

'Memory' in the Middle: Housing Price - Macroeconomic Interactions in the US

Abstract

Macroeconomic variations and housing price fluctuations are tightly interlinked. In this paper, we study the impact of (system) *long-memory* on a model of dynamic interactions between a housing market and a macroeconomy. We characterize housing price equilibrium via identification and quantification of distinct demand and supply responses to changes in macroeconomic conditions. We argue that the actual disequilibrium error corrections in the interactive system are slow and nonlinear inducing an undesirable interplay of many economy-wide shocks so many so that the expected dynamic stability of the system becomes a difficult objective to achieve. To resolve this issue, using a quarterly data (1975Q1-2016Q1) for the US, our fractionally cointegrated vector autoregressive estimations demonstrate that the housing market adjusts gradually towards the market clearing, while shocks in the system are featured with a long-memory, further indicating informational inefficiency in the housing market. We quantify memory-driven impacts of macroeconomic variables, and find that the impacts can be transmitted not only through either the housing demand or supply channel exclusively, but also through both the channels simultaneously. Overall impacts of macroeconomic variables are eventually derived by aggregating their possible impacts from both the channels. We conclude that a failure to identify the distinct demand and supply effect-transmission channels could result in an estimation bias of macroeconomic effects; disregarding the memory pattern of shocks in the system further leads to a mis-representation of macroeconomic policy effectiveness in an environment with persistent policy uncertainty. A forecasting exercise confirms the predictive power of the FCVAR model, and robustness checks support our baseline results.

Key Words: Equilibrium housing prices; Macroeconomic fundamentals; Long memory identification; Market inefficiency; Fractionally cointegrated VAR

JEL Classifications: R31; E32; C32

1 Introduction

"No memory is ever alone; it's at the end of a trail of memories, a dozen trails that each have their own associations." **Louis L'Amour (An American Author)**

A growing body of literature has emerged recently arguing that changes in macroeconomic conditions are not only veritable sources of (international) housing price fluctuations (Arestis and Gonzalez-Martinez, 2016; Carstensen, 2006), but their inherent impacts on equilibrium housing prices, if disregarded, would induce systematic over-estimation bias (Duan et al., 2018a,b). It is well-known that a given macroeconomic factor could impact housing prices through distinct demand and supply effect-transmission channels either exclusively through one channel or simultaneously through both of them; the equilibrium housing prices are eventually determined by the macroeconomic variables, which impacts are transmitted through these two channels. So far, prior literature reports a 'puzzling' housing price behaviour in the face of a macroeconomic shock. Given a clear dearth theory and empirical strategy, one potential argument can be the identifiable distinct while normally intertwined impacts of the same macroeconomic variable on shifting the housing demand and supply curves, respectively, and finally on equilibrium housing prices. For example, McCarthy and Peach (2002) observe that housing prices first rise over a short period and then fall given a positive shock to the federal funds rate.¹ Moreover, given a fixed housing demand, a rising economic policy uncertainty could induce a more cautious response of housing supply due to an intrinsic irreversibility of residential fixed investment (Tsatsaronis and Zhu, 2004); at the same time, it also tends to depress investors' intention of a house purchase given its large transaction-to-whole-wealth ratio and an increasing role of risk aversion. Hence, a primary research interest of this paper is to uncover the real macroeconomic impacts on equilibrium housing price determinations by identifying the distinct roles of macroeconomic variables through the housing demand and supply channels, respectively.

At the same time, we make one fundamental assumption related to the systemic characteristic of the macroeconomy-housing market interaction (in short, *MH*): not only each variable within the system but also the system as a whole, can possess a long-memory (loosely representing the ability of the system to remember shocks for a long time in the future). Through the perspective of information transmission, 'memory' describes a convergence speed to govern how fast the impacts of past shocks/information on the current value of a target variable decay over time. Governed by such a memory pattern, the system does not have to display as a white noise process (with no memory), a covariance stationary process (with a short memory), or a unit root process (with a perfect/permanent memory) as conventionally assumed.² Instead, we relax the above assumptions and expect our system to be featured with a long-memory that systemic shocks could

¹A more detailed discussion is provided in Section 2.

²Conventional strategy defines statistical features and memory patterns of a given time series as shown in the above sentence by assuming that the series integration order should be an integer. However, it neglects the case that the series integration order actually can be a fraction, indicating a long-memory patterns. Detailed descriptions of the statistical properties of the series with different integration orders and memory patterns are discussed in Section 3.

decay hyperbolically and converge towards a steady-state mean in the long-run, i.e. a fractionally integrated process. Importantly, a critical question emerges: Can shocks within a macroeconomy-housing market interaction depict a long-memory pattern? It is a possibility and close to the reality as the housing market is essentially governed by waves of buyer-induced or seller-driven transactions that are relatively predictable.³

Surprisingly, established housing-related literature fails to underline the existence of long memory shocks and their probable impacts on the co-evolutionary paths of macroeconomic variables and housing prices. Instead, by imposing a unit root assumption for variables in the interactive system, most of the extant literature conventionally employs a $I(1)/I(0)$ framework and accordingly draws empirical conclusions by simply using the transformed data with the potential unit root removed from the raw data. In our case, such a conventional strategy potentially assumes the housing market efficiency indicating that the current housing price contains all available information from the past and its future dynamics cannot be predicted by using the past information with its changes depicted as a white noise stochastic process.⁴ Moreover, in line with Fama's market efficient hypothesis (EMH), it further implies that shocks within such an 'efficient' system must exist permanently and never converge (i.e. a unit root process with a permanent memory).

However, such an efficient market assumption potentially imposed in the conventional literature tends to be too strict to achieve as the permanently-existing impacts of past shocks over-time characterized by an efficient market may not exist in reality. Instead, as researched by much related literature, the housing market could be characterized by inefficiencies (See, for example, Case and Shiller, 1989, 1990; Hjalmarsson and Hjalmarsson, 2009; Larsen and Weum, 2008; Leung et al., 2006, among others), indicating that the systemic shocks tend to be slowly-decayed over time with their convergence speeds vary with different memory patterns.⁵ In addition, simply removing the unit root by first differentiating the series in the conventional strategy would further lead to a serious information loss of the raw data. Hence, it can be concluded that stochastic shocks in an economic system could actually taper-off slowly towards a long-run constant mean, i.e. a long memory pattern (Jones et al., 2014), a failure to account for the impacts of such long-memory featured shocks in the conventional strategy (with an integer integration assumption, notably a unit root assumption) would be unrealistic and very difficult to identify the real co-movement structure between housing prices and macroeconomic variables, and therefore lead to an estima-

³A series tends to be more predictable with persistent volatility if its shocks can depict a long memory compared with a less predictable one with short-memory shocks (Nguyen et al., 2019). According to Peter's (1994) Fractal Market Hypothesis (FMH), economic theories have long recognized that there exist the market frictions lasting for a relatively long time period due to the existence of market irrationality and the non-linear price response to information governing dynamics of systemic shocks. Thus, the shocks tend to vanish asymptotically rather than vanish exponentially or exist permanently.

⁴As originally proposed by Fama (1970), informational efficiency is defined as a condition when prices can fully and instantly reflect all relevant and available information. According to this concept, price dynamics in an efficient market follow a martingale, or its special form, a random walk; price changes are completely random with no autoregressive dependence known as a white noise process (Larsen and Weum, 2008).

⁵Although some literature produces empirical results in the favor of the housing market efficiency (Schulz and Werwatz, 2011), it is still too strict to impose the market efficiency assumption at the very beginning.

tion bias. However, given the great importance, these have not yet been encapsulated in extant literature. More so, far little has gone beyond a mere empirical exercise of testing for the presence of long-memory and discusses its economic implications.

Furthermore, compelling evidence on both theoretical and empirical aspects has demonstrated that there is a gradual adjustment process towards a housing market equilibrium rather than an instant market clearing as conventionally assumed (See, for example, DiPasquale and Wheaton, 1994; Eubank Jr and Sirmans, 1979; Fair, 1972; Riddell, 2004, among others). Accordingly, to be commensurate with the broad literature, we account for the slow market adjustment towards the equilibrium by employing a long-memory cointegration framework, called fractionally cointegrated vector autoregressive model (FCVAR). Overall, our paper aims to provide a thorough quantitative assessment of the distinct impacts of macroeconomic fundamentals through the housing demand and supply channels on the equilibrium housing price determination by comprehensively accounting for all possible memory patterns of shocks in the interactive system, especially the long-memory featured shocks. Moreover, this paper considers the possibility that the housing market could be inefficient, and provides a novel way to identify and measure the degree of market inefficiency by allowing the existence of a slow-convergence of disequilibrium shocks (with a long-memory feature) to a steady-state value.⁶ A possibility of a slow mean-converging shock identified within a dynamic system, such as ours, can mimic real-life fluctuations of variables in the macroeconomy-housing market interaction and quantify equilibrium housing price determinations more accurately than conventional approaches/strategies that disregard the impact of the system memory pattern.

The contributions of our paper are summarized as follows. First, our paper both theoretically and empirically identifies the distinct macroeconomic effect-transmission channels through both the demand and supply sides, and finds that macroeconomic variables can affect equilibrium housing prices either exclusively through one side or simultaneously through both sides, respectively. This provides a precise and policy-informative interpretation of housing market responses to macroeconomic interventions. Simply estimating a single price determination function including both demand and supply factors altogether would otherwise mask the real macroeconomic effects, instead, only the aggregate effects are obtained. Second, this paper provides a novel way to examine if the housing market is efficient and further compare the degree of market (in)efficiency across markets through a measurement of memory patterns of target variables in the macroeconomy - housing market interactions. Third, our paper employs an innovative long-memory cointegration framework, i.e. FCVAR, to identify and quantify the role of 'memory' in affecting the dynamics of macroeconomy-housing market interactions. We find the mediating role of memory, which 'mediates' magnitudes of true macroeconomic effects on equilibrium housing price determinations through both the housing demand and supply functions.

Consistent with theoretical expectations discussed in our conceptual framework, using a quar-

⁶Our *MH* is essentially a memory-driven system, where due to varying degrees of path-dependence of each variable within this system, impacts of different system shocks are not necessarily limited to converge as the same mean-reverting process with the same variance.

terly data in the US (1975Q1-2016Q1), our FCVAR estimations demonstrate a gradual price adjustment towards the housing market clearing, and capture the existence of long-memory featured shocks in the macroeconomy - housing market interactions, indicating the housing market inefficiency. By invoking a complex interplay of 'memory' within the interactive system and quantifying the memory-driven macroeconomic impacts, we find that macroeconomic variables can not only affect equilibrium housing prices through either the housing demand or supply channel exclusively, but also through both these two channels simultaneously with a 'dual' role. An overall equilibrium housing price determination function reporting the aggregate effects of included macroeconomic variables is eventually derived by manually solving the simultaneous demand and supply functions in the equilibrium. Moreover, we find a significant bias of macroeconomic estimations if the distinct demand and supply channels are failed to be identified, indicating that separately estimating the two simultaneous functions can be more meaningful than estimating a single and combined function (following conventional proposition) for equilibrium housing price determinations. The five-year-ahead dynamics of the macroeconomy - housing market interactions have been forecast, and the better predictive performance of our employed FCVAR model against a naive CVAR model has been examined.

Our conclusions possess meaningful policy insights regarding an accurate comprehension of equilibrium housing price dynamics determined by macroeconomic fundamentals through both the strategies of identifying the distinct demand and supply channels and capturing the long-memory featured shocks in the interactive system. In addition, featured with a long-memory, the US housing market is checked to be inefficient. In our robustness exercise, we perform FCVAR estimations with prior restrictions, which are imposed according to the statistical significance of model equilibrium parameters. Our results provide robust evidence of the mediating role of 'memory' in understanding true effects of macroeconomic variables on the US housing market.

The rest of paper is structured as follows. Section 2 reviews existing literature. Section 3 presents a conceptual framework in housing price determination identified through both the housing demand and supply channels. Section 4 presents methodology and discusses estimation issues. Section 5 provides variable descriptions and corresponding data sources. Section 6 describes procedures of data transformation and preliminary observations. Section 7 contains detailed discussions of our empirical estimations. Finally, Section 8 summarizes main findings of the paper and discusses related policy implications.

2 Literature

Although recent research has made some headway regarding co-movement between housing prices and macroeconomic fundamentals (Arestis and Gonzalez-Martinez, 2016; Duan et al., 2018a,b), how the latter would influence demand and supply channels and finally the equilibrium housing price, is yet to be rigorously determined. A failure to disentangle real macroeconomic impacts from different channels may result in an imprecise interpretation of housing price movements.

Existing literature, although thin, has started realizing this importance.

McCarthy and Peach (2002) find ‘puzzling’ housing price behaviours in the face of a monetary policy shock in the US: after restructuring the housing finance system since the mid 1980s, tightening monetary policy (a positive shock to the federal funds rate) first heightens housing prices in the short-run, which decline in the long-run. They attribute such a short-run increase of housing prices to sellers’ willingness to sustain a high housing price to minimize loss particularly in a downturn, which suggests a decreasing housing supply given a rising cost of construction finance. On the other hand, buyers tend to expect a further fall in interest rates given a current monetary tightening policy, thanks to an extensive availability of adjustable-rate mortgages. Thereby maintaining a relatively strong housing demand, which also induces an increase of housing prices in the short-run. However, in the long-run, housing prices will witness a gradual decline due to an overwhelmingly negative impact of a slump in housing demand given an increasing mortgage financing expenditure.

2.1 Housing demand effect channel

Undoubtedly, housing prices are determined by many demand-driven factors. Among others, Muellbauer and Murphy (1997) focus on studying the dynamics of housing prices in the UK through an inverted housing demand function, and point out a dominant role of housing demand factors in the boom (in the late 1980s) and bust (in the 1990s) eras of housing prices. Our survey of the broad literature leads us the following conclusions. First, along with many existing studies (DiPasquale and Wheaton, 1994; Meen, 1996, 1990, 1993), they find an important role of individuals’ financial constraints, which dampen both housing demand and housing prices. A failure to account for this may overlook potential credit borrowing/down-payment limitations. Therefore, it fails to precisely describe the dynamic behaviour of housing buyers and the shifts of the housing demand curve. Moreover, the study of Muellbauer and Murphy (1997) is also consistent with Poterba (1984) who points out that a negative shock of user financing costs raises housing prices given fixed housing stocks in the short-run. Afterwards, housing prices experience a gradual decline in a market adjustment period along with increasing housing stocks until a new steady state is reached.⁷

Second, regarding the effects of macroeconomic factors on housing prices, Muellbauer and Murphy (1997) point out that income exerts a positive effect on housing prices by boosting housing demand. They also find that available housing stocks reveal the level of housing demand in the market and assume that the latter is essentially proportional to the former. New housing completion reflects a specific component of buyers’ house needs that has already been incorporated into the effective housing demand. This also represents the ability of the market’s effective supply to meet demand although there remains what we call a ‘demand gap’ (Heath, 2014). Thus, current

⁷The importance of credit on housing price movements has also been embraced by much of the recent literature (See representative studies among others, Abdallah and Lastrapes, 2013; Favara and Imbs, 2015; Gerlach and Peng, 2005; Ling et al., 2016; Mian and Sufi, 2009).

high housing stocks could indicate an upturn of housing demand leading to an acceleration of housing prices given the backdrop of a strong demand.

McCarthy and Peach (2002) echo the above discussion and recognize the role of housing stocks in determining the equilibrium housing prices through both demand and supply channels. Recent studies, such as Arestis and Gonzalez-Martinez (2016); Duan et al. (2018a,b), build a conceptual framework to attribute housing price dynamics to factors from both the demand and supply sides. They find that demand factors involving personal disposable income, interest rates and credit availability contribute theoretically expected effects to housing price changes. Other relevant studies including Fitzpatrick and McQuinn (2007); Gerlach and Peng (2005); Hwang and Quigley (2006); Senhadji and Collyns (2002) also discuss the determination of housing prices through the demand channel. Third, the effect of uncertainty cannot be neglected (Baker et al., 2016; Muellbauer and Murphy, 1997). Along with Meen (1990, 1993), the existence of housing market uncertainty can dampen individuals' intentions of property purchase and then depress housing prices (through the housing demand side), while it is also believed to affect housing prices through the housing supply side (Tsatsaronis and Zhu, 2004).

To summarise, three main flaws characterise the existing demand-related research on housing prices. First, although a gradual/slow price adjustment to market equilibrium has been widely recognized (DiPasquale and Wheaton, 1994), the traditional assumption of a faster housing market clearing is further away from reality. Second, extant literature does not account for the extent to which supply-driven (production) factors affect housing prices. However, important predictive power of factors governing housing production from the supply side cannot be ignored. Furthermore, although some research consider housing price determinants from both demand and supply sides, their empirical investigation invariably concentrates on 'final' equilibrium price determination, thus sidelining potentially important impacts of distinct macroeconomic factors separately on demand and supply functions. In the next section, we discuss relevant literature focusing on the supply side of the housing market.

2.2 Housing supply effect channel

Although literature that models macroeconomic effects of housing prices via demand channel is comparatively large, there are few studies that enrich our understanding on the supply side. Some prior research have underlined the following reasons on why the housing supply is hard to model. Quigley (1979) points out that considerable quality variations of each housing unit and indefinite dimensions of the housing quality evaluation inhibit accurate measurement of the total housing supply and outputs. Second, available housing stocks in the market are provided by different housing suppliers such as new housing builders and existing housing owners. It is nevertheless difficult to capture their individual behaviours due to the paucity of such micro-level data. Third, in addition to private sectors, the government can also exert a marked impact on shifting the supply curve by its implementation of public housing provision and a property tax levy, further raising uncertainty and fluctuations of the housing supply (DiPasquale, 1999).

As mentioned in Section 2.1, one popular approach to model the equilibrium housing prices using demand-driven factors and housing stocks is the stock-flow model. DiPasquale and Wheaton (1994) introduce an error correction structure, and enhance the traditional model by allowing a slow price adjustment towards a market clearing. Features of housing market operations are summarized in their paper as follows. First, housing prices exert a strong positive autocorrelation, denoting that future prices are moved with backward-looking expectations. This finding is qualitatively consistent with the feature of ‘lagged appreciation of current housing prices’ proposed by Abraham and Hendershott (1996) and Muellbauer and Murphy (2008) and the feature of ‘forward-looking’ of housing builders regarding current prices Murphy (2018). Second, the housing market appears to behave as a serious disequilibrium given the excess demand and insufficient supply in reality. Given a price shock, demand reacts more quickly and on a greater scale compared to supply, which tends to be unresponsive, particularly in the short-run. This is also in line with Poterba (1984) and Mankiw and Weil (1989).

Third, consistent with DiPasquale (1999), impacts of various (housing production) factor markets, particularly the land market, on the housing constructions are still elusive due to the data limitation. Poterba (1984) suggests that a buoyant demand of production factors is able to heighten the equilibrium housing prices due to a downward shift of supply curve induced by a rising supply expenditure. The author recognizes the importance of land, although it is neglected in the paper due to data constraints. This is also embraced by Knoll et al. (2017), in which the land price is found to be a key factor that determines the long-run housing price dynamics. In addition, DiPasquale and Wheaton (1994) also provide an explanation about the impact of land value on housing prices on the supply side. Given an initial housing price increase, it stimulates the rise of housing stocks, which implies an increase of housing supply and a decline of land availability for construction. Land value will then rise as a consequence, which tends to cause the housing supply to further falter by absorbing excess returns generated from the initial housing price increase, and further raise housing prices. Recent studies echo this viewpoint and find that the supply-driven factors such as construction cost and land value demonstrate marked impacts on housing prices (Glaeser et al., 2008; Green et al., 2005; Knoll et al., 2017; Saiz, 2010).

Although housing price drives housing stocks in a literature where ‘stock-flow model’ dominates empirical formulation, the latter in turn exert an impact on the former. In addition to its impact through the housing demand channel as discussed in Section 2.1, it is also able to affect housing prices through housing supply channel. Muellbauer and Murphy (1997) point out that an increase of available housing stocks is found to drive a slump in housing prices by positively shifting the supply curve. Furthermore, studies using one aggregated housing price determination function can indeed involve both demand and supply factors (Arestis and Gonzalez-Martinez, 2016; Duan et al., 2018a,b). However, although described theoretically, they nevertheless could not empirically disentangle explanatory powers of specific variables, which have dual impacts through both demand and supply functions, respectively. Instead, only aggregate effects of these factors are reported, which could still give rise to confusing conclusions. For example, through

a financing perspective in the housing market, it is possible that a positive shock to interest rates decreases housing prices by depressing housing demand, while it can also raise housing prices by a falling housing supply simultaneously. A similar mechanism is expected from other variables with the dual roles such as credit, housing stocks and uncertainty.

We do not apply the traditional two-equation stock-flow model in this paper for the following reasons. First, the model fails to account for impacts of supply-driven factors on housing prices even though these factors can contribute to important explanatory powers. Instead, only total housing stock is included to represent the housing supply in the price determination function. Second, there is no precise way to quantify the housing stock. Its conventional proxy is the aggregate housing supply, regardless of possible heterogeneous housing qualities and types (See, among others, DiPasquale and Wheaton, 1994; Hwang and Quigley, 2006; Muellbauer and Murphy, 1997), which is far from reality. Moreover, as explained in Section 2.1, the instant market clearing assumed in the traditional model is far from reality. The approach of an aggregate determination equation is not applicable to our research focus. Thus, to avoid these obstacles, we construct simultaneous housing demand and supply function systems to separately model how housing prices are determined through those two channels, and subsequently quantify which one dominates the equilibrium housing price determination.

3 Theoretical Underpinning

Following market equilibrium theory, we build a theoretical construct to describe the ways that macroeconomic fundamentals contribute to equilibrium housing prices. This incorporates both demand and supply functions, both of which consider a gradual price adjustment process towards a market clearing within a long-memory framework. We follow McCarthy and Peach (2002) and formulate the long-run equilibrium housing price levels through both the functions, respectively.⁸ From housing demand perspective, demand can drive equilibrium housing prices (RHP^{D*}) to clear the current stock of housing (HUC) (McCarthy and Peach, 2002). It in turn depends upon variables such as individuals' house purchase abilities (CD), purchase of financing cost (LIR), the price level of the economy (DEF), the current stock of housing (HUC), and (economic policy) uncertainty (EPC). In a demand-driven equilibrium, DEF , LIR , and EPU are all expected to exert a negative effect on RHP^{D*} , while the impacts of HUC and CD can be positive.

$$RHP^{D*} = \alpha_1 \underset{(-)}{DEF} + \alpha_2 \underset{(+)}{HUC} + \alpha_3 \underset{(-)}{LIR} + \alpha_4 \underset{(-)}{EPU} + \alpha_5 \underset{(+)}{CD} \quad (1)$$

Similarly, from the perspective of housing supply, assuming that there is a perfectly competitive environment in which housing suppliers make a zero profit in the long-run, the supply function determines equilibrium housing prices (RHP^S) that equates supply of housing to the effective buyers' housing demand in the market (HUC) (McCarthy and Peach, 2002). The HUC , in

⁸Detailed explanations of signs of all incorporated demand- and supply-driven factors have been discussed in Arestis and Gonzalez-Martinez (2016); Duan et al. (2018b).

turn depends upon suppliers' financing levels (CS), market value of land (RLV), financing cost of suppliers (LIR), the current stock of housing (HUC), and (economic policy) uncertainty (EPU). Therefore, in the supply-driven equilibrium, HUC and CS are expected to have a negative impact on RHP^{S*} , while EPU , LIR , and RLV may exert a positive effect. The detailed definitions of all variables and the corresponding data sources are summarized in Table 2 in Section 5.

$$RHP^{S*} = \lambda_1 \underset{(+)}{RLV} + \lambda_2 \underset{(-)}{HUC} + \lambda_3 \underset{(+)}{LIR} + \lambda_4 \underset{(+)}{EPU} + \lambda_5 \underset{(-)}{CS} \quad (2)$$

Assuming that the housing market clears, we can directly estimate both demand and supply functions simultaneously to arrive at the final housing market equilibrium: $RHP^* = RHP^{D*} = RHP^{S*}$. The assumption of market clearing depends crucially on how fast stochastic shocks within the system taper-off: the faster decay offers smaller probability of interaction with external shocks, whereas slow decay gives rise to higher probability of non-linear interaction with shocks. Eventually, holding other things constant, a longer duration of shock convergence produces an important feature: a gradual adjustment of housing prices towards equilibrium level. Very importantly, there is heterogeneous adjustment behaviour of shocks depending on whether it is the demand or supply channels. The final equilibrium, as we argue in this paper, is driven greatly by distinct adjustment speed of shocks in either demand or supply channels. Indeed, it will be wrong to assume that a stochastic shock will have a unique effect on housing price equilibrium. Identification of the exact effect of that shock in demand and supply dynamics is very important from a policy point of view as modelling of responses of demand and supply to respective shocks helps in targeted policy design.⁹

Moreover, there exists overwhelming evidence in supporting the disequilibrium status in the housing market particularly in the short-run due to aforementioned shocks. Thus, we explicitly consider such slow adjustments by employing an error correction framework in both (1) and (2). Specifically, we identify that a given shock to the equilibrium housing prices will generate wedges between current price level and RHP^{D*} as well as RHP^{S*} . Such a disequilibrium status in a housing market implied by these wedges will dissipate slowly towards the equilibrium level if there are no other shocks in the system.

Furthermore, there is compelling evidence in favour of persistent and slowly-converged shocks in the macroeconomy-housing market interaction instead of a unit root process as conventionally assumed. Thus, we identify a slow decay pattern (e.g. long-memory) of shocks in each target series in demand and supply functions by allowing for the existence of fractional integration.¹⁰ Thus, demand- and supply-driven housing price determination functions are constructed as fol-

⁹(see McCarthy and Peach, 2002, for a summary of related literature).

¹⁰Detailed mathematical explanations are developed in Section 4.

lows:

$$\Delta RHP_t = \Pi_D L_{d1}(RHP_t - RHP_t^{D*}) + \beta_1 \Delta HUC_t + \beta_2 \Delta DEF_t + \beta_3 \Delta LIR_t + \beta_4 \Delta EPU_t + \beta_5 \Delta CD_t + \varepsilon_D \quad (3)$$

$$\Delta RHP_t = \Pi_S L_{d2}(RHP_t - RHP_t^{S*}) + \gamma_1 \Delta HUC_t + \gamma_2 \Delta EPU_t + \gamma_3 \Delta LIR_t + \gamma_4 \Delta RLV_t + \gamma_5 \Delta CS_t + \varepsilon_S \quad (4)$$

where $L_{d1}(RHP_t - RHP_t^{D*})$ and $L_{d2}(RHP_t - RHP_t^{S*})$ represented in (3) and (4) denote error correction processes towards the equilibrium housing prices through demand and supply effect-transmission channels, respectively. L_d denotes the difference operator with an order d while d can be any real number. Π_D and Π_S are parameter matrices that form the existing cointegrating relationships in the demand and supply functions, respectively. Furthermore, both short-run disequilibrium corrections and long-run equilibrium relationships in the macroeconomy-housing market interaction system can be explicitly investigated separately from both the channels, respectively. Although our research emphasis is the determination of housing prices as modelled by (3) and (4), we also recognize and allow for potential multi-directional interactions among target variables. Moreover, as earlier noted in Section 1, this theoretical setting enables us to quantify effects of factors that impact exclusively through demand or supply functions, for instance, DEF and CD on the demand function; and RLV and CS on the supply function.

More importantly, it also disentangles possible dual roles of specific factors, for instance, HUC , LIR , and EPU , which can affect housing prices through both the functions. It is well known that these specific variables can demonstrate two distinct effects on housing prices simultaneously by shifting demand and supply curves, respectively. Thus, simply measuring their aggregate effects in a single equation can not precisely investigate their real roles. Instead, through a micro perspective, our theoretical construct provides an effective way to separately gauge their dual impacts and further study which one demonstrates a dominant role in determining housing prices. After separately estimating both (3) and (4), we can eventually derive the overall equilibrium housing price determination function. In the next section, we will focus on discussing how to gauge the fractional integration order implied in a target series, and how to identify the cointegrating relationship(s) in both demand and supply functions in a long-memory framework.

4 Methodology

Rather than using the conventional methodology, which imposes an implausible assumption that orders of integration and cointegration should be integer numbers (Engle and Granger, 1987), we relax this assumption and employ the fractionally cointegrated vector autoregressive (FCVAR) model. Following Johansen (2008) and Johansen and Nielsen (2012), the FCVAR model is able to identify potential long-memory properties in our target series by allowing for fractional integra-

tion orders, while it can further model both disequilibrium error corrections and cointegrating relationship(s) among target variables in a long-memory context. In particular, a clear identification of the long-memory shocks by using the FCVAR model is an innovative contribution in studying interactions between housing prices and macroeconomic fundamentals. Discussions surrounding both fractional integration and the FCVAR model are presented in this section.

4.1 Fractional integration

Given any time series, we start from a conventional expression of an integrated process of order d as follows given that $t = 1, \dots, T$.

$$(1 - L)^d y_t = \psi(L)\varepsilon_t \quad (5)$$

where $(1 - L)^d$ is the difference operator of order d . For example, if $d = 1$, $(1 - L)^1 y_t = y_t - y_{t-1} = \Delta y_t$. $\psi(L^j)$ is the coefficient of the error term (ε) at each specific time period $t - j$ with $\sum_{j=0}^{\infty} |\psi(L^j)| < \infty$, $j = 0, 1, 2, \dots$, and the error term (ε_t) is a white noise process with zero mean and constant variance, viz. $\varepsilon_t \sim iid(0, \sigma^2)$. Following Hamilton (1994), instead of abiding by the conventional assumption that order d should be an integer, a fractional integrated process allows a fractional value of d . Given that the inverse value of $(1 - L)^d$ exists subject to $d < 1/2$, (5) can be transformed into the following form.¹¹

$$y_t = (1 - L)^{-d} \psi(L)\varepsilon_t \quad (6)$$

Based on the technique of power series expansion, the operator $(1 - L)^{-d}$ can be demonstrated as

$$(1 - L)^{-d} = \sum_{j=0}^{\infty} \gamma_j L^j \quad (7)$$

where $\gamma_0 \equiv 1$ and

$$\gamma_j = \frac{(d + j - 1)(d + j - 2) \cdots (d + 2)(d + 1)(d)}{j!} \quad (8)$$

where $\gamma_j \cong (j + 1)^{d-1}$ given that $d < 1$ and j is large. Thus, the fractionally integrated process (6) can be re-formulated subject to (7) as a following infinite moving average (MA(∞)) representation.¹²

$$y_t = (1 - L)^{-d} \varepsilon_t = \gamma_0 \varepsilon_t + \gamma_1 \varepsilon_{t-1} + \gamma_2 \varepsilon_{t-2} + \cdots \quad (9)$$

where impulse response coefficients of y_t , γ_j , imply a slow decay pattern of shocks to the error

¹¹ As explained by Hamilton (1994), if $d > 1/2$, y_t will no longer be stationary as the inverse of $(1 - L)^d$ approaches infinity.

¹² We remove ψ_L in (9) and the coefficient of each $L^j \varepsilon_t$ is now depicted by γ_j as defined in (8).

terms of y_t . Indeed, it can capture the potential ‘long-memory’ property of a time series (Granger and Joyeux, 1980). In contrast, impulse response coefficients of a ‘short-memory’ time series decay more quickly. For example, impulse response coefficients (ρ^i) of a covariance-stationary AR(1) process, $y_t = \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i}$, decay geometrically.

To summarize, the series y_t presented in (9) is a mean-reverting process when the superscript $d - 1$ in $\gamma_j \cong (j + 1)^{d-1}$ is less than 0, i.e., $d < 1$. This indicates that the impacts of shocks from past periods on y_t will diminish gradually over time. Moreover, it can be also checked that y_t can have a finite variance only when $d < 1/2$, implying a square-summable error term coefficients in (9). This indicates the stationarity of y_t given that $d < 1/2$. Overall, Table 1 summarizes ‘memory properties’ of a time series y_t with different integration orders (d).

Table 1: Memory properties of y_t with different d values

d Value	Memory	Stationarity	Mean	Variance	Shock Duration
$d < 0$	Long	Stationary	Mean-reversion	Finite	Long-lived
$d = 0$	Short	Stationary	Mean-reversion	Finite	Short-lived
$0 < d < 0.5$	Long	Stationary	Mean-reversion	Finite	Long-lived
$0.5 \leq d < 1$	Long	Non-stationary	Mean-reversion	Infinite	Long-lived
$d = 1$	Permanent	Non-stationary, unit root process	No Mean-Reversion	Infinite	Permanent
$d > 1$	Permanent	Non-stationary	No Mean-Reversion	Infinite	Permanent, the effects increase over time

4.2 Fractional cointegrated VAR model

It is widely recognized that a fractional integration order of a series is able to demonstrate its long-memory property, while also providing an essential foundation when examining whether a specific group of variables are fractionally cointegrated or not. To estimate the long-memory in a system, we employ the fractionally cointegrated vector autoregressive (FCVAR) model developed by Johansen (2008) and Johansen and Nielsen (2012). It enables us to capture both error corrections and equilibrium relationship(s) in a system including specific target variables in a long-memory cointegration framework.

The FCVAR model is derived from the cointegrated vector autoregressive (CVAR) model, which only allows for an integer integration order, proposed by Johansen (1995). Assuming X_t is a K -dimensional $I(1)$ time series with $t = 1, 2, \dots, T$, the CVAR model with p lags can be expressed as

$$\Delta X_t = \alpha \beta' X_{t-1} + \sum_{i=1}^p \Gamma_i \Delta X_{t-i} + \varepsilon_t = \alpha \beta' L X_t + \sum_{i=1}^p \Gamma_i \Delta L^i X_t + \varepsilon_t \quad (10)$$

Based on (10), the FCVAR can be derived by replacing the difference operator (Δ) and the lag operator ($L = 1 - \Delta$) by their fractional counterparts, which are $\Delta^b = 1 - L_b = (1 - L)^b$ and

$L_b = 1 - \Delta^b$, respectively. L_b can be also re-expressed as: $L_b = 1 - \Delta^b = 1 - (1 - L)^b$. In addition, b should be positive to ensure that the order of target time series should not be affected by applying the fractional lag operator (L_b) (Tschernig et al., 2013). Thus, the FCVAR model specification is formulated as follows:

$$\Delta^b X_t = \alpha \beta' L_b X_t + \sum_{i=1}^p \Gamma_i \Delta^b L_b^i X_t + \varepsilon_t \quad (11)$$

where the error term (ε_t) is a K -dimensional independent identically distributed time series with zero mean and variance-covariance matrix ($\varepsilon_t \sim iid(0, \Omega)$) and Ω stands for a constant variance-covariance matrix. Indeed, the FCVAR model allows multiple time series integrated with fractional order d to be cointegrated to order $d - b$. We now assume that X_t is fractionally integrated with an order b : $(1 - L)^b X_t = \varepsilon_t$. When we apply $X_t = \Delta^{d-b} Y_t$, then $(1 - L)^b X_t = (1 - L)^b \Delta^{d-b} Y_t = (1 - L)^b (1 - L)^{d-b} Y_t = (1 - L)^d Y_t = \varepsilon_t$. Thus, the FCVAR model shown in (11) can be re-formulated as:

$$\Delta^d Y_t = \alpha \beta' L_b \Delta^{d-b} Y_t + \sum_{i=1}^p \Gamma_i \Delta^d L_b^i Y_t + \varepsilon_t \quad (12)$$

Model parameters in the FCVAR have the same interpretations as those in the CVAR model. Specifically, Π is a parameter that defines the cointegration relationship(s) and it can be further identified as two sub-parameters, viz. $\Pi = \alpha \beta'$. α and β are $K \times r$ matrices given that r is the rank of Y_t and $0 \leq r \leq K$. In addition, the value of r indicates the number of cointegration(s) in the model. β identifies the cointegrating relationship(s) among variables in Y_t , and α defines the adjustment speed towards the long-run equilibrium of each variable in Y_t . Γ_i describes the short-run dynamics of target variables. Overall, (12) implies that elements of Y_t are fractionally integrated to order d , and the model system is cointegrated to order $d - b$. The FCVAR model enables us to capture the long-run equilibrium relationship, viz. $\beta' L_b \Delta^{d-b} Y_t$, the short-run adjustment processes to deviations towards the equilibrium, and the short-run dynamics among variables in the system. Moreover, the FCVAR model also allows us to evaluate the model fit (viz., if the asymptotic distribution assumption of the model parameters is achieved) by testing the residuals. In our empirical research, we concentrate on the case when $d = b$ to ensure that a linear combination of variables depicted in cointegrating relationship(s) with a constant term δ tends to be stationary. The FCVAR can be further expressed as:

$$\Delta^d Y_t = \alpha (\beta' L_d Y_t + \delta') + \sum_{i=1}^p \Gamma_i \Delta^d L_d^i Y_t + \varepsilon_t \quad (13)$$

Our defined fractional integration is based on an infinite time series as shown in (9), while it is hard to achieve in reality. Although an assumption that values of any a given time series are zero before the start of our data sample allows us to measure the fractional difference, it is nevertheless too strict to be rational in reality and will cause an estimation bias (Johansen and Nielsen, 2016).

They point out that such bias can be corrected by introducing a drift term (ρ) that shifts each time series in Y_t by a constant value. Thus, the updated FCVAR model can be expressed as:

$$\Delta^d(Y_t - \rho) = \alpha\beta' L_d(Y_t - \rho) + \sum_{i=1}^p \Gamma_i \Delta^d L_d^i(Y_t - \rho) + \varepsilon_t \quad (14)$$

where $\beta' \rho = -\delta'$ represents the constant mean value of stationary cointegrating relationships. (14) is the FCVAR model specification that we will employ in the empirical section. In terms of the estimation technique, we follow Johansen and Nielsen (2012) and estimate the FCVAR model by using the maximum likelihood (ML) estimation. They find that the ML estimators of model parameters, such as \hat{d} , $\hat{\alpha}$ and $\hat{\Gamma}_i$, follow an asymptotically normal distribution, while other model parameters, viz. $\hat{\beta}$ and $\hat{\delta}$, follow an asymptotically normal distribution when $d < 1/2$ and an asymptotically mixed normal distribution when $d > 1/2$.

Importantly, these above properties imply that the asymptotic χ^2 inference can be applied to test the significance of parameters through the likelihood ratio (LR) tests. Although the asymptotic distribution of the drift parameter, $\hat{\rho}$, is still unknown, it is not that crucial for the estimation as $\hat{\rho}$ is only used to correct for the fact that all initial values of Y_t are not observed (Jones et al., 2014). In addition, the determination of the FCVAR model specification, its model estimation and correspondingly its forecasting exercise are executed using a Matlab program proposed by Nielsen and Popiel (2018), viz. FCVAR version 1.4.0a. It is also worth noting that identification problems of the FCVAR system raised in Johansen and Nielsen (2010) and Carlini and de Magistris (2017) have been considered and alleviated in the program.

Similar to the hypothesis testing in the CVAR model, the FCVAR model can also conduct a series of hypothesis tests on model parameters (Jones et al., 2014). In particular, the theoretical framework of hypothesis tests on β and α can be formulated below respectively.

$$\beta = \omega \lambda \quad (15)$$

$$\alpha = \tau \theta \quad (16)$$

In terms of the hypothesis test on β as shown in (15), ω is a $K \times q$ matrix identifying imposed restriction(s) on cointegrating relationship(s), and λ is a $q \times r$ matrix defining free varying parameter(s). K is the number of variables within the FCVAR system; q is the number of restriction(s) associated with β -related hypothesis tests; and r denotes the number of rank(s) of Y_t . In the case that each cointegrating relationship is imposed with the same restriction, the degree of freedom of the hypothesis test is equal to $(K - q)r$. If the number of cointegrating relationships is greater than one, viz. $r > 1$, different restrictions could be imposed on different columns of β . β can be then re-expressed as a row vector, i.e., $\beta = (\omega_1 \lambda_1, \omega_2 \lambda_2, \dots, \omega_r \lambda_r)$. Each column of β is the product between ω_i and λ_i , where ω_i is a $K \times q_i$ matrix and defines the imposed restriction on the column i of β ; and λ_i is a $q_i \times 1$ matrix and defines the free varying parameter on the column i of β . In that

case, the degree of freedom of the hypothesis test is $\sum_{i=1}^r (K - r - q_i + 1)$.

In terms of the hypothesis test on α as shown in (16), τ is a $K \times l$ matrix that defines restriction(s) on error corrections towards equilibrium of target variables, while θ is a $l \times r$ matrix representing free varying parameter(s) with $l \geq r$ where l stands for the number of restriction(s) associated with α -related hypothesis tests. Its degree of freedom is equal to $(K - l)r$.

Identification and endogeneity

Given that our employed FCVAR model is constructed based on a vector autoregressive structure, where the endogeneity issue becomes negligible as all target variables considered in the model are assumed to be endogenous. Moreover, in the FCVAR model, whether the included variables form the long-run equilibrium relationship and correct for short-run disequilibrium errors or not can be separately tested. In specific, endogenous impacts of the variables in the system can be well-identified by testing for zero restrictions on feedback coefficients in the α -matrix. If α coefficient of a given variable is restricted to be zero, the variable can be defined as weakly-exogenous, indicating that it contributes to no adjustment to restore the long-run equilibrium after disequilibrium has occurred in the system. Conversely, endogenous impacts of the variables are tested to be significant for error corrections.

Moreover, whether the variables in the system contribute to building the long-run cointegrating relationship(s) or not can be also well-identified by testing for zero restrictions on feedback coefficients in the β -matrix. If β coefficient of a given variable is restricted to be zero, it indicates that the variable would not enter the cointegrating relation(s). To summarize, thanks to the FCVAR model specification, the endogeneity issue is well ameliorated, while impacts of the included variables in the system both for the short-run disequilibrium error corrections and the long-run equilibrium constructions can be all clearly identified. Finally, super-exogeneity refers to the irrelevance of the Lucas critique of parameters depending on the policy regime.

4.3 Test of fractional cointegration

We now discuss how numbers of fractional cointegration ranks are tested through the likelihood ratio (LR) test statistic, which enables us to build the trace test of the null hypothesis (H_0): $\text{rank}(\Pi) = r$ against the alternative hypothesis (H_1): $\text{rank}(\Pi) = K$. Based on the FCVAR model specified in (14), let $\theta = (d, b, \rho)$ denote the model parameters set that numerically maximizes the likelihood of making the given observations. Let $L(d, b, r)$ be the profile likelihood function given a specified rank r , where other model parameters, viz. α, β and Γ , have been concentrated out by regression and reduced rank regression (Johansen and Nielsen, 2012). The LR test statistic can be calculated when the profile likelihood function is maximized under both hypotheses H_0 and H_1 as

$$LR_T(\tau) = 2 \log \left(\frac{L(\hat{\theta}_K, K)}{L(\hat{\theta}_r, r)} \right) \quad (17)$$

where $\tau = K - r$, $L(\hat{\theta}_K, K) = \max(L(\theta_K, K))$ and $L(\hat{\theta}_r, r) = \max(L(\theta_r, r))$. The asymptotic distribution of $LR_T(\tau)$ depends upon the parameter b . Thus, the cointegration is defined as ‘weak’ when $0 < b < 1/2$ and $LR_T(\tau)$ follows a standard asymptotic distribution (Johansen and Nielsen, 2012).

$$LR_T(\tau) \rightarrow \chi^2(\tau^2) \quad (18)$$

Moreover, when $1/2 < b \leq d$, the cointegration is defined as ‘strong’, while the asymptotic distribution is not standard (Nielsen and Popiel, 2018). It is then formulated as

$$LR_T(\tau) \rightarrow Tr \left\{ \int_0^1 dW(s)F(s)' \left(\int_0^1 F(s)F(s)' ds \right)^{-1} \int_0^1 F(s)dw(s)' \right\} \quad (19)$$

Following Nielsen and Popiel (2018), the vector process dW denotes the increment of ordinary vector standard Brownian motion of dimension $\tau = K - r$, and the vector process F relies on the deterministic, which is similar to the mechanism in the CVAR model discussed in Johansen (1995). Building the above LR cointegration rank test when $d = b$ involves calculations of both asymptotic critical values and corresponding P values. In our case, we measure both of them by employing computer programs provided by MacKinnon and Nielsen (2014) in the empirical section.

4.4 Forecasting from the FCVAR model

Following Nielsen and Shibaev (2018), we now focus on how to forecast our target series (Y_t) and obtained cointegrating relationships from the FCVAR model by using the best (minimum mean-squared error) linear predictor. First, regarding a one-step-ahead forecast of Y_{t+1} , we note that

$$\Delta^d(Y_{t+1} - \rho) = Y_{t+1} - \rho - (Y_{t+1} - \rho) + \Delta^d(Y_{t+1} - \rho) = Y_{t+1} - \rho - L_d(Y_{t+1} - \rho) \quad (20)$$

Based on (20), since $L_d = 1 - \Delta^d$ and $d = b$ as earlier defined, the FCVAR model, (14), is then re-formulated as:

$$Y_{t+1} = \rho + L_d(Y_{t+1} - \rho) + \alpha\beta' L_d(Y_{t+1} - \rho) + \sum_{i=1}^p \Gamma_i \Delta^d L_d^i(Y_{t+1} - \rho) + \varepsilon_{t+1} \quad (21)$$

This is the foundation of the FCVAR forecasting. Each item on the right hand side (RHS) of (21) is known at time t for $d > 0$ and $i \geq 1$. A conditional expectation of any variable Y_{t+1} given an available information set at time t can be defined as: $\hat{Y}_{t+1|t} = E_t(Y_{t+1})$. Similarly, a conditional expectation of the residuals ε_{t+1} given available information at time t is $\hat{\varepsilon}_{t+1|t} = E_t(\varepsilon_{t+1})$. Hence, by substituting the estimated values of FCVAR model coefficients, viz. $\hat{d}, \hat{\rho}, \hat{\alpha}, \hat{\beta}, \hat{\Gamma}_i$, which have

been obtained through the ML method, we re-express (21) as:

$$\hat{Y}_{t+1|t} = \hat{\rho} + L_{\hat{d}}(Y_{t+1} - \hat{\rho}) + \hat{\alpha}\hat{\beta}'L_{\hat{d}}(Y_{t+1} - \hat{\rho}) + \sum_{i=1}^p \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (Y_{t+1} - \hat{\rho}) \quad (22)$$

where (22) defines the model specification of a one-step-ahead forecasting of Y_{t+1} given available information at time t . Then, a multi-period forecasting can be simply derived based on that. We can similarly define a conditional expectation of Y_{t+j} given an available information set at time t as $\hat{Y}_{t+j|t} = E_t(Y_{t+j})$. Thus, a j -step ahead FCVAR forecasting can be formulated as:

$$\hat{Y}_{t+j|t} = \hat{\rho} + L_{\hat{d}}(\hat{Y}_{t+j|t} - \hat{\rho}) + \hat{\alpha}\hat{\beta}'L_{\hat{d}}(\hat{Y}_{t+j|t} - \hat{\rho}) + \sum_{i=1}^p \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (\hat{Y}_{t+j|t} - \hat{\rho}) \quad (23)$$

where $\hat{Y}_{z|t} = Y_z$ if $z \leq t$. Recursively, the j -step-ahead forecasting, $\hat{Y}_{t+j|t}$, are calculated from (23) for any a given $j \geq 1$. As examined by Nielsen and Shibaev (2018), the FCVAR model performs a minimised root-mean-squared forecast error, while its forecasting performance is superior to both the univariate fractional model and the cointegrated VAR model. We will apply its j -step-ahead forecasting in both our demand- and supply-driven determination functions.

Model forecasting performance evaluations

To evaluate the model forecasting performance and measure the improvement of forecasting accuracy of the FCVAR model over other model specifications, for instance, the CVAR model (a special case of the FCVAR model when $d = b = 1$), we follow Nielsen and Shibaev (2018) and examine the target model's forecasting performance by calculating its root mean squared forecasting errors (RMSFE). The specification of the RMSFE calculation is shown in (24).

$$RMSFE = \left\{ \frac{1}{K} \sum_{i=1}^K (\hat{Y}_{i,t+h|t} - Y_{i,t+h|t})^2 \right\}^{1/2} \quad (24)$$

where $\hat{Y}_{i,t+h|t}$ denotes forecasting values of variable Y_i over the time period $t + 1$ to $t + h$ given that an information set of Y_i at time t is available. i indicates specific included variables in our system, and $i=1, \dots, K$. h denotes specified forecasting horizons. Overall, the above formula can measure the RMSFE of the target multivariate model system, which is calculated as the averaged value of the RMSFE for each incorporated series in the system. It can report magnitudes of forecasting errors produced by the whole model system.

5 Data

Our empirical study uses a quarterly dataset for the US spanning more than four decades (1975-2016). Overall, a summarized data description of our target variables involving housing prices and macroeconomic fundamentals is reported in Table 2 below. In this section, we introduce

each of them in detail. Sequentially starting from the variable for bank credit, we discuss issues surrounding what each variable’s definition is and how we choose a rational proxy to represent each of them in the empirical analysis.

Table 2: Data Description

Variable Name and Abbreviation	Detailed Series	Time Period	Data Source
Credit to the Housing Demand (<i>CD</i>)	Mortgage debt outstanding for the residence purchase	1951Q3-2017Q2	Board of Governors of the Federal Reserve System (US)
Credit to the Housing Supply (<i>CS</i>)	Private residential fixed investment	1946Q4-2017Q2	US Bureau of Economic Analysis
Residential Land Value (<i>RLV</i>)	Aggregate market value of residential land	1975Q1-2016Q1	Lincoln Institute of Land Policy
Long-term Interest Rate (<i>LIR</i>)	10-year treasury constant maturity rate	1954Q2-2017Q3	Board of Governors of the Federal Reserve System
Inflation (<i>DEF</i>)	GDP deflator	1946Q4Q1-2017Q3	US Bureau of Economic Analysis
Residential Housing Stocks (<i>HUC</i>)	New privately-owned housing units completed	1967Q4-2017Q4	US Bureau of Census & US Department of Housing and Urban Development
Economic Policy Uncertainty (<i>EPU</i>)	US historical news-based policy index	1940Q1-2017Q4	Baker et al. (2016)
Residential Housing Prices (<i>RHP</i>)	S&P/Case-Shiller US National Home Price Index	1975Q1-2017Q3	S&P Dow Jones Indices LLC

Bank credit is defined as the net lending that is claimed by money issuers, while it also denotes the outstanding amounts that money borrowers are liable to repay. Money-issuing sectors are formed by monetary financial institutions (MFIs), while bank credit can be obtained from the asset side of MFIs’ consolidated balance sheets (Docker and Willoughby, 1999). MFIs are financial institutions whose businesses are to receive deposits and grant credit on their own account to entities other than MFIs (non-MFIs), such as households, non-profit institutions serving households, private non-financial corporations and other financial corporations (OFCs).¹³ On the basis of definitions from the European Central Bank (ECB), the Bank of England (BOE), and the International Monetary Fund (IMF), under the conceptual framework of the 2018 SNA (United Nations, 2008), MFIs stand for depository corporations including central bank and other deposit-taking corporations, such as commercial banks, credit unions, saving institutions and money market mutual funds, at the broadest level. In light of the definition of credit, we segregate bank credit to separately gauge how much credit is provided to the demand side (*CD*) and the supply side (*CS*), respectively, in the residential real estate market.

In our paper, the amount of *CD* is represented by outstanding mortgage debts for the home purchase (e.g. one- to four- family, and multifamily residences). It explicitly measures the amount of money/credit used to finance the housing demand in a given economy, which also indicates the households’ purchase power regarding the housing demand. It is collected from the Board of

¹³For simplicity, non-MFIs refer to private sectors in our paper. Moreover, according to United Nations (2008), OFCs are financial corporations other than depository corporations. They include non-MMF (Money market funds) investment funds, other financial intermediaries except insurance corporations and pension funds, financial auxiliaries, captive financial institutions and money lenders, insurance corporations, and pension funds.

Governors of the Federal Reserve System (US) covering the period 1951Q3-2017Q2. In terms of credit to the housing supply side (*CS*), there is a lack of data that explicitly represent the credit lending to each industry, including the real estate industry. Alternatively, we use the private residential fixed investment (*PRFI*) as a proxy. Instead of focusing on how many loans are issued by MFIs to the residential real estate market, as defined in the US National Income and Product Accounts (NIPA) handbook, *PRFI* describes the money spending by private sectors (e.g. private firms, households, and non-profit institutions serving households) for the construction and development of residential properties, such as an improvement of existing houses, a creation of new houses, and a replacement of worn out or obsolete houses, in the form of fixed investments. Thus, *PRFI* enables us to gauge the amount of money provided by private sectors for the provision of housing supply (*CS*). It is available from 1946Q4 to 2017Q2, and is provided by the US Bureau of Economic Analysis.

In addition to disaggregate bank credit, this paper further includes a series of variables, viz. residential land market value (*RLV*), long-term interest rate (*LIR*), residential housing stock (*HUC*), inflation (*DEF*), and economic policy uncertainty (*EPU*), combine to form the macroeconomic fundamentals of the housing demand and supply sides. In terms of *RLV*, following Davis and Heathcote (2007), we approximate it using the aggregate market value of residential land, which is measured based on the S&P/Case-Shiller U.S. National Home Price Index. *RLV* is able to describe the housing supply/construction expenditure through a land perspective, and data are provided by the Lincoln Institute of Land Policy (1975Q1 to 2016Q1).¹⁴ In terms of long-run interest rate (*LIR*), we approximate it using a 10-year treasury constant maturity rate. *LIR* measures the cost level of both housing buyers and developers in financing the house purchase and construction, respectively. The higher the *LIR*, the greater the borrowing costs are on both housing demand and supply sides. Then, housing prices will subsequently increase/decrease depending upon whether dominating shocks are from supply/demand sides. Its data are collected from the Board of Governors of the Federal Reserve System (US) and range from 1953Q1 to 2017Q4.

In terms of inflation (*DEF*), we employ the GDP deflator to describe the US price level of all domestic-produced final goods and services in a given time period. It explicitly demonstrates the inflationary and deflationary periods in the entire US economy, while its changes are directly linked to the dynamics of asset prices, such as housing prices. The data are collected from the US Bureau of Economic Analysis, and are from 1946Q4 to 2017Q3. In terms of residential housing stock (*HUC*), we use a series named the completion of new privately-owned housing units to approximate. This measures total amounts of completed residential properties that are currently available in the US real estate market. Available housing stocks (*HUC*) represent amounts of housing units that are required by the housing buyers, while they also demonstrate the amounts that are able to be provided by the housing suppliers. Thus, *HUC* can reflect and affect both housing demand and supply dynamics, respectively, and then impact housing prices. Its data are available from the US Bureau of Census and the US Department of Housing and Urban Development, and

¹⁴Data are available through <http://datatoolkits.lincolninstitute.edu/subcenters/land-values/>

range from 1967Q4 to 2017Q4.

In terms of economic policy uncertainty (*EPU*), it is an important indicator in depicting a level of uncertainty in an economy. As explained in Baker et al. (2016), this index is constructed to measure the uncertainty through three aspects, viz. newspaper coverage of policy and economic related uncertainty, the number of federal tax code provisions set to expire in forthcoming years, and the disagreement among economic forecasters. It is well known that the persistence of uncertainty can impact housing prices through both channels of housing demand and supply. To ensure a long time-series, we use the US historical news-based policy index as a proxy for uncertainty, and it ranges from 1900Q1 to 2017Q4.¹⁵ Moreover, one key variable in the paper is residential housing prices (*RHP*). In order to be consistent with the housing price series used in the calculation of *RLV*, we employ the S&P/Case-Shiller US National Home Price Index to approximate the US national residential real estate prices. Data are from S&P Dow Jones Indices LLC and range from 1975Q1 to 2017Q3. In addition, except for *RLV* and *EPU*, all aforementioned time series are retrieved from the Federal Reserve of St. Louis (FRED), US.

In addition to the above introduced variables, there exist some other variables, which also could potentially determine housing demand and supply functions, such as GDP, aggregate money, construction cost, and credit to other financial corporations (OFCs). However, we do not include them in our empirical research because of the following concerns, viz. a multicollinearity problem, a usage of a better proxy, and a limitation of available data. Specifically, for example, although GDP reflects the overall income level of domestic housing buyers, it is highly correlated with GDP deflator (*DEF*), which is a proxy for the inflation level of the entire economy and has already been included. Moreover, the purchase power of the households in buying residences can be better depicted by outstanding mortgage debts (*CD*); similarly, a better approximation of credit to the supply side is private residential fixed investment (*CS*) against credit to OFCs. Therefore, we decide to use disaggregate credit, which can disentangle impacts of credit on housing prices through demand and supply channels, respectively, rather than aggregate credit. In addition, although we intend to include total residential construction costs, its data are nevertheless only available from 1993Q1, which is too short a period to form our empirical dataset. Instead, we consider alternatives to measure the financing expenditure of the housing supply such as long-term interest rate (*LIR*) and residential land value (*RLV*).

6 Data transformation and preliminary observations

6.1 Seasonal adjustment

The presence of seasonal components, if left untreated, may obscure the true nature of persistence in a time series. A report in (UK Office for National Statistics, 2007) argues that short-term disturbances in the form of seasonal effects can induce certain amount of volatility in the dynamics of macroeconomic variables in the long-run. Thus, prior to our FCVAR estimation, we transform our

¹⁵Data are provided by Baker et al. (2016) and are available through www.policyuncertainty.com/index.html

raw time series by first removing potential seasonal effects. Following convention, we employ the popular X-13ARIMA-SEATS statistical package developed by the US Census Bureau to seasonally adjust all target variables.

Figure 1 presents patterns of seasonal components in variables such as *CD*, *RHP*, *RLV*, and *LIR*. It also explicitly describes short-run periodic fluctuations in those variables with a serrated shape. Indeed, removal of seasonal effects unfolds the true dynamics of each variable in the long-run. Other variables, such as *CS*, *HUC* and *DEF* have already been seasonally adjusted and hence we leave them untransformed.¹⁶

6.2 Business cycles removal

As Lucas (1981) notes, repeated fluctuations also exist in the mid/long-run movements of many aggregate economic variables, and are usually longer than a year. Such repeated dynamics within variables' growth patterns are defined as *business cycles*¹⁷ (Hodrick and Prescott, 1997). Thus, to free our variables from an inherent cyclical movement, which may further put a layer over identification of a true memory, we remove business cycles from each original series. Therefore, in an attempt to identify true nature of memory in our time series, we remove periodic disturbances in the short-run (seasonal effects) and the mid-/long-run (business cycles). For the latter, we employ the recently developed Hamilton filter (Hamilton, 2017). In contrast to the traditional Hodrick-Prescott (H-P) filter Hodrick and Prescott (1997), the Hamilton filter solves one of the most fundamental problems with regard to replication of a true Data Generation Process (DGP). Hamilton argues that due to the extra smoothness (the source of which depends on an a-theoretic value of the smoothing parameter), the degree of integration order arrived at by H-P filter is always higher.

The H-P filter decomposes a given series y_t into a trend (g_t) and a cyclical (c_t) component.

$$y_t = g_t + c_t \quad (25)$$

where $t = 1, \dots, T$. The H-P filter aims to minimize then the following objective function

$$\min_{g_t} \left\{ \sum_{t=1}^T (y_t - g_t)^2 + \lambda \sum_{t=1}^T \left[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2}) \right]^2 \right\} \quad (26)$$

where $\sum_{t=1}^T (y_t - g_t)^2$ denotes the sum of squares of c_t . $\sum_{t=1}^T \left[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2}) \right]^2$ denotes the sum of squares of the second difference of g_t , which is used to model the smoothness of the variation of g_t . The larger the value of parameter λ , the smoother the growth of g_t . In particular, the value of parameter λ is suggested to be 1600 given a quarterly dataset in our empirical research.

¹⁶Comparative plots between non-seasonally adjusted (SA) and SA series of each variable can further depict the role of the seasonal adjustment. They are available from the authors upon request.

¹⁷Specifically, business cycles describe periodic behaviours of a given variable that first start to increase/decrease from a reference time point until a peak point/trough point, and then decreases/increases until the end of a downturn/upturn.

In spite of a popularity of the H-P filter, its accuracy and rationale are still in dispute. The main criticism lies in its assumption of a white noise cyclical component, which is too strict and far from reality. More so, its decomposed cyclical term tends to demonstrate a strong autocorrelation property, which is a feature imposed by the H-P method instead of that possessed by the true data generating process (DGP).

The Hamilton filter (Hamilton, 2017) is able to avoid the weaknesses of the H-P filter. Given a non-stationary series (y_t), according to Hamilton, its cyclical component at time $t + h$, viz. c_{t+h} , can be defined as the difference between the real value of y at time $t + h$ and its expected value at time t conditional on the available information set prior to time t . c_{t+h} is formulated in (27). Thus, this method enables us to capture business cycles which encompass the shocks whose effects last over time h , while they are still transient and tend to disappear in the long-run.

$$c_{t+h} = y_{t+h} - g_t \quad (27)$$

where $h = 1, \dots, T - t$. $g_t = E(y_{t+h} \mid y_t, y_{t-1}, \dots, y_{t-p+1})$, and is defined as the trend component of y_{t+h} . In light of (27), y_{t+h} can be expressed as

$$y_{t+h} = \beta_0 + \sum_{i=0}^p \beta_{j+1} y_{t-i} + u_{t+h} \quad (28)$$

where the residuals in (28) represent the cyclical component, viz. u_{t+h} ; and the difference between y_{t+h} and u_{t+h} can be defined as the trend component. As suggested by Hamilton (2017), numbers of lags (p) are selected to be four to both ensure the stationarity of u_{t+h} and maximize available observations of specific series; a two-year standard setting is employed to capture business cycles indicating that h equals to 2, 8 and 24, for annual, quarterly, and monthly data, respectively. Thus, the cyclical term describes shocks that last within two years, while are still 'transient'. Overall, in contrast to the H-P filter, which imposes a strict assumption to obtain a smooth-varying trend term, the Hamilton's filter ensures the stationarity of the cyclical component for any given non-stationary series and better duplicates the real data generating process (DGP). Therefore we apply Hamilton's filter for business cycle removal in our empirical research.

Empirical identification of business cycles

In Figures 2 to 7, we present cyclical and trend components of each variable using both filtering methods. In each figure, movements of business cycles are shown in the left-hand side of panels, while both trend components and non-decomposed variables are depicted in the right-hand side of panels. Each variable is demeaned to eliminate common characteristics and make observations from each cumulative series comparable over time. Specifically, variables in levels are first used in Figures 2 to 4, where both cyclical and trend components of each target series decomposed by H-P and Hamilton's filters can be observed, although movements of each component particularly at an early period appear to be similar between both filters. Thus, to better illustrate and compare the

results of business cycle removal by using these two filters, we further transform each target series in a logarithmic format and multiply transformed variables by 100. Plots using corresponding transformed variables are depicted in Figures 5 to 7. Moreover, this transformation is also able to clearly demonstrate the growth of each variable, viz. a unit increase of a log-transformed variable is equivalent to a 1% unit increase of the variable in levels. In addition, regarding Hamilton's filter, a suggested two-year setting is applied to identify business cycles.¹⁸

Indeed, we observe that H-P filtered trend series demonstrate a more smoothly moving pattern compared with the ones filtered by the Hamilton's method. On the other hand, movements of business cycles obtained by the Hamilton's filter tend to be more volatile, while it appears to be a more sophisticated and precise way to replicate true DGP than the H-P filter, which artificially imposes a 'guarantee' of a smooth-varying growth path (Hamilton, 2017). In addition, it can also explain why de-cycled series filtered by the H-P method would express exceptional integration orders, which are hard to explain both empirically and theoretically. Overall, all of these motivate us to employ Hamilton's filter to remove business cycles from our target variables preceding with the further fractional cointegration VAR estimation.

To further observe notable disturbances of business cycles and to illustrate the excellent performance of Hamilton's filter, we carry out a comparison of autocorrelation function (ACF) plots between decomposed (de-cycled) and non-decomposed (non-de-cycled) series. Specifically, Figures 8 and 9 report comparative ACF plots of selected variables including logged credit to the housing demand, credit to the housing supply, land value and logged land value. All these variables are differenced once to remove potential non-stationary elements, following a conventional unit root assumption, in order to better observe effects of business cycles.¹⁹ De-cycled and non-de-cycled series are presented in the right- and left-hand side panels, respectively. Interestingly, instead of behaving as a general stationary series, ACF movements of non-de-cycled variables behave like a sine or cosine function. They first witness a gradual decay until zero and keep sinking negatively until a trough; then they turn to move back towards zero once again. Such periodic dynamics occur repeatedly although the amplitudes reduce gradually over time and are expected to eventually diminish towards zero in the long run. In contrast, the ACF movements of de-cycled series move like a stationary process without the aforementioned periodic fluctuations. Thus, the above comparisons further confirm that the Hamilton's filter can better minimise disturbances induced by business cycles in contrast to the H-P filter.

In light of existing studies related to the housing-macroeconomic cycles, durations of boom-bust cycles of housing prices and macroeconomic variables are normally longer than a two-year standard setting. Specifically, the duration of debt cycles is suggested to be five years (Hamilton, 2017). Cesa-Bianchi (2013) chooses five-year as the length of rolling window for the calculation

¹⁸We also use different time durations to capture business cycles, such as 5-, 8-, and 10-year settings. Qualitatively similar dynamics of both components of each variable are obtained. These plots are available from the authors upon request.

¹⁹We only plot the ACF for the variables that show the strongest periodic moving patterns. Corresponding comparisons for other variables are available from the authors upon request.

of cross-country average correlations of real housing prices and real GDP in different sub-country groups (e.g. advanced economies and emerging market economies.). Moreover, Igan and Loun-gani (2012) measure the length of housing cycles as well as durations of downturn and upturn of housing prices for different countries. They find that durations of housing cycles for the US are different in different time periods, e.g. 5.25 years (Peak: 1973Q4; Trough: 1975Q3), 10.75 years (Peak: 1979Q1; Trough: 1982Q4), and 17 years (Peak: 1989Q4; Trough: 1995Q3). In addition, given empirical observations in European cross-country real estate markets, both Duan et al. (2018a) and Duan et al. (2018b) suggest that cycles of housing prices and macroeconomic fundamentals on housing demand and supply sides tend to last for 10 years.

Overall, according to relevant literature and data availability in our case, we generally select 10-years as the cycle duration for variables with a relatively long time series. On the other hand, credit cycles for CD have been identified as five years following Hamilton (2017). Cycles of other housing-related variables, e.g. RHP , HUC , and RLV , are set to have a five-year duration due to their constrained data. Once periodic disturbances from both seasonal and cyclical components have been removed from raw data series, the real long-run dynamics of target variables can then be unfolded to guarantee a more precise empirical analysis. In addition, regarding the following empirical study, whether a variable is added in levels or a logarithmic format depends upon which one can better demonstrate the variable's long-memory property. Our dataset has been trimmed to be strongly balanced and runs from 1980Q1 to 2016Q1 for the FCVAR estimation. Next, we start the empirical section by first testing the existence of a fractional integration in each target series.

7 Results and discussions

7.1 Long-memory in individual series

Before proceeding with the FCVAR estimation, a series of tests needs to be conducted to demonstrate whether there exists a long-memory property in our target series. To do that, we apply three univariate analyses discussed as follows.

(1) Visual evidence of a long memory

As implied by (8), a fractionally integrated series behaves with a long-memory property indicating that its impulse response coefficients decay hyperbolically over time in contrast to a geometric decay of a short-memory stationary series for example, a stationary AR(1) process. Following Jones et al. (2014), the long memory can be checked by plotting the autocorrelation function (ACF) and the spectral density. If a specific series has a long-memory, its autocorrelation values should decay hyperbolically until zero in the long-run, in contrast to a geometric decay. In addition, evidence of a long-memory can be also captured by examining the zero frequency of its spectral density figure, where a fractionally integrated process will have mass densities near the zero frequency which are proportional to f^{-2d} . Parameter f stands for the frequency value.

Figures 10 to 13 depict movements of both ACF and spectrum of each series. In terms of ACF drawn until 100 lags, its values of each variable decay slowly towards zero by taking a long time period. Such ACF moving patterns with long-memory implications are consistent with existing applications (See for instance, Jones et al., 2014; Kumar and Okimoto, 2007; Tkacz, 2001, among others). In particular, although the ACF value of residential housing stocks depicted in Figure 11 dwindles towards zero for a long time period, it appears to fluctuate periodically while such patterns are less significant against its analogies with similar moving patterns presented in the left panels of Figures 8 and 9. In addition, this comparison speaks in favour of the strong performance of the Hamilton’s filter discussed in Section 6.2 regarding the removal of cyclical disturbances. Interestingly, the positive ACF value of residential land value shown in Figure 13 first witnesses a gradual decrease until zero for 40 lags, then it keeps decreasing beyond the zero line and moves negatively throughout rest of the periods. Moreover, as demonstrated from the spectrum figures, there exist mass values at around the zero frequency of each target variable, which provides further visual evidence in favour of a long-memory property in our dataset. In addition, we continue testing in the following subsections whether our target series are fractionally integrated or not.

(2) Stationarity and unit root tests

In theory, a fractionally integrated time series should reject the null hypotheses of both stationary test and unit root test. That is to say, if a given series is non-stationary while it does not have a unit root, it can be defined as a fractionally integrated series with a long-memory property. Thus, we carry out the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test and the Augmented Dickey-Fuller (ADF) test to examine the stationarity and the unit root of each series, respectively. Corresponding results are reported in Table 3.

All series reject the null hypothesis of the KPSS test, implying the existence of a non-stationary feature at the 5% significance level except for the residential land value (*RLV*) with a 10% significance level. In terms of the ADF test, except for credit to the housing demand side (*LCD*) and inflation (*LDEF*), all the other variables significantly reject the null hypothesis, implying no unit root. Thus, variables that reject both null hypotheses indicate a fractional integration, while both *LCD* and *LDEF* appear to have a unit root. In the next step, we estimate the integration order (*d*) of each series through different estimate techniques.

Table 3: The Stationarity and Unit Root Tests

	LCD	LCS	RHP	LHUC	LIR	LDEF	RLV	EPU
KPSS Test	0.201**	0.154**	0.255***	0.165**	0.694***	0.222***	0.129*	1.260***
ADF Test	-1.313	-3.785**	-4.032***	-3.959**	-3.247*	1.798	-3.242*	-3.722**

Note: (i) * significant at the 10% level; ** significant at the 5% level; *** significant at the 1% level; (ii) the logarithmic variables begin with a prefix ‘L’; (iii) numbers of lags for both tests are selected based on the information criteria (IC).

7.2 Estimates of memory pattern (d): Static and dynamic procedures

(1) Static estimates

We further proceed with univariate d estimates for each series in our macroeconomy-housing market system using a variety of estimates, viz. 'LW', '2ELW', and '2ELWdm'. Specifically, 'LW' denotes the local Whittle estimator (Kuensch, 1987; Robinson, 1995); '2ELW' denotes the two-step exact local Whittle estimator (Shimotsu, 2010; Shimotsu et al., 2005); and '2ELWdm' denotes the two-step exact local Whittle estimator with demeaning and detrending procedures. Univariate d estimates are executed using both static and dynamic types, respectively.

Table 4 reports results of the static estimates with different bandwidth values (B) from 0.4 to 0.8 with a 0.05 increment. Overall, d estimates with different bandwidths overwhelmingly support fractional integration in all series except for *LCD* and *LDEF*; their d values range between 0.5 and 1, viz. $0.5 < d < 1$. Interestingly, *LCD* displays $1 < d < 2$, except when the bandwidth (B) is very small or large. It helps explain the unit root conclusion of *LCD* as earlier suggested by the result of its ADF test. Indeed, a unit root assumption in the ADF test tends to be irrational due to its assumption of an integer value rather than a fractional value for the integration order. Overall, in light of all of these, *LCD* is assumed to be fractionally integrated with $1 < d < 2$, while it should contain a unit root.

Moreover, *LDEF* is also found to be fractionally integrated ($0.5 < d < 1$) with most moderate bandwidths (B), although its d values are approaching to 1. Its d values tend to equal to or greater than 1 and less than 2 when B is very small or large. Thus, this also helps explain the unit root conclusion in *LDEF* by the ADF test. In addition, it is also worth noting that our d estimates of *LIR* are consistent with the empirical findings in Tkacz (2001) and Jones et al. (2014). Specifically, as shown in Table 4, except for a unit root suggestion when $B = 0.40$, all its d values range from 0.5 to 1. In the case of both USA and Canada, Jones et al. (2014) find that d estimates of *LIR* are close to 1 when B is small, viz. $B = 0.4$, while its values witness a gradual decrease with an increase of B . In particular, the estimated d of *LIR* with $B = 0.6$ that Jones et al. (2014) calculate using the Geweke and Porter-Hudak (GPH) estimator is 0.886, and its value is roughly equal to our d estimates with the same B by using ELW, 2ELW, and 2ELWdm, which are 0.825, 0.833, and 0.837, respectively. Overall, the static d estimates confirm the existence of a fractional integration and indicate a long-memory property in our target series.

(2) Dynamic estimates

In addition to the static d estimates, we further proceed with the dynamic univariate d estimates in a 10-year rolling-window setting.²⁰ We start estimating d by using the first 10-year data of each variable, while data are then updated with a four-quarter (equivalent to one-year) increment, and d is accordingly re-estimated using an updated window until approaching the end of data sam-

²⁰We also calculate the dynamic estimates with different rolling windows, e.g. 5- and 15-year, and qualitatively consistent results are obtained. These results are available from the authors upon request.

Table 4: The Univariate 'd' Estimates

Bandwidth Variable	$B = 0.40$				$B = 0.45$				$B = 0.50$			
	LW	2ELW	2ELWdm	SD	LW	2ELW	2ELWdm	SD	LW	2ELW	2ELWdm	SD
LCD	0.444	0.900	1.466	0.167	0.723	1.446	1.434	0.146	1.146	1.546	1.518	0.127
LCS	0.490	0.945	0.537	0.167	0.510	0.890	0.511	0.146	0.600	0.890	0.575	0.127
RHP	0.751	0.845	0.749	0.148	0.669	0.803	0.653	0.127	0.588	0.729	0.578	0.109
LHUC	-0.037	0.200	-0.011	0.177	0.054	0.242	0.062	0.156	0.356	0.458	0.346	0.137
LIR	1.048	1.043	1.047	0.137	0.759	0.759	0.771	0.116	0.783	0.785	0.794	0.099
LDEF	1.108	1.186	1.154	0.167	0.994	0.995	0.871	1.146	0.929	0.977	0.846	0.127
RLV	0.760	1.293	1.320	0.186	1.237	1.336	1.362	0.164	1.190	1.328	1.349	0.145
EPU	0.668	0.841	0.809	0.131	0.787	0.858	0.838	0.111	0.713	0.763	0.734	0.094
Bandwidth Variable	$B = 0.55$				$B = 0.60$				$B = 0.65$			
	LW	2ELW	2ELWdm	SD	LW	2ELW	2ELWdm	SD	LW	2ELW	2ELWdm	SD
LCD	1.183	1.576	1.498	0.111	1.183	1.584	1.510	0.096	0.988	1.244	1.246	0.084
LCS	0.830	0.945	0.714	0.111	1.117	1.079	1.043	0.097	1.081	1.222	1.223	0.084
RHP	0.471	0.671	0.471	0.094	0.575	0.703	0.575	0.080	0.768	0.813	0.772	0.069
LHUC	0.444	0.544	0.464	0.120	0.590	0.647	0.624	0.106	0.532	0.592	0.568	0.093
LIR	0.821	0.823	0.829	0.084	0.825	0.833	0.837	0.071	0.750	0.758	0.762	0.061
LDEF	0.722	0.929	0.784	0.111	0.642	0.912	0.797	0.096	0.744	0.950	0.877	0.084
RLV	0.837	1.061	1.078	0.128	0.722	0.933	0.939	0.114	0.718	0.838	0.829	0.100
EPU	0.661	0.684	0.606	0.079	0.677	0.671	0.627	0.067	0.763	0.748	0.733	0.057
Bandwidth Variable	$B = 0.70$				$B = 0.75$				$B = 0.80$			
	LW	2ELW	2ELWdm	SD	LW	2ELW	2ELWdm	SD	LW	2ELW	2ELWdm	SD
LCD	1.016	1.189	1.205	0.073	1.020	1.175	1.182	0.064	0.886	1.015	1.050	0.057
LCS	0.921	1.019	0.989	0.073	0.823	0.937	0.874	0.064	0.684	0.870	0.780	0.058
RHP	0.756	0.801	0.774	0.059	0.850	0.896	0.889	0.051	0.833	0.913	0.909	0.044
LHUC	0.507	0.580	0.558	0.082	0.623	0.698	0.697	0.072	0.745	0.859	0.859	0.063
LIR	0.664	0.676	0.680	0.052	0.689	0.712	0.715	0.044	0.745	0.791	0.792	0.037
LDEF	0.816	1.031	1.000	0.073	0.915	1.169	1.165	0.064	0.922	1.257	1.258	0.056
RLV	0.661	0.766	0.752	0.088	0.590	0.729	0.713	0.078	0.608	0.717	0.697	0.069
EPU	0.659	0.633	0.623	0.048	0.736	0.710	0.700	0.040	0.788	0.787	0.782	0.034

Note: (i) the logarithmic transformed variables begin with a prefix 'L'; (ii) 'LW' stands for the local Whittle estimator, '2ELW' stands for the two-step ELW estimator, '2ELWdm' stands for the two-step ELW estimator used for the demeaned and detrended variable; (iii) stand errors of the estimates with different bandwidths (B) are saved in columns named 'SD'. SD is calculated as $(4\psi)^{-1/2}$, $\psi = N^B$. N is the number of observations and B represents the value of estimation bandwidth.

ple. A same rolling window setting is also employed in Kumar and Okimoto (2007). Through the dynamic estimates, we aim to confirm the long-memory property of our target series, test the sensitivity of static d estimates with the changing data sample, and study how the integration order of each variable evolves over time. As earlier applied in the static estimates, the same estimators, i.e. LW, 2ELW and 2ELWdm, are also employed in the rolling-window estimates. Corresponding results of each series with different B are illustrated in Figures 14 to 19.

Overall, in light of above overwhelming evidence from both static and dynamic d estimates, we confirm the existence of fractional integration in our target variables. In particular, some key patterns from the dynamic estimates emerge. First, the estimated d with a lower bandwidth, particularly when $B = 0.4$ and $B = 0.5$, tend to be more volatile and striking in contrast to the ones with a higher B , and this is in line with Kumar and Okimoto (2007). It implies that moderate bandwidth values, such as 0.6 and 0.7, appear to be more rational in our empirical case. Second, in terms of inflation, its persistence witnessed a gradual decline after the 1990s with many small fluctuations across the period. This finding is consistent with many existing studies (Cogley and Sargent, 2001, 2005; Kumar and Okimoto, 2007; Taylor, 2000). In addition, in terms of LCD , although its estimated d fluctuates over the unit root line ($d = 1$), its values mainly behave as

greater than 1 with different rolling-window periods and bandwidths.

In the next section, we proceed with the FCVAR estimation of both housing demand and supply functions involving the calculation of fractional cointegration order of the system, the determination of model specifications, the investigation of the long-run equilibriums between housing prices and macroeconomic factors through both the functions.

7.3 FCVAR estimation results

Given that the dynamics of housing prices are determined by macroeconomic shocks on both housing demand and supply sides, a fundamental assumption of the paper is that a macroeconomic determinant of housing prices can impact through the channel of either demand or supply; or both of them. For example, the shocks of economic policy uncertainty can depress both house buyers' purchase intentions and house constructors' development intentions, implying a simultaneous fall of both housing demand and supply, and then, depending upon which one is dominant, housing prices could shift negatively or positively. Thus, without a clear identification of the distinct channels of housing price determination on demand and supply sides, the real effects of our target variables, which could impact on both the sides, would not be disentangled but instead remain intertwined.

Overall, both housing demand- and supply-driven housing price determination functions are constructed, respectively, by using the FCVAR model. The estimates of both functions are presented in the following subsections. As assumed earlier in Section 4.2, $d = b$. It implies that the fractional order (d) of the group of our target variables is cointegrated to zero. That is to say, any long-run cointegrating relationship(s) among our target variables tends to be a short-memory stationary process. In addition, to both remove common unchanged characteristics and minimize non-stationary deterministic elements induced by 'the continued and inertial movements' over time, we further demean and detrend each target variable by using Shimotsu's (2010) method after the data transformation.

Furthermore, after the FCVAR estimations, a five-year-ahead forecasting of the future movements of both variables and obtained cointegrating relationship(s) from demand and supply functions are respectively executed to examine the validation of model estimates. Finally, by solving the simultaneous equilibrium relationships in these two functions, an overall housing determination equation involving macroeconomic factors from both the functions can be eventually derived. In addition, in light of the significance test of the cointegrating parameters (α and β) of each target variable, a robustness check by employing rational restricted FCVAR estimates further confirms our conclusions.

7.3.1 Determination of model specification

After selecting variables to form both demand and supply functions, the primary issue then is to determine the FCVAR model specification by choosing the optimal model parameters, such as the

lag order and the number of ranks, in the FCVAR system for each function. First of all, we need to gauge the highest lag order (p) to form the short-run corrections. We follow Jones et al. (2014) and select the optimal lag order by a series of Likelihood Ratio (LR) tests through a ‘*general to specific*’ strategy. The LR test starts from a very generous lag order, viz. $p = 8$, by assuming that potential short-run interdependence among target variables exists within eight quarters (equivalent to two years).

For each LR test, the null hypothesis is that the coefficient of the highest lag order (p) is not significant ($H_0 : \Gamma_p = 0$), in contrast to the alternative hypothesis in favour of the significance of Γ_p ($H_1 : \Gamma_p \neq 0$). If the null hypothesis is accepted, the highest lag order (p) should then be dropped and the model will be re-estimated until it can be significantly rejected. Within each LR test, we also perform a white noise test through the Ljung-Box Q-test to examine if the residuals are autocorrelated.²¹ If its null hypothesis of no autocorrelation is rejected, we have to drop that specified highest lag order and move one step back in the determination of model specification. Another important question is how to confirm the highest lag order (p) that we choose is the best among all qualified candidates, which all can significantly reject the H_0 and have no autocorrelation in the residuals of their LR tests. We can answer this question and eventually determine the optimal lag order through the information criteria (IC) technique, such as the Akaike information criteria (AIC) and the Bayesian information criteria (BIC), whose values with different p are reported during each LR test. The optimal order should have a minimum value of the IC.

Once the optimal p has been decided, we move on to determine the number of ranks (*rank*) in the FCVAR system, i.e. the number of the long-run cointegrating relationships. To do this, we follow Johansen (1995) and identify *rank* through a series of the Likelihood Ratio (LR) tests. The sequential constructed null hypotheses are $H_0 : rank = k$ for $k = 0, 1, \dots, K$ against the same alternative hypothesis implying the *full rank*, i.e. $H_1 : rank = K$ where K is the number of variables and equals to the *full rank* in the system. Finally, the selected rank order is the one that first accepts its corresponding null hypothesis. It represents the number of long-run equilibrium relationships among target variables. Once both lag orders and ranks of the FCVAR system are determined, we can then move forward to proceed with the FCVAR model estimation.

Furthermore, as pointed out by Johansen (1995), the parameters of cointegrating relationship(s), viz. α and β , cannot be separately identified without additional restrictions of the matrix normalization for Π in (14). Thus, in the following estimations, we impose an identification restriction that normalizes β regarding housing prices (*RHP*). The second variable selected to do the β normalization is residential housing stocks (*HUC*) but only in the case when model ranks are greater than one. This normalization setting is meaningful for the paper because it enables us to resolve our key research questions regarding how equilibrium housing prices are determined by and interact with macroeconomic factors from demand and supply sides, respectively.

²¹The number of lags in the test is chosen as 12 in the following estimations; we also tried other lag orders such as 4, 8, and 16, and the test results are qualitatively the same.

7.3.2 The demand-driven determination function of housing prices

The group of demand-driven factors includes residential housing stocks ($LHUC$), inflation ($LDEF$), long interest rate (LIR), credit to the housing demand (LCD), and economic policy uncertainty (EPU). They are either exclusive demand variables, which affect housing prices only on the demand side, viz. $LDEF$ and LCD , or dual-impact variables, which can impact on both demand and supply sides, viz. $LHUC$, LIR , and EPU . The target explanatory variable of the demand function is housing prices (RHP^D), where its superscript (D) indicates that it is the specific housing prices determined by the demand function instead of the supply function. In particular, in light of the discussions in Section 7.1, we further differentiate both $LDEF$ and LCD to remove their potential unit root and conveniently capture all target variables' long-memory properties in the specified FCVAR system.

Table 5: **Lag-order Selection - FCVAR (Demand Function)**

p	K	\hat{d}	$LogL$	LR	$P\text{-value}$	AIC	BIC	$PmvQ$
8	6	1.404	-2138.50	63.21	0.003	4938.99	5915.03	1.00
7	6	1.508	-2170.10	40.31	0.285	4930.20	5800.08	1.00
6	6	1.401	-2190.25	71.70	0.000	4898.51	5662.24	1.00
5	6	1.294	-2226.10	50.18	0.059	4898.21	5555.78	1.00
4	6	0.624	-2251.19	87.40	0.000	4876.38*	5427.80	0.98
3	6	1.224	-2294.89	87.11	0.000	4891.78	5337.04	1.00
2	6	1.209	-2338.45	83.69	0.000	4906.89	5246.00	0.40
1	6	0.856	-2380.29	86.39	0.000	4918.58	5151.53	0.00
0	6	0.784	-2423.48	0.00	0.000	4932.96	5059.76*	0.00

Note: (i) number of observations (T) in sample is 141; (ii) order of the white noise test is 12.

Table 6: **Rank Tests - FCVAR (Demand Function)**

$Rank$	\hat{d}	$LogL$	$LR\text{statistic}$	$P\text{-value}$
0	0.77	-2316.228	130.073	0
1	0.687	-2288.641	74.899	0.001
2	0.68	-2271.028	39.672	0.045
3	0.641	-2254.864	7.344	0.926
4	0.616	-2252.225	2.067	0.945
5	0.625	-2251.194	0.004	0.998
6	0.624	-2251.192	—	—

Note: (i) number of observations (T) in sample is 141; (ii) order of lags is 4.

Following procedures discussed in Section 7.3.1, to determine the model specification, we first perform the lag order selection with the corresponding results presented in Table 5. Each LR test with different highest lag orders (p) ranging from 0 to 8 demonstrates the significance of p at either the 5% or 1% significance levels except when $p = 5$ and $p = 7$. Meanwhile, for the autocorrelation check of the residuals in each LR test, it suggests no autocorrelation except when $p = 0$ and $p = 1$. In light of these results, the optimal p has thus been chosen as 4 due to its lowest AIC value.²² In addition, the estimated d of the FCVAR system equals to 0.624 when $p = 4$, which supports a fractional co-integration order of the group of demand-driven variables.

²²We do not follow the BIC in selecting the optimal p . This is because the residuals with the BIS's suggested p do not pass the white noise test, while its suggestion ($p = 0$) of no short-run corrections is counter-factual and hard to explain in reality.

ferring to Section 3, RHP^{D*} and $LHUC^{D*}$ respectively denote the level of housing prices (RHP^D) and residential housing stocks ($LHUC$) in the equilibrium condition achieved through the housing demand side. Regarding the demand function, the first relation shown in (30) demonstrates how housing demand factors drive the equilibrium housing prices given that $\nu_{1t} = 0$; the second one shown in (31) describes how these factors determine the equilibrium level of housing stock provisions given that $\nu_{2t} = 0$.

The cointegrating relationships derived from the demand function are consistent with theoretical expectations. As shown in (30), housing demand factors such as inflation ($LDEF$), long interest rate (LIR), and economic policy uncertainty (EPU) negatively affect RHP^{D*} , while the money supply to the housing demand side (LCD) exerts a positive impact on RHP^{D*} . Specifically, a 1% unit change of the growth of DEF induces a 14.238 unit decline of RHP^{D*} ; the downward impact of LIR on RHP^{D*} is greater than that of EPU , which are -2.415 and -0.865 respectively; and LCD positively affects RHP^{D*} (1.280). In terms of the second relation in (31), similar to the first relation regarding signs of the effect, $LDEF$, LIR , and EPU negatively affect $LHUC^{D*}$, while LCD provides a positive effect (0.012). In comparison between (30) and (31), impacts of demand factors on $LHUC^{D*}$ are much smaller than their counterparts on RHP^{D*} .

In theory, on the housing demand side, the stationary equilibrium relation (30) explains to what extent demand-driven factors determine the equilibrium housing prices, and the results are consistent with our theoretical expectations in light of the market equilibrium theory. Specifically, in an overheated economy with high inflation, interest rate tends to experience an upward pressure, implying an increasing cost of borrowing money. This depresses housing demand leading to a fall in housing prices. Similarly, an increase in the interest rate leads to a fall of housing prices by depressing the housing demand. In addition, the existence of high uncertainty in the economy potentially unnerves investors and subdues their intentions of consumption/investment; therefore housing prices tend to fall given that housing demand witnesses a downturn. More importantly, understanding the impact of money/credit supply in the long-run is one of our main focuses. Credit provided to the housing buyers, which is measured by using mortgage debt outstanding for the residential properties, can exert a positive impact on determining housing prices. The more money that housing buyers own, the greater their purchasing capabilities/powers should be.

Moreover, the second equilibrium equation demonstrates the long-run relation between available housing stocks and other demand-driven factors. As theoretically expected, their nexuses can be interpreted in detail through the housing demand channel. First, the appreciation of inflation depresses the housing demand by increasing the interest rate, while the intention of housing supply also dwindles, all of which indicate a decrease of both house buyers' required housing stocks and house suppliers' provision intentions. Eventually, available amounts of housing stock completion in the real estate market tend to decrease. A similar effect mechanism is also applied to other demand factors on housing stocks. An increase of long-term interest rates will similarly depress housing demand and then decrease available housing stock. Moreover, an exposure of the economy to a high-level of uncertainty stagnates housing demand by depressing house buy-

ers' purchase intentions, therefore the number of housing stocks for sale will decrease. Besides, a tightened constraint of credit supply will also restrict housing stocks by depressing the housing demand.

To evaluate the reliability of the above reported FCVAR estimation results and corresponding theoretical explanations, this chapter further conducts forecasting evaluation exercises to assess the predictive accuracy of the employed FCVAR model, while its predictive improvement over the traditional vector error correction model with strict assumption of integer integration order, i.e. CVAR model, is also examined accordingly. Following (24), the model forecasting performance is measured through the calculation of RMSFE. Based on the forecasting algorithms as discussed in Subsection 4.4, we conduct nine out-of-sample forecasting with h -step/ $\frac{h}{4}$ -year ahead where $h = 1, 5, 10, 15, 20, 25, 30, 35, 40$ for the price determination function on the housing demand side estimated by using the FCVAR and CVAR models, respectively.

Table 7: RMSFE calculations (Demand Function)

Model	Forecast horizon (h)								
	1 step	5 step	10 step	15 step	20 step	25 step	30 step	35 step	40 step
<i>(a) The magnitudes of RMSFE values</i>									
FCVAR	0.0069	0.0059	0.0148	0.0256	0.0302	0.0617	0.0466	0.0321	0.0184
CVAR	0.0046	0.0067	0.0284	0.0630	0.2075	0.1743	0.0633	0.2514	0.1449
<i>(b) Percentage change in RMSFE values</i>									
FCVAR versus CVAR	50.2906	-12.2085	-47.7410	-59.3619	-85.4452	-64.5979	-26.3334	-87.2188	-87.3032

Note: (i) forecasting performance of the overall demand-driven model system is measured by the RMSFE values. (ii) Section (a) reports the values of RMSFE for the multivariate model system of the FCVAR and CVAR. (iii) Section (b) reports the comparisons of RMSFE values between the FCVAR and CVAR; negative reported values favour the FCVAR model.

The magnitudes of RMSFE values for each h -step ahead until 40-step/10-year for both the two models are reported in Section (a) in Table 7. Specifically, it is clear that the FCVAR model outperforms the CVAR model throughout all nine forecasting evaluations with smaller RMSFE values except at the one-step ahead forecast horizon where the RMSFE values for both models are quite similar and the one from the CVAR is slightly greater than that from the FCVAR. The forecasting accuracy of the FCVAR model is increasingly much higher than that of the CVAR model with the increase of forecast horizons.

Furthermore, the improvement degree of forecasting accuracy of the FCVAR model over the CVAR model can be measured by reporting the percentage change in RMSFE values of the FCVAR model relative to the CVAR model following

$$100 \times \left\{ \frac{RMSFE_{FCVAR}}{RMSFE_{CVAR}} - 1 \right\} \quad (32)$$

where its reported negative results favour the superiority of the FCVAR model while positive results favour the superiority of the CVAR model in terms of the model predictive performance. Relevant results are accordingly reported in Section (b) in Table 7 and show that the RMSFE of the FCVAR model can be as much lower as 87% than that of the CVAR model. Overall, the results in

Section (a) are broadly consistent with that in Section (b), and both demonstrate that the FCVAR model behaves smaller FMSFE values than that of the CVAR model except at the one-step ahead forecast horizon where FMSFE values of both models are similar. We can finally conclude that the forecasting performance of the FCVAR model is checked to be more accurate than that of the CVAR model in the case of the demand-driven housing price determination function.

In the next, we carry out a five-year-ahead forecasting exercise for the incorporated series and its estimated equilibrium relationships. Their predictions can be clearly observed in Figure 20 in Appendix C. To summarize, the in-sample dynamics of each series fluctuate frequently over the zero line, while the out-of-sample forecast broadly predicts a consistent movement where each series gradually converges to the zero line. In particular, variables such as *RHP*, *LCD* and *EPU* witnessed a more striking movement from 2008 onwards probably due to the outbreak of the global financial crisis. They particularly experienced a significant decrease after 2010; their downward momentums are then expected to be curbed from 2016 while they start to recover gradually towards the zero point. In terms of the cointegrating relationships, the one normalized by *RHP* has a more volatile movement than the one normalized by *LHUC*, while both are also expected to converge towards zero in the forecast.

7.3.3 The supply-driven determination function of housing prices

The group of supply-driven factors includes variables such as residential housing stocks (*LHUC*), economic policy uncertainty (*EPU*), long interest rates (*LIR*), residential land value (*RLV*), and credit to the housing supply (*LCS*). They are either exclusive supply variables, which affect housing prices only on the supply side, viz. *RLV* and *LCS*, or dual-impact variables, which affect housing prices on both demand and supply sides, viz. *LHUC*, *LIR*, and *EPU*. The target explanatory variable is housing prices (*RHP^S*), where its superscript (*S*) indicates that it is determined by the supply function instead of the demand function. In addition, we decide to not include inflation (*LDEF*); otherwise there would be a multicollinearity problem in the supply-driven function as *LDEF* is highly correlated with *LCS*.²³

Table 8: Lag-order Selection - FCVAR (Housing Supply)

<i>p</i>	<i>K</i>	\hat{d}	<i>LogL</i>	<i>LR</i>	<i>P-value</i>	<i>AIC</i>	<i>BIC</i>	<i>PmvQ</i>
8	6	1.577	-2162.43	52.12	0.040	4986.86	5965.24	1.00
7	6	1.070	-2188.49	88.47	0.000	4966.98	5838.95	1.00
6	6	1.340	-2232.73	50.71	0.053	4983.45	5749.01	1.00
5	6	0.876	-2258.08	91.94	0.000	4962.16*	5621.31	1.00
4	6	1.129	-2304.05	98.29	0.000	4982.10	5534.84	0.96
3	6	0.927	-2353.19	37.52	0.399	5008.38	5454.71	0.59
2	6	0.010	-2371.95	125.13	0.000	4973.90	5313.82	0.00
1	6	0.066	-2434.52	220.35	0.000	5027.04	5260.55*	0.00
0	6	0.860	-2544.69	0.000	0.000	5175.39	5302.49	0.00

Note: (i) number of observations (T) in sample is 142; (ii) order for the white noise test is 12.

²³*LDEF* is approximated by the GDP deflator, which is calculated based on the nominal GDP, while parts of GDP form the private residential fixed investment, which proxies *LCS*. Therefore, both tend to be highly correlated.

Table 9: Rank Tests - FCVAR (Housing Supply)

<i>Rank</i>	\hat{d}	<i>LogL</i>	<i>LRstatistic</i>	<i>P-value</i>
0	0.873	-2305.240	94.318	0.040
1	0.872	-2287.005	57.849	0.263
2	0.907	-2274.238	32.313	0.676
3	0.827	-2263.226	10.290	0.962
4	0.874	-2259.783	3.404	0.984
5	0.877	-2258.085	0.007	1.000
6	0.876	-2258.081	—	—

Note: (i) number of observations (T) in sample is 142; (ii) order of lags is 5.

Similar to Section 7.3.2, we start the FCVAR estimation by first determining the model specification. In terms of the optimal highest lag order (p), it should be selected based on a series of tests including LR test, corresponding white noise test, and information criteria. As indicated in Table 8, we prefer the optimal $p = 5$ due to its minimum AIC value, while it also passes both the LR test and corresponding white noise test. In addition, the fractional order d with $p = 5$ equals to 0.876, implying the long-memory property in the group of supply-driven factors. In terms of the rank number, as shown in Table 9, the first null hypothesis of $rank = 0$ is rejected while the second one of $rank = 1$ is accepted against the same alternative hypothesis of $rank = 6$, viz. the *full rank*. Thus, it implies one cointegrating relationship among supply-driven factors. To sum up, regarding the FCVAR system of the supply function, its short-run correction terms are up to order 5, while its rank is equal to 1. In light of this, we then proceed with the FCVAR estimation and produce the results shown as follows.

Estimated Unrestricted FCVAR model:

$$\Delta^{\hat{d}} \begin{pmatrix} RHP^S \\ LHUC \\ EPU \\ LIR \\ RLV \\ LCS \end{pmatrix} - \begin{pmatrix} 6.234 \\ -1.509 \\ -14.985 \\ -9.547 \\ 3.446 \\ -19.121 \end{pmatrix} = L_{\hat{d}} \begin{pmatrix} -0.020 \\ -0.259 \\ 0.509 \\ 0.811 \\ 0.271 \\ -0.719 \end{pmatrix} \nu_t + \sum_{i=1}^5 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \quad (33)$$

$$\hat{d} = 0.872, Q_{\varepsilon}(12) = 347.802, LogL = -2287.005$$

(0.025) (0.999)

The Supply-driven Equilibrium Relationship (long-run):

$$RHP_t^{S*} = -0.5229 - 0.827 \times LHUC_t + 0.065 \times EPU_t + 0.143 \times LIR_t + 1.312 \times RLV_t - 0.174 \times LCS_t + \nu_t \quad (34)$$

With regard to the estimated FCVAR supply function presented in (33), both estimated drift term (ρ) and short-term adjustment speed parameter (α) have been shown on the left and right hand sides, respectively. The parameter, ν_t , viz. $\beta' L_d (Y_t - \rho)$, is expanded in (34), which demonstrates the long-run cointegrating relationship between housing prices (RHP^S) and supply-driven

variables given that $\nu_t = 0$, and it is normalized by RHP^S . Key parameters of the FCVAR estimation are presented below (33). The fractional integration order (d) of the system is 0.872. The statistic of the Ljung-Box Q-test is 347.802 with P value = 0.999, implying a strong indication of no autocorrelation in the residuals. Moreover, the long-run cointegrating relationship between housing prices and supply-driven factors is shown in (34).

Referring to Section 3, RHP^{S*} denotes the level of housing prices in the equilibrium condition that is achieved on the housing supply side. Specifically, residential housing stocks ($LHUC$) and credit to the housing supply (LCS) negatively affect housing prices (RHP^{S*}), while economic policy uncertainty (EPU), long interest rates (LIR), and residential land value (RLV) positively affect RHP^{S*} . A 1% unit change of HUC and CS can induce a 0.827 unit and a 0.174 unit change of RHP^{S*} in the opposite direction, respectively; the positive effects of EPU , LIR , and RLV on RHP^{S*} are 0.065, 0.143, and 1.312, respectively.

In terms of theoretical explanations, through the housing supply channel, our estimated long-run equilibrium relationship between housing prices and supply-driven factors shown in (34) is consistent with the theoretical expectations. Specifically, the excess provision of housing stocks implies the excess housing supply in circulation; this will directly discourage housing builders' intention of further housing supply, while housing prices then tend to slump. Moreover, the exposure of high-level economic policy uncertainty will depress not only the housing demand but also the housing supply, which further affects housing prices. An increasing uncertainty level indicates a heightening uncertainty of the investment return for the real estate construction; it will depress the housing supply and subsequently increase housing prices. In particular, regarding the dual effects of uncertainty on housing prices in our empirical estimation, its positive effect through the supply channel is much less than its negative impact through the demand channel. This suggests that the overall effects of uncertainty tend to be negative on housing prices.

Similarly, long interest rates also exert dual effects on housing prices through the demand and supply sides. Regarded as a proxy for levels of borrowing costs/expenditures for the housing construction/development, the larger the costs are, the much greater the constraints will be on housing supply; then housing prices will rise correspondingly. Thus, in light of its stronger negative effect on the demand side versus its relatively less negative effect on the supply side, the aggregate effect of long interest rates on housing prices tends to be negative. Indeed, our elaborations of specific factors with dual impacts, such as uncertainty and interest rates, offer a precise way to disentangle their real effects on housing prices. In terms of residential land (market) value, its increase enlarges the housing production costs and then dwindles the housing supply, therefore housing prices will then witness a rise. In addition, an increase in available money/credit provided to housing suppliers, which is represented by the levels of residential investment for the housing construction, can stimulate a rise in housing suppliers' intention in developing housing units. Therefore, the housing supply will increase while housing prices tend to decline.

In the next, to demonstrate the reliability of above-reported FCVAR estimations and the model validation, similar to the forecasting performance evaluation process conducted in previous sec-

tion regarding the demand-driven function, in the case of housing price determination function on the supply side, we follow (24) to calculate RMSFE values of the FCVAR model. Then its calculated RMSFE values are compared with that of the CVAR model. Through this, the forecasting accuracy of the FCVAR model can be well examined.

Corresponding results in Section (a) in Table 10 report that the RMSFE values of the FCVAR model are checked to be greater than that of the CVAR model except at 25-step and 30-step ahead forecast horizons where the two models possess similar RMSFE values. Moreover, as depicted in Section (b) in Table 10 and calculated based on (32), the improvement degree of forecasting accuracy of the FCVAR model can be as much as 99% compared with the CVAR model, while the results obtained from both Sections (a) and (b) are consistent. Overall, in the case of housing supply-driven function, we can conclude that the FCVAR model outperform the traditional model of vector error corrections with the assumption of integer integration order, i.e. CVAR model.

Table 10: RMSFE calculations (Supply Function)

Model	Forecast horizon (h)								
	1 step	5 step	10 step	15 step	20 step	25 step	30 step	35 step	40 step
<i>(a) The magnitudes of RMSFE values</i>									
FCVAR	0.0073	0.0084	0.0261	0.0532	0.0269	0.0900	0.0421	0.0273	0.0129
CVAR	0.0091	0.0227	0.0667	0.0579	0.0300	0.0457	0.0346	0.1552	1.5336
<i>(b) Percentage change in RMSFE values</i>									
FCVAR versus CVAR	-19.9531	-63.1356	-60.9260	-8.0968	-10.1478	96.7783	21.5241	-82.3900	-99.1620

Note: (i) forecasting performance of the overall supply-driven model system is measured by the RMSFE values. (ii) Section (a) reports the values of RMSFE for the multivariate model system of the FCVAR and CVAR. (iii) Section (b) reports the comparisons of RMSFE values between the FCVAR and CVAR; negative reported values favour the FCVAR model.

Finally, by using the FCVAR model, we carry out forecasting exercises to predict the future 5-year movements of the incorporated variables and the obtained cointegrating relationship in the housing supply-driven function. The corresponding results are accordingly plotted in Figure 21 in Appendix C. Overall, the in-sample movements of variables in the supply function tend to be similar to the ones in the demand function that fluctuate frequently across the zero line. The out-break of financial crisis markedly affects their dynamics, which induced a plunge after 2010. Their movements tend to gradually become stable during the out-of-sample forecast, while the forecast of dual-effect factors, such as *LHUC*, *EPU* and *LIR*, behave consistently in both demand and supply functions as observed in Figures 20 and 21, respectively. With regard to the cointegrating relationship, it witnessed a striking fluctuation after the financial crisis, and is expected to taper off and converge to zero during the forecast.

7.3.4 The manually-derived overall determination function of equilibrium housing prices

Thus far, we conclude that there exist factors that exclusively affect housing prices through demand or supply sides, and dual-effect factors that can impact on both sides. The equilibrium housing prices tend to be determined as a trade-off between impacts from distinct housing de-

mand and supply effect channels. According to Sections 7.3.2 and 7.3.3, we have gauged the extent of impacts that factors from both demand and supply channels separately exert in contributing to the equilibrium housing price determination. Indeed, as theoretically elaborated, the equilibrium housing prices can be achieved only when both demand and supply functions reach a level of market clearing. In that state, demand and supply curves intersect, viz. demand and supply are equal in the real estate market: that is, $RHP^* = RHP^{D*} = RHP^{S*}$.

Next, we investigate what aggregate impacts macroeconomic fundamentals can exert on the equilibrium housing price determination, having considered the two distinct demand and supply channels. The above question can be answered by manually solving the two simultaneous equilibrium relations as shown in (30) and (34), which are obtained from these two channels, respectively. In specific, the results from (30) and (34) are reported in the first and second rows in Table 11, where we can clearly compare between impacts from the demand and supply sides, particularly for the dual effect factors, on equilibrium housing price determinations. Then by solving the simultaneous (30) and (34), aggregated impacts of each of the included variables are measured and reported in the third row of Table 11. Eventually, an overall determination function reporting these aggregated impacts is derived and presented in (35).

Table 11: **Summary of estimates in demand- and supply-driven determination functions**

Impacts	Dual effect			Exclusive demand		Exclusive supply	
	LIR	EPU	LHUC	LCD	LDEF	RLV	LCS
Demand	-2.415	-0.865	-	1.280	-14.238		
Supply	0.143	0.065	-0.827			1.312	-0.174
Aggregate	-1.136	-0.400	-0.4135	0.640	-7.119	0.656	-0.087

Note: (i) the variable estimates are from the previously-reported demand- and supply-driven housing price determination functions; (ii) $LHUC$ is not included in the demand-driven function due to the identification requirement for its second cointegration relationship where $LHUC$ is the variable for normalization; (iii) LIR , EPU , and $LHUC$ are dual effect factors; LCD and $LDEF$ are exclusive factors that impact only through the demand side; RLV , and LCS are exclusive factors that impact only through the supply side.

The Equilibrium Relationship of the Overall Determination Function:

$$RHP_t^* = -1.48885 - 7.119 \times LDEF_t + 0.640 \times LCD_t - 0.4135 \times LHUC_t - 0.087 \times LCS_t + 0.656 \times RLV_t - 0.400 \times EPU_t - 1.136 \times LIR_t + \nu_t^* \quad (35)$$

where $\nu_t^* = (\nu_{1t} + \nu_t)/2 = 0$ in equilibrium. Following (35), with regard to factors with a dual-impact, such as economic policy uncertainty (EPU) and long interest rates (LIR), their aggregate impacts on housing prices are negative at -0.400 and -1.136, respectively. This is because both of their much stronger negative effects from the housing demand channel offset their relatively smaller positive impacts from the housing supply channel. It is noteworthy that dual-effect factors (e.g. EPU and LIR) in our case demonstrate negative aggregate impacts, which are smaller than their effects obtained from the demand channel while being greater than the counterparts exerted from the supply channel in absolute value. Hence, it is concluded that impacts of both EPU and LIR are driven by the housing demand side. In addition, due to the identification re-

quirement of cointegrating relationships in the demand function, the specific dual-effect factor, residential housing stocks (*LHUC*), does not enter the demand-driven equilibrium. Thus, its aggregate impact reported in (35) is actually from the supply channel, which is -0.4135.

At the same time, regarding the exclusive factors that impact housing prices only through either the demand channel, viz. *LDEF* and *LCD*, or the supply channel, viz. *LCS* and *RLV*, their aggregate impacts from the overall determination function tend to be smaller than that from their individual effect channels in absolute value, due to the calculation mechanism of the simultaneous equation system. Specifically, impacts of the variables estimated from the demand/supply function could be adjusted and offset directly through the same variables (for the variables with dual impacts) or indirectly through other variables (for the variables with exclusive impacts) from the supply/demand function. Hence, the significance of considering housing price determinations respectively through the demand and supply functions is well tested, having recognized obvious evidence from Table 11. Indeed, real impacts of macroeconomic factors would be distorted when being estimated directly through an overall determination function, while the construction of a simultaneous demand and supply function system can nevertheless help avoid this distortion.

Estimation of a single equilibrium housing price determination function

So far, by respectively estimating the simultaneous demand- and supply-driven functions for equilibrium housing price determinations, impacts of macroeconomic fundamentals including dual-effect factors, exclusive demand- and supply-driven factors have been well gauged. Moreover, as described in the above part of this section, an overall determination function including all incorporated macroeconomic variables from both the demand and supply channels has been manually derived by solving the simultaneous demand- and supply-driven functions.

Next, although the confirmed superiority of a simultaneous function system to a single and combined function for equilibrium housing price determinations, we nevertheless directly estimate the latter as a benchmark model, which together includes both demand and supply variables, by using the FCVAR method. Through this, impact estimates of the incorporated variables obtained from the previously-built simultaneous function system can be well compared with that of the combined function, which specification has been widely employed in much housing-related literature to date. To determine the FCVAR model specification of the combined function, both the lag order selection test and the rank test are conducted. According to the corresponding results shown in Tables 12 and 13, the combined equilibrium housing price determination function is constructed with 5 system lags and 2 ranks. Moreover, estimated by the FCVAR method, coefficient estimates of the combined function in both the short- and long-runs are reported from equations (36) to (38). In particular, (37) reports the long-run relationship between housing prices and demand/supply factors. In other words, equilibrium housing price determinations through a combined function including both demand and supply factors ('Aggregate1'). The significance of both the fractional integration order of the function system and the long-run parameters, viz. α and β , of all incorporated variables are confirmed. Corresponding descriptions and results of all

Table 14: Hypothesis Tests

H_D^d	The fractional order, d , equals to one.	H_{D1}^α	RHP is weakly exogenous.
H_{D1}^β	RHP and LHUC do not enter the cointegrating relationships.	H_{D2}^α	LHUC is weakly exogenous.
H_{D2}^β	EPU does not enter the cointegrating relationships.	H_{D3}^α	EPU is weakly exogenous.
H_{D3}^β	LIR does not enter the cointegrating relationships.	H_{D4}^α	LIR is weakly exogenous.
H_{D4}^β	RLV does not enter the cointegrating relationships.	H_{D5}^α	RLV is weakly exogenous.
H_{D5}^β	LCS does not enter the cointegrating relationships.	H_{D6}^α	LCS is weakly exogenous.
H_{D6}^β	LCD does not enter the cointegrating relationships.	H_{D7}^α	LCD is weakly exogenous.
H_{D7}^β	LDEF does not enter the cointegrating relationships.	H_{D8}^α	LDEF is weakly exogenous.

Table 15: Hypothesis Test Results

	H_D^d	H_{D1}^β	H_{D2}^β	H_{D3}^β	H_{D4}^β	H_{D5}^β	H_{D6}^β	H_{D7}^β
df	1	4	2	2	2	2	2	2
LR Statistic	187.910	29.488	583.383	435.384	566.184	511.349	459.583	514.855
P-Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	H_{D1}^α	H_{D2}^α	H_{D3}^α	H_{D4}^α	H_{D5}^α	H_{D6}^α	H_{D7}^α	H_{D8}^α
df	2	2	2	2	2	2	2	2
LR Statistic	521.734	426.986	438.975	516.672	497.818	509.056	440.17	509.443
P-Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: (i) *: significant at the 10% level, **: significant at the 5% level, ***: significant at 1% level; (ii) df denotes the degree of freedom; (iii) LR is the abbreviation for the Likelihood Ratio test.

Table 16: Comparison of estimates between the simultaneous function system and the combined function

Impacts	Dual effect			Exclusive demand		Exclusive supply	
	LIR	EPU	LHUC	LCD	LDEF	RLV	LCS
Aggregate	-1.136	-0.400	-0.4135	0.640	-7.119	0.656	-0.087
Aggregate1	-0.001	-0.530	-	0.305	0.238	1.322	-0.485
Difference	-1.135	0.13	-	0.335	-7.357	-0.666	0.398

Note: (i) ‘aggregate’ refers to the overall price determination function, which estimates are obtained by manually solving the demand- and supply-driven functions; ‘aggregate1’ refers to the combined determination function, which includes both demand and supply factors and is directly estimated using the FCVAR method; (ii) *LHUC* is not included in ‘aggregate1’ as it is used to for the identification purpose of the second cointegrating relationship obtained in ‘aggregate1’; (iii) *LIR*, *EPU*, and *LHUC* are dual effect factors; *LCD* and *LDEF* are exclusive factors that impact only through the demand side; *RLV*, and *LCS* are exclusive factors that impact only through the supply side.

of the previously-reported overall function (viz. ‘Aggregate’ depicted in (35)), which is manually derived from the simultaneous function system, with the combined function (viz. ‘Aggregate1’ depicted in (37)) for equilibrium housing price determinations. Then, estimates of each considered variable are further differenced between the two functions with results shown in the third row (‘Difference’) of Table 16. It can be clearly observed that magnitudes of the macroeconomic impacts would be seriously mis-quantified when directly estimating the combined function where the specific effect-transmission channels of each macroeconomic variable are failed to be identified, although signs of the macroeconomic impacts are all shown to be consistent between the two functions except for *LDEF*.

It is explainable regarding the seemingly contradictory estimates of *LDEF*: negative in ‘Aggregate’ versus positive in ‘Aggregate1’. As previously discussed in the theoretical section, the inflation level, viz. *LDEF*, has a dual role. *LDEF* tends to decline the housing demand and then housing prices; at the same time, it can also increase housing prices by decreasing the housing

supply. However, as $LDEF$ is not included in the supply channel of the simultaneous function system, therefore its negative impacts shown in the 'Aggregate' function are actually from the demand channel. In contrast, the positive estimate of $LDEF$ reported from 'Aggregate1' is an aggregated impact, which is measured by considering the impacts from both the demand and supply channels, and it indicates that the positive effects of $LDEF$ from the supply side are greater than its negative counterpart from the demand side. Overall, the above empirical analysis re-confirms the accuracy of our theoretical arguments regarding the importance of identifying both the demand and supply channels respectively. The impacts of macroeconomic fundamentals on equilibrium housing price determinations could be mis-estimated (either over- or under-estimated) if their potential effect-transmission channels are failed to be distinctly identified.

7.4 Robustness exercises: Sensitivity of unrestricted FCVAR results to restrictions

To examine the robustness of both unrestricted demand- and supply-driven FCVAR estimations presented in Subsections 7.3.2 and 7.3.3, respectively, we re-estimate both unrestricted functions with reasonable imposed restrictions that consider the significance of their cointegrating parameters, viz. β and α . To do that, we first conduct a series of hypothesis tests based on (15) and (16) by using the Likelihood Ratio (LR) test. In theory, if a given null hypothesis for β is rejected, the tested variable(s) can enter and form the long-run cointegrating relationship(s). If a given null hypothesis for α is rejected, the tested variable(s) can contribute to the adjustments/error corrections towards the equilibrium; if not, then it is long-run weakly exogenous. The null hypothesis of each test in demand and supply functions can be seen in Tables 17 and 19, respectively. Corresponding test results are depicted in Tables 18 and 20, respectively.

Specifically, results of the hypothesis testing are summarized as follows. First, the significant rejections of H_D^d and H_S^d indicates the validity of fractional integration setting in modelling both the demand- and supply-driven functions. In terms of the demand-driven function, β of EPU is restricted to be zero in favour of the result of H_{D5}^β in Table 18. In terms of the supply-driven function, α of RLV is restricted to be zero in favour of the result of H_{S5}^α in Table 20. In addition, it is worth noting that while the P value for the hypothesis test H_{D1}^α that RHP is weakly exogenous is 0.192, we do not impose this restriction in the demand function. This is because RHP is the key variable of this paper, therefore both its short-run corrections and long-run equilibrium behaviours cannot be ignored in both the housing demand and supply functions, respectively. Similarly, we also do not impose the restriction described in H_{S3}^β that support for $LHUC$ does not enter the cointegrating relationship in the supply function. Overall, the estimation results and obtained cointegrating relationships of restricted demand- and supply-driven functions are demonstrated in (39)-(43), respectively.

Overall, the results of both restricted demand- and supply-driven functions are highly consistent with their unrestricted counterparts, respectively, with regard to both the signs and magnitudes. That is to say, regarding both the housing demand and supply functions, restricted and unrestricted FCVAR estimations give rise to qualitatively the same conclusions. Particularly, in

terms of the restricted estimations, the specific variable with a dual role, e.g. *LIR*, demonstrates a negative impact on *RHP* through the demand channel in contrast to its far smaller positive effect through the supply channel. Speaking in favour of the unrestricted estimations, it implies that the aggregate effect of *LIR* on *RHP* should be negative. Thus, the robustness of our unrestricted FCVAR estimates can be explicitly and conveniently checked.

8 Conclusions

Understanding the real impacts of macroeconomic interventions on housing prices is very important, especially in the current global environment with persistent economic and policy uncertainty. Although there is a rapid rise in empirical work in this regard, a data-driven inference on the subject appears to neglect the crucial role of memory patterns of shocks in interpreting dynamics of the macroeconomy - housing market interactions. More so, the innately-existing distinct housing demand and supply channels through which macroeconomic impacts are transmitted to equilibrium housing price determinations are also failed to be clearly identified in extant literature. This paper addresses these practical issues and broadens our understanding of the inherent dynamic mechanism involved in the interactions.

To summarize, this paper sheds new light on investigating the real macroeconomic impacts on the equilibrium housing price determination through a clear identification of the distinct housing demand and supply effect-transmission channels, respectively. The paper finds a gradual disequilibrium error-correcting process while demonstrating a long-memory feature of variable dynamics in the interactive system, indicating the inefficient housing market in the US. Moreover, short-run error corrections and long-run equilibrium relationships between housing prices and macroeconomic variables on both the demand and supply channels have been separately gauged. The significance of cointegrating parameters in both the housing demand and supply functions has been accordingly examined. In addition, a five-year-ahead forecasting exercise was further conducted to forecast future movements of incorporated variables and obtained cointegrating relationship(s), and the predictive power of the FCVAR model has been checked to be stronger than that of its naive specification, i.e. traditional CVAR model.

Our FCVAR estimates are consistent with theoretical expectations. This paper not only measures impacts of the factors that affect housing prices exclusively through a demand or supply channel, but also provides a novel strategy in interpreting what real impacts the dual-effect factors could exert from both the two channels, respectively. In line with related literature, we find that the aggregate impacts of specific macroeconomic variables with 'a dual-effect', such as economic policy uncertainty and long-run interest rates, are negative, which are calculated by aggregating their much stronger negative impacts from the demand channel and relatively smaller positive impacts from the supply channel. A failure to identify these effect channels (on the demand and/or supply sides) through which macroeconomic impacts are transmitted to equilibrium housing price determinations would lead to an estimation bias of their real impacts. In addition, estimations

obtained from the restricted FCVAR models reassure our conclusions.

Policy implications

What policy insights are implied in light of our main findings? Since aggregate estimates of macroeconomic effects would result in confusing and unreliable conclusions, policy-makers could gain a clearer picture about the real macroeconomic impacts on the determination of housing price dynamics through a precise identification of the separate housing demand and supply effect-transmission channels. Thus, a major implication of our work is that policy-makers are able to minimize the micro-level information loss by recognizing the potentially-existing different roles of macroeconomic variables in altering the housing demand and supply curves, respectively.

Moreover, an accurate interpretation of the memory pattern of target variables in the macroeconomy-housing market interaction is also of great importance for meaningful policy implications. Specifically, if a given variable in the interactive system demonstrates a long memory, indicating a strong persistence and slow convergence of its movements (either increasing or decreasing), an effective policy implementations to control for its current moving tendencies could be achieved only when policymakers employ a radical regulatory strategy rather than a moderate one. At the same time, a relatively moderate strategy should be employed when regulating dynamics of a short-memory featured variable, which tends to move with a rather low persistence. In particular, given that housing price dynamics in the US are governed by a long-memory feature, therefore policymakers should implement a relatively radical policy strategy when they perceive that current housing prices may be either rising or dropping too fast and would like to accordingly regulate/control the 'abnormal' price changes.

In addition, policymakers can clearly understand if the market is efficient or not by measuring the memory pattern of the market price. In our case, existence of long-memory featured shocks in the interaction implies informational inefficiency in the housing market, indicating that past information can not be entirely and instantly reflected on the current housing price. This further demonstrates a predictive possibility of future dynamics of the interaction by using its past information.

Table 17: Hypothesis Tests of the Demand Function

H_D^d	The fractional order, d , equals to one.	H_{D1}^α	RHP is weakly exogenous.
H_{D1}^β	HPI and LHUC do not enter the cointegrating relationships.	H_{D2}^α	LHUC is weakly exogenous.
H_{D2}^β	All demand-driven variables except HUC do not enter the cointegrating relationships.	H_{D3}^α	LDEF is weakly exogenous.
H_{D3}^β	LDEF does not enter the cointegrating relationships.	H_{D4}^α	LIR is weakly exogenous.
H_{D4}^β	LIR does not enter the cointegrating relationships.	H_{D5}^α	EPU is weakly exogenous.
H_{D5}^β	EPU does not enter the cointegrating relationships.	H_{D6}^α	LCD is weakly exogenous.
H_{D6}^β	LCD does not enter the cointegrating relationships.		

Table 18: Hypothesis Test Results of the Demand Function

	H_D^d	H_{D1}^β	H_{D2}^β	H_{D3}^β	H_{D4}^β	H_{D5}^β	H_{D6}^β
df	1	4	8	2	2	2	2
LR Statistic	35.228	38.878	19.136	8.136	38.037	3.751	107.991
P-Value	0.000***	0.000***	0.014**	0.017**	0.000***	0.153	0.000***
	H_{D1}^α	H_{D2}^α	H_{D3}^α	H_{D4}^α	H_{D5}^α	H_{D6}^α	
df	2	2	2	2	2	2	
LR Statistic	6.098	108.636	16.580	228.568	278.629	192.560	
P-Value	0.192	0.000***	0.002***	0.000***	0.000***	0.000***	

Note: (i) *: significant at the 10% level, **: significant at the 5% level, ***: significant at 1% level; (ii) df denotes the degree of freedom; (iii) LR is the abbreviation for the Likelihood Ratio test.

Estimated Restricted Demand-driven FCVAR model:

$$\Delta^{\hat{d}} \begin{pmatrix} RHP^D \\ LHUC \\ LDEF \\ LIR \\ EPU \\ LCD \end{pmatrix} - \begin{pmatrix} 2.948 \\ 0.135 \\ -0.307 \\ -9.655 \\ -21.971 \\ -30.672 \end{pmatrix} = L_{\hat{d}} \begin{pmatrix} -0.032 & 2.056 \\ 0.053 & -2.868 \\ -0.049 & 1.103 \\ 0.039 & -0.615 \\ 0.002 & 0.860 \\ -0.018 & 2.054 \end{pmatrix} \begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix} + \sum_{i=1}^4 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\rho}) + \hat{\varepsilon}_t \quad (39)$$

$$\hat{d} = 0.693, Q_\varepsilon(12) = 351.949, \text{Log}L = -2272.903$$

(0.037) (0.998)

The Demand-driven Equilibrium Relationships (long-run):

$$RHP_t^{D*} = -3.130 - 61.142 \times LDEF_t - 8.219 \times LIR_t + 3.001 \times LCD_t + \nu_{1t} \quad (40)$$

$$LHUC_t^{D*} = 0.069 - 1.074 \times LDEF_t - 0.122 \times LIR_t + 0.047 \times LCD_t + \nu_{2t} \quad (41)$$

Table 19: Hypothesis Tests of the Supply Function

H_S^d	The fractional order, d , equals to one.	H_{S1}^α	RHP is weakly exogenous.
H_{S1}^β	RHP does not enter the cointegrating relationship.	H_{S2}^α	LHUC is weakly exogenous.
H_{S2}^β	All supply-driven variables do not enter the cointegrating relationship.	H_{S3}^α	EPU is weakly exogenous.
H_{S3}^β	LHUC does not enter the cointegrating relationship.	H_{S4}^α	LIR is weakly exogenous.
H_{S4}^β	EPU does not enter the cointegrating relationship.	H_{S5}^α	RLV is weakly exogenous.
H_{S5}^β	LIR does not enter the cointegrating relationship.	H_{S6}^α	LCS is weakly exogenous.
H_{S6}^β	RLV does not enter the cointegrating relationship.		
H_{S7}^β	LCS does not enter the cointegrating relationship.		

Table 20: Hypothesis Test Results of the Supply Function

	H_S^d	H_{S1}^β	H_{S2}^β	H_{S3}^β	H_{S4}^β	H_{S5}^β	H_{S6}^β	H_{S7}^β
df	1	5	1	1	1	1	1	1
LR Statistic	16.757	3.053	24.124	2.418	43.997	68.836	70.016	118.443
P-Value	0.000***	0.081*	0.000***	0.120	0.000***	0.000***	0.000***	0.000***
	H_{S1}^α	H_{S2}^α	H_{S3}^α	H_{S4}^α	H_{S5}^α	H_{S6}^α		
df	1	1	1	1	1	1		
LR Statistic	18.736	6.961	37.494	18.479	2.635	38.463		
P-Value	0.000***	0.031**	0.000***	0.000***	0.268	0.000***		

Note: (i) *: significant at the 10% level, **: significant at the 5% level, ***: significant at 1% level; (ii) df denotes the degree of freedom; (iii) LR is the abbreviation for the Likelihood Ratio test;

Estimated Restricted Supply-driven FCVAR model:

$$\Delta^{\hat{d}} \begin{pmatrix} RHP^S \\ LHUC \\ EPU \\ LIR \\ RLV \\ LCS \end{pmatrix} - \begin{pmatrix} 6.273 \\ -1.558 \\ -15.197 \\ -9.981 \\ 3.584 \\ -18.961 \end{pmatrix} = L_{\hat{d}} \begin{pmatrix} -0.204 \\ -0.288 \\ 0.506 \\ 0.811 \\ 0.000 \\ -0.672 \end{pmatrix} \nu_t + \sum_{i=1}^5 \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (X_t - \hat{\rho}) + \hat{\varepsilon}_t \quad (42)$$

$$\hat{d} = 0.868, Q_\varepsilon(12) = 350.796, \text{Log}L = -2288.045$$

(0.025) (0.998)

The Supply-driven Equilibrium Relationship (long-run):

$$RHP_t^{S*} = -0.631 - 0.816 \times LHUC_t + 0.060 \times EPU_t + 0.156 \times LIR_t + 1.282 \times RLV_t - 0.184 \times LCS_t + \nu_t \quad (43)$$

References

- Abdallah, C. S. and Lastrapes, W. D. (2013), 'Evidence on the relationship between housing and consumption in the United States: A state-level analysis', *Journal of Money, Credit and Banking* **45**(4), 559–590.
- Abraham, J. M. and Hendershott, P. H. (1996), 'Bubbles in metropolitan housing markets', *Journal of Housing Research* **7**(2), 191.
- Arestis, P. and Gonzalez-Martinez, A. R. (2016), 'House prices and current account imbalances in OECD countries', *International Journal of Finance & Economics* **21**(1), 58–74.
- Baker, S. R., Bloom, N. and Davis, S. J. (2016), 'Measuring economic policy uncertainty', *The Quarterly Journal of Economics* **131**(4), 1593–1636.
- Carlini, F. and de Magistris, P. S. (2017), 'On the identification of fractionally cointegrated VAR models with the $F(d)$ condition', *Journal of Business & Economic Statistics* pp. 1–13. Forthcoming.
- Carstensen, K. (2006), 'Stock market downswing and the stability of european monetary union money demand', *Journal of Business & Economic Statistics* **24**(4), 395–402.
- Case, K. E. and Shiller, R. J. (1989), 'The efficiency of the market for single-family homes', *The American Economic Review* **79**(1), 125–137.
- Case, K. E. and Shiller, R. J. (1990), 'Forecasting prices and excess returns in the housing market', *Journal of American Real Estate Urban Economic Association* **18**(3), 253–273.
- Cesa-Bianchi, A. (2013), 'Housing cycles and macroeconomic fluctuations: A global perspective', *Journal of International Money and Finance* **37**, 215–238.
- Cogley, T. and Sargent, T. J. (2001), 'Evolving post-world war II US inflation dynamics', *NBER Macroeconomics Annual* **16**, 331–373.
- Cogley, T. and Sargent, T. J. (2005), 'Drifts and volatilities: monetary policies and outcomes in the post WWII US', *Review of Economic Dynamics* **8**(2), 262–302.
- Davis, M. A. and Heathcote, J. (2007), 'The price and quantity of residential land in the United States', *Journal of Monetary Economics* **54**(8), 2595–2620.
- DiPasquale, D. (1999), 'Why don't we know more about housing supply?', *The Journal of Real Estate Finance and Economics* **18**(1), 9–23.
- DiPasquale, D. and Wheaton, W. C. (1994), 'Housing market dynamics and the future of housing prices', *Journal of Urban Economics* **35**(1), 1–27.

- Docker, S. and Willoughby, D. (1999), 'Monetary statistics and the monetary financial institutions consolidated balance sheet', *Bank of England Monetary and Financial Statistics* **July**, 7–13.
URL: webarchive.nationalarchives.gov.uk/20170823154433/www.bankofengland.co.uk/statistics/Documents/ms/articles/art2jl99.pdf
- Duan, K., Mishra, T. and Parhi, M. (2018a), 'Space matters: Understanding the real effects of macroeconomic variations in cross-country housing price movements', *Economics Letters* **163**, 130–135.
- Duan, K., Mishra, T., Parhi, M. and Wolfe, S. (2018b), 'How effective are policy interventions in a spatially-embedded international real estate market?', *The Journal of Real Estate Finance and Economics* . forthcoming.
- Engle, R. F. and Granger, C. W. (1987), 'Co-integration and error correction: Representation, estimation, and testing', *Econometrica: Journal of the Econometric Society* **55**(2), 251–276.
- Eubank Jr, A. A. and Sirmans, C. (1979), 'The price adjustment mechanism for rental housing in the united states', *The Quarterly Journal of Economics* pp. 163–168.
- Fair, R. C. (1972), 'Disequilibrium in housing models', *The Journal of Finance* **27**(2), 207–221.
- Fama, E. F. (1970), 'Efficient capital markets: A review of theory and empirical work', *The Journal of Finance* **25**(2), 383–417.
- Favara, G. and Imbs, J. (2015), 'Credit supply and the price of housing', *The American Economic Review* **105**(3), 958–992.
- Fitzpatrick, T. and McQuinn, K. (2007), 'House prices and mortgage credit: Empirical evidence for Ireland', *The Manchester School* **75**(1), 82–103.
- Gerlach, S. and Peng, W. (2005), 'Bank lending and property prices in Hong Kong', *Journal of Banking & Finance* **29**(2), 461–481.
- Glaeser, E. L., Gyourko, J. and Saiz, A. (2008), 'Housing supply and housing bubbles', *Journal of Urban Economics* **64**(2), 198–217.
- Granger, C. W. and Joyeux, R. (1980), 'An introduction to long-memory time series models and fractional differencing', *Journal of Time Series Analysis* **1**(1), 15–29.
- Green, R. K., Malpezzi, S. and Mayo, S. K. (2005), 'Metropolitan-specific estimates of the price elasticity of supply of housing, and their sources', *American Economic Review* **95**(2), 334–339.
- Hamilton, J. D. (1994), *Time Series Analysis*, Vol. 2, Princeton: Princeton University Press.
- Hamilton, J. D. (2017), 'Why you should never use the Hodrick-Prescott filter', *Review of Economics and Statistics*, *Forthcoming* . Forthcoming.

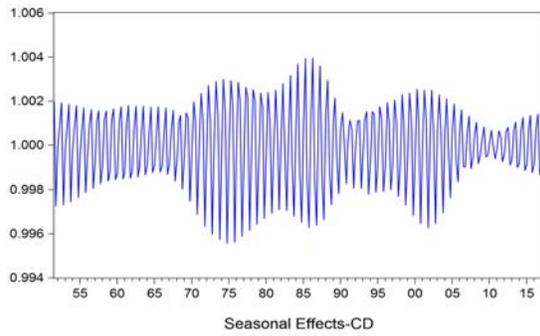
- Heath, S. (2014), 'Housing demand and need (England)', *England: House of Commons Library* .
- Hjalmarsson, E. and Hjalmarsson, R. (2009), 'Efficiency in housing markets: Which home buyers know how to discount?', *Journal of Banking & Finance* **33**(11), 2150–2163.
- Hodrick, R. J. and Prescott, E. C. (1997), 'Postwar US business cycles: An empirical investigation', *Journal of Money, Credit, and Banking* **29**(1), 1–16.
- Hwang, M. and Quigley, J. M. (2006), 'Economic fundamentals in local housing markets: Evidence from US metropolitan regions', *Journal of Regional Science* **46**(3), 425–453.
- Igan, D. and Loungani, P. (2012), 'Global housing cycles', *IMF Working Paper: Global Housing Cycles* **12**(217).
- Johansen, S. (1995), *Likelihood-based inference in cointegrated vector autoregressive models*, New York: Oxford University Press.
- Johansen, S. (2008), 'A representation theory for a class of vector autoregressive models for fractional processes', *Econometric Theory* **24**(03), 651–676.
- Johansen, S. and Nielsen, M. Ø. (2010), 'Likelihood inference for a nonstationary fractional autoregressive model', *Journal of Econometrics* **158**(1), 51–66.
- Johansen, S. and Nielsen, M. Ø. (2012), 'Likelihood inference for a fractionally cointegrated vector autoregressive model', *Econometrica* **80**(6), 2667–2732.
- Johansen, S. and Nielsen, M. Ø. (2016), 'The role of initial values in conditional sum-of-squares estimation of nonstationary fractional time series models', *Econometric Theory* **32**(5), 1095–1139.
- Jones, M. E., Nielsen, M. Ø. and Popiel, M. K. (2014), 'A fractionally cointegrated VAR analysis of economic voting and political support', *Canadian Journal of Economics/Revue canadienne d'économique* **47**(4), 1078–1130.
- Knoll, K., Schularick, M. and Steger, T. (2017), 'No price like home: Global house prices, 1870–2012', *American Economic Review* **107**(2), 331–53.
- Kuensch, H. R. (1987), Statistical aspects of self-similar processes, in 'Proceedings of the First World Congress of the Bernoulli Society', Vol. 1, VNU Science Press Utrecht, pp. 67–74.
- Kumar, M. S. and Okimoto, T. (2007), 'Dynamics of persistence in international inflation rates', *Journal of Money, Credit and Banking* **39**(6), 1457–1479.
- Larsen, E. R. and Weum, S. (2008), 'Testing the efficiency of the norwegian housing market', *Journal of Urban Economics* **64**(2), 510–517.
- Leung, C. K. Y., Leong, Y. C. F. and Wong, S. K. (2006), 'Housing price dispersion: An empirical investigation', *The Journal of Real Estate Finance and Economics* **32**(3), 357–385.

- Ling, D. C., Naranjo, A. and Scheick, B. (2016), 'Credit availability and asset pricing dynamics in illiquid markets: Evidence from commercial real estate markets', *Journal of Money, Credit and Banking* **48**(7), 1321–1362.
- Lucas, R. E. J. (1981), *Studies in business-cycle theory*, Cambridge, Mass: The MIT Press.
- MacKinnon, J. G. and Nielsen, M. Ø. (2014), 'Numerical distribution functions of fractional unit root and cointegration tests', *Journal of Applied Econometrics* **29**(1), 161–171.
- Mankiw, N. G. and Weil, D. N. (1989), 'The baby boom, the baby bust, and the housing market', *Regional Science and Urban Economics* **19**(2), 235–258.
- McCarthy, J. and Peach, R. W. (2002), 'Monetary policy transmission to residential investment', *Federal Reserve Bank of New York Economic Policy Review* **8**(1), 139–158.
- Meen, G. (1996), 'Ten propositions in UK housing macroeconomics: An overview of the 1980s and early 1990s', *Urban Studies* **33**(3), 425–444.
- Meen, G. P. (1990), 'The removal of mortgage market constraints and the implications for econometric modelling of UK house prices', *Oxford Bulletin of Economics and Statistics* **52**(1), 1–23.
- Meen, G. P. (1993), *The Treatment of House Prices in Macroeconometric Models: A Comparison Exercise*, Great Britain, Department of the Environment, Housing and Urban Monitoring & Analysis.
- Mian, A. and Sufi, A. (2009), 'The consequences of mortgage credit expansion: Evidence from the US mortgage default crisis', *The Quarterly Journal of Economics* **124**(4), 1449–1496.
- Muellbauer, J. and Murphy, A. (1997), 'Booms and busts in the UK housing market', *The Economic Journal* **107**(445), 1701–1727.
- Muellbauer, J. and Murphy, A. (2008), 'Housing markets and the economy: The assessment', *Oxford Review of Economic Policy* **24**(1), 1–33.
- Murphy, A. (2018), 'A dynamic model of housing supply', *American Economic Journal: Economic Policy* .
- Nguyen, D. B. B., Prokopczuk, M. and Sibbertsen, P. (2019), 'The memory of stock return volatility: Asset pricing implications', *Journal of Financial Markets* .
- Nielsen, M. Ø. and Popiel, M. K. (2018), A Matlab program and user's guide for the fractionally cointegrated VAR model, Technical report, Queen's Economics Department Working Paper No. 1330.
- Nielsen, M. Ø. and Shibaev, S. S. (2018), 'Forecasting daily political opinion polls using the fractionally cointegrated vector auto-regressive model', *Journal of the Royal Statistical Society: Series A (Statistics in Society)* **181**(1), 3–33.

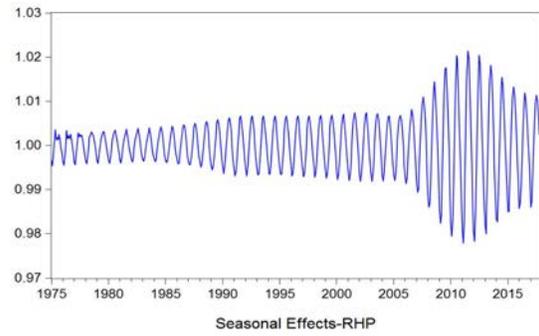
- Poterba, J. M. (1984), 'Tax subsidies to owner-occupied housing: An asset-market approach', *The Quarterly Journal of Economics* **99**(4), 729–752.
- Quigley, J. M. (1979), 'What have we learned about urban housing markets', *Current Issues in Urban Economics* pp. 391–429.
- Riddel, M. (2004), 'Housing-market disequilibrium: an examination of housing-market price and stock dynamics 1967–1998', *Journal of Housing Economics* **13**(2), 120–135.
- Robinson, P. M. (1995), 'Gaussian semiparametric estimation of long range dependence', *The Annals of statistics* **23**(5), 1630–1661.
- Saiz, A. (2010), 'The geographic determinants of housing supply', *The Quarterly Journal of Economics* **125**(3), 1253–1296.
- Schulz, R. and Werwatz, A. (2011), 'Is there an equilibrating relationship between house prices and replacement cost? empirical evidence from berlin', *Journal of Urban Economics* **69**(3), 288–302.
- Senhadji, A. S. and Collyns, C. (2002), Lending Booms, Real Estate Bubbles and The Asian Crisis, Technical report, IMF Working paper No. 02/20.
- Shimotsu, K. (2010), 'Exact local whittle estimation of fractional integration with unknown mean and time trend', *Econometric Theory* **26**(2), 501–540.
- Shimotsu, K., Phillips, P. C. et al. (2005), 'Exact local whittle estimation of fractional integration', *The Annals of Statistics* **33**(4), 1890–1933.
- Taylor, J. B. (2000), 'Low inflation, pass-through, and the pricing power of firms', *European Economic Review* **44**(7), 1389–1408.
- Tkacz, G. (2001), 'Estimating the fractional order of integration of interest rates using a wavelet OLS estimator', *Studies in Nonlinear Dynamics & Econometrics* **5**(1).
- Tsatsaronis, K. and Zhu, H. (2004), 'What drives housing price dynamics: cross-country evidence1', *BIS Quarterly Review* **March**, 65–78.
- Tschernig, R., Weber, E. and Weigand, R. (2013), *Fractionally Integrated VAR Models with a Fractional Lag Operator and Deterministic Trends: Finite Sample Identification and Two-Step Estimation*, Technical report, Regensburg Discussion Paper 471, University of Regensburg.
- UK Office for National Statistics (2007), *Guide to Seasonal Adjustment with X-12-ARIMA*, Draft, ONS Methodology and Statistical Development, UK Office for National Statistics.
- United Nations (2008), *System of national accounts 2008*, United Nations.

Figure 1: The Seasonal Effects of Variables in the US

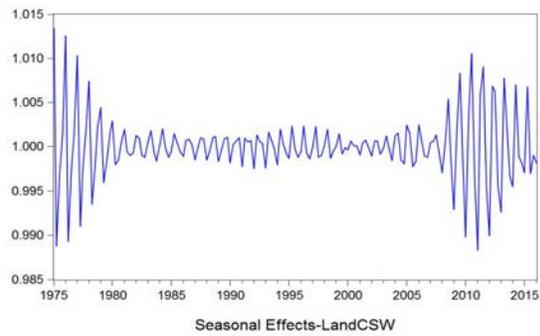
(a) Credit to the housing demand



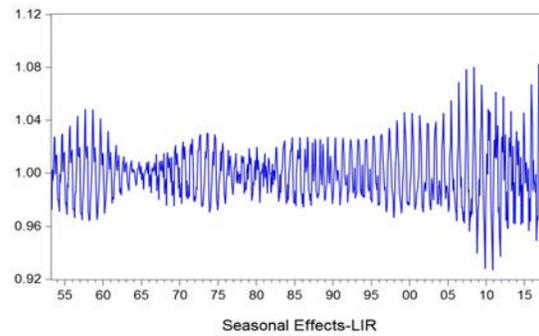
(b) Residential Housing Prices



(c) Residential land value



(d) Long-run interest rate



(e) Economic Policy Uncertainty

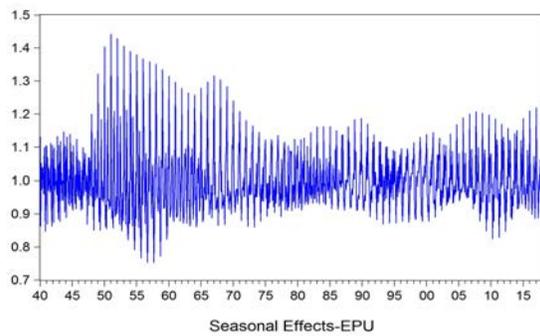
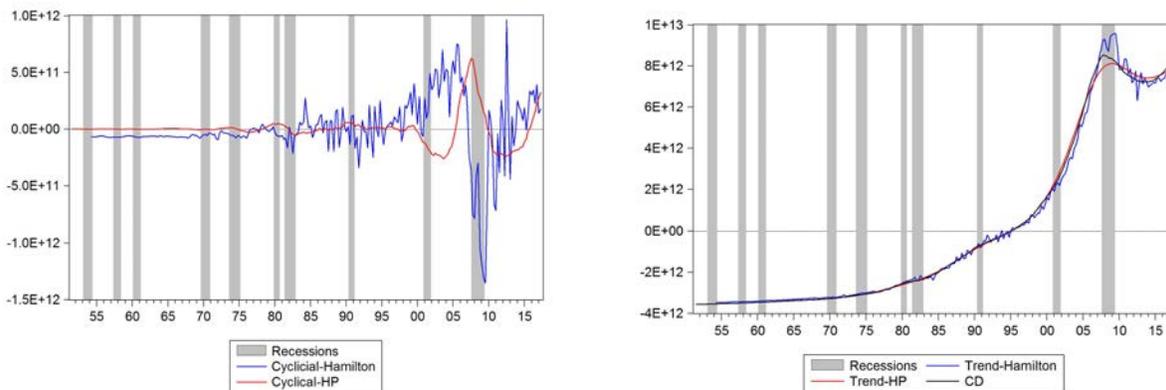
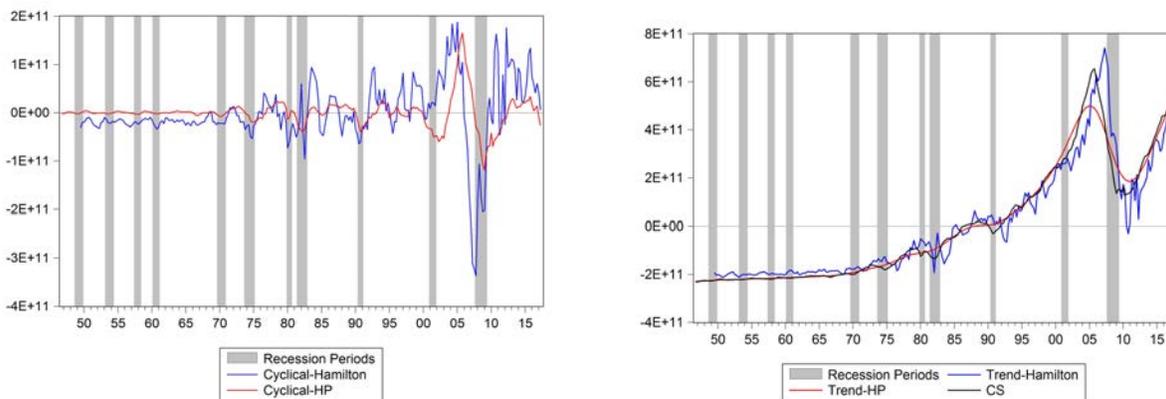


Figure 2: Cycles and Trends of Variables in the US (1)

(a) Credit to the housing demand



(b) Credit to the housing supply



(c) Residential housing prices

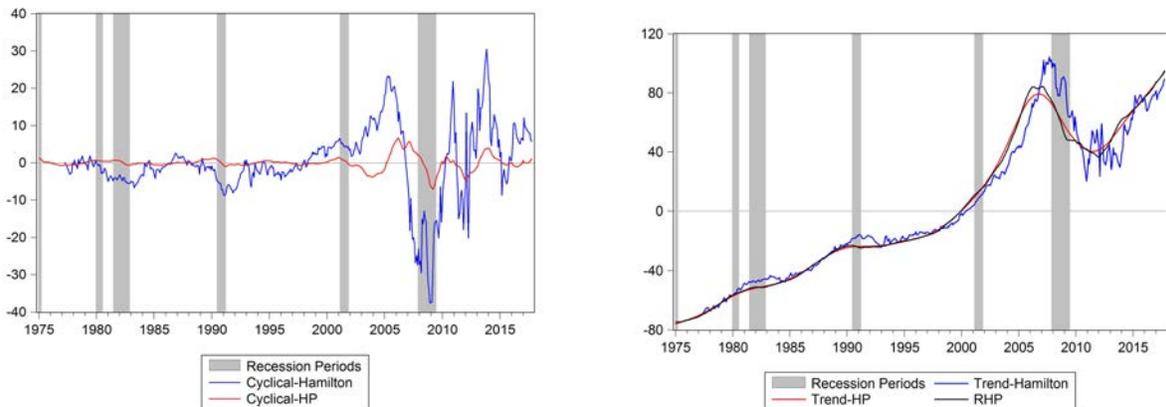
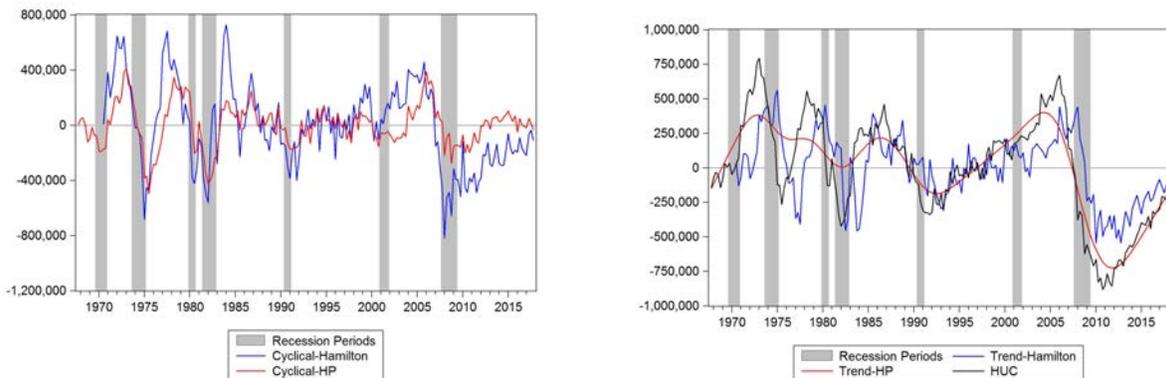
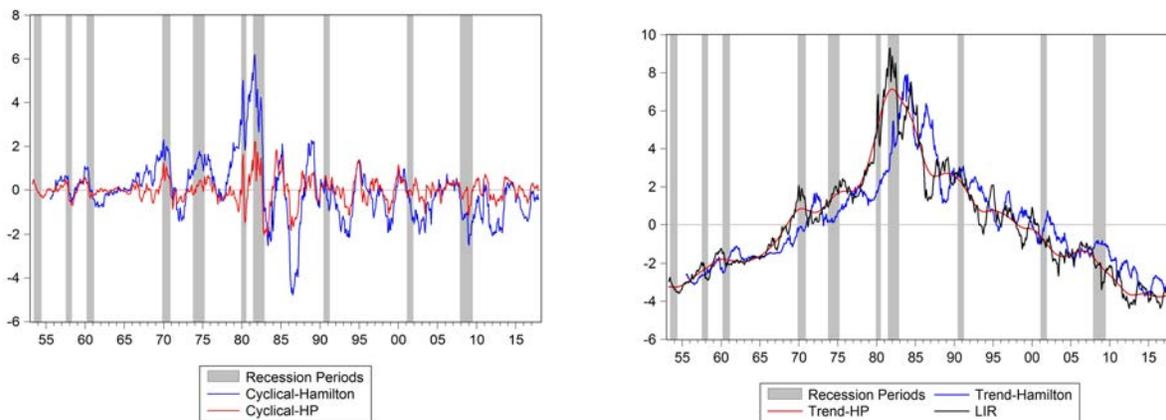


Figure 3: Cycles and Trends of Variables in the US (2)

(a) Residential housing stocks



(b) Long-term interest rate



(c) Inflation

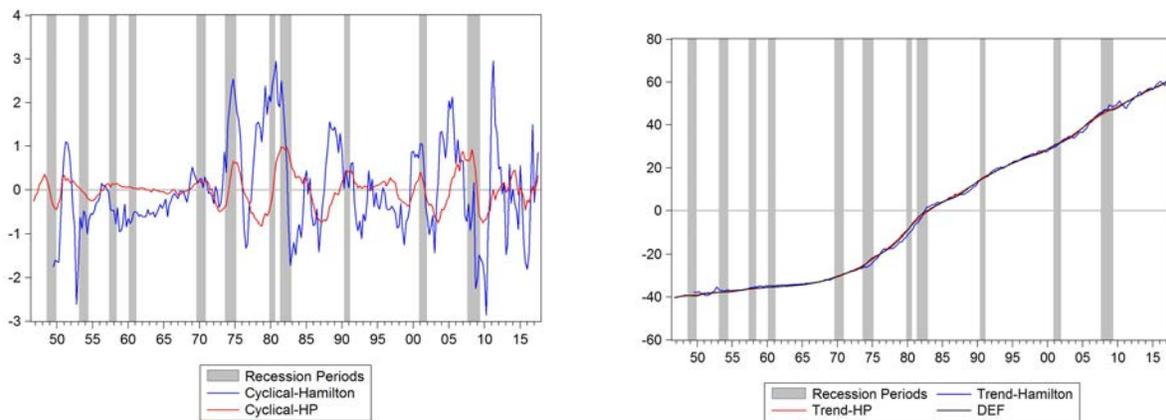
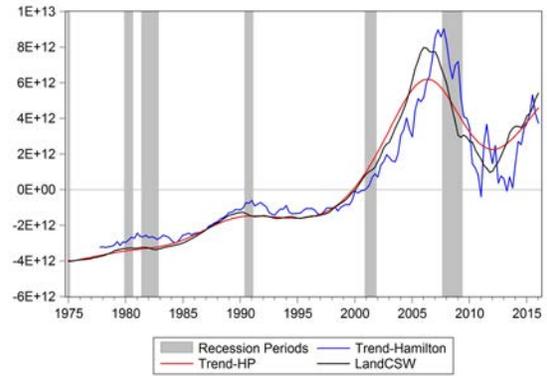
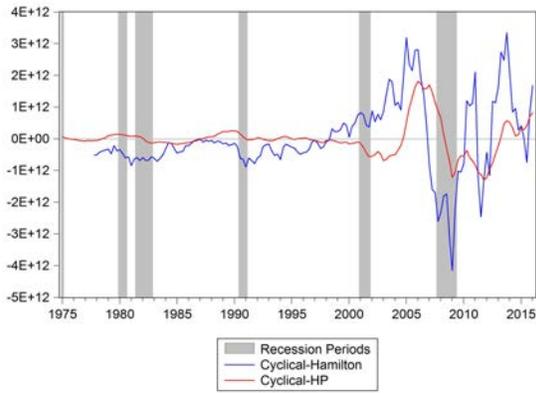


Figure 4: Cycles and Trends of Variables in the US (3)

(a) Residential land value



(b) Economic policy uncertainty

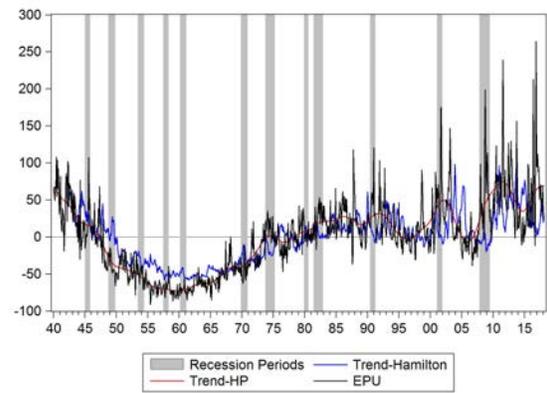
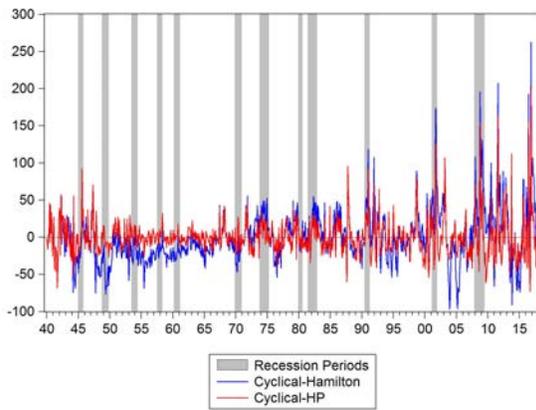
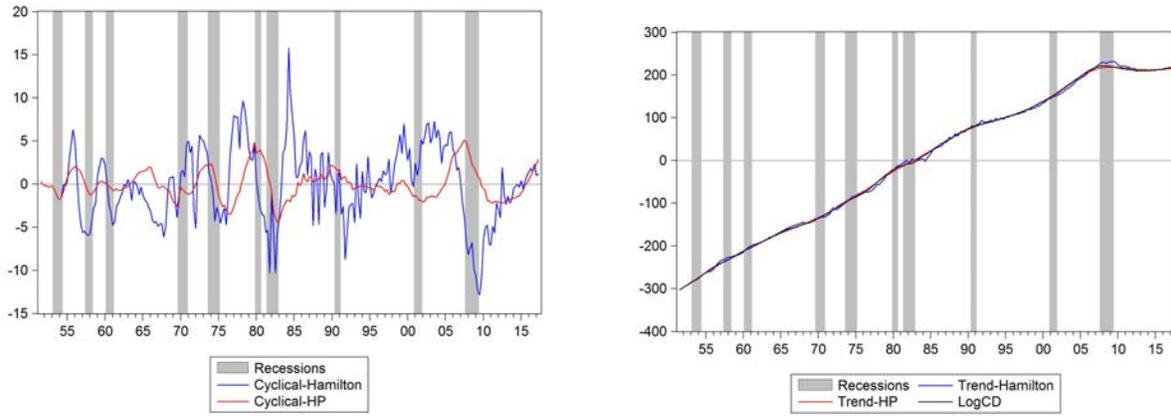
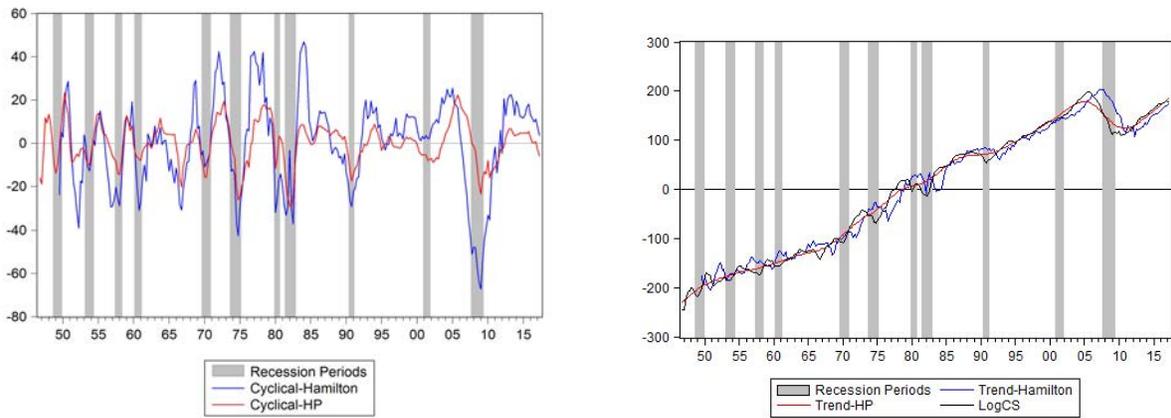


Figure 5: Cycles and Trends of Log-transformed Variables in the US (1)

(a) Credit to the housing demand



(b) Credit to the housing supply



(c) Residential housing prices

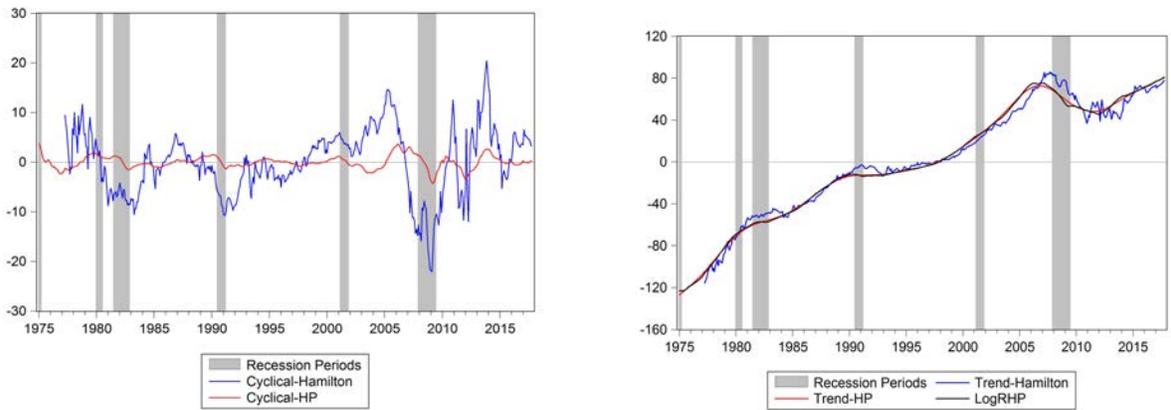
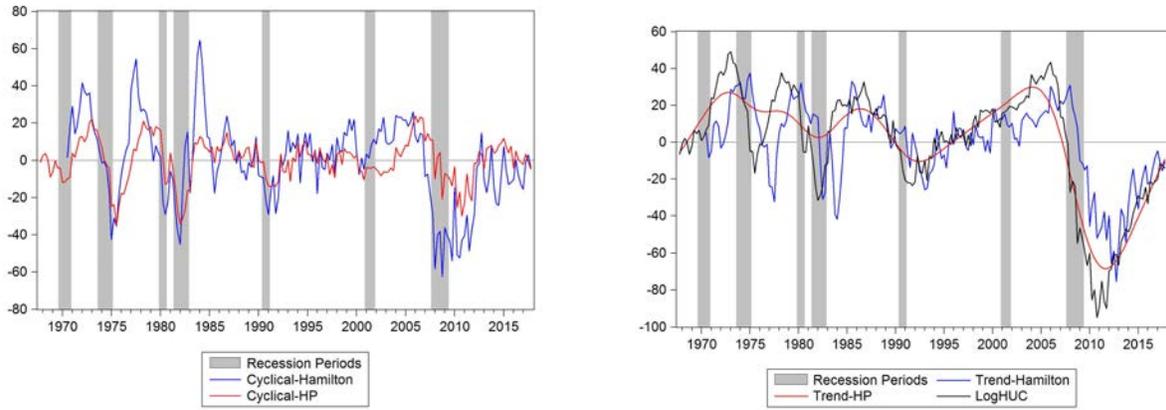
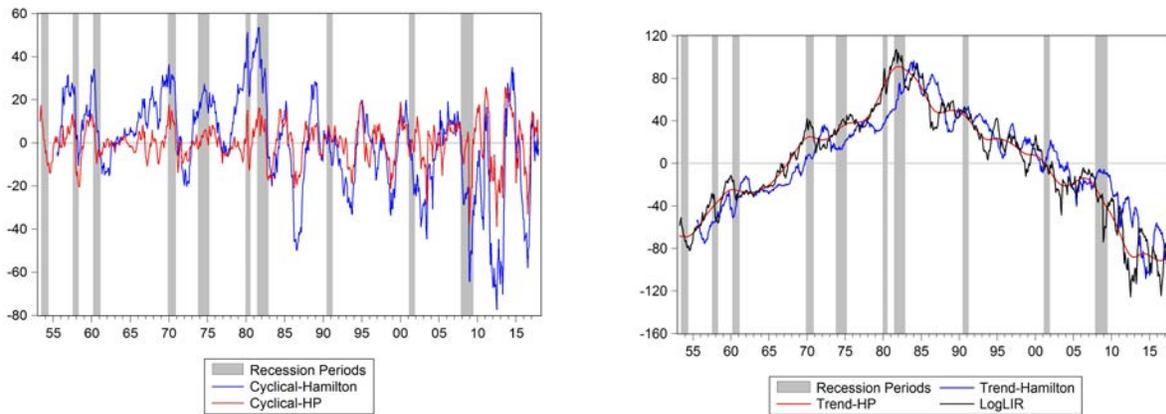


Figure 6: Cycles and Trends of Log-transformed Variables in the US (2)

(a) Residential housing stocks



(b) Long-term interest rate



(c) Inflation

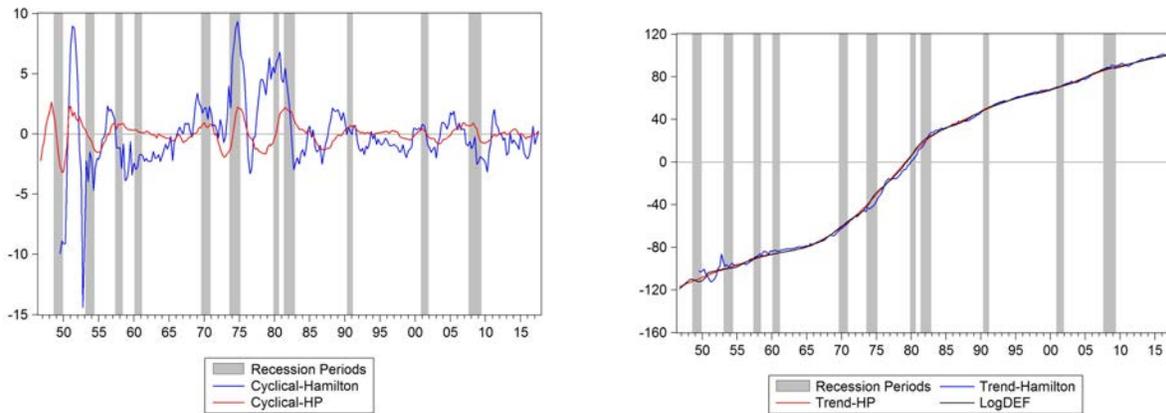
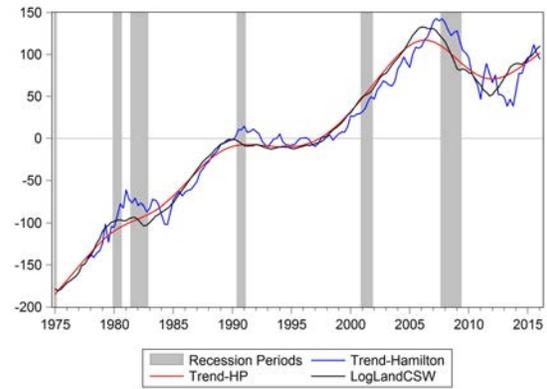
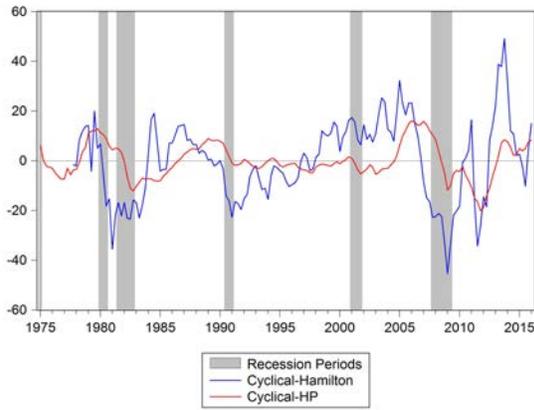


Figure 7: Cycles and Trends of Log-transformed Variables in the US (3)

(a) Residential land value



(b) Economic policy uncertainty

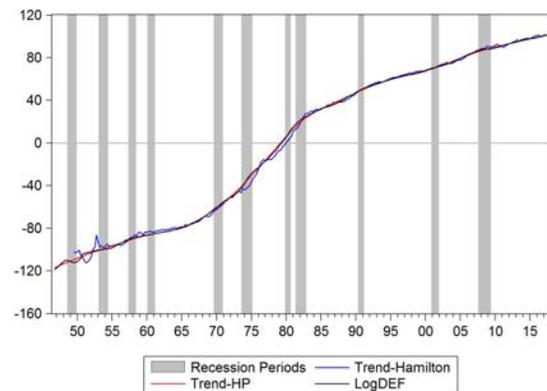
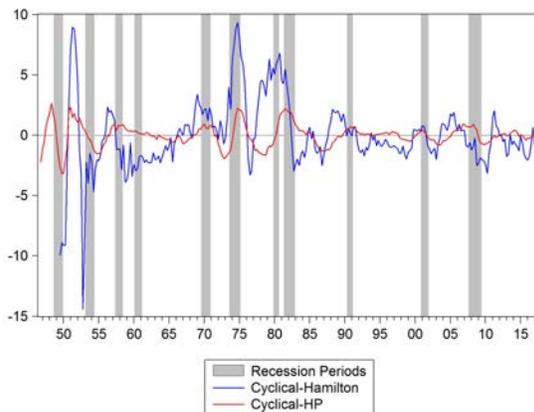
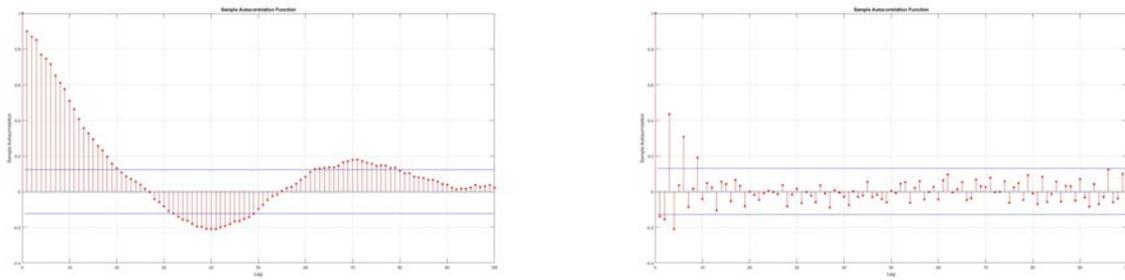


Figure 8: The Comparison between De-cycled and Non De-cycled Variables (1)

(a) Credit to the housing demand



(b) Credit to the housing supply

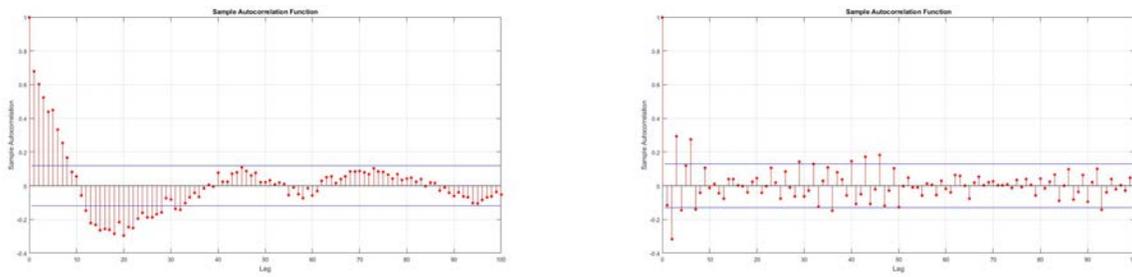
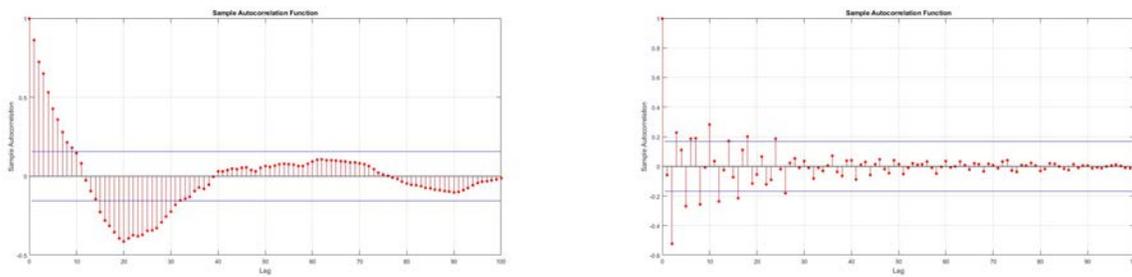


Figure 9: The Comparison between De-cycled and Non De-cycled Variables (2)

(a) Residential land market value



(b) Logged residential land market value

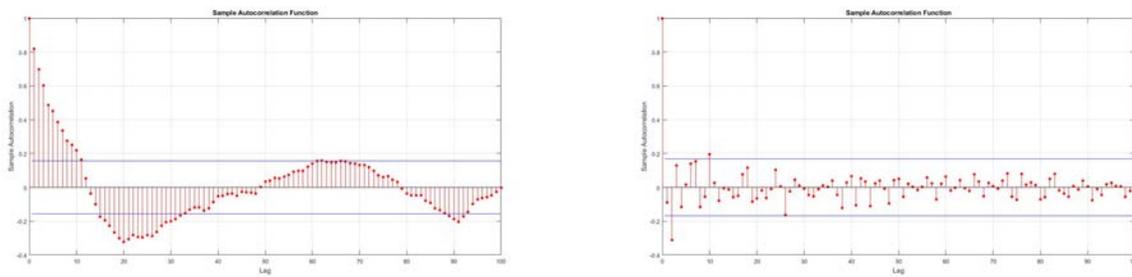
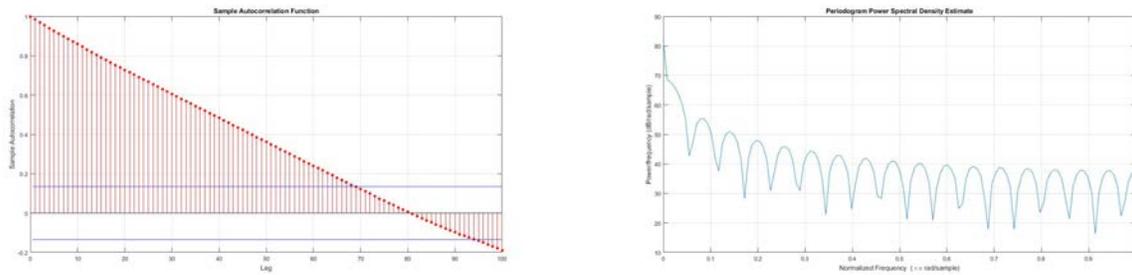


Figure 10: ACF and Spectral Figures (1)

(a) Credit to the demand side



(b) Credit to the supply side

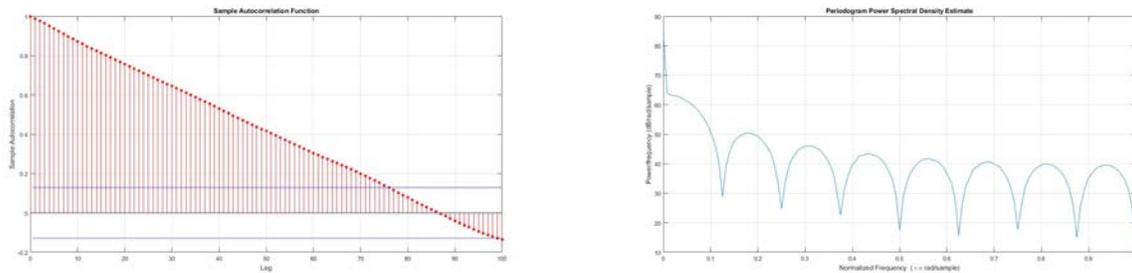
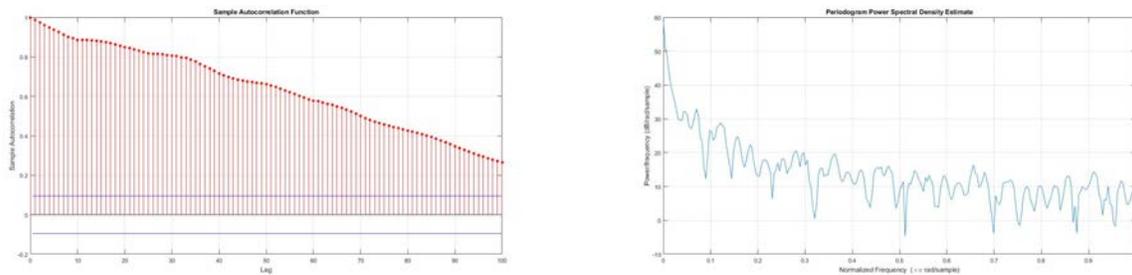


Figure 11: ACF and Spectral Figures (2)

(a) Residential housing prices



(b) Residential housing stocks

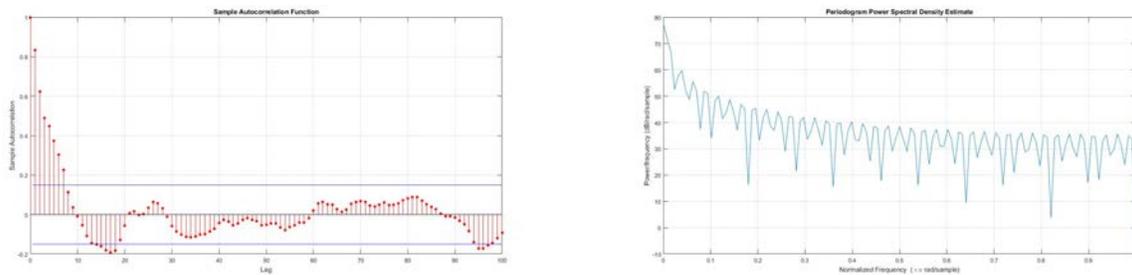
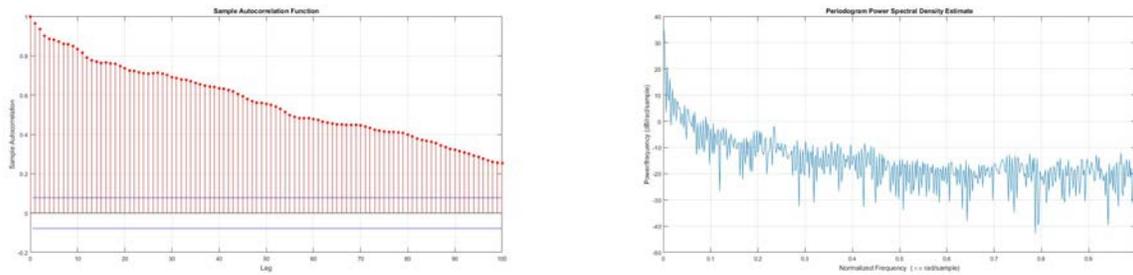


Figure 12: ACF and Spectral Figures (3)

(a) Long interest rate



(b) Inflation

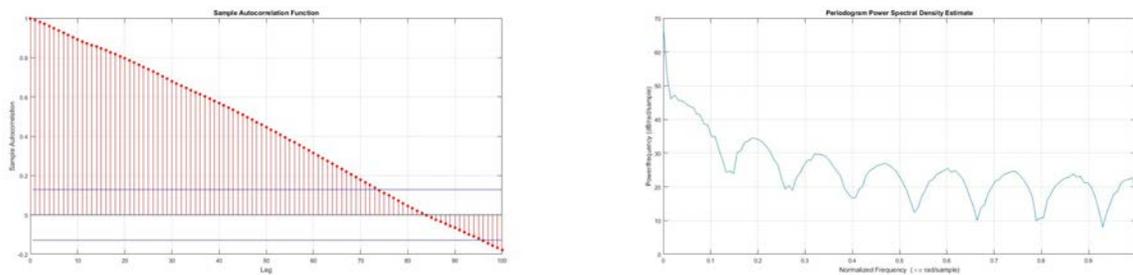
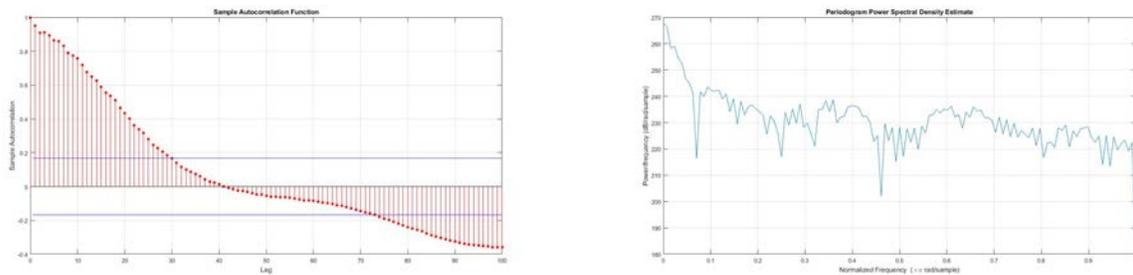


Figure 13: ACF and Spectral Figures (4)

(a) Residential land value



(b) Economic policy uncertainty

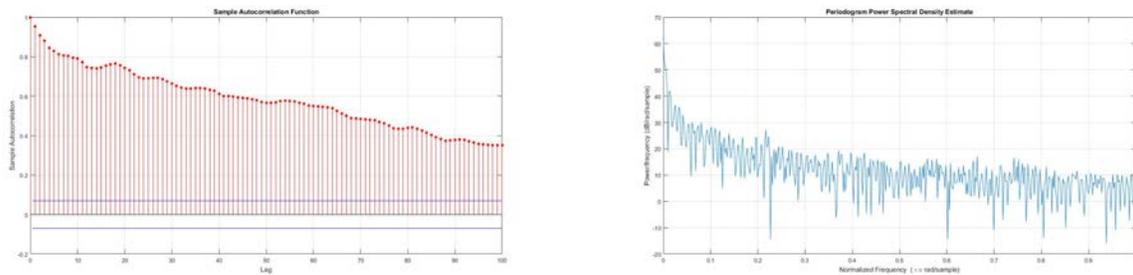
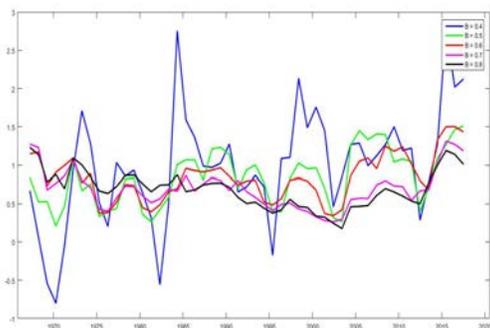
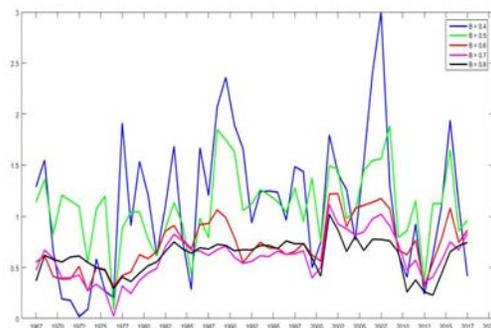


Figure 14: The Rolling-Window Univariate d Estimates-LW (1)

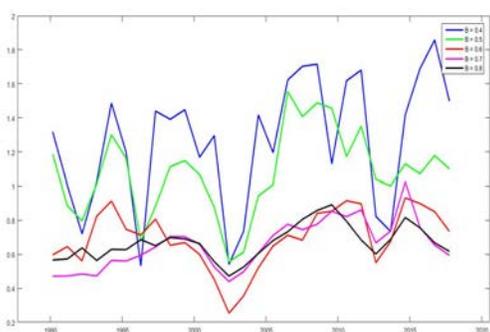
(a) Credit to the housing demand



(b) Credit to the housing supply



(c) Residential housing prices



(d) Residential housing stocks

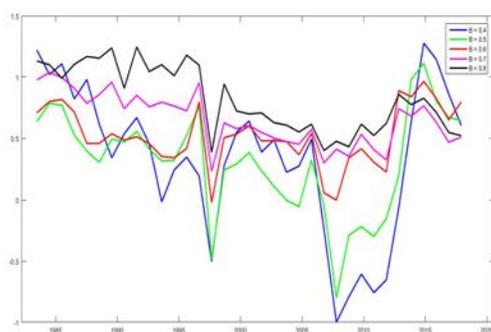
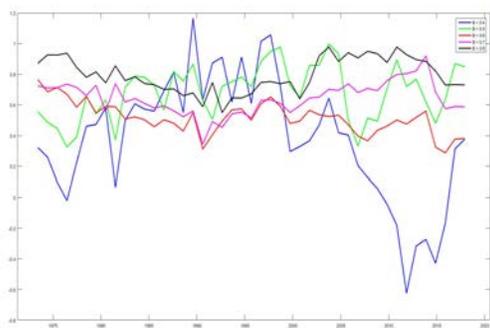
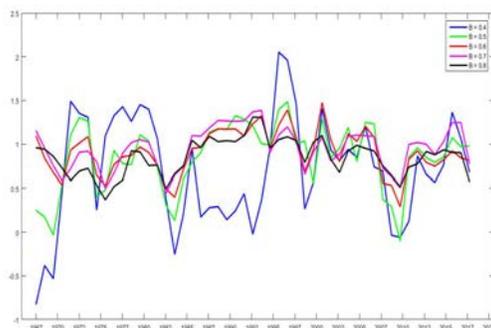


Figure 15: The Rolling-Window Univariate d Estimates-LW (2)

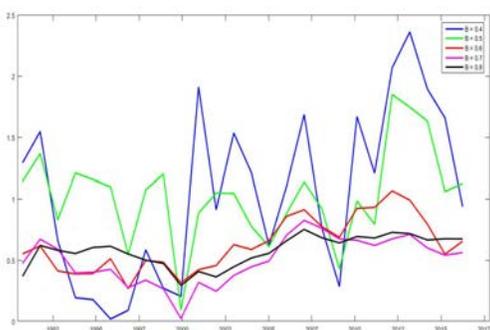
(a) Long-run interest rate



(b) Inflation



(c) Residential land value



(d) Economic policy uncertainty

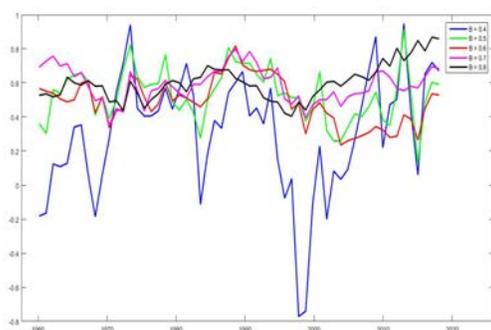
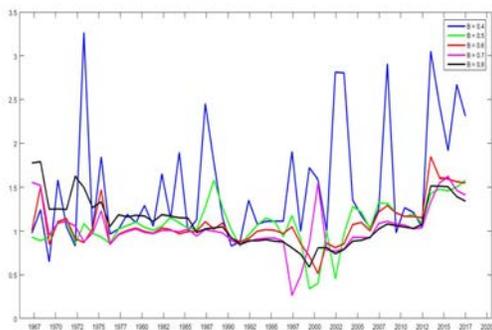
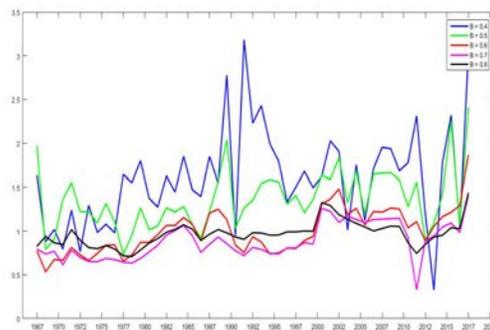


Figure 16: The Rolling-Window Univariate d Estimates-2ELW (1)

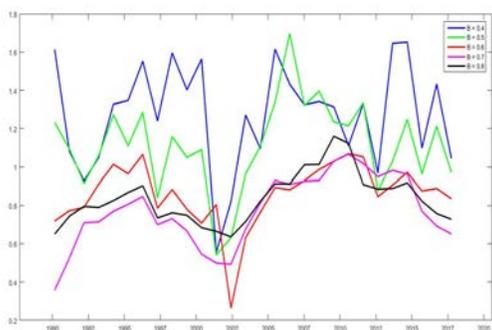
(a) Credit to the housing demand



(b) Credit to the housing supply



(c) Residential housing prices



(d) Residential housing stocks

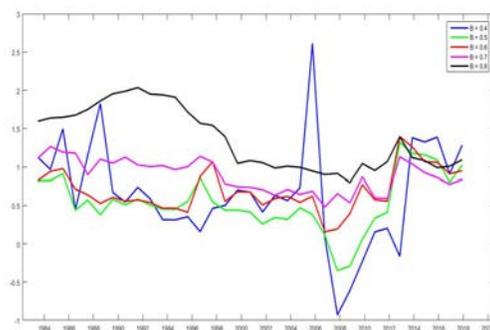
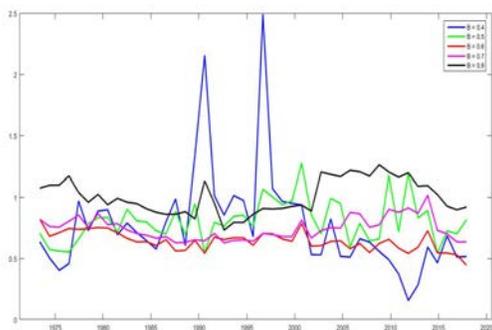
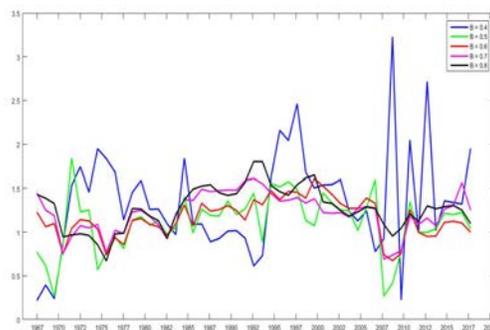


Figure 17: The Rolling-Window Univariate d Estimates-2ELW (2)

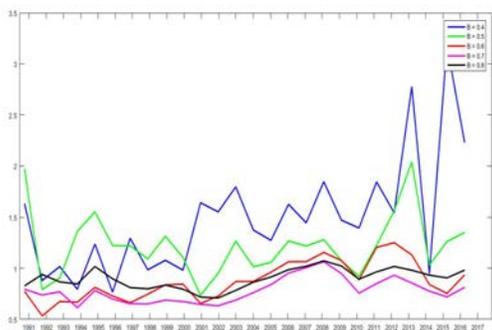
(a) Long-run interest rate



(b) Inflation



(c) Residential land value



(d) Economic policy uncertainty

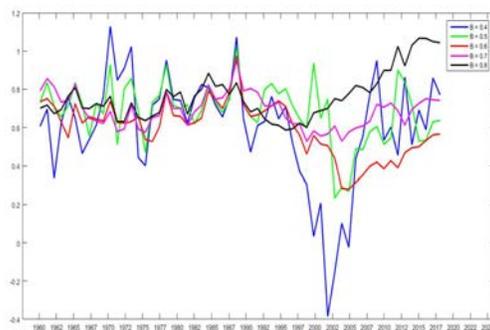
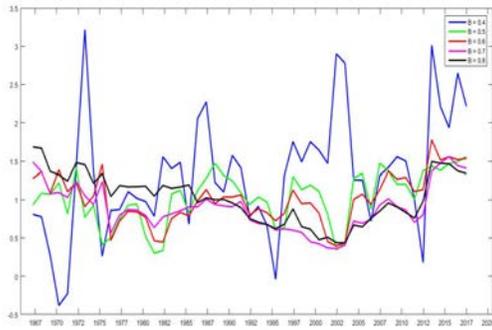
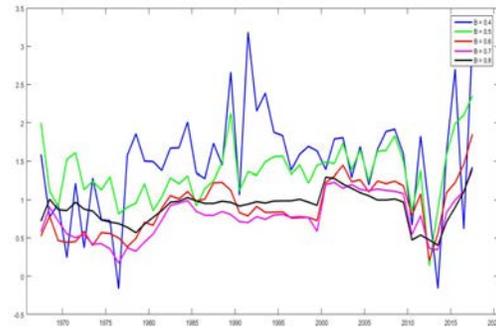


Figure 18: The Rolling-Window Univariate d Estimates-2ELWdm (1)

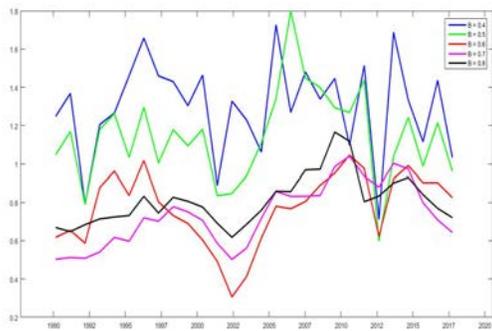
(a) Credit to the housing demand



(b) Credit to the housing supply



(c) Residential housing prices



(d) Residential housing stocks

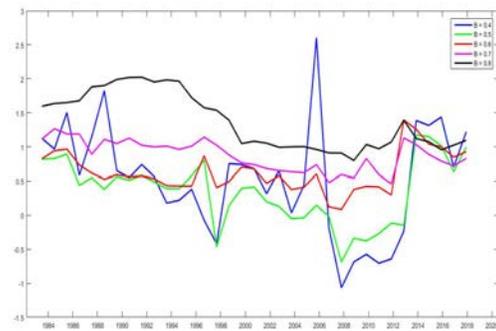
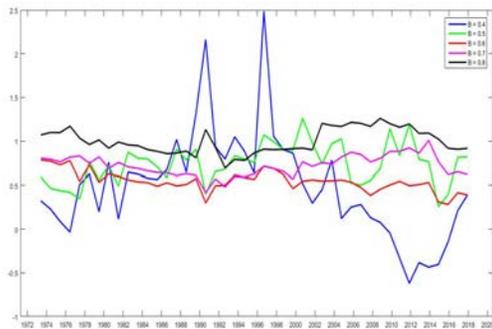
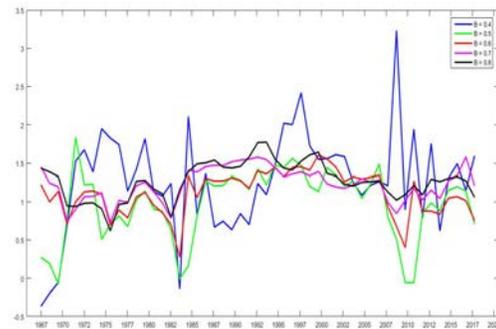


Figure 19: The Rolling-Window Univariate d Estimates-2ELWdm (2)

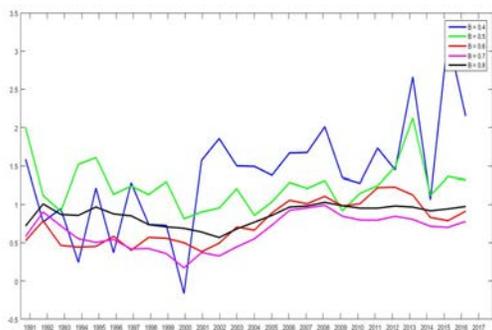
(a) Long-run interest rate



(b) Inflation



(c) Residential land value



(d) Economic policy uncertainty

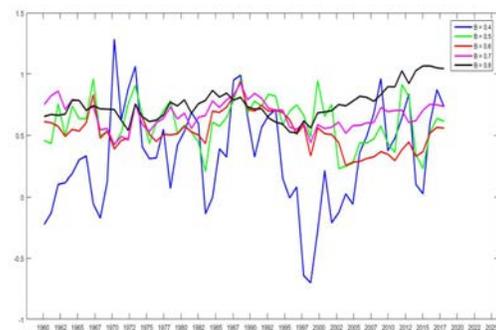


Figure 20: 5-year Ahead Forecasts from Demand Function

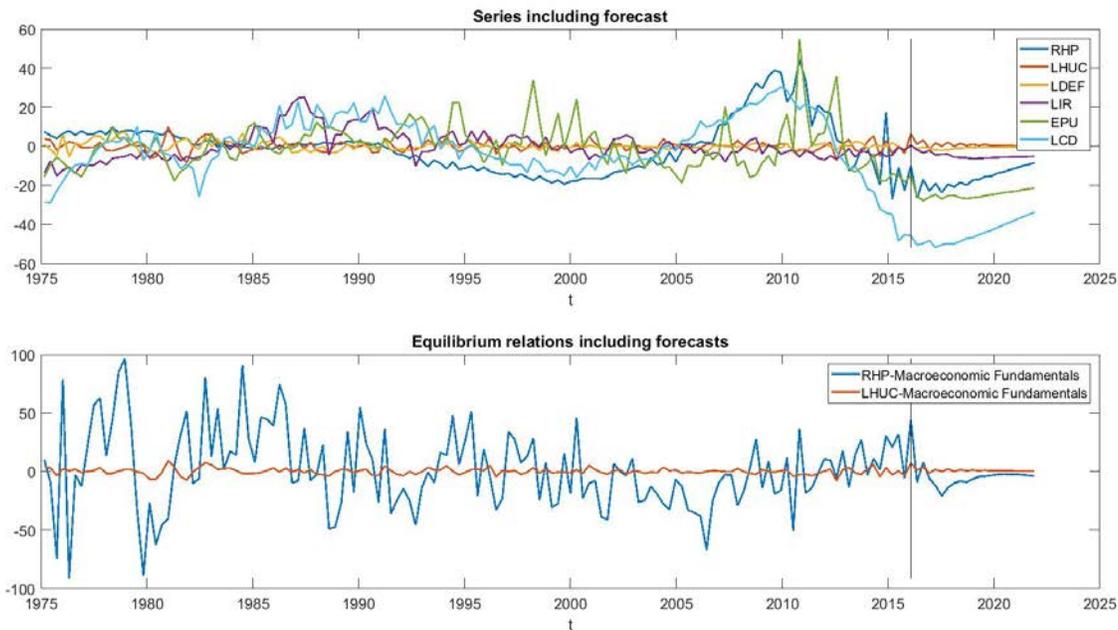


Figure 21: 5-year Ahead Forecasts from Supply Function

