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Liquidity and Default: A Continuous Time Approach

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Outline

- Motivation
- Related Literature
- Model
- Results

Motivation

- Bankruptcy laws impose a penalty to individuals, firms and even countries that decide to default.
- Question: Is there any trade-off between Monetary Policy and default penalties (Regulatory Policy)?
- If the answer is yes, are they *complements* or *substitutes* and how does this affect optimal welfare? What is the optimal policy mix?
- Why is this question important?

Related Literature

- Brunnermeier, M. K., Sannikov, Y. (2014). A macroeconomic model with a financial sector. American Economic Review, 104(2), 379-421.
- Dubey, P., Geanakoplos, J. (1992). The value of money in a finite horizon economy: a role for banks. Economic Analysis of Market and Games, MIT Press, Cambridge, 407-444.
- Dubey, P., Geanakoplos, J., Shubik, M. (2005). Default and punishment in general equilibrium 1. Econometrica, 73(1), 1-37.
- Shubik, M., Tsomocos, D. P. (1992). A strategic market game with a mutual bank with fractional reserves and redemption in gold. Journal of Economics, 55(2), 123-150.
- Kashyap, A. K., Tsomocos, D. P., Vardoulakis, A. (2017). Optimal bank regulation in the presence of credit and run risk.

Brunnermeier and Sannikov (2014)

- One salient feature of this class of models is that they able to analyse to analyze the full macroeconomic dynamics.
- In particular, they show that the long run behavior of an economy facing aggregate shocks under the presence of financial frictions can be in stark contrast with the one predicted by the steady state analysis that is typically used in mainstream linearised macroeconomic models
- The main reason for this divergence is the presence of system generated (endogenous) risk that may drive the equilibrium dynamics far away from the state in which the system would end up in the absence of such risk.
- Taking into account this feature is crucial, if one intends to assess welfare distortions or the impact of policy actions.

Dubey and Geanakoplos (1992)

- Provides microfoundations for the positive value of money by establishing a role for banks.
- The "banking sector" issues both *inside* and *outside* money .
- The value of money to the agents is derived from the transaction technology (see, Gurley and Shaw (1956))
- It also allows for endogenous default to be compatible with the orderly function of markets.

Model (1)

Heterogeneous Households

- 2 Types of Agents: Experts (Agents A) and Households (Agents B).
- Both types are endowed with outside money m_t^A and m_t^B each period.
- A (dummy) Central Bank. The objective of the CB is to withdraw all liquidity in the end of each period.
- Both types of agents can own capital, but experts have higher productivity.
- Denote the aggregate amount of capital in the economy by K_t and capital held by an individual agent by k_t
- The production function of experts is:

$$y_t = ak_t$$

Model (2)

Heterogeneous Households (cont.)

- When held by an expert, capital evolves according to:

$$dk_t = (\phi(i_t) - \delta)k_t dt + \sigma k_t dZ_t$$

- The productivity function of households is:

$$\underline{y_t} = \underline{a}\underline{k_t}$$

with $\underline{a} \leq a$

- When held by an households, capital evolves according to :

$$d\underline{k_t} = (\phi(\underline{i_t}) - \underline{\delta})\underline{k_t} dt + \sigma \underline{k_t} dZ_t$$

with $\underline{\delta} > \delta$

Model (3)

Monetary flows

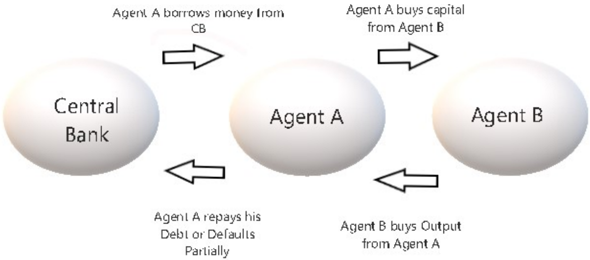


Figure: The Economy

Model (4)

Preferences

- Experts and less productive households are risk averse with logarithmic utilities.
- Discount rates are b_1 and b_2 respectively.
- Household utility function:

$$E[\int_0^\infty e^{-b_2 t} \ln c_t^B d\underline{t}]$$

- We assume that $b_1 > b_2$.
- Experts utility function:

$$E[\int_0^\infty e^{-b_1 t} \ln c_t^A d\underline{t}] - E[\int_0^\infty e^{-b_1 t} max(0, \frac{\tau(1-u_t)\mu_t}{q_t a_t})dt]$$

Model (5)

Market for Capital (A)

- Individual experts and households trade physical capital in a fully liquid market.
- We denote the equilibrium market price of capital q_t and postulate that its law of motion is of the form:

$$dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$$

- When an expert buys and holds k_t units of capital at price q_t , by Ito's Lemma the value of this capital evolves as follows:

$$\frac{dk_t q_t}{k_t q_t} = (\phi(i_t) - \delta + \mu_t^q + \sigma\sigma_t^q)dt + (\sigma + \sigma_t^q)dZ_t$$

Go to Appendix

Market for Capital (B)

- Thus, the total return that experts earn from capital (per unit of wealth invested) is:

$$dr_t^k = \underbrace{\frac{a - i_t}{q_t} dt}_{\text{divident yield}} + \underbrace{(\phi(i_t) - \delta + \mu_t^q + \sigma\sigma_t^q)dt + (\sigma + \sigma_t^q)dZ_t}_{\text{capital gains rate}}$$

- Similar for (less productive) Households.

$$d\underline{r}_t^k = \underbrace{\frac{\underline{a} - \underline{i}_t}{q_t} dt}_{\text{divident yield}} + \underbrace{(\phi(\underline{i}_t) - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q)dt + (\sigma + \sigma_t^q)dZ_t}_{\text{capital gains rate}}$$

Model (7)

Optimization Problems

- Experts and Households can trade physical capital in a fully liquid market.
- Formally, each expert solves:

$$\max_{x_t, c_t^A, i_t, \mu_t, v_t, b_t} E[\int_0^{\infty} e^{-b_1 t} \ln c_t^A d\underline{t}] - E[\int_0^{\infty} e^{-b_1 t} \frac{\tau(1-u_t)\mu_t}{q_t a_t} dt]$$

- Subject to her budget constraint (wealth accumulation):

$$\frac{dn_t}{n_t} = x_t dr_t^k + (x_t - 1) r dt - \frac{p_t^c c_t^A dt}{n_t}$$

and the Cash-in-Advance Constraints:

$$n_t(x_t - 1)dt \leq (e^{-r_t}\mu_t + m_t^A)dt$$

Model

Optimization Problems (cont.)

- $$v_t \mu_t dt \leq \frac{b_t(a-i)p_t^c n_t x_t}{q_t} dt$$

  and

- $$p_t^c c_t^A dt \leq \frac{(1-b_t)(a-i)p_t^c n_t x_t}{q_t} dt$$

- Likewise for Households.

Model

Markets

- Capital

$$\int_I k_t^i di + \int_J \underline{k}_t^j dj = K_t$$

- Money

$$e^{-r_t}\mu_t dt = M^{CB} dt$$

- Commodity

$$\int_j c_t^i di + \int_J \underline{c}_t^j dj = (\int_I (a - i_t^i) k_t^i di + \int_J (\underline{a} - \underline{i}_t^j) \underline{k}_t^j dj) dt$$

Equilibrium

- For any initial endowments of capital $\{k_0^i, \underline{k_0^j} : i \in I, j \in J\}$ such that:

$$\int_I k_0^i di + \int_J \underline{k_0^j} dj = K_0$$

an equilibrium is described by stochastic processes on the filtered probability space defined by the Brownian Motion $\{Z_t, t \geq 0\}$: the price processes of consumption capital p_t^c abd q_t, net worths $\{n_t^i, \underline{n}_t^i \geq 0\}$, capital holdings $\{k_t^i, \underline{k_t^j} \geq 0\}$, investment decisions $\{i_t^i, \underline{i_t^j} \in \mathbb{R}\}$,consumption choices $\{c_t^A, c_t^B\}$ of individual agents $i \in I, j \in J$: and repayment rate v_t and borrowing μ_t, such that
(i) initial net worths satisfy $n_0^i = k_0^i q_0$ and $\underline{n}_0^i = \underline{k}_0^i q_0$
(ii) each expert $i \in I$ and each household $j \in J$ solve their problems given prices and
(iii) markets for consumption, capital and money clear.

Results

- 1) Given parameters, there exists $\tau_{min} >> 0$ such that a Monetary Equilibrium exists only for values $\tau \geq \tau_{min}$. Furthermore, there exists a τ_{max} such that for every $\tau \geq \tau_{max}$ the repayment rate is always 1 (corner solution).
- 2) Interest rates decrease with increased penalty (Default Premium decreases).
- 3) Default suppresses investment and decreases capital accumulation of agent A.
- 4) For any given penalty, there exists an optimal money supply that maximizes welfare.
- 5) For any given money supply, there exists a finite positive default penalty, that maximizes Welfare.

1st Result

1) Monetary equilibrium exists ONLY for positive and finite values of the default penalty τ.

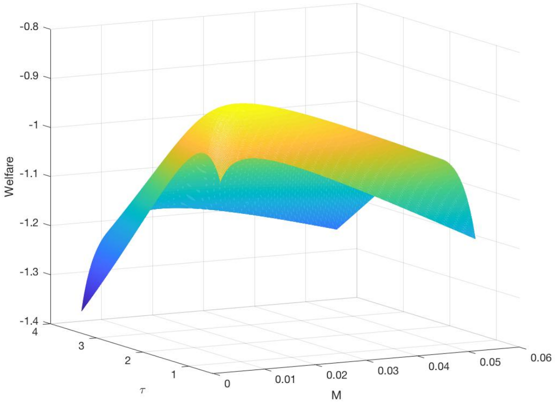


Figure: Different Regions

1st Result

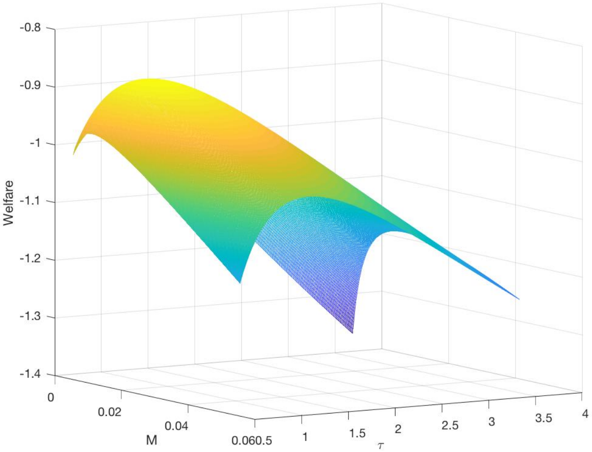


Figure: Optimal Welfare for different Regulatory and Monetary Policies

2nd Result

2) Interest rates decrease with increased penalty (Default Premium decreases).

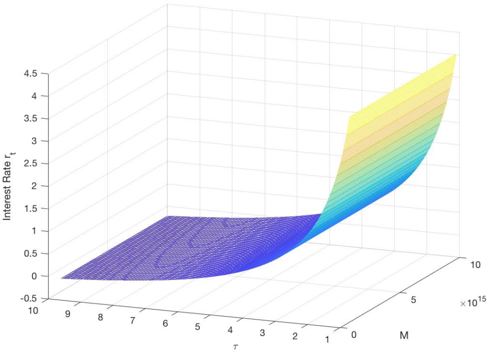


Figure: Interest Rate for different Regulatory and Monetary Policies

3rd Result

3) Default suppresses investment and decreases capital accumulation of agent A.

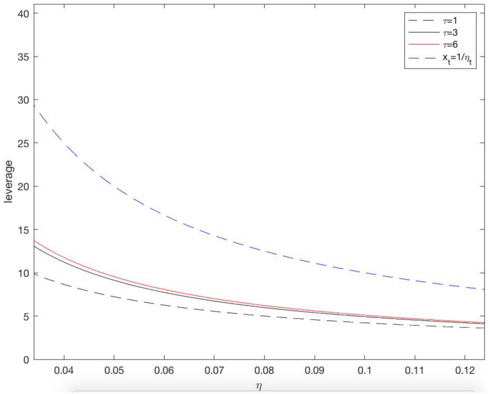


Figure: Leverage for different default penalties

3rd Result

3) Default suppresses investment and decreases capital accumulation of agent A.

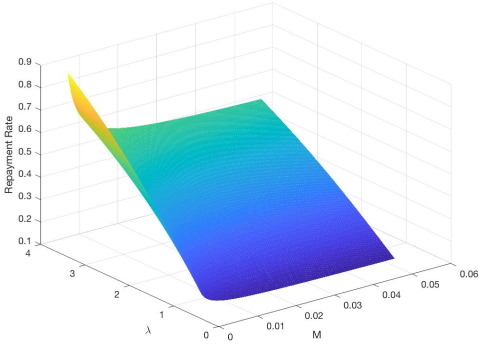


Figure: Leverage for different default penalties

4th Result

4) For any given penalty, there exists an optimal money supply that maximizes welfare.

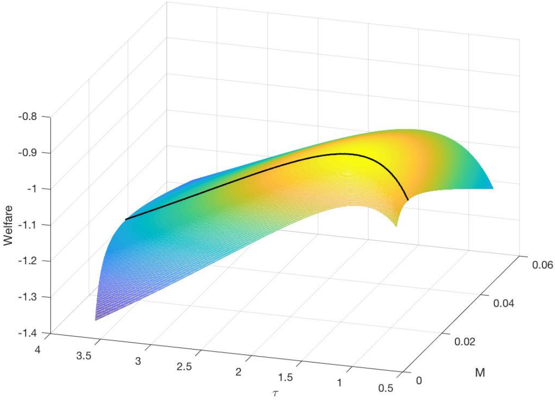
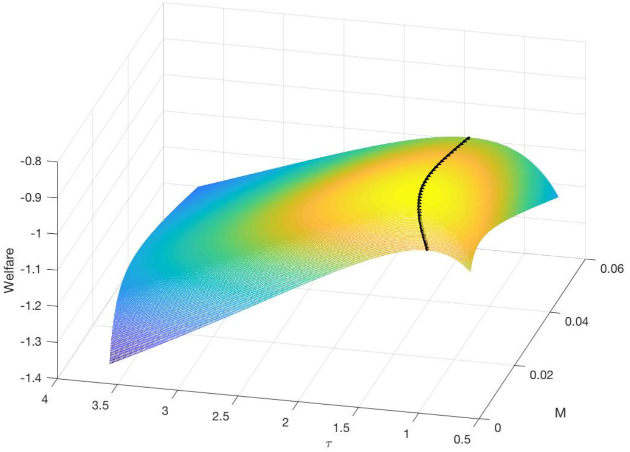


Figure: Optimal Welfare for given τ

5th Result

5) For any given money supply, there exists a finite positive default penalty, that maximizes Welfare.



Final Remarks

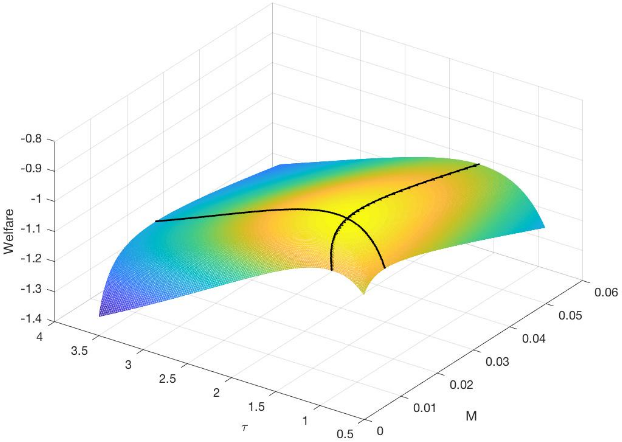


Figure: Trade-off between M and τ

Thank you!

Appendix

We use Ito's product rule, i.e. If $\frac{dX_t}{X_t} = \mu_t^X dt + \sigma_t^X dZ_t$ and $\frac{dY_t}{Y_t} = \mu_t^Y dt + \sigma_t^Y dZ_t$ then

$$d(X_t Y_t) = Y_t dX_t + X_t dY_t + (\sigma_t^X \sigma_t^Y)(X_t Y_t) dt$$

Applying this to

$$dk_t = (\phi(i_t) - \delta) k_t dt + \sigma k_t dZ_t$$

and to

$$dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$$

we obtain:

$$\frac{dk_t q_t}{k_t q_t} = (\phi(i_t) - \delta + \mu_t^q + \sigma \sigma_t^q) dt + (\sigma + \sigma_t^q) dZ_t$$

Go back