Pitfalls of Central Clearing in the Presence of Systematic Risk*

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Abstract

Through the lens of market participants' objective to minimize counterparty risk, we investigate central clearing in derivatives markets, and its interaction with systematic risk, portfolio directionality, and loss sharing. Previous studies suggest that central clearing always reduces counterparty risk for a sufficiently large number of clearing members. We show that this is not the case – mostly because of loss sharing. Central clearing can increase counterparty risk, particularly during extreme market events, for traders with directional portfolios, and because CCPs mutualize default losses. Our results are consistent with the reluctance to clear derivative trades in the absence of a clearing obligation.

Counterparty risk is the risk that counterparties do not fulfill their future obligations (e.g., when they default). It is one of the most important risks in derivatives markets and has been identified as a major contributor to the amplification of the 2007-08 financial crisis (e.g., Duffie, Li, and Lubke (2010), Acharya, Shachar, and Subrahmanyam (2011), Arora, Gandhi, and Longstaff (2012)). Lehman Brothers' default in particular demonstrated that the failure of an entity with large over-the-counter (OTC) derivative positions can result in substantial risk spillover to its counterparties, creating contagion and systemic risk. To mitigate such systemic risks from derivatives, regulators worldwide promoted the use of Central Clearing Counterparties (CCPs) to centrally clear OTC derivatives transactions (G20 (2009)).[1]

A primary task of CCPs is to insure counterparty risk (Koeppl and Monnet (2010)), which is especially needed during extreme negative macroeconomics conditions. In this paper, we examine how central clearing affects the level and distribution of counterparty risk from a market participant's perspective. We focus in particular on the presence of a systematic risk factor that affects derivatives prices (reflecting, e.g., macroeconomic conditions) and analyze the impact of central clearing on counterparty risk during extreme market events.[2] The main result of our analysis is that traders with directional positions face substantially larger counterparty risk with than without central clearing. However, traders with a flat portfolio substantially benefit from central clearing. The main reason is that directional traders are the one with larger initial margins and therefore face a larger fraction of loss sharing.

Central clearing primarily relies on two components: multilateral netting (i.e., offsetting of gains and losses across clearing members; see, e.g., Duffie and Zhu (2011) and Cont and Kokholm (2014)) and loss sharing (i.e., mutualization of default losses among clearing members; see, e.g., Huang (2018) and Capponi, Wang, and Zhang (2019)). Despite the increasing importance of central clearing in derivatives markets, research on the impact of these mechanisms on counterparty risk is still scarce. We contribute to the literature by isolating and decomposing the impact of (1)

[1] OTC derivatives markets are very large, with a worldwide outstanding notional amount of $542 trillion in 2017 (Bank for International Settlements (BIS)). Before the 2007-08 financial crisis, the derivatives market architecture was dominated by bilateral trades (Financial Stability Board (FSB) (2017)). The G20 initiative in 2009 was followed by the Dodd-Frank Wall Street Reform and Consumer Protection Act (DFA) in 2010 and the European Market Infrastructure Regulation (EMIR) in 2012, with a key element being the mandatory central clearing of standardized OTC derivatives through CCPs.

[2] According to Acharya and Yorulmazer (2008), Nier, Yang, Yorulmazer, and Alentorn (2007), and De Bandt and Hartmann (2000), systematic risk in the sense of exposure to common factors is a key element of systemic risk in financial markets.

systematic risk in derivative prices, (2) portfolio directionality, (3) extreme market events, and (4) loss sharing on the benefits and pitfalls of central clearing from a market participant's point of view. We show that those elements can fundamentally reverse previous results (e.g., Duffie and Zhu (2011), Cont and Kokholm (2014), and Lewandowska (2015)).

First, we consider systematic risk in derivative prices in the sense of a single risk factor that affects the entire derivatives market (reflecting, e.g., macroeconomic conditions). It is well-known that systematic (undiversifiable) risk is not insurable (e.g., Doherty and Dionne (1993)) and, in particular, impairs the ability of CCPs to insure counterparty risk, as hypothesized by Bernanke (1990). Nonetheless, CCPs are tasked to stabilize the financial system particularly in stress times, when macroeconomic conditions deteriorate and systematic shocks hit the financial system. Therefore, systematic risk is a key variable that needs to be considered when one examines the impact of central clearing on the level and distribution of counterparty risk across market participants.[3] In line with Bernanke (1990)'s intuition, our model formally demonstrates that systematic risk impairs the ability of CCPs to reduce counterparty risk – however, this result holds only for traders with directional portfolios, such as end-users. Instead, systematic risk increases the benefit of central clearing for dealers with flat portfolios, since central clearing enables them to harness substantial netting benefits.

Second, we show that, due to the presence of systematic risk, directionality in derivative portfolios becomes highly relevant for the trade-off between central clearing and non-central clearing. In particular, central clearing becomes substantially more favorable for dealers with flat derivative portfolios than for directional traders, e.g., end-users with directional portfolios. For example, market participants with only pay-fixed (or only pay-float) IRS positions (i.e., directional traders) benefit less from centrally clearing IRS than those with both pay-fixed *and* pay-float positions (i.e., dealers with flat positions). Directional portfolios are typically used by end-users (such as insurers, (non-dealer) banks, and hedge funds) to hedge business risks (Abad, Aldasorol, Aymanns, D'Errico, Rousová, Hoffmann, Langfield, Neychev, and Roukny (2016), Siriwardane (2018)). Our finding is

[3]Derivative prices are indeed highly correlated with systematic factors. For example, we empirically find that US index CDS returns exhibit a correlation of 43% with S&P 500 returns and that 19% of their variation is explained by the S&P 500. This finding is in line with other studies: Pan and Singleton (2008) find that over 96% of the variation in sovereign CDS spreads for one reference country (differing, for example, by maturity) is explained by a single factor. Longstaff, Pan, Pedersen, and Singleton (2011) find that 64% of variation in sovereign CDS spreads for different reference countries is explained by a single global factor.

consistent with the reluctance of these end-users to become clearing members in practice (see Bank for International Settlements (BIS) (2018)).

Third, we analyze macroeconomic stress times. For this purpose, we examine counterparty risk conditional on extreme realizations of the systematic risk factor. Our results show that (1) such extreme events amplify the bifurcation between market participants with flat and those with directional portfolios, while (2) multilateral netting opportunities become less relevant and, in most of the cases, are dominated by bilateral netting. More specifically, multilateral netting does *not* reduce but increases directional traders' counterparty risk regardless of the number of counterparties, compared to bilateral netting. For dealers with flat portfolios, the opposite result emerges: multilateral netting reduces their counterparty risk *particularly* during extreme events for any number of counterparties, since flat derivative portfolios mitigate exposure to the systematic risk factor. Therefore, during extreme market events multilateral netting opportunities (coming from the number of counterparties) are irrelevant in determining whether or not multilateral netting reduces counterparty risk: for dealers with flat portfolios, only two counterparties are sufficient for multilateral netting to dominate bilateral netting; however, for end-users with directional portfolios, bilateral netting stays dominant for any number of counterparties. The reason is that the number of counterparties (reflecting multilateral netting opportunities) are dominated by large expected portfolio gains and losses.

Finally, we examine loss sharing, which is at the heart of a CCP's insurance function but has not been examined by previous studies (such as Duffie and Zhu (2011) and Cont and Kokholm (2014)). Specifically, if the CCP's loss upon a clearing member's default exceeds the defaulter's pre-funded resources, surviving clearing members share the remaining loss.[4] For example, the default of a single trader at the Swedish clearinghouse *Nasdaq Clearing AB* caused EUR 107 million to be shared among surviving clearing members in September 2018 (Faruqui, Huang, and Takáts (2018)). Our main results are that loss sharing (1) substantially decreases the likelihood that central clearing reduces a market participant's counterparty risk, and (2) loss sharing amplifies the bifurcation between market participants with flat and those with directional portfolios.

More specifically, conditional on the majority (more than 80%) of realizations of the systematic

[4] Loss sharing is also required by post-crisis regulation as, e.g., in the European Market Infrastructure Regulation (EMIR).

risk factor in our calibrated model, central clearing with loss sharing does *not* reduce counterparty risk for *any* market participant, even those with flat positions, compared to bilateral netting.[5] Instead, a market participant may only benefit from loss sharing if it can off-load an extremely large (bilateral) risk to the CCP and, thereby, offset the additional exposure from sharing other market participants' risks. In less extreme situations, no market participant benefits from central clearing. The reason is that loss sharing smooths counterparty risk across states. In many states, this smoothing increases counterparty risk (similar to an insurance premium) and thereby diminishes multilateral netting benefits.

Overall, we find that, because of loss sharing, for any realization of the systematic risk factor there is a market participant that does not benefit from central clearing, compared to bilateral netting. Instead, as Figure 1 illustrates, in both positive and negative extreme market events, central clearing reduces counterparty risk for dealers and only one type of directional trader (e.g., which is long), while it increases counterparty risk for the other type of directional trader (e.g., which is short). This result even holds when all derivative trades are cleared through one single CCP. Thus, transforming derivatives markets into only centrally cleared trades at one CCP would still be unlikely to reduce counterparty risk compared to bilateral netting. This result contrasts previous studies. For example, not taking loss sharing into account, Duffie and Zhu (2011) suggest to increase the joint clearing of different types of derivatives in order to increase netting efficiency. For any such increase in clearing concentration, our results show that it remains unlikely for market participants to benefit from central clearing in all the cases.

[Place Figure 1 about here]

Overall, our analysis highlights important pitfalls of central clearing that emerge due to the presence of systematic risk, portfolio directionality, and loss sharing, and these were not addressed by previous literature. We identify a large number of realistic situations in which central clearing does not reduce but increase counterparty risk in comparison to bilateral netting, particularly in stress times, for market participants with directional portfolios, and regardless of netting opportu-

[5] Conditional on each realization of the systematic risk factor, we compute counterparty risk as the sum of a market participant's expected bilateral default loss and mutualized CCP contribution (a) when all derivatives are bilaterally netted and (b) when one (or all) derivatives classes are centrally cleared and non-pre-funded losses are shared among clearing members.

nities (i.e., number of clearing members). This result is consistent with the observation that market participants are generally reluctant to centrally clear derivative contracts in practice, unless forced (Financial Stability Board (FSB) (2017, 2018)).[6] Our results thus provide an explanation for low clearing rates based on the impact of clearing on market participants' counterparty risk.

Following previous studies, we take a single market participant's perspective. Although this perspective is only partial (i.e., from market participants' point of view conditional on existing trades), it provides important insights that support policymakers in specifying financial market infrastructure regulation to enhance financial stability.[7] While focusing on counterparty risk, we are aware that there are other important aspects that influence a market participant's decision whether to clear derivatives, such as margin costs, market liquidity, operational risk, or the cost of being a clearing member. Nonetheless, minimizing counterparty risk is a primary objective for market participants' risk management and for the decision whether to clear derivatives (Bellia et al. (2019), Financial Stability Board (FSB) (2018)). For example, Vuillemey (2019) documents that a spike in counterparty risk during the global coffee crisis in 1880-81 motivated a group of well-established coffee traders to create the CCP *Caisse de Liquidation des Affaires en Marchandises* specifically to mitigate counterparty risk.

The remainder of this paper is structured as follows. Section 1 describes the related literature and our contribution. Section 2 presents a stylized model that highlights the trade-off between bilateral and multilateral netting. In Section 3, we study the impact of central clearing on counterparty risk exposure. We add loss sharing to our analysis in Section 4. Section 5 revisits some empirical predictions and main policy implications, and Section 6 concludes. Propositions and proofs are provided in the Appendix.

[6] Central clearing is currently mandatory for standardized interest rate swaps (IRS) contracts and index credit default swaps (CDS) in the US and EU. Instead, clearing is still optional for single-name CDS, foreign exchange forwards, and commodity and equity derivatives, which largely remain uncleared (Abad et al. (2016), Office of the Comptroller of the Currency (2016), Financial Stability Board (FSB) (2017)). The Financial Stability Board (FSB) (2017) reports that only 28% of outstanding CDS notionals were cleared in December 2016 (compared to 5% in June 2009). The fraction of notionals cleared is even smaller than 20% for foreign exchange, commodity, and equity derivatives in 2016. In contrast, 61% of all IRS notionals outstanding were cleared in December 2016 (compared to 24% in December 2008), and 80% of new index CDS transactions in the US are cleared as of April 2017. In 2016, 48% of Italian, German and French Sovereign CDS transactions were cleared (Bellia, Panzica, Pelizzon, and Peltonen (2019)), while 81% of new IRS transactions in 2017 were cleared (Dalla Fontana, Holz auf der Heide, Pelizzon, and Scheicher (2019)).

[7] The ultimate effect of central clearing on financial stability also depends on its contribution to the transparency of derivative markets (Acharya and Bisin (2014)), potential reduction in loss concentration (Lewandowska (2015)), and tightening of financial market infrastructures' risk-management practices.

1 Literature Review

We contribute to a growing literature on central clearing and its role for financial stability. We are complementary to previous studies and, from a market participant's perspective, provide theoretical evidence for pitfalls of central clearing in four dimensions that other studies have not acknowledged until now. These are systematic risk, extreme market events, portfolio directionality, and loss sharing.

Previous studies have examined loss sharing and its interaction with CCP collateral and fee policies (Capponi, Cheng, and Sethuraman (2017), Capponi and Cheng (2018), Huang (2018)) as well as its impact on clearing members' propensity to engage in risk-shifting (Biais, Heider, and Hoerova (2016), Capponi et al. (2019)). In a simulation study, Lewandowska (2015) shows that loss sharing may reduce loss concentration and counterparty risk exposure compared to bilateral netting in the absence of systematic risk, extreme events, or heterogeneous portfolio directionality.

We add to these studies by comparing counterparty risk under central clearing with loss sharing to that with bilateral netting, focusing on (1) directionality of clearing members' portfolios, (2) systematic risk in derivatives prices, and (3) extreme events. We show that loss sharing has an extremely heterogeneous impact on clearing members and benefits mostly dealers with flat portfolios. Moreover, it substantially reduces the likelihood that central clearing reduces a market participant's counterparty risk compared to bilateral netting – regardless of portfolio directionality.

Duffie and Zhu (2011) and Lewandowska (2015) study the impact of multilateral versus bilateral netting on counterparty risk exposure when derivative prices are independently distributed. Their main result is that a sufficiently large number of clearing members guarantees that central clearing reduces counterparty risk. Duffie and Zhu (2011) also show that multilateral netting becomes relatively more beneficial compared to bilateral netting with larger correlation *across* derivative classes.[8] Cont and Kokholm (2014) follow this rationale and study the effect of correlation *across* derivative classes on the benefit of multilateral netting. They conclude that multilateral netting is likely to reduce counterparty risk exposure compared to bilateral netting, in practice.

We contribute to a deeper understanding of netting by shedding light on the role of system-

[8] In their appendix "A. Cross-class exposure correlation", Duffie and Zhu (2011) note that "Increasing the correlation between positions increases the relative netting benefits of a CCP, because between-entity netting is not as beneficial if cross-class exposures are positively correlated" (Duffie and Zhu (2011, p. 91-92)).

atic risk, extreme market events, portfolio directionality, and loss sharing, which have not been considered. We show that accounting for the joint effect of these components, results anticipated from previous studies reverse. Importantly, we show that the presence of systematic risk and loss sharing results in situations in which market participants face larger counterparty risk exposure in centrally cleared than in bilateral markets for *any* number of clearing members, which is not the case in previous studies that ignore systematic risk and loss sharing.

Extreme events are also studied by Huang, Menkveld, and Yu (2019) and Menkveld (2017), who take a CCP's perspective and identify extreme price movements as well as crowding in a small number of risk factors as important risks to CCP stability. Complementing their analyses, we take a market participant's perspective and compare central to bilateral netting with respect to counterparty risk, and show that extreme events and correlation with one risk factor reduce the benefit of central clearing.

Ghamami and Glasserman (2017) study the capital and collateral costs of central clearing and find that there is no cost incentive for single market participants to centrally clear derivatives, which is driven primarily by margin costs in their model. Their result is contrasted by the Financial Stability Board (FSB) (2018)'s assessment that central clearing reforms create an overall incentive to clear. We add to these studies by examining the sensitivity of counterparty risk toward loss sharing, systematic risk, portfolio directionality, and margins. Even without considering these capital and collateral costs, we find that multilateral netting does not always dominate bilateral netting.

Empirical evidence on the impact of central clearing on derivative markets has been growing only recently, fueled by the increasing availability of granular data. Examples, among others, are Loon and Zhong (2014), Duffie, Scheicher, and Vuillemey (2015), Du, Gadgil, Gordy, and Vega (2016), and Bellia et al. (2019) for single-name CDS, Menkveld, Pagnotta, and Zoican (2015) for equity, Mancini, Ranaldo, and Wrampelmeyer (2016) for interbank repo, and Cenedese, Ranaldo, and Vasios (2018) and Dalla Fontana et al. (2019) for IRS markets. In particular, Bellia et al. (2019) provide empirical evidence that dealers typically clear contracts with risky counterparties that result in small CCP margins being paid, i.e., contracts with large netting benefits. Hence, counterparty risk and netting considerations are highly relevant for decisions to centrally clear. This result is consistent with the historical evidence documented by Vuillemey (2019), who shows that a spike in counterparty risk during the global coffee crisis in 1880-81 motivated a group of well-established coffee traders to create the CCP *Caisse de Liquidation des Affaires en Marchandises* specifically to mitigate counterparty risk.

2 A model of central clearing with systematic risk

To identify the impact of netting and loss sharing on counterparty risk, we compare a central clearing architecture with a bilateral market from a market participant's perspective for a given set of derivative trades. Derivative trades are classified in K derivative classes. This classification might result from grouping derivatives according to the type of underlying, such as interest rate, credit, commodities, or equities. One could also more granularly distinguish between derivatives that are sufficiently standardized for central clearing and those that are not. This interpretation will be relevant because we will later assume that a CCP clears all derivative trades within a specific derivative class.

Counterparty risk results from replacement costs during the time between opening and settlement (i.e., close-out) of a derivative contract (Bank for International Settlements (BIS) (1998)). These costs result from changes in contract values during the settlement period, which is the time between the latest exchange of collateral (i.e., variation margin) and the liquidation (i.e., settlement) of a contract portfolio. Clearly, the length of the settlement period depends on the liquidity of contracts as well as the frequency of margin exchange. It typically ranges from 2 to 5 days for centrally cleared products, as these tend to be very liquid and margins are exchanged daily (Arnsdorf (2012)), but might be larger for non-centrally cleared and less liquid positions. Without loss of generality, we consider a one-period model. At time $t = 0$, contracts are exchanged (or, equivalently, all contracts are marked to market by the exchange of variation margin) and, subsequently, counterparties might default. At time $t = 1$, contracts are settled.

[Place Figure 2 about here]

As illustrated in Figure 2, we assume that, during the settlement period, the price change of contracts that market participant i traded with market participant j in derivative class k is given by $X_{ij}^k = v_{ij}^k r_{ij}^k$, where v_{ij}^k reflects the contract volume (i.e., the quantity traded) as well as the direction of trade (i.e., long vs short position). Market participants are called *entities* or *counterparties* hereafter.

r_{ij}^k is the return (at market value, scaled by contract size v_{ij}^k) of all contracts traded between entities i and j in derivative class k during the settlement period. We initially assume that all contract returns are normally distributed with zero mean, $\mathbb{E}[r_{ij}^k] = 0$.[9] Symmetry substantially reduces the dimension of our model and improves its tractability.[10] We consider a single-factor model for contract returns, such that

$$r_{ij}^k = \beta_{ij}^k M + \sigma_{ij}^k \varepsilon_{ij}^k, \tag{1}$$

where $\varepsilon_{ij}^k \sim \mathcal{N}(0,1)$ is idiosyncratic risk. It is $\varepsilon_{ij}^k = \varepsilon_{ji}^k$ (due to symmetry of trades), ε_{ij}^k and ε_{hl}^m are independent for different derivative classes $k \neq m$ and different entity pairs $(h,l) \notin \{(i,j),(j,i)\}$, and ε_{ij}^k is independent from M for all i,j,k.[11] The systematic risk factor $M \sim \mathcal{N}(0, \sigma_M^2)$ serves as a latent variable that reflects the state of the derivatives market (or, more generally, macroeconomic conditions), and β_{ij}^k is the systematic risk exposure of the portfolio of all contracts traded between i and j in derivative class k.[12]

It will be useful to reparametrize r_{ij}^k in terms of the total contract volatility, $\sigma_{X,ij}^k = \sqrt{\mathrm{var}(r_{ij}^k)}$, and correlation with M, $\rho_{X,M,ij}^k = \mathrm{cor}\left(r_{ij}^k, M\right)$, such that $\beta_{ij}^k = \rho_{X,M,ij}^k \frac{\sigma_{X,ij}^k}{\sigma_M}$ and $\sigma_{ij}^k = \sigma_{X,ij}^k \sqrt{1-(\rho_{X,M,ij}^k)^2}$. The correlation between two contracts in classes k and m, traded between i and j, and h and l, then equals $\mathrm{cor}\left(X_{ij}^k,\, X_{hl}^m\right) = sgn\left(v_{ij}^k\, v_{hl}^m\right) \rho_{X,M,ij}^k \rho_{X,M,hl}^m$, where $sgn(x) = |x|/x$ is the signum function. This correlation is positive if i and h have either both long or both short positions, and is negative otherwise. In the following, we will often focus on one market participant i's contract portfolio $\left\{X_{ij}^k : j \in \{1,...,\gamma\}\backslash\{i\},\ k \in \{1,...,K\}\right\}$.

[9] Due to the small time horizon of the settlement period, the risk-free rate and risk premium in derivative prices are negligible. Thus, we assume that they are equal to zero, i.e., $\mathbb{E}[r_{ij}^k] = 0$. Expected returns will, however, be non-zero when we condition on a specific realization of the systematic risk factor.

[10] The assumption of normally distributed bilateral exposures might not be justified for individual contracts, since these often exhibit heavily skewed and fat-tailed market values. However, due to diversification arising from aggregating across underlying names as well as long and short positions across derivatives traded in the same derivative class with the same counterparty, it is reasonable that exposures are substantially less skewed or fat-tailed, particularly for large dealers. The assumption of normality allows us to work with closed-form analytical solutions for the most part of the paper.

[11] Due to symmetry, the gain of i is the loss of j, such that $r_{ij}^k = r_{ji}^k$, and $v_{ij}^k = -v_{ji}^k$.

[12] In the absence of systematic risk, $\beta_{ij}^k \equiv 0$, then our model coincides with Duffie and Zhu (2011)'s baseline model. In their appendix, Duffie and Zhu (2011) also consider correlation across (but not within) asset classes, which corresponds to $\beta_{ij}^k \equiv 0$ and $\mathrm{cor}(\varepsilon_{ij}^k, \varepsilon_{hl}^m) \neq 0$ for $k \neq m$, but $\mathrm{cor}(\varepsilon_{ij}^k, \varepsilon_{hl}^m) = 0$ for $k = m$, $(i,j) \neq (h,l)$.

Throughout the paper, we assume a positive correlation between returns r_{ij}^k and the systematic risk factor M, $\beta_{ij}^k > 0$. This comes without loss of generality, since the final profit and loss, X_{ij}^k, ultimately depends on the long and short position of entities, reflected by the sign of v_{ij}^k. For example, if $v_{ij}^k > 0$, then entity i is long in the systematic risk factor, $\mathrm{cor}(X_{ij}^k, M) > 0$. Since symmetry implies that $v_{ji}^k = -v_{ij}^k$, market participant j is then short in the systematic risk factor, $\mathrm{cor}(X_{ji}^k, M) < 0$, if $v_{ij}^k > 0$. The absolute size $|v_{ij}^k|$ determines the contract volume and thus reflects the notional. Since we will be mainly interested in heterogeneity in portfolio directionality (i.e., the sign of v_{ij}^k), but not heterogeneity in absolute position size, throughout the paper we assume that $|v_{ij}^k| = 1$ for all $i \neq j$.

First, we begin with the model of a bilaterally netted (i.e., non-centrally cleared) market. We assume that all entity pairs have bilateral (close-out) netting agreements with each other. Netting agreements aggregate outstanding positions into one single claim (Bergman, Bliss, Johnson, and Kaufman (2004)) and are common market practice (e.g., Mengle (2010)). Bilateral netting offsets gains and losses of different derivative trades across different derivative classes (e.g., IRS and CDS) with a single counterparty. For example, suppose that entity i trades two contracts with entity j and the value of these contracts is $X_{ij}^1 = -100$ and $X_{ij}^2 = 100$. Without bilateral netting, entity j owes 100 to i on contract 2, and thus i loses 100 if j defaults. Moreover, i is still obligated to pay 100 to j for contract 1 if j defaults. With a bilateral netting agreement, the value of the two contracts is canceled out prior to default. In this example, neither counterparty i or j would suffer a net loss if one of them defaults. Thus, in general, the total counterparty loss of i given a default of j equals the positive value of the sum of contract value changes in the K derivative classes, $\max\left(\sum_{k=1}^K X_{ij}^k, 0\right)$. Trading with $\gamma - 1$ counterparties in a bilateral market with K derivative classes, the total loss of i given default of all its counterparties equals

$$E_i^{BN,K} = \sum_{j=1, j\neq i}^{\gamma} \max\left(\sum_{k=1}^{K} X_{ij}^k, 0\right). \tag{2}$$

Second, we introduce central clearing. If derivative class K is centrally cleared by a CCP, all entities $i = 1, ..., \gamma$ become clearing members at the CCP while the CCP becomes the single counterparty to all positions in this derivative class.[13] Thus, there is netting across counterparties,

[13]Note that we do not consider client clearing in our analysis. Nonetheless, the contractual relationship between a

which is called *multilateral netting*. For example, in Figure 3, A can reduce its total counterparty risk exposure from \$100 to \$40 with multilateral netting, as the exposure of \$100 to B is offset with a loss of \$60 to C.

[Place Figure 3 about here]

If a clearing member's default results in a loss for the CCP, this loss is offset by contributions from the surviving clearing members.[14] To calculate a market participant's counterparty risk toward a CCP, Duffie and Zhu (2011) propose to focus on the case that all other clearing members default, i.e., they are implicitly assuming no loss sharing. Then, if market participant i is the only surviving clearing member, the trades between all other pairs of clearing members exactly offset each other and the exposure of i toward the CCP equals

$$E_i^{MN} = \max\left(\sum_{j=1, j\neq i}^{\gamma} X_{ij}^K, 0\right). \tag{3}$$

As Equation (3) illustrates, in the situation that all counterparties except for the considered one (i) default, there is mechanically no loss sharing. In this case without loss sharing, E_i^{MN} depends only on market participant i's centrally cleared portfolio. By following this perspective, the counterparty risk toward a CCP only depends on multilateral netting of derivatives. In Section 3, we follow this approach.

However, the situation that all other clearing members default is clearly an extreme case. A market participant's contribution to loss sharing may substantially differ from E_i^{MN} if not *all* but only some clearing members default.[15] In Section 4 we therefore study a more general measure for counterparty risk, that also accounts for the possibility that any subset of a CCP's clearing members defaults. In this case, market participant i's counterparty risk toward the CCP is affected by the expected contribution to loss sharing.

client and dealer is often similar to a bilaterally netted contract in our model. In this case, our model is straightforward to apply.

[14] The most recent example for a clearing member default that triggered loss sharing is that of the Swedish CCP Nasdaq in September 2018, as discussed by Faruqui et al. (2018). For a detailed discussion of the use of a CCP's funds to cover losses, we refer to Armakolla and Laurent (2017) and Elliott (2013).

[15] This is because the contribution to loss sharing is not linear in clearing members defaults (see Equation (30)).

3 Bilateral versus multilateral netting in the absence of loss sharing

In this section, we take a similar perspective as Duffie and Zhu (2011) by looking at the situation that, for a given market participants, all its counterparties default. We will stepwise increase the complexity of our model in order to decompose the impact of central clearing on counterparty risk into different components, namely systematic risk, portfolio directionality, and extreme events. For this purpose, we distinguish between counterparty risk exposure before considering collateral, called *collateralized counterparty risk exposure*, and counterparty risk exposure exceeding collateral, called *uncollateralized counterparty risk exposure*. We start by studying an entity's collateralized counterparty risk exposure, which corresponds to the metric in Duffie and Zhu (2011) and Cont and Kokholm (2014). For simplicity, we sometimes just refer to it as *counterparty risk exposure*. Duffie and Zhu (2011) argue that counterparty risk exposure is a reasonable measure for the risk of loss from counterparty defaults and thus for a first-order consideration for systemic risk analysis.[16] Comparing counterparty risk exposure between a multilaterally and bilaterally netted position allows us to assess how central clearing changes the netting benefits, which have important effect on counterparty risk.

For simplicity and tractability, we consider a market that is as homogeneous as possible, while we relax several assumptions about homogeneity later. Homogeneity ensures that our baseline results are not driven by heterogeneity of market participants and contracts. For this purpose, we follow Duffie and Zhu (2011) and assume that all contracts exhibit the same distributional properties. We skip entity-specific indices where possible: $\beta \equiv \beta_{ij}^k$ and $\sigma \equiv \sigma_{ij}^k$ for all $i \neq j$ and $k = 1, ..., K$, which implies that $\rho_{X,M} \equiv \rho_{X,M,ij}^k = \beta \frac{\sigma_M}{\sqrt{\beta^2 \sigma_M^2 + \sigma^2}}$, and therefore a monotonic relationship between $\rho_{X,M}$ and β. We thus focus on a specific group of contracts, namely those with the same volatility and exposure to systematic risk, to identify the basic mechanisms that govern the interaction between central clearing and counterparty risk exposure.

The fragmentation into bilaterally and multilaterally netted portfolios is illustrated in Figure 4. Multilateral netting of derivative class K has two opposing effects: on one hand, it shrinks the

[16] The inverse of the collateralized counterparty risk exposure is called *netting efficiency* by Duffie and Zhu (2011).

bilateral netting portfolio with each counterparty by taking out derivative class K. This reduces the netting opportunities in bilateral portfolios. On the other hand, it creates a new portfolio across all counterparties, the multilateral netting portfolio. Clearly, if there is a large number of counterparties γ compared to the number of derivative classes K, the number of netting opportunities in the multilateral netting portfolio may be sufficiently large to offset the reduction in bilateral netting opportunities. The relation between multilateral and bilateral netting opportunities is thus very transparently reflected by the ratio of γ to K. We will fix K in the following and vary γ, i.e., the number of counterparties, to examine the interaction between the trade-off of bilateral and multilateral netting with systematic risk, extreme market events, margins, and portfolio directionality.

[Place Figure 4 about here]

3.1 Systematic risk and counterparty risk exposure

In this section, we begin the analysis by considering directional derivative portfolios where all positions equal unity, $v_{ij}^k \equiv 1$, as in Duffie and Zhu (2011).[17] We relax this assumption in Section 3.2.

We assess the benefit of multilateral netting by comparing the counterparty risk exposure of a given entity i with multilateral netting to that with bilateral netting. With the assumptions above, i's total counterparty risk exposure with bilateral netting of K derivative classes with $\gamma - 1$ counterparties is given by

$$\mathbb{E}[E_i^{BN,K}] = (\gamma - 1)\varphi(0)\sqrt{\sigma_M^2 K^2 \beta^2 + K\sigma^2}, \tag{4}$$

where $\varphi(\cdot)$ is the probability density function of the standard normal distribution.

Proof: See Proposition 1 in the Appendix.

If derivative class K is multilaterally netted, remaining $K - 1$ classes are bilaterally netted and i's

[17]Note that due to the symmetry of X, results (that are unconditional of M) also hold for $v_{ij}^k \equiv -1$.

total counterparty risk exposure is given by

$$\mathbb{E}[E_i^{BN+MN}] = \underbrace{\varphi(0)(\gamma-1)\sqrt{\sigma_M^2(K-1)^2\beta^2+(K-1)\sigma^2}}_{=\mathbb{E}[E_i^{BN,K-1}]\text{ (bilaterally netted)}} + \underbrace{\varphi(0)\sqrt{\sigma_M^2(\gamma-1)^2\beta^2+(\gamma-1)\sigma^2}}_{=\mathbb{E}[E_i^{MN}]\text{ (multilaterally netted)}}. \tag{5}$$

Proof: See Proposition 2 in the Appendix.

The first term of $\mathbb{E}[E_i^{BN+MN}]$ captures entity i's counterparty risk exposure resulting from bilateral netting agreements with $\gamma-1$ counterparties in $K-1$ derivative classes, which is $\mathbb{E}[E_i^{BN,K-1}]$. The second term is the counterparty risk exposure in the multilaterally netted derivative class K, which is $\mathbb{E}[E_i^{MN}]$.

In the following, we examine the impact of systematic risk on a market participant's counterparty risk exposure with multilateral netting relative to that with bilateral netting. Proposition 2 in the Appendix shows that there exists a positive lower bound for the multilaterally netted class-K exposure if, and only if, entities are exposed to systematic risk. Thus, a large number of counterparties cannot provide an arbitrarily low level of multilateral exposure in the presence of systematic risk. This is a main distinction to previous models without systematic risk (such as the one by Duffie and Zhu (2011)) and will drive many of our results.

Figure 5 (a) illustrates the relative change in counterparty risk exposure by moving from bilateral to multilateral netting of derivative class K, which is given by

$$\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K}]/\mathbb{E}[E_i^{BN,K}]. \tag{6}$$

If $\Delta E < 0$, then multilateral netting results in smaller counterparty risk exposure than bilateral netting. In Figure 5 (a), we vary the number of counterparty γ, which is the key variable of interest in previous studies and reflects the number of multilateral netting opportunities.[18] ΔE is positive for a small number of counterparties γ and negative for large γ. Thus, multilateral netting

[18]We hold the number of derivative classes fixed to $K = 10$, which does not qualitatively affect our results. Alternatively, we could fix γ and vary the degree of concentration in derivative portfolios. However, this would require more assumptions on the structure of the portfolio. Instead, we will first vary the number of counterparties as proxy for multilateral netting opportunities here, and portfolio directionality in Section 3.2.

reduces counterparty risk only for a large number of counterparties.[19] The intuition is as outlined above, namely that a larger number of counterparties increases the number of multilateral netting opportunities (γ) relative to bilateral netting opportunities (K). As a result, the average volatility per counterparty and thus counterparty risk exposure decrease as the number of counterparties increases.

[Place Figure 5 about here]

In Proposition 2 in the Appendix we show that there always exists a minimum number of counterparties such that multilateral netting reduces counterparty risk exposure compared to bilateral netting, $\gamma_{\min} = \min\{\gamma \in \mathbb{N} : \Delta E < 0\} < \infty$. Figure 5 (b) illustrates the impact of correlation $\rho_{X,M}$ between derivatives prices and the systematic risk factor M on $\gamma_{\min}$, with the calibration described below. Without systematic risk ($\rho_{X,M} = 0$), multilateral netting is only beneficial when at least 39 counterparties are present. As Figure 5 (b) shows, systematic risk radically changes the minimum number of counterparties: $\gamma_{\min}$ is steeply increasing with the correlation $\rho_{X,M}$ and for the calibrated correlation $\rho_{X,M} = 0.43$ it is equal to 121.[20]

The intuition for this result is that the benefit of an additional counterparty in the multilateral netting portfolio is decreasing with systematic risk, since

$$\frac{d}{d\gamma}\frac{\mathbb{E}[E_i^{MN}]}{\gamma-1} = \frac{\varphi(0)\sigma_X\left(1-\rho_{X,M}^2\right)}{2(\gamma-1)^2\sqrt{\rho_{X,M}^2(1-(\gamma-1)^{-1})+(\gamma-1)^{-1}}} \tag{7}$$

converges to zero when $|\rho_{X,M}|$ approaches unity. For bilaterally netted portfolios, $\frac{\mathbb{E}[E_i^{BN}]}{\gamma-1}$ is instead unaffected by γ. It thus requires a larger number of counterparties γ, such that netting opportunities in the multilateral portfolio are sufficient to offset the reduction in overall netting opportunities due to fragmented netting. As a result, the larger the systematic risk exposure, the larger is the minimum number of counterparties $\gamma_{\min}$.[21] At the most extreme, with perfect correlation across

[19] Indeed, it is straightforward to show that at least $\gamma = K + 2$ entities are needed such that multilateral netting of derivative class K may reduce counterparty risk exposure.

[20] This result differs from previous studies: Duffie and Zhu (2011) and Cont and Kokholm (2014) only study correlation across (but not within) derivative classes, which *reduces* the minimum number of counterparties, while systematic risk (which is correlation across and within derivative classes) *increases* the minimum number of counterparties.

[21] Nonetheless, note that a larger systematic risk exposure also reduces the inefficiency of multilateral netting - that is, it reduces ΔE if $\Delta E > 0$ for a small number of counterparties, as Figure 5 (a) shows. The reason is that systematic risk exposure does not only impact multilateral but also bilateral netting portfolios. The larger the correlation, the smaller the impact of different bilateral and multilateral netting opportunities.

contracts ($|\rho_{X,M}| = 1$), there are no netting opportunities with bilateral or multilateral netting, and thus no difference between bilateral and multilateral netting in terms of counterparty risk exposure for any number of counterparties.

Result 1. *There exists a minimum number of counterparties, $\gamma_{\min}$, such that multilateral netting reduces a directional trader's collateralized counterparty risk exposure compared to bilateral netting if $\gamma \geq \gamma_{\min}$. $\gamma_{\min}$ is increasing with the correlation between derivatives prices and the systematic risk factor.*

We calibrate the model in order to realistically reflect the characteristics of derivative markets. Although we vary the number of counterparties in most analyses, we use $\gamma = 16$ as a baseline calibration, which corresponds to the 16 largest dealers in European markets, that trade more than 50% (in terms of outstanding notional) of centrally and non-centrally cleared interest rate derivatives and 80% of single-name CDS in the European market (Abad et al. (2016)), and is close to the actual number of clearing members at US and European CCPs.[22]

We empirically calibrate contract returns based on 5-day returns of index CDS, which are already subject to a clearing obligation in the US and EU. The systematic risk factor M is proxied by the S&P 500. Then the empirically calibrated correlation between derivatives contract returns and systematic risk is $\rho_{X,M} = 0.43$. The detailed calibration procedure is documented in the Online Appendix.

For this baseline calibration, multilateral netting of one derivative class only reduces exposures in a market with at least 121 counterparties. This is unrealistically large compared to the high concentration among a small number of dealers, for example, in the IRS and CDS market (Abad et al. (2016), Peltonen, Scheicher, and Vuillemey (2014), Getmansky, Girardi, and Lewis (2016)), and the current number of clearing members at CCPs. It also largely exceeds the minimum number of counterparties in the absence of systematic risk (as in Duffie and Zhu (2011)), which is 39 with our calibration.

[22]According to their CPMI-IOSCO public quantitative disclosures for 2018 Q2, ICE Clear US has 35 general clearing members. LCH has 55 general clearing members for interest rates, 11 for OTC foreign exchange, 7 for fixed income, and 22 for equities-derivatives clearing.

3.2 Portfolio directionality

The previous section examines an entity with a directional portfolio: all trades exhibit the same positive correlation ($v\beta > 0$) with the systematic risk factor. However, market participants might engage in trades that offset each others' systematic risk exposure and, thus, reduce portfolio directionality. For example, taking a long position on an IRS with 5 years tenor and a short position on an IRS with 10 years tenor hedges systematic risk exposure if the 5 year and 10 year interest rates are correlated with the systematic risk factor (reflecting, e.g., macroeconomic conditions). Equation (8) illustrates differences in portfolio directionality across entities in a market with five traders, where a cell (i, j) is the derivative position of the entity i (depicted in a row) with counterparty j (depicted in a column).

$$(v_{ij}^k)_{i,j\in\{1,\ldots,\gamma\}} = \begin{pmatrix} & 1 & 1 & 1 & 1 & \text{(purely directional)} \\ -1 & & 1 & 1 & 1 & \\ -1 & -1 & & 1 & 1 & \text{(flat)} \\ -1 & -1 & -1 & & 1 & \\ -1 & -1 & -1 & -1 & & \text{(purely directional)} \end{pmatrix} \quad \text{for all } k = 1, .., K. \tag{8}$$

For simplicity, most of our analysis will focus on the most extreme entities that are illustrated in Equation (8):

(a) Market participants $i = 1$ and $i = \gamma$ are (purely) *directional* since all their derivatives positions have the same correlation with the systematic risk factor, respectively.

(b) Market participant $i = \lfloor(1 + \gamma)/2\rfloor$ is a *dealer* has a flat portfolio and is, thus, hedged against systematic risk within each derivative class k (as $\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k \approx 0$).

In the following, we examine the interaction between portfolio directionality and the impact of central clearing on counterparty risk. We first, very generally, consider an entity with arbitrary net volume $v_{i*}^K = \sum_{j=1,j\neq i}^{\gamma} v_{ij}^K$ in the centrally cleared derivative class. v_{i*}^K reflects i's net class-K systematic risk exposure and is the sum across columns for a given row in Equation (8). For simplicity, we assume that $v_{ij}^k \in \{-1, +1\}$ for all i, j, k and hold fixed the net bilateral volume with each counterparty, $v_i^* \equiv v_{ij}^* = \sum_{k=1}^{K} v_{ij}^K$. Then, from Propositions 1 and 2 in the Appendix, counterparty risk exposure with bilateral netting is

$$\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma - 1)\sqrt{\sigma_M^2 \beta^2 (v_i^*)^2 + \sigma^2 K} \tag{9}$$

and, if class-K is centrally cleared, the class-K counterparty risk exposure is

$$\mathbb{E}[E_i^{MN}] = \varphi(0)\sqrt{\sigma_M^2 \beta^2 (v_{i*}^K)^2 + \sigma^2(\gamma - 1)}. \tag{10}$$

The impact of multilateral netting on counterparty risk exposure (relative to bilateral netting) is now

$$\Delta E = \frac{(\gamma - 1)\sqrt{\sigma_M^2 \beta^2 (v_i^* - v_{i1}^K)^2 + \sigma^2(K-1)} + \sqrt{\sigma_M^2 \beta^2 (v_{i*}^K)^2 + \sigma^2(\gamma - 1)}}{(\gamma - 1)\sqrt{\sigma_M^2 \beta^2 (v_i^*)^2 + \sigma^2 K}} - 1, \tag{11}$$

assuming for simplicity that $v_{i1}^K \equiv v_{ij}^K$ for all j. If there were no systematic risk (i.e., if $\beta = 0$), then $\Delta E = \sqrt{\frac{K-1}{K}} + \sqrt{\frac{1}{(\gamma-1)K}} - 1$ for all entities. In this case, as in Duffie and Zhu (2011), a larger number of counterparties γ reduces ΔE and thus increases the multilateral netting benefit. Since derivative contracts are independent in this case, it is irrelevant whether an entity is long or short with any contract.

Result 2. *The directionality of an entity's portfolio affects collateralized counterparty risk exposure only in the presence of systematic risk.*

Portfolio directionality is relevant in the presence of systematic risk (i.e., with $\beta \neq 0$). Equation (11) implies that it is relatively less beneficial to multilaterally net class-K if the net class-K portfolio value is more directional (i.e., with larger $(v_{i*}^K)^2$): for $|\beta| > 0$ it is

$$\frac{\partial \Delta E}{\partial (v_{i*}^K)^2} = \frac{(\sigma_M^2 \beta^2 (v_{i*}^K)^2 + \sigma^2(\gamma - 1))^{-1/2} \sigma_M^2 \beta^2}{2(\gamma - 1)\sqrt{\sigma_M^2 \beta^2 (v_i^*)^2 + \sigma^2 K}} > 0. \tag{12}$$

This is the case, for example, for entities that take only IRS pay-fixed positions. In contrast, multilateral netting is more favorable for entities with a flat multilateral (class-K) portfolio (e.g., consisting of a similar amount of pay-fixed and pay-float IRS positions).

Equation (12) shows that, due to systematic risk in derivatives prices, multilateral netting has a heterogeneous impact on different entities, driven by the directionality of their portfolios.[23] It is relatively less favorable to move from bilateral to multilateral netting for entities that have a more directional portfolio in the cleared derivative class (with the CCP). Since end-users typically have directional portfolios within the same derivative class (to hedge other balance sheet risks), our result suggests that end-users have either a small or no benefit from multilateral netting. Instead, dealers that hedge derivative trades across counterparties can derive a large benefit from multilateral netting.

Result 3. *The less directional an entity's portfolio of multilaterally netted derivatives, the smaller is this entity's collateralized counterparty risk exposure with multilateral netting compared to bilateral netting.*

While the previous results are for general portfolio directionality, in the remaining paper we will focus on the most distinct types of entities, illustrated in Equation (8): (1) entities with a perfectly directional portfolio (as in Section 3.1), which we call *directional traders*, and (2) entities with the same positive and negative positions within one derivative class (i.e., $\sum_{j=1,j\neq i}^{\gamma} v_{ij}^{k} = 0$), which we call *dealers*.

Figure 6 illustrates the impact of multilateral netting compared to bilateral netting on a dealer's counterparty risk exposure. Multilateral netting becomes more beneficial the larger the correlation between derivatives prices and systematic risk factor. The reason is that systematic risk only affects the dealer's bilateral portfolios (which are directional) but not the multilateral portfolio (which is perfectly hedged). Thus, dealers' benefit from multilateral netting relative to that from bilateral netting increases with larger correlation between derivatives prices and systematic risk factor. Moreover, by comparison of Figure 6 for a dealer to Figure 5 for a directional trader, we find that multilateral netting is clearly more favorable for a dealer than for a directional trader in the presence of systematic risk ($\rho_{X,M} > 0$). The reason is that the dealer is perfectly hedged against systematic risk in the multilaterally netted portfolio while the directional trader is not.

[Place Figure 6 about here]

[23] Portfolio directionality is typically driven by business type of derivative market participants. For example, in the IRS market, dealers (intermediating trade) exhibit close-to-zero net exposure, while insurers and pension funds hedge a negative duration mismatch on their balance sheet by taking pay-float positions, and (non-dealer) banks hedge a positive duration mismatch on their balance sheet by taking pay-fixed positions (Abad et al. (2016)).

3.3 Multilateral netting during extreme events

The primary purpose of central clearing is to enhance financial stability during crisis times (e.g., G20 (2009), Financial Stability Board (FSB) (2017)). In these times, where counterparty defaults are more likely than in normal times, CCPs should ideally absorb losses arising from counterparty defaults and thereby decrease the spillover of risk in the overall financial system. Thus, it is of prevalent importance to examine the impact of central clearing on counterparty risk exposure during exactly these times.

We study counterparty risk exposure during extreme events by conditioning on extreme realizations of the systematic risk factor M. This rationale and approach is similar to the (marginal) expected shortfall of Acharya, Pedersen, Philippon, and Richardson (2017): while they examine the capital shortfall of financial institutions during crises, we study counterparty risk exposure. We start by considering a directional trader (as in Section 3.1) and then assess the impact of portfolio directionality on counterparty risk (analogously to Section 3.2).

Consider a directional trader whose derivative positions are positively correlated with the systematic risk factor ($v_{ij}^k \equiv 1$). For this trader, the total counterparty risk exposure with bilateral netting conditional on a realization $\bar{M}$ of the systematic risk factor M is given by

$$\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}] = (\gamma - 1)\sqrt{K}\left(\bar{M}\sqrt{K}\beta\Phi\left(\bar{M}\sqrt{K}\frac{\beta}{\sigma}\right) + \sigma\varphi\left(-\bar{M}\sqrt{K}\frac{\beta}{\sigma}\right)\right) \tag{13}$$

and with multilaterally netting derivative class K it is given by

$$\begin{aligned}\mathbb{E}[E_i^{BN+MN} \mid M = \bar{M}] &= \mathbb{E}[E_i^{BN,K-1} \mid M = \bar{M}] \\ &+\bar{M}(\gamma - 1)\beta\Phi\left(\bar{M}\sqrt{\gamma - 1}\frac{\beta}{\sigma}\right) + \sigma\sqrt{\gamma - 1}\varphi\left(-\bar{M}\sqrt{\gamma - 1}\frac{\beta}{\sigma}\right),\end{aligned} \tag{14}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Proof: See Proposition 3 in the Appendix.

The overall effect of multilateral netting crucially depends on the severity of realizations $\bar{M}$. We are particularly interested in extremely adverse events, and denote the severity of an event $\{M = \bar{M}\}$ by $q = \mathbb{P}(M \leq \bar{M})$, such that $\bar{M} = \sigma_M \Phi^{-1}(q)$, where $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of the standard normal distribution. For given q, $\bar{M}$ is then the largest among the $q \times 100\%$ worst possible realizations of the systematic risk factor M. The smaller q, the smaller (large and negative) $\bar{M}$, and thus the more adverse is the state of the world.

Figure 7 (a) depicts the change in counterparty risk exposure due to moving from bilateral to multilateral netting of class K. We show that, in more adverse realizations of the systematic risk factor (smaller q), multilateral netting is less favorable compared to bilateral netting. If an event is too extreme, then multilateral netting *increases* counterparty risk exposure relative to bilateral netting *regardless of the number of counterparties.* For our baseline calibration, this already holds for $q \leq 0.34$ (i.e., in the 34% most adverse realizations of M), as illustrated in Figure 7 (b). Our result thus implies that counterparty risk exposure during stress times is unambiguously smaller without multilateral netting regardless of the number of counterparties. Note that this result holds in very extreme states (such as the $q = 10\%$ worst possible states) but also in relatively moderate states (such as $q = 34\%$).

[Place Figure 7 about here]

The reason for our result is the dominance of extremely large expected gains and losses during extreme events. By rearranging Equation (13), the counterparty risk exposure with bilateral netting can be represented as

$$\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}] = (\gamma - 1)\mathbb{E}\left[\max\left(\bar{M}K\beta + \sqrt{K}\sigma\tilde{\varepsilon}, 0\right)\right], \tag{15}$$

where $\tilde{\varepsilon} \sim \mathcal{N}(0, 1)$. Clearly, if $\bar{M} \approx 0$, then $\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}]$ is increasing with the number of derivative classes K since it is proportional to $\sqrt{K}$. Thus, when one derivative class-K is taken out from bilateral portfolios, the bilaterally netted counterparty risk exposure decreases due to a smaller volatility in the remaining portfolio. This leaves room for the total counterparty risk exposure to be smaller after additionally multilaterally netting derivative class K, i.e., it is feasible that

$$\mathbb{E}[E_i^{BN,K-1} \mid M = 0] + \mathbb{E}[E_i^{MN} \mid M = 0] < \mathbb{E}[E_i^{BN,K} \mid M = 0]. \tag{16}$$

In contrast, if contracts have sufficiently large negative expected returns during extreme events (if $\bar{M}$ is large and negative), then the bilateral exposure in Equation (15) is not increasing but decreasing with K. The reason is that the effect of the number of derivative classes K on the expected value

$\bar{M}K\beta$ (making it very large and negative) dominates the effect on total volatility $\sqrt{K}\sigma$. In this case of large and negative realizations $\bar{M}$, excluding class-K from bilateral portfolios increases the expected portfolio gain and, thereby, the counterparty risk exposure in these portfolios. Then,

$$\mathbb{E}[E_i^{BN,K-1} \mid M = \bar{M}] > \mathbb{E}[E_i^{BN,K} \mid M = \bar{M}] \tag{17}$$

and, thus, counterparty risk exposure is smaller without multilateral netting of class K, i.e.,

$$\mathbb{E}[E_i^{BN,K-1} \mid M = \bar{M}] + \mathbb{E}[E_i^{MN} \mid M = \bar{M}] > \mathbb{E}[E_i^{BN,K} \mid M = \bar{M}]. \tag{18}$$

Interestingly, the impact of multilateral netting is symmetric in the right-tail of the distribution of M, i.e., for large q, as Figure 7 (b) shows. During these extremely large and positive realizations of M, the directional trader makes large expected gains on all contracts and, thus, has large counterparty risk. Then, for small γ, the small number of netting opportunities in the multilateral portfolio makes multilateral netting less beneficial, analogously to our baseline analysis in Section 3.1. For large γ, the expected value of contracts in the multilateral portfolio is extremely large, such that multilateral netting opportunities are negligible. However, removing class-K contracts from bilateral portfolios substantially reduces *bilateral* netting opportunities (since there are only K contract classes in bilateral portfolios compared to $\gamma >> K$ counterparties in the multilateral portfolio). This increases the per-contract bilateral counterparty risk exposure. As a result, if $\bar{M}$ is sufficiently large, multilateral netting is also not beneficial compared to bilateral netting regardless of the number of counterparties.

The analogous rationale holds for directional traders whose positions are negatively correlated with the systematic risk factor (i.e., with $v_{ij}^k < 0$). However, the relation with q is now reversed, since the positions of these traders make expected losses upon extremely *positive* realizations of the systematic risk factor (with large q). As the trade-off is exactly symmetric to the one that governs the impact of extreme events for a directional trader with $v_{ij}^k = 1$, a directional trader with $v_{ij}^k = -1$ does not benefit from multilateral netting during extremely large or small realizations of the systematic risk factor with any number of counterparties.

Result 4. *During sufficiently severe extreme events, multilateral netting of one derivative class* *does not reduce a directional trader's collateralized counterparty risk exposure compared to bilateral netting for any number of counterparties.*

Extreme events make it particularly unfavorable to exclude a derivative class from bilateral netting due to the dominance of large absolute contract values. By hedging across different contracts, entities may however reduce the unfavorable effect of extreme events. Figure 8 revisits the impact of extreme events $\bar{M} = \sigma_M \Phi^{-1}(q)$ for a dealer. In contrast to a directional trader, the dealer benefits from multilateral netting *particularly* during extreme events, i.e., both small and large q. During such events, multilateral netting allows a dealer to offset the extremely large expected gain from one trade with the exactly opposite loss from another trade. For these trades, multilateral netting thus eliminates the part of counterparty risk exposure that is driven by the systematic risk factor. Hence, multilateral netting is beneficial both in extremely positive and negative realizations of the systematic risk factor. For example, for the 10% most positive and most negative realizations of M, multilateral netting reduces a dealer's counterparty risk exposure in Figure 8 (a) for any number of counterparties. The minimum number of counterparties $\gamma_{\min}$ is decreasing with the severity of realizations of M, i.e., decreasing with $|q - 1/2|$, as Figure 8 (b) illustrates. Thus, the insight is similar to Section 3.2: the stronger systematic risk, the more dealers benefit from multilateral netting.

[Place Figure 8 about here]

Result 5. *During sufficiently severe extreme events, multilateral netting of one derivative class reduces a dealer's collateralized counterparty risk exposure compared to bilateral netting for any number of counterparties.*

3.4 Margin requirements and counterparty risk exposure

In this section, we examine the impact of margin requirements on the benefit of multilateral netting. Collateralizing exposures (also called *margining*) is a primary measure to reduce counterparty risk in derivative transactions (International Swaps and Derivatives Association (2014)). Typically, one distinguishes between initial and variation margin: initial margin is collateral available to the (central clearing) counterparty and posted at the beginning of a trade to cover potential future counterparty risk exposure. Variation margins are frequently (typically daily) exchanged to compensate for changes in market values. For simplicity, we assume in our model that initial margins were exchanged before the settlement period and contracts are marked to market (i.e., variation margin is exchanged) at the beginning of the settlement period. Then the remaining collateral available to compensate for losses from counterparty defaults is given by the initial margin.[24]

We only examine a directional trader in this section. It will be clear from our results below, that for dealers there exists a number of counterparties for any bilateral and clearing margin level such that multilateral netting leads to smaller counterparty risk exposure than bilateral netting, i.e., the insights from Section 3.1 qualitatively apply. This will not be the case for directional traders.

In line with recent regulations, we assume that the initial margin that j posts to i based on a bilateral netting agreement (referred to as *bilateral margin*) is given by the Value at Risk at the α_{BN} confidence level of the portfolio value of their trades,

$$C_{ij}^{BN,K} = VaR_{\alpha_{BN}}\left(\sum_{k=1}^{K} X_{ij}^{k}\right) = \Phi^{-1}(\alpha_{BN})\sqrt{\sigma_M^2 K^2\beta^2 + K\sigma^2}. \tag{19}$$

We refer to α_{BN} as the *bilateral margin (confidence) level.*

The *uncollateralized* counterparty risk exposure is the exposure in excess of collateral, and given by

$$\begin{aligned} \mathbb{E}\left[\tilde{E}_i^{BN,K}\right] &= \mathbb{E}\left[\sum_{j=1,j\neq i}^{\gamma} \max\left(\sum_{k=1}^{K} X_{ij}^{k} - C_j^{BN,K}, 0\right)\right] && (20)\\ &= (\gamma-1)\sqrt{\sigma_M^2 K^2\beta^2 + K\sigma^2}\,\xi(\alpha_{BN}), && (21) \end{aligned}$$

where $\xi(\alpha) = (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$ adjusts the counterparty risk exposure for margin.[25]
Proof: See Proposition 4 in the Appendix.

If derivative class K is multilaterally netted, then j posts collateral (referred to as *clearing*

[24] Note that CCPs also have pre-funded resources that can be employed in case of a loss. However, these are small compared to the collateral posted by clearing members. For example, for CDS clearing, pre-funded resources are 0.5% of initial margins at CME Clearing US, 2.8% at LCH Clearnet SA, and 8% ICE Clear Credit; for IRS clearing, pre-funded resources are 3.2% of initial margin at LCH Ltd. as of March 2016 (Armakolla and Laurent (2017)). Thus, we do not expect that accounting for pre-funded resources would substantially alter our results. Indeed, the default of just one single trader was enough to trigger a loss of $107 million in excess of margin and default fund contributions of the Swedish clearinghouse *Nasdaq Clearing AB* in September 2018 (Stafford and Sheppard (2018)).

[25] If $\alpha = 0.5$, then $\Phi^{-1}(\alpha) = \Phi^{-1}(1-\alpha) = C^{BN,K} = 0$, and uncollateralized counterparty risk exposure is equal to collateralized counterparty risk exposure.

margin) as given by the Value at Risk at the α_{MN} confidence level,

$$C_j^{MN} = \Phi^{-1}(\alpha_{MN})\sqrt{\sigma_M^2(\gamma-1)^2\beta^2 + (\gamma-1)\sigma^2}. \tag{22}$$

To compute the uncollateralized counterparty risk exposure of entity i in the multilaterally netted derivative class K, we assume that the collateral provided by clearing member j is available to i proportionally to the size of j's trades with i. Thus, $\frac{|v_{ij}^K|}{\sum_{h=1,h\neq j}^{\gamma}|v_{hj}^K|}C_j^{MN}$ is assigned to entity i.[26] With $|v_{ij}^K| \equiv 1$, the uncollateralized exposure of entity i is then given by

$$\mathbb{E}\left[\tilde{E}_i^{BN+MN}\right] = \sqrt{\sigma_M^2(\gamma-1)^2\beta^2 + (\gamma-1)\sigma^2}\xi(\alpha_{MN}) + \mathbb{E}[\tilde{E}_i^{BN,K-1}]. \tag{23}$$

Proof: See Proposition 4 in the Appendix.

Comparing the collateralized and uncollateralized counterparty risk exposure $\mathbb{E}[E_i^{BN+MN}]$ and $\mathbb{E}[\tilde{E}_i^{BN+MN}]$ in Equations (14) and (23), respectively, it becomes apparent that the only difference is the adjustment factor ξ. Hence, margins have an impact on the benefit of multilateral netting only if clearing and bilateral margins differ. The larger (smaller) the confidence level of the clearing margin α_{MN} relative to that of the bilateral margin α_{BN}, the larger (smaller) is the reduction of exposures due to multilateral netting of derivative class K (for a proof, see Proposition 5 in the Appendix).

Result 6. *The larger the margin for centrally cleared derivatives relative to that for bilaterally netted derivatives, the lower is uncollateralized counterparty risk exposure with multilateral netting relative to that with bilateral netting.*

While it is intuitive that margins reduce counterparty risk, the question we examine below is how sensitive benefits from multilateral netting are toward differences in bilateral and clearing margins. Interestingly, our results suggest that netting benefits are extremely sensitive toward margins: for small deviations between bilateral and clearing margin, the number of netting opportunities is irrelevant for the trade-off between multilateral netting. The reason for this high sensitivity is systematic risk, as it limits netting benefits.[27]

[26] By accounting for loss sharing in Section 4, we will study a more subtle allocation of collateral among a CCP's clearing members.

[27] In the Online Appendix, we provide a more detailed analysis of the impact of margins on counterparty risk

Result 7. *There exists a threshold for the difference between bilateral and clearing margin levels such that multilateral netting is beneficial for a directional trader's counterparty risk exposure compared to bilateral netting regardless of the number of counterparties, and vice versa. The larger the correlation between derivatives prices and systematic risk factor, the smaller is this threshold.*

4 Loss sharing

In the previous section, we examine the counterparty risk exposure of one entity in the absence of loss sharing, i.e., for a situation in which all other counterparties default. However, this is clearly an extreme case. The most general case is that a CCP that suffers losses due to *any* clearing member's default enters a recovery process by exploiting the resources of surviving clearing members, i.e, shares losses (Elliott (2013), Duffie (2015)).[28] However, the exposure toward a CCP is not linear in clearing member defaults, as we show below. As a result, while the previous section provides a clear analysis of trade-offs between bilateral and multilateral netting, it does not take into account that a CCP shares losses among surviving clearing members. Below, we provide a more detailed assessment of the impact of loss sharing on counterparty risk.

More specifically, we study the uncollateralized counterparty risk with loss sharing incurred by each clearing member. Counterparty risk $E_i^{*BN+MN,LS}$ is thereby defined as the sum of expected default losses in bilaterally netted trades (exceeding margins) in $K-1$ asset classes and the expected contributions to loss sharing of class-K at the CCP, e.g. for entity i it is

$$\mathbb{E}[E_i^{*BN+MN,LS}] = \sum_{j=1,j\neq i} \mathbb{P}(\text{default}_j)\big(\mathbb{E}[\text{bilateral exposure}_{ij}^{K-1}] + \mathbb{E}[\text{contribution to CCP}_{ij}^{K}]\big). \tag{24}$$

We compare this measure of counterparty risk under central clearing to the counterparty risk in a fully bilaterally netted market, which is measured as entities' probability of default times counterparty risk exposure,

$$\mathbb{E}[E_i^{*BN,K}] = \sum_{j=1,j\neq i} \mathbb{P}(\text{default}_j)\mathbb{E}[\text{bilateral exposure}_{ij}^{K}]. \tag{25}$$

exposure.

[28]The aim of this recovery process is to return to exactly balanced positions (a so-called *matched book*). The contribution of clearing members to a CCP's loss sharing arrangement is also the relevant exposure to calculate capital requirements of banks (Bank for International Settlements (BIS) (2014a)).

4.1 Model

We specify a complete network structure among entities' positions, which in particular requires that $v_{ij}^k = -v_{ji}^k$.[29] To refrain from further assumptions about the heterogeneity of entities, we impose the same network structure for each derivative class $v_{ij}^k \equiv v_{ij}$ for all $k = 1, ..., K$. This structure includes one entity that is long in the systematic risk factor with each trade (i.e., $v_{ij}^k > 0$), one that is short with each trade, and entities in between – as illustrated in Equation 8. All bilateral portfolios (i.e., derivative trades with the same counterparty) are directional but multilateral portfolios (i.e., derivative trades in the same derivative class with different counterparties) differ in directionality. While this assumption is clearly a simplification, it allows us to shed light on the trade-off between directionality in bilateral and multilateral portfolios.

In the following, we study whether central clearing as a combination of loss sharing and multilateral netting reduces counterparty risk compared to bilateral netting. To investigate loss sharing, we need to consider market participants' default probability. For this purpose, we also allow for the possibility of default clustering. Our model for defaults is inspired by Merton (1974)'s credit risk model and described in the Online Appendix in detail. We model each entity j's value of assets A_j. If A_j is below an exogenous debt threshold, j defaults. In the default model, the random value of entity j's value of assets at the settlement period begin is given by

$$A_j = \exp\left(\mu_{A_j} - \frac{\sigma_{A_j}^2}{2} + \sigma_{A_j} W_j\right), \tag{26}$$

where $(W_1, ..., W_\gamma)$ are jointly standard normally distributed and correlated according to the correlation matrix $(\rho_{A_j,A_h})_{j,h\in\{1,...,\gamma\}}$.[30] μ_{A_j} and σ_{A_j} are the drift and volatility of the asset value process, respectively.

We assume that $\rho_{A_j,A_h} > 0$ for $j \neq h$, which implies that market participants default in clusters.

[29] We choose a network structure of positions that seems to be realistic - for example, in the CDS market (Getmansky et al. (2016)). While it may be unknown whether a specific entity will be long or short in the future, the market structure is likely to be stable over time. Moreover, business models and strategies of many entities naturally lead to the direction of trader sides. For example, insurers take pay-float positions to hedge the negative duration mismatch on their balance sheet. Another example are asset managers (e.g., hedge funds), that have been replacing dealers as largest net sellers of CDS protection since the 2008 financial crisis (Siriwardane (2018)).

[30] In an earlier working paper version, we also allowed for correlation between asset values and systematic risk factor, which introduces a wedge between entities with a negative vs. positive correlation between their derivatives portfolio and the systematic risk factor. Here, we only focus on correlation among derivatives prices but not between derivatives prices and defaults. This allows us to isolate the additional impact of loss sharing compared to Section 3.

Correlation in defaults can result from interconnectedness between (financial) institutions, e.g., interbanking liabilities, such that the financial distress of one entity spills over to other entities. A prime example has been Lehman Brothers' default during the 2007-08 financial crisis, which triggered substantial losses at other financial institutions. For simplicity, in the following we assume that all market participants' assets have the same distributional parameters, and thus drop the parameter indices.

We define by D_j a binary random variable that equals one if market participant j defaults (i.e., if A_j breaches an exogenous debt threshold). Analogously to Lewandowska (2015), if all derivative classes are bilaterally netted, then counterparty risk is given by

$$\mathbb{E}\left[E_i^{*BN,K}\right] = \mathbb{E}\left[\sum_{j=1,j\neq i}^{\gamma} D_j \max\left(\sum_{k=1}^{K} X_{ij}^k - C_{ij}^{BN,K}, 0\right)\right], \tag{27}$$

where the bilateral collateral $C_{ij}^{BN,K}$ is given as in Section 3.4. Note that a loss is realized only in case a counterparty's default coincides with an adverse price movement in exceedance of collateral. We assume independence between defaults and derivatives prices, which substantially improves the tractability of our model. We then can combine the results from Sections 3.2 and 3.4 to rewrite Equation (27) as

$$\mathbb{E}\left[E_i^{*BN,K}\right] = \pi\xi(\alpha_{BN}) \sum_{j=1,j\neq i}^{\gamma} \sqrt{\sigma_M^2 \beta^2 \left(\sum_{k=1}^{K} v_{ij}^k\right)^2 + K\sigma^2}, \tag{28}$$

where $\pi \in (0,1)$ is the unconditional probability of each entity's default and $\xi(\alpha) = (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$. Thus, counterparty risk is equivalent to counterparty risk exposure (the variable of interest in Section 3) times the probability of counterparty default.

Now, we consider the central clearing of derivative class K. In line with loss-allocation rules in practice (e.g., Arnsdorf (2012), Elliott (2013), and Duffie (2015)), we assume that default losses at the CCP that exceed a defaulter's margin are shared among surviving clearing members. A clearing member suffers losses with the CCP only in case at least one other clearing member j defaults and the multilaterally netted contract value of j exceeds the collateral provided by j.[31] In contrast

[31] In practice, default losses are absorbed not only by the defaulter's collateral (and default fund contribution) but also by a share of the CCP's capital, its *skin-in-the-game* (SITG). However, SITG is very small in practice, typically

to the counterparty risk exposure measure of Duffie and Zhu (2011), we explicitly consider the cases that one or more clearing members default – but do not assume that necessarily all clearing members other than i default. The aggregate loss of the CCP is given by

$$\bar{L}^{CCP} = \sum_{j=1}^{\gamma} D_j \max\left(\sum_{\substack{g=1,\\ g\neq j}}^{\gamma} X_{gj}^{K} - C_j^{MN}, 0\right). \tag{29}$$

As suggested by Duffie (2015) and applied in practice, default losses are shared among surviving clearing members proportionally to the risk of their cleared trades as reflected by the initial margin C_i^{MN}. Then, the counterparty risk of clearing member i toward the CCP is given by[32]

$$\mathbb{E}[E_i^{*MN,LS}] = \mathbb{E}\left[\frac{(1-D_i)C_i^{MN}}{\sum_{g=1}^{\gamma}(1-D_g)C_g^{MN}}\bar{L}^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]. \tag{30}$$

The primary difference between $\mathbb{E}[E_i^{*MN,LS}]$ and $\mathbb{E}[\tilde{E}_i^{MN}]$, where the latter is the counterparty risk exposure toward default of *all* counterparties (as defined in Equation (3) for the case without margin and in Equation (23) including margin), is that $\mathbb{E}[E_i^{*MN,LS}]$ depends on both the defaulting entity j's portfolio (which determines the CCP's loss) as well as entity i's margin relative to other surviving entities' margins (which determines the loss allocation). Class-K counterparty risk $\mathbb{E}[E_i^{*MN,LS}]$ can be rewritten for entity i as

$$\mathbb{E}[E_i^{*MN,LS}] = \mathbb{E}\left[(1-D_i)f_i\xi(\alpha_{MN})\left(\frac{\sum_{j=1}^{\gamma} f_j}{f_i + \sum_{j=1,j\neq i}^{\gamma}(1-D_j)f_j} - 1\right) \mid \sum_{j=1}^{\gamma}(1-D_j) > 0\right] \tag{31}$$

with $f_i = \sqrt{\sigma_M^2\beta^2(\sum_{g=1,g\neq i}^{\gamma} v_{gi}^{K})^2 + \sigma^2(\gamma-1)}$. Simulations in the Online Appendix show that

$$\mathbb{E}\left[(1-D_i)f_i\left(\frac{\sum_{j=1}^{\gamma} f_j}{f_i + \sum_{j=1,j\neq i}^{\gamma}(1-D_j)f_j} - 1\right) \mid \sum_{j=1}^{\gamma}(1-D_j) > 0\right] \approx \frac{(1-\pi)f_i\pi}{1-\pi+\frac{f_i}{\sum_{j=1,j\neq i}^{\gamma} f_j}}$$

less than 5% of initial margins (Huang (2018)). Therefore, we exclude SITG from our considerations. We do not expect that including it would change our insights qualitatively.

[32] We condition on at least one entity surviving since it is extremely unlikely that all entities default at the same time, and in practice it seems likely that a government would bail out a CCP in the case that all clearing members default. Note also that the CCP does not default in our model since we assume that surviving clearing members are always able to fully absorb losses.

if π and ρ are small and γ large.[33] In this case, it is $\frac{f_i}{\sum_{j=1,j\neq i}^{\gamma} f_j} \approx 0$ and class-K counterparty risk with loss sharing is approximately equal to

$$\mathbb{E}[E_i^{*MN,LS}] \approx \pi \underbrace{\xi(\alpha_{MN}) \sqrt{\sigma_M^2 \beta^2 \left(\sum_{g=1,g\neq i}^{\gamma} v_{gi}^K \right)^2 + \sigma^2(\gamma - 1)}}_{=\mathbb{E}[\tilde{E}_i^{MN}]} . \tag{32}$$

The right-hand side in Equation (32) is the probability of counterparty default times the uncollateralized class-K counterparty risk exposure toward default of *all* counterparties, $\mathbb{E}[\tilde{E}_i^{MN}]$. Thus, if π and ρ are small and γ large, class-K counterparty risk exposure (times probability of default) is actually a reasonable indicator for class-K counterparty risk with loss sharing. The intuition is that, in this case, the loss allocation offsets the impact of other clearing members' portfolios on $\mathbb{E}[E_i^{*MN,LS}]$ and only market participant i's portfolio remains relevant for its expected contribution to the CCP.

Otherwise, these two risk measures, $\mathbb{E}[E_i^{*MS,LS}]$ and $\pi \times E[\tilde{E}_i^{MN}]$, may significantly differ – depending in particular on the distribution of f_i across clearing members. In the latter case, nonlinearity arising from loss sharing may substantially affect counterparty risk and the relationship between bilateral netting and central clearing.[34]

Result 8. *Uncollateralized counterparty risk exposure toward the default of all counterparties multiplied by the probability of counterparty default is a reasonable measure for a clearing member's expected contribution to CCP loss sharing,* $\mathbb{E}[E^{*MN,LS}] \approx \pi\mathbb{E}[\tilde{E}^{MN}]$*, if (1) default probability* π *and correlation of defaults* ρ *are small, and (2) the number of clearing members* γ *is large.*

As before, if derivative class K is centrally cleared, the remaining $K-1$ derivative classes are bilaterally netted with counterparty risk as in Equation (27). The total counterparty risk of entity

[33] We examine this approximation by drawing random weights $\{f_j : j = 1, ..., \gamma\}$ from a truncated Gaussian distribution on $[0, \infty)$. For $\gamma = 50$, $\pi = 0.1$, $\rho \in [0, 0.7]$, the median relative deviation between $\mathbb{E}\left[(1 - D_i) f_i \left(\frac{\sum_{j=1}^{\gamma} f_j}{f_i + \sum_{j=1,j\neq i}^{\gamma}(1-D_j) f_j} - 1 \right) \mid \sum_{j=1}^{\gamma}(1 - D_j) > 0 \right]$ and $\frac{(1-\pi) f_i \pi}{1-\pi+\frac{f_i}{\sum_{j=1,j\neq i}^{\gamma} f_j}}$ is roughly 4%.

[34] Throughout the remaining paper, we compute counterparty risk with loss sharing as defined in Equation (31) instead of using its approximation (32).

i is then given by

$$\mathbb{E}[E_i^{*BN+MN}] = \mathbb{E}\left[E_i^{*MN} + E_i^{*BN,K-1}\right].$$

Throughout the remaining analysis, we assume that the clearing and bilateral margins are both based on a 99% confidence level, since differing margins have similar effects as in Section 3.4. We compute the following results by using the same baseline calibration for derivative prices as described in Section 3.1. Moreover, if not specified differently, entities default with unconditional probability $\pi = 0.1$ and assets in the default model have correlation $\rho_{A,A} = 0.1$.[35] Market participants only differ in portfolio directionality, and thus we suppress entity indices where possible. The detailed calibration is reported in the Online Appendix. While we derive closed-form analytic expressions for bilateral counterparty risk, counterparty risk with loss sharing in Equation (31) is not analytic. Therefore, we evaluate (31) by using Monte-Carlo simulations with 100,000 realizations of default vectors $(D_j)_{j=1,\dots,\gamma}$.

4.2 Baseline results

We start with the benchmark case that derivatives prices have no systematic risk exposure, i.e., $\beta = \rho_{X,M} = 0$. In this case, the impact of loss sharing is the same for all entities, independently of portfolio directionality. For example, in Figures 9 (a) and (b) the effect of central clearing with loss sharing coincides for directional traders and dealers if $\rho_{X,M} = 0$. Central clearing reduces counterparty risk compared to bilateral netting in the presence of a sufficiently large number of counterparties. Similarly to the minimum number of counterparties for multilateral netting to be beneficial in Section 3.1, the minimum of counterparties for loss sharing to be beneficial is roughly $\gamma_{\min} = 40$.

Result 9. *In the absence of systematic risk, the impact of central clearing on counterparty risk is independent from portfolio directionality. If the number of counterparties is sufficiently large, central clearing with loss sharing reduces counterparty risk compared to bilateral netting.*

[Place Figure 9 about here]

[35] The results are robust toward other levels of default clustering.

In the following, we consider a positive correlation between derivatives prices and the systematic risk factor. In this case, portfolio directionality becomes relevant for counterparty risk. According to Result 8, counterparty risk exposure toward default of *all* counterparties (times probability of default) is a reasonable measure for counterparty risk *with* loss sharing if defaults are infrequent, uncorrelated, and there are many counterparties. In this situation, the insights from Section 3.2 suggest that, compared to bilateral netting, dealers benefit relatively more from central clearing than directional traders. Figure 9 is in line with this intuition: if derivative prices are correlated ($\rho_{X,M} > 0$), dealers derive a substantially larger benefit from central clearing with loss sharing vis-á-vis bilateral netting than directional traders. Instead, for a realistic number of counterparties central clearing does not reduce directional traders' counterparty risk.

Moreover, we find that the impact of central clearing with loss sharing on counterparty risk ($E_i^{*MN,LS}$ in Figure 9) is similar to the impact of multilateral netting (${E_i}^{MN}$ in Figures 5 and 6) compared to bilateral netting.[36] The primary, subtle difference is that loss sharing is slightly more beneficial than multilateral netting (without loss sharing) for directional traders than for dealers in the presence of a small number of counterparties and large systematic risk exposure.[37]

Result 10. *Compared to bilateral netting, the impact of central clearing with loss sharing on the average level of counterparty risk is similar to that of multilateral netting.*

4.3 Loss sharing during extreme events

In the following, we examine how the impact of central clearing differs *across* realizations of M. Applying the insights from Proposition 3 in the Appendix shows that counterparty risk in K bilaterally netted derivative classes conditional on realization $\bar{M}$ of the systematic risk factor is given by

$$\mathbb{E}[E_i^{*BN,K} \mid M = \bar{M}] = \pi \sum_{j=1, j\neq i}^{\gamma} \Bigg[\left(\bar{M}\beta \sum_{k=1}^{K} v_{ij}^k - C_j^{BN} \right) \Phi \left(\frac{\bar{M}\beta \sum_{k=1}^{K} v_{ij}^k - C_j^{BN}}{\sqrt{K}\sigma} \right) + \sqrt{K}\sigma\varphi \left(-\frac{\bar{M}\beta \sum_{k=1}^{K} v_{ij}^k - C_j^{BN}}{\sqrt{K}\sigma} \right) \Bigg]. \tag{33}$$

[36]Note that correlation of defaults has no impact on counterparty risk exposure, since the latter is linear in defaults.

[37]The reason is that loss sharing enables directional traders to benefit from the flat portfolios of dealers: if a dealer defaults, the CCP's loss is small due to its flat portfolio with the dealer and, thus, directional trader face relatively small losses.

If derivative class K is centrally cleared, the CCP's expected loss conditional on $\{M = \bar{M}\}$ and defaults $D = (D_1, ..., D_\gamma)$ is

$$\mathbb{E}[\bar{L}^{CCP} \mid M = \bar{M}, D] = \sum_{j=1}^{\gamma} D_j \Bigg[\left(\bar{M}\beta \sum_{g=1, g\neq j}^{\gamma} v_{gj}^K - C_j^{MN} \right) \Phi \left(\frac{\bar{M}\beta \sum_{g=1,g\neq j}^{\gamma} v_{gj}^K - C_j^{MN}}{\sqrt{\gamma - 1}\sigma} \right) + \sqrt{\gamma - 1}\sigma\varphi \left(-\frac{\bar{M}\beta \sum_{g=1,g\neq j}^{\gamma} v_{gj}^K - C_j^{MN}}{\sqrt{\gamma - 1}\sigma} \right) \Bigg]. \quad (34)$$

Then, i's counterparty risk with loss sharing in the centrally cleared derivative class K is

$$\mathbb{E}[E_i^{*MN,LS} \mid M = \bar{M}] = \mathbb{E}\left[\frac{(1 - D_i)C_i^{MN}}{\sum_{g=1}^{\gamma}(1 - D_g)C_g^{MN}} \mathbb{E}[\bar{L}^{CCP} \mid M = \bar{M}, D] \mid M = \bar{M}, \sum_{g=1}^{\gamma}(1 - D_g) > 0 \right],$$

which we evaluate by using Monte-Carlo simulations.

To provide an intuition for the impact of different realizations of the systematic risk factor on counterparty risk, Figure 10 compares the distribution of counterparty risk with loss sharing and with bilateral netting for different event severity $q \in (0, 1)$ (with $\bar{M} = \sigma_M \Phi^{-1}(q)$) across directional traders and dealers.[38] For example, the first column of figures describes a directional trader with $v_{ij}^k \equiv 1$ (e.g., entity $i = 1$ in Equation (8)). The positions of this trader have large expected gains upon large and positive realizations of the systematic risk factor (large q). Thus, bilateral counterparty risk in these states is large (see row $E^{*BN,K}$ in Figure 10). By participating in central clearing with loss sharing, the directional trader becomes exposed to default losses in bad states (small q) as well, e.g., when it must offset the CCP's loss due to another clearing member's default. Counterparty risk with loss sharing (see first column and row $E^{*MN,LS}$) is thus large for both extremely positive and negative realizations of the systematic risk factor, i.e., very large and small q. As a result, compared to bilateral netting, central clearing with loss sharing increases this directional trader's counterparty risk for small q and may reduce it only for very large q (see first column and row $E^{*BN+MN,LS} - E^{*BN,K}$). Directional traders' with $v_{ij}^k = -1$ have the opposite correlation between systematic risk factor and derivatives positions and, thus, their benefit from central clearing with loss sharing is flipped and occurs only for very small q (see third column in Figure 10). In contrast, as half of a dealer's positions make gains in good and bad

[38]We fix the number of counterparties in Figure 10 to $\gamma = 16$, but the result is qualitatively the same for different values.

states, respectively, dealers benefit from central clearing with loss sharing in any extreme state, i.e., extremely small and large q, compared to bilateral netting.

[Place Figure 10 about here]

Figure 11 depicts the relative impact of central clearing with loss sharing compared to bilateral netting conditional on a realization $\bar{M} = \sigma_M \Phi^{-1}(q)$ of the systematic risk factor M. The results are in line with the intuition above. Entities only benefit in those states in which they have an extremely large bilateral counterparty risk, i.e., for extremely large q for directional traders with $v = 1$, small q for directional traders with $v = -1$, and large $|\frac{1}{2} - q|$ for dealers. Upon less extreme realizations of M, the additional exposure from bearing other clearing members' shared losses increases counterparty risk compared to bilateral netting.

[Place Figure 11 about here]

Hence, our first insight is that, for any market participant, a benefit from loss sharing (compared to bilateral netting) occurs only in some very extreme realizations of the systematic risk factor. These are states in which the counterparty risk without loss sharing (but with bilateral netting) is particularly large and, thus, market participants can load-off the large bilateral counterparty risk to the CCP. For our baseline calibration in Figure 11, both types of directional traders and dealers do not reduce but increase counterparty risk by sharing losses in more than 80% of the systematic risk factor's realizations.[39] For example, the directional trader in Figure 11 (a) does not benefit from central clearing with loss sharing in the $q < 0.9 \times 100\%$ least extreme realizations of M for any number of counterparties. The dealer in Figure 11 (c) benefits from central clearing only in either extremely large or small realizations of M, since these are also the states in which he is exposed to large bilateral counterparty risk.[40]

Result 11. *Central clearing with loss sharing does not reduce counterparty risk during moderate market events compared to bilateral netting.*

The second insight is that, during sufficiently extreme events, central clearing with loss sharing is beneficial compared to bilateral netting for all dealers but only one type of directional trader.

[39] We do not find any $\gamma \leq 500$ for which this result does not hold.

[40] As we show in the Online Appendix, our results qualitatively also hold when a CCP clears more than one derivative class - even when it clears the whole derivatives market.

This benefit comes at the expense of the other type of directional trader, that is exposed to larger counterparty risk. Thus, only some market participants can benefit from loss sharing at the same time – at the expense of other market participants. In less extreme states, loss sharing exposes entities to default losses that they would not bear with bilateral netting and thus no market participant benefits from it.

Result 12. *Conditional on extreme realizations of the systematic risk factor M, loss sharing transfers counterparty risk from dealers and one type of directional traders to other directional traders. There exists no realization of M such that, conditional on this realization, central clearing with loss sharing reduces counterparty risk for ALL market participants compared to bilateral netting.*

In Section 3 we followed Duffie and Zhu (2011) and measure i's counterparty risk exposure conditional on all other counterparties defaulting. This measure only reflects the trade-off between bilateral and multilateral netting but does not take loss sharing into account. Our third insight is that the impact of loss sharing on counterparty risk (compared to bilateral netting) is different from the impact of multilateral netting on counterparty risk (compared to bilateral netting). We show this result by comparing loss sharing (as discussed above) to multilateral netting (as in Section 3.3) across different realizations of M. For example, consider a directional trader, for which Figure 11 (a) illustrates the impact of loss sharing and Figure 7 (a) illustrates the impact of multilateral netting *without taking loss sharing into account*, both compared to bilateral netting. This market participant does not benefit from multilateral netting compared to bilateral netting in any extreme realization of the systematic risk factor (see Figure 7), while it *only* benefits in extreme realizations when loss sharing is taken into account (see Figure 11). More generally, we find that loss sharing pushes benefits from central clearing (compared to bilateral netting) into tails of the systematic risk factor's distribution compared to the benefits of multilateral netting (compared to bilateral netting). For example, a directional trader does not benefit from loss sharing in $q < 0.9 \times 100\%$ realizations of M (Figure 11 (a)), but benefits from multilateral netting for $q > 0.4 \times 100\%$ (Figure 7 (a)). Thus, with probability 50% (0.9-0.4=0.5), this trader can reduce counterparty risk with multilateral netting but not by centrally clearing with loss sharing. The result is symmetric for a directional trader with $v = -1$. Similarly, a dealer may reduce counterparty risk with multilateral netting but not with loss sharing for $q \in (0.1, 0.9)$, i.e., with probability 80% (comparing Figures

11 (c) and 8).

Hence, while loss sharing has a small impact on the *average* level of counterparty risk (see Result 10), loss sharing pushes gains and losses from central clearing into the tails of the distribution of M: it reduces the likelihood that a market participants benefits from central clearing.[41]

Result 13. *Loss sharing makes it less likely (across states of the systematic risk factor) that central clearing reduces a given market participant's counterparty risk compared to central clearing without loss sharing.*

The reason for our result is the insurance function of CCPs, and loss sharing in particular. The primary purpose of insurance is to smooth wealth across states. Figure 12 illustrates that loss sharing indeed smooths a directional trader's counterparty risk *across realizations of the systematic risk factor*: in the first row of Figure 12, the class-K counterparty risk with loss sharing is more evenly distributed across realizations of M than the class-K counterparty risk with only multilateral netting. Hence, loss sharing is similar to insurance of directional trader's multilaterally netted counterparty risk against systematic risk. Since the distribution of counterparty risk with multilateral netting ($\tilde{E}^{MN}$) is highly skewed, loss sharing increases counterparty risk for moderate realizations of M compared to multilateral netting (see second row of Figure 12). This increase may be interpreted as an *insurance premium*: directional traders pay for a reduction of counterparty risk in extreme states via an increase in counterparty risk in moderate states.[42] As Result 13 shows, paying this insurance premium then completely removes the multilateral netting benefits of central clearing in a large number of realizations and thereby renders central clearing unfavorable compared to bilateral netting.

[Place Figure 12 about here]

Finally, in the Online Appendix we show that our results also hold if a CCP does not only clear one derivative class but even all derivative classes. Thus, "more central clearing" does not provide a remedy for the pitfalls our analysis highlights.

[41] Note, however, that for each realization of M loss sharing reduces the total (sum of) counterparty risk of all market participants. Thus, while the aggregate impact of loss sharing is beneficial, from each market participant's perspective it is beneficial only with a small probability.

[42] Loss sharing also increases counterparty risk for dealers, compared to multilateral netting. The reason is that loss sharing exposes dealers to the risk of directional traders, while the flat portfolio of dealers ensures a large multilateral netting benefit.

5 Empirical predictions and policy implications

Our results are helpful in understanding several characteristics of derivative markets. First, in practice, market participants are on average reluctant to centrally clear derivatives in the absence of a clearing mandate. E.g., only 28% of CDS trades and less than 1% of foreign exchange derivatives were cleared, as of December 2016 (Wooldridge (2017)). From the perspective of providing netting opportunities and reducing counterparty risk, we show that central clearing is indeed not necessarily beneficial compared to bilateral netting in all cases. In contrast, central clearing may expose market participants to additional (instead of less) counterparty risk. Loss sharing amplifies this result by allocating the benefits of central clearing to only a few states of the world.

Second, CCP clearing members are dominantly dealers and large banks, while only a few investment funds and non-financial firms participate in central clearing and loss sharing (Bank for International Settlements (BIS) (2018)).[43] Indeed, large end-users such as Blackrock claim that loss sharing at CCPs "unfairly penalizes end-investors, who in general hold directional positions, vs. CMs [clearing members] or dealers, who generally manage to a flat market position" (Novick, De Jesus, Fisher, Kiely, Osman, and Hsu (2018)). Consistent with this statement, our results show that entities with directional positions (such as end-users) have a smaller or no benefit from central clearing - while this may not hold for dealers with flat positions. This heterogeneous impact of central clearing thus provides a possible explanation for the tendency of end-users not to become clearing members in practice.

Finally, previous studies have highlighted the benefit of concentrating multilateral netting at - preferably - one large CCP that clears the whole derivatives market (e.g., Duffie and Zhu (2011)). However, in this setting the dynamics of loss sharing are still the same as we describe in Section 4. Indeed, our results in the Online Appendix show that loss sharing still diminishes multilateral netting benefits and makes it less likely that central clearing reduces a market participant's counterparty risk. Thus, *more* central clearing does not resolve the pitfalls highlighted in our analysis.

Despite the pitfalls we find, central clearing may still improve overall financial stability in other dimensions than counterparty risk, e.g., by increasing transparency (Acharya and Bisin (2014)) and

[43]To avoid participation in loss sharing, entities may use client clearing by clearing through another clearing member.

facilitating fast auctioning of defaulting members' portfolios. Indeed, the cleared share of Lehman's derivative trades was hedged and closed out within three weeks of Lehman's failure, suggesting that central clearing may stabilize derivatives markets.[44]

6 Conclusion

The recent financial crises exposed vulnerabilities in the derivatives market architecture, which was dominated by bilaterally netted trades. The introduction of mandatory central clearing clearly increased the transparency of derivative markets, but would it decrease a market participant's counterparty risk exposure in crises?

We present a theoretical analysis of the impact of central clearing on counterparty risk in the presence of systematic risk. Our main result is that the effect of central clearing is highly sensitive toward (1) different levels of systematic risk exposure, (2) extreme market events, (3) directionality in market participants' derivatives portfolios, and (4) loss sharing. In many realistic situations, central clearing actually results in larger counterparty risk than bilateral netting.

In particular, our results show that traders with flat portfolios substantially benefit from central clearing compared to bilateral netting - but only during extreme market events and at the expense of traders with directional portfolios. This result emerges in particular due to sharing of a CCP's default losses among surviving clearing members. The prediction that dealers with flat portfolios may derive a much larger benefit from central clearing than market participants with directional portfolios is also consistent with the reluctance of end-users to become clearing members in practice.

Our analysis mainly concentrates on counterparty risk. We are not considering other advantages and disadvantages of central clearing, such as capital requirement benefits, margin costs, transparency, and market liquidity. If market participants only considered margin costs in their decision to clear, then there would obviously be an incentive to clear resulting from smaller margins for cleared relative to uncleared derivatives. If, however, the decision to clear is driven by a market participant's objective to minimize counterparty risk (which is highlighted as an important determinant for the decision to clear derivatives by Bellia et al. (2019) and the Financial Stability Board (FSB) (2018), as well as for the creation of the first CCP in Europe by Vuillemey (2019)), then

[44] See Sir Jon Cunliffe's speech from 5 June 2018, *Central clearing and resolution - learning some of the lessons of Lehmans*, available at `www.bankofengland.co.uk`.

our results show that there is no incentive to clear for some market participants in various realistic situations. This is consistent with the current situation in practice, wherein market participants are reluctant to centrally clear derivatives in several markets (like single-name CDS) unless they are forced.

Proofs

For the following proofs, we will make extensive use of the following property of the Normal distribution: for $Y \sim \mathcal{N}(\mu, \sigma^2)$ the truncated expectation is given by $\mathbb{E}[Y \mid Y > 0] = \mu + \sigma \frac{\varphi(-\mu/\sigma)}{\Phi(\mu/\sigma)}$, and thus $\mathbb{E}[\max(Y, 0)] = \mathbb{E}[Y \mid Y > 0]\Phi(\mu/\sigma) = \mu\Phi(\mu/\sigma) + \sigma\varphi(-\mu/\sigma)$.

Proposition 1 (Collateralized bilateral counterparty risk exposure)**.** *The collateralized counterparty risk exposure with bilateral netting is given by*

$$\mathbb{E}[E_i^{BN,K}] = \varphi(0) \sum_{j=1, j\neq i}^{\gamma} \bar{\sigma}_{ij}^{BN,K}, \tag{35}$$

where $\left(\bar{\sigma}_{ij}^{BN,K}\right)^2 = \sigma_M^2 \left(\sum_{k=1}^K v_{ij}^k \beta_{ij}^k\right)^2 + \sum_{k=1}^K \left(v_{ij}^k\right)^2 \left(\sigma_{ij}^k\right)^2$*. If* $v_{ij}^k \equiv 1$ *or* $v_{ij}^k \equiv -1$, $\beta_{ij}^k \equiv \beta$, *and* $\sigma_{ij}^k \equiv \sigma$ *for all* $j = 1, ..., \gamma$, $k = 1, .., K$, *then* $\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma - 1)\sqrt{\sigma_M^2 \beta^2 K^2 + K\sigma^2}$.

Proof of Proposition 1:

Proof. The counterparty risk exposure equals

$$\mathbb{E}\left[E_i^{BN,K}\right] = \sum_{j=1, j\neq i}^{\gamma} \mathbb{E}\left[\max\left(\sum_{k=1}^K X_{ij}^k, 0\right)\right] \tag{36}$$

Define

$$\begin{aligned}
\bar{\mu}_{ij}^{BN,K} &= \mathbb{E}\left[\sum_{k=1}^K X_{ij}^k\right] = \sum_{k=1}^K \mathbb{E}\left[X_{ij}^k\right] = 0, \\
\left(\bar{\sigma}_{ij}^{BN,K}\right)^2 &= \mathrm{var}\left(\sum_{k=1}^K X_{ij}^k\right) = \mathrm{var}\left(\sum_{k=1}^K v_{ij}^k (\beta_{ij}^k M + \sigma_{ij}^k \varepsilon_{ij}^k)\right) \\
&= \sigma_M^2 \left(\sum_{k=1}^K v_{ij}^k \beta_{ij}^k\right)^2 + \sum_{k=1}^K \left(v_{ij}^k\right)^2 \left(\sigma_{ij}^k\right)^2.
\end{aligned}$$

The counterparty risk exposure of i to j is then given by $\varphi(0)\bar{\sigma}_{ij}^{BN,K}$, and the total counterparty risk exposure is given by

$$\mathbb{E}[E_i^{BN,K}] = \varphi(0) \sum_{j=1, j\neq i}^{\gamma} \bar{\sigma}_{ij}^{BN,K}. \tag{37}$$

Simplification in the case of homogeneous entities and trades is straightforward. □

Proposition 2 (Collateralized multilateral counterparty risk exposure)**.** *The collateralized counterparty risk exposure with multilateral netting of derivative class K is given by*

$$\mathbb{E}[E_i^{BN+MN}] = \mathbb{E}[E_i^{BN,K-1}] + \varphi(0)\sqrt{\sigma_M^2\left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K\right)^2 + \sum_{j=1,j\neq i}^{\gamma}(v_{ij}^K)^2(\sigma_{ij}^K)^2}. \tag{38}$$

1. *If $v_{ij}^k \equiv -1$ or $v_{ij}^k \equiv 1$, $\sigma_{ij}^k \equiv \sigma$, and $\beta_{ij}^k \equiv \beta$ for all $j = 1, ..., \gamma$, $k = 1, .., K$, then* $\mathbb{E}[E_i^{BN+MN}] = \mathbb{E}[E_i^{BN,K-1}] + \varphi(0)\sqrt{\sigma_M^2\beta^2(\gamma-1)^2 + (\gamma-1)\sigma^2}$.

2. *If $\rho_{X,M} > 0$, either $v_{ij}^k \equiv 1$ or $v_{ij}^k \equiv -1$, $\beta_{ij}^k = \beta$, and $\sigma_{ij}^k = \sigma$, then $\mathbb{E}[E_i^{MN}] > (\gamma - 1)|\rho_{X,M}|\sigma_X\varphi(0)$ for all $\gamma > 1$.*
3. *If $\rho_{X,M} \in (-1, 1)$, $v_{ij}^k \equiv v_{ij} \in \{-1, 1\}$, $\beta_{ij}^k \equiv \beta$, and $\sigma_{ij}^k \equiv \sigma$, there exists $\gamma_{\min} < \infty$ such that $\mathbb{E}[E_i^{BN+MN}] < \mathbb{E}[E_i^{BN,K}]$ for all $\gamma \geq \gamma_{\min}$.*

Proof. In a multilaterally netted derivative class K, the collateralized counterparty risk exposure is $\mathbb{E}[E_i^{MN}] = \mathbb{E}\left[\max\left(\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K, 0\right)\right]$. Define

$$\begin{aligned}
\bar{\mu}_i^{MN} &= \mathbb{E}\left[\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K\right] = 0, \\
(\bar{\sigma}_i^{MN})^2 &= \mathrm{var}\left(\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K\right) = \mathrm{var}\left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^K(\beta_{ij}^K M + \sigma_{ij}^K \varepsilon_{ij}^K)\right) \\
&= \sigma_M^2\left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^K\beta_{ij}^K\right)^2 + \sum_{j=1,j\neq i}^{\gamma}(v_{ij}^K)^2(\sigma_{ij}^K)^2.
\end{aligned}$$

Then, it is $\mathbb{E}\left[\max\left(\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K, 0\right)\right] = \varphi(0)\bar{\sigma}_i^{MN}$ and thus

$$\mathbb{E}[E_i^{BN+MN}] = \mathbb{E}[E_i^{BN,K-1}] + \varphi(0)\sqrt{\sigma_M^2\left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K\right)^2 + \sum_{j=1,j\neq i}^{\gamma}(v_{ij}^K)^2(\sigma_{ij}^K)^2}. \tag{39}$$

1. Simplification in the case of homogeneous entities and trades is straightforward.

2. Assume that either $v_{ij}^k \equiv 1$ or $v_{ij}^k \equiv -1$, $\beta_{ij}^k \equiv \beta$ and $\sigma_{ij}^k \equiv \sigma$. Then,

$$\mathbb{E}[E_i^{MN}]/(\gamma-1) \quad = \varphi(0)\sqrt{\sigma_M^2 \frac{\left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K\right)^2}{(\gamma-1)^2} + \frac{\sum_{j=1,j\neq i}^{\gamma}(v_{ij}^K)^2}{(\gamma-1)^2}(\sigma_{ij}^K)^2} \tag{40}$$

$$= \varphi(0)\sqrt{\sigma_M^2\beta^2 + (\gamma-1)^{-1}\sigma^2}, \tag{41}$$

where the RHS is strictly monotonically decreasing in γ for all $\gamma > 1$ and

$$\lim_{\gamma\to\infty} \mathbb{E}[E_i^{MN}]/(\gamma-1) = \varphi(0)\sigma_M|\beta|,$$

it holds that $\mathbb{E}[E_i^{MN}]/(\gamma-1) > \varphi(0)\sigma_M|\beta| = |\rho_{X,M}|\sigma_X\varphi(0)$ for all $\gamma > 0$.

3. Let $\rho_{X,M} \in (-1,1)$. Then, it is $\beta = \rho_{X,M}\frac{\sigma}{\sigma_M\sqrt{1-\rho_{X,M}^2}}$ with $|\beta| < \infty$.

   First, assume that either $v_{ij}^k \equiv 1$ or $v_{ij}^k \equiv -1$, $\beta_{ij}^k = \beta$ and $\sigma_{ij}^k = \sigma$. Then, $\mathbb{E}[E_i^{MN}] = \varphi(0)\sqrt{\sigma_M^2\beta^2(\gamma-1)^2 + (\gamma-1)\sigma^2} \searrow \varphi(0)\sqrt{\sigma_M^2\beta^2}(\gamma-1)$ for $\gamma \to \infty$ and thus there exists $\gamma_{\min}$ such that $\mathbb{E}[E_i^{MN} + E_i^{BN,K-1}] < \mathbb{E}[E_i^{BN,K}]$ if

$$\varphi(0)\sqrt{\sigma_M^2\beta^2}(\gamma-1) + \mathbb{E}[E_i^{BN,K-1}] < \mathbb{E}[E_i^{BN,K}] \tag{42}$$

$$\Leftrightarrow \sqrt{\sigma_M^2\beta^2} + \sqrt{\sigma_M^2(K-1)^2\beta^2 + (K-1)\sigma^2} < \sqrt{\sigma_M^2K^2\beta^2 + K\sigma^2} \tag{43}$$

$$\Rightarrow \sqrt{\sigma_M^2\beta^2(\sigma_M^2(K-1)^2\beta^2 + (K-1)\sigma^2)} < \sigma_M^2\beta^2(2K-2) + \sigma^2 \tag{44}$$

$$\Rightarrow (K-1)\sigma^2\sigma_M^2\beta^2 < 3\sigma_M^4\beta^4(K-1)^2 + \sigma^4 + 4\sigma^2\sigma_M^2\beta^2(K-1) \tag{45}$$

$$\Leftrightarrow 0 < 3\sigma_M^4\beta^4(K-1)^2 + \sigma^4 + 3\sigma^2\sigma_M^2\beta^2(K-1), \tag{46}$$

   which holds for any $K \geq 1$. Since $(\mathbb{E}[E_i^{MN} + E_i^{BN,K-1}] - \mathbb{E}[E_i^{BN,K}])/(\gamma-1)$ is decreasing with γ, $\mathbb{E}[E_i^{MN} + E_i^{BN,K-1}] < \mathbb{E}[E_i^{BN,K}]$ for all $\gamma \geq \gamma_{\min}$.

   Second, assume that $v_{ij}^k \equiv v_{ij} \in \{-1,1\}$ for all i,j,k. Thus, it still holds that $\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma-1)\sqrt{\sigma_M^2\beta^2K^2 + K\sigma^2}$. Since

$$\mathbb{E}[E_i^{MN}] \leq \varphi(0)\sqrt{\sigma_M^2\beta^2(\gamma-1)^2 + (\gamma-1)\sigma^2}, \tag{47}$$

   $\mathbb{E}[E_i^{MN} + E_i^{BN,K-1}] < \mathbb{E}[E_i^{BN,K}]$ for all $\gamma \geq \gamma_{\min}$ with $\gamma_{\min}$ as defined above.

$\square$

Proposition 3 (Collateralized exposure during extreme events)**.** *If $v_{ij}^k \equiv 1$, $\sigma_{ij}^k \equiv \sigma$, and $\beta_{ij}^k \equiv \beta$ for all $j = 1, ..., \gamma$, $k = 1, .., K$, then the collateralized counterparty risk exposure conditional on a realization of systematic risk $\bar{M}$ with bilateral netting is given by*

$$\mathbb{E}[E_{ij}^{BN} \mid M = \bar{M}] = \quad \bar{M} \sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k \Phi\left(\bar{M} \frac{\sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{k=1}^{K} (v_{ij}^k)^2 (\sigma_{ij}^k)^2}}\right) \tag{48}$$

$$+ \sqrt{\sum_{k=1}^{K} (v_{ij}^k)^2 (\sigma_{ij}^k)^2} \varphi\left(-\bar{M} \frac{\sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{k=1}^{K} (v_{ij}^k)^2 (\sigma_{ij}^k)^2}}\right) \tag{49}$$

and with multilateral netting in class K, it is

$$\mathbb{E}[E_i^{MN} \mid M = \bar{M}] = \quad \bar{M} \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \Phi\left(\frac{\bar{M} \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}}\right) \tag{50}$$

$$+ \sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2} \varphi\left(-\frac{\bar{M} \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}}\right) \tag{51}$$

in this class.

Proof. For the collateralized counterparty risk exposure with bilateral netting conditional on state M, define

$$\bar{\mu}_{ij|M}^{BN} = \mathbb{E}\left[\sum_{k=1}^{K} X_{ij}^k \mid M = \bar{M}\right] = \bar{M} \sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k,$$

$$\left(\bar{\sigma}_{ij|M}^{BN}\right)^2 = \operatorname{var}\left(\sum_{k=1}^{K} X_{ij}^k \mid M = \bar{M}\right) = \operatorname{var}\left(\sum_{k=1}^{K} v_{ij}^k \sigma_{ij}^k \varepsilon_{ij}^k\right) = \sum_{k=1}^{K} \left(v_{ij}^k\right)^2 \left(\sigma_{ij}^k\right)^2.$$

Then, with bilateral netting, the collateralized exposure to j conditional on $\{M = \bar{M}\}$ is given by

$$\mathbb{E}[E_{ij}^{BN} \mid M = \bar{M}] = \bar{\mu}_{ij|M}^{BN} \Phi(\bar{\mu}_{ij|M}^{BN} / \bar{\sigma}_{ij|M}^{BN}) + \bar{\sigma}_{ij|M}^{BN} \varphi(-\bar{\mu}_{ij|M}^{BN} / \bar{\sigma}_{ij|M}^{BN}) \tag{52}$$

$$= \bar{M} \sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k \Phi\left(\bar{M} \frac{\sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{k=1}^{K} (v_{ij}^k)^2 (\sigma_{ij}^k)^2}}\right) + \sqrt{\sum_{k=1}^{K} (v_{ij}^k)^2 (\sigma_{ij}^k)^2} \varphi\left(-\bar{M} \frac{\sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{k=1}^{K} (v_{ij}^k)^2 (\sigma_{ij}^k)^2}}\right),$$

and the total counterparty risk exposure is $\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}] = \sum_{j=1,j\neq i}^{\gamma} \mathbb{E}[E_{ij}^{BN} \mid M = \bar{M}]$.

If class K is multilaterally netted, the counterparty risk exposure in this class conditional on $\{M = \bar{M}\}$ is given by $\mathbb{E}[E_i^{MN} \mid M = \bar{M}] = \mathbb{E}\left[\max\left(\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K, 0\right) \mid M = \bar{M}\right]$. Define

$$
\begin{aligned}
\bar{\mu}_{i|M}^{MN} &= \mathbb{E}\left[\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K \mid M = \bar{M}\right] = \bar{M} \sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \\
(\bar{\sigma}_{i|M}^{MN})^2 &= \operatorname{var}\left(\sum_{j=1,j\neq i}^{\gamma} X_{ij}^K \mid M = \bar{M}\right) = \operatorname{var}\left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \sigma_{ij}^K \varepsilon_{ij}^K\right) = \sum_{j=1,j\neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2.
\end{aligned}
$$

Thus, it is

$$
\begin{aligned}
\mathbb{E}[E_i^{MN} \mid M = \bar{M}] &= \bar{\mu}_{i|M}^{MN} \Phi(\bar{\mu}_{i|M}^{MN}/\bar{\sigma}_{i|M}^{MN}) + \bar{\sigma}_{i|M}^{MN} \varphi(-\bar{\mu}_{i|M}^{MN}/\bar{\sigma}_{i|M}^{MN}) \\
&= \bar{M} \sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \Phi\left(\frac{\bar{M} \sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K}{\sqrt{\sum_{j=1,j\neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}}\right) \\
&\quad + \sqrt{\sum_{j=1,j\neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2} \varphi\left(-\frac{\bar{M} \sum_{j=1,j\neq i}^{\gamma} v_{ij}^K \beta_{ij}^K}{\sqrt{\sum_{j=1,j\neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}}\right).
\end{aligned}
\tag{53}
$$

Simplification in the case of homogeneous entities and trades is straightforward. □

Proposition 4 (Uncollateralized counterparty risk exposure)**.** *Assume that* $|v_{ij}^k| \equiv 1$*,* $\beta_{ij}^k \equiv \beta$*, and* $\sigma_{ij}^k \equiv \sigma$ *for all* $j = 1, ..., \gamma$*,* $k = 1, .., K$*. Then, the uncollateralized counterparty risk exposure with bilateral netting equals*

$$
\mathbb{E}\left[\tilde{E}_i^{BN,K}\right] = (\gamma - 1)\sqrt{\sigma_M^2 K^2 \beta^2 + K\sigma^2}\,\xi(\alpha_{BN}),
\tag{54}
$$

and with class-K multilateral netting it is

$$
\mathbb{E}\left[\tilde{E}_i^{BN+MN}\right] = \mathbb{E}\left[\tilde{E}_i^{BN,K-1}\right] + \sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1)\sigma^2}\,\xi(\alpha_{MN}),
\tag{55}
$$

where $\xi(\alpha) = (1 - \alpha)\Phi^{-1}(1 - \alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$*.*

Proof. First, we consider the bilateral case. Define

$$(\bar{\sigma}_{ij}^{BN})^2 = \mathrm{var}\left(\sum_{k=1}^{K} X_{ij}^{k}\right) = \sigma_M^2 \left(\sum_{k=1}^{K} v_{ij}^{k}\beta_{ij}^{k}\right)^2 + \sum_{k=1}^{K}(v_{ij}^{k}\sigma_{ij}^{k})^2,$$

$$\bar{\mu}_{ij}^{BN} = -C_{ij}^{BN,K} = -\bar{\sigma}_{ij}^{BN}\Phi^{-1}(\alpha_{BN}).$$

Then, it is

$$\begin{aligned}
\mathbb{E}\left[\tilde{E}_i^{BN,K}\right] &= \bar{\mu}_{ij}^{BN}\Phi(\bar{\mu}_{ij}^{BN}/\bar{\sigma}_{ij}^{BN}) + \bar{\sigma}_{ij}^{BN}\varphi(-\bar{\mu}_{ij}^{BN}/\bar{\sigma}_{ij}^{BN}) \\
&= \sum_{j=1,j\neq i}^{\gamma}\left(-\Phi^{-1}(\alpha_{BN})\bar{\sigma}_{ij}^{BN}\Phi\left(\frac{-\Phi^{-1}(\alpha_{BN})\bar{\sigma}_{ij}^{BN}}{\sigma_{ij}^{\bar{B}N}}\right) + \bar{\sigma}_{ij}^{BN}\varphi\left(-\frac{-\Phi^{-1}(\alpha_{BN})\bar{\sigma}_{ij}^{BN}}{\sigma_{ij}^{\bar{B}N}}\right)\right) \\
&= \sum_{j=1,j\neq i}^{\gamma}\left(-\Phi^{-1}(\alpha_{BN})\bar{\sigma}_{ij}^{BN}\Phi\left(\Phi^{-1}(1-\alpha_{BN})\right) + \bar{\sigma}_{ij}^{BN}\varphi\left(\Phi^{-1}(\alpha_{BN})\right)\right). \qquad (56)
\end{aligned}$$

If entities and trades are homogeneous, then the exposure equals

$$\mathbb{E}\left[\tilde{E}_i^{BN,K}\right] = (\gamma-1)\sqrt{\sigma_M^2K^2\beta^2 + K\sigma^2}\left((1-\alpha_{BN})\Phi^{-1}(1-\alpha_{BN}) + \varphi\left(\Phi^{-1}(\alpha_{BN})\right)\right). \qquad (57)$$

Second, we consider the multilateral case, with derivative class K being multilaterally netted. It is $\mathbb{E}[\tilde{E}_i^{MN}] = \mathbb{E}\left[\max\left(\sum_{j=1,k\neq i}^{\gamma} X_{ij}^{K} - \frac{|v_{ij}^K|}{\sum_{h=1,h\neq j}^{\gamma}|v_{hj}^K|}C_j^{MN}, 0\right)\right]$ the exposure in derivative class K. Define

$$(\bar{\sigma}_{i}^{MN})^2 = \mathrm{var}\left(\sum_{j=1}^{\gamma} X_{ij}^{K}\right) = \sigma_M^2 \left(\sum_{j=1,j\neq i}^{\gamma} v_{ij}^{K}\beta_{ij}^{K}\right)^2 + \sum_{j=1,j\neq i}^{\gamma}(v_{ij}^{K}\sigma_{ij}^{K})^2,$$

$$C_j^{MN} = \bar{\sigma}_j^{MN}\Phi^{-1}(\alpha_{MN}),$$

$$\bar{\mu}_i^{MN} = -\sum_{j=1,j\neq i}^{\gamma}\frac{|v_{ij}^K|}{\sum_{h=1,h\neq j}^{\gamma}|v_{hj}^K|}C_j^{MN}.$$

Assuming that $|v_{ij}^K| \equiv 1$, $\beta_{ij}^K \equiv \beta$, and $\sigma_{ij}^K \equiv \sigma$, it is $\bar{\sigma}_j^{MN} \equiv \bar{\sigma}_1^{MN}$ for all i, j and

$$\begin{aligned}
\mathbb{E}\left[\tilde{E}_i^{MN}\right] &= \bar{\mu}_i^{MN}\Phi\left(\bar{\mu}_i^{MN}/\bar{\sigma}_i^{MN}\right) + \bar{\sigma}_i^{MN}\varphi\left(-\bar{\mu}_i^{MN}/\bar{\sigma}_i^{MN}\right) \\
&= -\sum_{j=1,j\neq i}^{\gamma}\frac{1}{\gamma-1}\Phi^{-1}(\alpha_{MN})\bar{\sigma}_j^{MN}\Phi\left(\frac{-\sum_{j=1,j\neq i}^{\gamma}\frac{1}{\gamma-1}\Phi^{-1}(\alpha_{MN})\bar{\sigma}_j^{MN}}{\bar{\sigma}_i^{MN}}\right) \\
&\quad + \bar{\sigma}_i^{MN}\varphi\left(-\frac{-\sum_{j=1,j\neq i}^{\gamma}\frac{1}{\gamma-1}\Phi^{-1}(\alpha_{MN})\bar{\sigma}_j^{MN}}{\bar{\sigma}_i^{MN}}\right) \\
&= \bar{\sigma}_1^{MN}\left(\Phi^{-1}(1-\alpha_{MN})(1-\alpha_{MN}) + \varphi\left(\Phi^{-1}(1-\alpha_{MN})\right)\right) \qquad (58) \\
&= \sqrt{\sigma_M^2(\gamma-1)^2\beta^2 + (\gamma-1)\sigma^2}\left(\Phi^{-1}(1-\alpha_{MN})(1-\alpha_{MN}) + \varphi\left(\Phi^{-1}(1-\alpha_{MN})\right)\right).
\end{aligned}$$

As before, $\mathbb{E}[\tilde{E}_i^{BN+MN}] = \mathbb{E}\left[\tilde{E}_i^{BN,K-1}\right] + \mathbb{E}\left[\tilde{E}_i^{MN}\right]$. □

Proposition 5. *Assume that $v_{ij}^k \equiv 1$, $\beta_{ij}^k \equiv \beta$, and $\sigma_{ij}^k \equiv \sigma$ for all $j = 1, ..., \gamma$, $k = 1, .., K$.*

1) *If $\alpha_{BN} = \alpha_{MN}$, the relative benefit of multilateral netting is independent from the margin level.*
2) *The smaller the clearing margin level compared to the bilateral margin level, the larger the uncollateralized counterparty risk exposure with multilateral netting of one derivative class relative to exposure with full bilateral netting, and vice versa.*

Proof.

1) Assume that $\alpha_{BN} = \alpha_{MN}$. Then,

$$\frac{E\left[\tilde{E}_i^{BN+MN}\right]}{\mathbb{E}\left[\tilde{E}_i^{BN,K}\right]} = \frac{\sqrt{\sigma_M^2(\gamma-1)^2\beta^2 + (\gamma-1)\sigma^2}\frac{\xi(\alpha_{MN})}{\xi(\alpha_{BN})} + (\gamma-1)\sqrt{\sigma_M^2K^2\beta^2 + K\sigma^2}}{(\gamma-1)\sqrt{\sigma_M^2K^2\beta^2 + K\sigma^2}} \qquad (59)$$

is independent from the margin level α_{BN}.

2) $\xi(\alpha)$ is decreasing with the margin requirement α. Thus, the smaller $\frac{\alpha_{MN}}{\alpha_{BN}}$ is, the larger $\frac{\xi(\alpha_{MN})}{\xi(\alpha_{BN})}$ and the ratio of counterparty risk in Equation (59) are, and thus the smaller the benefit of multilateral netting is.

□

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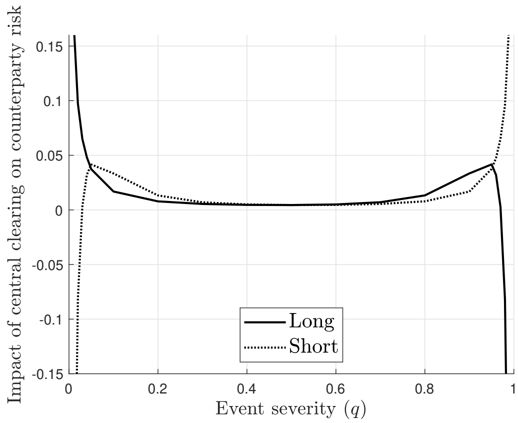
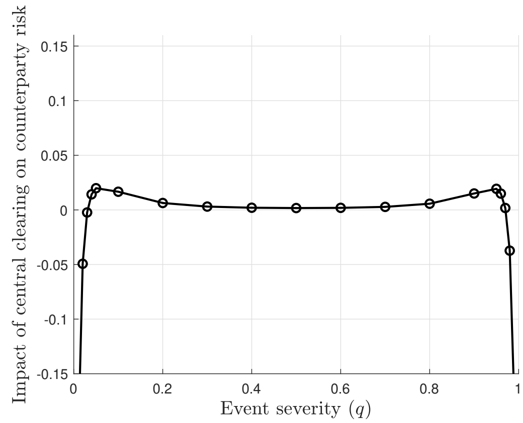


(a) Directional portfolio.



(b) Flat portfolio.

Figure 1. Change in counterparty risk due to central clearing (with loss sharing) compared to bilateral netting conditional on different realizations of the systematic risk factor.

The figure depicts the relative change in counterparty risk with central clearing and loss sharing of one derivative class compared to bilateral netting of all derivative classes, conditional on different levels of the systematic risk factor $M = \bar{M} = \sigma_M \Phi^{-1}(q)$, where $\Phi^{-1}(\cdot)$ is the inverse cdf of the standard normal distribution. If the change in counterparty risk is negative, central clearing reduces counterparty risk. The calibration is described in Section 4. A long (short) position is a position whose profit and loss is positively (negatively) correlated with the systematic risk factor.

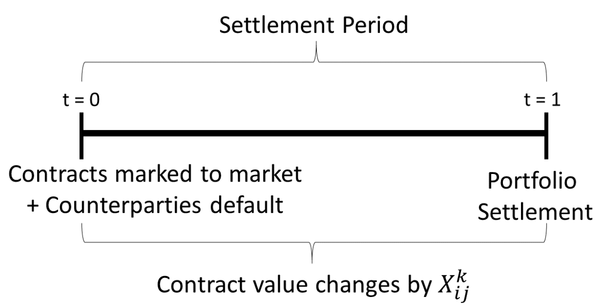
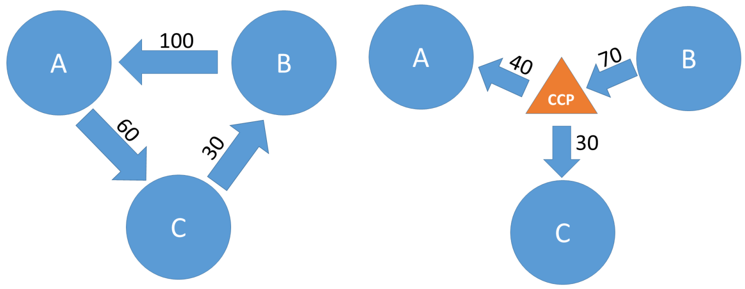


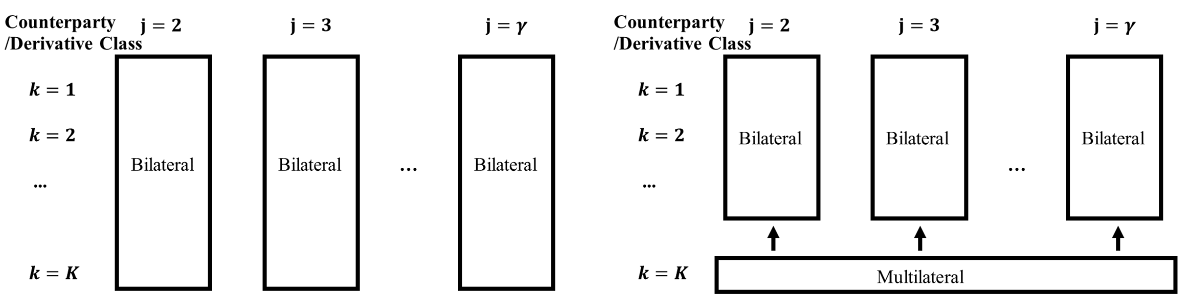
Figure 2. Timeline of the model.

Losses due to counterparty default occur between time $t = 0$, the most recent date where contracts have been marked to market and counterparties might default, and time $t = 1$, at which time the portfolio is settled.



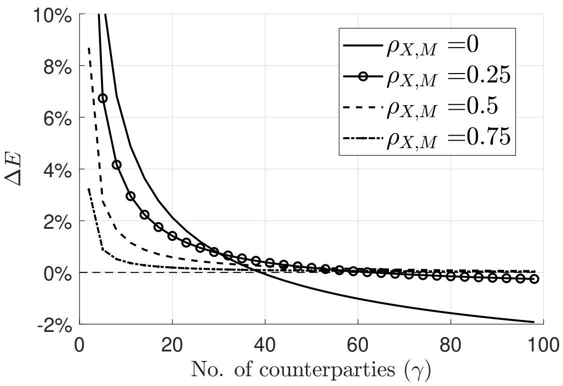
(a) Bilateral netting. (b) Multilateral netting.

Figure 3. Illustration of bilateral and multilateral netting.
(a) Bilateral netting and (b) multilateral netting across counterparties. Arrows illustrate the flow of profits and losses (e.g., B owes $100 to A).

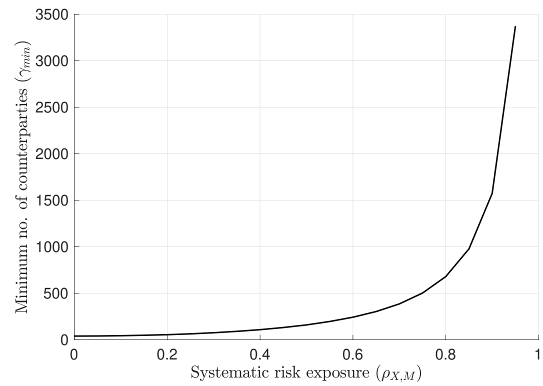


(a) Full bilateral netting. (b) Multilateral netting of derivative class K.

Figure 4. Bilateral and multilateral netting portfolios.
K is the number of derivative classes and γ the number of counterparties trading with each other. The illustration is from the perspective of entity i, which trades with $\gamma - 1$ counterparties.



(a) Change in exposure due to multilateral netting.



(b) Minimum number of counterparties such that multilateral netting of derivative class K reduces exposures.

Figure 5. Impact of systematic risk for directional traders.
(a) Change in collateralized counterparty risk exposure due to multilateral netting of derivative class K compared to bilateral netting, $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K}]/\mathbb{E}[E_i^{BN,K}]$ for different levels of systematic risk exposure, i.e., correlation between derivative prices and systematic risk factor $\rho_{X,M}$. If $\Delta E < 0$, multilateral netting reduces counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties $\gamma_{\min}$, such that multilateral netting of derivative class K reduces collateralized counterparty risk exposure compared to bilateral netting with respect to systematic risk exposure $\rho_{X,M}$. We assume $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, and volatility of the systematic risk factor $\sigma_M = 0.03$.

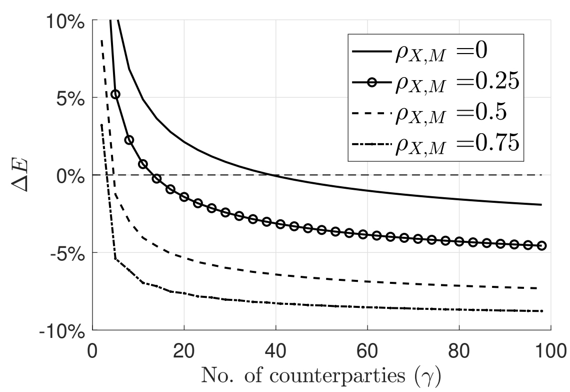
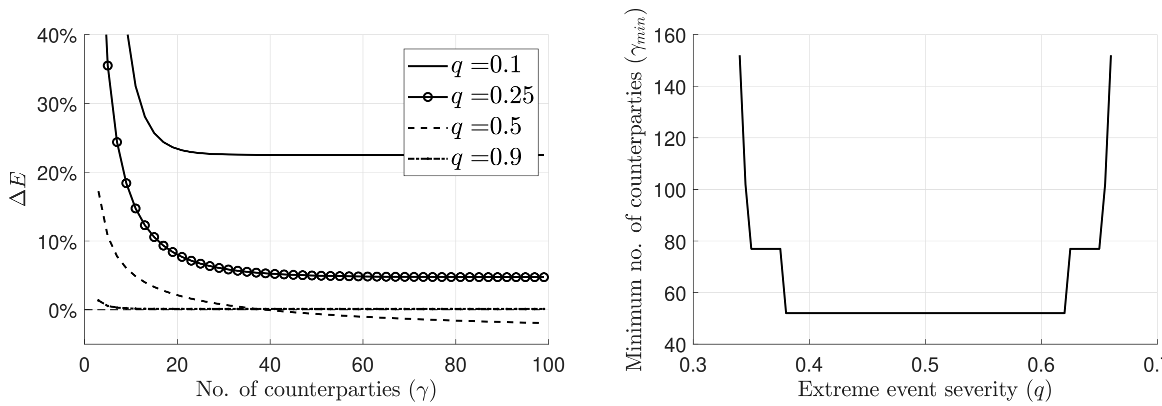


Figure 6. Impact of systematic risk for dealers.
Change in collateralized counterparty risk exposure due to multilateral netting of derivative class K compared to bilateral netting, $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K}]/\mathbb{E}[E_i^{BN,K}]$ for different levels of systematic risk exposure, i.e., correlation between derivative prices and systematic risk factor $\rho_{X,M}$. If $\Delta E < 0$, multilateral netting reduces counterparty risk exposure compared to bilateral netting. We assume $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, and volatility of the systematic risk factor $\sigma_M = 0.03$.

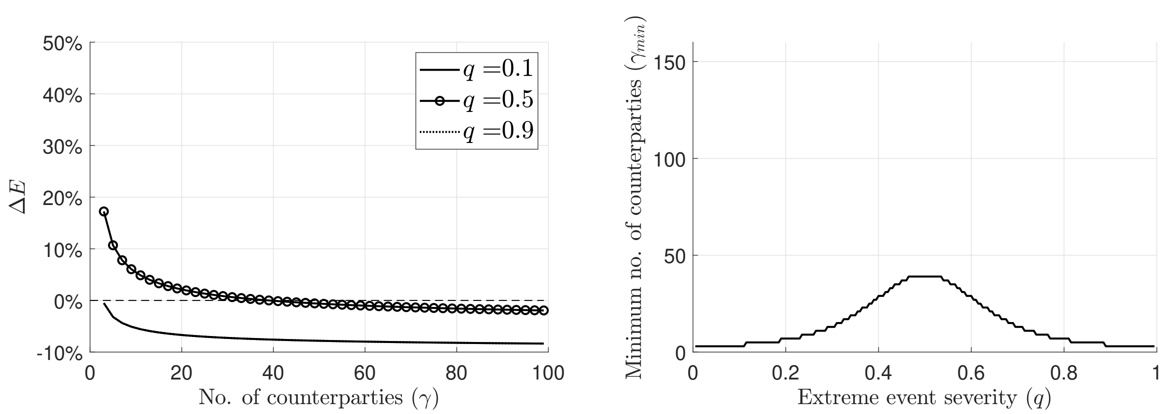


(a) Change in exposure due to multilateral netting.

(b) Minimum number of counterparties.

Figure 7. Impact of extreme events for a directional trader ($v_{ij}^k = 1$).

(a) Change in collateralized counterparty risk exposures due to multilateral netting of derivative class K compared to bilateral netting, $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K} \mid M = \bar{M}]/\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}]$ conditional on extreme event $\bar{M} = \sigma_M \Phi^{-1}(q)$. The smaller the severity parameter q, the more adverse the event. If $\Delta E < 0$, multilateral netting reduces counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties $\gamma_{\min}$, such that multilateral netting of derivative class K reduces collateralized counterparty risk exposure compared to bilateral netting with respect to the severity of extreme events. We assume $K = 10$ derivative classes, correlation between contract returns and systematic risk $\rho_{X,M} = 0.43$, total contract volatility $\sigma_X = 0.01$, and volatility of the systematic risk factor $\sigma_M = 0.03$.

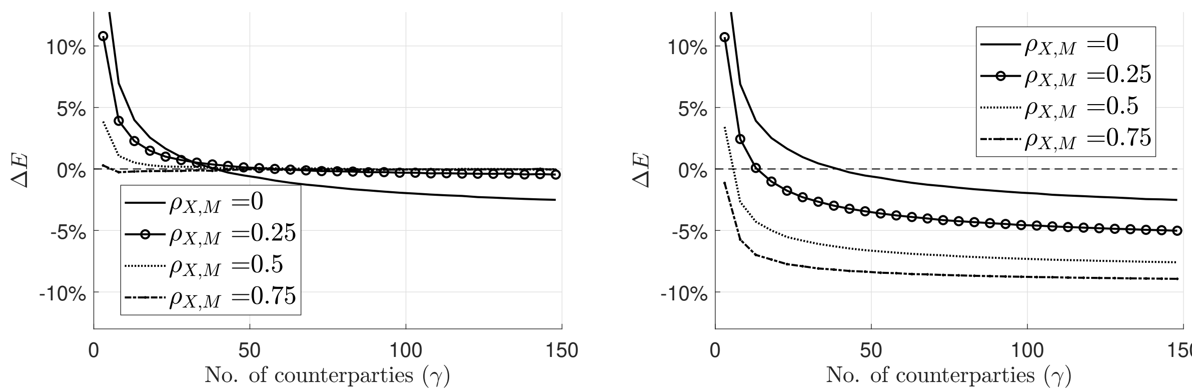


(a) Change in exposure due to multilateral netting.

(b) Minimum number of counterparties.

Figure 8. Impact of extreme events for a dealer.

(a) Change in collateralized counterparty risk exposures due to multilateral netting of derivative class K compared to bilateral netting, $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K} \mid M = \bar{M}]/\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}]$ conditional on extreme event $\bar{M} = \sigma_M \Phi^{-1}(q)$. The smaller the severity parameter q, the more adverse the event. If $\Delta E < 0$, multilateral netting reduces counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties $\gamma_{\min}$, such that multilateral netting of derivative class K reduces collateralized counterparty risk exposure compared to bilateral netting with respect to the severity of extreme events. We assume $K = 10$ derivative classes, correlation between contract returns and systematic risk $\rho_{X,M} = 0.43$, total contract volatility $\sigma_X = 0.01$, and volatility of the systematic risk factor $\sigma_M = 0.03$.



(a) Directional trader.

(b) dealer.

Figure 9. Impact of systematic risk on counterparty risk with loss sharing.

Change in counterparty risk due to central clearing with loss sharing of derivative class K compared to bilateral netting, $\Delta E = \mathbb{E}[E_i^{*BN+MN,LS} - E_i^{*BN,K}]/\mathbb{E}[E_i^{*BN,K}]$, for different levels of systematic risk exposure $\rho_{X,M}$ of derivatives prices. If $\Delta E < 0$, then central clearing reduces counterparty risk compared to bilateral netting. We assume the correlation between entities' log asset values, i.e., default clustering, to be $\rho_{A,A} = 0.1$, with $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, volatility of the systematic risk factor $\sigma_M = 0.03$, entity default probability $\pi = 0.1$, log-asset value volatility $\sigma_A = 1$, and clearing and bilateral margin level $\alpha_{MN} = \alpha_{BN} = 0.99$.

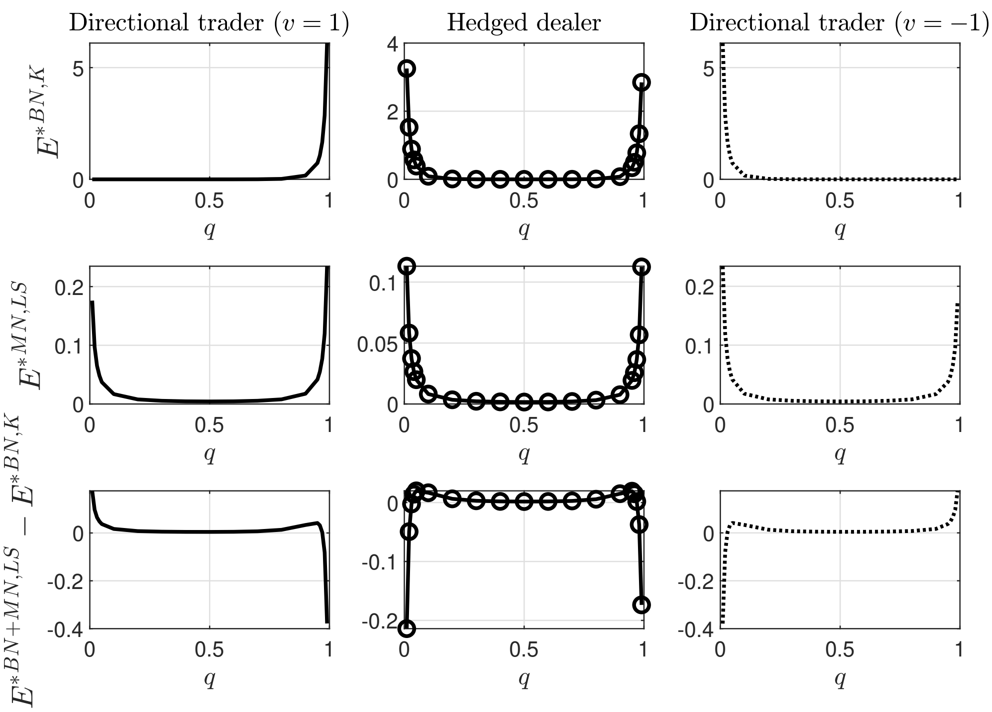
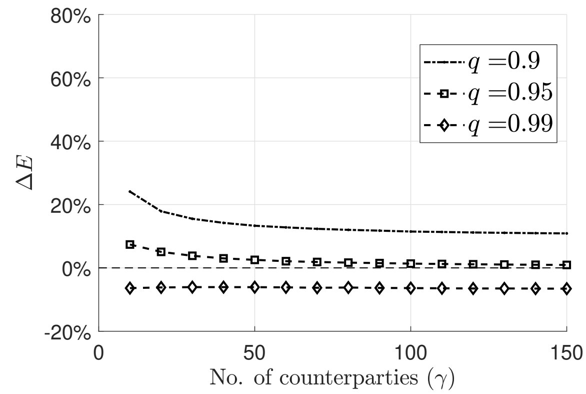
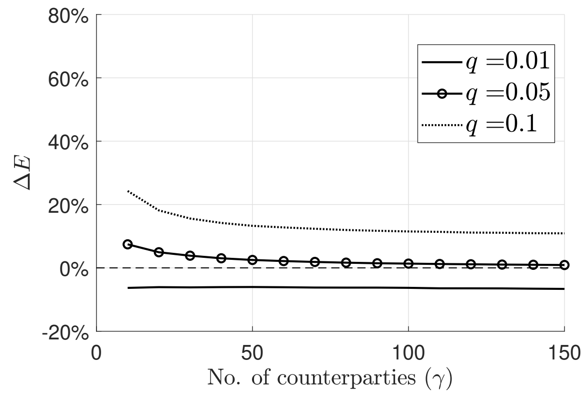


Figure 10. Counterparty risk with central clearing and loss sharing vs. bilateral netting for different realizations of the systematic risk factor.

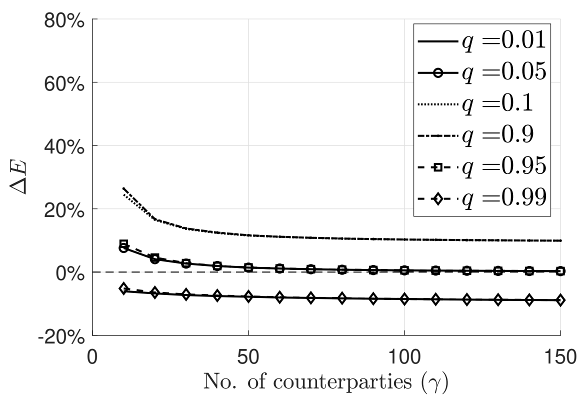
Counterparty risk with loss sharing conditional on different levels of the systematic risk factor $M = \bar{M} = \sigma_M \Phi^{-1}(q)$. $E^{*BN,K}$ is the counterparty risk if all derivative classes are bilaterally netted, $E^{*MN,LS}$ is the counterparty risk from central clearing with loss sharing of class K, and $E^{*BN+MN,LS}$ is the counterparty risk from central clearing with loss sharing of class K and bilateral netting of remaining derivative classes. We assume the correlation between entities' log asset values, i.e., default clustering, to be $\rho_{A,A} = 0.1$, with $\gamma = 16$ counterparties, $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, volatility of the systematic risk factor $\sigma_M = 0.03$, entity default probability $\pi = 0.1$, log-asset value volatility $\sigma_A = 1$, and clearing and bilateral margin level $\alpha_{MN} = \alpha_{BN} = 0.99$. The figure does qualitatively not change with a larger number of counterparties γ.



(a) Directional trader ($v = 1$).



(b) Directional trader ($v = -1$).



(c) dealer.

Figure 11. Central clearing with loss sharing vs. bilateral netting during extreme market events.
Change in counterparty risk with central clearing and loss sharing of derivative class K compared to bilateral netting, $\Delta E = \mathbb{E}[E_i^{*BN+MN,LS} - E_i^{*BN,K} \mid M = \bar{M}]/\mathbb{E}[E_i^{*BN,K} \mid M = \bar{M}]$, for different levels of systematic risk exposure $\rho_{X,M}$ of derivatives prices and conditional on different levels of the systematic risk factor $M = \bar{M} = \sigma_M \Phi^{-1}(q)$. If $\Delta E < 0$, central clearing reduces counterparty risk compared to bilateral netting. We assume the correlation between entities' log asset values, i.e., default clustering, to be $\rho_{A,A} = 0.1$, with $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, volatility of the systematic risk factor $\sigma_M = 0.03$, entity default probability $\pi = 0.1$, log-asset value volatility $\sigma_A = 1$, and clearing and bilateral margin level $\alpha_{MN} = \alpha_{BN} = 0.99$. Note that in Figure (c) the lines for $q = 0.01$ and $q = 0.99$, $q = 0.05$ and $q = 0.95$, $q = 0.1$ and $q = 0.9$ coincide, respectively.

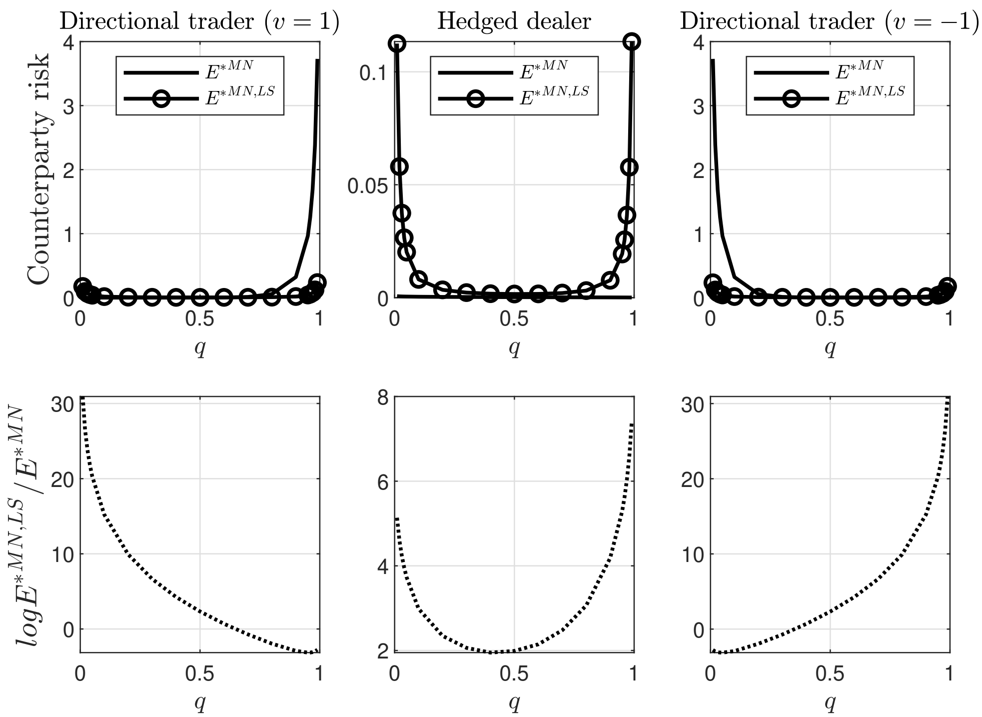


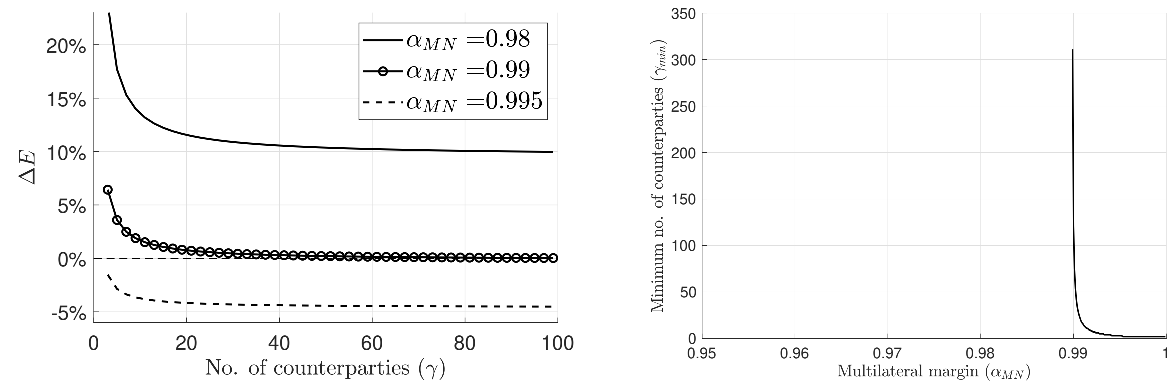
Figure 12. Counterparty risk with central clearing and loss sharing vs. central clearing without loss sharing for different realizations of the systematic risk factor.

Counterparty risk with central clearing and loss sharing, and central clearing without loss sharing conditional on different levels of the systematic risk factor $M = \bar{M} = \sigma_M \Phi^{-1}(q)$. E^{*MN} is the counterparty risk in one derivative class if it is centrally cleared without loss sharing, $E^{*MN,LS}$ is the counterparty risk in one derivative class if it is centrally cleared wit loss sharing. We assume the correlation between entities' log asset values, i.e., default clustering, to be $\rho_{A,A} = 0.1$, with $\gamma = 16$ counterparties, $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, volatility of the systematic risk factor $\sigma_M = 0.03$, entity default probability $\pi = 0.1$, log-asset value volatility $\sigma_A = 1$, and clearing and bilateral margin level $\alpha_{MN} = \alpha_{BN} = 0.99$. The figure does qualitatively not change with a larger number of counterparties γ.

Online Appendix

Counterparty risk exposure and margin requirements

Figure 13 (a) depicts the change in uncollateralized exposures due to multilateral netting. Clearly, if the clearing margin confidence level α_{MN} is small relative to the bilateral margin confidence level α_{BN}, multilateral netting increases uncollateralized counterparty risk exposure compared to bilateral netting.[45]



(a) Change in exposure due to multilateral netting.

(b) Minimum number of counterparties.

Figure 13. Impact of margins for directional traders.
(a) Change in uncollateralized counterparty risk exposure due to multilateral netting of class K compared to bilateral netting, $\Delta\tilde{E} = \mathbb{E}[\tilde{E}_i^{BN+MN} - \tilde{E}_i^{BN,K}]/\mathbb{E}[\tilde{E}_i^{BN,K}]$, for different levels of the clearing margin confidence level α_{MN}. If $\Delta\tilde{E} < 0$, multilateral netting reduces uncollateralized counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties $\gamma_{\min}$, such that multilateral netting of derivative class K reduces uncollateralized exposure with respect to the clearing margin level. The bilateral margin confidence level is $\alpha_{BN} = 0.99$. We assume $K = 10$ derivative classes, correlation between contract returns and systematic risk $\rho_{X,M} = 0.43$, total contract volatility $\sigma_X = 0.01$, and volatility of the systematic risk factor $\sigma_M = 0.03$.

If the clearing margin is small, then multilateral netting does not reduce counterparty risk exposure *for any number of counterparties* (e.g., with $\alpha_{MN} = 0.98$ in Figure 13). This is the case in Figure 13 (b), as we do not find any number of counterparties γ that reduces uncollateralized exposures for $\alpha_{MN} \leq 0.9897$ compared to the bilateral margin confidence level $\alpha_{BN} = 0.99$. Hence, uncollateralized exposures are extremely sensitive toward small discrepancies between margins for centrally and bilaterally netted derivatives.

The reason for this high sensitivity is systematic risk. Due to systematic risk, the average counterparty risk per counterparty under multilateral netting is bounded from below by (see Proposition

[45]Note that this result trivially also holds for a dealer.

2 in the Appendix)[46]

$$\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma - 1) > |\rho_{X,M}|\sigma_X \xi(\alpha_{MN}). \tag{60}$$

Larger correlation between derivatives prices and systematic risk factor ($\rho_{X,M}$) and lower margins (α_{MN}) increase this lower bound for multilateral exposure.[47] Eventually, if $|\rho_{X,M}|$ is sufficiently large (or α_{MN} is sufficiently low compared to α_{BN}), the lower bound for the exposure in multilaterally netted class-K exceeds the exposure from including class-K into bilateral netting pools.

Based on this observation, in the following we examine the sensitivity of netting benefits toward differences in margins more closely. For this purpose, we derive a condition for the clearing confidence level α_{MN}, such that, for a given α_{BN}, multilateral netting leads to a reduction in uncollateralized counterparty risk exposure compared to bilateral netting. From the lower bound of $\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma - 1)$, we derive the following threshold for the clearing margin: multilateral netting does not reduce uncollateralized exposures for any finite number of counterparties $\gamma < \infty$ if $\alpha_{MN} \leq \mathcal{H}_{MN}$ with

$$\mathcal{H}_{MN} = \xi^{-1}\left(\frac{\xi(\alpha_{BN})}{|\rho_{X,M}|}\left(\sqrt{K}\sqrt{1+\rho_{X,M}^2(K-1)} - \sqrt{K-1}\sqrt{1+\rho_{X,M}^2(K-2)}\right)\right). \tag{61}$$

It is straightforward to show that $\mathcal{H}_{MN}$ is increasing with systematic risk exposure $|\rho_{X,M}|$, $\frac{d\mathcal{H}_{MN}}{d|\rho_{X,M}|} > 0$, which mainly results from ξ, and thus ξ^{-1}, being decreasing functions (see Proposition 5 in the Appendix). Hence, the more extreme (positive or negative) the systematic risk exposure $\rho_{X,M}$ is, the larger must be the clearing margin for multilateral netting to reduce counterparty risk exposure. Moreover, $\mathcal{H}_{MN}$ is bounded from above by α_{BN}, $\mathcal{H}_{MN} \leq \alpha_{BN}$, since

$$\lim_{\rho_{X,M}^2 \to 1} \mathcal{H}_{MN} = \xi^{-1}(\xi(\alpha_{BN})) = \alpha_{BN}, \tag{62}$$

and bounded from below by zero, $0 \leq \mathcal{H}_{MN}$, since

$$\lim_{\rho_{X,M} \to 0} \mathcal{H}_{MN} = \xi^{-1}(\infty) = 0. \tag{63}$$

[46]This result only holds for entities with a directional multilateral portfolio. If the multilaterally netted portfolio is not exposed to systematic risk (e.g., for a dealer), $\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma - 1) \to 0$ for $\gamma \to \infty$.

[47]This results resembles the finding of Menkveld (2017), who stresses that current CCP margin practices are inefficient in that they do not account for correlation across clearing members.

Thus, in the case of no systematic risk ($\rho_{X,M} = 0$), for any confidence levels α_{BN} and α_{MN}, there exists a number of counterparties γ, such that multilateral netting is beneficial. However, the larger the systematic risk exposure, the smaller is the acceptable difference between the margin for cleared and non-centrally cleared derivatives. For example, in our baseline calibration, multilateral netting is not beneficial compared to bilateral netting for any number of counterparties if the bilateral margin is $\alpha_{BN} = 0.99$ and the clearing margin is below $\alpha_{MN} \leq 0.98$, as Figure 13 (a) shows. This is in line with the upper bound we derived above, which is $\mathcal{H}_{MN} = 0.9897$ for our baseline calibration.

Result 14. *For every bilateral margin confidence level $\alpha_{BN} \in (0,1)$, there exists a threshold $\mathcal{H}_{MN} \leq \alpha_{BN}$, such that a directional trader's uncollateralized counterparty risk exposure is larger with multilateral netting than with bilateral netting for any number of counterparties if the clearing margin is $C^{MN} \leq VaR_{\mathcal{H}_{MN}}(\sum_{i=1,i\neq j}^{\gamma} X_{ij}^k)$. The threshold $\mathcal{H}_{MN}$ is increasing with the absolute value of correlation between derivatives prices and systematic risk factor, $|\rho_{X,M}|$, such that a larger clearing margin is necessary for more extreme systematic risk exposure.*

Vice versa,a sufficiently large clearing margin results in an unambiguously smaller counterparty risk exposure with multilateral netting: if $\alpha_{MN} \geq \mathcal{U}_{MN}$, then the counterparty risk exposure is smaller with multilateral than bilateral netting for any number of counterparties $\gamma \geq 2$, where[48]

$$\mathcal{U}_{MN} = \xi^{-1}\left(\xi(\alpha_{BN})\left(\sqrt{K}\sqrt{(K-1)\rho_{X,M}^2 + 1} - \sqrt{K-1}\sqrt{(K-2)\rho_{X,M}^2 + 1}\right)\right). \tag{64}$$

This is the case, for example, with $\alpha_{BN} = 0.99$ and $\alpha_{MN} = 0.995$ in Figure 13 (a), since $\mathcal{U}_{MN} = 0.995$ for our baseline calibration. From Equation (64), it is clear that $\mathcal{U}_{MN}$ is decreasing with $|\rho_{X,M}|$ and converging to α_{BN} for $|\rho_{X,M}| \to 1$. Hence, the larger the absolute value of correlation between derivatives prices and systematic risk factor, the smaller the necessary clearing margin $VaR_{\mathcal{U}_{MN}}$, such that multilateral netting is beneficial for any number of counterparties. The necessary clearing margin is always larger than the bilateral margin, $VaR_{\mathcal{U}_{MN}} > VaR_{\alpha_{BN}}$ for $|\rho_{X,M}| < 1$.

Result 15. *For every bilateral margin confidence level $\alpha_{BN} \in (0,1)$, there exists a threshold $\mathcal{U}_{MN} \geq \alpha_{BN}$, such that a directional trader's uncollateralized counterparty risk exposure is lower*

[48]This results from $\xi(\alpha)$ being strictly positive for any $\alpha \in (0,1)$ and ξ^{-1} with full support on $(0,\infty)$.

with multilateral netting than with bilateral netting for any number of counterparties if the clearing margin is $C^{MN} \geq VaR_{\mathcal{U}_{MN}}(\sum_{i=1,i\neq j}^{\gamma} X_{ij}^{k})$. *The threshold* $\mathcal{U}_{MN}$ *is decreasing with the absolute value of systematic risk exposure* $|\rho_{X,M}|$, *such that a smaller clearing margin is sufficient for more extreme systematic risk exposure.*

Eventually, our results divide possible margin confidence levels into three disjoint intervals:

1. $\alpha_{MN} \in (0, \mathcal{H}_{MN}]$ with $\mathcal{H}_{MN} \leq \alpha_{BN}$: Multilateral netting is not beneficial for any number of counterparties γ.
2. $\alpha_{MN} \in (\mathcal{H}_{MN}, \alpha_{BN}] \cup (\alpha_{BN}, \mathcal{U}_{MN})$: Multilateral netting is beneficial if the number of counterparties γ is sufficiently large.
3. $\alpha_{MN} \in [\mathcal{U}_{MN}, 1)$ with $\mathcal{U}_{MN} > \alpha_{BN}$: Multilateral netting is beneficial for any number of counterparties $\gamma \geq 2$.

Regulation for non-centrally cleared derivatives requires initial margins to account for a 99% confidence interval over at least a 10-day horizon of market price changes (Bank for International Settlements (BIS) (2015)). CCPs are required to establish a single-tailed confidence interval level of at least 99% of future exposure, while the margin period is typically 5 days (Bank for International Settlements (BIS) (2012, 2014b), Duffie et al. (2015), Ghamami and Glasserman (2017)). These requirements result in a smaller margin for centrally than bilaterally netted trades, which is intended by policymakers to incentivize market participants to make use of central clearing (Duffie et al. (2010)).[49] The difference of 10 and 5 days in calculation horizon for the margin relates to a volatility ratio of $\sqrt{2}$, such that $\sqrt{2}C^{MN} = C^{BN,K}$. Fixing $\alpha_{BN} = 0.99$, the equivalent clearing margin confidence level is $\alpha_{MN} = \Phi(\Phi^{-1}(\alpha_{BN})/\sqrt{2}) = 0.88$, i.e., $\alpha_{MN} = 0.88$ reflects the 99% Value-at-Risk for a five-day margin period and $\alpha_{BN} = 0.99$ that for a 10-day margin period.

In our baseline calibration with $\alpha_{BN} = 0.99$, multilateral netting with $\alpha_{MN} = 0.88$ never leads to a reduction in uncollateralized counterparty risk exposure, but increases exposures for any number of counterparties γ. Indeed, $\alpha_{MN} = 0.88 < \mathcal{H}_{MN}$, since $\mathcal{H}_{MN} = 0.9897$. Thus, for our

[49] In practice, CCPs have some flexibility in setting margins. However, according to industry information, margins for cleared transactions are clearly smaller than those for non-centrally cleared transactions. Discrepancies and heterogeneity in margin requirements might also result from CCP funding cost and clearing member fundamentals. Huang (2018) links margin requirements to the capitalization of for-profit CCPs, and shows that better-capitalized CCPs require larger margins. Capponi and Cheng (2018) examine the trade-off between larger margins and decreased market volume that results from larger margins raising margin cost for clearing members and, thereby, reducing trading volume but also simultaneously protecting the CCP against default losses.

calibration and current margin requirements, counterparty risk exposure is unambiguously smaller with bilateral than multilateral netting. Instead, a confidence level α_{MN} of more than 98.97% is needed for multilateral netting to be able to achieve a reduction in counterparty risk exposure with a sufficient number of clearing members.[50] If the clearing margin confidence level was at least $\mathcal{U}_{MN} = 99.5\%$, then multilateral netting would be beneficial for any number of clearing members.

Model for correlated defaults

In order to allow for correlation of entity defaults, we employ a credit model based on the Merton model (Merton (1974)). In particular, we assume that each counterparty i defaults at the start of the settlement period if the random value of its assets is below a given bankruptcy threshold, $A_i < B_i$.

The value of assets at the start of the settlement period is given by

$$A_i = \exp\left(\mu_{A_i} - \frac{\sigma_{A_i}^2}{2} + \sigma_{A_i} W_i\right), \tag{65}$$

where $(W_1, ..., W_\gamma)$ are jointly standard normally distributed and correlated with pairwise correlation ρ_{A_i,A_j}. The log value of assets is normally distributed with $\log A_i \sim \mathcal{N}\left(\mu_{A_i} - \frac{\sigma_{A_i}^2}{2}, \sigma_{A_i}^2\right)$. The pairwise correlation of two entities' log assets is given by

$$\tilde{\rho}_{A_i,A_j} = cor\left(\log A_i, \log A_j\right) = \frac{\sigma_{A_i}\sigma_{A_j}\rho_{A_i,A_j}}{\sigma_{A_i}\sigma_{A_j}}. \tag{66}$$

The individual (unconditional) default probability of entity i is given by

$$\pi_i = \mathbb{P}\left(A_i < B_i\right) = \Phi\left(\frac{\log B_i - \mu_{A_i} + \frac{\sigma_{A_i}^2}{2}}{\sigma_{A_i}}\right). \tag{67}$$

Without loss of generality, we assume that $\mu_{A_i} \equiv 0$. Then, the default intensity is given by $\bar{d}_i = \frac{\log B_i}{\sigma_{A_i}} + \frac{\sigma_{A_i}}{2}$. We define by $D = (D_1, ..., D_\gamma)$ a vector of binary random variables $D_i = \delta_{A_i < B_i}$

[50] For this result, we assume that the actual liquidation period is the same for non-centrally cleared and cleared transactions. However, the infrastructure of CCPs enables them to auction assets faster than individual market participants. If the liquidation period of cleared trades is smaller than of non-centrally cleared ones, then a smaller clearing margin $\mathcal{H}_{MN}$ is acceptable.

that signal the default of entity $i \in \{1, ..., \gamma\}$. The joint distribution of two entities' default state is determined by

$$\mathbb{P}\left(D_i = 1, D_j = 1\right) = \mathbb{P}\left(\bar{Z}_i < \bar{d}_i, \bar{Z}_j < \bar{d}_j\right) = \Phi_{2,\Sigma}(\bar{d}_i, \bar{d}_j), \tag{68}$$

where (Z_i, Z_j) are multi-normally distributed with zero mean, unit variance, and correlation matrix Σ, with $\Sigma_{ij} = \rho$, $i \neq j$, and $\Sigma_{ii} = 1$, and

$$\mathbb{P}\left(D_i = 1, D_j = 0\right) = \mathbb{P}\left(Z_i < \bar{d}_i, Z_j \geq \bar{d}_j\right) = \mathbb{P}\left(Z_i < \bar{d}_i, -Z_j < -\bar{d}_j\right) \tag{69}$$

$$= \mathbb{P}\left(Z_i < \bar{d}_i, \tilde{Z}_j < -\bar{d}_j\right) = \Phi_{2,\tilde{\Sigma}}(\bar{d}_i, -\bar{d}_j) \tag{70}$$

where $(Z_i, \tilde{Z}_j)$ is multi-normally distributed, $(Z_i, \tilde{Z}_j) \sim \mathcal{N}_2(0, \tilde{\Sigma})$ with correlation matrix $\tilde{\Sigma}_{ij} = -\tilde{\rho}$, $i \neq j$ and $\tilde{\Sigma}_{ii} = 1$, $i, j \in \{1, 2\}$. Iteration yields the general distribution of default states as

$$\mathbb{P}(D = d) = \Phi_{\gamma,\tilde{\Sigma}}\left(\tilde{d}\right), \tag{71}$$

where $\tilde{d}_i = \begin{cases} \bar{d}_i, & d_i = 1 \\ -\bar{d}_i, & d_i = 0 \end{cases}$, $\tilde{\Sigma}_{ii} = 1$, and $\tilde{\Sigma}_{ij} = \begin{cases} \tilde{\rho}, & d_i = d_j \\ -\tilde{\rho}, & d_i \neq d_j \end{cases}$, $i \neq j$. Thus, $\tilde{\Sigma}$ has a unit diagonal and 4 blocks of $\tilde{\rho}$ and $-\tilde{\rho}$:

$$\tilde{\Sigma} = \begin{pmatrix} 1 & \tilde{\rho} & ... & \tilde{\rho} & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ \tilde{\rho} & 1 & \tilde{\rho} & \tilde{\rho} & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ & & \ddots & & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ \tilde{\rho} & ... & \tilde{\rho} & 1 & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & 1 & \tilde{\rho} & ... & \tilde{\rho} \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & \tilde{\rho} & 1 & ... & \tilde{\rho} \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & & & \ddots & \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & \tilde{\rho} & ... & \tilde{\rho} & 1 \end{pmatrix} \tag{72}$$

Assuming homogeneous counterparties (i.e., $\bar{d} \equiv \bar{d}_i$), the number of defaulting counterparties,

$N_D = \sum_{i=1}^{\gamma} D_i$, is distributed as

$$\mathbb{P}\left(N_D = k\right) = \binom{\gamma}{k} \Phi_{\gamma,\tilde{\Sigma}}(\underbrace{\bar{d}, ..., \bar{d}}_{k}, \underbrace{-\bar{d}, ..., -\bar{d}}_{\gamma-k}), \tag{73}$$

where $\bar{d} > 0$ is the individual default intensity, and $\tilde{\Sigma}$ is defined as before.

As a benchmark case, consider independent defaults (i.e., $\tilde{\rho} = 0$). Then, the distribution of joint defaults is given by

$$\Phi_{\gamma,\tilde{\Sigma}}(\underbrace{\bar{d}, ..., \bar{d}}_{k}, \underbrace{-\bar{d}, ..., -\bar{d}}_{\gamma-k}) = \Phi(\bar{d})^k \Phi(-\bar{d})^{\gamma-k} = \Phi(\bar{d})^k (1 - \Phi(\bar{d}))^{\gamma-k}. \tag{74}$$

Thus, if defaults are independent, the number of defaults is binomially distributed,
$N_D \sim Binom(\gamma, \Phi(\bar{d}))$. As Figure 14 shows, increasing the correlation $\tilde{\rho}$ yields larger tails of the distribution of N_D. Then, it is more likely that counterparties default together (i.e., a large or small number of counterparties defaults).

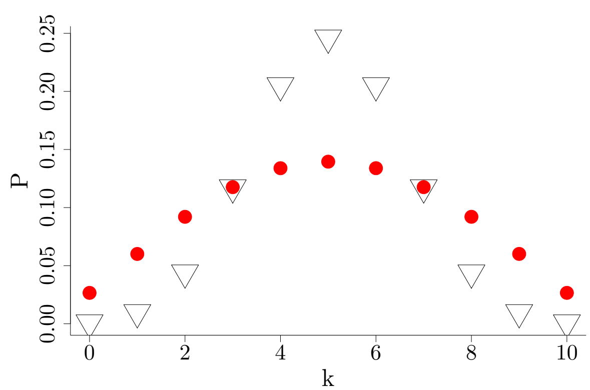


Figure 14. Probability distribution of the number of defaults, N_D, for $\gamma = 10$ entities and individual probability of default $\pi = 0.5$ if defaults are uncorrelated (triangles) or correlated with $\tilde{\rho} = 0.25$ (filled dots).

Figure 14 depicts the distribution of N_D for exemplary parameters. Clearly, increasing the total correlation $\tilde{\rho}$ yields larger tails of the distribution. Then it is more likely that entities default in clusters (i.e., that either many or few counterparties default together).

Figure 15 depicts the relative error when approximating loss allocation rules by using counterparty risk exposure, i.e.,

$$\frac{\mathbb{E}[\frac{\sum_{j=1}^{\gamma} f_j}{\sum_{j=1}^{\gamma}(1-D_j)f_j} \mid \sum_{j=1}^{\gamma}(1-D_j)>0]}{\frac{(1-\pi)\pi f_1}{\frac{f_1}{\sum_{i=2}^{\gamma} f_i}+1-\pi}} - 1. \tag{75}$$

The figures show that counterparty risk exposure is a good approximation if the default probability and the correlation between defaults is small, and the number of counterparties large (see also Section 4).

Calibration

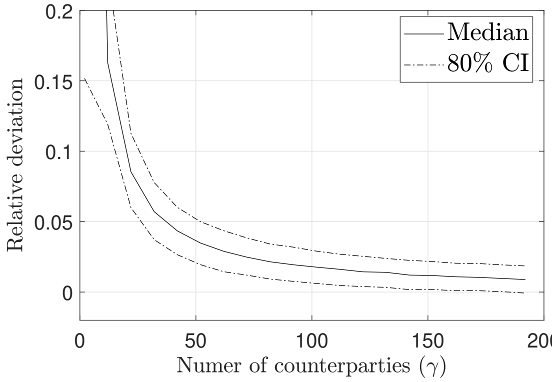
We calibrate the volatility of contract values based on index CDS, since these are already subject to clearing obligations in the US and EU. For this purpose, we retrieve data about the performance of the North American family of index CDS, the *CDX family*, from January 2006 to 2010, from Markit. We choose this period because it covers the 2007-08 financial crisis. Table 1 reports the names of index CDS included in our sample. Starting with the assumption of a five-day settlement period, the descriptive statistics in Table 2 show that the average standard deviation of index CDS prices' five-day log returns roughly equals $\sigma_X = 0.01$, which we use as an estimate for total contract volatility. During the same time period, the standard deviation of S&P 500 five-day log returns is roughly $\sigma_M = 0.03$, which we use as an estimate for volatility of the systematic risk factor.[51]

To calibrate the correlation between contract returns and systematic risk, we employ a one-factor model, regressing CDS index returns on five-day S&P 500 log returns between 2006 to 2010,

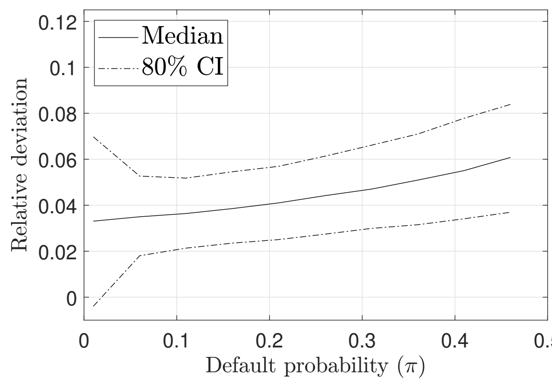
$$CDX_{name,tenor,series,version,t} = \alpha + \beta SP_t + \varepsilon_{name,tenor,series,version,t}, \tag{76}$$

where $CDX_{name,tenor,series,version,t}$ is the five-day CDS index log returns for different family names, tenors, series, and versions at day t, and SP_t is the five-day S&P 500 log return at day t. The estimated OLS coefficients are in Table 3. The implied correlation between CDS and S&P 500 returns roughly equals $\rho_{X,M} = 0.43$, which we use as a baseline calibration. It is larger for indices that

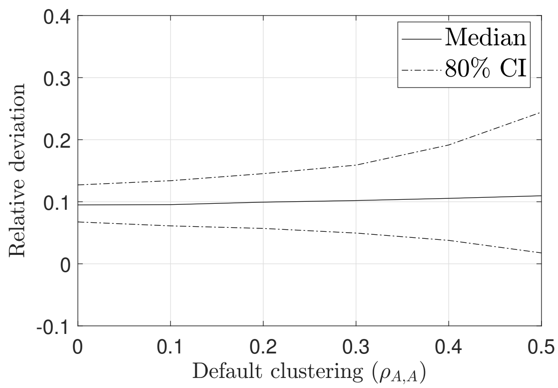
[51] We approximate the discrete returns r_{ij}^k in our model by using empirically calibrated log returns $\tilde{r}_{ij}^k$ (i.e., $\log(1 + r_{ij}^k) \approx \tilde{r}_{ij}^k$). The calibration, in particular the standard deviation and correlation of S&P 500 and index CDS returns, is robust to using either empirical discrete returns or log returns.



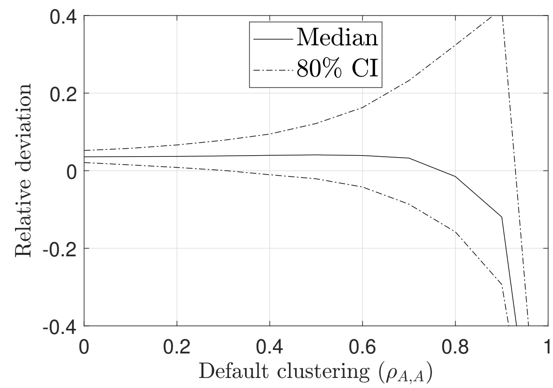
(a) Sensitivity toward γ.



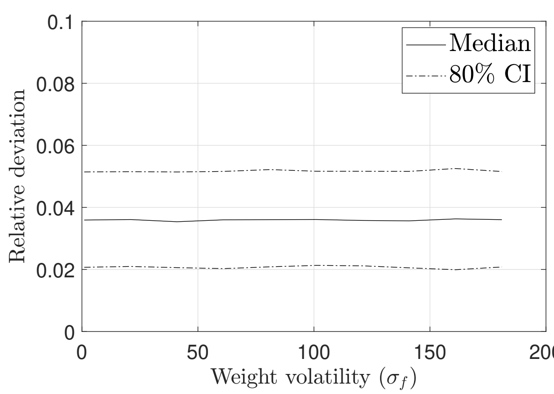
(b) Sensitivity toward π.



(c) Sensitivity toward $\rho_{A,A}$ ($\gamma = 20$).



(d) Sensitivity toward $\rho_{A,A}$ ($\gamma = 50$).



(e) Sensitivity toward σ_F.

Figure 15. Approximation of loss allocation.

Figures depict the relative deviation between $\eta = \mathbb{E}[\frac{\sum_{j=1}^{\gamma} f_j}{\sum_{j=1}^{\gamma}(1-D_j) f_j} \mid \sum_{j=1}^{\gamma}(1-D_j) > 0]$ and $\hat{\eta} = \frac{(1-\pi)\pi f_1}{\frac{f_1}{\sum_{i=2}^{\gamma} f_i} + 1 - \pi}$. For each observation, we draw $M = 3000$ values for γ weights $f_n = |\hat{f}_n|$, where $\hat{f}_n \sim \mathcal{N}(0, \sigma_F^2)$ and independent, $n = 1, ..., \gamma$, and for each weight realization s we draw 4000 values for D from the default model specified above to estimate $\eta = \eta(s)$ and $\hat{\eta} = \hat{\eta}(s)$. The figures depict the median and 10% and 90% percentiles of $\{\eta(s)/\hat{\eta}(s) - 1, s \in \{1, ..., M\}\}$. The baseline calibration uses $\pi = 0.1$, $\rho_{A,A} = 0$, $\gamma = 50$, and $\sigma_F = 1$.

CDX name	Description
CDX NA.HY	North American High Yield CDSs
CDX NA.HY.B	Rating sub-index of CDX NA.HY
CDX NA.HY.BB	Rating sub-index of CDX NA.HY
CDX NA.HY.HB	Sub-index of CDX NA.HY (high beta)
CDX NA.IG	North American investment-grade CDSs
CDX NA.IG.CONS	Sub-index of CDX NA.IG (consumer cyclical)
CDX NA.IG.ENRG	Sub-index of CDX NA.IG (energy)
CDX NA.IG.FIN	Sub-index of CDX NA.IG (financials)
CDX NA.IG.TMT	Sub-index of CDX NA.IG (telecom, media and technology)
CDX NA.IG.INDU	Sub-index of CDX NA.IG (industrial)
CDX NA.IG.HVOL	Sub-index of CDX NA.IG (high volatility)
CDX NA.XO	Sub-index of CDX NA.IG (crossover between grade and junk)
CDX.EM	Emerging market CDSs
CDX EM.DIV	Emerging market CDSs (diversified)

Table 1. Names and descriptions of index CDS included in our data sample. *Source*: Markit (2015).

Statistic	N	Min	Pctl(25)	Median	Pctl(75)	Max	Mean	St. Dev.
S&P 500	1,021	−0.203	−0.013	0.002	0.015	0.175	−0.001	0.031
CDX (all)	590,706	−0.288	−0.002	0.0003	0.004	0.291	0.001	0.012
CDX (CDX.NA.HY)	131,945	−0.096	−0.004	0.002	0.010	0.095	0.003	0.015
CDX (CDX.NA.HY.B)	27,921	−0.090	−0.003	0.0005	0.005	0.146	0.002	0.013
CDX (CDX.NA.HY.BB)	19,474	−0.064	−0.003	0.0004	0.003	0.056	0.0005	0.009
CDX (CDX.NA.HY.HB)	38,254	−0.163	−0.005	0.002	0.011	0.215	0.005	0.024
CDX (CDX.NA.IG)	83,264	−0.288	−0.001	0.0001	0.002	0.291	0.0002	0.006
CDX (CDX.NA.IG.CONS)	29,007	−0.046	−0.001	0.000	0.001	0.027	−0.0001	0.005
CDX (CDX.NA.IG.ENRG)	29,007	−0.039	−0.001	−0.00001	0.001	0.032	−0.00003	0.004
CDX (CDX.NA.IG.FIN)	47,653	−0.095	−0.003	0.0003	0.005	0.045	0.0003	0.011
CDX (CDX.NA.IG.TMT)	31,953	−0.056	−0.002	0.00001	0.002	0.078	0.0001	0.006
CDX (CDX.NA.IG.INDU)	35,790	−0.049	−0.002	0.0001	0.002	0.037	0.00002	0.005
CDX (CDX.NA.IG.HVOL)	56,996	−0.073	−0.002	0.0001	0.002	0.048	0.0001	0.008
CDX (CDX.NA.XO)	30,508	−0.081	−0.005	0.001	0.006	0.067	0.001	0.012
CDX (CDX.EM)	14,372	−0.180	−0.003	−0.00001	0.004	0.192	−0.0002	0.018
CDX (CDX.EM.DIV)	14,562	−0.144	−0.002	0.0002	0.003	0.149	0.0002	0.014

Table 2. Descriptive statistics of five-day log returns of index CDS and the S&P 500.
The statistics are based on date-tenor-series-version observations for different index CDS families (see Table 1 for descriptions), all family-date-tenor-series-version observations for CDS (all), and date observations for the S&P 500 from January 2006 to December 2009. *Source*: Markit.

are on-the-run (0.63) and slightly smaller for indices that are off-the-run (0.4).[52] The methodology is equivalent to estimating the correlation between an equally weighted basket of index CDS and the S&P 500. We do not allow for different factor loadings β for different indices, since we are interested in only one parameter for the correlation $\rho_{X,M}$. The level of correlation is similar when estimating the single-factor model for individual index CDS for the baseline period from 2006 to 2010 as well as for the period from 2010 to 2018, confirming the robustness of our estimate.[53]

[52]index CDS are frequently updated. The most recently updated index is called *on-the-run* and typically exhibits the highest liquidity. Older versions of the indices are called *off-the-run* and are often still traded but exhibit lower liquidity.

[53]Correlation estimates are available on request. The correlation can be substantially smaller for single reference

	Dependent variable: five-day CDX return		
	Full	On-the-run	Off-the-run
	(1)	(2)	(3)
S&P 500	0.148	0.235	0.148
	t = 370.284***	t = 23.845***	t = 369.824***
Observations	590,706	856	589,850
R^2	0.188	0.400	0.188
Adjusted R^2	0.188	0.399	0.188
Residual Std. Error	0.011 (df = 590704)	0.007 (df = 854)	0.011 (df = 589848)
Implied correlation $\rho_{X,M}$	0.43	0.63	0.43
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3. Calibration of the correlation of contract values with systematic risk.
OLS regression of five-day index CDS log returns on S&P 500 five-day returns between January 2006 and December 2009: $CDX_{name,t,tenor,series,version} = \alpha + \beta SP_t + \varepsilon_{name,t,tenor,series,version}$ for all index CDS at days t. The methodology is equivalent to estimating a single-factor model for an equally weighted basket of all index CDS. $\rho_{X,M}$ is the implied correlation coefficient between index CDS and S&P 500 returns. *Source:* Markit and own calculations.

Based on these empirical results, Table 4 and 5 describe the final calibration of our model.

Variable	**Value**	**Description**
γ	16	Number of counterparties
K	10	Number of derivative classes
σ_X	0.01	Total contract volatility
$\rho_{X,M}$	0.43	Correlation between contract value and systematic risk M
σ_M	0.03	volatility of the systematic risk factor
β	0.1433	Implied beta-factor contracts
σ	0.009	Implied idiosyncratic contract volatility
v	1	Initial market value
$\text{cor}\left(r_{ij}^k, r_{hl}^m\right)$	0.185	Implied pair-wise correlation of contracts

Table 4. Baseline calibration. We assume the same calibration for each entity and derivative class.

entities, as these do not diversify across entities' idiosyncratic default risk. For example, the correlation of the S&P 500 with Wells Fargo's five-year tenor spreads is -0.06; with Goldman Sachs's, it is -0.12; with Deutsche Bank's, it is -0.1; with General Electric's, it is -0.18; with AIG's, it is -0.16, and with Metlife's, it is -0.42. The correlation is almost identical when using three-year tenors. Note that the negative sign of the correlation coefficient reflects the protection buyer's perspective in spreads, while we account for the difference between buyer and seller with the sign of the contract size v. Thus, we use the absolute value of the correlation.

Variable	**Value**	**Description**
pd	0.1	Individual probability of default
$\rho_{A,A}$	0.25	Correlation of log assets conditional on M
σ_A	1	Log-asset value volatility
α_{BN}	0.99	Bilateral margin level
α_{MN}	0.99	Multilateral (clearing) margin level

Table 5. Baseline calibration of the default model. We assume the same calibration for each entity and derivative class.

Cross-netting and a single CCP for the whole market

To address the failure of multilateral netting of one derivative class to reduce counterparty risk exposure in sufficiently extreme events, one might increase the overall degree of netting. A natural extension is to net across not only one but several derivative classes. We refer to such netting across all $\gamma - 1$ counterparties and $\kappa > 1$ derivative classes as *cross-netting*. It occurs when one CCP offers clearing of several derivative classes within the same portfolio.[54]

The counterparty risk exposure in κ cross-netted derivative classes with $\gamma - 1$ counterparties conditional on systematic risk is given by

$$\bar{M}(\gamma-1)\kappa\beta\Phi\left(\frac{\bar{M}\sqrt{(\gamma-1)\kappa}\beta}{\sigma}\right)+\sqrt{(\gamma-1)\kappa}\sigma\varphi\left(-\frac{\bar{M}\sqrt{(\gamma-1)\kappa}\beta}{\sigma}\right). \tag{77}$$

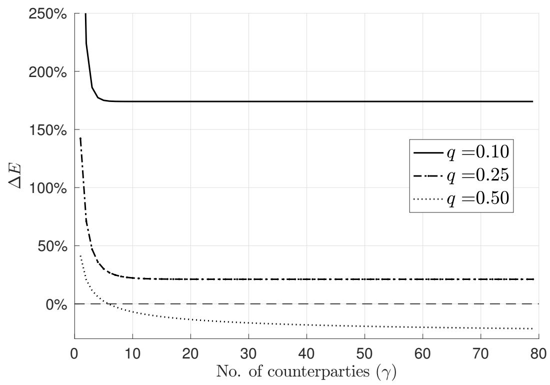
Figure 16 illustrates the benefit of cross-netting for counterparty risk exposure during extreme events with severity q (defined as before). Figure 16 (a) considers an intermediate case that half of all derivative classes are cleared, i.e., the CCP nets across all counterparties and $\kappa = 5 = K/2$ derivative classes. The figure shows that even with cross-netting of $\kappa = 5$ derivative classes, the counterparty risk exposure is larger in sufficiently extreme states (such as $q = 0.25$ or $q = 0.1$) than with bilateral netting.

Figure 16 (b) depicts the minimum number of counterparties for cross-netting to be beneficial compared to bilateral netting. We find that cross-netting essentially needs to net across all K derivative classes and $\gamma - 1$ counterparties (i.e., $\kappa = K$), in order to be beneficial in all realizations of systematic risk, a case we refer to as *Mega CCP*. In other words, only a *Mega CCP* that clears with all counterparties in all derivative classes can unambiguously reduce counterparty risk exposure for all realizations of systematic risk compared to bilateral netting.[55] The reason is that central clearing with a *Mega CCP* does not fragment the derivatives market into bilateral and multilateral (cross-) netting but all derivatives are cross-netted.

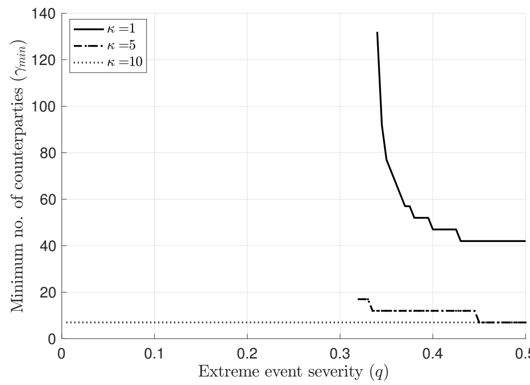
Result 16. *Only a Mega CCP, netting across all derivative classes and counterparties, reduces*

[54] For example, Eurex offers netting across several derivative classes such as money-market and interest rate derivatives, including margining for a clearing member's entire portfolio. Cross-netting is promoted by interoperability arrangements that create linkages between different CCPs (Garvin (2012)).

[55] Similarly, Result 1, about multilateral netting of one derivative class, qualitatively carries over to the case of cross-netting with $\kappa < K$.



(a) Change in exposure due to cross-netting $\kappa = 5$ classes.



(b) Minimum number of counterparties.

Figure 16. Impact of cross-netting during extreme events.
(a) Effect of netting across counterparties and $\kappa = 5$ derivative classes on collateralized counterparty risk exposure during an extreme event $\bar{M} = \sigma_M \Phi^{-1}(q)$, $\Delta E = \mathbb{E}[E_i^{BN+CN} - E_i^{BN,K} \mid M = \bar{M}]/\mathbb{E}[E_i^{BN,K} \mid M = \bar{M}]$. The smaller q, the more adverse the event. If $\Delta E < 0$, cross-netting reduces counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties $\gamma_{\min}$, such that cross-netting of κ derivative classes reduces counterparty risk exposure. In case $\kappa = K = 10$, we refer to the CCP as *Mega CCP*. We assume $K = 10$ derivative classes, correlation between contract returns and systematic risk $\rho_{X,M} = 0.43$, total contract volatility $\sigma_X = 0.01$, and volatility of the systematic risk factor $\sigma_M = 0.03$.

counterparty risk exposure for all realizations of the systematic risk factor compared to bilateral netting.

Can a *Mega CCP* compensate for the adverse effect of a small clearing margin? The uncollateralized exposure in cross-netted κ derivative classes is given by

$$\mathbb{E}[\tilde{E}^{CN,\kappa}] = \sqrt{\sigma_M^2 \kappa^2 (\gamma-1)^2 \beta^2 + \kappa(\gamma-1)\sigma^2}\,\xi(\alpha_{CN}), \tag{78}$$

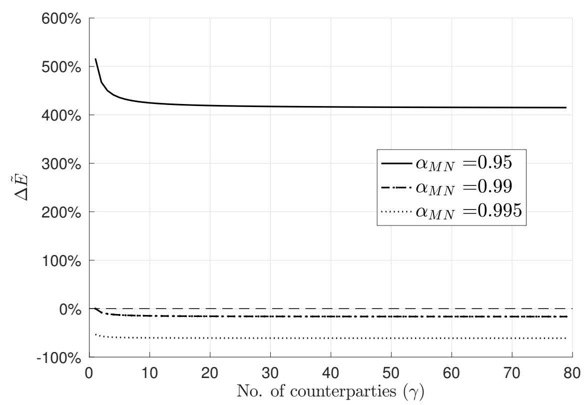
where $\xi(\alpha)$ is defined as above and α_{CN} is the margin level for cross-netting.

A *Mega CCP* underlies the same dynamics as the multilateral netting of one derivative class with respect to margins: the smaller (larger) the clearing margin, the larger (smaller) is the uncollateralized exposure. If the clearing margin is sufficiently small, then cross-netting is not beneficial for any number of counterparties, and vice versa.

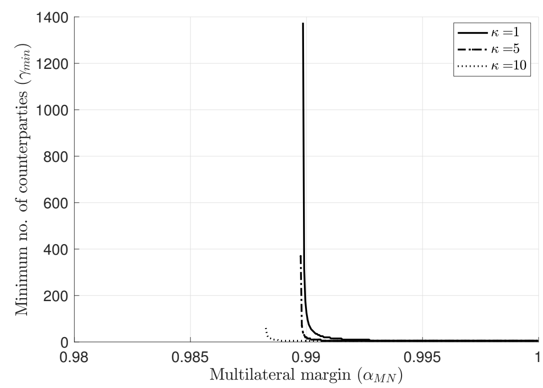
Analogously to multilateral netting, we derive the smallest acceptable margin confidence level $\mathcal{H}_{CN}$, such that cross-netting is not beneficial for any number of counterparties if $\alpha_{CN} \leq \mathcal{H}_{CN}$:

$$\mathcal{H}_{CN} = \xi^{-1}\left(\frac{\xi(\alpha_{BN})}{|\rho_{X,M}|} \frac{\sqrt{1+\rho_{X,M}^2(K-1)}}{\sqrt{K}}\right). \tag{79}$$

Similarly to $\mathcal{H}_{MN}$, $\mathcal{H}_{CN}$ is increasing with $|\rho_{X,M}|$. A *Mega CCP* is, however, associated with a larger degree of netting. This reduces the smallest acceptable clearing margin compared to the multilateral netting of one derivative class: it is straightforward to show that $\mathcal{H}_{CN} < \mathcal{H}_{MN}$ for any $\rho_{X,M} \in (0, 1)$. This effect, however, is very small. For example, with $\alpha_{BN} = 0.99$, the smallest acceptable confidence level is reduced only by 0.15 percentage points: from $\mathcal{H}_{MN} = 98.97\%$ with multilateral netting of class K to $\mathcal{H}_{CN} = 98.82\%$ with cross-netting of all classes $k = 1, ..., K$. Figure 17 illustrates the result. This effect seems still negligible in light of the large difference of 11 percentage points, in practice, between $\alpha_{BN} = 99\%$ and $\alpha_{MN} = 88\%$. We conclude that the degree of netting is only of minor importance if the margin for cleared derivatives is not sufficiently large.



(a) Change in exposure due to cross-netting across $\kappa = 10$ derivative classes.



(b) Minimum number of counterparties.

Figure 17. Impact of cross-netting on uncollateralized counterparty risk exposure.
(a) Change in uncollateralized counterparty risk exposure due to netting across $\kappa = 10$ derivative classes and across counterparties on uncollateralized counterparty risk exposure, $\Delta\tilde{E} = \mathbb{E}[\tilde{E}_i^{BN+CN} - \tilde{E}_i^{BN,K}]/\mathbb{E}[\tilde{E}_i^{BN,K}]$, for bilateral margin confidence level $\alpha_{BN} = 0.99$, and $\gamma = 16$ entities. If $\Delta\tilde{E} < 0$, cross-netting reduces uncollateralized exposure compared to bilateral netting. (b) Minimum number of counterparties $\gamma_{\min}$, such that cross-netting reduces uncollateralized exposure compared to bilateral netting. The bilateral margin is $\alpha_{BN} = 0.99$. We assume $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, volatility of the systematic risk factor $\sigma_M = 0.03$, correlation between contract returns and systematic risk $\rho_{X,M} = 0.43$, entity default probability $\pi = 0.05$, correlation between entities' log asset values $\rho_{A,A} = 0.05$, and total log asset volatility $\bar{\sigma}_A = 1$.

Result 17. *For every bilateral margin confidence level $\alpha_{BN} \in (0, 1)$, there exists a threshold $\mathcal{H}_{CN} \leq \alpha_{BN}$, such that the counterparty risk exposure is larger with a Mega CCP than with bilateral netting for any number of counterparties if the clearing margin $C^{CN} \leq VaR_{\mathcal{H}_{CN}}(\sum_{i=1,i\neq j}^{\gamma} X_{ij}^k)$. The threshold $\mathcal{H}_{CN}$ is increasing with the absolute value of systematic risk exposure $|\rho_{X,M}|$, such that a larger clearing margin is necessary for more extreme systematic risk exposure.*

Finally, Figure 18 illustrates the impact of loss sharing at a Mega CCP, i.e., loss sharing across all derivative classes and counterparties, conditional on different realizations of the systematic risk factor. The figure shows that the impact of loss sharing on counterparty risk with loss sharing qualitatively remains the same with a Mega CCP as with loss sharing of only one derivative class. The main difference is the magnitude, which is now substantially larger (in absolute terms): as more derivative trades are shared among entities, the absolute change in counterparty risk with loss sharing is larger. As with loss sharing of only one derivative class, loss sharing of all derivative trades reduces directional traders' counterparty risk only in one (extremely positive or negative) tail of realizations, while dealers benefit in both tails.

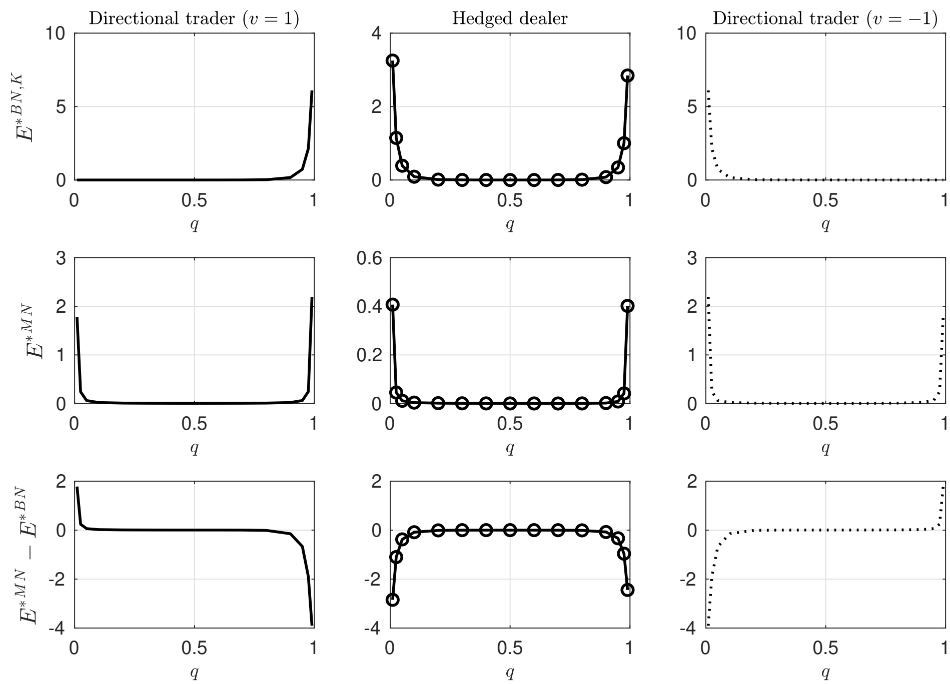


Figure 18. Counterparty risk with loss sharing and a Mega CCP for different realizations of the systematic risk factor.

Counterparty risk with loss sharing conditional on different levels of the systematic risk factor $M = \bar{M} = \sigma_M \Phi^{-1}(q)$. We assume the correlation between entities' log asset values, i.e., default clustering, to be $\rho_{A,A} = 0.1$, with $\gamma = 16$ counterparties, $K = 10$ derivative classes, total contract volatility $\sigma_X = 0.01$, volatility of the systematic risk factor $\sigma_M = 0.03$, entity default probability $\pi = 0.1$, log-asset value volatility $\sigma_A = 1$, and clearing and bilateral margin level $\alpha_{MN} = \alpha_{BN} = 0.99$.