# Optimal Currency Exposure Under Risk and Ambiguity Aversion

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## Motivation

- Investors tend to hold portfolios with **global** exposure primarily for **diversification** benefits
- Recent studies of foreign currency exposure show that **full hedging** is **not optimal**
- In addition to market risk, agents face **model uncertainty** of the probability laws governing the stochastic processes of asset and currency **returns**
- This paper:
  - Explores the implications of **currency exposure** under **ambiguity** and sheds new light on optimal currency allocations
  - Builds a bridge between the literatures on **currency hedging** and **ambiguity aversion**

## Model

- For a fully hedged portfolio return $R^{fh}_{t+1}$, currency exposure $\psi_{c,t}$ , foreign exchange rate return $e_{c,t+1}$ and forward premium $f_{c,t}$, we derive

$$R^{h}_{t+1} = R^{fh}_{t+1} + \sum_{c=2}^{n+1} \psi_{c,t}(e_{c,t+1} - f_{c,t})$$

- This expression is **model-free**! No underlying dynamics for asset or currency returns are assumed
- **Model uncertainty**: The situation in which an investor is **uncertain** about the true **probabilistic model** governing the occurrence of different states
- For a coefficient of risk aversion $\lambda$ and a coefficient of ambiguity aversion $\theta$, a **risk** and **ambiguity averse** investor **maximizes** her **utility**

$$\max_{\mathbf{\Psi}_t} U(R^h_{t+1}) = \max_{\mathbf{\Psi}_t} \left\{ \mathrm{E}_{\bar{\mathbb{Q}}}[R^h_{t+1}] - \frac{\lambda}{2}\mathrm{Var}_{\bar{\mathbb{Q}}}(R^h_{t+1}) - \frac{\theta}{2}\mathrm{Var}_{\mu}(\mathrm{E}_{\mathbb{Q}}[R^h_{t+1}]) \right\}$$

- The argument $\mathbf{\Psi}^*_t$ which maximizes the above expression is the **optimal currency exposure** in the presence of risk and ambiguity and is given by

$$\mathbf{\Psi}^*_t = -\Big[\lambda \mathrm{Var}_{\bar{\mathbb{Q}}}(\mathbf{e}_{t+1} - \mathbf{f}_t) + \theta \mathrm{Var}_{\mu}(\mathrm{E}_{\mathbb{Q}}[\mathbf{e}_{t+1} - \mathbf{f}_t])\Big]^{-1} \cdot$$
$$\cdot \Big[\lambda \mathrm{Cov}_{\bar{\mathbb{Q}}}(R^{fh}_{t+1}, \mathbf{e}_{t+1} - \mathbf{f}_t) + \theta \mathrm{Cov}_{\mu}(\mathrm{E}_{\mathbb{Q}}[R^{fh}_{t+1}], \mathrm{E}_{\mathbb{Q}}[\mathbf{e}_{t+1} - \mathbf{f}_t]) - \mathrm{E}_{\bar{\mathbb{Q}}}[\mathbf{e}_{t+1} - \mathbf{f}_t]\Big]$$

### Example:

- Solve an optimal currency allocation problem by looking at the **domestic** assets position as purely **risky** and an exposure to **foreign** currencies as **ambiguous**
- The **optimal** currency **exposure** is obtained as

$$\mathbf{\Psi}^*_{t,expl} = -\Big[\mathrm{Var}_{\bar{\mathbb{Q}}}(\mathbf{e}_{t+1} - \mathbf{f}_t) + \frac{\theta}{\lambda}\mathrm{Var}_{\mu}(\mathrm{E}_{\mathbb{Q}}[\mathbf{e}_{t+1} - \mathbf{f}_t])\Big]^{-1} \cdot$$
$$\cdot \Big[\mathrm{Cov}_{\bar{\mathbb{Q}}}(R^{fh}_{t+1}, \mathbf{e}_{t+1} - \mathbf{f}_t) - \frac{1}{\lambda}\mathrm{E}_{\bar{\mathbb{Q}}}[\mathbf{e}_{t+1} - \mathbf{f}_t]\Big]$$

- In the limit when $\lambda \to \infty$, the optimal currency exposure converges to the **minimum variance** case
- When $\theta \to \infty$, the optimal currency exposure converges to zero (**full hedging**) and the entire currency exposure is kept solely in the **domestic** currency
- The puzzle of insufficient currency diversification (**home-currency bias**) may be driven by investors' **ambiguity aversion**

## References

[1] Britten-Jones, Mark, 1999, The Sampling Error in Estimates of Mean-Variance Efficient Portfolio Weights, *The Journal of Finance* 54, 655-671.

[2] Campbell, John Y., Karine Serfaty-de Medeiros, and Luis M. Viceira, 2010, Global Currency Hedging, *The Journal of Finance* 65, 87-121.

[3] Froot, Kenneth A., 2019, Currency Hedging Over Long Horizons, *Annals of Economics and Finance* 20, 37-66.

[4] Maccheroni, Fabio, Massimo Marinacci, and Doriana Ruffino, 2013, Alpha as Ambiguity: Robust Mean-Variance Portfolio Analysis, *Econometrica* 81, 1075-1113.

## In-Sample Analysis

- **Aim**: Investigate historical optimality and the role of **sampling error** in the construction of the ex-post **efficient** currency exposures
- Here, we work with the demeaned historical returns and define a **loss function** as $\mathcal{L}(R^h_{t+1}) := -U(R^h_{t+1})$
- For a matrix of demeaned currency excess returns $\mathbf{X}$, vector of demeaned fully hedged portfolio returns $\mathbf{y}$, weighting matrix $\mathbf{W}$, ambiguity matrix $\mathbf{Z} = \theta\mathrm{Var}_{\mu}(\mathrm{E}_{\mathbb{Q}}[R^h_{t+1}])$, optimal infinitely ambiguity averse currency exposure $\mathbf{\Psi}^*_{t,amb}$, and a weighted $L^2$-norm squared $\|\mathbf{\Psi}_t\|^2_{\mathbf{D}} = \mathbf{\Psi}'_t\mathbf{D}\mathbf{\Psi}_t$, we prove that the in-sample **efficient** currency **exposure** can be found as a **generalized ridge regression**

$$\operatorname*{argmin}_{\mathbf{\Psi}_t} \mathcal{L}(R^h_{t+1}) = \operatorname*{argmin}_{\mathbf{\Psi}_t} \|\mathbf{y} - \mathbf{X}(-\mathbf{\Psi}_t)\|^2_{\mathbf{W}} + \|(-\mathbf{\Psi}_t) - (-\mathbf{\Psi}^*_{t,amb})\|^2_{\mathbf{Z}}$$

- **Ambiguity** induces **shrinkage** (regularization) towards the infinitely ambiguity averse optimal exposure $\mathbf{\Psi}^*_{t,amb}$ distorted by the **level** and **structure** of **uncertainty** from matrix $\mathbf{Z}$
- The optimal in-sample currency weights produce a **pure** currency exposure which is **closest** in terms of penalized least squares **distance** to the fully hedged portfolio returns
- The generalized **penalty** term corresponds to the **utility loss** arising from **model uncertainty**. It geometrically implies a non-zero centered, **ellipsoid** parameter **constraint**

## Empirical Analysis

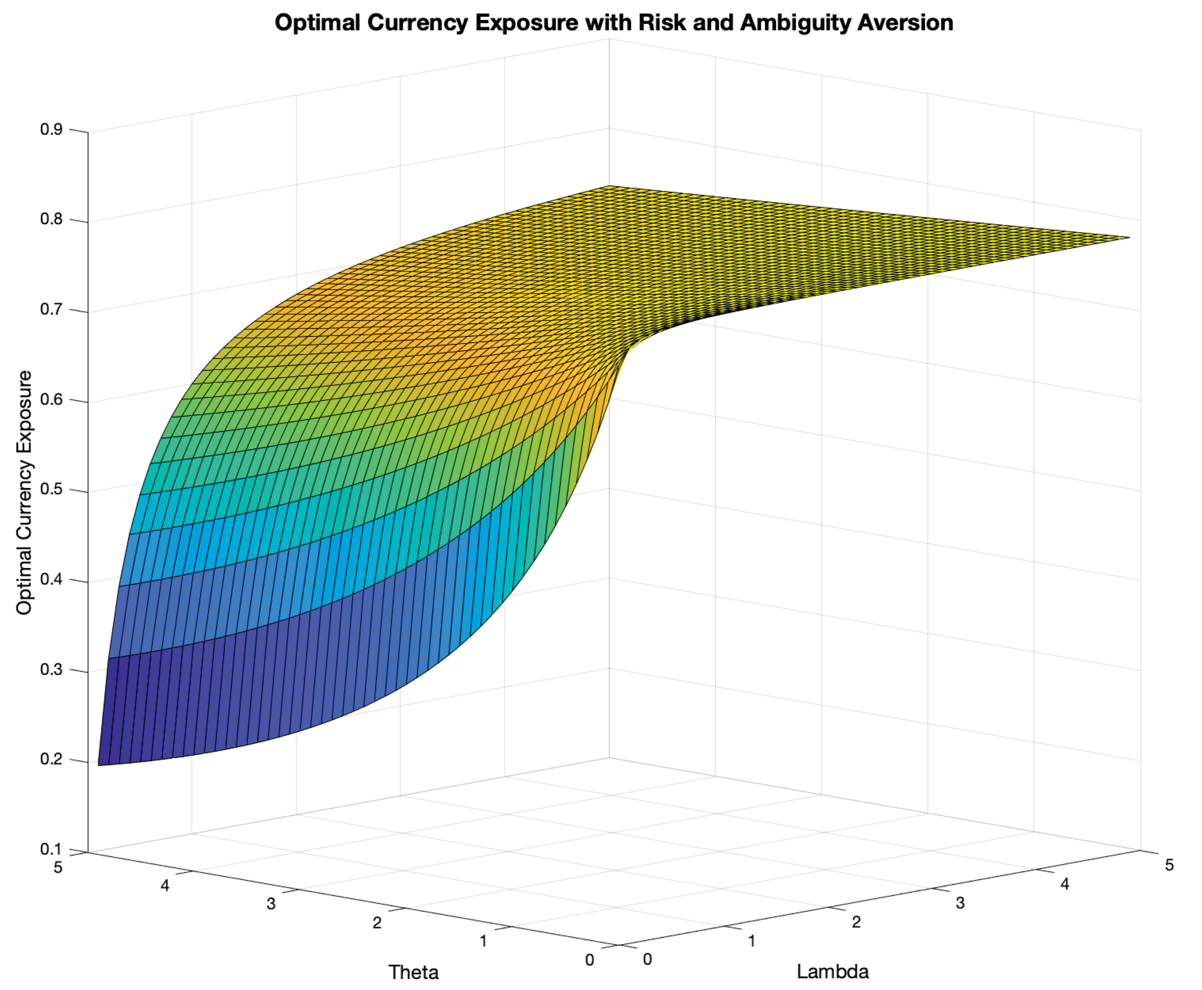


Figure 1: Optimal currency exposure in CHF (for a EUR based investor) in dependence of risk and ambiguity aversion parameters is plotted here. We assume independent prediction models and the uncovered interest rate parity to hold.

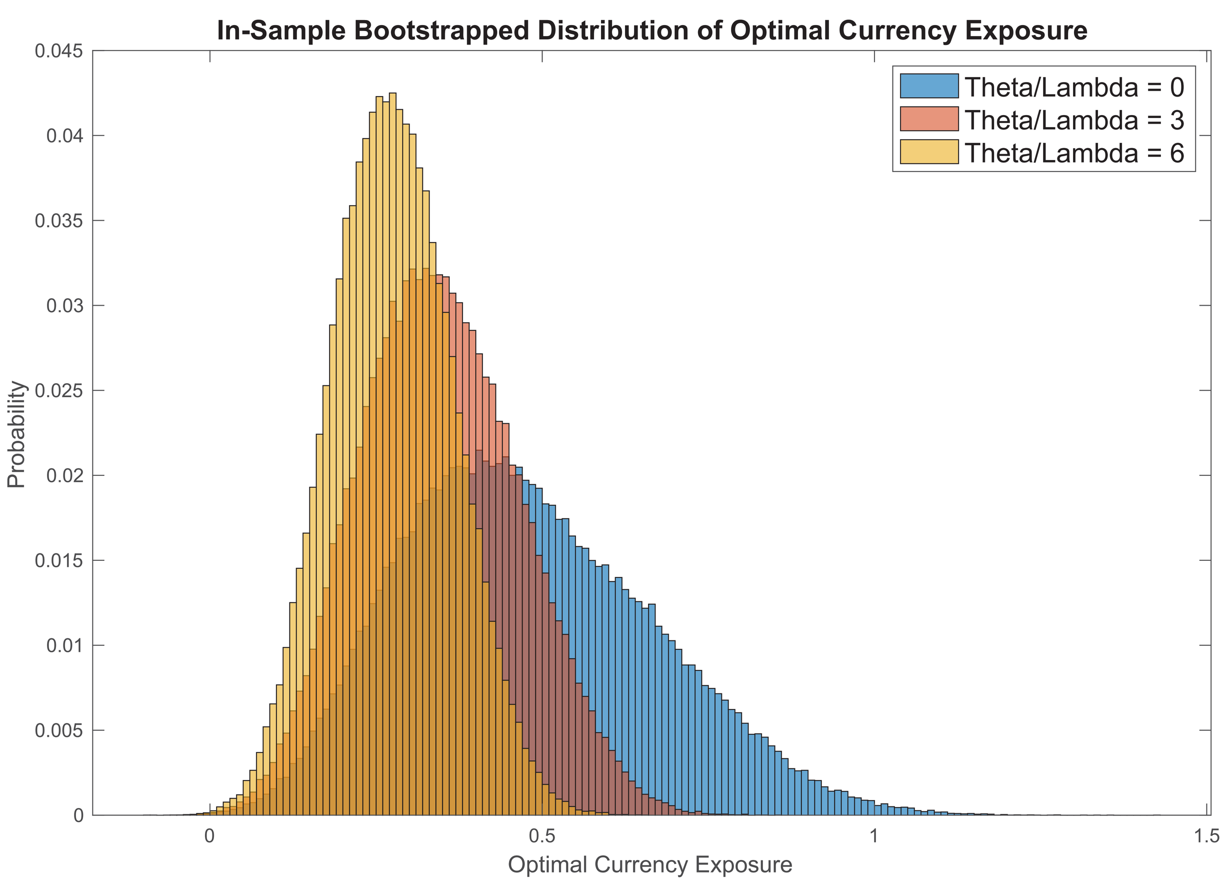


Figure 2: Bootstrapped distribution of optimal currency exposure in CHF (for a USD based investor) for different values of risk and ambiguity aversion parameters is plotted here.

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## Data

- The **empirical analysis** employs the **data** of: exchange rates, short-term interest rates, equity broad market indices, and fixed income total return indices (for various maturities)
- The data series for seven **developed** economies: Australia, Canada, Switzerland, Eurozone, United Kingdom, Japan and United States, are available at a **daily** frequency
- The sample period starts in January 1999, when the euro was introduced to the world financial markets, and ends in June 2018

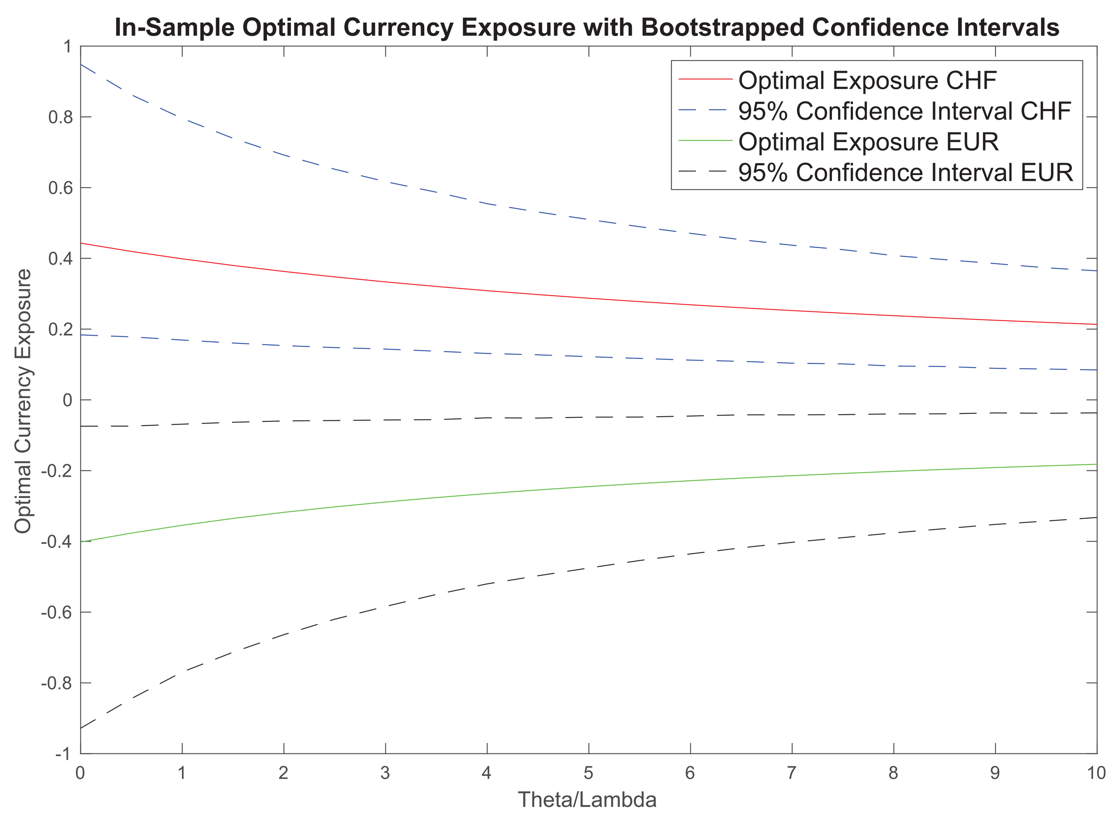


Figure 3: Optimal currency exposure and the corresponding bootstrapped 95% confidence intervals for CHF and EUR (for a USD based investor) in dependence of risk and ambiguity aversion parameters are plotted here.

**Volatility and Sharpe ratios of Hedged Global Equity Portfolios with Ambiguity Aversion**

| Base Country | No Hedge | Half Hedge | Full Hedge | Opt Min Var Hedge | Opt Mean Var Hedge | Opt Robust Amb Hedge |
|---|---|---|---|---|---|---|
| **Volatility** | | | | | | |
| Australia | 11.88% | 11.67% | 13.12% | 11.52% | 12.35% | 11.65% |
| Canada | 12.65% | 12.43% | 13.12% | 11.52% | 12.35% | 11.65% |
| Switzerland | 16.31% | 14.39% | 13.12% | 11.52% | 12.35% | 11.65% |
| Eurozone | 13.97% | 13.28% | 13.12% | 11.52% | 12.35% | 11.65% |
| UK | 13.86% | 13.13% | 13.12% | 11.52% | 12.35% | 11.65% |
| Japan | 19.27% | 15.80% | 13.12% | 11.52% | 12.35% | 11.65% |
| USA | 15.79% | 14.17% | 13.12% | 11.52% | 12.35% | 11.65% |
| **Sharpe Ratio** | | | | | | |
| Australia | 0.26 | 0.38 | 0.43 | 0.26 | 0.48 | 0.41 |
| Canada | 0.40 | 0.43 | 0.42 | 0.21 | 0.48 | 0.39 |
| Switzerland | 0.37 | 0.40 | 0.41 | 0.14 | 0.41 | 0.32 |
| Eurozone | 0.46 | 0.45 | 0.42 | 0.16 | 0.45 | 0.35 |
| UK | 0.48 | 0.46 | 0.42 | 0.19 | 0.48 | 0.39 |
| Japan | 0.44 | 0.44 | 0.41 | 0.10 | 0.40 | 0.31 |
| USA | 0.40 | 0.42 | 0.42 | 0.16 | 0.44 | 0.36 |

Table 1: This table reports annualized standard deviations and Sharpe ratios of portfolios featuring different uses of currencies for risk management. An equally weighted global equity portfolio and hedging at a quarterly horizon are assumed.

## Main Results

- **Closed form** expressions of **optimal currency exposure** for a **risk** and **ambiguity averse** investor are derived in a **model-free** setting
- The in-sample **efficient** currency exposure capturing agent's dislike for **risk** as well as **model uncertainty** are found by a **generalized ridge regression**
- The **penalty term** corresponds to the **utility loss** arising from **model uncertainty**
- Empirically, **ambiguity** induces a **bias-variance trade-off** which leads to an **improved** in-sample **estimator** of optimal currency exposure
- Realized volatility and Sharpe ratios for the **ambiguity** adjusted **currency overlay** strategy lie between the minimum variance and mean-variance cases
- The investigated link between **model uncertainty** and **penalized regression** formally connects the areas of **financial economics** (asset allocation) and **statistical learning** (regularization)