Sequential Persuasion

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The Paper

A Bayesian Persuasion Model

- one receiver and multiple senders
- senders move sequentially

Simple Equilibrium Characterization

- one-step equilibrium
- convex polytope for equilibrium outcome

Applications on Communication Protocol Design

- the effect of adding senders
- the value of multiple rounds of rebuttals
- simultaneous vs sequential

SFFA vs Harvard

The Harvard Crimson:

> *"The trial and lawsuit unleashed mountains of classified* ***Harvard admissions data****. Both the University and SFFA employed statistical experts to analyze the data and testify about their results in court ... SFFA paid Duke economics professor Peter S. Arcidiacono to create a model of the College's admissions process. He claims his model proves Harvard does discriminate against Asian Americans. Harvard, though, paid University of California, Berkeley economics professor David E. Card to create his own model of the admissions process. He claims his model proves the College does not discriminate ...*

Literature

Bayesian Persuasion

- Kamenica and Gentzkow (2011), Lipnowski and Mathevet (2017)
- Gentzkow and Kamenica (2016, 2017), Li and Norman (2018)
- Boleslavsky and Cotton (2016), Au and Kawai (2017a,b)
- Board and Lu (2017), Wu (2018)

Other Models with Multiple Senders

- Hu and Sobel (2019)
- Battaglini (2002), Ambrus and Takahashi (2008)
- Kawai (2015), Krishna and Morgan (2001)

Model

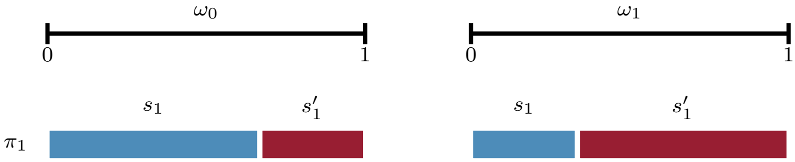
Model

- Senders $1, ..., n$ persuade a receiver d.
- The state is drawn from a finite set Ω.
- Players' common prior is $\mu_0 \in \Delta(\Omega)$.
- The receiver chooses an action from a finite set A.
- The utility of player i is

$$u_i : A \times \Omega \to \mathbb{R},$$

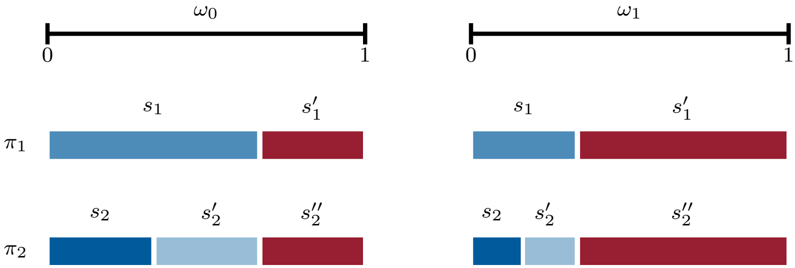
  for every $i = 1, ..., n, d$.
- Senders post experiments to disclose information.

Experiments



Define $p_1 : \Omega \to \Delta(\{s_1, s_1'\})$ by the measure of each $\pi_1(s|\omega)$.

Experiments



$p_2 : \Omega \to \Delta(\{s_2, s_2', s_2''\})$ is more informative than p_1 in the sense of Blackwell.

Extensive Form

- Sender 1 creates a partition π_1
- Sender i observes $\pi_1, ..., \pi_{i-1}$ and chooses π_i.
- Nature randomly decides ω.
- The signal profile $s_1, ..., s_n$ is realized.
- The receiver observes $\pi_1, ..., \pi_n, s_1, ..., s_n$ and chooses a.
- Information is symmetric, so we solve for SPE.

On the Information Environment

Equilibrium Characterization

Simplifying the Problem

Definition

Consider a strategy profile σ and let h_i denote the implied outcome path before the move by sender i. We say that σ is **one step** if $\vee_{j=1}^{n} \sigma_i(h_i) = \sigma_1$.

Proposition

For any SPE, there exists an outcome equivalent SPE in which senders play a one step continuation strategy profile after any history of play.

- A revelation-principle like characterization
- Trivialize information disclosure dynamics

In a one-step equilibrium,

- sender 1 replicates the joint experiment $(\pi_1, ..., \pi_n)$ on the original equilibrium path,
- IC is ensured by the threat of the punishment in the original equilibrium, and
- the corresponding sender replicates the continuation experiments off the path.

It results from

- complete information
- frictionless information design

Equilibrium Construction: Receiver

- Her choice depends on his derived posterior belief $\mu \in \Delta(\Omega)$.
- Divide $\Delta(\Omega)$ into convex polytopes $\{M(a)\}_{a \in A}$.
- Break the tie to favor sender n.

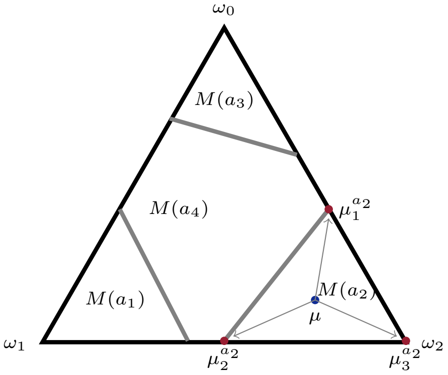


Figure: $\Omega = \{\omega_0, \omega_1, \omega_2\}$ and $A = \{a_1, a_2, a_3, a_4\}$.

Equilibrium Construction: The Last Sender

- Each signal profile of $\pi_1, ..., \pi_{n-1}$ induces an "interim" belief μ.
- Sender n's experiment generates a MPS of every interim belief μ.
- He splits every interim belief into a MPS separately.
- It's without loss to focus on MPS onto vertices of $\{M(a)\}_{a \in A}$.
  - Refine an interior belief $\mu \in M(a)$ onto the vertices.
  - If the MPS induces the same action, no one cares.
  - If the action differs at some vertex, sender n is better off.
- Let X_n collect vertices that sender n has no incentive to split.
- Assume sender n does nothing for $\mu \in X_n$.

Equilibrium Construction: Induction

- Sender $n-1$ also splits his interim belief μ.
- It's without loss to focus on MPS onto X_n.
  - Whenever $\mu \in X_n$ is induced, sender n does nothing.
  - Any $\mu \notin X_n$ will be further refined onto X_n.
- $X_{n-1} \subseteq X_n$ collects vertices that he has no incentive to split.
- Repeat the process and recursively define

$$X_{n-2} \supseteq X_{n-3} ... \supseteq X_1.$$

- X_1 is the set of **stable beliefs** that no sender wants to split.

Existence

Proposition

There exists a one-step equilibrium where

- *On the path, sender 1 splits μ_0 onto X_1, and other senders do nothing.*
- *Off the path, sender i splits an interim belief onto X_i, and subsequent senders do nothing.*

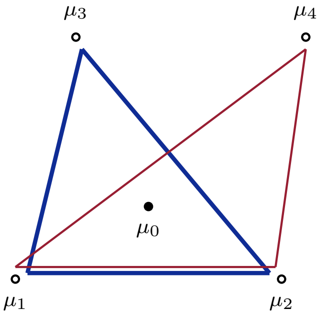
- stable belief is crucial
- the equilibrium is Markov
- There is non-essential multiplicity

Outcome Uniqueness

Proposition

All SPE are outcome equivalent for generic preferences.

- In a one-step eq, sender 1 picks a MPS of μ_0 on stable beliefs.
- The uniqueness fails if he is indifferent between multiple MPS, requiring non-generic linearly dependent $u_1(a, \omega)$.



- Substantial Non-Markov eq also needs enough indifferences.

Applications

Consultation Organization

What affects information revelation?

- the number of senders
- information sharing among senders
- multiple rounds of rebuttals and counter-rebuttals

We study some comparative statics including

- adding a new sender
- compare simultaneous vs sequential persuasion
- letting a sender to speak multiple times

Focus on results holding for arbitrary but generic preferences.

Information Criteria

Definition

π is **essentially less informative** than π' if the finest signal that is outcome equivalent to π is less informative than the finest signal that is outcome equivalent to π' in the Blackwell order

- Evaluate information revelation by the resulting dist. on $\Omega \times A$.
- The finest signal puts probability one on X_1.

Adding Senders

Proposition

*Adding a new sender does **not cause information reduction** if and only if the new sender speaks before all other senders.*

Adding the new sender after some senders *may* reduce information.

- These senders may disclose less information to avoid more radical disclosure by the new sender (Li and Norman, 2018).

Adding the new sender before all others *never* reduce information.

- the continuation game after the new sender's move is essential the original one with another prior
- whatever being disclosed in the original game cannot be hidden in the new game

Multiple Moves by the Same Sender

Proposition

Consider a game with n senders and each of them moves only once. Add a move for a sender that ***precedes his move in the original game*** *does not affect the set of stable beliefs.*

- Whatever being disclosed gradually can be disclosed at the end.
- Allowing one move multiple times matters only by changing the position of his last move.
- It does benefit to let a sender to speak before everyone else. He decides which beliefs in X_1 to induce.

Simultaneous vs Sequential Persuasion

Proposition

There exists no equilibrium in the simultaneous game that is essentially less informative than the equilibrium in the sequential game.

- In simultaneous game, each sender can unilaterally induce any mean-preserving spread of any beliefs resulting from the strategy profile of all senders.
- In sequential game, only the last sender has such power.
- Less vertix beliefs survive deviations in simultaneous game.

Take-Home Messages

To Understand Persuasion Games

- Rich information structure trivializes the disclosure dynamics.
- What matters is the set of stable beliefs.
- Focusing on finite models is rewarding.

On Consultation Structure

- Adding a sender never cause less information if he speaks first.
- Strategic consideration does not justify multiple rounds of disclosure by one sender.
- Simultaneous persuasion cannot be less informative than sequential persuasion.

Thank You!

On the Modeling Choice

Partition Representation

- Transparently combine multiple experiments:
  - In sequential game, a sender responds upon previous senders' experiments signal by signal.
  - In simultaneous game, a sender chooses upon everyone else's experiments signal by signal.
- Easy to modify and compare different the extensive forms.

Observability of Signals

- Strategically equivalent to a model where nature moves first.
- No need to keep tracking of the history of signal realizations.
- Convenient to discuss the unconditional distribution over outcomes.

Back