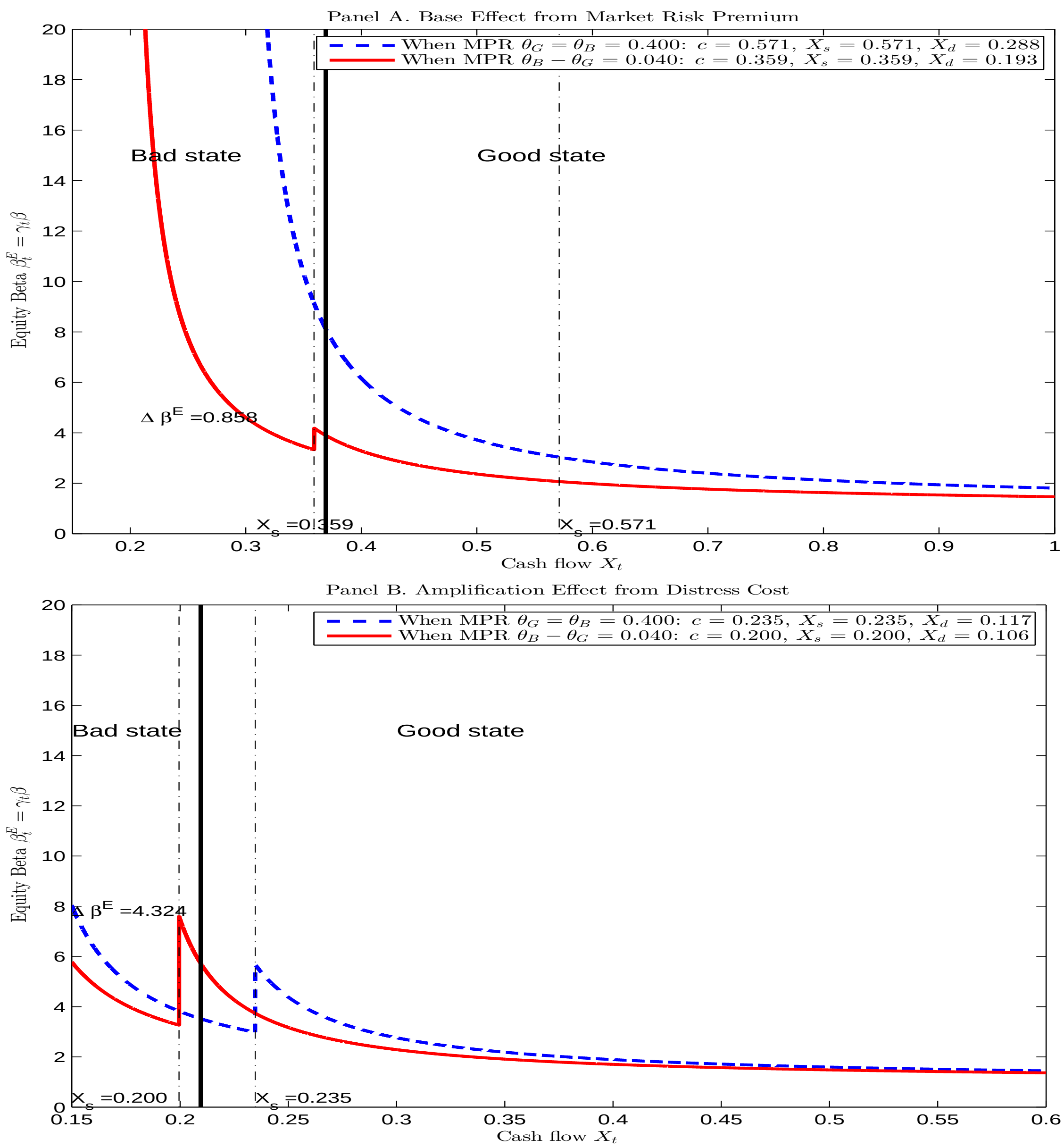

Figure 4. Equity Beta in the Simplified Model

This figure plots equity betas against cash flow X_t, with optimal policies as shown in the legend. We set nominal Interest rate r to 0.04, market price of risk θ to 0.40, systematic volatility σ^m to 0.1, debt issue cost ϕ to 0.75%, liquidation cost α to 0.3, effective tax rate τ to 0.175, cash flow growth rate $\hat{\mu}_H = \hat{\mu}_D$ to 0.04, cash flow beta β to 1, and idiosyncratic volatility $\sigma^{i,X}$ to 0.17. At time 0, $X_0 = 1$, the firm chooses optimal debt P and coupon c. They liquidate their firms at the threshold of X_d if the corrective action does not save their firms. In Panel A, we consider a firm is operating across two aggregate states, and exogenously change the market price of risk from 0.40 to 0.44 by 10% when the economy switches from the good to the bad state. Debt is fixed after initial issuance. Panel B, the firm incurs distress costs, $\eta = 0.01$, when the cash flow falls below the low distress threshold of X_s, which is the coupon c.

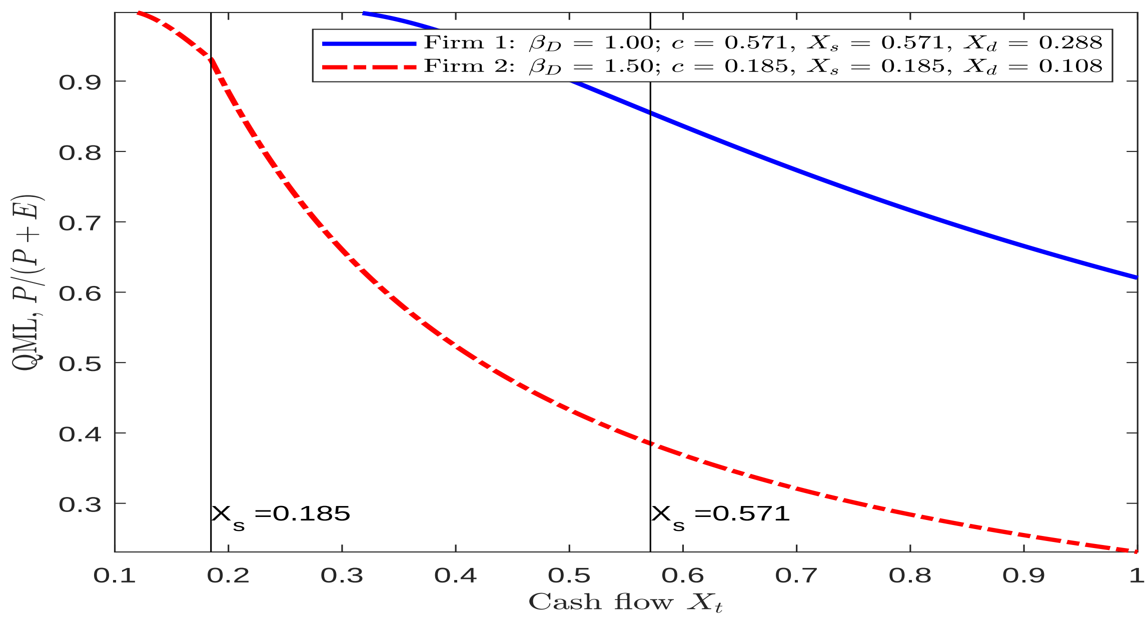

Figure 5. Financial Leverages in the Simplified Model

This figure plots financial leverage against cash flow X_t, with optimal policies as shown in the legend. We set nominal Interest rate r to 0.04, market price of risk θ to 0.40, systematic volatility σ^m to 0.1, debt issue cost ϕ to 0.75%, liquidation cost α to 0.3, effective tax rate τ to 0.175, cash flow growth rate $\hat{\mu}_H = \hat{\mu}_D$ to 0.04, and idiosyncratic volatility $\sigma^{i,X}$ to 0.17. The cash flow beta β are set to 1 and 1.5. At time 0, $X_0 = 1$, the firm chooses different level of debt P and coupon c, given their different exposure to the market risk (i.e., cash flow beta β).

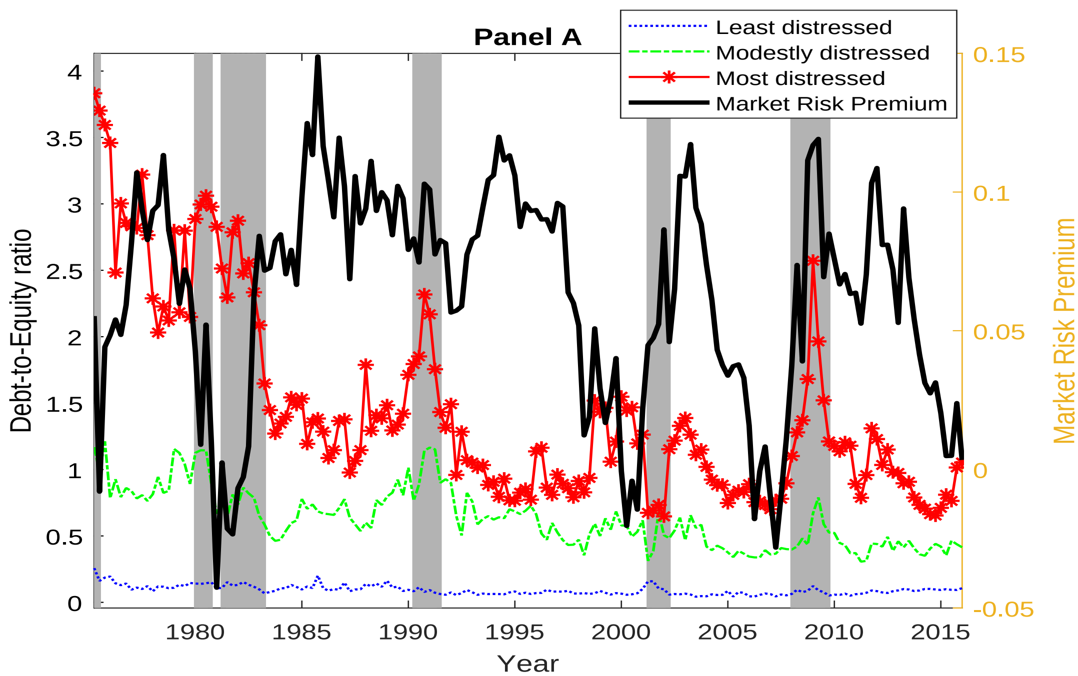

Figure 6. Time Series of Debt-to-Equity Ratios in the Data

This figure plots the time series of value-weighted debt-to-equity ratio from 1974 to 2015. The black line with stars is for the expected market risk premium and the gray shade areas are for the NBER recessions. The dashed, dotted, and solid with stars lines are for the least, modestly, and most distressed portfolios, respectively.

Table 1. Parameter Estimates

This table presents the parameter values and optimal policies solved from the model. Panel A lists the predetermined parameters from the existing literature. Panel B presents the optimal policies, given the predetermined parameter values. The optimal policies include coupon $c(s_0)$, the refinance threshold $X_u(s_t)$, and the default threshold $X_d(s_t)$ and the distress threshold $X_s(s_t)$ in the states $s_t = B, G$ when the firm enters at the initial state $s_0 = G$. The last row reports the average values.

Panel A. Parameter Values from the Literature		
	s_t = B	s_t = G
Probability of leaving current state s_t, $\hat{p}_{s_t}$	0.500	0.100
Aggregate state-switching risk premium, κ_{s_t}	1/2	2
Nominal interest rate, r_{s_t}	0.040	0.040
Market price of risk, θ_{s_t}	0.380	0.220
Systematic volatility, $\sigma^m_{s_t}$	0.120	0.100
Debt issue cost (%), ϕ_{s_t}	0.75	0.75
Liquidation cost, α_{s_t}	0.100	0.100
Effective tax rate, τ_{s_t}	0.175	0.175
Idiosyncratic volatility, $\sigma^{i,X}_{s_t,w}$	0.170	0.170
Low- and high-cash flow beta, β_{s_t}	0.6, 1.1	0.6, 1.1
Physical growth rate of healthy firms, $\hat{\mu}_{s_t,H}$ (%)	−4.19	8.33
Distress Cost, η_{s_t} (%)	1.50	1.00

Panel B. Optimal Policies								
	$c(B)$	$c(G)$	$X_u(B;G)$	$X_u(G;G)$	$X_d(B;G)$	$X_d(G;G)$	$X_s(B;G)$	$X_s(G;G)$
Low beta Firms	0.183	0.581	18.895	1.948	0.191	0.154	0.581	0.581
High beta Firms	0.176	0.417	5.474	1.989	0.191	0.155	0.417	0.417
Average	0.180	0.499	12.184	1.969	0.191	0.155	0.499	0.499

Table 2. Moments of Generated Samples

This table reports the model generated moments from 100 model-generated samples. Each sample contains the quarterly observations for 4000 firms over 150 years (The first 100 years observations have been discarded). The statistics are averaged across all the samples. In Panel A, financial leverage moments include the average of the median, mean, and interquarterile of quasi-market leverage (QML) in each quarter, as well as the first order autocorrelation (AC(1)) of the mean of QML. Cash flow moments include the growth rate of cash flow $\hat{\mu}_{s_t}$ in the good and bad aggregate states, and total cash flow volatility. Additionally, we report average market excess stock returns. 'Data' is for the moments from Data and 'Model' is for the average of the moments across all the samples generated from the full model, and 'M–D' is for the difference between Model and Data. We also report the 25th, 50th and 75th percentiles of moments from the 100 samples.

Panel. Moments								
	D(ata)	M(odel)	5th	25th	50th	75th	95th	M–D
Median of QML	0.303	0.294	0.258	0.280	0.294	0.308	0.330	−0.009
Mean of QML	0.313	0.314	0.270	0.295	0.317	0.332	0.359	0.001
Interquartile of QML	0.167	0.145	0.107	0.128	0.144	0.161	0.190	−0.022
AC(1) of the Mean of QML	0.920	0.929	0.856	0.900	0.944	0.957	0.975	0.009
Cash flow growth rate in the bad states $\hat{\mu}_B$	−0.046	−0.041	−0.074	−0.063	−0.049	−0.032	0.012	0.005
Cash flow growth rate in the good states $\hat{\mu}_G$	0.081	0.085	0.060	0.077	0.087	0.095	0.108	0.004
Total volatility of cash flow rate $\sigma^{T,X}$	0.212	0.198	0.194	0.196	0.197	0.199	0.202	−0.014
Market risk premium	0.051	0.050	0.014	0.037	0.048	0.070	0.084	−0.001

Table 3. Portfolios Sorted on the Failure Probability

We report the cross-sectional key moments in Panel A and stock returns in Panel B for decile portfolios sorted on the probability of failure (Campbell et al., 2008), using the simulated data panels. The key moments include financial leverage (QML), default probability of Merton (1974), and the fraction of distressed firms within each decile of firms. Following Andrade and Kaplan (1998), a firm is classified as "distressed" if its interest coverage is less than one. We simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each data panel there are 4000 firms. In each panel, the failure probability is calculated by running logit regressions as in Campbell et al. (2008). Then, we sort simulated firms at the end of each year, based on the failure probability, and rebalance the portfolios each year.

Panel A. Leverage and Default Probabilities

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H–L
QML	0.30	0.31	0.34	0.37	0.40	0.45	0.51	0.60	0.80	0.99	0.69
Merton Default Prob (%)	0.02	0.02	0.04	0.08	0.17	0.34	0.77	1.81	4.71	20.88	20.86
Fraction of Distressed Firms (%)	0.15	0.29	0.48	0.61	0.82	1.33	2.59	5.59	14.01	46.22	46.07

Panel B. Cross Section of Stock Returns

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H–L
r^{ex} (%)	7.26	6.30	5.75	5.68	5.57	5.32	5.02	4.24	2.01	−2.94	−10.20
(t)	(2.84)	(2.62)	(2.50)	(2.48)	(2.40)	(2.28)	(2.13)	(1.76)	(0.86)	(−0.68)	(−4.62)
α (%)	1.60	0.94	0.61	0.53	0.39	0.12	−0.23	−1.14	−3.51	−9.16	−10.76
(t)	(2.92)	(2.16)	(1.74)	(1.63)	(1.24)	(0.28)	(−0.43)	(−1.48)	(−3.18)	(−5.34)	(−5.60)
β	1.08	1.01	0.98	0.99	1.00	1.00	1.00	1.03	1.07	1.26	0.18
(t)	(51.16)	(59.77)	(64.41)	(66.35)	(60.52)	(48.56)	(37.77)	(27.68)	(20.21)	(9.53)	(0.58)
$Adj.R^2$	0.97	0.97	0.98	0.98	0.98	0.96	0.94	0.92	0.84	0.74	0.14

Table 4. Portfolios Sorted on the Implied Distress Risk Premium

We report the cross-sectional key moments in Panel A and stock returns in Panel B for decile portfolios sorted on the probability of failure (Campbell et al., 2008), using the simulated data panels. The key moments include financial leverage (QML), default probability of Merton (1974), and the fraction of distressed firms within each decile of firms. Following Andrade and Kaplan (1998), a firm is classified as "distressed" if its interest coverage is less than one. We simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each data panel there are 4000 firms. The implied default risk premium is calculated as the log difference between the risk-neutral and the physical default probabilities. Then, we sort simulated firms at the end of each year, based on the implied default risk premium, and rebalance the portfolios each year.

Panel A. Leverage and Default Probabilities											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H–L
QML	0.55	0.42	0.36	0.32	0.30	0.28	0.27	0.26	0.25	0.23	−0.33
Merton Default Prob (%)	21.10	4.97	1.90	0.79	0.36	0.18	0.10	0.06	0.04	0.02	−21.40
Fraction of Distressed Firms (%)	47.12	13.71	4.97	2.10	1.02	0.68	0.61	0.62	0.70	0.51	−47.29

Panel B. Cross Section of Stock Returns											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H–L
r^{ex} (%)	−1.54	2.89	4.74	5.42	5.93	5.95	5.98	6.09	6.22	6.44	8.04
(t)	(−0.28)	(1.16)	(1.98)	(2.33)	(2.54)	(2.56)	(2.56)	(2.49)	(2.45)	(2.46)	(3.75)
α (%)	−7.84	−2.57	−0.58	0.22	0.69	0.71	0.74	0.68	0.67	0.79	8.73
(t)	(−4.87)	(−2.50)	(−0.78)	(0.36)	(1.73)	(1.91)	(2.15)	(1.70)	(1.46)	(1.50)	(4.68)
β	1.27	1.05	1.01	0.99	1.00	1.00	1.00	1.02	1.05	1.07	−0.20
(t)	(9.09)	(16.43)	(25.63)	(35.21)	(43.47)	(65.98)	(74.14)	(55.55)	(33.85)	(32.98)	(−0.54)
$Adj.R^2$	0.74	0.84	0.92	0.93	0.97	0.98	0.98	0.98	0.96	0.96	0.10

Table 5. Counterfactual Analysis

We report two counterfactual analysis for stock returns. We set the distress cost to zero in Panels A1 and A2 and set the cash flow betas to their average in Panels B1 and B2. We report excess returns sorted on the probability of failure (Campbell et al., 2008) in Panels A1 and B1, and those sorted on the implied distress risk premium in Panels A2 and B2, using the simulated data panels. We simulate the model and generate 100 artificial panels of data at the quarterly frequency for a period of 150 years. The first 100 years of observations are discarded to reduce the dependence on initial values. In each data panel there are 4000 firms. In each panel, the failure probability is calculated by running logit regressions as in Campbell et al. (2008), and the implied default risk premium is calculated as the log difference between the risk-neutral and the physical default probabilities. Then, we sort simulated data at the end of each year, based on the failure probability and on the implied default risk premium, and rebalance the portfolios each year.

Panel A1. Sorted on Failure Probability When Distress Costs $\eta_{s_t} = 0$

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$r^{ex}(\%)$	7.07	6.43	6.13	6.02	6.14	6.11	6.10	6.03	5.62	3.10	-3.97
(t)	2.79	2.69	2.64	2.58	2.57	2.49	2.39	2.23	1.90	0.96	-2.31

Panel A2. Sorted on Implied Distressed Risk Premium When Distress Costs $\eta_{s_t} = 0$

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$r^{ex}(\%)$	3.68	5.96	6.24	6.17	6.29	6.22	6.27	6.55	7.06	7.27	3.59
(t)	1.11	1.99	2.30	2.42	2.52	2.54	2.58	2.60	2.69	2.74	1.92

Panel B1. Sorted on Failure Probability Without Heterogeneity in Cash Flow Betas

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$r^{ex}(\%)$	6.43	6.70	6.76	6.80	6.94	6.81	6.69	6.40	5.43	0.79	-5.64
(t)	2.76	2.77	2.72	2.67	2.62	2.52	2.39	2.21	1.79	0.40	-2.85

Panel B2. Sorted on Implied Distressed Risk Premium Without Heterogeneity in Cash Flow Betas

	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$r^{ex}(\%)$	1.30	5.38	6.47	6.82	7.01	6.98	6.85	6.80	6.67	6.45	5.15
(t)	0.42	1.79	2.23	2.44	2.58	2.64	2.68	2.73	2.76	2.75	2.80

Table 6. Debt-to-Equity Ratios and the Market Premium in the Data
This table reports results from time series regressions of quarterly debt-to-equity ratios on the market risk premium (MRP), measured by predicted market excess returns (Haddad et al., 2017) in Panel A and actual market excess returns in Panel B, at the portfolio level. The debt-to-equity ratio is total debt (DLCQ + DLTTQ) divided by equity, which is the product of the stock price (PRCCQ) and share outstanding (CSHOQ). We form the portfolio by sorting firms into deciles based on the failure probability of Campbell et al. (2008) at the end of the previous quarter. We include the standard control variables in the capital structure literature, such as logarithm of book assets (Compustat item ATQ), profitability (OIBDPQ/ATQ) and Tobin's Q ((PRCCQ×CSHOQ + DLCQ + DLTTQ)/ATQ).

Panel A. Predicted Market Excess Returns										
	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Expansions	0.09	0.15	0.23	0.33	0.45	0.60	0.77	0.93	1.12	1.35
(t)	(32.68)	(40.47)	(41.29)	(39.25)	(37.92)	(35.58)	(32.09)	(32.93)	(26.28)	(23.67)
Recessions	0.12	0.20	0.31	0.43	0.58	0.79	0.95	1.18	1.55	2.08
(t)	(14.00)	(13.74)	(16.33)	(15.55)	(17.17)	(14.35)	(14.78)	(13.02)	(13.70)	(12.34)
Intercept	0.13	0.24	0.40	0.94	1.31	1.33	1.88	1.63	1.98	3.47
(t)	(2.77)	(1.90)	(6.00)	(5.90)	(7.52)	(8.82)	(9.92)	(7.11)	(6.67)	(9.58)
MRP_t	−0.14	−0.23	−0.22	−0.59	−0.78	−0.50	−0.73	−0.15	−1.50	−5.31
(t)	(−1.62)	(−1.70)	(−1.49)	(−1.91)	(−1.92)	(−1.15)	(−1.14)	(−0.19)	(−1.28)	(−3.50)
$log(BA)_{t-1}$	−0.00	−0.01	−0.03	−0.06	−0.09	−0.10	−0.11	−0.08	−0.00	−0.03
(t)	(−0.23)	(−1.30)	(−2.73)	(−3.77)	(−4.63)	(−3.75)	(−3.48)	(−2.66)	(−0.04)	(−0.33)
$Profit_{t-1}$	0.71	1.69	2.46	−1.20	−2.17	4.39	−0.96	9.09	12.97	9.09
(t)	(1.50)	(1.27)	(3.29)	(−0.59)	(−0.87)	(2.68)	(−0.52)	(2.96)	(4.12)	(2.80)
$Tobin'sQ_{t-1}$	−0.03	−0.04	−0.06	−0.11	−0.16	−0.19	−0.35	−0.34	−0.59	−1.09
(t)	(−4.65)	(−3.74)	(−3.36)	(−2.76)	(−2.80)	(−2.90)	(−4.24)	(−3.21)	(−4.92)	(−9.21)
$Adj.R^2$	0.43	0.54	0.62	0.50	0.54	0.50	0.50	0.52	0.64	0.68

Panel B. Actual Market Excess Returns										
	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Intercept	0.09	0.17	0.35	0.81	1.13	1.25	1.80	1.73	1.90	2.83
(t)	(1.71)	(1.44)	(5.20)	(7.02)	(8.96)	(7.86)	(9.53)	(7.95)	(8.10)	(9.25)
MRP_t	−0.02	−0.02	−0.03	−0.04	−0.06	−0.09	−0.18	−0.28	−0.35	−0.27
(t)	(−2.15)	(−2.21)	(−2.66)	(−1.96)	(−2.19)	(−2.82)	(−4.81)	(−5.18)	(−4.76)	(−2.12)
$log(BA)_{t-1}$	0.00	−0.01	−0.02	−0.05	−0.08	−0.09	−0.09	−0.08	0.03	0.03
(t)	(0.47)	(−0.68)	(−2.27)	(−2.91)	(−3.75)	(−3.41)	(−3.04)	(−2.84)	(0.54)	(0.37)
$Profit_{t-1}$	0.90	2.03	2.77	−0.56	−0.91	4.74	−0.77	8.27	10.39	9.69
(t)	(1.73)	(1.52)	(3.69)	(−0.30)	(−0.40)	(2.48)	(−0.45)	(2.34)	(3.39)	(2.58)
$Tobin'sQ_{t-1}$	−0.03	−0.04	−0.06	−0.11	−0.15	−0.19	−0.35	−0.37	−0.64	−1.02
(t)	(−3.69)	(−3.04)	(−2.98)	(−2.58)	(−2.49)	(−2.80)	(−4.19)	(−3.64)	(−5.06)	(−7.20)
$Adj.R^2$	0.43	0.54	0.62	0.48	0.52	0.51	0.53	0.59	0.67	0.63

Table 7. The Failure Risk Premium in the Data

This table reports results from unconditional CAPM regressions in Panel A, and annualized excess stock returns $r^{ex}_{i,t}$ and unconditional alphas α^u implied by the conditional CAPM in Panel B, for portfolios sorted on the failure probability. In Panel A, we report annualized excess stock returns $r^{ex}_{i,t}$ and unconditional alphas α^u and β^u from the unconditional time series regressions for each portfolio. In Panel B, we report the value-weighted average of the time-varying CAPM beta $\beta^E_{i,t}$ from the data in the first row. They are obtained from the regression of daily returns on the daily excess market returns, month by month, and are adjusted by the procedure of Lewellen and Nagel (2006). Then, we report the two main components, $\mathbf{E}[\beta^E_{i,t}]\mathbf{E}[r^m_t]$ and $\mathbf{cov}(\beta^E_{i,t}, r^m_t)$ in the next two rows. Finally, we follow Lewellen and Nagel (2006) and calculate the unconditional expected excess return as $r^{ex}_{i,t} = \mathbf{E}[\beta^E_{i,t}]\mathbf{E}[r^m_t] + \mathbf{cov}(\beta^E_{i,t}, r^m_t)$ and the unconditional CAPM alpha $\alpha^u \approx \mathbf{cov}(\beta^E_{i,t}, r^m_t) - \frac{\mathbf{E}[r^m_t]}{(\mathbf{E}[\sigma^m_t])^2}\mathbf{cov}\left(\beta^E_t, (\sigma^m_t)^2\right)$.

Panel A. Excess Return and Alphas from Data											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$r^{ex}_{i,t}$	9.53	7.74	7.28	6.80	7.70	7.10	7.61	3.34	0.66	−0.10	−9.63
(t)	(3.71)	(3.23)	(2.83)	(2.54)	(2.68)	(2.23)	(2.18)	(0.82)	(0.15)	(−0.02)	(−3.46)
α^u	3.02	1.33	0.32	−0.31	−0.00	−1.11	−1.21	−6.33	−9.79	−11.16	−14.2
(t)	(2.68)	(1.58)	(0.43)	(−0.31)	(−0.00)	(−0.80)	(−0.85)	(−2.66)	(−3.80)	(−3.17)	(−4.85)
β^u	0.94	0.92	1.00	1.02	1.11	1.18	1.27	1.39	1.50	1.59	0.66
(t)	(25.99)	(42.93)	(38.59)	(50.34)	(45.19)	(29.18)	(27.08)	(19.01)	(22.22)	(17.46)	(5.73)

Panel B. Model-Implied Excess Returns and Alphas											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H-L
$\beta^E_{i,t}$	0.99	0.96	0.99	1.04	1.08	1.14	1.22	1.30	1.36	1.29	0.30
$E[\beta^E_{i,t}]E[r^m_t]$	6.89	6.66	6.88	7.24	7.50	7.90	8.47	9.07	9.49	9.00	2.11
$cov(\beta^E_{i,t}, r^m_t)$	0.62	0.02	0.04	−0.60	0.18	−1.90	−1.09	−3.20	−3.95	−5.96	−6.58
$r^{ex}_{i,t}$	7.50	6.67	6.92	6.64	7.68	6.00	7.37	5.87	5.54	3.04	−4.47
α^u	1.19	0.52	0.41	−0.49	0.12	−2.29	−1.72	−4.28	−5.30	−7.96	−9.15

Online Appendixes of "A Unified Model of Distress Risk Puzzles"

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- Section A: Boundary Conditions and Asset Valuations
- Section B: Closed-Form Solutions of Stock Returns in a Simplified Model
- Section C: Construction of Ranking Variables
- Section D: Robustness on Empirical Results

A Model

We list the boundary conditions, present the value functions of equity and debt for the firm over the healthy and distress status over the business cycle.

A.1 Boundary conditions

Given the setup of the model, we list the boundary conditions to solve for the model. The closed-form solutions are presented in the appendix.

A.1.1 Boundary Conditions of Equity Value Functions

When the firm is distressed, $w = D$, we have the following conditions:

$$\lim_{X_t \downarrow X_d(B;s_0)} E(X_t, B, D; s_0) = 0, \tag{A1}$$

$$\lim_{X_t \downarrow X_d(G;s_0)} E(X_t, G, D; s_0) = 0, \tag{A2}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} E(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} E(X_t, G, D; s_0), \tag{A3}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} E^{'}(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} E^{'}(X_t, G, D; s_0). \tag{A4}$$

Equations (A1) and (A2) state equity holders receive nothing at bankruptcy at both aggregate state, $s_t = B, G$. Equations (A3) and (A4) are to ensure that the equity value function $E(X_t, G, D; s_0)$ be continuous and smooth at $X_d(B; s_0)$.

Before the firm goes bankrupt or restructure its debt, it switches between the healthy and distress financial status in both aggregate states. We impose the following conditions for equity value functions :

$$\lim_{X_t \uparrow X_s(B;s_0)} E(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B;s_0)} E(X_t, B, H; s_0), \tag{A5}$$

$$\lim_{X_t \uparrow X_s(G;s_0)} E(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G;s_0)} E(X_t, G, H; s_0). \tag{A6}$$

$$\lim_{X_t \uparrow X_s(B;s_0)} E^{'}(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B;s_0)} E^{'}(X_t, B, H; s_0), \tag{A7}$$

$$\lim_{X_t \uparrow X_s(G;s_0)} E^{'}(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G;s_0)} E^{'}(X_t, G, H; s_0). \tag{A8}$$

Equations (A5) and (A6) are value matching conditions, which state the equity value are identical at the distress threshold $X_s(s_t; s_0)$ for the same state, s_t. Equations (A7) and (A8) are smooth pasting conditions, respectively.

When the firm is currently in a healthy status, i.e., $w = H$ in both aggregate states, it restructures its debt upward. We impose the boundary conditions as follows:

$$\lim_{X_t \uparrow X_u(B;s_0)} E(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(B;s_0)} \frac{X_t}{X_0}[(1-\phi)D(X_0, B, H; B) + E(X_0, B, H; B)] - D(X_0, s_0, H; s_0), \tag{A9}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} E(X_t, G, H; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} \frac{X_t}{X_0}[(1-\phi)D(X_0, G, H; G) + E(X_0, G, H; G)] - D(X_0, s_0, H; s_0), \tag{A10}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} E(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} E(X_t, B, H; s_0), \tag{A11}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} E^{'}(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} E^{'}(X_t, B, H; s_0). \tag{A12}$$

Equations (A9) and (A10) are value matching conditions at the restructuring threshold, $X_u(s_t; s_0)$, which states that equity holders retire debt at par $D(X_0, s_0, H; s_0)$, which was issued at the initial state s_0, and issue more debt $D(X_t, s_t, H; s_t)$ at the current aggregate state $s_t = B, G$. *The scaling property applies only within the same aggregate state, s_t.* That is, if the firm starts at an initial state $s_0 = B$ but refinance at $s_t = G$, we scale up the firm value to $X_t/X_0[(1-\phi)D(X_0, G, H; G) + E(X_0, G, H; G)]$ as if it starts at $s_0 = G$. Equations (A11) and (A12) are to ensure that equity value function $E(X_t, B, H; s_0)$ is continuous and smooth at $X_u(G; s_0)$.

A.1.2 Boundary Conditions of Debt Value Functions

When the firm is in distress, $w = D$, we have the following conditions for debt value functions:

$$\lim_{X_t \downarrow X_d(B;s_0)} D(X_t, B, D; s_0) = (1-\alpha)A(X_b, B, D; s_0), \tag{A13}$$

$$\lim_{X_t \downarrow X_d(G;s_0)} D(X_t, G, D; s_0) = (1-\alpha)A(X_b, G, D; s_0), \tag{A14}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} D(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} D(X_t, G, D; s_0), \tag{A15}$$

$$\lim_{X_t \uparrow X_d(B;s_0)} D^{'}(X_t, G, D; s_0) = \lim_{X_t \downarrow X_d(B;s_0)} D^{'}(X_t, G, D; s_0). \tag{A16}$$

Equations (A13) to (A14) states that debt holders receive the asset value after liquidation cost α in both states $s_t = B, G$. Equations (A15) and (A16) are to ensure that the debt value function $D(X_t, G, D; s_0)$ be continuous and smooth at $X_d(B; s_0)$.

We impose the following conditions for debt value functions before the firm goes bankrupt or restructure its debt:

$$\lim_{X_t \uparrow X_s(B;s_0)} D(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B;s_0)} D(X_t, B, H; s_0), \tag{A17}$$

$$\lim_{X_t \uparrow X_s(G;s_0)} D(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G;s_0)} D(X_t, G, H; s_0). \tag{A18}$$

$$\lim_{X_t \uparrow X_s(B;s_0)} D^{'}(X_t, B, D; s_0) = \lim_{X_t \downarrow X_s(B;s_0)} D^{'}(X_t, B, H; s_0), \tag{A19}$$

$$\lim_{X_t \uparrow X_s(G;s_0)} D^{'}(X_t, G, D; s_0) = \lim_{X_t \downarrow X_s(G;s_0)} D^{'}(X_t, G, H; s_0). \tag{A20}$$

$$\tag{A21}$$

The interpretations for equations (A17) to (A20) are similar to those for equations (A5) to (A8).

When the firm restructures its debt upward, we have the following conditions:

$$\lim_{X_t \uparrow X_u(B;s_0)} D(X_t, B, H; s_0) = P(X_0; s_0), \tag{A22}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D(X_t, G, H; s_0) = P(X_0; s_0), \tag{A23}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} D(X_t, B, H; s_0), \tag{A24}$$

$$\lim_{X_t \uparrow X_u(G;s_0)} D^{'}(X_t, B, H; s_0) = \lim_{X_t \downarrow X_u(G;s_0)} D^{'}(X_t, B, H; s_0). \tag{A25}$$

Equations (A22) and (A23) are value matching condition which indicate that debt holder receive par value at the debt refinancing threshold $X_u(s_t; s_0)$ at both states, respectively. Regardless of which the current aggregate state s_t is, debt holders receive the par value $P(X_0; s_0)$ determined at the initial state s_0 where debt is issued. Equations (A24) and (A25) are to ensure that debt value function $D(X_t, B, H; s_0)$ is continuous and smooth at $X_u(G; s_0)$.

A.2 Asset Valuations

The standard no-arbitrage condition implies that the value function $J_{s_t,w}$ of any security that receives a cash flow $CF_{s_t,w}$ is a function of two state variables, such as the cash flow X_t and the economy sates s_t.[22] It can be expressed as follows:

$$(r + p_B)J_{B,w} = CF_{B,w} + \mu_{B,w} X_t J^{'}_{B,w} + \frac{1}{2}\sigma^2_{B,w} X_t J^{''}_{B,w} + p_B J_{G,w}, \tag{A26}$$

$$(r + p_G)J_{G,w} = CF_{G,w} + \mu_{G,w} X_t J^{'}_{G,w} + \frac{1}{2}\sigma^2_{G,w} X_t J^{''}_{G,w} + p_G J_{B,w}. \tag{A27}$$

[22]Note that the financial status w is not an another state variable, because it is entirely determined by the cash flow X_t.

In the matrix form,

$$\left[\begin{pmatrix} r_B + p_B & -p_B \\ -p_G & r_G + p_G \end{pmatrix} - \begin{pmatrix} \mu_{B,w} & 0 \\ 0 & \mu_{G,w} \end{pmatrix} X_t \frac{\partial}{\partial X_t} - \frac{1}{2} \begin{pmatrix} \sigma^2_{B,w} & 0 \\ 0 & \sigma^2_{G,w} \end{pmatrix} X_t^2 \frac{\partial^2}{\partial X_t^2}\right] \begin{pmatrix} J_{B,w} \\ J_{G,w} \end{pmatrix} = \begin{bmatrix} D_{B,w} \\ D_{G,w} \end{bmatrix} \tag{A28}$$

We successively characterize the values of equity and debt for each region. For each initial state, s_0, there are a total of four cash flow regions for both equity and debt, as shown in Figure 2. The cash flow regions are divided as follows:

$$\mathbb{R}_1 : X_d(G; s_0) \leq X_t < X_d(B; s_0); \tag{A29}$$

$$\mathbb{R}_2 : X_d(B; s_0) \leq X_t < c(s_0); \tag{A30}$$

$$\mathbb{R}_3 : c(s_0) \leq X_t < X_u(G; s_0); \tag{A31}$$

$$\mathbb{R}_4 : X_u(G; s_0) \leq X_t < X_u(B; s_0). \tag{A32}$$

For regions of $\mathbb{R}_1$ and $\mathbb{R}_2$, firms are distressed, $w = D$ in both states $s_t = G, B$. For regions of $\mathbb{R}_3$ and $\mathbb{R}_4$, firms are healthy, $w = H$ in both states. We assume the distress threshold is the same in both aggregate states and is dependent on the initial coupon $c(s_0)$, i.e., $X_s(G; s_0) = X_s(B; s_0) = c(s_0)$.

A.2.1 Equity Value Functions

$\mathbb{R}_1 = \mathbf{X_d(G; s_0) \leq X_t < X_d(B; s_0)}$

After becoming distressed, $w = D$, the firm has already gone bankrupt in the bad state, but not yet in the good state. Because equity holders receive nothing at bankruptcy, $E(X_t, B, D; s_0) = 0$, in the bad state. They still receive the residuals after interest and taxes before bankruptcy in the good state. In addition, a sudden switch of the economy from the good state to the bad state will cause the firm to go bankrupt immediately.

The value function of $E(X_t, G, D; s_0)$ satisfies the following ODE:

$$(r_G + p_G)E(X_t, G, D; s_0) = (1-\tau)(X_t - c(s_0)) + \mu_{G,D} X_t E'(X_t, G, D; s_0) + \frac{1}{2}\sigma^2_{G,D} X_t^2 E''(X_t, G, D; s_0) \tag{A33}$$

Assume that the function form of the equity value is

$$E(X_t, G, D; s_0) = (A(X_t, G, D) - C_G(s_0))(1-\tau) + \sum_{i=1}^{2} a^E_{G,D,i} X_t^{\psi_{D,i}}, \tag{A34}$$

where $\psi_{D,i}$ is the roots of

$$\frac{1}{2}\sigma^2_{G,D}\psi_D(\psi_D - 1) + \mu_{G,D}\psi_D - r_G - p_G = 0. \tag{A35}$$

We can easily verify that the particular parts of the above function form are, respectively,

$$A(X_t, G, D) = \frac{X_t}{r_G + p_G - \mu_{G,D}}, \tag{A36}$$

and

$$C_G(s_0) = \frac{c(s_0)}{r_G + p_G}. \tag{A37}$$

It is evident that the unleveled asset value $A(X_t, G, D)$ is decreasing with the probability of leaving the good state for the bad state, p_G, in line with our intuition. While $A(X_t, G, D)$ is independent of initial state s_0, $C_G(s_0)$ is dependent on the initial state where the firm enters market and issue debt.

$\mathbb{R}_2 = \mathbf{X_d(B; s_0) \leq X_t < c(s_0)}$

In this region, the firm has become distressed, i.e., $w = D$, but has not gone bankrupt in both states. Equity holders receive $(1-\tau)(X_t - c(s_0))$ in both states so that $E(X_t, B, D; s_0)$ and $E(X_t, G, D; s_0)$ satisfy the following system of ODEs:

$$\begin{aligned}(r_B + p_B)E(X_t, B, D; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{B,D} X_t E'(X_t, B, D; s_0) \\ &\quad + \frac{1}{2}\sigma_{B,D}^2 X_t^2 E''(X_t, B, D; s_0) + p_B E(X_t, G, D; s_0) \qquad \text{(A38)} \\ (r_G + p_G)E(X_t, G, D; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{G,D} X_t E'(X_t, G, D; s_0) \\ &\quad + \frac{1}{2}\sigma_{G,D}^2 X_t^2 E''(X_t, G, D; s_0) + p_G E(X_t, B, D; s_0). \qquad \text{(A39)}\end{aligned}$$

Assume the functional form of the solution in state s_t is

$$E(X_t, s_t, D; s_0) = (A(X_t, s_t, D) - C_{s_t}(s_0))(1-\tau) + \sum_{i=1}^{4} e_{s_t,D,i}^E X_t^{\omega_{D,i}}. \tag{A40}$$

Plugging (A40) into the ODEs (A38) and (A39), we obtain the solutions to the particular parts as follows:

$$\begin{bmatrix} A(X_t, B, D) \\ A(X_t, G, D) \end{bmatrix} = \begin{bmatrix} r_B - \mu_{B,D} + p_B & -p_B \\ -p_G & r_G - \mu_{G,D} + p_G \end{bmatrix}^{-1} \begin{bmatrix} X_t \\ X_t \end{bmatrix} \tag{A41}$$

and

$$\begin{bmatrix} C_B(s_0) \\ C_G(s_0) \end{bmatrix} = \begin{bmatrix} r_B + p_B & -p_B \\ -p_G & r_G + p_G \end{bmatrix}^{-1} \begin{bmatrix} c(s_0) \\ c(s_0) \end{bmatrix} \tag{A42}$$

For the homogenous part of the solution, we verify that, for each pair of $e_{B,D,i}^E X_t^{\omega_{D,i}}$ and $e_{G,D,i}^E X_t^{\omega_{D,i}}$, we have

$$\left[\begin{pmatrix} r_{B,D} + p_B & -p_B \\ -p_G & r_{G,D} + p_G \end{pmatrix} - \begin{pmatrix} \mu_{B,D} & 0 \\ 0 & \mu_{G,D} \end{pmatrix} \omega_{D,i} - \frac{1}{2} \begin{pmatrix} \sigma_{B,D}^2 & 0 \\ 0 & \sigma_{G,D}^2 \end{pmatrix} \omega_{D,i}(\omega_{D,i} - 1)\right] \begin{pmatrix} e_{B,D,i}^E \\ e_{G,D,i}^E \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A43}$$

Moreover, $e^E_{B,D,i} = g_{D,i}e^E_{G,D,i}$, where

$$g_{D,i} = \frac{1}{p_G}(\frac{1}{2}\sigma^2_{G,D}\omega_{D,i}(\omega_{D,i}-1) + \mu_{G,D}\omega_{D,i} - r_G - p_G), \tag{A44}$$

and $\omega_{D,i}$ is one of two positive roots and two negative roots of the following function

$$(\frac{1}{2}\sigma^2_{B,D}\omega_D(\omega_D-1) + \mu_{B,D}\omega_D - r_B - p_B)(\frac{1}{2}\sigma^2_{G,D}\omega_D(\omega_D-1) + \mu_{G,D}\omega_D - r_G - p_G) = p_B p_G. \tag{A45}$$

$\mathbb{R}_3 = \mathbf{c}(\mathbf{s_0}) \leq \mathbf{X_t} < \mathbf{X_u}(\mathbf{G};\mathbf{s_0})$

The firm is healthy in both states in this region. Hence, equity value functions $E(X_t, G, H; s_0)$ and $E(X_t, B, H; s_0)$ satisfy the following system of ODEs

$$\begin{aligned}(r_G + p_G)E(X_t, G, H; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{G,H}X_t E^{'}(X_t, G, H; s_0)\\ &\quad + \frac{1}{2}\sigma^2_{G,H}X_t^2 E^{''}(X_t, G, H; s_0) + p_G E(X_t, B, H; s_0), \quad \text{(A46)}\\ (r_G + p_B)E(X_t, B, H; s_0) &= (1-\tau)(X_t - c(s_0)) + \mu_{B,H}X_t E^{'}(X_t, B, H; s_0)\\ &\quad + \frac{1}{2}\sigma^2_{B,H}X_t^2 E^{''}(X_t, B, H; s_0) + p_B E(X_t, G, H; s_0). \quad \text{(A47)}\end{aligned}$$

Assume the functional form of the value function is

$$E(X_t, s_t, H; s_0) = (A(X_t, s_t, H) - C_{s_t}(s_0))(1-\tau) + \sum_{i=1}^{4} e^E_{s_t,H,i}X_t^{\omega_{H,i}}. \tag{A48}$$

Plugging (A48) into ODEs (A46) and (A47), we obtain its particular solutions $A(X_t, s_t, H)$ and $C_{s_t}(s_0)$ in the matrix form are as follows:

$$\begin{bmatrix} A(X_t, B, H) \\ A(X_t, G, H) \end{bmatrix} = \begin{bmatrix} r_B - \mu_{B,H} + p_B & -p_B \\ -p_G & r_G - \mu_{G,H} + p_G \end{bmatrix}^{-1} \begin{bmatrix} X_t \\ X_t \end{bmatrix}, \tag{A49}$$

and

$$\begin{bmatrix} C_B(s_0) \\ C_G(s_0) \end{bmatrix} = \begin{bmatrix} r_B + p_B & -p_B \\ -p_G & r_G + p_G \end{bmatrix}^{-1} \begin{bmatrix} c(s_0) \\ c(s_0) \end{bmatrix}. \tag{A50}$$

We can verify for each item $e^E_{B,H,i}X_t^{\omega_{H,i}}$ and $e^E_{G,H,i}X_t^{\omega_{H,i}}$ of the homogenous solution is

$$\left[\begin{pmatrix} r_{B,H} + p_B & -p_B \\ -p_G & r_{G,H} + p_G \end{pmatrix} - \begin{pmatrix} \mu_{B,H} & 0 \\ 0 & \mu_{G,H} \end{pmatrix}\omega_{H,i} - \tfrac{1}{2}\begin{pmatrix} \sigma^2_{B,H} & 0 \\ 0 & \sigma^2_{G,H} \end{pmatrix}\omega_{H,i}(\omega_{H,i}-1)\right]\begin{pmatrix} e^E_{B,H,i} \\ e^E_{G,H,i} \end{pmatrix} = \begin{bmatrix} 0 \\ 0. \end{bmatrix} \tag{A51}$$

Additionally, $e^E_{B,H,i} = g_{H,i}e^E_{G,H,i}$, where

$$g_{H,i} = \frac{1}{p_G}(\frac{1}{2}\sigma^2_{G,H}\omega_{H,i}(\omega_{H,i}-1) + \mu_{G,H}\omega_{H,i} - r_G - p_G), \tag{A52}$$

and $\omega_{H,i}$ is two positive roots and two negative roots of the following function

$$(\frac{1}{2}\sigma^2_{B,H}\omega_H(\omega_H-1)+\mu_{B,H}\omega_H-r_B-p_B)(\frac{1}{2}\sigma^2_{G,H}\omega_H(\omega_H-1)+\mu_{G,H}\omega_H-r_G-p_G)=p_Bp_G. \quad \text{(A53)}$$

$\mathbb{R}_4 = \mathbf{X_u(G;s_0) \leq X_t < X_u(B;s_0)}$

In this region, the firm in the good state has already refinanced their debt upward, but not yet in the bad state. By retiring the existing debt at par $D(X_0,s_0,H;s_0)$ and issuing new debt $D(X_u,G,H;G)$ at a fraction cost ϕ, equity holders increase their own wealth to $E(X_t,G,H;s_0) = (1-\phi)\,D(X_t,G,H;G)+E(X_t,G,H;G)-D(X_0,s_0,H;s_0)$. By scaling property, we have the following equity value at the refinancing threshold $X_u(G;s_0)$:

$$E(X_u,G,H;s_0)=\frac{X_u(G;s_0)}{X_0}\left((1-\phi)\,D(X_0,G,H;G)+E(X_0,G,H;G)\right)-D(X_0,s_0,H;s_0).$$

In contrast, equity holders in the bad state have not refinanced their debt yet. However, an exogenous switch from the bad state to the good state induces equity holders to refinance their debt immediately. Hence, the equity value function in the bad state, $E(X_t,B,H;s_0)$, satisfies the following ODE

$$\begin{aligned}(r_B+p_B)E(X_t,B,H;s_0) &= (1-\tau)(X_t-c(s_0))+\mu_{B,H}X_tE^{'}_{B,H}+\frac{1}{2}\sigma^2_{B,H}X_tE^{''}_{B,H} \\ &+p_B\left(\frac{X_t}{X_0}\left((1-\phi)\,D(X_0,G,H;G)+E(X_0,G,H;G)\right)-D(X_0,s_0,H;s_0)\right).\end{aligned} \quad \text{(A54)}$$

Its solution is

$$E(X_t,B,H;s_0)=A(X_t,B,H)-C_B(s_0)+\sum_{i=1}^{2}a^E_{B,H,i}X_t^{\psi_{H,i}} \quad \text{(A55)}$$

where $\psi_{H,i}$ is the negative and positive roots of

$$\frac{1}{2}\sigma^2_{B,H}\psi_H(\psi_H-1)+\mu_{B,H}\psi_H-r_B-p_B=0. \quad \text{(A56)}$$

We can verify the particular parts of the value function are as follows:

$$A(X_t,B,H)=\frac{X_t(1-\tau)+p_B\frac{X_t}{X_0}\left((1-\phi)\,D(X_0,G,H;G)+E(X_0,G,H;G)\right)}{r_B+p_B-\mu_{B,H}}, \quad \text{(A57)}$$

and

$$C_B(s_0)=\frac{c(s_0)(1-\tau)+p_BD(X_0,s_0,H;s_0)}{r_B+p_B}. \quad \text{(A58)}$$

In total, we have 12 unknown coefficients for equity value function for an initial state s_0.

A.2.2 Debt Value Functions

$\mathbb{R}_1 = \mathbf{X_d(G; s_0) \leq X_t < X_d(B; s_0)}$

In this region, the firm has gone bankrupt in the bad state. Debt holders take over the assets and receive the residual value after the liquidation cost, i.e., $D(X_t, B, D; s_0) = (1-\alpha_B)A(X_{B,d}, B, D)(1-\tau)$. In the good state, debt holders still receive the fixed coupon $c(s_0)$ before bankruptcy. Hence, its value function $D(X_t, G, D; s_0)$ satisfies the following ODE:

$$(r_G + p_G)D(X_t, G, D; s_0) = c(s_0) + \mu_{G,D} X_t D^{'}(X_t, G, D; s_0) \\ + \frac{1}{2}\sigma^2_{G,D} X_t^2 D^{''}(X_t, G, D; s_0) + p_G(1-\alpha_B)A(X_{B,d}, B, D)(1-\tau) \quad \text{(A59)}$$

The solution of the debt value function is

$$D(X_t, G, D; s_0) = C_G(s_0) + \sum_{i=1}^{2} a^D_{G,D,i} X_t^{\psi_{D,i}} + a_d p_G(1-\alpha_B)A(X_{B,d}, B, D)(1-\tau), \quad \text{(A60)}$$

where $C_G(s_0)$ is defined in equation (A37), $\psi_{D,i}$ in (A35),

$$a_d = \frac{1}{r_G + p_G - \mu_{G,D}}, \quad \text{(A61)}$$

and

$$A(X_t, B, D) = \frac{X_t}{r_B + p_B - \mu_{B,D}}. \quad \text{(A62)}$$

$\mathbb{R}_2 = \mathbf{X_d(B; s_0) \leq X_t < c(s_0)}$

In this region, the firm is distressed and its debt holders receive a stream of fixed coupon $c(s_0)$ in both states. $D(X_t, B, D; s_0)$ and $D(X_t, G, D; s_0)$ satisfy the following system of ODEs:

$$(r_B + p_B)D(X_t, B, D; s_0) = c(s_0) + \mu_{B,D} X_t D^{'}(X_t, B, D; s_0) \\ + \frac{1}{2}\sigma^2_{B,D} X_t^2 D^{''}(X_t, B, D; s_0) + p_B D(X_t, G, D; s_0) \quad \text{(A63)}$$

$$(r_G + p_G)D(X_t, G, D; s_0) = c(s_0) + \mu_{G,D} X_t D^{'}(X_t, G, D; s_0) \\ + \frac{1}{2}\sigma^2_{G,D} X_t^2 D^{''}(X_t, G, D; s_0) + p_G D(X_t, B, D; s_0), \quad \text{(A64)}$$

The debt value function in state s_t is

$$D(X_t, s_t, D; s_0) = C_{s_t}(s_0) + \sum_{i=1}^{4} e^D_{s_t,D,i} X_t^{\omega_{D,i}} \quad \text{(A65)}$$

where $C_{s_t}(s_0)$ is shown in (A42) and $\omega_{D,i}$ in (A45). Similar to the equity value function in the same region, $e^D_{B,D,i} = g_{D,i} e^D_{G,D,i}$, where $g_{D,i}$ is in equation (A44).

$\mathbb{R}_3 = \mathbf{c(s_0) \leq X_t < X_u(G; s_0)}$

The firm is healthly in both states. Debt value functions $D(X_t, G, H; s_0)$ and $D(X_t, B, H; s_0)$ satisfy the following system of ODEs:

$$(r_G + p_G)D(X_t, G, H; s_0) = c(s_0) + \mu_{G,H} X_t D^{'}(X_t, G, H; s_0) + \frac{1}{2}\sigma^2_{G,H} X_t^2 D^{''}(X_t, G, H; s_0) + p_G D(X_t, B, H; s_0), \quad \text{(A66)}$$

$$(r_G + p_B)D(X_t, B, H; s_0) = c(s_0) + \mu_{B,H} X_t D^{'}(X_t, B, H; s_0) + \frac{1}{2}\sigma^2_{B,H} X_t^2 D^{''}(X_t, B, H; s_0) + p_B D(X_t, G, H; s_0). \quad \text{(A67)}$$

And the solution function in both states is

$$D(X_t, s_t, H; s_0) = C_{s_t}(s_0) + \sum_{i=1}^{4} e^D_{s_t,H,i} X_t^{\omega_{H,i}}. \quad \text{(A68)}$$

where $C_{s_t}(s_0)$ is shown in (A50) and $\omega_{H,i}$ in (A53). Similar to the equity value function in the same region, $e^D_{B,H,i} = g_{H,i} e^D_{G,H,i}$, where $g_{H,i}$ is in equation (A52).

$\mathbb{R}_4 = \mathbf{X_u(G; s_0) \leq X_t < X_u(B; s_0)}$

Because the firm refinances earlier in the good state than in the bad state, debt holders have already redeemed the par value, $D(X_t, G, H; s_0) = D(X_0, s_0, H; s_0)$, in the good state. Because debt holders have not received the payment at par in the bad state, the debt value function, $D(X_t, B, H; s_0)$, satisfies the following ODE:

$$(r_B + p_B)D(X_t, B, H; s_0) = c(s_0) + \mu_{B,H} X_t D^{'}(X_t, B, H; s_0) + \frac{1}{2}\sigma^2_{B,H} X_t^2 D^{''}(X_t, B, H; s_0) + p_B D(X_0, s_0, H; s_0) \quad \text{(A69)}$$

Its solution is

$$D(X_t, B, H; s_0) = C_B(s_0) + \sum_{i=1}^{2} a^D_{B,H,i} X_t^{\psi_{H,i}} \quad \text{(A70)}$$

where $\psi_{H,i}$ is in (A56) and

$$C_B(s_0) = \frac{c(s_0) + p_B D(X_0, s_0, H; s_0)}{r_B + p_B}. \quad \text{(A71)}$$

In total, we have 12 unknown coefficients for debt value function for an initial state s_0.

A.3 Equity Returns

We apply Ito's Lemma to equation (3) and obtain

$$\frac{dE_{s_t,w}}{E_{s_t,w}} = \frac{\partial E_{s_t,w}}{\partial X_t E_{s_t,w}} dX_t + \frac{1}{2}\frac{\partial^2 E_{s_t,w}}{\partial X_t^2}\frac{(dX_t)^2}{E_{s_t,w}} + \frac{E_{s_t^+,w} - E_{s_t,w}}{E_{s_t,w}}(\hat{p}_{s_t} dt + dM_{s_t,t}) \tag{A72}$$

$$= \frac{X_t \partial E_{s_t,w}}{E_{s_t,w}\partial X_t}\left(\hat{\mu}_{s_t} dt + \sigma^m_{s_t} d\hat{W}^m_t + \sigma^{i,X}_{s_t,w} d\hat{W}^i_t\right) + \frac{1}{2}\frac{X_t^2 \partial^2 E_{s_t,w}}{E_{s_t,w}\partial X_t^2}\sigma^2_{s_t,w} dt + \frac{E_{s_t^+,w} - E_{s_t,w}}{E_{s_t,w}}(dM_{s_t,t} + \hat{p}_{s_t} dt) \tag{A73}$$

$$= \frac{1}{E_{s_t,w}}\left(\frac{X_t \partial E_{s_t,w}}{\partial X_t}\hat{\mu}_{s_t} + \frac{X_t^2 \sigma^2_{s_t,w}}{2}\frac{\partial^2 E_{s_t,w}}{\partial X_t^2} + \frac{E_{s_t^+,w} - E_{s_t,w}}{E_{s_t,w}}\hat{p}_{s_t}\right) dt + \frac{E_{s_t^+,w} - E_{s_t,w}}{E_{s_t,w}} dM_{s_t,t}$$

$$+ \frac{X_t}{E_{s_t,w}}\frac{\partial E_{s_t,w}}{\partial X_t}\left(\sigma^m_{s_t} d\hat{W}^m_t + \sigma^{i,X}_{s_t,w} d\hat{W}^i_t\right) \tag{A74}$$

For two aggregate states, $s_t \in (G, B)$, and two levels of financial status, $w \in (H, D)$, the excess equity return is given by

$$r^e_{s_t,w}(X_t) = \mathbb{E}_t[r^E_{s_t,w}] - r dt \tag{A75}$$

$$= -\mathbb{E}\left[\frac{dm_{s_t,t}}{m_{s_t,t}}, \frac{dE_{s_t,w}}{E_{s_t,w}}\right] = \left[(\frac{X_t \partial E_{s_t,w}}{E_{s_t,w}\partial X_t})(\sigma^m_{s_t}\theta_{s_t}) - (\frac{E_{s_t^+,w}}{E_{s_t,w}} - 1)(\kappa_{s_t} - 1)\hat{p}_{s_t}\right] dt. \tag{A76}$$

Let $\frac{X_t \partial E_{s_t,w,s_0}}{E_{s_t,w,s_0}\partial X_t} = \gamma_{s_t,w}$ and $(\frac{E_{s_t^+,w,s_0}}{E_{s_t,w,s_0}} - 1) = \psi_{s_t,w}$, we have

$$r^e_{s_t,w} = \gamma_{s_t,w}\lambda^m_{s_t} dt + \psi_{s_t,w}(1 - \kappa_{s_t})\hat{p}_{s_t} dt. \tag{A77}$$

B Simplified Model

For the simplified model, we list boundary conditions, present the value functions for equity and debt, and then finally the closed-form solution of the expected stock return.

B.1 Boundary Conditions

Equity Boundary Conditions

The boundary conditions for equity are as follows:

$$\lim_{X_t \uparrow X_d} E(X_t, D) = 0; \tag{B1}$$

$$\lim_{X_t \uparrow X_s} E(X_t, H) = \lim_{X_t \downarrow X_s} E(X_t, D). \tag{B2}$$

Equation (B1) states that equity holders of a distressed firm (i.e., $w = D$) receive nothing at bankruptcy. Equations (B2) is the value-matching condition at the distress threshold X_s.

Debt Boundary Conditions

The boundary conditions for debt are as follows:

$$\lim_{X_t \uparrow X_d} D(X_t, D) = \lim_{X_t \downarrow X_d} (1-\alpha) A(X_t, D)(1-\tau); \tag{B3}$$

$$\lim_{X_t \uparrow X_s} D(X_t, H) = \lim_{X_t \downarrow X_s} D(X_t, D). \tag{B4}$$

Equation (B3) shows that debt holders take over the assets and receive the residual value of assets $A(X_d, D)(1-\tau)$ after the liquidation cost α. Equations (B4) is the value-matching condition at the distress threshold X_s.

B.2 Asset Valuations

In this simplified model, the firm has two financial status of w. That is, $w = H$ for $X_t \geq X_s$ and $w = D$ for $X_t < X_s$. Similar to equation (A28), standard dynamic programming suggests that $J(X_t, w) \equiv J_{t,w}$ in this single-state economy satisfies the ordinary differential equation

$$\mu_w X J'_{t,w} + \frac{\sigma_w^2}{2} X^2 J''_{t,w} - r J_{t,w} + CF_t = 0, \tag{B5}$$

where $J(X_t, w)$, $J'_{t,w}$ and $J''_{t,w}$ denote the first and second-order derivatives of $J_{t,w}$ with respect to X_t, respectively.

When the firm is healthy, the dividend d_t accruing to equity holders is $(X_t - c)(1-\tau)$. The dividend becomes $(X_t - c)(1-\tau)$ when the firm is distressed. Hence, the value function of equity is

$$E(X_t, w) = (1-\tau)\left(A_{t,w} - \frac{c}{r}\right) + e_{w,1} X_t^{\omega_{w,1}} + e_{w,2} X_t^{\omega_{w,2}}, \tag{B6}$$

where $A_{t,w}$ is the value of a unlevered, "perpetual" firm[23]

$$A_{t,w} \equiv A(X_t, w) = \frac{X_t}{r - \mu_w}. \tag{B7}$$

The $\omega_{w,1} < 0$ and $\omega_{w,2} > 1$ are the two roots of the characteristic equation

$$\frac{1}{2}\sigma_w^2 \omega_w(\omega_w - 1) + \mu_w \omega_w - r = 0. \tag{B8}$$

The two roots are:

$$\omega_{w,1} = \frac{1}{2} - \frac{\mu_w}{\sigma_w^2} - \sqrt{\left(\frac{\mu_w}{\sigma_w^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma_w^2}} < 0, \tag{B9}$$

and

$$\omega_{w,2} = \frac{1}{2} - \frac{\mu_w}{\sigma_w^2} + \sqrt{\left(\frac{\mu_w}{\sigma_w^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma_w^2}} > 1. \tag{B10}$$

The cash flow accruing to debt holders is the coupon c, respectively. Hence, the value function of debt is

$$D(X_t, w) = \frac{c}{r} + d_{w,1} X_t^{\omega_{w,1}} + d_{w,2} X_t^{\omega_{w,2}}. \tag{B11}$$

B.3 Equity Returns

Similar to the full model, we start with the general formula for the equity return and then use the boundary conditions. Ito's lemma implies that the equity value $E(X_t, w) \equiv E_{t,w}$ satisfies

$$\frac{dE_{t,w}}{E_{t,w}} = \frac{1}{E_{t,w}}\left(\frac{\partial E_{t,w}}{\partial t} + \hat{\mu}_w X_t \frac{\partial E_{t,w}}{\partial X_t} + \frac{\sigma_w^2}{2} X_t^2 \frac{\partial^2 E_{t,w}}{\partial X_t^2}\right) dt + (\sigma^m d\hat{W}_t^m + \sigma_w^{i,X} d\hat{W}_t^i) \frac{X_t}{E_{t,w}} \frac{\partial E_{t,w}}{\partial X_t}. \tag{B12}$$

The standard asset pricing argument gives

$$\mathbb{E}\left[\frac{dE_{t,w} + d_t dt}{E_{t,w}}\right] - rdt = -\mathbb{E}\left(\frac{dE_{t,w}}{E_{t,w}}, \frac{dm_t}{m_t}\right) = \frac{X_t}{E_{t,w}} \frac{\partial E_{t,w}}{\partial X_t} \beta_w(\sigma^m \theta) dt. \tag{B13}$$

Denoting $(dE_{t,w} + d_t dt)/E_{t,w}$ by $r_{t,w}^E$ and $(X_t \partial E_{t,w})/(E_{t,w} \partial X_t)$ by $\gamma_{t,w}$, we have the excess equity return

$$r_{t,w}^e(X_t) = \mathbb{E}_t[r_{t,w}^E] - rdt = (\gamma_{t,w}\beta_w)\sigma^m \theta dt = \beta_w^E \lambda^m dt. \tag{B14}$$

The elasticity of the equity to the underlying cash flows $\gamma_{t,w} = \frac{X_t \partial E_{t,w}}{E_{t,w} \partial X_t}$ can be obtained by differ-

[23] Assuming the firm is perpetual, we follow Goldstein et al. (2001) and label $A_{t,w}$ the unlevered asset value. If the unlevered firm hits the default threshold X_d at the time t_d, the unlevered asset value can be calculated as the difference between the perpetuity value of $A(X_t, D)$ at X_t and the perpetuity value of $A(X_d, D)$ at X_d discounted by the Arrow-Debreu price $\pi_t = (\frac{X}{X_d})^{\omega_{D,1}}$ as follows:

$$A_{t,w} = \mathbb{E}_t^Q\left[\int_t^{t_d} X_s e^{-r(s-t)} ds\right] = \mathbb{E}_t^Q[\int_t^\infty X_t e^{-r(s-t)} ds] - \mathbb{E}_t^Q[\int_{t_d}^\infty X_d e^{-r(s-t)} ds] = A(X_t, w) - A(X_d, w)\pi_t.$$

entiating (B6), and is as follows:

$$\gamma_{t,w} = 1 + \frac{c(1-\tau)}{rE_{t,w}} + \frac{(\omega_{w,1}-1)}{E_{t,w}} e_{w,1} X_t^{\omega_{w,1}} + \frac{(\omega_{w,2}-1)}{E_{t,w}} e_{w,2} X_t^{\omega_{w,2}} \tag{B15}$$

Because we assume the firm does not have the refinancing option in the further simplified model, the no-bubble condition implies $e_{w,2} = 0$. The condition in equation (B1) implies:

$$e_{H,1} = \left(\frac{E(X_s, D)}{(1-\tau)} - \left(A(X_s, H) - \frac{c}{r} \right) \right) (1-\tau) (\frac{X}{X_s})^{\omega_{H,1}}. \tag{B16}$$

When the firm is distressed, $w = D$, the value-matching conditions in equation (B2) determine

$$e_{D,1} = \left(\frac{c(1-k)}{r} - A(X_d, D) \right) (1-\tau) \pi_t, \tag{B17}$$

where pi_t is the risk-neutral default probability, i.e., $\pi_t = \left(\frac{X_t}{X_s}\right)^{\omega_{D,1}}$. Substituting $e_{H,1}$, and $e_{D,1}$ into equations (B6), we obtain the equity value of a distressed firm[24]

$$E_{t,D} = \left[\underbrace{\left(A(X_t, D) - \frac{c(1-k)}{r} \right)}_{\text{Equity-in-Place}} + \underbrace{\left(\frac{c(1-k)}{r} - A(X_d, D) \right) \pi_t}_{\text{Put Option of Going Bankruptcy}} \right] (1-\tau). \tag{B18}$$

and the equity value of a healthy firm[25]

$$E_{t,H} = \left[\underbrace{\left(A(t, H) - \frac{c}{r} \right)}_{\text{Equity-in-Place}} + \underbrace{\left(\frac{E(X_s, D)}{(1-\tau)} - \left(A(X_s, H) - \frac{c}{r} \right) \right) (\frac{X}{X_s})^{\omega_{H,1}}}_{\text{Put Option of deleveraging}} \right] (1-\tau). \tag{B19}$$

[24] Alternatively, the equity value of a distressed firm is the present value of accumulated after-tax dividends, $(X_t - c)(1-\tau)$, between time t and τ_d, where τ_d is the first passage time of hitting the default threshold X_d. Using the unlevered asset value in Footnote 23, we have the following expression:

$$\begin{aligned} E_{t,D} &= \left(\mathbb{E}_t^Q \left[\int_t^{t_d} X_s e^{-r(s-t)} ds \right] - \mathbb{E}_t^Q \left[\int_t^{t_d} c e^{-r(s-t)} ds \right] \right) (1-\tau) \\ &= \left(A(X_t, D) - A(X_d, D) \pi_t \right) (1-\tau) - \frac{c(1-k)}{r} (1-\pi_t)(1-\tau). \end{aligned}$$

[25] Similar to that of a distressed firm, the equity value of a healthy firm is the sum of the present value of accumulated after-tax dividends between time t and τ_s, where τ_s is the first passage time of hitting the distress threshold X_s, and the present value of the equity value $E(X_s, D)$ after becoming distressed, which is discounted by the Arrow-Debreu price $(\frac{X}{X_s})^{\omega_{H,1}}$. Using the unlevered asset value in footnote 23, we have the following expression for the equity value:

$$\begin{aligned} E_{t,H} &= \mathbb{E}_t^Q \left[\int_t^{t_s} X_s e^{-r(s-t)} ds \right] (1-\tau) - \mathbb{E}_t^Q \left[\int_t^{t_s} c e^{-r(s-t)} ds \right] (1-\tau) + E(X_s, D) (\frac{X}{X_s})^{\omega_{H,1}} \\ &= \left(A(X_t, H) - A(X_s, H) (\frac{X}{X_s})^{\omega_{H,1}} \right) (1-\tau) - \frac{c}{r} (1 - (\frac{X}{X_s})^{\omega_{H,1}})(1-\tau) + E(X_s, D) (\frac{X}{X_s})^{\omega_{H,1}}. \end{aligned}$$

Finally, we insert $e_{H,1}$, and $e_{D,1}$ into (B15) and obtain the equity-cash flow elasticity as follows:

$$\gamma_{t,H} = \frac{\partial E_{t,H}/E_{t,H}}{\partial X_t/X_t}, \tag{B20}$$

$$= 1 + \underbrace{\frac{\frac{c}{r}(1-\tau)}{E_{t,H}}}_{\text{Leverage}} + \underbrace{(1-\omega_{H,1})\frac{(A(X_s,H) - \frac{c}{r} - \frac{E(X_s,D)}{1-\tau})}{E_{t,H}}(\frac{X_t}{X_s})^{\omega_{H,1}}(1-\tau)}_{\text{Put Option of Deleveraging } (-)}, \tag{B21}$$

and

$$\gamma_{t,D} = \frac{\partial E_{t,D}/E_{t,D}}{\partial X_t/X_t}, \tag{B22}$$

$$= 1 + \underbrace{\frac{\frac{c}{r}(1-\tau)}{E_{t,D}}}_{\text{Leverage}} + \underbrace{\frac{(\frac{c}{r} - A(X_d,D))\pi_t}{E_{t,D}}(1-\tau)}_{\text{Put Option of Going Bankruptcy } (-)} . \tag{B23}$$

C Appendix C: Construction of Ranking Variables

We discuss the construction of the two ranking variables used in the paper, and provide theoretical justification for the implied default risk premium.

C.1 Failure Probability

When examining the actual data, we compute failure probability as in the third column of Table 4 of Campbell et al. (2008):

$$\begin{aligned} \textit{F-Probability} = & -9.164 - 20.264NIMTAAVG + 1.416TLMTA - 7.129EXRETAVG \\ & +1.411SIGMA - 0.045RSIZE + 0.075MB - 0.058PRICE - 2.132CASHMTA. \end{aligned} \tag{C1}$$

$NIMTAAVG$ is the moving average of the net income

$$NIMTAAVG_{t-1,t-12} = \frac{1-\phi^3}{1-\phi^{12}}(NIMTA_{t-1,t-3} + \ldots + \phi^9 NIMTA_{t-10,t-12}),$$

and *EXRETAVG* is the moving average of the relative excess returns

$$EXRETAVG_{t-1,t-12} = \frac{1-\phi}{1-\phi^{12}}(EXRET_{t-1} + \cdots + \phi^{11} EXRET_{t-12}),$$

where *NIMTA* is net income divided by the sum of market equity and total liabilities; *EXRET* is monthly log excess return on each firm's equity relative to market excess return ($EXRET = \log(1+R_{it}) - \log(1+R_{mkt,t})$); *TLMTA* is the ratio of total liabilities divided by the sum of market equity and total liabilities; *SIGMA* is the volatility of stock returns; *RSIZE* is the relative size measured as the log ratio of the firm's market equity to that of the total market; *CASHMTA* is the ratio of cash and short-term investments divided by the sum of market equity and total liabilities; MB is the market-to-book equity; and *PRICE* is the log price per share.

When applying logit regressions to our simulated data panels, we do not include $CASHMTA$ because we do not model cash holdings. Nor do we include *Price* because we do not have the number of shares in the model. These two items have no significant effect on our results because their estimated coefficients and t-statistics from Campbell et al. (2008) are relatively small. However, we include both of them when we use the actual data.

C.2 Proxy of Distress Risk Premium

When simulating the model, we know the distress risk premium parameters *ex ante*, but, in the empirical tests, we do not. Friewald et al. (2014) infer the distress risk premium from CDS *ex post*. We follow them and infer the distress risk premium from the simulated data *ex post* by "pretending" that we do not know the distress risk premium to assess the empirical procedure.[26]

[26]Friewald et al. (2014) demonstrate the distress risk premium is the difference between the inverse function of the risk-neutral default probability and the physical probability in the framework of Merton (1974) and back out the risk

Inspired by Almeida and Philippon (2007), we propose to proxy for the distress risk premium using the logarithmic difference between the risk-neutral and objective default probability as follows:[27]

$$\text{Implied Distress Risk Premium} = \log(\pi_t/\hat{\pi}_t) = \log(\pi_t) - \log(\hat{\pi}_t). \tag{C2}$$

Following the empirical literature, we apply the KMV procedure to the simulated data. Denoting $\mathbf{N}(\cdot)$, the cumulative probability function of a standard normal distribution, the one-year objective default probability is given by:

$$\hat{\pi}_t = \mathbf{N}(-\hat{DD}_t), \tag{C3}$$

in which $\hat{DD}_t$ denotes the distance to default as follows:

$$\hat{DD}_t = \frac{\log[P_t/(E_t + P_t)] + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}, \tag{C4}$$

where $\hat{\mu}$ is the actual asset growth rate, σ is the annual asset volatility, E_t is the equity value, P_t is the book value of debt, and T is the time to maturity. We assume the maturity $T = 1$ as in the standard KMV procedure.

C.3 Theoretical Justification for the Implied Default Risk Premium

We motivate our proxy of the distress risk premium, the difference in the logarithm of risk-neutral and physical probabilities. The following proposition connects the cash flow risk premium with risk-neutral and physical default probabilities explicitly.[28]

If the risk-neutral rate $\mu \to r$, the cash flow risk premium λ_t is:

$$\lambda_t \;=\; \left(\frac{\log(\pi_t) - \log(\hat{\pi}_t)}{\log(X_t) - \log(X_d)} + 1\right)\frac{\sigma_t^2}{2}dt, \tag{C5}$$

where $\pi_t \equiv (\frac{X_t}{X_d})^{\omega}$ and $\hat{\pi}_t \equiv (\frac{X_t}{X_d})^{\hat{\omega}}$ is the risk-neutral and physical default probability, respectively. The exponents $\omega < 0$ and $\hat{\omega} < 0$ are defined in the Appendix B.

Because the risk-free rate is lower than the objective growth rate $\hat{\mu}$ by the risk premium λ_t, the risk-neutral probability of default exceeds its actual counterpart for a risk-averse agent, i.e.,

premium from CDS. Compared with the procedure of Friewald et al. (2014), our proxy is easy to implement.

[27] Note that the proxy we use is based on the simplified model and can be easily implemented using the Merton's model. If we allow the distressed firm to rebound, the probability of default is

$$\mathscr{L}_t = \frac{X_t^{\omega_{D,1}}(X_s)^{\omega_{D,2}} - X_t^{\omega_{D,2}}(X_s)^{\omega_{D,1}}}{(X_s)^{\omega_{D,2}}(X_d)^{\omega_{D,1}} - (X_s)^{\omega_{D,1}}(X_d)^{\omega_{D,2}}}$$

and the probability of rebounding

$$\mathscr{H}_t = \frac{X_t^{\omega_{D,2}}(X_d)^{\omega_{D,1}} - X_t^{\omega_{D,1}}(X_d)^{\omega_{D,2}}}{(X_s)^{\omega_{D,2}}(X_d)^{\omega_{D,1}} - (X_s)^{\omega_{D,1}}(X_d)^{\omega_{D,2}}}.$$

[28] We drop the subscript of w for ease of notation.

$\log(\pi_t) - \log(\hat{\pi}_t) \geq 0$. Combined with the condition of $X_t \geq X_d$, the model implies a positive risk premium λ_t. Hence, we use $\log(\pi_t) - \log(\hat{\pi}_t)$ to proxy for the distress risk premium.

Although the risk-neutral and physical probabilities of default are positively correlated, the difference, $\log(\pi_t) - \log(\hat{\pi}_t)$, can be negatively related to $\hat{\pi}_t$ or π_t. In other words, a high physical or risk-neutral probability of default does not guarantee a high risk premium, λ_t, as the two probabilities can be negatively correlated.

By directly applying the property of hitting time distribution of a geometric Brownian motion according to equation (11) of p.14 on Harrison (1985), we obtain the cumulative physical default probability $\hat{\pi}$ for the firm issuing a bond with the time-to-maturity T:

$$\hat{\pi}_t = \mathbb{N}\left(h(T)\right) + \left(\frac{X_t}{X_d}\right)^{-2\xi/\sigma_i^2} \mathbb{N}\left(h(T) + \frac{2\xi T}{\sigma_i\sqrt{T}}\right), \tag{C6}$$

where $\xi = \hat{\mu} - 0.5\sigma^2 > 0$ and $h(T) = \frac{log(X_d/X_t) - \xi T}{\sigma_i\sqrt{T}}$.

For the perpetual bond in our model, $T \to \infty$. Therefore,

$$\hat{\pi}_t = (\frac{X_t}{X_d})^{\hat{\omega}}. \tag{C7}$$

where $\hat{\omega} = -2(\hat{\mu} - 0.5\sigma^2)/\sigma^2$.

When $\mu \to r$, equation (B9) implies $\omega \to -2r/\sigma^2$. Therefore,

$$\pi_t = (\frac{X_t}{X_d})^{\omega} \to (\frac{X_t}{X_d})^{-2r/\sigma^2}. \tag{C8}$$

By taking logarithm of $\hat{\pi}_t$ and π_t, we can easily obtain:

$$\lambda = \hat{\mu} - \mu \to \hat{\mu} - r = \left(\frac{log(\pi_t) - log(\hat{\pi}_t)}{log(X_t) - log(X_d)} + 1\right)\frac{\sigma^2}{2}. \tag{C9}$$

D Appendix D: Additional Empirical Results

D.1 Different Measures for Debt-to-Equity Ratios

To ensure our results are robust, we use different proxy for debt and different weight in calculating the average debt ratios.

First, we replace total debt with total liability (Compustat LTQ), which we use to construct the failure probability by following Campbell et al. (2008), and calculate liability-to-equity ratio. As shown in Panel A of Table D1, the estimated coefficients are increasingly negative from –0.32 to –8.34. The difference is 8.02, greater than the difference of 5.17 reported in the main table 6. We observe similar patterns in Panel B. Therefore, we obtain stronger results when using this alternative measure.

[Insert Table D1 Here]

Second, we replace value-weighted debt-to-equity ratios with equal-weighted ratios. Table D2 shows that the results are slightly weaker results, although the negative association between the debt-to-equity ratios is still increasingly negative with the failure probability in both panels.

[Insert Table D2 Here]

D.2 Debt Financing and the Market Risk Premium

Table D3 reports time series regression results for debt financing behavior across ten decile portfolios. When we use the scaled change in total debt to proxy for debt financing in Panel A, the estimated coefficients of MRP_t decrease from 0.05 (t-statistic = 1.38) to –0.23 (t-statistic = –2.50), indicating that the firms decrease their debt when the market risk premium increases the discount rate and lowers their continuation value. Alternatively, when using the scaled change in total liability in Panel B, we find that the negative responses are even stronger. That is, the estimated coefficients of MRP_t are all negative and greater than their counterparts in Panel A. More importantly, they become increasingly negative from –0.00 (t-statistic = –0.02) to –0.36 (t-statistic = –3.43).

[Insert Table D3 Here]

Taken together, we demonstrate that firms decrease their debt in response to the market risk premium, using two different measures of debt financing.

Table D1. Value-Weighted Liability-to-Equity Ratio and the Market Premium in the Data

This table reports results from time series regressions of quarterly liability-to-equity ratios on the market risk premium (MRP), measured by predicted market excess returns (Haddad et al., 2017) in Panel A and actual market risk returns in Panel B, at the portfolio level. The liability-to-equity ratios are calculated as total liabilities (Compustat item LTQ), divided by equity, respectively. The equity is the product of the stock price (PRCCQ) and share outstanding (CSHOQ). We form the portfolio by sorting firms into deciles based on the failure probability of Campbell et al. (2008) at the end of the previous quarter. Control variables include the equal-weighted logarithm of assets, *log(BA)* as well as value-weighted profitability (*profit*) and *Tobin's Q*.

Panel A. Predicted Market Excess Returns										
	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Expansions	0.26	0.36	0.51	0.71	0.97	1.29	1.64	1.96	2.24	2.73
(t)	(32.68)	(40.47)	(41.29)	(39.25)	(37.92)	(35.58)	(32.09)	(32.93)	(26.28)	(23.67)
Recessions	0.33	0.49	0.69	1.02	1.25	1.69	1.99	2.45	3.08	4.05
(t)	(14.00)	(13.74)	(16.33)	(15.55)	(17.17)	(14.35)	(14.78)	(13.02)	(13.70)	(12.34)
Intercept	0.44	0.63	1.02	2.52	2.66	2.73	4.74	4.15	4.45	7.68
(t)	(3.64)	(2.79)	(10.04)	(3.69)	(4.17)	(7.68)	(5.50)	(4.87)	(6.84)	(11.78)
MRP_t	−0.32	−0.50	−0.54	−2.08	−2.62	−1.63	−4.28	−2.96	−4.93	−8.34
(t)	(−1.94)	(−1.85)	(−1.86)	(−1.90)	(−1.82)	(−1.27)	(−1.66)	(−1.20)	(−1.85)	(−3.12)
$log(BA)_{t-1}$	−0.01	−0.03	−0.06	−0.11	−0.16	−0.22	−0.22	−0.19	−0.00	−0.26
(t)	(−1.01)	(−1.63)	(−3.07)	(−3.00)	(−3.55)	(−3.40)	(−2.73)	(−2.21)	(−0.01)	(−1.87)
$Profit_{t-1}$	1.96	3.40	3.47	−10.21	−0.09	14.83	−7.34	16.75	24.80	17.17
(t)	(1.64)	(1.47)	(3.25)	(−1.12)	(−0.01)	(3.98)	(−0.93)	(1.64)	(2.75)	(3.10)
$Tobin'sQ_{t-1}$	−0.09	−0.11	−0.15	−0.38	−0.44	−0.48	−1.09	−1.07	−1.46	−2.16
(t)	(−5.53)	(−4.52)	(−4.18)	(−2.40)	(−2.55)	(−2.80)	(−3.48)	(−2.93)	(−4.18)	(−9.69)
$Adj.R^2$	0.65	0.67	0.70	0.36	0.49	0.50	0.42	0.48	0.60	0.69

Panel B. Actual Market Excess Returns										
	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Intercept	0.35	0.48	0.88	2.02	1.99	2.44	4.04	3.77	3.94	6.79
(t)	(2.82)	(2.27)	(9.75)	(4.69)	(5.17)	(6.79)	(6.86)	(5.63)	(7.19)	(11.67)
MRP_t	−0.05	−0.07	−0.08	−0.07	−0.07	−0.22	−0.44	−0.58	−0.65	−0.73
(t)	(−3.78)	(−4.04)	(−4.25)	(−0.84)	(−0.88)	(−2.64)	(−4.15)	(−3.36)	(−3.39)	(−4.03)
$log(BA)_{t-1}$	−0.00	−0.01	−0.05	−0.08	−0.12	−0.19	−0.15	−0.14	0.10	−0.19
(t)	(−0.19)	(−0.87)	(−2.52)	(−1.89)	(−2.51)	(−2.81)	(−1.65)	(−1.60)	(0.69)	(−1.40)
$Profit_{t-1}$	2.44	4.23	4.29	−7.25	5.27	16.11	−5.43	17.67	20.21	18.77
(t)	(2.00)	(1.78)	(3.83)	(−0.94)	(0.69)	(3.82)	(−0.86)	(1.68)	(2.48)	(3.00)
$Tobin'sQ_{t-1}$	−0.09	−0.11	−0.15	−0.34	−0.37	−0.48	−1.04	−1.06	−1.53	−2.05
(t)	(−4.42)	(−3.77)	(−3.69)	(−2.34)	(−2.28)	(−2.72)	(−3.50)	(−3.09)	(−4.39)	(−7.89)
$Adj.R^2$	0.67	0.68	0.72	0.32	0.43	0.51	0.42	0.51	0.61	0.68

Table D2. Equal-Weighted Debt-to-Equity Ratios and the Market Premium in the Data

This table reports results from time series regression of quarterly equal-weighted debt-to-equity ratios on the market risk premium (MRP), measured by predicted market excess returns (Haddad et al., 2017) in Panel A and actual market risk returns in Panel B, at the portfolio level. The debt-to-equity ratios are calculated as total debt (DLCQ + DLTTQ) divided by equity, which is the product of the stock price (PRCCQ) and share outstanding (CSHOQ). The equity is the product of the stock price (PRCCQ) and share outstanding (CSHOQ). We form the portfolio by sorting firms into deciles based on the failure probability of Campbell et al. (2008) at the end of the previous quarter. Control variables include the equal-weighted logarithm of assets, *log(BA)* as well as value-weighted profitability (*profit*) and *Tobin's Q*.

Panel A. Predicted Market Excess Returns

	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Expansions	0.08	0.13	0.20	0.28	0.37	0.49	0.62	0.76	0.93	1.20
(t)	(28.51)	(30.02)	(31.40)	(30.14)	(29.37)	(28.15)	(26.52)	(27.26)	(23.65)	(21.60)
Recessions	0.11	0.18	0.28	0.41	0.51	0.70	0.86	1.09	1.43	1.91
(t)	(9.96)	(10.78)	(12.32)	(12.15)	(14.45)	(13.34)	(17.32)	(15.68)	(15.01)	(13.68)
Intercept	0.11	0.24	0.52	0.94	1.33	1.55	2.08	2.45	2.23	2.65
(t)	(2.10)	(2.47)	(4.28)	(7.91)	(9.63)	(9.61)	(10.80)	(10.23)	(7.19)	(10.88)
MRP_t	−0.10	−0.29	−0.46	−0.96	−1.35	−1.51	−1.85	−1.89	−1.90	−3.46
(t)	(−1.62)	(−2.71)	(−2.42)	(−3.46)	(−3.65)	(−2.96)	(−2.64)	(−2.34)	(−1.97)	(−3.85)
$log(BA)_{t-1}$	−0.00	−0.02	−0.03	−0.05	−0.08	−0.08	−0.11	−0.10	0.02	0.09
(t)	(−0.84)	(−2.05)	(−2.82)	(−3.39)	(−4.33)	(−3.49)	(−4.53)	(−3.18)	(0.31)	(2.22)
$Profit_{t-1}$	1.33	2.39	1.85	0.20	−0.83	1.13	−0.29	−1.30	5.22	13.21
(t)	(2.76)	(2.34)	(1.29)	(0.15)	(−0.52)	(0.50)	(−0.14)	(−0.57)	(2.18)	(6.13)
$Tobin'sQ_{t-1}$	−0.04	−0.06	−0.10	−0.19	−0.28	−0.38	−0.53	−0.75	−0.86	−0.97
(t)	(−5.33)	(−4.10)	(−4.91)	(−5.59)	(−6.84)	(−5.90)	(−6.08)	(−7.21)	(−7.59)	(−7.29)
$Adj.R^2$	0.62	0.69	0.69	0.64	0.67	0.65	0.66	0.70	0.79	0.84

Panel B. Actual Market Excess Returns

	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Intercept	0.08	0.14	0.39	0.68	0.94	1.21	1.72	2.14	1.94	2.19
(t)	(1.62)	(1.38)	(3.61)	(6.46)	(8.01)	(7.85)	(9.85)	(8.79)	(6.42)	(7.33)
MRP_t	−0.01	−0.02	−0.05	−0.07	−0.08	−0.13	−0.19	−0.26	−0.29	−0.33
(t)	(−1.71)	(−2.01)	(−3.32)	(−2.48)	(−2.95)	(−3.23)	(−4.94)	(−5.87)	(−4.15)	(−4.52)
$log(BA)_{t-1}$	−0.00	−0.01	−0.02	−0.03	−0.05	−0.05	−0.08	−0.06	0.05	0.14
(t)	(−0.26)	(−0.97)	(−2.10)	(−2.04)	(−2.64)	(−2.20)	(−3.10)	(−2.30)	(1.14)	(2.80)
$Profit_{t-1}$	1.48	2.87	2.66	1.77	1.91	3.12	1.30	−0.46	5.85	14.05
(t)	(3.14)	(2.64)	(1.99)	(1.35)	(1.26)	(1.39)	(0.59)	(−0.18)	(2.29)	(4.71)
$Tobin'sQ_{t-1}$	−0.04	−0.05	−0.09	−0.17	−0.23	−0.35	−0.49	−0.72	−0.83	−0.90
(t)	(−4.60)	(−2.90)	(−3.51)	(−4.03)	(−4.10)	(−4.45)	(−4.78)	(−5.64)	(−6.85)	(−5.27)
$Adj.R^2$	0.62	0.68	0.68	0.60	0.62	0.64	0.66	0.72	0.81	0.84

Table D3. Debt Financing and the Market Premium in the Data

This table reports the results of time series regression of quarterly debt financing variables on the market risk premium (MRP), proxied by market risk returns. Debt financing (Panel A) and liability financing (Panel B) are calculated as the change in total debt (DLCQ + DLTTQ) and the change in total liabilities (Compustat item LTQ), divided by total assets (ATQ) of the last quarter, respectively. We form the portfolio by sorting firms into deciles based on the failure probability of Campbell et al. (2008) at the end of the previous quarter. Then, we run the time series regression as follows: $y_{j,t} = a_j + b_{j,Y} MRP_t + cX_{j,t} + e_{j,t}$, where $y_{j,t}$ is the value-weighted average of the debt financing variables, and the quarterly expected market risk premium MRP_t is obtained from the website of Erik Loualiche. We include the standard control variables in the capital structure literature, such as logarithm of book assets (Compustat item ATQ), profitability (OIBDPQ/ATQ) and Tobin's Q ((PRCCQ×CSHOQ + DLCQ + DLTTQ)/ATQ).

Panel A. Debt Financing										
	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Intercept	0.02	−0.00	−0.01	−0.03	−0.02	−0.05	−0.03	−0.02	0.01	−0.01
(t)	(0.78)	(−0.08)	(−0.67)	(−0.81)	(−0.75)	(−2.42)	(−1.60)	(−1.01)	(0.81)	(−0.35)
MRP_t	−0.00	0.00	0.00	−0.00	0.00	0.01	−0.00	0.01	0.00	0.00
(t)	(−0.26)	(0.48)	(0.28)	(−0.21)	(0.80)	(1.52)	(−0.06)	(1.23)	(0.61)	(0.26)
$log(BA)_{t-1}$	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00	0.00	−0.00	−0.01	−0.00
(t)	(−0.80)	(−0.62)	(−0.41)	(−0.49)	(−1.40)	(−0.52)	(0.19)	(−1.15)	(−2.18)	(−0.43)
$Profit_{t-1}$	0.03	0.52	0.70	1.15	1.28	1.57	1.09	1.39	0.77	−0.11
(t)	(0.10)	(1.66)	(3.78)	(2.04)	(2.04)	(5.06)	(2.73)	(3.64)	(2.43)	(−0.46)
$Tobin'sQ_{t-1}$	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.02
(t)	(1.70)	(1.68)	(1.23)	(1.16)	(2.23)	(2.03)	(2.40)	(3.23)	(3.40)	(2.24)
$Adj.R^2$	0.01	0.01	0.06	0.10	0.09	0.21	0.08	0.09	0.07	0.06

Panel B. Liability Financing										
	L(ow)	2	3	4	5	6	7	8	9	H(igh)
Intercept	−0.01	0.06	0.08	0.02	0.03	0.05	0.04	0.03	0.07	0.07
(t)	(−0.20)	(1.53)	(2.61)	(0.46)	(0.93)	(1.70)	(1.38)	(0.95)	(2.29)	(2.26)
MRP_t	−0.00	−0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.01	0.02
(t)	(−0.20)	(−1.06)	(0.14)	(0.58)	(0.73)	(1.29)	(0.55)	(1.83)	(0.74)	(1.62)
$log(BA)_{t-1}$	−0.00	−0.01	−0.01	−0.01	−0.01	−0.01	−0.00	−0.01	−0.02	−0.02
(t)	(−0.71)	(−2.71)	(−2.04)	(−1.65)	(−1.95)	(−2.39)	(−0.76)	(−1.79)	(−2.61)	(−2.20)
$Profit_{t-1}$	1.28	1.17	0.82	2.17	2.14	1.73	1.15	2.14	1.34	0.24
(t)	(2.94)	(3.12)	(2.11)	(3.28)	(2.86)	(2.94)	(1.98)	(4.54)	(3.35)	(0.74)
$Tobin'sQ_{t-1}$	0.02	0.02	0.01	0.02	0.02	0.02	0.01	0.03	0.04	0.03
(t)	(4.19)	(4.54)	(1.61)	(2.29)	(1.97)	(2.03)	(1.51)	(3.48)	(4.57)	(2.76)
$Adj.R^2$	0.30	0.41	0.34	0.38	0.36	0.30	0.25	0.28	0.22	0.19